Abdullah MEMİŞOĞLU 171024001

Bonus HW # 14

Q1: Work on the following problem in order to exercise your skills on the induction Method of proof. You must have computed before the moments of a random variable $X \sim N(0,1)$ through the moment generating function $mgf_X(t)$. Now show that the moments you computed then are correct through an induction-oriented proof. $\int_{-\infty}^{+\infty} pdf_X(x)dx = 1$ is the ~~easy~~ trivial proof of your base case.

Base case.

$$\int_{-\infty}^{+\infty} pdf_X(x)dx = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) = 1$$ o bunu kabul ediyoruz.

$$mgf_X(t) = E[\exp(tX)] = \int_{-\infty}^{+\infty} \exp(tx) \cdot \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left(-\frac{(\cancel{x}^2 - 2t\cancel{x})}{2}\right) dz. \qquad \underbrace{z^2 - 2tz + t^2}_{(z-t)^2} - t^2$$

$$mgf_X(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left(\frac{t^2}{2}\right) \cdot \exp\left(-\frac{(x-t)^2}{2}\right) dx$$

$$= \exp\left(\frac{t^2}{2}\right)\Big[\underbrace{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left(-\frac{(x-t)^2}{2}\right) dx}_{1 \;\to\; \text{base case}}\Big] \qquad mgf_X(t) = \exp\left(\frac{t^2}{2}\right)$$

$$\frac{d^n}{dt^n}(mgf_X(t))\Big|_{t=0} = E[X^n] \quad \to n=1 \text{ için} \quad \frac{d}{dt} mgf_X(t) = \frac{d(\exp(t^2/2))}{d(t^2/2)} \cdot \frac{d(t^2/2)}{dt}$$

$$\Big|_{t=0} \underbrace{\exp(t^2/2)}\cdot \frac{2t}{2}$$

$$= 0$$

$\mu = 0$ olduğundan

$E[X] = 0$

n=1 için doğru

n = k için

$$\frac{d^k}{dt^k} mgf_X(t)\Big|_{t=0} = E[X^k]$$

$\boxed{n \to k}$ $\frac{d^k}{dt^k}(mgf_x(t))\Big|_{t=0} = E\{X^k\}$ $mgf_x(t) = exp\left(\frac{t^2}{2}\right)$

$n \to k$ $\frac{d}{dt^k}(mgf_x(t))\Big|_{t=0} = E[X^k] = \int_{-\infty}^{+\infty} x^k \cdot \frac{1}{\sqrt{2\pi}} \cdot exp\left(\frac{-x^2}{2}\right)$

$mgf_x(t) = exp(t^2/2)$

$\frac{d}{dt^k}\left(exp\left(\frac{t^2}{2}\right)\right)\Big|_{t=0} = E[X^k] = \int_{-\infty}^{+\infty} x^k \frac{1}{\sqrt{2\pi}} \cdot exp\left(\frac{-x^2}{2}\right)$ olduğunu kabul ediyoruz

bu durumda ~~...~~ $n=k+1$ için de sağlanmalı

$\frac{d}{dt^{k+1}}\left(exp\left(\frac{t^2}{2}\right)\right)\Big|_{t=0} = E[X^{k+1}] = \int_{-\infty}^{+\infty} x \cdot x^k \cdot \frac{1}{\sqrt{2\pi}} \cdot exp\left(-\frac{x^2}{2}\right)dx$

$$\frac{d}{dt}\Bigg[\underbrace{\frac{d}{dt^k}\left(exp\left(\frac{t^2}{2}\right)\right)\Big|_{t=0}}_{\int_{-\infty}^{+\infty} x^k \cdot \frac{1}{\sqrt{2\pi}} \cdot exp\left(-\frac{x^2}{2}\right)dx}\Bigg] = \int_{-\infty}^{+\infty} x \cdot x^k \frac{1}{\sqrt{2\pi}} \cdot exp\left(-\frac{x^2}{2}\right)$$

Q2: Look up these words "demotivational" "arbitrage", "bookmaker" "bookie"

$\boxed{\text{demotivational}}$: Tending or intended to demotivate.

$\boxed{\text{arbitrage}}$: the practice of buying something, such as shares or currency, in one place and selling them in another where you can get a higher price at the same time.

$\boxed{\text{bookmaker}}$: a person who accepts and pays out amounts of money risked on a particular result.

