Heterogeneous Agent Trade

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ABSTRACT

This paper studies the implications of household heterogeneity for trade. I develop a model where household heterogeneity is induced via the standard incomplete markets model and results in heterogenous price elasticities. Even if exposure to trade is similar across households, heterogenous price elasticities imply that different households value price changes differently and, thus, rich and poor households experience different gains from trade. I calibrate the model to match bilateral trade flows and micro-facts about household-level expenditure patterns and price elasticities. The gains from trade are pro-poor with a ten percent reduction in trade costs delivering welfare gains equivalent to a permanent 600 dollar transfer of income for the poorest households. These gains are four and a half times larger than those for the richest households; the average gains are three times larger than representative agent benchmarks.

Email: michael.e.waugh@gmail.com. The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. This project was developed with research support from the National Science Foundation (NSF Award number 1948800). Thomas Hasenzagl provided excellent research assistance. My github repository provides the code and supplementary work behind this paper at `https://github.com/mwaugh0328/heterogeneous-agent-trade`.

This paper studies the implications of household heterogeneity for trade. From the perspective of trade, household heterogeneity is interesting because of the notion that some benefit from trade and others don't. One aspect of these unequal gains relates to the idea that rich and poor consumers have different sensitivities to price and, thus, they shape the gains from trade. I develop this idea in a model that results in heterogenous price elasticities and I study it's implications for trade qualitatively and quantitatively.

The core issue in my model is that heterogeneity in price sensitivity reflects heterogeneity in the marginal utility of consumption across households. Then even if rich and poor households are equally exposed to changes in prices — heterogeneity in price sensitivity implies that they value a price change differently. Thus, poor, high marginal utility households — who are very sensitive to price — benefit more from trade than rich households. Quantitatively, I find that this mechanism is powerful with the poorest households gaining four and a half times more than the richest. And the average gains from trade are nearly three times larger than standard, representative agent benchmarks.

The model that I develop builds upon workhorse frameworks. Trade in goods follows the Armington tradition with producers in each country producing a national variety. The important twist is that households have random utility over these varieties and they make a discrete choice over the varieties to consume (McFadden (1974)). Household heterogeneity is induced via the standard incomplete markets model (Bewley (1979), Imrohoroğlu (1989), Huggett (1993), Aiyagari (1994)) with households facing incomplete insurance against idiosyncratic productivity and taste shocks. This setting naturally leads to dispersion the marginal utility of consumption.

Together, the discrete choice and incomplete markets model interact with the key force being household-level trade (price) elasticities that endogenously vary with income and wealth. Income and wealth matter because a household's price elasticity, in essence, is about the marginal utility of consumption. For poor high marginal utility households, a price reduction for a variety delivers a lot of utility — on the margin — and, thus, this induces strong substitution into that variety. In contrast, rich households' marginal utility is low, a price reduction for delivers little incremental gain in utility and, thus, this induces weak substitution into that variety. In aggregate, the distribution of households — how many rich and poor people are in a country — then determines the aggregate response of economy to changes in trade frictions and the aggregate pattern of trade.

The issues behind heterogeneity in price sensitivity leads to new perspectives on the welfare gains from trade. I show how one aspect of the gains from trade reflect the expected, discounted stream of changes in a household's home choice probability, similar in spirit to the result of Arkolakis, Costinot, and Rodríguez-Clare (2012). Unpacking this component reveals that the change in the home choice probability is essentially about two forces: (i) how exposed a household is to trade through its expenditure shares and (ii) its own price elasticity. Because the elasticity part reflects the marginal utility of consumption, the elasticity effect delivers the intuitive idea that one aspect of the gains from trade is a households' individual valuation of the price reduction. So even if a rich and poor household have similar expenditure patterns, the reduction in price is more valuable on the margin for the poor, high marginal utility households.

Before moving on to the quantitative work, I explore two special cases to highlight the role that market incompleteness and preferences play in shaping these results. The first case is the efficient allocation where a planner can reallocate resources and overcome market incompleteness. In this case, I recover "first-best intuition" with the gains from trade only reflecting the direct savings associated with a reduction in trade costs. In this allocation, changes in expenditure patterns are not relevant via an envelope theorem argument — the planner already sources goods from the correct places so there are no gains from expenditure switching. And heterogeneity in a household's valuations of gains are irrelevant because marginal utility is equated. While my economy is about heterogeneity on the household side, this result is reminiscent of Atkeson and Burstein (2010) and the irrelevance of firm heterogeneity in an economy where the allocation is efficient. Thus, the core issues at play in my model are not household heterogeneity per se, but inefficiencies induced by market incompleteness.

The second special case is when the utility function over the physical commodity is $\log$. With $\log$ utility, I obtain a separation result where aggregate trade outcomes "separate" from household heterogeneity and all households gain through lower commodity prices in the same way.[1] Trade takes a constant elasticity. gravity form with the trade elasticity pinned down by the dispersion parameter on the taste shocks similar to Eaton and Kortum (2002). The welfare impact of lower commodity prices is common across households and takes the same form as in Arkolakis et al. (2012) with the trade elasticity and the change in the share of home purchases summarizing the gains from trade. The reason behind these results is that the marginal gain in utility from a percent change in consumption — $u'(c)c$ — is independent of the level of consumption with $\log$ preferences. Thus, both rich and poor households substitute in the same way and they gain the same amount from lower prices.

Quantitatively, I make a contribution by computing and calibrating the model at a scale typically reserved for static trade models. As a testing ground, I focus on the data set of Eaton and Kortum (2002). The 19 countries in this data set is about the right size to easily illustrate how a very rich model like this can work in a multi-country setting. Moreover, the Eaton and

[1]This case is also interesting because Anderson, De Palma, and Thisse (1987) showed that in a static model with log utility and additive logit shocks, the economy behaves *as if* there were a representative agent CES consumer. In my economy, my suspicion was that market incompleteness and intertemporal behavior would nullify Anderson et al.'s (1987) result—it does not.

Kortum (2002) data set provides a well defined benchmark disciplined by bilateral trade flows and gravity variables—so it's a nice laboratory to explore new issues in.

The calibration challenge is the following. The model does not admit a gravity representation that allows researchers to invert trade frictions and productivity levels from trade flows as done in Eaton and Kortum (2002) and many subsequent papers. Similarly, the model does not admit the use of exact-hat algebra which allows the research to construct counterfactuals without the knowledge of primitives like trade frictions or productivity (see, e.g., Costinot and Rodríguez-Clare (2014) or the dynamic extension in Caliendo, Dvorkin, and Parro (2019)).

My solution is to use the insight that the regressions employed in gravity frameworks provide very accurate descriptions of the data generating process. Rather than treating the gravity regression as a structural relationship, I use it as a "guide" and use an indirect inference procedure where I estimate parameters of the model so that the regression coefficients from a standard gravity regression run on my model's data match that seen in the data. This procedure works well and, thus, the model is able to match spatial distribution of economic activity in the data—just as well as standard, constant elasticity gravity models.

I then illustrate the quantitative implications of the model by working through several counterfactual changes in trade costs and studying the welfare gains from trade. Figure 2 illustrates the main qualitative takeaway showing that gains from trade are strongly pro-poor with the poorest households gaining four and a half times more from trade than the richest households. And because the poor gain a lot, while rich households look a lot like Arkolakis et al. (2012) type households, the average gains from trade are three times larger than representative agent benchmarks. I show that this general theme — pro-poor and large on average — holds both for bilateral reductions and global reductions.

Behind these large, pro-poor gains are the force I emphasize in the theory section of the paper — heterogenous price elasticities. This is validated in two ways. First, my calibration scheme builds on the evidence of Borusyak and Jaravel (2021) by ensuring household-level import expenditure shares are roughly the same across rich and poor households, so differences in exposure do not account for the pro-poor gains found. In contrast, the model was designed and calibrated to match the facts of Auer, Burstein, Lein, and Vogel (2022) — that poor households are very elastic with respect to price and, thus, poor households strongly value a price reduction. This point is further validated by exploiting the example of the $\log$ preference model with households substituting in a common way. In this specification, micro-level heterogeneity plays little role and the gains from trade are nearly uniform across rich and poor households.

Motivating my work has been a sequence of papers focusing on measuring the heterogenous impacts of trade on the consumer side. Fajgelbaum and Khandelwal (2016), Carroll and Hur (2020), Borusyak and Jaravel (2021), Jaccard (2023) are recent examples that measure heterogeneity in import exposure. Auer, Burstein, Lein, and Vogel (2022) and Colicev, Hoste, and Konings (2022) go a step further measuring heterogeneity in price sensitivity across the income distribution and this type of evidence is very much the launching point for my paper.

While this work motivates my paper, I take a conceptually different approach. Rather than focusing on measurement, I develop a model of household heterogeneity that endogenously delivers heterogeneity in price elasticities and study its implications. In this sense, my papers approach is most similar to Fajgelbaum, Grossman, and Helpman (2011) who study how inequality and non-homotheticities shape trade in vertically differentiated products. A unique aspect of my work is that I start with a theory behind the distribution of income and wealth. This theory plus a demand system with heterogenous price elasticities then breaks aggregation in the goods market. And because aggregation is broken, it opens the door to new insights about trade, the interaction of trade and financial markets, and how market incompleteness shapes the aggregate pattern of trade.

My paper also relates to a recent series of papers that combines trade models with heterogenous agent, incomplete market models. Some of this is my own work in Lyon and Waugh (2018), Lyon and Waugh (2019), Waugh (2019); Ferriere, Navarro, and Reyes-Heroles (2022) and Carroll and Hur (2020) are important contributions as well. This class of papers primarily focuses on how heterogenous exposure through the labor market passes through to consumption and, thus, welfare. In this paper, I'm doing something different and it is an attempt to answer the question: does heterogeneity and market incompleteness matter for trade?

This paper also relates to a body of work focusing on the pricing implications in the presence of heterogenous price sensitivity. Nakamura and Zerom (2010) is an early example of a macro-style model with an IO-style demand system similar in spirit to my paper and they focuses on the implications for the incomplete pass through of shocks to prices. My own work in Mongey and Waugh (2023) is very much a companion piece to this paper with imperfect competition in product markets and it focuses on the heterogenous pass-through of supply and demand shocks into prices for different types of consumers. Nord (2022) takes a search-theoretic approach, but the core issue is the same — how demand composition affects pricing decisions. With that said, this model simplifies matters by focusing on a world with perfect competition and, hence, I turn my focus on how household heterogeneity matters for trade.

1. The Heterogeneous Agent Trade Model

This section describes the model and then defines the decentralized competitive equilibrium. Trade is in the Armigton tradition with each country producing a nationally differentiated variety. Households face the "income fluctuations problem" as in the standard incomplete markets tradition (see, e.g., Chapter 17 of Ljungqvist and Sargent (2012)).

The key twist is that I do not employ modeling techniques with aggregation at household level across national varieties. Instead, I lean into the household heterogeneity and have households make a discrete choice over the varieties they consume in addition to their savings decisions. Aggregate trade flows, trade elasticities, and the gains from trade are then defined by the explicit aggregation of household-level decisions to purchase different varieties, their elasticity of demand, and their gains from trade.

1.1. Production and Trade

There are M locations which I call a country. Each country produces a nationally differentiated product. In country i, competitive firms' production technology to produce variety i is:

$$Q_i = A_i N_i, \tag{1}$$

where A_i is total factor productivity and N_i are the efficiency units of labor supplied by households in country i.[2]

I focus on only one type of barrier to trade: there are iceberg trade costs $d_{ij} > 1$ for a good to go from supplier j to buyer i.

Profit maximization of the producers in location i results in the wage per efficiency unit reflecting the value of the marginal product of labor

$$w_i = p_i A_i. \tag{2}$$

Given iceberg trade costs, the unit cost for country i to purchase a good from location j is

$$p_{ij} = \frac{d_{ij} w_j}{A_j}. \tag{3}$$

This is the trade and production side of the model. While sparse, it's worth reminding you that with a representative agent and a constant elasticity Armigton aggregator much comes out of this model. There is a gravity equation relating bilateral trade flows to country characteristics with a constant trade elasticity. And there are two sufficient statistics (the trade elasticity and home trade share) that globally characterize the welfare gains from trade. In the next section, I give up on the representative agent.

[2]Note that lack of physical capital in the model. Households here are saving in via pure exchange of non-state contingent IOUs as in Huggett (1993) rather than in physical capital as in Aiyagari (1994).

1.2. Households

There is a mass of L_i households in each country. Households are immobile across countries. They are infinite lived and have time-sparable preferences over consumption of varieties:

$$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{u}(\{c_{ijt}\}_M), \tag{4}$$

where the notation $\{c_{ijt}\}_M$ means that the household has preferences over all j varieties supplied by M countries in the world. Here I'm indexing things by ij to denote the variety j that is consumed in location i at date t.

Households' period utility function is of the random utility class and each period households can only consume one variety.[3] The utility associated with the choice of variety j is

$$\tilde{u}(c_{ijt}) = u(c_{ijt}) + \epsilon_{jt}. \tag{5}$$

where the ϵ_{jt} are iid random variables across time, households, and countries. For the analysis, I assume that these shocks are distributed Type 1 Extreme Value with CDF

$$F(\epsilon) = \exp(-\exp(-\sigma_\epsilon^{-1}\epsilon)) \tag{6}$$

where σ_ϵ is the dispersion parameter. A useful generalization of this setting to a multi-sector model is the "infinite shopping isle" approach of Mongey and Waugh (2023) where these shocks take on a Generalized Extreme Value representation and then households choose the sector and then the variety each period.

For now, all I assume is that the utility function over the physical good c_{ijt} is well behaved. In the analysis below I explore different specifications of the utility function u over the physical commodity. The canonical case for product markets (Anderson et al. (1987)) or the spatial literature is where u is $\log$ utility. Below, I highlight the rather curious properties of this case.

A household's efficiency units are stochastic and they evolve according to a Markov chain. So, z is a household's efficiently units and $\mathcal{P}(z, z')$ describes the probability of a household with state z efficiency units transiting to state z'. Again, I assume that $\mathcal{P}$ is well behaved in the necessary ways.

Households can save and borrow in a non-state contingent asset a that is denominated in the units of the numeraire. One unit of the asset pays out with gross interest rate R_i next period. I discuss this more in depth below, but the determination of R_i is that which clears the bond

[3] A more formal statement of preferences is that they $\tilde{u}(\{c_{ijt}\}_M) = \sum_j \imath_j \tilde{u}(c_{ijt})$ where $\imath_j$ is an indicator function taking the value one if the consumer chooses variety j and zero otherwise.

market (local or global). A country specific, exogenous debt limit ϕ_i constrains borrowing so:

$$a_{t+1} \geq -\phi_i. \tag{7}$$

All these pieces come together in the household's budget constraint, conditional on choosing variety j to consume, and focusing on a stationary setting where prices are constant:

$$p_{ij} c_{ijt} + a_{t+1} \leq R_i a_t + w_i z_t. \tag{8}$$

The value of asset purchases and consumption expenditures must be less than or equal to asset payments and labor earnings.

1.3. The Household Problem

The state variables of a individual household are its asset holdings and efficiency units. As alluded to above, for now I focus on a stationary setting where aggregates are not changing and, thus, I abstract from carrying the notation associated with them around.[4]

The value function of a household in country i, after the variety shocks are realized, is

$$v_i(a, z) = \max_j \left\{ \ v_i(a, z, j) \ \right\} \tag{9}$$

which is the maximum across the value functions associated with the discrete choices of different national varieties. The value function conditional on a choice of variety is

$$v_i(a, z, j) = \max_{a'} \left\{ u(c_{ij}) + \epsilon_j + \beta \, \mathbb{E}[v_i(a', z')] \right\} \tag{10}$$

subject to (7) and (8)

where households choose asset holdings and the level of consumption is residually determined through the budget constraint. Associated with the solution to this problem is a policy function $g_i(a, z, j)$ which solves (10) and maps current states into asset holdings tomorrow a' contingent upon the variety choice j. Correspondingly, there is a consumption function $c_i(a, z, j)$ mapping states into consumption today, contingent upon the variety choice j.

The continuation value function on the right-hand side of (10) is the expectation over (9) with respect to (i) efficiency units next period, z' and (ii) the variety taste shocks. An implication of

[4] If you *do* want to carry them around, notice that all that households in each country care about are prices (today and in the future). The distributions of households in other countries, per se, don't matter. Thus, the relevant aggregate states in country i are $\left[\ \{w_i\}_M, R_i \right]$ which is the collection wage per efficiency units and the interest rate.

this is that households understand that their may be situations where they really desire, say, a high priced imported good and, hence, save accordingly.

The Type 1 extreme value distribution on the taste shocks give rise to the following choice probabilities for each differentiated good:

$$\pi_{ij}(a,z) = \exp\left(\frac{v_i(a,z,j)}{\sigma_\epsilon}\right) \Big/ \Phi_i(a,z), \tag{11}$$

$$\text{where} \quad \Phi_i(a,z) := \sum_{j'} \exp\left(\frac{v_i(a,z,j')}{\sigma_\epsilon}\right). \tag{12}$$

which is the probability that a household with assets a and efficiency units z chooses country variety j. The term in the denominator, $\Phi_i(a,z)$, has a "price-index" interpretation and is very similar in spirit to the same term in Eaton and Kortum (2002). And then the expectation of (9) with respect to the taste shocks takes the familiar log-sum form

$$v_i(a,z) = \sigma_\epsilon \log\left\{\Phi_i(a,z)\right\}. \tag{13}$$

Associated with this problem in (10) for non-borrowing-constrained households is an Euler Equation for each variety choice j:

$$\frac{u'(c_i(a,z,j))}{p_{ij}} = \beta R_i \mathrm{E}_{z'}\left[\sum_{j'} \pi_{ij'}(a',z')\frac{u'(c_i(a',z',j'))}{p_{ij'}}\right]. \tag{14}$$

This has a very natural interpretation: a household equates marginal utility of consumption today with expected discounted marginal utility of consumption tomorrow adjusted by the return on delaying consumption. The interesting feature here is that the expected value of the marginal utility of consumption reflects the uncertainty over one's preference over different varieties tomorrow via the choice probabilities. And note that households has some control over these probabilities as the asset choice today influence the choice probabilities tomorrow.

