ELM-365 MATLAB ÖDEVİ

ANALİTİK ÇIKARIMLAR

(a) (Eşit Olasılıklı Bitler)

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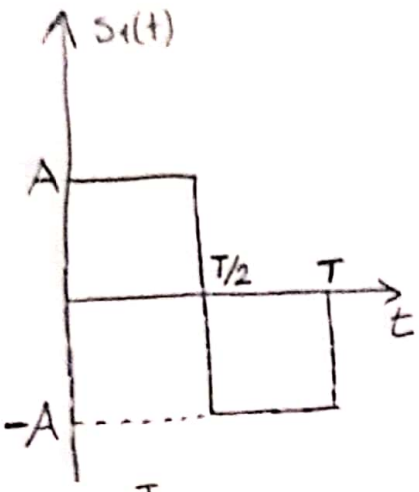

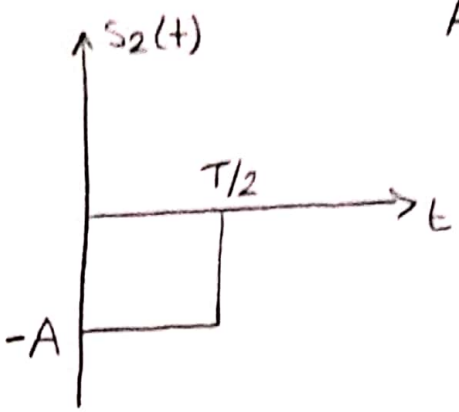

$$a_i(t) = \int_0^T [s_1(t) - s_2(t)] \cdot s_i(t) \rightarrow a_1(T) = \int_0^{T/2} (A-(-A)).A\,dt + \int_{T/2}^{T} (-A).(-A)\,dt$$

$$= 2A^2\frac{T}{2} = A^2T \qquad = A^2\left(T-\frac{T}{2}\right) = \frac{A^2T}{2}$$

$$a_2(T) = \int_0^{T/2} (A-(-A)).(-A)\,dt = \int_0^{T/2} -2A^2\,dt = -2A^2.\frac{T}{2} = -A^2T = a_2(T)$$

$$a_1(T) = \frac{3A^2T}{2}$$

$$\gamma_0 = \frac{\sigma_0^2}{a_1 - a_2}\ln\left(\frac{P(S_2)}{P(S_1)}\right) + \frac{a_1+a_2}{2}$$

*Eşit olasılıklı bitler için $\ln\left(\frac{P(S_2)}{P(S_1)}\right) = \ln(1) = 0$ olacağından $\gamma_0 = \frac{a_1+a_2}{2}$ olarak yazılır.

$$\gamma_0 = \frac{a_1+a_2}{2} = \frac{\frac{3A^2T}{2} - A^2T}{2} = \frac{A^2T}{4} = \gamma_0$$

$$\gamma_0 = \frac{A^2T}{4} \qquad a_1(T) = \frac{3A^2T}{2} \qquad a_2(T) = -A^2T$$

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$$E_{S_1} = \int_0^T |s_1(t)|^2 dt = \int_0^{T/2} |(A)|^2 dt + \int_{T/2}^T |(-A)|^2 dt$$

$$= A^2(T/2 - 0) + A^2(T - T/2) = \boxed{A^2T = E_{S_1}}$$

$$E_{S_2} = \int_0^T |S_2(t)|^2 dt = \int_0^{T/2} (-A)^2 dt = A^2(T/2 - 0) = \boxed{\frac{A^2T}{2} = E_{S_2}}$$

Eb = 1 kabulü ile A^2T değeri bulunması:

$$E_b = E_{S_1} \cdot P(S_1) + E_{S_2} \cdot P(S_2) = A^2T \cdot \frac{1}{2} + \frac{A^2T}{2} \cdot \frac{1}{2} = \frac{3A^2T}{4} = 1$$

$$\boxed{A^2T = \frac{4}{3}}$$

$A^2T = \frac{4}{3}$ ise;

