"Ödevi başka bir öğrenciden kopyalamadım, çözümleri kendim yaptım."

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SORU-1 2.20

SORU-1.A

$$y[n] + \frac{1}{a} y[n-1] = x[n-1] \quad \text{İmpulse response için} \quad x[n] \to \delta[n], \; y[n] \to h[n]$$

$$h[n] + \frac{1}{a} h[n-1] = \delta[n-1]$$

$n=0 \to h[0] + \frac{1}{a} \cancel{h[-1]}^{0} = \delta[-1] = 0 \to \boxed{h[0]=0}$

$n=1 \to h[1] + \frac{1}{a} h[0] = \delta[0] = \boxed{1 = h[1]}$

$n=2 \to h[2] + \frac{1}{a} h[1] = \delta[1] \Rightarrow h[2] = -\frac{1}{a} h[1] \to \boxed{h[2] = -\frac{1}{a}}$

$n=3 \to h[3] + \frac{1}{a} h[2] = 0 \Rightarrow h[3] = \frac{1}{a^2}$

$n=4 \to h[4] + \frac{1}{a} h[3] = 0 \Rightarrow h[4] = -\frac{1}{a^3}$

$\vdots$

$\frac{1}{a^4}$

$-\frac{1}{a^5}$

$\vdots$

$n=1 \to 1 \to a^{n-1}$ //

$n=1$'e kadar sıfır $u[n-1]$

n çift iken negatif n tek iken pozitif.

$$\boxed{h[n] = (-1)^{n+1} \cdot \left(\frac{1}{a}\right)^{n-1} \cdot u[n-1]}$$

SORU 1-B

$$\sum_{n-\infty}^{+\infty} |h[n]| = \sum_{n=-\infty}^{+\infty} \left(\frac{1}{a}\right)^{n-1} u[n-1] \Rightarrow n-1 \to k \Rightarrow \sum_{k=-\infty}^{+\infty} \left(\frac{1}{a}\right)^{k} u[k] = \underbrace{\sum_{k=0}^{+\infty} \left(\frac{1}{a}\right)^{k}}_{A}$$

A toplamı geometrik seri açılım toplamıdır. $\frac{1}{a} = \alpha$ için $|\alpha| < 1$ ise $A = \frac{1}{1-\alpha}$

$|\alpha| > 1$ ise $A \to +\infty$ olur. Bu durumda kararlılık şartı vardır.

$$\sum_{k=0}^{+\infty} \left(\frac{1}{a}\right)^{k} = \begin{cases} \frac{1}{1-\frac{1}{a}}, & \left|\frac{1}{a}\right| < 1 \\ \infty, & \left|\frac{1}{a}\right| > 1 \end{cases}$$

$\underbrace{\left|\frac{1}{a}\right| < 1}_{\boxed{|a|>1}}$ bölgesinde kararlıdır.

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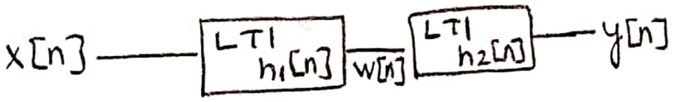
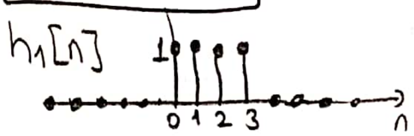
SORU-2

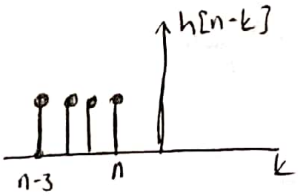


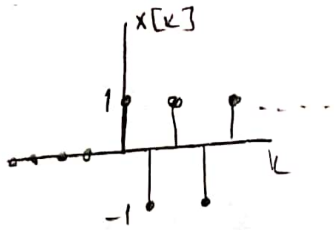
SORU-2A



ⓐ → $x[n] = (-1)^n u[n]$ $w[n] = ?$ $y[n]$

$$w[n] = x[n] * h_1[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h_1[n-k]$$





(∗) eğer sağ yan üst sınır $n<0$ ise $h[n-k]$ negatif bölgede, $x[k]$ pozitif bölgede tanımlıdır. Kesiştikleri bir nokta olmadığından $w[n]=0$ bu noktalardaki değerdir.

(∗) Eğer sağ yan üst sınır $n \geq 0$, sol yan alt sınır $n-3<0$ ise ilgili değerler hesaplanmalı $n=0$ için, $w[0] = \sum_{k=-\infty}^{+\infty} x[k]\, h[-k] = 1$ → $n=0$'da ikisi de başka ortak noktada yok.

