We begin with the coupled differential equations:

$$\frac{d\tilde{a}}{d\tau} = -\frac{16}{5}\frac{1}{\tilde{a}^3}\frac{1}{(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

$$\frac{de}{d\tau} = -\frac{76}{15}\frac{1}{\tilde{a}^4}\frac{e}{(1-e^2)^{5/2}}\left(1+\frac{121}{304}e^2\right)$$

Where $\tilde{a}(\tau)$ is the dimensionless quantity $\tilde{a}(\tau) = \frac{a}{R_*}$, and $R_*^3 = 4G^3\mu m^2/c^6$ and $\tau = \frac{ct}{R_*}$ is a dimensionless time unit.

To decouple this, we solve $\frac{d\tilde{a}}{de}$:

$$\frac{d\tilde{a}}{de} = \frac{\left(\frac{d\tilde{a}}{d\tau}\right)}{\left(\frac{de}{d\tau}\right)} = \frac{12}{19}\tilde{a}\,\frac{\left[1+\frac{73}{24}e^2+\frac{37}{96}e^4\right]}{e(1-e^2)(1+\frac{121}{304}e^2)}$$

$$\int\frac{d\tilde{a}}{\tilde{a}} = \int\frac{12}{19}\,\frac{\left[1+\frac{73}{24}e^2+\frac{37}{96}e^4\right]}{e(1-e^2)(1+\frac{121}{304}e^2)}\,de$$

$$\ln(\tilde{a}) = \frac{12}{19}\int\frac{\left[1+\frac{73}{24}e^2+\frac{37}{96}e^4\right]}{e(1-e^2)(1+\frac{121}{304}e^2)}\,de,$$ letting $u = e^2$; $du = 2e\,de$:

$$\ln(\tilde{a}) = \frac{12}{38}\int\frac{\left[1+\frac{73}{24}u+\frac{37}{96}u^2\right]}{e^2(1-u)(1+\frac{121}{304}u)}\,du.$$

$$\frac{38}{12}\ln(\tilde{a}) = \int\frac{\left[1+\frac{73}{24}u+\frac{37}{96}u^2\right]}{u(1-u)(1+\frac{121}{304}u)}\,du$$

$$\int \frac{\left[1+\frac{73}{24}u+\frac{37}{46}u^2\right]}{u(1-u)(1+\frac{121}{304}u)}du = \int \frac{A}{u} + \frac{B}{(1-u)} + \frac{C}{1+\frac{121}{304}u}\,du$$

$$1+\frac{73}{24}u+\frac{37}{46}u^2 = \left(\frac{A}{u} + \frac{B}{(1-u)} + \frac{C}{1+\frac{121}{304}u}\right)\left(u(1-u)(1+\frac{121}{304}u)\right)$$

$$\begin{aligned}1+\frac{73}{24}u+\frac{37}{46}u^2 = {} & A(1-u)(1+\frac{121}{304}u) \\ & + B(u)(1+\frac{121}{304}u) \\ & + C(u)(1-u)\end{aligned}$$

$$\begin{aligned}1+\frac{73}{24}u+\frac{37}{46}u^2 = {} & A(1-\frac{183}{304}u-\frac{121}{304}u^2) \\ & + B(u+\frac{121}{304}u^2) \\ & + C(u-u^2)\end{aligned}$$

$$\begin{aligned}1+\frac{73}{24}u+\frac{37}{46}u^2 = {} & A+u(-\frac{183}{304}A+B+C) \\ & + u^2(-\frac{121}{304}A+\frac{121}{304}B-C)\end{aligned}$$

yielding the system of equations:

$$A = 1$$

$$-\frac{183}{304}A + B + C = \frac{73}{24}$$

$$-\frac{121}{304}A + \frac{121}{304}B - C = \frac{37}{46}$$

This can be solved for:

$$A = 1 \quad B = \frac{19}{6} \quad C = \frac{145}{304}$$, so:

$$\frac{38}{12}\ln(\tilde{a}) = \int \frac{du}{u} + \frac{19}{6}\int \frac{du}{1-u} + \frac{145}{304}\int \frac{du}{1+\frac{121}{304}u}$$

$$\frac{19}{6}\ln(\tilde{a}) = \ln(u) - \frac{19}{6}\ln(1-u) + \frac{145}{121}\ln\left(1+\frac{121}{304}u\right) + C$$

$$\ln(\tilde{a}) = \frac{6}{19}\ln(u) - \ln(1-u) + \frac{870}{2{,}299}\ln\left(1+\frac{121}{304}u\right) + C$$

$$\ln(\tilde{a}) = \ln\left[\frac{u^{\frac{6}{19}}\left(1+\frac{121}{304}u\right)^{\frac{870}{2299}}}{1-u}\right] + C$$

$$\tilde{a}(e) = \frac{C_0 u^{\frac{6}{19}}\left(1+\frac{121}{304}u\right)^{\frac{870}{2299}}}{1-u}$$, and recalling that $u = e^2$:

$$\tilde{a}(e) = \frac{C_0 e^{\frac{12}{19}}\left(1+\frac{121}{304}e^2\right)^{\frac{870}{2299}}}{1-e^2}$$

Now, we can decouple our second equation:

$$\frac{de}{d\tau} = -\frac{76}{15}\frac{1}{\tilde{a}^4}\frac{e}{(1-e^2)^{5/2}}\left(1+\frac{121}{304}e^2\right)$$

Where $\tilde{a} = \dfrac{c_0 e^{12/19}\left(1+\frac{121}{304}e^2\right)^{870/2299}}{1-e^2}$; so:

$$\frac{de}{d\tau} = -\frac{76}{15c_0^4}\cdot\frac{(1-e^2)^{\cancel{4}\,3/2}}{e^{48/19}\left(1+\frac{121}{304}e^2\right)^{3480/2299}}\cdot\frac{\cancel{e}}{\cancel{(1-e^2)^{5/2}}}\cancel{\left(1+\frac{121}{304}e^2\right)}$$

$$\frac{de}{d\tau} = -\frac{76}{15c_0^4}\cdot\frac{(1-e^2)^{3/2}}{e^{29/19}\cdot(1+e^2)^{1189/2299}}$$

$$d\tau = -\frac{15c_0^4}{76}\cdot\frac{e^{29/19}\cdot(1+e^2)^{\frac{1189}{2299}}}{(1-e^2)^{3/2}}\,de$$

This is non-integrable; we will try to solve it with a fourth order Runga Kutta method; we will examine what we do at each step. Our time step will be Δt; then:

$$k_1 = \Delta t\left(-\frac{76}{15c_0^4}\cdot\frac{(1-e_n^2)^{3/2}}{e_n^{29/19}\cdot(1+e_n^2)^{1189/2299}}\right) \quad \text{(Euler Step)}$$

$$k_2 = \Delta t\Bigg(-\frac{76}{15c_0^4}\cdot\frac{(1-(e_n+\frac{1}{2}k_1)^2)^{3/2}}{(e_n+\frac{1}{2}k_1)^{29/19}\cdot(1+(e_n+\frac{1}{2}k_1)^2)^{1189/2299}}$$

$$k_3 = \Delta t\, f(e_n + \tfrac{1}{2}k_2)$$

$k_4 = \Delta t\, f(e_n + \tfrac{1}{2}k_3)$, and finally:

$$e_{n+1} = e_n + \frac{1}{6}k_1 + \frac{1}{3}k_2 + \frac{1}{3}k_3 + \frac{1}{6}k_4 + O(h^5)$$