GEBZE TEKNİK ÜNİVERSİTESİ
ELEKTRONİK MÜHENDİSLİĞİ

ELM218
Probability and Randomness

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HW EXAM BONUS 1
Questions and Answers

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Mid Term-1 Solutions Extra Bonus HW#1

1) 6 boys and 4 girls,

→ In how many ways can these 10 people sit if the boys are to sit next to each other in a group

$\boxed{5! \cdot 6!}$

6 boys (G) (G) (G) (G)

→ if no two girls are to sit next to each other.

$\boxed{E_1 K_1 E_2 K_2 E_3 K_3 E_4 K_4 E_5 E_6}$ $\boxed{K_1 E_1 K_2 E_2 K_3 E_3 K_4 E_4 E_5 E_6}$

kızların ve erkeklerin kendi aralarında değişmesine engel yok burada 2 farklı durum var $\boxed{2 \cdot 6! \cdot 4!}$

→ if no two boys are to sit next to each other.

$E_1 K_1 E_2 K_2 E_3 K_3 E_4 K_4 E_5 K_5 E_6$ Toplam kız sayısı yetersiz olduğu için bu durum gerçekleşemez.

→ Committee of 2 boys and 2 girls to be selected. Ayşe and Fatma are cross with each other, won't be selected into the same committee. How many ways can the committee be selected.

erkeklerde kısıtlama yok → $C\binom{6}{2}$

kızlarda Ayşe ve Fatma yan yana gelmeyecek → A F G_3 G_4

Ayşe veya Fatmadan 1'i ve diğer ikisinden 1i $\left(\binom{2}{1}\binom{2}{1}\right)$

Ayşe ve Fatmayı almadan; $\binom{2}{2}$

$$\binom{6}{2} \cdot \left(\binom{2}{1} \cdot \binom{2}{1}\right) + \binom{6}{2} \cdot \binom{2}{2}$$

$$15 \cdot 2 \cdot 2 + 15 \cdot 1 = 75 //$$

2 → 3 coins and 3 associated events $\{C_i = H\} = \left\{\begin{array}{l}\text{Heads shows up on}\\ \text{face on the } i\text{th coin}\end{array}\right\}$
for $i = 1, 2, 3, \ldots$ which equalities should be checked (and How many are they) to prove that these three events are independent.

$\boxed{E_1, E_2, E_3 = \text{Events}}$

3 para için → $E_1 = \{C_1 = H\}$
$E_2 = \{C_2 = H\}$
$E_3 = \{C_3 = H\}$ olmak üzere

A set of events $\underbrace{\{E_1, E_2, E_3\}}_{N=3}$ constitues an independent set of events

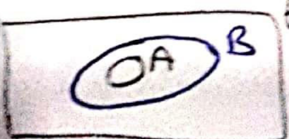
the total of # checks necessary then is (for $N=3$ events)

$$\binom{3}{2} + \binom{3}{3} = 2^3 - 3 - 1 = \underline{\underline{4}} \Rightarrow \begin{array}{l} P\{C_1 \cap C_2\} \overset{?}{=} P\{C_1\} . P\{C_2\} \\ P\{C_2 \cap C_3\} \overset{?}{=} P\{C_2\} P\{C_3\} \\ P\{C_1 \cap C_3\} \overset{?}{=} P\{C_1\} P\{C_3\} \\ P\{C_1 \cap C_2 \cap C_3\} \overset{?}{=} P\{C_1\} P\{C_2\} P\{C_3\} \end{array}$$

3- "If a building does not have a strong enough foundation, it will collapse"
Denote the two events in this statement by A and B, and illustrate the relation between them in a VENN diagram. And express this statement in a different way.
Hint: We have "A implies B" prove that this is equivalent to saying "B^c implies A^c"

A = {"Building does not have strong foundation"}, B = {"It collapse"}

$A \xrightarrow{\text{implies}} B$



this is equivalent to saying $B^c \xrightarrow{\text{implies}} A^c$

"The building does not collapse because it has strong foundation"

4- Two events : {"You passing this course} {you failing this course}
Why are these disjoint? If which condition is also satisfied, then these two events will also become independent.

$A = \{$ You passing this course $\}$

$B = \{$ you failing this course $\}$

$A \cup B = E$ (all inclusive [sample space içerisinde bu iki olay dışında hiçbir durum yok]

$A \cap B = \emptyset$ (Mutually exclusive) [iki olayın kesişim kümesi boş küme]

bu iki durumu birlikte sağlayan olaylara "complementary events" denir ve "complementary events" aynı zamanda disjoint'dir.