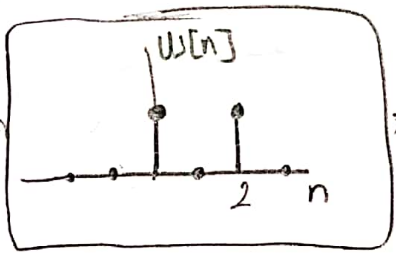
$n=1$ için, $w[1] = \sum_{k=-\infty}^{+\infty} x[k]\, h[1-k] = \underbrace{1}_{x[0]h[1]} - \underbrace{1}_{x[1]h[0]} = 0$ → $n=0$'da 1 $n=1$'de -1 kesişim sonucu

$n=2$ için, $w[2] = \sum_{k=-\infty}^{+\infty} x[k]\, h[2-k] = \underbrace{1}_{x[0]h[2]} - \underbrace{1}_{x[1]h[1]} + \underbrace{1}_{x[2]h[0]} = 1 \rightarrow w[2] = 1$

$n \geq 3$ ise n her ne olursa olsun $h[n-k]$'nın bulunan 4 değeri de $x[k]$ ile çakışacak

$$\sum_{k=-\infty}^{+\infty} x[k]\, h[n-k] = 1-1+1-1 = 0 \text{ olacaktır.}$$

$$w[n] = \begin{cases} 0, & n<0 \\ 1, & n=0 \\ 0, & n=1 \\ 1, & n=2 \\ 0, & n \geq 3 \end{cases}$$



2.A devamı

$$y[n] = w[n] * h_2[n] = \sum_{k=-\infty}^{+\infty} h_2[k]\, w[n-k]$$

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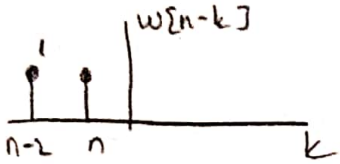


$w[n-k]$ sağ uç sınır $n < -3$ ise çakışma yok $y[n] = 0$

$w[n-k]$ " " " $n > -3$ ve sol uç sınır $n-2 < -3$ ise değer incelenir.

$$y[-3] = \sum_{k=-\infty}^{+\infty} w[-3-k] \cdot h[k] = 1\,(w[-3] \cdot h[0]) = 1$$

$$n=-2 \quad y[-2] = \sum_{k=-\infty}^{+\infty} w[-2-k]\, h[k] = 1 \text{ (tek kesişim)}$$

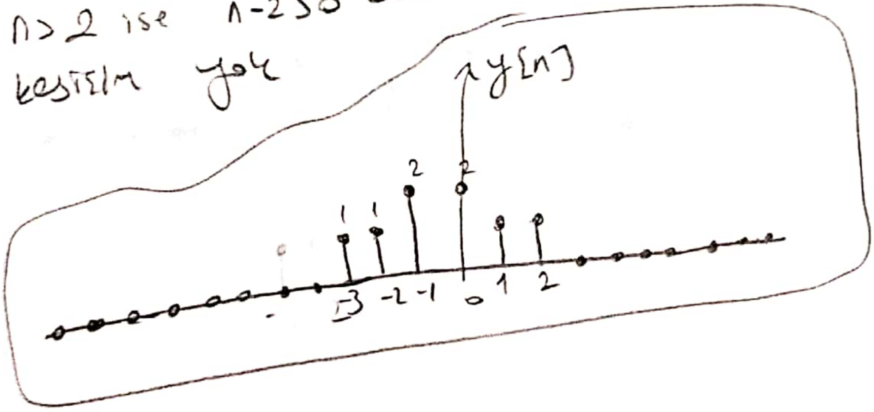
$$n=-1 \quad y[-1] = \sum_{k=-\infty}^{+\infty} w[-1-k]\, h[k] = 2 \text{ (} h[k] \text{ ile } w[n-k]\text{'da çift kesişim)}$$

$$n=0 \quad y[0] = \sum_{k=-\infty}^{+\infty} w[-k]\, h[k] = 2 \text{ (çift kesişim)}$$

$$n=1 \rightarrow y[1] = \sum_{k=-\infty}^{+\infty} w[1-k]\, h[k] = 1 \text{ (tek kesişim)}$$

$$n=2 \rightarrow y[2] = \sum_{k=-\infty}^{+\infty} w[2-k]\, h[k] = 1 \text{ (" ")}$$

