Heterogeneous Agent Trade

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What am I doing?

To trade economists, household heterogeneity is interesting because of the notion that some benefit from trade and others don't.

One mechanism behind this notion is heterogeneity in **elasticities**.

- Auer, Burstein, Lein, and Vogel (2022) is a nice example. In the context of the 2015 Swiss appreciation, they find that poor households are more price elastic.

This paper:

- A model of household heterogeneity in price elasticities at the micro level *and* they arise because of a market failure, i.e., the lack of insurance against life's circumstances.

- And I use it as a laboratory to think about aggregate trade, the gains from trade and how they are distributed, and the normative implications, e.g., what *should* the pattern of trade look like.

How I do it. . .

Two ingredients:

- Trade as in Armington, but households have random utility over varieties — McFadden (1974)
- Standard incomplete markets model with households facing incomplete insurance against idiosyncratic productivity and taste shocks — Bewley (1979)

Qualitatively I characterize. . .

- How price elasticities vary at the micro-level and when micro-heterogeneity shapes aggregates.
- The welfare gains from trade at the micro and macro level.
- The efficient allocation and, thus, how market incompleteness shapes these outcomes.

How I do it...

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- Trade as in Armington, but households have random utility over varieties — McFadden (1974)
- Standard incomplete markets model with households facing incomplete insurance against idiosyncratic productivity and taste shocks — Bewley (1979)

Quantitatively...

- I compute and calibrate a 19 country model (the Eaton and Kortum (2002) data set) to match trade flow data using "gravity as a guide."
- Illustrate some workings of the model through gains from trade type calculations.
- Compare trade in the efficient allocation vs. the decentralized allocation.

Model: Production and Trade

M countries. Each country produces a nationally differentiated product as in Armington.

In country i, competitive firms' produce variety i with:

$$Q_i = A_i N_i,$$

where A_i is TFP; N_i are efficiency units of labor supplied by households.

Cross-country trade faces obstacles:

- iceberg trade costs $d_{ij} > 1$ for one unit from supplier j to go to buyer i.

This structure leads to the following prices that households face

$$p_{ij} = \frac{d_{ij} w_j}{A_j}.$$

Model: Households I

Continuum of households $k \in [0,\ L_i]$ in each country i. Household preferences:

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t \ \tilde{u}_{ijt}^k$$

where conditional direct utility for good j is

$$\tilde{u}_{ijt}^k = u(c_{ijt}^k) + \epsilon_{jt}^k, \quad j = 1, \ldots, M$$

Assumptions:

- discrete-continuous choice. . . so first chose one variety, then continuous choice over quantity.
- ϵ_{jt}^ks are iid across hh and time; distributed Type 1 Extreme Value with dispersion parameter σ_ϵ.
- For now, only that u is well behaved.

Multiple sectors? We do this with an "infinite shopping aisle" in Mongey and Waugh (2022).

Model: Households II

Household k's efficiency units z_t evolve according to a Markov Chain. They face the wage per efficiency unit w_{it}.

Households borrow or accumulate a non-state contingent asset, a, with gross return R_i. Household's face the debt limit

$$a_{t+1} \geq -\phi_i.$$

Conditional on a variety choice, a household's budget constraint is

$$p_{ij} c_{ijt} + a_{t+1} \leq R_i a_t + w_{it} z_t.$$

What Households Do I

Focus on a stationary setting. A hh's state are its asset holdings a and shock z.

1. Condition on variety choice their problem is:

$$v_i(a, z, j) = \max_{a',\ c_{ij}} \left\{ u(c_{ij}) + \epsilon_j + \beta\, \mathbb{E}[v_i(a', z')] \right\}$$

$$\text{subject to} \quad p_{ij} c_{ij} + a' \leq R_i a + w_i z \quad \text{and} \quad a' \geq -\phi_i.$$

2. The ex-post value function of a household in country i is

$$v_i(a, z) = \max_j \left\{ \ v_i(a, z, j) \ \right\}.$$

What Households Do II

Two equations characterizing the commodity choice, consumption / savings...

