Heterogeneous Agent Trade

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What am I doing?

Big picture — these are the questions that interest me. . .

1. What are distributional consequences of trade?

2. Is there a role for trade policy to improve outcomes?

One mechanism behind **1.** is heterogeneity in expenditure shares on traded goods and **elasticities**.

- Auer, Burstein, Lein, and Vogel (2022) is a nice example. In the context of the 2015 Swiss appreciation, they find that poor households are more price elastic.

Behind **2.** are inefficiencies arising from market incompleteness that feed into product markets. . . so lack of insurance against life's circumstances distorts the pattern of trade.

This paper:

GE, heterogenous agent model of trade delivering ABLV-like facts. I work out the implications for aggregate trade, the gains from trade, and the normative implications.

How I do it...

Two ingredients:

- Trade as in Armington, but households have random utility over these varieties.
- Standard incomplete markets model with households facing incomplete insurance against idiosyncratic productivity and taste shocks.

Qualitatively I characterize...

- How price elasticities vary at the micro-level and when micro-heterogeneity shapes aggregates.
- The welfare gains from trade at the micro and macro level.
- The efficient allocation and, thus, how market incompleteness shapes these outcomes.

Quantitatively, I compute a 19 country model (the Eaton and Kortum (2002) data) and (today) study the pattern of trade in the decentralized vs. efficient allocation.

Model: Production and Trade

M countries. Each country produces a nationally differentiated product as in Armington.

In country i, competitive firms' produce variety i with:

$$Q_i = A_i N_i,$$

where A_i is TFP; N_i are efficiency units of labor supplied by households.

Cross-country trade faces obstacles:

- iceberg trade costs $d_{ij} > 1$ for one unit from supplier j to go to buyer i.

This structure leads to the following prices that households face

$$p_{ij} = \frac{d_{ij} w_j}{A_j}.$$

Model: Households I

Mass of L_i households in each country i.

Household-level preferences:

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t \ \tilde{u}(\{c_{ijt}, \epsilon_{jt}\}_M)$$

$$\text{where} \quad \tilde{u}(c_{ijt}, \epsilon_{jt}) = u(c_{ijt}) + \epsilon_{jt}.$$

- ϵ_{jt} is iid (across time and households) taste shocks over national varieties.

Assumptions:

- For most of the analysis, I'll only assume u is well behaved.
- ϵ_{jt}s are distributed Type 1 Extreme Value with dispersion parameter σ_ϵ.

Model: Households II

A household's efficiency units z_t evolve according to a Markov Chain. They face the wage per efficiency unit w_{it}.

Households borrow or accumulate a non-state contingent asset, a, with gross return R_i. Household's face the debt limit

$$a_{t+1} \geq -\phi_i.$$

Conditional on a variety choice, a household's budget constraint is

$$p_{ij} c_{ijt} + a_{t+1} \leq R_i a_t + w_{it} z_t.$$

What Households Do I

Focus on a stationary setting. A hh's state are its asset holdings a and shock z.

1. Condition on variety choice (e.g. a US or Italian variety) their problem is:

$$v_{ij}(a,z) = \max_{a',\ c_{ij}} \left\{ u(c_{ij}) + \epsilon_j + \beta\, \mathbb{E}[v_i(a',z')] \right\}$$

$$\text{subject to} \quad p_{ij}c_{ij} + a' \leq R_i a + w_i z \quad \text{and} \quad a' \geq -\phi_i.$$

2. The ex-post value function of a household in country i is

$$v_i(a,z) = \max_j \{\ v_{ij}(a,z)\ \}.$$

What Households Do II

Two equations characterizing the commodity choice, consumption / savings...

