HW-10

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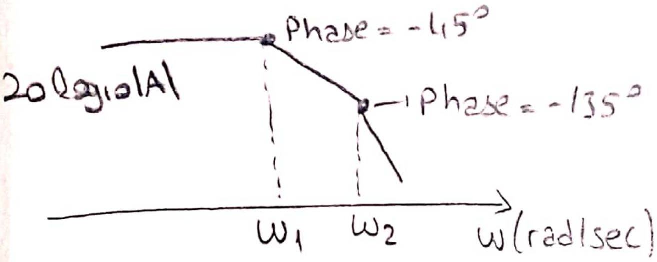
Q1. If the phase margin of an amplifier is $PM = 40°$ and the magnitude of the open loop gain is $|A(jw)| = 50$, find the magnitude of the closed-loop gain $|A_f(jw)|$.

β is real and open-loop amp. is a two pole system.

$$A(jw) = \frac{A \rightarrow 50}{\left(1+\frac{jw}{w_1}\right)\left(1+\frac{jw}{w_2}\right)} \qquad \underline{w_1 \ll w_2}$$

$$\rightarrow 20\log_{10}|\beta . A(jw)| = 20\log_{10}|A(jw)| - 20\log_{10}\left|\frac{1}{\beta}\right|$$



$$\rightarrow A(jw_1) \cong \frac{A}{\left(1+\frac{jw_1}{w_1}\right).1} \Rightarrow 45° \text{ phase}$$

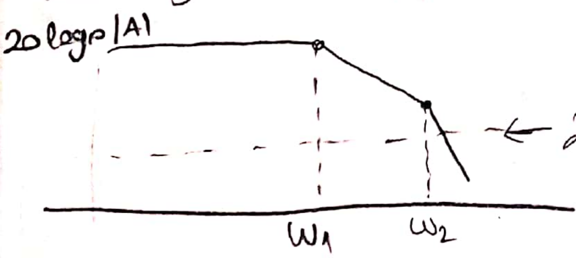
$$\rightarrow A(jw_2) \cong \frac{A}{\frac{jw_2}{w_1}\left(1+\frac{jw_2}{w_2}\right)} \Rightarrow 135° \text{ phase}$$

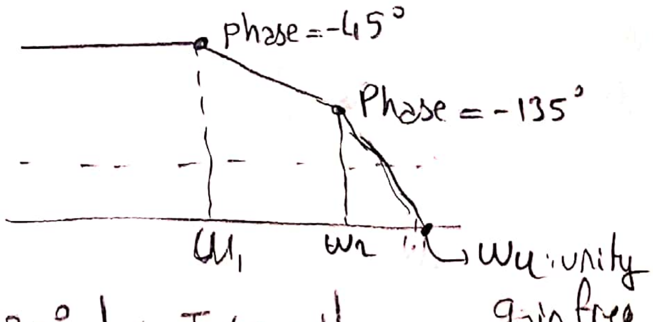
$PM = 40°$ by designing β

$PM = 180° - |\angle A(jw_u)\beta|$ w_u: unity gain frequency for loop gain

$T_L(jw) = A(jw).\beta \rightarrow \beta > 0$ and β real

$$20\log_{10}|A(jw).\beta| = 20\log_{10}|A(jw)| - 20\log_{10}\left|\left(\frac{1}{\beta}\right)\right|$$





$$PM = 180° - |\angle A(jw)\beta| = 180° - |\angle T_L(jw_u)|$$

$$|T_L(jw_u)| = 140° \quad \angle T_L(jw_u) = -140°$$

$$T_L(jw_u) = \frac{A\beta}{\left(1+\frac{jw_u}{w_1}\right)\cdot\left(1+\frac{jw_u}{w_2}\right)} \cong \frac{A\beta}{\left(\frac{jw_u}{w_1}\right)\left(1+\frac{jw_u}{w_2}\right)}$$

