Heterogeneous Agent Trade

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What am I doing?

To trade economists, household heterogeneity is interesting because of the notion that some benefit from trade and others don't.

One mechanism behind this notion is heterogeneity in **elasticities**.

- Auer, Burstein, Lein, and Vogel (2022) is a nice example. In the context of the 2015 Swiss appreciation, they find that poor households are more price elastic.

This paper:

- A model of household heterogeneity in price elasticities at the micro level *and* they arise because of a market failure, i.e., the lack of insurance against life's circumstances.
- And I use it as a laboratory to think about aggregate trade, the gains from trade and how they are distributed, and the normative implications, e.g., what *should* the pattern of trade look like.

How I do it...

Two ingredients:

- Trade as in Armington, but households have random utility over varieties — McFadden (1974)
- Standard incomplete markets model with households facing incomplete insurance against idiosyncratic productivity and taste shocks — Bewley (1979)

Qualitatively I characterize:

- How price elasticities vary at the micro-level and when micro-heterogeneity shapes aggregates.
- The welfare gains from trade at the micro and macro level.
- The efficient allocation and, thus, how market incompleteness shapes these outcomes.

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Quantitatively:

- I work at a scale typically reserved for static frameworks—I calibrate a 19 country model (the Eaton and Kortum (2002) data set) to match trade flow data using "gravity as a guide."
- Illustrate some workings of the model through gains from trade type calculations.
- Compare trade in the efficient allocation vs. the decentralized allocation.

Model: Production and Trade

M countries. Each country produces a nationally differentiated product as in Armington.

In country i, competitive firms' produce variety i with:

$$Q_i = A_i N_i,$$

where A_i is TFP; N_i are efficiency units of labor supplied by households.

Cross-country trade faces obstacles:

- iceberg trade costs $d_{ij} > 1$ for one unit from supplier j to go to buyer i.

This structure leads to the following prices that households face

$$p_{ij} = \frac{d_{ij} w_j}{A_j}.$$

Model: Households I

Continuum of households $k \in [0,\ L_i]$ in each country i. Household preferences:

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t \ \tilde{u}_{ijt}^k,$$

where conditional direct utility for good j is

$$\tilde{u}_{ijt}^k = u(c_{ijt}^k) + \epsilon_{jt}^k, \quad j = 1, \ldots, M.$$

Assumptions:

- discrete-continuous choice. . . so first chose one variety, then continuous choice over quantity.
- ϵ_{jt}^ks are iid across hh and time; distributed Type 1 Extreme Value with dispersion parameter σ_ϵ.
- For now, u is well behaved.

Multiple sectors? We do this with an "infinite shopping aisle" in Mongey and Waugh (2023).

Model: Households II

Household k's efficiency units z_t evolve according to a Markov Chain. They face the wage per efficiency unit w_{it}.

Households borrow or accumulate a non-state contingent asset, a, with gross return R_i. Household's face the debt limit

$$a_{t+1} \geq -\phi_i.$$

Conditional on a variety choice, a household's budget constraint is

$$p_{ij} c_{ijt} + a_{t+1} \leq R_i a_t + w_{it} z_t.$$

What Households Do I

Focus on a stationary setting. A hh's state are its asset holdings a and shock z.

1. Condition on variety choice their problem is:

$$v_i(a,z,j) = \max_{a',\ c_{ij}} \left\{ u(c_{ij}) + \epsilon_j + \beta\,\mathbb{E}[v_i(a',z')] \right\},$$

$$\text{subject to} \quad p_{ij}c_{ij} + a' \leq R_i a + w_i z \quad \text{and} \quad a' \geq -\phi_i.$$

2. The ex-post value function of a household in country i is

$$v_i(a,z) = \max_j \left\{\ v_i(a,z,j)\ \right\}.$$

What Households Do II

Two equations characterizing the commodity choice, consumption / savings. . .

