CS 474 Image Processing and Interpretation

Assignment 3 Discrete Fourier Transform

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Experiment 1

(1.a)

Compute the DFT of the signal f = [2, 3, 4, 4].

In image processing the discrete Fourier transform converts an image from the spatial domain to the frequency domain. This enables techniques for filtering and feature analysis. Computing the DFT has a time complexity of $O(N^2)$. To improve this computation, we will use the fast Fourier transform (FFT). The FFT uses a divide and conquer approach by dividing the problem into sub-problems and solving each recursive. This approach is more efficient with a time complexity of $O(Nlog(N))$. Calculating the forward DFT is given by:

$$F(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x)e^{\frac{-j2\pi ux}{N}}$$

Computing the DFT of our input signal will produce an imaginary part and a real part. The complex numbers allow the DFT to represent the amplitude and phase of the frequency. The real part of our input signal is: $(3.25, -0.5, -0.25, -0.5)$ and the imaginary part is: $(0, -0.25, 0, 0.25)$. Calculating the magnitude of the DFT represents the strength of each frequency component. A plot of the real part, imaginary part, and the magnitude is shown in Fig.1.

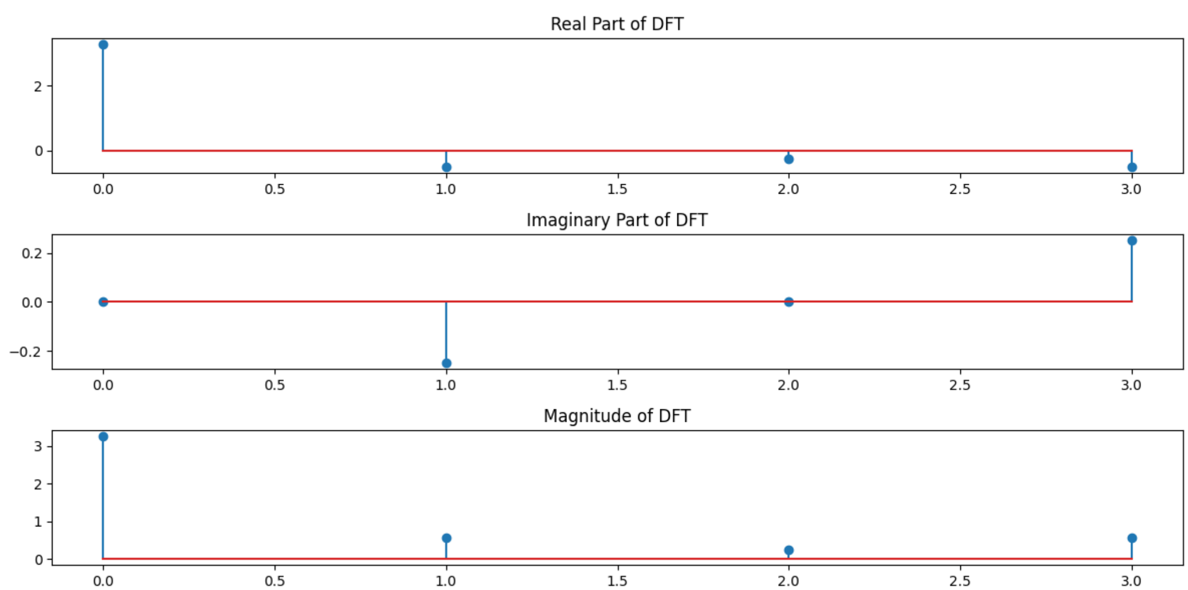

Figure 1. Plot of the real part, imaginary, and magnitude of the input signal f= [2,3,4,4]

(1.b)

Generate a one-dimensional cosine wave with 128 samples that make 8 cycles over a period: $f(x) = \cos(2\pi ux)$ where $u = 8$ and $N = 128$. Compute the DFT of $f(x)$.

Fig.2 is a graph of the generated cosine wave.