$w_u >> w_1$
$w_u > w_2$

$$\angle T_L(jw_u) = -90^\circ - \arctan\left(\frac{w_u}{w_2}\right) = -140^\circ$$

$$\arctan\left(\frac{w_u}{w_2}\right) = 50^\circ \longrightarrow \frac{w_u}{w_2} = 1.1917 \rightarrow \boxed{w_u = 1.1917\, w_2}$$

$$|T_L(jw_u)| = 1 \cong \frac{\overset{\to 50}{A}\beta}{\left(\frac{w_u}{w_1}\right)\sqrt{1+\frac{w_u^2}{w_2^2}}}$$

$$\frac{w_u}{w_1}\cdot\sqrt{1+(1.1917)^2} = 50\beta \qquad \boxed{\beta = \frac{w_u}{w_1}\cdot 0.0311}$$

$$\frac{w_u}{w_1} = \frac{w_2}{w_1}\cdot\frac{w_u}{w_2} = 1.1917\cdot$$

$$\boxed{\beta = \frac{w_2}{w_1}\cdot 0.0370}$$

$$\boxed{A_f = \frac{A}{1+\beta A}} = \boxed{A_f = \frac{50}{1+1.853\cdot\frac{w_2}{w_1}}}$$

Q2. The open loop gain of an amplifier has break frequencies at $f_{P1} = 10kHz$ $f_{P2} = 100kHz$, $f_{P3} = 1MHz$. The low-frequency gain is $A_o = 250$, and the feedback factor is $\beta = 0.9$. Calculate the gain margin GM and the phase margin PM.

$$A(jw) = \frac{A_o}{\left(1+\frac{jf}{f_1}\right)\cdot\left(1+\frac{jf}{f_2}\right)\cdot\left(1+\frac{jf}{f_3}\right)} = \frac{250}{\underbrace{\left(1+\frac{jf}{10kHz}\right)\cdot\left(1+\frac{jf}{100kHz}\right)\left(1+\frac{jf}{1MHz}\right)}_{I}}$$

$$A(jf)\cdot\beta = \frac{250\cdot 0.9}{I} = \frac{225}{I}$$

$$|A(jf)\cdot\beta| = 1 = \frac{225}{\left|1+\frac{jf_u}{10k}\right|\cdot\left|1+\frac{jf_u}{100k}\right|\left|1+\frac{jf_u}{1MHz}\right|}$$

$$|A(jf_u)\beta| = 1 = \frac{225}{\sqrt{1+\frac{f_u^2}{(10k)^2}} \cdot \sqrt{1+\frac{f_u^2}{(100k)^2}} \cdot \sqrt{1+\frac{f_u^2}{(1MHz)^2}}} = 1$$

$$(225)^2 = \left(1+\frac{f_u^2}{(10k)^2}\right) \cdot \left(1+\frac{f_u^2}{(100k)^2}\right) \cdot \left(1+\left(\frac{f_u}{10^6}\right)^2\right)$$

$$5.062\times10^4 = \frac{(10^8+f_u^2)}{10^8} \cdot \frac{(10^{10}+f_u^2)}{10^{10}} \cdot \frac{(10^{12}+f_u^2)}{10^{12}}$$

$$5.062\times10^{34} = (10^8+f_u^2).(10^{10}+f_u^2).(10^{12}+f_u^2)$$

$$5.062\times10^{34} = 10^8\left(1+\frac{f_u^2}{10^8}\right).10^8\left(100+\frac{f_u^2}{10^8}\right).10^8\left(10000+\frac{f_u^2}{10^8}\right)$$

$$\boxed{\frac{f_u^2}{10^8} = a}$$

$$5.062\times10^{10} = (a+1).(a+100).(a+10000)$$

$$'' \quad = a^3 + 10\cancel{0}a^2 + 100\cancel{00}a^2 + 10^6a + \cancel{a^2} + 100a + 10^4a + 10^6$$

