GEBZE TECHNICAL UNIVERSITY
ENGINEERING FACULTY
ELECTRONICS ENGINEERING

ELM 218

PROBABILITY AND RANDOMNESS

HWBONUS 13

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HWBONUS # 13

Q1: Go through the account in proving the "if" part of statement 1 and explain each step in your own words.

Prove the "if" part $A \xrightarrow{\text{implies}} B \Rightarrow$

$P\{A,B\} = P\{A\}$ olmalı. $\boxed{eq.1}$

Bunun sağlandığını kabul ediyoruz

eq1 ile $\boxed{P\{X+Y=i+j,\ X=i\} = P\{Z=k,\ X=i\}}$

$Z = X+Y$, $k = i+j$ ise

$$P\{X=i, Y=j\} = P\{X+Y=i+j, X=i\}$$

bu durumda $P\{X=i;\ Y=j\} = P\{Z=k, X=i\}$

$X \sim Poisson(\lambda_1)$

$Y \sim Poisson(\lambda_2)$

$Z \sim poisson(\lambda_1+\lambda_2)$

$$P\{X=i, Y=j\} \overset{\text{independency}}{=\!=} \boxed{P\{X=i\}P\{Y=j\} = \exp(-(\lambda_1+\lambda_2))\frac{(\lambda_1+\lambda_2)^{i+j}}{(i+j)!}} ?$$

$$\exp(-(\lambda_1+\lambda_2)).\frac{\lambda_1^i\lambda_2^j}{i!\,j!}.\underbrace{\left(\frac{(\lambda_1+\lambda_2)^{i+j}}{(\lambda_1+\lambda_2)^{i+j}}.\frac{(i+j)!}{(i+j)!}\right)} = \exp(-(\lambda_1+\lambda_2)).\frac{\lambda_1^i\lambda_2^j}{i!.j!} =$$

$$= \underbrace{\frac{(i+j)!}{i!\,j!}}_{\binom{i+j}{i}}\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^i\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^j \overset{1}{=\!=} \binom{i+j}{i}.\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^i.\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^j$$

$$\xrightarrow{i+j=k} \binom{k}{i}.\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^i.\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{k-i}.\exp(-(\lambda_1+\lambda_2)).\frac{(\lambda_1+\lambda_2)^k}{k!}$$

Buraya kadar olan kısımda X ve Y RV'ların bağımsızlığından bir poisson RV'in pdf formülünden yararlandık

$$P\{Z=k, X=i\} = \binom{k}{i}\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^i \cdot \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{k-i} \cdot \exp(-(\lambda_1+\lambda_2)) \cdot \frac{(\lambda_1+\lambda_2)^k}{k!}$$

$(0,1, \ldots k)$ değerlerinin hepsi i'nin alabileceği değerlerdir. Bu değerler için $\{X=i\}$ all inclusive ve mutually exclusive dir.

bu durumda $A \xrightarrow{\text{implies}} B$ için

$$\sum_{i=0}^{k} P\{Z=k, X=i\} = P\{Z=k\}$$ olmasını bekleriz.

$$\underbrace{\sum_{i=0}^{k}\left(\binom{k}{i}\cdot\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^i\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{k-i}\right)}_{\sum_{i=0}^{k}\binom{k}{i} \cdot p^i \cdot q^{k-i}} \cdot \exp(-(\lambda_1+\lambda_2))\frac{(\lambda_1+\lambda_2)^k}{k!}$$

$\sum_{i=0}^{k}\binom{k}{i} \cdot p^i \cdot q^{k-i} \rightarrow$ support$(X) = (0,1 \ldots k)$ ise $(p+q)^k = 1^k = \underline{\underline{1}}$

$$\boxed{\sum_{i=0}^{k} P\{Z=k, X=i\} = \exp(-(\lambda_1+\lambda_2)) \cdot \frac{(\lambda_1+\lambda_2)^k}{k!}}$$

$Z \sim \text{Poisson}(\lambda_1+\lambda_2)$ $\quad i+j=k \quad X+Y=Z$ old. sürece

$$P\{Z=k\} = \exp(-(\lambda_1+\lambda_2)) \cdot \frac{(\lambda_1+\lambda_2)^k}{k!}$$ (pmf of poisson R.V)

bu iki terimin eşitliği ile "if" $A \xrightarrow{\text{implies}} B$ kanıtlanmış olur.