$$a_1 = \frac{3}{2} \cdot A^2T = \frac{3}{2} \cdot \frac{4}{3} = 2$$

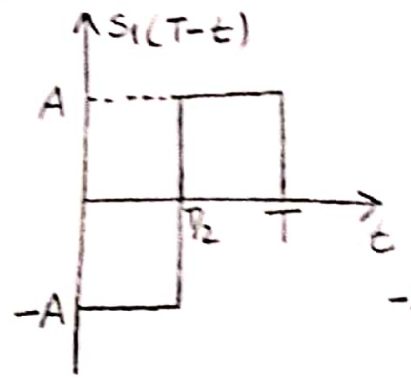
$$a_2 = -A^2T = -\frac{4}{3},$$

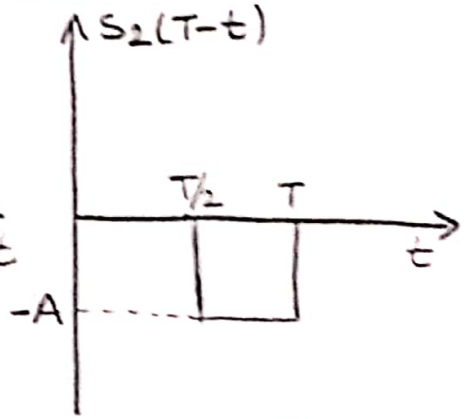
$$\gamma_0 = \frac{1}{4} \cdot A^2T = \frac{1}{4} \cdot \frac{4}{3} = \frac{1}{3}$$

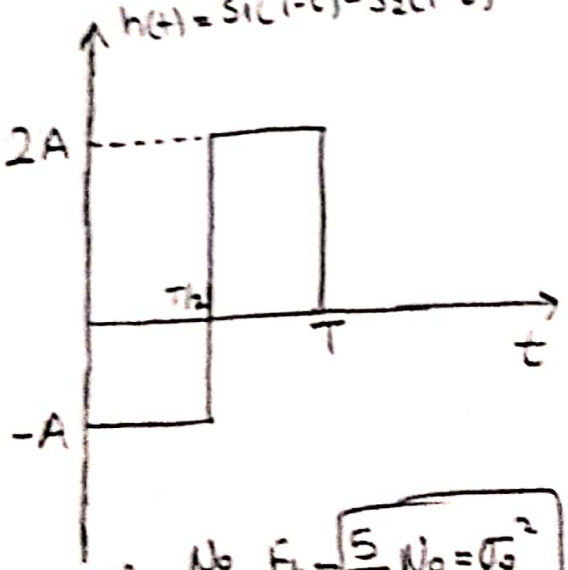
$$\boxed{a_1 = 2} \quad \boxed{a_2 = -\frac{4}{3}} \quad \boxed{\gamma_0 = \frac{1}{3}}$$

$\sigma_0^2 = \frac{N_0}{2} \cdot E_h$ olduğundan E_h değeri için $h(t)$ bulunmalıdır.

$$\boxed{h(t) = S_1(T-t) - S_2(T-t)}$$







$$E_h = \int_0^T |h(t)|^2 dt = \int_0^{T/2} |(-A)|^2 dt + \int_{T/2}^T |(2A)|^2 dt$$

$$E_h = A^2(T/2 - 0) + 4A^2(T - T/2)$$

$$E_h = \frac{A^2T}{2} + 2A^2T$$

$$E_h = \frac{5A^2T}{2} \Rightarrow \frac{5}{2} \cdot \frac{4}{3} = \boxed{\frac{10}{3} = E_h} \quad (*)$$

(*) $\sigma_0^2 = \frac{N_0}{2} \cdot E_h = \boxed{\frac{5}{3} N_0 = \sigma_0^2}$

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(b) $P(1)=\frac{1}{5}$, $P(0)=\frac{4}{5}$ için;

Bu durumda $s_1(t)$ ve $s_2(t)$ sinyalleri değişmeyeceğinden;

$E_h=\frac{5A^2T}{2}$, $a_1(T)=\frac{3A^2T}{2}$, $a_2=-A^2T$, $E_{s1}=A^2T$, $E_{s2}=\frac{A^2T}{2}$

yukarıdaki 5 terim A^2T'ye bağlı değerleri olasılıklardan bağımsızdır.

$E_b=1$ eşitliği ile A^2T değerinin hesaplanması:

$$E_b=E_{s1}P(s_1)+E_{s2}P(s_2)=A^2T\cdot\frac{1}{5}+\frac{A^2T}{2}\cdot\frac{4}{5}=\frac{3A^2T}{5}=1$$

$$\Rightarrow \boxed{A^2T=\frac{5}{3}}$$

$$\boxed{a_1(T)=\frac{3}{2}\cdot\frac{5}{3}=\frac{5}{2}},\quad \boxed{a_2(T)=-\frac{5}{3}}$$

$$\boxed{E_{s1}=\frac{5}{3}}\quad \boxed{E_{s2}=\frac{1}{2}\cdot A^2T=\frac{1}{2}\cdot\frac{5}{3}=\frac{5}{6}}\quad \boxed{E_h=\frac{5}{2}\cdot A^2T=\frac{5\cdot5}{2\cdot3}=\frac{25}{6}}$$