1. The choice probability is:

$$\pi_{ij}(a,z) = \exp\left(\frac{v_i(a,z,j)}{\sigma_\epsilon}\right) \Big/ \Phi_i(a,z),$$

$$\text{where} \quad \Phi_i(a,z) := \sum_{j'} \exp\left(\frac{v_i(a,z,j')}{\sigma_\epsilon}\right).$$

2. Away from the constraint, consumption and asset choices must respect this Euler Equation:

$$\frac{u'(c_i(a,z,j))}{p_{ij}} = \beta R_i \mathrm{E}_{z'}\left[\sum_{j'} \pi_{ij'}(a',z') \frac{u'(c_i(a',z',j'))}{p_{ij'}}\right].$$

Aggregation

Aggregates arise from explicit aggregation of hh-level actions. Two examples:

1. Aggregate, bilateral imports and exports are

$$M_{ij} = L_i \int_z \int_a p_{ij} c_i(a,z,j) \pi_{ij}(a,z) \lambda_i(a,z), \qquad X_{ji} = L_j \int_z \int_a p_{ji} c_j(a,z,i) \pi_{ji}(a,z) \lambda_j(a,z),$$

where λ_i is the *endogenous* distribution of hhs across states. Here trade flows take on a mixed-logit form similar to Berry, Levinsohn, and Pakes (1995), but everything is tied down in equilibrium.

2. The national income accounting identity (GDP = C + I + G + X - M) . . .

$$p_i Y_i = \underbrace{L_i \sum_j \int_z \int_a p_{ij} c_i(a,z,j) \pi_{ij}(a,z) \lambda_i(a,z)}_{\widetilde{P_i C_i}} + \underbrace{\left[\sum_{j \neq i} X_{ji} - \sum_{j \neq i} M_{ij} \right]}_{-R_i A_i + A_i'}.$$

Equilibrium

Definition 1 (The Decentralized Stationary Equilibrium)

A Decentralized Stationary Equilibrium are asset policy functions and commodity choice probabilities $\{\ g_i(a,z,j), \pi_{ij}(a,z)\ \}_i$, probability distributions $\{\ \lambda_i(a,z)\ \}_i$ and positive real numbers $\left\{w_i, p_{ij}, R_i\right\}_{i,j}$ such that

- i Prices (w_i, p_{ij}) satisfy (**??**) and (**??**);
- ii The policy functions and choice probabilities solve the household's optimization problem in (**??**) and (**??**);
- iii The probability distribution $\lambda_i(a,z)$ induced by the policy functions, choice probabilities, and primitives satisfies (**??**) and is stationary;
- iv Goods market clears:

$$p_i Y_i - \sum_j X_{ji} = 0, \quad \forall i \tag{1}$$

- v Bond market clears with either

$$\mathrm{A_i'} = 0, \quad \forall i \quad \text{or} \quad \sum_i \mathrm{A_i'} = 0 \tag{2}$$

Proposition 1 (The HA Trade Elasticity)

The trade elasticity between country i and country j is:

$$\theta_{ij} = 1 + \int_{z,a} \left\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \right\} \omega_{ij}(a,z) da\ dz - \int_{z,a} \left\{ \theta_{ii,j}(a,z)^I + \theta_{ii,j}(a,z)^E \right\} \omega_{ii}(a,z) da\ dz$$

which is an expenditure-weighted average of micro-level elasticities. The micro-level elasticities are decomposed into an intensive margin and extensive margin

$$\theta_{ij}(a,z)^I = \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}, \qquad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and $\omega_{ij}(a,z)$ are the expenditure weights.

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and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^I = \left[-\frac{\partial g_i(a,z,j)/p_{ij}c_i(a,z,j)}{\partial p_{ij}/p_{ij}} - 1 \right] \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}.$$

The idea: a Δ in trade costs relaxes the budget constraint and then the division of new resources between assets and expenditure determines the intensive margin.

In absolute value, this is larger for the poor, smaller for the rich.

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and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^E = \frac{1}{\sigma_\epsilon} \frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}}.$$

Now assume the number of countries is large. . .

