Heterogeneous Agent Trade

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What am I doing?

Big picture — these are the questions that interest me...

1. What are distributional consequences of trade?

2. Is there a role for trade policy to improve outcomes?

One mechanism behind **1.** is heterogeneity in expenditure shares on traded goods and **elasticities**.

- **?** is a nice example. In the context of the 2015 Swiss appreciation, they find that poor households are more price elastic.

This paper:

GE, heterogenous agent model of trade delivering ABLV-like facts. I work out the implications for aggregate trade, the gains from trade, and the normative implications for trade policy.

How I do it. . .

Two ingredients:

- Standard incomplete markets model with households facing incomplete insurance against idiosyncratic productivity and taste shocks.
- Trade as in Armington (national varieties), but households have random utility over these varieties.

Qualitatively I characterize. . .

- How price elasticities vary at the micro-level and how (and if) micro-heterogeneity shapes aggregate trade and trade elasticities.
- The welfare gains from trade at the micro and macro level.
- The efficient allocation and, thus, how market incompleteness shapes these outcomes.

Quantitatively, I compute the a 19 country model (the **?** data) and (today) study the welfare gains to small reductions in trade costs.

Model: Production and Trade

M countries. Each country produces a nationally differentiated product as in Armington.

In country i, competitive firms' produce variety i with:

$$Q_i = A_i N_i,$$

where A_i is TFP; N_i are efficiency units of labor supplied by households.

Cross-country trade faces obstacles:

- iceberg trade costs $d_{ij} > 1$ for one unit from supplier j to go to buyer i.

This structure leads to the following prices that households face

$$p_{ij} = \frac{d_{ij} w_j}{A_j}.$$

Model: Households I

Mass of L_i households in each country i.

Household-level preferences:

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t\ \tilde{u}(\{c_{ijt},\epsilon_{jt}\}_M)$$

$$\text{where}\quad \tilde{u}(c_{ijt},\epsilon_{jt}) = u(c_{ijt}) + \epsilon_{jt}.$$

- ϵ_{jt} is iid (across time and households) taste shocks over national varieties.

Assumptions:

- For most of the analysis, I'll only assume u is well behaved.
- ϵ_{jt}s are distributed Type 1 Extreme Value with dispersion parameter σ_ϵ.

Model: Households II

A household's efficiency units z_t evolve according to a Markov Chain. They face the wage per efficiency unit w_{it}.

Households borrow or accumulate a non-state contingent asset, a, with gross return R_i. Household's face the debt limit

$$a_{t+1} \geq -\phi_i$$

Conditional on a variety choice, a household's budget constraint is

$$p_{ij} c_{ijt} + a_{t+1} \leq R_i a_t + w_{it} z_t.$$

What Households Do. . .

Focus on a stationary setting. A hh's state are its asset holdings a and shock z.

1. The hh makes a variety choice (e.g. a US or Italian variety) and how much to consume. The choice probability is:

$$\pi_{ij}(a,z) = \exp\left(\frac{v_{ij}(a,z)}{\sigma_\epsilon}\right) \Bigg/ \sum_{j'} \exp\left(\frac{v_{ij'}(a,z)}{\sigma_\epsilon}\right),$$

where $v_{ij}(a,z)$ is the hh's value function conditional on a choice.

2. The hh makes an asset choice. Away from the constraint, asset choices (conditional on a variety choice) must respect this Euler Equation:

$$\frac{u'(c_{ij}(a,z))}{p_{ij}} = \beta \mathrm{E}_{z'}\left\{ -\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + \frac{u'(c_{ii}(a',z'))R_i}{p_{ii}} \right\},$$

where I'm exploiting an ACR-like feature that ex-ante value functions can be expressed in terms of i, i home choices.

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Aggregation

Aggregates arise from explicit aggregation of hh-level actions. Two examples:

Aggregate, bilateral imports and exports are

$$M_{ij} = L_i \int_z \int_a p_{ij} c_{ij}(a,z) \pi_{ij}(a,z) \lambda_i(a,z), \qquad X_{ji} = L_j \int_z \int_a p_{ji} c_{ji}(a,z) \pi_{ji}(a,z) \lambda_i(a,z),$$

where λ_i is the distribution of hhs across states and $c_{ij}(a,z)$ is the consumption function. Here trade flows take on a mixed logit formulation as in **?**.

