Hw 1.

Q1. Determine the output impedance of each circuit shown in fig 46. Assume $\beta >> 1$. Explain which ones are considered cascode stages.

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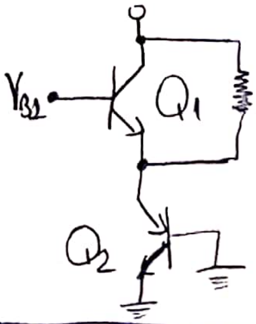
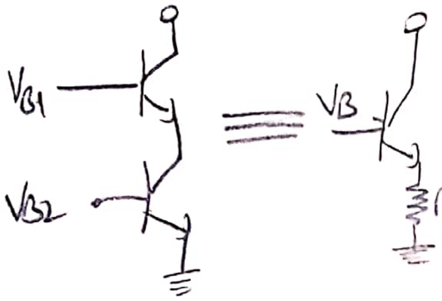


fig. 9.46

Cascode tipi için



$\rightarrow R_{out} = [1 + g_{m1}(r_{O2} \| r_{\pi 1})] r_{O1} + r_{O2} \| r_{\pi 1}$

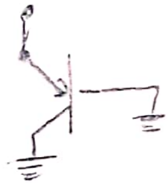
$\hookrightarrow g_{m1}(r_{O2} \| r_{\pi 1}) >> 1$ tipik değerlerden

$R_{out} \cong g_{m1} r_{O1} (r_{O2} \| r_{\pi 1})$

Burada fig 9.46 için $V_{B2} = 0$ ve r_{O1}'e paralel R_P eklenmiştir.

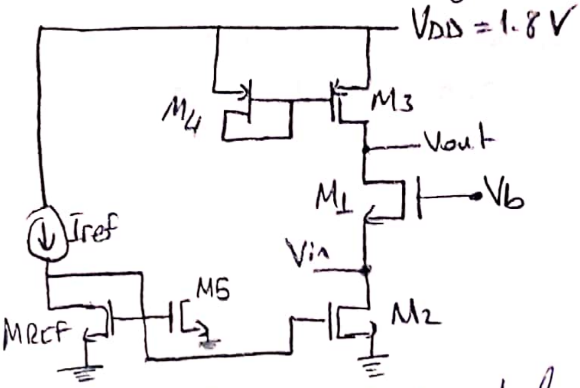
Buna göre $\longrightarrow$ $\boxed{R_{out} = (1 + g_m \underbrace{[r_{O1} \| R_P]}_{R_P \text{ paraleli ekledik}}) \underbrace{[r_{\pi 1} \| \frac{1}{g_{m2}}]}_{V_{B2}=0 \text{ olduğundan}} + [r_{O1} \| R_P]}$

$V_{B2} = 0$ durumunda



} devre parçası cascode tipine uygun değildir.

Q2: The common-gate stage of Fig 9.83 employs the current source M_3 as the load to achieve



a high voltage gain. For simplicity, neglect channel-length modulation in M_1. Assuming $(W/L)_3 = 40/0.18$, $\lambda_n = 0.1V^{-1}$, and $\lambda_p = 0.2V^{-1}$, design the circuit for a voltage gain of 20, an input impedance of 50Ω, and a power budget of 13mW. (You may not need all of the power budget.)

$$\frac{V_{out}}{V_{in}} = g_{m1} \cdot r_{o3} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_{D1}} \cdot \frac{1}{\lambda_p \cdot I_D} = 20 \text{ (voltage gain)}$$

$$r_{in} = \frac{1}{g_{m1}} \| r_{o2} \longrightarrow \frac{1}{g_{m1}} \ll r_{o2} \longrightarrow \frac{1}{g_{m1}} \| r_{o2} \approx \frac{1}{g_{M1}} = 50\Omega \text{ (input impedance)}$$