Q_2 : Explain every step of the proof of the "only if" part.

We know from Lecture 8 (sum of independent RVs and moment generating function)

Two $\underline{\text{independent}}$ RVs X and Y both of them $\underline{\text{discrete}}$

then $\boxed{Z = X+Y}$ $\quad mgf_Z(t) = E[\exp(tZ)] = \sum_{i=-\infty}^{+\infty} \exp(tz)\, pmf_Z(z_i)$

$$mgf_Z(t) = E[\exp(t(X+Y))] = \sum_{\substack{i,j \\ -\infty < i < +\infty \\ -\infty < j < +\infty}} \sum \underbrace{[\exp(t(x_i+y_j))]}_{\text{separable function}} \cdot \underbrace{pmf_{X,Y}(x_i,y_j)}_{X \text{ and } Y \text{ independent}}$$

$$= \sum_{\substack{-\infty < i < +\infty \\ -\infty < j < +\infty}} \sum \exp(tx_i)\exp(ty_j)\, pmf_X(x_i)\, pmf_Y(y_j)$$

$$= \underbrace{\left[\sum_{i=-\infty}^{+\infty} \left(\exp(tx_i) \cdot pmf_X(x_i)\right)\right]}_{Mgf_X(t)} \cdot \underbrace{\left[\sum_{j=-\infty}^{+\infty} \left(\exp(ty_j) \cdot pmf_Y(y_j)\right)\right]}_{Mgf_Y(t)}$$

for all if X and Y independent and $Z = X+Y$ $\quad$ $Mgf_Z(t) = Mgf_X(t) \cdot Mgf_Y(t)$

$Mgf_Z(t) = Mgf_X(t) \cdot mgf_Y(t) = Mgf_Y(t) \cdot Mgf_X(t) \rightarrow$ eq. 1

We know eq. 1 is correct if X and Y independent and $Z = X+Y$

then the proof of the "only if" part is correct.

Q3: Look up how poisson random variables can be utilized for traffic modeling.

- One of the most widely used and oldest traffic models is the Poisson Model. The memoryless Poisson distribution is the predominant model used for analyzing traffic in traditional telephony networks. The poisson process characterized as a renewal process.

The poisson distribution can be visualized as a limiting form of the binomial distribution, and is also used widely in queueing models.

- The two primary assumptions that the Poisson model makes are:
  1. the number of sources is infinite
  2. the traffic arrival pattern is random.

Q4: Show that this is correct. ["a" in eq. 2 is such that $a>0$ otherwise $P\{X \geq a\} = 1$ for $a \leq 0$]

$P\{X \geq a\} = E[\mathbb{1}_{\{X \geq a\}}]$ $\quad \mathbb{1}_{\{A\}}$ if A occurs then assume $\underline{1}$
if A doesnot occurs then assume $\underline{0}$

we know support $(X) \cap (-\infty, 0) = \phi$ then if $a<0$ it means

$\{X \geq 0\} \xrightarrow{\text{implies}} \{X \geq a\}$ $\quad \mathbb{1}\underbrace{\{X \geq 0\}}_{A} = 1$ because A occurs then $E[1] = \underline{\underline{1}}$

$\to \underline{\mathbb{1}\{X \geq a\} = 1}$ for all $a \leq 0$

Q5: Does Markov's inequality work for both discrete and continuous random variables?

Discrete RV

$$E[X] = \sum_{support(x)} x . P\{X=x\} = \sum_{x \geq a} x . P\{X=x\} + \sum_{x<a} x P\{X=x\}$$

$$\geq \sum_{x \geq a} a . P\{X=x\} + 0 \longrightarrow E[X] \geq a . \sum_{x \geq a} P\{X=x\},$$

$$E[X] \geq a . P\{X \geq a\}, \quad \boxed{P\{X \geq a\} \leq \frac{E[X]}{a}}$$

Continous RV

$$E[X] = \int_{support} x . P\{X=x\} = \int_{x \geq a} x . P\{X=x\} + \sum_{x<a} x P\{X=x\} \leq$$