1. The choice probability is:

$$\pi_{ij}(a,z) = \exp\left(\frac{v_i(a,z,j)}{\sigma_\epsilon}\right) \Big/ \Phi_i(a,z),$$

$$\text{where} \quad \Phi_i(a,z) := \sum_{j'} \exp\left(\frac{v_i(a,z,j')}{\sigma_\epsilon}.\right)$$

2. Away from the constraint, consumption and asset choices must respect this Euler Equation:

$$\frac{u'(c_i(a,z,j))}{p_{ij}} = \beta R_i \mathrm{E}_{z'}\left[\sum_{j'} \pi_{ij'}(a',z') \frac{u'(c_i(a',z',j'))}{p_{ij'}}\right].$$

Aggregation

Aggregates arise from explicit aggregation of hh-level actions. Two examples:

1. Aggregate, bilateral imports and exports are

$$M_{ij} = L_i \int_z \int_a p_{ij} c_i(a, z, j) \pi_{ij}(a, z) \lambda_i(a, z), \qquad X_{ji} = L_j \int_z \int_a p_{ji} c_j(a, z, i) \pi_{ji}(a, z) \lambda_j(a, z),$$

where λ_i is the *endogenous* distribution of hhs across states. Here trade flows take on a mixed-logit form similar to Berry, Levinsohn, and Pakes (1995).

2. The national income accounting identity (GDP = C + I + G + X - M) . . .

$$p_i Y_i = \underbrace{L_i \sum_j \int_z \int_a p_{ij} c_i(a, z, j) \pi_{ij}(a, z) \lambda_i(a, z)}_{\widetilde{P_i C_i}} + \underbrace{\left[\sum_{j \neq i} X_{ji} - \sum_{j \neq i} M_{ij} \right]}_{-R_i A_i + A_i'}.$$

Equilibrium

The Decentralized Stationary Equilibrium. A Decentralized Stationary Equilibrium are asset policy functions and commodity choice probabilities $\{\ g_i(a, z, j), \pi_{ij}(a, z)\ \}_{ij}$, probability distributions $\{\ \lambda_i(a, z)\ \}_i$ and positive real numbers $\{w_i, p_{ij}, R_i\}_{ij}$ such that

i Prices (w_i, p_{ij}) satisfy the firms problem;

ii The policy functions and choice probabilities solve the household's optimization problem;

iv The probability distribution $\lambda_i(a, z)$ induced by the policy functions, choice probabilities, and primitives satisfies the law of motion and is stationary;

v Goods market clears:

$$p_i Y_i - \sum_j^M X_{ji} = 0, \quad \forall i$$

v Bond market clears with either

$$\mathrm{A_i'} = 0, \quad \forall i \quad \text{in the case of finacial autarky, or}$$

$$\sum_i \mathrm{A_i'} = 0, \text{in the case of finacial globalization and } R_i = R\ \forall i$$

The H-A Trade Elasticity

Proposition #1: The H-A Trade Elasticity. The trade elasticity between country i and country j is:

$$\theta_{ij} = 1 + \int_a \int_z \left\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \right\} \omega_{ij}(a,z) - \left\{ \theta_{ii}(a,z)^I + \theta_{ii}(a,z)^E \right\} \omega_{ii}(a,z),$$

which is the difference between ij and ii expenditure-weighted micro-level elasticities. The micro-level elasticities for households with states a, z are an intensive and extensive elasticity

$$\theta_{ij}(a,z)^I = \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}, \qquad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and $\omega_{ij}(a,z)$ are the expenditure weights.

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and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^I = \left[-\frac{\partial g_i(a,z,j)/p_{ij} c_i(a,z,j)}{\partial p_{ij}/p_{ij}} - 1 \right] \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}.$$

The idea here is that reduction in trade costs relaxes the hh's budget constraint and then the division of new resources between assets and expenditure determines the intensive margin elasticity.