1. The choice probability is:

$$\pi_{ij}(a,z) = \exp\left(\frac{v_{ij}(a,z)}{\sigma_\epsilon}\right) \Bigg/ \sum_{j'} \exp\left(\frac{v_{ij'}(a,z)}{\sigma_\epsilon}\right).$$

2. Away from the constraint, consumption and asset choices must respect this Euler Equation:

$$\frac{u'(c_{ij}(a,z))}{p_{ij}} = \beta R_i \mathrm{E}_{z'}\left[\sum_{j'} \pi_{ij'}(a',z')\frac{u'(c_{ij'}(a',z'))}{p_{ij'}}\right].$$

Aggregation

Aggregates arise from explicit aggregation of hh-level actions. Two examples:

1. Aggregate, bilateral imports and exports are

$$M_{ij} = L_i \int_z \int_a p_{ij} c_{ij}(a,z)\pi_{ij}(a,z)\lambda_i(a,z), \qquad X_{ji} = L_j \int_z \int_a p_{ji} c_{ji}(a,z)\pi_{ji}(a,z)\lambda_j(a,z),$$

where λ_i is the distribution of hhs across states and $c_{ij}(a,z)$ is the consumption function. Here trade flows take on a mixed logit formulation similar to Berry, Levinsohn, and Pakes (1995).

2. The national income accounting identity (GDP = C + I + G + X - M) . . .

$$p_i Y_i = \underbrace{L_i \sum_j \int_z \int_a p_{ij} c_{ij}(a,z)\pi_{ij}(a,z)\lambda_i(a,z)}_{\widetilde{P_i C_i}} + \underbrace{\left[\sum_{j \neq i} X_{ji} - \sum_{j \neq i} M_{ij} \right]}_{-R_i A_i + A_i'}.$$

Equilibrium

The Decentralized Stationary Equilibrium. A Decentralized Stationary Equilibrium are asset policy functions and commodity choice probabilities $\{ \ g_{ij}(a,z), \pi_{ij}(a,z) \ \}_{ij}$, probability distributions $\{ \ \lambda_i(a,z) \ \}_i$ and positive real numbers $\{w_i, p_{ij}, R_i\}_{ij}$ such that

- i Prices (w_i, p_{ij}) satisfy the firms problem;
- ii The policy functions and choice probabilities solve the household's optimization problem;
- iv The probability distribution $\lambda_i(a,z)$ induced by the policy functions, choice probabilities, and primitives satisfies the law of motion and is stationary;
- v Goods market clears:

$$p_i Y_i - \sum_j^M X_{ji} = 0, \quad \forall i$$

- v Bond market clears with

$$\mathrm{A}_\mathrm{i}' = 0, \quad \forall i.$$

The H-A Trade Elasticity

Proposition #1: The H-A Trade Elasticity. The trade elasticity between country i and country j is:

$$\theta_{ij} = 1 + \int_a \int_z \left\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \right\} \omega_{ij}(a,z) - \left\{ \theta_{ii}(a,z)^I + \theta_{ii}(a,z)^E \right\} \omega_{ii}(a,z),$$

which is the difference between ij and ii expenditure-weighted micro-level elasticities. The micro-level elasticities for households with states a, z are an intensive and extensive elasticity

$$\theta_{ij}(a,z)^I = \frac{\partial c_{ij}(a,z)/c_{ij}(a,z)}{\partial d_{ij}/d_{ij}}, \qquad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and $\omega_{ij}(a,z)$ are the expenditure weights.

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and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^I = \left[- \frac{\partial g_{ij}(a,z)/p_{ij}c_{ij}(a,z)}{\partial p_{ij}/p_{ij}} - 1 \right] \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}.$$

The idea here is that reduction in trade costs relaxes the hh's budget constraint and then the division of new resources between assets and expenditure determines the intensive margin elasticity.

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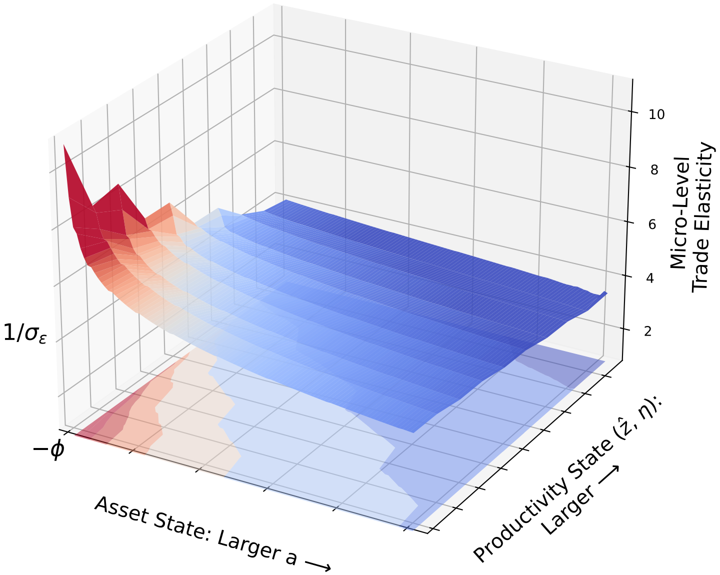
and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^E = -\frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}} + \frac{1}{\sigma_\epsilon} \frac{\partial v_{ij}(a,z)}{\partial d_{ij}/d_{ij}}.$$

