

Зад 6, правило 1)

$$\arctan\left(\frac{y}{x}\right) = \ln \sqrt{x^2 + y^2}$$

$$\frac{d}{dx} \left( \arctan\left(\frac{y}{x}\right) \right) = \frac{d \arctan^{-1}(u)}{du} \cdot \frac{du}{dx}, \quad u = \frac{y}{x}$$

$$\frac{d}{du} \left( \arctan^{-1}(u) \right) = \frac{1}{1+u^2}$$

$$\frac{1}{1+\frac{y^2}{x^2}} \cdot \left( \frac{y'x - y}{x^2} \right) = \frac{1}{2\sqrt{x^2+y^2}} (2x + 2yy')$$

$$\frac{1}{x^2+y^2} (y'x - y) = \frac{x + yy'}{\sqrt{x^2+y^2}}$$

$$y' \cdot \frac{x}{x^2+y^2} - \frac{y}{x^2+y^2} = \frac{x}{\sqrt{x^2+y^2}} + \frac{y}{\sqrt{x^2+y^2}} \cdot y'$$

$$y' = \frac{\frac{x}{\sqrt{x^2+y^2}} + \frac{y}{x^2+y^2}}{\frac{x}{x^2+y^2} - \frac{y}{\sqrt{x^2+y^2}}} = \frac{x\sqrt{x^2+y^2} + y}{x - y\sqrt{x^2+y^2}}$$