Heterogeneous Agent Trade

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ABSTRACT

This paper develops a model of heterogenous agents and international trade. Heterogenous agents are modeled as in the standard incomplete markets tradition with households facing incomplete insurance against idiosyncratic productivity and taste shocks. Trade in goods follows the Armington tradition but is derived from the "bottom up" with micro-level heterogeneity shaping aggregate trade. I show how micro-level trade elasticities, trade, and the gains from trade vary with a household's wealth and how self-insurance motives shape these outcomes. In aggregate, the pattern of trade is distorted relative to the efficient allocation and the gains from trade deviate from standard benchmarks. Quantitatively, I compute the multi-country, asymmetric global economy and calibrate it to match bilateral trade flows for 19 countries. I use the model to measure the gains from trade and the first-best pattern of trade.

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This paper develops a model of heterogenous households and international trade. From the perspective of trade, household heterogeneity is interesting because of the notion that some benefit from trade and others don't. One aspect of these unequal gains relates to the idea that rich and poor consumers have different sensitivities to price and, thus, they shape the expenditure patterns and the gains from trade at the micro and macro level. I develop this idea and study the role that household heterogeneity plays in shaping aggregate trade, the gains from trade, and the model's normative implications.

The model that I develop builds upon workhorse frameworks. Household heterogeneity is induced via the standard incomplete markets model (Bewley (1979), Imrohoroğlu (1989), Huggett (1993), Aiyagari (1994)) with households facing incomplete insurance against idiosyncratic productivity and taste shocks. Trade in goods follows the Armington tradition with producers in each country producing a national variety. The important twist is that households have random utility over these varieties and they make a discrete choice over the varieties to consume (McFadden (1974)). The explicit aggregation of household-level decisions then determines aggregate trade flows, trade elasticities, and the gains from trade.

The key force through which heterogeneity matters are household-level trade (price) elasticities that vary with income and wealth. Income and wealth matter because a household's price sensitivity, in essence, is about the marginal utility of consumption. Poor, high marginal utility households strongly value extra consumption independent of the variety and, thus, they are very sensitive to price and then concentrate their expenditure on the cheapest commodity available. In contrast, rich households' marginal utility is low, are less sensitive to changes in prices, and likely consume their ideal variety independent of price. The distribution of households — how many rich and poor people are in a country — then determines the aggregate pattern of trade and response of economy to changes in trade frictions.

My model and the issues behind heterogeneity in price sensitivity leads to new perspectives on the welfare gains from trade. First, I show how one aspect of the gains from trade reflect the expected, discounted stream of changes in a household's home choice probability, similar in spirit to the result of Arkolakis, Costinot, and Rodríguez-Clare (2012). Unpacking this component reveals that the change in the home choice probability is essentially about two forces: (i) how exposed a household is to a market and (ii) its own price elasticity. Because the elasticity part reflects the marginal utility of consumption, the elasticity effect delivers the intuitive idea that one aspect of the gains from trade is a households' individual valuation of the price reduction. So if a rich and poor household are equally exposed, the poor (high marginal utility) household will gain more as the reduction in price is more valuable for them.

A second implication that comes out of my model is that part of the gains from trade reflect how a liberalization changes a household's valuation of its net asset position. The issue is that a trade liberalization will change interest rates and lead to winners and losers depending upon a households' net asset position. For example, if a trade liberalization leads to an increase in interest rates, net debtors suffer since their terms to borrow deteriorated, while net savers benefit. Why might a trade liberalization change interest rates? It's because of heterogenous effects on expenditure patterns and savings. If the liberalization disproportionately benefits the rich /savers in the economy, they shed assets to consume a bit more and interest rates must increase to clear financial markets impacting the poor / debtors in the economy. The novelty here is how heterogeneity on the trade side links up with the financial market—not separate as they are typically treated.

Before moving on to the quantitative work, I explore two special cases. The first case is the efficient allocation where a planner can reallocate resources and overcome market incompleteness. In this case, I recover "first-best intuition" with the gains from trade only reflecting the direct savings associated with a reduction in trade costs. In this allocation, changes in expenditure patterns are not relevant via an envelope theorem argument—the planner already sources goods from the correct places so there are no gains from expenditure switching. And heterogeneity in a household's valuations of these direct gains are irrelevant because marginal utility is properly equated. While my economy is about heterogeneity on the household side, this result is reminiscent of Atkeson and Burstein (2010) and the irrelevance of firm heterogeneity in an economy where the allocation is efficient. Thus, the core issues at play in my model are not household heterogeneity per se, but inefficiencies induced by market incompleteness.

The second special case that I consider is when the utility function over the physical commodity is $\log$. With $\log$ utility, I obtain a separation result where aggregate trade outcomes "separate" from household heterogeneity. Trade takes a constant elasticity form with the trade elasticity pinned down by the dispersion parameter on the taste shocks similar to Eaton and Kortum (2002). And the trade elasticity and the share of home purchases summarize the gains from trade like in Arkolakis et al. (2012). This case is also interesting because Anderson, De Palma, and Thisse (1987) showed that in a static model with log utility and additive logit shocks, the economy behaves *as if* there were a representative agent CES consumer. In my economy, my suspicion was that market incompleteness and intertemporal behavior would nullify Anderson et al.'s (1987) result—it does not.

Quantitatively, I make a contribution by computing and calibrating the model at a scale typically reserved for static trade models. As a testing ground, I focus on the data set of Eaton and Kortum (2002). The 19 countries in this data set is about the right size to easily illustrate how a very rich model like this can work in a multi-country setting. Moreover, the Eaton and Kortum (2002) data set provides a well defined benchmark disciplined by bilateral trade flows and gravity variables—so it's a nice laboratory to explore new issues in.

The calibration challenge is the following. The model does not admit a "gravity" representation that allows researchers to invert trade frictions and productivity levels from trade flows as done in Eaton and Kortum (2002) and many subsequent papers. Similarly, the model does not admit the use of "exact-hat algebra" which allows the research to construct counterfactuals without the knowledge of primitives like trade frictions or productivity (see, e.g., Costinot and Rodríguez-Clare (2014) or the dynamic extension in Caliendo, Dvorkin, and Parro (2019)).

My solution is to use the insight that the regressions employed in gravity frameworks provide very accurate descriptions of the data generating process. Rather than treating the gravity regression as a structural relationship, I use it as a "guide" and use an indirect inference procedure where I estimate parameters of the model so that the regression coefficients from a standard gravity regression run on my model's data match that seen in the data. This procedure works well and, thus, the model is able to match spatial distribution of economic activity in the data—just as well as standard, constant elasticity gravity models.

An outcome of the calibration is that aggregate trade elasticities decrease with the competitiveness of a source country in a destination. In other words, expensive products are more elastic relative to cheaper products — in aggregate. Behind this is what some might call a super-elasticity of trade within the model *endogenously* delivering the result that elasticities increases with price and a pattern conforming with "Marshall's Second Law of Demand" or in the language of Mrázová and Neary (2017) aggregate demand functions turn out to be "subconvex" relative to CES demand.

The final part of the quantitative analysis explores how trade behaves in the efficient allocation relative to that seen in the baseline model (and data). I should emphasize that this is a novel exercise than typical executed in the trade literature. Unlike the industry standard, I'm *not* changing spatial, technological relationships and looking at how trade changes. What I'm doing is completing markets with the planner facing the same technological restrictions as private market participants and then asking what happens.

Two interesting things happen. Generally, the planner reduces trade in relationships that were already small. These are trading relationships that have the feature of being a luxury in the sense that it's mostly rich people engage in them. So when the planner redistributes within the country, trade "catering to the rich" is reduced. In contrast, the planner increases trade within already competitive relationships. An example of this is US imports with Japan, where the planner increases trade by nearly 5 percentage points. In this case, the planner uses trade of competitive partners to "service the masses" as it redistributes. In general, this model delivers the interesting answer that a better pattern of trade would be with more of some types of trade (nearby, competitive partners) and less of others (faraway, uncompetitive partners).

Motivating my work has been a sequence of papers focusing on measuring the heterogenous impacts of trade on the consumer side. Fajgelbaum and Khandelwal (2016), Carroll and Hur (2020), Borusyak and Jaravel (2021), Jaccard (2023) are recent examples that measure heterogeneity in import exposure.[1] Auer, Burstein, Lein, and Vogel (2022) and Colicev, Hoste, and Konings (2022) go a step further measuring heterogeneity in price sensitivity across the income distribution and this type of evidence is very much the launching point for my paper.

While this work motivates my paper, I take a conceptually different approach. Rather than focusing on measurement, I develop a model of household heterogeneity that endogenously delivers heterogeneity in price elasticities and study its implications. In this sense, my papers approach is most similar to Fajgelbaum et al. (2011) who study how inequality and non-homotheticicities shape trade in vertically differentiated products. A unique aspect of my work is that I start with a theory behind the distribution of income and wealth. This theory plus a demand system with heterogenous price elasticities then breaks aggregation in the goods market. And because aggregation is broken, it opens the door to new insights about trade, the interaction of trade and financial markets, and how market incompleteness shapes the aggregate pattern of trade.

My paper also relates to a recent series of papers that combines trade models with heterogenous agent, incomplete market models. Some of this is my own work in Lyon and Waugh (2018), Lyon and Waugh (2019), Waugh (2019); Ferriere, Navarro, and Reyes-Heroles (2022) and Carroll and Hur (2020) are important contributions as well. This class of papers primarily focuses on how heterogenous exposure through the labor market passes through to consumption and, thus, welfare. In this paper, I'm doing something different and it is an attempt to answer the question: does heterogeneity and market incompleteness matter for trade?

This paper also relates to a body of work focusing on the pricing implications in the presence of heterogenous price sensitivity. Nakamura and Zerom (2010) is an early example of a macro-style model with an IO-style demand system similar in spirit to my paper and they focuses on the implications for the incomplete pass through of shocks to prices. My own work in Mongey and Waugh (2022) is very much a companion piece to this paper with imperfect competition in product markets and it focuses on the heterogenous pass-through of supply and demand shocks into prices for different types of consumers. Nord (2022) takes a search-theoretic approach, but the core issue is the same — how demand composition affects pricing decisions. With that said, this model simplifies matters by focusing on a world with perfect competition and, hence, I turn my focus on how household heterogeneity matters for trade.

[1] Its important for the reader to distinguish the pattern of heterogeneity across sectors vs. within a sector (Cravino and Levchenko (2017) makes this distinction very clear). The evidence suggests that the patterns work in different directions with poor households consuming more traded goods, but within traded goods the poor consume lower price varieties and less imported content. My model is of one sector and about expenditure and substitution within that one sector.

1. The Heterogeneous Agent Trade Model

This section describes the model and then defines the decentralized competitive equilibrium. Trade is in the Armigton tradition with each country producing a nationally differentiated variety. Households face the "income fluctuations problem" as in the standard incomplete markets tradition (see, e.g., Chapter 17 of Ljungqvist and Sargent (2012)).

The key twist is that I do not employ modeling techniques with aggregation at household level across national varieties. Instead, I lean into the household heterogeneity and have households make a discrete choice over the varieties they consume in addition to their savings decisions. Aggregate trade flows, trade elasticities, and the gains from trade are then defined by the explicit aggregation of household-level decisions to purchase different varieties, their elasticity of demand, and their gains from trade.

1.1. Production and Trade

There are M locations which I call a country. Each country produces a nationally differentiated product. In country i, competitive firms' production technology to produce variety i is:

$$Q_i = A_i N_i, \tag{1}$$

where A_i is total factor productivity and N_i are the efficiency units of labor supplied by households in country i.[2]

I focus on only one type of barrier to trade: there are iceberg trade costs $d_{ij} > 1$ for a good to go from supplier j to buyer i.

Profit maximization of the producers in location i results in the wage per efficiency unit reflecting the value of the marginal product of labor

$$w_i = p_i A_i. \tag{2}$$

Given iceberg trade costs, the unit cost for country i to purchase a good from location j is

$$p_{ij} = \frac{d_{ij} w_j}{A_j}. \tag{3}$$

This is the trade and production side of the model. While sparse, it's worth reminding you that with a representative agent and a constant elasticity Armigton aggregator much comes out of this model. There is a gravity equation relating bilateral trade flows to country characteristics

[2]Note that lack of physical capital in the model. Households here are saving in via pure exchange of non-state contingent IOUs as in Huggett (1993) rather than in physical capital as in Aiyagari (1994).

with a constant trade elasticity. And there are two sufficient statistics (the trade elasticity and home trade share) that globally characterize the welfare gains from trade. In the next section, I give up on the representative agent.

1.2. Households

There is a mass of L_i households in each country. Households are immobile across countries. They are infinite lived and have time-sparable preferences over consumption of varieties:

$$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{u}(\{c_{ijt}\}_M), \tag{4}$$

where the notation $\{c_{ijt}\}_M$ means that the household has preferences over all j varieties supplied by M countries in the world. Here I'm indexing things by ij to denote the variety j that is consumed in location i at date t.

Households' period utility function is of the random utility class and each period households can only consume one variety.[3] The utility associated with the choice of variety j is

$$\tilde{u}(c_{ijt}) = u(c_{ijt}) + \epsilon_{jt}. \tag{5}$$

where the ϵ_{jt} are iid random variables across time, households, and countries. For the analysis, I assume that these shocks are distributed Type 1 Extreme Value with CDF

$$F(\epsilon) = \exp(-\exp(-\sigma_\epsilon^{-1}\epsilon)) \tag{6}$$

where σ_ϵ is the dispersion parameter. A useful generalization of this setting to a multi-sector model is the "infinite shopping isle" approach of Mongey and Waugh (2022) where these shocks take on a Generalized Extreme Value representation and then households choose the sector and then the variety each period.

For now, all I assume is that the utility function over the physical good c_{ijt} is well behaved. In the analysis below I explore different specifications of the utility function u over the physical commodity. The canonical case for product markets (Anderson et al. (1987)) or the spatial literature is where u is $\log$ utility. Below, I highlight the rather curious properties of this case.

A household's efficiency units are stochastic and they evolve according to a Markov chain. So, z is a household's efficiently units and $\mathcal{P}(z, z')$ describes the probability of a household with state z efficiency units transiting to state z'. Again, I assume that $\mathcal{P}$ is well behaved in the necessary ways.

[3] A more formal statement of preferences is that they $\tilde{u}(\{c_{ijt}\}_M) = \sum_j \imath_j \tilde{u}(c_{ijt})$ where $\imath_j$ is an indicator function taking the value one if the consumer chooses variety j and zero otherwise.

Households can save and borrow in a non-state contingent asset a that is denominated in the units of the numeraire. One unit of the asset pays out with gross interest rate R_i next period. I discuss this more in depth below, but the determination of R_i is that which clears the bond market (local or global). A country specific, exogenous debt limit ϕ_i constrains borrowing so:

$$a_{t+1} \geq -\phi_i. \tag{7}$$

All these pieces come together in the household's budget constraint, conditional on choosing variety j to consume, and focusing on a stationary setting where prices are constant:

$$p_{ij}c_{ijt} + a_{t+1} \leq R_i a_t + w_i z_t. \tag{8}$$

The value of asset purchases and consumption expenditures must be less than or equal to asset payments and labor earnings.

1.3. The Household Problem

The state variables of a individual household are its asset holdings and efficiency units. As alluded to above, for now I focus on a stationary setting where aggregates are not changing and, thus, I abstract from carrying the notation associated with them around.[4]

The value function of a household in country i, after the variety shocks are realized, is

$$v_i(a, z) = \max_j \left\{ \ v_i(a, z, j) \ \right\} \tag{9}$$

which is the maximum across the value functions associated with the discrete choices of different national varieties. The value function conditional on a choice of variety is

$$v_i(a, z, j) = \max_{a'} \left\{ u(c_{ij}) + \epsilon_j + \beta \, \mathbb{E}[v_i(a', z')] \right\} \tag{10}$$

$$\text{subject to (7) and (8)}$$

where households choose asset holdings and the level of consumption is residually determined through the budget constraint. Associated with the solution to this problem is a policy function $g_i(a, z, j)$ which solves (10) and maps current states into asset holdings tomorrow a' contingent upon the variety choice j. Correspondingly, there is a consumption function $c_i(a, z, j)$ mapping

[4]If you *do* want to carry them around, notice that all that households in each country care about are prices (today and in the future). The distributions of households in other countries, per se, don't matter. Thus, the relevant aggregate states in country i are $\left[\ \{w_i\}_M, R_i \right]$ which is the collection wage per efficiency units and the interest rate.

states into consumption today, contingent upon the variety choice j.

The continuation value function on the right-hand side of (10) is the expectation over (9) with respect to (i) efficiency units next period, z' and (ii) the variety taste shocks. An implication of this is that households understand that their may be situations where they really desire, say, a high priced imported good and, hence, save accordingly.

The Type 1 extreme value distribution on the taste shocks give rise to the following choice probabilities for each differentiated good:

$$\pi_{ij}(a,z) = \exp\left(\frac{v_i(a,z,j)}{\sigma_\epsilon}\right) \Bigg/ \Phi_i(a,z), \tag{11}$$

$$\text{where} \quad \Phi_i(a,z) := \sum_{j'} \exp\left(\frac{v_i(a,z,j')}{\sigma_\epsilon}\right). \tag{12}$$

which is the probability that a household with assets a and efficiency units z chooses country variety j. The term in the denominator, $\Phi_i(a,z)$, has a "price-index" interpretation and is very similar in spirit to the same term in Eaton and Kortum (2002). And then the expectation of (9) with respect to the taste shocks takes the familiar log-sum form

$$v_i(a,z) = \sigma_\epsilon \log\left\{\Phi_i(a,z)\right\}. \tag{13}$$

Associated with this problem in (10) for non-borrowing-constrained households is an Euler Equation for each variety choice j:

$$\frac{u'(c_i(a,z,j))}{p_{ij}} = \beta R_i \mathrm{E}_{z'}\left[\sum_{j'} \pi_{ij'}(a',z') \frac{u'(c_i(a',z',j'))}{p_{ij'}}\right]. \tag{14}$$

This has a very natural interpretation: a household equates marginal utility of consumption today with expected discounted marginal utility of consumption tomorrow adjusted by the return on delaying consumption. The interesting feature here is that the expected value of the marginal utility of consumption reflects the uncertainty over one's preference over different varieties tomorrow via the choice probabilities. And note that households has some control over these probabilities as the asset choice today influence the choice probabilities tomorrow.

Before moving on to aggregation, I make one useful observation that assists the analysis. Stare at (182) and (13) long enough, one can arrive at a dynamic, sufficient statistic representation of

$v_i(a, z)$. Appendix C works through the individual steps, but (13) can be summarized as

$$v_i(a,z) = -\sigma_\epsilon \log \pi_{ii}(a,z) + u(c_i(a,z,i)) + \beta \mathbb{E}_{z'} v_i(a',z'). \tag{15}$$

Here the ex-ante value function (prior to the realization of the preference shocks) is expressed as a sum of the log home choice probability, utility over physical consumption of the home good, and recursively the expected value function tomorrow. What's going on here is that the home choice probability π_{ii} summarizes the expected value of the taste shocks, their benefits, and how households respond to them in the future.[5]

Equation (15) together with (14) also provides more insight about how households' savings motives interact with the variety choice. Focusing on a household consuming the home good (and note that the left-hand-side below could be for any variety choice), the Euler Equation in (14) becomes:

$$\frac{u'(c_i(a,z,i))}{p_{ii}} = \beta \mathrm{E}_{z'} \left\{ -\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + \frac{u'(c_i(a',z',i))R_i}{p_{ii}} \right\}, \tag{16}$$

which says that an unconstrained household should be indifferent between the marginal utility of consumption forgone to hold some more assets and two components: (i) the benefit from how a change in assets changes in their variety choice in the future and this is summarized by the change in the home choice probability and (ii) the direct benefit of the returns on the assets evaluated at the marginal utility of consumption.

