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THE CES IS A DISCRETE CHOICE MODEL?

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The CES demand function is a special case of a nested logit model whose second-stage is deterministic.

Consider an individual who has to choose a certain amount of one good among n possible goods. His choice is seen as a two-stage process: (i) he chooses which good to buy, and (ii) the quantity of that good.

Suppose that the outcome of the first stage is good i. Then the individual's conditional direct utility is assumed to be

$$u_i = \ln q_i, \qquad i = 1, \ldots, n, \tag{1}$$

where q_i is the quantity of good i. Let y denote his available income and p_i the price of good i. Maximizing (1) under the budget constraint yields the demand

$$q_i^* = \frac{y}{p_i}, \qquad i = 1, \ldots, n. \tag{2}$$

The conditional indirect utility is therefore

$$V(p_i) = -\ln p_i + \ln y, \qquad i = 1, \ldots, n. \tag{3}$$

Let us now describe the first stage. It is assumed that the choice of good i follows the stochastic utility approach used in discrete choice theory, i.e.,

$$U_i = V(p_i) + \mu\epsilon_i, \qquad i = 1, \ldots, n, \tag{4}$$

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where U_i is the stochastic utility associated with i, μ is a positive constant and ϵ_i a random variable with zero mean and unit variance. The probability for the individual to choose i is given by

$$P_i = \text{Prob}\left[U_i = \max_{j=1\ldots n} U_j\right], \qquad i = 1, \ldots, n. \tag{5}$$

Assuming that the ϵ_is are identically, independently Gumbel distributed, (5) becomes the (multinomial) logit:

$$P_i = \frac{e^{V(p_i)/\mu}}{\sum_{j=1}^{n} e^{V(p_j)/\mu}}, \qquad i = 1, \ldots, n. \tag{6}$$

Substituting (3) into (6) yields

$$P_i = \frac{p_i^{-1/\mu}}{\sum_{j=1}^{n} p_j^{-1/\mu}}, \qquad i = 1, \ldots, n. \tag{7}$$

Given (2), the expected demand of the individual for good i is

$$D_i = \frac{p_i^{-1/\mu-1}}{\sum_{j=1}^{n} p_j^{-1/\mu}} y, \qquad i = 1, \ldots, n. \tag{8}$$

In (8), we recognize the demand system generated by a CES direct utility function

$$\mathscr{U} = \left[\sum_{j=1}^{n} q_j^{\rho}\right]^{1/\rho} \tag{9}$$

with $0 \leq \rho \leq 1$, provided that $\mu = (1-\rho)/\rho$.

Thus, the demand model derived from the CES utility function (9) is equivalent to the solution of a nested logit model in which the second stage is deterministic with conditional direct utility function given by (1).