Gravitational Waves

Gravitational wave astronomy

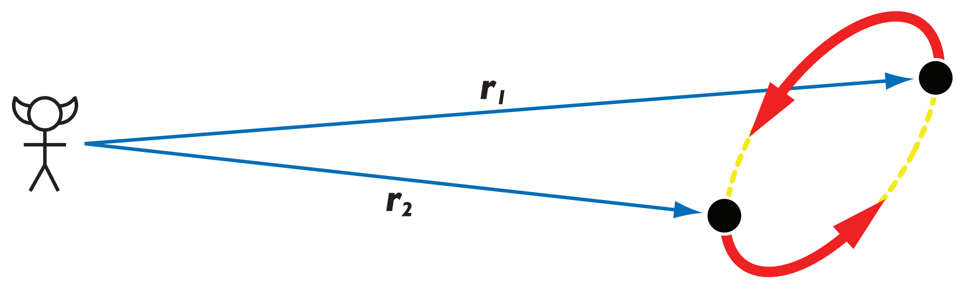
• Virtually everything you and I know about the Cosmos has been discovered via electromagnetic observations.

• Some information has recently been gleaned from astro-particle observations (neutrinos and cosmic rays)

• We are now entering an era where we are using a wide variety of detectors to probe the Cosmos with *gravity*.

• A full relativistic derivation of the existence and action of gravitational waves is beyond our scope here, but can be obtained by linearizing the field equations of general relativity. Here let's convince our intuition.

• The idea that gravitational information can *propagate* is a consequence of special relativity: nothing can travel faster than the ultimate speed limit, c



• Imagine observing a distant binary star and trying to measure the gravitational field at your location. It is the sum of the field from the two individual components of the binary, located at distances r_1 and r_2 from you.

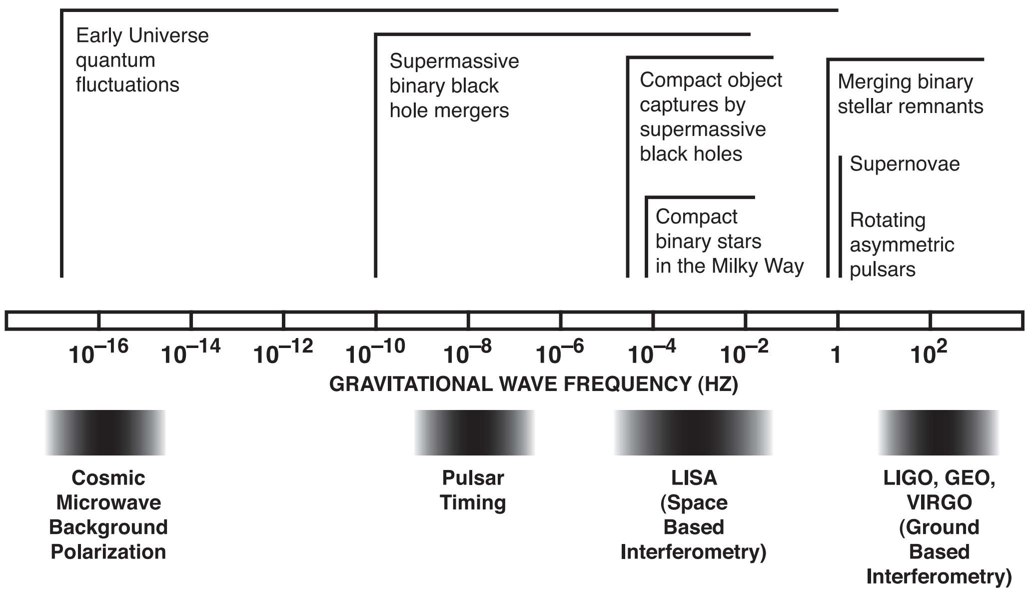
• As the binary evolves in its orbit, the masses change their position with respect to you, and so the gravitational field must change. It takes time for that information to propagate from the binary to you — $t_{propagate} = d/c$, where d is the luminosity distance to the binary.

