Урок 5, задание 6

$y = x \ln(x+\sqrt{x^2+1}) - \sqrt{x^2+1}$

$\left(\frac{d}{dx}\left(\sqrt{1+x^2}\right)\right) + \frac{d}{dx}\left(x \ln\left(x+\sqrt{x^2+1}\right)\right) \rightarrow$

$\rightarrow \frac{d}{dx}\left(\sqrt{x^2+1}\right) = \frac{d\sqrt{a}}{da}\,\frac{da}{dx}, \; a = x^2+1 \rightarrow \frac{d}{da}(\sqrt{a}) = \frac{1}{2\sqrt{a}} \rightarrow$

$\rightarrow \frac{d}{dx}\left(x \ln\left(x+\sqrt{x^2+1}\right)\right) - \left(\frac{\frac{d}{dx}(x^2+1)}{2\sqrt{x^2+1}}\right) \rightarrow$

$\rightarrow \frac{d}{dx}\left(x \ln\left(x+\sqrt{x^2+1}\right)\right) - \left(\frac{d}{dx}(1) + \frac{d}{dx}(x^2)\right)\frac{1}{2\sqrt{x^2+1}} \rightarrow$

$\rightarrow \frac{d}{dx}\left(x \ln\left(x+\sqrt{x^2+1}\right)\right) - \frac{\frac{d}{dx}(x^2)+0}{2\sqrt{x^2+1}} \rightarrow$

$\rightarrow \frac{d}{dx}(x^n) = n x^{n-1}, \; n=2 \rightarrow \frac{d}{dx}(x^2) = 2x \rightarrow$

$\rightarrow \frac{d}{dx}\left(x \ln\left(x+\sqrt{x^2+1}\right)\right) - 2x\,\frac{1}{2\sqrt{x^2+1}} \rightarrow$

$\rightarrow -\frac{x}{\sqrt{x^2+1}} + \frac{d}{dx}\left(x \ln\left(x+\sqrt{x^2+1}\right)\right) \rightarrow$

① ②

$\rightarrow \frac{d}{dx}(ab) = b\frac{da}{dx} + a\frac{db}{dx}, \; a = x; \; b = \ln\left(x+\sqrt{x^2+1}\right) \rightarrow$

$\rightarrow -\frac{x}{\sqrt{x^2+1}} + \left(\ln\left|x+\sqrt{x^2+1}\right|\right)\left(\frac{d}{dx}(x)\right) + x\left(\frac{d}{dx}\left(\ln\left(x+\sqrt{x^2+1}\right)\right)\right) \rightarrow$

$\rightarrow -\frac{x}{\sqrt{x^2+1}} + x\left(\frac{d}{dx}\left(\ln\left(x+\sqrt{x^2+1}\right)\right)\right) + 1\ln\left(x+\sqrt{x^2+1}\right) \rightarrow$

$\rightarrow \frac{d}{dx}\left(\ln\left|x+\sqrt{x^2+1}\right|\right) = \frac{d\ln a}{da}\,\frac{da}{dx}, \; a = x+\sqrt{x^2+1}$ и $\frac{d}{da}\ln a = \frac{1}{a}$

$\rightarrow -\frac{x}{\sqrt{x^2+1}} + \ln\left(x+\sqrt{x^2+1}\right) + \left(\frac{\frac{d}{dx}\left(x+\sqrt{x^2+1}\right)}{x+\sqrt{x^2+1}}\right) \times \rightarrow$

$\rightarrow -\frac{x}{\sqrt{x^2+1}} + \ln\left(x+\sqrt{x^2+1}\right) + \left(\frac{d}{dx}(x) + \frac{d}{dx}\left(\sqrt{x^2+1}\right)\right)\frac{x}{x+\sqrt{x^2+1}}$

$\rightarrow -\frac{x}{\sqrt{x^2+1}} + \ln\left(x+\sqrt{x^2+1}\right) + \frac{x\left(\frac{d}{dx}\left(\sqrt{x^2+1}\right)+1\right)}{x+\sqrt{x^2+1}} \rightarrow$

$\rightarrow \frac{d}{dx}\left(\sqrt{x^2+1}\right) = \frac{d\sqrt{a}}{da}\,\frac{da}{dx}, \; a = x^2+1, \; \frac{d}{da}\sqrt{a} = \frac{1}{2\sqrt{a}}$

$\rightarrow -\frac{x}{\sqrt{x^2+1}} + \ln\left(x+\sqrt{x^2+1}\right) + \frac{x\left(1+\left(\frac{\frac{d}{dx}(x^2+1)}{2\sqrt{x^2+1}}\right)\right)}{x+\sqrt{x^2+1}} \rightarrow$

$\rightarrow -\frac{x}{\sqrt{x^2+1}} + \ln\left(x+\sqrt{x^2+1}\right) + \frac{x\left(1+\left(\frac{d}{dx}(1)+\frac{d}{dx}(x^2)\right)\frac{1}{2\sqrt{x^2+1}}\right)}{x+\sqrt{x^2+1}}$

$\rightarrow -\frac{x}{\sqrt{x^2+1}} + \ln\left(x+\sqrt{x^2+1}\right) + \frac{x\left(1+\frac{\frac{d}{dx}(x^2)+0}{2\sqrt{x^2+1}}\right)}{x+\sqrt{x^2+1}} \rightarrow$

$\rightarrow \frac{d}{dx}(x^2) = 2x \rightarrow -\frac{x}{\sqrt{x^2+1}} + \ln\left(x+\sqrt{x^2+1}\right) + \frac{x\left(1+\frac{2x}{2\sqrt{x^2+1}}\right)}{x+\sqrt{x^2+1}}$

$\rightarrow -\frac{x}{\sqrt{x^2+1}} + \ln\left(x+\sqrt{x^2+1}\right) + \frac{x\left(1+\frac{x}{\sqrt{x^2+1}}\right)}{x+\sqrt{x^2+1}} \rightarrow$

$\rightarrow \ln\left(x+\sqrt{x^2+1}\right)$ ответ

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