$$\frac{1}{g_{m1}} \| r_{o2} \approx \frac{1}{g_{m1}}$$

$$r_{in} = \frac{1}{\underbrace{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_{D1}}}_{\to g_{m1}}} = 50\Omega \qquad \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_{D1}} = \frac{1}{50\Omega} = g_{m1}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{50} \cdot \frac{1}{\lambda_p \cdot I_{D1}} = \frac{1}{50} \cdot \frac{1}{0.2 \cdot I_{D1}} = 20 \to \boxed{I_{D1} = \frac{1}{1000} \cdot \frac{1}{0.2} = 5mA}$$

$$\boxed{I_{D1} = I_{D2} = I_{D3} = 5mA} \to \frac{1}{\lambda_n I_{D,2}} = \frac{1}{\lambda_n \cdot I_{D,1}} = \frac{1}{0.1 \cdot 5mA} = 2k\Omega$$

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_{D1}} = \frac{1}{50} \to 2 \cdot \mu_n C_{ox}\left(\frac{W}{L}\right) I_{D1} = \frac{1}{2500}$$

$$2 \cdot 100 \cdot 10^{-6} \frac{W}{L} 5mA = \frac{1}{2500}$$

$\mu_n C_{ox} = 100\mu A/V^2$

$\mu_p C_{ox} = 50\mu A/V^2$

$$\frac{W}{L} = \frac{1000}{2500} \cdot \frac{1}{200} \cdot 10^{6} = 400$$

$$\boxed{\frac{W}{L} = 400}$$

$L_1 = 0.18$ $W_1 = 72$

2.2 $I_{TOT} = \frac{Power}{V_{DD}} = \frac{13mV}{1.8V} = 7.222mA$

$I_{D1} = I_{out} = 5mA \qquad I_{D4} = I_{D5} = I_{ref} = I_1$

$I_{in} + I_{out} = I_{TOT}$; $I_{in} = (7.222-5)mA = 2.222mA$

$I_{in} = 2I_1 \Rightarrow I_1 = I_{D4} = I_{D5} = I_{REF} = 1.11\ mA$

$I_{REF} = 1.11mA = \frac{1}{2}\underbrace{\mu_n C_{ox}}\left(\frac{W}{L}\right)_{ref} \cdot \underbrace{(\underbrace{V_{GS}}_{0.6} - \underbrace{V_{TH}}_{0.4})^2}_{HP}$

$I_{REF} = 1.11mA = \frac{1}{2} \cdot \overset{100 \cdot 10^{-6}}{100} \cdot \left(\frac{W}{L}\right) \cdot (0.2)^2 \rightarrow \left(\frac{W}{L}\right)_{ref} = \frac{1.11 \cdot 10^{-3} \cdot 2 \cdot 10^{6}}{\frac{4}{100} \cdot 100} = \frac{1.11}{2} \cdot 1000 = 555$

$\left(\frac{W}{L}\right)_{ref} = 555 \Rightarrow \frac{W}{0.18} \Rightarrow W = 100 \rightarrow \left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_{ref}$

$\left(\frac{100}{0.18}\right) = \left(\frac{W}{L}\right)_{ref}$

$\boxed{\left(\frac{W}{L}\right)_{ref} = \left(\frac{100}{0.18}\right)} \quad \boxed{\left(\frac{W}{L}\right)_1 = \left(\frac{72}{0.18}\right)}$

$M_3 = 5mA$

$M_4 = 1.11\ mA$

$I_{D4} = \frac{1}{2}\underbrace{\mu_p \cdot C_{ox}}_{50\mu}\underbrace{\left(\frac{W}{L}\right)_4}_{\frac{8.88}{0.18}}(V_{GS} - V_{TH})^2$

$1mA$

$(V_{GS} - V_{TH})^2 = 0.810 = (V_{GS} - 0.4)^2$

$0.9004 = V_{GS} - 0.4$

$\boxed{V_{GS_4} = 1.3004V}$

M3 node

$V_{GS,4} = V_{GS3}$ olmalı (current mirror yapısı)

$V_{DS,3} < 0.5 - 1.300 \Rightarrow V_{DS,3} < -0.800V \rightarrow$ satürasyonda olması için.