Before moving on to aggregation, I make one useful observation that assists the analysis. Stare at (11) and (13) long enough, one can arrive at a dynamic, sufficient statistic representation of $v_i(a,z)$. Appendix B works through the individual steps, but (13) can be summarized as

$$v_i(a,z) = -\sigma_\epsilon \log \pi_{ii}(a,z) + u(c_i(a,z,i)) + \beta \mathbb{E}_{z'} v_i(a',z'). \tag{15}$$

Here the ex-ante value function (prior to the realization of the preference shocks) is expressed as a sum of the log home choice probability, utility over physical consumption of the home good, and recursively the expected value function tomorrow. What's going on here is that the home choice probability π_{ii} summarizes the expected value of the taste shocks, their benefits, and how households respond to them in the future.[5]

Equation (15) together with (14) also provides more insight about how households' savings motives interact with the variety choice. Focusing on a household consuming the home good (and note that the left-hand-side below could be for any variety choice), the Euler Equation in (14) becomes:

$$\frac{u'(c_i(a,z,i))}{p_{ii}} = \beta \mathrm{E}_{z'} \Big\{ -\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + \frac{u'(c_i(a',z',i))R_i}{p_{ii}} \Big\}, \tag{16}$$

which says that an unconstrained household should be indifferent between the marginal utility of consumption forgone to hold some more assets and two components: (i) the benefit from how a change in assets changes in their variety choice in the future and this is summarized by the change in the home choice probability and (ii) the direct benefit of the returns on the assets evaluated at the marginal utility of consumption.

1.4. Aggregation

Aggregation. At the core of aggregation is a probability distribution $\lambda_i(a,z)$ describing the measure of households across the individual states. This distribution evolves according to

$$\lambda_i(a',z') = \sum_j \int_z \int_{a:a'=g_i(a,z,j)} \pi_{ij}(a,z)\mathcal{P}(z,z')\lambda_i(a,z)\, da\, dz. \tag{17}$$

where the inner most term describes the mass of households choosing variety j, multiplied by the probability that z transits to z', multiplied by the existing measure of households with states a and z. This is integrated with respect to those actually choosing asset holdings a', over all z's, and then summed over the different variety choices.

Given this distribution, everything else follows. First focusing on trade, aggregate bilateral imports are

$$M_{ij} = L_i \int_z \int_a p_{ij} c_i(a,z,j)\pi_{ij}(a,z)\lambda_i(a,z)\, da\, dz. \tag{18}$$

Here imports take on a mixed logit formulation that very much mimics that used in the industrial organization literature, e.g, Berry, Levinsohn, and Pakes (1995). There are, however, several interesting differences. First, there is an active intensive margin, not unit demand. Sec-

[5] Home choice probabilities are not necessarily the same as home trade shares, but this is closely related to Equation (15), Footnote 42 of Eaton and Kortum (2002) and I'm heading towards situations where this result plus restrictions on u give rise to the result in Arkolakis et al. (2012).

ond, inside the choice probability $\pi_{ij}(a,z)$ is the non-linear value function from (9).[6] Because the choice probability reflects the value function, it embeds forward looking behavior of the household.

The third interesting feature is that the mixing distribution (the λ) over which demands are aggregated is endogenous. Through the law of motion in (17) household behavior determines the distribution of wealth. In other words, this model imposes cross-equation restrictions between aggregate demand and individual demands through the distribution. So it's not a free parameter and it will change with changes in primitives of the environment.

Similar to imports, aggregate bilateral exports from country i to country j are

$$X_{ji} = L_j \int_z \int_a p_{ji} c_j(a,z,i) \pi_{ji}(a,z) \lambda_i(a,z) da\, dz. \tag{19}$$

The value of aggregate consumption is

$$\widetilde{P_i C_i} = L_i \sum_j \int_z \int_a p_{ij} c_i(a,z,j) \pi_{ij}(a,z) \lambda_i(a,z) da\, dz \tag{20}$$

In (20), one can see both a bug and a feature of this model. Here there is an "index number problem" in the sense that there is not an ideal price index for which one can decompose aggregate values in to a price and quantity component. This is in contrast to, e.g., a model where households consume a CES bundle of goods.

Finally, the aggregate quantity of asset holdings integrates across the asset choices of individual households

$$\mathrm{A}_i' = L_i \sum_j \int_z \int_a g_i(a,z,j) \pi_{ij}(a,z) \lambda_i(a,z) da\, dz. \tag{21}$$

which integrates over the asset choices—given the policy function $g_i(a,z,j)$ and variety choices $\pi_{ij}(a,z)$. And then sum's across the different varieties available.

National Accounting. From here, I reconstruct national income and product identities. Starting from the production side, aggregate efficiency units are

$$N_i = L_i \int_z \int_a z \lambda_i(a,z) da\, dz. \tag{22}$$

and from here we can connect the value of aggregate production must equal aggregate pay-

[6]A good contrast is Nevo (2000) where inside the choice probability is an indirect utility function of the form $\eta \times (y - p_{ij})$ where y and p are in logs and η is a parameter to be estimated and it stands in for the marginal utility of consumption. And related to my next comment, λ is just "read from the data" and treated as policy invariant.

ments to labor so

$$p_i Y_i = p_i A_i N_i = L_i \int_z \int_a w_i \, z \, \lambda_i(a, z) da \, dz, \tag{23}$$

Then by summing over individual consumers' budget constraint and substituting in (23), the aggregated budget constraint is:

$$p_i Y_i = \widetilde{P_i C_i} + \Big[- R_i \mathrm{A_i} + \mathrm{A_i'} \Big], \tag{24}$$

where national income equals the value of aggregate consumption $\widetilde{P_i C_i}$ and the country's net factor payments and net asset position. To arrive at the standard national income accounting identity, simply work with the relationship between production, exports, and aggregate consumption in (20) and imports gives rise to

$$p_i Y_i = \widetilde{P_i C_i} + \Big[\sum_{j \neq i} X_{ji} - \sum_{j \neq i} M_{ij} \Big], \tag{25}$$

where national production or GDP equals consumption plus exports minus imports. A comparison of (24) and (25) then makes clear that the trade imbalance is connected with a countries net factor payments and net asset position.

Beyond accounting, this last observation shows how trade flows are interlinked with financial flows. Inspection of the individual elements in (18), (21), and the households' budget constraint reveal that household's asset positions are intertwined with trade flows through both the intensive (how much to consume and, hence, save) and the extensive margins (which variety to consume). Thus, a feature of this model is that the trade side is interlinked with the financial side of the economy in a non-trivial way.

1.5. The Decentralized Equilibrium

In this section, I discuss the market clearing conditions that an equilibrium must respect and then define the Decentralized Equilibrium of this economy.

The Goods Market. Goods market clearing equates the value of production of commodity i with global demand for country i's commodity:

$$p_i Y_i = \sum_j X_{ji}, \tag{26}$$

where the left hand side is production and the right hand side is world demand for the commodity (via exports) from (19).

The Bond Market. The second market clearing condition is the bond market. There are two case worth thinking about here. One is of "financial autarky" in which there is a local bond market that facilitates within country asset trade, but not across countries. In this case, there is an interest rate R_i for each country and the associated market clearing condition is

$$\mathrm{A}_{\mathrm{i}}' = 0, \quad \forall i \tag{27}$$

which says that net asset demand within each country i must be zero. As is common in the trade literature, this condition implies that trade is balanced—just stare at (24) and (25). Yet, even with balanced trade, there is still within country trade of financial assets. Some households are savers, others are borrowers and the interest rate is that which the net asset position is zero.

The second case is of "financial globalization" where there is a global bond market that facilitates both within country asset trade, and across countries. In this case, there is a single interest rate R and the associated market clearing condition is

$$\sum_i \mathrm{A}_{\mathrm{i}}' = 0 \tag{28}$$

In this case trade need not be balanced for each country. Here a specific country might run, say, a trade deficit because at the given prices, the total amount of borrowing within a country is larger than the total amount of saving. However, across all countries total borrowing must equal total saving.

Below I formally define the Decentralized Stationary Equilibrium where private market participants taking prices as given solve their problems, the distribution of households is stationary, and prices are consistent with market clearing.

Definition 1 (The Decentralized Stationary Equilibrium) A Decentralized Stationary Equilibrium are asset policy functions and commodity choice probabilities $\{\ g_i(a,z,j), \pi_{ij}(a,z)\ \}_i$, probability distributions $\{\ \lambda_i(a,z)\ \}_i$ and positive real numbers $\{w_i, p_{ij}, R_i\}_{i,j}$ such that

i Prices (w_i, p_{ij}) satisfy (2) and (3);

ii The policy functions and choice probabilities solve the household's optimization problem in (9) and (10);

iii The probability distribution $\lambda_i(a,z)$ induced by the policy functions, choice probabilities, and primitives satisfies (17) and is stationary;

iv Goods market clears:

$$p_i Y_i - \sum_j X_{ji} = 0, \ \ \forall i \tag{29}$$

v Bond market clears with either

$$\mathrm{A}_{\mathrm{i}}' = 0, \ \ \forall i \quad \text{or} \quad \sum_i \mathrm{A}_{\mathrm{i}}' = 0 \tag{30}$$

1.6. Outline of the rest of paper

This model above has households making individual choices over national varieties, savings, all while facing productivity and taste shocks. Explicit aggregation of household behavior determines the pattern of trade and this is linked with trade in financial assets. The remaining sections of the paper work through the following questions:

1. **What are the model's implications for trade elasticities and the gains from trade in decentralized allocation?** Here I characterize micro and macro level trade elasticities and connect them with households' inter-temporal motives and the heterogeneity in the marginal utility of consumption that is induced by market incompleteness. I also work-out a knife edge case when utility is $\log$ over consumption and how it delivers complete separation between the heterogenous agent side of the economy and trade.

2. **What are the quantitative implications of this model?** I then compute and calibrate the model with a key featuring being that the model matches the spatial distribution of economic activity in the data—just as well as static, standard, constant elasticity gravity models. I then perform several counterfactuals to illustrate the mechanics of the model and what the pattern of trade should look like in the centralized allocation.

2. Trade Elasticities and the Gains from Trade

This section focuses on the decentralized equilibrium and works towards understanding core trade outcomes — trade elasticities (Proposition 1) and the gains from trade (Proposition 2) and how micro-level heterogeneity determines them. I then contrast these results with how elasticities behave in the efficient allocation (Proposition 3) and the $\log$ preference case when micro-level heterogeneity does not affect aggregate trade outcomes (Corollary 1).

2.1. Trade Elasticities

My definition of the trade elasticity is the partial equilibrium response of imports from j relative to domestic consumption due to a permanent change in trade costs.[7] By partial equilibrium, I mean that wages, interest rates, and the distribution of agents are fixed at their initial equilibrium values. This is consistent with the definition of the trade elasticity in say, Arkolakis et al. (2012) or Simonovska and Waugh (2014). By permanent, I mean that the change in trade costs is for the indefinite future and that households correctly understand this.

Given this discussion, my mathematical definition of the aggregate trade elasticity is

$$\theta_{ij} = \frac{\partial M_{ij}/M_{ij}}{\partial d_{ij}/d_{ij}} - \frac{\partial M_{ii}/M_{ii}}{\partial d_{ij}/d_{ij}}. \tag{31}$$

Then working from the definition of imports in (18), Proposition 1 connects the aggregate trade elasticity with micro-level behavior:

Proposition 1 (The HA Trade Elasticity) *The trade elasticity between country i and country j is:*

$$\theta_{ij} = 1 + \int_{z,a} \left\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \right\} \omega_{ij}(a,z) da\, dz - \int_{z,a} \left\{ \theta_{ii,j}(a,z)^I + \theta_{ii,j}(a,z)^E \right\} \omega_{ii}(a,z) da\, dz \tag{32}$$

which is an expenditure-weighted average of micro-level elasticities. The micro-level elasticities are decomposed into an intensive margin and extensive margin

$$\theta_{ij}(a,z)^I = \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}, \qquad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and the expenditure weights are defined as

$$\omega_{ij}(a,z) = \frac{p_{ij} c_i(a,z,j) \pi_{ij}(a,z) \lambda_i(a,z) L_i}{M_{ij}}.$$

and the notation $\theta^I_{ii,j}$, $\theta^E_{ii,j}$ represents how home choice probabilities on the intensive and extensive margin respond to the ij change in trade costs.

Proposition 1 says that the aggregate trade elasticity is an expenditure weighted average of micro-level trade elasticities. And these elasticities are decomposed into two components: an intensive margin trade elasticity $\theta_{ij}(a,z)^I$ which is the change in spending by a household on variety from j and an extensive margin trade elasticity $\theta(a,z)^E_{ij}$ reflecting how households substitute across varieties. And this is all relative to how these margins adjust home choices given the change in j, hence, the subscripts ii, j in the second part of equation (32).

Proposition 1 is derived only off the aggregation of imports at the micro level—no market clear-

[7] Because the aggregate distribution of households will adjust—even with prices fixed—the elasticities that I derive are in a sense "short-run" elasticities.

ing, functional forms, etc. It's essentially an identity that could be applied to any model. The next step inserts my model of household behavior. From the household's budget constraint, I can say more about the intensive margin elasticity. Then with the Type 1 extreme value assumption and the household's problem, I can say more about the extensive margin elasticity.

The Intensive Margin Elasticity. The intensive margin elasticity is about how do quantities change, conditional on a choice. Starting from the budget constraint in (8) I express the intensive margin elasticity as:

$$\underbrace{\frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}}_{\theta_{ij}(a,z)^I} = \left[-\frac{\partial g_i(a,z,j)/p_{ij}c_i(a,z,j)}{\partial p_{ij}/p_{ij}} - 1 \right] \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}, \tag{33}$$

where $g_i(a,z,j)$ is the policy function mapping states into asset holdings next period a'.

The inside bracket of Equation (33) connects the intensive margin elasticity with the household's savings decision, i.e., how it adjusts its wealth relative to expenditure when prices change.[8] A way to think about (33) is that it answers the question: If a household faced with lower prices, how much would go to extra consumption, how much to savings? And this division of resources determines the intensive margin elasticity.

Heterogeneity matters here because this division is not invariant to a household's state a, z and it works through their incentives to save. For example, if a households is constrained, assets can't adjust, and the intensive margin becomes -1. In contrast, wealthy households save some stuff from a reduction in prices and the intensive margin for these households will be less than one in absolute value. The result is that poor households are more price sensitive than rich households — on the intensive margin — and the mechanism works through their savings motives.

The Extensive Margin Elasticity. The extensive margin elasticity is about how households substitute across varieties. The elasticity of the choice probability with respect to a change in trade costs is

$$\underbrace{\frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}}}_{\theta_{ij}(a,z)^E} = \frac{1}{\sigma_\epsilon}\frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}} \tag{34}$$

The second term is how the value of all variety options change (recalling the definition of $\Phi_i(a,z)$ in 12). This is very much similar to how CES models behave except that the price-

[8]Outside of the bracket in (33) is how prices change with trade costs which is also known as "pass-through." In the competitive environment here, it is always one, even though there is an is an active super-elasticity in the background. In the non-competitive environment of Mongey and Waugh (2023), the super-elasticity matters for price responses and pass-through deviates from one.

index-like term is state a, z specific. The leading term is how the choice specific value function changes multiplied by the taste shock parameter.[9] In other words, how elastic or the extensive margin is depends on how much more valuable choice j becomes.

Now assume the number of countries is large. Then the effects on the Φ term are negligible. Thus the second term in (34) is negligible. Moreover, because the future impacts in the value function are just functions of Φ, the only non-negligible term in the value function moving around is the effect of the change in utility today and, hence:

$$\theta_{ij}(a,z)^E \approx -\frac{1}{\sigma_\epsilon}\Bigg[u'(c_i(a,z,j))c_i(a,z,j)\Bigg]. \tag{35}$$

The term in the brackets is a core piece of this paper and it shows up repeatedly. Mathematically, this term is the semi-elasticity of utility with respect to a percent change in consumption.[10] So it determines how many more incremental utils does a household get, given a percent change in consumption. The intuition as to why this matters for the extensive margin elasticity is that if a household receives a lot of utils, on the margin, from the change in trade costs, then the substitution to that variety is stronger. In contrast, if that variety does not yield a lot of utils, on the margin, than substitution is weak.

How does this elasticity depend upon a household's circumstances? Differentiate (35) with respect to assets. The thought experiment here is if a household was a bit wealthier how much more elastic would the household be:

$$\frac{\partial(u'(c_i(a,z,j))c_i(a,z,j))}{\partial a} = u'(c_i(a,z,j)) \times \mathbf{MPC}_i(a,z,j) \times \Bigg[-\gamma_i(a,z,j)+1\Bigg], \tag{36}$$

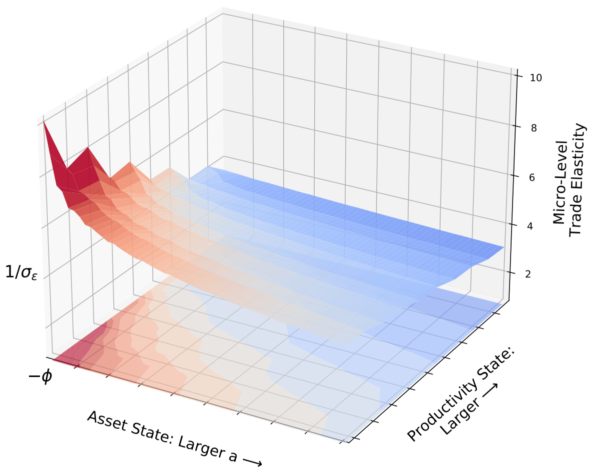
where $\mathbf{MPC}_i(a,z,j)$ is the household's marginal propensity to consume and $\gamma_i(a,z,j)$ is the Arrow-Pratt measure of relative risk aversion. With constant relative risk aversion (CRRA) preferences then $\gamma_i(a,z,j)$ becomes a constant γ and log preferences is when $\gamma = 1$. Equation (36) implies that if $\gamma > 1$, then poor, high marginal utility households who are also likely high MPC households are *more elastic relative* to rich households on the extensive margin.

