### 1 Computational Geometr

58

59

# 1.1 Geometry.cpp

```
1 const double PI=atan2(0.0,-1.0);
                                                 61
   template<typename T>
   struct point{
                                                 62
    T x, y;
                                                 63
    point(){}
    point(const T&x,const T&y):x(x),y(y){}
    point operator+(const point &b)const{
                                                 64
      return point(x+b.x,y+b.y); }
                                                 65
    point operator-(const point &b)const{
                                                 66
      return point(x-b.x,y-b.y); }
                                                 67
    point operator*(const T &b)const{
11
       return point(x*b,v*b); }
12
                                                 69
13
    point operator/(const T &b)const{
                                                 70
      return point(x/b,v/b): }
14
                                                 71
    bool operator==(const point &b)const{
15
                                                 72
      return x==b.x&&y==b.y; }
16
                                                 73
17
    T dot(const point &b)const{
                                                 74
18
      return x*b.x+y*b.y; }
                                                 75
19
    T cross(const point &b)const{
                                                 76
20
      return x*b.y-y*b.x; }
                                                 77
    point normal()const{//求法向量
                                                 78
       return point(-y,x); }
                                                 79
23
    T abs2()const{//向量長度的平方
                                                 80
      return dot(*this); }
24
                                                 81
                                                 82
    T rad(const point &b)const{//兩向量的弧度
   return fabs(atan2(fabs(cross(b)),dot(b))); }
                                                 83
    T getA()const{//對x軸的弧度
      T A=atan2(y,x);//超過180度會變負的
29
      if(A<=-PI/2)A+=PI*2;
30
      return A;
                                                 85
   template<typename T>
                                                 86
   struct line{
    line(){}
    point<T> p1,p2;
    T a,b,c;//ax+by+c=0
    line(const point<T>&x,const point<T>&y):p1
         (x),p2(y){}
    void pton(){//轉成一般式
39
40
      a=p1.v-p2.v:
41
      b=p2.x-p1.x;
42
      c = -a*p1.x-b*p1.y;
43
    T cross(const point<T> &p)const{//點和有向
          直線的關係, >0左邊、=0在線上<0右邊
       return (p2-p1).cross(p-p1);
45
46
    bool point_on_segment(const point<T>&p)
47
         const{//點是否線段上
48
      return cross(p) == 0&&(p1-p).dot(p2-p) <= 0;
49
                                                100
    T dis2(const point<T> &p,bool is segment
50
                                                101
         =0) const { // 點 跟 直 線 / 線 段 的 距 離 平 方
       point<T> v=p2-p1,v1=p-p1;
                                                102
52
       if(is segment){
                                                103
         point<T> v2=p-p2;
53
                                                104
         if(v.dot(v1)<=0)return v1.abs2();</pre>
```

```
if(v.dot(v2)>=0)return v2.abs2();
  T tmp=v.cross(v1);
  return tmp*tmp/v.abs2();
                                          107
                                          108
T seg dis2(const line<T> &1)const{//兩線段 109
  return min({dis2(1.p1,1),dis2(1.p2,1),1. 111
      dis2(p1,1),1.dis2(p2,1)});
point<T> projection(const point<T> &p)
                                          113
    const{//點對直線的投影
                                          114
  point<T> n=(p2-p1).normal();
  return p-n*(p-p1).dot(n)/n.abs2();
point<T> mirror(const point<T> &p)const{
  //點對直線的鏡射·要先呼叫pton轉成一般式 116
  point<T> R;
 T d=a*a+b*b:
                                          118
 R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
 R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
bool equal(const line &1)const{//直線相等
  return cross(1.p1)==0&&cross(1.p2)==0;
                                          124
                                          125
bool parallel(const line &1)const{
                                          126
 return (p1-p2).cross(l.p1-l.p2)==0;
                                          127
bool cross_seg(const line &1)const{
                                          128
 return (p2-p1).cross(l.p1-p1)*(p2-p1).
                                          129
      cross(1.p2-p1)<=0;//直線是否交線段
char line_intersect(const line &1)const{// ^{131}
     直線相交情況,-1無限多點、1交於一點、0132
  return parallel(1)?(cross(1.p1)==0?-1:0)
       :1:
                                          135
                                          136
char seg_intersect(const line &l)const{//
     線段相交情況,-1無限多點、1交於一點、
 T c1=(p2-p1).cross(l.p1-p1);
 T c2=(p2-p1).cross(1.p2-p1);
 T c3=(1.p2-1.p1).cross(p1-1.p1);
  T c4=(1.p2-1.p1).cross(p2-1.p1);
                                          141
  if(c1==0&&c2==0){
                                          142
    if(p1==1.p1&&(p2-p1).dot(1.p2)<=0)
        return 1;
                                          143
    if(p1==1.p2&&(p2-p1).dot(1.p1) <=0)
        return 1;
                                          144
    if(p2==1.p1&&(p1-p2).dot(1.p2)<=0)
        return 1:
    if(p2==1.p2&&(p1-p2).dot(1.p1)<=0)
                                          146
        return 1:
                                          147
    return -1:
                                          148
  }else if(c1*c2<=0&&c3*c4<=0)return 1;</pre>
                                          149
 return 0:
                                          150
point<T> line_intersection(const line &l)
    const{/*直線交點*/
  point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
                                          152
  //if(a.cross(b)==0)return INF;
  return p1+a*(s.cross(b)/a.cross(b));
```

```
point<T> seg intersection(const line &1)
          const{//線段交點
                                                  156
       T c1=(p2-p1).cross(l.p1-p1);
       T c2=(p2-p1).cross(1.p2-p1);
                                                  157
       T c3=(1.p2-1.p1).cross(p1-1.p1);
                                                  158
                                                  159
       T c4=(1.p2-1.p1).cross(p2-1.p1);
       if(c1==0&&c2==0){
                                                  160
                                                  161
          if(p1==1.p1&&(p2-p1).dot(1.p2)<=0)
                                                  162
              return p1;
         if(p1==1.p2&&(p2-p1).dot(1.p1)<=0)
                                                  163
              return p1;
                                                  164
          if(p2==1.p1&&(p1-p2).dot(1.p2)<=0)</pre>
                                                  165
              return p2:
                                                  166
          if(p2==1.p2&&(p1-p2).dot(1.p1) <=0)
              return p2;
                                                  167
       }else if(c1*c2<=0&&c3*c4<=0)return</pre>
                                                  168
            line intersection(1);
                                                  169
       //return INF;
                                                  170
120 template<typename T>
                                                  171
   struct polygon{
     polygon(){}
                                                  172
     vector<point<T> > p;//逆時針順序
                                                  173
     T area()const{//面積
       T ans=0:
                                                  174
       for(int i=p.size()-1,j=0;j<(int)p.size()</pre>
            ;i=j++)
                                                  175
          ans+=p[i].cross(p[j]);
                                                  176
       return ans/2;
                                                  177
                                                  178
     point<T> center_of_mass()const{//重心
                                                  179
       T cx=0, cy=0, w=0;
       for(int i=p.size()-1,j=0;j<(int)p.size()</pre>
            ;i=j++){
         T a=p[i].cross(p[j]);
          cx+=(p[i].x+p[j].x)*a;
                                                  182
          cy+=(p[i].y+p[j].y)*a;
                                                  183
                                                  184
       return point<T>(cx/3/w,cy/3/w);
                                                  185
139
     char ahas(const point<T>& t)const{//點是否
140
          在簡單多邊形內,是的話回傳1、在邊上回 187
                                                  188
           傳-1、否則回傳@
       bool c=0:
                                                  189
       for(int i=0,j=p.size()-1;i<p.size();j=i 190</pre>
          if(line<T>(p[i],p[j]).point_on_segment 192
              (t))return -1;
                                                 194
          else if((p[i].y>t.y)!=(p[j].y>t.y)&&
145
          t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j]
              ].y-p[i].y)+p[i].x)
                                                  195
                                                  196
            c=!c;
                                                  197
       return c:
                                                  198
     char point in convex(const point<T>&x)
                                                  199
          const{
       int l=1,r=(int)p.size()-2;
                                                  200
       while(1 < = r){//點是否在凸多邊形內,是的話 201
                                                  202
             回傳1、在邊上回傳-1、否則回傳0
                                                  203
          int mid=(1+r)/2;
153
         T a1=(p[mid]-p[0]).cross(x-p[0]);
```

```
T a2=(p[mid+1]-p[0]).cross(x-p[0]);
    if(a1>=0&&a2<=0){
      T res=(p[mid+1]-p[mid]).cross(x-p[
           mid]);
      return res>0?1:(res>=0?-1:0);
    }else if(a1<0)r=mid-1;</pre>
    else l=mid+1:
  return 0;
vector<T> getA()const{//凸包邊對x軸的夾角
  vector<T>res://一定是遞增的
  for(size t i=0;i<p.size();++i)</pre>
    res.push_back((p[(i+1)%p.size()]-p[i])
         .getA());
  return res;
bool line intersect(const vector<T>&A,
     const line<T> &l)const{//O(logN)
  int f1=upper_bound(A.begin(),A.end(),(1.
       p1-1.p2).getA())-A.begin();
  int f2=upper_bound(A.begin(),A.end(),(1.
       p2-1.p1).getA())-A.begin();
  return 1.cross_seg(line<T>(p[f1],p[f2]))
polygon cut(const line<T> &l)const{//△包
     對直線切割,得到直線 L左側的凸包
  polygon ans;
  for(int n=p.size(),i=n-1,j=0;j<n;i=j++){</pre>
    if(1.cross(p[i])>=0){
      ans.p.push back(p[i]);
      if(1.cross(p[j])<0)
        ans.p.push back(1.
             line intersection(line<T>(p[i
             ],p[j])));
    }else if(1.cross(p[j])>0)
      ans.p.push back(1.line intersection(
           line<T>(p[i],p[i]));
  return ans;
static bool graham cmp(const point<T>& a.
     const point<T>& b){//凸包排序函數
  return (a.x<b.x)||(a.x==b.x&&a.y<b.y);</pre>
void graham(vector<point<T> > &s){// □ 包
  sort(s.begin(),s.end(),graham_cmp);
  p.resize(s.size()+1);
  int m=0;
  for(size t i=0;i<s.size();++i){</pre>
    while (m>=2&&(p[m-1]-p[m-2]).cross(s[i
         ]-p[m-2])<=0)--m;
    p[m++]=s[i];
  for(int i=s.size()-2,t=m+1;i>=0;--i){
    while (m>=t&&(p[m-1]-p[m-2]).cross(s[i
         ]-p[m-2])<=0)--m;
    p[m++]=s[i];
  if(s.size()>1)--m;
  p.resize(m);
T diam(){//直徑
```

```
int n=p.size(),t=1;
                                                    258
206
        T ans=0; p.push back(p[0]);
        for(int i=0;i<n;i++){</pre>
207
208
          point<T> now=p[i+1]-p[i];
                                                    260
          while(now.cross(p[t+1]-p[i])>now.cross 261
209
               (p[t]-p[i]))t=(t+1)%n;
210
          ans=max(ans,max((p[i]-p[t]).abs2(),(p[262]
               i+1]-p[t+1]).abs2()));
211
212
        return p.pop_back(),ans;
                                                    264
213
214
      T min_cover_rectangle(){//最小覆蓋矩形
                                                    265
        int n=p.size(),t=1,r=1,l;
215
                                                    266
        if(n<3)return 0;//也可以做最小周長矩形
216
                                                    267
        T ans=1e99; p. push back(p[0]);
217
                                                    268
        for(int i=0;i<n;i++){</pre>
218
                                                    269
219
          point<T> now=p[i+1]-p[i];
                                                    270
          while(now.cross(p[t+1]-p[i])>now.cross 271
220
               (p[t]-p[i]))t=(t+1)%n;
          while (now.dot(p[r+1]-p[i]) > now.dot(p[r_{273}))
221
               ]-p[i]))r=(r+1)%n;
                                                    274
          if(!i)l=r;
222
                                                    275
223
          while (now.dot(p[l+1]-p[i]) \le now.dot(p[276])
               l]-p[i]))l=(l+1)%n;
224
          T d=now.abs2();
          T tmp=now.cross(p[t]-p[i])*(now.dot(p[_{278}
225
               r]-p[i])-now.dot(p[l]-p[i]))/d;
          ans=min(ans,tmp);
226
                                                    280
227
                                                    281
228
        return p.pop_back(),ans;
                                                    282
229
                                                    283
      T max_triangle(){//最大內接三角形
230
                                                    284
231
        int n=p.size(),a=1,b=2;
                                                    285
232
        if(n<3)return 0;</pre>
                                                    286 };
233
        T ans=0,tmp;p.push back(p[0]);
        for(int i=0;i<n;++i){</pre>
234
          while((p[a]-p[i]).cross(p[b+1]-p[i])>( 289
235
               tmp=(p[a]-p[i]).cross(p[b]-p[i])))290
               b=(b+1)%n;
          ans=max(ans,tmp);
236
          while((p[a+1]-p[i]).cross(p[b]-p[i])>( 292
237
               tmp=(p[a]-p[i]).cross(p[b]-p[i])))293
               a=(a+1)%n;
                                                    294
          ans=max(ans,tmp);
                                                    295
238
239
                                                    296
240
        return p.pop_back(),ans/2;
                                                    297
241
                                                    298
     T dis2(polygon &pl){//凸包最近距離平方
242
                                                    299
        vector<point<T> > &P=p,&O=pl.p;
243
                                                    300
        int n=P.size(),m=0.size(),l=0,r=0;
244
                                                    301
      for(int i=0;i<n;++i)if(P[i].y<P[l].y)l=i;</pre>
245
246
      for(int i=0;i<m;++i)if(Q[i].y<Q[r].y)r=i;</pre>
247
        P.push_back(P[0]),Q.push_back(Q[0]);
                                                    303
248
        T ans=1e99:
                                                    304
        for(int i=0;i<n;++i){</pre>
249
          while((P[1]-P[1+1]).cross(Q[r+1]-Q[r]) _{305}
250
               <0)r=(r+1)%m;
                                                    306
          ans=min(ans,line<T>(P[1],P[1+1]).
251
                                                    307
               seg_dis2(line<T>(Q[r],Q[r+1])));
                                                    308
252
          l=(l+1)%n;
253
                                                    309
254
