HW-13

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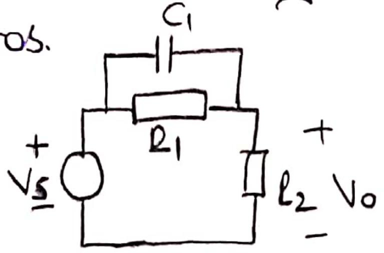
Q1. Determine (a) the transfer function of the network shown in fig P12.2 and (b) its pole and zeros.



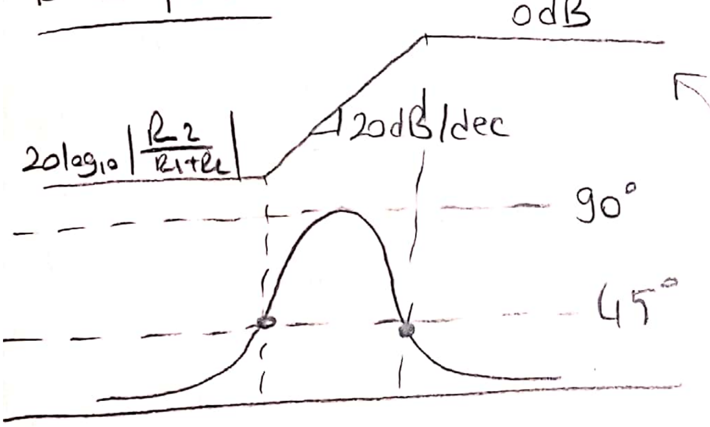
$$\frac{V_o}{V_{ss}} = \frac{R_2}{R_2 + R_1 \parallel \frac{1}{sC_1}} = \frac{R_2}{R_2 + \frac{R_1 \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}}}$$

$$= \frac{R_2}{R_2 + \frac{R_1}{1+sC_1R_1}} = \frac{R_2(1+sR_1C_1)}{R_1+R_2+R_1.R_2C_1s} = \underbrace{\frac{R_2}{R_1+R_2}}_{\text{low freq. gain}} \cdot \frac{1+\frac{s}{1/R_1C_1}}{1+\frac{s}{1/C_1(R_1\parallel R_2)}}$$

$1/R_1C_1 \rightarrow$ zero

$1/C_1.(R_1 \parallel R_2) \rightarrow$ pole

Bode plot



$$\frac{R_2}{R_1+R_2} \cdot \frac{R_1.C_1}{(R_1\parallel R_2)C_1} = \frac{R_2}{R_1+R_2} \cdot \frac{R_1}{R_1.R_2/R_1+R_2}$$

$$= \frac{\cancel{R_2.R_1}}{\cancel{R_1+R_2}} \cdot \frac{\cancel{R_1+R_2}}{\cancel{R_1.R_2}} = 1$$

Q2. Determine (a) the pole and zero quality factors Q_p and Q_z, (b) the Pole and zero resonant frequecies w_p and w_z, (c) the pole factor β_p, and (d) the pole angle ϕ_p. The transfer function has the general form as given by

$$H(s) = \frac{5s^2+15s+100}{s^2+20s+200}$$

$$H(s) = A \cdot \frac{s^2 + 2\delta_z w_{o,z} s + w_{o,z}^2}{s^2 + 2\delta_p w_{o,p} s + w_{o,p}^2}$$

$\boxed{Bw_{,p} = 20}$

$$Bw_{,p} = 2\delta_p w_{op} \rightarrow Q_p = \frac{w_{o,p}}{2\delta_p w_{op}} = \frac{\sqrt{200}}{20} = \boxed{\frac{\sqrt{2}}{2} = Q_p}$$

$$Bw_{,z} = 2\delta_{z} w_{o,z} \Rightarrow Q_z = \frac{w_{o,P}}{2\delta_z w_{o,z}} = \frac{\sqrt{20}}{3} = \boxed{\frac{2\sqrt{5}}{3} = Q_z}$$

$\boxed{Bw_{,z} = 3}$

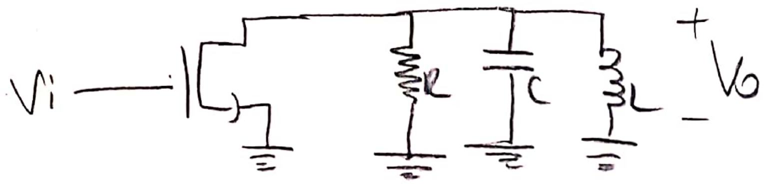
$$s^2 + 20s + 200 = (s + \alpha_p + j\beta_p)(s + \alpha_p - j\beta_p)$$
$$= s^2 + 2\delta_p w_{op} + w_{o,p}^2$$