$$= \int_{x \geq a} x P\{X=x\} + 0 \quad E[X] \geq a \underbrace{\int_{x \geq a} P\{X=x\}}_{P\{X \geq a\}} \rightarrow E[X] \geq a . P\{X \geq a\}$$

$$\boxed{P\{X \geq a\} \leq \frac{E[X]}{a}}$$

Then yes it does work for both discrete and continuous random variables

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Continous RV (X ~ Continous)

$$E[X] = \int_0^a x \; P\{X=x\} dx + \int_a^{+\infty} x \; P\{X=x\} dx \geq \int_a^{+\infty} a \; P\{X=x\} dx = a \int_a^{+\infty} P\{X=x\} dx$$

$$E[X] \geq a . P\{X \geq a\} \qquad P\{X \geq a\} \leq \frac{E[X]}{a} \; ✓$$

Q6: Let $X \sim N(0,1)$ be a standert normal RV. for X, experiment with different values of "k" in (eq.6)

$X \sim N(0,1)$ $\qquad E[X] = \mu = 0$

$\qquad Var(X) = \sigma^2 = 1^2 = 1$

$$P\{(X-E[X])^2 \geq k^2\} \leq \frac{E[(X-E[X])^2]}{k^2}$$

$$P\{(X-0)^2 \geq k^2\} \leq \frac{E[(X-E[X])^2]}{k^2} \Big\} Var(X)$$

$$P\{|X-E[X]| \geq k\} \leq \frac{Var(X)}{k^2} \longrightarrow E[X]=0 \quad Var(X)=1$$

$$P\{|X| \geq k\} \leq \frac{1}{k^2} \xRightarrow[\substack{\text{support}(X) \cap \\ (-\infty,0)=\phi}]{} P\{X \geq k\} \leq \frac{1}{k^2}$$

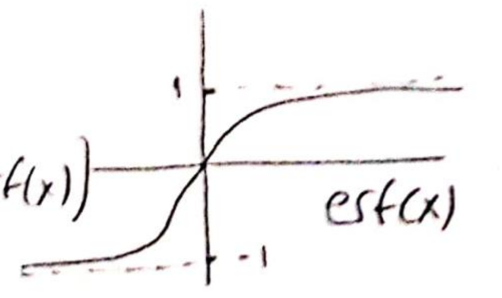
$$P\{X \geq k\} = 1 - P\{X \leq k\} = 1 - cdf_X(k) \qquad cdf_X(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{k} \exp\left(-\frac{u^2}{2}\right) du$$

$$cdf_X(k) = \frac{\sqrt{\pi}\left(erf\left(\frac{k}{\sqrt{2}}\right)+1\right)}{\sqrt{2}}$$

$$1 - cdf_X(k) \leq \frac{1}{k^2} \qquad 1 - \frac{1}{k^2} \leq cdf_X(k), \ \boxed{1 - \frac{1}{k^2} \leq \frac{\sqrt{\pi}}{\sqrt{2}}\left(erf\left(\frac{k}{\sqrt{2}}\right)+1\right)} \text{ eq.1}$$

if $k > 0$ then $\boxed{0 \leq 1 - \frac{1}{k^2} \leq 1}$

$\longrightarrow 0 \leq erf\left(\frac{k}{\sqrt{2}}\right) \leq 1$ (from graph of $erf(x)$)



$$1 \leq erf\left(\frac{k}{\sqrt{2}}\right) + 1 \leq 2$$

$$\frac{\sqrt{\pi}}{\sqrt{2}} \leq \frac{\sqrt{\pi}}{\sqrt{2}}\left(erf\left(\frac{k}{\sqrt{2}}\right)+1\right) \leq 2\frac{\sqrt{\pi}}{\sqrt{2}}$$ then we can say

for $k \in (0, +\infty)$ $\boxed{1 - \frac{1}{k^2} \leq cdf_X(k)}$ is correct

Q8: Show that this representation is correct in view of (eq.1, 2a)

$$\left(eq.1 \rightarrow Z_n = \frac{X_1+X_2 \dots +X_n - n\mu}{\sigma\sqrt{n}} \right)$$

