Heterogeneous Agent Trade

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Heterogenous Price Elasticities and Trade

Two ingredients:

- Trade as in Armington, but households have random utility over varieties — McFadden (1974).
- Standard incomplete markets model with households facing incomplete insurance against idiosyncratic productivity and taste shocks — Bewley (1979).

Two core results:

- A household's price elasticity, in essence, is about the marginal gain in utility from a percent change in consumption $\Rightarrow$ the poor are the most price sensitive.
- The gains from trade — to a first order — depend on these elasticities. And can $\Rightarrow$ pro-poor gains from trade.

Quantitatively, these forces are powerful — the poorest households gain 4.5X more than the richest; the average gains from trade are 3X than representative agent benchmarks.

Outline

1. Illustrative model

- Static framework — intended to illustrate how everything works.

2. Main model

- Dynamic framework — production side + the standard incomplete markets model.

3. Quantitative results

- Calibrated to bilateral trade flows and micro evidence. Gains from trade calculations.

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Illustrative Model — Environment

Armington model with two countries home and foreign.

Continuum of agents with names $k \in [0,\ 1]$ in the home country. Agents are heterogenous in wealth w^k. Agents have preferences:

$$u(c_{hh}^k) + \epsilon_h^k \quad \text{and} \quad u(c_{hf}^k) + \epsilon_f^k,$$

- discrete choice. . . so each household chooses only the home or foreign variety.
- ϵ_j^ks are iid across agents and distributed Type 1 Extreme Value with parameter σ_ϵ.
- For now, u is well behaved.

Agents **maximize expected utility** by formulating state contingent plans of variety choice based upon all possible realizations of ϵ_h, ϵ_f.

Illustrative Model — Trade

Micro-level trade

$$\pi_{hf}^{k} = \exp\left(\frac{u(w^k/p_{hf})}{\sigma_\epsilon}\right) \Big/ \Phi_h^k, \quad \text{where} \quad \Phi_h^k := \sum_{j \in h,f} \exp\left(\frac{u(w^k/p_{hj})}{\sigma_\epsilon}\right),$$

which is the standard result with the Type 1 EV taste shocks.

Aggregate trade aries from the **explicit aggregation** of these decisions

$$M_{hf} = \int w^k \pi_{hf}^k dk.$$

The **aggregate trade elasticity** is just a expenditure weighted average of all agent ks elasticities.

- Micro-elasticities. . . next slide

Agent k's trade elasticity

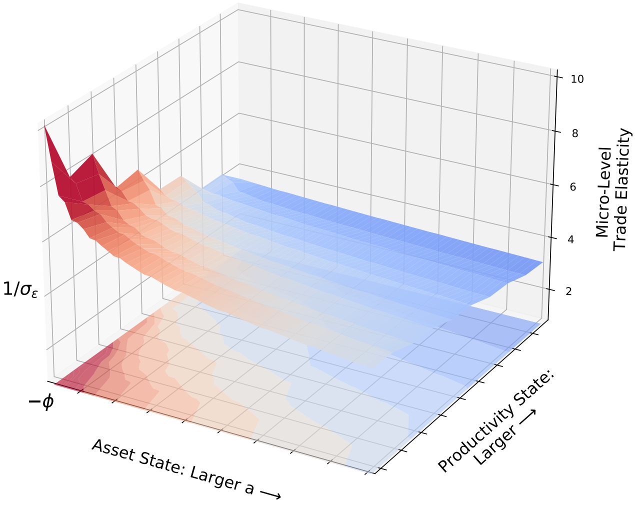
$$-\theta^k_{hf,f} = \frac{1}{\sigma_\epsilon} \times \left[u'(c^k_{hf}) c^k_{hf} \right]$$

Lesson #1: This model easily leads to heterogeneous price sensitivity and specifically the idea that **the poor are the most price sensitive**.

- Generally depends on how curved u is ... with CRRA and curvature parameter $> 1 \Rightarrow$ elasticities are larger for the poor.
- Intuition: A price reduction delivers a lot of extra utility for high marginal utility (poor) households and this induces strong substitution by the poor.

A second interesting case is when u is log ... elasticities are constant and this (*and* the Type 1 EV taste shocks) leads to the Anderson et al. (1987) CES aggregation result.

Example: Micro-Level Trade Elasticities by Income and Wealth



What are the gains from a reduction in the price of the foreign good?

In equivalent variation units, the first order gains from trade are...

$$\mathrm{EV}^{\mathrm{k}} \approx -\pi^{k}_{hf} \times \left[\frac{\theta^{k}_{hf,f}}{\sum_{j} \pi^{k}_{hj} \times \theta^{k}_{hj,j}} \right] \times \Delta \log p_{hf}$$

The gains are about (i) exposure (Deaton (1989), Borusyak and Jaravel (2021)) and (ii) elasticities (Auer, Burstein, Lein, and Vogel (2022)).

