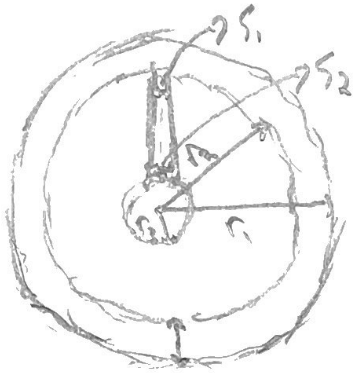
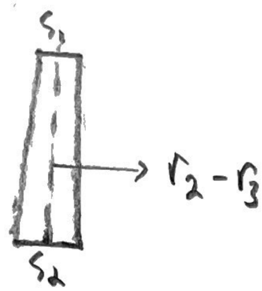
We assume uniform density of the wheel. As it is of uniform thickness, the geometry reduces as follows:



Where only one of five spokes is shown. We first work to determine the total area of the wheel:

Area of Hoop: $(\pi r_1^2) - (\pi r_2^2) = \pi(r_1^2 - r_2^2)$

Area of spoke:



$$\frac{s_1 + s_2}{2}(r_2 - r_3)$$

The total surface area is then $A = \pi(r_2^2 - r_1^2) + 5\left(\frac{s_1+s_2}{2}\right)(r_2-r_3)$ and the density is:

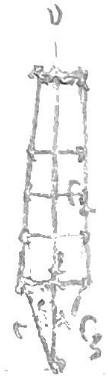
$$\rho = \frac{M}{A} = \frac{M}{\pi(r_1^2 - r_2^2) + 5\left(\frac{s_1+s_2}{2}\right)(r_2 - r_3)}$$

Now, we determine the moment of inertia I. First, we treat the hoop:

We have $I = \int r^2 dm$, where $dm = \rho dA = \rho \cdot 2\pi r \cdot dr$

$$I = 2\pi\rho\int_{r_2}^{r_1} r^3 dr = \frac{1}{2}\pi\rho r^4\Big|_{r_2}^{r_1} = \frac{1}{2}\pi\rho(r_1^4 - r_2^4)$$

We next address the inertia of a spoke, treated as a trapezoid:



It will be approximated that each point on a cross section is r from the center. Then, an area element is:

$$2\left(\frac{\frac{1}{2}s_2 - \frac{1}{2}s_1}{r_2 - r_3}\right)\cdot r$$

$$dA = l\,dr,$$

First, we notice that the width changes linearly with r with a slope $m = -\frac{s_2 - s_1}{r_2 - r_3}$. At $r = r_3$, $l = s_2$, so:

$$l = s_2 - \left(\frac{s_2 - s_1}{r_2 - r_3}\right)(r - r_3)$$

Therefore:

$$I = \int r^2\,dm = \int r^2 \rho\, dA = \int r^2 \rho \left(s_2 - \left(\frac{s_2 - s_1}{r_2 - r_3}\right)(r - r_3)\right) dr$$

$$= \rho s_2 \int_{r_3}^{r_2} r^2\,dr - \rho\left(\frac{s_2 - s_1}{r_2 - r_3}\right)\int_{r_3}^{r_2} r^3\,dr$$

$$+ \rho\, r_3 \left(\frac{s_2 - s_1}{r_2 - r_3}\right)\int_{r_3}^{r_2} r^2\,dr$$

$$= \rho\left[\left(s_2 + r_3\left(\frac{s_2 - s_1}{r_2 - r_3}\right)\right)\frac{r^3}{3}\Bigg|_{r_3}^{r_2} - \frac{s_2 - s_1}{r_2 - r_3}\,\frac{r^4}{4}\Bigg|_{r_3}^{r_2}\right]$$

The total inertia is then:

$$I_{net} = I_{hoop} + 5 I_{stroke}$$

$$= \frac{1}{2}\pi\rho(r_1^4 - r_2^4) + \rho\left[\frac{1}{3}\left(s_2 + r_3\left(\frac{s_2 - s_1}{r_2 - r_3}\right)\right)(r_2^3 - r_3^3) - \frac{s_2 - s_1}{4(r_2 - r_3)}(r_2^4 - r_3^4)\right]$$

$$= \frac{M}{\pi(r_2^2 - r_1^2) + 5\left(\frac{s_1 + s_2}{2}\right)(r_2 - r_3)}\left[\frac{1}{2}\pi(r_1^4 - r_2^4) + \frac{5}{3}\left(s_2 + r_3\left(\frac{s_2 - s_1}{r_2 - r_3}\right)\right)(r_2^3 - r_3^3) - \frac{5(s_2 - s_1)}{4(r_2 - r_3)}(r_2^4 - r_3^4)\right]$$

With $r_1 = 94.97 \pm 0.05$ mm, $r_1 - r_2 = 20.00$ mm, $r_3 = 0.15$ mm, $s_1 = 8.66 \pm 0.02$ mm, and $s_2 = 11.59 \pm 0.02$ mm, ~~and~~ $M = 240.30 \pm 0.05$ g:

$$r_2 = r_1 - (r_1 - r_2) = 94.97 \pm 0.05 - 20.00 = 74.97 \pm 0.05 \text{ mm}$$

and:

$$\rho_{best} = \frac{240.30\,g}{\pi(94.97^2 - 74.97^2)\,mm^2 + 5\left(\frac{8.66 + 11.59}{2}\right)mm\,(74.97 - 15.00)\,mm}$$

$$= 0.017\ g/mm^2$$