The national income accounting identity (GDP = C + I + G + X - M) . . .

$$p_i Y_i = \underbrace{L_i \sum_j \int_z \int_a p_{ij} c_{ij}(a,z) \pi_{ij}(a,z) \lambda_i(a,z)}_{\widetilde{P_i C_i}} + \underbrace{\left[\sum_{j \neq i} X_{ji} - \sum_{j \neq i} M_{ij} \right]}_{-R_i A_i + A_i'}.$$

Notice how trade is non-trivially connected to a county's capital account.

Equilibrium

The Decentralized Stationary Equilibrium. A Decentralized Stationary Equilibrium are asset policy functions and commodity choice probabilities $\{\ g_{ij}(a,z), \pi_{ij}(a,z)\ \}_{ij}$, probability distributions $\{\ \lambda_i(a,z)\ \}_i$ and positive real numbers $\{w_i, p_{ij}, R_i\}_{ij}$ such that

- i Prices (w_i, p_{ij}) satisfy the firms problem;
- ii The policy functions and choice probabilities solve the household's optimization problem;
- iv The probability distribution $\lambda_i(a,z)$ induced by the policy functions, choice probabilities, and primitives satisfies the law of motion and is stationary;
- v Goods market clears:

$$p_i Y_i - \sum_j^M X_{ji} = 0, \quad \forall i$$

- v Bond market clears with

$$\mathrm{A}_\mathrm{i}' = 0, \quad \forall i.$$

The H-A Trade Elasticity

Proposition #1: The H-A Trade Elasticity. The trade elasticity between country i and country j is:

$$\theta_{ij} = 1 + \int_a \int_z \Big\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \Big\} \omega_{ij}(a,z) - \Big\{ \theta_{ii}(a,z)^I + \theta_{ii}(a,z)^E \Big\} \omega_{ii}(a,z),$$

which is the difference between ij and ii expenditure-weighted micro-level elasticities. The micro-level elasticities for households with states a, z are an intensive and extensive elasticity

$$\theta_{ij}(a,z)^I = \frac{\partial c_{ij}(a,z)/c_{ij}(a,z)}{\partial d_{ij}/d_{ij}}, \qquad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and $\omega_{ij}(a,z)$ are the expenditure weights.

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and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^I = \left[-\frac{\partial g_{ij}(a,z)/p_{ij}c_{ij}(a,z)}{\partial p_{ij}/p_{ij}} - 1 \right] \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}.$$

The idea here is that reduction in trade costs relaxes the hh's budget constraint and then the division of new resources between assets and expenditure determines the intensive margin elasticity.

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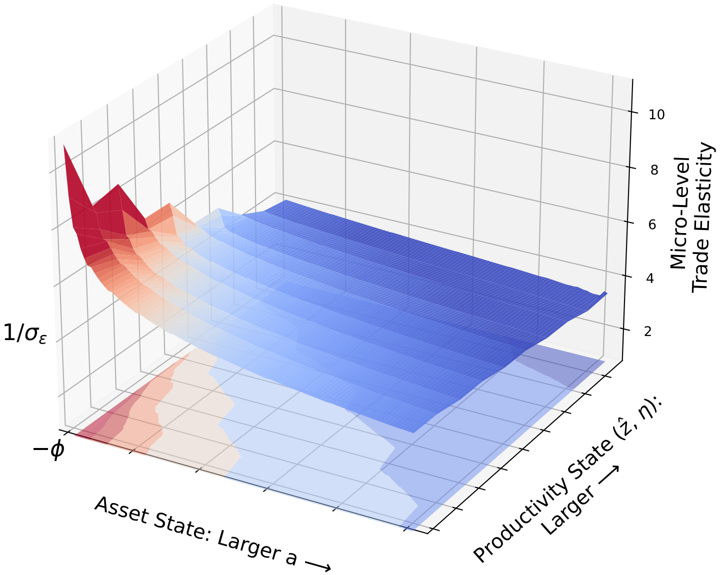
and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^E = -\frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}} + \frac{1}{\sigma_\epsilon} \frac{\partial v_{ij}(a,z)}{\partial d_{ij}/d_{ij}}.$$