Intuitively, what this means is given a price reduction on some variety, it delivers a lot of utility — on the margin — for poor households. So this induces strong substitution into that variety by poor households. For rich households a price reduction for delivers little incremental gain in utility and, thus, this induces weak substitution into that variety.

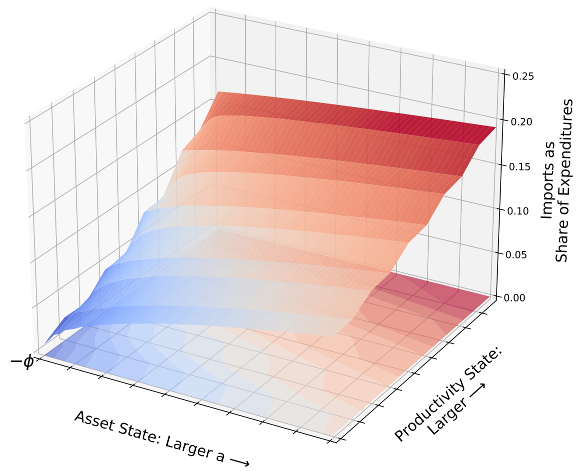
Two more points about this that foreshadow several results. It makes clear the specific role that preferences play. For example, with $\log$ preferences the semi-elasticity of utility with respect to

[9]As I show in the Appendix, a third term reflecting the effect of borrowing constraints would also be here, but via envelope theorem type arguments, they zero out for small changes in trade costs.

[10]If this is not clear, note that this semi-elasticity is $\partial u(c)/(\partial c/c) = u'(c)c$.



(a) Household-Level Elasticities, $-\theta_{ij}(a, z)$



(b) Household-Level Import Shares

a percent change in consumption is always one ,i.e., the term in brackets in (35) is one. Even though market incompleteness generates heterogeneity in marginal utility, a percent change in consumption delvers the same change in utils for rich and poor households alike (this can be seen also in (36)). Then rich and poor households substitute on the extensive margin in the same way.

Second, equation (35) mimics the trade elasticity expression in the socially optimal, efficient allocation in equation (43). In the efficient allocation, the planner's elasticity reflects the incremental social gain in utils from the change in trade costs. What is different in (35), is that households' private valuations associated with the change in trade costs differ and, thus, they substitute differently given their own specific circumstances (outside of the knife edge case of $\log$ preferences). Here, heterogeneity in elasticities are a symptom of a conflict between social and private valuations of the change in trade costs.

Figure 1a and 1b illustrates how this works in a two country economy. Figure 1a plots the absolute value of the trade elasticity (intensive and extensive margin) by household state (assets are on the x-axis, productivity state on the y-axis) and the borrowing constraint ϕ is in the south-west corner. This shows is how the trade elasticity systematically varies with assets and income: Poor households—especially those near the borrowing constraint—are very price elastic with a trade elasticity of around -10. Richer households are less price elastic with this elasticity declining towards -3.

This pattern of trade elasticities has a strong intuitive feel and there is evidence in support of it.[11]. This result comes out of estimates in Berry et al. (1995) and in a more macro context,

[11]This idea goes back to Harrod's (1936) "Law of Diminishing Elasticity of Demand" that says that price sensitivity declines with income which is not to be confused with Marshall's Second Law of Demand which I discuss

Nakamura and Zerom (2010). Sangani (2022) is a recent paper that provides evidence in support of this fact from the Kilts-Nielsen data set. The evidence in Auer, Burstein, Lein, and Vogel (2022) most closely relates to the patterns in Figure 1a with poorer households having higher price elasticities; Colicev, Hoste, and Konings (2022) finds similar results.

One more implication of this result: because rich and poor households face the same prices, differences in elasticities lead to different expenditure shares. Figure 1b illustrates this point by plotting expenditure on the foreign good relative to total expenditure. Because of trade costs and symmetry across countries, the home good is the relatively cheaper good. Thus, poor, high-price-elastic households spend more on the cheap home good versus the expensive foreign good. In fact, for those near the borrowing constraint in this example, it's near zero. This pattern appears counterfactual per the evidence of Borusyak and Jaravel (2021) and, hence, it motivates my introduction of non-price product attributes (quality) in the quantitative application to match micro-level expenditure shares and elasticities.

2.2. The Gains from Trade

In this section I compute how welfare changes due to a change in trade costs. The purpose here is to illustrate mechanics and where the gains from trade arise from. To that end, I derive these gains across steady states where I'm thinking a situation where the change is small and there is an immediate jump to the new steady state. Unlike the trade elasticity, I take total derivatives that encompass general equilibrium changes in wages and interest rates.

The analysis proceeds in several steps before stating the main result in Proposition 2. First, I focus only on country i and study a change in trade costs d_{ij}. I then start from the top down with the following social welfare function:

$$W_i = \int_z \int_a v_i(a,z) L_i \lambda_i(a,z) \tag{37}$$

where $v_i(a,z)$ is a households ex-ante value function in country i, with states a, z. The total change in total welfare is

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} \approx \int_z \int_a \left\{ \frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} + v_i(a,z) \frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \right\} L_i \lambda_i(a,z) \, da \, dz. \tag{38}$$

where the $\approx$ symbol reinforces that this is a heuristic, thinking about a small change and there is an immediate jump to the new steady state.

At a high-level, (38) illustrates that the gains from trade come through two forces. The first component reflects changes in household-level welfare. So conditional on a distribution of

later.

households across states, this computes if households are better or worse off. The second component of (38) is about reallocation. It says: take the old v's and compute how the change in the distribution (that arise because of behavioral responses of households) effects social welfare. So does trade make it more or less likely that households are in "good" parts of the distribution.

The key issue is how household-level welfare changes. Here, I make the use of the observation in Equation (15) that I can express the ex-ante value function in only in terms of home choice ii values and then recursively push forward. In other words, I can compute the change in the ex-ante value function *as if* the household only consumed the home good for the infinite future. Appendix B provides the details and leads to the following expression:

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \underbrace{-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{A(a,z)} \tag{39}$$

$$+ \underbrace{u'(c_i(a,z,i))\left[\frac{\mathrm{d}w_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}} z + \frac{\mathrm{d}R_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}} a\right]}_{B(a,z)}$$

$$+ \underbrace{\left\{ - \frac{u'(c_i(a,z,i))}{p_{ii}} + \beta \mathbb{E}_{z'}\left[- \sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + \frac{u'(c_i(a',z',i))R_i}{p_{ii}}\right]\right\} \frac{\mathrm{d}g_i(a,z,i)}{\mathrm{d}d_{ij}/d_{ij}}}_{C(a,z)}$$

$$+ \beta \mathbb{E}_{z'}\Bigg\{ \underbrace{-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a',z')/\pi_{ii}(a',z')}{\mathrm{d}d_{ij}/d_{ij}}}_{A(a',z')} + \underbrace{u'(c_i(a',z',i))\left[\frac{\mathrm{d}w_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}} z' + \frac{\mathrm{d}R_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}} a'\right]}_{B(a',z')} \ \ldots$$

Let me walk through the interpretation of each term:

Gains from substitution: $A(a,z)$ The $A(a,z)$ term here is a household-specific gains from substitution term and is summarized by the change in the home choice probability and the dispersion parameter on tastes. The change in the home share summarizes two forces: (i) how exposed a household is to the change through the choice probabilities and then (ii) elasticities. To see this, define $\bar{\theta}(a,z)^{E}_{ij',j}$ as the extensive margin, cross-price elasticity, in total derivative form (and it's derivation follows what is done in 34). As I show in the Appendix B, the change in the home share can be expressed as:

$$-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \sigma_\epsilon \sum_{j'} \pi_{ij'}(a,z) \times \left[\bar{\theta}(a,z)^{E}_{ii,j} - \bar{\theta}(a,z)^{E}_{ij',j}\right], \tag{40}$$

$$\approx -\sigma_\epsilon \times \pi_{ij}(a,z) \times \bar{\theta}(a,z)^{E}_{ij,j}. \tag{41}$$

In words, the top line says that the change in home choice probability is equivalent to a weighted average of relative cross-price elasticities with the weights being the choice probabilities.

The next line assumes that all cross-terms are small, then the gains from substitution depend upon the initial exposure of a household to market j and their own-price elasticity (the total derivative analog to (34). And because the own-price elasticity is intimately connected to marginal utility of consumption, the elasticity effect picks up the intuitive idea that one aspect of the gains from trade is a household's individual valuation of the price reduction, in addition to the household's exposure.

This expression also connects with two related papers. Borusyak and Jaravel (2021) (following an approach dating back to at least Deaton (1989)) consider an environment where, to a first order, only exposure matters, similar to the exposure term in Equation (41). Auer et al. (2022) work out second order effects with non-homothetic CES preferences and additional effects from elasticities show up, similar to the elasticity term in (41). Here both are present in a first-order formulation. The interpretation here has a different flavor than articulated in Auer et al. (2022). Because elasticities are intimately connected with the marginal utility of consumption, this relationship is saying that the reason high elasticity households gain (conditional on exposure) more, is because the price reduction from trade is more valuable, on the margin.

Gains from factor prices: $B(a, z)$ The second $B(a, z)$ terms is essentially how a reduction in trade costs affects factor prices — the wage relative to the price of the home good and the interest rate relative to the price of the home good. And these effects are all valued at that household's marginal utility of consumption. I can simplify this term in two ways: in the perfect competition world I consider $\frac{w_i}{p_{ii}} = A_i$ from (2) and, thus, $\frac{\mathrm{d}w_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}} = 0$, so households are perfectly "hedged" from any effect of trade on labor earnings.

The second simplification is that $\frac{\mathrm{d}R_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}} = \frac{\mathrm{d}R_i/w_i}{\mathrm{d}d_{ij}/d_{ij}}$ and so the $B(a, z)$ term becomes

$$B(a, z) = u'(c_i(a, z, i)) \times a \times \frac{\mathrm{d}R_i/w_i}{\mathrm{d}d_{ij}/d_{ij}}. \tag{42}$$

Two issue that this term presents (i) how does the ratio of the gross interest rate relative to the wage rate change and (ii) how exposed is the household to this change. Unlike labor earnings, households are not perfectly hedged against these changes and because households can have negative positions in a changes in relative factor prices have distributional effects. For example, if a trade liberalization leads to an increase in R/w, net debtors suffer since their terms to borrow deteriorated, while net savers benefit. This force is very much analogous to forces identified in Auclert (2019).

Gains from changes in asset holdings: $C(a, z)$ The third term which I'm labeling as $C(a, z)$ is about changes in asset holdings. For the small / local changes that I'm considering it should zero out, but for larger changes this term could be relevant. Let me expand upon this.

First, notice that the inside the bracket term is the Euler Equation from (16) and its multiplied by the change in policy function. Now if the household is unconstrained, the inside the bracket term is zero as there is no gain through changes in asset behavior. Asset holdings are already chosen optimally so that margins are equated, thus, on the margin any benefit of lower trade costs on changes in asset behavior is zero. This inside the bracket term may not be zero because of borrowing constrained households. However, the bracket is multiplied by the change in the asset policy function. What this picks up is that if the household is constrained, then assets can't adjust so the outside term is zero and, thus, overall the second term is zero.

Via this logic, the only people that benefit (and contribute to social welfare) through these are those on the margin between constrained and not-constrained. But if they are on the margin between being constrained and not-constrained, then they are on their Euler equation anyways.

Finally, these repeat themselves into the expected future, appropriately discounted. Proposition 2 summarizes the result below.

Proposition 2 (HA Gains from Trade) *The welfare gains from trade are given by*

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} = \int_a \int_z \left\{ \frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} + v_i(a,z)\frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \right\} \lambda_i(a,z).$$

which reflects the change in household level gains and how the distribution of households changes. Household level gains are given by

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \Big\{ A(a_t,z_t) + B(a_t,z_t) + C(a_t,z_t) \Big\}$$

where each term represents:

- *Gains from substitution:* $A(a_t, z_t) = -\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}}$.
- *Gains from factor prices:* $B(a_t, z_t) = u'(c_i(a_t, z_t, i)) \frac{\mathrm{d}R_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}} a$
- *Gains from changes in asset holdings:*

$$C(a_t,z_t) = \left\{ -\frac{u'(c_i(a,z,i))}{p_{ii}} + \beta \mathbb{E}_{z'} \left[-\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + \frac{u'(c_i(a',z',i))R_i}{p_{ii}} \right] \right\} \frac{\mathrm{d}g_i(a,z,i)}{\mathrm{d}d_{ij}/d_{ij}}$$

  which is zero for small changes.

2.3. Elasticities and Gains in the Efficient Allocation

One issue behind the results above is market incompleteness. Households are imperfectly insured against the risks they face, this leads to heterogeneity in the marginal utility of consumption and, in turn, heterogeneity in price sensitivity, expenditure patterns, and the gains from trade. Building on work in from my related paper in Waugh (2023), I contrasts the previous results with the elasticities and gains from trade in an allocation where a social planner can overcome market incompleteness. Appendix C provides a self-contained discussion of the planning problem and derivation of the results.

The starting point is a utilitarian social welfare function which is analogous to (37). In Waugh (2023), I fully characterize a Planner's choice of consumption allocations $c_i(z, j, t)$ and choice probabilities $\pi_{ij}(z, t)$ for all i, j pairs, z states, and dates t to maximize social welfare. Given a characterization of the optimal allocation, I can compute trade elasticities and the welfare gains from a change in trade costs and study how welfare changes across the two stationary allocations.[12]

Proposition 3 describes the result, Appendix C works out the details.

Proposition 3 (Trade Elasticities and Welfare Gains in the Efficient Allocation) *The elasticity of trade to a change in trade costs between ij in the efficient allocation is:*

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon}\Big[u'(c_i(j))c_i(j)\Big]. \tag{43}$$

And the welfare gains from a reduction in trade costs between i, j are

$$\frac{\mathrm{d}W}{\mathrm{d}d_{ij}/d_{ij}} = \frac{\sigma_\epsilon\, \theta_{ij}\, \pi_{ij}\, L_i}{1-\beta}, \tag{44}$$

which is the discounted, direct effect from relaxing the aggregate resource constraint. And this can be expressed as

$$= -\sigma_\epsilon \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}} \times \frac{L_i}{1-\beta}. \tag{45}$$

Proposition 3 highlights a couple of things. Consistent with intuition from Eaton and Kortum's (2002), the dispersion parameter matters inversely. If σ_ϵ is small, national varieties are "as if they are near substitutes" and thus trade flows will respond a lot.

Very similar to the household-level extensive margin elasticity in (35), the aggregate trade elasticity has a term with the marginal utility of consumption times consumption showing up. Like in the discussion above, this term matters for the elasticity in a very intuitive way—country pairs that deliver a lot of utility, on the margin, are the pairs where the planner will be most responsive to changes in trade costs.

[12] Unlike the previous section, the move across stationary allocations is no consequence as there is no moving aggregate state variable, so the jump across stationary equilibrium is instantaneous.

The second part of Proposition 3 summarizes the gains from trade. It says that the total change in welfare only reflects those eating that commodity $\pi_{ij} \times L_i$ which is the share of households consuming commodity j times the number of households in country i. This is then converted into utils using the elasticity which, as discussed above, is something about the dispersion in shocks and then the rate at which utils are being delivered at current quantities.[13] This is then discounted for the infinite future, hence the $1/(1-\beta)$ term.

This is just the direct effect from a reduction in trade costs relaxing the resource constraint and converted to utils appropriately. Behind this result is an envelope-type argument with direct effects only mattering because I'm evaluating the change in welfare at the optimized allocation and any benefits of adjusting consumption and choice probabilities are zero—on the margin.

This result is reminiscent of Atkeson and Burstein (2010) who make a similar claim in the context of a model with rich firm heterogeneity. They show that the only first order effect of lower trade costs on welfare is the direct consumption effect and that indirect effects are second order. This is similar, but with household heterogeneity, by saying that, in the efficient allocation all margins are properly equated heterogeneity is irrelevant and the welfare gains only the direct benefits.

The final part of Proposition 3 connects with Arkolakis et al. (2012). As I show in the appendix, in the efficient allocation the percent change in the home choice probability exactly equals the ij choice probability times the trade elasticity

$$\frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}} = -\theta_{ij} \times \pi_{ij}, \tag{46}$$

then inserting (46) into (44) delivers the final line of Proposition 3. Now the form of (44) is closely related to Arkolakis et al. (2012). Interestingly, and much like in the decentralized allocation, the change in the home choice probability summarizes a lot. Moreover, now there is an equivalence between Atkeson and Burstein (2010)-like logic and Arkolakis et al. (2012)-style formulas.

There are two details: choice probabilities do not necessarily correspond with expenditure shares and the σ_ϵ is not the inverse of the trade elasticity However, with $\log$ the trade elasticity becomes $1/\sigma_\epsilon$, choice probabilities are proportional to expenditure shares, and the correspondence between the gains from trade under efficiency and Arkolakis et al. (2012) becomes exact. The case of $\log$ has an additional implication that heterogeneity and market incompleteness does not matter for trade outcomes and I turn to this case next.