        return P.pop_back(),Q.pop_back(),ans;
255
                                                    310
     static char sign(const point<T>&t){
256
257
        return (t.y==0?t.x:t.y)<0;</pre>
```

```
312
 static bool angle cmp(const line<T>& A,
                                               313
       const line<T>& B){
                                               314 };
    point<T> a=A.p2-A.p1,b=B.p2-B.p1;
    return sign(a)<sign(b) | | (sign(a) == sign(b 316 |
         )&&a.cross(b)>0);
                                               317
 int halfplane_intersection(vector<line<T> 319
      > &s){//半平面交
    sort(s.begin(),s.end(),angle_cmp);//線段 320
         左側為該線段半平面
                                               322
    int L,R,n=s.size();
                                               323
    vector<point<T>> px(n);
                                               324
    vector<line<T> > q(n);
                                               325
    q[L=R=0]=s[0];
                                               326
    for(int i=1;i<n;++i){</pre>
                                               327
     while(L<R&&s[i].cross(px[R-1])<=0)--R;</pre>
     while(L<R&&s[i].cross(px[L])<=0)++L;</pre>
                                               329
      q[++R]=s[i];
                                               330
      if(q[R].parallel(q[R-1])){
                                               331
        if(q[R].cross(s[i].p1)>0)q[R]=s[i];
      if(L<R)px[R-1]=q[R-1].</pre>
           line intersection(q[R]);
                                               336
    while(L<R&&q[L].cross(px[R-1])<=0)--R;</pre>
    p.clear();
    if(R-L<=1)return 0;</pre>
                                               337
    px[R]=q[R].line_intersection(q[L]);
                                               338 };
    for(int i=L;i<=R;++i)p.push back(px[i]); 339</pre>
    return R-L+1;
                                               341
                                               342
template<typename T>
                                               343
struct triangle{
 point<T> a,b,c;
  triangle(){}
 triangle(const point<T> &a, const point<T> 345
      &b, const point <T> &c):a(a),b(b),c(c){}<sub>346</sub>
 T area()const{
                                               347
   T t=(b-a).cross(c-a)/2;
                                               348
    return t>0?t:-t;
                                               349
                                               350
  point<T> barycenter()const{//重心
                                               351
   return (a+b+c)/3;
                                               352
                                               353
                                               354
 point<T> circumcenter()const{//外心
   static line<T> u,v;
   u.p1=(a+b)/2;
                                               355
   u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-356
         b.x);
                                               357
    v.p1=(a+c)/2;
                                               358
   v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-
                                               359
   return u.line_intersection(v);
                                               360
  point<T> incenter()const{//內心
                                               361
   T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2 362
         ()),C=sqrt((a-b).abs2());
                                               363
    return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+ 364
         B*b.y+C*c.y)/(A+B+C);
                                               365
 point<T> perpencenter()const{//垂心
                                               366
```

```
return barycenter()*3-circumcenter()*2; 367|};
315 template<typename T>
   struct point3D{
                                                 371
    T x, y, z;
     point3D(){}
     point3D(const T&x,const T&y,const T&z):x(x 373
          ),y(y),z(z){}
     point3D operator+(const point3D &b)const{
       return point3D(x+b.x,y+b.y,z+b.z);}
     point3D operator-(const point3D &b)const{
                                                 376
       return point3D(x-b.x,y-b.y,z-b.z);}
     point3D operator*(const T &b)const{
       return point3D(x*b,y*b,z*b);}
                                                 378
     point3D operator/(const T &b)const{
                                                 379
       return point3D(x/b,y/b,z/b);}
                                                 380
     bool operator==(const point3D &b)const{
       return x==b.x&&y==b.y&&z==b.z;}
                                                 381
     T dot(const point3D &b)const{
       return x*b.x+y*b.y+z*b.z;}
                                                 382
     point3D cross(const point3D &b)const{
       return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x
            *b.y-y*b.x);}
                                                 383
                                                 384
     T abs2()const{//向量長度的平方
       return dot(*this);}
     T area2(const point3D &b)const{//和b、原點 385
          圍成面積的平方
       return cross(b).abs2()/4;}
                                                 387
   template<typename T>
                                                 388
   struct line3D{
                                                 389
     point3D<T> p1,p2;
                                                 390
     line3D(){}
     line3D(const point3D<T> &p1,const point3D< 391</pre>
          T> &p2):p1(p1),p2(p2){}
     T dis2(const point3D<T> &p,bool is_segment 393
          =0) const { // 點 跟 直 線 / 線 段 的 距 離 平 方
       point3D<T> v=p2-p1,v1=p-p1;
       if(is segment){
          point3D<T> v2=p-p2;
                                                 396
          if(v.dot(v1)<=0)return v1.abs2();</pre>
         if(v.dot(v2)>=0)return v2.abs2():
                                                 397
       point3D<T> tmp=v.cross(v1);
       return tmp.abs2()/v.abs2();
                                                 398
                                                 399
     pair<point3D<T>.point3D<T> > closest pair(
          const line3D<T> &1)const{
       point3D<T> v1=(p1-p2), v2=(1.p1-l.p2);
       point3D<T> N=v1.cross(v2),ab(p1-l.p1);
       //if(N.abs2()==0)return NULL;平行或重合
       T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//
             最近點對距離
       point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.
            cross(d2),G=1.p1-p1;
       T t1=(G.cross(d2)).dot(D)/D.abs2();
                                                 406
       T t2=(G.cross(d1)).dot(D)/D.abs2();
                                                 407
       return make pair(p1+d1*t1,1.p1+d2*t2);
                                                 408
                                                 409
     bool same_side(const point3D<T> &a,const
                                                 410
          point3D<T> &b)const{
                                                 411
       return (p2-p1).cross(a-p1).dot((p2-p1).
            cross(b-p1))>0;
```

```
template<typename T>
struct plane{
  point3D<T> p0,n;//平面上的點和法向量
  plane(){}
  plane(const point3D<T> &p0,const point3D<T</pre>
      > &n):p0(p0),n(n){}
 T dis2(const point3D<T> &p)const{//點到平
       面距離的平方
    T tmp=(p-p0).dot(n);
    return tmp*tmp/n.abs2();
  point3D<T> projection(const point3D<T> &p)
    return p-n*(p-p0).dot(n)/n.abs2();
  point3D<T> line intersection(const line3D<
      T> &1)const{
    T tmp=n.dot(1.p2-1.p1);//等於0表示平行或
         重合該平面
    return 1.p1+(1.p2-1.p1)*(n.dot(p0-1.p1)/
        tmp);
  line3D<T> plane_intersection(const plane &
      pl)const{
    point3D<T> e=n.cross(pl.n),v=n.cross(e);
    T tmp=pl.n.dot(v);//等於 Ø表示平行或重合
    point3D < T > q = p0 + (v*(pl.n.dot(pl.p0-p0))/
        tmp);
    return line3D<T>(q,q+e);
};
template<typename T>
struct triangle3D{
  point3D<T> a,b,c;
  triangle3D(){}
  triangle3D(const point3D<T> &a,const
      point3D<T> &b, const point3D<T> &c):a(a
      ),b(b),c(c){}
  bool point_in(const point3D<T> &p)const{//
       點在該平面上的投影在三角形中
    return line3D<T>(b,c).same_side(p,a)&&
        line3D<T>(a,c).same side(p,b)&&
        line3D<T>(a,b).same_side(p,c);
template<typename T>
struct tetrahedron{//四面體
  point3D<T> a,b,c,d;
  tetrahedron(){}
  tetrahedron(const point3D<T> &a,const
      point3D<T> &b, const point3D<T> &c,
      const point3D<T> &d):a(a),b(b),c(c),d(
      d){}
 T volume6()const{//體積的六倍
    return (d-a).dot((b-a).cross(c-a));
  point3D<T> centroid()const{
    return (a+b+c+d)/4;
  bool point in(const point3D<T> &p)const{
    return triangle3D<T>(a,b,c).point in(p)
        &&triangle3D<T>(c,d,a).point_in(p);
```

```
typedef const point < double > cp;
414
   };
                                                           p x;
    template<typename T>
415
                                                           double r2;
    struct convexhull3D{
                                                           bool incircle(cp &c)const{return (x-c).
     static const int MAXN=1005;
                                                                abs2()<=r2;}
417
418
     struct face{
                                                     8 };
419
       int a.b.c:
       face(int a,int b,int c):a(a),b(b),c(c){} 10 | Circle TwoPointCircle(Circle::cp &a, Circle
420
421
     };
                                                           Circle::p m=(a+b)/2;
422
     vector<point3D<T>> pt;
                                                           return (Circle){m,(a-m).abs2()};
423
     vector<face> ans;
                                                    12
     int fid[MAXN][MAXN];
                                                    13 }
424
     void build(){
425
                                                    14
426
       int n=pt.size():
                                                    15 Circle outcircle(Circle::p a, Circle::p b,
427
        ans.clear();
                                                            Circle::p c) {
       memset(fid,0,sizeof(fid));
                                                           if(TwoPointCircle(a,b).incircle(c))
428
                                                                return TwoPointCircle(a,b);
        ans.emplace_back(0,1,2);//注意不能共線
429
                                                           if(TwoPointCircle(b,c).incircle(a))
                                                    17
        ans.emplace back(2,1,0);
                                                                return TwoPointCircle(b,c);
431
        int ftop = 0:
                                                           if(TwoPointCircle(c,a).incircle(b))
432
        for(int i=3, ftop=1; i<n; ++i,++ftop){</pre>
                                                                return TwoPointCircle(c,a);
433
         vector<face> next;
                                                           Circle::p ret:
          for(auto &f:ans){
434
                                                           double a1=b.x-a.x, b1=b.y-a.y, c1=(a1*a1
            T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[
                                                   20
435
                                                                +b1*b1)/2;
                 f.a]).cross(pt[f.c]-pt[f.a]));
                                                           double a2=c.x-a.x, b2=c.y-a.y, c2=(a2*a2
436
            if(d<=0) next.push back(f);</pre>
                                                                +b2*b2)/2;
            int ff=0;
437
                                                           double d = a1*b2 - a2*b1;
            if(d>0) ff=ftop;
                                                    22
                                                           ret.x=a.x+(c1*b2-c2*b1)/d;
                                                    23
439
            else if(d<0) ff=-ftop;</pre>
                                                           ret.y=a.y+(a1*c2-a2*c1)/d;
            fid[f.a][f.b]=fid[f.c]=fid[f.c
                                                    ^{24}
440
                                                    25
                                                           return (Circle){ret,(ret-a).abs2()};
                 ][f.a]=ff;
                                                    26 }
441
                                                    27 //rand required
442
          for(auto &f:ans){
                                                    28 Circle SmallestCircle(std::vector<Circle::p>
            if(fid[f.a][f.b]>0 && fid[f.a][f.b
443
                                                             &p){
                 ]!=fid[f.b][f.a])
                                                           int n=p.size();
444
              next.emplace back(f.a.f.b.i):
                                                    29
                                                           if(n==1) return (Circle){p[0],0.0};
445
            if(fid[f.b][f.c]>0 && fid[f.b][f.c
                                                    30
                                                           if(n==2) return TwoPointCircle(p[0],p
                 ]!=fid[f.c][f.b])
                                                    31
              next.emplace back(f.b,f.c,i);
446
                                                           random shuffle(p.begin(),p.end());
            if(fid[f.c][f.a]>0 && fid[f.c][f.a
                                                    32
447
                                                           Circle c = \{p[0], 0.0\};
                 ]!=fid[f.a][f.c])
                                                    33
                                                           for(int i=0;i<n;++i){</pre>
                                                    34
              next.emplace back(f.c,f.a,i);
448
                                                                if(c.incircle(p[i])) continue;
                                                    35
449
                                                    36
                                                                c=Circle{p[i],0.0};
450
         ans=next;
                                                               for(int j=0;j<i;++j){</pre>
                                                    37
451
                                                    38
                                                                    if(c.incircle(p[j])) continue;
452
                                                    39
                                                                    c=TwoPointCircle(p[i],p[j]);
453
     point3D<T> centroid()const{
                                                                    for(int k=0;k<j;++k){</pre>
       point3D<T> res(0,0,0);
                                                    40
454
                                                                        if(c.incircle(p[k]))
                                                    41
       T vol=0;
455
                                                                             continue;
        for(auto &f:ans){
456
                                                                        c=outcircle(p[i],p[j],p[k]);
         T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c 42
457
         res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
458
         vol+=tmp;
459
                                                    46
                                                           return c;
460
461
       return res/(vol*4);
462
463 };
```

