Heterogeneous Agent Trade

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What am I doing?

Big picture — these are the questions that interest me...

1. What are distributional consequences of trade?

2. Is there a role for trade policy to improve outcomes?

One mechanism behind **1.** is heterogeneity in expenditure shares on traded goods and **elasticities**.

- Auer, Burstein, Lein, and Vogel (2022) is a nice example. In the context of the 2015 Swiss appreciation, they find that poor households are more price elastic.

This paper:

GE, heterogenous agent model of trade delivering ABLV-like facts. I work out the implications for aggregate trade, the gains from trade, and the normative implications for trade policy.

How I do it. . .

Two ingredients:

- Standard incomplete markets model with households facing incomplete insurance against idiosyncratic productivity and taste shocks.
- Trade as in Armington (national varieties), but households have random utility over these varieties.

Qualitatively I characterize. . .

- How price elasticities vary at the micro-level and how (and if) micro-heterogeneity shapes aggregate trade and trade elasticities.
- The welfare gains from trade at the micro and macro level.
- The efficient allocation and, thus, how market incompleteness shapes these outcomes.

Quantitatively, I compute the a 19 country model (the Eaton and Kortum (2002) data) and (today) study the welfare gains to small reductions in trade costs.

Model: Production and Trade

M countries. Each country produces a nationally differentiated product as in Armington.

In country i, competitive firms' produce variety i with:

$$Q_i = A_i N_i,$$

where A_i is TFP; N_i are efficiency units of labor supplied by households.

Cross-country trade faces obstacles:

- iceberg trade costs $d_{ij} > 1$ for one unit from supplier j to go to buyer i.

This structure leads to the following prices that households face

$$p_{ij} = \frac{d_{ij} w_j}{A_j}.$$

Model: Households I

Mass of L_i households in each country i.

Household-level preferences:

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t \; \tilde{u}(\{c_{ijt}, \epsilon_{jt}\}_M)$$

$$\text{where} \quad \tilde{u}(c_{ijt}, \epsilon_{jt}) = u(c_{ijt}) + \epsilon_{jt}.$$

- ϵ_{jt} is iid (across time and households) taste shocks over national varieties.

Assumptions:

- For most of the analysis, I'll only assume u is well behaved.
- ϵ_{jt}s are distributed Type 1 Extreme Value with dispersion parameter σ_ϵ.

Model: Households II

A household's efficiency units z_t evolve according to a Markov Chain. They face the wage per efficiency unit w_{it}.

Households borrow or accumulate a non-state contingent asset, a, with gross return R_i. Household's face the debt limit

$$a_{t+1} \geq -\phi_i$$

Conditional on a variety choice, a household's budget constraint is

$$p_{ij} c_{ijt} + a_{t+1} \leq R_i a_t + w_{it} z_t.$$

What Households Do...

Focus on a stationary setting. A hh's state are its asset holdings a and shock z.

1. The hh makes a variety choice (e.g. a US or Italian variety) and how much to consume. The choice probability is:

$$\pi_{ij}(a,z) = \exp\left(\frac{v_{ij}(a,z)}{\sigma_\epsilon}\right) \Big/ \sum_{j'} \exp\left(\frac{v_{ij'}(a,z)}{\sigma_\epsilon}\right),$$

where $v_{ij}(a,z)$ is the hh's value function conditional on a choice.

2. The hh makes an asset choice. Away from the constraint, asset choices (conditional on a variety choice) must respect this Euler Equation:

$$\frac{u'(c_{ij}(a,z))}{p_{ij}} = \beta \mathrm{E}_{z'}\left\{ -\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + \frac{u'(c_{ii}(a',z'))R_i}{p_{ii}} \right\},$$

where I'm exploiting an ACR-like feature that ex-ante value functions can be expressed in terms of i, i home choices.