Q3: Note that E[G] comes out to be independent of "k" the partitioning parameter.

$$E[G] = \{A \text{ olayının gerçekleşmesi}\} . \{A \text{ olayından gelen kazanç}\} +$$
$$\{B \text{ '' '' }\} . \{B \text{ '' '' '' }\}$$

$$E[G] = [k.B.a - B] . \frac{b}{a+b} + [(1-k).B.b - B] \frac{a}{a+b} \rightarrow eq.4a$$

sadeleştirmeler yapıldığında

$$E[G] = B\left[\frac{ab}{a+b} - 1\right]$$ bulunur. $\boxed{B = A^c}$

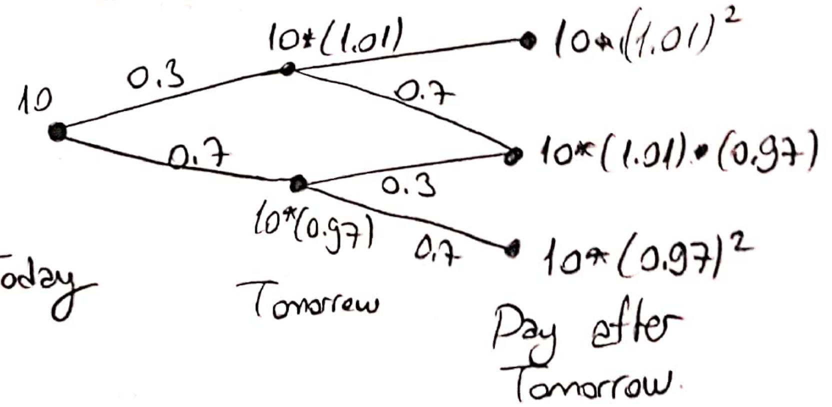
Burada k teriminin sadeleştiği gözlenir. Bunun sebebi bu değerin sadece A ve B olaylarının $(B = A^c)$ gerçekleşmesiyle oluşan toplam kazancı bölmek için varolmasıdır.
k değeri A ve B'nin kazançlarını bölmek kullanılır. toplam kazanca bakılırken k değeri kaybolacaktır.

Q.: What e the condition that must be satisfied by the ...

Q4: for your information look up temporary arbitrage opportunities in currency exchange schemes, and the method for pinpointing such opportunities through graph algorithms.

A binomial market where it is possible to find an investment strategy that yields a positive profit with positive probability but without any downside risk. Such a strategy is commonly known as an arbitrage opportunity.

They define formally an arbitrage opportunity as a self-financing trading strategy (x, y) such that the value of the initial portfolio (x_1, y_1) at time 0 is less than or equal to 0, but the value of final portfolio (x_T, y_T) at time T is nonnegative with probability 1 and positive with positive probability.



Q5: What is the condition that must be satisfied by the odds of disjoint and all inclusive events in bookmakers' tables of odds? Such events may be more than just two in number.

disjoint and all inclusive events in bookmakers' tables of odds

$P\{A\} = \frac{b}{a+b}$, $P\{A^c\} = \frac{a}{a+b}$

$E[G] = B\left[\frac{ab}{a+b} - 1\right] = -B\left[1 - \frac{ab}{a+b}\right]$ $\quad \frac{ab}{a+b} = P\{A\}.P\{A^c\}$

$E[G] = -B\left[1 - P\{A\}.P\{A^c\}\right]$

$P\{A\} < 1$ $\quad P\{A^c\} < 1$ then $P\{A\}.P\{A^c\} < 1$ then $1 - P\{A\}.P\{A^c\} > 0$

for all $\{A\}$ and $\{A^c\}$

then $E[G] = -B.\underbrace{\left[1 - P\{A\}.P\{A^c\}\right]}_{>0 \text{ is the condition}}$ B must be bigger than zero

so $E[G] < 0$ for all $\{A\}$

eq.5c $\rightarrow$ $\frac{ab}{a+b} < 1$ because $\underbrace{P\{A\}.P\{A^c\}}_{\substack{P\{A\}<1 \\ P\{A^c\}<1}} = \frac{ab}{a+b} < 1$

eq.5d $\rightarrow$ $\frac{1}{a} + \frac{1}{b} > 1$ because

$$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{1}{\left.\frac{ab}{a+b}\right\} X} = \frac{1}{X} > 1$$

$0 < X < 1$

$X = P\{A\}.P\{A^c\}$

Q6: What ~~does~~ do we mean by causality, how do we define it?

