Q1 HWTC # 12 Abdullah MEMİŞOĞLU

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In order to check Y(t) is WSS:

$\longrightarrow E[Y(t)] \overset{?}{=}$ constant in time

$\longrightarrow \tilde{R}_{YY}(t_1,t_2) = E[Y(t_1)\,Y(t_2)] \overset{?}{=} R_{YY}(t_1-t_2)$

$\star\; Y(t) = \int_{-\infty}^{+\infty} h(t-\tau)\cdot N(\tau)\,d\tau$ linearity

$E[Y(t)] = E\left[\int_{-\infty}^{+\infty} h(t-\tau)\cdot N(\tau)\,d\tau\right] = \int_{-\infty}^{+\infty} E[h(t-\tau)\cdot N(\tau)\,d\tau]$

$\int_{-\infty}^{+\infty} h(t-\tau)\cdot \underbrace{E[N(\tau)]}_{0 \to N(t) \text{ ise zero mean}}\,d\tau \longrightarrow$ constant in time ✓ $\longrightarrow$ eq.1

$Y(t) = \int_{-\infty}^{+\infty} h(\tau)N(t-\tau)\,d\tau, \quad Y(t+\tau) = \int_{-\infty}^{+\infty} h(r)X(t+\tau-r)\,dr$

$E[Y(t)\cdot Y(t+\tau)] = \tilde{R}_{YY}(t,t+\tau) = E\left[\int_{-\infty}^{+\infty} dr\, h(r) \int_{-\infty}^{+\infty} d\tau\cdot h(\tau)\right.$

$= \int_{-\infty}^{+\infty} dr\, h(r)\cdot \int_{-\infty}^{+\infty} d\tau\, h(\tau)\cdot \underbrace{E[N(t-\tau)N(t+\tau-r)]}_{R_{NN}(\tau-r+T)}\; N(t-\tau)\cdot N(t+T-r)$

$V = -\tau$

$dV = -d\tau$

$\int_{+\infty}^{-\infty} [-dv]\,h(-v)\,R_{NN}(T-r-v) = \int_{-\infty}^{+\infty} dv\,\tilde{h}(v)\,R_{NN}(T-r-v)$

$\tilde{h}(v) \downarrow h(-v)$

$= \underbrace{R_{NN}(T-r) * \tilde{h}(T-r)}_{G(T-r)}$

$\tilde{R}_{YY}(t,t+T) = \int_{-\infty}^{+\infty} dr\,(h(r))\,G(T-r) = h(T) * R_{NN}(T) * h(-T)$

$\tilde{R}_{YY}(t,t+T) = R_{YY}(T)$ ✓

Q1.2 then we can say $Y(t)$ is WSS

for finding PSD

$$F\{h(\tau)\} = H(\omega) = \int_{-\infty}^{+\infty} h(t)\, \exp(-j\omega\tau)\, d\tau)$$

$$= \int_{-\infty}^{+\infty} \left[\delta(\tau) - \omega_0 \cdot \exp(-\omega_0 \tau) u(\tau)\right] \exp(-j\omega\tau)\, d\tau)$$

$$= \underbrace{\int_{-\infty}^{+\infty} \left[\delta(\tau) \cdot \exp(-j\omega\tau)\, d\tau\right]}_{\underbrace{\exp(-j\omega 0)}_{1}} - \underbrace{\int_{0}^{+\infty} -\omega_0 \cdot \exp(-j\omega\tau - \omega_0\tau)\, d\tau)}_{I_2}$$