$$a^3 + 10000a^2 + 10^6a - 5.062\times10^{10} = 0$$

MATLAB CODE

```
syms f(a)
f(a) = a^3 + 10000*a^2 + 10^6*a - 5.062*10^10
sol = vpasolve(f)
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$\longrightarrow$ $a_1 = -9308.3490\ldots$

$a_2 = -2703.3080386$

$a_3 = 2011.657060\ldots$

$a>0$ olacağından $a = \frac{f_u^2}{10^8}$

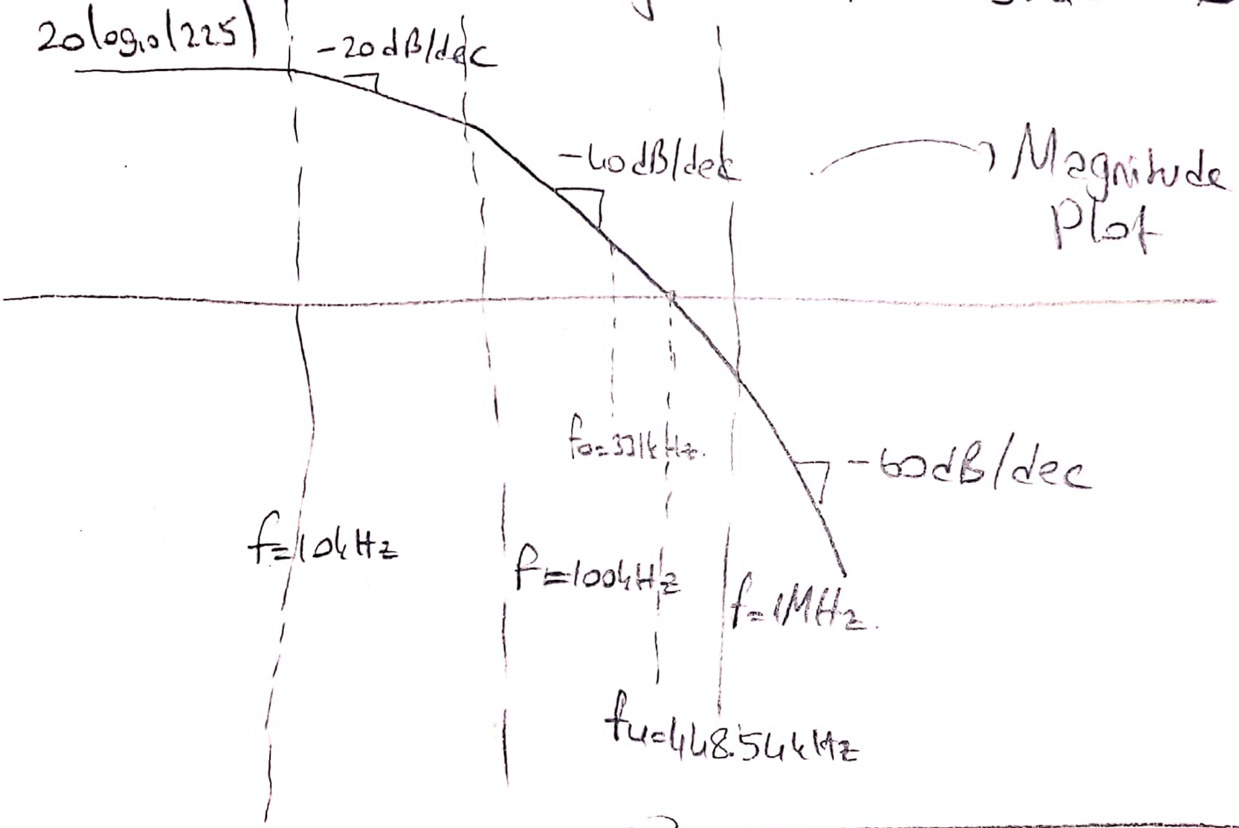
$$a = \frac{f_u^2}{10^8} = 2011.657 \longrightarrow f_u^2 = 2011.657\times10^8 \rightarrow \boxed{f_u = 448.51kHz}$$

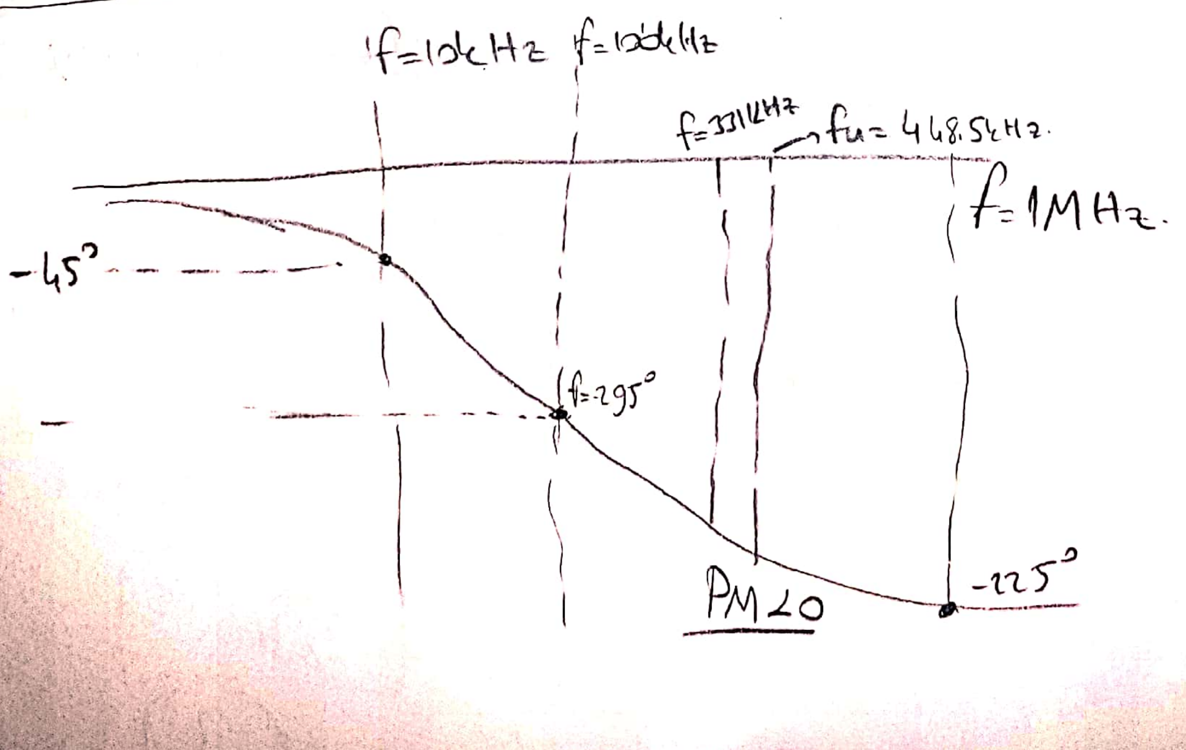
$$\angle A(jf_u)\beta \cong -190.25 < -180^\circ \quad PM = -10.25^\circ$$

$$\angle A(jf_0).\beta = -180^\circ \quad f_0 \cong 331kHz$$

$$|A(jf_0)\beta| = \frac{225}{\underbrace{\sqrt{1+\left(\frac{331kHz}{10kHz}\right)^2}}_{33.115}\underbrace{\sqrt{1+\left(\frac{331k}{100k}\right)^2}}_{3.457}\cdot\underbrace{\sqrt{1+\left(\frac{331k}{1M}\right)^2}}_{1.053}} \cong 1.8665$$

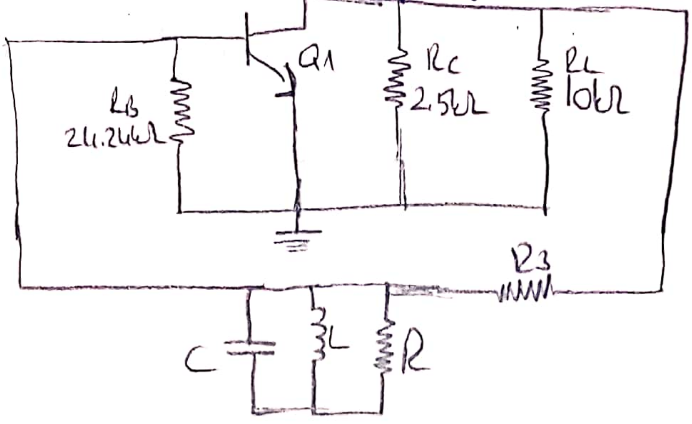
