Abdullah MEMİŞOĞLU 171024001

Q1: Contrive a similar example for yourself and solve it. You may work on alternatives for questions.

☆ Elimde 10 adet bilye var ve belirli uzaklıktan küçük bir kuyuya bilyeleri sokmaya çalışıyorum. Her atışım diğerinden bağımsız ve p olasılığa sahiptir. Yaptığım 10 atışın son üçünde art arda başarılı olma ihtimalim nedir?

☆☆ deneme sayıları

							p	p	p
1	2	3	4	5	6	7	8	9	10

$$P\{\underbrace{\{\text{İlk 7 deneme sonucu}\}}_{\text{event A}}, \underbrace{\{\text{son 3 atış başarılı}\}}_{\text{event B}}\} \to P\{A,B\} \overset{\text{bağımsız}}{=\!=} P\{A\}P\{B\}$$

$X \sim \text{Binomial}(n=7, p)$, $Y \sim \text{Binomial}(n=3, p)$ X ve y başarı sayısı indexi olmak üzere,

$$P\{A\} = P\{\{x=0\} \cup \{x=1\} \cup \{x=2\} \cdots \{x=6\} \cup \{x=7\}\}$$

$$\boxed{P\{A\} \overset{\text{axiom 3}}{=\!=} \sum_{k=0}^{7} P\{x=k\}}$$

$$P\{B\} = P\{y=3\}$$

3ü de başarılı

$$\boxed{P\{A\} = \sum_{k=0}^{7} \binom{7}{k} \cdot p^k q^{7-k} \overset{\text{binomial}}{\underset{\text{expansion}}{=\!=}} (p+q)^7 = 1}$$

$$P\{B\} = P\{y=3\} = \sum_{k=3}^{3} \binom{3}{k} p^k q^{3-k} = p^3 /\!/$$

$$P\{A,B\} = P\{A\} \cdot P\{B\} = 1 \cdot p^3 = p^3 /\!/$$

Q2 : What is the Result of the following probability? Let X_5 be the 5th Bernoulli RV embedded in the setup

$$\sum_{k=1}^{2} P\{C, X_5 = k\} = ?$$

$$\sum_{k=1}^{2} P\{C, X_5 = k\} \overset{\text{independence}}{=\!=} P\{C\} \cdot \sum_{k=1}^{2} P\{X_5 = k\}$$

$$P\{C\} \cdot \Big(\underbrace{P\{X_5 = 1\}}_{p} + \underbrace{P\{X_5 = 2\}}_{0} \Big)$$ (Bernoulli RV için $X_5 = 0$ (q) veya $X_5 = 1$ (p) alabilir.)

$$= p \cdot P\{C\}$$

Q3 : State why ~~independence~~ independence applies in eq. 1.

$$eq.1 = P\{A, B\} = P\{Z = 1, X_3 = 1\}$$

$P\{A, B\} = P\{$ 2 of 4 trials end in success with the 3rd a success $\}$

$Z \sim$ Binomial $(n = 3, p) \rightarrow n = 3 \rightarrow$ 1., 2. ve 4. denemedir.

$A = \{Z = 1\}$, $B = \{X_3 = 1\}$ 4 atıştan her biri birbirinden bağımsız ise 1., 2. ve 4. atışlardan 3. atıştan bağımsız olması gerekir. bu yüzden,

$$P\{A, B\} = \boxed{\text{independence}} \; P\{A\} \cdot P\{B\}$$ denebilir.

Q4 : Compute $P\{B|A\}$ in the question

$A = \{$ the 3rd trial ends in success $\}$, $B = \{$ 2 out of the first 4 trials are successful $\}$

$$P\{B|A\} \overset{\text{cond.}}{\underset{\text{prob.}}{=\!=}} \frac{P\{B, A\}}{P\{A\}}$$

$$P\{A, B\} = P\{B, A\} = 3p^2q^2$$

$$P\{B, A\} \overset{\text{indep.}}{=\!=} P\{B\} \cdot P\{A\}$$

$$P\{B\} = p$$

$$P\{A\} = \binom{3}{1} \cdot p^1 \cdot q^{3-1}$$

$$\frac{P\{B, A\}}{P\{A\}} = \frac{3p^2q^2}{\binom{3}{1} \cdot p \cdot q^2} = \frac{\cancel{3}p^{\cancel{2}}\cancel{q^2}}{\cancel{3}\cancel{p}\cancel{q^2}} = p$$

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Q5: Look up the compound verb "to reek of"

If an event or situation reeks of an unpleasant quality, it seems to be caused by or connected to that quality.