$u(t)$

$$I_2 = \omega_0 \int_{0}^{+\infty} \exp \tau(-j\omega - \omega_0)\, d\tau$$

$$= \omega_0 \Big|_0^{+\infty} \frac{-1}{j\omega + \omega_0} \cdot \exp(\tau(-j\omega - \omega_0))\, d\tau$$

$$0 - \left(-\frac{1}{j\omega + \omega_0}\right) \rightarrow \text{for } \underline{j\omega + \omega_0 > 0}$$

$$= \frac{\omega_0}{j\omega + \omega_0} = I_2, \quad \boxed{H(\omega) = 1 - \frac{\omega_0}{j\omega + \omega_0}}$$

$$S_{YY} = H(\omega) \cdot H^*(\omega) \cdot S_{NN}(\omega)$$

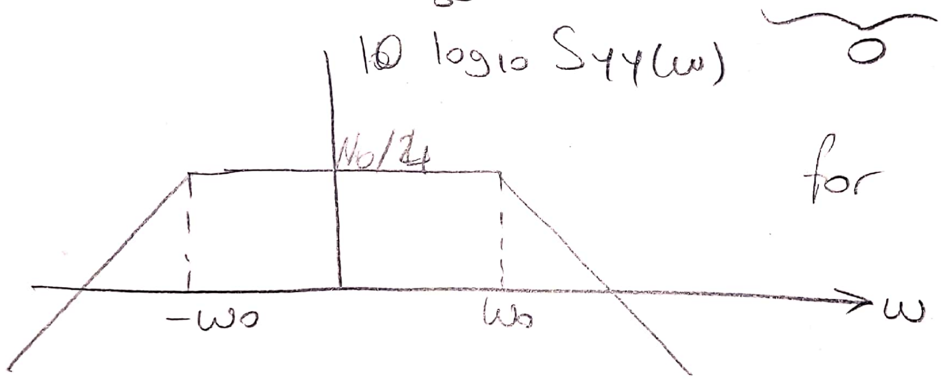
$$S_{YY} = \left(1 - \frac{\omega_0}{j\omega + \omega_0}\right) \cdot \left(1 - \frac{\omega_0}{-j\omega + \omega_0}\right) \cdot \frac{N_0}{2}$$

$$\left(\frac{j\omega + \omega_0 - \omega_0}{j\omega + \omega_0}\right) \cdot \left(\frac{-j\omega_0 + \omega_0 - \omega_0}{-j\omega + \omega_0}\right) \frac{N_0}{2} = \frac{(j\omega) \cdot (-j\omega)}{(j\omega + \omega_0)(\omega_0 - j\omega)}$$

Q1.3

$$\frac{(j\omega)\cdot(-j\omega)}{(\omega_0+j\omega)\cdot(\omega_0-j\omega)}\cdot\frac{N_0}{2}=\boxed{\left(\frac{\omega^2}{\omega_0^2+\omega^2}\right)\frac{N_0}{2}=S_{YY}(\omega)}$$

$$E\{Y(t)\}=\int_{-\infty}^{+\infty}h(t-\tau)\cdot\underbrace{E\{N(\tau)\}}_{0}\,d\tau=\underline{\underline{0}}$$



for $\omega\to\omega_0\to S_{YY}=\frac{N_0}{4}$

$\omega\to-\omega_0\to S_{YY}=\frac{N_0}{4}$

HWTC#12

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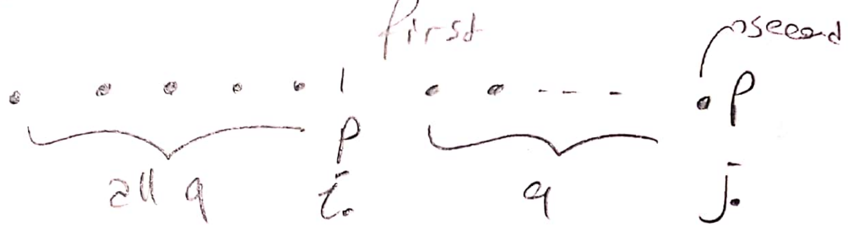
Q2

(a) S_1 : index of the bernoulli trial first successfull attempt

S_2 : index of the second successfull Bernoulli trial.

$S_1 \sim$ Geometric (P)



$P\{S_1 < 6, S_2 > 10\}$

$Pmf_{S_1,S_2}(i,j) = q^{j-2}p^2$, $Pmf_{S_1}(i) = q^{i-1}p$

$Pmf_{S_2}(j) = (j-1) \cdot q^{j-2}p^2$

$P\{$

(b) $P\{S_1 < 6, S_2 < 11\} = 1 - P\{S_1 < 6, S_2 > 10\}$