#### 1.3 最近點對.cpp

# 1.2 SmallestCircle.cpp

```
1 #include "Geometry.cpp"
2 struct Circle{
      typedef point < double > p;
```

```
1 template < typename IT = point < T > * >
2 T cloest pair( IT L, IT R){
    if(R-L <= 1) return INF;</pre>
     IT mid = L+(R-L)/2;
    T x = mid -> x;
```

```
T d = min(cloest pair(L,mid),cloest pair(
         mid,R));
                                                   30
     inplace_merge(L, mid, R, ycmp);
     static vector<point> b; b.clear();
                                                   31
     for(auto u=L;u<R;++u){</pre>
       if((u->x-x)*(u->x-x)>=d) continue;
       for(auto v=b.rbegin();v!=b.rend();++v){
                                                   33
         T dx=u->x-v->x, dy=u->y-v->y;
                                                   34
         if(dy*dy>=d) break;
         d=min(d,dx*dx+dy*dy);
                                                   35
                                                   36
       b.push back(*u);
                                                   37
     return d:
20
  T closest pair(vector<point<T>> &v){
                                                   39
    sort(v.begin(),v.end(),xcmp);
                                                   40
    return closest_pair(v.begin(), v.end());
                                                   41
                                                   42
```

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## Data Structure

### 2.1 DLX.cpp

10

11

13

14

15

16

17

18

19

21

22

```
1 const int MAXN=4100, MAXM=1030, MAXND=16390;
2 struct DLX{
                                               54
    int n,m,sz,ansd;//高是n · 寬是m的稀疏矩陣
    int S[MAXM],H[MAXN];
    int row[MAXND], col[MAXND]; //每個節點代表的
    int L[MAXND],R[MAXND],U[MAXND],D[MAXND];
    vector<int> ans,anst;
    void init(int n,int m){
                                               61
      n = n, m = m;
                                               62
      for(int i=0;i<=m;++i){</pre>
                                               63
11
        U[i]=D[i]=i,L[i]=i-1,R[i]=i+1;
                                               64
12
        S[i]=0;
                                               65
13
                                               66
14
      R[m]=0,L[0]=m;
      sz=m, ansd=INT MAX; //ansd存最優解的個數
15
16
      for(int i=1;i<=n;++i)H[i]=-1;</pre>
                                               69
17
                                               70
    void add(int r,int c){
                                               71
      ++S[col[++sz]=c];
      row[sz]=r;
20
      D[sz]=D[c],U[D[c]]=sz,U[sz]=c,D[c]=sz;
21
      if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
      else R[sz]=R[H[r]],L[R[H[r]]]=sz,L[sz]=H
23
           [r],R[H[r]]=sz;
24
    #define DFOR(i,A,s) for(int i=A[s];i!=s;i=
    void remove(int c){//刪除第c行和所有當前覆
26
         蓋到第c行的列
27
      L[R[c]]=L[c],R[L[c]]=R[c];//這裡刪除第c
                                               83
           行,若有些行不需要處理可以在開始時呼 84
      DFOR(i,D,c)DFOR(j,R,i){U[D[j]]=U[j],D[U[
           i]]=D[i],--S[col[i]];}
```

```
void restore(int c){//恢復第c行和所有當前
     覆 蓋 到 第 c 行 的 列 · remove 的 逆 操 作
  DFOR(i,U,c)DFOR(j,L,i){++S[col[j]],U[D[j
      1]=j,D[U[j]]=j;}
  L[R[c]]=c,R[L[c]]=c;
void remove2(int nd){//刪除nd所在的行當前
    所有點(包括虛擬節點),只保留nd
 DFOR(i,D,nd)L[R[i]]=L[i],R[L[i]]=R[i];
void restore2(int nd){//刪除nd所在的行當前
    所 有 點 · 為 remove 2 的 逆 操 作
 DFOR(i,U,nd)L[R[i]]=R[L[i]]=i;
bool vis[MAXM];
int h(){//估價函數 for IDA*
 int res=0;
  memset(vis,0,sizeof(vis));
 DFOR(i,R,0)if(!vis[i]){
   vis[i]=1;
   ++res;
   DFOR(j,D,i)DFOR(k,R,j)vis[col[k]]=1;
 return res;
bool dfs(int d){//for精確覆蓋問題
 if(d+h()>=ansd)return 0;//找最佳解用,找
      任意解可以刪掉
  if(!R[0]){ansd=d;return 1;}
 int c=R[0];
 DFOR(i,R,0)if(S[i]<S[c])c=i;</pre>
 remove(c);
 DFOR(i,D,c){
   ans.push_back(row[i]);
   DFOR(j,R,i)remove(col[j]);
   if(dfs(d+1))return 1;
   ans.pop back();
   DFOR(j,L,i)restore(col[j]);
  restore(c);
 return 0;
void dfs2(int d){//for最小重複覆蓋問題
 if(d+h()>=ansd)return;
 if(!R[0]){ansd=d;ans=anst;return;}
 int c=R[0];
 DFOR(i,R,0)if(S[i]<S[c])c=i;</pre>
 DFOR(i,D,c){
   anst.push back(row[i]);
   remove2(i);
   DFOR(j,R,i)remove2(j),--S[col[j]];
   dfs2(d+1);
   anst.pop_back();
   DFOR(j,L,i)restore2(j),++S[col[j]];
   restore2(i);
bool exact cover(){//解精確覆蓋問題
 return ans.clear(), dfs(0);
void min_cover(){//解最小重複覆蓋問題
 anst.clear();//暫存用,答案還是存在ans裡
```

```
dfs2(0);
                                                         std::nth element(A.begin()+1,A.begin()+ 109
                                                                                                            if(erase(cmp(x,u->pid)?u->l:u->r,(k+1)% 166
                                                                                                                                                              bool d=erase(root,0,p);
                                                                                                                                                              if(root&&root->s<alpha*maxn)rebuild();</pre>
                                                              mid,A.begin()+r+1,cmp);
                                                                                                                 kd,x)
                                                                                                                                                       167
    #undef DFOR
                                                          node *ret=A[mid];
                                                  52
                                                                                                              return --u->s, 1;
                                                                                                                                                              return d;
                                                                                                    110
                                                                                                                                                       168
                                                         ret->l = build(k+1,1,mid-1);
                                                                                                    111
                                                                                                            return 0;
                                                                                                                                                       169
                                                  54
                                                         ret->r = build(k+1,mid+1,r);
                                                                                                    112
                                                                                                                                                       170
                                                                                                                                                            void rebuild(){
                                                                                                                                                              if(root)rebuild(root,0);
                                                         ret->up();
                                                                                                          T heuristic(const T h[])const{
                                                                                                    113
                                                                                                                                                       171
                                                  56
                                                         return ret:
                                                                                                    114
                                                                                                                                                       172
                                                                                                                                                              maxn=root->s:
                                                  57
                                                                                                            for(size t i=0;i<kd;++i)ret+=h[i];</pre>
                                                                                                    115
                                                                                                                                                       173
  2.2 Dynamic KD_tree.cpp
                                                        bool isbad(node*o){
                                                                                                                                                            T nearest(const point &x,int k){
                                                                                                    116
                                                                                                            return ret;
                                                                                                                                                       174
                                                         return size(o->1)>alpha*o->s||size(o->r) 117
                                                                                                                                                       175
                                                                                                                                                              gM=k;
                                                              >alpha*o->s;
                                                                                                    118
                                                                                                                                                       176
                                                                                                                                                              T mndist=INF,h[kd]={};
1 template<typename T, size_t kd>//有kd個維度
                                                                                                          std::priority queue<std::pair<T.point > >
                                                                                                                                                              nearest(root.0.x.h.mndist):
2 struct kd tree{
                                                       void flatten(node *u,typename std::vector<</pre>
                                                                                                                                                              mndist=p0.top().first;
                                                  61
    struct point{
                                                            node*>::iterator &it){
                                                                                                          void nearest(node *u,int k,const point &x, 179
                                                                                                                                                              pO=std::priority queue<std::pair<T.point
      T d[kd]:
                                                  62
                                                         if(!u)return:
                                                                                                               T *h.T &mndist){
      T dist(const point &x)const{
                                                  63
                                                         flatten(u->1,it);
                                                                                                    121
                                                                                                            if(u==0||heuristic(h)>=mndist)return;
                                                                                                                                                              return mndist;//回傳離x第k近的點的距離
                                                                                                                                                       180
                                                         *it=u:
                                                                                                            T dist=u->pid.dist(x),old=h[k];
                                                                                                    122
                                                                                                                                                       181
         for(size t i=0;i<kd;++i)ret+=std::abs(</pre>
                                                                                                            /*mndist=std::min(mndist,dist);*/
                                                         flatten(u->r,++it);
                                                                                                    123
                                                                                                                                                             const std::vector<point> &range(const
              d[i]-x.d[i]);
                                                                                                            if(dist<mndist){</pre>
                                                  66
                                                                                                    124
                                                                                                                                                                 point&mi, const point&ma){
         return ret;
                                                                                                              pQ.push(std::make pair(dist,u->pid));
                                                        void rebuild(node*&u,int k){
                                                  67
                                                                                                    125
                                                                                                                                                               in range.clear();
                                                  68
                                                         if((int)A.size()<u->s)A.resize(u->s);
                                                                                                    126
                                                                                                              if((int)p0.size()==qM+1)
                                                                                                                                                              range(root,0,mi,ma);
       bool operator==(const point &p){
                                                         typename std::vector<node*>::iterator it 127
                                                                                                                mndist=pQ.top().first,pQ.pop();
                                                                                                                                                              return in_range;//回傳介於mi到ma之間的點
                                                                                                                                                       185
         for(size t i=0;i<kd;++i)</pre>
                                                              =A.begin():
                                                                                                    128
                                                                                                                                                                    vector
           if(d[i]!=p.d[i])return 0;
12
                                                  70
                                                         flatten(u,it);
                                                                                                    129
                                                                                                            if(x.d[k]<u->pid.d[k]){
                                                                                                                                                       186
13
         return 1;
                                                         u=build(k,0,u->s-1);
                                                                                                              nearest(u->1,(k+1)%kd,x,h,mndist);
                                                  71
                                                                                                    130
                                                                                                                                                       187
                                                                                                                                                            int size(){return root?root->s:0;}
14
                                                  72
                                                                                                    131
                                                                                                              h[k]=std::abs(x.d[k]-u->pid.d[k]);
       bool operator<(const point &b)const{</pre>
                                                       bool insert(node*&u,int k,const point &x,
                                                                                                              nearest(u->r,(k+1)%kd,x,h,mndist);
                                                                                                   132
         return d[0]<b.d[0];</pre>
                                                            int dep){
                                                                                                    133
                                                                                                            }else{
                                                         if(!u) return u=new node(x), dep<=0;</pre>
                                                                                                              nearest(u->r,(k+1)%kd,x,h,mndist);
                                                  74
                                                                                                    134
18
    };
                                                                                                              h[k]=std::abs(x.d[k]-u->pid.d[k]);
                                                  75
                                                         ++u->s;
                                                                                                    135
                                                                                                                                                           2.3 kd tree replace segment tr
   private:
                                                  76
                                                         cmp.sort id=k:
                                                                                                    136
                                                                                                              nearest(u->1,(k+1)%kd,x,h,mndist);
    struct node{
                                                         if(insert(cmp(x,u->pid)?u->1:u->r,(k+1)% 137)
       node *1,*r;
                                                              kd,x,dep-1)){
                                                                                                            h[k]=old;
                                                                                                    138
                                                                                                                                                        1 /*kd樹代替高維線段樹*/
22
       point pid;
                                                            if(!isbad(u))return 1;
                                                                                                    139
                                                                                                                                                        2 struct node{
       int s:
                                                                                                          std::vector<point>in_range;
                                                           rebuild(u,k);
                                                                                                    140
                                                                                                                                                             node *1.*r:
       node(const point &p):1(0),r(0),pid(p),s
                                                                                                          void range(node *u,int k,const point&mi.
                                                                                                                                                            point pid,mi,ma;
                                                         return 0;
                                                                                                               const point&ma){
                                                  81
                                                                                                                                                             int s:
       ~node(){delete l,delete r;}
                                                                                                            if(!u)return;
                                                  82
                                                                                                    142
                                                                                                                                                             int data;
26
       void up()\{s=(1?1->s:0)+1+(r?r->s:0);\}
                                                       node *findmin(node*o.int k){
                                                                                                    143
                                                                                                            bool is=1:
                                                                                                                                                             node(const point &p,int d):1(0),r(0),pid(p
     }*root:
                                                         if(!o)return 0;
                                                                                                            for(int i=0;i<kd;++i)</pre>
                                                                                                    144
                                                                                                                                                                  ),mi(p),ma(p),s(1),data(d),dmin(d),
     const double alpha,loga;
                                                         if(cmp.sort id==k)return o->l?findmin(o 145
                                                                                                              if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid
                                                                                                                                                                  dmax(d){}
     const T INF: //記得要給INF,表示極大值
                                                               ->1,(k+1)%kd):o;
                                                                                                                   .d[i]){
                                                                                                                                                            void up(){
                                                         node *l=findmin(o->l,(k+1)%kd);
30
     int maxn;
                                                                                                                is=0;break;
                                                                                                    146
                                                                                                                                                              mi=ma=pid:
31
    struct cmp{
                                                  87
                                                         node *r=findmin(o->r,(k+1)%kd);
                                                                                                    147
                                                                                                                                                              s=1;
                                                         if(1&&!r)return cmp(1,0)?1:0;
                                                                                                            if(is)in_range.push_back(u->pid);
32
       int sort_id;
                                                                                                    148
                                                                                                                                                              if(1){
                                                         if(!1&&r)return cmp(r,o)?r:o;
                                                                                                            if(mi.d[k] \le u \rightarrow pid.d[k]) range(u \rightarrow l,(k+1))
       bool operator()(const node*x,const node*
                                                                                                    149
                                                                                                                                                                 for(int i=0;i<kd;++i){</pre>
                                                         if(!1&&!r)return o;
                                                                                                                 %kd,mi,ma);
                                                                                                                                                                   mi.d[i]=min(mi.d[i],l->mi.d[i]);
         return operator()(x->pid,y->pid);
                                                         if(cmp(1,r))return cmp(1,o)?1:o;
                                                                                                            if(ma.d[k])=u-pid.d[k])range(u-r,(k+1)
34
                                                                                                                                                                   ma.d[i]=max(ma.d[i],1->ma.d[i]);
                                                         return cmp(r,o)?r:o;
                                                                                                                 %kd,mi,ma);
35
                                                                                                                                                        15
       bool operator()(const point &x,const
36
                                                                                                    151
                                                                                                                                                        16
                                                                                                                                                                 s+=1->s;
            point &y)const{
                                                       bool erase(node *&u,int k,const point &x){ 152 public:
                                                                                                         kd_tree(const T &INF, double a=0.75):root
         if(x.d[sort id]!=y.d[sort id])
                                                  95
                                                         if(!u)return 0;
                                                                                                                                                               if(r){
                                                                                                                                                       18
                                                                                                               (0), alpha(a), loga(log2(1.0/a)), INF(INF
           return x.d[sort id]<y.d[sort id];</pre>
                                                         if(u->pid==x){
                                                                                                                                                                 for(int i=0;i<kd;++i){</pre>
         for(size_t i=0;i<kd;++i)</pre>
                                                                                                               ), maxn(1){}
                                                           if(u->r);
                                                                                                                                                                   mi.d[i]=min(mi.d[i],r->mi.d[i]);
                                                            else if(u - > 1) u - > r = u - > 1, u - > 1 = 0;
           if(x.d[i]!=y.d[i])return x.d[i]<y.d[</pre>
                                                                                                          ~kd tree(){delete root;}
                                                                                                                                                                   ma.d[i]=max(ma.d[i],r->ma.d[i]);
                                                                                                                                                        21
                i];
                                                            else{
                                                                                                          void clear(){delete root, root=0, maxn=1;}
                                                                                                          void build(int n,const point *p){
         return 0;
                                                 100
                                                             delete u;
41
                                                                                                                                                        23
                                                                                                                                                                 s+=r->s;
                                                              return u=0, 1:
                                                                                                            delete root, A.resize(maxn=n);
42
                                                 101
                                                                                                    157
                                                                                                                                                        24
                                                                                                            for(int i=0;i<n;++i)A[i]=new node(p[i]);</pre>
                                                 102
                                                                                                    158
     int size(node *o){return o?o->s:0;}
                                                                                                            root=build(0,0,n-1);
                                                 103
                                                            --u->s:
                                                                                                    159
                                                                                                                                                            void up2(){
                                                                                                                                                        26
     std::vector<node*> A;
                                                            cmp.sort id=k;
                                                 104
                                                                                                    160
                                                                                                                                                              //其他懶惰標記向上更新
                                                                                                                                                        27
    node* build(int k,int l,int r){
                                                 105
                                                           u->pid=findmin(u->r,(k+1)%kd)->pid;
                                                                                                          void insert(const point &x){
                                                                                                            insert(root,0,x,__lg(size(root))/loga);
       if(l>r) return 0;
                                                 106
                                                            return erase(u->r,(k+1)%kd,u->pid);
                                                                                                    162
                                                                                                                                                        28
                                                                                                                                                             void down(){
       if(k==kd) k=0;
                                                                                                            if(root->s>maxn)maxn=root->s;
                                                                                                                                                        29
                                                 107
                                                                                                    163
                                                                                                                                                               //其他懶惰標記下推
       int mid=(1+r)/2;
                                                 108
                                                         cmp.sort_id=k;
                                                                                                    164
                                                                                                          bool erase(const point &p){
       cmp.sort id = k;
```

```
if(o->r&&range include(o->r,L,R))update(o
32 }*root:
33
                                                        ->r,L,R,data);
                                                   o->up2();
   /*檢查區間包含用的函數*/
                                               83
                                               84 }
  inline bool range_include(node *o,const
       point &L, const point &R){
                                               85
                                                  /*區間查詢,以總和為例*/
    for(int i=0;i<kd;++i){</pre>
      if(L.d[i]>o->ma.d[i]||R.d[i]<o->mi.d[i])
                                                 int query(node *o,const point &L,const point
                                                       &R){
           return 0;
    }//只要(L,R)區間有和o的區間有交集就回傳
                                                   if(!o)return 0;
         true
                                                   o->down();
    return 1;
                                                   if(range in range(o,L,R))return o->sum;
39
                                                   int ans=0;
40
                                                   if(point_in_range(o,L,R))ans+=o->data;
41 inline bool range_in_range(node *o,const
       point &L, const point &R){
                                                   if(o->1&&range include(o->1,L,R))ans+=
    for(int i=0;i<kd;++i){</pre>
                                                        query(o->1,L,R);
                                                   if(o->r&&range include(o->r,L,R))ans+=
      if(L.d[i]>o->mi.d[i]||o->ma.d[i]>R.d[i]) 94
                                                        query(o->r,L,R);
           return 0;
                                                   return ans;
    }//如果(L,R)區間完全包含o的區間就回傳true
45
    return 1;
46
   inline bool point_in_range(node *o,const
       point &L, const point &R){
    for(int i=0;i<kd;++i){</pre>
                                                  2.4 reference point.cpp
49
      if(L.d[i]>o->pid.d[i]||R.d[i]<o->pid.d[i
           ])return 0;
    }//如果(L,R)區間完全包含o->pid這個點就回傳
                                                1 | template<typename T>
         true
                                               2 struct RefC{
    return 1;
                                                   T data;
52
                                                   int ref:
                                                   _RefC(const T&d=0):data(d),ref(0){}
   /*單點修改,以單點改值為例*/
   void update(node *u,const point &x,int data,
                                                  template<typename T>
       int k=0){
                                               8 struct _rp{
    if(!u)return;
                                                    RefC<T> *p;
    u->down();
57
                                                   T *operator->(){return &p->data;}
58
    if(u->pid==x){
                                                   T & operator*() { return p->data; }
59
      u->data=data;
                                                   operator _RefC<T>*(){return p;}
                                               12
60
      u->up2();
                                                   rp &operator=(const rp &t){
      return;
61
                                                     if(p&&!--p->ref)delete p;
                                               14
62
                                                     p=t.p,p&&++p->ref;
                                               15
    cmp.sort id=k;
                                                     return *this;
                                               16
    update(cmp(x,u->pid)?u->l:u->r,x,data,(k
                                               17
         +1)%kd);
                                                   rp( RefC<T> *t=0):p(t){p&&++p->ref;}
    u->up2();
                                                   _rp(const _rp &t):p(t.p){p&&++p->ref;}
                                               19
66
                                                   ~_rp(){if(p&&!--p->ref)delete p;}
                                              21 };
   /*區間修改*/
                                               22 template<typename T>
   void update(node *o,const point &L,const
                                               23 inline rp<T> new rp(const T&nd){
       point &R, int data){
                                                  return _rp<T>(new _RefC<T>(nd));
    if(!o)return;
    o->down();
    if(range in range(o,L,R)){
73
      //區間懶惰標記修改
74
      o->down();
                                                 2.5 skew heap.cpp
75
      return:
76
    if(point_in_range(o,L,R)){
                                                1 | node *merge(node *a, node *b){
      //這個點在(L,R)區間,但是他的左右子樹不
                                                   if(!a||!b) return a?a:b;
           一定在區間中
                                                   if(b->data<a->data) swap(a,b);
      //單點懶惰標記修改
79
                                                   swap(a->1,a->r);
80
                                                   a->1=merge(b,a->1);
    if(o->1&&range_include(o->1,L,R))update(o
                                                   return a:
         ->1,L,R,data);
```

```
2.6 undo_disjoint_set.cpp
```

```
1 | struct DisjointSet {
    // save() is like recursive
     // undo() is like return
     int n, fa[MXN], sz[MXN];
     vector<pair<int*,int>> h;
     vector<int> sp;
     void init(int tn) {
       for (int i=0; i<n; i++) sz[fa[i]=i]=1;</pre>
       sp.clear(); h.clear();
10
11
     void assign(int *k, int v) {
12
13
       h.PB({k, *k});
14
       *k=v;
15
16
     void save() { sp.PB(SZ(h)); }
     void undo() {
18
       assert(!sp.empty());
19
       int last=sp.back(); sp.pop back();
20
       while (SZ(h)!=last) {
21
         auto x=h.back(); h.pop_back();
22
         *x.F=x.S;
23
24
25
     int f(int x) {
26
       while (fa[x]!=x) x=fa[x];
27
       return x;
28
29
     void uni(int x, int y) {
30
       x=f(x); y=f(y);
       if (x==y) return ;
31
       if (sz[x]<sz[y]) swap(x, y);</pre>
32
       assign(&sz[x], sz[x]+sz[y]);
33
       assign(&fa[y], x);
34
35
36 }djs;
```