Q6: Look up the English ~~that~~ verb "to resort to"

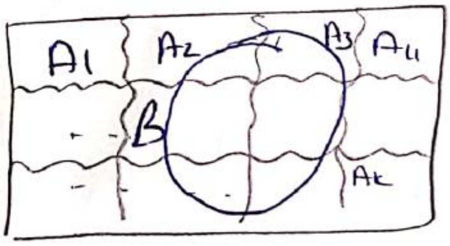
to do something that you do not want to do because you cannot find any other way of achieving something

Q7: Look up the word "saviour"

A person who saves someone from danger or harm

* In our case not person, Mutually exclusive and all inclusive events are our saviour.

Q8: Remind us once again how we made use of axiom 3 in writing down eq. 24. examining eq. 25



B eventi $\bigcup_{k=1}^{+\infty} (B \cap A_k)$ ile elde edilmektedir

ve burada $A_i \neq A_j$ for $i \neq j$ ve $i, j \in \{1, 2 \ldots k\}$ (mutually exclusive)

$\bigcup_{n=1}^{k} A_n = E$ (all inclusive)

A eventi Mutually exclusive olduğu sürece axiom 3 kullanılır.

$$P\left\{\bigcup_{k=1}^{+\infty} (B \cap A_k)\right\} = \sum_{k=1}^{+\infty} P\{B \cap A_k\}$$

Abdullah MEMİŞOĞLU 171024001

Q9. Is S_2 a geometric RV? How can you show that is not? If you have a access to its pmf, you can. If you have a access to its mgf you can?

S_1 = index of the Bernoulli trial at which the first successful attempt

S_2 = index of the second successful Bernoulli trial.

$S_1 \sim \text{Geometric}(p)$ $\quad \boxed{S_2 \text{ is not geometric RV}}$

$P\{S_1 = i, S_2 = j\} = ?$

trials index: $1 \quad 2 \quad \cdots \quad i-1 \quad \overset{p}{i} \quad i+1 \quad \cdots \quad j-1 \quad \overset{p}{j}$

$\underbrace{1 \quad 2 \quad \cdots \quad i-1}_{q} \qquad \underbrace{i+1 \quad \cdots \quad j-1}_{q}$

$$\boxed{P\{S_1 = i, S_2 = j\} = q^{j-2} \cdot p^2} = Pmf_{S_1,S_2}(i,j)$$

$$Pmf_{S_1}(i) = \sum_{j=i+1}^{+\infty} q^{j-2} p^2 = p^2 q^{i-1} (\underbrace{1 + q + q^2 + \cdots}_{})$$

$|q| < 1$ old. sürece $\dfrac{1 - \overbrace{q^{-n}}^{0}}{1-q} \quad n \to \infty \quad = \dfrac{1}{1-q}$

$\boxed{1 - q = p}$

$$\boxed{Pmf_{S_1}(i) = q^{i-1} \cdot p}$$

$$\boxed{Pmf_{S_2}(j) = \sum_{i=1}^{j-1} q^{j-2} p^2 = (j-1) \cdot q^{j-2} \cdot p^2}$$

$$Pmf_{S_1,S_2}(i,j) \neq Pmf_{S_1}(i) \cdot Pmf_{S_2}(j)$$

not independence

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Q10 what is the survival function of a Random Variable? How is Related to the cdf?

★ T is a continuous RV with probability density function (Pdf) $pdf(t)$ and cumulative distribution function (cdf) $cdf(t) = P\{T < t\}$ giving the Probability that the event has occured by duration t.

Then survival function,

$$\boxed{S(t) = 1 - cdf(t) = \int_t^{+\infty} pmf(t)\,dt}$$

Q11: Look up these words "encumbrance", "to entail"

entail → to make something necessary

encumbrance → something that makes it difficult for you to do something

Q12: Look up the word "unwieldy".

unwieldy → (of a system) difficult to manage, usually because it is too big or badly organized.