• The propagating effect of that information is known as *gravitational radiation*, which you should think of in analogy with the perhaps more familiar *electromagnetic radiation*

• Far from a source (like the aforementioned binary) we see the gravitational radiation field oscillating and these propagating oscillating disturbances are called *gravitational waves*.

• Like electromagnetic waves

- ▷ Gravitational waves are characterized by a wavelength λ and a frequency f
- ▷ Gravitational waves travel at the speed of light, where $c = \lambda \cdot f$
- ▷ Gravitational waves come in two polarization states (called $+$ and $\times$)

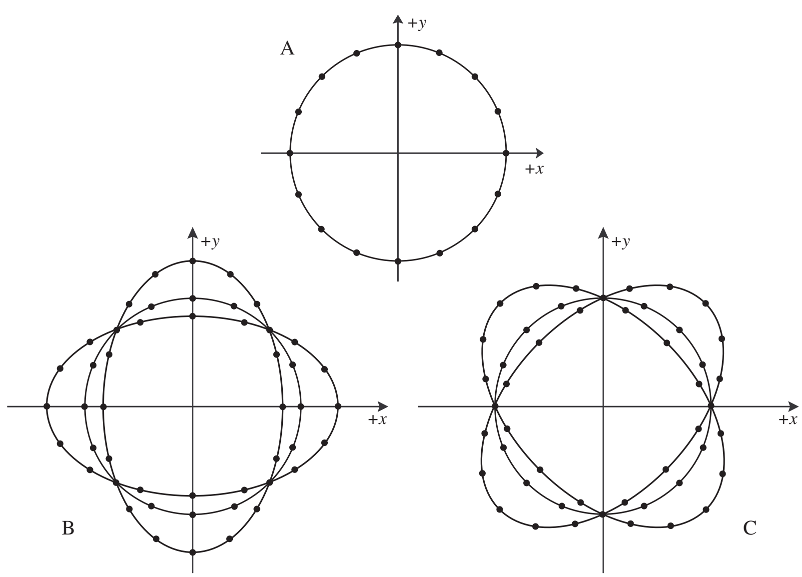


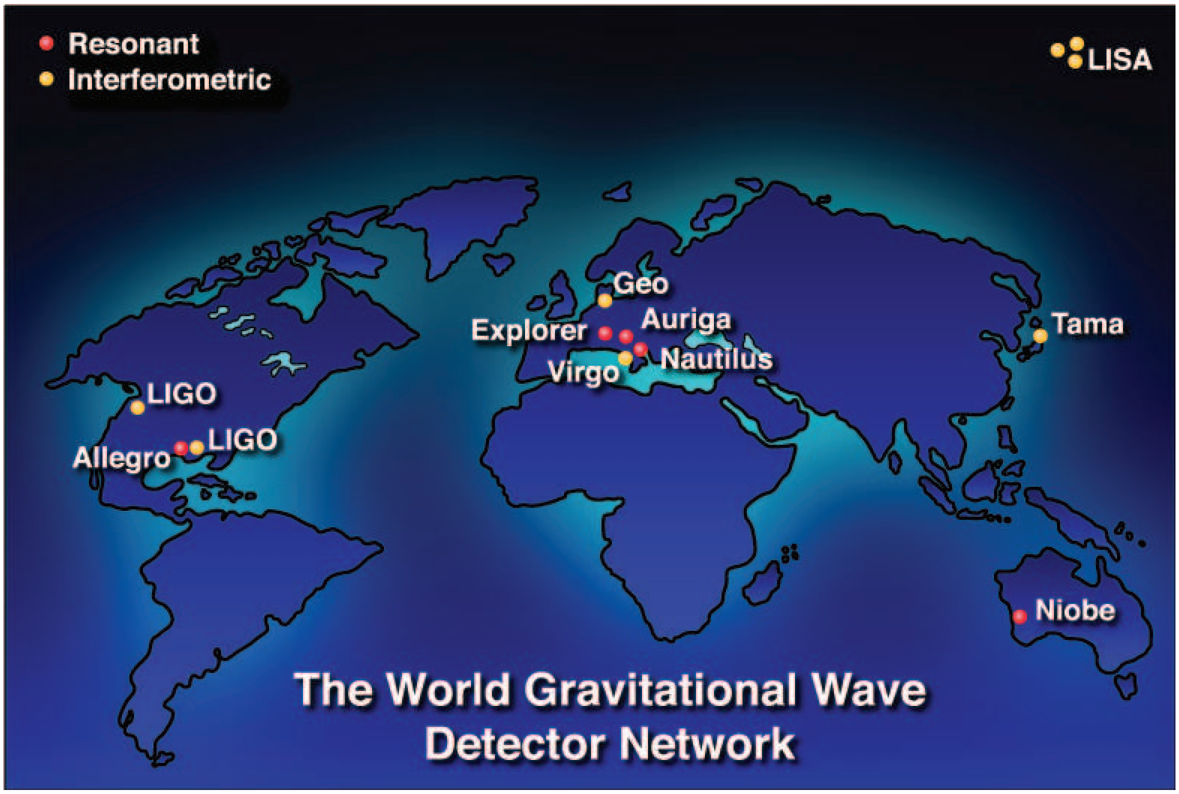
Gravitational wave observatories

• If you want to detect gravitational waves, you have to know what their physical effect on matter is — what do they do to things you can tape together and call a "detector"

• The fundamental influence of gravitational waves is to *change the proper distance between points in spacetime*. That means if you have two free particles, and monitor the spacetime interval between them, a gravitational wave will change that interval as it passes.

• The different polarization states have different effects on an array of particles. Imagine a ring of free test particles. The + and × states are named by the distortions they produce on the ring. For a gravitational wave propagating into or out of the page, the effects are shown below for a + polarization (in A) and for a × polarization (in B)





• Gravitational wave detectors then are devices that enable you to measure these kinds of distortions. Since the 1960's man-made detectors have focused on three fundamental technologies

▷ ***Bar detectors.*** Here the elements of the ring should be thought of as the atoms in a large aluminum cylinder (or, in the limiting case, the mass elements on the ends of the cylinder). Roughly sensitive to gravitational waves with frequencies near the resonant frequencies of the bar.