$\frac{1}{g_{m1}} = 50\Omega = \frac{V_{GS} - V_{TH}}{2I_{D1}} \rightarrow V_{GS} = 0.9V$ olmalı //

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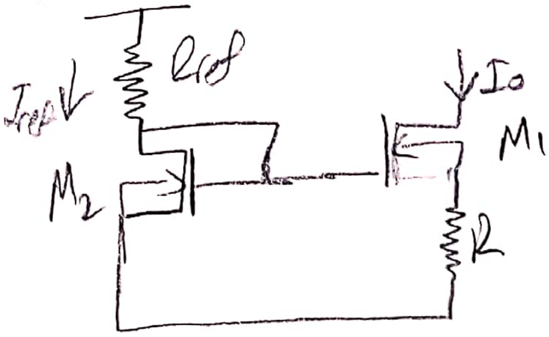
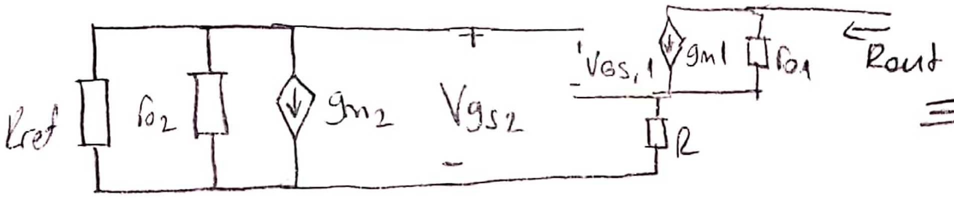
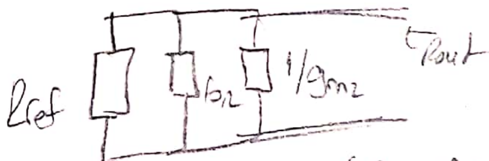
Q3. The Widlar current source shown in Fig. P9.6 has $I_{ref} = 50\mu A$, $R = 2k\Omega$ and $V_{DD} = 12V$. The MOS parameters are $k_n = 100\mu A/V^2$, $V_t = 1V$, $|V_M| = 100V$, and $(W/L)_1 = (W/L)_2 = 20$. Determine ⓐ the output current I_o, ⓑ the output resistance r_{o2}, and ⓒ the value of R_{ref}





≡ small signal eşdeğer devresi

$V_{GS,1} = V_{GS,2} = 0$ için small signal eşdeğeri aşağıdaki gibi çizilir



$R_{out} = (1 + g_{m1} r_{o1}) R + r_{o1} = \boxed{R + g_{m1} r_{o1} R + r_{o1}}$

Kontrol için (R_{out}):

$V_{test} = (i_{test} - g_{m1} V_{GS1}) r_{o1} - V_{GS1} \rightarrow V_{GS1} = -i_{test} \cdot R$

$V_{test} = i_{test} [1 + g_{m1} \cdot R] r_{o1} + R \cdot i_{test}$

$\frac{V_{test}}{i_{test}} = R_{out} = (1 + g_{m1} R) r_{o1} + R = \boxed{r_{o1} + g_{m1} R \cdot r_{o1} + R}$

kontrol edildi.

$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot (V_{GS} - V_{TH})^2$

$g_m = \frac{\partial I_D}{\partial V_{GS}} = 2 \cdot \frac{1}{2} \cdot \mu_n C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH}) = \boxed{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})} = \boxed{g_m}$

Q3.2 DC Analiz (M_2)

Gevrim $\quad V_{DD} - R_{ref} \cdot \underbrace{I_{ref}}_{I_{D2}} - V_{GS,2} = 0$

$$V_{DD} = R_{ref} \cdot \left(\underbrace{\tfrac{1}{2}\mu_n C_{ox}}_{} \cdot (V_{GS}-V_{TH})^2 \right) + V_{GS,2} \rightarrow g_{m2} = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \cdot (V_{GS}-V_{TH})$$

$\boxed{r_{o2} = \frac{V_m}{I_{D2}}}$

$100\mu A/V^2 \cdot (V_{GS}-V_{TH})^2 = 50\mu A \quad (V_{GS}-V_{TH}) = \frac{1}{\sqrt{2}} = 0.707$