Causation indicates a relationship between two events where one event is affected by the other. In statistics, when the value of one event or variable increases or decreases as a result of other events, it is said there is a causation.

Q7: What is the Markovian property of a stochastic process being Markov? Is this property implied in (eq.1)?

★ A markov process is a stochastic process that satisfies the Markov property (sometimes characterized as "memorylessness") In simpler terms, it is a process for which predictions can be made regarding future outcomes based solely on its present state and - most importantly - such predictions are just as good as the ones that could be made knowing the process's full history. In other words, conditional on the present state of the system, its future and past system are independent

by definition $cdf_{X_{t_1}, X_{t_2}, X_{t_3} \dots X_{t_n}}(a_1, a_2, a_3 \dots a_n) = P\{X_{t_1} \leq a_1, X_{t_2} \leq a_2 \dots X_{t_n} \leq a_n\}$

↳ independency of X_{t_1}, X_{t_2}

~~Q9: Based on~~

Q9: Based on what you know about probability and randomness, can you come up with a stochastic process that allows you always to express the cdf in eq.1 as a product of probabilities due to independence?

Markovian Property (X_k is independent on X_{k-1})

Multivariate Joint CDF

$$cdf_{X_1,X_2 \dots X_n}(a_1,a_2,\dots a_n) = P\{X_1 \le a_1, X_2 \le a_2 \dots X_n \le a_n\}$$

Markovian Property of stochastic process

$$cdf_{X_1,X_2 \dots X_n}(a_1,a_2,\dots a_n\} = \underbrace{P\{X_1 \le a_1\} \cdot P\{X_2 \le a_2\} \cdots P\{X_n \le a_n\}}_{\text{Product of probabilities.}}$$

Markovian property; conditional on the present state of the system, its future and past system are independent.

Q9: Think of a stochastic process in discrete time and discrete state with discrete time indices $n \in \{0,1,\dots\}$. It might be more meaningful to express the probabilistic law for such a process not as in eq.1 but as joint pmf. Please express it as such. Also impose the following condition. X_k is dependent on X_{k-1}, but on none of the past history before $n=k-1$. How can you express the pmf now

$X_{n_1}, X_{n_2} \dots X_{n_N} \in \{0,1,2,\dots\}$, $a_1, a_2 \dots a_N \in \{0,1,2,\dots\}$

$$pmf_{X_{n_1},X_{n_2} \dots X_{n_N}}\{a_1,a_2 \dots a_N\} = P\{X_{n_1}=a_1, X_{n_2}=a_2 \dots X_{n_N}=a_N\}$$

X_k is dependent on X_{k-1} none of the past history befor $n=k-1$

$$= P\{X_{n_1}=a_1\} \cdot P\{X_{n_2}=a_2\} \cdots P\{X_{n_{N-1}}=a_{N-1}, X_{n_N}=a_N\}$$

Q10: In view of (eq.7) and (eq.8), Show that these two equations are correct.

We know that S_{xx} is fourier transform of R_{xx}

usual fourier transform

$$R_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{xx}(\omega) \exp(+j\omega\tau) d\tau$$

then change the argument

$\omega = 2\pi f$

$$R_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{S}_{xx}(2\pi f) \exp(+j2\pi f \tau) d\tau$$

$S_{xx}(\omega) = 2\pi \tilde{S}_{xx}(f)$ so

$$R_{xx}(\tau) = \frac{2\pi}{2\pi} \int_{-\infty}^{+\infty} \tilde{S}_{xx}(f) \exp(j2\pi f \tau) d\tau$$

$\boxed{\text{fourier inversion theorem says}}$

$$F(f) = \int_{-\infty}^{+\infty} \exp(-j2\pi f y) f(y) dy \quad \text{Be inversion theorem}$$

$$f(x) = \int_{-\infty}^{+\infty} \exp(+j2\pi f x) . F(f) . df \quad \text{'tir}$$

bu bilgiye göre

by fourier inversion theorem

$$\tilde{S}_{xx}(f) = \int_{-\infty}^{+\infty} R_{xx}(\tau) . \exp(-j2\pi f \tau) d\tau$$