$$\alpha_p^2 + \beta_p^2 = w_{op}^2 = 200$$
$$2\alpha_p = 2\delta w_{o,p} = 20 \quad \boxed{\alpha_p = 10}$$
$$\beta_p = \sqrt{200 - 10^2} = \boxed{10 = \beta_p}$$

$$\boxed{\begin{aligned} P_1 &= -\alpha_p - j\beta_p = -10 - 10j \\ P_2 &= -\alpha_p + j\beta_p = -10 + 10j \end{aligned}}$$

$$\phi_p = \cos^{-1}\left(\frac{\alpha_p}{\sqrt{\alpha_p^2 + \beta_p^2}}\right) = \cos^{-1}\left(\frac{10}{\sqrt{200}}\right) = \boxed{\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ}$$

Q3 A coil having an inductance of 10MH is intended for applications around 1-MHz frequency. Its Q is specified to be 200. Find the equivalent parallel resistence Rp. What is the value of the capacitor required to produce resonance at 1MHz? What additional parallel resistance is requierd to produce a 3dB bandwith of 10kHz



$$\frac{V_o}{V_i} = -g_m\left[R \parallel \frac{1}{sC} \parallel sL\right] = -g_m \frac{1}{\frac{1}{R} + sC + \frac{1}{sL}}$$

$$= \frac{-g_m sL}{s\frac{L}{R} + s^2LC + 1} = \frac{-g_m s}{\frac{s}{R} + s^2C + \frac{1}{L}} = \frac{-g_m s}{C} \cdot \frac{1}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$(\text{res. freq.})^2 = w_o^2 = \frac{1}{LC} \qquad 2\delta w_o = \frac{1}{RC} \qquad 2\delta \frac{1}{\sqrt{LC}} = \frac{1}{RC}$$

$$\boxed{\delta = \sqrt{\frac{L}{C}} \cdot \frac{1}{2R}}$$

$$\frac{V_o}{V_i} = \frac{-g_m}{C} \cdot \frac{1}{2\delta w_o} = \frac{-g_m}{C} \cdot RC = -g_m R$$

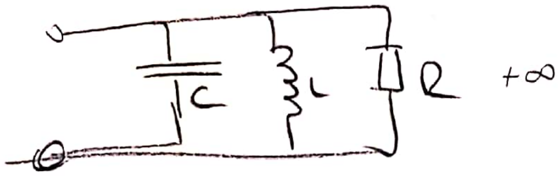
$$G(jw) = \frac{2\delta w_o \cdot jw}{(w_o^2 - w^2) + j2\delta w_o w} \Rightarrow \left|G(jw_{3dB})\right|^2 = \frac{(2\delta w_o)^2 w_{-3dB}^2}{(w_o^2 - w_{-3dB}^2)^2 + (2\delta w_o w_{-3dB})^2} = \frac{1}{2}$$

$$W_L = w_o\left[-\delta + \sqrt{1+\delta^2}\right]$$

$$W_H = w_o\left[\delta + \sqrt{1+\delta^2}\right]$$

$$\boxed{BW = W_H - W_L = 2\delta w_o = \frac{1}{RC}}$$

$R \to +\infty \quad \underline{BW = 0}$



$$Q = \frac{W_0}{BW} = \frac{1/\sqrt{LC}}{1/RC} = R\sqrt{\frac{C}{L}}$$

$R \to +\infty \quad Q \to +\infty$

~~R~~ $R \to +\infty$ Q için ideal değer olsa da bu durumda $BW = 0$ olduğundan kullanılamamaktadır. Trade-off

$L = 10\mu H \quad W_0 = \frac{1}{\sqrt{LC}} = 2\pi\, 1MHz$
$= 2\pi \times 10^6$

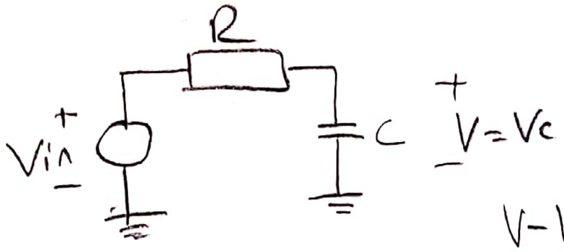
$$2\pi \times 10^6 = \frac{1}{\sqrt{LC}} \quad \sqrt{C} = \frac{1}{2\pi \times 10^6 \cdot \sqrt{L}} \longrightarrow C = \frac{1}{4\pi^2 \cdot 10^{12} \cdot L} \quad L = 10 \times 10^{-6}$$

$$Q = R_P\sqrt{\frac{C}{L}} = 200 \qquad C = \frac{1}{4\pi^2 \cdot 10^{12} \cdot 10 \cdot 10^{-6}} = \frac{10^{-7}}{4\pi^2}$$