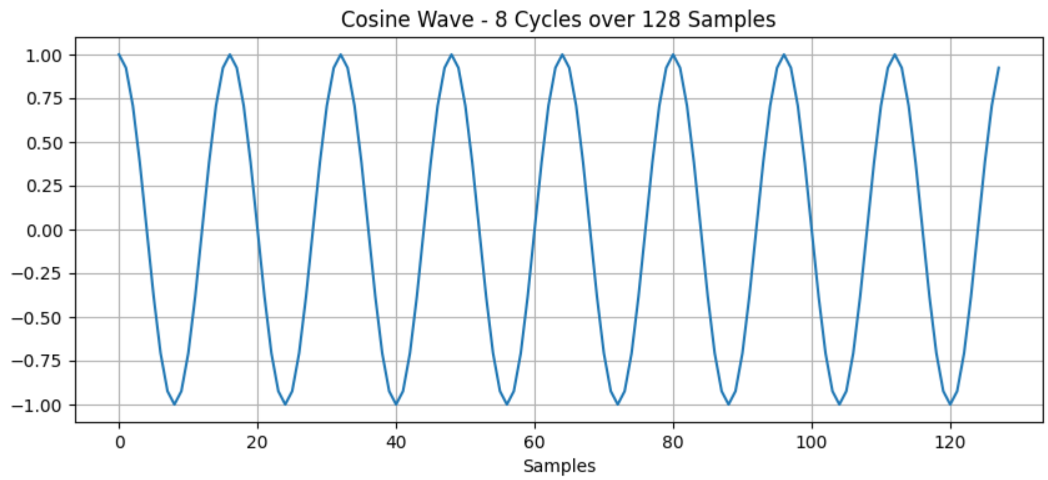

Figure 2. Generated cosine wave. f(x)=cos(2πux).

To plot the magnitude of the wave it must be shifted to the center of the frequency domain. To do so we can use the property $f(x)(-1)^x \leftrightarrow F(u - \frac{N}{2})$. Fig 3. Plots the real, imaginary, magnitude, and phase of the DFT.

