

Q1

Expectation of Exponentially distributed.

$$pdf_x = \lambda \cdot \exp(-\lambda x)$$

$$E[x] = \int_1^{+\infty} x \cdot \lambda \cdot \exp(-\lambda x) dx$$

\hookrightarrow ex. at \perp support $(x) = \{1, 2, \dots\}$

$$x = u \quad \lambda \exp(-\lambda x) dx = dv \quad uv - v du \quad v = -\exp(-\lambda x) \quad du = dv$$

$$= -\lambda \cdot \exp(-\lambda x) \Big|_{x=1}^{+\infty} - \left[\int_1^{+\infty} \exp(-\lambda x) dx \right]$$

$$\exp(-\lambda x) \Big|_{x \rightarrow +\infty} = 0$$

$$\underbrace{0 - (-1 \cdot \exp(-\lambda))}_{\exp(-\lambda)}$$

$$\underbrace{-\frac{1}{\lambda} \exp(-\lambda x) \Big|_{x=1}^{x \rightarrow +\infty}}$$

$$x \rightarrow +\infty \quad \exp(-\lambda x) \rightarrow 0$$

$$0 - \left(-\frac{1}{\lambda} \cdot \exp(-\lambda)\right)$$

$$= E[x] = \exp(-\lambda) + \frac{1}{\lambda} \cdot \exp(-\lambda) = \exp(-\lambda) \cdot \left(1 + \frac{1}{\lambda}\right)$$

$$\exp(-0.1) \cdot 11 \Rightarrow \boxed{E[x] = 11 \cdot \exp(-0.1)}$$

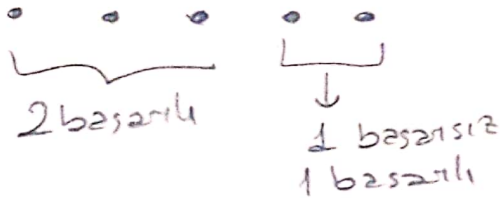
$$\lambda = 0.1$$

$$E[x] = 9.953$$

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HW TC # 06

Q2: $A = \{5 \text{ bağımsız atışın 3'ü başarılı}\}$
 $B = \{ \text{son iki denemede yalnızca 1 başarısızlık bulunması} \}$

$$P\{B|A\} = ? = \frac{P\{B, A\}}{P\{A\}}$$



$A \sim \text{Binomial}(n=5, p)$
 $B \sim \text{Binomial}(n=2, p)$

İlk 3 atıştan ikisi ve son 2 atıştan 1'i başarılı
C D

$$P\{B|A\} = P\{C \cap D\} \stackrel{\text{Independence}}{=} P\{C\} \cdot P\{D\}$$

$$P\{C\} = \sum_{k=2}^3 \binom{3}{k} \cdot p^k \cdot q^{3-k} = \binom{3}{2} \cdot p^2 \cdot q^{3-2} = 3p^2q = P\{C\}$$

$$P\{D\} = \sum_{k=1}^2 \binom{2}{k} \cdot p^k \cdot q^{2-k} = \binom{2}{1} \cdot p^1 \cdot q^{2-1} = 2pq = P\{D\}$$

$$P\{B|A\} = P\{C\} \cdot P\{D\} = 6p^3 \cdot q^2 = 6(0.6)^3 \cdot (0.4)^2 = 0.207$$

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$$Q_3 \quad \int_{-\infty}^{+\infty} \underbrace{\left[x + \sin^5(99x) + 6x^3 \right]}_{f(x)} \cdot \underbrace{\left[72 \sin^2(27x) + 61 \cos(32x) + x^2 \right]}_{g(x)} dx$$

$f(x)$ tek fonksiyon

$g(x)$ çift fonksiyon

$T(x)$

$T(x) = f(x) \cdot g(x) \Rightarrow$ tek fonksiyon

Tek fonksiyon özelliği:

$M(x)$ tek fonksiyon olmak üzere

$$\int_{-a}^{+a} M(x) dx = 0 \quad (\text{negatif ve poz. alanda altında kalan alanlar eşit})$$

$$\int_{-\infty}^{+\infty} T(x) dx = \underline{\underline{0}}$$

Q4: $\lambda = 7$ (poisson ortalaması)

$$E[X] = n \cdot p = \lambda$$

$$\{10 \leq X \leq 50\}$$

$$A = \{-\infty < X \leq 10\}$$

$$B = \{-\infty < X \leq 50\}$$

$$\{10 < X \leq 50\} = A^c \cap B$$

$$B = A \cup \{A^c \cap B\}$$

$$P\{B\} \stackrel{2 \times 3}{=} P\{A\} + P\{A^c \cap B\}$$

$$P\{A^c \cap B\} = P\{B\} - P\{A\} = \text{cdf}(50) - \text{cdf}(10)$$

$A = \{120 \text{ dolardan fazla geliri}\}$

$B = \{Araba \text{ sayısı } 5'ten az\}$

$$P\{B|A\} = \frac{P\{B, A\}}{P\{A\}} \stackrel{\text{independen}}{=}$$

$$\sum_{k=3}^5 \exp(-7) \cdot \frac{7^k}{k!} \} P\{B\}$$

$k=3$ (en az 3 araba 50+50+50)

$P\{B, A\} = P\{Araba \text{ sayısı } 5'ten az \text{ iken } 120 \text{ dolardan fazla geliri}\}$
 en az 24 dolar almali her bir iken

$$\frac{50-24}{50-10} = 0.65$$

$$P\{B|A\} \stackrel{\text{independen}}{\text{cond. prob}} \frac{P\{B, A\}}{P\{A\}} = \frac{P\{B\} P\{A\}}{P\{A\}} = P\{B\}$$

$$P\{B\} = \sum_{k=3}^5 \exp(-7) \cdot \frac{7^k}{k!}$$