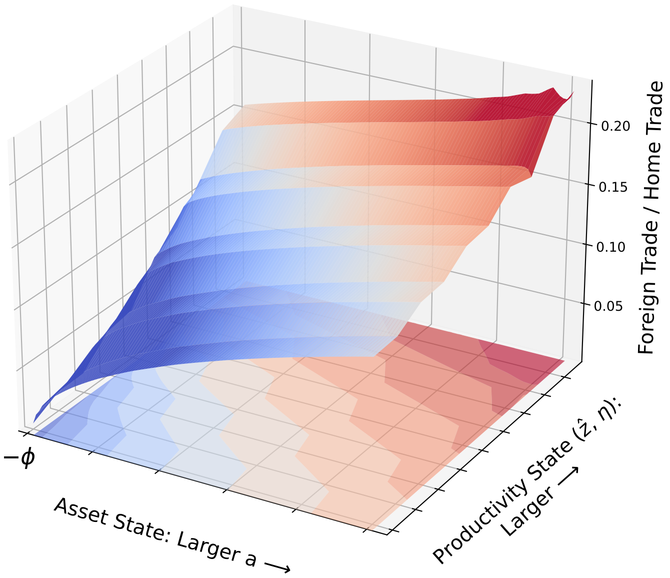
Key is $\frac{\partial v_{ij}(a,z)}{\partial d_{ij}/d_{ij}}$.

In the paper, I show that if relative risk aversion > 1 than hh's with (i) high $u'(c)$ and (ii) high MPCs are more price elastic. **So poor hh's are the most price sensitive.**

Trade Elasticities by HH-Level State



Trade Shares: $M_{ij}(a,z)/M_{ii}(a,z)$, by HH-Level State



H-A Gains from Trade

Proposition #2: H-A Welfare Gains from Trade. The gains from trade under a utilitarian social welfare function are

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} \approx \int_z \int_a \Bigg\{ \underbrace{\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{\text{gains to hh}} + \underbrace{v_i(a,z)\frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{\text{gains to reallocation}} \Bigg\} L_i \lambda_i(a,z),$$

where v_i is value function before realization of taste shocks; $\approx$ is about abstracting from transition.

Household-level gains are

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \approx \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \Bigg\{ -\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a_t,z_t)/\pi_{ii}(a_t,z_t)}{\mathrm{d}d_{ij}/d_{ij}} + u'(c_{ii}(a_t,z_t))a_t \frac{\mathrm{d}R_i}{\mathrm{d}d_{ij}/d_{ij}} \Bigg\}.$$

HH-level gains pick up two effects:

- An ACR-like term reflecting how it's home choice changes. . . basically the gains from substitution.
- How hh's wealth ($+$ or $-$) changes through GE effects on interest rates.

H-A Gains from Trade: log Preferences $\Rightarrow$ Separation of Trade and Heterogeneity

Proposition #3: Separation of Trade and Micro-Heterogeneity. In the dynamic, heterogenous agent trade model where preferences are logarithmic over the physical commodity

$$\tilde{u}(c_{ijt}, \epsilon_{jt}) = \log(c_{ijt}) + \epsilon_{jt},$$

the trade elasticity is

$$\theta = -\frac{1}{\sigma_\epsilon},$$

and is independent of household heterogeneity. And the welfare gains from trade are

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} = -\frac{1}{\theta(1-\beta)} \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}}.$$

and is (i) independent of the household heterogeneity and (ii) summarized by the trade elasticity and the change in the home choice probability (and home share).

Mimics the results of Anderson et al. (1987) and Arkolakis et al. (2012)— but stronger in the sense that this is a far more complex economy.