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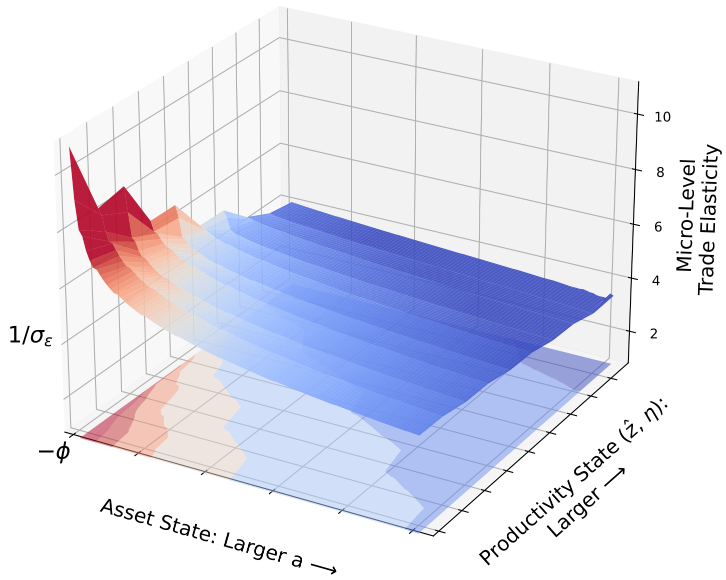
and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^E = -\frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}} + \frac{1}{\sigma_\epsilon} \frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}}.$$

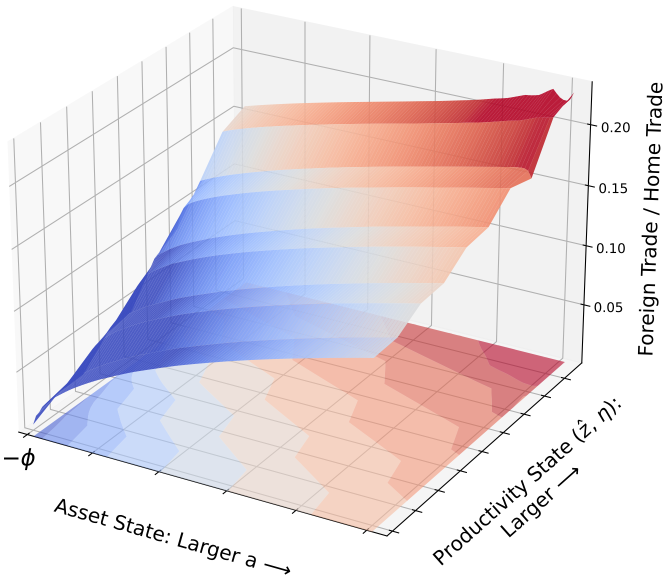
The interesting bit is $\frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}}$ ...

In the paper, I show that if Arrow-Pratt measure of relative risk aversion > 1 than hh's with (i) high $u'(c)$ and (ii) high MPCs are more price elastic. **So poor hh's are the most price sensitive.**

Trade Elasticities by HH-Level State



Trade Shares: $M_i(a, z, j)/M_i(a, z, i)$, by HH-Level State



H-A Gains from Trade I

Proposition #2: H-A Welfare Gains from Trade. The gains from trade are

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} \approx \int_z \int_a \Bigg\{ \underbrace{\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{\text{gains to hh}} + \underbrace{v_i(a,z)\frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{\text{gains to reallocation}} \Bigg\} L_i \lambda_i(a,z),$$

where v_i is value function before realization of taste shocks; $\approx$ is about abstracting from transition.

Household-level gains are

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \approx \sigma_\epsilon \frac{\mathrm{d}\Phi_i(a,z)/\Phi_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}$$

Like standard trade models! It's all about how this price-index-like thing changes.

H-A Gains from Trade II

Proposition #2: H-A Welfare Gains from Trade. The gains from trade are

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} \approx \int_z \int_a \left\{ \underbrace{\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{\text{gains to hh}} + \underbrace{v_i(a,z)\frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{\text{gains to reallocation}} \right\} L_i \lambda_i(a,z),$$

where v_i is value function before realization of taste shocks; $\approx$ is about abstracting from transition.

Household-level gains are

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \approx \sum_j \pi_{ij}(a,z)\frac{\mathrm{d}v_i(a,z,j)}{\mathrm{d}d_{ij}/d_{ij}}$$

The change in the $\Phi_i(a,z)$ thing (previous slide if you fell asleep) is share-weighted average of choice-specific value functions.

Next step. . . one algebra trick.

H-A Gains from Trade . . . the algebra trick

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where v_i is value function before realization of taste shocks; $\approx$ is about abstracting from transition.