$$R_P \cdot \underbrace{\sqrt{\frac{2{,}533 \times 10^{-9}}{10 \cdot 10^{-6}}}}_{0.0159} = 200 \qquad \boxed{C = 2{,}533\ nF}$$

$$\boxed{R_P = 4000\pi = 12{,}566 k\Omega}$$

Q4. Derive the response of an RC low-pass filter to a sinusoidal wave form



$V_{in}(t) = A\sin(wt)$

$$\frac{V - V_{in}}{R} + C\frac{dv}{dt} = 0$$

$$\frac{V - V_{in}}{RC} + \frac{dv}{dt} = 0$$

$$\frac{dv}{dt} + \frac{1}{RC}V = \frac{1}{RC}V_{in}(t)$$

$$sV(s) - V(0^-) + \frac{1}{RC}V(s) = \frac{1}{RC}V_{in}(s)$$

$$= \frac{A}{RC}\frac{w}{s^2+w^2} \longrightarrow \left(s + \frac{1}{RC}\right)V(s) = V(0^-) + \frac{A}{RC}\frac{w}{s^2+w^2}$$

$$V(s) = \frac{V(0^-)}{s+\frac{1}{RC}} + \frac{A}{RC}\cdot\frac{w}{(s^2+w^2)(s+\frac{1}{RC})}$$

$$\frac{1}{(s^2+w^2)\cdot(s+\frac{1}{RC})} = \frac{Fs+G}{s^2+w^2} + \frac{H}{s+\frac{1}{RC}} \rightarrow (Fs+G)\cdot\left(s+\frac{1}{RC}\right) + H(s^2+w^2) = 1$$

$$Fs^2 + \frac{G}{RC} + Gs + \frac{F}{RC}s + Hs^2 + Hw^2 = 1$$

$F + H = 0 \quad G + \frac{F}{RC} = 0$

$G = \frac{H}{RC} \quad H = -F$

$$\frac{G}{RC} + Hw^2 = 1 \rightarrow \frac{H}{(RC)^2} + Hw^2 = 1$$

$$H\left[1 + (RCw)^2\right] = (RC)^2$$

$$\boxed{H = \frac{(RC)^2}{1+(RCw)^2}} \quad \boxed{G = \frac{H}{RC} = \frac{RC}{1+(RCw)^2}}$$

$$\boxed{F = -\frac{(RC)^2}{1+(RCw)^2}}$$

$$V_{per}(s) = \frac{AW}{RC}\left[\frac{FS+G}{s^2+w^2}\right] = \frac{AFw}{RC}\cdot\frac{s}{s^2+w^2} + \frac{AG}{RC}\cdot\frac{w}{s^2+w^2}$$

$$V_{per}(t) = \frac{AFw}{RC}\cdot\cos(wt) + \frac{AG}{RC}\sin wt$$

$$K\cdot\cos\phi = \frac{AG}{RC} \qquad K\cdot\sin\phi = \frac{AFw}{RC}$$

$$\tan\theta = \frac{K\sin\theta}{K\cos\theta} = \frac{F\cdot w}{G} = -RCw$$

$$(K\cos\theta)^2 + (K\sin\theta)^2 = K^2\underbrace{(\sin^2\theta+\cos^2\theta)}_{1} = K^2 = \left(\frac{AG}{RC}\right)^2 + \left(\frac{AFw}{RC}\right)^2$$

$$K^2 = \left(\frac{A}{RC}\right)^2\cdot\left(G^2+(FW)^2\right) = \left(\frac{A}{RC}\right)^2\cdot\left[\frac{(RC)^4\cdot w^2}{[1+(RCw)^2]^2} + \frac{(RC)^2}{[1+(RCw)^2]^2}\right]$$

$$K^2 = \left(\frac{A}{RC}\right)^2\cdot(RC)^2\cdot\left[\frac{(RCw)^2+1}{[1+(RCw)^2]^2}\right] = A^2\cdot\frac{1}{1+(RCw)^2} \rightarrow \boxed{K = \frac{A}{\sqrt{1+(RCw)^2}}}$$

$$s_{per}(t) = K\sin(wt+\phi) = \frac{A}{\sqrt{1+(RCw)^2}}\sin(wt+\arctan(-RCw))$$

$$W = W_0 = \frac{1}{RC} \qquad K = \frac{A}{\sqrt{1+\left(RC\cdot\frac{1}{RC}\right)^2}} = \boxed{\frac{A}{\sqrt{2}} = K}$$

$$\phi = \arctan\left(-RC\cdot\frac{1}{RC}\right) = \arctan(-1) = -\frac{\pi}{4}$$