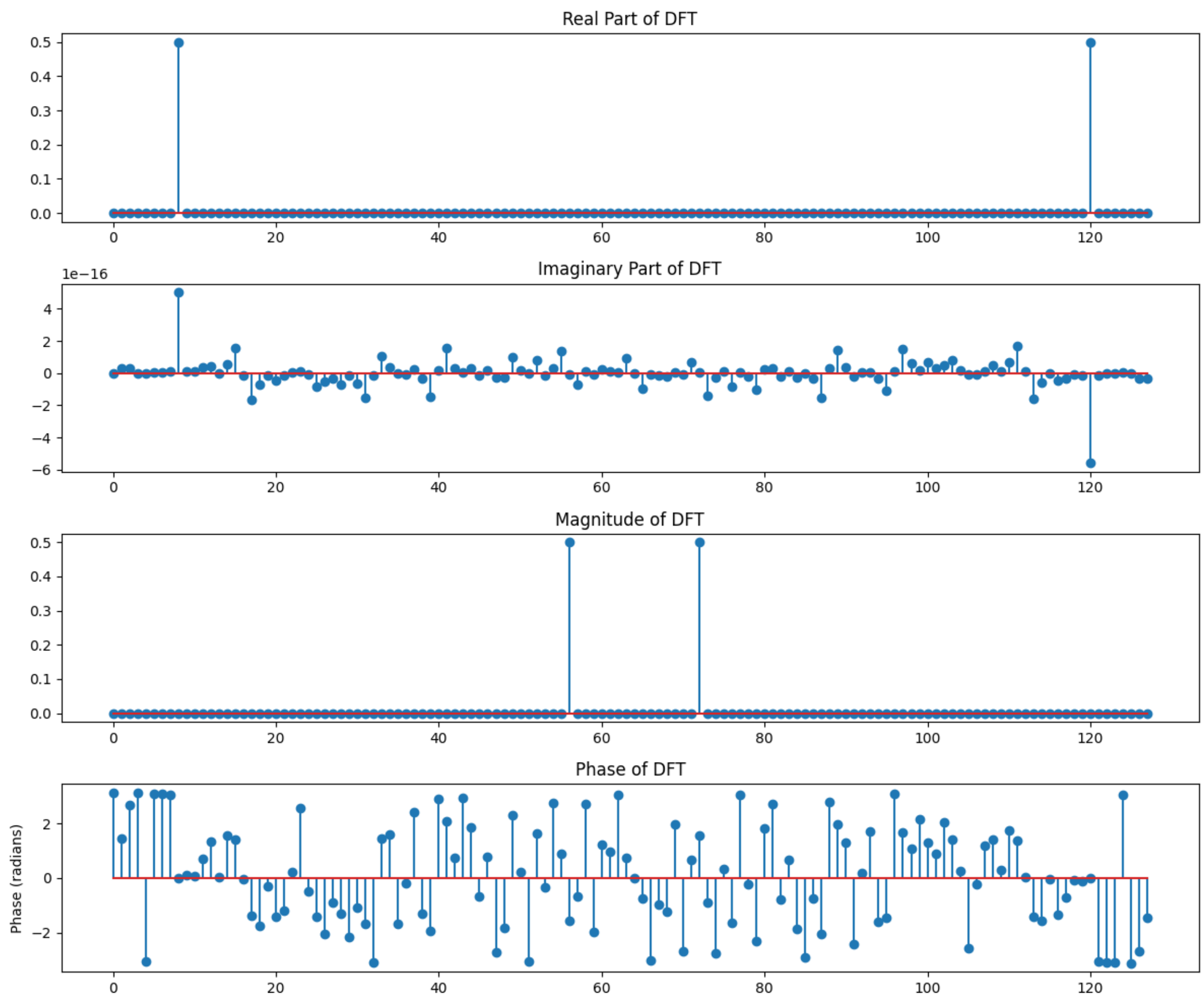

Figure 3. Real, Imaginary, phase, and the shifted magnitude of the DFT.

(1.c)

Repeat part 1.b with a rectangle functions.

Fig 4. Is a plot of the rectangle function. We will also compute the DFT and plot the real, imaginary, shifted magnitude, and phase shown in Fig 5.

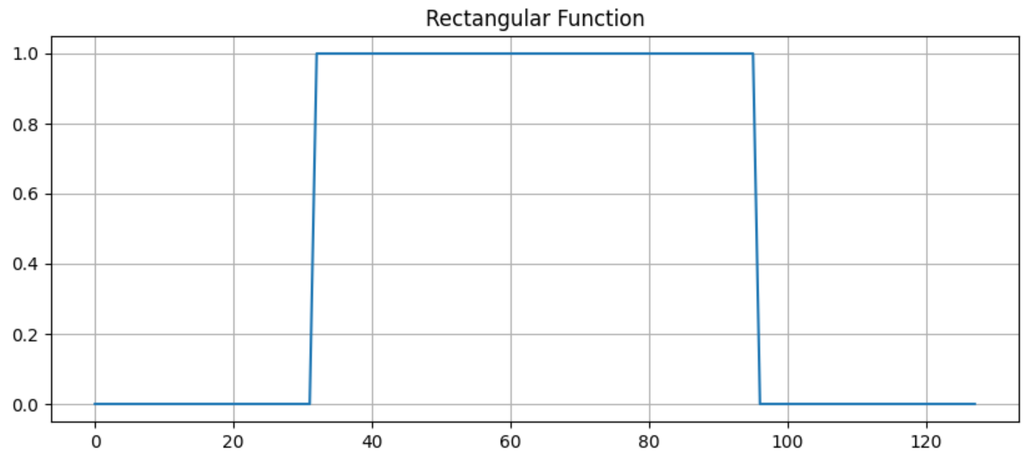

Figure 4. Rectangle function.