H-A Trade under Efficiency

Proposition #4: The Centralized (Efficient) Allocation. The allocation satisfying the Centralized Planning Problem (with a utilitarian social welfare function) is:

1. An allocation of consumption satisfying:

$$u'(c_{ij}(z,t)) = \chi_j(t) d_{ij}$$

where $\chi_j(t)$ is the multiplier on j resource constraint for variety j,

2. And variety choice probabilities:

$$\pi_{ij}(t) = \exp\left(\frac{u(c_{ij}(t)) - u'(c_{ij}(t))c_{ij}(t)}{\sigma_\epsilon}\right) \Bigg/ \sum_{j'} \exp\left(\frac{u(c_{ij'}(t)) - u'(c_{ij'}(t))c_{ij'}(t)}{\sigma_\epsilon}\right).$$

1. is like a Backus and Smith (1993) condition. **2.** is new—trade should reflect the net social benefit of buying that commodity.

H-A Gains from Trade under Efficiency

Proposition #5: Trade Elasticities and Welfare Gains in the Efficient Allocation The trade elasticity between i, j in the efficient allocation is:

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon}\left[u'(c_{ij})c_{ij}\right].$$

And the welfare gains from a reduction in trade costs between i, j are

$$\frac{\mathrm{d}W}{\mathrm{d}d_{ij}/d_{ij}} = \frac{\partial W}{\partial d_{ij}/d_{ij}} = \frac{1}{1-\beta} \times u'(c_{ij})c_{ij}\pi_{ij}L_i,$$

which is the discounted, direct effect from relaxing the resource constraint.

Mimics the results of Atkeson and Burstein (2010) but with household (not firm) heterogeneity. With log preferences the direct effect is equivalent to Arkolakis et al. (2012).

Quantitative Analysis

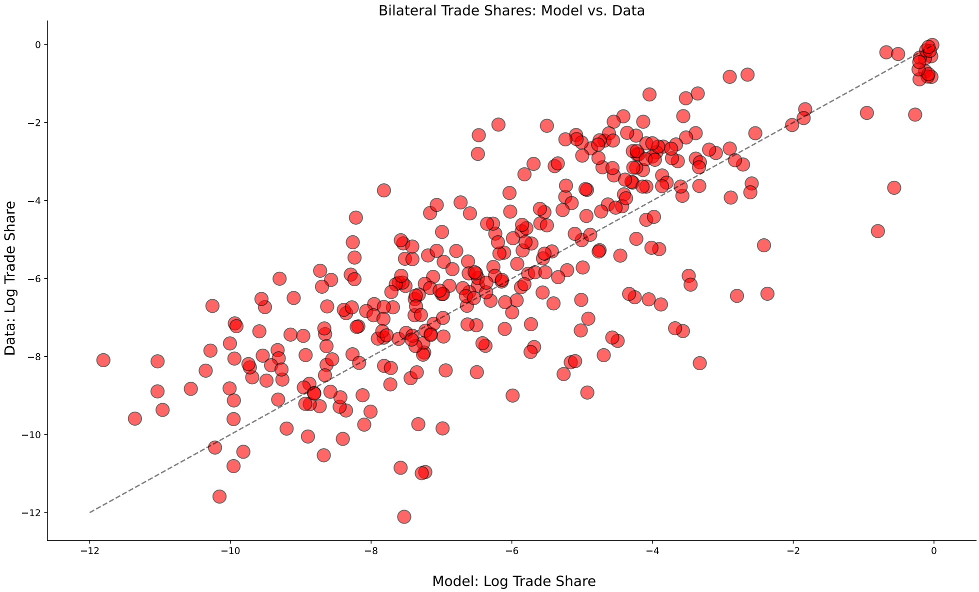
Still preliminary. This is what I'm going to do:

- Grab trade costs and productivity estimates from 19 country world of Eaton and Kortum (2002) and compute an equilibrium.
- Explore bilateral reduction in trade costs...I'll explain in two slides.

Other important parameters and how I set them for today.

- Taste shock parameter so $1/\sigma_{\epsilon} = 4.0$. CRRA for u with relative risk aversion $= 1.5$.
- Earnings process is a mixture of a persistent and transitory component and calibrated as in Krueger, Mitman, and Perri (2016).
- Borrowing constraint is set $\approx 2\times$ earnings for US. Discount factor set so $R \approx 2\%$ for US.