$n > 2$ ise $n-2 > 0$ olur w sol uç sınırı h sağ uç sınırından büyük olduğundan kesişim yok



3

SORU-2B $x[n] = \delta[n]$ ise $y[n] = h[n] = ?$

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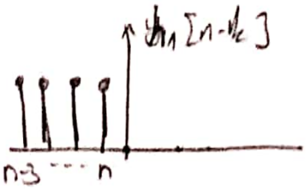
Konvolüsyonda $\delta[n]$ birim eleman olduğundan

$$h[n] = h_1[n] * h_2[n] = \sum_{k=-\infty}^{+\infty} h_1[n-k].h_2[k]$$



$n<-3$ ise h_1 sağ uç sınırı h_2 sol uç sınırından küçük $n<-3 \rightarrow h[n]=0$

$n=-3$ $h[-3] = 1$ $(k=-3)$

$n=-2$ $h[-2] = 2$ $(k=-3$ ve $k=-2)$

$n=-1$ $h[-1] = 3$ $(k=-3, k=-2, k=-1)$

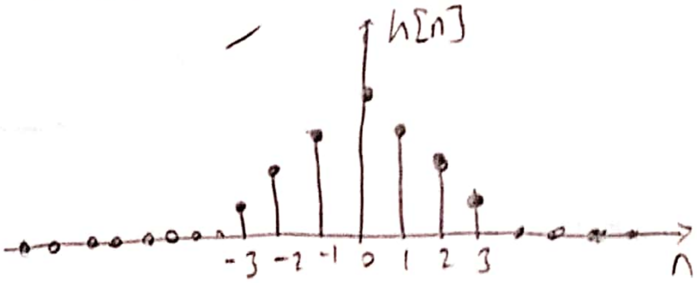
$n=0$ $h[0] = 4$ $(k=-3, k=-2, k=-1, k=0)$

$n=1$ $h[1] = 3$ $(k=-2, k=-1, k=0)$

$n=2$ $h[2] = 2$ $(k=-1, k=0)$

$n=3$ $h[3] = 1$ $(k=0)$

$n>3$ için çapraz uç sınırlar çakışmadığından $h[n]=0$



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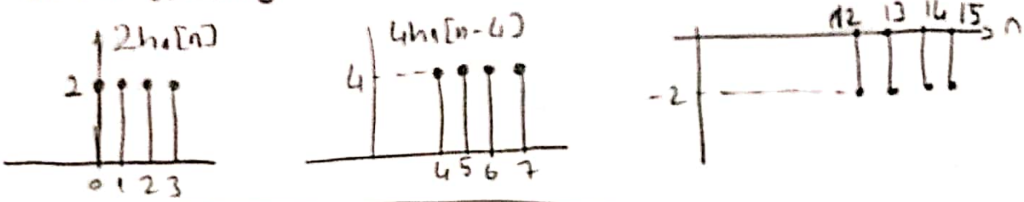
SORU-2C

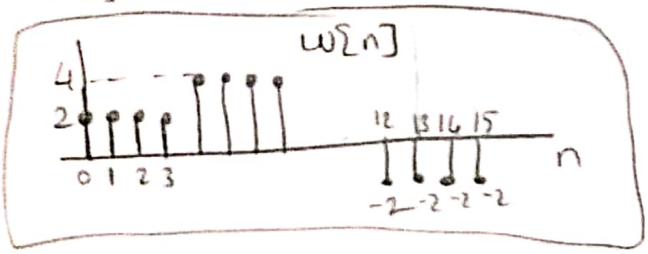
$x[n] = 2\delta[n] + 4\delta[n-4] - 2\delta[n-12]$

$w[n] = ?$ $\quad w[n] = x[n] * h_1[n]$

$w[n] \stackrel{\text{Lineerlik}}{=} 2\delta[n] * h_1[n] + 4\delta[n-4] * h_1[n] - 2\delta[n-12] * h_1[n]$