Mutually exclusive = disjoint böylece bu iki olay "disjoint"dir

Bu iki olayın "independent" olma şartını inceleyelim: 2 "independent" olay için;

A ve B iki "independent" olay olsun,

$P\{A \cap B\} = P\{A\} \cdot P\{B\}$ 'dir bu iki olay aynı zamanda disjoint olduğu için,

$\boxed{P\{A \cap B\} = 0}$ → böylece $P\{A\} \cdot P\{B\} = 0$'dır o zaman,
iki olayın ayrı ayrı olasılıkları da sıfır olmalı ki "disjoint" olan iki olay aynı zamanda "independent" olsun

$P\{A\} > 0$ $P\{B\} > 0$ olduğundan olamaz.

5- A coin selected out of a set of a fair and a two headed coin is thrown twice; heads comes up twice, in both throws?

$P\{\text{Picked coin is fair} \mid \text{Heads comes up twice}\}$

$E_1 = \{\text{Picked coin is fair}\}$

$E_2 = \{\text{Picked coin is two headed}\}$

$E_H = \{\text{Head comes up twice}\}$

$E_1 \cup E_2 = E$ (sample space) all inclusive

$E_1 \cap E_2 = \emptyset$ mutually exclusive so; $E_2 = \bar{E_1}$

$P\{E_1\} = P\{E_2\}$ $\quad P\{E_1\} + P\{E_2\} = 1$ $\quad \boxed{P\{E_1\} = \frac{1}{2} \quad P\{E_2\} = \frac{1}{2}}$

$$P\{E_1 \mid E_H\} = \frac{P\{E_H \mid E_1\} . P\{E_1\}}{P\{E_H \mid E_1\} P\{E_1\} + P\{E_H \mid E_2\} P\{E_2\}} = ?$$

$P\{E_H \mid E_1\} \longrightarrow$ İki para normalse bir birinden bağımsız olduğundan

$P\{C_1 = H\} = \frac{1}{2}$ $\quad P\{C_2 = H\} = \frac{1}{2}$

Independence $P\{\{C_1 = H\} \cap \{C_2 = H\}\} = \underbrace{P\{C_1 = H\}}_{\frac{1}{2}} . \underbrace{P\{C_2 = H\}}_{\frac{1}{2}} = \boxed{\frac{1}{4} = P\{E_H \mid E_1\}}$

$\boxed{P\{E_H \mid E_2\} = 1 \text{ (hileli zarda "head" gelmesi)}}$

$$P\{E_1 \mid E_H\} = \frac{\frac{1}{4} \cdot \frac{1}{2}}{\frac{1}{4} \cdot \frac{1}{2} + 1 . \frac{1}{2}} = \frac{\frac{1}{4} \cdot \frac{1}{2}}{\frac{1}{2}\left(\frac{1}{4} + 1\right)} = \frac{\frac{1}{8}}{5/8} = \frac{1}{5}$$

6- Prove that disjoint events are not necessarily independent.

A, B iki olay olmak üzere,

A ve B "disjoint" olaylar ise ve "disjoint" = "mutually exclusive" olduğundan

$A \cap B = \emptyset$ 'dir / $\emptyset = E^c$ olduğunu kabul ediyoruz

$\boxed{P\{A \cap B\} = P\{\emptyset\} = 0}$

$$\underbrace{P\{E\}}_{\substack{\| \\ \text{axiom 2} \\ \| \\ 1}} + P\{\emptyset\} = 1$$

$$1 + P\{\emptyset\} = 1 \qquad \boxed{P\{\emptyset\} = 0}$$

A ve B "independent" ise;

$P\{A \cap B\} = P\{A\}.P\{B\} \rightarrow P\{A\} > 0 \quad P\{B\} > 0$ olduğuna göre

$P\{A\}.P\{B\} > 0$ olmalı bu yüzden iki olay hem "disjoint" hem "independent" olamaz.

7- Two dice thrown, results are independent of each other. You know the definitions for R_1 and R_2, define another random variable $X = \min\{R_1, R_2\}$. Compute following probabilities.

$P\{X \leq 3 \mid R_1 = 4\} = ?$ $\quad P\{R_1 = 3 \mid X \geq 4\} = ?$

$$P\{X \leq 3 \mid R_1 = 4\} = \frac{3}{6} = \frac{1}{2} \,//$$

$$P\{R_1 = 3 \mid X \geq 4\} = \frac{0}{9} = 0 \,//$$

$R_2 \backslash R_1$	1	2	3	4	5	6
1	1	1	1	1	1	1
2	1	2	2	2	2	2
3	1	2	3	3	3	3
4	1	2	3	4	4	4
5	1	2	3	4	5	5
6	1	2	3	4	5	6