$\boxed{V_{GS} = V_{TH} + 0.707 \quad V_{GS} = 1.707V}$

$\boxed{12 = R_{ref} \cdot 50\mu A + V_{GS,2}}$

DC Analiz (M_1)

Düğüm

$$\frac{V_{GS,2}-V_{GS,1}}{R} - I_{D1} = 0 \qquad \frac{V_{GS2}-V_{GS1}}{R} = I_{D1} = \frac{1}{2}\mu_n C_{ox}(V_{GS1}-V_{TH})^2$$

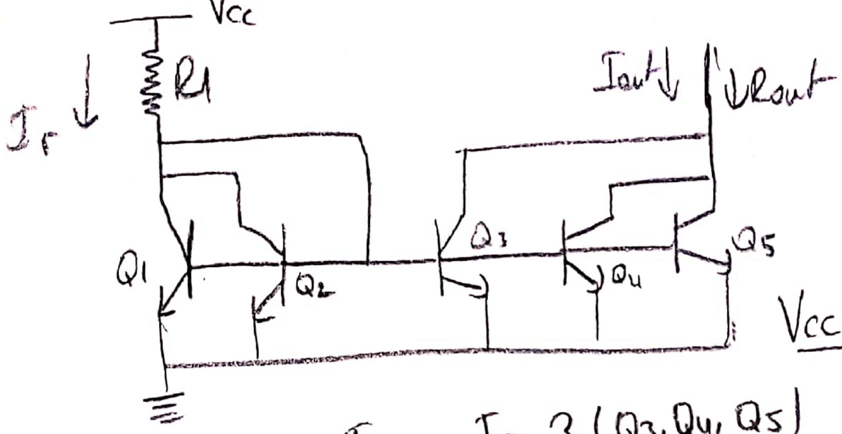
$I_{D1} = I_{out}$ için M_1 satürasyonda varsayım yapılır

$V_{DS1} \geq V_{GS,1} - V_{TH} \rightarrow V_{D1} - V_{S1} \geq V_{GS1} - V_{TH} \rightarrow V_{D1} - R.I_{out} \geq V_{GS1} - V_{TH}$

$$V_{D1} - I_{out}.R \geq \underbrace{V_{GS1}}_{V_{GS2}-RI_{out}} - V_{TH} \rightarrow V_{D1} - I_{out}R \geq V_{GS2} - RI_{out} - V_{TH}$$

$\boxed{V_{D1} \geq V_{GS2} - V_{TH}}$

(Q4) The multiple transistors of the current source in Fig P9.26 have $\beta_f = 150$, $R_1 = 10k\Omega$, $V_{CC} = 15V$ and $V_A = 100V$. The B-E voltages are equal, $V_{BE} = 0.7V$. Calculate (a) the output current I_{out}, (b) the output resistance R_{out} (c) Thevenin equivalent voltage V_{TH} and (d) The collector current ratio if $V_{CE2} = 15V$



if $\beta >>$ ise $I_B \approx 0$

$I_{B_1} \cong I_{B_2} \approx I_{B_3} \approx I_{B_4} \approx I_{B_5} = 0$

ise

$\dfrac{V_{CC} - V_{BE}}{R} = I_r = \dfrac{15-0.7}{10k} = \underline{1.43mA}$

$I_r = 1.43mA$, $I_o = I_r \cdot \dfrac{3\,(Q_3, Q_4, Q_5)}{2\,(Q_1, Q_2)}$

(a) $I_o = \dfrac{3}{2} I_r = 2.14mA$

$R_o = \dfrac{V_A}{I_o/3} \,//\, \dfrac{V_A}{I_o/3} \,//\, \dfrac{V_A}{I_o/3} = \dfrac{V_A/3}{I_o/3} = \dfrac{V_A}{I_o}$