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Aggregation

Aggregates arise from explicit aggregation of hh-level actions. Two examples:

Aggregate, bilateral imports and exports are

$$M_{ij} = L_i \int_z \int_a p_{ij} c_{ij}(a,z) \pi_{ij}(a,z) \lambda_i(a,z), \qquad X_{ji} = L_j \int_z \int_a p_{ji} c_{ji}(a,z) \pi_{ji}(a,z) \lambda_i(a,z),$$

where λ_i is the distribution of hhs across states and $c_{ij}(a,z)$ is the consumption function. Here trade flows take on a mixed logit formulation as in Berry, Levinsohn, and Pakes (1995).

The national income accounting identity (GDP = C + I + G + X - M) . . .

$$p_i Y_i = \underbrace{L_i \sum_j \int_z \int_a p_{ij} c_{ij}(a,z) \pi_{ij}(a,z) \lambda_i(a,z)}_{\widetilde{P_i C_i}} + \underbrace{\left[\sum_{j \neq i} X_{ji} - \sum_{j \neq i} M_{ij} \right]}_{-R_i A_i + A_i'}.$$

Notice how trade is non-trivially connected to a county's capital account.

Equilibrium

The Decentralized Stationary Equilibrium. A Decentralized Stationary Equilibrium are asset policy functions and commodity choice probabilities $\{ \ g_{ij}(a,z), \pi_{ij}(a,z) \ \}_{ij}$, probability distributions $\{ \ \lambda_i(a,z) \ \}_i$ and positive real numbers $\{w_i, p_{ij}, R_i\}_{ij}$ such that

- i Prices (w_i, p_{ij}) satisfy the firms problem;
- ii The policy functions and choice probabilities solve the household's optimization problem;
- iv The probability distribution $\lambda_i(a,z)$ induced by the policy functions, choice probabilities, and primitives satisfies the law of motion and is stationary;
- v Goods market clears:

$$p_i Y_i - \sum_j^M X_{ji} = 0, \quad \forall i$$

- v Bond market clears with

$$\mathrm{A}_{\mathrm{i}}' = 0, \quad \forall i.$$

Proposition #1: The H-A Trade Elasticity. The trade elasticity between country i and country j is:

$$\theta_{ij} = 1 + \int_a \int_z \left\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \right\} \omega_{ij}(a,z) - \left\{ \theta_{ii}(a,z)^I + \theta_{ii}(a,z)^E \right\} \omega_{ii}(a,z),$$

which is the difference between ij and ii expenditure-weighted micro-level elasticities. The micro-level elasticities for households with states a, z are an intensive and extensive elasticity

$$\theta_{ij}(a,z)^I = \frac{\partial c_{ij}(a,z)/c_{ij}(a,z)}{\partial d_{ij}/d_{ij}}, \qquad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and $\omega_{ij}(a,z)$ are the expenditure weights.

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and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^I = \left[- \frac{\partial g_{ij}(a,z)/p_{ij} c_{ij}(a,z)}{\partial p_{ij}/p_{ij}} - 1 \right] \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}.$$

The idea here is that reduction in trade costs relaxes the hh's budget constraint and then the division of new resources between assets and expenditure determines the intensive margin elasticity.

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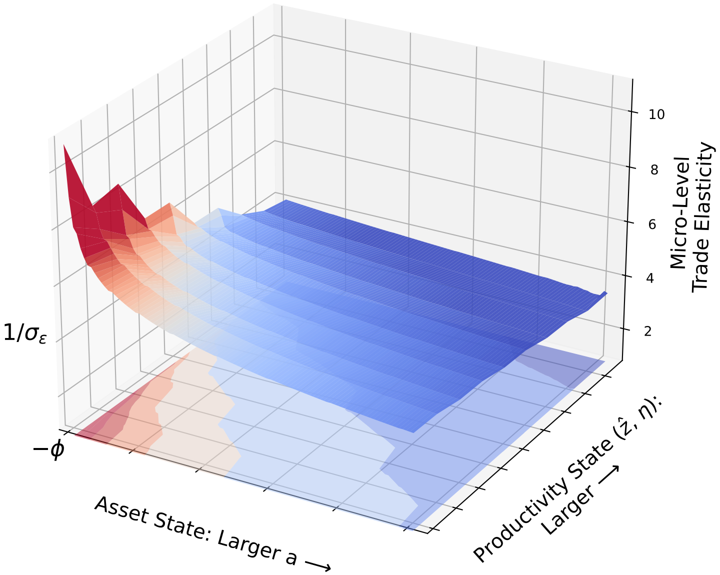
and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^E = -\frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}} + \frac{1}{\sigma_\epsilon} \frac{\partial v_{ij}(a,z)}{\partial d_{ij}/d_{ij}}.$$