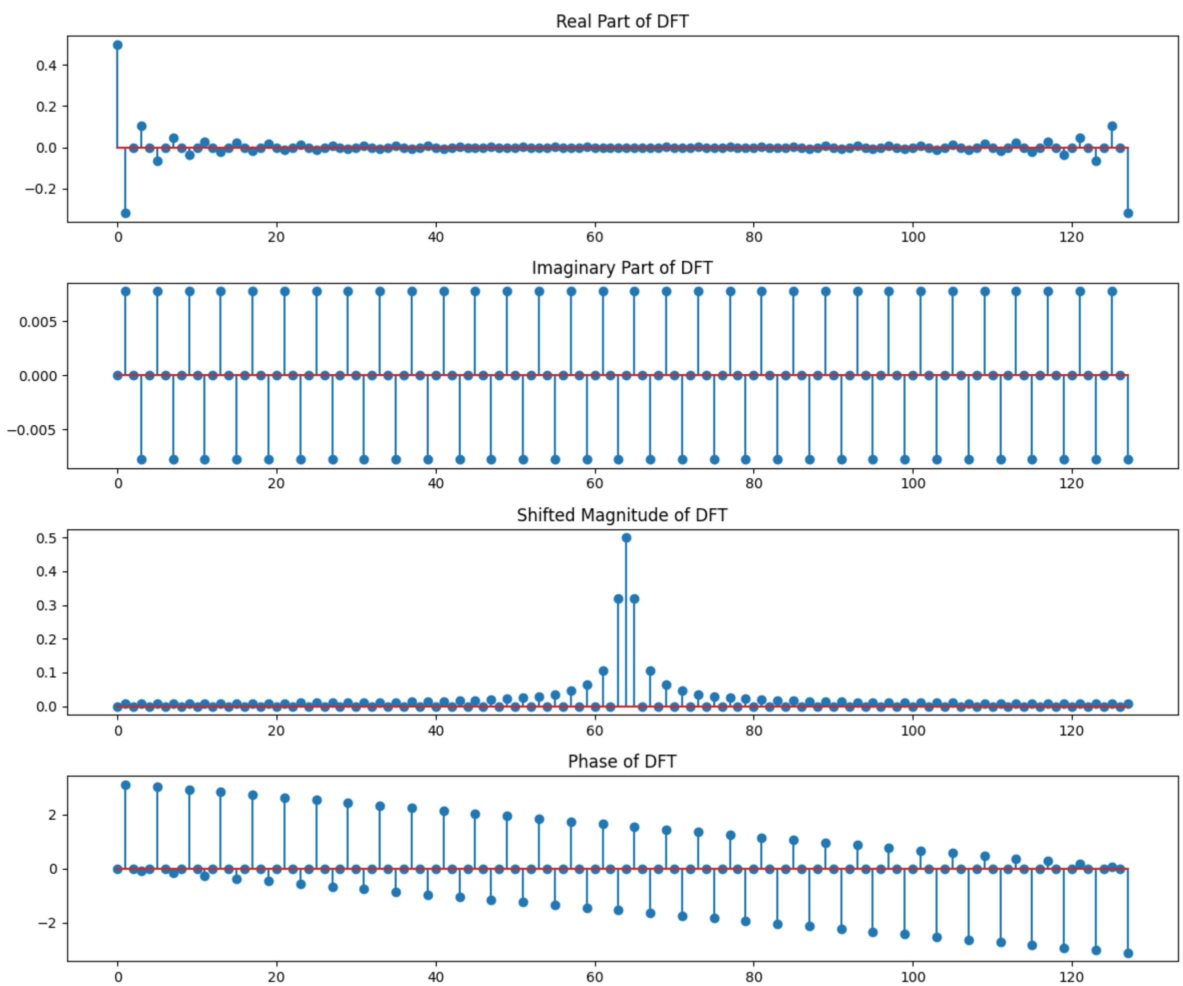

Figure 5. Rectangle function. Real, Imaginary, shifted magnitude, and phase plots.

Experiment 2

For the next experiment we will implement the 2-D DFT and its inverse. To do this we take advantage of the separability property which says we can compute a 2D DFT by first computing the 1D DFT on the rows followed by the 1D DFT of the columns. The 2D DFT is as follows:

$$F(u,v) = \frac{1}{N}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(\frac{ux+vy}{N})}$$

(2.a)

Generate a 512 x 512 image which contains a 32 x 32 square placed at the center of the image and compute the DFT.

Fig 6(a). Is the generated image. After taking the DFT of the image we will plot the magnitude before shifting it to the center. This will produce an image with subtle bright spots in the corners as seen in Fig 6(b). To see the full period of the DFT we will need to translate it to the origin. This can be done with the property $\left|F(u-\frac{N}{2}, v-\frac{N}{2})\right|$. To further visualize the centered DFT we will stretch the transformations. This will not change any information contained in the spectrum, it only improves visualization. Fig 6(c). Is the DFT after centering and stretching.

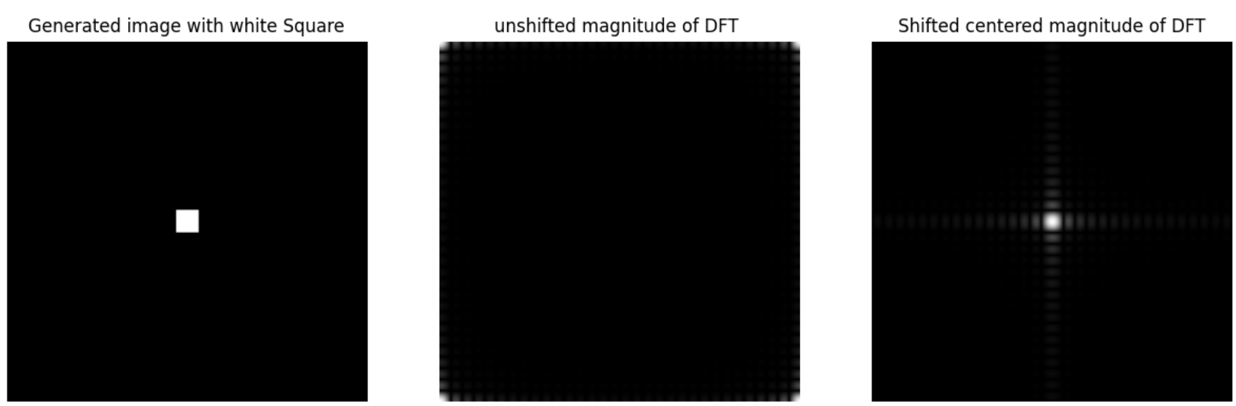

Figure 6(a)(b)(c). (a) Generated image. (b) Unshifted magnitude DFT. (c) Shifted and centered DFT.