#### 3.2 ext.cpp

### 2.7 整體二分.cpp

```
void totBS(int L, int R, vector<Item> M){
if(Q.empty()) return; //維護全域B陣列
if(L=R) 整個M的答案=r, return;
int mid = (L+R)/2;
vector<Item> mR;
do_modify_B_with_divide(mid,M);
//讓B陣列在遞迴的時候只會保留[L~mid]的資訊
undo_modify_B(mid,M);
totBS(L,mid,mL);
totBS(mid+1,R,mR);
]
```

#### 3 default

## 3.1 debug.cpp

### 3.3 IncStack.cpp

```
2 #pragma GCC optimize "Ofast"
3 //stack resize, change esp to rsp if 64-bit
  asm("mov \%0,\%/esp\n" :: "q"(mem+10000000));
   -Wl,--stack,214748364 -trigraphs
6 //linux stack resize
  #include<sys/resource.h>
  void increase stack(){
    const rlim_t ks=64*1024*1024;
    struct rlimit rl;
    int res=getrlimit(RLIMIT STACK,&rl);
    if(!res&&rl.rlim cur<ks){</pre>
      rl.rlim cur=ks;
      res=setrlimit(RLIMIT STACK,&rl);
14
15
16 }
```

#### 3.4 input.cpp if(u==t)return cur flow; 38 int v,pre; T tf=cur flow,df; 39 return 0; 26 T cap, cost; for(int &i=cur[u];~i;i=e[i].pre){ 27 edge(int v,int pre,\_T cap,\_T cost):v(v), 40 1 inline int read(){ T dfs(int u,int t,T cur flow=INF){ if(e[i].r&&d[u]==d[e[i].v]+1){ 41 28 pre(pre), cap(cap), cost(cost){} int x=0; bool f=0; char c=getchar(); if(u==t)return cur flow; 29 df=dfs(e[i].v,s,t,min(tf,e[i].r)); 42 while (ch<0'|1'9'<ch) f|=ch=='-', ch=getchar 43 T df: 30 e[i].flow+=df; int n,S,T; T dis[MAXN],piS,ans; 44 for(int &i=cur[u];~i;i=e[i].pre){ 31 e[i^1].flow-=df: 11 while ('0' <= ch&&ch <= '9') x = x\*10 - '0' + ch, ch =if(level[e[i].v]==level[u]+1&&e[i].r){ e[i].r-=df; bool vis[MAXN]; 45 getchar(); e[i^1].r+=df; if(df=dfs(e[i].v,t,min(cur flow,e[i 33 vector<edge> e; return f?-x:x; if(!(tf-=df)||d[s]==n)return 1.r))){ int g[MAXN]; void init(int \_n){ e[i].flow+=df; cur flow-tf; 15 47 // #!/bin/bash e[i^1].flow-=df: memset(g, -1, sizeof(int)\*((n= n)+1));48 35 // g++ -std=c++11 -02 -Wall -Wextra -Wnoe[i].r-=df; e.clear(); 49 36 17 unused-result -DDEBUG \$1 && ./a.out e[i^1].r+=df: 50 37 int mh=n: 18 -fsanitize=address -fsanitize=undefined 51 return df: 38 for(int i=cur[u]=g[u];~i;i=e[i].pre){ 19 void add edge(int u,int v, T cap, T cost, -fsanitize=return 52 39 if(e[i].r&&d[e[i].v]<mh)mh=d[e[i].v];</pre> bool directed=false){ e.push\_back(edge(v,g[u],cap,cost)); 53 40 20 g[u]=e.size()-1; **if**(!--gap[d[u]])d[s]=n; 54 41 21 else ++gap[d[u]=++mh]; 55 return level[u]=0; 42 22 e.push\_back(edge(u,g[v],directed?0:cap,return cur flow-tf; 56 43 cost)); Flow g[v]=e.size()-1; 57 dinic(int s,int t,bool clean=true){ 44 23 T isap(int s,int t,bool clean=true){ 58 if(clean){ 45 24 for(size t i=0;i<e.size();++i){</pre> 59 46 memset(d,0,sizeof(int)\*(n+1)); 25 T augment(int u, T cur flow){ dinic.cpp 60 e[i].flow=0; 47 memset(gap,0,sizeof(int)\*(n+1)); if(u==T||!cur flow)return ans+=piS\* memcpy(cur,g,sizeof(int)\*(n+1)); cur flow, cur flow; 61 e[i].r=e[i].cap; 48 62 if(clean) for(size t i=0;i<e.size();++i)</pre> vis[u]=1: 1 template < typename T> 63 T r=cur flow,d; struct DINIC{ 64 T ans=0, mf=0: e[i].flow=0: 29 for(int i=g[u];~i;i=e[i].pre){ while(bfs(s,t))while(mf=dfs(s,t))ans+=mf if(e[i].cap&&!e[i].cost&&!vis[e[i].v]) static const int MAXN=105; 65 e[i].r=e[i].cap; 30 51 static const T INF=INT MAX; 52 int n, level[MAXN], cur[MAXN]; 66 return ans: 53 T max flow=0: d=augment(e[i].v,min(r,e[i].cap)); struct edge{ 67 54 for(gap[0]=n;d[s]<n;)max\_flow+=dfs(s,s,t</pre> e[i].cap-=d; int v,pre; 68 }; e[i^1].cap+=d: ); T cap,flow,r; if(!(r-=d))break; 55 return max flow; 34 edge(int v,int pre,T cap):v(v),pre(pre), 56 35 cap(cap),flow(0),r(cap){} 36 vector<int> cut\_e;//最小割邊集 4.2 ISAP with cut.cpp return cur\_flow-r; 37 bool vis[MAXN]; int g[MAXN]; void dfs cut(int u){ 38 12 vector<edge> e; 39 bool modlabel(){ vis[u]=1;//表示u屬於source的最小割集 1 | template < typename T> void init(int \_n){ 40 for(int u=0;u<=n;++u)dis[u]=INF;</pre> for(int i=g[u];~i;i=e[i].pre) 61 memset(g, -1, sizeof(int)\*((n= n)+1)); 2 struct ISAP{ static deque<int>q; 14 41 if(e[i].flow<e[i].cap&&!vis[e[i].v])</pre> static const int MAXN=105; 15 e.clear(); 42 dis[T]=0,q.push back(T); dfs cut(e[i].v); static const T INF=INT MAX; while(q.size()){ 43 16 63 void add edge(int u,int v,T cap,bool int n://點數 44 int u=q.front();q.pop\_front(); 17 64 T min cut(int s,int t){ int d[MAXN],gap[MAXN],cur[MAXN]; directed=false){ 45 T ans=isap(s,t); e.push\_back(edge(v,g[u],cap)); struct edge{ 46 for(int i=g[u];~i;i=e[i].pre){ memset(vis,0,sizeof(bool)\*(n+1)); 19 g[u]=e.size()-1; int v,pre; if(e[i^1].cap&&(dt=dis[u]-e[i].cost) dfs cut(s), cut e.clear(); 20 e.push\_back(edge(u,g[v],directed?0:cap)) T cap,flow,r; <dis[e[i].v]){ for(int u=0;u<=n;++u)</pre> edge(int v,int pre,T cap):v(v),pre(pre), if((dis[e[i].v]=dt)<=dis[q.size()?</pre> if(vis[u])for(int i=g[u];~i;i=e[i].pre cap(cap),flow(0),r(cap){} 21 g[v]=e.size()-1; q.front():S]){ q.push\_front(e[i].v); 22 70 if(!vis[e[i].v])cut e.push back(i); int g[MAXN]; }else q.push back(e[i].v); 23 int bfs(int s,int t){ 12 71 return ans; memset(level,0,sizeof(int)\*(n+1)); vector<edge> e; 51 72 memcpy(cur,g,sizeof(int)\*(n+1)); void init(int \_n){ 52 } 73 }; 26 queue<int> q; 15 memset(g, -1, sizeof(int)\*((n= n)+1));53 16 e.clear(); for(int u=0;u<=n;++u)</pre> q.push(s); 54 for(int i=g[u];~i;i=e[i].pre) level[s]=1; 17 void add edge(int u,int v,T cap,bool while(q.size()){ 18 e[i].cost+=dis[e[i].v]-dis[u]; 4.3 MinCostMaxFlow.cpp int u=q.front();q.pop(); directed=false){ return piS+=dis[S], dis[S]<INF;</pre> for(int i=g[u];~i;i=e[i].pre){ e.push back(edge(v,g[u],cap)); 32 **if**(!level[e[i].v]&&e[i].r){ g[u]=e.size()-1; T mincost(int s,int t){ 33 level[e[i].v]=level[u]+1; 21 e.push\_back(edge(u,g[v],directed?0:cap)) 1 template<typename T> S=s,T=t; q.push(e[i].v); 2 struct MCMF{ piS=ans=0: static const int MAXN=440; 35 if(e[i].v==t)return 1; g[v]=e.size()-1; while(modlabel()){ static const T INF=999999999; do memset(vis,0,sizeof(bool)\*(n+1)); 36 23 T dfs(int u,int s,int t,T cur flow=INF){ struct edge{ while(augment(S,INF));

```
}return ans;
                                                           for(size t j=0;j<match list[v].size()</pre>
                                                                                                                int l=lca(y,x);
                                                                                                                                                       3 int n;// 1-base, 0表示沒有匹配
                                                                                                    38
                                                                                                                                                         int g[MAXN][MAXN], lx[MAXN], ly[MAXN], pa[MAXN
                                                                ;++i){
                                                                                                    39
                                                                                                                flower(y,x,l,q),flower(x,y,l,q);
                                                             int next_u=match_list[v][j];
67 };
                                                                                                                                                               ],slack_y[MAXN];
                                                  17
                                                                                                    40
                                                             if(dfs(next u))
                                                  18
                                                                                                    41
                                                                                                                                                          int match y[MAXN], match x[MAXN];
                                                               return match_list[v][j]=u, true;
                                                                                                                                                          bool vx[MAXN],vy[MAXN];
                                                  19
                                                                                                    42
                                                  20
                                                                                                    43
                                                                                                         return 0;
                                                                                                                                                          void augment(int y){
                                                  21
                                                         }
                                                                                                    44
                                                                                                                                                           for(int x,z;y;y=z){
        Graph
                                                                                                                                                             x=pa[y],z=match_x[x];
                                                  22
                                                                                                    45
                                                                                                       int blossom(){
                                                  23
                                                       return false;
                                                                                                    46
                                                                                                         int ans=0;
                                                                                                                                                             match_y[y]=x,match_x[x]=y;
                                                                                                         for(int i=1;i<=n;++i)</pre>
                                                  24
                                                                                                                                                       11
                                                     int max_match(){
                                                                                                    48
                                                                                                           if(!match[i]&&bfs(i))++ans;
                                                                                                                                                       12
  5.1 Augmenting Path.cpp
                                                       for(int i=0;i<n2;++i)match list[i].clear()</pre>
                                                                                                                                                         void bfs(int st){
                                                                                                         return ans:
                                                                                                    49
                                                                                                                                                           for(int i=1;i<=n;++i)slack_y[i]=INF,vx[i]=</pre>
                                                  27
                                                       int cnt=0:
                                                                                                                                                                vy[i]=0;
1 #define MAXN1 505
                                                  28
                                                       for(int u=0;u<n1;++u){</pre>
                                                                                                                                                       15
                                                                                                                                                            queue<int> q;q.push(st);
2 #define MAXN2 505
                                                  29
                                                         memset(vis,0,sizeof(bool)*n2);
                                                                                                                                                       16
                                                                                                                                                           for(;;){
3 int n1,n2;//n1 個點連向n2個點
                                                                                                       5.4 graphISO.cpp
                                                                                                                                                              while(q.size()){
                                                  30
                                                         if(dfs(u))++cnt;
                                                                                                                                                       17
                                                                                                                                                                int x=q.front();q.pop();
4 int match[MAXN2]; //屬於n2的點匹配了哪個點
                                                  31
                                                                                                                                                       18
5 vector<int > g[MAXN1];//

                                                                                                                                                       19
                                                                                                                                                                vx[x]=1;
                                                  32
                                                       return cnt;
                                                                                                                                                       20
                                                                                                                                                                for(int y=1;y<=n;++y)if(!vy[y]){</pre>
6 bool vis[MAXN2];//是否走訪過
                                                                                                     1 const int MAXN=1005, K=30; // K要 夠 大
                                                                                                                                                       21
                                                                                                                                                                  int t=lx[x]+ly[y]-g[x][y];
   bool dfs(int u){
                                                                                                     2 const long long A=3,B=11,C=2,D=19,P=0
     for(size_t i=0;i<g[u].size();++i){</pre>
                                                                                                                                                       22
                                                                                                                                                                  if(t==0){
                                                                                                            xdefaced;
                                                                                                                                                       23
                                                                                                                                                                    pa[y]=x;
       int v=g[u][i];
                                                                                                     3 long long f[K+1][MAXN];
                                                            blossom matching.cpp
                                                                                                                                                                    if(!match_y[y]){augment(y);return
                                                                                                                                                       24
10
       if(vis[v])continue;
                                                                                                     4 vector<int> g[MAXN],rg[MAXN];
       vis[v]=1;
                                                                                                     5 int n;
                                                                                                                                                                    vy[y]=1,q.push(match_y[y]);
       if(match[v]==-1||dfs(match[v]))
                                                                                                                                                       25
12
                                                                                                       void init(){
                                                   1 #define MAXN 505
                                                                                                                                                                  }else if(slack_y[y]>t)pa[y]=x,
                                                                                                                                                       26
13
         return match[v]=u, 1;
                                                                                                         for(int i=0;i<n;++i){</pre>
                                                   vector<int>g[MAXN];
                                                                                                                                                                       slack_y[y]=t;
14
                                                                                                           f[0][i]=1;
                                                   int pa[MAXN], match[MAXN], st[MAXN], S[MAXN], v[
                                                                                                                                                       27
15
    return 0;
                                                                                                           g[i].clear(), rg[i].clear();
                                                                                                                                                       28
16
                                                    int t,n;
                                                                                                                                                       29
                                                                                                                                                              int cut=INF:
   inline int max match(){
                                                                                                    11
                                                    int lca(int x,int y){
                                                                                                                                                              for(int y=1;y<=n;++y){</pre>
                                                                                                                                                       30
                                                                                                       void add_edge(int u,int v){
                                                       for(++t;;swap(x,y)){
                                                                                                                                                                if(!vy[y]&&cut>slack_y[y])cut=slack_y[
                                                                                                                                                       31
                                                                                                         g[u].push_back(v), rg[v].push_back(u);
    memset(match,-1,sizeof(int)*n2);
                                                         if(x==0)continue;
                                                                                                                                                                    у];
    for(int i=0;i<n1;++i){</pre>
                                                                                                    14
                                                         if(v[x]==t)return x;
       memset(vis,0,sizeof(bool)*n2);
                                                                                                                                                       32
                                                                                                       long long point_hash(int u){//O(N)
                                                         v[x]=t;
                                                                                                                                                       33
                                                                                                                                                              for(int j=1;j<=n;++j){</pre>
22
       if(dfs(i))++ans;
                                                                                                         for(int t=1;t<=K;++t){</pre>
                                                         x=st[pa[match[x]]];
                                                                                                                                                                if(vx[j])lx[j]-=cut;
                                                                                                                                                       34
23
                                                                                                    17
                                                                                                           for(int i=0;i<n;++i){</pre>
                                                  11
                                                                                                                                                       35
                                                                                                                                                                if(vy[j])ly[j]+=cut;
^{24}
    return ans;
                                                                                                              f[t][i]=f[t-1][i]*A%P;
                                                                                                    18
                                                  12 }
                                                                                                                                                                else slack y[j]-=cut;
                                                                                                                                                       36
                                                                                                              for(int j:g[i])f[t][i]=(f[t][i]+f[t
                                                                                                    19
                                                  13 #define qpush(x) q.push(x),S[x]=0
                                                                                                                                                       37
                                                                                                                   -1][j]*B%P)%P;
                                                  14 void flower(int x,int y,int l,queue<int> &q)
                                                                                                                                                              for(int y=1;y<=n;++y){</pre>
                                                                                                              for(int j:rg[i])f[t][i]=(f[t][i]+f[t
                                                                                                                                                                if(!vy[y]&&slack_y[y]==0){
                                                                                                                                                       39
                                                                                                                   -1][j]*C%P)%P;
                                                       while(st[x]!=1){
                                                                                                                                                                  if(!match_y[y]){augment(y);return;}
         Augmenting Path multiple
                                                                                                              if(i==u)f[t][i]+=D;//如果圖太大的話
                                                                                                    21
                                                                                                                                                                  vy[y]=1,q.push(match_y[y]);
                                                         pa[x]=y;
                                                                                                                  把這行刪掉,執行一次後f[K]就會是所
                                                         if(S[y=match[x]]==1)qpush(y);
                                                                                                                   有點的答案
                                                         st[x]=st[y]=1, x=pa[y];
                                                                                                                                                       43
                                                                                                              f[t][i]%=P;
1 #define MAXN1 1005
                                                  19
                                                                                                    22
                                                                                                                                                       44
2 #define MAXN2 505
                                                  20 }
                                                                                                    23
                                                                                                                                                       45
                                                                                                    24
3 int n1, n2; // n1 個點連向 n2 個點,其中 n2 個點可以
                                                     bool bfs(int x){
                                                                                                    25
                                                                                                         return f[K][u];
                                                       for(int i=1;i<=n;++i)st[i]=i;</pre>
                                                                                                                                                            memset(match y,0,sizeof(int)*(n+1));
        匹配很多邊
                                                                                                    26
                                                       memset(S+1,-1,sizeof(int)*n);
                                                                                                                                                            memset(ly,0,sizeof(int)*(n+1));
4 vector<int> g[MAXN1];// \bigsim
                                                                                                       vector<long long> graph_hash(){
                                                       queue<int>q; qpush(x);
                                                                                                                                                            for(int x=1;x<=n;++x){</pre>
5 int c[MAXN2]; //每個屬於n2點最多可以接受幾條
                                                                                                         vector<long long> ans;
                                                       while(q.size()){
                                                                                                                                                             1x[x]=-INF;
                                                                                                         for(int i=0;i<n;++i)ans.push_back(</pre>
                                                         x=q.front(),q.pop();
                                                                                                                                                              for(int y=1;y<=n;++y)</pre>
6 | vector<int> match_list[MAXN2];//每個屬於n2的
                                                                                                              point_hash(i));//0(N^2)
                                                         for(size_t i=0;i<g[x].size();++i){</pre>
                                                                                                                                                       52
                                                                                                                                                                1x[x]=max(1x[x],g[x][y]);
        點匹配了那些點
                                                                                                         sort(ans.begin(),ans.end());
                                                           int y=g[x][i];
                                                                                                    31
                                                                                                         return ans;
7 bool vis[MAXN2];//是否走訪過
                                                           if(S[v]==-1){
                                                                                                                                                            for(int x=1;x<=n;++x)bfs(x);</pre>
                                                  29
                                                                                                    32
  bool dfs(int u){
                                                             pa[y]=x,S[y]=1;
                                                                                                                                                            long long ans=0;
                                                  30
    for(size_t i=0;i<g[u].size();++i){</pre>
                                                  31
                                                             if(!match[y]){
                                                                                                                                                            for(int y=1;y<=n;++y)ans+=g[match_y[y]][y</pre>
       int v=g[u][i];
                                                               for(int lst;x;y=lst,x=pa[y])
                                                                                                                                                                ];
       if(vis[v])continue;
                                                                 lst=match[x],match[x]=y,match[y
                                                                                                                                                       57
                                                                                                                                                            return ans;
                                                                                                              KM.cpp
12
       vis[v]=true;
                                                                      ]=x;
       if((int)match list[v].size()<c[v]){</pre>
                                                               return 1;
                                                  34
         return match_list[v].push_back(u),
                                                  35
              true:
                                                  36
                                                             qpush(match[y]);
                                                                                                     1 #define MAXN 405
       }else{
                                                           }else if(!S[y]&&st[y]!=st[x]){
                                                                                                     2 #define INF 0x3f3f3f3f
```