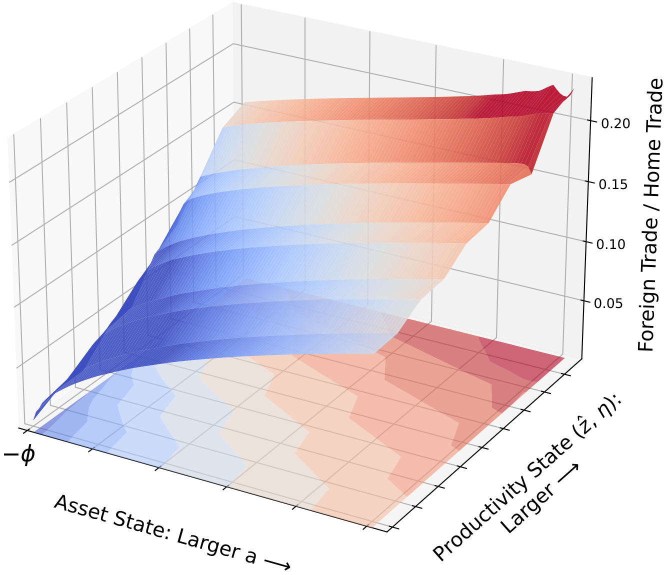
Key is $\frac{\partial v_{ij}(a,z)}{\partial d_{ij}/d_{ij}}$.

In the paper, I show that if relative risk aversion > 1 than hh's with (i) high $u'(c)$ and (ii) high MPCs are more price elastic. **So poor hh's are the most price sensitive.**

Trade Elasticities by HH-Level State



Trade Shares: $M_{ij}(a,z)/M_{ii}(a,z)$, by HH-Level State



H-A Gains from Trade

Proposition #2: H-A Welfare Gains from Trade. The gains from trade under a utilitarian social welfare function are

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} = \int_z \int_a \Bigg\{ \underbrace{\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{\text{gains to hh}} + \underbrace{v_i(a,z)\frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{\text{gains to reallocation}} \Bigg\} L_i \lambda_i(a,z),$$

where v_i is a hh's value function before taste shocks are realized.

Household-level gains are

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \Bigg\{ -\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a_t,z_t)/\pi_{ii}(a_t,z_t)}{\mathrm{d}d_{ij}/d_{ij}} + u'(c_{ii}(a_t,z_t))a_t \frac{\mathrm{d}R_i}{\mathrm{d}d_{ij}/d_{ij}} \Bigg\}.$$

HH-level gains pick up two effects:

- An ACR-like term reflecting how it's home choice changes... basically the gains from substitution.
- How the value of a hh's wealth changes through GE effects on interest rates.

Proposition #3: Separation of Trade and Micro-Heterogeneity. In the dynamic, heterogenous agent trade model where preferences are logarithmic over the physical commodity

$$\tilde{u}(c_{ijt}, \epsilon_{jt}) = \log(c_{ij,t}) + \epsilon_{j,t},$$

the trade elasticity is

$$\theta = -\frac{1}{\sigma_\epsilon},$$

and is independent of household heterogeneity. And the welfare gains from trade are

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} = -\frac{1}{\theta(1-\beta)} \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}}.$$

and is (i) independent of the household heterogeneity and (ii) summarized by the trade elasticity and the change in the home choice probability (and home share).

Mimics the results of Anderson et al. (1987) and Arkolakis et al. (2012), remarkable as this is a far more complex economy...