[13] An alternative perspective is to divide through both sides of (44) by the marginal utility of consumption and then the welfare is in money metric units.

2.4. The Case of log preferences

The case of log preferences over the physical commodity displays some unique features. This very common) preference structure leads to an interesting result where micro-level heterogeneity, market incompleteness completely sperate from the trade side of the economy. So in this one case, trade behaves "as if" there were a representative agent Armington-CES consumer.

Consider the following preference structure:

$$\tilde{u}(c_{ij,t}) = \log(c_{ij,t}) + \epsilon_{j,t}.$$

There is essentially one insight and then everything follows. Examining the problem in (10) and substituting in the households budget constraint from (8), then the observation is that the optimal a' conditional on a choice j is **independent** of the choice j. And this observation implies that choice probabilities become independent of states a and z. Everything follows from these observations and Proposition 1 and Proposition 2 can be applied. Corollary 1 states the result and Appendix D works through this logic step-by-step.

Corollary 1 (Separation of Trade and Heterogeneity) *In the dynamic, heterogenous agent trade model where preferences are logarithmic over the physical commodity: The trade elasticity is*

$$\theta = -\frac{1}{\sigma_\epsilon},$$

and trade flows satisfy a standard gravity relationship

$$\frac{M_{ij}}{M_{ii}} = \left(\frac{w_j/A_j}{w_i/A_i}\right)^{\frac{-1}{\sigma_\epsilon}} d_{ij}^{\frac{-1}{\sigma_\epsilon}},$$

and both are independent of the household heterogeneity. And the welfare gains from trade for an individual household are

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \frac{1}{\theta(1-\beta)} \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}} \;+\; \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \Big\{ B(a_t, z_t) + C(a_t, z_t) \Big\}$$

where the gains from substitution are (i) independent of the household heterogeneity and (ii) summarized by the trade elasticity and the change in the home choice probability and (iii) the other sources of gains are as in Proposition 4.

The first part of this result is heterogeneity plays no role in determining the aggregate trade elasticity. Like in the CES-Armington model or Eaton and Kortum (2002), it's just about how innately substitutable national varieties are. Similarly, aggregate trade satisfies a gravity relationship which no role for household heterogeneity.

Welfare is a bit more subtle. Choice probabilities are independent of states (and proportional to expenditure shares), then applying Proposition 2 and inserting the expression for the trade elasticity allows me to express the gains from substitution component of Proposition 2 mimic the results of Arkolakis et al. (2012). What is still present from the heterogenous agent part of the model is how changes in factor prices influence welfare.

To be honest, I found this result surprising. By looking at the choice probabilities in (11) and noting how the value functions determine choices, not period utility functions, one would suspect that the household's income fluctuations problem would shape aggregate trade outcomes. Corollary 1 shows that is not the case but that micro-outcomes and aggregate trade outcomes "separate."

Proposition 1 is also interesting because it generalizes the results of Anderson, De Palma, and Thisse (1987) and Anderson et al. (1992) to a far more complicated economy. They showed that in a static model with log utility and additive logit shocks, the economy behaves *as if* there were a representative agent CES consumer. I recover their result, but I must emphasize the complexity of the economy at the micro-level for which this result stands—households are froward looking, face productivity and taste shocks in the presence of incomplete markets and borrowing constraints. Yet, these details don't matter when the magic of $\log$ kicks in.

3. Calibration

This section focuses on my approach to calibrating the model. The next two subsections discuss the preference specification, income and taste shock process, borrowing constraints and how I scale things so the model can deliver balanced growth like properties.

The final section follows the trade literature by picking country specific TFP and trade cost parameters to match bilateral trade flows. How I do this is a novelty — I use "gravity as a guide" to overcome the fact that my model does not admit a closed form map from trade flows to parameters as static, gravity models do. I describe this approach below and then discuss how the remaining non-country specific parameters are chosen.

3.1. Functional Forms, Quality Shifters, Balanced Growth Scaling

Utility over the physical commodity is CRRA with relative risk aversion γ. On top of these preferences I do several things to ensure that micro and macro facts can be matched given that there is a sense in which preferences are non-homothetic on the extensive margin.

The first feature I introduce are household-specific quality shifters. This is important because it helps the model match facts regarding how import shares vary across rich and poor households (see, e.g., Borusyak and Jaravel (2021)). The necessity of quality shifters relates to the discussion around Figure 1b — that prices and price elasticities determine how shares vary across households and these forces lead to a pattern of sorting with poor, high-elasticity households concentrating their expenditures on the cheapest commodities available. Quality shifters that vary with household-specific characteristics are one way to match shares, yet allow for heterogeneity in price elasticities. Berry et al. (1995) make this point and it motivates their modeling of demand with interactions between attributes of the product and household characteristics; Auer et al. (2022) allows for this force as well in both their model and empirical specification.

Mechanically, I implement quality shifters by introducing a home bias term $\psi_{ii}(z)$ which additively shifts period utility when consuming the home good i and it is indexed by the households productivity state z. This last point implies that rich and poor households may have different valuations quality between home and foreign goods. To reduce ψs dimensionality, I assume that it's a log-linear function of a households permanent productivity state and this function is the same across countries. The slope of this relationship is calibrated to match the fact from Borusyak and Jaravel (2021)) that import expenditure shares are essentially the same between US poor (below median income) and rich (above median income) households.

The second feature is that I want things to scale and deliver a "balanced-growth-like" property. Specifically, the want-operator here is that if there were two countries one with high TFP and one with low TFP, elasticities (both at the micro and macro level) in the two countries are the same. The way to do this is to make the Type 1 Extreme value parameter country specific and scaled so that $\sigma_{\epsilon,i} = \sigma_{\epsilon} A_i^{1-\gamma}$ and similarly for the quality shifters above.

3.2. Shocks and Constraints

The income shock process is set up to be a mixture of a AR(1) persistent component and an iid transitory component and this is calibrated using results from Krueger, Mitman, and Perri (2016). I use their exact parameter values. It is assumed to be the same across countries.

The borrowing constraint is set in the following way. First, I scale it by a country's autarky level of average real labor income. Then it is set so that a household can borrow up to fifty percent of it's autarky level of income. The scaling here is done to deliver a balanced-growth-like property of the model so a households debt capacity is invariant to a country's autarky level of income.

In my main calibration, I work with the case of financial globalization. In this case there is one interest rate clearing the global asset market. All countries are assumed to have the same discount factor and it is set so the equilibrium world interest rate is 1 percent.

Per the discussion above, the taste shock parameter is set in the following way. TFP in the United States is normalized to one and then, per the discussion above, I set this so that $1/\sigma_{\epsilon} = 4.0$ and, thus, $1/\sigma_{\epsilon,us} = 4.0$. And again, per the discussion above, all other countries shock parameters are pinned down by σ_ϵ and their level of TFP A_i.

3.3. Using Gravity as a Guide to Match Trade Data

My calibration strategy is to use the gravity regression as a guide in an indirect inference procedure where I estimate parameters of the model so that the regression coefficients from a standard gravity regression run on my model's data match the coefficients when the same regression is ran on the data.

Here are the details. The bilateral trade flows that I use are from Eaton and Kortum (2002). The 19 countries in this data set is a nice size to do what I want to do in about an afternoon. Moreover, the Eaton and Kortum (2002) data set provides a well defined benchmark disciplined by bilateral trade flows and gravity variables.

In the 19 country model, the parameters I need to choose are $19-1$ country-specific TFP parameters (the A_is) and then $(19-1)\times(19-1)$ trade costs (with the minus one since the ii trade costs is normalized to one) to infer from the bilateral trade data. This leaves me under-identified with only $(19-1)\times(19-1)$ bilateral trade shares and $19-1$ TFP parameters.

Step 0. I'll reduce the number of parameters to estimate by placing a restriction on trade costs that are a function of observable data. Specifically, I assume that trade costs take the form as in Eaton and Kortum (2002) with

$$\log d_{ij} = d_k + b + l + e_h + m_i, \tag{47}$$

where trade costs are a logarithmic function of distance, where d_k with $k = 1, 2, ..., 6$, is the effect of distance between country i and j lying in the k-th distance interval.[14] The b term is the effect of a shared border in which $b = 1$ if country i and j share a border and zero otherwise. Similarly l is a dummy variable if country's i and j share a language, and e_h represent two dummy variable for different indicators of European integration. The final part is an importer fixed effect shifting trade costs up or down depending upon the identity of the importer.

At this point, I've reduced the parameter space to the coefficients on the trade cost function rather than the complete matrix of trade costs and then the TFP terms.

Step 1. The next step is to run the following gravity regression on the data

$$\log\left(\frac{M_{ij}}{M_{ii}}\right) = im_i + ex_j + d_k + b + l + e_h + \delta_{ij}, \tag{48}$$

which projects imports between country i and j (normalized relative to domestic expenditures) on an importer effect, exporter effect and then the gravity variables relating to distance, bor-

[14]Intervals are in miles: $[0, 375)$; $[375, 750)$; $[750, 1500)$; $[1500, 3000)$; $[3000, 6000)$; and $[6000, \text{maximum}]$.

der, language, etc. and finally there is an error term δ_{ij} that reflects other factors not in this specification.

This is the canonical representation of trade flows—the gravity model. In a standard Armington-CES, Eaton and Kortum (2002), or Melitz (2003) style model, the importer effects and exporter effects have specific interpretations. And given the point estimates from (48), productivity and the importer fixed effects on the trade cost function are easily recovered.

In my model, this is not the case. However, the idea is to use the point estimates from (48) as moments for my model to match. The next step constructs model analogs to (48).

Step 2. To construct model analogs to (48), I guess TFP parameters and coefficients on the trade cost function in (47). Define this parameter vector as Θ.

Given Θ, I compute an equilibrium of the world economy. This amounts to: (i) solving for households' dynamic problems—in each country (ii) constructing the stationary distribution of wealth and expenditure patterns—in each country (iii) aggregating and then (iv) finding a vector of prices so goods markets and financial markets clear world wide.

Once I find an equilibrium, I run the same regression as in (48) on the model generated data. As some notation, the model constructed moments are defined as, e.g., $im_i(\Theta)$ which is the importer effect estimated on model generated data under the parameter vector Θ.

Step 3. The final step constructs moment conditions which provide the foundation for estimation. Define $\mathbf{y}(\Theta)$ as a set of moments conditions comparing the point estimates from the data with the point estimates from the model under the parameter vector Θ. For example, $im_i - im_i(\Theta)$ or $d_k - d_k(\Theta)$, etc.

My estimation procedure is based on the moment condition

$$E\left[\mathbf{y}(\Theta_o)\right] = 0, \tag{49}$$

where Θ_o is the true value of Θ. Thus, my method of moments estimator is:

$$\hat{\Theta} = \arg\min_{\Theta}\left[\mathbf{y}(\Theta)'\,\mathbf{y}(\Theta)\right], \tag{50}$$

At a mechanical level, finding the minimum to (50) amounts to returning to **Step 2.** each time and smartly updating parameter guess for Θ. One of the nice features of this set-up and the dimensionality reduction that I did, is that now this is an exactly identified problem and standard root-finding techniques can be applied to update Θ and a minimum found.

3.4. Calibration Results

Table 1: Preferences, Shocks, and Constraints — Calibrated Parameters

Description	Value	Target
Discount Factor, β	0.92	Global Interest Rate of 1%
CRRA parameter, γ	1.50	Micro elasticities of Auer et al. (2022)
Slope of Quality Shifter, $\psi_{ii}(z)$	0.60	Micro import share facts of Borusyak and Jaravel (2021)
Type One E-V parameter, $1/\sigma_\epsilon$	4.0	—
Borrowing Constraint ϕ_i	—	50% of i's autarky labor income
Income Process on z	—	Krueger et al. (2016)

3.4.A. Micro Expenditure Shares, Elasticities, and MPCs

Table 2 reports household-level expenditure patterns, trade elasticities, and marginal propensities to consume (MPC). I focus on households in US and I brake down the statistics by household income.[15]

The first column of Table 2 2 reports the share of imports out of total expenditure for households below median income (poor), at the median, and above the median (rich). An outcome is that the model replicates Borusyak and Jaravel (2021) facts that import expenditure shares are the same between US poor (below median income) and rich (above median income) households. As discussed above, a key force behind this result are household-specific quality shifters with poor households having stronger perceived quality of imported goods than rich households.

The second column of 2 reports household-level trade elasticities. These are constructed at the micro-level as in Proposition 1 for each type of household and import partner. The import partner elasticities are then aggregated into one number using the household's expenditure weights.

Table 2 illustrates how price elasticities systematical fall with income. For the poor households with income below the median, elasticities are a bit above 6; for the richest households they are a bit below 4. The middle row reports the elasticity for a household in the middle of the distribution and this is 4.8. Auer et al. (2022) report price elasticities for the median and at points above and below in the income distribution. Similar to Table 2, they find that the household in the middle of the income distribution has an elasticity of 4.8 and those at the top and bottom (which are more extreme points than what I report here) they have elasticities of 3.0 and 6.6. Thus, the elasticities in my model at the micro-level are quantitatively consistent with those

[15]In the model, my income measure is labor income — income including asset payments or expenditures deliver similar results since all three are highly correlated.

Table 2: Model: Shares, Elasticities, and MPCs by Income of US Households

	Import Shares	Trade Elasticities	MPCs
Below Median Income	0.08	−6.24	0.50
Median	0.08	−4.88	0.28
Above Median Income	0.08	−3.97	0.17

found in Auer et al. (2022).

The final column in Table 2 reports marginal propensities to consume in the model. Recalling the discussion in Section 2.1 — how trade elasticities vary across households relates to MPCs in equation (36). MPCs in the model are computed by endowing households a one time, unanticipated cash transfer of 1,000 USD and then computing how consumption changes relative to the transfer. As with the elasticities, household level MPCs are aggregated across the different consumption baskets by the household's expenditure weights.

MPCs are right in the ballpark of what is typically thought plausible with the median annual MPC being a little under 0.30 implying that a household spends about 30 cents per dollar of transfer on consumption; see, e.g., **?**.[16] Not surprisingly, poorer households have substantially higher MPCs and richer households lower. And together with the second column, Table 2 confirms the connection between how sensitive a household is to prices and how sensitive a household is to cash transfers.

To summarize, the model is quantitatively matching salient facts about (i) similar import expenditure shares between rich and poor households as in Borusyak and Jaravel (2021)), that (ii) poor households have higher price elasticities relative to rich households as in Auer et al. (2022), and (iii) is able to mimic patterns of marginal propensities to consume as seen in micro data and surveyed **?**.

3.4.B. Aggregate Trade and Trade Elasticities

Figure 2 provides a sense of model fit after running my "gravity as guide procedure." The y-axis reports bilateral trade data and the x-axis reports the outcome from my model. The fit is very high, and nearly indistinguishable from, for example, how a standard trade model perform would perform. Or the log preference model which per Proposition 1 should (and it does) operate just like a standard trade model.

[16] With that said, this calibration achieves high MPCs essentially by having very little wealth in the economy. This model feature is consistent with the small quantity of liquid wealth observed in the US economy, but leaves out large amounts of illiquid wealth.

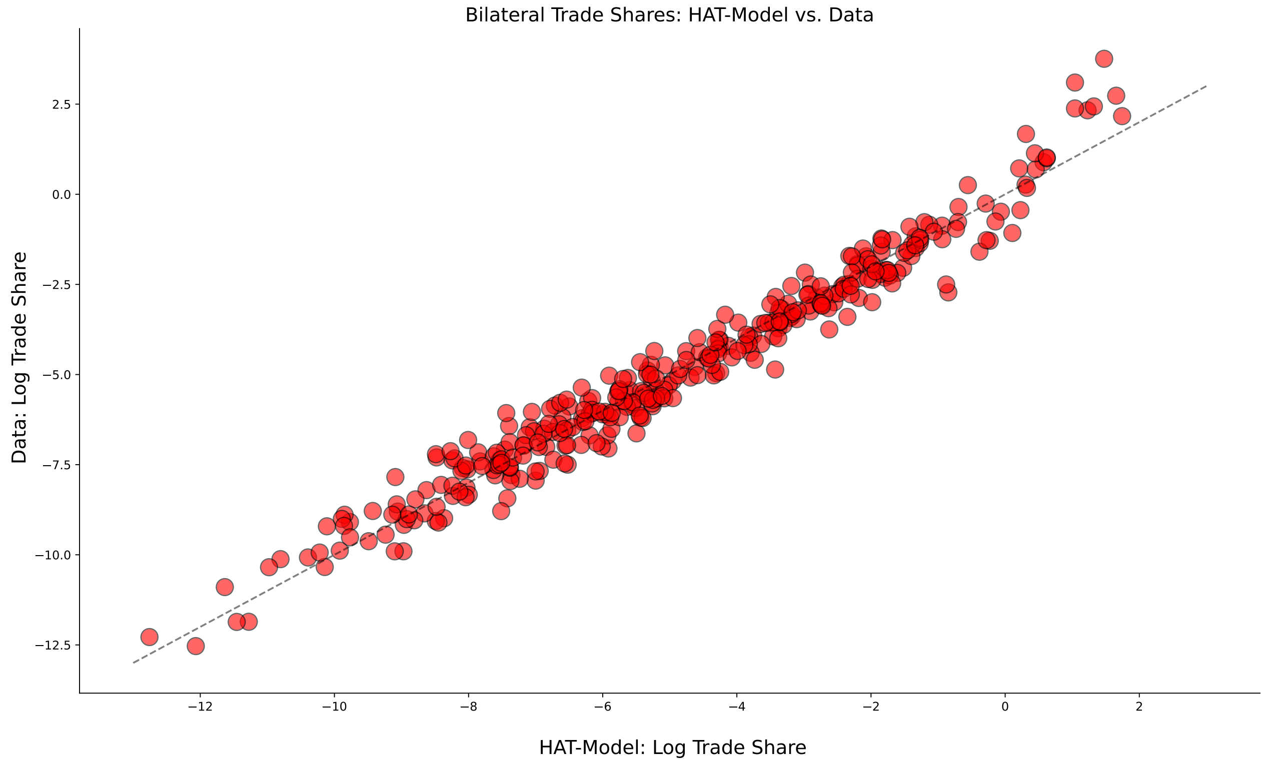


Figure 2: Bilateral Trade: Model vs. Data

Table 3 reports another measure of fit and some of the resulting parameter values. The first column are the distance, border, etc. moments from the gravity regression in (48) (and note they exactly correspond with those in the top panel of Table 3 of Eaton and Kortum (2002)). The second column reports the moments from the model. Here they exactly line up and are consistent with the argument in Figure 2, the fit is good and the model is replicating geographic pattern of activity seen in the data.