Bilateral Trade: Model vs. Data



Quantitative Experiment

Two ideas I want to illustrate:

1. You pick the market, you pick a person. . .

- In this calibration, the rich are more likely to consume all varieties; the poor are likely to consume the cheapest (which is typically the home good).

2. Modern day Stolper-Samuelson. . .

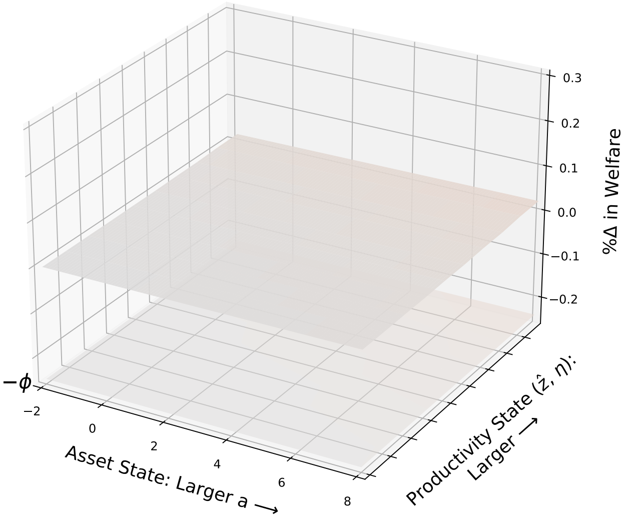
- Who directly benefits from **1.** effects $R \Rightarrow$ shapes the extent to which their are winners and losers.

Next slides: 10% reduction to US import trade cost on different source markets. . . Australia, Japan, Canada. Focus on US welfare and break it down by

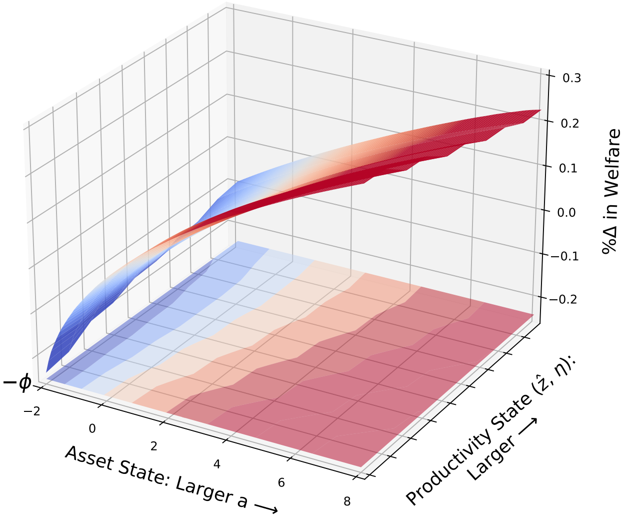
A. Fix R & w, so what is direct effect of change in trade cost,

B. R & w adjust to clear goods and asset markets.

U.S. Welfare: 10% Reduction to a Small Market (Australia), Fixed R & w



U.S. Welfare: 10% Reduction to a Small Market (Australia), GE

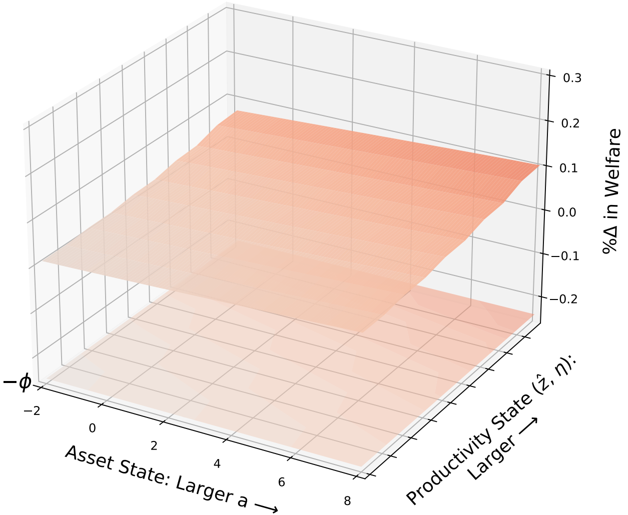


U.S. Welfare: 10% Reduction to a Small Market (Australia)