Lesson #2: In this model, elasticities matter for the gains from trade and how they are distributed.

- The standard result is that elasticities don't matter to a first order.
- **Here they do**. And under certain conditions, these gains are pro-poor.

Why? Next slide...

Why do elasticities matter?

The same equivalent variation formula can be rewritten as

$$\mathrm{EV}^{\mathrm{k}} \approx -\pi_{hf}^{k} \times \left[\frac{u'(c_{hf}^{k})/p_{hf}}{\sum_{j} \pi_{hj}^{k} \times u'(c_{hj}^{k})/p_{hj}} \right] \times \Delta \log p_{hf}$$

Lesson #3: Elasticities matter because **markets are incomplete**. Agents lack markets to insure against the desire to consume a good. An efficient allocation / complete markets yields

$$u'(c_{hj}^{k})/p_{hj} = u'(c_{hj'}^{k})/p_{hj'} \quad \forall\, j,\ j'$$

and the term in brackets disappears.

Intuition: If the price reduction occurs on the high price good, this helps equate marginal utility across goods $\Rightarrow$ an additional, first order gain from trade.

And the gains can be pro-poor when the poor are more "misallocated."

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Connections and implications:

- **Theory:** Relates to the spatial analysis of Donald, Fuku, and Miyauchi (2024) and complete markets decentralization of discrete choice models in Mongey and Waugh (2023).
- **Measurement:** Assumptions when micro-level shares (the π^ks) are constructed?

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Motivating the Main Model

Two enrichments:

1. Production side to connect prices with trade flows as in quantitative trade models.

2. The standard incomplete markets model on the household side. This is important because...

- In the illustrative model, there was no ability to insure. The SIM model provides an **intermediate setting** with self-insurance, and thus works against the elasticity effects.
- In the illustrative model, types are fixed. In the SIM model, household-level dynamics $\Rightarrow$ poor people today are rich in the future, and this also works against the elasticity effects.
- Less important reasons (i) I like it (ii) I started there (iii) provides an interface with large body of research in macro (iv) a theory as to why there are rich and poor people.

Main Model: Overview

M countries. Each country produces a nationally differentiated product as in Armington; competitive firms with linear technologies in labor and trade costs d_{ij}.

Continuum of households $k \in [0,\ L_i]$ in each country i. Household preferences:

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t\ \tilde{u}_{ijt}^k,$$

where conditional direct utility for good j is

$$\tilde{u}_{ijt}^k = u(c_{ijt}^k) + \epsilon_{jt}^k, \quad j = 1, \ldots, M.$$

- discrete-continuous choice. . . so first chose one variety, then continuous choice over quantity.
- ϵ_{jt}^ks are iid across hh and time; distributed Type 1 Extreme Value with parameter σ_ϵ.

Household k's efficiency units z_t evolve according to a Markov Chain, and can borrow or accumulate a non-state contingent asset, a, with gross return R_i.

The HA Trade Elasticity

How do imports change relative to domestic consumption due to a permanent change in d_{ij}?

Proposition 1 (The HA Trade Elasticity)

The trade elasticity between country i and country j is:

$$\theta_{ij} = 1 + \int_{z,a} \left\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \right\} \omega_{ij}(a,z) da\ dz - \int_{z,a} \left\{ \theta_{ii,j}(a,z)^I + \theta_{ii,j}(a,z)^E \right\} \omega_{ii}(a,z) da\ dz$$

which is an expenditure-weighted average of micro-level elasticities. The micro-level elasticities are decomposed into an intensive margin and extensive margin

$$\theta_{ij}(a,z)^I = \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}, \qquad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and $\omega_{ij}(a,z)$ are the expenditure weights.

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$$\theta_{ij}(a,z)^I = \left[- \frac{\partial a_i'(a,z,j)/p_{ij}c_i(a,z,j)}{\partial p_{ij}/p_{ij}} - 1 \right] \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}.$$

The idea: a lower d_{ij} relaxes the bc and then the division of new resources between assets and expenditure determines the intensive margin. **This is larger for the poor, smaller for the rich.**

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and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^E = \frac{1}{\sigma_\epsilon} \frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}}.$$

Now assume the number of countries is large. . .

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and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^E \approx -\frac{1}{\sigma_\epsilon} \Big[u'(c_i(a,z,j)) c_i(a,z,j) \Big].$$

Exactly the same as in the illustrative model. With CRRA and relative risk aversion > 1 then **poor hh's are the most price sensitive on the extensive margin.**

HA Gains from Trade

How does utility change under the heuristic of an immediate jump to the new steady state?