Q13: Look up this word "progression"

Progression :→ the process of changing or developing towards an improved situation

Q14: Look up this word "intact"

intact: Complete and in the original state

Q15: Look up these expressions and words (" to come to an understanding ") (" to commiserate with ")

" come to an understanding " → Compromise with someone else, agree on

" to commiserate with " → to express sympathy to someone about some bad luck.

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Q16: Just for your Information: Have you heard of "Louis the Just"? King of France

Q17: Look up "~~[illegible]~~ " confounded fool"

→ When somebody is confused and perplexed by a problem the guy is trying to solve, this is an example of when the guy is confounded.

Q18 You May want to have Matlab or Python compute the numerical values of eq.43 and eq.44 for you (Pg 141 in notes)

```
SUM1=0;
SUM2=0;
for k=5:10
  SUM1 = SUM1 + comb(10,k)* Power(0.1,k) * power(0.9,10-k);
end
SUM1
for k=5:100
  SUM2 = sum2+ comb(100,k) * power(0.01,k)* Power(0.99, 100-k);
end
sum2
function output = comb (x,y)
output = factorial (x)/(factorial (x-y) * factorial(y));

end
```

ÇIKTI

SUM 1 = 0.0016
SUM 2 = 0.0034

Q19: You may want to look up the term BER.

The bit error rate is the number of bit errors per unit time.

The bit error probability P_e is the expectation value of the bit error ratio.

The BER may be evaluated using Monte Carlo simulations.

(We search that 3-4 weeks ago)

Q20: Illustrate that regarding Y in eq.11 $P\{Y \le \frac{-A}{\sigma}\} = P\{Y \ge \frac{+A}{\sigma}\}$ due to symmetricity.

$M \sim N(A, \frac{N_0}{2})$

$$P\{M \le 0\} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{0} \exp\left(-\frac{(\alpha-A)^2}{2\sigma^2}\right) d\alpha$$

$y = \frac{\alpha - A}{\sigma}$ $\quad dy = \frac{d\alpha}{\sigma}$ $\quad \alpha \to 0 \Rightarrow y = -\frac{A}{\sigma}$ $\quad A > 0$

$\alpha \to -\infty \Rightarrow y \to -\infty$ $\quad \sigma > 0$

$$P\{M \le 0\} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{-\frac{A}{\sigma}} \exp\left(-\frac{y^2}{2}\right) [\sigma\, dy] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-A/\sigma} \exp\left(\frac{-y^2}{2}\right) dy$$

$$\boxed{P\{M \le 0\} = P\{Y \le -\frac{A}{\sigma}\}}$$

$-y = \frac{\alpha - A}{\sigma}$ $\quad -dy = \frac{d\alpha}{\sigma}$ $\quad \alpha \to 0 \quad y = \frac{A}{\sigma}$

$\alpha \to -\infty \quad y \to +\infty$

$$P\{M \le 0\} = \frac{1}{\sigma\sqrt{2\pi}} \int_{+\infty}^{\frac{A}{\sigma}} \exp\left(-\frac{(-y)^2}{2}\right) \cdot (-\sigma\, dy) =$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \cdot (-1) \cdot \int_{+\infty}^{\frac{A}{\sigma}} \exp\left(\frac{-y^2}{2}\right) dy \;\xrightarrow[\text{sınırlarını değiştirir}]{-1 \text{ int}}\; \frac{1}{\sqrt{2\pi}} \int_{\frac{A}{\sigma}}^{+\infty} \exp\left(\frac{-y^2}{2}\right) dy$$

$$= \boxed{P\{y \ge \frac{A}{\sigma}\} = P\{M \le 0\} = P\{y \le -\frac{A}{\sigma}\}}$$

Q21: Look up the functions called error function ($erf(.)$) and error function complementary ($erfc(\circ)$)

In mathematics, the error function (also called Gauss error function) is a special function of sigmoid shape which occurs in probability and statistics.

$$erf(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-t^2} dt = \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t^2} dt$$

for a random variable Y that is normally distributed with mean 0 and Variance 1/2, erf x is the probability that Y falls in the range [-x, x]