▷ ***Interferometers.*** Here the elements of the ring should be thought of as the end mirrors in a traditional Michelson interferometer. Roughly sensitive to gravitational waves with wavelengths comparable to the length of the interferometer arms.

▷ ***Astrophysical detectors.*** using astrophysical systems, such as pulsars and the Cosmic Microwave Background, to detect extremely low-frequency gravitational waves.

• For the past 40 years, a world network of gravitational wave detectors has been growing up.

▷ **Bar Network.** The first network consisted of resonant bar detectors around the world, including `Niobe`, `Explorer`, `Nautilus`, `Auriga` and `Allegro`. With the advent of broadband interferometric detectors, these instruments have all but shut down operations.

▷ **Interferometer Network.** Beginning in the late 1980s, a network of ground based interferometric observatories sprang up and is still growing. Currently it is comprised of two LIGO detectors in the United States, the VIRGO detector in Italy, and the GEO detector in Germany. The TAMA detector in Tokyo has been shut-down, but the Japanese are now ramping up to build a new detector called LCGT (Large Cryogenic Gravitational-wave Telescope).

▷ **Space-based Interferometers.** Space based gravitational wave detectors like LISA or OMEGA are still at least a decade away.

▷ **Pulsar Timing/CMB Polarization.** There is a growing effort to search for low frequency gravitational waves. Most notably this includes the radio pulsar timing collaboration, `NanoGrav`. There is also a dedicated collaboration of microwave astronomers building telescopes in Chile (`ACTPol`) and the South Pole (SPT) capable of detecting gravitational wave imprints in the polarization of the cosmic microwave background.