Proposition 2 (HA Gains from Trade)

Household level gains are given by

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \Big\{ A(a_t, z_t) + B(a_t, z_t) + C(a_t, z_t) \Big\}.$$

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This term is what I call the "gains from substitution":

$$A(a,z) = -\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}}$$

$$\approx \sigma_\epsilon \times \pi_{ij}(a,z) \times \bar{\theta}(a,z)^{E}_{ij,j}$$

Same idea as in illustrative model — the last line says these gains are about (i) exposure and (ii) elasticities.

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This term is what I call "valuation effects":

$$B(a,z) = u'(c_i(a,z,i)) \times a \times \frac{\mathrm{d}R_i/w_i}{\mathrm{d}d_{ij}/d_{ij}}$$

How hh's real wealth ($+$ or $-$) change through GE effects on prices — all evaluated at the hh's marginal utility of home consumption.

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This term is what I call the "gains from changes in asset holdings"

$$C(a,z) = \Big\{ \underbrace{-\frac{u'(c_i(a,z,i))}{p_{ii}} + \beta \mathbb{E}_{z'} \Big[-\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + \frac{u'(c_i(a',z',i))R_i}{p_{ii}} \Big]}_{\text{Euler equation}} \Big\} \frac{\mathrm{d}g_i(a',z',i)}{\mathrm{d}d_{ij}/d_{ij}}$$

which is zero for small changes as hh's are either (i) on their Euler equation or (ii) constrained and can't adjust their asset position.

HA Gains from Trade: log Preferences ⇒ Separation of Trade and Heterogeneity

Proposition 3 (Separation of Trade and Micro-Heterogeneity)

In the heterogenous agent trade model where preferences are logarithmic over the physical commodity, the trade elasticity is

$$\theta = -\frac{1}{\sigma_\epsilon},$$

and trade flows satisfy a standard gravity relationship

$$\frac{M_{ij}}{M_{ii}} = \left(\frac{w_j/A_j}{w_i/A_i}\right)^{\frac{-1}{\sigma_\epsilon}} d_{ij}^{\frac{-1}{\sigma_\epsilon}},$$

and both are independent of the household heterogeneity. And the welfare gains from trade for an individual household are

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \frac{1}{\theta(1-\beta)} \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}} \quad + \quad \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \Big\{ B(a_t, z_t) + C(a_t, z_t) \Big\}.$$

This mimics the results of Anderson, De Palma, and Thisse (1987). This was not obvious to me given the environment . . . risk, market incompleteness, borrowing constraints, etc.

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And we are back to Arkolakis et al. (2012) + what's going on with factor prices and borrowing constraints.

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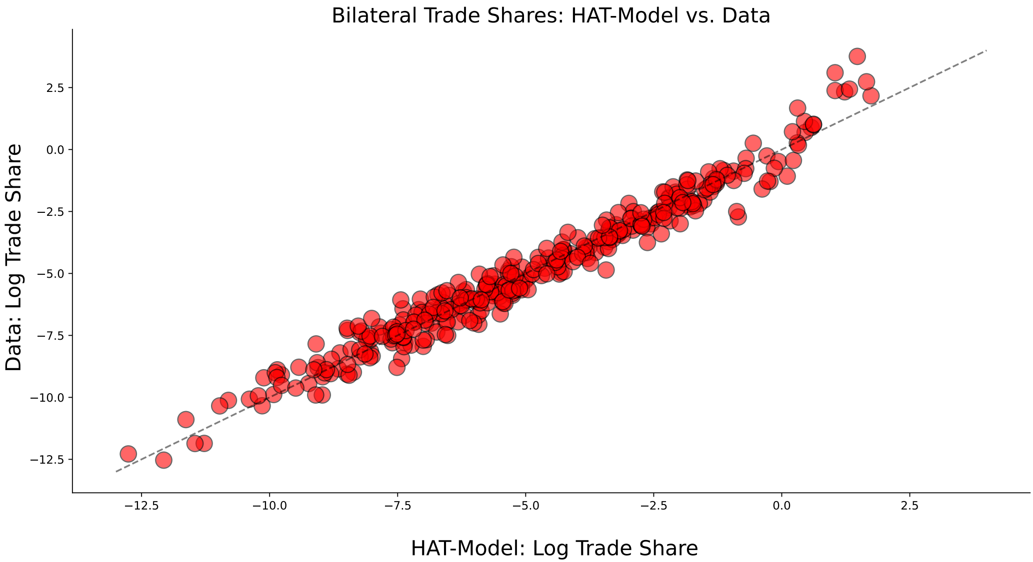
Quantitative Analysis

This is what I'll do...

