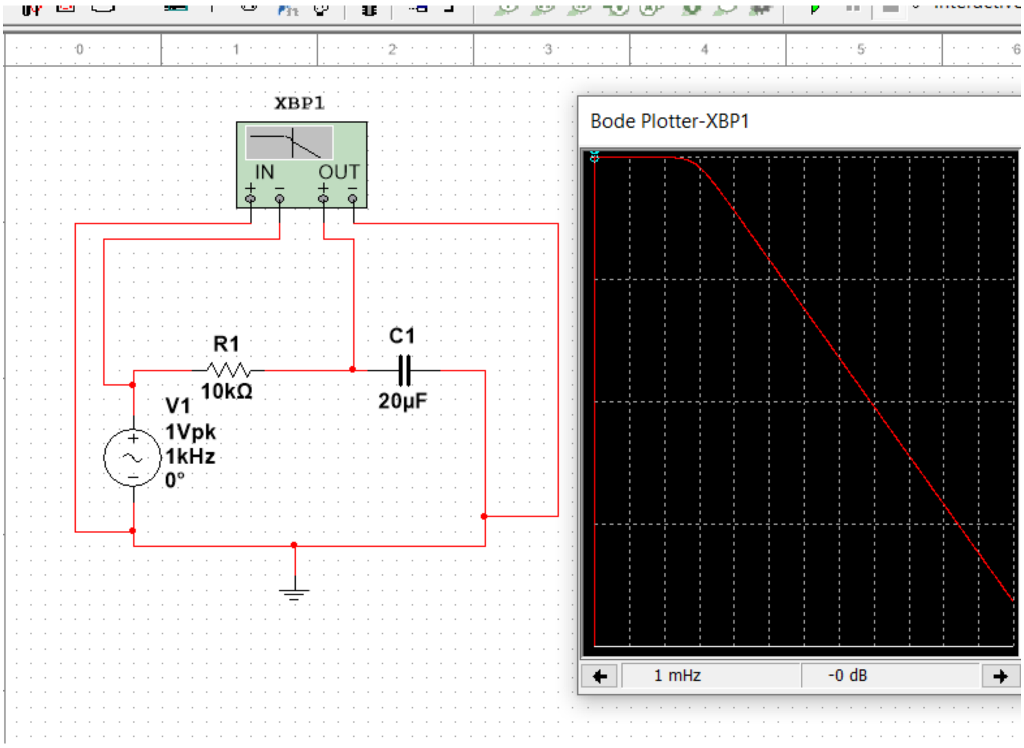
GEBZE TECHNICAL UNIVERSITY
ENGINEERING FACULTY
ELECTRONICS ENGINEERING

ELEC 218

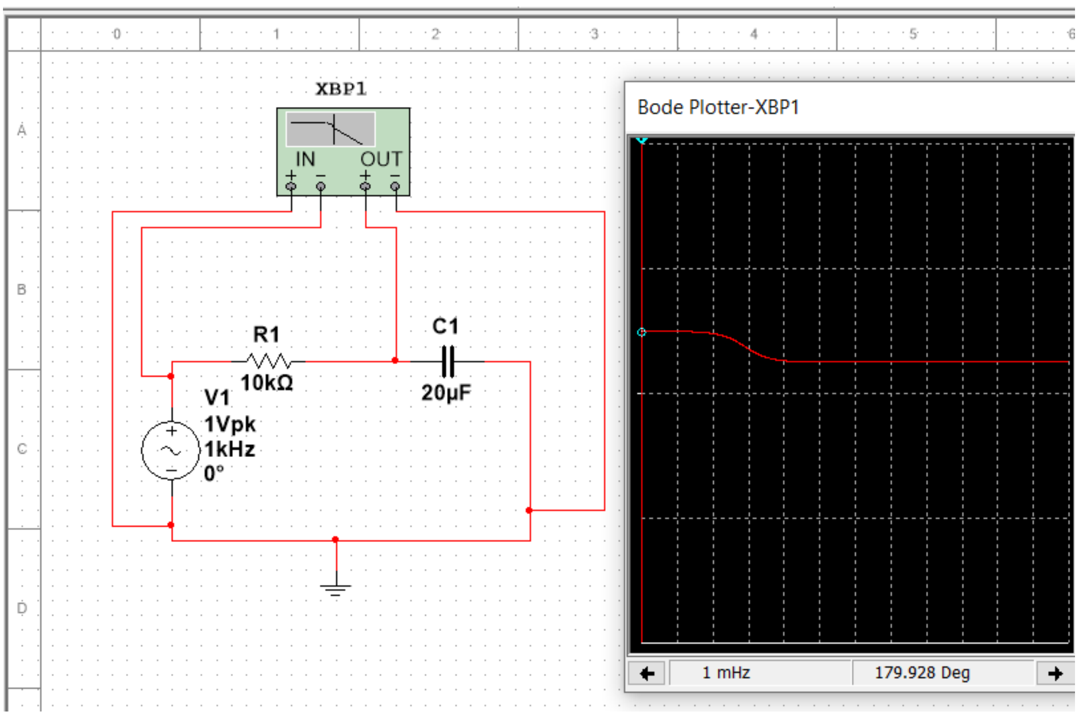
PROBABILITY AND RANDOMNESS
HWBONUS 15

Name - Surname	Abdullah Memişoğlu
Student ID	171024001

EK – 1 SORU 2 için Örnekleme (BODE PLOT)