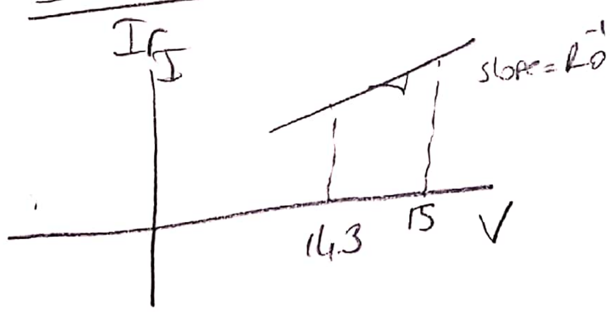
$V_A = 100V$ $I_o = 2.14mA$ (b)

$R_o = \dfrac{100V}{2.14mA} = 46.728k\Omega$

(c) $V_{TH} = I_o \cdot R_o = 100V$

(d) collector current ratio

$\dfrac{I_o + \Delta I_c}{I_r} = \dfrac{2.14mA + \Delta I_c}{1.43mA} = 1.5 + \dfrac{\Delta I_c}{1.43mA}$



$\dfrac{1}{R_o} = \dfrac{\Delta I_c}{0.7V}$ $\Delta I_c = \dfrac{0.7V}{46.72k\Omega} = \underline{0.014mA}$

$\Delta I_c = 0.014mA$ //

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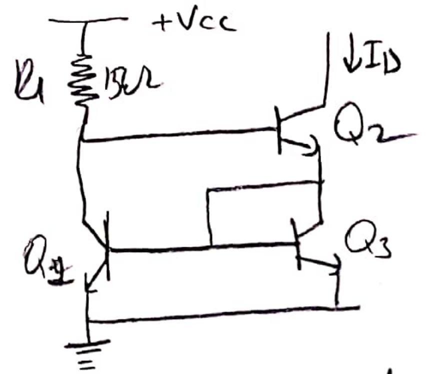
Q5. For the Wilson current source in fig. P9.31, determine the output current I_O and the output resistance R_o, Assume $V_{CC}=20V$, $V_{BE}=0.7V$, $V_T=26mV$, $V_A=150V$ and $\beta_F=150$



Sol

$$I_{R_1} = \frac{V_{CC}-(V_{BE1}+V_{BE2})}{R_1}$$

$$I_{R1} = \frac{20-(0.7+0.7)}{15k\Omega} = 1.24mA$$

BJT'ler eşdeğer olmalı.

$$I_{E2} = I_{C2}\cdot\left(\frac{\beta+1}{\beta}\right) = \left(\frac{\beta+2}{\beta}\right)\cdot I_{C1} \Rightarrow \boxed{I_{C3} = I_{C2}\cdot\left(\frac{\beta+1}{\beta}\right)\left(\frac{\beta}{\beta+2}\right)}$$

$$\rightarrow I_{C3} = I_{C1}$$

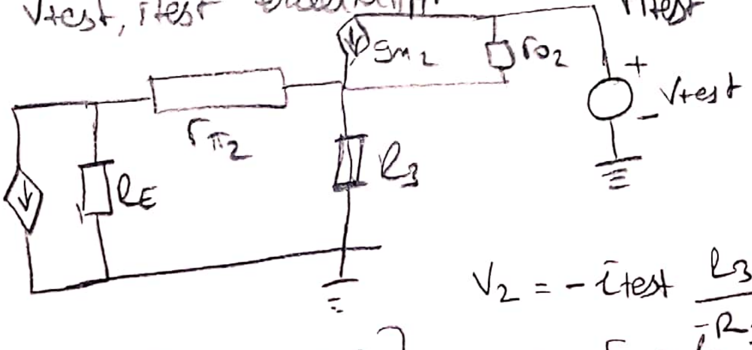
$$I_R = I_{C1} + I_{B2}$$

$$I_R = I_{C2}\cdot\left(\frac{\beta+1}{\beta}\right)\cdot\left(\frac{\beta}{\beta+2}\right) + \underbrace{I_{B2}}_{\frac{I_{C2}}{\beta}} = I_{C2}\left(\left(\frac{\beta+1}{\beta}\right)\cdot\left(\frac{\beta}{\beta+2}\right)+\frac{1}{\beta}\right)$$