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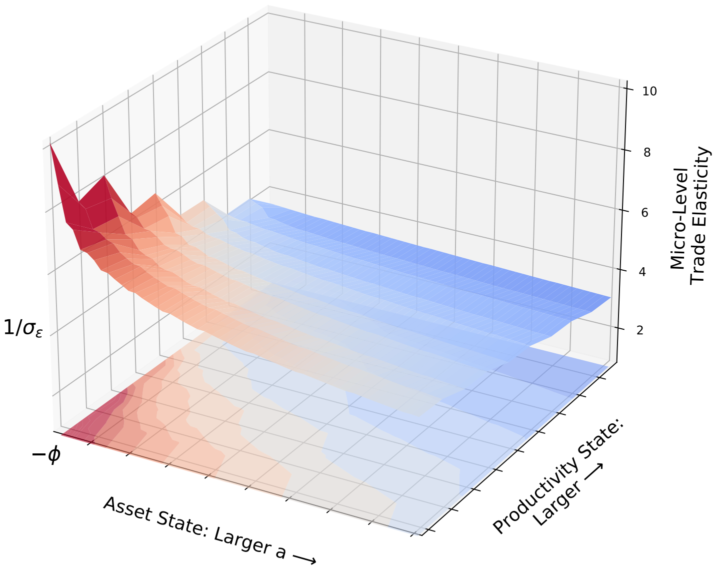
and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^E \approx -\frac{1}{\sigma_\epsilon} \Big[u'(c_i(a,z,j)) c_i(a,z,j) \Big].$$

With CRRA and relative risk aversion > 1 then **poor hh's are the most price sensitive.**

Trade Elasticities by HH-Level State



HA Gains from Trade

Proposition 2 (HA Gains from Trade)

The welfare gains from trade are given by

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} = \int_a \int_z \left\{ \frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} + v_i(a,z) \frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \right\} \lambda_i(a,z).$$

which reflects the change in household level gains and how the distribution of households changes. Household level gains are given by

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \Big\{ A(a_t, z_t) + B(a_t, z_t) + C(a_t, z_t) \Big\}$$

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This term is what I call the “gains from substitution”:

$$A(a,z) = -\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}}$$

$$\approx -\sigma_\epsilon \times \pi_{ij}(a,z) \times \bar{\theta}(a_t, z_t)^E_{ij}$$

Where the last line says these gains from substitution are about (i) exposure (Deaton (1989), Borusyak and Jaravel (2021)) and (ii) elasticities (Auer, Burstein, Lein, and Vogel (2022)).

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This term is what I call the "gains from changes in factor prices":

$$B(a,z) = u'(c_i(a,z,i)) \times a \times \frac{\mathrm{d}R_i/w_i}{\mathrm{d}d_{ij}/d_{ij}}$$

How hh's real wealth (+ or −) change through GE effects on prices — all evaluated at the hh's marginal utility of home consumption.

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This term is what I call the "gains from changes in asset holdings"

$$C(a,z) = \Bigg\{ \underbrace{-\frac{u'(c_i(a,z,i))}{p_{ii}} + \beta \mathbb{E}_{z'} \left[-\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + \frac{u'(c_i(a',z',i))R_i}{p_{ii}} \right]}_{\text{Euler equation}} \Bigg\} \frac{\mathrm{d}g_i(a',z',i)}{\mathrm{d}d_{ij}/d_{ij}}$$

which is zero for small changes as hh's are either (i) on their Euler equation or (ii) constrained and can't adjust their asset position.