Şekil 1 – Magnitude Plot

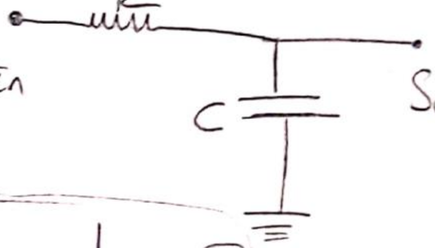


Şekil 2 – Phase Plot

Q1: Look up, if you have not still done so, the ~~expression~~ expression "to delve into"

* to delve into : to examine something carefully in order to discover more information about someone or something.

Q2: Plot the magnitude and phase Bode plots for $H(\omega)$ as in eq.8



$$\frac{S_{out}}{S_{in}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega CR + 1}$$

$$= \boxed{\frac{1}{CR}\left[\frac{1}{j\omega + \frac{1}{CR}}\right]} \rightarrow \left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{1}{j\omega CR + 1}\right| = \text{Magnitude}$$

$$= \boxed{\frac{1}{\sqrt{\omega^2 C^2 R^2 + 1}}} \text{ magnitude}$$

$$\boxed{\text{Magnitude on dB} = 20\log_{10}\frac{1}{\sqrt{\omega^2 C^2 R^2 + 1}}} \rightarrow \boxed{\text{eq.1}}$$

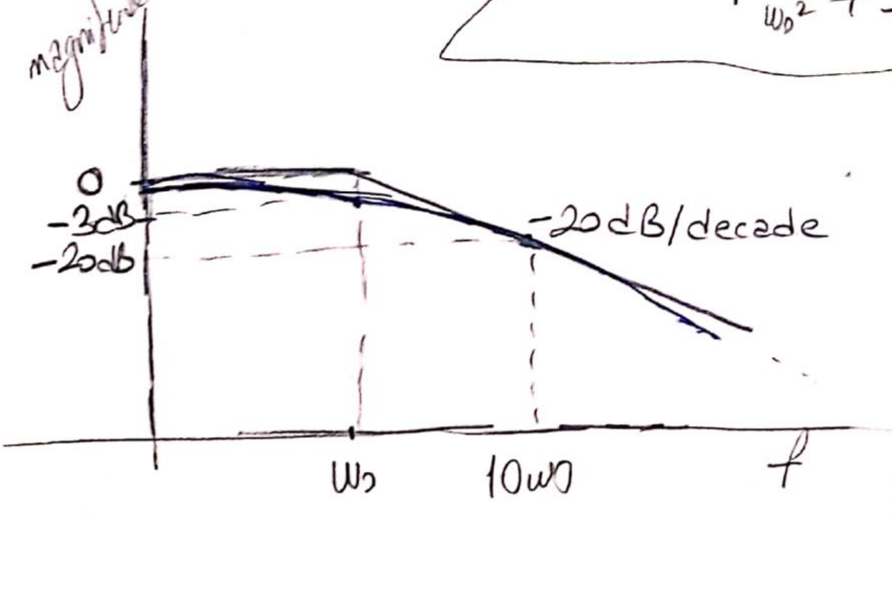
$$\omega^2 = \frac{1}{C^2R^2}, \quad \boxed{\omega = \frac{1}{CR}} \rightarrow 20\log_{10}\frac{1}{\sqrt{2}} = 20\log_{10}(0.707)$$

$1+1$

$$\underline{-3.01\,dB}$$

$$\boxed{\omega = \frac{1}{CR} = \omega_0}$$

eq.1 can be modified as $\boxed{20\log_{10}\frac{1}{\sqrt{\frac{\omega^2}{\omega_0^2} + 1}}}$ ☆

