

HW TC # 10

Abdullah MEMİSOĞLU

171024001

duQ1: X is a random variable.

$$E[X] = 10$$

$$P\{|X - E[X]| \leq 5\} = 0.25$$

X 'in beklenen değerinden en fazla 5 birim uzakta olma olasılığı 0.25 ise.

$$\text{Var}(X) = ?$$

with a Chebyshev inequality? $\text{Var}(X)$

$$P\{(X - E[X])^2 \geq k^2\} \leq \frac{E[(X - E[X])^2]}{k^2} \quad \left. \vphantom{P\{(X - E[X])^2 \geq k^2\}} \right\} \text{Using Markov's inequality}$$

$$P\{|X - E[X]| \geq k\} \leq \frac{\text{Var}(X)}{k^2}$$

$$P\{|X - E[X]| \geq k\} = 1 - P\{|X - E[X]| \leq k\}$$

$$1 - P\{|X - E[X]| \leq k\} \leq \frac{\text{Var}(X)}{k^2} \quad \begin{array}{l} \text{In this case} \\ k = 5 \end{array}$$

0.25

$$1 - 0.25 \leq \frac{\text{Var}(X)}{5^2} \rightarrow 0.75 \leq \frac{\text{Var}(X)}{25}$$

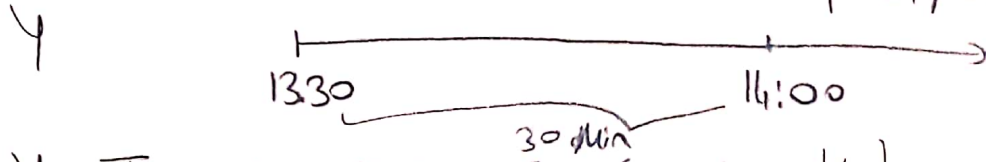
$$\frac{75}{4} \leq \text{Var}(X) \quad \text{Var}(X) \geq \frac{75}{4}$$

$$\text{Var}(X) = \frac{75}{4} \quad \text{Var}(X) \text{ için alt sınırdır.}$$

$$\text{Alt sınır} = 18.75 = \text{Var}(X)$$

HW TC#10 Abdullah MEMİSOĞLU 171024001

Q2: $Y \sim \text{Exponential}(\lambda = \frac{1}{3 \text{ min}})$ and start 14:00 since var
 $Y \sim \text{Exponential}$ denez.

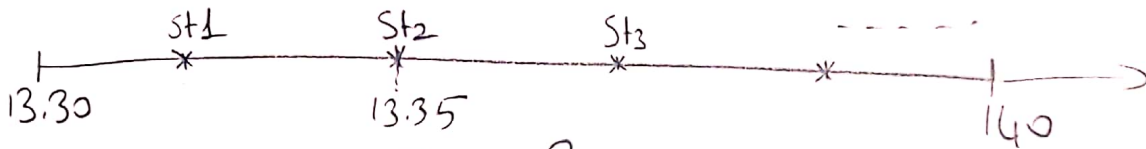


$Y \sim \text{Truncated Exponential}(\lambda = \frac{1}{3 \text{ min}}, K)$

$$K = (35 - 30) \cdot \lambda = \frac{25}{3}$$

$$K = 25 \cdot \frac{1}{3} = \frac{25}{3} \quad \left\{ \begin{array}{l} \text{ilk 2 ögrenici} \\ \text{1. ve 2.} \end{array} \right.$$

$T_1, T_2, T_3, T_4 \sim \text{Truncated Exponential}(\lambda = \frac{1}{3 \text{ min}}, K)$



$$P\{T_1 + T_2 \leq 5, T_3 + T_4 > 5\}$$

independency,

$$P\{T_1 + T_2 \leq 5\} \cdot P\{T_3 + T_4 > 5\}$$

$$\underbrace{\text{cdf}_{T_1+T_2}}_{\frac{1 - \exp(-\lambda x)}{1 - \exp(-K)}} \cdot \underbrace{1 - \text{cdf}_{T_2}}_{1 - \frac{1 - \exp(-\lambda x)}{1 - \exp(-K_2)}}$$

$$= \left(\frac{1 - \exp(-\frac{1}{3} \cdot 5)}{1 - \exp(-\frac{25}{3})} \right) \cdot \left(1 - \left(\frac{1 - \exp(-\frac{1}{3} \cdot 5)}{1 - \exp(-\frac{10}{3})} \right) \right)$$

$$K_2 = (10 - 30) \cdot \lambda$$

$$K_2 = \frac{10}{3}$$

Q3: S: Bernoulli random Variable. Her bir deneyi satış ile sonlandırma olasılığı $P\{S=1\}=0.3$, $P\{S=0\}=0.7$

X_k Uniform (\$10, \$14) (The money earned at kth successful sales)

H: Alinin başarısızlıkla karşılaşma olasılığı Bernoulli

$$P\{H=1\}=0.05, P\{H=0\}=0.95$$

R: Deterministic value of sum of money = \$150

her 15 dakikalık satış deneyinin indeksi $k \in \{0, 1, 2, \dots, 32\}$

X_k for $k \in \{0, 1, 2, \dots, 32\}$

$$P\{Q(N) \leq R\} \geq 1$$

Ali'nin $P\{$

$$11 \leq N \leq 15$$

k	$\text{cdf}_X\left(\frac{R}{k}\right)$
0	$\text{cdf}_X(+\infty) = 1$
1	$\text{cdf}_X(150) = 1$
2	$\text{cdf}_X(75) = 1$
3	$\text{cdf}_X(50) = 1$
...	...
11	$\text{cdf}_X\left(\frac{150}{11}\right) < 1$
...	...
15	$\text{cdf}_X\left(\frac{150}{15}\right) > 0$