Then complementary error function, erfc, defined as

$$\boxed{erfc\, x = 1 - erf\, x} \qquad \boxed{erfc\, x = \frac{2}{\sqrt{\pi}} \int_{x}^{+\infty} e^{-t^2} dt}$$

Q22: what is the parametric expression for the probability in (eq.15)

$$eq.15 = P\{R \geq 0\} = P\left\{ \begin{matrix} \text{Misinterpreted} \\ \text{as bit 1} \end{matrix} \middle| \begin{matrix} \text{bit 0} \\ \text{is sent} \end{matrix} \right\}$$

$$P\{R \geq 0\} = P\left\{ \begin{matrix} \text{Misinterpreted} \\ \text{as bit 1} \end{matrix} \middle| \begin{matrix} \text{bit 0} \\ \text{is sent} \end{matrix} \right\} = P\{Z \geq R + A\}$$

$$= \frac{1}{\sigma . \sqrt{2\pi}} \int_{-R+A}^{+\infty} \exp\left(-\frac{\rho^2}{2\sigma^2} d\rho\right)$$

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Q23: Look up the English word "profound."

Profound → showing a clear and deep understanding of serious matters

Q24: In view of the account I have just been conducting, show that the equivalence relation for the events in eq.11 holds.

$$\{-\infty < x < -y\} \equiv \{-\infty < x \le y\} \rightarrow \text{eq.11}$$

eq11'in gerçeklendiğini gösterebilmek için event'lere isim verelim.

$A = \{-\infty < x < -y\}$, $B = \{-\infty < x \le y\}$ bu eşitlik için

B eventini parçalara ayıralım. Ondan önce X continous RV olduğu sürece $P\{X=-y\}=0$ olduğu ders notlarında eq.12 olarak verilmiş

$$B = \{-\infty < x \le -y\} = \underbrace{\{-\infty < x < -y\}}_{B_1} \cup \underbrace{\{x = y\}}_{B_2}$$

$B = B_1 \cup B_2$ $\boxed{B_1 \text{ and } B_2 \text{ mutually exclusive events}}$

$\Downarrow$

$$P\{B\} = P\{B_1 \cup B_2\} \xrightarrow{\text{Axiom 3}} P\{B_1\} + P\{B_2\}$$

$$\boxed{P\{B_1\} = P\{-\infty < x < -y\} = P\{A\}}$$

$P\{B_2\} = P\{x = -y\} = 0$ (eq.12)

bu durumda $P\{A\} = \underbrace{P\{B_1\}}_{P\{A\}} + \underbrace{P\{B_2\}}_{0} \Rightarrow A = \{B_1 \cup B_2\}$

$$\boxed{\{-\infty < x \le -y\} = \{-\infty < x \le y\}}$$

Bonus Q25: Show that for the continous RV in the previous example, The RHS of (eq.14) can be expressed as follows

$$cdf_Y(y) = \int_{-y}^{+y} pdf_X(\varrho)\, d\varrho \quad (eq.18)$$

if we had eq.18 how would the derivative with respect to y, through the $\frac{d}{dy}[\cdot]$ operator, of the integral on the RHS of eq.18 be calculated?

$$cdf_X(y) - cdf_X(-y) = \int_{-y}^{+y} pdf_X(\varrho)\, d\varrho$$ olduğunu göster.

$\frac{d}{dy}$

$$\frac{d}{dy}\left[cdf_X(y) - cdf_X(-y)\right] = \underbrace{\frac{d}{dy}\int_{-y}^{+y} pdf_X(\varrho)\, d\varrho}_{\text{fundamental law of calculus}}$$

ardışık türev ve integral uzar.

$$\underbrace{pdf_X(y).1 - (-1).pdf_X(-y)}_{pdf_X(y) + pdf_X(-y)}$$

$$(+y)' . pdf_X(+y) - (-y)' . pdf_X(-y)$$

$$\underbrace{1.pdf_X(+y) - (-1).pdf_X(-y)}_{pdf_X(+y) + pdf_X(-y)}$$

$$\boxed{pdf_X(y) + pdf_X(-y) = pdf_X(y) + pdf_X(-y)}$$

... from that (eq. 29) yields the result

Q2b: Since we must have gone through this setup before, reiterate $Pmf_{S_1}(x_1)$, $Pmf_{S_2}(x_2)$, $cdf_{S_1}(x_1)$, $cdf_{S_2}(x_2)$ and also $E[S_1]$.