$$GM = 20\log_{10}|1| - 20\log_{10}|1.8665| = -5.420dB$$





Q3. Find the values of R, R_3, C and L for the phase shift oscillator in Fig. P13.6 so that the oscillation frequency is $f_0 = 5kHz$.

$h_{ie} = 1.3k\Omega$
$h_{fe} = 100$



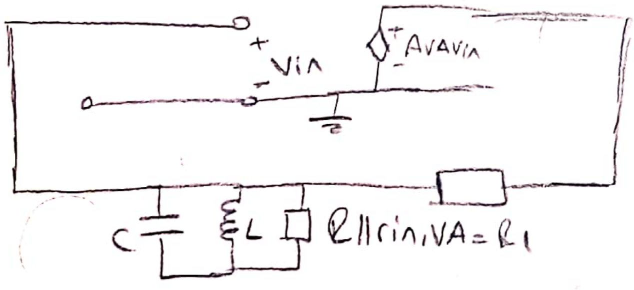
analyze the amplifier

$r_{in,VA} = R_B \parallel r_\pi$

$r_{out,VA} = R_C \parallel R_L \parallel r_o \cong R_C \parallel R_L \quad r_o \to +\infty$

r_π, β given $\to$ $g_m = \frac{\beta}{r_\pi} = \frac{100}{1.3k\Omega} = 0.0769$

$A_{VA} = -0.0769(2.5k\Omega \parallel 10k\Omega) = \boxed{-153.8 = A_{VA}}$



feedback Network

$$B(j\omega) = \frac{V_i}{V_o} = \frac{j\omega L \parallel \frac{1}{j\omega C} \parallel R_1}{j\omega L \parallel \frac{1}{j\omega C} \parallel R_1 + R_F}$$

$$\frac{j\omega L \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1 - \omega^2 LC}$$

$$B(j\omega) = \frac{\frac{j\omega L R_1}{j\omega L + (1-\omega^2 LC)R_1}}{\frac{j\omega L R_1}{j\omega L + (1-\omega^2 LC)R_1} + R_F} = \frac{j\omega L R_1}{j\omega L R_1 + j\omega L R_F + (1-\omega^2 LC)R_1 R_F} \quad \text{eq.1}$$

