Liam Keeley
ENWT Chapter 2 Summary
August 5, 2022

Chapter two of Emmy Noether's Wonderful Theorem introduces the functional as a mapping from a set of functions $x(t)$ to the real numbers. The chapter gives examples, such as the distance functional, Fermat's Principle, and the earnings functional from finance.

One especially interesting example is free fall in gravity-free space-time. This example states that a formulation of special relativity states that of all the possible paths that a particle could take from point A in space-time to point B, the path actually taken is that which maximizes the proper time. That is:

$$\begin{aligned}\Delta\tau &= \int_a^b d\tau, \text{Where } d\tau \text{ is the invariant space time interval:}\\ c^2 d\tau^2 =& c^2 dt^2 - dx^2 - dy^2 - dz^2\end{aligned}$$

The chapter ends with the following formal definition of a functional:

Definition: A **functional**, Γ, is a mapping from a well-defined set of functions to the real numbers. The domain of the mapping is the set of all twice differentiable functions $q^\mu(t)$ on the closed interval $[a, b]$, where the label μ distinguishes N dependent variables. The mapping is given by a definite integral of the form:

$$\Gamma = \int_a^b L(t, q^\mu, \dot{q^\mu})dt$$

Where the function $L(t, q^\mu, \dot{q^\mu})$ is called the **Lagrangian** of the functional.

It is interesting to note that this definition is obviously anticipating Hamilton's principle and the need for Euler-Lagrange equations. For example, why should the domain of a functional need to be the set of twice differentiable functions? No reason except that the Euler-Lagrange equations require that each $q^\mu(t)$ be twice differentiable.