13

14

15

20

21

27 };

32 33 }

37

38 }

vector<bit node> bit;

for(:i:i-=i&(-i)){

bit[i];

].x,p[i].y);

return x.id;

5.6 MaximumClique.cpp

#### 1 struct MaxClique{ static const int MAXN=105: int N,ans; int g[MAXN][MAXN], dp[MAXN], stk[MAXN][MAXN int sol[MAXN],tmp[MAXN];//sol[0~ans-1]為答 void init(int n){ N=n;//0-base memset(g,0,sizeof(g)); void add\_edge(int u,int v){ g[u][v]=g[v][u]=1;12 int dfs(int ns,int dep){ **if**(!ns){ if(dep>ans){ ans=dep; memcpy(sol,tmp,sizeof tmp); return 1; }else return 0; 19 20 for(int i=0;i<ns;++i){</pre> if(dep+ns-i<=ans)return 0;</pre> 22 int u=stk[dep][i],cnt=0; if(dep+dp[u]<=ans)return 0;</pre> for(int j=i+1; j<ns;++j){</pre> int v=stk[dep][j]; **if**(g[u][v])stk[dep+1][cnt++]=v; tmp[dep]=u; if(dfs(cnt,dep+1))return 1; return 0; 32 33 int clique(){ int u,v,ns; for(ans=0,u=N-1;u>=0;--u){ for (ns=0, tmp[0]=u, v=u+1; v<N; ++v) **if**(g[u][v])stk[1][ns++]=v; dfs(ns,1),dp[u]=ans; 40 41 return ans; 42 43 };

# 5.7 MinimumMeanCycle.cpp

```
1| #include < cstdint > // for DBL_MAX
2 int dp[maxN+1][maxN+1];
3 double mnc(int n){
    int u,v,w;
    const int inf=0x7f7f7f7f;
    memset(dp,0x7f,sizeof(dp));
    memset(dp[0],0,sizeof(dp[0]));
    for(int i=0;i<n;++i){</pre>
       for(auto e:E){
10
         tie(u,v,w)=e;
         if(dp[i][u]!=inf)
```

```
dp[i+1][v]=min(dp[i+1][v],dp[i][u]+w); 43
     double res = DBL MAX;
     for(int i=1;i<=n;++i){</pre>
       double val = DBL MIN;
       for(int j=0;j<n;++j)</pre>
         val=max(val,double(dp[n][i]-dp[i][j
              ])/(n-j));
       res=min(res,val);
   return res:
 5.8 Rectilinear MST.cpp
1 //平面曼哈頓最小生成樹構造圖(去除非必要邊)
  T x,y;
   int id;//從0開始編號
   point(){}
   T dist(const point &p)const{
     return abs(x-p.x)+abs(y-p.y);
```

45

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58

```
2 #define T int
3 #define INF 0x3f3f3f3f
4 struct point{
10
11
  };
12 bool cmpx(const point &a,const point &b){
    return a.x<b.x||(a.x==b.x&&a.y<b.y);
14
   struct edge{
    int u,v;
    T cost:
     edge(int u,int v,T c):u(u),v(v),cost(c){}
19
    bool operator<(const edge&e)const{</pre>
      return cost<e.cost;</pre>
20
21
22 };
23 struct bit node{
    T mi;
     int id:
     bit node(const T&mi=INF,int id=-1):mi(mi),
          id(id){}
```

```
13
                                                   14
                                                   20
   void bit update(int i,const T&data,int id){
      if(data<bit[i].mi)bit[i]=bit node(data,</pre>
34 int bit find(int i,int m){
     for(;i<=m;i+=i&(-i)) if(bit[i].mi<x.mi)x=</pre>
39 vector<edge> build graph(int n,point p[]){
    vector<edge> e;//edge for MST
     for(int dir=0;dir<4;++dir){//4種座標變換
      if(dir%2) for(int i=0;i<n;++i) swap(p[i</pre>
```

```
else if(dir==2) for(int i=0;i<n;++i) p[i</pre>
          ].x=-p[i].x;
      sort(p,p+n,cmpx);
                                                10
      vector<T> ga(n), gb;
                                                1.1
      for(int i=0;i<n;++i)ga[i]=p[i].y-p[i].x; 12</pre>
      gb=ga, sort(gb.begin(),gb.end());
      gb.erase(unique(gb.begin(),gb.end()),gb.
                                                14
          end());
                                                15
      int m=gb.size();
                                                16
      bit=vector<bit node>(m+1);
      for(int i=n-1;i>=0;--i){
                                                18
        int pos=lower bound(gb.begin(),gb.end
             (),ga[i])-gb.begin()+1;
                                                20
        int ans=bit find(pos,m);
        if(~ans)e.push back(edge(p[i].id,p[ans
            ].id,p[i].dist(p[ans])));
                                                22
        bit_update(pos,p[i].x+p[i].y,i);
                                                24
    return e;
                                                25
                                                26
                                                27
                                                28
 5.9 treeISO.cpp
                                                29
                                                30
                                                31
1 const int MAXN=100005:
                                                32
const long long X=12327,P=0xdefaced;
                                                33
3 vector<int> g[MAXN];
                                                34
4 bool vis[MAXN];
                                                35
5 long long dfs(int u){//hash ver
                                                36
   vis[u]=1:
                                                37
   vector<long long> tmp;
                                                38
    for(auto v:g[u])if(!vis[v])tmp.PB(dfs(v));
    if(tmp.empty())return 177;
   long long ret=4931;
                                                41
    sort(tmp.begin(),tmp.end());
                                                42
    for(auto v:tmp)ret=((ret*X)^v)%P;
                                                43
    return ret;
 string dfs(int x,int p){
   vector<string> c;
                                                47
   for(int y:g[x])
                                                48
     if(y!=p)c.emplace_back(dfs(y,x));
                                                49
    sort(c.begin(),c.end());
   string ret("(");
    for(auto &s:c)ret+=s;
   ret+=")";
   return ret;
                                                53
 5.10 一般圖最小權完美匹配.cpp
```

```
1 struct Graph {
   // Minimum General Weiahted Matchina (
        Perfect Match) 0-base
   static const int MXN = 105;
   int n, edge[MXN][MXN];
   int match[MXN], dis[MXN], onstk[MXN];
   vector<int> stk;
   void init(int n) {
```

65 | }graph;

```
for (int i=0; i<n; i++)</pre>
    for (int j=0; j<n; j++)</pre>
      edge[i][j] = 0;
void add edge(int u, int v, int w) {
  edge[u][v] = edge[v][u] = w;
bool SPFA(int u){
 if (onstk[u]) return true;
  stk.push back(u);
  onstk[u] = 1:
  for (int v=0; v<n; v++){</pre>
   if (u != v && match[u] != v && !onstk[
         v]){
      int m = match[v];
      if (dis[m] > dis[u] - edge[v][m] +
           edge[u][v]){
        dis[m] = dis[u] - edge[v][m] +
             edge[u][v];
        onstk[v] = 1;
        stk.push back(v);
        if (SPFA(m)) return true;
        stk.pop_back();
        onstk[v] = 0;
  onstk[u] = 0;
  stk.pop_back();
  return false;
int solve() {
  // find a match
  for (int i=0; i<n; i+=2){</pre>
    match[i] = i+1, match[i+1] = i;
  for(;;){
    int found = 0:
    for (int i=0; i<n; i++) dis[i] = onstk</pre>
         [i] = 0;
    for (int i=0; i<n; i++){</pre>
      stk.clear();
      if (!onstk[i] && SPFA(i)){
        found = 1;
        while (stk.size()>=2){
          int u = stk.back(); stk.pop_back
          int v = stk.back(); stk.pop_back
               ();
          match[u] = v;
          match[v] = u;
    if (!found) break;
  int ret = 0:
  for (int i=0; i<n; i++)</pre>
   ret += edge[i][match[i]];
  ret /= 2;
  return ret;
```

## 5.11 全局最小割.cpp

```
1 const int INF=0x3f3f3f3f;
2 template<typename T>
  struct stoer wagner{// 0-base
     static const int MAXN=150;
     T g[MAXN][MAXN], dis[MAXN];
     int nd[MAXN],n,s,t;
     void init(int n){
       n = n;
       for(int i=0;i<n;++i)</pre>
10
         for(int j=0;j<n;++j)g[i][j]=0;</pre>
     void add_edge(int u,int v,T w){
12
13
       g[u][v]=g[v][u]+=w;
14
15
     T min_cut(){
       T ans=INF:
16
17
       for(int i=0;i<n;++i)nd[i]=i;</pre>
       for(int ind,tn=n;tn>1;--tn){
19
         for(int i=1;i<tn;++i)dis[nd[i]]=0;</pre>
         for(int i=1;i<tn;++i){</pre>
20
21
           ind=i;
22
            for(int j=i;j<tn;++j){</pre>
              dis[nd[j]]+=g[nd[i-1]][nd[j]];
24
              if(dis[nd[ind]]<dis[nd[j]])ind=j;</pre>
25
26
           swap(nd[ind],nd[i]);
27
         if(ans>dis[nd[ind]])ans=dis[t=nd[ind
              ]],s=nd[ind-1];
29
         for(int i=0;i<tn;++i)</pre>
           g[nd[ind-1]][nd[i]]=g[nd[i]][nd[ind
                 -1]]+=g[nd[i]][nd[ind]];
31
32
       return ans;
33
34 };
```

## 5.12 平面圖判定.cpp

```
1 static const int MAXN = 20;
2 struct Edge{
    int u, v;
    Edge(int s, int d) : u(s), v(d) {}
  bool isK33(int n, int degree[]){
    int t = 0, z = 0;
    for(int i=0;i<n;++i){</pre>
       if(degree[i] == 3)++t;
       else if(degree[i] == 0)++z;
       else return false;
12
     return t == 6 && t + z == n:
14
   bool isK5(int n, int degree[]){
    int f = 0, z = 0;
    for(int i=0;i<n;++i){</pre>
       if(degree[i] == 4)++f;
       else if(degree[i] == 0)++z;
       else return false;
20
```

```
22 return f == 5 && f + z == n;
23 }
24 // it judge a given graph is Homeomorphic
        with K33 or K5
25 bool isHomeomorphic(bool G[MAXN][MAXN],
        const int n){
26
     for(::){
27
       int cnt = 0;
       for(int i=0;i<n;++i){</pre>
28
29
         vector<Edge> E;
30
         for(int j=0;j<n&E.size()<3;++j)</pre>
           if(G[i][j] && i != j)
31
              E.push_back(Edge(i, j));
32
         if(E.size() == 1){
33
34
           G[i][E[0].v] = G[E[0].v][i] = false;
35
         }else if(E.size() == 2){
           G[i][E[0].v] = G[E[0].v][i] = false;
36
37
            G[i][E[1].v] = G[E[1].v][i] = false; 40
38
            G[E[0].v][E[1].v] = G[E[1].v][E[0].v 41
                 ] = true;
            ++cnt;
39
40
41
42
       if(cnt == 0)break;
43
     static int degree[MAXN];
44
     fill(degree, degree + n, 0);
45
     for(int i=0;i<n;++i){</pre>
46
47
       for(int j=i+1; j<n; ++j){</pre>
         if(!G[i][j])continue;
48
49
         ++degree[i];
50
         ++degree[j];
51
52
    }
53
     return !(isK33(n, degree) || isK5(n,
          degree));
```

