Урок 10 решение 0

$y = e^x$ ряд Тейлора в точке 0

$$1 + x + \frac{x^2}{2} + O(x^3)$$

$$e + e(x-1) + \frac{e(x-1)^2}{2} + O\left((x-1)^3; x \to x\right).$$

в точке 1

$y = e^x$ ряд Фурье $[-\pi, \pi]$

$$f(x) = \frac{a_0}{2} + \sum a_n \cos\left(\frac{\pi n x}{T}\right) + b_n \sin\left(\frac{\pi n x}{T}\right)$$

$$a_0 = \frac{1}{T}\int_{-T}^{T} f(x)dx, \quad a_n = \frac{1}{T}\int_{-T}^{T} f(x)\cos\left(\frac{\pi n x}{T}\right)dx$$

$$b_n = \frac{1}{T}\int_{-T}^{T} f(x)\sin\left(\frac{\pi n x}{T}\right)dx$$

$\to T = 2\pi$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x dx = \frac{1}{\pi}(e^x)\Big|_{-\pi}^{\pi} = \frac{1}{\pi}\left(e^{\pi} - (e^{-\pi})\right) = 2\frac{\sinh(\pi)}{\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \cos\left(\frac{\pi n x}{2\pi}\right)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \cos(nx)dx =$$

②

$$= \frac{1}{\pi}\left(ne^x\frac{\sin nx}{n^2+1} + e^x\frac{\cos nx}{n^2+1}\right)\Big|_{-\pi}^{\pi} = \frac{1}{\pi}(a_n)$$

$$\to \left(ne^{\pi}\frac{\sin \pi n}{n^2+1} + e^{\pi}\frac{\cos \pi n}{n^2+1} - \left(-n\frac{\sin \pi n}{(n^2+1)e^{\pi}} + \frac{\cos \pi n}{(n^2+1)e^{\pi}}\right)\right) =$$

$$= \frac{-(-1)^n + (-1)^n e^{2\pi}}{\pi(n^2+1)e^{\pi}}$$

при $n = 2k$ $a_n = 0$

при $n = 2k-1$; $a_n = \frac{(-1)^{2k-1} e^{2\pi} + 1}{\pi((2k-1)^2+1)e^{\pi}}$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \sin\left(\frac{\pi n x}{2\pi}\right)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \sin((2k-1)x)dx =$$

$$= \frac{1}{\pi}\left(-ne^x\frac{\cos(nx)}{n^2+1} + e^x\frac{\sin(nx)}{n^2+1}\right)\Big|_{-\pi}^{\pi} = \frac{1}{\pi}$$

$$\left(-ne^{\pi}\frac{\cos \pi n}{n^2+1} + e^{\pi}\frac{\sin \pi n}{n^2+1} - \left(-n\frac{\cos \pi n}{(n^2+1)e^{\pi}} - \frac{\sin \pi n}{(n^2+1)e^{\pi}}\right)\right) =$$

$$= \frac{-(-1)^n n e^{2\pi} + (-1)^n n}{\pi(n^2+1)e^{\pi}}$$

$$y(x) = \frac{\sinh(\pi)}{\pi} + \sum\left(\frac{(-1)^{2k-1} e^{2\pi} + 1}{\pi((2k-1)^2+1)e^{\pi}}\cos((2k-1)x) + \frac{-(-1)^n n e^{2\pi} + (-1)^n n}{\pi(n^2+1)e^{\pi}}\sin((2k-1)x)\right)$$