$$I_R = I_{C2}\left(\frac{\beta+1}{\beta+2}+\frac{1}{\beta}\right)$$

$$I_R = I_{C2}\left(1-\frac{1}{\beta+2}+\frac{1}{\beta}\right) = I_{C2}\left(1-\frac{2\beta+2}{(\beta+2).\beta}\right)$$ //

small-signal eşdeğer
tipik değerlerde $r_{o1} >> R_1$, $R_1 >> r_{\pi,2}$ ve $g_m R_1 >> 1$ olduğu için basitleştirilmiş bir eşdeğer devre çizilmiştir ve çözüm yapabilmek için Vtest, itest eklenmiştir



Çevrim

$$\boxed{V_{test} = (i_{test}-g_{m2}V_2)r_{o2} + i_{test}[R_3 \| R_e]}$$

$$V_2 = -i_{test}\frac{R_3}{R_3+R_e}\cdot r_{\pi2}$$

$$V_{test} = i_{test}[r_{o2}+R_3 \| R_e] - g_{m2}r_{o2}\left[-i_x\frac{R_3}{R_3+R_e}r_{\pi2}\right]$$

$$R_{out} = \frac{V_{test}}{i_{test}} = r_{o2} + \frac{1}{R_3}+\frac{1}{R_2} + \beta_2\frac{R_3.r_{o2}}{R_3+R_e}$$

$$R_{out} = r_{o2} + \frac{R_3\cdot(R_e+\beta r_{o2})}{R_3+R_e}$$

$$R_{out} = r_{o2} + \underbrace{R_s \parallel R_e}_{\ll} + r_{o2}\beta_2 \cdot \underbrace{\frac{R_e}{R_3 + R_e}}_{\frac{1}{2}} \qquad R_3 \approx R_e$$

$$R_{out} = r_{o2} + r_{o2} \cdot \beta_2 \cdot \frac{1}{2}$$

$$R_{out} = r_{o2}\left(1 + \frac{\beta_2}{2}\right) \qquad g_m = \frac{I_C}{V_T} \qquad g_m r_\pi = \beta \qquad r_\pi = \frac{\beta}{g_m} = \frac{\beta \cdot V_T}{I_C}$$

$$\boxed{r_o = \frac{V_A}{I_C}}$$

$V_A = 150V$ $\quad \beta_f = 150$ $\quad V_T = 26mV$ $\quad V_{BE} = 0.7V$

$V_{CC} = 20V$

$I_R = 1.24mA$ $\qquad I_R = I_{C2} \cdot \left(1 - \frac{2\beta + 2}{(\beta+2) \cdot \beta}\right)$

$$\boxed{I_C = \frac{1.24 \times 10^{-3}}{1 - \frac{302}{(302) \cdot 152}} \approx 1.24 \times 10^{-3}}$$

$$\boxed{r_o = \frac{150}{1.24 \times 10^{-3}} = 120.96k\Omega}$$

$$\boxed{r_\pi = \frac{150 \cdot 26 \times \cancel{10^{-3}}}{1.24 \times \cancel{10^{-3}}} = 3145.16\Omega}$$