1.4. Aggregation

Aggregation. At the core of aggregation is a probability distribution $\lambda_i(a, z)$ describing the measure of households across the individual states. This distribution evolves according to

$$\lambda_i(a',z') = \sum_j \int_z \int_{a:a'=g_i(a,z,j)} \pi_{ij}(a,z)\mathcal{P}(z,z')\lambda_i(a,z)\, da\, dz. \tag{17}$$

where the inner most term describes the mass of households choosing variety j, multiplied by the probability that z transits to z', multiplied by the existing measure of households with states a and z. This is integrated with respect to those actually choosing asset holdings a', over all z's, and then summed over the different variety choices.

Given this distribution, everything else follows. First focusing on trade, aggregate bilateral

[5] Home choice probabilities are not necessarily the same as home trade shares, but this is closely related to Equation (15), Footnote 42 of Eaton and Kortum (2002) and I'm heading towards situations where this result plus restrictions on u give rise to the result in Arkolakis et al. (2012).

imports are

$$M_{ij} = L_i \int_z \int_a p_{ij} c_i(a,z,j) \pi_{ij}(a,z) \lambda_i(a,z) \, da \, dz. \tag{18}$$

Here imports take on a mixed logit formulation that very much mimics that used in the industrial organization literature, e.g, Berry, Levinsohn, and Pakes (1995). There are, however, several interesting differences. First, there is an active intensive margin, not unit demand. Second, inside the choice probability $\pi_{ij}(a,z)$ is the non-linear value function from (9).[6] Because the choice probability reflects the value function, it embeds forward looking behavior of the household.

The third interesting feature is that the mixing distribution (the λ) over which demands are aggregated is endogenous. Through the law of motion in (17) household behavior determines the distribution of wealth. In other words, this model imposes cross-equation restrictions between aggregate demand and individual demands through the distribution. So it's not a free parameter and it will change with changes in primitives of the environment.

Similar to imports, aggregate bilateral exports from country i to country j are

$$X_{ji} = L_j \int_z \int_a p_{ji} c_j(a,z,i) \pi_{ji}(a,z) \lambda_i(a,z) da \, dz. \tag{19}$$

The value of aggregate consumption is

$$\widetilde{P_i C_i} = L_i \sum_j \int_z \int_a p_{ij} c_i(a,z,j) \pi_{ij}(a,z) \lambda_i(a,z) da \, dz \tag{20}$$

In (20), one can see both a bug and a feature of this model. Here there is an "index number problem" in the sense that there is not an ideal price index for which one can decompose aggregate values in to a price and quantity component. This is in contrast to, e.g., a model where households consume a CES bundle of goods.

Finally, the aggregate quantity of asset holdings integrates across the asset choices of individual households

$$\mathrm{A}'_i = L_i \sum_j \int_z \int_a g_i(a,z,j) \pi_{ij}(a,z) \lambda_i(a,z) da \, dz. \tag{21}$$

which integrates over the asset choices—given the policy function $g_i(a,z,j)$ and variety choices $\pi_{ij}(a,z)$. And then sum's across the different varieties available.

[6]A good contrast is Nevo (2000) where inside the choice probability is an indirect utility function of the form $\eta \times (y - p_{ij})$ where y and p are in logs and η is a parameter to be estimated and it stands in for the marginal utility of consumption. And related to my next comment, λ is just "read from the data" and treated as policy invariant.

National Accounting. From here, I reconstruct national income and product identities. Starting from the production side, aggregate efficiency units are

$$N_i = L_i \int_z \int_a z\lambda_i(a,z)da\, dz. \tag{22}$$

and from here we can connect the value of aggregate production must equal aggregate payments to labor so

$$p_i Y_i = p_i A_i N_i = L_i \int_z \int_a w_i\, z\, \lambda_i(a,z)da\, dz, \tag{23}$$

Then by summing over individual consumers' budget constraint and substituting in (23), the aggregated budget constraint is:

$$p_i Y_i = \widetilde{P_i C_i} + \Big[- R_i \mathrm{A_i} + \mathrm{A_i'} \Big], \tag{24}$$

where national income equals the value of aggregate consumption $\widetilde{P_i C_i}$ and the country's net factor payments and net asset position. To arrive at the standard national income accounting identity, simply work with the relationship between production, exports, and aggregate consumption in (20) and imports gives rise to

$$p_i Y_i = \widetilde{P_i C_i} + \Big[\sum_{j \neq i} X_{ji} - \sum_{j \neq i} M_{ij} \Big], \tag{25}$$

where national production or GDP equals consumption plus exports minus imports. A comparison of (24) and (25) then makes clear that the trade imbalance is connected with a countries net factor payments and net asset position.

Beyond accounting, this last observation shows how trade flows are interlinked with financial flows. Inspection of the individual elements in (18), (21), and the households' budget constraint reveal that household's asset positions are intertwined with trade flows through both the intensive (how much to consume and, hence, save) and the extensive margins (which variety to consume). Thus, a feature of this model is that the trade side is interlinked with the financial side of the economy in a non-trivial way.

1.5. The Decentralized Equilibrium

In this section, I discuss the market clearing conditions that an equilibrium must respect and then define the Decentralized Equilibrium of this economy.

The Goods Market. Goods market clearing equates the value of production of commodity i

with global demand for country i's commodity:

$$p_i Y_i = \sum_j X_{ji}, \tag{26}$$

where the left hand side is production and the right hand side is world demand for the commodity (via exports) from (19).

The Bond Market. The second market clearing condition is the bond market. There are two case worth thinking about here. One is of "financial autarky" in which there is a local bond market that facilitates within country asset trade, but not across countries. In this case, there is an interest rate R_i for each country and the associated market clearing condition is

$$\mathrm{A}_\mathrm{i}' = 0, \quad \forall i \tag{27}$$

which says that net asset demand within each country i must be zero. As is common in the trade literature, this condition implies that trade is balanced—just stare at (24) and (25). Yet, even with balanced trade, there is still within country trade of financial assets. Some households are savers, others are borrowers and the interest rate is that which the net asset position is zero.

The second case is of "financial globalization" where there is a global bond market that facilitates both within country asset trade, and across countries. In this case, there is a single interest rate R and the associated market clearing condition is

$$\sum_i \mathrm{A}_\mathrm{i}' = 0 \tag{28}$$

In this case trade need not be balanced for each country. Here a specific country might run, say, a trade deficit because at the given prices, the total amount of borrowing within a country is larger than the total amount of saving. However, across all countries total borrowing must equal total saving.

Below I formally define the Decentralized Stationary Equilibrium where private market participants taking prices as given solve their problems, the distribution of households is stationary, and prices are consistent with market clearing.

The Decentralized Stationary Equilibrium. A Decentralized Stationary Equilibrium are asset policy functions and commodity choice probabilities $\{\ g_i(a,z,j), \pi_{ij}(a,z)\ \}_i$, probability distributions $\{\ \lambda_i(a,z)\ \}_i$ and positive real numbers $\{w_i, p_{ij}, R_i\}_{i,j}$ such that

i Prices (w_i, p_{ij}) satisfy (2) and (3);

ii The policy functions and choice probabilities solve the household's optimization problem in (9) and (10);

iv The probability distribution $\lambda_i(a, z)$ induced by the policy functions, choice probabilities, and primitives satisfies (17) and is stationary;

v Goods market clears:

$$p_i Y_i - \sum_{j}^{M} X_{ji} = 0, \ \ \forall i \tag{29}$$

v Bond market clears with either

$$\mathrm{A}_{\mathrm{i}}' = 0, \ \ \forall i \quad \text{or} \quad \sum_i \mathrm{A}_{\mathrm{i}}' = 0 \tag{30}$$

1.6. Outline of the rest of paper

This model above has households making individual choices over national varieties, savings, all while facing productivity and taste shocks. Explicit aggregation of household behavior determines the pattern of trade and this is linked with trade in financial assets. The remaining sections of the paper work through the following questions:

1. **What does the efficient, centralized allocation look like?** The model features an inefficiency arising from market incompleteness. Households would like insurance against future shocks to income and taste shocks, but only have a partial ability to provide themselves this insurance. Endowing a central planner with the ability to overcome this friction and studying the resulting allocation helps illustrate the behavior of model in the decentralized allocation.

2. **What are the model's implications for trade elasticities and the gains from trade in decentralized allocation?** Here I characterize micro and macro level trade elasticities and connect them with households' inter-temporal motives and the heterogeneity in the marginal utility of consumption that is induced by market incompleteness. I also workout a knife edge case when utility is $\log$ over consumption and how it delivers complete separation between the heterogenous agent side of the economy and trade.

3. **What are the quantitative implications of this model?** I then compute and calibrate the model with a key featuring being that the model matches the spatial distribution of economic activity in the data—just as well as static, standard, constant elasticity gravity models. I then perform several counterfactuals to illustrate the mechanics of the model and what the pattern of trade should look like in the centralized allocation.

2. The Centralized Allocation

This section describes the Centralized (Efficient) Allocation. The starting point is a stance on the social welfare function. I focus on a additive social welfare:

$$W = \sum_{t=0}^{\infty} \sum_{i} \int_{z} \beta^t \psi_i v_i(z,t) L_i \lambda_i(z,t),$$

and here $v_i(z,t)$ is a households ex-ante utility (before preference shocks are realized) with state z, time t, in country i. The ψ_i terms are country specific Pareto weights. And $\lambda_i(z,t)$ is the distribution of households in country i across productivity states at date t. Now unpack the benefits from physical commodity and the preference shock in the following way:

$$W = \sum_{t=0}^{\infty} \sum_{i} \sum_{j} \int_{z} \beta^t \psi_i \left\{ u(c_i(z,j,t)) + \mathrm{E}[\, \epsilon \mid \pi_{ij}(z,t)] \right\} \pi_{ij}(z,t) L_i \lambda_i(z,t) \tag{31}$$

so the inner term is period utility given the associated consumption allocation and then the expected value of the preference shock conditional on the choice probability $\pi_{ij}(z,t)$. This inner term is then weighted by the number of households that receive that utility, i.e., the choice probability times the mass of households with shock z at date t. The sum across j adds up all households in country i. Then the sum across i reflects that this is global welfare.

The Planner chooses consumption allocations $c_i(z,j,t)$ and choice probabilities $\pi_{ij}(z,t)$ for all i,j pairs, z states, and dates t to maximize (31). This maximization problem is subject to two constraints. The first is the resource constraint:

$$Y_{it} \geq \sum_{j} \int_{z} d_{ji} c_j(z,i,t) \pi_{ji}(z,t) L_j \lambda_j(z,t), \tag{32}$$

which says that production of variety i must be greater than or equal to world consumption of variety i inclusive of trade costs d_{ji}. The second constraint is that the choice probabilities are probabilities and sum to one:

$$1 = \sum_{j} \pi_{ij}(z,t). \tag{33}$$

Given these constraints I define the **Centralized Planner's Problem** as the following:

$$\max_{c_i(z,j,t),\ \pi_{i,j}(z,t)} \sum_{t=0}^{\infty}\sum_{i}\sum_{j}\beta^t\psi_i\int_z \Big\{u(c_i(z,j,t))+\mathrm{E}[\ \epsilon \mid \pi_{ij}(z,t)]\Big\}\pi_{ij}(z,t)L_i\lambda_i(z,t) \tag{34}$$

$$\text{subject to } (32),\ (33) \text{ and an inital condition } \lambda_i(z,0). \tag{35}$$

In Appendix A, I derive the solution to this problem. Proposition 1 describes the allocation that maximizes social welfare and I will call this allocation the Efficient Allocation.

Proposition 1 (The Centralized (Efficient) Allocation) *The allocation that satisfies the Centralized Planning Problem in (34) is:*

1. *A consumption allocation satisfying:*

$$\psi_i u'(c_i(z,j,t)) = \chi_j(t)d_{ij} \tag{36}$$

where $\chi_j(t)$ is the shadow price of variety j.

2. *The choice probabilities are*

$$\pi_{ij}(t) = \exp\left(\frac{u(c_i(j,t)) - u'(c_i(j,t))c_i(j,t)}{\sigma_\epsilon}\right)\Big/\sum_{j'}\exp\left(\frac{u(c_i(j',t)) - u'(c_i(j',t))c_i(j',t)}{\sigma_\epsilon}\right) \tag{37}$$

and are independent of z because of (36).

Proposition 1 has some neat features. First, the consumption allocation takes on the natural feature that the (Pareto-weighted) marginal utility of consumption equals its shadow price adjusted by the trade friction. Here the shadow price is the multiplier $\chi_j(t)$ on the resource constraint in (32). Because the shadow price only reflects the scarcity of the commodity, the marginal utility of consumption (and consumption) then does not depend upon the household's state z. So if a household is productive or unproductive at date, t, the Planner equates the marginal utility of consumption across these states—within variety. This is the sense in which the planner is providing full insurance across households.

Another implication of (36) is that the marginal utility of consumption is *not* equated across all households within a country as one would typically expect in a one good model. (36) implies that the ratio of the marginal utility of consumption—adjusted by relative shadow prices—should be equal. This is like a within-country Backus and Smith (1993) condition.

The second part of Proposition 1 prescribes the mass of households that consume variety j. Per the distributional assumption on the taste shocks, the choice probabilities in (37) take the form of ratio's of $\exp$ relative to sum of $\exp$ functions with the dispersion parameter σ_ϵ showing up in the natural way.

The interpretation of the terms within the $\exp$ function take on a very intuitive form: it's the net social benefit of assigning a household to consume that variety.[7] Net is here because it's the difference between the utility received $u(c_i(j,t))$ minus the social cost $u'(c_i(j,t))c_i(j,t))$. Why is that the social cost? From (36), the marginal utility of consumption exactly reflects the scarcity of providing $c_i(j,t)$. Thus, the interpretation is that variety assignment is based upon the social benefit net of providing that benefit.

Given Proposition 1, I can compute the welfare gains from a change in trade costs. I do this by focusing on stationary allocations, so t's are not relevant. And then study how welfare changes across the two stationary allocations. Here it is of no consequence as there is no moving aggregate state variable in the allocation, so the jump across stationary equilibrium is instantaneous.

Appendix A works out the details, Proposition 2 describes the result.

Proposition 2 (Trade Elasticities and Welfare Gains in the Centralized Allocation) *The elasticity of trade to a change in trade costs between i, j in the centralized efficient is:*

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon}\Big[u'(c_i(j))c_i(j)\Big]. \tag{38}$$

And the welfare gains from a reduction in trade costs between i, j are

$$\frac{\mathrm{d}W}{\mathrm{d}d_{ij}/d_{ij}} = \frac{\partial W}{\partial d_{ij}/d_{ij}} = -\frac{\psi_i}{1-\beta} \times \ \sigma_\epsilon \times \theta_{ij} \times \pi_{ij} \times L_i \tag{39}$$

which is the discounted, direct effect from relaxing the resource constraint in (32).

Proposition 2 highlights a couple of things. First, consistent with intuition from Eaton and Kortum's (2002) Ricardian model, the dispersion parameter matters inversely. So if σ_ϵ is small, national varieties are "as if they are near substitutes" and thus trade flows will respond a lot.

The non-standard part is that the marginal utility of consumption times consumption shows up. One way to view this part of the elasticity is that it's the rate of conversion from quantities into utils. That is how many utils, on the margin, a particular quantity of level of consumption delivers. And this matters for the elasticity in a very intuitive way—country pairs that deliver a

[7] A precursor to this result are the migration probabilities derived in Lagakos, Mobarak, and Waugh (2023) in the planing problem for their economy.

lot of utility, on the margin, are the pairs where the planner will be most responsive to changes in trade costs.

The second part of Proposition 2 says that the total change in welfare only reflects the direct effect from relaxing the resource constraint, then discounted for the infinite future, hence the $1/(1-\beta)$ term. Behind this result is an envelope-type argument with direct effects only mattering because I'm evaluating the change in welfare at the optimized allocation and any benefits of adjusting consumption and choice probabilities are zero—on the margin.

What is the direct effect? It equals those eating that commodity $\pi_{ij} \times L_i$ which is the share of households consuming commodity j times the number of households in country i. They directly gain because less resources are destroyed by trade costs. This is then converted into utils using the elasticity which, as discussed above, is something about the dispersion in shocks and then the rate at which utils are being delivered at current quantities.

This result is reminiscent of Atkeson and Burstein (2010) who make a similar claim in the context of a model with rich firm heterogeneity. More specifically, they argue that the only first order effect lower trade costs have on welfare is the direct consumption effect and that indirect effects are second order. This is similar, but with household heterogeneity, by saying that, in the efficient allocation, the welfare gains are these direct consumption benefits.

Before moving on, let me connect the results in Proposition 2 with Arkolakis et al. (2012). As I show in the appendix, in the efficient allocation we have that

$$\frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}} = \pi_{ij} \tag{40}$$

As mentioned above, the non-standard thing is that the trade elasticity is i, j specific and reflects the marginal utility of consumption times consumption. When preferences are $\log$

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon} \quad \forall \ i, \ j, \tag{41}$$

the trade elasticity is *not* i, j specific. Per the discussion above, what is going on here is that with log preferences, the rate of conversion from quantities into utils is constant and thus, the bilateral pair does not matter. The $\log$ case also connects with Anderson et al. (1987) and Anderson et al. (1992), in that trade elasticity is constant and behaves as if there were a representative agent Armington-CES consumer. I discuss this case in the decentralized allocation more below in Section 3.3.

Proposition 2 is useful because it sets a benchmark for thinking about the role that incomplete markets is playing. It also highlights an interesting observation, that outside of $\log$ preferences, the random utility model naturally introduces a form of non-homotheticity. And, even in the efficient allocation, this is shaping how aggregate trade is responding to changes in trade costs.

3. Trade Elasticities and the Gains from Trade

This section focuses on the decentralized equilibrium and works towards understanding outcomes like trade elasticities and gains from trade (as in Proposition 2). I show how micro-level decisions shape the aggregate trade elasticity and then the gains from trade and the sources of those gains.

3.1. Trade Elasticities

My definition of the trade elasticity is the partial equilibrium response of imports from j relative to domestic consumption due to a permanent change in trade costs.[8] By partial equilibrium, I mean that wages, interest rates, and the distribution of agents are fixed at their initial equilibrium values. This is consistent with the definition of the trade elasticity in say, Arkolakis et al. (2012) or Simonovska and Waugh (2014). By permanent, I mean that the change in trade costs is for the indefinite future and that households correctly understand this.

Given this discussion, my mathematical definition of the aggregate trade elasticity is

$$\theta_{ij} = \frac{\partial M_{ij}/M_{ij}}{\partial d_{ij}/d_{ij}} - \frac{\partial M_{ii}/M_{ii}}{\partial d_{ij}/d_{ij}}. \tag{42}$$

Then working from the definition of imports in (18), Proposition 3 connects the trade elasticity with micro-economic behavior:

Proposition 3 (The H-A Trade Elasticity) *The trade elasticity between country i and country j is:*

$$\theta_{ij} = 1 + \int_{z,a} \Big\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \Big\} \omega_{ij}(a,z) da\, dz - \int_{z,a} \Big\{ \theta_{ii,j}(a,z)^I + \theta_{ii,j}(a,z)^E \Big\} \omega_{ii}(a,z)\, da\, dz \tag{43}$$

which is an expenditure-weighted average of micro-level elasticities. The micro-level elasticities are decomposed into an intensive margin and extensive margin

$$\theta_{ij}(a,z)^I = \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}, \qquad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and the expenditure weights are

$$\omega_{ij}(a,z) = \frac{p_{ij} c_i(a,z,j) \pi_{ij}(a,z) \lambda_i(a,z) L_i}{M_{ij}}.$$

and the notation $\theta^I_{ii,j}, \quad \theta^E_{ii,j}$ represents how home choices on the intensive and extensive margin respond to the i, j change in trade costs.