dirac konvolüsyonun birim deneni $\Big\} = 2h_1[n] + 4h_1[n-4] - 2h_1[n-12]$

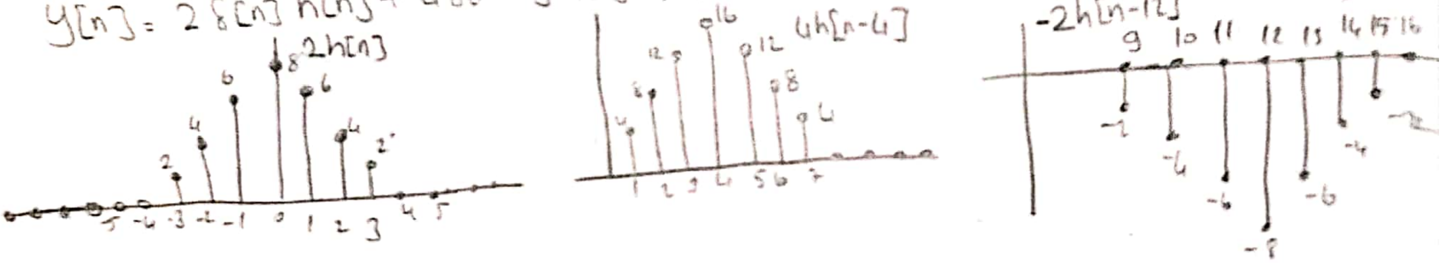


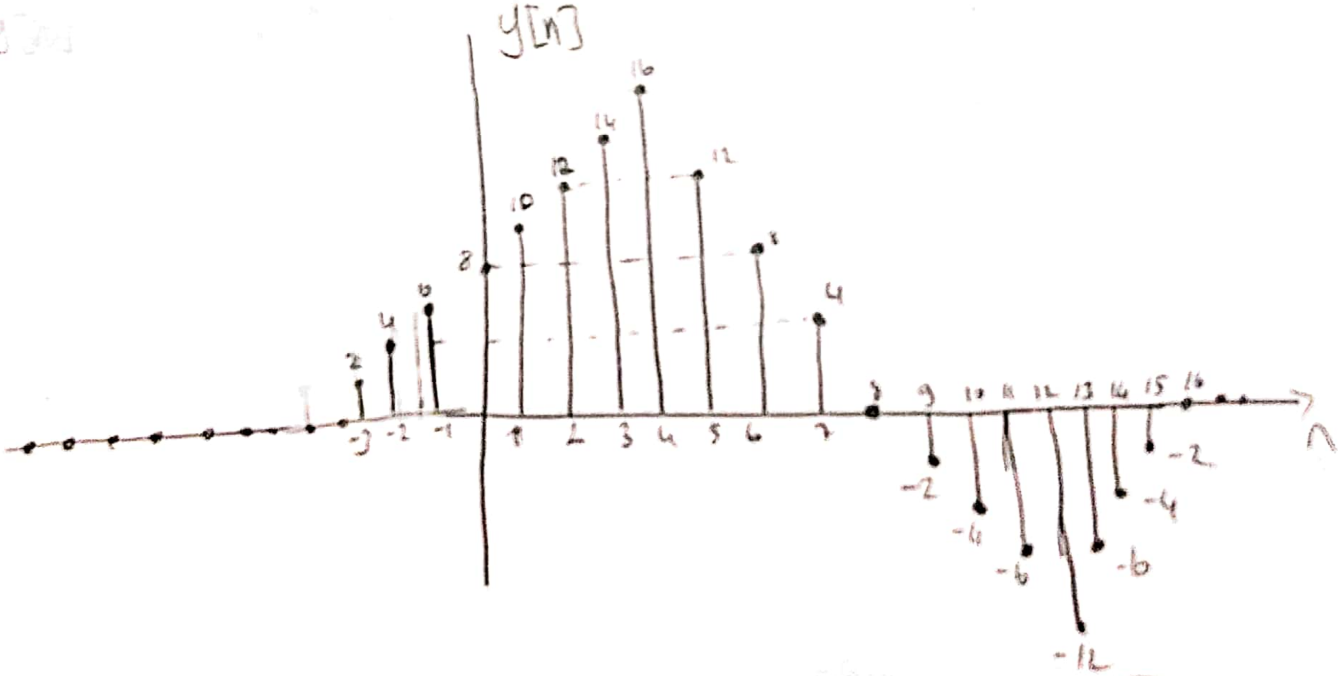


SORU-2D

c şıkkı $y[n]$ için tekrarlanırsa

$y[n] = 2\delta[n]\,h[n] + 4\delta[n-4]\,h[n] - 2\delta[n-12]\,h[n] = 2h[n] + 4h[n-4] - 2h[n-12]$