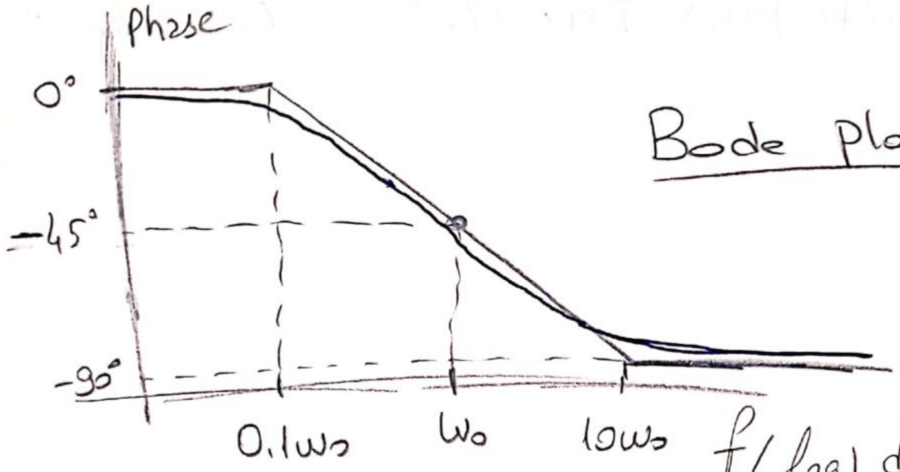


$$\text{Phase} = -\tan^{-1}\left(\frac{\omega CR}{1}\right)$$

$$= -\tan^{-1}(\omega CR)$$

$$\omega_0 = \frac{1}{CR}, \quad -\tan^{-1}(\omega CR) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

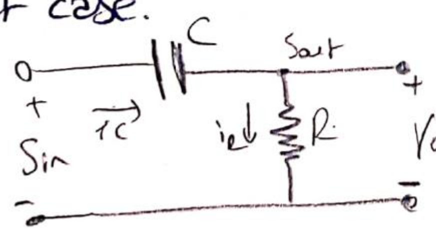
$$\text{Phase}(\omega_0) = -\tan^{-1}\left(\frac{\omega_0}{\omega_0}\right) = -\tan^{-1}(1) = -45^\circ$$



Bode plot

Bode plot: A bode plot is a graph of the Magnitude (in dB) or phase of the transfer function versus frequency.

Q3: In fig. 1, exchange the locations of R and C, and go through derivations similar to those in this write up to reach S_{out}, $V_{out}(\omega)$ in that case.



$S_{out}(t) = h(t) * S_{in}(t)$ → usual input-output relation

$$i_R = i_C = \frac{d}{dt}\left[C.\left(S_{in} - S_{out}\right)\right]$$

$$= i_R = i_C = C.\left[\frac{d}{dt}S_{in}(t) - \frac{d}{dt}S_{out}(t)\right] \qquad \boxed{S_{out} = i_R . R}$$

eq.1 eq.2

eq.1'deki i_R değeri eq.2'de yerine yazılırsa

$$S_{out} = R.C.\left[\frac{d}{dt}\sin(t) - \frac{d}{dt}S_{out}(t)\right] \longrightarrow \frac{1}{RC} \text{ ile çarpalım.}$$

$$\frac{1}{RC} S_{out} = \frac{d}{dt}S_{in}(t) - \frac{d}{dt}S_{out}(t)$$

Q_3 $\xrightarrow{\text{devamı}}$ Q3.2

$$\frac{1}{RC}\cdot S_{out}(t) + \frac{d}{dt} S_{out}(t) = \frac{d}{dt}\sin(t) \xrightarrow{\exp\left(\frac{t}{RC}\right) \text{ ile çarpalım}}$$

$$\underbrace{\exp\left(\frac{t}{RC}\right)\cdot\frac{1}{RC}\cdot S_{out}(t) + \exp\left(\frac{t}{RC}\right)\cdot\frac{d}{dt} S_{out}(t)} = \exp\left(\frac{t}{RC}\right)\cdot\frac{d}{dt}\sin(t)$$

$$\frac{d}{dt}\left[\exp\left(\frac{t}{RC}\right)\cdot S_{out}(t)\right] = \exp\left(\frac{t}{RC}\right)\cdot\frac{d}{dt}\sin(t)$$

$$\underbrace{\int_0^t d\left[\exp\left(\frac{\tau}{RC}\right) S_{out}(t)\right]\Big|_t}_{\text{fund. theorem of calculus.}} = \int_0^t \exp\left(\frac{\tau}{RC}\right)\cdot\frac{d}{d\tau}\sin(\tau)\,d\tau)$$

$$\exp\left(\frac{t}{RC}\right)\cdot S_{out}(t) - S_{out}(0) = \int_0^t \exp\left(\frac{\tau}{RC}\right)\cdot\frac{d}{d\tau}\sin(\tau)\,d\tau$$

$$\exp\left(\frac{t}{RC}\right)\cdot S_{out}(t) = S_{out}(0) + \int_0^t \exp\left(\frac{\tau}{RC}\right)\cdot\frac{d}{d\tau}\sin(\tau)\,d\tau$$

$$S_{out}(t) = \exp\left(\frac{-t}{RC}\right)\cdot S_{out}(0) + \exp\left(\frac{-t}{RC}\right)\cdot\int_0^t \exp\left(\frac{\tau}{RC}\right)\cdot\frac{d}{d\tau}\sin(\tau)\,d\tau$$

$$S_{out}(t) = \exp\left(\frac{-t}{RC}\right)\cdot S_{out}(0) + \int_0^t \exp\left(\frac{\tau-t}{RC}\right)\cdot\frac{d}{d\tau}\sin(\tau)\,d\tau$$

$\neg\{S_{n+1} \le b\}$ implies $\{S_1 \le b\}$

Q3.2: In fig. 1, exchange the location of R and C, and go through derivations similar to those in this write up to reach $S_{V_{out}V_{out}}(\omega)$ in that case