$$\boxed{g_m = \frac{1.24 \times 10^{-3}}{26 \times 10^{-3}} = 0.047}$$

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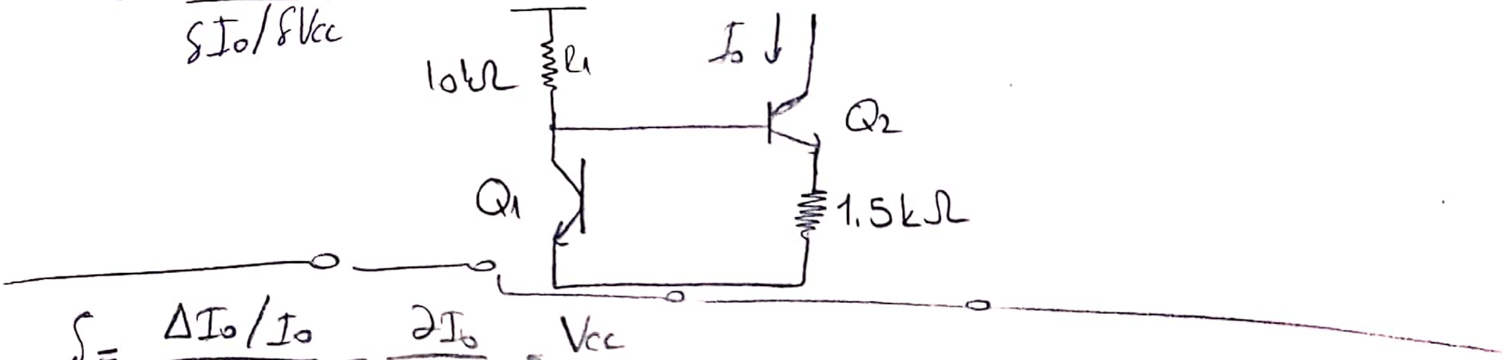
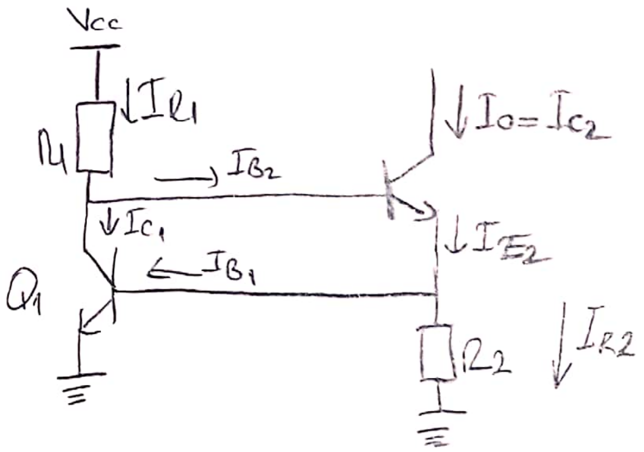
Q6. Determine the sensitivity S of output current I_o to supply voltage V_{cc} for the circuit in Fig P.9.34. S is defined as

$$S = \frac{V_{cc}/I_o}{\delta I_o / \delta V_{cc}}$$



$$S = \frac{\Delta I_o / I_o}{\Delta V_{cc}/V_{cc}} = \frac{\partial I_o}{\partial V_{cc}} \cdot \frac{V_{cc}}{I_o}$$



$$\frac{V_{cc} - (V_{BE1} + V_{BE2})}{R_1} = I_{R1}$$

$I_{R1} = I_{B2} + I_{C1} \rightarrow$ node

$I_o = I_{C2} = I_{B2} \cdot \beta$

$I_{R1} = \frac{I_o}{\beta} + I_{C1}$ $\qquad I_{C1} = \beta I_{B1}$

$$I_{R1} = \frac{I_o}{\beta} + \beta\left(I_o\left(\frac{\beta+1}{\beta}\right) - \frac{V_{BE1}}{R_2}\right)$$

$I_{E2} - \frac{V_{BE1}}{R_2} = I_{B1}$

$I_o \frac{\beta+1}{\beta} - \frac{V_{BE1}}{R_2} = I_{B1}$

$\beta\left[I_o \frac{\beta+1}{\beta} - \frac{V_{BE1}}{R_2}\right] = I_{C1}$

$V_{cc} = I_{R1} R_1 + V_{BE1} + V_{BE2}$

$$V_{cc} = R_1\left[\frac{I_o}{\beta} + \beta\left(I_o\left(\frac{\beta+1}{\beta}\right) - \frac{V_{BE1}}{R_2}\right)\right] + V_{BE1} + V_{BE2}$$

$$\frac{V_{cc} - V_{BE1} - V_{BE2}}{R_1} + \frac{\beta \cdot V_{BE1}}{R_2} = \frac{I_o}{\beta} + \frac{\cancel{\beta} \cdot I_o \cdot (\beta+1)}{\cancel{\beta}} \rightarrow I_o\left(\frac{1}{\beta} + \beta + 1\right)$$

$$I_o = \left[\frac{V_{cc} - V_{BE1} - V_{BE2}}{R_1} + \frac{\beta V_{BE1}}{R_2}\right] \cdot \frac{1}{\frac{1}{\beta} + \beta + 1}$$