Q11: Show that this is true. (if the process $X(t)$ is real, then the autocorrelation function $R_{xx}(s)$, or write dummy back to τ, $R_{xx}(\tau)$ should be real)

We know $S_{xx}(w)$ is real in w

then

$S^*_{xx}(w)$ should be real too.

$$S^*_{xx}(w) = \int_{-\infty}^{+\infty} R^*_{xx}(\tau)\exp(jw\tau)d\tau)$$

$s \to -\tau$

$$S^*_{xx}(w) = \int_{+\infty}^{-\infty} R^*_{xx}(-s)\exp(jws)\cdot[-ds] = \int_{-\infty}^{+\infty} R^*_{xx}(-s)\exp(-jws)ds$$

$$R^*_{xx}(-s) \xrightarrow{F} S^*_{xx}(w)$$

We know from begining $S_{xx}(w)$ real so $R_{xx}(\tau)$ is real

then we can see how $S^*_{xx}(w)$ is real because $S_{xx}(w)$ is real

then $R^*_{xx}(-s)$ is real too.

Q12: Show that this is true through eq.16

$$S^*_{xx}(w) = \int_{-\infty}^{+\infty} R^*_{xx}(-s)\exp(-jws)ds \quad \text{eq.16}$$

$$S_{xx}(w) = \int_{-\infty}^{+\infty} R_{xx}(\tau)\exp(-jw\tau)d\tau \xrightarrow{\tau \to -s} \int_{-\infty}^{+\infty} R_{xx}(-s).\exp(jws).[-ds]$$

$$= +\int_{-\infty}^{+\infty} R_{xx}(+s).\exp(jws)ds = S_{xx}(w)$$

$$S^*_{xx}(-w) = \int_{-\infty}^{+\infty} R^*_{xx}(-s).\exp(jws)ds$$

$$\left.\begin{aligned} S_{xx}(w) &= \int_{-\infty}^{+\infty} R_{xx}(+s).\exp(jws)ds \\ S^*_{xx}(-w) &= \int_{-\infty}^{+\infty} R^*_{xx}(-s).\exp(jws)ds \end{aligned}\right\} R_{xx}(s) = R^*_{xx}(-s) \quad \text{eq. 18}$$

so equation is true

by equation $S_{xx}(w) = S^*_{xx}(-w)$ then $R_{xx}(s) = R^*_{xx}(-s)$

Q13: One can also show that $Re\{S_{xx}(w)\}$ is an even function and $Im\{S_{xx}(w)\}$ is an odd function.

$S^*_{xx}(w) = S_{xx}(w)$ eq.19 , $S^*_{xx}(-w) = S_{xx}(w)$ eq.21

then $S^*_{xx}(w) = S^*_{xx}(-w)$ so $S^*_{xx}(w)$ is even function

so for $S_{xx}(w)$ $Re\{S_{xx}(w)\} = Re\{S^*_{xx}(w)\}$ = even function

$Im\{S^*_{xx}(w)\}$ is even function $S_{xx}(w) = (S^*_{xx}(w))^*$

$Im\{(S^*_{xx}(w))^*\}$ is odd function

Q14: We always assumed in this write up that the stochastic process X(t) is real. What happens if X(t) can assume complex values. How should the equations and representations in this write up be modified?

we find for x(t) is real $R_{xx}(\tau) = R_{xx}(-\tau)$ → eq.1 then we accept R_{xx} is even function when x(t) is real.

now x(t) is not real function then

from eq.14 and eq.16 $R_{xx}(\tau) = R^*_{xx}(-\tau)$ eq.2

then we can't say R_{xx} is a even function by eq.2 → Hermitian function

$S_{xx}(w) = \int_{-\infty}^{+\infty} R_{xx}(\tau) \exp(-jw\tau) d\tau$

$S^*_{xx}(w) = \int_{-\infty}^{+\infty} R^*_{xx}(\tau) \exp(jw\tau) d\tau$ $s \to -\tau$

$S^*_{xx}(w) = \int_{-\infty}^{+\infty} R^*_{xx}(-s) \cdot \exp(-jws) ds$

$R^*_{xx}(-s) \xrightarrow{F} S^*_{xx}(w)$

since $R_{xx}(\tau)$ is not real function

$S^*_{xx}(-w) \neq S_{xx}(w)$

$S_{xx}(w)$ is not even function.