S_1: index of the Bernoulli trial at which the first successful attempt

S_2: index of the second successful bernoulli trial.

$S_1 \sim$ Geometric (p) $\quad$ S_2 is not geometric

$$\boxed{Pmf_{S_1}(x_1) = q^{x_1-1} \cdot p} \to \text{geometric RV} \to$$

$$Pmf_{S_1}(x_1) = P\{S_1 = x_1\} = \sum_{j=x_1+1}^{+\infty} \underbrace{q^{j-2} p^2}_{Pmf_{S_1,S_2}(i,j)}$$

$$p^2 q^{x_1-1} \underbrace{(1+q+q^2 \cdots)}_{\frac{1}{1-q}}$$

$$Pmf_{S_2}(x_2) = P\{S_2 = x_2\}$$

$$= \sum_{x_1=1}^{x_2-1} \underbrace{q^{x_2-2} p^2}_{Pmf_{S_1,S_2}(x_1 x_2)} = \boxed{(x_2-1)\, q^{x_2-2} p^2 = Pmf_{S_2}(x_2)}$$

$$\underline{cdf_{S_1}(x_1)} = P\{X \le x_1\} = 1 - q^{x_1+1} = \frac{1-(1-p)^{x_1+1}}{}$$

$$p.(x_1-1)\, q^{x_1-2}$$

$$E[S_1] = \sum_{k=1}^{+\infty} k.(1-p)^{k-1} \cdot p =$$

$$E[S_1] = P \cdot \left(\frac{1}{p} + \frac{1-p}{p} + \frac{(1-p)^2}{p} \cdots\right) = \frac{\cancel{p}}{\cancel{p}}\left(1+(1-p)+(1-p)^2 \cdots\right)$$

$$E[S_1] \overset{1-p=q}{=\!=} (1+q+q^2 \cdots) = \frac{1}{1-q} = \boxed{\frac{1}{p} = E[S_1]}$$

$$\sum_{\text{support}(x)} Pmf_{S_2}(x_2) = \sum_{\text{support}}^{+x_2} q^{j-2} p^2 (j-1) = p^2 + 2q.p^2 + 3p^2q^2 \cdots$$

$$= p^2 \underbrace{(1+2q+3q^2+ \cdots)}_{A}$$

$$A = 1+2q+3q^2 \cdots \cdots q^{x_2-2}$$

$$-\ qA = q+2q^2+3q^3 \cdots \cdot q^{x_2-1}$$

$$A - qA = \underbrace{1+q+q^2+ \cdots}_{\frac{1-q^{x_2}}{1-q}} \cdots q^{x_2-1}$$

$$A \cdot (1-q) = \frac{1-q^{x_2}}{(1-q)} \to A = \frac{(1-q^{x_2})}{(1-q)^2}$$

$$\boxed{cdf_{S_2}(x_2) = p^2 \cdot \frac{1-q^{x_2}}{(1-q)^2} = 1-q^{x_2}}$$

Q27: How to transition between these two proceeds.

$$p^2 q^{i-1}[1+q+q^2 \cdots] = p^2 q^{i-1} \frac{1}{1-q}$$

$$[1+q+q^2 \cdots] = \frac{1}{1-q} \quad \underline{|q|<1}$$

$|q|<1$ ise $\quad 1-q^n = (1-q).(1+q+q^2 \cdots)$

$$(1+q+q^2 \cdots) = \frac{1-q^n}{1-q} \rightarrow \lim_{n\to\infty} \frac{1-q^n}{1-q} \quad |q|<1 \text{ old. sürece}$$

$$q^n \to 0$$

$$= \frac{1}{1-q} = (1+q+q^2 \cdots)$$

Q28: Recall the alternative method to compute to compute the result in eq.8

$$Pmf_{S_2} = (j-1).q^{j-2}.p^2$$

$\text{support}(i|j) = \underline{1+2 \cdots j-1}$ bu durumda $\underline{\sum_{i=1}^{j-1} Pmf_{S_1|S_2}(i|j) = 1}$

$$\sum_{i=1}^{j-1} \frac{Pmf_{S_1,S_2}(i,j)}{Pmf_{S_2}(j)} = 1 = \sum_{i=1}^{j-1} \frac{q^{j-2}.p^2}{Pmf_{S_2}(j)} \Rightarrow 1 = \frac{(j-1).p^2 q^{j-2}}{Pmf_{S_2}(j)}$$

$$\boxed{Pmf_{S_2}(j) = (j-1)\, p^2.q^{j-2}}$$

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Q.29 How do you interpret the conditional distribution in eq.11?