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SORU-3

impulse response $h[n]=?$ $\quad y[n]=-2x[n]+4x[n-1]-2x[n-2]$

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SORU-3A

$y[n]$, $x[n]$ bağıntısı verildiğinde, $x[n]=\delta[n]$ için $y[n]$ çıkısı $h[n]$ olur. Buna göre,

$$\boxed{h[n]=-2\,\delta[n]+4\,\delta[n-1]-2\,\delta[n-2]}$$

SORU-3B

Frequency Response $=H(e^{jw})$

$$F\{h[n]\}=H(e^{jw})=\underbrace{F\{-2\delta[n]\}}_{-2}+\underbrace{F\{4\delta[n-1]\}}_{4.1.e^{-jw}}+\underbrace{F\{-2\delta[n-2]\}}_{-2.1.e^{-2jw}}$$

$\delta[n]\xleftrightarrow{F}1$

$\delta[n-1]\xleftrightarrow{F}1.e^{-jw}$ (time shifting)

$$\boxed{H(e^{jw})=-2+4e^{-jw}-2e^{-j2w}}$$

$$H(e^{jw})=e^{-jw}(-2.e^{jw}-2e^{-jw}+4)$$

$$H(e^{jw})=-2e^{-jw}(\underbrace{e^{jw}+e^{-jw}}_{2\cos(w)}-2)$$

$$\frac{e^{jw}+e^{-jw}}{2}=\cos(w)\Rightarrow e^{jw}+e^{-jw}=2\cos(w)$$

$$H(e^{jw})=-2e^{-jw}(2\cos(w)-2)$$

$$\cos(w)=2\cos^2\left(\tfrac{w}{2}\right)-1=1-2\sin^2\left(\tfrac{w}{2}\right)$$

$$H(e^{jw})=4e^{-jw}(1-\cos(w))$$

$$\cos(w)\to 1-2\sin^2\left(\tfrac{w}{2}\right)$$

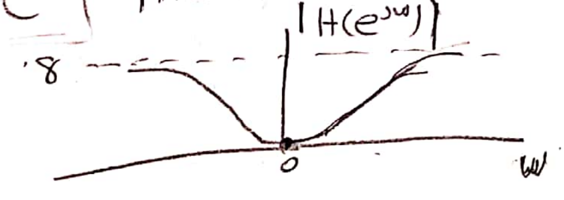
$$H(e^{jw})=4e^{-jw}.2.\sin^2\left(\tfrac{w}{2}\right)=\boxed{\underbrace{8\sin^2\left(\tfrac{w}{2}\right)}_{\text{Real function of } w}.e^{-jw}=H(e^{jw})}$$

$nd=1$

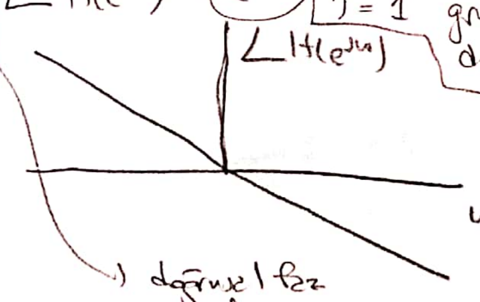
$$e^{-jwnd}=e^{-jw}\rightarrow\boxed{nd=1}$$

SORU-3C

$|H(e^{jw})|=|8\sin^2(w/2)|$ $\longrightarrow$ çift fonk. y-simetrik, max değer 8, min değer 0



$\angle H(e^{jw})=\angle e^{-jw}$ $\quad e^{-jw\tau}$ $\quad \boxed{\tau=1}$ group delay



doğrusal faz grafiği getirir.

CamScanner ile tarandı

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SORU-3D

$$x_1[n] = 1 + e^{j0.5\pi n}, \quad -\infty < n < \infty$$

→ Öz fonksiyon tanımı kullanabilmek için,

$$x_1[n] = e^{j0n} + e^{j0.5\pi n}$$

$$y_1[n] = H(e^{j0}).e^{j0n} + H(e^{j0.5\pi n}).e^{j0.5\pi n}$$

$$y_1[n] = 8\sin^2(0).e^0.e^0 + 8.\sin^2\left(\frac{\pi}{4}\right)e^{-j0.5\pi}.e^{j0.5\pi n}$$

$$y_1[n] = 8.\frac{1}{2}.e^{j(0.5\pi n - 0.5\pi)} \qquad \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$$

$$\boxed{y_1[n] = 4e^{j0.5\pi(n-1)} \quad -\infty < n < \infty}$$

SORU-3E

$$x_2[n] = (1 + e^{j0.5\pi n})u[n] \quad -\infty < n < \infty$$

use difference equation. or discrete convolution.