q_{15}: What if $X(t)$ is a complex vector of stochastic process indexed by t. How should the equations and representations in this write up be modified.

for $X(t) = Z(t) + jy(t)$

$$R_{yy}(t_1, t_2) = R_{zz}(t_1, t_2) \rightarrow \quad R_{yz}(t_1, t_2) = -R_{zy}(t_1, t_2)$$

from these equations

$$E[X(t_1)X^*(t_2)] = 2R_{zz}(t_1, t_2) + 2j \cdot R_{yz}(t_1, t_2)$$

and $E[X(t_1)X(t_2)] = 0$ for all t_1, t_2

This implies that all of the joint second order statistics for the complex process $Z(t)$ are represented in the function

$$R_{xx}(t_1, t_2) = E[X(t_1)X^*(t_2)]$$

$$S_{xx}(w_1, w_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} R_{xx}(t_1, t_2)\, e^{-jw_1 t_1} e^{+jw_2 t_2}\, dt_1 dt_2$$

$R_{xx}(t_1, t_2)$ and $S_{xx}(w_1, w_2)$ is Hermitian function

$$R_{xx}(t_1, t_2) = R^*_{xx}(t_2, t_1), \quad S_{xx}(w_1, w_2) = S^*_{xx}(w_2, w_1)$$

Q16: What if the mean as defined in (eq.29) is constant for all t? How should eq.31 be expressed then?

$$\tilde{R}_{xx}(t_1, t_2) = R_{xx}(t_1 - t_2) = E[x(t_1)\cdot x(t_2)] \rightarrow \text{eq.1}$$

mean$(x(t)) = E[x(t)] = a$ for all t then

$$\text{autocov}_{xx}(t_1,t_2) = \underbrace{E[x(t_1)\,x(t_2)]}_{R_{xx}(t_1-t_2)} - \underbrace{E[x(t_1)]}_{a}\cdot\underbrace{E[x(t_2)]}_{a}$$

$$\boxed{\text{autocov}_{xx}(t_1,t_2) = R_{xx}(t_1-t_2) - a^2}$$

Q17: What if the mean as defined in eq 29 is zero for all t. What would be the relation between the auto covariance and the autocorrelation through (eq.31)

from Q16 → $\text{autocov}_{xx}(t_1,t_2) = R_{xx}(t_1-t_2) - a^2$

$a \to 0$ $E[x(t)] = 0$ for all t → $\boxed{\text{autocov}_{xx} = R_{xx}(t_1-t_2)}$

Q18: Look up this word "to uphold"

to uphold → to defend or keep a principle or law, or to say that a decision that has already been made, especially a legal one is correct.

Q19: Look up this word "to govern"

to govern: to control and direct the public business of a country, city, group of people, etc...

Q20: Look up the word "restrictive"

restrictive: limiting the freedom of someone or preventing something from growing.

Q21: Look up the word "to impose"

to impose: to officially force a rule, tax, punishment etc. to be obeyed.

Q22: Look up the term AWGN(additive white Gaussian noise)]

Additive white Gaussian Noise is a basic noise model used in information theory to mimic the effect of many random processes that occur in nature. The modifiers denote the specific characteristics:

Additive: Because it is added to any noise that might be intrinsic to the information system.

White: refers to the idea that it has uniform power accross the frequency band for the information system. It is an analogy to the collor white which has uniform emmissions at all frequencies in the visible

Gaussian: because it has a normal distribution in the domain with an average time domain value of zero.

$\boxed{Q23}$ → Look up the word "outset"

outset: the start or begining.

CamScanner ile tarandı

HWTC # 10

Q1: $E[x] = 10 \quad k = 5 \quad X$ is a Random Variable

$$\underbrace{P\{|X - E[X]| \le k\}}_{\text{x'in beklenen değerinden en fazla 5 br uzakta olması olasılığı}} \le 0.25$$

$Var(x) = ?$

by Chebyshev inequality;

$$\left[P\{(X - E[X])^2 \ge k^2\} \le \frac{E[(X - E(X))^2]}{k^2} \right]$$

$$P\{|X - E[X]| \ge k\} \le \frac{Var(x)}{k^2}$$

$$1 - \underbrace{P\{|x - E[x]| \le k\}}_{0.25} \le \frac{Var(x)}{k^2} \longrightarrow 0.75 \le \frac{Var(x)}{5^2}$$

$$\boxed{Var(x) \ge \frac{75}{4}}$$

HW Bonus if we had "...≥0.25" in eq. 1

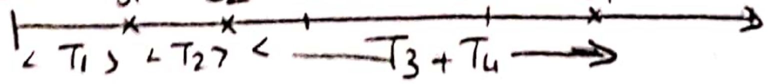
$Var(x) = ?$ using chebyshev's inequality.