[8] Because the aggregate distribution of households will adjust—even with prices fixed—the elasticities that I derive are in a sense "short-run" elasticities.

Proposition 3 says that the aggregate trade elasticity is an expenditure weighted average of micro-level trade elasticities. And these micro-level trade elasticities are decomposed into two components: an intensive margin trade elasticity $\theta_{ij}(a,z)^I$ reflecting the change in spending by a household on variety from j as trade costs change and an extensive margin trade elasticity $\theta(a,z)^E_{ij}$ reflecting how households substitute across varieties as trade costs change. And this is all relative to how these margins adjust home choices given the change in j, hence, the subscripts ii, j in the second part of equation (43).

Proposition 3 is derived only off the aggregation of imports at the micro level—no market clearing, functional forms, etc. It's essentially an identity that could be applied to any model. The next step is to insert my model of household behavior. From the household's budget constraint, I can say more about the intensive margin elasticity. Then with the Type 1 extreme value assumption and the household's problem, I can say more about the extensive margin elasticity.

The Intensive Margin. The intensive margin elasticity reflects, conditional on a choice, how do quantities change. By using the households budget constraint in (8) I can express the intensive margin elasticity as:

$$\underbrace{\frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}}_{\theta_{ij}(a,z)^I} = \left[-\frac{\partial g_i(a,z,j)/p_{ij}c_i(a,z,j)}{\partial p_{ij}/p_{ij}} - 1\right]\frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}, \tag{44}$$

where $g_i(a,z,j)$ is the policy function mapping states into asset holdings next period a'. The inside bracket of Equation (44) connects the intensive margin elasticity with the household's savings decision, i.e., how it adjusts its wealth relative to expenditure when prices change.[9] A way to think about (44) is that it answers the question: If a household faced with lower prices, how much would go to extra consumption, how much to savings? And this division of resources determines the intensive margin elasticity.

Heterogeneity matters here because this division is not invariant to a household's state a, z and it works through their incentives to save. For example, if a households is constrained, assets can't adjust, and the intensive margin becomes -1. In contrast, wealthy households save some stuff from a reduction in prices and the intensive margin for these households will be less than one in absolute value. The result is that poor households are more price sensitive than rich households — on the intensive margin — and the mechanism works through their savings motives.

The Extensive Margin. The idea here is to understand how households adjust their choice of

[9] Outside of the bracket in (44) is how prices change with trade costs which is also known as "pass-through." In the competitive environment here, it is always one, even though there is an is an active super-elasticity in the background. In the non-competitive environment of Mongey and Waugh (2022), the super-elasticity matters for price responses and pass-through deviates from one.

variety. I'm going to work in several steps by first deriving the elasticity of the choice probability and then assume that the number of countries is large to isolate the key mechanism.

The elasticity of the choice probability with respect to a change in trade costs is

$$\underbrace{\frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}}}_{\theta_{ij}(a,z)^E} = -\frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}} + \frac{1}{\sigma_\epsilon}\frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}}. \tag{45}$$

The first term reflects the fact that the change choice j depends upon a comparison relative to the overall change in the value of options across variety. This is very much similar to how CES models behave except that this term is state a, z specific. The second term is how the choice specific value function changes multiplied by the taste shock parameter.[10] In other words, how elastic or inelastic the extensive margin is depends on how much more valuable choice j becomes.

Now assume the number of countries is large. Then the effects on the Φ term are negligible and the effects working through the value function lead to

$$\theta_{ij}(a,z)^E \approx -\frac{1}{\sigma_\epsilon}\Big[u'(c_i(a,z,j))c_i(a,z,j)\Big]. \tag{46}$$

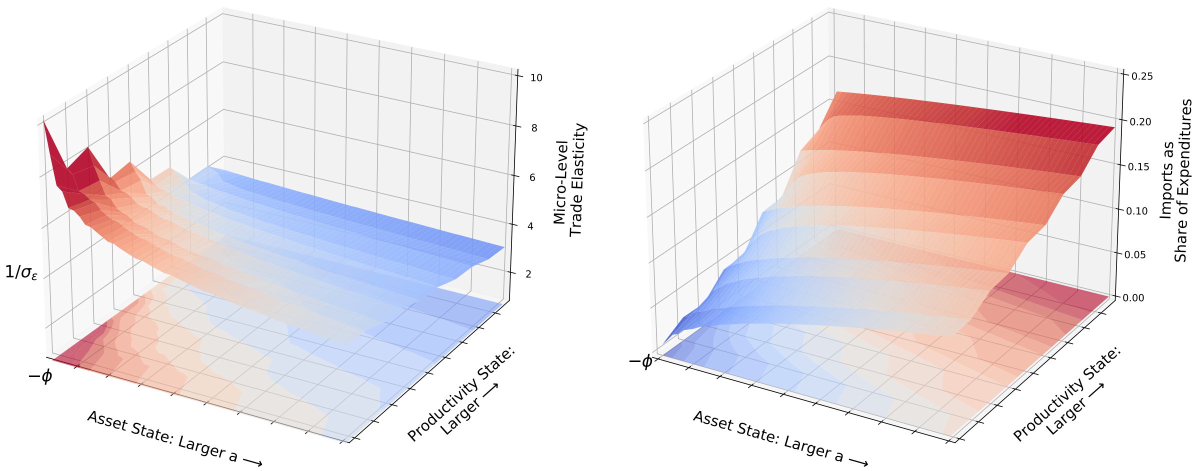
This says that how elastic a household is on the extensive margin relates to how many more incremental utils does a household get from the change in trade costs. What this means is that if a household receives a lot of utils, on the margin, from the change in trade costs then the extensive margin responds much more.

This term also mimics the trade elasticity expression in the efficient allocation, equation (38). In the efficient allocation, the planner's elasticity reflects the incremental social gain in utils from the change in trade costs. What is different in (46), is that households' private valuations associated with the change in trade costs differ and, thus, they substitute differently given their own specific circumstances. Here, heterogeneity in elasticities are a symptom of a conflict between social and private valuations of the change in trade costs.

How does this elasticity depend upon a household's circumstances? Differentiate (46) with respect to assets. The thought experiment here is if a household was a bit wealthier how much more elastic would the household be:

$$\frac{\partial(u'(c_i(a,z,j))c_i(a,z,j))}{\partial a} = u'(c_i(a,z,j)) \times \mathbf{MPC}_i(a,z,j) \times \Big[-\gamma_i(a,z,j)+1\Big], \tag{47}$$

[10]As I show in the Appendix, a third term reflecting the effect of borrowing constraints would also be here, but via envelope theorem type arguments, they zero out for small changes in trade costs.



(a) Household-Level Elasticities, $-\theta_{ij}(a, z)$

(b) Household-Level Import Shares

where $\mathbf{MPC}_i(a, z, j)$ is the household's marginal propensity to consume and $\gamma_i(a, z, j)$ is the Arrow-Pratt measure of relative risk aversion.

With constant relative risk aversion (CRRA) preferences then $\gamma_i(a, z, j)$ becomes a constant γ and log preferences is when $\gamma = 1$. Equation (47) implies that if $\gamma > 1$, then poor, high marginal utility households who are also likely high MPC households are *more elastic relative* to rich households on the extensive margin. This is the sense in which wealthy or poor households will have systematically different extensive margin elasticities.

I find (47) clarifying because it makes clear the specific role that preferences play vs. the more general environment, i.e., savings, shocks, market incompleteness. For things like savings motives to play a role, for heterogeneity to matter, a necessary condition seems to be a departure from log preferences.[11] Then conditional on a departure from log preferences, the incomplete markets model starts to play a role. And the incomplete markets model provides me an equilibrium theory of how marginal utility and marginal propensities to consume vary across households.

Figure 1a and 1b illustrates how this works in a two country economy. Figure 1a plots the absolute value of the trade elasticity (intensive and extensive margin) by household state (assets are on the x-axis, productivity state on the y-axis) and the borrowing constraint ϕ is in the south-west corner. This shows is how the trade elasticity systematically varies with assets and income: Poor households—especially those near the borrowing constraint—are very price elastic with a trade elasticity of around -10. Richer households are less price elastic with this elasticity declining towards -3.

[11] I directly tackle this case in Section 3.3. A careful reader may anticipate that while log implies no heterogeneity in the extensive margin, what about the intensive margin? Amazingly, it turns out that it is constant as well.

This pattern of trade elasticities has a strong intuitive feel and there is evidence in support of it.[12]. This result comes out of estimates in Berry et al. (1995) and in a more macro context, Nakamura and Zerom (2010). Sangani (2022) is a recent paper that provides evidence in support of this fact from the Kilts-Nielsen data set. The evidence in Auer, Burstein, Lein, and Vogel (2022) most closely relates to the patterns in Figure 1a with poorer households having higher price elasticities; Colicev, Hoste, and Konings (2022) finds similar results.

One more implication of this result: because rich and poor households face the same prices, differences in elasticities lead to different expenditure shares. Figure 1b illustrates this point by plotting expenditure on the foreign good relative to total expenditure. Because of trade costs and symmetry across countries, the home good is the relatively cheaper good. Thus, poor, high-price-elastic households spend more on the cheap home good versus the expensive foreign good. In fact, for those near the borrowing constraint in this example, it's near zero.

3.2. The Gains from Trade

In this section I compute how social welfare changes due to a change in trade costs. I derive these gains across steady states, the idea here is that I'm thinking a situation where the change is small and there is an immediate jump to the new steady state. The purpose here is to heuristically illustrate mechanics and where the gains from trade arise from. Unlike the trade elasticity, I take total derivatives that encompass general equilibrium changes in wages and interest rates.

The analysis proceeds in several steps before stating the main result in Proposition 4. First, I focus only on country i and study a change in trade costs d_{ij}. To simplify the algebra, I choose w_i to be the numeraire and normalize $A_i = 1$. This implies is that p_{ii} equals one and it's derivative with respect to a change in trade costs is zero as it's pinned down by my choice of normalizations.

I focus on country i with a utilitarian social welfare function:

$$W_i = \int_z \int_a v_i(a, z) L_i \lambda_i(a, z) \tag{48}$$

where $v_i(a, z)$ is a households ex-ante (before preference shocks are realized) value function in country i, with states a, z. The total change in total welfare is

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} \approx \int_z \int_a \left\{ \frac{\mathrm{d}v_i(a, z)}{\mathrm{d}d_{ij}/d_{ij}} + v_i(a, z) \frac{\mathrm{d}\lambda_i(a, z)/\lambda_i(a, z)}{\mathrm{d}d_{ij}/d_{ij}} \right\} L_i \lambda_i(a, z) \, da \, dz. \tag{49}$$

where the $\approx$ symbol reinforces that this is a heuristic, thinking about a small change and there

[12]This idea goes back to Harrod's (1936) "Law of Diminishing Elasticity of Demand" that says that price sensitivity declines with income which is not to be confused with Marshall's Second Law of Demand which I discuss later.

is an immediate jump to the new steady state.

At a high-level, (49) illustrates that the gains from trade come through two forces. The first component reflects changes in household-level welfare. So conditional on a distribution of households across states, this computes if households are better (or worse!) off from the change. Relative to the efficient allocation this force is always present, however, the planner equalizes things across households such that individual states do not matter and, thus, distributional issues do not matter.

The second component of (49) is about reallocation. It says: take the old v's and compute how the change in the distribution (that arise because of behavioral responses of households) effects social welfare. So does trade make it more or less likely that households are in "good" parts of the distribution. This force is unique to the decentralized allocation and symptomatic of an inefficiency in the initial allocation.

In total, the change in social welfare is then the weighted average of these two forces with the weights being those at the initial distribution.

A key issue is how household-level welfare changes. Here, I make the use of the observation in Equation (15) that the ex-ante value function can be represented in only in terms of home choice ii values and then recursively push forward. In other words, I can compute $\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}$ *as if* one only consumed the home good for the infinite future. This observation gives rise to the following:

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = -\,\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \;+\; u'(c_i(a,z,i))a\,\frac{\mathrm{d}R_i}{\mathrm{d}d_{ij}/d_{ij}} \tag{50}$$

$$+\,\beta\mathbb{E}\Bigg\{-\,\sigma_\epsilon\frac{\mathrm{d}\pi_{ii}(a',z')/\pi_{ii}(a',z')}{\mathrm{d}d_{ij}/d_{ij}} + u'(c_i(a',z',i))\frac{\mathrm{d}R_i}{\mathrm{d}d_{ij}/d_{ij}}a' \quad \ldots \text{and into the future.}$$

The first term: $-\sigma_\epsilon\frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}}$ is what I'll call *gains from substitution*. The second term is what I'll term as *gains to asset trade*.[13]

Finally, these repeat themselves into the expected future, appropriately discounted. Proposition 4 summarizes the result below.

Proposition 4 (H-A Gains from Trade) *The welfare gains from trade are given by*

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} \approx \int_z\int_a \left\{\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} + v_i(a,z)\frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}\right\}L_i\lambda_i(a,z).$$

which reflects the change in household level gains and how the distribution of households changes. House-

[13]As discussed previously and worked out in the Appendix, the general expression has terms reflecting the effect of borrowing constraints. But again, envelope arguments for small changes mean that these terms are zero.

hold level gains are given by

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \bigg\{ -\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a_t,z_t)/\pi_{ii}(a_t,z_t)}{\mathrm{d}d_{ij}/d_{ij}} + u'(c_i(a_t,z_t,i))a_t \times \frac{\mathrm{d}R_i}{\mathrm{d}d_{ij}/d_{ij}} \bigg\}$$

where each term represents:

- *Gains from substitution:* $-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a_t,z_t)/\pi_{ii}(a_t,z_t)}{\mathrm{d}d_{ij}/d_{ij}}$.
- *Gains to asset trade:* $u'(c_i(a_t,z_t,i))a_t \times \frac{\mathrm{d}R_i}{\mathrm{d}d_{ij}/d_{ij}}$

Before moving on, I unpack the components in this result and provide some intuition.

Gains from substitution. The change in the home share summarizes two forces: (i) how exposed a household is to the change through the choice probabilities and then (ii) elasticities. To see this, define $\bar{\theta}(a,z)^E_{ij',j}$ as the extensive margin, cross-price elasticity, in total derivative form (and it's derivation follows what is done in 45). As I show in the Appendix C, the change in the home share can be expressed as:

$$-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \sigma_\epsilon \sum_{j'} \pi_{ij'}(a,z) \times \Big[\bar{\theta}(a,z)^E_{ii,j} - \bar{\theta}(a,z)^E_{ij',j} \Big], \tag{51}$$

$$\approx -\sigma_\epsilon \times \pi_{ij}(a,z) \times \bar{\theta}(a,z)^E_{ij,j}. \tag{52}$$

In words, the top line says that the change in home choice probability is equivalent to a weighted average of relative cross-price elasticities with the weights being the choice probabilities.

The next line assumes that all cross-terms are small, then the gains from substitution depend upon the initial exposure of a household to market j and their own-price elasticity (the total derivative analog to (45). And because the own-price elasticity is intimately connected to marginal utility of consumption, the elasticity effect picks up the intuitive idea that one aspect of the gains from trade is a household's individual valuation of the price reduction, in addition to the household's exposure.

Equation (52) is interesting for several reasons. First it is analogous to the gains from trade formula in the efficient allocation (Proposition 2). The difference is that in that market incompleteness leads to heterogeneity in both exposure *and* elasticities giving rise to heterogenous gains from trade.

This expression also connects with two related papers. Borusyak and Jaravel (2021) (following an approach dating back to at least consider an environment where, to a first order, only exposure matters, similar to the exposure term in Equation (52). Auer et al. (2022) work out second order effects with non-homothetic CES preferences and additional effects from elasticities show up, similar to the elasticity term in (52). The interpretation here has a different flavor than articulated in Auer et al. (2022). Because elasticities are intimately connected with the marginal utility of consumption, this relationship is saying that the reason high elasticity households gain (conditional on exposure) more, is because the price reduction from trade is more valuable, on the margin.

Gains to asset trade. The idea here is that changes in trade costs affect the equilibrium interest rate in the country and then benefit (or hurt) households depending upon their net asset position. For example, if a trade liberalization leads to an increase in interest rates, net debtors suffer since their terms to borrow deteriorated, while net savers benefit.

Why might a trade liberalization change interest rates? It's because the liberalization has heterogenous effects on expenditure patterns and savings through the forces discussed in Section 3.1. So if the liberalization disproportionately benefits rich /savers in the economy, they shed assets to consume a bit more of the now cheaper goods. Then interest rates must increase to clear financial markets impacting the poor / debtors in the economy. The novelty here is how the goods trade and the financial market are interlinked—not separate as they are typically treated.

3.3. The Case of $\log$ preferences

There is one special case worth working through, it's with $\log$ preferences over the physical commodity. This (very common) preference structure leads to an interesting result where micro-level heterogeneity, market incompleteness completely sperate from the trade side of the economy. So in this one case, trade behaves "as if" there were a representative agent Armington-CES consumer and then the gains from trade take the form in Arkolakis et al. (2012)—even though individual households are facing shocks, borrowing constraints, and in general trying to deal with life's circumstances.

Consider the following preference structure:

$$\tilde{u}(c_{ij,t}) = \log(c_{ij,t}) + \epsilon_{j,t}.$$

There is essentially one insight and then everything follows. Examining the problem in (10) and substituting in the households budget constraint from (8), then leads to the observation that the optimal a' conditional on a choice j is **independent** of the price and the choice j. In Appendix D, I employ a formal "guess and verify" approach off the Euler equation and show it leads to the same conclusion and this is verified on the computer as well.

Everything follows from this observation.

Because assets don't adjust to changes in prices, from (44) the intensive margin elasticity for

ij is minus one. On the extensive margin, things are more involved as at the micro level they are varying by income and assets. But across different destinations, within states, they vary in the same exact way. Given that the trade elasticity is always defined relative to home choices, heterogenous effects wash out and the end result is that everything collapses to $-\frac{1}{\sigma_\epsilon}$. Similarly, aggregate trade satisfies a gravity-like relationship (which no role for household heterogeneity) as in a CES-Armington model or Eaton and Kortum (2002).

Working from Proposition 4 one can see how the gains only depend on aggregates. The reallocation term in (49) is zero because asset holdings don't change. The gains to asset trade are zero because R does not move. Thus, the gains from trade are only about the discounted stream of changes in the home choice probability π_{ii}—which is the same for rich and poor households per the argument in the previous paragraph.

Appendix D works through this logic step-by-step. Below I state the result:

Corollary 1 (Separation of Trade and Micro-Heterogeneity) *In the dynamic, heterogenous agent trade model where preferences are logarithmic over the physical commodity: The trade elasticity is*

$$\theta = -\frac{1}{\sigma_\epsilon},$$

and relative trade flows satisfy a gravity relationship

$$\frac{M_{ij}}{M_{ii}} = \left(\frac{w_j/A_j}{w_i/A_i}\right)^{\frac{-1}{\sigma_\epsilon}} d_{ij}^{\frac{-1}{\sigma_\epsilon}},$$

and are independent of the household heterogeneity. Ant the welfare gains from trade are

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} = -\frac{1}{\theta(1-\beta)} \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}}.$$

and is (i) independent of the household heterogeneity and (ii) summarized by the trade elasticity and the change in the home choice probability.

To be honest, I found this result surprising. By looking at the choice probabilities in (182) and noting how the value functions determine choices, not period utility functions, one would suspect that the household's income fluctuations problem would shape aggregate trade outcomes. Corollary 1 shows that is not the case but that micro-outcomes and aggregate trade outcomes "separate."

