Analysis of Cooperative Games with Matlab and Mathematica

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[1] This chapter is based on Meinhardt (2018b).
[2] This chapter is based on Meinhardt (2018c).

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