## 5.13 弦圖完美消除序列.cpp

```
1 | struct chordal{
     static const int MAXN=1005;
     int n;// 0-base
     vector<int>G[MAXN];
     int rank[MAXN],label[MAXN];
     bool mark[MAXN];
     void init(int _n){n=_n;
      for(int i=0;i<n;++i)G[i].clear();</pre>
     void add edge(int u,int v){
11
      G[u].push_back(v);
12
      G[v].push back(u);
13
     vector<int> MCS(){
       memset(rank,-1,sizeof(int)*n);
       memset(label,0,sizeof(int)*n);
17
       priority queue<pair<int,int> > pq;
       for(int i=0;i<n;++i)pq.push(make pair(0,</pre>
19
       for(int i=n-1;i>=0;--i)for(;;){
         int u=pq.top().second;pq.pop();
20
         if(~rank[u])continue;
21
         rank[u]=i;
```

```
for(auto v:G[u])if(rank[v]==-1){
           pg.push(make pair(++label[v],v));
         break;
       vector<int> res(n);
       for(int i=0:i<n:++i)res[rank[i]]=i;</pre>
       return res;
     bool check(vector<int> ord){//弦圖判定
       for(int i=0;i<n;++i)rank[ord[i]]=i;</pre>
       memset(mark,0,sizeof(bool)*n);
       for(int i=0;i<n;++i){</pre>
         vector<pair<int,int> > tmp;
         for(auto u:G[ord[i]])if(!mark[u])
           tmp.push back(make pair(rank[u],u));
         sort(tmp.begin(),tmp.end());
         if(tmp.size()){
           int u=tmp[0].second;
           set<int> S;
           for(auto v:G[u])S.insert(v);
           for(size t j=1;j<tmp.size();++j)</pre>
             if(!S.count(tmp[i].second))return
         mark[ord[i]]=1;
       return 1;
51 };
```

23

 $^{24}$ 

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### 5.14 最小斯坦納樹 DP.cpp

```
1 //n個點,其中r個要構成斯坦納樹
 2 //答案在max(dp[(1<<r)-1][k]) k=0~n-1
 3 //p表示要構成斯坦納樹的點集
 4 //0 (n^3 + n*3^r + n^2*2^r)
 5 #define REP(i,n) for(int i=0;i<(int)n;++i)</pre>
 6 const int MAXN=30, MAXM=8;// 0-base
  const int INF=0x3f3f3f3f;
 8 int dp[1<<MAXM][MAXN];</pre>
 9 int g[MAXN][MAXN];// 🗟
void init(){memset(g,0x3f,sizeof(g));}
void add_edge(int u,int v,int w){
    g[u][v]=g[v][u]=min(g[v][u],w);
13 }
void steiner(int n,int r,int *p){
    REP(k,n)REP(i,n)REP(j,n)
       g[i][j]=min(g[i][j],g[i][k]+g[k][j]);
17
     REP(i,n)g[i][i]=0;
     REP(i,r)REP(j,n)dp[1<<i][j]=g[p[i]][j];</pre>
     for(int i=1;i<(1<<r);++i){</pre>
19
20
       if(!(i&(i-1)))continue;
21
       REP(j,n)dp[i][j]=INF;
       REP(j,n){
23
         int tmp=INF:
         for(int s=i&(i-1);s;s=i&(s-1))
24
25
           tmp=min(tmp,dp[s][j]+dp[i^s][j]);
26
         REP(k,n)dp[i][k]=min(dp[i][k],g[j][k]+
              tmp);
27
28
    }
```

## 5.15 最小樹形圖 朱劉.cpp

static const int MAXN=110, MAXM=10005;

1 | template<typename T>

struct zhu liu{

struct node{

int u,v;

T w,tag;

29 }

```
node *1,*r;
       node(int u=0, int v=0, T w=0): u(u), v(v), w(v)
            w), tag(0), 1(0), r(0){}
       void down(){
         w+=tag;
         if(1)1->tag+=tag;
11
         if(r)r->tag+=tag;
12
         tag=0;
13
14
    }mem[MAXM];//靜態記憶體
15
     node *pq[MAXN*2],*E[MAXN*2];
     int st[MAXN*2],id[MAXN*2],m;
17
     void init(int n){
18
19
       for(int i=1;i<=n;++i){</pre>
20
         pq[i]=E[i]=0, st[i]=id[i]=i;
       }m=0:
21
22
23
    node *merge(node *a, node *b){//skew heap
24
       if(!a||!b)return a?a:b;
25
       a->down(),b->down();
       if(b->w<a->w)return merge(b,a);
26
27
       swap(a->1,a->r);
28
       a->1=merge(b,a->1);
29
       return a;
30
31
     void add edge(int u,int v,T w){
       if(u!=v)pq[v]=merge(pq[v],&(mem[m++]=
           node(u,v,w)));
33
     int find(int x,int *st){
34
35
       return st[x]==x?x:st[x]=find(st[x],st);
36
37
    T build(int root, int n){
       T ans=0; int N=n, all=n;
       for(int i=1;i<=N;++i){</pre>
         if(i==root||!pq[i])continue;
40
41
         while(pq[i]){
           pq[i]->down(),E[i]=pq[i];
42
43
           pq[i]=merge(pq[i]->1,pq[i]->r);
           if(find(E[i]->u,id)!=find(i,id))
44
                break;
45
         if(find(E[i]->u,id)==find(i,id))
46
              continue:
         ans+=E[i]->w:
47
         if(find(E[i]->u,st)==find(i,st)){
48
49
           if(pq[i])pq[i]->tag-=E[i]->w;
           pq[++N]=pq[i];id[N]=N;
           for(int u=find(E[i]->u,id);u!=i;u=
                find(E[u]->u,id)){
             if(pq[u])pq[u]->tag-=E[u]->w;
             id[find(u,id)]=N;
53
             pq[N]=merge(pq[N],pq[u]);
```

```
st[N]=find(i,st);
          id[find(i,id)]=N;
57
        }else st[find(i,st)]=find(E[i]->u,st)
59
      return all==1?ans:-INT_MAX;//圖不連通就
61
62 };
```

```
1 | queue < int > Q;
2 for ( i: 所有考生 ) {
   設定在第0志願;
   Q.push(考生i);
6 while(Q.size()){
   當前考生=Q.front();Q.pop();
   while ( 此考生未分發 ) {
     指標移到下一志願:
    if (已經沒有志願 or 超出志願總數)
     計算該考生在該科系加權後的總分;
     if (不符合科系需求) continue;
12
    if (目前科系有餘額) {
      依加權後分數高低順序將考生id加入科系錄
          取名單中:
15
      break;
16
    if (目前科系已額滿) {
17
      if ( 此考生成績比最低分數還高 ) {
18
       依加權後分數高低順序將考生id加入科系
19
           錄取名單;
       Q.push(被踢出的考生);
21
22
23
```

# language

# 6.1 CNF.cpp

```
1 #define MAXN 55
 struct CNF{
    int s,x,y;//s->xy \mid s->x, if y==-1
    int cost;
    CNF(){}
    CNF(int s, int x, int y, int c):s(s), x(x), y(y)
         ),cost(c){}
s int state; //規則數量
```

```
9 | map<char, int> rule; //每個字元對應到的規則
                                                          if(\sim c.y) relax(1,r,c,dp[1][k][c.x]+ 48
       小寫字母為終端字符
                                                              dp[k+1][r][c.v]+c.cost);
                                                      bellman(l,r,tok.size());
10 vector<CNF> cnf:
                                              61
                                              62
11 void init(){
                                              63
                                                  }
    state=0;
                                              64
    rule.clear();
    cnf.clear();
16 void add_to_cnf(char s,const string &p,int
    //加入一個s -> 的文法,代價為cost
```

## 5.16 穩定婚姻模板.cpp

```
for(auto c:p)if(rule.find(c)==rule.end())
19
         rule[c]=state++;
     if(p.size()==1){
20
21
       cnf.push_back(CNF(rule[s],rule[p[0]],-1,
           cost));
     }else{
22
      int left=rule[s];
23
24
       int sz=p.size();
       for(int i=0;i<sz-2;++i){</pre>
25
         cnf.push back(CNF(left,rule[p[i]],
26
             state,0));
27
         left=state++;
28
       cnf.push_back(CNF(left,rule[p[sz-2]],
29
           rule[p[sz-1]],cost));
30
31
32 vector<long long> dp[MAXN][MAXN];
33 vector (bool > neg_INF[MAXN][MAXN];//如果花費
        是負的可能會有無限小的情形
34 void relax(int 1,int r,const CNF &c,long
       long cost,bool neg c=0){
     if(!neg_INF[1][r][c.s]&&(neg_INF[1][r][c.x 18])
         ]||cost<dp[1][r][c.s])){
       if(neg_c||neg_INF[1][r][c.x]){
         dp[1][r][c.s]=0;
         neg INF[1][r][c.s]=true;
      }else dp[1][r][c.s]=cost;
40
41 }
   void bellman(int l,int r,int n){
```

if(rule.find(s)==rule.end())rule[s]=state

#### 46 } void cyk(const vector<int> &tok){ 47 for(int i=0;i<(int)tok.size();++i){</pre> 49 for(int j=0;j<(int)tok.size();++j){</pre> 50

44

45

51

52

53

54

12

15 }

```
dp[i][j]=vector<long long>(state+1,
        INT MAX);
    neg_INF[i][j]=vector<bool>(state+1,
         false);
 dp[i][i][tok[i]]=0;
 bellman(i,i,tok.size());
for(int r=1;r<(int)tok.size();++r){</pre>
 for(int l=r-1;l>=0;--1){
    for(int k=1;k<r;++k)</pre>
      for(auto c:cnf)
```

if(c.y==-1)relax(l,r,c,dp[l][r][c.x]+c

for(int k=1;k<=state;++k)</pre>

.cost.k==n):

for(auto c:cnf)

# 7 Linear Programming

1 typedef double T;//POJ 3155

2 const int MAXN=105:

3 struct edge{

```
int u,v;
     edge(int u=0, int v=0, T w=0):u(u),v(v),w(w)
7 };
8 vector<edge> E;
9 int n,m;// 1-base
10 | T de [MAXN], pv [MAXN]; // 每 個 點 的 邊 權 和 和 點 權 (
        有些題目會給)
  void init(){
    E.clear();
    for(int i=1;i<=n;++i)de[i]=pv[i]=0;</pre>
14 }
void add edge(int u,int v,T w){
    E.push_back(edge(u,v,w));
    de[u]+=w,de[v]+=w;
19 T U;//二分搜的最大值
  void get_U(){
21
    U=0:
     for(int i=1;i<=n;++i)U+=2*pv[i];</pre>
     for(size_t i=0;i<E.size();++i)U+=E[i].w;</pre>
24 }
25 | ISAP<T> isap;//網路流
26 int s,t;//原匯點
  void build(T L){
    isap.init(n+2);
     for(size_t i=0;i<E.size();++i)</pre>
       isap.add_edge(E[i].u,E[i].v,E[i].w);
     for(int v=1; v<=n; ++v){</pre>
      isap.add_edge(s,v,U);
       isap.add edge(v,t,U+2*L-de[v]-2*pv[v]);
34
35 }
  int main(){
     while(~scanf("%d%d",&n,&m)){
       if(!m){
         puts("1\n1");
         continue;
       init();
       int u,v;
       for(int i=0;i<m;++i){</pre>
         scanf("%d%d",&u,&v);
         add_edge(u,v,1);
46
```

## 7.1 最大密度子圖.cpp

```
build(1);
  isap.min cut(s,t);
 vector<int> ans;
  for(int i=1:i<=n:++i)</pre>
   if(isap.vis[i])ans.push_back(i);
  printf("%d\n",ans.size());
  for(size_t i=0;i<ans.size();++i)</pre>
    printf("%d\n",ans[i]);
return 0;
```

get U();

52

53

s=n+1, t=n+2;

T l=0, r=U, k=1.0/(n\*n);

T mid=(1+r)/2;

if(res>0)l=mid;

build(mid);

else r=mid;

while(r-1>k){//二分搜最大值

T res=(U\*n-isap.isap(s,t))/2;