$Pmf_{S_1|S_2}(i|j) = \frac{1}{j-1}$ Bu koşullu olasılıkta başarı oranının hiçbir önemi olmaması özelliği dikkat çekmektedir.

$$Pmf_{S_1|S_2}(i|j) = \frac{Pmf_{S_1,S_2}(i,j)}{Pmf_{S_2}(j)} = \frac{\cancel{q^{j-2}}\cancel{p^2}}{(j-1).\cancel{p^2}\cancel{q^{j-2}}} = \frac{1}{j-1}$$

başarı oranı ne olursa olsun bu koşullu olasılık tanımları 2. başarı indeksine bağlı

Q30 What is a discrete uniform mass function?

A uniform distribution is a probability distribution that has constant probability. The distribution has two types:

Common type → continous uniform distribution

second type → discrete uniform distribution

It still resembles a rectangle. but instead of a line, a series of dots represent of outcomes. T..

It takes $\frac{1}{n}$

Q31: Check if $Pmf_{S_2|S_1}(j|i) \stackrel{?}{=} Pmf_{S_2}(j)$

$$Pmf_{S_2|S_1}(j|i) = \frac{Pmf_{S_2,S_1}(j,i)}{Pmf_{S_1}(i)} = \frac{p^2 q^{j-2}}{p.q^{i-1}} = p.q^{j-i-1} \neq p^2 q^{j-2}(j-1)$$

Q32 Alternative method to compute the result in eq.18 through the result in eq.18

$$eq.18 = cdf_{S_1|S_2}(m|j) = \frac{m}{j-1}$$

$$cdf_{S_1|S_2}(m|j) = \sum_{i=1}^{m} pdf_{S_1|S_2}(i|j)$$

$$\boxed{cdf_{S_1|S_2}(m|j) = \sum_{i=1}^{m} \frac{1}{j-1} = \frac{m}{j-1}}$$

Q33: The parametric result in (eq.21) can be computed also through the conditional pmf given in (eq 14)

$eq.21 = cdf_{S_2|S_1}(n|i) = 1-q^{n-i}$, eq.14 $(pmf_{S_2|S_1}(j|i) = q^{j-i-1}p$

$$cdf_{S_2|S_1}(n|i) = \sum_{j=1}^{n} pmf_{S_2|S_1}(j|i)$$

$$cdf_{S_2|S_1}(n|i) = \sum_{j=1}^{n} q^{j-i-1}p = p \cdot \underbrace{\sum_{j=1}^{n} q^{j-i-1}}$$

$j-i-1 = t$

$$\frac{1-q^{t+1}}{1-q} \quad (1+q+q^2 \dots 1-q^{j-i-1})$$

$$= \cancel{p} \cdot \left(\frac{1-q^{n-i-1+1}}{\underbrace{1-q}_{\cancel{p}}} \right) = \boxed{1-q^{n-i}} \checkmark$$

$$\boxed{cdf_{S_2|S_1}(n|i) = 1-q^{n-i}}$$

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Q34: You may want to confirm that (eq. 29) yields the result in (eq. 28)

$$eq.29 = \sum_{j=2}^{+\infty} \sum_{i=1}^{j-1} (j-i)\, q^{j-2} p^2 \qquad eq.28 = \frac{1}{p}$$

$$eq29 = \sum_{j=2}^{+\infty} p^2 q^{j-2} \left(j-1 + j-2 + \dots 1\right) \qquad i < j$$

