Hw Bonus # 17 Solutions of Hw Tc # 13

Q1: $A_1, A_2$: Amount of gas in litres bought at the 1st and 2nd visits to the service station, that is if the second visit occurs

$A_1, A_2 \sim \text{Uniform}(15\,lt, 30\,lt)$

V: Bernoulli RV reporting whether the second visit occurs or not $V \sim \text{Bernoulli}(P=0.6)$

R: Deterministic value 40 lt

then Compute $P\{A_1 + A_2 V > R\}$

Through the conditional expectations formula the probability in (eq.1) conditioning on V, can be written as

$$P\{A_1 + A_2 V > R\} = E[P\{A_1 + A_2 V > R \mid V\}] =$$

$$= P\{A_1 + A_2 V > R \mid V=0\} \cdot P\{V=0\} + P\{A_1 + A_2 V > R \mid V=1\} P\{V=1\}$$

$$\boxed{\begin{array}{l} P\{V=1\} = p \\ P\{V=0\} = q \end{array}}$$

Since $A_1$ and $A_2$ are independent we will have to compute $P\{A_1 > R\}$, $P\{A_1 + A_2 > R\}$

$$P\{A_1 + A_2 > R\} = E[P\{A_1 + A_2 > R \mid A_2\}]$$

$$P\{A_1 > R\} = 1 - P\{A_1 \leq R\} = 1 - cdf_{A_1}(R) = 0$$

if $R < a_L$ $cdf_{A_1}(R) = 0$

if $R > a_H$ $\underline{cdf_{A_1}(R) = 1}$ ✓

we need to compute:

$$\underbrace{P\{A_1 + A_2 > R\}}_{?} \cdot \underbrace{P\{V_1\}}_{P}$$

$$E[P\{\underbrace{A_1 + A_2 > R}_{X} \mid \underbrace{V=1}_{Y}\}] \overset{\text{independency}}{=} E[P\{A_1 + A_2 > R\}] = ?$$

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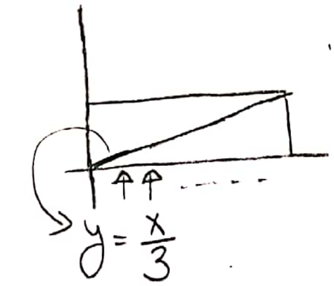
$Q_2$: Graphically we have the following for the computational domain which the indicated probability has to be computed.




HW Bonus: ~~Verify~~ (eq. 7) through (eq. 6)

Part (a) $\int_0^2 \int_0^1 \underbrace{pdf_{X,Y}(x,y)}_{cxy}\, dy\, dx$

$$\int_0^2 \Big|_0^1 \frac{cxy^2}{2}\, dx = \int_0^2 \frac{cx}{2}\, dx = \Big|_0^2 \frac{cx^2}{4} = 1 \rightarrow \boxed{c=1}$$

end of bonus.

Part (b)

$$P\{X > 3Y\} = ? \longrightarrow \int_0^2 \int_0^{x/3} pdf_{X,Y}\, dy\, dx = \int_0^{2/3} \int_{3y}^{2} pdf_{X,Y}(x,y)\, dx\, dy$$

HW bonus Verify eq. 9 through both equations

$$\int_0^2 \int_0^{x/3} \cancel{x}y\, dy\, dx = \int_0^2 \Big|_0^{x/3} \frac{xy^2}{2}\, dy = \int_0^2 \frac{x^3}{18}\, dy = \Big|_0^2 \frac{x^4}{72} = \frac{16}{72} = \frac{2}{9}$$

$$\int_0^{2/3} \int_{3y}^{2} xy\, dx\, dy = \int_0^{2/3} \Big|_{3y}^{2} y\frac{x^2}{2} = \int_0^{2/3} y\left(2 - \frac{9y^2}{2}\right) dy = \Big|_0^{2/3} y^2 - \frac{9y^4}{8}$$

end of bonus

$$\frac{4}{9} - \frac{9}{8}\cdot\frac{16}{81} = \frac{2}{9}$$

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$Q_3$: Note that $cdf_X(a) = cdf_Y(a) = 1 - \exp(-\lambda a) = 1 - \exp(-a)$

$pdf_X(a) = pdf_Y(a) = \lambda \cdot \exp(-\lambda a) = \exp(-a)$

$$P\{Z > 2 \mid X < 1\} = ? = P\{X+Y > 2 \mid X < 1\} = \boxed{\frac{P\{X+Y>2, X<1\}}{P\{X<1\}} = ?}$$

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Hw bonus. Explain each step of eq.13

$= \int_0^1 P\{X+Y>2 \mid X=a\} \, pdf_X(a) \, da \rightarrow$ condition properti $P\{A|B\}.P\{B\}=P\{A\}$

$B=b$ değilde $B<b$ durumu olduğundan $X<1$ için $\int_0^1 P\{X=x\}$ integrali alınır.

