Dehn surgeon school: Nathan's HW 1 Wednesday, July 17, 2019.

References and links for the second computational session, which demonstrated a wide variety of tools for studying Dehn surgery, hyperbolic geometry, Floer homology, foliations, character varieties, and Heegaard splittings, all of which can be used together via Python/SageMath via the computop/sage Docker image.

- http://snappy.computop.org
- http://www.sagemath.org
- http://bitbucket.org/t3m/sagedocker
- http://doi.org/10.7910/DVN/LCYXPO
- http://github.com/bzhan/bfh_python
- http://regina-normal.github.io

1. Get SageMath and SnapPy working together on your laptop, for example using the computop Docker image. Alternatively, from any of the physical ICERM terminals you can access it via http://icerm2.icerm.brown.edu:8888.

2. You can get a knot of 14 or fewer crossings in SnapPy by doing:

   ```
   knots = snappy.HTLinkExteriors(cusps=1)
   E = knots.random()
   ```

   Use verified computation as described here: http://snappy.computop.org/verify.html to prove it is hyperbolic and to compute its volume to a provably correct 250 decimal places. By Mostow rigidity this number is an invariant of the knot exterior and hence of the knot itself. (There are a handful of non-hyperbolic links in this range, so you're very unlike to pick one of them and so be unable to complete this problem!)

3. Look at the documentation for HTLinkExteriors by typing

   ```
   ?snappy.HTLinkExteriors
   ```

   to figure out how to pick a random **10 crossing** knot. Download the software of http://doi.org/10.7910/DVN/LCYXPO and use it to find coorientable taut/Reebless foliations on at least one Dehn surgery of your random knot.

4. Python programming practice:

   Use http://snappy.computop.org/spherogram.html to write a Python function to produce a link projection of the $(a_1, a_2, \ldots, a_k)$ pretzel link. For the $(-2,3,7)$ pretzel knot, write a procedure that searches for the two slopes of the two len space Dehn surgeries discovered by Fintushel-Stern. Use Regina to determine which lens spaces these are. Can you find lens space surgeries on other pretzel knots?

5. Look at the list of software that is part of the computop Docker image. See if you can compute something interesting with one of them.

6. The webpage http://computop.org lists a wide variety of computational tools in low-dimensional topology. Find one that is relevant to your own work and try to get it working in your Docker container.