Household-level gains are

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \approx \underbrace{\sum_j \pi_{ij}(a,z)\left\{\frac{\mathrm{d}v_i(a,z,j)}{\mathrm{d}d_{ij}/d_{ij}} - \frac{\mathrm{d}v_i(a,z,i)}{\mathrm{d}d_{ij}/d_{ij}}\right\}}_{\text{how relative valuations change}} + \frac{\mathrm{d}v_i(a,z,i)}{\mathrm{d}d_{ij}/d_{ij}}$$

H-A Gains from Trade . . . the algebra trick

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where v_i is value function before realization of taste shocks; $\approx$ is about abstracting from transition.

Household-level gains are

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \approx \underbrace{\sum_j \pi_{ij}(a,z)\Bigg\{\frac{\mathrm{d}v_i(a,z,j)}{\mathrm{d}d_{ij}/d_{ij}} - \frac{\mathrm{d}v_i(a,z,i)}{\mathrm{d}d_{ij}/d_{ij}}\Bigg\}}_{=\text{ how the home choice, } \pi_{ii}\text{, changes}} + \frac{\mathrm{d}v_i(a,z,i)}{\mathrm{d}d_{ij}/d_{ij}}$$

Now recursively iterate forward in time given how v_{ii} connects with v_i next period.

H-A Gains from Trade III

Proposition #2: H-A Welfare Gains from Trade. The gains from trade are

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} \approx \int_z \int_a \Bigg\{ \underbrace{\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{\text{gains to hh}} + \underbrace{v_i(a,z)\frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}}_{\text{gains to reallocation}} \Bigg\} L_i \lambda_i(a,z),$$

where v_i is value function before realization of taste shocks; $\approx$ is about abstracting from transition.

Household-level gains are

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \approx \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \Bigg\{ -\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a_t,z_t)/\pi_{ii}(a_t,z_t)}{\mathrm{d}d_{ij}/d_{ij}} + u'(c_i(a_t,z_t,i)) \Bigg[a_t \, \frac{\mathrm{d}R_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}} \Bigg] \Bigg\}.$$

HH-level gains pick up two effects:

- A state contingent, ACR-like term summarizing how relative valuations across choices change.
- How hh's real wealth ($+$ or $-$) change through GE effects on prices — all evaluated at the hh's marginal utility of home consumption.

Proposition #3: Separation of Trade and Micro-Heterogeneity. When preferences are logarithmic over the physical commodity, choice probabilities are independent of household heterogeneity

$$\pi_{ij}(a, z) = \exp\left(\frac{-\log p_{ij}}{\sigma_\epsilon}\right) \Bigg/ \sum_{j'} \exp\left(\frac{-\log p_{ij'}}{\sigma_\epsilon}\right),$$

and the trade elasticity is

$$\theta = -\frac{1}{\sigma_\epsilon}.$$

And hh-level gains from trade

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \approx \underbrace{-\frac{1}{\theta(1-\beta)} \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}}}_{\text{ACR}} + \underbrace{\mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \left[u'(c_i(a_t, z_t, i))\, a_t\, \frac{\mathrm{d}R_i/p_{ii}}{\mathrm{d}d_{ij}/d_{ij}} \right]}_{\text{exposure to finacial market}}.$$

This mimics the results of Anderson, De Palma, and Thisse (1987). This was not obvious to me given the environment . . . risk, market incompleteness, borrowing constraints, etc.

H-A Gains from Trade: log Preferences $\Rightarrow$ Separation of Trade and Heterogeneity

Proposition #3: Separation of Trade and Micro-Heterogeneity. When preferences are logarithmic over the physical commodity, choice probabilities are independent of household heterogeneity

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And we are back to Arkolakis et al. (2012) + what's going on with the financial market.