$\xrightarrow[X=a]{\text{given that}} \int_0^1 \underbrace{P\{Y>2-a \mid X=a\}}_{P\{a+Y>2 \mid X=a\}} \, pdf_X(a) \, da$

$\xrightarrow{\text{independency}} \int_0^1 \frac{P\{Y>2-a, X=a\}.\cancel{P\{X=a\}}}{\cancel{P\{X=a\}}} . pdf_X(a)$

$= \int_0^1 P\{Y>2-a\} . pdf_X(a) \, da =$ end of bonus.

Bonus Hw Verify (eq.14) through (eq.13) and whatever else is necessary.

$P\{Y>2-a\} = 1 - P\{Y \leq 2-a\} = 1 - cdf_Y(2-a) = 1 - \underbrace{[1-\exp(-(2-a))]}_{\exp(a-2)}$

$\boxed{pdf_X(a) = \exp(-a)}$

$P\{X<1\} \overset{cont.}{=} P\{X \leq 1\} = cdf_X(1) = \underline{\exp(-1)}$ $\frac{1}{e^2}$

$\Rightarrow \frac{P\{X+Y>2, X<1\}}{P\{X<1\}} = \frac{\int_0^1 \exp(a-2).\exp(-a) \, da}{\exp(-1)} = \frac{\Big|_0^1 a.\exp(-2)}{\exp(-1)} = \frac{1}{e}$

?

end of bonus.

Q4: Notice that the question requests from us the probability $P\{|G-\mu| \leq 2\sigma\}$

$$P\left\{\left|\frac{G-\mu}{\sigma}\right| \leq 2\right\} = P\left\{-2 \leq \frac{G-\mu}{\sigma} \leq +2\right\}$$

since $G \sim N(\mu, \sigma^2)$, $Z \sim N(0,1)$ Then we may rightfully set

$Z = \dfrac{G-\mu}{\sigma}$ for some $Z$

Hw Bonus (eq.18) → Verify $G \sim N(\mu, \sigma^2) \rightarrow pdf_G(x) = \dfrac{1}{\sigma\sqrt{2\pi}} \exp\left(\dfrac{(x-\mu)^2}{2\sigma^2}\right)$

$Z \sim N(0,1) \rightarrow pdf_Z(z) = \dfrac{1}{\sqrt{2\pi}} \exp\left(\dfrac{-z^2}{2}\right)$

$$z = \frac{G-\mu}{\sigma} \longrightarrow G = \sigma z + \mu$$
$$x = \sigma z + \mu$$

$$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\sigma z + \mu - \mu)^2}{2\sigma^2}\right) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\cancel{\sigma^2} z^2)}{2\cancel{\sigma^2}}\right)$$

$G \sim N(\mu, \sigma^2)$ ve $Z \sim N(0,1)$ ise $G = \sigma z + \mu$ sağlanır.

end of bonus.

Hw bonus Verify eq.19

then $P\{-2 \leq Z \leq 2\} = P\{\{-2 \leq Z\} \cup \{Z \leq 2\}\} \underset{\text{Axiom 3}}{=} P\{-2 \leq Z\} + P\{Z \leq 2\}$

$P\{Z \leq 2\} = cdf_Z(2)$, $P\{-2 \leq Z\} = 1 - P\{Z \leq -2\} = 1 - cdf_Z(-2)$

$$\Rightarrow cdf_Z(2) + 1 - \underbrace{cdf_Z(-2)}_{cdf_Z(2)}$$

$$cdf_Z(z) = \underbrace{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{*} \exp\left(-\frac{z^2}{2}\right)}$$

$\Rightarrow 2\, cdf_Z(2) + 1$

end of bonus

☆ integral sonucu tek fonksiyon çıkacağından ☆

☆ $cdf_Z(-2) = -cdf_Z(2)$ ☆