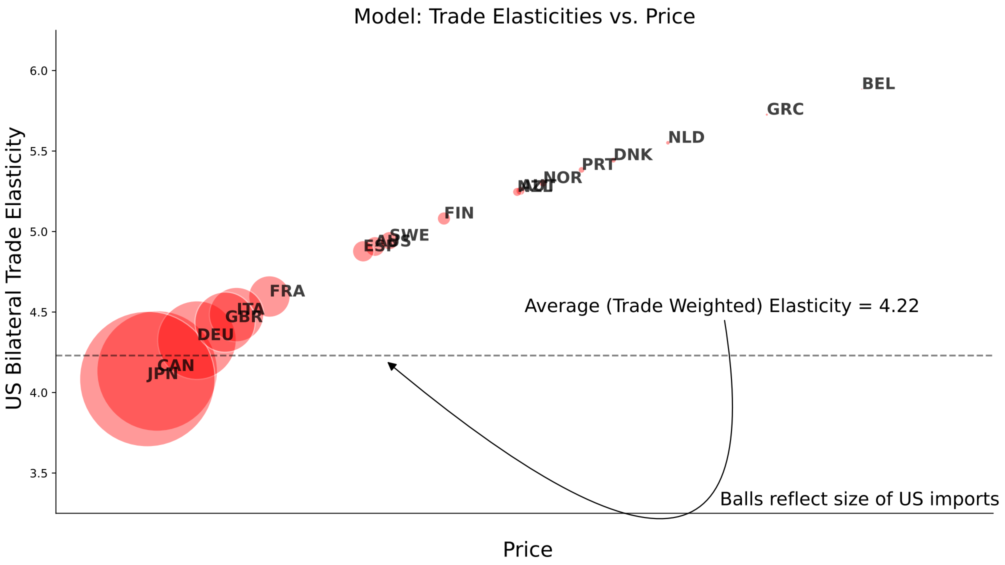
1. Calibrate my model using my "gravity as a guide" approach on the 19 country data set of Eaton and Kortum (2002) and targeting micro-evidence from Borusyak and Jaravel (2021) and Auer et al. (2022).