The final column reports the primitive estimates on the trade cost function. Each value reports the level effect of being in a distance bin or sharing a border etc. So if two countries are measured to be in the smallest distance bin and share a border, the trade cost between these two countries is 2.35×0.91 (first row times seventh row). Or if a country is in the furthest distance bin, its trade costs is 5.69.

How would this compare to a standard model? It's a bit hard since one needs to take a stand on the trade elasticity in the standard model to translate estimates in column one into levels of the trade costs. But an approach is the following: find the trade elasticity so the cost of the nearest distance bin is the same as in my model and then look at how things relate in other bins. In an Eaton and Kortum (2002) world, this would correspond with an trade elasticity of about 3.6. Then, for example, one takes the moment in the first column, last distance bin and compares $\exp(-1/3.6 \times -6.56)$ vs. 5.69.

Table 3: Estimation Results

Barrier	Moment	HAT-Model Model Fit	HAT-Model Parameter
$[0, 375)$	-3.10	-3.10	2.35
$[375, 750)$	-3.67	-3.67	2.81
$[750, 1500)$	-4.03	-4.03	3.09
$[1500, 3000)$	-4.22	-4.22	3.23
$[3000, 6000)$	-6.06	-6.06	4.88
$[6000, \text{maximum}]$	-6.56	-6.56	5.69
Shared border	0.30	0.30	0.91
Language	0.51	0.51	0.87
EFTA	0.04	0.04	0.98
European Community	0.54	0.54	0.89

Note: The first column reports data moments the HAT-model targets. The second reports the model moments. The third column reports the estimated parameter values.

What comes out of is that closer relationship are a bit more expensive then what a constant elasticity model would predict. And the furthest destinations are meaningfully less expensive, seven and ten percent less, for the last two distance bins. This is picking up a model outcome where trade elasticities are increasing with cost. So far away destinations are relatively elastic destinations, so the cost need not be as large to deter trade.

Figure 3 provides an example of the of trade elasticities that come of this model. In this figure, I focus on the US and plot each bilateral trade elasticity versus the price a consumer in the US faces when importing a variety from that country. The balls represent the relative size of US imports from that destination. And these elasticities are constructed from the bottom up via the formula in (32).

The feature that stands out very clearly in Figure 3 is that trade elasticities systematically increase with price and decrease with the volume of trade.

At the micro-level, there are two opposing forces giving rise to this aggregate relationship. Per the arguments discussed above around Proposition 1, the aggregate bilateral trade elasticity reflects both household level elasticities $\theta(a, z)$s and a composition effect that works through the expenditure weights $\omega(a, z)$s. Thus, when prices increase as one moves from one source to a less competitive source, there are two competing forces at work: (i) how do micro-level elasticities change and (ii) how does composition change?

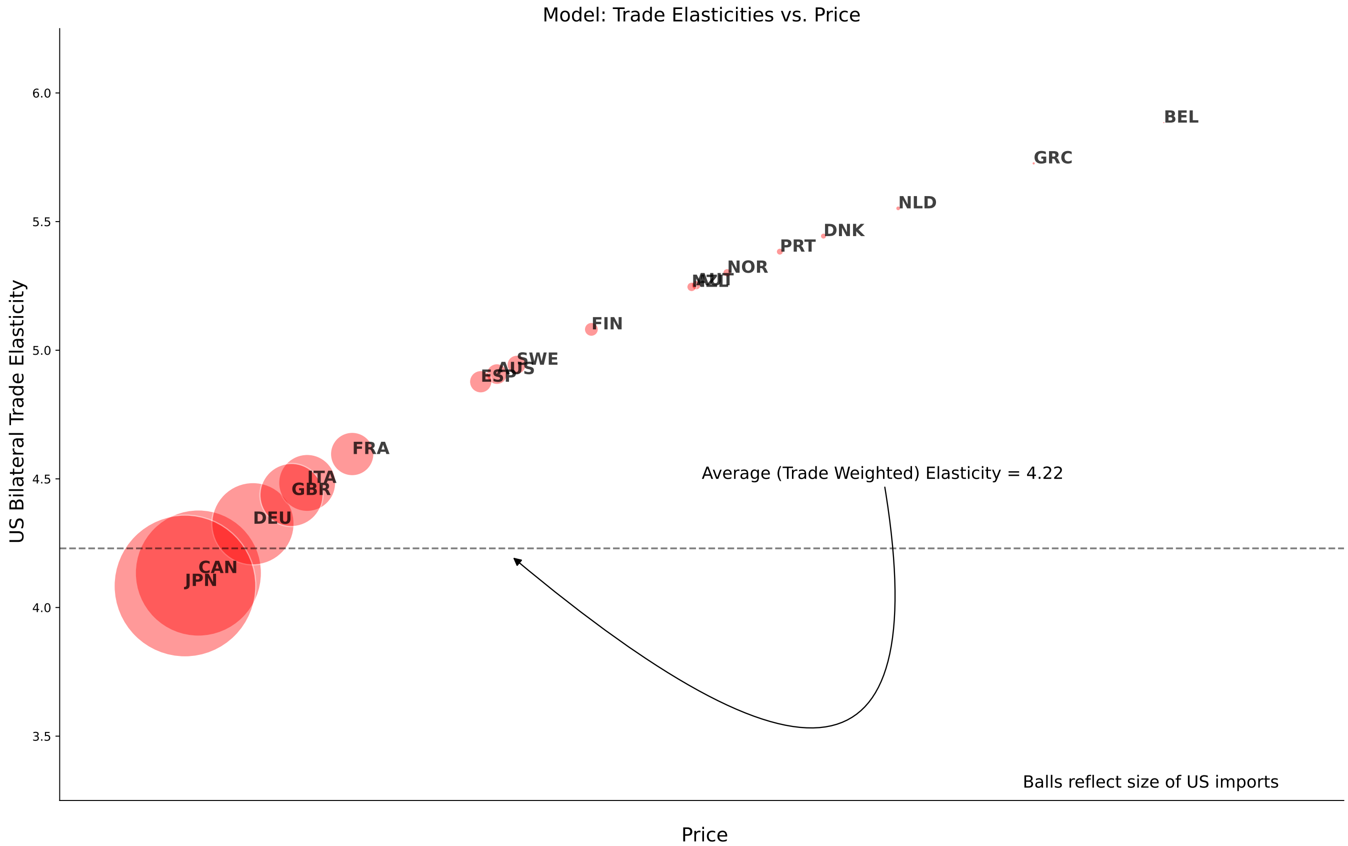


Figure 3: Trade Elasticities $-\theta_{us,j}$

The first force is that as prices increase *both* rich and poor households' elasticities increase. In other words, everyone is more elastic when contemplating a purchase from a more expensive destination. This is a force pushing the model to have elasticities *increase* with price.

The second force — the composition effect— generally works in the opposite direction. As one moves from more cheaper to more expensive destinations, less price sensitive households sort into those varieties. Thus, the composition of households purchasing more expensive varieties are the rich, relatively inelastic households. This is a force pushing the model to have elasticities *decrease* with price. In more standard, BLP-like settings, Nakamura and Zerom (2010) and Head and Mayer (2021) highlight this composition effect in shaping pass-through.

Which one wins? Figure 3 shows that the first force dominates the composition effect. One way to view this result is through the lens of Mrázová and Neary's (2017) language that demand in this model endogenously turns out to be "subconvex" relative to CES demand and which is equivalent to "Marshall's Second Law of Demand." The endogenous part is important as it's not parameterized as in, say, Kimball Demand which has become a popular tool to allow for non-constant elasticities. Did the model have to deliver this? Per the arguments above, it's not obvious as composition effects could have dominated.[17]

In the trade literature, there is evidence suggesting that trade elasticities conform to what comes out of my model. Both Novy (2013) and Carrère, Mrázová, and Neary (2020) find that proxy's for the trade elasticity are larger, the less trade there is between two countries. Chen and Novy (2022) further confirm this idea by finding that trade cost effects are strong for small bilateral relationships weak or even zero for large trading relationships. Mapping these ideas back into outcomes from my model, a currency union between the US and Canada would likely have a small effect since this is a high volume / low elasticity relationship.

4. The Welfare Gains From Trade

In this section, I measure the gains from trade and study how they are distributed across households, the role of the asset market, and how the gains vary as trade costs change with different trading partners.

4.1. Measuring Welfare

In the previous section, I focused on what amount to level changes in utils. Here I define an equivalent variation measure that has more interpretable units and I use in the quantitative results.

As a quick refresher, equivalent variation does the following: given that some price change delivers utility level u', equivalent variation asks "at the old prices, p_0, how much extra income must be provided to be indifferent between u' and u." To implement this in my model, define the value function of a household at base period prices as

$$v_i(a, z;\ p, R_i, w_i). \tag{51}$$

And the value function for the same states, but at counterfactual prices

$$v_i'(a, z;\ p', R_i', w_i'), \tag{52}$$

where I'm evaluating this with the prices prevailing at the new steady state and, hence, there are no t subscripts. I focus on across steady states, not transition paths. Those may be important, but it adds an additional computational challenge which I've decided to abstract from.

Given these definitions, my equivalent variation measure is a permanent, proportional increase

[17] Head and Mayer (2021), using Berry et al.'s (1995) estimated model, illustrate that composition does indeed dominate with pass-through greater than one when heterogeneity in the valuation of product characteristics are shut down.

in income (asset and labor) $\tau_{i,a,z}$, at the old prices such that the new level of utility v_i' is achieved:

$$v_i'(a, z;\ p', R_i', w_i') - v_i(a, z;\ p, R_i\tau_{i,a,z}, w_i\tau_{i,a,z})) = 0. \tag{53}$$

To be clear, this says a household living in country i with states a, z must have their income increased today (and for the infinite future) by the number $\tau_{i,a,z}$. The underscore notation indexes this value by the original-type of household I'm looking at, and so there is one number for a household with a, z states in country i. The τs that solve (53) are my primary measure of welfare at the household level.[18]

4.2. 10% Reduction in US Trade Costs

This section studies a ten percent reduction in all US trade costs. That is $d_{us,j}$ is shifted down for all j partners by ten percent.

Figure 4 reports the welfare gains, in equivalent variation units, across households. Households are binned by quantiles of the initial distribution of consumption expenditure and the y-axis reports the average gains within each bin. The red bars report the baseline model and the blue bars report the $\log$ preference case when, per Corollary 1, heterogenous price sensitivity is turned off.

The main takeaway is that gains from trade are strongly pro-poor — across income brackets the poorest households gain four and a half times more from trade than the richest households (2.93 percent vs. 0.65 percent). If one were to convert this to dollar amounts, a US household in the bottom 20th percentile has income around \$20,000. My model implies that a 10 percent reduction in trade costs is equivalent to a — permanent — transfer of \$600 to the poorest households in the economy at the old prices.

A secondary implication is that the average gains across are much larger than representative agent benchmarks. The dashed line in Figure 4 show that the average gain is 1.35 percent. In contrast, a calculation of Arkolakis et al. (2012) off of the change in the US import share with an elasticity equal to the average (trade-weighted) elasticity implies a gain of only 0.45 percent.[19] This is a third of the size (1.35 vs. 0.45) of the average gains in my baseline model.

The reason why my model delivers larger average gains is because of the large gains in the bottom part of the income distribution. Looking at Figure 4, the gains from trade for the rich households look a lot like those that would come out of Arkolakis et al. (2012)-style calculation.

[18]To be clear, there are alternatives like a Lucas-style consumption equivalent variation — this however confronts questions about which consumption. I also explored a lump sum transfer version of 53 as well. The proportional increase measure is my preference because it's essentially the same as in Auer et al. (2022).

[19]I show in the appendix, the formula in Arkolakis et al. (2012) is equivalent variation measure as I compute and, hence, the large gains I'm finding are not because I add curvature to the utility function or having infinitely lived agents — these do not matter when computing equivalent variation.

Figure 4: Welfare Gains by Household Expenditure: 10% Reduction in $d_{us,j}$

However, for poor households, the gains for the poor are big relative to the rich. Then when averaging over ACR-like gains for rich households and large gains for poor households, the average gains in my model are three times larger.

The blue bars in Figure 4 help answer why the gains are pro-poor. These bars are the gains in the $\log$ preference model where, per Corollary 1, the gains from substitution are the same across households and any heterogeneity arises from changes in factor prices and potential effects from people coming off their borrowing constraint. The key point that the blue bars in Figure 4 illustrate is that now the gains from trade are nearly uniform across the distribution. Overall, the level of gains is lower as well, but this is because trade increases by less.

The comparison between the red and blue bars illustrates that heterogeneity in the gains from substitution is the force behind the pro-poor gains from trade. As discussed around equation (41) there are essentially two issues are at play determining how this aspect of the model leads to heterogenous gains across households: (i) how exposed a household to trade and (ii) how households value a price reduction. The model was calibrated so that exposure was equal across the income distribution, thus (i) is not leading to heterogenous gains from trade. In contrast, the model was designed and calibrated to match the fact that poor households are very elastic with respect to price and, thus, they strongly value a price reduction. So the second force is the key component driving the pro-poor aspect of these gains from trade.[20]

[20]For example, I calibrated the model without quality shifters delivering a pattern of micro-level expenditure

Per the results in Proposition 2, what is the role of factor prices and how the change in the interest rate relative to the wage rate influences the gains from trade. In the counterfactual, the ratio of R/w increases by about 4.6 percent. The reason for the decline is that demand for US goods (and hence labor) falls as households substitute into foreign goods and so US wages fall. The interest rate is essentially globally determined and increases only slightly. Then thinking through this the implication is that there is a counteracting pro-rich force behind Figure 4. That is for those holding positive assets, these moves in factor prices are beneficial and for debtors these moves hurt.

To isolate this force, I resolved the households problem and constructed the value function in (52) with the new equilibrium prices, but assuming that the factor prices entering the households budget constraint (the wage and the interest rate) are the same. So this partials out the effect from these moves in factor prices.

4.3. Heterogeneity by Trading Partner

4.4. The Role of the Asset Market

5. Conclusion

What do you find interesting? Email me.

shares with the poor unexposed to trade similar Figure 1b. In this case, the equivalent variation gains are flat across the income distribution except for the vary poorest.

Appendix

A. The H-A Trade Elasticity

My definition of the trade elasticity is the partial equilibrium response of imports from j relative to domestic consumption due to a permanent change in trade costs. By partial equilibrium, I mean that wages, interest rates, and the distribution of agents are fixed at their initial equilibrium values. This is consistent with the definition of the trade elasticity in say, Arkolakis et al. (2012) and Simonovska and Waugh (2014). By permanent, I mean that the change in trade costs is for the indefinite future and that households correctly understand this. Consistent with this discussion and the notation below, I compute the partial derivatives (not total) of objects with respect to trade costs.

Mathematically, the trade elasticity equals the difference between the elasticities for how trade between i and j change minus how home trade changes:

$$\frac{\partial(M_{ij}/M_{ii})}{\partial d_{ij}} \times \frac{d_{ij}}{(M_{ij}/M_{ii})} = \frac{\partial M_{ij}/M_{ij}}{\partial d_{ij}/d_{ij}} - \frac{\partial M_{ii}/M_{ii}}{\partial d_{ij}/d_{ij}}. \tag{54}$$

The change in imports between i and j with respect to a change in trade costs is:

$$\frac{\partial M_{ij}}{\partial d_{ij}} = \int_{a,z} \left\{ \frac{\partial p_{ij}}{\partial d_{ij}} c_i(a,z,j)\pi_{ij}(a,z) + \frac{\partial c_i(a,z,j)}{\partial d_{ij}} p_{ij}\pi_{ij}(a,z) + \frac{\partial \pi_{ij}(a,z)}{\partial d_{ij}} p_{ij} c_i(a,z,j) \right\} L_i \lambda_i(a,z) da\, dz. \tag{55}$$

Divide the stuff inside the brackets by household level imports, $p_{ij}c_i(a,z,j)\pi_{ij}(a,z)$ and multiply on the outside giving

$$\frac{\partial M_{ij}}{\partial d_{ij}} = \int_{a,z} \left\{ \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}} + \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}} + \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}} \right\} p_{ij} c_i(a,z,j)\pi_{ij}(a,z) L_i \lambda_i(a,z) da\, dz. \tag{56}$$

Define the following "weight" which is the share of goods that those with states a, z account for in total expenditures from j as

$$\omega_{ij}(a,z) = \frac{p_{ij} c_i(a,z,j)\pi_{ij}(a,z) L_i \lambda_i(a,z)}{M_{ij}}, \tag{57}$$

where the sum of $\omega_{ij}(a,z)$ over states a, z equals one. This gives a nice expression for the import elasticity

$$\frac{\partial M_{ij}/M_{ij}}{\partial d_{ij}/d_{ij}} = 1 + \int_{a,z} \Bigg\{ \underbrace{\frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}}_{\theta_{ij}(a,z)^I} + \underbrace{\frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}}}_{\theta_{ij}(a,z)^E} \Bigg\} \omega_{ij}(a,z) da\, dz, \tag{58}$$

or more succinctly as

$$\frac{\partial M_{ij}/M_{ij}}{\partial d_{ij}/d_{ij}} = 1 + \int_{a,z} \Big\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \Big\} \omega_{ij}(a,z) da\, dz. \tag{59}$$

where the elasticity of aggregate imports into i from j is a weighted average of several effects. The value of one out in front arises from the complete pass-through of changes in trade costs to changes in prices. Then the first term within the brackets represent the intensive margin $\theta_{ij}(a,z)^I$, so how much do quantities change conditional on choosing to consume variety j. The next term $\theta_{ij}(a,z)^E$ represents the extensive margin, so how the choice probabilities change.