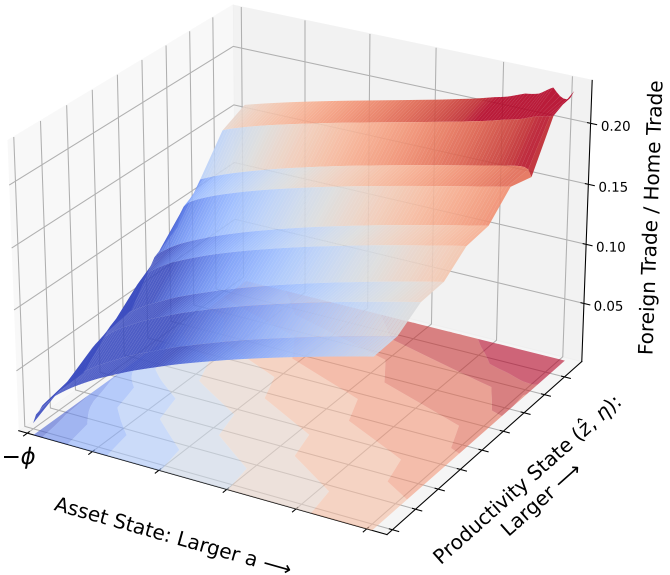
Key is $\frac{\partial v_{ij}(a,z)}{\partial d_{ij}/d_{ij}}$.

In the paper, I show that if relative risk aversion > 1 than hh's with (i) high $u'(c)$ and (ii) high MPCs are more price elastic. **So poor hh's are the most price sensitive.**

Trade Elasticities by HH-Level State



Trade Shares: $M_{ij}(a,z)/M_{ii}(a,z)$, by HH-Level State



H-A Gains from Trade

Proposition #2: H-A Welfare Gains from Trade. The gains from trade under a utilitarian social welfare function are

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} = \int_z \int_a \Bigg\{ \underbrace{\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{\text{gains to hh}} + \underbrace{v_i(a,z)\frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{\text{gains to reallocation}} \Bigg\} L_i \lambda_i(a,z),$$

where v_i is a hh's value function before taste shocks are realized.

Household-level gains are

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \Bigg\{ -\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a_t,z_t)/\pi_{ii}(a_t,z_t)}{\mathrm{d}d_{ij}/d_{ij}} + u'(c_{ii}(a_t,z_t))a_t \frac{\mathrm{d}R_i}{\mathrm{d}d_{ij}/d_{ij}} \Bigg\}.$$

HH-level gains pick up two effects:

- An ACR-like term reflecting how it's home choice changes... basically the gains from substitution.
- How the value of a hh's wealth changes through GE effects on interest rates.

H-A Gains from Trade: log Preferences $\Rightarrow$ Separation of Trade and Heterogeneity

Proposition #3: Separation of Trade and Micro-Heterogeneity. In the dynamic, heterogenous agent trade model where preferences are logarithmic over the physical commodity

$$\tilde{u}(c_{ijt}, \epsilon_{jt}) = \log(c_{ij,t}) + \epsilon_{j,t},$$

the trade elasticity is

$$\theta = -\frac{1}{\sigma_\epsilon},$$

and is independent of household heterogeneity. And the welfare gains from trade are

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} = -\frac{1}{\theta(1-\beta)} \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}}.$$

and is (i) independent of the household heterogeneity and (ii) summarized by the trade elasticity and the change in the home choice probability (and home share).

Mimics the results of **?** and **?**, remarkable as this is a far more complex economy. . .

H-A Gains from Trade under Efficiency

Proposition #4: Trade Elasticities and Welfare Gains in the Efficient Allocation The trade elasticity between i, j in the efficient allocation is:

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon}\Big[u'(c_{ij})c_{ij}\Big].$$

And the welfare gains from a reduction in trade costs between i, j are

$$\frac{\mathrm{d}W}{\mathrm{d}d_{ij}/d_{ij}} = \frac{\partial W}{\partial d_{ij}/d_{ij}} = \frac{1}{1-\beta} \times u'(c_{ij})c_{ij}\pi_{ij}L_i,$$

which is the discounted, direct effect from relaxing the resource constraint.

Mimics the results of **?** but with household (not firm) heterogeneity.

