Урок 7, задание 6

$U = x^2y + \frac{1}{3}y^3 + 2x^2 + 3y^2 - 1$

$\begin{cases} U'_x = 2xy + 4x = 0 \\ U'_y = x^2 + y^2 + 6y = 0 \end{cases}$ $\begin{cases} U''_{xx} = 2y + 4 \\ U''_{yy} = 2y + 6 \end{cases}$ $\begin{cases} U''_{xy} = 2 \\ U''_{yx} = 0 \end{cases}$

① $x_1 = -\sqrt{-y(y+6)} \rightarrow -2y\sqrt{-y(y+6)} - 4\sqrt{-y(y+6)} = 0$

$x_2 = \sqrt{-y(y+6)} \rightarrow 2y\sqrt{-y(y+6)} + 4\sqrt{-y(y+6)} = 0$

$\Rightarrow y_1 = -6;\ y_2 = -2;\ y_3 = 0;\ y_4 = -6;\ y_5 = -2;\ y_6 = 0$

$x_1 = 0;\ x_2 = -2\sqrt{2};\ x_3 = 0;\ x_4 = 0;\ x_5 = 2\sqrt{2};\ x_6 = 0$

② $y = -2;\ x^2 - 8 = 0 \Rightarrow x_1 = -2\sqrt{2};\ x_2 = 2\sqrt{2}$

$y_1 = -2;\ y_2 = -2$

Критические точки: $A_1(0; -6)$ $A_3(0; 0)$

$A_2(-2\sqrt{2}; -2)$ $A_4(2\sqrt{2}; -2)$

максимум в точке $A_1(0; -6)$

минимум в точке $A_3(0; 0)$