Not only does heterogeneity not matter, aggregate outcomes essentially mimic the results of Arkolakis et al. (2012) and, thus, my heterogenous agent model *with log preferences* delivers the "same old gains." Trade flows take a constant elasticity form with the trade elasticity pinned down by the dispersion parameter on the trade shocks. And then the total change in welfare is summarized by the trade elasticity and how the share of home purchases changes to any change in trade costs.

Proposition 1 is also interesting because it generalizes the results of Anderson, De Palma, and Thisse (1987) and Anderson et al. (1992) to a far more complicated economy. They showed that in a static model with log utility and additive logit shocks, the economy behaves *as if* there were a representative agent CES consumer. I recover their result, but I must emphasize the complexity of the economy at the micro-level for which this result stands—households are froward looking, face productivity and taste shocks in the presence of incomplete markets and borrowing constraints. Yet, these details don't matter when the magic of $\log$ kicks in.

4. Calibration

This section focuses on my approach to calibrating the model.

At a high-level, I follow the gravity literature by picking country specific parameters to match bilateral trade flows. How I do this is a novelty of my paper—I use "gravity as a guide" to overcome the fact that my model does not admit a closed form map from trade flows to parameters as static, gravity models do. I describe this approach below and then discuss how the remaining non-country specific parameters are chosen.

In the quantitative work below, I always focus on the case of Financial Globalization where there is a international bond market and a global interest rate.

4.1. Using Gravity as a Guide

My calibration strategy is to use the gravity regression as a guide in an indirect inference procedure where I estimate parameters of the model so that the regression coefficients from a standard gravity regression run on my model's data match that seen in the data.

Here are the details. The bilateral trade flows that I use are from Eaton and Kortum (2002). The 19 countries in this data set is about the right size to do what I want to do below in about an afternoon. Moreover, the Eaton and Kortum (2002) data set provides a well defined benchmark disciplined by bilateral trade flows and gravity variables—so it's a nice laboratory to explore new issues within.

In the 19 country model, the parameters I need to choose are 19 country-specific TFP parameters (the A_is) and then $(19-1)\times(19-1)$ trade costs (with the minus one since the ii trade costs is normalized to one) to infer from the bilateral trade data. This leaves me under-identified with $(19-1)\times(19-1)$ bilateral trade shares.

Step 0. Like most of the literature, I'll reduce the number of parameters to estimate by placing a restriction on trade costs and assume they are some function of observable data. Specifically, I assume that trade costs take the form as in Eaton and Kortum (2002) with

$$\log d_{ij} = d_k + b + l + e_h + m_i, \tag{53}$$

where trade costs are a logarithmic function of distance, where d_k with $k = 1, 2, ..., 6$, is the effect of distance between country i and j lying in the k-th distance interval.[14] The b term is the effect of a shared border in which $b = 1$ if country i and j share a border and zero otherwise. Similarly l is a dummy variable if country's i and j share a language, and e_h represent two dummy variable for different indicators of European integration. The final part is an importer fixed effect shifting trade costs up or down depending upon the identity of the importer.

At this point, I've reduced the parameter space to the 29 (or if your not following $6{+}1{+}1{+}2{+}19$) coefficients on the trade cost function rather than the complete matrix of trade costs and then the 19 TFP terms.

Step 1. Then I'm going to run the following gravity regression on the data

$$\log\left(\frac{M_{ij}}{M_{ii}}\right) = im_i + ex_j + d_k + b + l + e_h + \delta_{ij}, \tag{54}$$

which projects imports between country i and j (normalized relative to domestic expenditures) on an importer effect, exporter effect and then the gravity variables relating to distance, border, language, etc. and finally there is an error term δ_{ij} that reflects other factors not in this specification.

This is the canonical representation of trade flows—the gravity model. In a standard Armington-CES, Eaton and Kortum (2002), or Melitz (2003) style model, the importer effects and exporter effects have specific interpretations. And given the point estimates from (54), productivity and the importer fixed effects on the trade cost function are easily recovered.

In my model, this is not the case. However, what I can do is use the point estimates from (54) as moments for my model to match. Abstracting from normalizations, I now have $19 + 29$ point estimates to match up with the $19 + 29$ parameters I need to estimate.[15]

The next step constructs model analogs to (54).

Step 2. To construct model analogs to (54), I guess TFP parameters and coefficients on the trade cost function in (53). Define this parameter vector as Θ.

[14] Intervals are in miles: $[0, 375)$; $[375, 750)$; $[750, 1500)$; $[1500, 3000)$; $[3000, 6000)$; and $[6000, \text{maximum}]$.

[15] To be clear about normalizations: I really only have 18 TFP parameters to match and then a normalization on the importer fixed effects is that they sum to zero as in Eaton and Kortum (2002). The fixed effects in 54 satisfy the normalization that each of their sum is zero. Thus I have $18 + 28$ parameters to match $18 + 28$ moments

Given Θ, I compute an equilibrium of the world economy. This amounts to: (i) solving for households' dynamic problems—in each country (ii) constructing the stationary distribution of wealth and expenditure patterns—in each country (iii) aggregating and then (iv) finding a vector of prices so goods markets and financial markets clear world wide.

Once I find an equilibrium, I run the same regression as in (54) on the model generated data. As some notation, the model constructed moments are defined as, e.g., $M_i(\Theta)$ which is the importer effect estimated on model generated data under the parameter vector Θ.

Step 3. The final step constructs moment conditions which provide the foundation for estimation. Define $\mathbf{y}(\Theta)$ as a set of moments conditions comparing the point estimates from the data with the point estimates from the model under the parameter vector Θ. For example, $im_i - im_i(\Theta)$ or $d_k - d_k(\Theta)$, etc.

My estimation procedure is based on the moment condition

$$E\left[\mathbf{y}(\Theta_o)\right] = 0, \tag{55}$$

where Θ_o is the true value of Θ. Thus, my method of moments estimator is:

$$\hat{\Theta} = \arg\min_{\Theta} \left[\mathbf{y}(\Theta)' \, \mathbf{y}(\Theta)\right], \tag{56}$$

At a mechanical level, finding the minimum to (56) amounts to returning to **Step 2.** each time and smartly updating parameter guess for Θ. One of the nice features of this set-up and the dimensionality reduction that I did, is that now this is an exactly identified problem and standard root-finding techniques can be applied to update Θ and a minimum found.

4.2. Remaining Parameter Values

The final parameters to calibrated in the following way. First, utility over the physical commodity is CRRA with relative risk aversion $\gamma = 1.5$, so just a bit above $\log$ and per the discussion above will lead to a pattern (at least qualitatively) or elasticities of substitution across rich and poor households that are consistent with Auer, Burstein, Lein, and Vogel (2022).

The income shock process is set up to be a mixture of a persistent and transitory component and calibrated as in Krueger, Mitman, and Perri (2016). I use their exact parameter values. And it is assumed to be the same across countries.

The borrowing constraint is set in the following way. First, I scale it by a countries autarky level of average real labor income. Then it is set so that a household can borrow up to fifty percent of it's autarky level of income. The scaling here is done to deliver a "balanced-growth-like" property of the model so a households debt capacity is invariant to a country's level of income.

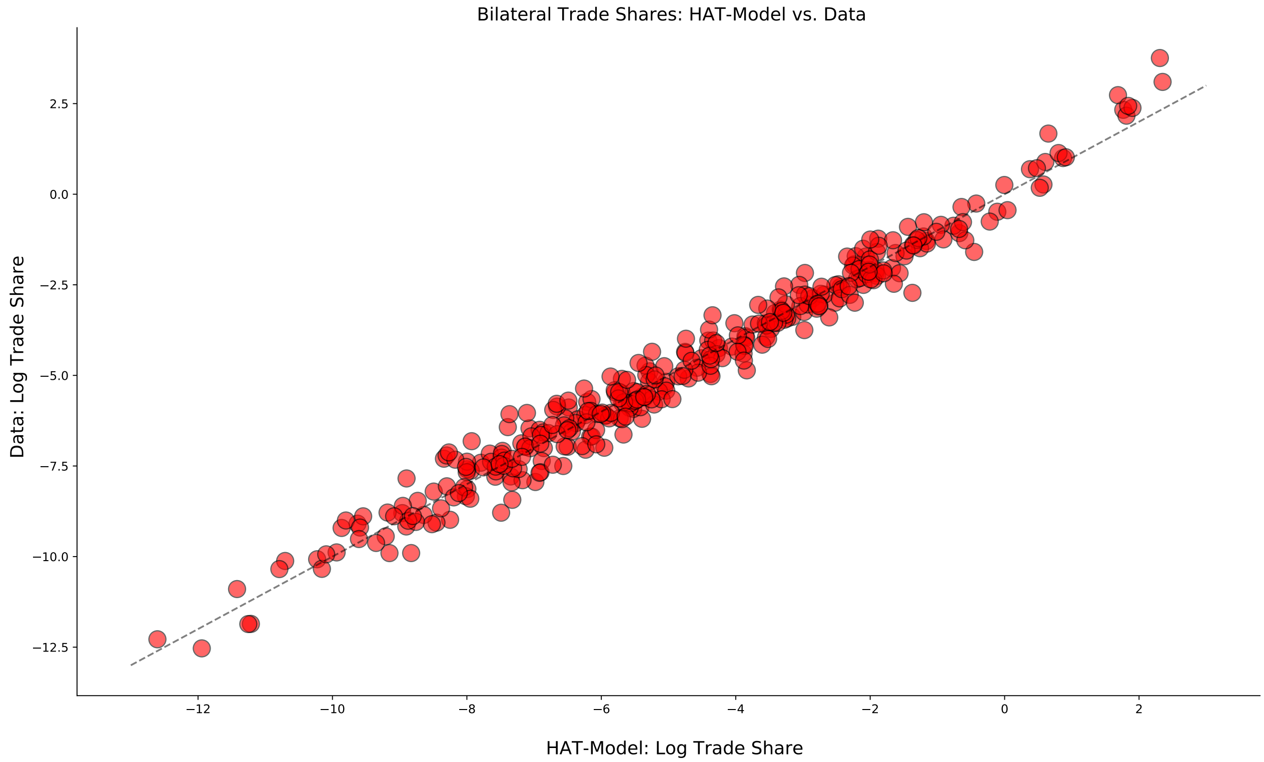


Figure 2: Bilateral Trade: Model vs. Data

Since the level of income is endogenous, a solution to this is to pin things down my connecting it with the autarky labor income.

The discount factor is jiggled around so the equilibrium world interest rate is about 1.5 percent.

The taste shock parameter is set in the following way. First, I set it so that $1/\sigma_\epsilon = 4.0$. Second, I then scale this parameter country by country to deliver a "balanced-growth-like" property of the model. Specifically, each country will have it's own parameter and it is scaled so that $\sigma_{\epsilon,i} = \sigma_\epsilon A_i^{1-\gamma}$. What this implies is that if there were two countries one with high TFP and one with low TFP in a closed economy, this scaling ensures that trade elasticities in the two countries are the same, but one is just richer than the other.

4.3. Calibration Results

Figure 2 provides a sense of model fit after running my "gravity as guide procedure." The y-axis reports bilateral trade data and the x-axis reports the outcome from my model. The fit is very high, and nearly indistinguishable from, for example, how a standard trade model perform would perform. Or the $\log$ preference model which per Proposition 1 should (and it does) operate just like a standard trade model.

Table 1 reports another measure of fit and some of the resulting parameter values. The first column are the distance, border, etc. moments from the gravity regression in (54) (and note

Table 1: Estimation Results

		HAT-Model	
Barrier	Moment	Model Fit	Parameter
$[0, 375)$	-3.10	-3.10	2.35
$[375, 750)$	-3.67	-3.67	2.81
$[750, 1500)$	-4.03	-4.03	3.09
$[1500, 3000)$	-4.22	-4.22	3.23
$[3000, 6000)$	-6.06	-6.06	4.88
$[6000, \text{maximum}]$	-6.56	-6.56	5.69
Shared border	0.30	0.30	0.91
Language	0.51	0.51	0.87
EFTA	0.04	0.04	0.98
European Community	0.54	0.54	0.89

Note: The first column reports data moments the HAT-model targets. The second reports the model moments. The third column reports the estimated parameter values.

they exactly correspond with those in the top panel of Table 3 of Eaton and Kortum (2002)). The second column reports the moments from the model. Here they exactly line up and are consistent with the argument in Figure 2, the fit is good and the model is replicating geographic pattern of activity seen in the data.

The final column reports the primitive estimates on the trade cost function. Each value reports the level effect of being in a distance bin or sharing a border etc. So if two countries are measured to be in the smallest distance bin and share a border, the trade cost between these two countries is 2.35×0.91 (first row times seventh row). Or if a country is in the furthest distance bin, its trade costs is 5.69.

How would this compare to a standard model? It's a bit hard since one needs to take a stand on the trade elasticity in the standard model to translate estimates in column one into levels of the trade costs. But an approach is the following: find the trade elasticity so the cost of the nearest distance bin is the same as in my model and then look at how things relate in other bins. In an Eaton and Kortum (2002) world, this would correspond with an trade elasticity of about 3.6. Then, for example, one takes the moment in the first column, last distance bin and compares $\exp(-1/3.6 \times -6.56)$ vs. 5.69.

What comes out of is that closer relationship are a bit more expensive then what a constant elasticity model would predict. And the furthest destinations are meaningfully less expensive,

Figure 3: Trade Elasticities $-\theta_{us,j}$

seven and ten percent less, for the last two distance bins. This is picking up a model outcome where trade elasticities are increasing with cost. So far away destinations are relatively elastic destinations, so the cost need not be as large to deter trade.

Figure 3 provides an example of the of trade elasticities that come of this model. In this figure, I focus on the US and plot each bilateral trade elasticity versus the price a consumer in the US faces when importing a variety from that country. The balls represent the relative size of US imports from that destination. And these elasticities are constructed from the bottom up via the formula in (43).

The feature that stands out very clearly in Figure 3 is that trade elasticities systematically increase with price and decrease with the volume of trade.

At the micro-level, there are two opposing forces giving rise to this aggregate relationship. Per the arguments discussed above around Proposition 3, the aggregate bilateral trade elasticity reflects both household level elasticities $\theta(a, z)$s and a composition effect that works through the expenditure weights $\omega(a, z)$s. Thus, when prices increase as one moves from one source to a less competitive source, there are two competing forces at work: (i) how do micro-level elasticities change and (ii) how does composition change?

The first force is that as prices increase *both* rich and poor households' elasticities increase. In other words, everyone is more elastic when contemplating a purchase from a more expensive destination. This is a force pushing the model to have elasticities *increase* with price.

The second force — the composition effect— generally works in the opposite direction. As one moves from more cheaper to more expensive destinations, less price sensitive households sort into those varieties. Thus, the composition of households purchasing more expensive varieties are the rich, relatively inelastic households. This is a force pushing the model to have elasticities *decrease* with price. In more standard, BLP-like settings, Nakamura and Zerom (2010) and Head and Mayer (2021) highlight this composition effect in shaping pass-through.

Which one wins? Figure 3 shows that the first force dominates the composition effect. One way to view this result is through the lens of Mrázová and Neary's (2017) language that demand in this model endogenously turns out to be "subconvex" relative to CES demand and which is equivalent to "Marshall's Second Law of Demand." The endogenous part is important as it's not parameterized as in, say, Kimball Demand which has become a popular tool to allow for non-constant elasticities. Did the model have to deliver this? Per the arguments above, it's not obvious as composition effects could have dominated.[16]

In the trade literature, there is evidence suggesting that trade elasticities conform to what comes out of my model. Both Novy (2013) and Carrère, Mrázová, and Neary (2020) find that proxy's for the trade elasticity are larger, the less trade there is between two countries. Chen and Novy (2022) further confirm this idea by finding that trade cost effects are strong for small bilateral relationships weak or even zero for large trading relationships. Mapping these ideas back into outcomes from my model, a currency union between the US and Canada would likely have a small effect since this is a high volume / low elasticity relationship.

5. Gains From Trade

Sorry, I'm still filling this in.

6. Trade in the Efficient Allocation

This section computes what the pattern of trade *should* look like were we living in a first-best world.

Before getting into the results, an important input into computing the allocation in Proposition 1 are the Pareto weights. I set the Pareto weight for a country to be proportional to it's level of TFP. My rationale behind this choice of weights is to eliminate the Planners incentives

[16] Head and Mayer (2021), using Berry et al.'s (1995) estimated model, illustrate that composition does indeed dominate with pass-through greater than one when heterogeneity in the valuation of product characteristics are shut down.

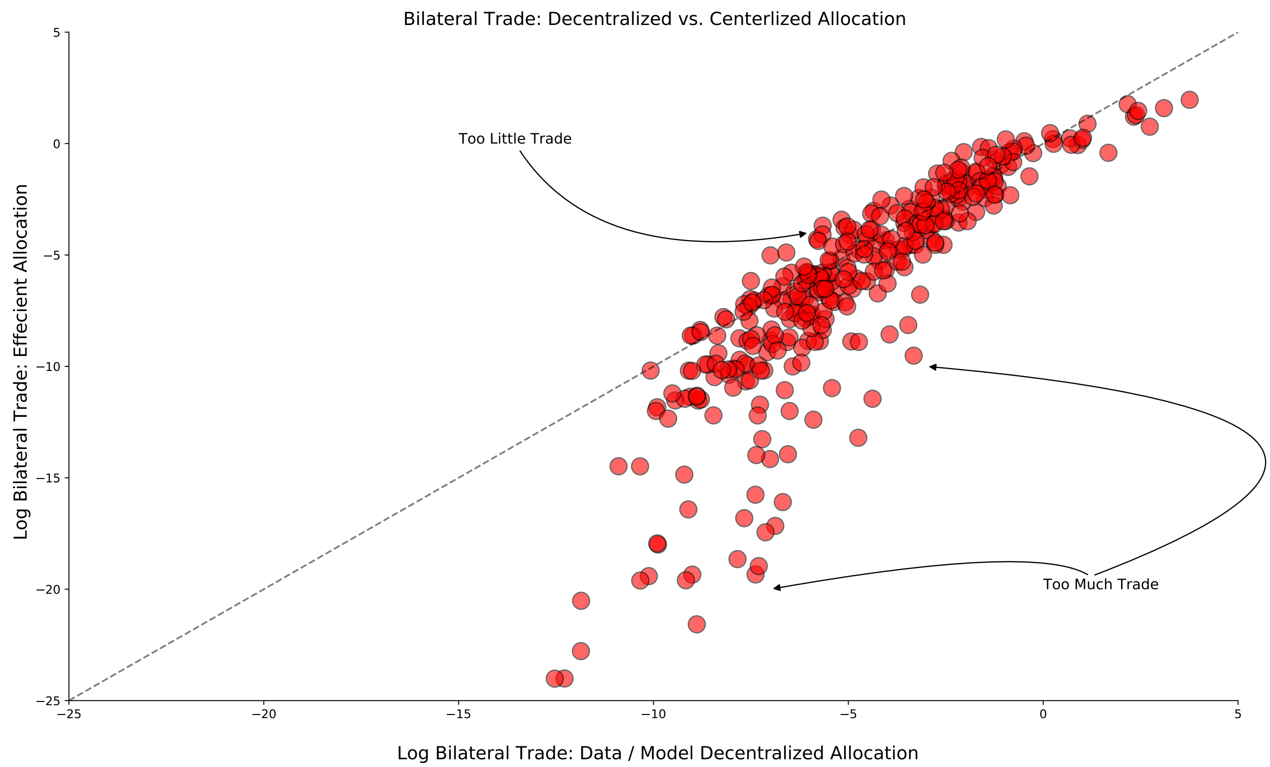


Figure 4

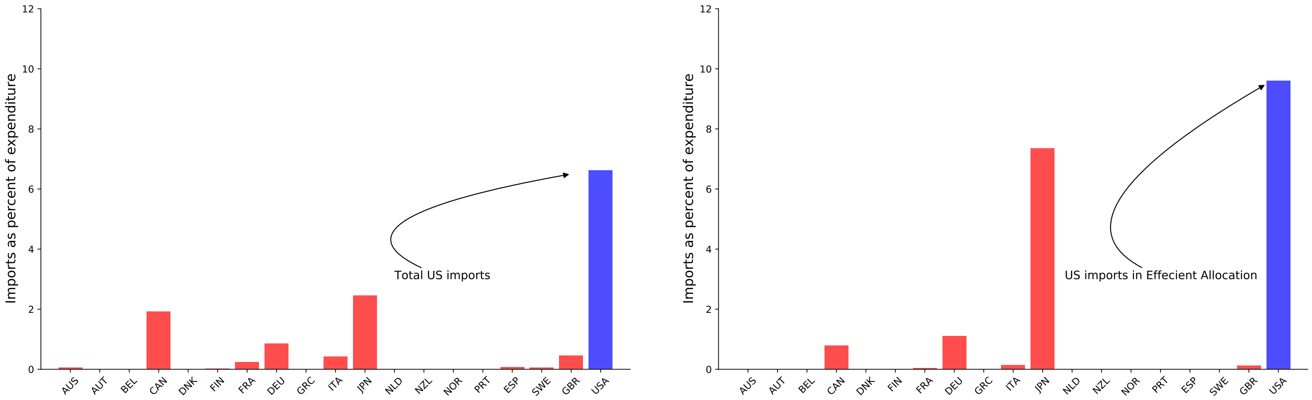
to shift consumption across countries vs. margins within the country. More specifically, this specification limits the planners desire to redistribute consumption from say, the US (a rich country) to Greece (a poor country in the data set) and focuses the exercise on within country redistribution.