$A_{VA}\,\beta(j\omega) = [-g_m(R_C \parallel R_L)]\,\beta(j\omega)$

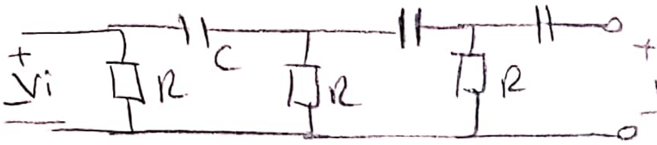
Phase cond. bulmak için eq. 1'de $1-\omega^2 LC = 0$ verilmeli

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

CamScanner ile tarandı

$$W_0 = \frac{1}{\sqrt{LC}} \longrightarrow A_{VA}\,\beta(j\omega)\Big|_{W=W_0} = \frac{\cancel{j\omega L}\,(-g_m(R_C\|R_L))}{\cancel{j\omega L}\,(1+\frac{R_F}{R_1})} \angle 0$$

$W=W_0$ için 180° phase cond.



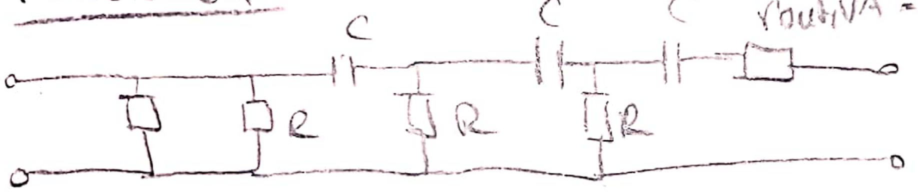
$$\beta(j\omega) = \frac{V_i}{V_o} = \frac{1}{1 - j\left(\frac{1}{wRC}\right) - \frac{5}{w^2R^2C^2} + \frac{j}{w^3R^3C^3}}$$

$$A_{VA}\,\beta(j\omega) = [-g_m(R_C\|R_L)]\,\beta(j\omega)$$

$$\frac{1}{w^3R^3C^3} - \frac{6}{wRC} = 0 \qquad 1 - 6w^2R^2C^2 = 0 \qquad W_0 = \frac{1}{\sqrt{6}RC}$$

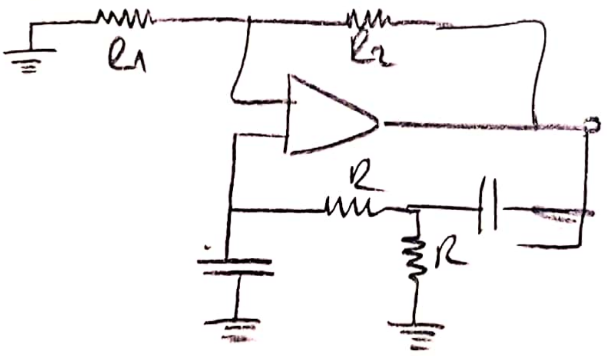