H-A Gains from Trade under Efficiency

Proposition #4: Trade Elasticities and Welfare Gains in the Efficient Allocation The trade elasticity between i, j in the efficient allocation is:

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon}\left[u'(c_{ij})c_{ij}\right].$$

And the welfare gains from a reduction in trade costs between i, j are

$$\frac{\mathrm{d}W}{\mathrm{d}d_{ij}/d_{ij}} = \frac{\partial W}{\partial d_{ij}/d_{ij}} = \frac{1}{1-\beta} \times u'(c_{ij})c_{ij}\pi_{ij}L_i,$$

which is the discounted, direct effect from relaxing the resource constraint.

Mimics the results of Atkeson and Burstein (2010) but with household (not firm) heterogeneity.

Quantitative Analysis

Preliminary. This is what I'm going to do:

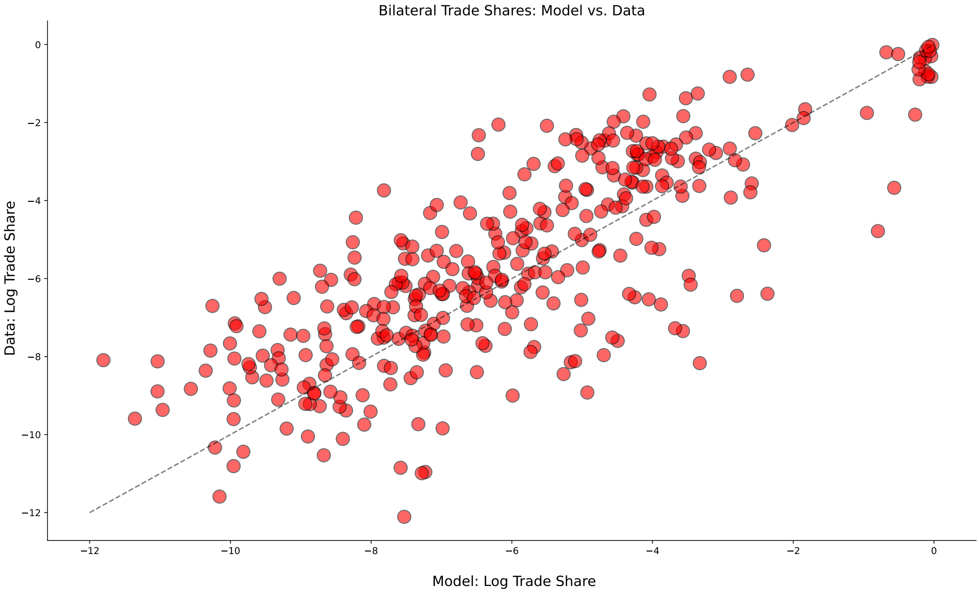
- Grab trade costs and productivity estimates from 19 country world of Eaton and Kortum (2002) and compute an equilibrium.
- Explore small, global reduction in trade costs. No transition path today, ran out of time!

Other important parameters and how I set them for today.

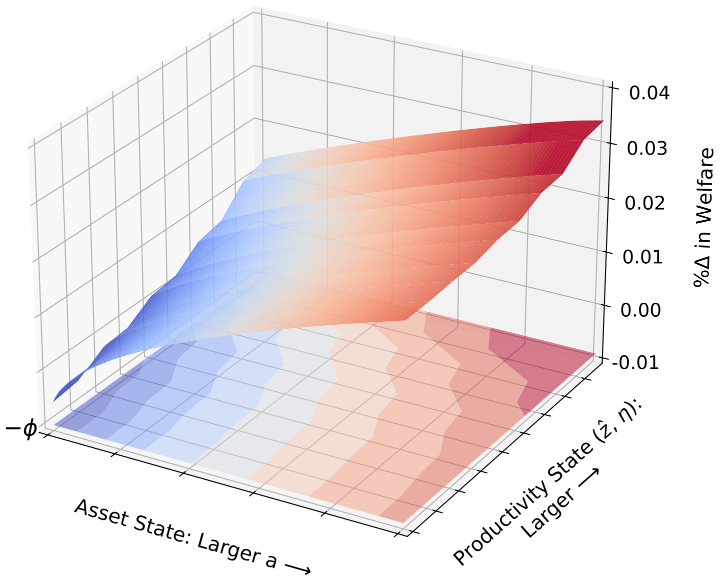
- Taste shock parameter so $1/\sigma_\epsilon = 4.0$. CRRA for u with relative risk aversion $= 1.5$.
- Earnings process is a mixture of a persistent and transitory component and calibrated as in Krueger, Mitman, and Perri (2016).
- Borrowing constraint is set $\approx 2\times$ earnings for US. Discount factor set so $R \approx 2\%$ for US.