Detector Sensitivity

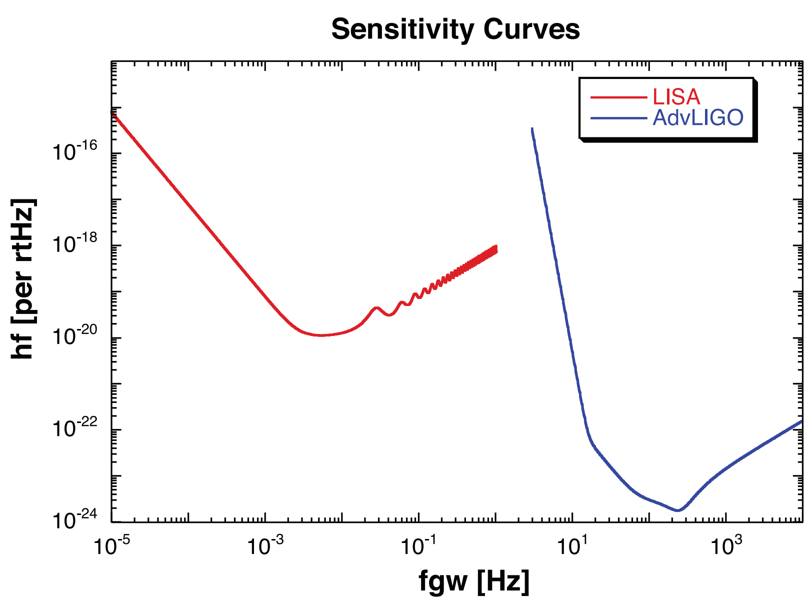
• If you build a detector, the principle goal is to determine what gravitational waves the instrument will be sensitive to. We characterize the noise in the instrument and the instrument's response to gravitational waves using a *sensitivity curve*.

• Sensitivity curves plot the strength a source must have, as a function of gravitational wave frequency, to be detectable. There are two standard curves used by the community:

▷ **Strain Sensitivity.** This plots the gravitational wave strain amplitude h versus gravitational wave frequency f.

▷ **Strain Spectral Amplitude.** This plots the square root of the power spectral density, $h_f = \sqrt{S_h}$ versus gravitational wave frequency f. The power spectral density is the power per unit frequency and is often a more desirable quantity to work with because gravitational wave sources often evolve dramatically in frequency during observations.

• The sensitivity for LIGO and LISA are shown below. Your own LISA curves can be created using the online tool at `www.srl.caltech.edu/~shane/sensitivity/MakeCurve.html`.



• If you are talking about observing sources that are evolving slowly (the are approximately monochromatic) then the spectral amplitude and strain are related by

$$h_f = h\sqrt{T_{obs}}$$

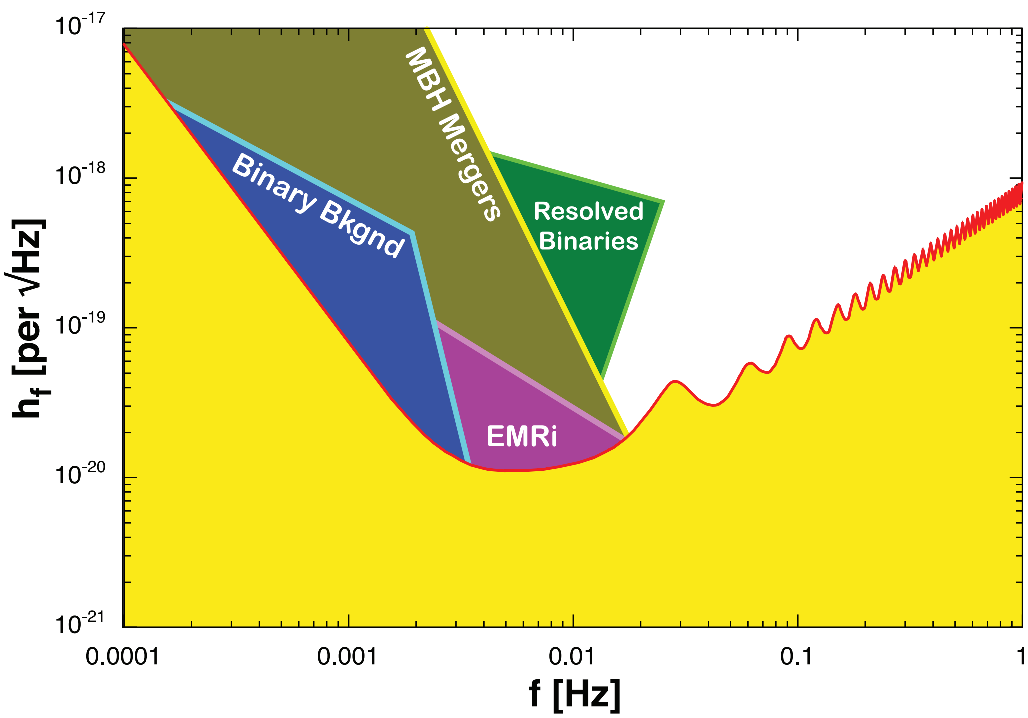
• If you are talking about a short-lived ("bursting") source with a characteristic width τ, then to a good approximation the bandwidth of the source in frequency space is $\Delta f \sim \tau^{-1}$ and the spectral amplitude and strain are related by