$$P\{(x - E[x])^2 \ge k^2\} \le \frac{E[(x - E[x])^2]}{k^2}$$

$$\underbrace{P\{|x - E[x]| \ge k\}}_{0.25} \le \frac{Var(x)}{k^2}$$

$$0.25 \le \frac{Var(x)}{25} \longrightarrow Var(x) \ge \frac{25}{4}$$

$Q2:$ 13.30 S_1 S_2 13.35 13.40 S_4



$S_k = \sum_{i=1}^{k} T_i$, $S_1 = T_1$ $S_2 = T_1 + T_2$ $S_3 = T_1 + T_2 + T_3$, $S_4 = T_1 + T_2 + T_3 + T_4$

$S_2 < S_3 < S_4$ $P\{S_2 < a = 5min, S_4 > b = 10min\}$ eq.1

eq.1 $\Rightarrow P\{T_1 + T_2 < a, T_1 + T_2 + T_3 + T_4 > b\} \rightarrow T_1 + T_2 < a$ old. göre $T_1 + T_2 + T_3 + T_4 > b$ ve $T_3 + T_4 > b - a$ olmalı

Bonus: Make use of the equivalence of stated events to show that this is true!

if $T_1 + T_2 < a$ and $Max(T_1 + T_2) = a^-$ then if $T_1 + T_2 + T_3 + T_4 > b$

$T_3 + T_4 + a^- > b$ $T_3 + T_4 \geq b - a^- \xrightarrow{a^- \neq a \text{ so}} \underline{T_3 + T_4 > b - a}$

end of bonushw

$P\{T_1 + T_2 < a, T_3 + T_4 > b - a\}$ $T_1 + T_2 = S_2$, $T_3 + T_4 = S_4 - S_2$

$P\{S_2 < a, S_4 - S_2 > b - a\} = P\{S_2 < 5, S_4 - S_2 > 5\}$

$\xrightarrow{\text{independency}} P\{S_2 < 5\} . P\{S_4 - S_2 > 5\}$

$S_2 \sim Erlang(\lambda = \frac{1}{3min}, k = 2)$

$S_4 - S_2 \sim Erlang(\lambda = \frac{1}{3min}, k = 2)$

$cdf_{S_2}(x) = 1 - \sum_{n=0}^{k-1} \frac{1}{n!} \cdot e^{-\lambda x} (\lambda x)^n$

$\xrightarrow{\text{cont. RV}} P\{S_2 \leq 5\} = cdf_{S_2}(5) =$

$= 1 - \sum_{n=0}^{1} \frac{1}{n!} \cdot e^{-\frac{1}{3}x} \cdot \left(\frac{1}{3}x\right)^n \xrightarrow{x=5}$

$= 1 - \sum_{n=0}^{1} \frac{1}{n!} \cdot e^{-\frac{5}{3}} \cdot \left(\frac{5}{3}\right)^n = P\{S_2 < 5\}$

$\boxed{cdf_{S_2} = cdf_{S_4 - S_2}}$

$P\{S_4 - S_2 > 5\} = 1 - P\{S_4 - S_2 \leq 5\} = 1 - cdf_{S_4 - S_2}(5) \Rightarrow$

$= 1 - \left(1 - \sum_{n=0}^{1} \frac{1}{n!} e^{-5/3} \left(\frac{5}{3}\right)^n\right) = \sum_{n=0}^{1} \frac{1}{n!} e^{-5/3} \left(\frac{5}{3}\right)^n$

$$P\{S_2 < a; S_4 - S_2 > b-a\} = P\{S_2 \le 5\}\cdot\left(1 - P\{S_4 - S_2 \le 5\}\right)$$

$$= \left[1 - \sum_{n=0}^{1} \frac{1}{n!}\cdot \exp\left(\frac{-5}{3}\right)\cdot\left(\frac{5}{3}\right)^n\right]\cdot \sum_{n=0}^{1}\frac{1}{n!}\exp\left(\frac{-5}{3}\right)\left(\frac{5}{3}\right)^n$$