2. Gains from trade calculations.

Bilateral Trade: Model vs. Data



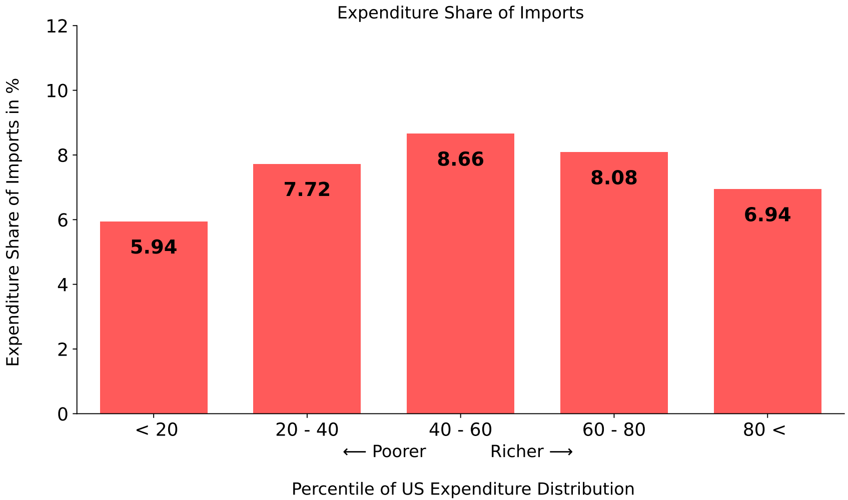
US Trade Elasticities: $-\theta_{us,j}$



Micro Moments — Model Consistent with HH-Level Elasticities

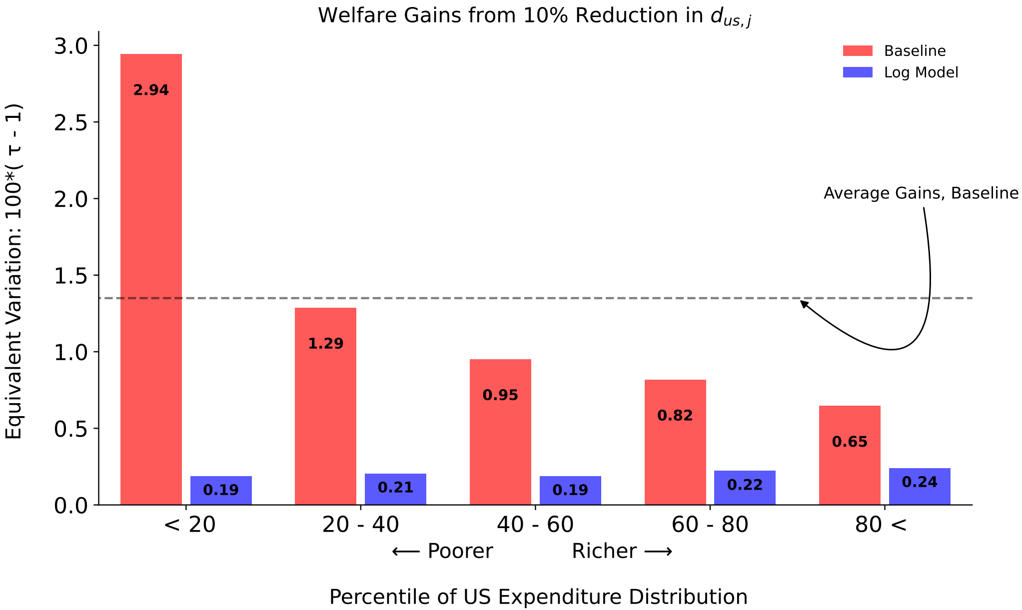
- Household-level elasticities consistent with those in Auer, Burstein, Lein, and Vogel (2022), i.e. rich less elastic than the poor.

Micro Moments — Model Consistent with HH-Level Expenditure Patterns



- Household-level import shares consistent with facts from Borusyak and Jaravel (2021), i.e. rich and poor do not spend unequally on imports.

U.S. Welfare: 10% Reduction in $d_{us,j}$



Final Thoughts...

This paper has prompted even more questions...

- The efficient pattern of trade? In a companion paper, I show that near-shoring is an outcome that a global planner likes.
- Can trade policy improve outcomes? Put in tariffs and redistribute.
- The interaction between trade goods and trade in assets?

One more thing: My github repository provides the code and supplementary work behind this paper at `https://github.com/mwaugh0328/heterogeneous-agent-trade`.

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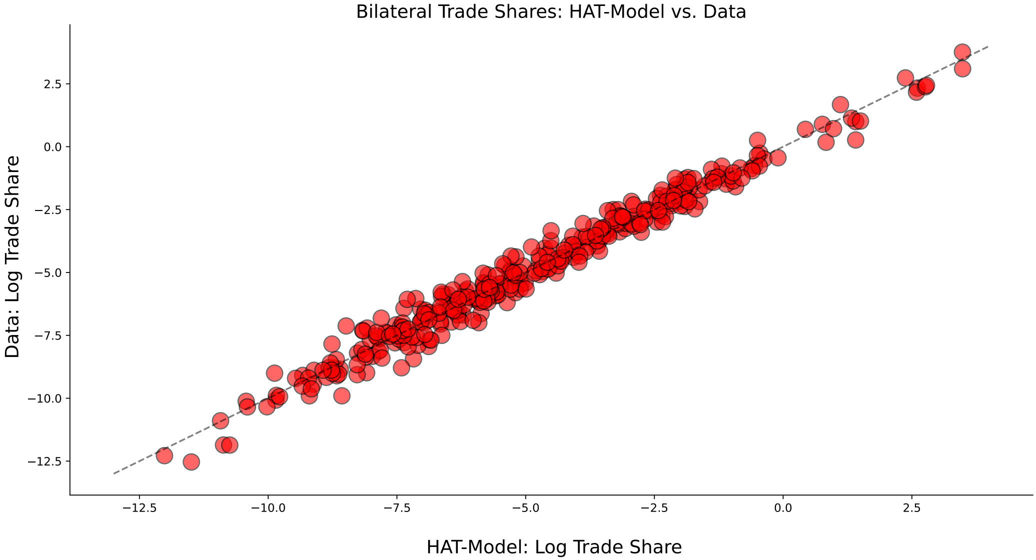
Log Model — Fit of Trade Data



Model: Production and Trade

M countries. Each country produces a nationally differentiated product as in Armington.

In country i, competitive firms' produce variety i with:

$$Q_i = A_i N_i,$$

where A_i is TFP; N_i are efficiency units of labor supplied by households.

Cross-country trade faces obstacles:

- iceberg trade costs $d_{ij} > 1$ for one unit from supplier j to go to buyer i.

This structure leads to the following prices that households face

$$p_{ij} = \frac{d_{ij} w_j}{A_j}.$$

Model: Households I

Continuum of households $k \in [0, \ L_i]$ in each country i. Household preferences:

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t \ \tilde{u}_{ijt}^k,$$

where conditional direct utility for good j is

$$\tilde{u}_{ijt}^k = u(c_{ijt}^k) + \epsilon_{jt}^k, \quad j = 1, \ldots, M.$$

Assumptions:

- discrete-continuous choice. . . so first chose one variety, then continuous choice over quantity.
- ϵ_{jt}^ks are iid across hh and time; distributed Type 1 Extreme Value with dispersion parameter σ_ϵ.
- For now, u is well behaved.