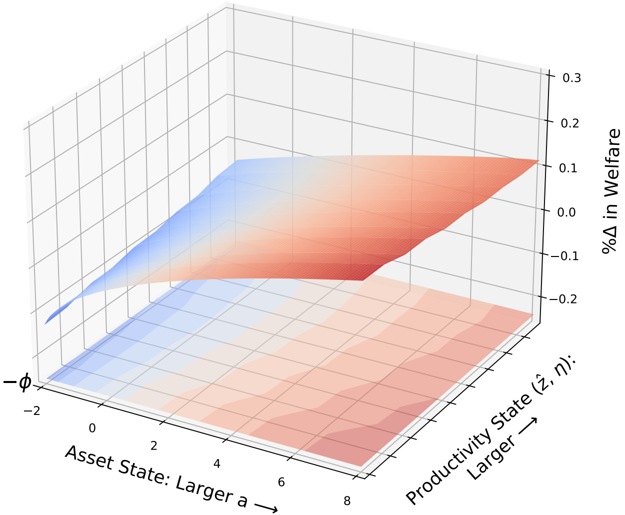
Welfare by Wealth — Australia 10% Reduction

	Fixed R & w	GE: Prices Adjust
Asset Quartile	Welfare (% Change)	Welfare (% Change)
Bottom quartile	0.0013	-0.163
Median	0.0023	-0.078
Upper quartile	0.0052	0.084
Aggregate	0.0031	-0.056
% losers	0.0	72.6

U.S. Welfare: 10% Reduction to a Medium Market (Japan), Fixed R & w



U.S. Welfare: 10% Reduction to a Medium Market (Japan), GE

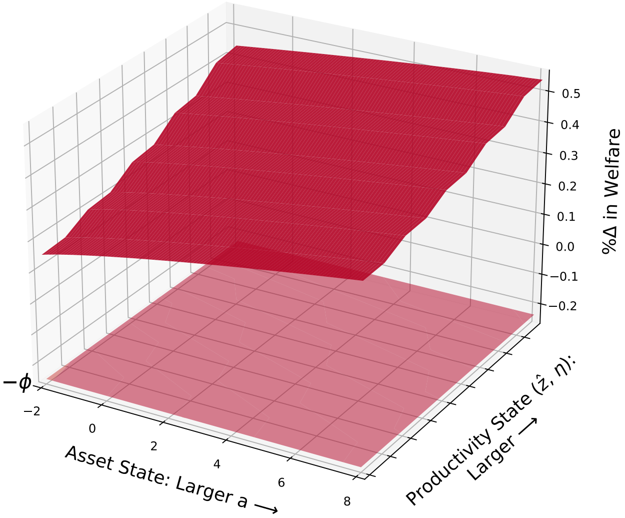


U.S. Welfare: 10% Reduction to a Medium Market (Japan)

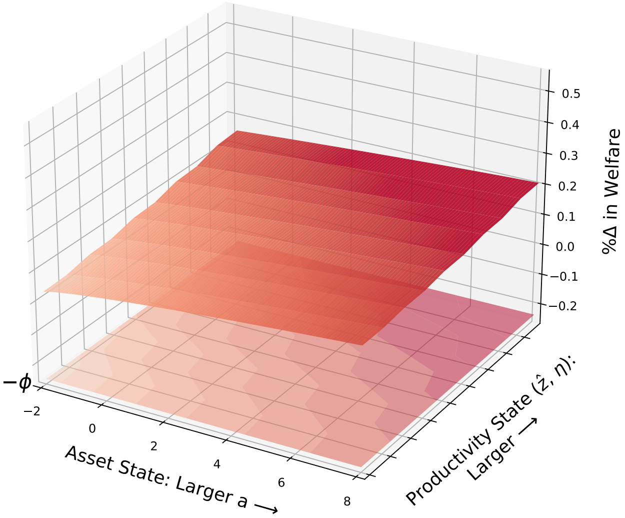
Welfare by Wealth — Japan 10% Reduction

	Fixed R & w	GE: Prices Adjust
Asset Quartile	Welfare (% Change)	Welfare (% Change)
Bottom quartile	0.0193	-0.095
Median	0.0306	-0.052
Upper quartile	0.0535	0.030
Aggregate	0.0031	-0.040
% losers	0.0	77.8