My RA Thomas Hasenzagl and I have Julia and Python code to compute things pretty fast.

Bilateral Trade: Model vs. Data



U.S. Welfare Gains to a Global 1% Reduction in Trade Costs



Welfare Gains to a Global 1% Reduction in Trade Costs

Welfare Gains to a Global 1% Reduction in Trade Costs

	Baseline Model	Rep. Agent Model
USA	0.0075 [83]	0.025 [−−]
Germany	0.14 [100]	0.31 [−−]
Japan	0.004 [80]	0.014 [−−]
Canada	0.09 [100]	0.18 [−−]

Note: Numbers in brackets are % of population who gain. Rep. Agent Model uses ACR calculation with trade elasticity = 4.0

Where I'm headed next...

Lot's to do, but "big picture" this is where I'm aiming:

1. Can trade policy improve outcomes?

- This is a useful laboratory to think about policy because (i) there is scope for it and (ii) have a direct representation of utility (not an indirect representation).

2. How financial globalization relates globalization in goods trade?

- The model provides a coherent account of both trade in goods and assets. I think it'd be interesting to see what happens.

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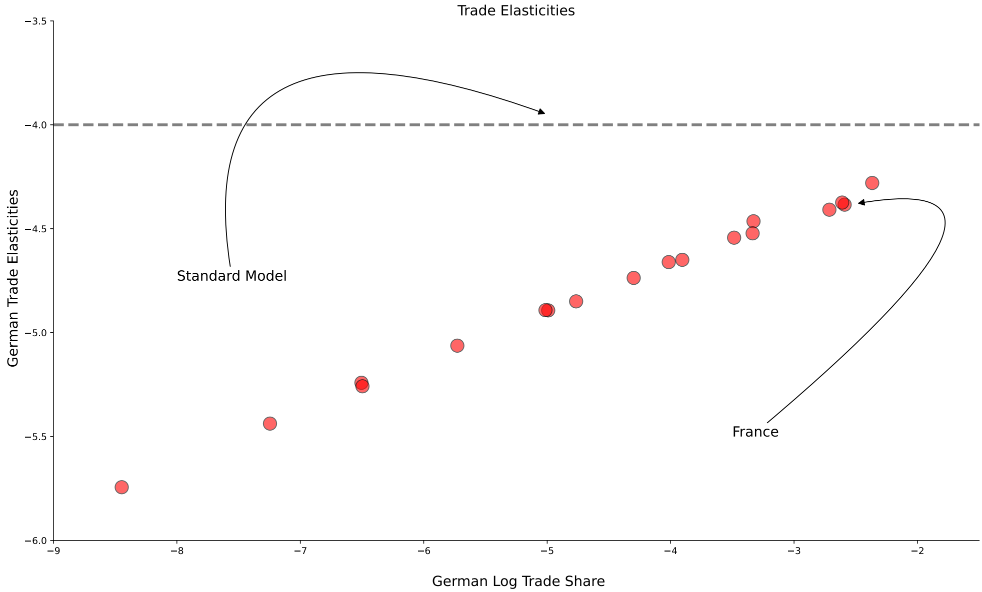
Bilateral Trade Elasticities: German Example



Welfare Gains to a Global 10% Reduction in Trade Costs

Welfare Gains to a Global 10% Reduction in Trade Costs

	Baseline Model	Rep. Agent Model
USA	0.21 [100]	0.28 [−−]
Germany	1.6 [100]	3.5 [−−]
Japan	0.14 [100]	0.21 [−−]
Canada	1.13 [100]	1.9 [−−]

Note: Numbers in brackets are % of population who gains. Rep. Agent Model uses ACR calculation with trade elasticity = 4.0