Given the Pareto weights, I take the parameters of the model and then compute consumption allocations and trade flows that solve planning problem described in Section 2. To be clear, there is no change in trade costs — all the planner is doing is redistributing and overcoming market incompleteness that is present in the decentralized allocation.

6.1. Efficient Trade

Figure 4 illustrates what the pattern of trade should look like from the planners perspective. The x-axis of Figure 4 plots log bilateral trade flows in the model (and data) in the decentralized equilibrium. So the same thing as in Figure 2. The y-axis reports the trade flows in the efficient allocation. The 45-degree line is drawn so if the red-dots lie on this line, this means that trade in the efficient allocation lines up with the decentralized allocation.

Generally they do not line up on the 45-degree line. Two types of deviations stand out. First, small trade flows tends to be made smaller. From the planners perspective these are not efficient trading relationships. In contrast, large trading relationships are generally not large enough,



(a) Trade in the Decentralized Allocation

(b) Trade in the Efficient Allocation

Figure 5: US imports: Decentralized vs. Centralized Pattern of Trade

this is the set of dots lying from about -5 to 0. So there is a meaningful sense that the Planner wants a reallocation of how households source goods.

Figure 5 further illustrates this reallocation by focusing on how US imports change. The left panel reports import shares in the model (and data). Here you can see that the aggregate import share is about 7 percent with Canada, Japan, and some other European countries contributing to the bulk of US imports.

In the efficient allocation the planner simultaneously wants the US to trade *more*, but concentrate US imports on it's most productive (and large) partner which is Japan, a bit more from Germany, and less from Canada. And small trade flows with the potpourri of the remaining countries are made much smaller consistent with Figure 4.

The issue is this: uncompetitive source countries are "luxuries" from the planners perspective. In the decentralized equilibrium, the varieties from these countries are only purchased from rich households (and have the appearance of being luxuries as they are also high elasticity goods). Then, when the planner equalizes consumption within a country, the demand for these goods is eliminated.

Why does trade expand in certain directions? There is an optimistic message here: the US is under-utilizing other productive countries via trade as a means to increase consumption for the masses. This is why trade expands with Japan which in the calibrated economy is very productive with a large labor force.

7. Conclusion

What do you find interesting? Email me.

Appendix

A. The Planning Problem

I focus on a utilitarian social welfare function with Pareto weights that vary across countries:

$$W = \sum_{t=0}^{\infty} \sum_{i} \int_{z} \beta^t \psi_i v_i(z,t) L_i \lambda_i(z,t), \tag{57}$$

and here v_i a households utility in country i and ψ_i is the Planners weight on those residing in country i. Now, I'm going to place the social welfare function in sequence space and then unpack the benefits from the preference shock in the following way:

$$W = \sum_{t=0}^{\infty} \sum_{i} \sum_{j} \int_{z} \beta^t \ \psi_i \Big\{ u(c_i(z,j,t)) + \mathrm{E}[\ \epsilon \mid \pi_{ij}(z,t)] \Big\} \pi_{ij}(z,t) L_i \lambda_i(z,t) \tag{58}$$

so the inner term is period utility given the associated consumption allocation c_{ij} and then the expected value of the preference shock conditional on the choice probability $\pi_{ij}(z,t)$. This inner term is then weighted by the number of households that receive that utility, i.e. the choice probability times the mass of households with shock z at date t. The sum across j adds up all households in country i and all of this is weighted by the Pareto weight ψ_i. Then the sum across i reflects that this is global welfare.

One more point about the inner term in (58), my claim is that with the Type 1 extreme value shocks:

$$\mathrm{E}[\ \epsilon \mid \pi_{ij}(z,t)] = -\sigma_\epsilon \log \pi_{ij}(z,t) \tag{59}$$

where this is like the "selection correction" where if π becomes smaller, the expected value of the taste shock becomes larger. So only those with the largest relative shocks are chosen and higher utility for those, conditional on being selected, is felt.

Given this formulation, the planner does the following: he chooses consumption and choice probabilities for all country pair combinations, state by state, for the infinite future. The Lagrangian associated with the Planning Problem is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \sum_i \sum_j \int_z \beta^t \ \psi_i \Big\{ u(c_i(z,j,t)) + \mathrm{E}[\ \epsilon \mid \pi_{ij}(z,t)] \Big\} \pi_{ij}(z,t) L_i \lambda_i(z,t), \tag{60}$$

$$+ \sum_{t=0}^{\infty} \sum_i \beta^t \chi_i(t) \Big\{ Y_{it} \ - \ \sum_j \int_z d_{ji} c_j(z,i,t) \pi_{ji}(z,t) L_j \lambda_j(z,t) \Big\}$$

$$+ \sum_{t=0}^{\infty} \sum_i \int_z \beta^t \chi_{2i}(z,t) \Big\{ 1 - \sum_j \pi_{ij}(z,t) \Big\} L_i \lambda_i(z,t),$$

where the first term is the objective function; the second line is the resource constraint saying that output from country i must equal the consumption of commodity i globally including the transport costs. Then the third line ensures that choice probabilities are probabilities and sum to one. The final thing I'm doing is that I'm scaling the multipliers by β^t so that the algebra is easier.

Associated with this problem are two first order conditions. The first one is consumption in i with respect to variety j so:

$$\frac{\partial \mathcal{L}}{\partial c_i(z,j,t)} = \beta^t \psi_i u'(c_i(z,j,t)) \pi_{ij}(z,t) L_i \lambda_i(z,t) - \beta^t \chi_j(t) d_{ij} \pi_{ij}(z,t) L_i \lambda_i(z,t) = 0 \tag{61}$$

$$\Rightarrow \quad \psi_i u'(c_i(z,j,t)) = \chi_j(t) d_{ij}, \tag{62}$$

which says that the weighted marginal utility of consumption of (i consuming variety j) should equal it's costs. What is this cost, it's j's multiplier adjusted by the trade cost d_{ij} which gives this "price" like interpretation of the multiplier.

This first order condition has several implications. First, because the right hand size is independent of z, then $\psi_i u'(c_i(z,j,t)) = \psi_i u'(c_i(z',j,t)) = \psi_i u'(c_i(j,t))$ for all z, z' combinations. So within choice, the marginal utility of consumption is equated. Second, across variety choice, within a country (so Pareto weights cancel), the ratio of marginal utility across variety j is:

$$\frac{u'(c_i(j,t))}{u'(c_i(j',t))} = \frac{\chi_j(t) d_{ij}}{\chi_{j'}(t) d_{ij'}} \tag{63}$$

which says that consumption differences relate to the ratio of the multipliers on the resource constraint and the trade costs. This again gives the price like interpretation of the multipliers, i.e. that differences in the ratio of the marginal utility of consumption reflect relative scarcity or the shadow values. This is essentially a within country Backus and Smith (1993) condition.

The second first order condition is with respect to the choice probabilities. So

$$\frac{\partial \mathcal{L}}{\partial \pi_{ij}(z,t)} = \beta^t \psi_i \Big\{ u(c_i(z,j,t)) + \mathrm{E}[\, \epsilon \mid \pi_{ij}(z,t)] \Big\} L_i \lambda_i(z,t) + \beta^t \psi_i \frac{\partial \mathrm{E}[\, \epsilon \mid \pi_{ij}(z,t)]}{\partial \pi_{ij}} \pi_{ij}(z,t) L_i \lambda_i(z,t)$$

$$- \beta^t \chi_j(t) d_{ij} c_i(z,j,t) L_i \lambda_i(z,t) - \beta^t \chi_{2i}(z,t) L_i \lambda_i(z,t) = 0 \tag{64}$$

where the first two terms are the marginal, social benefit of changing the choice probabilities. This reflects how utility shifts by adding more households to that choice and then how the expected preference shock changes. The last two terms reflect the costs of changing the choice probabilities which are the resources to provide the additional consumption and the cost of moving households out of other choices which is what the multiplier $\chi_{2i}(z,t)$ represents.

Further simplifying gives:

$$\frac{\partial \mathcal{L}}{\partial \pi_{ij}(z,t)} = \psi_i \left[u(c_i(z,j,t)) + \mathrm{E}[\, \epsilon \mid \pi_{ij}(z,t)] + \frac{\partial \mathrm{E}[\, \epsilon \mid \pi_{ij}(z,t)]}{\partial \pi_{ij}} \pi_{ij}(z,t) \right] - \chi_j(t) d_{ij} c_i(z,j,t) - \chi_{2i}(z,t) = 0. \tag{65}$$

Then inserting the observations made above about the Type 1 extreme value distribution and then connecting things with the consumption allocation in (62) one has

$$\psi_i \left[u(c_i(j,t)) - \sigma_\epsilon \log \pi_{ij}(z,t) - \sigma_\epsilon - u'(c_i(j,t)) c_i(j,t) \right] = \chi_{2i}(z,t). \tag{66}$$

Then some algebra gets toward a closed-form expression for the choice probability

$$-\sigma_\epsilon \log \pi_{ij}(z,t) = -u(c_i(j,t)) + u'(c_i(j,t)) c_i(j,t) + \sigma_\epsilon + \chi_{2i}(z,t) \psi_i^{-1}, \tag{67}$$

$$\pi_{ij}(z,t) = \exp\left(\frac{u(c_i(j,t)) - u'(c_i(j,t)) c_i(j,t)}{\sigma_\epsilon} - 1 \right) \Big/ \exp\left(\chi_{2i}(z,t) \psi_i^{-1} / \sigma_\epsilon \right). \tag{68}$$

The choice probabilities must sum to one, so we can set the multiplier so

$$\exp\left(\chi_{2i}(z,t) \psi_i^{-1} / \sigma_\epsilon \right) = \sum_{j'} \exp\left(\frac{u(c_i(j,t)) - u'(c_i(j,t)) c_i(j,t)}{\sigma_\epsilon} - 1 \right), \tag{69}$$

which implies

$$\chi_{2i}(z,t) = \sigma_\epsilon \psi_i \log\left\{ \sum_{j'} \exp\left(\frac{u(c_i(t,j)) - u'(c_i(j,t))c_i(j,t)}{\sigma_\epsilon} - 1 \right) \right\}. \tag{70}$$

Then the choice probability is

$$\pi_{ij}(t) = \exp\left(\frac{u(c_i(j,t)) - u'(c_i(j,t))c_i(j,t)}{\sigma_\epsilon} \right) \Big/ \sum_{j'} \exp\left(\frac{u(c_i(j',t)) - u'(c_i(j',t))c_i(j',t)}{\sigma_\epsilon} \right), \tag{71}$$

which is independent of z and the Pareto weight does not appear. This has the standard Type 1 shape, but the inner most term says that the choice probability should reflect the "net" social benefit of having someone chose that good. The net part is the utility a household receives net of the cost of providing that amount of the consumption. And the cost is, well, c converted into utils which is what the marginal utility bit is doing. Because of that this is all net, the Pareto weight does not directly appear. Given this discussion, the multiplier $\chi_{2i}(t)$ has the interpretation as the expected, net contribution to social welfare of households in country i.

The formal statement is below:

Proposition 5 (The Efficient Allocation) *The allocation that satisfies the Centralized Planning Problem in (61) is:*

1. *A consumption allocation satisfying:*

$$\psi_i u'(c_{ij}(z,t)) = \chi_j(t) d_{ij} \tag{72}$$

where $\chi_j(t)$ is the shadow price of variety j.

2. *The choice probabilities are*

$$\pi_{ij}(t) = \exp\left(\frac{u(c_{ij}(t)) - u'(c_{ij}(t))c_{ij}(t)}{\sigma_\epsilon} \right) \Big/ \sum_{j'} \exp\left(\frac{u(c_{ij'}(t)) - u'(c_{ij'}(t))c_{ij'}(t)}{\sigma_\epsilon} \right) \tag{73}$$

and are independent of z because of (72).

The Gains from Trade. Let's compute the social gain to a change in trade costs. First, I going to express social welfare depending directly upon the trade costs d, and then indirectly as the allocations of c and π depend upon d as well.

$$W(d, c_i(j;d), \pi_{ij}(d)) \tag{74}$$

And then totally differentiate social welfare, so

$$\frac{\mathrm{d}W}{\mathrm{d}d} = \frac{\partial W}{\partial d} + \frac{\partial W}{\partial c_i(j;d)}\frac{\partial c_i(j;d)}{\partial d} + \frac{\partial W}{\partial \pi_{ij}(d)}\frac{\partial \pi_{ij}(d)}{\partial d} \tag{75}$$

and then I invoke the Envelope Theorem. What this means here is that I evaluate this derivative at the optimal allocation. But the optimal allocation is optimal, so on the margin any gain from changing consumption or choice probabilities is zero and these indirect effects (at the optimal allocation) are zero. Then computing the direct effect gives

$$\partial W = -\sum_{t=0}^{\infty} \beta^t \, \chi_j(t) c_i(j,t) \pi_{ij}(t) L_i \partial d_{ij}, \tag{76}$$

$$= -\sum_{t=0}^{\infty} \beta^t \, \psi_i \, u'(c_i(j,t)) c_i(j,t) \pi_{ij}(t) L_i \partial d_{ij}/d_{ij}, \tag{77}$$

where the last line inserts the relationship between the multiplier and the marginal utility of consumption in (72). This says that the change in social welfare equals essentially how much the resource constraint is relaxed by the change in trade costs. So it's like the $c_{ij}(t)\pi_{ij}(t)L_i$ is how much stuff people in i get from j and $\partial d_{ij}/d_{ij}$ perturbs it by the percent change in trade costs, then $u'(c_{ij}(t))$ converts it into utils and ψ_i weights it appropriately.

The Elasticity of Trade. Here is my argument.

Claim #1: The intensive margin trade elasticity is minus one, i.e. any change in d_{ij} results in a one for one increase, $c_i(j)$ So I don't need to mess with this.

Claim #2: Then all I need to compute is the extensive margin elasticity. So I'm going to note that

$$\frac{\partial \pi_{ij}/\pi_{ij}}{\partial d_{ij}/d_{ij}} = \frac{1}{\sigma_\epsilon}\left[u'(c_i(j,t)\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}} - u''(c_i(j,t)\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}}c_i(j,t) - u'(c_i(j,t)\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}}\right] - \frac{\partial \Phi_i(t)/\Phi_i(t)}{\partial d_{ij}/d_{ij}} \tag{78}$$

$$= \frac{-1}{\sigma_\epsilon}\left[u''(c_i(j,t)\frac{\partial c_i(j,t)}{\partial d_{ij}/d_{ij}}c_i(j,t)\right] - \frac{\partial \Phi_i(t)/\Phi_i(t)}{\partial d_{ij}/d_{ij}} \tag{79}$$

which the first line follows from the quotient rule and where $\Phi_i(t)$ is the part of the denominator in the choice probability. Recall the trade elasticity is relative to own trade so

$$\frac{\partial \pi_{ii}/\pi_{ii}}{\partial d_{ij}/d_{ij}} = -\frac{\partial \Phi_i(t)/\Phi_i(t)}{\partial d_{ij}/d_{ij}} \tag{80}$$

Then using my H-A Trade Elasticity formula in Proposition 3 and canceling terms and noticing as well that the expenditure weights don't matter since they are common across households, I have that:

$$\theta_{ij} =1 + \left[\theta^I_{ij} + \theta^E_{ij}\right] - \left[\theta^I_{ii,j} + \theta^E_{ii,j}\right] \tag{81}$$

$$=1 + -1 + \frac{-1}{\sigma_\epsilon}\left[u''(c_{ij}(t)\frac{\partial c_{ij}(t)}{\partial d_{ij}/d_{ij}}c_{ij}(t)\right] - \frac{\partial \Phi_i(t)/\Phi_i(t)}{\partial d_{ij}/d_{ij}} - 0 - \frac{\partial \Phi_i(t)/\Phi_i(t)}{\partial d_{ij}/d_{ij}} \tag{82}$$

$$=\frac{-1}{\sigma_\epsilon}\left[u''(c_{ij}(t)\frac{\partial c_{ij}(t)}{\partial d_{ij}/d_{ij}}c_{ij}(t)\right] \tag{83}$$

$$\tag{84}$$

Then here is what is interesting. I mention this fact below

$$u'(c_{ij}(t)) = \chi_j(t) d_{ij} \quad \Rightarrow \quad u''(c_{ij}(t))\frac{\partial c_{ij}(t)}{\partial d_{ij}/d_{ij}} = \chi_j(t) d_{ij} \tag{85}$$

so this means that the trade elasticity is:

$$\theta_{ij} = \frac{-1}{\sigma_\epsilon}\left[u'(c_{ij}(t))c_{ij}(t)\right] \tag{86}$$

where here you can already see the "standard result" with log preferences that the trade elasticity is independent of i, j and is the one over the dispersion parameter like in Eaton and Kortum (2002). The next interesting piece is if this is combined with the gains from trade formula which is (in a stationary setting)

$$\frac{\partial W}{\partial d_{ij}/d_i j} = \frac{\sigma_\epsilon \theta_{ij} \pi_{ij} L_i}{1-\beta}, \tag{87}$$

in other words, the gains from trade are how many people are buying i, j times the trade elasticity, discounted for the indefinite future.

Proposition 6 (Trade Elasticities and Welfare Gains in the Efficient Allocation) *The elasticity of trade to a change in trade costs between i, j in the efficient allocation is:*

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon}\left[u'(c_{ij})c_{ij}\right]. \tag{88}$$

And the welfare gains from a reduction in trade costs between i, j are

$$\frac{\mathrm{d}W}{\mathrm{d}d_{ij}/d_{ij}} = \frac{\partial W}{\partial d_{ij}/d_{ij}} = \frac{1}{1-\beta} \times \psi_i u'(c_{ij}) c_{ij} \pi_{ij} L_i \tag{89}$$

which is the discounted, direct effect from relaxing the resource constraint in (32).