Q3.3

Gerilim bölme.

$$S_{out} = \frac{R}{R+Z_C} \times S_{in} = \frac{R}{R+\frac{1}{j\omega C}} \cdot S_{in}$$

$\omega \to +\infty$, $S_{out} \to S_{in}$

$\omega \to 0$, $S_{out} \to 0$

$$\frac{S_{out}}{S_{in}} = \frac{R}{\frac{j\omega CR+1}{j\omega C}} \rightarrow \frac{S_{out}}{S_{in}} = \frac{j\omega CR}{j\omega CR+1} = 1 - \frac{1}{j\omega CR+1}$$

$$= 1 - \frac{1/RC}{j\omega + 1/RC} = H_2(\omega)$$ That is High Pass Filter

$H_1(\omega) + H_2(\omega) = 1$

$$S_{out}V_{out}(\omega) = |H(\omega)|^2 S_{NN}(\omega)$$

$$= \left|\frac{j\omega CR}{j\omega CR+1}\right|^2 \left[\frac{N_0}{2}\right] = \left(\frac{j\omega CR}{j\omega CR+1}\right) \cdot \left(\frac{-j\omega CR}{-j\omega CR+1}\right) = \frac{\omega^2 C^2 R^2}{1+\omega^2 C^2 R^2} \cdot N_0/2$$

$$S_{V_{out}V_{out_1}}(\omega) + S_{V_{out}V_{out_2}}(\omega) = \frac{N_0}{2} = \frac{N_0/2}{1+\omega^2C^2R^2} + \frac{\omega^2C^2R^2 \cdot N_0/2}{1+\omega^2C^2R^2} = \frac{N_0}{2}$$

??

$\{S_{n+1} \leq b\}$ implies $\{S_1 \leq b\}$

Q4: Look up the Lorentzian function.

★ The Lorentzian is well used peak function with the form

$$I(2\theta) = \frac{w^2}{w^2 + (2\theta - 2\theta_0)^2}$$

where w is equal to half of the peak width ($w = 0.5H$).

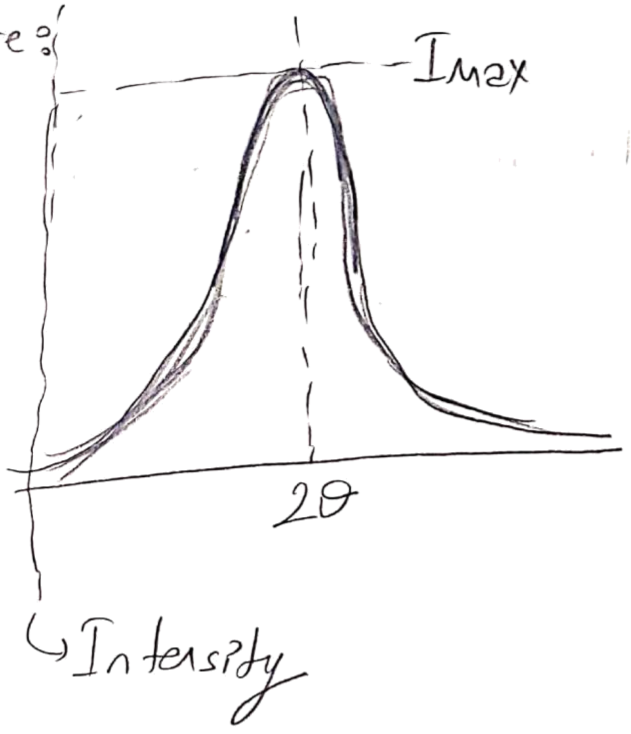
The main features Lorentzian function are:

- That is also easy to calculate
- that relative to the Gaussian function It emphasises the tails of the peak
- It's integral breadth $\beta = \pi H/2$

★★ It has a convenient convolution property

★ It is symetrical.

- Instrumental peak shapes are not normally Lorentzian except at high angles where wavelength dispersion is dominant



HW TC # 11 Bonus

Q1: We have the following stochastic process $X(t) = A(t) + N(t)$ with $N(t)$ a zero mean white noise process with $S_{NN}(\omega) = \frac{N_0}{2}$ as its PSD. In order to check if $X(t)$ is WSS, We have to compute $\tilde{R}_{XX}(t_1,t_2) = E[(X(t_1))(X(t_2))]$ then we have the following

$$E[X(t)] = E[At + N(t)] \rightarrow eq. 2a$$

HW Bonus: What is going to be the end result in eq2-a result going to be constant as a function of t.

$$E[X(t)] = E[At + N(t)] = E[At] + E[N(t)] \rightarrow eq.3$$

$E[N(t)] = 0$ → $N(t)$ is zero mean

for a stochastic process $X(t)$ to be called wide sense stationary (WSS) the following condition should hold

- $Mean(X(t)) = E[X(t)] =$ Constant in time

So eq3 $\longrightarrow$ $E[At] + \underbrace{E[N(t)]}_{0} = A.\underbrace{E[t]}_{\text{constant for WSS}}$ } $E[X(t)]$ constant in time

end of Bonus.