Quantitative Analysis

Preliminary. This is what I'm going to do:

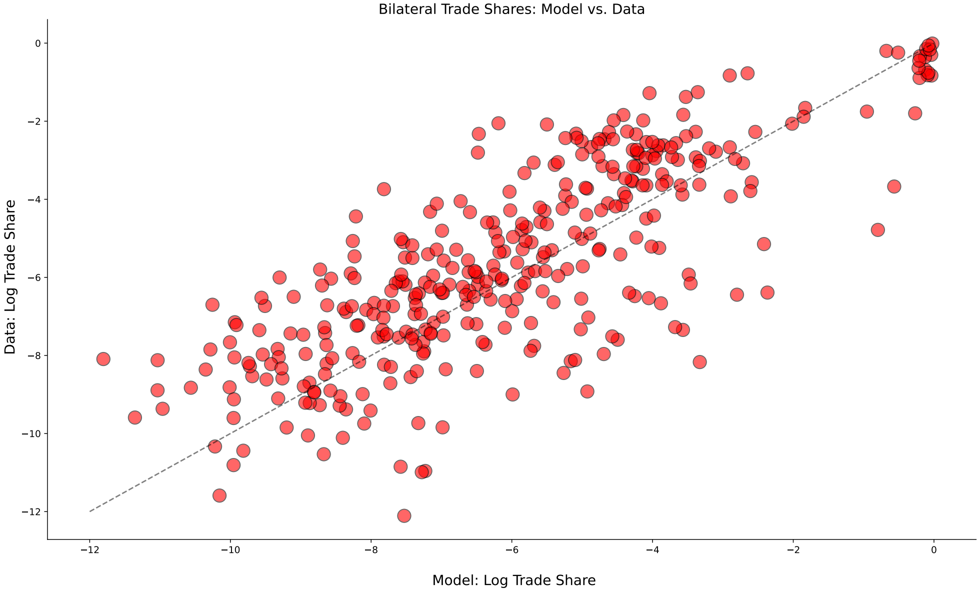
- Grab trade costs and productivity estimates from 19 country world of **?** and compute an equilibrium.
- Explore small, global reduction in trade costs. No transition path today, ran out of time!

Other important parameters and how I set them for today.

- Taste shock parameter so $1/\sigma_\epsilon = 4.0$. CRRA for u with relative risk aversion $= 1.5$.
- Earnings process is a mixture of a persistent and transitory component and calibrated as in **?**.
- Borrowing constraint is set $\approx 2\times$ earnings for US. Discount factor set so $R \approx 2\%$ for US.

My RA Thomas Hasenzagl and I have Julia and Python code to compute things pretty fast.

Bilateral Trade: Model vs. Data



U.S. Welfare Gains to a Global 1% Reduction in Trade Costs

Trade and Welfare by Income — Global 1% Reduction

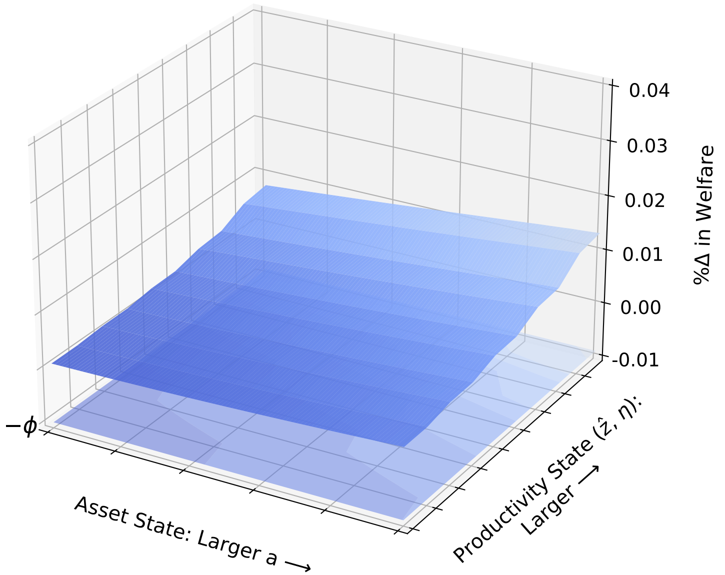
	Baseline	1% reduction	
Income Quartile	$1-\pi_{ii}$	$1-\pi_{ii}$	Welfare (% Change)
Bottom quartile	0.0179	0.0182	0.0006
Median	0.0509	0.0515	0.0061
Upper quartile	0.0960	0.0972	0.0137
Aggregate	0.0744	0.0753	0.0078
Interest rate	2.14	2.15	
% losers	—	17.9	