$y_2[n] = ?$

$$y_2[n] = \sum_{k=-\infty}^{+\infty} h[k]x_2[n-k] = \sum_{k=-\infty}^{+\infty} h[k]\left(1 + e^{j0.5\pi(n-k)}\right)u[n-k]$$

$$= \sum_{k=-\infty}^{n} h[k]\left(1 + e^{j0.5\pi(n-k)}\right)$$

↳ $u[n-k]$ dan dolayı üst sınır değişti.

sistem nedensel ise $h[k]=0$, for $k<0$ bu durumda

$y_2[n] = 0$ for, $n<0$

$$y_2[n] = \sum_{k=0}^{n} \underbrace{h[k]\left(1 + e^{j0.5\pi(n-k)}\right)}_{I}, \text{ for } n \geq 0$$

$$\star\, y_2[n] = \sum_{k=0}^{+\infty} I - \sum_{k=n+1}^{+\infty} I \,\star$$

$$y_2[n] = \underbrace{\underbrace{\sum_{k=0}^{+\infty} h[k]}_{H(e^{j0})} + \underbrace{\sum_{k=0}^{+\infty} h[k]e^{j0.5\pi(n-k)}}_{H(e^{j0.5\pi(n-k)}).e^{j\frac{\pi}{2}(n-k)}}}_{y_{SS}[n]} \underbrace{- \left[\sum_{k=n+1} h[k]\left(e^{j0.5\pi(n-k)} + 1\right)\right]}_{y_{TR}[n]}$$

$n \geq 2$ için $h[n] = -2\delta[n] + 4\delta[n-1] - 2\delta[n-2] = 0$

bu durumda $y_{TR}[n] = 0$ olur. transient durum sıfırlanıyorsa

$y_1[n] = y_2[n]$ olur. Böylece;

$n > 2$ için $y_{TR}[n] = 0 \rightarrow y_1[n] = y_2[n]$ alkarmı yapılır

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SORU-4

$x[n] \rightarrow$ LTI system $h_1[n], H_1(e^{jw})$ $\xrightarrow{w[n]}$ LTI system $h_2[n], H_2(e^{jw})$ $\rightarrow y[n]$

SORU-4A

$$H_1(e^{jw}) = e^{-jw} \begin{cases} 0 & |w| \leq 0.25\pi \\ 1 & 0.25\pi < |w| \leq \pi \end{cases} \qquad h_2[n] = 2 \cdot \frac{\sin(0.5\pi n)}{\pi n}$$

SORU-4A

$$H(e^{jw}) = H_1(e^{jw}) \cdot H_2(e^{jw})$$

$$H_2(e^{jw}) = F\{h[n]\} \Rightarrow F\Big\{ \underbrace{\frac{\sin(0.5\pi n)}{0.5\pi n}}_{\text{sinc}(0.5n)} \Big\}$$

$$F\{\text{sinc}(0.5n)\} = \frac{1}{|0.5|} \cdot \text{rect}\left(\frac{f}{0.5}\right)$$

$$\text{sinc}(\cancel{\alpha t}) \xrightarrow{F} \frac{1}{|\alpha|} \text{rect}\left(\frac{f}{|\alpha|}\right)$$

$$F\{\text{sinc}(0.5n)\} = 2 \cdot \text{rect}\left(\frac{f}{0.5}\right)$$

$-\frac{1}{4} < f < \frac{1}{4}$ bölgesinde 2 diğer bölgelerde sıfır olan kare dalga

$$w = 2\pi f = 2\pi \cdot \frac{1}{4} = \frac{\pi}{2}$$

$$F\{\text{sinc}(0.5n)\} = 2, \text{ for } w < \frac{\pi}{2}$$

$$H_2(e^{jw}) = \begin{cases} 2, & |w| < \pi/2 \\ 0, & \pi/2 < |w| \leq \pi \end{cases}$$