$$p^2 (\underbrace{1+3q+6q^2+10q^3+\dots}_{S})$$

$$S = 1+3q+6q^2 \dots$$

$$q \cdot S = q + 3q^2 + 6q^3 \dots$$

$$S - qS = 1 + 2q + 3q^2 + \dots + \infty$$

$$q.(S-qS) = q + 2q^2 + 3q^3 + \dots + \infty$$

$$S - qS - q(S-qS) = \underbrace{1+q+q^2+q^3 \dots}$$

$$S(1-q) - S(1-q)q = \frac{1}{1-q}$$

$$S(1-q).(1-q) = \frac{1}{1-q} \longrightarrow S = \frac{1}{(1-q)^3} = \frac{1}{p^3}$$

$$eq.29 = p^2.S = p^2.\frac{1}{p^3} = \frac{1}{p} = eq.28$$

HW TC #05 BONUS

Q1 A = { İlk 3 denemeden 1'inde başarılı olma olasılığı }

B = { son 2 denemeden 1'inin başarılı olması }

$P\{A \cap B\} = ?$

$P\{A \cap B\} \overset{\text{independence}}{=\!=\!=} P\{A\}.P\{B\}$

$P\{A\} = \sum_{k=1}^{1} \binom{3}{k} \cdot q^{3-k} \cdot p^{k} = \underline{3.p.q^2}$

$P\{B\} = \sum_{k=1}^{1} \binom{2}{k} \cdot q^{2-k} p^{k} = 2pq$

$\underline{P\{A\}.P\{B\} = 6p^2q^3} = \boxed{6p^2(1-p)^3}$

Q2 S_1: Index of the 1st successfull attempt in a sequence of iid Bernoulli trials

S_2: Index of the 2nd succesful trial.

$S_1 \sim$ Geometric $(p) \longrightarrow Pmf_{S_1}(k) = P\{S_1 = k\} = q^{k-1}p$

S_2 is not geometric RV because,

$Pmf_{S_2}(j) = P\{S_2 = j\} = \sum_{i=1}^{j-1} q^{j-2}p^2 = (j-1)\, q^{j-2}p^2$ geometric RV

formuna uymamaktadır.

HW TC #05 Bonus Q1

the probability that there successes occurs 2 in the first 3 [illegible] are found among the last 3 trials.

Q3:

$T_1 \sim$ Exponential $(\lambda_1 = 1^2)$

$T_2 \sim$ Exponential $(\lambda_2 = 2^2)$

$\vdots$

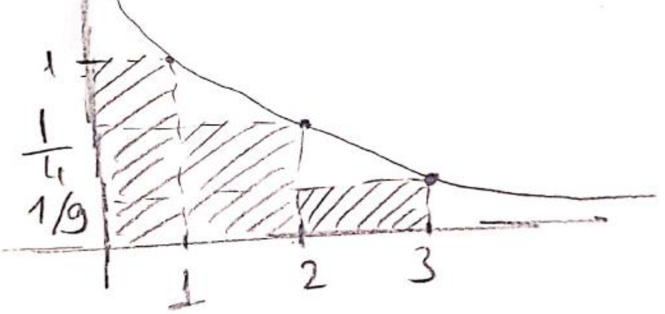
$T_n \sim$ Exponential $(\lambda_n = n^2)$

T_k's with $k \in \{1, 2, \dots\}$ are independent.

The question is $E\left[\sum_{k=1}^{+\infty} T_k\right] \overset{?}{<} +\infty$

linearity $\rightarrow \sum_{k=1}^{+\infty} \underbrace{E[T_k]}_{\text{Exponential RV}} = \underbrace{\sum_{k=1}^{+\infty} \frac{1}{k^2}}_{y(k)}$

$f(k) = \frac{1}{k^2}$ ise $y(k) \rightarrow f(k)$'nın altında kalan alandır.



$$\int_1^{+\infty} \frac{1}{k^2}\,dk = \left.\frac{-1}{k}\right|_{k=1}^{+\infty} = 0 - (-1) = \underline{\underline{1 < +\infty}}$$

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Q4: Baye's Rule kullanılacak,

$A_1 = \{$ John'un 1. kadını seçmesi $\}$

$A_2 = \{$ " 2. kadını " $\}$

$A_3 = \{$ " Marie seçmesi $\}$

$A_4 = \{$ " 4. kadını " $\}$

$B \{$ seçilen kadının 5 mağaza gezmesi $\}$

$P\{A_3|B\} = ?$

$$P\{A_3|B\} = \frac{P\{B|A_3\} \cdot P\{A_3\}}{P\{B|A_1\} \cdot P\{A_1\} + P\{B|A_2\} \cdot P\{A_2\} + \cdots + P\{B|A_4\} \cdot P\{A_4\}}$$