Model: Households II

Household k's efficiency units z_t evolve according to a Markov Chain. They face the wage per efficiency unit w_{it}.

Households borrow or accumulate a non-state contingent asset, a, with gross return R_i. Household's face the debt limit

$$a_{t+1}^k \geq -\phi_i.$$

Conditional on a variety choice, a household's budget constraint is

$$p_{ij} c_{ijt}^k + a_{t+1}^k \leq R_i a_t^k + w_{it} z_t^k.$$

What Households Do I

Focus on a stationary setting. A hh's state are its asset holdings a and shock z.

1. Condition on variety choice their problem is:

$$v_i(a,z,j) = \max_{a',\ c_{ij}} \Big\{ u(c_{ij}) + \beta\,\mathbb{E}[v_i(a',z')] \Big\},$$

$$\text{subject to} \quad p_{ij}c_{ij} + a' \leq R_i a + w_i z \quad \text{and} \quad a' \geq -\phi_i.$$

2. The ex-post value function of a household in country i is

$$\max_j \{\ v_i(a,z,j) + \epsilon_j\ \}.$$

What Households Do II

Three equations characterizing the commodity choice, value functions, consumption / savings. . .

1. The choice probability is:

$$\pi_{ij}(a, z) = \exp\left(\frac{v_i(a, z, j)}{\sigma_\epsilon}\right) \Big/ \Phi_i(a, z),$$

$$\text{where} \quad \Phi_i(a, z) := \sum_{j'} \exp\left(\frac{v_i(a, z, j')}{\sigma_\epsilon}\right).$$

2. The ex-ante value function of a household in country i is

$$v_i(a, z) = \sigma_\epsilon \log\left\{\Phi_i(a, z)\right\}.$$

3. Away from the constraint, consumption and asset choices must respect this Euler equation:

$$\frac{u'(c_i(a, z, j))}{p_{ij}} = \beta R_i \mathrm{E}_{z'}\left[\sum_{j'} \pi_{ij'}(a', z') \frac{u'(c_i(a', z', j'))}{p_{ij'}}\right].$$

Definition 1 (The Decentralized Stationary Equilibrium)

A Decentralized Stationary Equilibrium are asset policy functions and commodity choice probabilities $\{ g_i(a,z,j), \pi_{ij}(a,z) \}_i$, probability distributions $\{ \lambda_i(a,z) \}_i$ and positive real numbers $\left\{w_i, p_{ij}, R_i\right\}_{i,j}$ such that

i Prices (w_i, p_{ij}) satisfy firms problem;

ii The policy functions and choice probabilities solve the household's problem;

iii The probability distribution $\lambda_i(a,z)$ induced by the policy functions, choice probabilities, and primitives satisfies the law of motion and is stationary;

iv Goods market clears:

$$p_i Y_i - \sum_j X_{ji} = 0, \quad \forall i$$

v Bond market clears with either

$$\mathrm{A}_\mathrm{i}' = 0, \quad \forall i \quad \text{or} \quad \sum_i \mathrm{A}_\mathrm{i}' = 0$$

The HA Trade Elasticity

How do i's imports from j change relative to domestic consumption due to a permanent change in d_{ij}?

Proposition 4 (The HA Trade Elasticity)

The trade elasticity between country i and country j is:

$$\theta_{ij} = 1 + \int_{z,a} \Big\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \Big\} \omega_{ij}(a,z) da\ dz - \int_{z,a} \Big\{ \theta_{ii,j}(a,z)^I + \theta_{ii,j}(a,z)^E \Big\} \omega_{ii}(a,z) da\ dz$$

which is an expenditure-weighted average of micro-level elasticities. The micro-level elasticities are decomposed into an intensive margin and extensive margin

$$\theta_{ij}(a,z)^I = \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}, \qquad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and $\omega_{ij}(a,z)$ are the expenditure weights.

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and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^I = \left[- \frac{\partial a_i'(a,z,j)/p_{ij}c_i(a,z,j)}{\partial p_{ij}/p_{ij}} - 1 \right] \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}.$$

The idea: a lower d_{ij} relaxes the bc and then the division of new resources between assets and expenditure determines the intensive margin. **This is larger for the poor, smaller for the rich.**

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and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^E = \frac{1}{\sigma_\epsilon} \frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}}.$$

Now assume the number of countries is large. . .