U.S. Welfare: 10% Reduction to a Large Market (Canada), Fixed R & w



U.S. Welfare: 10% Reduction to a Large Market (Canada), GE

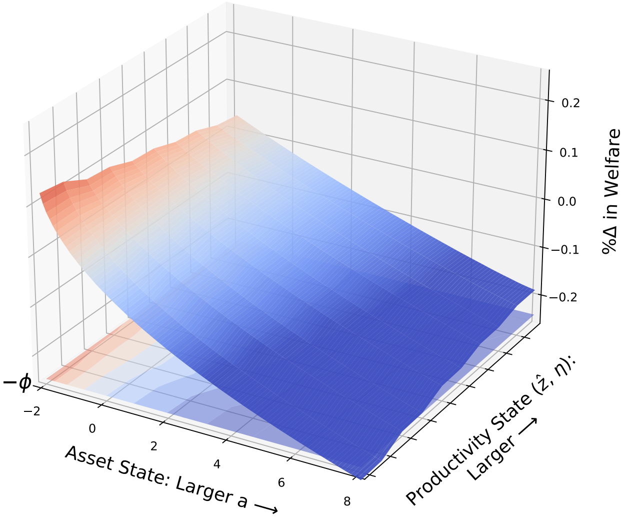


U.S. Welfare: 10% Reduction to a Large Market (Canada)

Welfare by Wealth — Canada 10% Reduction

	Fixed R & w	GE: Prices Adjust
Asset Quartile	Welfare (% Change)	Welfare (% Change)
Bottom quartile	0.21	0.06
Median	0.28	0.09
Upper quartile	0.39	0.14
Aggregate	0.30	0.09
% losers	0.0	0.0

U.S. Welfare: 10% Increase to a Small Market (Australia), GE



Where I'm headed next...

Lot's to do, but "big picture" this is where I'm aiming:

1. Can trade policy improve outcomes?

- This is a useful laboratory to think about policy because (i) there is scope for it and (ii) have a direct representation of utility (not an indirect representation).

2. How financial globalization relates globalization in goods trade?

- The model provides a coherent account of both trade in goods and assets. I think it'd be interesting to see what happens.

Problems...

These are some issues I'm mulling over:

1. Differences in hh-level shares too different (at least for US).

2. How to think about China?

3. Sectors?

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Micro-Elasticities I: The Intensive Margin

How do households respond on the **intensive** margin to a change in trade costs?

$$\theta_{ij}(a,z)^{I} := \frac{\partial c_{ij}(a,z)/c_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

$$= \left[-\frac{\partial g_{ij}(a,z)/p_{ij}c_{ij}(a,z)}{\partial p_{ij}/p_{ij}} - 1 \right] \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}.$$

The idea: A reduction in trade costs relaxes the hh's budget constraint, so the intensive margin elasticity depends on the division of new resources between assets and expenditure.

Micro-Elasticities II: The Extensive Margin

How do households respond on the **extensive** margin?

$$\theta_{ij}(a,z)^E := \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

$$= -\frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}} - \frac{1}{\sigma_\epsilon}\Big[u'(c_{ij}(a,z))c_{ij}(a,z)\Big] + \beta\mathbb{E}\frac{1}{\sigma_\epsilon}\frac{\partial v_i(a',z')}{\partial d_{ij}/d_{ij}}.$$

To get a sense of things, vary the second term by wealth...

$$\frac{\partial(u'(c_{ij}(a,z))c_{ij}(a,z))}{\partial a} = u'(c_{ij}(a,z)) \times \text{MPC}_{ij}(a,z) \times \Big[-\rho_{ij}(a,z)+1\Big],$$

where $\rho_{ij}(a,z)$ is the Arrow-Pratt measure of relative risk aversion.

With CRRA, if risk aversion >1, then poor, high marginal utility households (who are also high MPC households) are *more elastic relative* to rich households on the extensive margin.

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Bilateral Trade Elasticities: German Example