$V_{B1} = V_{BE2} = 0.7V$

$$I_o = \left[\frac{V_{cc} - 1.4}{10k\Omega} + \frac{0.7\beta}{R_2}\right] \cdot \frac{\beta}{\beta^2 + \beta + 1} \qquad \text{eq.1}$$

$$\frac{\partial I_o}{\partial I_o} = \frac{\partial\left[\left(\frac{V_{cc} - 1.4}{10k\Omega} + \frac{0.7\beta}{R_2}\right) \cdot \beta/\beta^2 + \beta + 1\right]}{\partial V_{cc}} = \boxed{1 = \frac{1}{10k\Omega} \cdot \frac{\beta}{\beta^2 + \beta + 1} \cdot \frac{V_{cc}}{I_o}}$$

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Q6.2

$$I_o = \left[\frac{V_{cc}-1.4}{10k\Omega} - \frac{0.7\beta}{R_2}\right]\frac{\beta}{\beta^2+\beta+1}$$

$$I_o \cdot \left(\frac{\beta^2+\beta+1}{\beta}\right) = \frac{V_{cc}}{10k\Omega} - \frac{1.4}{10k\Omega} - \frac{0.7\beta}{1.5k\Omega}$$

$$\left[I_o \cdot \left(\frac{\beta^2+\beta+1}{\beta}\right) + \frac{1.4}{10k\Omega} + \frac{0.7\beta}{1.5k\Omega}\right] \cdot 10k\Omega = V_{cc}$$

Q9.1 $\rightarrow$ $1 = \frac{1}{10k\Omega} \cdot \left(\frac{\beta}{\beta^2+\beta+1} \cdot \frac{V_{cc}}{I_o}\right)$

$$10k\Omega \cdot \left(\frac{\beta^2+\beta+1}{\beta}\right) = \frac{I_o \cdot \left(\frac{\beta^2+\beta+1}{\beta}\right) + \frac{1.4}{10k\Omega} + \frac{0.7\beta}{1.5k\Omega}}{I_o}$$

$$10k\Omega \cdot \left(\frac{\beta^2+\beta+1}{\beta}\right) = \frac{\beta^2+\beta+1}{\beta} + \frac{1.4}{10k\Omega \cdot I_o} + \frac{0.7\beta}{1.5k\Omega \cdot I_o}$$

$$= 9999\left(\frac{\beta^2+\beta+1}{\beta}\right) = \frac{2100\cancel{I_o} + 7000\beta\cancel{I_o}}{15\times10^6 \cdot I_o^{\cancel{2}}} = \frac{1}{I_o} \cdot \frac{9100}{15\times10^6}$$

$$I_o = \frac{9100}{15\times10^6} \cdot \left(\frac{\beta}{\beta^2+\beta+1}\right) \cdot \frac{1}{9999 \approx 10000}$$

$$\boxed{I_o = 0.0606\times10^{-6} \cdot \left(\frac{\beta}{\beta^2+\beta+1}\right) A}$$

I_o yerine yazılarak V_{cc} bulunur.

$$V_{cc} = \left[I_o\left(\frac{\beta^2+\beta+1}{\beta}\right) + \frac{1.4}{10k\Omega} + \frac{0.7\beta}{1.5k\Omega}\right] \cdot 10k\Omega$$

$$V_{cc} = \left[0.0606\times10^{-6} + \frac{1.4}{10k\Omega} + \frac{0.7\beta}{1.5k\Omega}\right] \cdot 10k\Omega$$

$$V_{cc} = \left[0.140\times10^{-3} + \frac{0.7\beta}{1.5k\Omega}\right] \cdot 10k\Omega = \boxed{1.400 + 4.666\beta = V_{cc}}$$