$$A_{VA}\,\beta(j\omega)\Big|_{W=W_0} = \frac{-g_m(R_C\|R_L)}{1-30} = 1 \longrightarrow g_m(R_C\|R_L) = 29$$

Forward amp

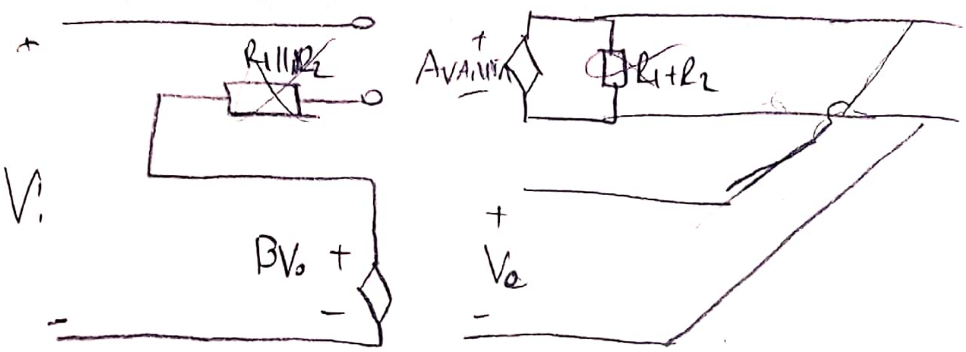


$$\left|\frac{1}{j\omega_0 C}\right| \geq 10\cdot(R_C\|R_L) \qquad W_0 = \frac{1}{\sqrt{6}RC} \qquad r_{in,VA} = R_B \| r_\pi \geq 10R$$

Q4. for the circuit in Fig. P13.13 find $L(s)$, $L(j\omega)$ the frequency for zero loop phase, and R_2/R_1 for oscillation



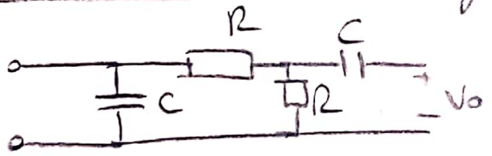
Opamp bir voltage amp.'dır. voltage-series bağlantı tipi inceleriz.



$$\frac{V_o}{V_i} = \frac{A}{1+A\frac{R_1}{R_1+R_2}} \cong \frac{R_1+R_2}{R_1} = 1+\frac{R_2}{R_1}$$

$A \to +\infty$ $r_{in,f} \to +\infty$ $r_{out,f}=0$

<u>feedback network Analysis</u>



$$\frac{V_i}{V_o} = \frac{\left(R+\frac{1}{Cs}\right) \| R}{\left(R+\frac{1}{Cs}\right) \| R + \frac{1}{Cs}} \cdot \frac{1/Cs}{1/Cs + R}$$

$$\frac{V_+ - V_i}{R} = \frac{V_i}{1/Cs} = Cs \cdot V_i$$

$V_+ = V_i + RCsV_i = (1+RCs)V_i$

$\frac{V_+}{R} + CsV_i + Cs(V_+ - V_o) = 0$ $\quad V_+(1+RCs) + RCsV_i = RCsV_o$