And the autocorrelation function is

$$\tilde{R}_{XX}(t_1,t_2) = E[X(t_1)X(t_2)] = E[[At_1 + N(t_1)].[At_2 + N(t_2)]]$$

$$\tilde{R}_{XX}(t_1,t_2) = \underbrace{E[A^2 t_1 t_2]}_{a} + \underbrace{E[At_1 N(t_2)]}_{b} + \underbrace{E[At_2 . N(t_1)]}_{c} + \underbrace{E[N(t_1)N(t_2)]}_{d} \quad eq.4$$

HW Bonus: What is the result of each of the four terms in here? Will any of them turn out to be zero.

$b = A\, E[t_1].E[N(t_2)]$

$N(t_1) = N(t_2) = 0$ so $b = c = d = 0$ in eq.4

$$\tilde{R}_{XX}(t_1,t_2) = E[At_1 . At_2] = E[X(t_1).X(t_2)]$$

$E[At_1].E[At_2]$

$E[At_1] = E[X(t_1)]$

end of bonus.

$$= R_{XX}(t_1 - t_2) = E[X(t_1).X(t_2)]$$

$R_{XX}(0) = E[X(t).X(t)]$

→ $t - t = 0$

Bu durumda $\tilde{R}_{xx}(t_1,t_2) = E[X(t_1).X(t_2)] = R_{xx}(t_1-t_2)$ bulunur.

- $E[X(t)]$ is constant in time
- $\tilde{R}_{xx}(t_1,t_2) = R_{xx}(t_1-t_2)$

$\Big\}$ $X(t)$ is WSS

$$\tilde{R}_{xx}(t_1,t_2) = E[X(t_1).X(t_2)] = R_{xx}(\underbrace{t_1-t_2}_{\tau}) = \frac{N_0}{2}\delta(t_1-t_2)$$

$$\tilde{R}_{xx}(t_1,t_2) = \frac{N_0}{2}\delta(t_1-t_2) \quad S_{NN}(w) = F\{R_{xx}(\tau)\} = \frac{N_0}{2}$$

Q2: S_k: arrival time in minutes of kth passenger, after 14.00. This is a random variable. $k=1,2,3,\dots$ And $\boxed{S_0 = 0 \text{ min}}$

T_k: The interarrival time between the $(k-1)$st and kth passenge

$T_k = S_k - S_{k-1} \quad \dots \quad T_1 = S_1 - S_0$

$T_k \sim \text{Exponential}(\lambda)$ a,b: deterministic times $a<b$ $a=4$min $b=8$min

$$P\{S_1 \leq a, S_3 > b\} = \int_0^a P\{S_3 > b \mid S_1 = \sigma\} \, pdf_{S_1}(\sigma)\, d\sigma$$

$$= P\{T_1+T_2+T_3 > b \mid T_1 = \sigma\} \, pdf_{S_1}(\sigma)\, d\sigma = \int_0^a P\{T_2+T_3 > b - T_1 \mid T_1 = \sigma\} \, pdf_{S_1}(\sigma) d\sigma$$

$$= \int_0^a P\{T_2+T_3 > b-\sigma \mid T_1 = \sigma\} \, pdf_{S_1}(\sigma)\, d\sigma$$

(T_2+T_3) and T_1 independent. (T_2+T_3) is identically distributed with S_2

$$\int_0^a P\{S_2 > b-a\} . pdf_{S_1}(\sigma)\, d\sigma = \int_0^a [1 - cdf_{S_2}(b-\sigma)] . \underbrace{pdf_{S_1}(\sigma)}_{\lambda . \exp(-\lambda\sigma)} d\sigma \quad \boxed{eq.1}$$

$$cdf_{S_2}(b-\sigma) = 1 - \sum_{n=0}^{1} \frac{1}{n!} . \exp(-\lambda(b-\sigma)) . (\lambda(b-a))^n$$

HW Bonus: Do this one. (Pg 414 eq. 9g) $S_2 \sim \text{Erlang}(\lambda, k=2)$

$$cdf_{S_2}(b-\sigma) = 1 - \exp(-\lambda(b-\sigma)) - 1 . \exp(-\lambda(b-\sigma)) . \lambda(b-\sigma))$$

$$1 - cdf_{S_2}(b-\sigma) = \exp(-\lambda(b-\sigma)) + \lambda(b-\sigma) . \exp(-\lambda(b-\sigma))$$

$$eq.1 = \left[\exp(-\lambda(b-\sigma)) + \lambda(b-\sigma).\exp(-\lambda(b-\sigma))\right] \cdot \lambda(\exp(-\lambda\sigma))$$

$$b=8 \longrightarrow \lambda = \frac{1}{2\,min} \longrightarrow \left[\exp\left(-\frac{1}{2}(8-\sigma)\right) + \frac{1}{2}(8-\sigma).\exp\left(-\frac{1}{2}(8-\sigma)\right)\right]$$