$$h_f = \frac{h}{\sqrt{\Delta f}} = h\sqrt{\tau}$$

• The strain sensitivity of a detector, h^D, builds up over time. If you know the observation time T_{obs} and the spectral amplitude curve (like those plotted above) you can convert between the two via

$$h_f^D = h^D\sqrt{T_{obs}}$$

• Sensitivity curves are used to determine whether or not a source is detectable. Rudimentarily, if the strength of the source places it above the sensitivity curve, it can be detected!



• The fundamental metric for detection is the SNR ρ (signal to noise ratio) defined as

$$\rho \sim \frac{h_f^{src}}{h_f^D}$$

• To use this you need to know how to compute h_f^{src}.

Pocket Gravitational Waves: Source Basics

• Binaries are expected to be among the most prevalent of gravitational wave sources. It is useful to have a set of pocket formulae for quickly estimating their characteristics.

• Consider a gravitational wave binary with masses m_1 and m_2, in a circular orbit with gravitational wave frequency $f = 2f_{orb}$, then:

$$\text{chirp mass} \qquad \mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

$$\text{scaling amplitude} \qquad h_o = 4\frac{G}{c^2}\frac{\mathcal{M}_c}{D}\left(\frac{G}{c^3}\pi f \mathcal{M}_c\right)^{2/3}$$

$$\text{chirp} \qquad \dot{f} = \frac{96}{5}\frac{c^3}{G}\frac{f}{\mathcal{M}_c}\left(\frac{G}{c^3}\pi f \mathcal{M}_c\right)^{8/3}$$

• The *chirp* indicates that as gravitational waves are emitted, they carry energy away from the binary. The gravitational binding energy decreases, and the orbital frequency increases. The *phase* $\phi(t)$ of the orbit evolves in time as

$$\phi(t) = 2\pi\left(f\ t + \frac{1}{2}\dot{f}\ t^2\right) + \phi_o\ ,$$

where $\dot{f}$ is the chirp given above, and ϕ_o is the initial orbital phase of the binary.

Luminosity Distance from Chirping Binaries

Suppose I can measure the chirp $\dot{f}$ and the gravitational wave amplitude h_o. The chirp can be inverted to give the chirp mass:

$$\mathcal{M}_c = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3}\dot{f}\right]^{3/5}$$

If this chirp mass is used in the amplitude equation, one can solve for the *luminosity distance* D:

$$D = \frac{5}{24\pi^2}\frac{c}{h_o}\frac{\dot{f}}{f^3}$$

This is a method of measuring the luminosity distance using *only gravitational wave observables*! This is extremely useful as an independent distance indicator in astronomy.

Working with chirping binaries

• Chirping binaries are neither monochromatic, nor are they bursting. They are leisurely evolving through the observing band. How can they be plotted on a sensitivity curve?