To complete the derivation, I derive the own-imports term which is similar with

$$\frac{\partial M_{ii}}{\partial d_{ij}} = \int_{a,z} \Bigg\{ \underbrace{\frac{\partial p_{ii}}{\partial d_{ij}} c_i(a,z,i) \pi_{ii}(a,z)}_{=0} + \frac{\partial c_i(a,z,i)}{\partial d_{ij}} p_{ii} \pi_{ii}(a,z) + \frac{\partial \pi_{ii}(a,z)}{\partial d_{ij}} p_{ii} c_i(a,z,i) \Bigg\} L_i \lambda_i(a,z) da\, dz, \tag{60}$$

where the first-term is zero because this is a partial equilibrium elasticity. Then after constructing the proper weights and converting everything to elasticity form we have

$$\frac{\partial M_{ii}/M_{ii}}{\partial d_{ij}/d_{ij}} = \int_{a,z} \Big\{ \theta_{ii,j}(a,z)^I + \theta_{ii,j}(a,z)^E \Big\} \omega_{ii}(a,z) da\, dz, \tag{61}$$

where the ii, j notation means that $\theta_{ii,j}(a,z)^I$ reflects how the intensive margin adjusts, conditional on a ii choice, given a change in ij price. Similarly, $\theta_{ii,j}(a,z)^E$ represents how the ii choice probability changes given the ij change in price.

Proposition 1 then follows:

Proposition 3 (The H-A Trade Elasticity) *The trade elasticity between country i and country j is:*

$$\theta_{ij} = 1 + \int_{z,a} \Big\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \Big\} \omega_{ij}(a,z) da\, dz - \int_{z,a} \Big\{ \theta_{ii,j}(a,z)^I + \theta_{ii,j}(a,z)^E \Big\} \omega_{ii}(a,z) da\, dz \tag{62}$$

which is an expenditure-weighted average of micro-level elasticities. The micro-level elasticities are *decomposed into an intensive margin and extensive margin*

$$\theta_{ij}(a,z)^I = \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}, \qquad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and the expenditure weights are defined as

$$\omega_{ij}(a,z) = \frac{p_{ij}c_i(a,z,j)\pi_{ij}(a,z)\lambda_i(a,z)L_i}{M_{ij}}.$$

1.1. Connecting Elasticities with Household Behavior

To derive the **intensive margin elasticity**, start from the households budget constraint and differentiate consumption of variety j with respect to price p_{ij} and one gets

$$\underbrace{\frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}}_{\theta_{ij}(a,z)^I} = \left[- \frac{\partial g_i(a,z,j)/p_{ij}c_i(a,z,j)}{\partial p_{ij}/p_{ij}} - 1 \right], \tag{63}$$

where recall that $g_i(a,z,j)$ is the policy function mapping states into asset holdings next period a'. To derive the **extensive margin elasticity**, start from the definition of the choice probability and

$$\underbrace{\frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}}}_{\theta_{ij}(a,z)^E} = \frac{1}{\sigma_\epsilon}\frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}}. \tag{64}$$

I then use the following arguments to unpack how the value function $v_i(a,z,j)$ changes:

$$\frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} = -\, u'(c_i(a,z,j))c_i(a,z,j) + \left[- \frac{u'(c_i(a,z,j))}{p_{ij}} \frac{\partial g_i(a,z,j)}{\partial p_{ij}/p_{ij}} \right] \tag{65}$$

$$+\, \beta \mathrm{E} \left\{ \frac{\partial v}{\partial a'} \frac{\partial g_i(a,z,j)}{\partial p_{ij}/p_{ij}} \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}} + \frac{\partial v(g_i(a,z,j),z')}{\partial p_{ij}/p_{ij}} \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}} \right\} \tag{66}$$

which can then be further expressed in terms of the Euler Equation (derived below in Equation (154))

$$\frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} = -\, u'(c_i(a,z,j))c_i(a,z,j) \tag{67}$$

$$+ \underbrace{\left\{ -\frac{u'(c_i(a,z,j))}{p_{ij}} + \beta \mathbb{E}\Big[-\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + u'(c_i(a',z',i))R_i \Big] \right\}}_{\text{Euler equation in (154)}} \frac{\partial g_i(a,z,j)}{\partial p_{ij}/p_{ij}} \tag{68}$$

$$+ \beta \mathbb{E} \frac{\partial v_i(a',z')}{\partial p_{ij}/p_{ij}} \Big\} \tag{69}$$

The term in the second line is the Euler equation multiplied by how assets change. This term should be zero for small changes. I discuss this more in depth below around the welfare gains calculation, but the argument is that either the Euler equation holds and thus this term is zero, or it does not hold, but then households can't adjust asset holdings and then the outside part is zero. And for small changes households on the margin of a binding constraint or not are on the margin and don't matter.

To add some clarity to this expression assume the number of countries is large. This assumption implies that the $\partial\Phi$ term in (64) is zero or approximately so. The next implication is that because the ex-ante value function next period $v_i(a',z')$ is just a function of Φ (see its definition in (12)). Hence, the large number of countries implies future effects don't matter or approximately so. All together using these observations under this assumption in (64) gives

$$\theta_{ij}(a,z)^E \approx -\frac{1}{\sigma_\epsilon}\Big[u'(c_i(a,z,j))c_i(a,z,j) \Big]. \tag{70}$$

From here, I can connect (70) with things like relative risk aversion and the marginal propensity to consume. The thought experiment here is ignore all the future effects and ask if a household was a bit wealthier what would the effect be on the $u'(c_i(a,z,j))c_i(a,z,j)$:

$$\frac{\partial(u'(c_i(a,z,j))c_i(a,z,j))}{\partial a} = u''(c_i(a,z,j))\frac{\partial c_{ij}}{\partial a}c_i(a,z,j) + u'(c_i(a,z,j))\frac{\partial c_{ij}}{\partial a} \tag{71}$$

$$= \frac{\partial c_{ij}}{\partial a}\Bigg[u''(c_i(a,z,j))c_i(a,z,j) + u'(c_i(a,z,j))\Bigg] \tag{72}$$

$$= u'(c_i(a,z,j)) \times \mathbf{MPC}_{ij}(a,z,j) \times \Bigg[-\rho_i(a,z,j) + 1\Bigg]. \tag{73}$$

And just to emphasize how this works, it's a derivative of $u'(c_i(a,z,j))c_i(a,z,j)$. So as assets go up, with $\rho > 1$ this implies that $u'(c_i(a,z,j))c_i(a,z,j)$ goes down. And this is a force for things to be less elastic for rich guys. As assets go down, this implies that $u'(c_i(a,z,j))c_i(a,z,j)$ goes up, and this is a force for poor guys to be more elastic.

The final elasticity I want to derive is how home choices respond to changes in trade frictions. This is a term that shows up all the time (in the calculations above) and in the welfare expressions, so it's worth computing as well:

$$\frac{\partial \pi_{ii}(a,z)/\pi_{ii}(a,z)}{\partial d_{ij}/d_{ij}} = \frac{1}{\sigma_\epsilon}\frac{\partial v_i(a,z,i)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}}. \tag{74}$$

Then the derivative of the term Φ takes on a unique property where

$$\frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}} = \sum_j \pi_{ij}(a,z)\frac{1}{\sigma_\epsilon}\frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} \tag{75}$$

which takes on this flavor of exposure (which are the choice probabilities) times how the household's valuations across the goods change (as represented by the value functions). Then expressing things all relative to how the home valuation changes we have

$$\frac{\partial \pi_{ii}(a,z)/\pi_{ii}(a,z)}{\partial d_{ij}/d_{ij}} = \frac{1}{\sigma_\epsilon}\sum_j \pi_{ij}(a,z)\Bigg[\frac{\partial v_i(a,z,i)}{\partial d_{ij}/d_{ij}} - \frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}}\Bigg]. \tag{76}$$

So what this says is that the change in the home choice completely summarizes how things change (in a relative sense).

B. The Welfare Gains from Trade

This section derives the gains from a permanent change in trade costs, across steady states. Like the discussion above, the idea here is that I'm thinking a situation where the change is small and there is an immediate jump to the new steady state. Unlike the trade elasticity, I'm going to take total derivatives encompassing general equilibrium changes in wages and interest rates.

The analysis proceeds in several steps. First, I'll focus on country i and study a change in trade costs d_{ij} with respect to partner j. Second, I start by working a utilitarian social welfare function and then drill down into how welfare of an individual household changes.

Social welfare from the perspective of country i is

$$W_i = \int_a \int_z v_i(a, z)\lambda_i(a, z), \tag{77}$$

Then the total change in total welfare is

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} = \int_a \int_z \left\{ \frac{\mathrm{d}v_i(a, z)}{\mathrm{d}d_{ij}/d_{ij}} + v_i(a, z)\frac{\mathrm{d}\lambda_i(a, z)/\lambda_i(a, z)}{\mathrm{d}d_{ij}/d_{ij}} \right\} \lambda_i(a, z). \tag{78}$$

The first component reflects changes in household-level welfare. The second component is about reallocation, i.e., if—at the old v's—does the distribution change so that social welfare gets better or worse. The change in social welfare is then the weighted average of these two forces with the weights being those at the initial distribution.

How does household-level welfare change? Recall that the value function (with the expectation taken over the different preference shocks) is

$$v_i(a, z) = \sigma_\epsilon \log \left\{ \sum_{j'} \exp\left(\frac{v_i(a, z, j')}{\sigma_\epsilon} \right) \right\}, \tag{79}$$

and then I'm going to make the observation that I can substitute out the sum part (79) with the $\exp$ of the home value function relative to the micro-level "home choice" so

$$\pi_{ii}(a, z) = \exp\left(\frac{v_i(a, z, i)}{\sigma_\epsilon} \right) \Big/ \sum_{j'} \exp\left(\frac{v_i(a, z, j')}{\sigma_\epsilon} \right), \tag{80}$$

$$\pi_{ii}(a, z) \times \sum_{j'} \exp\left(\frac{v_i(a, z, j')}{\sigma_\epsilon} \right) = \exp\left(\frac{v_i(a, z, i)}{\sigma_\epsilon} \right), \tag{81}$$

$$\sum_{j'} \exp\left(\frac{v_i(a, z, j')}{\sigma_\epsilon} \right) = \exp\left(\frac{v_i(a, z, i)}{\sigma_\epsilon} \right) \Big/ \pi_{ii}(a, z). \tag{82}$$

Then substituting (82) into the value function in (79) gives:

$$v_i(a,z) = \sigma_\epsilon \log \left\{ \frac{\exp\left(\frac{v_i(a,z,i)}{\sigma_\epsilon}\right)}{\pi_{ii}(a,z)} \right\} \tag{83}$$

and recall that the home choice value function that enters into (83) is

$$v_i(a,z,i) = u(c_i(a,z,i)) + \beta \mathbb{E} v_i(g_i(a,z,i), z) \tag{84}$$

where the expectation operator is over the zs and the v_i is the same value function as in (79) so the taste shocks are integrated out. Taking logs and exp's of the left hand size of (83) allows for the v_i value function to be represented as

$$v_i(a,z) = -\sigma_\epsilon \log \pi_{ii}(a,z) + u(c_i(a,z,i)) + \beta \mathbb{E} v_i(g_i(a,z,i), z). \tag{85}$$

Now everything is written with respect to the home choice. What is going on is that the home choice π_{ii} summarizes the expected value of those shocks and their benefits. No need to explicitly carry around the v_{ij}s. This is essentially the dynamic analog to Equation (15), Footnote 42 of Eaton and Kortum (2002) and Arkolakis et al. (2012).

One more detail, to facilitate interpretation, it will be useful to compute the Euler equation associated with asset holdings when the borrowing constraint does not bind. This euler equation is:

$$\frac{u'(c_i(a,z,i))}{p_{ii}} = \beta \mathrm{E}_{z'} \left[-\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + \frac{u'(c_i(a',z',i))R_i}{p_{ii}} \right]$$

This equation is derived below in (158).

Now the strategy is to totally differentiate (85) with respect to trade costs and use the recursive structure to iterate forward and construct the change across time. Totally differentiating the value function gives

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} =$$

$$-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} + u'(c_i(a,z,i)) \left[\frac{\mathrm{d}w_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}} z + \frac{\mathrm{d}R_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}} a \right] \tag{86}$$

$$-\frac{u'(c_i(a,z,i))}{p_{ii}} \frac{\mathrm{d}g_i(a,z,i)}{\mathrm{d}d_{ij}/d_{ij}} + \beta \mathbb{E}_{z'} \frac{\mathrm{d}v_i(g_i(a,z,i),z')}{\mathrm{d}d_{ij}/d_{ij}} \tag{87}$$

Then the derivative of the continuation value function is

$$\frac{\mathrm{d}v_i(g(a,z,i),z')}{\mathrm{d}d_{ij}/d_{ij}} = \underbrace{\left[-\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + \frac{u'(c_i(a',z',i))R_i}{p_{ii}}\right]}_{\frac{\partial v_i(g_i(a,z,i),z')}{\partial a}} \frac{\mathrm{d}g_i(a,z,i)}{\mathrm{d}d_{ij}/d_{ij}} \; + \tag{88}$$

$$-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a',z')/\pi_{ii}(a',z')}{\mathrm{d}d_{ij}/d_{ij}} + u'(c_i(a',z',i))\left[\frac{\mathrm{d}w_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}}z' + \frac{\mathrm{d}R_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}}a'\right] + \tag{89}$$

$$-\frac{u'(c_i(a',z',i))}{p_{ii}} \frac{\mathrm{d}g_i(a',z',i)}{\mathrm{d}d_{ij}/d_{ij}} + \beta \mathbb{E}_{z'} \frac{\mathrm{d}v_i(g_i(a',z',i),z'')}{\mathrm{d}d_{ij}/d_{ij}} \tag{90}$$

And now collect terms so

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \underbrace{-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{A(a,z)} \tag{91}$$

$$+ \underbrace{u'(c_i(a,z,i))\left[\frac{\mathrm{d}w_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}}z + \frac{\mathrm{d}R_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}}a\right]}_{B(a,z)} \tag{92}$$

$$+ \underbrace{\left\{-\frac{u'(c_i(a,z,i))}{p_{ii}} + \beta \mathbb{E}_{z'}\left[-\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + \frac{u'(c_i(a',z',i))R_i}{p_{ii}}\right]\right\} \frac{\mathrm{d}g_i(a,z,i)}{\mathrm{d}d_{ij}/d_{ij}}}_{C(a,z)} \tag{93}$$

$$+ \beta \mathbb{E}_{z'}\left\{-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a',z')/\pi_{ii}(a',z')}{\mathrm{d}d_{ij}/d_{ij}} + u'(c_i(a',z',i))\left[\frac{\mathrm{d}w_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}}z' + \frac{\mathrm{d}R_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}}a'\right] \; \dots \right. \tag{94}$$

Let me walk through the interpretation of each term:

A(a,z) - The term here is $-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}}$ is a gains from substitution term. I discuss this below, but it summarizes two effects (i) how exposed the household is to market j and an effect from a elasticity term.

B(a,z) - $\left[\frac{\mathrm{d}w_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}}z + \frac{\mathrm{d}R_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}}a\right]$ is essentially how a reduction in trade costs affects factor prices — the wage relative to the price of the home good and the interest rate relative to the price of the home good. And these effects are all valued at that household's marginal utility of consumption.

Two more observations. First, in the perfect competition world I consider $\frac{w_i}{p_{ii}} = A_i$ from (2). And thus, $\frac{\mathrm{d}w_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}} = 0$, so households are perfectly "hedged" from any effect on labor earnings.

Second, there is an effect from how the "real" interest rate changes. Here real is in quotes because this is real in units of the home good (and per the observation above, this boils down to the change in the interest rate relative to the wage rate). And because the a can take on positive or negative values, this is a force that could in principal lead to losers from trade.

C(a,z) - The third term which I'm labeling as $C(a, z)$ is about changes in asset holdings. For the small / local changes that I'm considering it should zero out, but for larger changes this term should be relevant. Let me expand upon this.

First, notice that the inside the bracket term is the Euler Equation from (158) and its multiplied by the change in policy function.

The idea is that if the household is unconstrained, then this term is zero as there is no gain through changes in asset behavior. Asset holdings are already chosen optimally so that margins are equated, thus, on the margin any benefit of lower trade costs on changes in asset behavior is zero. Essentially an application of the Envelope Theorem.

Now in this economy, this term may not be zero because of borrowing constrained households, thus the inside-the-bracket term is positive. However, notice how the outside brackets is multiplied by the change in the asset policy function. What this picks up is that if the household is constrained, then assets can't change so the outside term is zero and, thus, overall the second term is zero.

Final point, then the only people that benefit and contribute to social welfare through these effects are those on the margin between constrained and not-constrained. But if they are on the margin between being constrained and not-constrained, then they are on their euler equation.

- The final term is about this continuing on into the infinite future.