$$eq.2a \rightarrow \mu = E[X_k] \text{ for } k \in \{1,2 \dots\}$$

$$eq.2b \rightarrow \sigma = \sqrt{var(X_k)} \text{ for } k \in \{1,2 \dots\}$$

$$Z_n = \frac{(X_1 + \dots + X_n) - E[X_1+X_2 \dots +X_n]}{\sqrt{Var(X_1+X_2+\dots+X_n)}}$$

$$E[X_1+X_2+\dots+X_n] \overset{\text{linearity}}{=} \underbrace{E[X_1]}_{\mu} + \underbrace{E[X_2]}_{\mu} + \dots + \underbrace{E[X_n]}_{\mu} = n.\mu$$

$$\sqrt{var(X_1+X_2 \dots X_n)} = \sqrt{var(X_1)+var(X_2) \dots} = \sqrt{\underbrace{\sigma^2+\sigma^2+\sigma^2 \dots}_{n \text{ times}}} = \sqrt{n.\sigma^2} = \underline{\sigma\sqrt{n}}$$

$$Z_n = \frac{X_1+X_2+\dots+X_n - n\mu}{\sigma\sqrt{n}}$$ doğrulanmış olur.

Q7: eq.(3b) pg. 314.

$$\sum_{k=1}^{n} Var\left(\frac{X_k}{n}\right) \overset{?}{=} \sum_{k=1}^{n} \frac{1}{n^2} Var(X_k)$$

for X_k, $Var(X_k) = \sigma^2$

for $\frac{X_k}{n}$, $Var\left(\frac{X_k}{n}\right) = \left(\frac{\sigma}{n}\right)^2$

$Var\left(\frac{X_k}{n}\right) = \frac{\sigma^2}{n^2}$, $\underline{Var(X_k) = \sigma^2}$

$$\underline{Var\left(\frac{X_k}{n}\right) = \frac{1}{n^2} \cdot Var(X_k)}$$

Q9: Show that $E[Z_n]=0$ (eq.4a) $Var(Z_n)=1$ (eq.4b)

$$E[Z_n] = E\left[\frac{(X_1+X_2+X_3\cdots X_n)-E[X_1+X_2+\cdots X_n]}{\sqrt{Var(X_1+X_2+\cdots+X_n)}}\right]$$

$$\overset{iid}{=} E\left[\frac{n.X_1 - E[n.X_1]}{\sqrt{n.Var(X_1)}}\right] \overset{\text{linearity of } E[.]}{=} E\left[\frac{nX_1 - nE[X_1]}{\sqrt{n.Var(X_1)}}\right]$$

$$\overset{\text{linearity of } E[.]}{=} n.E\left[\frac{X_1-E[X_1]}{\sqrt{n.Var(X_1)}}\right] \overset{\text{homogeneity}}{=} \frac{E[X_1-E[X_1]]}{E[n.Var(X_1)]}$$

from old lectures we know for Gaussian RV $E[X-E[X]]=0$

then $E[Z_n]=0$

$$Z_n \overset{iid}{=} \frac{n.X_1 - n.E[X_1]}{\sqrt{Var(X_1+X_2\cdots)}} =$$

$Var(X_1+X_2+\cdots X_1) = n^2.Var(X_1)$
doubtful

$$= \frac{nX_1 - nE[X_1]}{\sqrt{n^2.Var(X_1)}} = \frac{n(X_1-E[X_1])}{n\sqrt{Var(X)}} = \frac{X_1-\mu_1}{\sigma_1}$$

$$E\left[\frac{x-\mu}{\sigma}\right]=0, \quad Var\left[\frac{x-\mu}{\sigma}\right]=1$$

$E[x-E[x]] = E[x-\mu] = 0$ (pg. 99 we did it as Bonus HW)

$$Var(X) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{-\infty} y^2 \exp\left(-\frac{y^2}{2\sigma^2}\right)dy \quad y=\frac{x-\mu}{\sigma}$$

$$=\sigma^2\left[\frac{1}{\sigma^2}\underbrace{\int_{-\infty}^{+\infty}\frac{1}{\sigma\sqrt{2\pi}}\exp\left(\frac{y^2}{2\sigma^2}\right)}_{1}\right] = 1$$ def. of $Var(x)$ for Gaussian RV

when $y = x-\mu \longrightarrow Var(X) = \sigma^2$

$y = \frac{x-\mu}{\sigma} \longrightarrow Var(X) = 1$

Q10: How is the chain rule of differentiation at work in here?