The one issue with this last result is that it's a bit opaque relative to the other welfare results that I derived. Let me connect them. So per the arguments above, I can write the ex-ante utility and inserting the allocations associated with the planner as

$$v_i(t) = -\sigma_\epsilon \log \pi_{ii} + u(c_{ii}(t)) \tag{90}$$

and then

$$\frac{\partial v_i(t)}{\partial d_{ij}/d_{ij}} = -\sigma_\epsilon \frac{\partial \pi_{ii}/\pi_{ii}}{\partial d_{ij}/d_{ij}} \tag{91}$$

so the gains only work through the home share. Now this elasticity is

$$\frac{\partial \pi_{ii}/\pi_{ii}}{\partial d_{ij}/d_{ij}} = -\frac{\pi_{ij}}{\sigma_\epsilon} \left\{ u'(c_{ij}(t)) \frac{\partial c_{ij}(t)}{\partial d_{ij}/d_{ij}} - \left[u'(c_{ij}(t)) \frac{\partial c_{ij}(t)}{\partial d_{ij}/d_{ij}} + u''(c_{ij}(t)) \frac{\partial c_{ij}(t)}{\partial d_{ij}/d_{ij}} c_{ij}(t) \right] \right\} \tag{92}$$

$$= -\frac{\pi_{ij}}{\sigma_\epsilon} u''(c_{ij}(t)) \frac{\partial c_{ij}(t)}{\partial d_{ij}/d_{ij}} c_{ij}(t) \tag{93}$$

Then notice the following from the consumption allocation that

$$u'(c_{ij}(t)) = \chi_j(t) d_{ij} \quad \Rightarrow \quad u''(c_{ij}(t)) \frac{\partial c_{ij}(t)}{\partial d_{ij}/d_{ij}} = \chi_j(t) d_{ij} \tag{94}$$

where I'm just differentiating both sides by d_{ij} and the dividing through by d to make an elasticity. Then this implies that

$$\frac{\partial \pi_{ii}(t)/\pi_{ii}(t)}{\partial d_{ij}/d_{ij}} = \frac{\pi_{ij}(t)}{\sigma_\epsilon} u'(c_{ij}(t)) c_{ij}(t) \tag{95}$$

and then

$$\frac{\partial v_i(t)}{\partial d_{ij}/d_{ij}} = \pi_{ij} u'(c_{ij}(t)) c_{ij}(t) \tag{96}$$

which means that the total change in social welfare equals

$$\frac{\partial W}{\partial d_{ij}/d_{ij}} = \frac{\partial W_i}{\partial d_{ij}/d_{ij}} = \frac{1}{1-\beta} \times u'(c_{ij}(t))c_{ij}(t)\pi_{ij}(t)L_i \tag{97}$$

and I'm done.

Computing the Efficient Allocation. Here I talk through an algorithm to actually compute the efficient allocation in a stationary setting.

1. First guess a level of home consumption c_{ii} for each country i.

2. From first part of the efficient allocation, we can recover the country specific multipliers from

$$u'(c_{ii}) = \chi_i \tag{98}$$

   and then recover the consumption levels for every country i, j pairs

$$c_{ij} = u'^{-1}(\chi_j d_{ij}) \tag{99}$$

   so what we have is now an allocation of consumption that satisfies the first part of 5.

3. Construct choice probabilities from (73)

4. Check if the resource constraint holds. Specifically

$$Y_i \; - \; \sum_j d_{ji}c_{ji}\pi_{ji}L_j = 0 \tag{100}$$

   where notice that Y_i is predetermined given that there is no labor supply or capital. If this condition is not satisfied, for every country, then update the guess on consumption.

5. Once this is converged, we can construct trade flows in the following way:

$$\frac{M_{ij}}{M_{ii}} = \frac{u'(c_{jj})d_{ij}c_{ij}\pi_{ij}L_i}{u'(c_{ii})c_{ii}\pi_{ii}L_i} \tag{101}$$

   where I'm using the observation that $u'(c_{ii}) = \chi_i$ and is the shadow price of variety i, not inclusive of the trade costs. So this is the as if value of imports in country i from country j relative to home trade.

B. The H-A Trade Elasticity

My definition of the trade elasticity is the partial equilibrium response of imports from j relative to domestic consumption due to a permanent change in trade costs.6 By partial equilibrium, I mean that wages, interest rates, and the distribution of agents are fixed at their initial equilibrium values. This is consistent with the definition of the trade elasticity in say, Arkolakis et al. (2012) and Simonovska and Waugh (2014). By permanent, I mean that the change in trade costs is for the indefinite future and that households correctly understand this. Consistent with this discussion and the notation below, I compute the partial derivatives (not total) of objects with respect to trade costs.

Mathematically, the trade elasticity equals the difference between the elasticities for how trade between i and j change minus how home trade changes:

$$\frac{\partial(M_{ij}/M_{ii})}{\partial d_{ij}} \times \frac{d_{ij}}{(M_{ij}/M_{ii})} = \frac{\partial M_{ij}/M_{ij}}{\partial d_{ij}/d_{ij}} - \frac{\partial M_{ii}/M_{ii}}{\partial d_{ij}/d_{ij}}. \tag{102}$$

The change in imports between i and j with respect to a change in trade costs is:

$$\frac{\partial M_{ij}}{\partial d_{ij}} = \int_{a,z} \left\{ \frac{\partial p_{ij}}{\partial d_{ij}} c_i(a,z,j)\pi_{ij}(a,z) + \frac{\partial c_i(a,z,j)}{\partial d_{ij}} p_{ij}\pi_{ij}(a,z) + \frac{\partial \pi_{ij}(a,z)}{\partial d_{ij}} p_{ij}c_i(a,z,j) \right\} L_i\lambda_i(a,z)da\, dz. \tag{103}$$

Divide the stuff on the inside of the brackets by household level imports, $p_{ij}c_i(a,z,j)\pi_{ij}(a,z)$ and multiply on the outside giving,

$$\frac{\partial M_{ij}}{\partial d_{ij}} = \int_{a,z} \left\{ \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}} + \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}} + \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}} \right\} p_{ij}c_i(a,z,j)\pi_{ij}(a,z)L_i\lambda_i(a,z)da\, dz. \tag{104}$$

Define the following "weight" which is the share of goods that those with states a, z account for in total expenditures from j as

$$\omega_{ij}(a,z) = \frac{p_{ij}c_i(a,z,j)\pi_{ij}(a,z)L_i\lambda_i(a,z)}{M_{ij}}, \tag{105}$$

where the sum of $\omega_{ij}(a,z)$ over states a, z equals one. This gives a nice expression for the import elasticity

$$\frac{\partial M_{ij}/M_{ij}}{\partial d_{ij}/d_{ij}} = 1 + \int_{a,z} \Bigg\{ \underbrace{\frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}}_{\theta_{ij}(a,z)^I} + \underbrace{\frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}}}_{\theta_{ij}(a,z)^E} \Bigg\} \omega_{ij}(a,z)da\, dz, \tag{106}$$

or more succinctly as

$$\frac{\partial M_{ij}/M_{ij}}{\partial d_{ij}/d_{ij}} = 1 + \int_{a,z} \Big\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \Big\} \omega_{ij}(a,z) da\ dz. \tag{107}$$

where the elasticity of aggregate imports into i from j is a weighted average of several effects. The value one out in front arises from the complete pass-through of changes in trade costs to changes in prices and this is a bug of perfect competition market structure. Then the first term within the brackets represent the intensive margin $\theta_{ij}(a,z)^I$, so how much do quantities change conditional on choosing to consume variety j. The next term $\theta_{ij}(a,z)^E$ represents the extensive margin, so how the choice probabilities change.

To complete the derivation, I'll derive the own-imports term which is similar with

$$\frac{\partial M_{ii}}{\partial d_{ij}} = \int_{a,z} \Bigg\{ \underbrace{\frac{\partial p_{ii}}{\partial d_{ij}} c_{ii}(a,z)\pi_{ii}(a,z)}_{=0} + \frac{\partial c_{ii}(a,z)}{\partial d_{ij}} p_{ii}\pi_{ii}(a,z) + \frac{\partial \pi_{ii}(a,z)}{\partial d_{ij}} p_{ii}c_{ii}(a,z) \Bigg\} L_i \lambda_i(a,z) da\ dz, \tag{108}$$

where the first-term is zero because this is a partial equilibrium elasticity. Then after constructing the proper weights and converting everything to elasticity form we have

$$\frac{\partial M_{ii}/M_{ii}}{\partial d_{ij}/d_{ij}} = \int_{a,z} \Big\{ \theta_{ii,j}(a,z)^I + \theta_{ii,j}(a,z)^E \Big\} \omega_{ii}(a,z) da\ dz, \tag{109}$$

where the ii, j notation is that $\theta_{ii,j}(a,z)^I$ reflects how the intensive margin adjusts, conditional on a ii choice, given a change in ij price. Similarly, $\theta_{ii,j}(a,z)^E$ represents how the ii choice probability changes given the ij change in price.

Proposition 3 then follows:

Proposition 3 (The H-A Trade Elasticity) *The trade elasticity between country i and country j is:*

$$\theta_{ij} = 1 + \int_{z,a} \Big\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \Big\} \omega_{ij}(a,z) da\ dz - \int_{z,a} \Big\{ \theta_{ii,j}(a,z)^I + \theta_{ii,j}(a,z)^E \Big\} \omega_{ii}(a,z) da\ dz \tag{110}$$

which is an expenditure-weighted average of micro-level elasticities. The micro-level elasticities are decomposed into an intensive margin and extensive margin

$$\theta_{ij}(a,z)^I = \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}, \qquad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and the expenditure weights are defined as

$$\omega_{ij}(a,z) = \frac{p_{ij} c_i(a,z,j) \pi_{ij}(a,z) \lambda_i(a,z) L_i}{M_{ij}}.$$

2.1. Connecting Household Behavior with Elasticities

To derive the **intensive margin elasticity**, simply start from the households budget constraint and differentiate consumption of variety j with respect to price p_{ij} and one gets

$$\underbrace{\frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}}_{\theta_{ij}(a,z)^I} = \left[-\frac{\partial g_i(a,z,j)/p_{ij}c_i(a,z,j)}{\partial p_{ij}/p_{ij}} - 1\right]\frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}, \tag{111}$$

where recall that $g_{ij}(a,z)$ is the policy function mapping states into asset holdings next period a'. To derive the **extensive margin elasticity**, start from the definition of the choice probability and one finds

$$\underbrace{\frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}}}_{\theta_{ij}(a,z)^E} = \frac{1}{\sigma_\epsilon}\frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}}. \tag{112}$$

And then I use the following arguments to unpack how the value function $v_i(a,z,j)$ changes:

$$\frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} = -\,u'(c_i(a,z,j))c_i(a,z,j) + \left[-\frac{u'(c_i(a,z,j))}{p_{ij}}\frac{\partial g_i(a,z,j)}{\partial p_{ij}/p_{ij}}\right] \tag{113}$$

$$+\,\beta \mathrm{E}\left\{\frac{\partial v}{\partial a'}\frac{\partial g_i(a,z,j)}{\partial p_{ij}/p_{ij}}\frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}} + \frac{\partial v(g_i(a,z,j),z')}{\partial p_{ij}/p_{ij}}\frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}\right\} \tag{114}$$

which can then be further expressed in terms of the Euler Equation (derived below in Equation (133))

$$\frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} = -\,u'(c_i(a,z,j))c_i(a,z,j) \tag{115}$$

$$+\underbrace{\left\{-\frac{u'(c_i(a,z,j))}{p_{ij}} + \beta\mathbb{E}\left[-\,\sigma_\epsilon\frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + u'(c_i(a',z',i))R\right]\right\}}_{\text{Euler equation in (133)}}\frac{\partial g_i(a,z,j)}{\partial p_{ij}/p_{ij}} \tag{116}$$

$$+\,\beta\mathbb{E}\frac{\partial v_i(a',z')}{\partial p_{ij}/p_{ij}}\Big\} \tag{117}$$

The term in the second line: the Euler equation multiplied by how assets change should be zero for small changes. I discuss this more in depth below around the welfare gains calculation, but the key issue is that either the Euler equation holds and thus this term is zero, or it does not hold, but then households can't adjust asset holdings and then the outside part is zero. And for small changes households on the margin of a binding constraint or not are on the margin and don't matter.

The final step is to connect the first term in (115) with things like the relative risk aversion and the marginal propensity to consume. So specifically, thought experiment here is ignore all the future effects and ask if a household was a bit wealthier what would the effect be on the $u'(c_{ij}(a,z))c_{ij}(a,z)$ and hence how one component of the extensive margin elasticity changes:

$$\frac{\partial(u'(c_{ij}(a,z))c_{ij}(a,z))}{\partial a} = u''(c_{ij}(a,z))\frac{\partial c_{ij}}{\partial a}c_{ij}(a,z) + u'(c_{ij}(a,z))\frac{\partial c_{ij}}{\partial a} \tag{118}$$

$$= \frac{\partial c_{ij}}{\partial a}\Bigg[u''(c_{ij}(a,z))c_{ij}(a,z) + u'(c_{ij}(a,z))\Bigg] \tag{119}$$

$$= u'(c_{ij}(a,z)) \times \mathbf{MPC}_{ij}(a,z) \times \Bigg[-\rho_{ij}(a,z) + 1\Bigg]. \tag{120}$$

And just to emphasize how this works, it's a derivative of $u'(c_{ij}(a,z))c_{ij}(a,z)$. So as assets go up, with $\rho > 1$ this implies that $u'(c_{ij}(a,z))c_{ij}(a,z)$ goes down! And this is a force for things to be less elastic for rich guys. As assets go down, this implies that $u'(c_{ij}(a,z))c_{ij}(a,z)$ goes up, and this is a force for poor guys to be more elastic.

The final elasticity I want to derive is how home choices respond to changes in trade frictions. This is a term that shows up all the time (in the calculations above) and in the welfare expressions, so it's worth computing as well:

$$\frac{\partial \pi_{ii}(a,z)/\pi_{ii}(a,z)}{\partial d_{ij}/d_{ij}} = \frac{1}{\sigma_\epsilon}\frac{\partial v_i(a,z,i)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}}. \tag{121}$$

Then the derivative of the term Φ takes on a unique property where

$$\frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}} = \sum_j \pi_{ij}(a,z)\frac{1}{\sigma_\epsilon}\frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} \tag{122}$$

which takes on this flavor of exposure (which are the home choice probabilities) times how the household's valuations across the goods change (as represented by the value functions). Then expressing things all relative to how the home valuation changes we have

$$\frac{\partial \pi_{ii}(a,z)/\pi_{ii}(a,z)}{\partial d_{ij}/d_{ij}} = \frac{1}{\sigma_\epsilon}\sum_j \pi_{ij}(a,z)\Bigg[\frac{\partial v_i(a,z,i)}{\partial d_{ij}/d_{ij}} - \frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}}\Bigg]. \tag{123}$$

So what this says is that the change in the home choice completely summarizes how things change (in a relative sense).

C. The Welfare Gains from Trade

This section derives the gains from a permanent change in trade costs, across steady states. Like the discussion above, the idea here is that I'm thinking a situation where the change is small and there is an immediate jump to the new steady state. Unlike the trade elasticity, I'm going to take total derivatives encompassing general equilibrium changes in wages and interest rates.

The analysis proceeds in several steps.

First, I'll focus on country i and study a change in trade costs d_{ij}. To simplify the algebra, I choose w_i to be the numeraire and normalize $A_i = 1$. This implies is that $p_{ii} = \frac{w_i}{A_i}$ equals one and it's derivative with respect to things is zero.

Second, To compute how social welfare changes, I focus on a utilitarian social welfare function (Pareto weights across households, within a country, are the same):

$$W_i = \int_a \int_z v_i(a,z)\lambda_i(a,z) \tag{124}$$

Then the total change in total welfare is

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} = \int_a \int_z \left\{ \frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} + v_i(a,z)\frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \right\} \lambda_i(a,z). \tag{125}$$

The first component reflects changes in household-level welfare. The second component is about reallocation, i.e., if—at the old v's—does the distribution change so that social welfare gets better or worse. The change in social welfare is then the weighted average of these two forces with the weights being those at the initial distribution.

How does household-level welfare change? I'm going to walk through this in several steps.

First, I show how I can express everything relative to the home country i. Recall that the value function (with the expectation taken over the different preference shocks) is

$$v_i(a,z) = \sigma_\epsilon \log \left\{ \sum_{j'} \exp\left(\frac{v_i(a,z,j')}{\sigma_\epsilon} \right) \right\}, \tag{126}$$

and then I'm going to make the observation that I can substitute out the sum part (126) with the exp of the home value function relative to the micro-level "home choice" so

$$\pi_{ii}(a,z) = \exp\left(\frac{v_i(a,z,i)}{\sigma_\epsilon}\right) \Bigg/ \sum_{j'} \exp\left(\frac{v_i(a,z,j')}{\sigma_\epsilon}\right), \tag{127}$$

$$\pi_{ii}(a,z) \times \sum_{j'} \exp\left(\frac{v_i(a,z,j')}{\sigma_\epsilon}\right) = \exp\left(\frac{v_i(a,z,i)}{\sigma_\epsilon}\right), \tag{128}$$

$$\sum_{j'} \exp\left(\frac{v_i(a,z,j')}{\sigma_\epsilon}\right) = \exp\left(\frac{v_i(a,z,i)}{\sigma_\epsilon}\right) \Bigg/ \pi_{ii}(a,z). \tag{129}$$

Then substituting (129) into the value function in (126) gives:

$$v_i(a,z) = \sigma_\epsilon \log \left\{ \frac{\exp\left(\frac{v_i(a,z,i)}{\sigma_\epsilon}\right)}{\pi_{ii}(a,z)} \right\} \tag{130}$$

and recall that the home choice value function is

$$v_i(a,z,i) = u(c_i(a,z,i)) + \beta \mathbb{E} v_i(g_i(a,z,i),z) \tag{131}$$

where the expectation operator is over the zs and the v_i is the same value function as in (126) so the taste shocks are integrated out. Taking logs and exp's of (130) allows for the v_i value function to be represented as

$$v_i(a,z) = -\sigma_\epsilon \log \pi_{ii}(a,z) + u(c_i(a,z,i)) + \beta \mathbb{E} v_i(g_i(a,z,i),z). \tag{132}$$

Now everything is written with respect to the home choice. What is going on is that the home choice π_{ii} summarizes the expected value of those shocks and their benefits. No need to explicitly carry around the v_{ij}s. This is essentially the dynamic analog to Equation (15), Footnote 42 of Eaton and Kortum (2002) and Arkolakis et al. (2012).

Now the strategy is to totally differentiate (132) with respect to trade costs and use the recursive structure to iterate forward and construct the change across time. One more detail, to facilitate interpretation, it will be useful to compute the Euler equation associated with asset holdings when the borrowing constraint does not bind. This euler equation is:

$$-u'(c_i(a,z,i)) = \beta \mathbb{E} \bigg\{ -\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + u'(c_i(a',z',i))R \bigg\}, \tag{133}$$

which says that the agent should be indifferent between the marginal utility of consumption forgone to hold some more assets and two components (i) the benefit from how a change in assets changes in their variety choice and (ii) the direct benefit of the returns on the assets evaluated at the marginal utility of consumption.