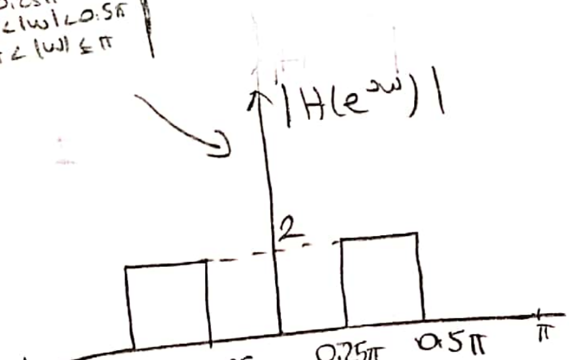
$$H_1(e^{jw}) = e^{-jw} \begin{cases} 0, & |w| \leq 0.25\pi \\ 1, & 0.25\pi \leq |w| \leq \pi \end{cases}$$

$$H(e^{jw}) = H_1(e^{jw}) \cdot H_2(e^{jw})$$

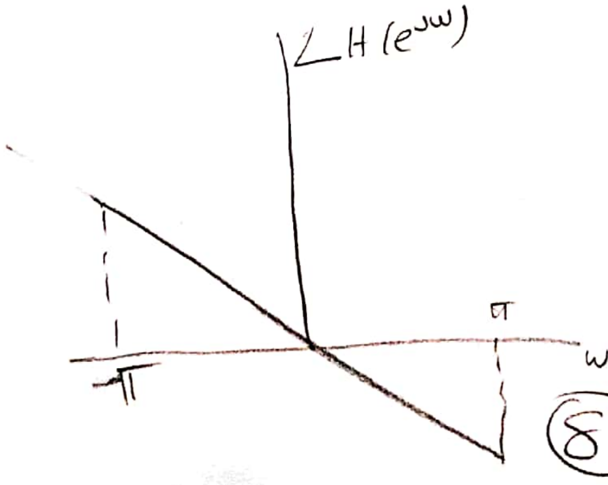
$$H(e^{jw}) = e^{-jw} \cdot \begin{cases} 0, & |w| < 0.25\pi \\ 2, & 0.25\pi < |w| < 0.5\pi \\ 0, & 0.5\pi < |w| \leq \pi \end{cases}$$

SORU-4B

$$|H(e^{jw})| = \underbrace{|e^{-jw}|}_{1} \cdot \left| \begin{cases} 0, & |w| < 0.25\pi \\ 2, & 0.25\pi < |w| < 0.5\pi \\ 0, & 0.5\pi < |w| \leq \pi \end{cases} \right|$$



$\angle H(e^{jw}) = e^{-jw}$, $H(e^{jw}) = A(w) e^{-jwT}$ formatı durumlarında $A(w)$ real valued function, T, group delay olarak gecer. wının artışı ile fazda azalan değer gözlenir ve doğrusal bir değişim görülür.



SORU-4C

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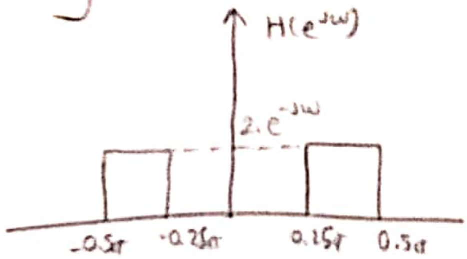
$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{jw})\, e^{jwn}\, dw$$

$$H(e^{jw}) = e^{-jw} \cdot \begin{cases} 0, & |w| < 0.25\pi \\ 2, & 0.25\pi < |w| < 0.5\pi \\ 0, & 0.5\pi < |w| < \pi \end{cases}$$



$$h[n] = \frac{1}{2\pi}\left[\int_{-0.5\pi}^{-0.25\pi} 2 \cdot e^{-jw} \cdot e^{jwn}\, dw + \int_{0.25\pi}^{0.5\pi} 2 \cdot e^{-jw} \cdot e^{jwn}\, dw\right]$$

$$h[n] = \frac{1}{2\pi}\cdot 2\left[\int_{-0.5\pi}^{-0.25\pi} e^{jw(n-1)}\, dw + \int_{0.25\pi}^{0.50\pi} e^{jw(n-1)}\, dw\right]$$

$$h[n] = \frac{1}{\pi}\left[2\int_{0.25\pi}^{0.5\pi}\left(\frac{e^{jw(n-1)} + e^{-jw(n-1)}}{2}\right) dw\right]$$

$w \to -w$ sınırlar bir önceki integral sınırı ile aynı olduğundan ortaklandı.

$$h[n] = \frac{1}{\pi}\left[2\int_{0.25\pi}^{0.5\pi} \cos(w(n-1))\, dw\right]$$

$$h[n] = \frac{2}{\pi}\left[\int_{0.25\pi}^{0.5\pi} \cos(w(n-1))\, dw\right] = \frac{2}{\pi}\cdot\left[\frac{1}{n-1}\cdot \sin(w(n-1))\right]\Bigg|_{0.25\pi}^{0.50\pi}$$

$$\boxed{h[n] = \frac{2}{\pi(n-1)}\cdot\left[\sin(0.5\pi(n-1)) - \sin(0.25\pi(n-1))\right]} \star$$

$$\star\quad \boxed{h[n] = \frac{2\sin(0.5\pi(n-1))}{\pi(n-1)} - \frac{2\sin(0.25\pi(n-1))}{\pi(n-1)}}$$ olarak bulunur