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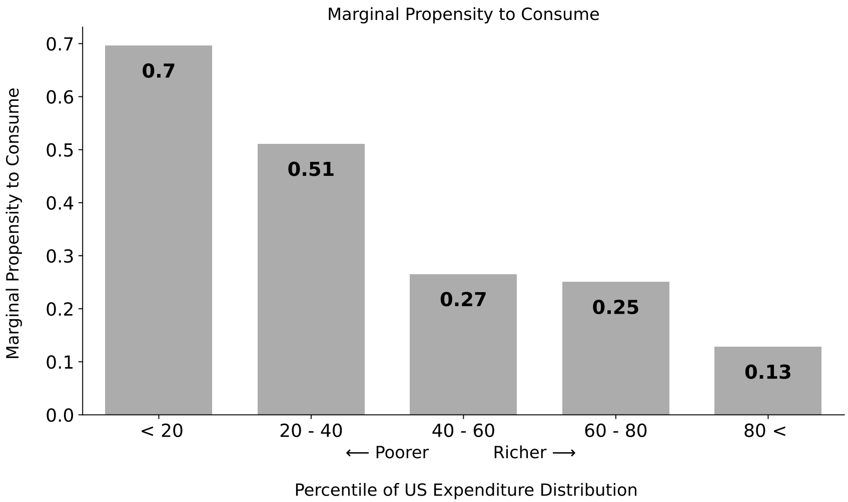
and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^{E} \approx -\frac{1}{\sigma_{\epsilon}} \left[u'(c_i(a,z,j)) c_i(a,z,j) \right].$$

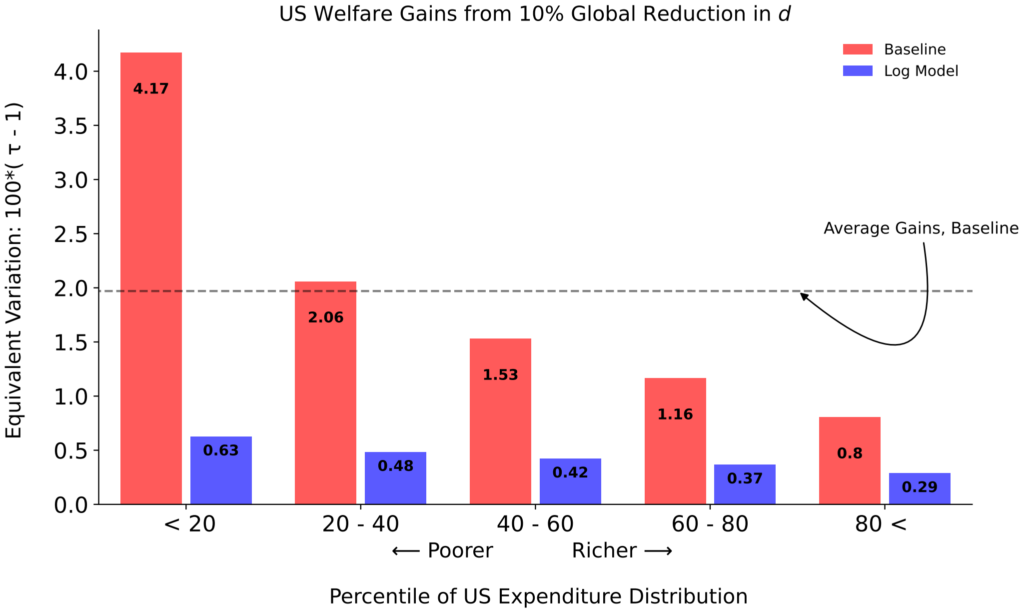
With CRRA and relative risk aversion > 1 then **poor hh's are the most price sensitive on the extensive margin.**

Micro Moments — Model Consistent with HH-Level MPCs



- Household MPCs consistent with Kaplan and Violante (2022).

U.S. Welfare: Global 10% Reduction in d



Measuring Welfare

Want is a measure of welfare in interpretable units. I'm going to focus on equivalent variation.

Reminder: Given some price change delivering utility level v', equivalent variation asks "at the old prices, p, how much extra income must be provided to be indifferent between v' and $v(p)$?"

My measure is a permanent, proportional increase in wealth $\tau_{i,a,z}$, at the old prices such that the new level of utility v_i' is achieved:

$$v_i'(a, z; \mathbf{p}') = v_i(a, z; \mathbf{p}, \tau_{i,a,z}).$$

Also, I'm doing this across steady states, not transitions.
Sorry :(

Preferences, Shocks, and Constraints — Calibrated Parameters

Description	Value	Target
Discount Factor, β	0.92	Global Interest Rate of 1%
CRRA parameter, γ	1.45	Micro elasticities of Auer et al. (2022)
Type One E-V parameter, $1/\sigma_\epsilon$	3.0	
Slope of Quality Shifter, $\psi_{ii}(z)$	0.72	Micro moments of Borusyak and Jaravel (2021)
Borrowing Constraint ϕ_i	—	50% of i's autarky labor income
Income Process on z	—	Krueger, Mitman, and Perri (2016)

- Everything is done under financial globalization case.