Totally differentiating the value function gives

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} =$$

$$-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} + u'(c_i(a,z,i))\frac{\mathrm{d}R}{\mathrm{d}d_{ij}/d_{ij}}a - u'(c_i(a,z,i))\frac{\mathrm{d}g_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} + \beta E\frac{\mathrm{d}v_i(g_i(a,z,i),z')}{\mathrm{d}d_{ij}/d_{ij}} \tag{134}$$

Then the derivative of the continuation value function is:

$$\frac{\mathrm{d}v_i(g(a,z,i),z')}{\mathrm{d}d_{ij}/d_{ij}} = \underbrace{\Big[-\sigma_\epsilon \frac{\partial \pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + u'(c_i(a',z',i))R\Big]}_{\frac{\partial v_i(g_i(a,z,i),z')}{\partial a}}\frac{\mathrm{d}g_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \;+ \tag{135}$$

$$-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a',z')/\pi_{ii}(a',z')}{\mathrm{d}d_{ij}/d_{ij}} + u'(c_i(a',z',i))\frac{\mathrm{d}R}{\mathrm{d}d_{ij}/d_{ij}}a' - u'(c_i(a',z',i))\frac{\mathrm{d}g_i(a',z',i)}{\mathrm{d}d_{ij}/d_{ij}} + \beta E\frac{\mathrm{d}v_i(g_i(a',z',i),z'')}{\mathrm{d}d_{ij}/d_{ij}} \tag{136}$$

And now collect terms so

$$\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = -\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \tag{137}$$

$$+\underbrace{u'(c_i(a,z,i))\frac{\mathrm{d}R}{\mathrm{d}d_{ij}/d_{ij}}a}_{\gamma_{ii}(a,z)} \tag{138}$$

$$+\underbrace{\Big\{-u'(c_{ii}(a,z)) + \beta\mathbb{E}_z\big[-\sigma_\epsilon\frac{\partial\pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + u'(c_i(a',z',i))R\big]\Big\}\frac{\mathrm{d}g_i(a,z,i)}{\mathrm{d}d_{ij}/d_{ij}}}_{\delta_{ii}(a,z)} \tag{139}$$

$$+\beta\mathbb{E}_z\Big\{-\sigma_\epsilon\frac{\mathrm{d}\pi_{ii}(a',z')/\pi_{ii}(a',z')}{\mathrm{d}d_{ij}/d_{ij}} + u'(c_i(a',z',i))\frac{\mathrm{d}R}{\mathrm{d}d_{ij}/d_{ij}}a' \quad \ldots \tag{140}$$

Let me walk through the interpretation of each term:

- $-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}}$ is a gains from trade term. I expand more on this term below.
- $u'(c_i(a,z,i))\frac{\partial R}{\partial d_{ij}/d_{ij}}a$ or what I'm labeling as $\gamma_{ii}(a,z)$ is what I would call how changes in goods trade facilitates leads to changes in asset trade.
- The third term which I'm labeling as $\delta_{ii}(a,z)$ is the Euler equation for assets. Honestly, it's pretty cool the way this shows up here. But it should also zero out through some basic arguments. Let me expand on this.

  The idea is that if the household is unconstrained, then this term is zero as there is no gain through changes in asset behavior. Asset holdings are already chosen optimally so that margins are equated, thus, on the margin any benefit of lower trade costs on changes in asset behavior is zero. Essentially an application of the Envelope Theorem.

  Now in this economy, this term may not be zero because of borrowing constrained households, thus this term is positive. However, notice how the outside brackets is multiplied by the change in the asset policy function. Again, this is super cool. What this picks up is that if the household is constrained, then assets can't change so the outside term is zero and, thus, overall the second term is zero.

  Final point, then the only people that benefit and contribute to social welfare through these effects are those on the margin between constrained and not-constrained. But if they are on the margin between being constrained and not-constrained, then they are on their euler equation. More formally, these agents are those where, from the Generalized Euler equation in (175), both terms under the max operator are equated, so

$$\beta R_i \mathrm{E}_{z'}\left[\sum_{j'} \pi_{ij}(a',z')\frac{u'(c_{ij}(a',z'))}{p_{ij}}\right] = u'\left(\frac{R_i a + w_i - \phi_i}{p_{ij}}\right) \tag{141}$$

- The final term is about this continuing on into the infinite future.

Iterating on (140) into the future, the gains from trade for a household with states a, z today are

$$\frac{\partial v_i(a,z)}{\partial d_{ij}/d_{ij}} = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left\{-\sigma_\epsilon\frac{\mathrm{d}\pi_{ii}(a_t,z_t)/\pi_{ii}(a_t,z_t)}{\mathrm{d}d_{ij}/d_{ij}} + \gamma_{ii}(a_t,z_t) + \delta_{ii}(a_t,z_t)\right\} \tag{142}$$

Where the first component is the expected discounted gains from substitution, gains from asset trade and the relaxation of borrowing constraints. Combining (142) and (125) yields the following proposition for the gains from trade

Proposition 4 (The Welfare Gains from Trade) *The welfare gains from trade are given by*

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} = \int_a\int_z\left\{\frac{\mathrm{d}v_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}} + v_i(a,z)\frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}\right\}\lambda_i(a,z).$$

which reflects the change in household level gains and how the distribution of households changes. Household level gains are given by

$$\frac{\partial v_i(a,z)}{\partial d_{ij}/d_{ij}} = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big\{-\sigma_\epsilon\frac{\mathrm{d}\pi_{ii}(a_t,z_t)/\pi_{ii}(a_t,z_t)}{\mathrm{d}d_{ij}/d_{ij}} + \gamma_{ii}(a_t,z_t) + \delta_{ii}(a_t,z_t)\Big\}$$

where each term represents:

- *Gains from substitution:* $-\sigma_\epsilon\frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}}$.
- *Gains from asset trade:* $\gamma_{ij}(a,z) = u'(c_{ii}(a,z))\frac{\mathrm{d}R}{\mathrm{d}d_{ij}/d_{ij}}a$
- *Gains from relaxing borrowing constraints:*

$$\delta_{ii}(a,z) = \Big\{-u'(c_{ii}(a,z)) + \beta\mathbb{E}\big[-\sigma_\epsilon\frac{\partial\pi_{ii}(a',z')/\pi_{ii}(a',z')}{\partial a'} + u'(c_{ii}(a',z'))R\big]\Big\}\frac{\mathrm{d}g_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = 0$$

which equals zero for small changes.

The final step is to unpack the gains from substitution term. Now from the elasticity discussion we can simply convert (123) into a total derivative

$$\frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \frac{1}{\sigma_\epsilon}\sum_{j'}\pi_{ij'}(a,z)\Bigg[\frac{\mathrm{d}v_i(a,z,i)}{\partial d_{ij}/d_{ij}} - \frac{\mathrm{d}v_i(a,z,j')}{\mathrm{d}d_{ij}/d_{ij}}\Bigg]. \tag{143}$$

so change in the home choice summarizes two forces: (i) how exposed a household is to the change through the choice probabilities and then (ii) how value functions change.

Now the value function component is where elasticities enter. To see this, define $\bar\theta(a,z)^E_{ij',j}$ as the extensive margin, cross-price elasticity, and in total derivative form. Following the derivation of (112) this is

$$\theta_{ij',j}(a,z)^E = \frac{1}{\sigma_\epsilon}\frac{\mathrm{d}v_i(a,z,j')}{\partial d_{ij}/d_{ij}} - \frac{\mathrm{d}\Phi_i(a,z)/\Phi_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}, \tag{144}$$

which then noticing that the $\mathrm{d}\Phi$ term is independent of option j' and so they difference out, we can express in terms of cross-price elasticities

$$-\sigma_\epsilon\frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} = \sigma_\epsilon\sum_{j'}\pi_{ij'}(a,z)\Big[\bar\theta(a,z)^E_{ii,j} - \bar\theta(a,z)^E_{ij',j}\Big], \tag{145}$$

Now there is one more way to express things that is also informative. **Assume that all cross-**

terms are near zero. Then we have

$$-\sigma_\epsilon \frac{\mathrm{d}\pi_{ii}(a,z)/\pi_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}} \approx -\sigma_\epsilon \times \pi_{ij}(a,z) \times \bar{\theta}(a,z)^E_{ij} \tag{146}$$

This expression is interesting because now it is analogous to the gains from trade formula in the efficient allocation. The pure gains from trade component comes from (i) how the taste shock are valued (and this is standard) (ii) a households exposure and (iii) the household's elasticity. And this last part, more or less reflects how sensitive the value function is with respect to price.

D. Log Preferences

This example is interesting because it retains an aggregate constant trade elasticity, but at the micro-level it is not quite with things canceling in a way during aggregation. Second, the welfare gains from trade formula looks like ACR kind of thing. Because this is a bit more involved I'm going to be super systematic about this.

Step 1: Individual Choices. With log preferences the j choice value function is

$$v_{ij}(a,z) = \max_{a' \in \mathcal{A}} \left\{ \log\left(\frac{Ra + wz - a'}{p_{ij}}\right) + \beta\, \mathbb{E}[v_i(a',z')] \right\} \tag{147}$$

which is then

$$v_{ij}(a,z) = \max_{a' \in \mathcal{A}} \left\{ \log(Ra + wz - a') + \beta\, \mathbb{E}[v_i(a',z')] \right\} - \log p_{ij} \tag{148}$$

which then leads to the observation that the optimal a' conditional on a choice j is **independent** of the price and the choice j. So what is going on is if you consume an expensive or cheap good, then consumption simply scales up or down so that assets next period are exactly the same. This observation has the implication that expenditures on consumption are the same across choices. Compare households expenditures with the same state a, z but different choices. Equation (148) implies

$$p_{ij} c_{ij}(a,z) = p_{ii} c_{ii}(a,z) \tag{149}$$

so within states, people always spend the same amount. This fact will be useful below.

Finally, this observation implies that the choice probabilities are independent of the state only prices matter so

$$\pi_{ij}(a,z) = \exp\left(\frac{v_{ij}(a,z)}{\sigma_\epsilon}\right) \bigg/ \sum_{j'} \exp\left(\frac{v_{ij'}(a,z)}{\sigma_\epsilon}\right) \tag{150}$$

$$\pi_{ij} = \exp\left(\frac{-\log p_{ij}}{\sigma_\epsilon}\right) \bigg/ \sum_{j'} \exp\left(\frac{-\log p_{ij'}}{\sigma_\epsilon}\right) \tag{151}$$

which is exactly the same as discussed above in the static model. These observations are all consistent with the Generalized Euler Equation below. To see this

$$\frac{u'(c_{ij}(a,z))}{p_{ij}} = \max\left\{\beta R_i \mathbb{E}\left[\sum_{j'} \pi_{ij}(a',z')\frac{u'(c_{ij}(a',z'))}{p_{ij}}\right] \,,\ u'\left(\frac{R_i a + w_i - \phi_i}{p_{ij}}\right)\right\} \tag{152}$$

and then impose log preferences and notice that

$$(Ra + wz - a')^{-1} = \max\left\{\beta R \mathbb{E}\left[\sum_{j'} \pi_{ij}(a',z')(Ra' + wz - a'')^{-1}\right] \,,\ (Ra + w - \phi_i)^{-1}\right\} \tag{153}$$

and then because the term multiplying the π_{ij}'s does not depend upon j it can be pulled out and

$$(Ra + wz - a')^{-1} = \max\left\{\beta R \mathbb{E}(Ra' + wz' - a'')^{-1} \,,\ (Ra + w - \phi_i)^{-1}\right\} \tag{154}$$

and thus the asset choice is independent from the variety choice j.

Step 2: Micro Trade Elasticities. I'm going to be super systematic about this:

- Starting with (111) and because the asset choice is independent of prices, the intensive margin elasticity $\theta_{ij}(a,z)^I$ is -1 and $\theta_{ii}(a,z)^I$ is zero as there are no partial effects on prices in ii.

- The extensive margin elasticity is:

$$\theta_{ij}(a,z)^E = \frac{1}{\sigma_\epsilon}\frac{\partial v_{ij}(a,z)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i/\Phi_i}{\partial d_{ij}/d_{ij}} \tag{155}$$

$$= -\frac{1}{\sigma_\epsilon}\frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}} + \beta\mathbb{E}\frac{\partial v_i(a',z')}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(z)/\Phi_i(z)}{\partial d_{ij}/d_{ij}} \tag{156}$$

$$= -\frac{1}{\sigma_\epsilon} + \beta\mathbb{E}\frac{\partial v_i(a',z')}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i/\Phi_i}{\partial d_{ij}/d_{ij}} \tag{157}$$

  where the first line removes the a, z indexing of Φ_i because they don't shape the choice probabilities. The next line then partially differentiates the value function with respect to the change in trade costs and I'm exploiting how with log preferences one can pull out the price term. And then the final line notes that the price elasticity is minus one. One more fact that:

$$\theta_{ii}(a,z)^E = \beta\mathbb{E}\frac{\partial v_i(a',z')}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i/\Phi_i}{\partial d_{ij}/d_{ij}} \tag{158}$$

  where a key thing to notice is that the i, i elasticity is the same as the second and third terms above in (157).

It's worth emphasizing that the micro trade elasticities are **not** constant across states a, z. Unlike the static model with log preferences, they are varying by income and assets as the derivative of the value function is showing up. But what **is** occurring is that across different destinations, within states, they are varying in the same exact way. This is one aspect of this case that facilitates aggregation. However, the necessary aspect is that the expenditure weights work out in the right way, I show this next.

Step 3: Expenditure Weights. Recall that the micro level trade elasticities when aggregated are weighted by

$$\omega_{ij}(a,z) = \frac{p_{ij}c_{ij}(a,z)\pi_{ij}(a,z)\lambda_i(a,z)}{M_{ij}}. \tag{159}$$

and note that we can relabel $p_{ij}c_{ij}(a,z) = x_i(a,z)$ given (149), that expenditures are independent of the source. With the choice probabilities independent of a, z the weights become

$$\omega_{ij}(a,z) = \frac{x_i(a,z)\pi_{ij}\lambda_i(a,z)}{\sum_{a,z} x_i(a,z)\pi_{ij}\lambda_i(a,z)}, \tag{160}$$

$$= \frac{x_i(a,z)\lambda_i(a,z)}{\sum_{a,z} x_i(a,z)\lambda_i(a,z)} \tag{161}$$

which are independent of source j. This is the second important observation that will facilitate aggregation.

Step 4: The Trade Elasticity. Now just mechanically follow Proposition 3:

$$\begin{aligned}\theta_{ij} =& 1 + \int_{a,z} \left\{ -1 + -\frac{1}{\sigma_\epsilon} + \beta\mathbb{E}\frac{\partial v_i(a',z')}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i/\Phi_i}{\partial d_{ij}/d_{ij}} \right\}\omega_i(a,z) \\ & - \int_{a,z} \left\{\beta\mathbb{E}\frac{\partial v_i(a',z')}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i/\Phi_i}{\partial d_{ij}/d_{ij}}\right\}\omega_i(a,z) \\ =& -\frac{1}{\sigma_\epsilon}\end{aligned} \tag{162}$$

where the last line follows because the a, z terms in the micro level trade elasticities exactly cancel given that expenditure weights are source independent. And the aggregate trade elasticity is constant and parameterized by the dispersion in tastes.

Step 5: Gravity. Following the arguments in **Step 3** about expenditures being independent of the destination, bilayer imports are

$$M_{ij} = \pi_{ij} \sum_{a,z} x_i(a,z)\lambda_i(a,z) \tag{163}$$

where the last term does not depend upon the source. Dividing by home consumption, using (151) and subsisting prices with technology and wages we have

$$\frac{M_{ij}}{M_{ii}} = \left(\frac{w_j/A_j}{w_i/A_i}\right)^{\frac{-1}{\sigma_\epsilon}} d_{ij}^{\frac{-1}{\sigma_\epsilon}} \tag{164}$$

which is the same form as in a Armington model or Eaton and Kortum (2002).

Step 5: The Grains From Trade. Then from here I can just follow Proposition 4. First the individual gains are

$$\frac{\partial v_i(a,z)}{\partial d_{ij}/d_{ij}} = \underbrace{-\frac{1}{\theta(1-\beta)} \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}}}_{ACR} + \mathbb{E}\sum_{t=0}^{\infty} \beta^t \Big\{ \gamma_{ii}(a_t, z_t) + \delta_{ii}(a_t, z_t) \Big\}$$

where the first term is exactly in the static model except for the discounting bit. But what facilitates this is that the choice probabilities are independent of a, z and it can be pulled out of the expected discounted sum stuff. Then the subsequent terms take a slightly cleaner form:

- Gains from asset trade: $\gamma_{ij}(a,z) = \frac{\mathrm{d}R}{\mathrm{d}d_{ij}/d_{ij}} \frac{a}{c_{ii}(a,z)}$

Step 6: Assets don't matter. Claim is that because the asset policy function g is independent of the price of varieties, then any change in trade costs will not affect R and thus the total derivative $\frac{\mathrm{d}R}{\mathrm{d}d_{ij}/d_{ij}}$ equals zero and then the total derivative $\frac{\mathrm{d}g_{ii}(a,z)}{\mathrm{d}d_{ij}/d_{ij}}$ on the asset policy function is zero. And then $\frac{\mathrm{d}\lambda_i(a,z)/\lambda_i(a,z)}{\mathrm{d}d_{ij}/d_{ij}}$ is zero as well.

Corollary 2 (Separation of Trade and Heterogeneity) *In the dynamic, heterogenous agent trade model where preferences are logarithmic over the physical commodity: The trade elasticity is*

$$\theta = -\frac{1}{\sigma_\epsilon},$$

and relative trade flows satisfy a gravity relationship

$$\frac{M_{ij}}{M_{ii}} = \left(\frac{w_j/A_j}{w_i/A_i}\right)^{\frac{-1}{\sigma_\epsilon}} d_{ij}^{\frac{-1}{\sigma_\epsilon}},$$

and are independent of the household heterogeneity. Ant the welfare gains from trade are

$$\frac{\mathrm{d}W_i}{\mathrm{d}d_{ij}/d_{ij}} = -\frac{1}{\theta(1-\beta)} \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}}.$$

and is (i) independent of the household heterogeneity and (ii) summarized by the trade elasticity and the change in the home choice probability.

To visualize some of this on the computer, Figures (6) and (7) plot elasticities by state and trade shares. No short-cuts are involved in this calculation, the same exact algorithm is applied. The result is apparent, with log preference, trade elasticities and trade shares become independent of what is going on with the household and, thus, there is pure separation between them.