Iterating on (94) into the future, the gains from trade for a household with states a, z today are

$$\frac{\partial v_i(a,z)}{\partial d_{ij}/d_{ij}} = \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \Bigg\{ -\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a_t,z_t)/\pi_{ii}(a_t,z_t)}{\mathrm{d}d_{ij}/d_{ij}} + B_t(a_t,z_t) + C(a_t,z_t) \Bigg\} \tag{95}$$

Where the first component is the expected discounted gains from substitution, revaluation of asset holdings, and changes in asset holdings. Combining (95) and (78) yields the following proposition for the gains from trade.

Proposition 4 (The Welfare Gains from Trade) *The welfare gains from trade are given by*

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} = \int_a \int_z \left\{ \frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} + v_i(a,z) \frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \right\} \lambda_i(a,z).$$

which reflects the change in household level gains and how the distribution of households changes. Household level gains are given by

$$\frac{\partial v_i(a,z)}{\partial d_{ij}/d_{ij}} = \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \left\{ -\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a_t,z_t)/\pi_{ii}(a_t,z_t)}{\mathrm{d}d_{ij}/d_{ij}} + B(a_t,z_t) + C(a_t,z_t) \right\}$$

where each term represents:

- *Gains from substitution:* $-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}}$.
- *Gains from asset revaluations:* $B(a_t,z_t) = u'(c_{ii}(a_t,z_t)) \frac{\mathrm{d}R_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}} a$
- *Gains from changes in asset holdings:*

$$C(a_t,z_t) = \left\{ -\frac{u'(c_i(a,z,i))}{p_{ii}} + \beta \mathbb{E}_{z'} \left[-\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + \frac{u'(c_i(a',z',i))R_i}{p_{ii}} \right] \right\} \frac{\mathrm{d}g_i(a,z,i)}{\mathrm{d}d_{ij}/d_{ij}} = 0$$

which is zero for small changes.

The final step is to unpack the gains from substitution term. Now from the elasticity discussion I can convert (76) into a total derivative form

$$\frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \frac{1}{\sigma_\epsilon} \sum_{j'} \pi_{ij'}(a,z) \left[\frac{\mathrm{d}v_i(a,z,i)}{\partial d_{ij}/d_{ij}} - \frac{\mathrm{d}v_i(a,z,j')}{\mathrm{d}d_{ij}/d_{ij}} \right]. \tag{96}$$

so change in the home choice summarizes two forces: (i) how exposed a household is to the change through the choice probabilities and then (ii) how value functions change.

Now the value function component is where elasticities enter. Define $\bar{\theta}(a,z)^E_{ij',j}$ as the extensive margin, cross-price elasticity (how ij' changes with respect to the j change), and in total derivative form (this is what the bar notation denotes). Following the derivation of (64) this is

$$\theta_{ij',j}(a,z)^E = \frac{1}{\sigma_\epsilon} \frac{\mathrm{d}v_i(a,z,j')}{\partial d_{ij}/d_{ij}} - \frac{\mathrm{d}\Phi_i(a,z)/\Phi_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}, \tag{97}$$

which then noticing that the $\mathrm{d}\Phi$ term is independent of option j'. This last observation implies that when the cross-price elasticities are substituted into (96) the $\mathrm{d}\Phi$s difference out. Thus we we can express the change in the home choice in terms of cross-price elasticities

$$-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \sigma_\epsilon \sum_{j'} \pi_{ij'}(a,z) \left[\bar{\theta}(a,z)^E_{ii,j} - \bar{\theta}(a,z)^E_{ij',j} \right], \tag{98}$$

Now let's make the approximation where that all cross-price elasticity terms are zero. Then we have

$$-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \approx -\sigma_\epsilon \times \pi_{ij}(a,z) \times \bar{\theta}(a,z)^{E}_{ij,j} \tag{99}$$

This expression is interesting because now it is analogous to the gains from trade formula in the efficient allocation. The pure gains from trade component comes from (i) how the taste shock is valued (ii) a households exposure and (iii) the household's elasticity. And this last part, per the arguments above, is about how sensitive the value function is with respect to price.

C. Gains in the Efficient Allocation

This section of the appendix presents abbreviated results from my related paper in Waugh (2023). Below I discus the planning problem, I state the solution to it, then discuss how I arrive at the gains from trade calculations in Proposition 3.

I focus on a utilitarian social welfare function:

$$W = \sum_{t=0}^{\infty} \sum_{i} \int_{z} \beta^t v_i(z,t) L_i \lambda_i(z,t), \tag{100}$$

and here v_i is a households value function in country i at date t. Now, I'm going to place the social welfare function in sequence space and then unpack the benefits from the preference shock in the following way:

$$W = \sum_{t=0}^{\infty} \sum_{i} \sum_{j} \int_{z} \beta^t \left\{ u(c_i(z,j,t)) + \mathrm{E}[\, \epsilon \mid \pi_{ij}(z,t)] \right\} \pi_{ij}(z,t) L_i \lambda_i(z,t) \tag{101}$$

so the inner term is period utility given the associated consumption allocation $c_i(z,j,t)$ and then the expected value of the preference shock conditional on the choice probability $\pi_{ij}(z,t)$. This inner term is then weighted by the number of households that receive that utility, i.e. the choice probability times the mass of households with shock z at date t. The sum across j adds up all households in country i. Then the sum across i reflects that this is global welfare.

One more point about the inner term in (101), my claim is that with the Type 1 extreme value shocks:

$$\mathrm{E}[\, \epsilon \mid \pi_{ij}(z,t)] = -\sigma_\epsilon \log \pi_{ij}(z,t) \tag{102}$$

where this is like the "selection correction" where if π becomes smaller, the expected value of the taste shock becomes larger. So only those with the largest relative shocks are chosen and higher utility for those, conditional on being selected, is felt.

Given this formulation, the planner does the following: he chooses consumption and choice probabilities for all country pair combinations, state by state, for the infinite future. The Lagrangian associated with the Planning Problem is:

$$\mathcal{L} = \sum_{t=0}^{\infty}\sum_i\sum_j\int_z \beta^t \Big\{u(c_i(z,j,t)) + \mathrm{E}[\,\epsilon \mid \pi_{ij}(z,t)]\Big\}\pi_{ij}(z,t)L_i\lambda_i(z,t), \tag{103}$$

$$+\sum_{t=0}^{\infty}\sum_i \beta^t\chi_i(t)\Big\{Y_{it} \;-\; \sum_j\int_z d_{ji}c_j(z,i,t)\pi_{ji}(z,t)L_j\lambda_j(z,t)\Big\}$$

$$+\sum_{t=0}^{\infty}\sum_i\int_z \beta^t\chi_{2i}(z,t)\Big\{1-\sum_j \pi_{ij}(z,t)\Big\}L_i\lambda_i(z,t),$$

where the first term is the objective function; the second line is the resource constraint saying that output from country i must equal the consumption of commodity i globally including the transport costs. Then the third line ensures that choice probabilities are probabilities and sum to one. The final thing I'm doing is that I'm scaling the multipliers by β^t so that the algebra is easier.

The statement below characterizes the allocation that solves (103):

Proposition 5 (The Efficient Allocation) *The allocation that satisfies the Centralized Planning Problem in (103) is:*

1. *A consumption allocation satisfying:*

$$u'(c_{ij}(z,t)) = \chi_j(t)d_{ij} \tag{104}$$

where $\chi_j(t)$ is the shadow price of variety j.

2. *The choice probabilities are*

$$\pi_{ij}(t) = \exp\left(\frac{u(c_{ij}(t)) - u'(c_{ij}(t))c_{ij}(t)}{\sigma_\epsilon}\right)\Big/\sum_{j'}\exp\left(\frac{u(c_{ij'}(t)) - u'(c_{ij'}(t))c_{ij'}(t)}{\sigma_\epsilon}\right) \tag{105}$$

The Gains from Trade. Given this allocation, I want to compute the social gain to a change in trade costs. First, I express social welfare depending directly upon the trade costs d, and then indirectly as the allocations of c and πs depend upon d as well.

$$W(d, c_i(j;d), \pi_{ij}(d)) \tag{106}$$

And then totally differentiate social welfare, so

$$\frac{\mathrm{d}W}{\mathrm{d}d} = \frac{\partial W}{\partial d} + \frac{\partial W}{\partial c_i(j;d)}\frac{\partial c_i(j;d)}{\partial d} + \frac{\partial W}{\partial \pi_{ij}(d)}\frac{\partial \pi_{ij}(d)}{\partial d} \tag{107}$$

and then I invoke the Envelope Theorem. That is I evaluate this derivative at the optimal allocation. But the optimal allocation is optimal, so on the margin any gain from changing consumption or choice probabilities is zero and these indirect effects (at the optimal allocation) are zero. Computing the direct effect gives

$$\partial W = -\sum_{t=0}^{\infty} \beta^t \, \chi_j(t) c_i(j,t) \pi_{ij}(t) L_i \partial d_{ij}, \tag{108}$$

$$= -\sum_{t=0}^{\infty} \beta^t \;\, u'(c_i(j,t)) c_i(j,t) \pi_{ij}(t) L_i \partial d_{ij}/d_{ij}, \tag{109}$$

where the first line is how the resource constraint in (103) changes with respect to trade costs. Then the second line inserts the relationship between the multiplier and the marginal utility of consumption. Breaking it down, this says: $c_{ij}(t)\pi_{ij}(t)L_i$ term is how much stuff people in i eat from j and $\partial d_{ij}/d_{ij}$ perturbs it by the percent change in trade costs, then $u'(c_{ij}(t))$ converts it into utils. Imposing stationarity delivers

$$\frac{\mathrm{d}W}{\mathrm{d}d_{ij}/d_{ij}} = \frac{\partial W}{\partial d_{ij}/d_{ij}} = -\frac{u'(c_i(j))c_i(j)\pi_{ij}L_i}{1-\beta} \tag{110}$$

The Elasticity of Trade. Now I compute the trade elasticity in this allocation. I essentially follow the formulas outlined in Proposition 1. They apply because they don't depend upon specifics about the environment, just accounting.

Claim #1: The intensive margin trade elasticity is minus one, i.e. any change in d_{ij} results in a one-for-one increase, $c_i(j)$. This follows from the planner directly controlling things and assets are not held or used.

Claim #2: Next I need to compute the extensive margin elasticity. So I'm going to note that

$$\frac{\partial \pi_{ij}/\pi_{ij}}{\partial d_{ij}/d_{ij}} = \frac{1}{\sigma_\epsilon}\left[u'(c_i(j,t)\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}} - u''(c_i(j,t)\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}}c_i(j,t) - u'(c_i(j,t)\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}}\right] - \frac{\partial \Phi_i(t)/\Phi_i(t)}{\partial d_{ij}/d_{ij}} \tag{111}$$

$$= -\frac{1}{\sigma_\epsilon}\left[u''(c_i(j,t)\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}}c_i(j,t)\right] - \frac{\partial \Phi_i(t)/\Phi_i(t)}{\partial d_{ij}/d_{ij}} \tag{112}$$

which the first line follows from the quotient rule and where $\Phi_i(t)$ is the part of the denominator in the choice probability. Recall the trade elasticity is relative to own trade so

$$\frac{\partial \pi_{ii}/\pi_{ii}}{\partial d_{ij}/d_{ij}} = -\frac{\partial \Phi_i(t)/\Phi_i(t)}{\partial d_{ij}/d_{ij}} \tag{113}$$

Then using my H-A Trade Elasticity formula in Proposition 1 and canceling terms and noticing as well that the expenditure weights don't matter since they are common across households, I have that:

$$\theta_{ij} = 1 + \left[\theta^I_{ij} + \theta^E_{ij}\right] - \left[\theta^I_{ii,j} + \theta^E_{ii,j}\right] \tag{114}$$

$$= 1 + -1 + \frac{-1}{\sigma_\epsilon}\left[u''(c_i(j,t)\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}}c_i(j,t)\right] - \frac{\partial \Phi_i(t)/\Phi_i(t)}{\partial d_{ij}/d_{ij}} - 0 - \frac{\partial \Phi_i(t)/\Phi_i(t)}{\partial d_{ij}/d_{ij}} \tag{115}$$

$$= -\frac{1}{\sigma_\epsilon}\left[u''(c_i(j,t))\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}}c_i(j,t)\right]. \tag{116}$$

Then here is a fact I exploit. Starting from the first order condition for consumption and then (i) differentiating both sides with respect to d_{ij} and then multiplying both sides by d_{ij} gives

$$u'(c_i(j,t)) = \chi_j(t)d_{ij} \Rightarrow \tag{117}$$

$$u''(c_i(j,t))\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}} = \chi_j(t)d_{ij}, \tag{118}$$

which implies that at the optimal allocation

$$u'(c_i(j,t)) = u''(c_i(j,t))\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}}. \tag{119}$$

Then the trade elasticity is:

$$\theta_{ij}(t) = -\frac{1}{\sigma_\epsilon}\left[u'(c_i(j,t))c_i(j,t)\right]. \tag{120}$$

where I'll note that the $u'(c)c$ term is marginal utility in semi-elasticity form. So given a percent change in consumption, how much does utility change. Combining the trade elasticity with the gains from trade formula in (110) gives

$$\frac{\partial W}{\partial d_{ij}/d_i j} = \frac{\sigma_\epsilon\, \theta_{ij}\, \pi_{ij}\, L_i}{1-\beta}, \tag{121}$$

in other words, the gains from trade are how many people are buying ij times the trade elasticity, discounted for the indefinite future. To be clear about signs here, θ_{ij} is a negative number, all other values are positive. A decline in trade costs means $\partial d_{ij}/d_{ij}$ is negative and hence ∂W is positive and there are gains from trade.

Proposition 6 (Trade Elasticities and Welfare Gains in the Efficient Allocation) *The elasticity of trade to a change in trade costs between ij in the efficient allocation is:*

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon}\left[u'(c_i(j))c_i(j)\right]. \tag{122}$$

And the welfare gains from a reduction in trade costs between i, j are

$$\frac{\mathrm{d}W}{\mathrm{d}d_{ij}/d_{ij}} = \frac{\partial W}{\partial d_{ij}/d_{ij}} = \frac{\sigma_\epsilon\, \theta_{ij}\, \pi_{ij}\, L_i}{1-\beta} \tag{123}$$

which is the discounted, direct effect from relaxing the aggregate resource constraint.

As a final step, I connect Proposition 6 with Arkolakis et al. (2012) where the change in the home choice and the dispersion parameter summarize everything. To do, so I first derive the elasticity of home choice probability

$$\frac{\partial \pi_{ii}/\pi_{ii}}{\partial d_{ij}/d_{ij}} = -\frac{\pi_{ij}}{\sigma_\epsilon}\left\{u'(c_i(j,t))\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}} - \left[u'(c_i(j,t))\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}} + u''(c_i(j,t))\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}}c_i(j,t)\right]\right\} \quad (124)$$

$$= \frac{\pi_{ij}}{\sigma_\epsilon}u''(c_i(j,t))\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}}c_i(j,t) \quad (125)$$

$$= \frac{\pi_{ij}}{\sigma_\epsilon}u'(c_i(j))c_i(j) \quad (126)$$

$$= -\,\theta_{ij} \times \pi_{ij}. \quad (127)$$

The jump from the second to the third line follows from (119) and the fourth line follows from the definition of the trade elasticity. To be clear on signs here, $-\theta_{ij}$ is positive, the π_{ij} is positive. Then a decline in trade costs means $\partial d_{ij}/d_{ij}$ is negative and hence the probability of choosing the home good must decline. Then inserting (127) into (123) I have

$$\frac{\mathrm{d}W}{\mathrm{d}d_{ij}/d_{ij}} = -\sigma_\epsilon \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}} \times \frac{L_i}{1-\beta}. \quad (128)$$

This says a sufficient statistic for the direct effect of the gains in the efficient allocation is how the home choice probability changes multiplied by the dispersion parameter. Again, regarding the signs here, the change in the home choice probability is positive (declines with decline in trade costs), its multiplied by a negative sign, so welfare goes up with a decline in trade costs.

D. Log Preferences

This example is interesting because it retains an aggregate constant trade elasticity and the welfare gains from trade formula looks like ACR kind of thing and the expression in the efficient allocation (128).

Step 1: Individual Choices. With log preferences the j choice value function is

$$v_i(a,z,j) = \max_{a' \in \mathcal{A}}\left\{\log\left(\frac{Ra + wz - a'}{p_{ij}}\right) + \beta\,\mathbb{E}[v_i(a',z')]\right\} \quad (129)$$

which is then

$$v_i(a,z,j) = \max_{a' \in \mathcal{A}}\left\{\log(Ra + wz - a') + \beta\,\mathbb{E}[v_i(a',z')]\right\} - \log p_{ij} \quad (130)$$

which then leads to the observation that the optimal a' conditional on a choice j is **independent** of the price and the choice j. So what is going on is if you consume an expensive or cheap good, then consumption simply scales up or down so that assets next period are exactly the same. This observation has the implication that expenditures on consumption are the same across choices. Compare households expenditures with the same state a, z but different choices. Equation (130) implies

$$p_{ij} c_i(a, z, j) = p_{ii} c_i(a, z, i) \tag{131}$$

so within states, people always spend the same amount. This observation implies that the choice probabilities are independent of the state only prices matter so

$$\pi_{ij}(a, z) = \exp\left(\frac{v_{ij}(a, z)}{\sigma_\epsilon}\right) \Bigg/ \sum_{j'} \exp\left(\frac{v_{ij'}(a, z)}{\sigma_\epsilon}\right) \tag{132}$$

$$\pi_{ij} = \exp\left(\frac{-\log p_{ij}}{\sigma_\epsilon}\right) \Bigg/ \sum_{j'} \exp\left(\frac{-\log p_{ij'}}{\sigma_\epsilon}\right) \tag{133}$$

These observations are all consistent with the Generalized Euler Equation below. To see this

$$\frac{u'(c_i(a, z, j))}{p_{ij}} = \max\left\{\beta R_i \mathbb{E}_{z'}\left[\sum_{j'} \pi_{ij}(a', z') \frac{u'(c_i(a', z', j'))}{p_{ij}}\right] \ , \ u'\left(\frac{R_i a + w_i - \phi_i}{p_{ij}}\right)\right\} \tag{134}$$

and then impose log preferences and notice that

$$(Ra + wz - a')^{-1} = \max\left\{\beta R \mathbb{E}\left[\sum_{j'} \pi_{ij} (Ra' + wz - a'')^{-1}\right] \ , \ (Ra + w - \phi_i)^{-1}\right\} \tag{135}$$

and then π_{ij}'s do not depend upon a or z, and then $(Ra' + wz - a'')^{-1}$ not depend upon j either, so simplifying we have

$$(Ra + wz - a')^{-1} = \max\left\{\beta R \mathbb{E}_{z'} (Ra' + wz' - a'')^{-1} \ , \ (Ra + w - \phi_i)^{-1}\right\}. \tag{136}$$

The variety choice j does not appear at all in this equation, thus the asset choice is independent from the variety choice j.