$$= \int_0^a \left[\exp(-4)\cdot\exp\left(\frac{\sigma}{2}\right) + \left(4-\frac{\sigma}{2}\right)\cdot\exp(-4)\cdot\exp\left(\frac{\sigma}{2}\right)\right] \cdot \frac{1}{2} \cdot \exp\left(-\frac{\sigma}{2}\right)$$

$$= \int_0^a \left[\exp(-4) + \frac{8-\sigma}{2}\cdot\exp(-4)\right]\cdot\frac{1}{2}$$

$$= \int_0^a \left[\frac{\exp(-4)}{2} + \frac{8-\sigma}{4}\cdot\exp(-4)\right]$$

$$= \exp(-4)\cdot\int_0^a \frac{1}{2} + \frac{8-\sigma}{4} = \int_0^a \left(\frac{1}{2} + 2 - \frac{\sigma}{4}\right) d\sigma$$

$$= \exp(-4)\cdot \Big|_0^a \frac{5}{2}\sigma - \frac{\sigma^2}{8} = \exp(-4)\cdot\left(\frac{5a}{2} - \frac{a^2}{8}\right)$$

$$a=4 \text{ o.ü} \longrightarrow \exp(-4)\cdot(10-2) = 8\exp(-4) = \underline{0.146}$$

end of Bows.

CamScanner ile tarandı

HWTC # 11. Bonus

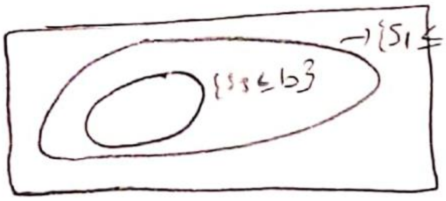
Q2 Part (b)

$P\{S_3 \le b\} \rightarrow$ for computing this check the following.

$$P\{S_1 \le a, S_3 \le b\} = \underbrace{P\{S_1 \le a\}}_{1-\exp(-\lambda a)} - P\{S_1 \le a, S_3 > b\}$$

$$\lim_{a \to b^-} P\{S_1 \le a, S_3 \le b\}' = P\{S_1 \le b, S_3 \le b\} = P\{S_3 \le b\}$$

HW Bonus: Which one is true once again $\{S_1 \le b\} \xrightarrow{imp} \{S_3 \le b\}$ or $\{S_3 \le b\} \xrightarrow{imp} \{S_1 \le b\}$



$\{S_3 \le b\} \xrightarrow{\text{implies}} \{S_1 \le b\}$

$P\{S_1 \le b, S_3 \le b\}$ ↳ intersection

In Venn diagram $\{S_1 \le b\}$ and $\{S_3 \le b\}$ intersection area is $\{S_3 \le b\}$ then $P\{S_1 \le b, S_3 \le b\} = P\{S_3 \le b\}$

$S_1 = T_1$, $S_3 = T_1 + T_2 + T_3 \le b$ if $T_1 + T_2 + T_3 \le b$ then $T_1 \le b$

end of bonus

HW Bonus! Compute the probability in eq. 10 through the technique in eq. 11 -

eq. 12a,b $\quad P\{S_3 \le b\} \overset{?}{=} \lim_{a \to b^-} P\{S_1 \le a, S_3 \le b\}$

$S_1 = T_1 \quad S_3 = T_1 + T_2 + T_3$

$$\lim_{a \to b^-} P\{T_1 \le a, T_1 + T_2 + T_3 \le b\} = P\{T_1 \le a, T_2 + T_3 \le b - a\}$$

$$P\{S_1 \le b, S_3 - S_1 \le 0\} \rightarrow \lim_{a \to b^-} P\{S_1 \le b, S_3 \le S_1\} \equiv P\{S_3 \le b\}$$

end of bonus.

$S_3 \sim Erlang(\lambda, k=3)$

$$P\{S_3 \le b\} = cdf_{S_3}(b) = 1 - \sum_{n=0}^{2} \frac{1}{n!} \exp(-\lambda b)\cdot(\lambda b)^n$$

$\lambda = \frac{1}{2\,min}$ $\quad b = 8$

$$= 1 - \exp(-\lambda b) - \exp(-\lambda b)\cdot\lambda b - \frac{1}{2}\exp(-\lambda b)(\lambda b)^2$$

$$= 1 - \exp(-4) - \exp(-4)\cdot 4 - \frac{1}{2}\cdot 16\cdot\exp(-4) = 1 - 13\exp(-4)$$

$$= 0.7618$$

Part(C) Q2. HWTC#11 Bonus.