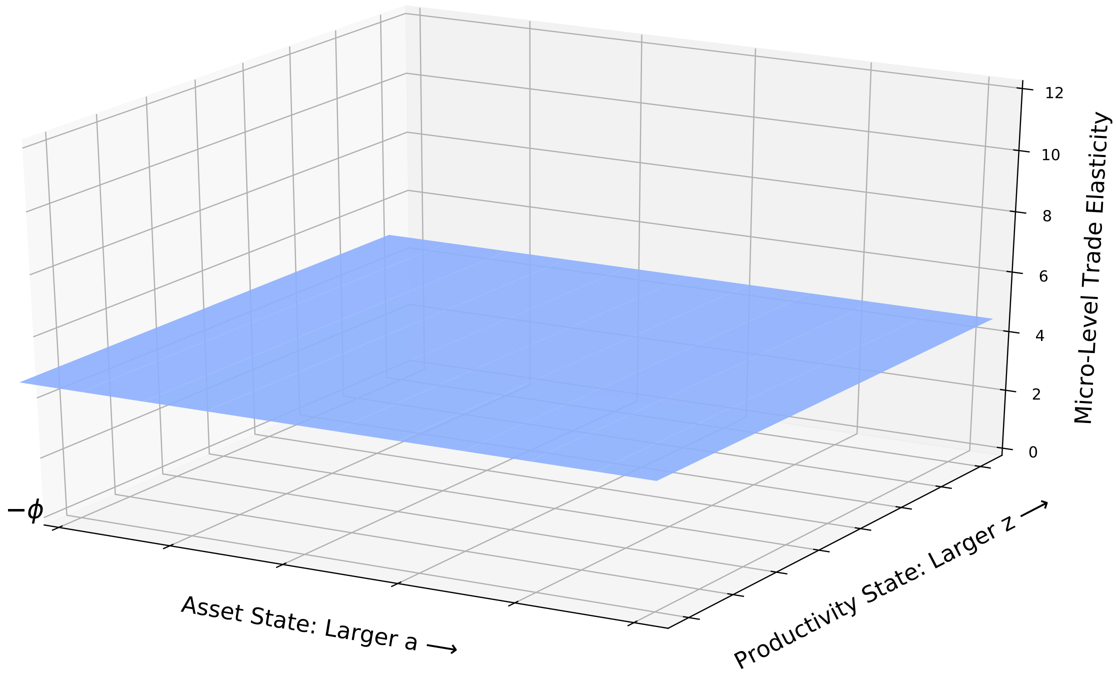


Figure 6: Log Preferences, Trade Elasticities, $-\theta_{ij}(a, z)$

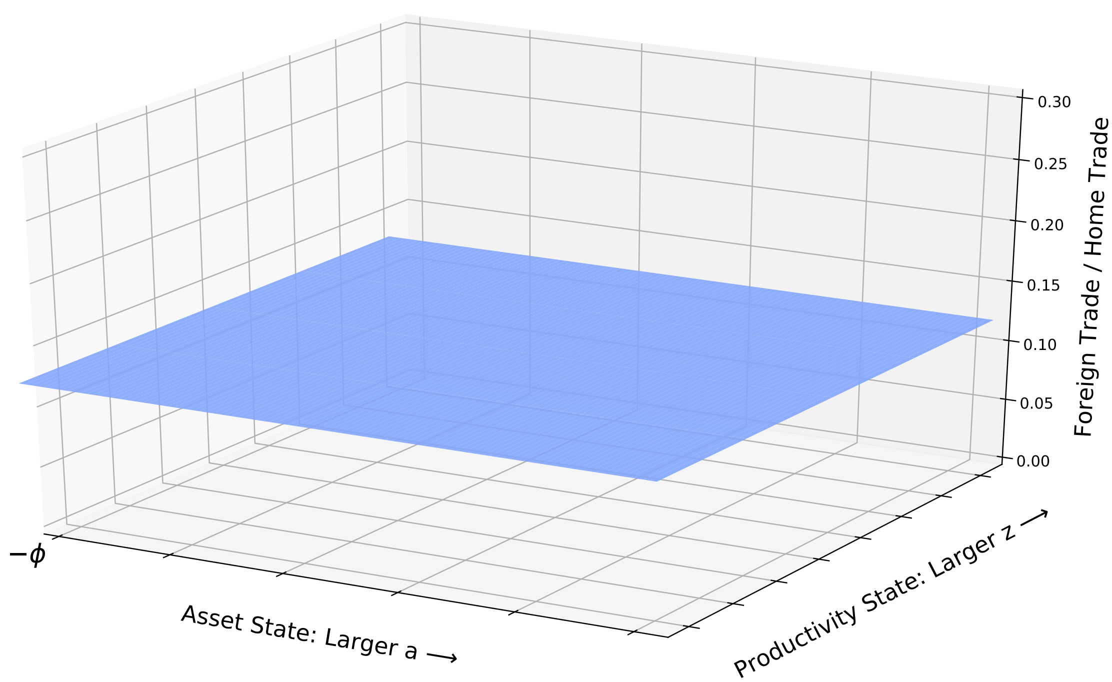


Figure 7: Log Preferences, Trade, $M_{ij}(a, z)/M_{ii}(a, z)$

E. Computing Welfare

First the issue. Given the structure of the value function, a simple Lucas style consumption equivalent measure is not easy to construct. And the key issue that there are multiple value functions for each good consumed and they are aggregated in a non-linear way.

I'm going to construct the following equivalent variation measure. First define the value function of a household in the base period as

$$v_i(a, z;\ p_{ij}, R_i, w_i) \tag{165}$$

And then define the value function for the same states, but at some counter factual prices

$$v_i(a, z;\ p'_{ij}, R'_i, w'_i) \tag{166}$$

Then my equivalent variation measure is a lump sum transfer (in the units of the numeraire or asset), in the base period, such that

$$v_i(a, z;\ p_{ij}, R_i, w_i, \tau) - v_i(a, z; p'_{ij}, R'_i, w'_i) = 0 \tag{167}$$

where the first part with the argument τ means that the household gets τ at the base prices. Then once this τ is solved, this measure can be converted into interesting economics units as one wishes.

Numerically, this is done by simply solving for τ at the old prices such that (167) is satisfied. Notice, this is a transfer given permanently to people of all states. But what this is asking is what τ makes the a, z guy indifferent. Then do this state by state, so the result is a distribution of τs which are a, z specific.

F. Appendix: Endogenous Grid Method

First, I'm going to derive the Euler equation for this model. I'll abstract from the situation in which the HH is at the borrowing constraint.

Focus on the within a variety choice component, the households value function can be written as:

$$v_{ij}(a,z) = \max_{a'} u\left(\frac{R_i a + w_i z - a'}{p_{ij}}\right) + \beta \mathrm{E} v(a', z') \tag{168}$$

then the first order condition associated with this problem is:

$$\frac{u'(c_{ij}(a,z))}{p_{ij}} = \beta \mathrm{E} \frac{\partial v(a',z')}{\partial a'} \tag{169}$$

which is saying that, conditional on a variety choice the left hand side is the loss in consumption units which is $1/p_{ij}$ evaluated at the marginal utility of consumption and then this is set equal to the marginal gain from saving a bit more which is how the value function changes with respect to asset holdings. Now we can arrive at the $\frac{\partial v(a',z')}{\partial a'}$ in the following way, so start from the log-sum expression for the expected value function

$$\mathbb{E}_\epsilon v(a',z') = \sigma_\epsilon \log \left\{ \sum_{j'} \exp\left(\frac{v_{ij}(a',z')}{\sigma_\epsilon}\right) \right\} \tag{170}$$

and then differentiate this with respect to asset holdings which gives:

$$\frac{\partial \mathbb{E}_\epsilon v(a',z')}{\partial a'} = \left(\frac{\sigma_\epsilon}{\sum_{j'} \exp\left(\frac{v_{ij}(a',z')}{\sigma_\epsilon}\right)}\right) \left[\sum_{j'} \exp\left(\frac{v_{ij}(a',z')}{\sigma_\epsilon}\right) \frac{1}{\sigma_\epsilon} \frac{\partial v_{ij}(a',z')}{\partial a'}\right] \tag{171}$$

Then if you look at this carefully and notices how the choice probabilities from (182) are embedded in here, we have:

$$\frac{\partial \mathbb{E}_\epsilon v(a',z')}{\partial a'} = \sum_{j'} \pi_{ij}(a',z) \frac{\partial v_{ij}(a',z')}{\partial a'} \tag{172}$$

and then we can just apply the Envelop theorem to the value functions associated with the discrete choices across the options:

$$\frac{\partial \mathbb{E}_\epsilon v(a',z')}{\partial a'} = \sum_{j'} \pi_{ij}(a',z') \frac{u'(c_{ij}(a',z'))R_i}{p_{ij}} \tag{173}$$

So then putting everything together we have:

$$\frac{u'(c_{ij}(a,z))}{p_{ij}} = \beta R_i \mathrm{E}_{z'}\left[\sum_{j'} \pi_{ij}(a',z')\frac{u'(c_{ij}(a',z'))}{p_{ij}}\right] \tag{174}$$

where this has a very natural form: you set the marginal utility of consumption today equal to the marginal utility of consumption tomorrow adjusted by the return on delaying consumption, and the expected value of the marginal utility of consumption which reflects how the uncertainty over both ones' preference over different varieties and shocks to efficiency units. Taking into account the borrowing constraint then gives the generalized Euler equation from which the endogenous grid method will exploit:

$$\frac{u'(c_{ij}(a,z))}{p_{ij}} = \max\left\{\beta R_i \mathrm{E}_{z'}\left[\sum_{j'} \pi_{ij}(a',z')\frac{u'(c_{ij}(a',z'))}{p_{ij}}\right]\ ,\ u'\left(\frac{R_i a + w_i - \phi_i}{p_{ij}}\right)\right\} \tag{175}$$

6.1. EGM-Discrete Choice Algorithm

Here is a proposed approach. This focuses on just the consumer side in one country i.

0. Set up an asset grid as usual. Then guess (i) a consumption function $g_{c,ij}(a,z)$ for each a, z, and product choice j and (ii) choice specific value function $v_{ij}(a,z)$.

1. Compute the choice probabilities from (182) for each (a,z) combination, given the guessed value functions.

1. Given the consumption function and choice probabilities compute the RHS of (14) first.

2. Then invert to find the new updated consumption choice so

$$c_{ij}(\tilde{a},z) = u^{'-1}\left\{p_{ij}\max\left\{\beta R_i \mathrm{E}_{z'}\left[\sum_{j'} \pi_{ij}(a',z')\frac{u'(c_{ij}(a',z'))}{p_{ij}}\right]\ ,\ u'\left(\frac{R_i a + w_i - \phi_i}{p_{ij}}\right)\right\}\right\} \tag{176}$$

where $u^{'-1}$ is the inverse function of the marginal utility of consumption.

Side note: One of the interesting things about this equation is that the direct j component on the RHS that only affects the consumption choice is through the price. Can this be exploited? We also know the choice probabilities need to sum to one, so is there a way to map the consumption choice into the choice probabilities? Also, can interpolation be done once how p scales things...

3. The key issue in this method is that we have found $c_{ij}(\tilde{a}, z)$ where the consumption function is associated with some asset level that is not necessarily on the grid. The solution is to (i) use the budget constraint and infer $\tilde{a}$ given that a' was chosen above (that's where we started), z, and $c_{ij}(\tilde{a}, z)$. Now we have a map from $\tilde{a}$ to a' for which one can use interpolation to infer the a' chosen given a where a is on the grid.

- Do steps **2.** and **3.** for each j variety choice. This then makes the function $g_{ij}(a, z)$ mapping each state and j choice (today) into a', z' states and then from the budget constraint we have an associated consumption function $g_{c,ij}(a, z)$

4. Compute the $\mathrm{E}\left[v(g_{a,ij}(a, z), z')\right]$. This is performed in the `make_Tv_upwind!` function. It fixes a country j, then works through shocks and asset states today and from the policy function $g_{a,ij}(a, z)$ figures out the asset choice tomorrow. Then the $\mathrm{E}\left[v(g_{a,ij}(a, z), z')\right]$ is (13) over the different variety choices tomorrow (this is the integration over ϵ) multiplied by the probability of z' occurring (this is the integration over z).

5. Given **4.** update the value function using the bellman equation evaluated at the optimal policies:

$$Tv_{ij}(a, z) = u(g_{c,ij}(a, z)) + \beta \mathrm{E}\left[v(g_{a,ij}(a, z), z')\right] \tag{177}$$

6. Compare old and new policy functions, old and new value functions, and then update accordingly.

G. Quality Version of the Model

The utility associated with the choice of variety j is

$$u(c_{ijt}) + \psi_j + \epsilon_{jt}, \tag{178}$$

now there is a shifter ψ_j in utility that depends upon the commodity j chosen. Now I'm going to make the assumption that the quality valuation of a household varies with it's assets and efficiency units. In particular, the assumption will be something along the lines that

$$\psi(a, z, j) \tag{179}$$

So what this means is that a household, depending upon its situation, may have different valuations for a particular commodity. Then, given this assumption on quality, now we are back to the case where the state variables of a individual household are its asset holdings and efficiency units.

Then I'm going to write the value function of a household in country i, after the variety shocks are realized, as

$$v_i(a, z) = \max_j \left\{ \; v_i(a, z, j) + \psi(a, z, j) + \epsilon_j \; \right\} \tag{180}$$

And here, I've pulled out the quality term and the shock term to be more consistent with the code. Specifically, solution methods will work on the $v_i(a, z, j)$s and then reconstruct $v_i(a, z)$ given the shocks and quality specification. The value function conditional on a choice of variety is

$$v_i(a, z, j) = \max_{a'} \left\{ u(c_{ij}) + \beta \, \mathbb{E}[v_i(a', z')] \right\} \tag{181}$$

subject to (7) and (8)

Associated with this are the following choice probabilities for each differentiated good:

$$\pi_{ij}(a, z) = \exp\left(\frac{v_i(a, z, j) + \psi(a, z, j)}{\sigma_\epsilon} \right) \Big/ \Phi_i(a, z), \tag{182}$$

$$\text{where} \quad \Phi_i(a, z) := \sum_{j'} \exp\left(\frac{v_i(a, z, j') + \psi(a, z, j')}{\sigma_\epsilon} \right). \tag{183}$$

And then the expectation of (9) with respect to the taste shocks takes the familiar log-sum form

$$v_i(a, z) = \sigma_\epsilon \log \{\Phi_i(a, z)\} . \tag{184}$$

Or the equivalent representation of this which I'm now using in the code is

$$v_i(a, z) = \sum_{j'} \pi_{ij'}(a, z)\Big[v_i(a, z, j') + \psi(a, z, j') - \sigma_\epsilon \log(\pi_{ij'}(a, z))\Big] \tag{185}$$

how is the true again? Then there is an Euler Equation for each variety choice j. This (I believe) takes a different form so

$$\frac{u'(c_i(a, z, j))}{p_{ij}} = \beta R_i \mathrm{E}_{z'} \left[\sum_{j'} \pi_{ij'}(a', z')\left(\frac{u'(c_i(a', z', j'))}{p_{ij'}} + \frac{\partial \psi(a', z', j')}{\partial a'}\right)\right] . \tag{186}$$

where now the household needs to take into account how its quality valuation of the different commodities change. Now in my current code, this term is not there. I don't, however, have quality be a function of assets, it is only a function of the shock z and, hence, the agent does not think about this.

References

AIYAGARI, S. R. (1994): "Uninsured idiosyncratic risk and aggregate saving," *The Quarterly Journal of Economics*, 109, 659–684.

ANDERSON, S. P., A. DE PALMA, AND J.-F. THISSE (1987): "The CES is a discrete choice model?" *Economics Letters*, 24, 139–140.

——— (1992): *Discrete choice theory of product differentiation*, MIT press.

ARKOLAKIS, C., A. COSTINOT, AND A. RODRÍGUEZ-CLARE (2012): "New Trade Models, Same Old Gains?" *American Economic Review*, 102, 94–130.

ATKESON, A. AND A. T. BURSTEIN (2010): "Innovation, Firm Dynamics, and International Trade," *Journal of Political Economy*, 118, 433–484.

AUER, R., A. BURSTEIN, S. M. LEIN, AND J. VOGEL (2022): "Unequal expenditure switching: Evidence from switzerland," Tech. rep., National Bureau of Economic Research.

BACKUS, D. K. AND G. W. SMITH (1993): "Consumption and real exchange rates in dynamic economies with non-traded goods," *Journal of International Economics*, 35, 297 – 316.

BERRY, S., J. LEVINSOHN, AND A. PAKES (1995): "Automobile prices in market equilibrium," *Econometrica: Journal of the Econometric Society*, 841–890.

BEWLEY, T. (1979): "The optimum quantity of money," Tech. rep., Discussion Paper.

BORUSYAK, K. AND X. JARAVEL (2021): "The distributional effects of trade: Theory and evidence from the united states," Tech. rep., National Bureau of Economic Research.

CALIENDO, L., M. DVORKIN, AND F. PARRO (2019): "Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock," *Econometrica*, 87, 741–835.

CARRÈRE, C., M. MRÁZOVÁ, AND J. P. NEARY (2020): "Gravity without apology: the science of elasticities, distance and trade," *The Economic Journal*, 130, 880–910.

CARROLL, D. R. AND S. HUR (2020): "On the heterogeneous welfare gains and losses from trade," *Journal of Monetary Economics*, 109, 1–16.

CHEN, N. AND D. NOVY (2022): "Gravity and heterogeneous trade cost elasticities," *The Economic Journal*, 132, 1349–1377.

COLICEV, A., J. HOSTE, AND J. KONINGS (2022): "The Impact of a Large Depreciation on the Cost of Living of Rich and Poor Consumers," .

COSTINOT, A. AND A. RODRÍGUEZ-CLARE (2014): "Trade theory with numbers: Quantifying the consequences of globalization," in *Handbook of international economics*, Elsevier, vol. 4, 197–261.

CRAVINO, J. AND A. A. LEVCHENKO (2017): "The distributional consequences of large devaluations," *American Economic Review*, 107, 3477–3509.

EATON, J. AND S. KORTUM (2002): "Technology, geography, and trade," *Econometrica*, 70, 1741–1779.

FAJGELBAUM, P., G. M. GROSSMAN, AND E. HELPMAN (2011): "Income distribution, product quality, and international trade," *Journal of political Economy*, 119, 721–765.

FAJGELBAUM, P. D. AND A. K. KHANDELWAL (2016): "Measuring the unequal gains from trade," *The Quarterly Journal of Economics*, 131, 1113–1180.

FERRIERE, A., G. NAVARRO, AND R. REYES-HEROLES (2022): "Escaping the Losses from Trade: The Impact of Heterogeneity on Skill Acquisition," Tech. rep.

HARROD, R. F. (1936): *Trade cycle. An essay*, Oxford University Press, London.

HEAD, K. AND T. MAYER (2021): *Poor Substitutes?: Counterfactual Methods in IO and Trade Compared*.

HUGGETT, M. (1993): "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of economic Dynamics and Control*, 17, 953–969.

IMROHOROĞLU, A. (1989): "Cost of business cycles with indivisibilities and liquidity constraints," *Journal of Political economy*, 97, 1364–1383.

JACCARD, T. (2023): "Import Demand in Differentiated Products," Tech. rep., PhD Thesis, University of Toronto.

KRUEGER, D., K. MITMAN, AND F. PERRI (2016): "Macroeconomics and Household Heterogeneity," *Handbook of Macroeconomics*, 2, 843–921.

LAGAKOS, D., M. MOBARAK, AND M. E. WAUGH (2023): "The Welfare Effects of Encouraging Rural-Urban Migration," *Econometrica*.

LJUNGQVIST, L. AND T. J. SARGENT (2012): *Recursive macroeconomic theory*, MIT press.

LYON, S. AND M. WAUGH (2019): "Quantifying the Losses from International Trade," .

LYON, S. G. AND M. E. WAUGH (2018): "Redistributing the gains from trade through progressive taxation," *Journal of International Economics*, 115, 185–202.

McFadden, D. (1974): ",Conditional logit analysis of qualitative choice behavior," in *Frontiers in Econometrics*, Academic Press New York, NY, USA.

Melitz, M. J. (2003): "The impact of trade on intra-industry reallocations and aggregate industry productivity," *Econometrica*, 71, 1695–1725.

Mongey, S. and M. Waugh (2022): "Pricing Inequality," .

Mrázová, M. and J. P. Neary (2017): "Not so demanding: Demand structure and firm behavior," *American Economic Review*, 107, 3835–3874.

Nakamura, E. and D. Zerom (2010): "Accounting for incomplete pass-through," *The review of economic studies*, 77, 1192–1230.

Nevo, A. (2000): "A practitioner's guide to estimation of random-coefficients logit models of demand," *Journal of economics & management strategy*, 9, 513–548.

Nord, L. (2022): "Shopping, Demand Composition, and Equilibrium Prices," *Available at SSRN 4178271*.

Novy, D. (2013): "International trade without CES: Estimating translog gravity," *Journal of International Economics*, 89, 271–282.

Sangani, K. (2022): "Markups Across the Income Distribution: Measurement and Implications," *Available at SSRN 4092068*.

Simonovska, I. and M. E. Waugh (2014): "The Elasticity of Trade: Estimates and Evidence," *Journal of International Economics*, 92, 34–50.

Waugh, M. E. (2019): "The Consumption Response to Trade Shocks: Evidence from the US-China Trade War," Working Paper 26353, National Bureau of Economic Research.