$$\left[1 - \left(\frac{5}{3}\exp\left(\frac{-5}{3}\right) + \exp\left(\frac{-5}{3}\right)\right)\right]\cdot\left(\frac{5}{3}\exp\left(\frac{-5}{3}\right) + \exp\left(\frac{-5}{3}\right)\right)$$

$$\left(1 - \frac{8}{3}\exp\left(\frac{-5}{3}\right)\right)\cdot\left(\frac{8}{3}\exp\left(\frac{-5}{3}\right)\right) = \underline{0.2499}$$

!: Bonus: Recall how exponential and Erlang distributions are related to each other.

The Erlang distribution with shape parameter $k=1$ simplifies to the exponential distribution. It is the distribution of a sum of k independent exponential variables with mean $1/\lambda$ each.

* It is a special case of gamma distribution.

Gamma distribution:

In probability theory and statistics, the gamma distribution is a two parameter family of continous probability distributions.

Q3: Bonus Questions.

Bonus 1: Explain the rationale behind eq. 7a,b

$g(1) = (A_1 . S_1) . V_1$

$A_1 \rightarrow$ Alının ilk satıştan kazandığı para

$S_1 \rightarrow$ Alının satış yapabilme olasılığı

$V_1 \rightarrow$ Paranın çalınma olasılığı.

V_1 ve S_1 değerleri ya 1 ya sıfır olacaktır.

eğer $V_1 = 0$ ise paralar çalınmıştır bu durumda $g(1) = 0$ olacaktır

veya $S_1 = 0$ ise satış gerçekleşmedi bu durumda yine $g(1) = 0$

A_1 kazandığı değer ise $g(1) = A_1$ olması için satış gerçekleşmeli

$\underline{A_1 = 1}$ ve para çalınmamalı $\underline{V_1 = 1}$

eq.7b

$g(k+1) = [g(k) + A_{k+1} . S_{k+1}] V_{k+1}$

burada eğer satış gerçekleştiyse k. adıma kadar biriken para $g(k)$

$g(k) + A_{k+1} . S_{k+1} \rightarrow (k+1)$. adımda satış geliri $g(k)$'ya eklenir.

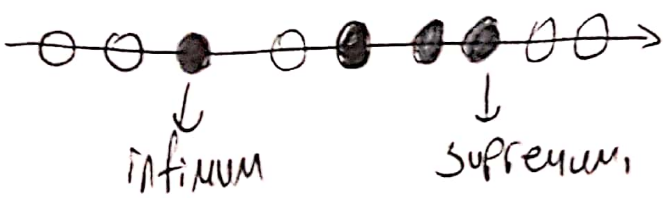
V_{k+1} neden parantez dışında çünkü paralar çalınırsa $(k+1)$. gelir

değil $(k+1)$ satıştaki toplam gelir sıfırlanacaktır.

Bonus 2: The operator "inf(.) and "sup(.)" are meaning what?

inf(.) = infimum → of a subset S of a partially ordered set T is the greatest element in T that is less than or equal to all elements of S, if such an element exists. the term greatest lower bound

sup(.) → supremum → of a subset S of a partially ordered set T is the least element in T that is greater than or equal to all elements of S, if such an element exist. Term → least upper bound.



T seti içi boş ve dolu olan tüm çemberleri, S seti ise sadece dolu çemberleri belirtsin. Tanıma göre infimum and supremum belirtildiği gibidir.

Bonus 3: Explain the meaning of eq.g

$$N-1 = \sup\{n \in \mathbb{Z}^+ : P\{g(n) \geq R\} = 1\}$$

Bu sette N-1 değerinin supremum belirlenmesi durumunda her n pozitif tam sayısı için $g(n) \leq R = E$ belirtmektedir.

Q3: $A_k \sim \text{Uniform}(a_L, a_H)$ for $k \in \{1, \dots, 32\}$

$S_k \sim \text{Bernoulli}(p_{sold})$ for $k \in \{1, 2, \dots 32\}$ $\quad p_{sold} + q_{sold} = 1$

$V_k \sim \text{Bernoulli}(q_{robbed})$ for $k \in \{1, 2, \dots 32\}$ $\quad q_{robbed} + p_{robbed} = 1$

R : deterministic target sum of money

g(k): random variable denoting the money Ali has got at the kth vase, after the sale and robber attempts.