$P\{B|A_3\} = \exp(-6) \cdot \frac{6^5}{5!}$

$P\{B|A_1\} = \exp(-2) \cdot \frac{2^5}{5!}$

$P\{B|A_2\} = \exp(-4) \cdot \frac{4^5}{5!}$

$P\{B|A_4\} = \exp(-8) \cdot \frac{8^5}{5!}$

$P\{A_1\} = P\{A_2\} = P\{A_3\} = P\{A_4\} = \frac{1}{4}$ all inclusive, mutually exclusive and

$$\boxed{\frac{\frac{1}{4}\left(\exp(-6) \frac{6^5}{5!} \right)}{\frac{1}{4}\left(\exp(-6) \frac{6^5}{5!} + \exp(-2) \frac{2^5}{5!} + \exp(-4) \frac{4^5}{5!} + \exp(-8) \frac{8^5}{5!} \right)}}$$

HW TC #05 Bonus Q1

• What is the probability that there successes occurs 2 in the first 3 trials and again 2 successful attempts are found among the last 3 trials.

$$\underbrace{1 \quad 2 \quad 3}_{\text{2 successful}} \quad 4 \quad 5 \qquad 1 \quad 2 \quad \underbrace{3 \quad 4 \quad 5}_{\text{2 successful}}$$

ortak deneme başarısız olsun.

P { [{ ilk 2 si başarılı } ∩ { son iki başarılı }] ∪ [{ ilk 2'den 1'i } ∩ { 3. başarılı } ∩ { son ikiden 1'i }] } olduğu durumlar da independence 3 eventler mutually exclusive 2 ayrı kesişimlerin

P{ilk 2'si başarılı} . P{son iki başarılı} + P{2'den 1'i} . P{3. başarılı} P{son ikiden 1'i}

$$\binom{2}{2} \cdot p^2 \cdot q^0 \cdot \binom{2}{2} \cdot p^2 q^0 + \binom{2}{1} \cdot p^1 q^1 \cdot \binom{1}{1} \cdot p^1 q^0 \cdot \binom{2}{1} \cdot q^1 \cdot p^1$$

$$4p^2q^2 \cdot p$$

$$= \underline{p^4 + 4p^3q^2}$$

HWTC #05 Bonus Q2

Can S_2 be interpreted as the sum of two i.i.d random variables? Can you at least obtain expectation of S_2, $E[S_2]$ through this alternative setup? And would it again be easier to obtain the moment generating function $Mgf_{S_2}(t)$

$$E[S_2] = \sum_{k=0}^{+\infty} k \cdot pmf_{S_2}(k) = \underbrace{\sum_{k=0}^{+\infty} k(k-1) \cdot q^{k-2} \cdot p^2}_{\substack{k=0 \to 0 \\ k=1 \to 1}} = \sum_{k=2}^{+\infty} k(k-1) q^{k-2} p^2$$

$$= 1.2p^2 + 3.2qp^2 + 4.3q^2p^2$$

$$p^2(2 + 6q + 12q^2 + 20q^3 \dots) = 2p^2(\underbrace{1 + 3q + 6q^2 + 10q^3 \dots}_{A})$$

$$A = 1 + 3q + 6q^2 \dots$$

$$q.A = q + 3q^2 + 6q^3 \dots$$

$$A - qA = 1 + 2q + 3q^2 \dots$$

$$q(A - qA) = q + 2q^2 + 3q^3$$

$$A - qA - q(A - qA) = (\underbrace{1 + q + q^2 + \dots}_{\frac{1}{1-q}})$$

$$A(1-q) - qA(1-q) = \frac{1}{1-q}$$

$$\boxed{(1-q)A(1-q) = \frac{1}{1-q}}$$

$$\boxed{A = \frac{1}{(1-q)^3}}$$

$$\boxed{E[S_2] = 2p^2 \cdot \frac{1}{(1-q)^3} = \frac{2}{p}}$$

$$\boxed{E[S_2] = \frac{2}{p}}$$

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