Step 2: Micro Trade Elasticities. Starting with (63) and because the asset choice is independent of prices, the intensive margin elasticity $\theta_{ij}(a, z)^I$ is -1 and $\theta_{ii,j}(a, z)^I$ is zero as there are no partial effects on prices in ii.

The extensive margin elasticity is:

$$\theta_{ij}(a,z)^E = \frac{1}{\sigma_\epsilon}\frac{\partial v_{ij}(a,z)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i/\Phi_i}{\partial d_{ij}/d_{ij}} \tag{137}$$

$$= -\frac{1}{\sigma_\epsilon}\frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}} + \beta\mathbb{E}\frac{\partial v_i(a',z')}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i/\Phi_i}{\partial d_{ij}/d_{ij}} \tag{138}$$

$$= -\frac{1}{\sigma_\epsilon} + \beta\mathbb{E}\frac{\partial v_i(a',z')}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i/\Phi_i}{\partial d_{ij}/d_{ij}} \tag{139}$$

where the first line removes the a, z indexing of Φ_i because only prices matter for choice probabilities, not state variables of the household. The next line then partially differentiates the value function with respect to the change in trade costs and I'm exploiting how with log preferences one can pull out the price term. And then the final line notes that the price elasticity is minus one. One more fact that:

$$\theta_{ii,j}(a,z)^E = \beta\mathbb{E}\frac{\partial v_i(a',z')}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i/\Phi_i}{\partial d_{ij}/d_{ij}} \tag{140}$$

where a key thing to notice is that the ii, j elasticity is the same as the second and third terms above in (139).

Step 3: Expenditure Weights. Recall that the micro level trade elasticities when aggregated are weighted by

$$\omega_{ij}(a,z) = \frac{p_{ij}c_{ij}(a,z)\pi_{ij}(a,z)\lambda_i(a,z)}{M_{ij}}. \tag{141}$$

and I can relabel $p_{ij}c_{ij}(a,z) = x_i(a,z)$ given (131), that expenditures are independent of the source. With the choice probabilities independent of a, z the weights become

$$\omega_{ij}(a,z) = \frac{x_i(a,z)\pi_{ij}\lambda_i(a,z)}{\int_z\int_a x_i(a,z)\pi_{ij}\lambda_i(a,z)da\, dz}, \tag{142}$$

$$= \frac{x_i(a,z)\lambda_i(a,z)}{\int_z\int_a x_i(a,z)\lambda_i(a,z)da\, dz} \tag{143}$$

which is independent of source j.

Step 4: The Aggregate Trade Elasticity. Now mechanically follow Proposition 1:

$$
\begin{aligned}
\theta_{ij} =& 1 + \int_z \int_a \Big\{ -1 + -\frac{1}{\sigma_\epsilon} + \beta \mathbb{E} \frac{\partial v_i(a', z')}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i / \Phi_i}{\partial d_{ij}/d_{ij}} \Big\} \omega_i(a,z) da \, dz \\
& - \int_z \int_a \Big\{ \beta \mathbb{E} \frac{\partial v_i(a', z')}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i / \Phi_i}{\partial d_{ij}/d_{ij}} \Big\} \omega_i(a,z) da \, dz \\
=& -\frac{1}{\sigma_\epsilon}
\end{aligned}
\tag{144}
$$

where the last line follows because the a, z terms in the micro level trade elasticities exactly cancel given that expenditure weights are source independent. And the aggregate trade elasticity is constant and parameterized by the dispersion in tastes.

Step 5: Gravity. Following the arguments that expenditures are independent of the source, bilateral imports are

$$
M_{ij} = \pi_{ij} \int_z \int_a x_i(a,z) \lambda_i(a,z) da \, dz \tag{145}
$$

where the last term does not depend upon the source. Dividing by home consumption, using (133), and substituting in prices with technology and wages we have

$$
\frac{M_{ij}}{M_{ii}} = \left(\frac{w_j / A_j}{w_i / A_i} \right)^{\frac{-1}{\sigma_\epsilon}} d_{ij}^{\frac{-1}{\sigma_\epsilon}} \tag{146}
$$

which is the same form as in a Armington model or Eaton and Kortum (2002).

Step 5: The Grains From Trade. Then from here I can just follow Proposition 4. The individual gains are

$$
\frac{\partial v_i(a,z)}{\partial d_{ij}/d_{ij}} = \underbrace{\frac{1}{\theta(1-\beta)} \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}}}_{ACR} + \mathbb{E} \sum_{t=0}^{\infty} \beta^t \Big\{ B(a_t, z_t) + C(a_t, z_t) \Big\}
$$

where the first term is exactly what would arise in the static, representative agent model except for the discounting bit. What facilitates this is that the choice probabilities are independent of a, z and it can be pulled out of the expected discounted sum stuff.

Corollary 2 (Separation of Trade and Heterogeneity) *In the dynamic, heterogenous agent trade model where preferences are logarithmic over the physical commodity: The trade elasticity* *is*

$$\theta = -\frac{1}{\sigma_\epsilon},$$

and trade flows satisfy a standard gravity relationship

$$\frac{M_{ij}}{M_{ii}} = \left(\frac{w_j/A_j}{w_i/A_i}\right)^{\frac{-1}{\sigma_\epsilon}} d_{ij}^{\frac{-1}{\sigma_\epsilon}},$$

and both are independent of the household heterogeneity. And the welfare gains from trade for an individual household are

$$\frac{\partial v_i(a,z)}{\partial d_{ij}/d_{ij}} = \underbrace{\frac{1}{\theta(1-\beta)} \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}}}_{ACR} + \mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big\{B(a_t,z_t) + C(a_t,z_t)\Big\}$$

where the gains from substitution are (i) independent of the household heterogeneity and (ii) summarized by the trade elasticity and the change in the home choice probability and the other sources of gains are as in Proposition 4.

E. Appendix: The Euler Equation and Endogenous Grid Method

First, I'm going to derive the Euler equation for this model when the household is away from it's borrowing limit. Focus on the within a variety choice component, the households value function can be written as:

$$v_{ij}(a,z) = \max_{a'} u\left(\frac{R_i a + w_i z - a'}{p_{ij}}\right) + \beta \mathrm{E} v(a',z') \tag{147}$$

then the first order condition associated with this problem is:

$$\frac{u'(c_i(a,z,j))}{p_{ij}} = \beta \mathrm{E}\frac{\partial v(a',z')}{\partial a'} \tag{148}$$

which says that, conditional on a variety choice the left hand side is the loss in consumption units which is $1/p_{ij}$ evaluated at the marginal utility of consumption and this is set equal to the marginal gain from saving a bit more which is how the value function changes with respect to asset holdings. Now we can arrive at the $\frac{\partial v(a',z')}{\partial a'}$ in the following way, so start from the log-sum expression for the expected value function

$$\mathbb{E}_\epsilon v(a', z') = \sigma_\epsilon \log \left\{ \sum_{j'} \exp\left(\frac{v_i(a', z', j')}{\sigma_\epsilon} \right) \right\}, \tag{149}$$

and then differentiate this with respect to asset holdings

$$\frac{\partial \mathbb{E}_\epsilon v(a', z')}{\partial a'} = \left(\frac{\sigma_\epsilon}{\sum_{j'} \exp\left(\frac{v_i(a', z', j')}{\sigma_\epsilon}\right)} \right) \left[\sum_{j'} \exp\left(\frac{v_i(a', z', j')}{\sigma_\epsilon} \right) \frac{1}{\sigma_\epsilon} \frac{\partial v_i(a', z', j')}{\partial a'} \right]. \tag{150}$$

Then if you look at this carefully and notice how the choice probabilities are embedded in here, I have

$$\frac{\partial \mathbb{E}_\epsilon v(a', z')}{\partial a'} = \sum_{j'} \pi_{ij}(a', z) \frac{\partial v_i(a', z', j')}{\partial a'}, \tag{151}$$

and then apply the Envelop theorem to the value functions associated with the discrete choices across the options

$$\frac{\partial \mathbb{E}_\epsilon v(a', z')}{\partial a'} = \sum_{j'} \pi_{ij}(a', z') \frac{u'(c_i(a', z', j'))R_i}{p_{ij}}, \tag{152}$$

So then putting everything together we have

$$\frac{u'(c_i(a, z, j))}{p_{ij}} = \beta R_i \mathrm{E}_{z'} \left[\sum_{j'} \pi_{ij'}(a', z') \frac{u'(c_i(a', z', j'))}{p_{ij'}} \right], \tag{153}$$

where this has a very natural form: set the marginal utility of consumption today equal to the marginal utility of consumption tomorrow adjusted by the return on delaying consumption, and the expected value of the marginal utility of consumption reflects how the uncertainty over both ones' preference over different varieties and shocks to efficiency units. The final step is the generalized version that incorporates the fact that some households are constrained

$$\frac{u'(c_i(a, z, j))}{p_{ij}} = \max \left\{ \beta R_i \mathrm{E}_{z'} \left[\sum_{j'} \pi_{ij'}(a', z') \frac{u'(c_i(a', z', j'))}{p_{ij}} \right] \ , \ u'\left(\frac{R_i a + w_i - \phi_i}{p_{ij}} \right) \right\} \tag{154}$$

To arrive at the representation of the Euler Equation in home choice form, I make the following observations. As mentioned above, the elasticity of the home choice probability with respect to a change in assets is

$$\frac{\partial \pi_{ii}(a,z)/\pi_{ii}(a,z)}{\partial a} = \frac{1}{\sigma_\epsilon}\frac{\partial v_i(a,z,i)}{\partial a} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial a}. \tag{155}$$

And then the change in the Φ index is

$$\frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial a} = \sum_j \pi_{ij}(a,z)\frac{1}{\sigma_\epsilon}\frac{\partial v_i(a,z,j)}{\partial a} \tag{156}$$

And now notice how this is connected with how the value function changes with respect to assets above. Specifically, inserting (156) into (155) and rearranging we have

$$-\sigma_\epsilon \frac{\partial \pi_{ii}(a,z)/\pi_{ii}(a,z)}{\partial a} + \frac{\partial v_i(a,z,i)}{\partial a} = \sum_j \pi_{ij}(a,z)\frac{\partial v_i(a,z,j)}{\partial a} \tag{157}$$

and from here we can insert the expression above into 151 and apply the envelope theorem giving

$$\frac{u'(c_i(a,z,j))}{p_{ij}} = \beta \mathrm{E}_{z'}\left[-\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + \frac{u'(c_i(a',z',i))R_i}{p_{ii}}\right] \tag{158}$$

which **any** variety choice on the left hand side must respect the ii representation on the right hand side. Note the way the home choice probability enters here, it has a flavor like an asset / shock specific price index.

5.1. EGM-Discrete Choice Algorithm

My computational approach exploits the Euler Equation derived above. Below, I describe my algorithm. This focuses on just the consumer side in one country i.

0. Set up an asset grid. Then guess (i) a consumption function $g_{c,i}(a,z,j)$ for each a, z, and product choice j and (ii) choice specific value function $v_i(a,z,j)$.

1. Compute the choice probabilities from (11) for each (a,z) combination, given the guessed value functions.

2. Given the consumption function and choice probabilities compute the RHS of (153) first.

3. Then invert to find the new updated consumption choice so

$$c_i(\tilde{a},z,j) = u'^{-1}\left\{p_{ij}\max\left\{\beta R_i \mathrm{E}_{z'}\left[\sum_{j'} \pi_{ij'}(a',z')\frac{u'(c_i(a',z',j'))}{p_{ij'}}\right],\ u'\left(\frac{R_i a + w_i - \phi_i}{p_{ij'}}\right)\right\}\right\} \tag{159}$$

where u'^{-1} is the inverse function of the marginal utility of consumption.

4. The key issue in this method is that we have found $c_i(\tilde{a}, z, j)$ where the consumption function is associated with some asset level that is not necessarily on the grid. The solution is to (i) use the budget constraint and infer $\tilde{a}$ given that a' was chosen above (that's where we started), z, and $c_i(\tilde{a}, z, j)$. Now we have a map from $\tilde{a}$ to a' for which one can use interpolation to infer the a' chosen given a where a is on the grid.

- Do steps **3.** and **4.** for each j variety choice. This then makes the function $g_i(a, z, j)$ mapping each state and j choice (today) into a', z' states and then from the budget constraint we have an associated consumption function $g_{c,i}(a, z, j)$.

5. Compute the $\mathrm{E}\left[v(g_{a,ij}(a, z), z')\right]$. This is performed in the `make_Tv.!` function. It fixes a country j, then works through shocks and asset states today and from the policy function $g_{a,ij}(a, z)$ figures out the asset choice tomorrow. Then the $\mathrm{E}\left[v(g_{a,ij}(a, z), z')\right]$ is (13) over the different variety choices tomorrow (this is the integration over ϵ) multiplied by the probability of z' occurring (this is the integration over z). This step and the next step is a key difference relative to the traditional approaches using Euler Equations. Here, I need to reconstruct the value function to construct choice probabilities; in traditional approaches the value function is not a required object.

6. Given **4.** update the value function using the bellman equation evaluated at the optimal policies:

$$Tv_i(a, z, j) = u(g_{c,i}(a, z, j)) + \beta \mathrm{E}\left[v(g_{a,i}(a, z), z', j')\right] \tag{160}$$

7. Compare old and new policy functions, old and new value functions, and then update accordingly.

F. Appendix: Quality Version of the Model

To match micro-level expenditure shares, I introduce are household-specific quality shifters. Mechanically, I implement quality shifters in the following way. Utility associated with the choice of variety j is

$$u(c_{ijt}) + \psi_j + \epsilon_{jt}, \tag{161}$$

now there is a shifter ψ_j in utility that depends upon the commodity j chosen. Now I'm going to make the assumption that the quality valuation of a household varies with it's and efficiency units. In particular, the assumption will be something along the lines that

$$\psi(z, j) \tag{162}$$

So what this means is that a household, depending upon its situation, may have different valuations for a particular commodity. Then, given this assumption on quality, now we are back to the case where the state variables of a individual household are its asset holdings and efficiency units.

Then I'm going to write the value function of a household in country i, after the variety shocks are realized, as

$$v_i(a, z) = \max_j \left\{ \ v_i(a, z, j) + \psi(z, j) + \epsilon_j \ \right\} \tag{163}$$

And here, I've pulled out the quality term and the shock term to be more consistent with the code. Specifically, solution methods will work on the $v_i(a, z, j)$s and then reconstruct $v_i(a, z)$ given the shocks and quality specification. The value function conditional on a choice of variety is

$$v_i(a, z, j) = \max_{a'} \left\{ u(c_{ij}) + \beta \, \mathbb{E}[v_i(a', z')] \right\} \tag{164}$$

$$\text{subject to (7) and (8).}$$

Associated with this are the following choice probabilities for each differentiated good:

$$\pi_{ij}(a, z) = \exp\left(\frac{v_i(a, z, j) + \psi(z, j)}{\sigma_\epsilon} \right) \Bigg/ \Phi_i(a, z), \tag{165}$$

$$\text{where} \quad \Phi_i(a, z) := \sum_{j'} \exp\left(\frac{v_i(a, z, j') + \psi(z, j')}{\sigma_\epsilon} \right). \tag{166}$$

And then the expectation of (9) with respect to the taste shocks takes the familiar log-sum form

$$v_i(a, z) = \sigma_\epsilon \log \left\{ \Phi_i(a, z) \right\}. \tag{167}$$

Or the equivalent representation of this which I'm now using in the code is

$$v_i(a, z) = \sum_{j'} \pi_{ij'}(a, z) \Big[v_i(a, z, j') + \psi(z, j') - \sigma_\epsilon \log(\pi_{ij'}(a, z)) \Big] \tag{168}$$

how is the true again? Then there is an Euler Equation for each variety choice j. This (I believe) takes a different form so

$$\frac{u'(c_i(a,z,j))}{p_{ij}} = \beta R_i \mathrm{E}_{z'}\left[\sum_{j'} \pi_{ij'}(a',z')\left(\frac{u'(c_i(a',z',j'))}{p_{ij'}}\right)\right]. \tag{169}$$

So fundamentally, nothing really changed by introducing quality shifters.

Now to reduce the dimensionality of these parameters, I set this up as a home bias term $\psi_i(z,i)$ which takes on some number and for all other js this is where $\psi_i(z,i) = 0$. Then assume that it's a log-linear function of a households permanent productivity state and this function is the same across countries. Slightly abusing notation, this is where $\psi_i(z,i) = \tilde{\psi} \times z$. The slope of this relationship is calibrated to match the fact from Borusyak and Jaravel (2021)) that import expenditure shares are essentially the same between US poor (below median income) and rich (above median income) households.

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