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SORU-5 $\quad \left(\frac{1}{2}\right)^n u[n] = x[n]$

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(a) $X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n} = \sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^n . z^{-n} = \sum_{n=0}^{+\infty} \underbrace{\left(\frac{1}{2z}\right)}_{\alpha}^n = \frac{1}{1-\frac{1}{2z}}$ $\quad \left|\frac{1}{2z}\right| < 1$ $\quad \frac{1}{2|z|} < 1$ $\quad \underbrace{\boxed{|z| > \frac{1}{2}}}_{ROC}$

(b) $X(z) = -\sum_{n=-\infty}^{+\infty} \left(\frac{1}{2}\right)^n u[-n-1] z^{-n} = -\sum_{n=-\infty}^{-1} \left(\frac{1}{2}\right)^n z^{-n} = -\sum_{n=1}^{+\infty} 2^n . z^n = -\left[\sum_{n=0}^{+\infty} 2^n z^n - 1\right]$

$= 1 - \sum_{n=0}^{+\infty} (2z)^n = 1 - \frac{1}{1-(2z)}$, if $|2z| < 1$ $\quad \boxed{|z| < \frac{1}{2}}$ ROC

$\frac{-2z}{1-2z} = \boxed{\frac{1}{1-\frac{1}{2z}}} = X(z)$ $\quad \boxed{x[n] = -\left(\frac{1}{2}\right)^n u[-n-1]}$

(c) $x[n] = \left(\frac{1}{2}\right)^n u[-n]$ $\quad X(z) = \sum_{n=-\infty}^{+\infty} \left(\frac{1}{2}\right)^n u[-n] z^{-n} = \sum_{n=-\infty}^{0} \underbrace{\left(\frac{1}{2}\right)^n . z^{-n}}_{\left(\frac{1}{2z}\right)^n} = \sum_{n=0}^{+\infty} (2z)^n = \frac{1}{1-2z}$ $\quad |2z| < 1$ $\quad |z| < \frac{1}{2}$

$\boxed{X(z) = \frac{1}{1-2z}, \ \underbrace{\boxed{|z| < \frac{1}{2}}}_{ROC}}$

(d) $x[n] = \delta[n]$ $\quad X(z) = \sum_{n=-\infty}^{+\infty} \delta[n] z^{-n} = z^0 = 1$ $\quad \underbrace{\text{for all } z}_{ROC}$

(e) $x[n] = \delta[n-1]$ $\quad X(z) = \sum_{n=-\infty}^{+\infty} \delta[n-1] . z^{-n} = z^{-1} = \frac{1}{z}$, $\quad |z| > 0$ $\quad \underbrace{\text{for all } z \text{ but } z \neq 0}_{ROC}$

(f) $x[n] = \delta[n+1]$ $\quad X(z) = \sum_{n=-\infty}^{+\infty} \delta[n+1] z^{-n} = z^{+1} = z$, $\quad \underbrace{\text{for all } z}_{ROC}$

(g) $x[n] = \left(\frac{1}{2}\right)^n \left(u[n] - u[n-10]\right)$, $\quad X(z) = \sum_{n=-\infty}^{+\infty} \left(\frac{1}{2}\right)^n \left(u[n] - u[n-10]\right) . z^{-n}$

$X(z) = \sum_{n=0}^{9} \left(\frac{1}{2}\right)^n . z^{-n} = \sum_{n=0}^{9} \left(\frac{1}{2z}\right)^n = \frac{1-\left(\frac{1}{2z}\right)^{10}}{\left(1-\frac{1}{2z}\right)}$ $\quad$ for $\left|\frac{1}{2z}\right| < 1$ $\quad \underbrace{\boxed{|z| > \frac{1}{2}}}_{ROC}$

$\sum_{n=0}^{k} \alpha^n = \frac{1-\alpha^{k+1}}{1-\alpha}$, for $|\alpha| < 1$

$$\boxed{X(z) = \frac{1-\left(\frac{1}{2z}\right)^{10}}{1-\frac{1}{2z}}}$$

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