$$\sigma_0^2=\frac{N_0}{2}\cdot E_h=\frac{25}{6}\cdot\frac{N_0}{2}=\boxed{\frac{25}{12}N_0=\sigma_0^2}$$

$$\gamma_0=\frac{\sigma_0^2}{(a_1-a_2)}\cdot\ln\left(\frac{P(s_2)}{P(s_1)}\right)+\frac{a_1+a_2}{2}$$

$$\gamma_0=\frac{\frac{25}{12}N_0}{\frac{5}{2}+\frac{5}{3}}\cdot\ln(4)+\frac{\frac{5}{2}-\frac{5}{3}}{2}=\boxed{0.6931\cdot N_0+0.4166=\gamma_0}$$

Eşit Olasılıklı Bitler Çıkarım Sonuçları

$a_1(T)=2$
$a_2(T)=-4/3$
$\gamma_0=1/3$
$E_{s1}=4/3$
$E_{s2}=2/3$
$E_h=10/3$
$\sigma_0^2=\frac{5}{3}N_0$

$P(1)=\frac{1}{5}$, $P(0)=\frac{4}{5}$ için Çıkarım Sonuçları

$a_1(T)=5/2$
$a_2(T)=-5/3$
$\gamma_0=0.6931\cdot N_0+0.4166$
$E_{s1}=5/3$
$E_{s2}=5/6$
$E_h=25/6$
$\sigma_0^2=\frac{25}{12}N_0$

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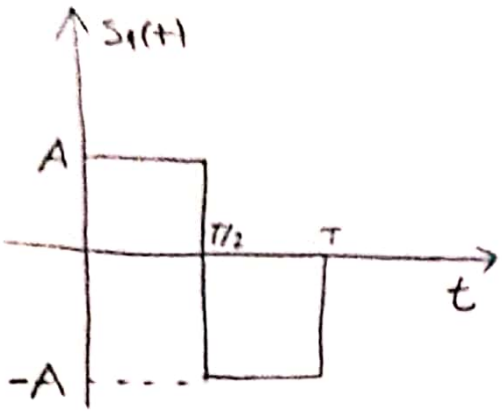

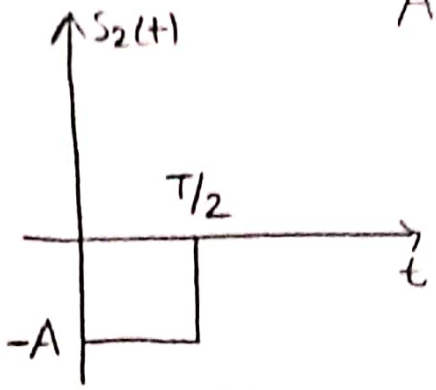

$$E_d = \int_0^T S_1(t)^2 dt + \int_0^T S_2(t)^2 dt - 2\int_0^T S_1(t).S_2(t)\,dt$$

$$E_d = \int_0^{T/2} A^2 dt + \int_{T/2}^{T} (-A)^2 dt + \int_0^{T/2} (-A)^2 dt - 2\int_0^{T/2} A.(-A)dt - \cancel{2\int_{T/2}^{T} 0\,dt}$$

$$E_d = \frac{A^2T}{2} + A^2(T - T/2) + \frac{A^2T}{2} - 2(-A)^2\frac{T}{2}$$

$$\boxed{E_d = \frac{5A^2T}{2}}$$

$E_d \rightarrow$	Eşit Olasılıklı bitler	$P(0)=\frac{4}{5}$, $P(1)=\frac{1}{5}$
	$\frac{5}{2}.A^2T = \frac{5}{2}.\frac{4}{3} = \frac{10}{3}$	$\frac{5}{2}.A^2T = \frac{5}{2}.\frac{5}{3} = \frac{25}{6}$
$P_b = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) \rightarrow$	$Q\left(\sqrt{\frac{5}{3}.\frac{1}{N_0}}\right)$	$Q\left(\sqrt{\frac{25}{12}.\frac{1}{N_0}}\right)$

Burada $\frac{1}{N_0}$ ayrı yazılmasının sebebi $E_b = 1$ kabulüdür. $E_b = 1$ olduğundan Q fonksiyonu içerisinde E_b/N_0 oranı bulunur. Bu oran kritiktir ve Matlab simülasyonunda Q fonksiyonu içerisindeki $\frac{5}{3}$ ve $\frac{25}{12}$ katsayıları ilgili alanlarda kullanılacak katsayılardır.