A. Product Space

There are M products. A firm has the following production technology to produce variety j:

$$q_j = A_j N_j, \tag{1}$$

where N_j are the efficiency units of labor supplied by households to produce product j.

B. Households

There is a mass of L_i households in each location i. Households are immobile across countries. They are infinite lived and have time-sparable preferences over non-durable consumption varieties:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(\{c_{jt}\}_M), \tag{2}$$

where the notation $\{c_{jt}\}_M$ means that the household has preferences over all j varieties. Households each period receive additive Type 1 extreme value shocks $\epsilon(j)$ with dispersion parameter σ_ϵ and then households make a discrete choice about which variety j to each period and a continuous choice about how much. More specifically, the utility associated with the choice of variety j is

$$u(c_{jt}) = \frac{c_{jt}^{1-\gamma}}{1-\gamma} + \epsilon(j)_t. \tag{3}$$

where consumption mapped into utils with a standard CRRA function, $\epsilon(j)$ is the taste shock.

A household's efficiency units are stochastic and they evolve according to a discrete state Markov chain. Mathematically, z is a households efficiently units and $\mathcal{P}(z, z')$ describes the probability of a household with state z efficiency units transiting to state z'.

Households can save and borrow in a non-state contingent asset a. The units of the asset are chosen to be the numeraire and pays out with gross interest rate R. I discuss this more in depth below, but the determination of R_i is either exogenously given or the rate that clears the bond market. An country specific, exogenous debt limit ϕ constrains borrowing so:

$$a_{t+1} \geq -\phi. \tag{4}$$

All these pieces come together in the household's budget constraint, conditional on choosing variety j to consume, and focusing on a stationary setting where prices and transfers are constant:

$$a_{t+1} + p_{ij} c_{jt} \leq R_i a_t + w_i z_t + \Pi_{i,\tau}. \tag{5}$$

The value of asset purchases and consumption expenditures must be less than or equal to asset payments, labor earnings, and transfers arising from firm profits.