$V_i\left[R^2C^2s^2 + 3RCs + 1\right] = RCsV_o$

$$\frac{V_i}{V_o} = \frac{RCs}{R^2C^2s^2 + 3RCs + 1} \Rightarrow \beta(j\omega) = \frac{V_i(j\omega)}{V_o(j\omega)} = \frac{1}{3+j\left(\omega CR - \frac{1}{\omega CR}\right)}$$

$$A_{VA}\,\beta(j\omega) = \frac{1+\frac{R_2}{R_1}}{3+j\left(\omega CR - \frac{1}{\omega CR}\right)}$$

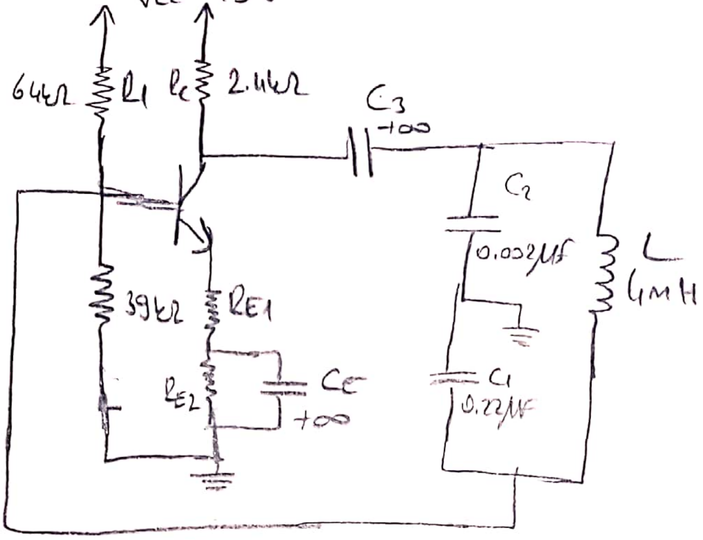
$\omega CR - \frac{1}{\omega CR} = 0$

$\omega_0 = \frac{1}{RC}$

$|A_{VA}\beta(j\omega_0)| = 1$ $\quad 1+\frac{R_2}{R_1} = 3$

$$\frac{R_2}{R_1} = 2$$

Q5. A Colpitts BJT oscillator is shown in Fig. P13.17. Calculate the frequency of oscillation f_o and the value of R_{E1} required to sustain the oscillation

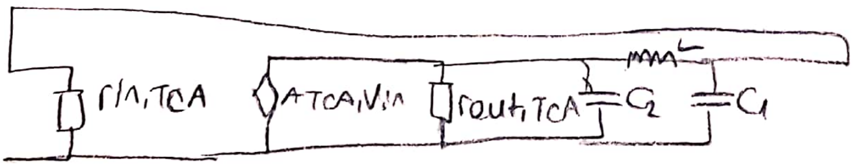


Model Common Emitter Amp. as Transconductance Amp.

$$r_{in,TCA} \cong R_1 // R_2 // \left[r_{\pi 1}(1+g_m R_{E1})\right]$$

$$r_{out,TCA} \cong R_C // \left[(1+g_{m1} r_{o1}) \cdot \frac{R_{E1}\, r_{\pi 1}}{R_{E1}+r_{\pi 1}+R_1//R_2}\right]$$