We suspect that $P\{S_n \leq b\}$ has a certain form, and we would like to confirm the indicated form through the induction method of proof.

$$h_n(b) = P\{S_n \leq b\} = 1 - \sum_{k=0}^{n-1} \frac{1}{k!} \exp(-\lambda b).(\lambda b)^k$$ Base Case

$h_n(b) \to n = 1,2,3 \ldots$ for $a < b$

$$P\{S_1 \leq a, S_1 \leq b\} = P\{S_1 \leq a\} - \underbrace{P\{S_1 \leq a, S_1 > b\}}_{\text{this must be zero}}$$

$$\Rightarrow P\{S_1 \leq a, S_1 \leq b\} = P\{S_1 \leq a\}$$

$$\lim_{a \to b^-} P\{S_1 \leq a, S_1 \leq b\} = P\{S_1 \leq b\} = 1 - \exp(-\lambda b) = h_1(b)$$

Inductive Step

$$P\{S_1 \leq a, S_{n+1} > b\} = \int_0^a P\{S_{n+1} > b \mid S_1 = x\} \, pdf_{S_1}(x) \, dx$$

$$= \int_0^a P\{T_1 + T_2 + \cdots T_{n+1} > b \mid S_1 = x\} \, pdf_{S_1}(x) \, dx$$

$$= \int_0^a P\{\underbrace{T_2 + \cdots T_{n+1}}_{A \text{ dersek}} > b - x\}. \underbrace{pdf_{S_1}(x)}_{pdf_{T_1}(x)} \, dx$$

$$= \int_0^a P\{S_n > b - x\} \, pdf_{S_1}(x) \, dx$$

$B = \{T_1 + \cdots T_n\} \to S_n$

T_i ve T_j identically distributed olduğundan $A = B$ denebilir

$P\{S_n \leq b\} = h_n(b)$

$1 - h_n(b) = P\{S_n > b\} \to \boxed{1 - h_n(b-x) = P\{S_n > b - x\}}$

$$P\{S_1 \leq a, S_{n+1} \leq b\}$$

$$= P\{S_1 \leq a\} - P\{S_1 \leq a, S_{n+1} > b\} \quad \text{let } a \to b^-$$

Q2. Part(c).2

$$\lim_{a \to b^-} P\{S_1 \leq a ; S_{n+1} \leq b\} \xlongequal{\{S_{n+1} \leq b\} \text{ implies } \{S_1 \leq b\}} P\{S_{n+1} \leq b\}$$

Last Step

$$P\{S_{n+1} \leq b\} \overset{?}{=} h_{n+1}(b)$$

$$\underbrace{P\{S_1 \leq a\}} = \underbrace{P\{S_1 \leq a, S_{n+1} > b\}}_{1-h_n(b-x)} + \underbrace{P\{S_1 \leq a, S_{n+1} \leq b\}}_{P\{S_1 \leq a\} - P\{S_{n+1} \leq b\}}$$

$$P\{S_{n+1} \leq b\} = 1 - h_n(b-x) \overset{?}{=} h_{n+1}(b)$$

Q3 Ali's financial belongings be uncorrelated for consecutive 15 minute intervals

A_n for $n \in \{1, 2, \dots 32\} \sim$ Uniform (a_L, a_H),
S_n for $n \in \{1, 2, \dots 32\} \sim$ Bernoulli (p_{sold})
V_n for $n \in \{1, 2, \dots 32\} \sim$ Bernoulli (q_{robbed})
R deterministic value

$g(k)$ = money Ali has got at the kth interval after sale and robbery attempts.

$$g(n+1) = \left[g(n) + A_{n+1} S_{n+1} \right] V_{n+1}$$

→ Bir önceki ödevde açıklandı (V_{n+1} parantez dışında çünkü $V_{n+1} = 0$ olduğunda para kalmayacak)

$$\text{cov}(g(n), g(n+1)) = E\left[g(n) . \left[g(n) + A_{n+1} S_{n+1} \right] . V_{n+1} \right]$$

$$= E[g(n)] . E[(g(n) + A_{n+1} S_{n+1}) V_{n+1}]$$

$Q_{3.2}$ HWTC # 11

$\rightarrow$ eq.21

$$E\left[[g(n) \cdot [g(n) + A_{n+1} S_{n+1}] V_{n+1}\right] - E[g(n)] \cdot E\left[[g(n) + A_{n+1} S_{n+1}] V_{n+1}\right]$$

HW Bonus: Fill in the necessary steps between eq. 21 and eq. 22

$$E\left[[g^2(n) + A_{n+1} g(n) S_{n+1}] \cdot V_{n+1}\right] - E[g(n)]\, E\left[[g(n) + A_{n+1} S_{n+1}] V_{n+1}\right]$$

linearity $\rightarrow$ $\left[E[g^2(n)] + A_{n+1} S_{n+1} E[g(n)]\right] \cdot E[V_{n+1}] -$

$$- E[g(n)] \cdot \left[E[g(n)] + A_{n+1} S_{n+1}\right] \cdot E[V_{n+1}]$$

$\xrightarrow{E[V_{n+1}] \text{ paranterinde}}$ $E[V_{n+1}] \left[E[g^2(n)] + A_{n+1} \cancel{S_{n+1} E[g(n)]} - \right.$

$$\left. \left(\left(E[g(n)]\right)^2 + A_{n+1} \cancel{S_{n+1} E[g(n)]}\right) \right]$$

$$= E[V_{n+1}] \cdot \underbrace{\left[E[g^2(n)] - \left[E[g(n)]\right]^2 \right]}_{Var(g(n))} = E[V_{n+1}] \cdot Var(g(n))$$

end of bonus.