$$L'(0) = \frac{d}{dt}\left[\ln(mgf_{X_k}(t)\right]\Big|_{t=0}$$

$$\underbrace{\frac{d(\ln(mgf_{X_k}(t)))}{d(mgf_{X_k}(t))}}_{\frac{1}{mgf_{X_k}(t)}\big|_{t=0}} \cdot \underbrace{\frac{d(mgf_{X_k}(t))}{dt}}_{E[X_k]} = \frac{E[X_k]}{mgf_{X_k}(t)\big|_{t=0}} = \frac{E[X_k]}{1}$$

Q11: Show that $E[X_1^*]=0$, $\sqrt{Var(X_1^*)}=1$ $\quad X_1^* = \frac{X_1-\mu}{\sigma}$

$$E[X_1^*] = E\left[\frac{X_1-\mu}{\sigma}\right] = E\left[\frac{X_1-E[X_1]}{\sigma}\right] \overset{\text{homogeneity}}{=} \frac{E[X_1-E[X_1]]}{E[\sigma]} = 0$$

$E[X_1-E[X_1]]=0$ for gaussian RV we did it before. pg 99.

$$Var\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left(\frac{x-\mu}{\sigma}\right)^2 \cdot \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)dx$$

$$y=\frac{x-\mu}{\sigma}, \quad = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{+\infty} y^2 \cdot \exp\left(\frac{-y^2}{2}\right)dy \cdot \frac{1}{\sigma}$$

$$= \sigma^2 \cdot \left(\frac{1}{\sigma^2}\int_{-\infty}^{+\infty}\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{y^2}{2\sigma^2}\right)\right) = \frac{\sigma^2}{\sigma^2} = 1$$

we did it before

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Bonus HW TC #09

Q1: $\text{pdf}_X(x) = C\,[x+\sqrt{x}]$ for $0 \le x \le 2$

$$\int_{-\infty}^{+\infty} \text{pdf}_X(x)\,dx = \int_{x \in \text{support}(x)} \text{pdf}_X(x)\,dx = 1 \Rightarrow \int_0^2 C\,[x+\sqrt{x}]\,dx = 1$$

$$C \cdot \Big|_0^2 \frac{x^2}{2} + \frac{2}{3}x^{3/2} = 1 = C \cdot \left(\frac{4}{2} + \frac{4\sqrt{2}}{3} - 0\right) = C \cdot \left(\frac{12+8\sqrt{2}}{6}\right) = 1$$

$$\boxed{C = \frac{6}{12+8\sqrt{2}} = \frac{3}{6+4\sqrt{2}}}$$

Q2: $Y = \min(X_1, X_2, \ldots X_N)$ since $\{X_1, X_2 \ldots X_N\}$ iid

$$\underbrace{P\{X_1 \geq x\}}_{} = \underbrace{P\{X_2 \geq x\}}_{} \cdots P\{X_N \geq x\} \rightarrow 1 - P\{X_N < x\} = 1 - P\{X_N \leq x\}$$

cont. RV

$$\underbrace{1 - P\{X_1 < x\}}_{\text{cont. RV}} = 1 - P\{X_2 < x\}$$

$$\underbrace{1 - P\{X_1 \leq x\}}_{} = 1 - P\{X_2 \leq x\} \ldots = 1 - P\{X_N \leq x\}$$

$$1 - cdf_{X_1}(x) = 1 - cdf_{X_2}(x) \ldots\ldots$$

$$1 - cdf_Y(x) = P\{Y > x\}$$ (Eğer bir setteki minimum sayı bir sayıdan büyükse diğer her sayı o sayıdan büyüktür)

$$1 - cdf_Y(x) = P\{Y \geq x\} = P\{\min(X_1, X_2 \cdots X_N) \geq x\} = P\{X_1 \geq x, X_2 \geq x \ldots X_N \geq x\}$$