$$A_{TCA} \cong \frac{g_{m1}}{1+g_{m1}R_{E1}}$$

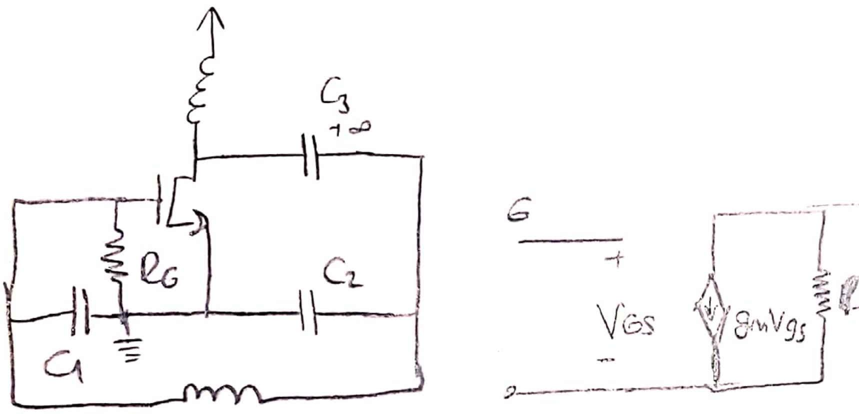


$$W_o = \frac{1}{\sqrt{L\frac{C_1C_2}{C_1+C_2}}}$$

$$A_{TCA} \cdot r_{out,TCA} \geq \frac{C_1}{C_2}$$

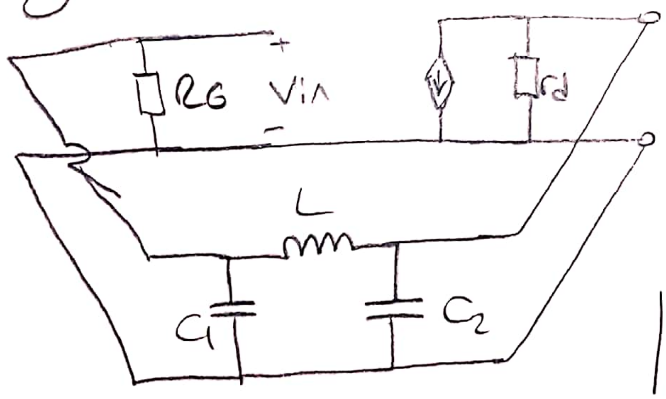
$$\left|\frac{1}{jW_oC_1}\right| \ll r_{in,TCA}$$

Q6. Determine the frequency of oscillation for the Collipts MOSFET oscillator in fig P.13.20 (a). The MOSFET can be replaced by its transconductance model, shown in fig P.13.20(b). The parameters are $r_d = 25k\Omega$; $g_m = 5mA/V$ $R_G = 1M\Omega$; $L = 1.5mH$, $C_1 = 10nF$, $C_2 = 10nF$, Calculate the frequency of oscillation and check to make sure the condition for oscillation satisfied.



RF Chocke short cir. at DC open circuit for higher frequencies

intrinsic gain of the MOS applies. MOS modelled as TCA



$$W_0 = \frac{1}{\sqrt{L \frac{C_1 \cdot C_2}{C_1 + C_2}}}$$

$$g_m r_d \geq \frac{C_1}{C_2}$$

$$\left| \frac{1}{j w_0 C_1} \right| << R_G$$