N: the first index in $\{1, 2, \dots 32\}$ for $P\{g(N) < R\} < 1$

$g(1) = (A_1 . S_1) V_1$

$g(k+1) = [g(k) + A_{k+1} \cdot S_{k+1}] \cdot V_{k+1}$ $\qquad$ A_k is bounded between $[a_L, a_H]$

$N-1 = \sup\{n \in \mathbb{Z}^+ : P\{g(n) < R\} = 1\}$

$k = \sup\{n \in \mathbb{Z}^+ : (a_H) \cdot (n) < R\}$ $\qquad N = k+1$

$\boxed{N = k+1}$ the index

(b) Condition $\left\{ \begin{array}{l} \text{all } S_k\text{'s are } 1, \\ \text{all } V_k\text{'s are } 1, \\ \text{for } k \in \{1, \dots N-1\} \end{array} \right\} \rightarrow$ her seferinde satış yapacak ve A_k kadar para kazanacak.

$A = \{(\max(g(N-1)) + A_N S_N) V_N < R\}$ $\rightarrow$ Max gelirin R'den küçük olması (N kere denendiğinde)

$\underbrace{\qquad\qquad}_{A_k = a_H}$

$B = \{(g(N-1) + A_N S_N) V_N \leq R\}$ $\rightarrow$ a_L a_H arasında başarılı her satıştan kazanılan toplam paranın R'den küçük olması

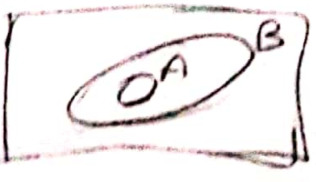
HW Explain the rationale behind eq. (3, a, b)

Yorum ile açıklarsak

her adımda a_L, a_H arasında kazandığı ve N kez tekrarladığı durumda kazanç R'den düşük olması olasılığı $P\{B\}$, her adımda a_H kazanarak N kez kazandığı paranın R'den küçük olması $P\{A\}$

eğer A olayı gerçekleşiyorsa B olayı zaten gerçekleşmiştir

$P\{A\} \leq P\{B\}$ $\qquad A \xrightarrow{\text{imp}} B$



$\boxed{a_L < x < a_H} \longrightarrow \underline{x \cdot N < a_H \cdot N < R}$

end of bonus

CamScanner ile tarandı

$B = \{ g(N) \leq R \}$

$P\{ g(N) > R\} = 1 - P\{ g(N) \leq R\} = P\{g(N) > R \mid \text{condition}\}.P\{\text{condition}\}$
$+ P\{ g(N) > R \mid (\text{condition})^c\}.P\{(\text{condition})^c\}$

$P\{A \mid \text{condition}\} = E[P\{A\} \mid \text{Condition}, V_N\}]$

$P\{A \mid \text{condition}\} \leq P\{B \mid \text{condition}\}$ $\underline{\text{eq.17}}$

Bonus. Justify (eq. 17) in view of (eq. 13 a, b).

$A \xrightarrow{\text{implies}} B$ then $\{A, \text{condition}\} \xrightarrow{\text{implies}} \{B, \text{Condition}\}$

$P\{A, \text{condition}\} \leq P\{B, \text{condition}\}$

Show $P\{A \mid \text{Condition}\} \leq P\{B \mid \text{condition}\}$ eq. 18

$$\frac{P\{A, \text{condition}\}}{P\{\text{Condition}\}} \leq \frac{P\{B, \text{Condition}\}}{P\{\text{Condition}\}}$$

since eq. 18 is true then $P\{A \mid \text{Condition}\} \leq P\{B \mid \text{condition}\}$
end of bonus.

Bonus: Justify (eq. 18)

$P\{B^c \mid \text{Condition}\} = 1 - P\{B \mid \text{Condition}\}$

we know $\{B\} + \{B^c\} = E$

$$\frac{P\{B^c, \text{Condition}\}}{P\{\text{condition}\}} + \frac{P\{B, \text{Condition}\}}{P\{\text{condition}\}} \stackrel{?}{=} 1$$

$S_1 + S_2 = P\{\text{Condition}\}$

$$\hookrightarrow \frac{P\{\text{condition}\}}{P\{\text{condition}\}} = 1$$