independency

$$P\{X_1 \geq x\}\, P\{X_2 \geq x\} \cdots P\{X_N \geq x\}$$

$$= \prod_{k=1}^{N} P\{X_k \geq x\} = \prod_{k=1}^{N} 1 - cdf_{X_k}(x) \overset{iid}{=} \left[1 - cdf_{X_1}(x)\right]^N$$

$$1 - cdf_Y(x) = \left[1 - cdf_{X_1}(x)\right]^N \rightarrow \boxed{cdf_Y(x) = 1 - \left[1 - cdf_{X_1}(x)\right]^N}$$

$$pdf_Y(x) = \frac{d}{dx}\left[cdf_Y(x)\right] = \frac{d}{dx}\left[1 - (1 - cdf_{X_1}(x))^N\right]$$

$$= \underbrace{\frac{d\left(1 - (1 - cdf_{X_1}(x))^N\right)}{d(1 - cdf_{X_1}(x))}}_{-N.(1 - cdf_{X_1}(x))^{N-1}} \cdot \underbrace{\frac{d(1 - cdf_{X_1}(x))}{dx}}_{-pdf_{X_1}(x)}$$

$$\boxed{pdf_Y(x) = N\,(1 - cdf_{X_1}(x))^{N-1} \cdot pdf_{X_1}(x)}$$

Q_3: A: The sum of money Thelma robs the post office of, if she makes it.

$A \sim \text{Uniform}(a_L = \$100, a_H = \$500)$

S: Bernoulli random variable accounting for the status of the robbery attempt

$P\{S=1\} = 0.3$, $P\{S=0\} = 0.7$

$\mathbb{1}_{\{S=1\}} = \begin{cases} 1 & \text{if } \{S=1\} \text{ occurs} \\ 0 & \text{if } \{S=1\} \text{ does not occur} \end{cases}$

$\mathbb{1}_{\{S=1\}} = 1 - \mathbb{1}_{\{S=0\}}$

R: Deterministic value of money $R = \$400$

$P\{A.S > R\} = P\{A.\mathbb{1}_{\{S=1\}} > R\}$

$- P\{A.S > R\} = E[\mathbb{1}_{\{A.S>R\}}] = E[P\{A.S > R \mid S\}]$

$= P\{\underbrace{A.0}_{0} > R \mid S=0\}.P\{S=0\} + P\{A.1 > R \mid S=1\}.P\{S=1\}$

independent

$\dfrac{P\{A.0 > R\}.\cancel{P\{S=0\}}}{\cancel{P\{S=0\}}}.P\{S=0\} + \dfrac{P\{A.1>R\}.\cancel{P\{S=1\}}}{\cancel{P\{S=1\}}}.P\{S=1\}$

$= \cancel{P\{0 > R\}.P\{S=0\}}^{\to 0} + \underbrace{P\{A > R\}}_{1-\underbrace{P\{A < R\}}_{cdf_A(R)}} P\{S=1\}$

$= [1 - cdf_A(R)].\underbrace{P\{S=1\}}_{0.3}$

$cdf_A(R) = \dfrac{400-100}{500-100} = \dfrac{3}{4}$

$= \boxed{\left(1 - \frac{3}{4}\right).0.3 = \frac{3}{40} = 0.075}$ A

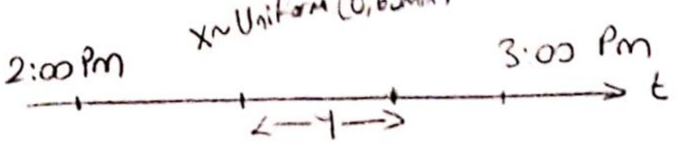
Q4: Bonus HWTC #09

X: arrival time in minutes after 2:00 pm of Ayse at the cafe

$X \sim \text{Uniform}(0\,\text{min}, 60\,\text{min})$

$Y \sim \text{Exponential}\left(\lambda = \frac{1}{10\,\text{min}}\right)$

Y cannot have an exponential distribution



Y is not allowed to assume values greater than [60-X] minutes

$Y \sim \text{Truncated Exponential}\left(\lambda = \frac{1}{10\,\text{min}}, k\right)$

$$\boxed{k = [60 - X] \cdot \lambda = 6 - \frac{X}{10}}$$

$$P\{Y > 10\,\text{min}\} = \underbrace{P\{Y > 10\,\text{min}, X > 50\,\text{min}\}} + P\{Y > 10\,\text{min}, X \leq 50\,\text{min}\} \quad \text{eq.18}$$

HW: Why is the 1st term on the RHS of eq.18 zero?