U.S. Welfare Gains to a Global 10% Reduction in Trade Costs

Trade and Welfare by Income — Global 10% Reduction

	Baseline	1% reduction	
Income Quartile	$1-\pi_{ii}$	$1-\pi_{ii}$	Welfare (% Change)
Bottom quartile	0.0179	0.0208	0.1247
Median	0.0509	0.0583	0.1988
Upper quartile	0.0960	0.1094	0.2963
Aggregate	0.0744	0.0848	0.2185
Interest rate	2.14	2.17	
% losers	—	0.0	

U.S. Welfare Gains to a Global 1% Reduction in Trade Costs



Welfare Gains to a Global 1% Reduction in Trade Costs

Welfare Gains to a Global 1% Reduction in Trade Costs

	Baseline Model	Rep. Agent Model
USA	0.0075 [83]	0.025 [−−]
Germany	0.14 [100]	0.31 [−−]
Japan	0.004 [80]	0.014 [−−]
Canada	0.09 [100]	0.18 [−−]

Note: Numbers in brackets are % of population who gain. Rep. Agent Model uses ACR calculation with trade elasticity = 4.0

Where I'm headed next. . .

Lot's to do, but "big picture" this is where I'm aiming:

1. Can trade policy improve outcomes?

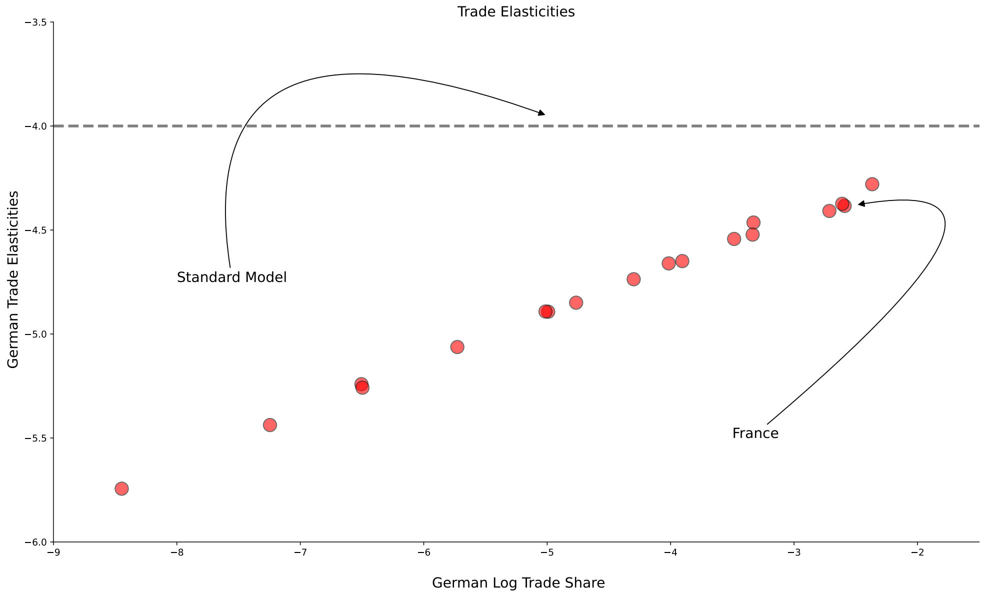
- This is a useful laboratory to think about policy because (i) there is scope for it and (ii) have a direct representation of utility (not an indirect representation).

2. How financial globalization relates globalization in goods trade?

- The model provides a coherent account of both trade in goods and assets. I think it'd be interesting to see what happens.

References I

Bilateral Trade Elasticities: German Example



Welfare Gains to a Global 10% Reduction in Trade Costs

Welfare Gains to a Global 10% Reduction in Trade Costs

	Baseline Model	Rep. Agent Model
USA	0.21 [100]	0.28 [−−]
Germany	1.6 [100]	3.5 [−−]
Japan	0.14 [100]	0.21 [−−]
Canada	1.13 [100]	1.9 [−−]

Note: Numbers in brackets are % of population who gains. Rep. Agent Model uses ACR calculation with trade elasticity = 4.0