Proposition 3 (Separation of Trade and Micro-Heterogeneity)

In the heterogenous agent trade model where preferences are logarithmic over the physical commodity, the trade elasticity is

$$\theta = -\frac{1}{\sigma_\epsilon},$$

and trade flows satisfy a standard gravity relationship

$$\frac{M_{ij}}{M_{ii}} = \left(\frac{w_j/A_j}{w_i/A_i}\right)^{\frac{-1}{\sigma_\epsilon}} d_{ij}^{\frac{-1}{\sigma_\epsilon}},$$

and both are independent of the household heterogeneity. And the welfare gains from trade for an individual household are

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \frac{1}{\theta(1-\beta)} \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}} \quad + \quad \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \Big\{ B(a_t, z_t) + C(a_t, z_t) \Big\}.$$

This mimics the results of Anderson, De Palma, and Thisse (1987). This was not obvious to me given the environment . . . risk, market incompleteness, borrowing constraints, etc.

HA Gains from Trade: log Preferences ⇒ Separation of Trade and Heterogeneity

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And we are back to Arkolakis et al. (2012) + what's going on with factor prices and asset holdings.

HA Gains from Trade under Efficiency

Proposition 4 (Trade Elasticities and Welfare Gains in the Efficient Allocation)

The elasticity of trade to a change in trade costs between ij in the efficient allocation is:

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon}\left[u'(c_i(j))c_i(j)\right].$$

And the welfare gains from a reduction in trade costs between i, j are

$$\frac{\mathrm{d}W}{\mathrm{d}d_{ij}/d_{ij}} = \frac{\sigma_\epsilon\ \theta_{ij}\ \pi_{ij}\ L_i}{1-\beta},$$

which is the discounted, direct effect from relaxing the aggregate resource constraint. And this can be expressed as

$$= -\sigma_\epsilon \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}} \times \frac{L_i}{1-\beta}.$$

Same idea as in decentralized allocation, but now everyone substitutes in a common way...

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Mimics the results of Atkeson and Burstein (2010) but with household (not firm) heterogeneity.

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And we are back to Arkolakis et al. (2012)-like expression and with log its exact.

Quantitative Analysis

This is what I'll do...

1. Calibrate my model using my "gravity as a guide" approach on the 19 country data set of Eaton and Kortum (2002) and targeting micro-evidence from Borusyak and Jaravel (2021) and Auer et al. (2022).

2. Gains from trade calculations.

Household Parameters

Parameters common across countries:

- CRRA for u with relative risk aversion γ — varied to fit elasticities in Auer et al. (2022).
- Earnings process as in Krueger, Mitman, and Perri (2016).
- Discount factor β jiggled to target a world interest rate of 1.0% in financial globalization case.

Parameters scaled across countries to deliver balanced-growth-like properties.

- Set $\sigma_{\epsilon,i} = \sigma_\epsilon \times A_i^{1-\gamma}$, — σ_ϵ varied to fit elasticities in Auer et al. (2022).
- Set the borrowing constraint so $\phi_i = \phi \times A_i$ where $\phi = 0.50$.

Household-specific quality shifters — a home bias term $\psi_{ii}(z)$ which additively shifts utility

- Without this prices and price elasticities determine shares, so to flexibly fit the data there needs to be interactions between quality and household characteristics; same idea as in Berry et al. (1995).
- Slope of $\psi_{ii}(z)$ wrt z varied to fit Borusyak and Jaravel (2021) facts.

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County Specific Parameters — Using Gravity as a Guide

The problem: no closed form map from trade flows to parameters as in standard trade models. But I want the model to replicate the geographic pattern of activity seen in the data.

- Step 0. Impose a trade cost function to reduce the parameter space

$$\log d_{ij} = d_k + b + l + e_h + m_i.$$

- Step 1. Run this gravity regression on the data

$$\log \left(\frac{M_{ij}}{M_{ii}} \right) = Im_i + Ex_j + d_k + b + l + e_h + \delta_{ij}.$$

- Step 2. Guess TFP terms and coefficients on the trade cost function, compute an equilibrium, run the same regression from above on model generated data.
- Step 3. Evaluate difference between model and data and update parameters until convergence.

County Specific Parameters — Using Gravity as a Guide

The solution: use the gravity regression "as a guide" where I estimate parameters of the model so that the regression coefficients run on my model's data match that seen in the data.