(2.b)

Generate a 512 x 512 image which contains a 64 x 64 square placed at the center of the image and compute the DFT.

Computing the DFT on this image produces similar results to (2.a). However, the unshifted DFT is much harder to see in Fig 7(b). The shifted DFT in Fig 7(c) is smaller compared to (2.a).

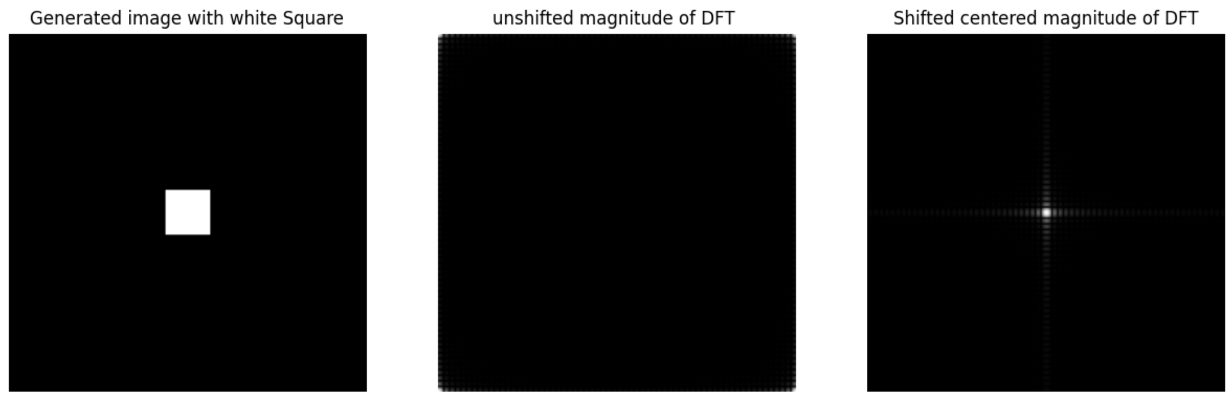

Figure 7(a)(b)(c). (a) Generated image with 64x64 white square. (b) Unshifted magnitude is much harder to see. (c) Shifted DFT is smaller.

(2.c)

Generate a 512 x 512 image which contains a 128 x 128 square placed at the center of the image and compute the DFT.

Computing the DFT on this image produces very small images. The unshifted DFT cannot be seen in Fig 8(b). The shifted DFT has been reduced to nearly a find point in Fig 8(c).

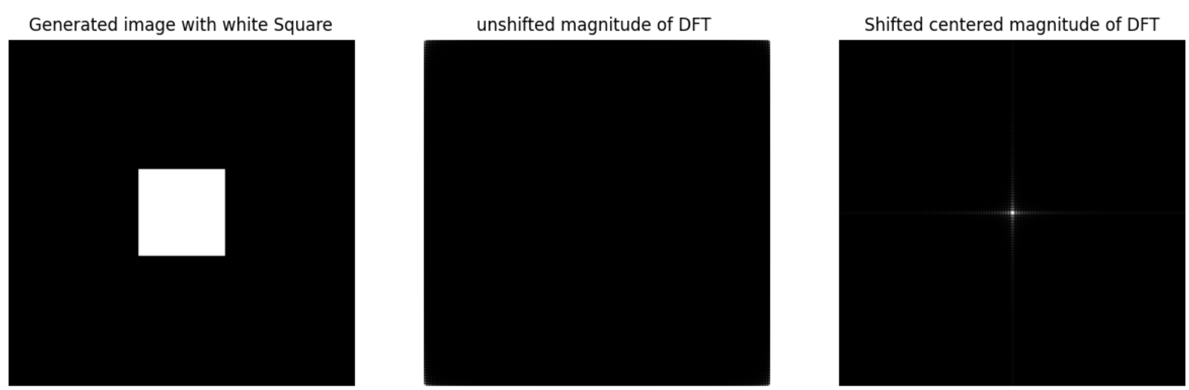

Figure 8(a)(b)(c). (a) Generated image with 128x128 white square. (b) Unshifted magnitude which cannot be seen. (c) Shifted magnitude is a small bright spot.