County Specific Parameters — Using Gravity as a Guide

The problem: no closed form map from trade flows to parameters as in standard trade models. But I want the model to replicate the geographic pattern of activity seen in the data.

- Step 0. Impose a trade cost function to reduce the parameter space

$$\log d_{ij} = d_k + b + l + e_h + m_i.$$

- Step 1. Run this gravity regression on the data

$$\log\left(\frac{M_{ij}}{M_{ii}}\right) = Im_i + Ex_j + d_k + b + l + e_h + \delta_{ij}.$$

- Step 2. Guess TFP terms and coefficients on the trade cost function, compute an equilibrium, run the same regression from above on model generated data.
- Step 3. Evaluate difference between model and data and update parameters until convergence.

County Specific Parameters — Using Gravity as a Guide

The solution: use the gravity regression "as a guide" where I estimate parameters of the model so that the regression coefficients run on my model's data match that seen in the data.

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Aggregation

Aggregates arise from explicit aggregation of hh-level actions. Two examples:

1. Aggregate, bilateral imports are

$$M_{ij} = L_i \int_z \int_a p_{ij} c_i(a, z, j) \pi_{ij}(a, z) \lambda_i(a, z)$$

where λ_i is the *endogenous* distribution of hhs across states. Here trade flows take on a mixed-logit form similar to Berry, Levinsohn, and Pakes (1995), but everything is tied down in equilibrium.

2. The national income accounting identity (GDP = C + I + G + X - M) . . .

$$p_i Y_i = \underbrace{L_i \sum_j \int_z \int_a p_{ij} c_i(a, z, j) \pi_{ij}(a, z) \lambda_i(a, z)}_{\widetilde{P_i C_i}} + \underbrace{\left[\sum_{j \neq i} X_{ji} - \sum_{j \neq i} M_{ij} \right]}_{-R_i A_i + A_i'} .$$

HA Gains from Trade under Efficiency

Proposition 5 (Trade Elasticities and Welfare Gains in the Efficient Allocation)

The elasticity of trade to a change in trade costs between ij in the efficient allocation is:

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon}\Big[u'(c_i(j))c_i(j)\Big].$$

And the welfare gains from a reduction in trade costs between i, j are

$$= \sigma_\epsilon \times \theta_{ij} \times \pi_{ij} \times \frac{L_i}{1-\beta},$$

which is the discounted, direct effect from relaxing the aggregate resource constraint. And this can be expressed as

$$= -\sigma_\epsilon \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}} \times \frac{L_i}{1-\beta}.$$

Same idea as in decentralized allocation, but now everyone substitutes in a common way...

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Mimics the results of Atkeson and Burstein (2010) but with household (not firm) heterogeneity.

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And — again— we are back to an Arkolakis et al. (2012)-like expression and with log its exact.

Household Parameters

Parameters common across countries:

- CRRA for u with relative risk aversion γ — varied to fit elasticities in Auer et al. (2022).
- Earnings process as in Krueger, Mitman, and Perri (2016).
- Discount factor β jiggled to target a world interest rate of 1.0% in financial globalization case.

Parameters scaled across countries to deliver balanced-growth-like properties.

- Set $\sigma_{\epsilon,i} = \sigma_{\epsilon} \times A_i^{1-\gamma}$, — σ_{ϵ} varied to fit elasticities in Auer et al. (2022).
- Set the borrowing constraint so $\phi_i = \phi \times A_i$ where $\phi = 0.50$.

Household-specific quality shifters — a home bias term $\psi_{ii}(z)$ which additively shifts utility

- Without this prices and price elasticities determine shares, so to fit the data interactions between quality and household characteristics is a way; same idea as in Berry et al. (1995).
- Slope of $\psi_{ii}(z)$ wrt z varied to fit Borusyak and Jaravel (2021) facts.

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Table 2: Estimation Results

		HAT-Model	
Barrier	Moment	Model Fit	Parameter
$[0, 375)$	-3.10	-3.10	1.92
$[375, 750)$	-3.67	-3.67	2.39
$[750, 1500)$	-4.03	-4.03	2.64
$[1500, 3000)$	-4.22	-4.22	2.74
$[3000, 6000)$	-6.06	-6.06	4.10
$[6000, \text{maximum}]$	-6.56	-6.56	4.83
Shared border	0.30	0.30	0.92
Language	0.51	0.51	0.85
EFTA	0.04	0.04	0.96
European Community	0.54	0.54	0.91

Note: The first column reports data moments the HAT-model targets. The second reports the model moments. The third column reports the estimated parameter values.