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ENWT Chapter 5 Summary
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5.1 Section 5.1 introduces Noether's Theorem. From the alternate version of the Invariance Identity:

$$-(\zeta^\mu - \dot{q}^\mu \tau)\left[\frac{\partial L}{\partial q^\mu} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}^\mu}\right] = \frac{d}{dt}\left[p_\mu \zeta^\mu - H\tau\right]$$

and if the functional is also extremal so that $\frac{\partial L}{\partial q^\mu} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}^\mu} = 0$, then it is clear, if we account for the divergence term, that:

$$\frac{d}{dt}\left[p_\mu \zeta^\mu - H\tau - F\right] = 0$$

This is **Noether's Theorem**, stated formally as:

If under the infintesimal transformation:

$$\begin{aligned} t' =& t + \epsilon\tau + ... \\ q^{\mu\prime} =&^\mu + \epsilon\zeta^\mu + ... \end{aligned}$$

The functional:

$$\Gamma = \int_a^b L(t, q^\mu, \dot{q^\mu})dt$$

is both invariant and extremal, then the following conservation law holds:

$$p_\mu \zeta^m u - H\tau - F = C$$

for some constant C.

5.4 Section 5.4 gives a method for determining the generators of infinitesimal transformations that result in invariance given a Lagrangian. The method essentially follows by assuming the invariance identity; then, because the generators depend on the generalized coordinates and independent variable, but not the generalized velocities, the identity must hold for any generalized velocities. Upon writing out all explicit and implicit generalized velocities, the resulting terms can be factored in terms of all distinct terms with generalized velocities. Because these velocities are unconstrained, each factored term must separately be zero. These equations, called **Killing equations**, can be solved for the generators τ and ζ^{μ}.

It should be noted that the generalized velocities may show up implicitly in some or all of: the generalized momenta, the Hamiltonian, the Lagrangian and its derivatives, and the time derivatives of the generators τ and ζ^{μ}.