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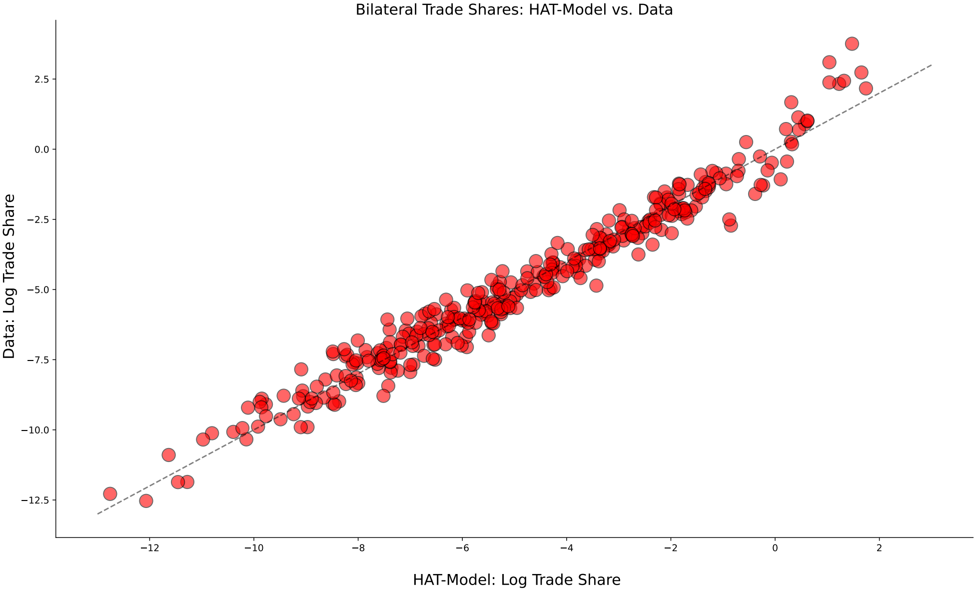
Bilateral Trade: Model vs. Data