H-A Trade under Efficiency

Proposition #4: The Centralized (Efficient) Allocation. The allocation satisfying the Centralized Planning Problem (with a utilitarian SWF and country-specific Pareto weights ψ_i) is:

1. An allocation of consumption satisfying:

$$\psi_i u'(c_i(z, j, t)) = \chi_j(t) d_{ij}$$

where $\chi_j(t)$ is the multiplier on j resource constraint for variety j,

2. And variety choice probabilities:

$$\pi_{ij}(t) = \exp\left(\frac{u(c_i(j, t)) - u'(c_i(j, t))c_i(j, t)}{\sigma_\epsilon}\right) \Big/ \sum_{j'} \exp\left(\frac{u(c_i(j', t)) - u'(c_i(j', t))c_i(j', t)}{\sigma_\epsilon}\right).$$

1. is a Backus and Smith (1993)-like condition.

2. is new — trade should reflect the net social benefit of buying that commodity.

H-A Gains from Trade under Efficiency

Proposition #5: Trade Elasticities and Welfare Gains in the Efficient Allocation The trade elasticity between i, j in the efficient allocation is:

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon}\left[u'(c_{ij})c_{ij}\right].$$

And the welfare gains from a reduction in trade costs between i, j are

$$\frac{\mathrm{d}W}{\mathrm{d}d_{ij}/d_{ij}} = \frac{\partial W}{\partial d_{ij}/d_{ij}} = \frac{\psi_i}{1-\beta} \times u'(c_{ij})c_{ij}\pi_{ij}L_i,$$

which is the discounted, weighted, direct effect from relaxing the resource constraint.

Mimics the results of Atkeson and Burstein (2010) but with household (not firm) heterogeneity.

With log preferences the direct effect is equivalent to Arkolakis et al. (2012).

Quantitative Analysis

This is what I'll go over today...

1. Calibrate my model using my "gravity as a guide" approach on the 19 country data set of Eaton and Kortum (2002).

2. Some gains from trade type calculations.

3. Trade in the efficient allocation vs. the decentralized allocation.

My Calibration Approach: Use Gravity as a Guide

The problem: no closed form map from trade flows to parameters as in standard trade models. But I want the model to replicate the geographic pattern of activity seen in the data.

- Step 0. Impose a trade cost function to reduce the parameter space

$$\log d_{ij} = d_k + b + l + e_h + m_i.$$

- Step 1. Run this gravity regression on the data

$$\log\left(\frac{M_{ij}}{M_{ii}}\right) = Im_i + Ex_j + d_k + b + l + e_h + \delta_{ij}.$$

- Step 2. Guess TFP terms and coefficients on the trade cost function, compute an equilibrium, run the same regression from above on model generated data.
- Step 3. Evaluate difference between model and data and update parameters until convergence.

My Calibration Approach: Use Gravity as a Guide

The solution: use the gravity regression "as a guide" where I estimate parameters of the model so that the regression coefficients run on my model's data match that seen in the data.

- Step 0. Impose a trade cost function to reduce the parameter space

$$\log d_{ij} = d_k + b + l + e_h + m_i.$$

- Step 1. Run this gravity regression on the data

$$\log\left(\frac{M_{ij}}{M_{ii}}\right) = Im_i + Ex_j + d_k + b + l + e_h + \delta_{ij}.$$

- Step 2. Guess TFP terms and coefficients on the trade cost function, compute an equilibrium, run the same regression from above on model generated data.
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Other Parameters

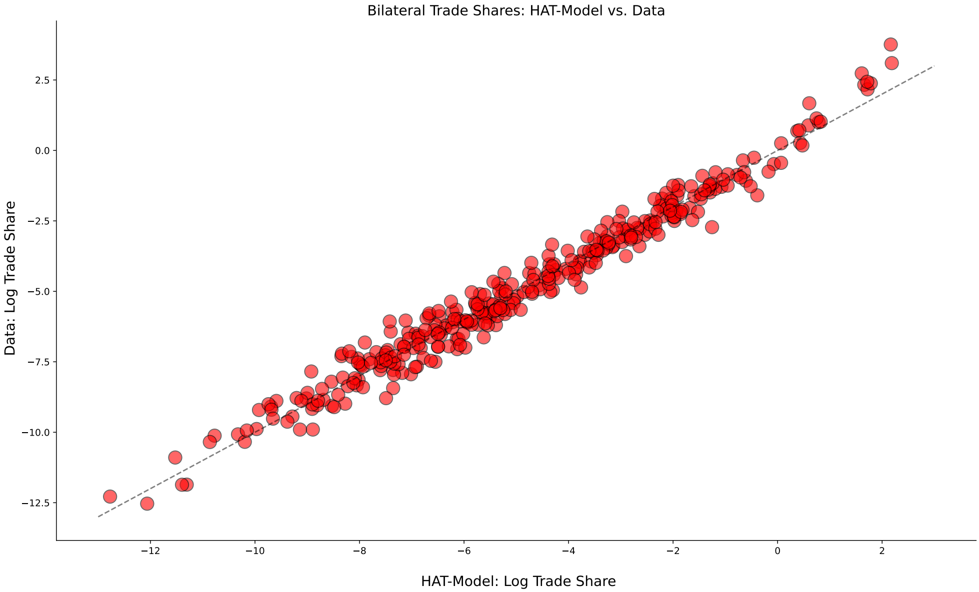
Parameters common across countries:

- CRRA for u with relative risk aversion $= 1.5$.
- Earnings process is a mixture of a persistent and transitory component and calibrated as in Krueger, Mitman, and Perri (2016).
- Discount factor β jiggled to target a world interest rate of 1.5% in financial globalization case.

Parameters scaled across countries to deliver "balanced growth" like properties.

- Set $\sigma_{\epsilon,i} = \sigma_\epsilon \times A_i^{1-\gamma}$ and $\sigma_\epsilon = 0.25$.
  In a log model corresponds with a trade elasticity of 4.
- I set the borrowing constraint so $\phi_i = \phi \times A_i$ where $\phi = 0.50$.
  Interpretation is the constraint $=$ 50 % of average, autarky labor income.

Bilateral Trade: Model vs. Data



Estimates of Geographic Barriers

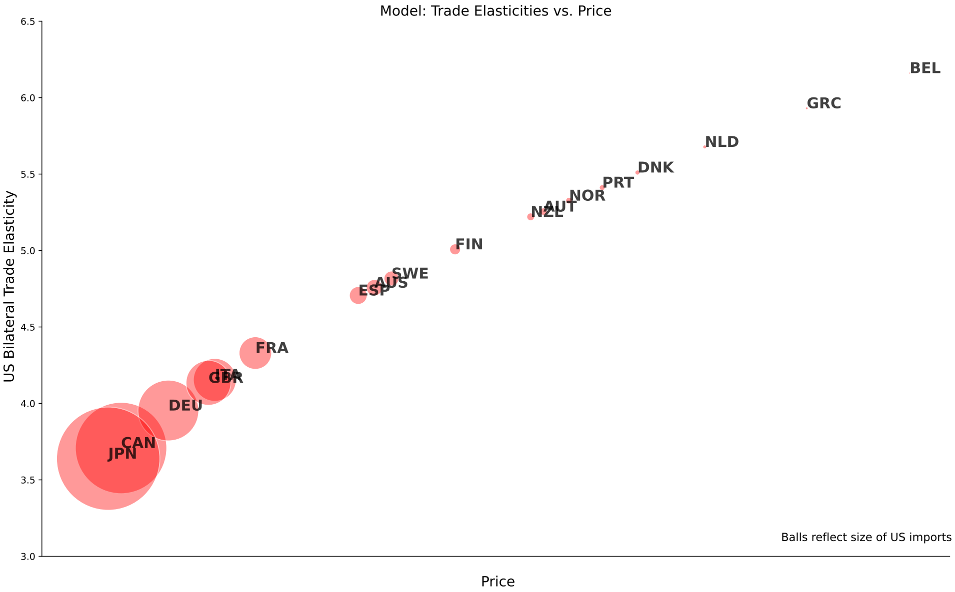
Table 1: Estimation Results

		HAT-Model	
Barrier	Moment	Model Fit	Parameter
$[0, 375)$	-3.10	-3.10	2.35
$[375, 750)$	-3.67	-3.67	2.81
$[750, 1500)$	-4.03	-4.03	3.09
$[1500, 3000)$	-4.22	-4.22	3.23
$[3000, 6000)$	-6.06	-6.06	4.88
$[6000, \text{maximum}]$	-6.56	-6.56	5.69
Shared border	0.30	0.30	0.91
Language	0.51	0.51	0.87
EFTA	0.04	0.04	0.98
European Community	0.54	0.54	0.89