• First we must estimate what chirping is even detectable. We can use the chirp $\dot{f} = df/dt$ to estimate the time it takes a circularized binary to evolve between any two frequencies f_1 and f_2:

$$\Delta t = \kappa \left[f_1^{-8/3} - f_2^{-8/3} \right] \qquad \text{where} \qquad \kappa = \frac{5}{256} \left(\frac{G}{c^3} \mathcal{M}_c \right)^{-5/3} \pi^{-8/3}$$

• The *frequency resolution* of the detector is $\delta f = 1/T_{obs}$. A chirp can be detected if the frequency f_2 at the end of the observation is

$$f_2 \gtrsim f_1 + \delta f$$

• The frequency at which a source spends $\Delta t = T_{obs}$ crossing δf is called the *stationary frequency*, f_s.

• To find the stationary frequency, set:

- ▷ $\Delta t = T_{obs}$
- ▷ $f_2 = f_1 + \delta f = f_1(1 + \delta f / f_1)$

then

$$\Delta t = \kappa \left[f_1^{-8/3} - f_2^{-8/3} \right] \qquad \rightarrow \qquad T_{obs} = \kappa \left[f_1^{-8/3} - f_1^{-8/3} (1 + \delta f / f_1)^{-8/3} \right]$$

$$\rightarrow \qquad T_{obs} = \kappa f_1^{-8/3} \left[1 - (1 + \delta f / f_1)^{-8/3} \right] \simeq \kappa f_1^{-8/3} \left[1 - \left(1 - \frac{8}{3} \frac{\delta f}{f_1} \right) \right]$$

$$\rightarrow \qquad T_{obs} \simeq \kappa f_1^{-8/3} \left[\frac{8}{3} \frac{\delta f}{f_1} \right] = \kappa f^{-11/3} \left[\frac{8}{3} \frac{1}{T_{obs}} \right]$$

$$\rightarrow \qquad f_1 = f_s \simeq \left[\frac{8}{3} \frac{\kappa}{T_{obs}^2} \right]^{3/11}$$

• For all sources with frequency $f \lesssim f_s$, they appear monochromatic over the observing time. If $f \gtrsim f_s$, then the time it spends between f_1 and $f_2 = f_1 + \delta f$ (the *time in a frequency bin*, t_{bin}) is given by

- ▷ $\Delta t = T_{obs}$
- ▷ $\delta f = 1/T_{obs}$
- ▷ $f_2 = f_1 + \delta f$

$$t_{bin} = \frac{8}{3} \kappa \frac{f_1^{-11/3}}{T_{obs}}$$

where we have assumed $\delta f \ll f$

• At a given frequency, the number of cycles N_{cy} of gravitational radiation that a source emits depends on the time t_{bin} that a source spends in the frequency bin around that frequency

$$N_{cy} = f \cdot t_{bin} \simeq \frac{8}{3}\frac{\kappa}{T_{obs}} f^{-8/3}$$

Strength of Gravitational Wave Binaries

• We have all the tools in hand at this point to make some basic estimates of source strength for binaries.

• At this stage, our goal is to determine the strain h^{src} that could be plotted on a strain spectral amplitude sensitivity curve (h_f^D vs. f)

• There are three basic cases we can address at this point:

- ▶ **Monochromatic binaries.** This are binaries for which $f \lesssim f_s$, so they do not chirp appreciably during an observation of length T_{obs}. In this case the strength is

$$h_f^{mono} = h_o \cdot \sqrt{T_{obs}}$$

- ▶ **Chirping binaries.** These sources have $f \gtrsim f_s$ and evolve across many frequencies during an observation time. For these sources, the strain *must be calculated and plotted at each frequency!* The value to plot depends on the number of cycles N_{cy} emitted in a give frequency bin, so

$$h_f^{chirp} = h_o \cdot \sqrt{t_{bin}}$$

- ▶ **Bursting sources.** For short events with time profiles of width τ_b and maximum burst amplitude h_b the central frequency of the burst is $f \sim 1/\tau_b$. For these sources, the strength is given by

$$h_f^{burst} = h_b \cdot \sqrt{\tau_b}$$