Estimates of Geographic Barriers

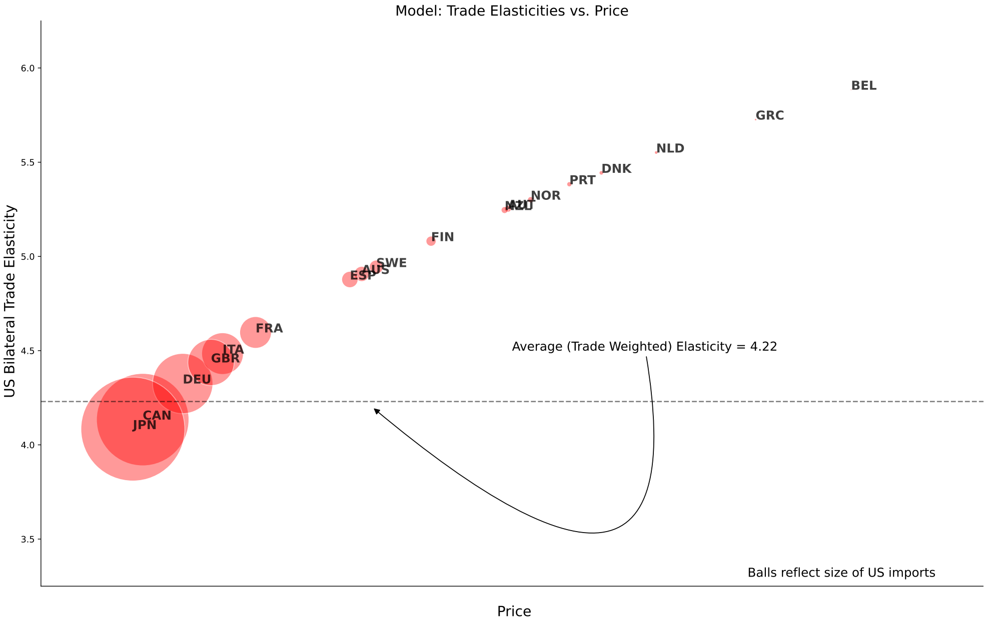
Table 1: Estimation Results

Barrier	Moment	HAT-Model Model Fit	HAT-Model Parameter
$[0, 375)$	-3.10	-3.10	2.35
$[375, 750)$	-3.67	-3.67	2.81
$[750, 1500)$	-4.03	-4.03	3.09
$[1500, 3000)$	-4.22	-4.22	3.23
$[3000, 6000)$	-6.06	-6.06	4.88
$[6000, \text{maximum}]$	-6.56	-6.56	5.69
Shared border	0.30	0.30	0.91
Language	0.51	0.51	0.87
EFTA	0.04	0.04	0.98
European Community	0.54	0.54	0.89

Note: The first column reports data moments the HAT-model targets. The second reports the model moments. The third column reports the estimated parameter values.

Far distances $\approx$ 10 percent less expensive than standard models would predict.

US Trade Elasticities: $-\theta_{us,j}$



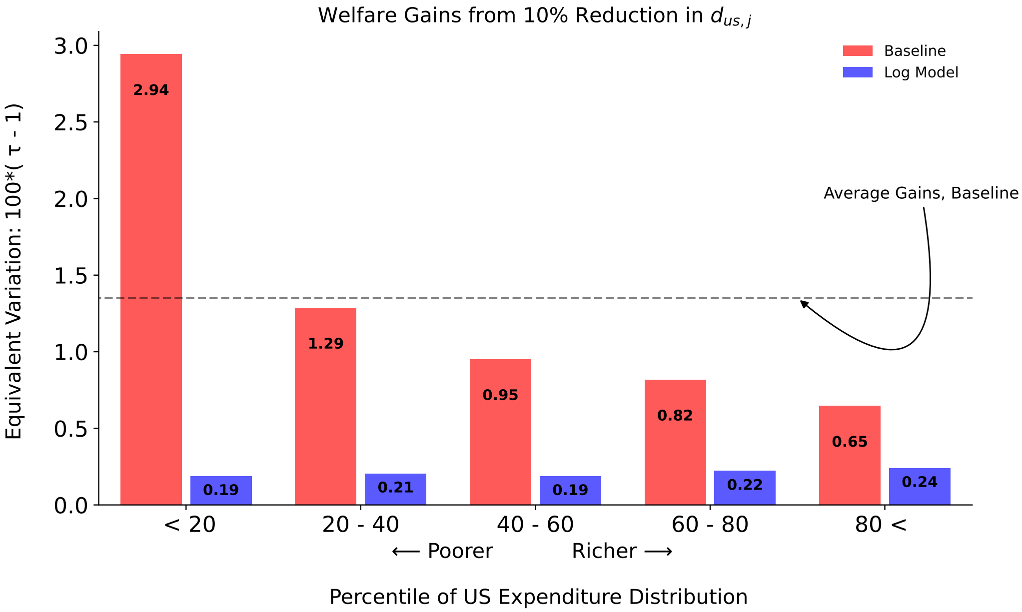
Micro Moments

Model: Shares, Elasticities, and MPCs by Income of US Households

	Import Shares	Trade Elasticities	MPCs
Below Median Income	0.08	−6.24	0.50
Median	0.08	−4.88	0.28
Above Median Income	0.08	−3.97	0.17

Measuring Welfare

U.S. Welfare: 10% Reduction in $d_{us,j}$



Where I'm headed next...

On my todo list:

- Can trade policy improve outcomes? Put in tariffs and redistribute.
- Quality or "residual demand shifters" and calibration to line up with the micro-evidence of papers like Auer et al. (2022).

Any thing else? Email me!

One more thing: My github repository provides the code and supplementary work behind this paper at `https://github.com/mwaugh0328/heterogeneous-agent-trade`.

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Log Model — Fit of Trade Data