Çünkü $X + Y \leq 60\,\text{min}$ olmak zorunda. end of hw

$$P\{Y > 10\,\text{min} \mid X = 50\} = \int_0^{50} P\{Y > 10\,\text{min} \mid X = a\} \cdot \underbrace{\text{Pdf}_X(a)}_{1/60}\, da$$

$$= \frac{1}{60} \cdot \int_0^{50} P\{Y > 10\,\text{min} \mid X = a\}\, da \stackrel{\text{independency}}{=} \frac{1}{60} \int_0^{50} \frac{P\{Y > 10\,\text{min}\} \cdot \cancel{P\{X = a\}}}{\cancel{P\{X = a\}}}\, da$$

$$= P\{Y > 10\,\text{min}\} \cdot \frac{1}{60} \int_0^{50} da = \frac{50}{60} P\{Y > 10\,\text{min}\}$$

$$= \frac{50}{60}\left(1 - P\{Y \leq 10\,\text{min}\}\right) = \frac{50}{60}\left(1 - \text{cdf}_Y(10\,\text{min})\right) \Rightarrow \text{cdf}_Y(10\,\text{min}) = \frac{1 - \exp(-\frac{1}{10} \cdot 10)}{1 - \exp(-k)}$$

$$\text{cdf}_Y(10\,\text{min}) = \frac{1 - \exp(-\frac{1}{10} \cdot 10)}{1 - \exp(-k)} = \frac{1 - \exp(-1)}{1 - \exp(-5)} \qquad k = 6 - \frac{10}{10} = 5$$

$$P\{Y > 10\,\text{min}\} = \frac{50}{60} \cdot \underbrace{\left(\frac{1 - \exp(-1)}{1 - \exp(-5)}\right)}_{0.6364} = \underline{0.5303}$$

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HWTC #09 Q3 – Bonus 1

* Explain how (eq.8) works. Can the two indicator random variable assume the same value together?

$\mathbb{1}_{A^c} = 1 - \mathbb{1}_A$ } sağlanır çünkü $\mathbb{1}_A$ demek A olayı gerçekleşirse 1, A olmazsa 0

A olayı gerçekleşirse $\mathbb{1}_A = 1$, $\mathbb{1}_{A^c} = 0$ böylece $0 = 1 - 1$ sağlanır

A olayı gerçekleşmezse $\mathbb{1}_A = 0$ $\mathbb{1}_{A^c} = 1$ böylece $1 = 1 - 0$ sağlanır

başka bir durum olmadığından bu eşitlik her zaman çalışır.

* Explain again how these formulations are justified.

$$P\{A\} = E[\mathbb{1}_{\{A\}}] = \underbrace{E[\underbrace{E[\mathbb{1}_{\{A\}} \mid \underbrace{Y}_{\text{rand. var}}]}_{\text{rand. var}}]}_{\text{deterministic value}} = \underbrace{E[P\{\underbrace{A}_{\text{event}} \mid \underbrace{Y}_{\text{rand. var.}}\}]}_{\text{deterministic value}}$$

random variable

from last lecture

then $P\{AS > R\} = E[\mathbb{1}_{\{AS > R\}}] = E[E[\mathbb{1}_{\{AS>R\}} \mid S]]$

$= E[P\{AS > R \mid S\}]$ eşitliği doğrudur.

CamScanner ile tarandı

HWTC #09 Q4

* Explain in your own words what I'm saying here.

Exponential RV'ın sınırlandırılmış halidir. Biz Exponential RV'i x ekseninde sonsuza giden bir alan için hesaplıyorduk. Verilen R.V. bunu gerektiriyordu. Eğer Exponential R.V olarak verilen bir durum aynı zamanda spesifik bir kısıtlama ile karşımıza çıkarsa bu durumda Trucated Exponential kullanılmalıdır