HW Bonus. (Answer these questions)

$\rightarrow$ How does one make the covariance in eq.22 zero?

for bernoulli RV $E[X] = P$ so if $V_{n+1} = q_{robbed} = 0$ then $cov(\dots) = 0$

$g(n) = 0$ olursa $cov(\dots) = 0$ bunun için $V_n = 0$ (Paralar çalındıysa)

diger durumlard ($A_n = 0$, $S_n = 0$) bunlar $g(n) = 0$'ı garanti etmez.

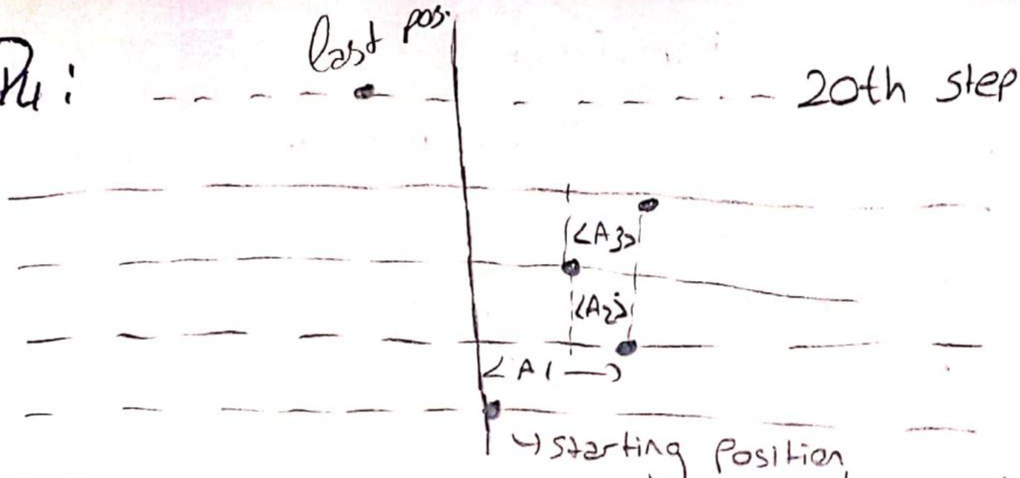
$\rightarrow$ What does $q_{robbed} = 0$ mean.

$q_{robbed} = 0$ 0. ana kadar toplanan paraların tümünü kaybetmek demektir.:

if $q_{robbed} = 0$ then $\{V_{n+1} = 0\}$ if $\{V_{n+1} = 0\}$ then $\{Var(g(n+1))\} = 0$

$\{V_{n+1} = 0\}$ imply $\{Var(g(n+1)) = 0\}$ not $g(n)$

Q4:



Deviations along the vertical are not taken into account.

$A_k \sim \text{Uniform}\,(a_L = 10cm,\ a_H = 25cm)$ for $k = 1, 2 \dots 20$

$V_k \sim$ with $P\{V_k = +1\} = P_R$ and $P\{V_k = -1\} = P_L$ $\quad A_k$'s are iid

direction of the kth step defined for $k = 1, 2, \dots 20$ $\{V_k\}$s are iid.

Also $\{A_k\}$s and $\{V_k\}$s are independent.

R: Deterministic value $\rightarrow R = 40cm$

$$\boxed{P\{|A_1V_1 + A_2V_2 + A_3V_3 \dots + A_{20}V_{20}| \le R\} = ?}$$

$g(n)$: Cumulative horizontal deviation at the end of the nth step $g(n)$ is a random variable

$g(n+1) = g(n) + A_{n+1}V_{n+1}$, $\quad h_n(R) = P\{|g(n)| \le R\}$

$h_{20}(R) = P\{|g(20)| \le R\} = P\{|\sum_{k=1}^{20} A_k V_k| \le R\}$

* HW Bonus: Why are these equations correct

$h_1(R) = P\{|A_1V_1| \le R\} = 1$

$h_2(R) = P\{|A_1V_1 + A_2V_2| \le R\} < 1$

1 adımdaki maximum yalpalama 25 cm

25 cm ≤ 40 cm $\quad P\{\dots\} = 1$

2 adımda yalpalama X

$X \in (20, 50) \quad X \le 40$

$P\{X\} \ne 1$

Çünkü bir adım attığında çizgiden maksimum 25cm uzağa basacak a_H bu durumda tek bir adımda $X < R$ for all $X \in (a_L, a_H)$

Ancak 2 adımda max değer

Max $X_2 = 25 + 25 = 50cm$ bu yüzden

$h_2(R) = P\{|A_1V_1 + A_2V_2| \le R\} < 1$

olmalıdır.

end of bonus.

CamScanner ile tarandı