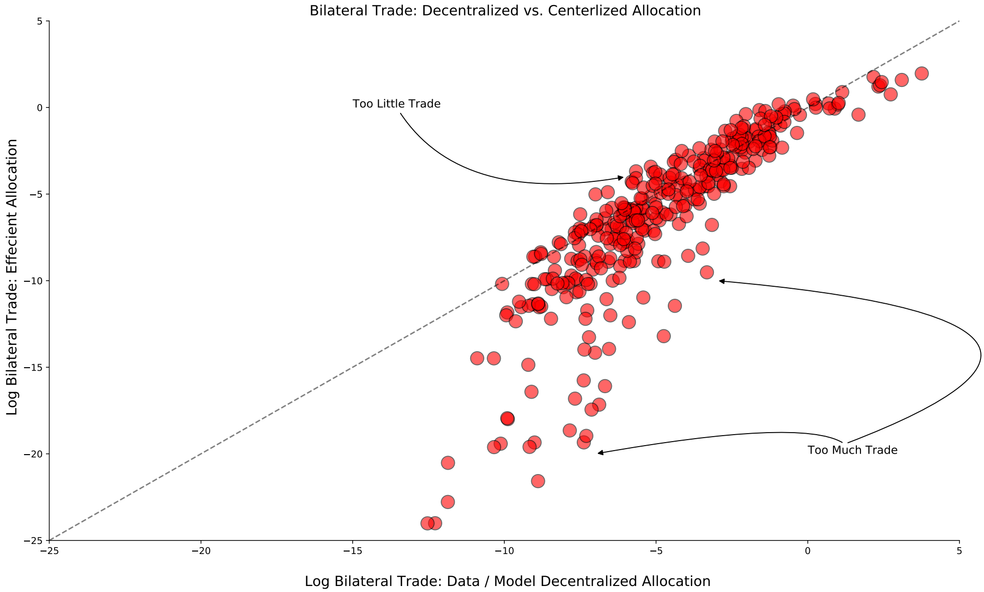
Note: The first column reports data moments the HAT-model targets. The second reports the model moments. The third column reports the estimated parameter values.

Far distances $\approx$ 10 percent less expensive than standard models would predict.

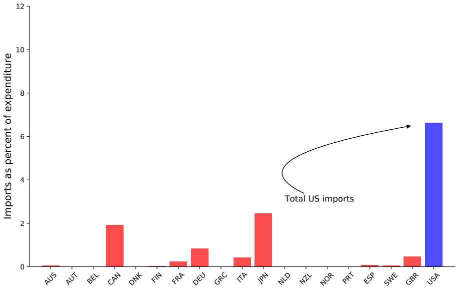
US Trade Elasticities: $-\theta_{us,j}$

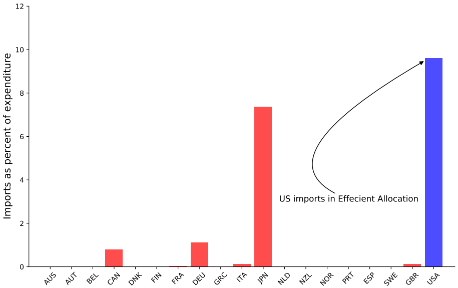


Efficient Trade



US Imports: Decentralized vs. Efficient Allocation





What is going on here?

Where I'm headed next...

Lot's to do, but "big picture" this is where I'm aiming:

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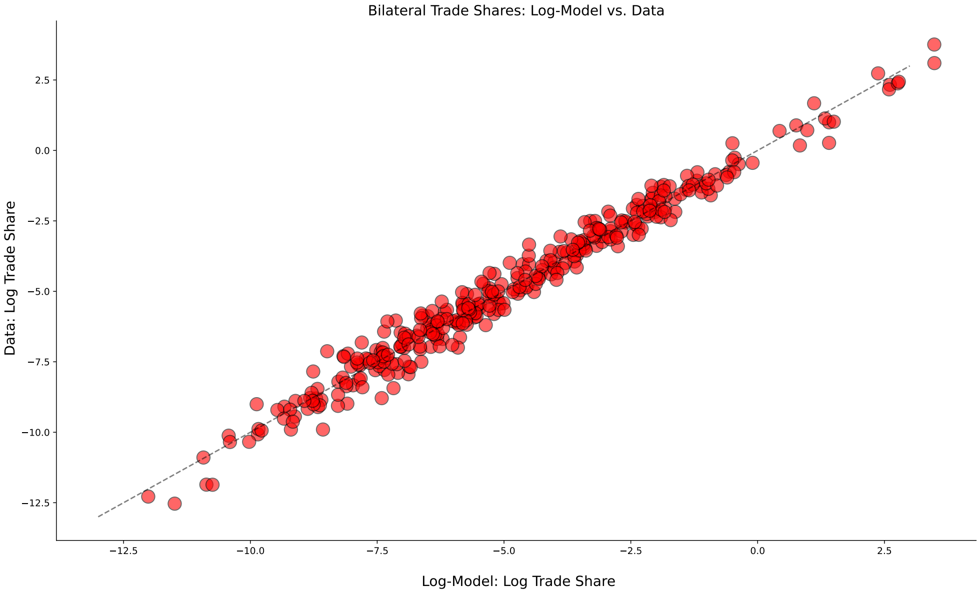
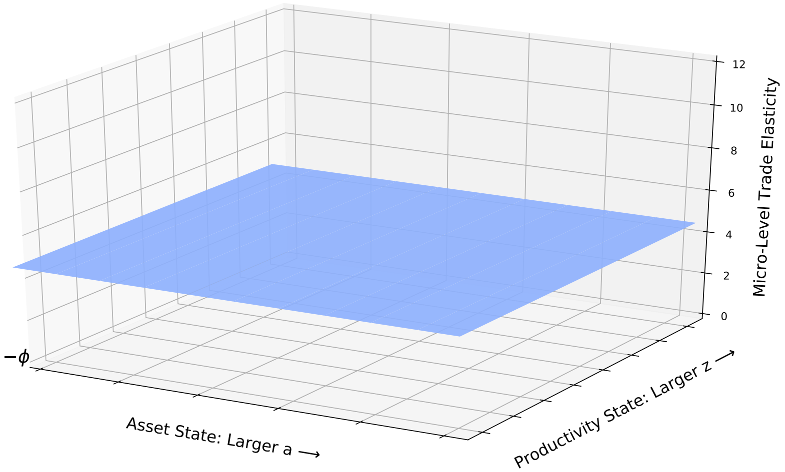
Log Model — Fit of Trade Data



Log Model — Micro Elasticities



Micro-Elasticities I: The Intensive Margin

How do households respond on the **intensive** margin to a change in trade costs?

$$\theta_{ij}(a,z)^{I} := \frac{\partial c_{ij}(a,z)/c_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

$$= \left[-\frac{\partial g_{ij}(a,z)/p_{ij}c_{ij}(a,z)}{\partial p_{ij}/p_{ij}} - 1\right]\frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}.$$

The idea: A reduction in trade costs relaxes the hh's budget constraint, so the intensive margin elasticity depends on the division of new resources between assets and expenditure.

Micro-Elasticities II: The Extensive Margin

How do households respond on the **extensive** margin?

$$\theta_{ij}(a,z)^E := \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

$$= -\frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}} - \frac{1}{\sigma_\epsilon}\Big[u'(c_{ij}(a,z))c_{ij}(a,z)\Big] + \beta\mathbb{E}\frac{1}{\sigma_\epsilon}\frac{\partial v_i(a',z')}{\partial d_{ij}/d_{ij}}.$$

To get a sense of things, vary the second term by wealth...

$$\frac{\partial(u'(c_{ij}(a,z))c_{ij}(a,z))}{\partial a} = u'(c_{ij}(a,z)) \times \mathrm{MPC}_{ij}(a,z) \times \Big[-\rho_{ij}(a,z)+1\Big],$$

where $\rho_{ij}(a,z)$ is the Arrow-Pratt measure of relative risk aversion.

With CRRA, if risk aversion $>$ 1 , then poor, high marginal utility households (who are also high MPC households) are *more elastic relative* to rich households on the extensive margin.

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With CRRA, if risk aversion > 1, then poor, high marginal utility households (who are also high MPC households) are *more elastic relative* to rich households on the extensive margin.