# Number Theory

#### 8.1 basic.cpp

```
1 template < typename T>
  void gcd(const T &a,const T &b,T &d,T &x,T &
     if(!b) d=a,x=1,y=0;
    else gcd(b,a%b,d,y,x), y-=x*(a/b);
  long long int phi[N+1];
   void phiTable(){
    for(int i=1;i<=N;i++)phi[i]=i;</pre>
     for(int i=1;i<=N;i++)for(x=i*2;x<=N;x+=i)</pre>
         phi[x]-=phi[i];
  void all divdown(const LL &n) {// all n/x
    for(LL a=1;a<=n;a=n/(n/(a+1))){
      // dosomething;
14
15 }
16 const int MAXPRIME = 1000000;
  int iscom[MAXPRIME], prime[MAXPRIME],
       primecnt;
  int phi[MAXPRIME], mu[MAXPRIME];
  void sieve(void){
    memset(iscom,0,sizeof(iscom));
    primecnt = 0;
     phi[1] = mu[1] = 1;
     for(int i=2;i<MAXPRIME;++i) {</pre>
      if(!iscom[i]) {
         prime[primecnt++] = i;
         mu[i] = -1;
27
         phi[i] = i-1;
       for(int j=0;j<primecnt;++j) {</pre>
         int k = i * prime[i];
         if(k>=MAXPRIME) break;
31
         iscom[k] = prime[j];
```

```
if(i%prime[j]==0) {
                                                   94 LL Tonelli Shanks(const LL &n, const LL &p) 154 //求sqrt(N)的連分數
                                                                                                                                                                for(int j=i+1;j<n;++j)</pre>
                                                                                                     155 public static void Pell(int n){
           mu[k] = 0;
                                                                                                                                                         11
                                                                                                                                                                  if(s[i]<s[i])++t;
                                                        // x^2 = n \pmod{p}
35
           phi[k] = phi[i] * prime[j];
                                                                                                           BigInteger N,p1,p2,q1,q2,a0,a1,a2,g1,g2,h1 12
                                                                                                                                                                res+=t*factorial[n-i-1];
                                                        if(n==0) return 0;
36
           break;
                                                                                                                ,h2,p,q;
                                                                                                                                                         13
                                                                                                           g1=q2=p1=BigInteger.ZERO;
37
         } else {
                                                        if(Legendre(n,p)!=1) while(1) { puts("SORT 157
                                                                                                                                                              return res;
                                                                                                                                                         14
                                                              ROOT does not exist"); }
                                                                                                           h1=q1=p2=BigInteger.ONE;
           mu[k] = -mu[i];
                                                                                                                                                         15
39
           phi[k] = phi[i] * (prime[j]-1);
                                                                                                           a0=a1=BigInteger.valueOf((int)Math.sqrt
                                                                                                                                                         16
                                                                                                                                                            vector<int> decode(int a.int n){
40
                                                        LL 0 = p-1;
                                                                                                                (1.0*n));
                                                                                                                                                              vector<int> res;
                                                   99
                                                        while( !(Q&1) ) { Q>>=1; ++S; }
                                                                                                           BigInteger ans=a0.multiply(a0);
                                                                                                                                                              vector<bool> vis(n,0);
41
                                                  100
                                                                                                     160
42
                                                  101
                                                        if(S==1) return modexp(n\%p,(p+1)/4,p);
                                                                                                           if(ans.equals(BigInteger.valueOf(n))){
                                                                                                                                                              for(int i=n-1;i>=0;--i){
                                                                                                     161
43
                                                  102
                                                        LL z = 2;
                                                                                                     162
                                                                                                             System.out.println("No solution!");
                                                                                                                                                         20
                                                                                                                                                                int t=a/factorial[i],j;
                                                        for(;Legendre(z,p)!=-1:++z)
                                                                                                             return :
                                                                                                                                                                for(i=0:i<n:++i)</pre>
44
                                                  103
                                                                                                     163
                                                                                                                                                         21
                                                                                                                                                                  if(!vis[j]){
   bool g_test(const LL &g, const LL &p, const
                                                        LL c = modexp(z,0,p);
                                                                                                                                                         22
                                                  104
                                                                                                     164
                                                                                                           while(true){
       vector<LL> &v) {
                                                        LL R = modexp(n\%p,(Q+1)/2,p), t = modexp(n_{165})
                                                                                                                                                         23
                                                                                                                                                                    if(t==0)break:
                                                                                                             g2=a1.multiply(h1).substract(g1);
    for(int i=0;i<v.size();++i)</pre>
                                                             %p,Q,p);
                                                                                                     166
                                                                                                                                                         24
                                                                                                                                                                    --t:
47
       if(modexp(g,(p-1)/v[i],p)==1)
                                                  106
                                                        int M = S;
                                                                                                     167
                                                                                                             h2=N.substract(g2.pow(2)).divide(h1);
                                                                                                                                                         25
         return false:
                                                        while(1) {
                                                                                                             a2=g2.add(a0).divide(h2):
                                                                                                                                                                res.push back(j);
48
                                                  107
                                                                                                     168
                                                                                                                                                         26
    return true;
                                                          if(t==1) return R;
                                                                                                             p=a1.multiply(p2).add(p1);
                                                                                                                                                         27
49
                                                  108
                                                                                                     169
                                                                                                                                                                vis[j]=1;
                                                          LL b = modexp(c,1L << (M-i-1),p);
                                                                                                             q=a1.multiply(q2).add(q1);
50
                                                  109
                                                                                                     170
                                                                                                                                                         28
                                                                                                                                                                a%=factorial[i];
                                                          R = LLmul(R,b,p);
   LL primitive root(const LL &p) {
                                                                                                             if(p.pow(2).substract(N.multiply(q.pow
                                                  110
                                                                                                                                                         29
52
    if(p==2) return 1;
                                                  111
                                                          t = LLmul( LLmul(b,b,p), t, p);
                                                                                                                  (2))).compareTo(BigInteger.ONE)==0)
                                                                                                                                                         30
                                                                                                                                                              return res;
    vector<LL> v:
                                                          c = LLmul(b,b,p);
53
                                                  112
                                                                                                                  break:
54
    Factor(p-1.v):
                                                  113
                                                          M = i:
                                                                                                     172
                                                                                                             g1=g2:h1=h2:a1=a2:
55
    v.erase(unique(v.begin(), v.end()), v.end
                                                                                                     173
                                                                                                             p1=p2;p2=p;
                                                 114
          ());
                                                  115
                                                        return -1;
                                                                                                     174
                                                                                                             q1=q2;q2=q;
    for(LL g=2;g<p;++g)</pre>
                                                                                                                                                            8.4 FFT.cpp
                                                  116
                                                                                                     175
56
57
      if(g_test(g,p,v))
                                                                                                           System.out.println(p+" "+q);
                                                  117
                                                                                                     176
         return g;
58
                                                      template<tvpename T>
                                                                                                     177
    puts("primitive_root NOT FOUND");
                                                      T Euler(T n){
59
                                                                                                                                                          1 | template < typename T, typename VT = vector <
60
    return -1;
                                                  120
                                                        T ans=n;
                                                                                                                                                                 complex<T>>>
61
                                                  121
                                                        for(T i=2:i*i<=n:++i){</pre>
                                                                                                                                                            struct FFT{
   int Legendre(const LL &a, const LL &p) {
                                                  122
                                                          if(n%i==0){
                                                                                                                                                              const T pi;
                                                                                                                bit set.cpp
       return modexp(a%p,(p-1)/2,p); }
                                                  123
                                                            ans=ans/i*(i-1);
                                                                                                                                                              FFT(const T pi=acos((T)-1)):pi(pi){}
                                                  124
                                                            while(n%i==0)n/=i;
                                                                                                                                                              unsigned bit reverse(unsigned a,int len){
   LL inv(const LL &a, const LL &n) {
                                                  125
                                                                                                                                                                a=((a\&0x55555555U)<<1)|((a\&0xAAAAAAAAU)
                                                                                                       1 void sub set(int S){
    LL d,x,y;
65
                                                  126
                                                                                                                                                                     >>1);
                                                                                                           int sub=S;
    gcd(a,n,d,x,y);
                                                  127
                                                        if(n>1)ans=ans/n*(n-1);
                                                                                                                                                                a=((a&0x33333333U)<<2)|((a&0xCCCCCCCU)
                                                                                                           do{
    return d==1 ? (x+n)%n : -1;
                                                        return ans;
                                                  128
                                                                                                             //對某集合的子集合的處理
                                                  129
                                                                                                                                                                a = ((a\&0x0F0F0F0FU) < <4) | ((a\&0xF0F0F0F0U)
                                                                                                             sub=(sub-1)&S;
                                                  130
                                                                                                           }while(sub!=S);
   int inv[maxN];
                                                      //Chinese remainder theorem
                                                                                                                                                                a=((a&0x00FF00FFU)<<8)|((a&0xFF00FF00U)
                                                  131
   LL invtable(int n,LL P){
                                                      template<typename T>
                                                                                                         void k sub set(int k,int n){
    inv[1]=1;
                                                                                                                                                                a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)
                                                      T pow mod(T n, T k, T m){
                                                                                                           int comb=(1<<k)-1,S=1<<n;</pre>
73
    for(int i=2:i<n:++i)</pre>
                                                        T ans=1:
                                                  134
                                                                                                                                                                     >>16);
                                                                                                           while(comb<S){</pre>
       inv[i]=(P-(P/i))*inv[P%i]%P;
                                                        for (n=(n)=m?n\%m:n);k;k>>=1){
74
                                                  135
                                                                                                                                                                return a>>(32-len);
                                                                                                                                                         11
                                                                                                             //對大小為k的子集合的處理
75
                                                  136
                                                         if(k&1)ans=ans*n%m;
                                                                                                                                                         12
                                                                                                             int x=comb&-comb,y=comb+x;
                                                                                                      12
                                                  137
                                                         n=n*n%m;
                                                                                                                                                              void fft(bool is inv,VT &in,VT &out,int N)
76
                                                                                                             comb = ((comb\&\sim y)/x>>1)|y;
                                                                                                      13
   LL log mod(const LL &a, const LL &b, const
                                                  138
                                                                                                      14
                                                                                                                                                                int bitlen=__lg(N),num=is_inv?-1:1;
       LL &p) {
                                                  139
                                                        return ans;
                                                                                                                                                         14
                                                                                                      15 }
                                                                                                                                                                for(int i=0;i<N;++i)out[bit reverse(i,</pre>
     // a ^ x = b \pmod{p}
                                                  140 }
                                                                                                                                                         15
                                                                                                                                                                     bitlen)]=in[i];
    int m=sqrt(p+.5), e=1;
                                                      template<typename T>
    LL v=inv(modexp(a,m,p), p);
                                                      T crt(vector<T> &m, vector<T> &a){
                                                                                                                                                                for(int step=2;step<=N;step<<=1){</pre>
    map<LL.int> x:
                                                        T M=1.tM.ans=0:
                                                                                                                                                                  const int mh=step>>1:
                                                                                                                                                         17
                                                                                                         8.3 cantor expansion.cpp
                                                        for(int i=0;i<(int)m.size();++i)M*=m[i];</pre>
                                                                                                                                                                  for(int i=0;i<mh;++i){</pre>
82
    x[1]=0;
                                                  144
     for(int i=1;i<m;++i) {</pre>
                                                  145
                                                        for(int i=0;i<(int)a.size();++i){</pre>
                                                                                                                                                                    complex<T> wi=exp(complex<T>(0,i*num
      e = LLmul(e,a,p);
                                                  146
                                                          tM=M/m[i];
                                                                                                                                                                          *pi/mh));
85
       if(!x.count(e)) x[e] = i;
                                                  147
                                                          ans=(ans+(a[i]*tM%M)*pow mod(tM,Euler(m[
                                                                                                       1 int factorial[MAXN];
                                                                                                                                                                    for(int j=i;j<N;j+=step){</pre>
                                                                                                       2 void init(){
86
                                                               i])-1,m[i])%M)%M;
                                                                                                                                                         21
                                                                                                                                                                      int k=i+mh;
                                                          /*如果m[i]是質數·Euler(m[i])-1=m[i]-2
                                                                                                           factorial[0]=1;
                                                                                                                                                                       complex<T> u=out[j],t=wi*out[k];
     for(int i=0;i<m;++i) {</pre>
                                                                                                                                                         22
                                                  148
                                                                                                           for(int i=1:i<=MAXN:++i)factorial[i]=</pre>
      if(x.count(b)) return i*m + x[b];
                                                                                                                                                         23
                                                                                                                                                                      out[i]=u+t:
                                                               就不用算Euler了*/
                                                                                                                factorial[i-1]*i;
89
       b = LLmul(b,v,p);
                                                                                                                                                         24
                                                                                                                                                                       out[k]=u-t;
                                                  149
90
    }
                                                                                                                                                         25
                                                  150
                                                        return ans;
    return -1;
                                                                                                        int encode(const vector<int> &s){
                                                                                                                                                         26
                                                  151 }
92
                                                                                                           int n=s.size(),res=0;
                                                                                                                                                         27
                                                  152
                                                                                                           for(int i=0;i<n;++i){</pre>
                                                                                                                                                         28
                                                                                                                                                                if(is inv)for(int i=0;i<N;++i)out[i]/=N;</pre>
93
                                                  153 //java code
                                                                                                             int t=0;
```

#### 8.6 FWT.cpp 30 }; 1 int mod fact(int n,int &e){ for(int y=0;y<r;y++)</pre> e=0; 46 for(int x=0;x<c;++x) if(n==0)return 1; m[y][x]=t.m[y][c+x]/t.m[y][y];47 1 | vector<int> F OR T(vector<int> f, bool int res=mod fact(n/P,e); 48 return true; 8.5 find real root.cpp inverse){ 49 for(int i=0; (2<<i)<=f.size(); ++i)</pre> if((n/P)%2==0)return res\*fact[n%P]%P; T gas(){ for(int j=0; j<f.size(); j+=2<<i)</pre> return res\*(P-fact[n%P])%P; 51 vector<T> lazv(r,1); for(int k=0; k<(1<<i); ++k)</pre> $1 // an*x^n + ... + a1x + a0 = 0;$ 52 bool sign=false; f[j+k+(1<<i)] += f[j+k]\*(inverse)2 int sign(double x){ int Cmod(int n,int m){ for(int i=0;i<r;++i){</pre> **return** x < -eps ? -1 : x > eps; ?-1:1); int a1,a2,a3,e1,e2,e3; 54 if( m[i][i]==0 ){ return f: a1=mod fact(n,e1); 55 int j=i+1; a2=mod fact(m.e2): while(j<r&&!m[j][i])j++; vector<int> rev(vector<int> A) { double get(const vector<double>&coef, double a3=mod fact(n-m,e3); if(j==r)continue; 57 for(int i=0; i<A.size(); i+=2)</pre> if(e1>e2+e3)return 0: m[i].swap(m[j]); swap(A[i],A[i^(A.size()-1)]); double e = 1, s = 0; return a1\*inv(a2\*a3%P,P)%P; 59 sign=!sign; for(auto i : coef) s += i\*e, e \*= x; 11 return A: 60 12 } return s; for(int j=0;j<r;++j){</pre> 61 vector<int> F\_AND\_T(vector<int> f, bool 10 if(i==j)continue; 62 inverse){ 63 lazy[j]=lazy[j]\*m[i][i]; double find(const vector<double>&coef, int n return rev(F\_OR\_T(rev(f), inverse)); 8.9 Matrix.cpp 64 T mx=m[j][i]; 15 } , double lo, double hi){ 65 for(int k=0;k<c;++k)</pre> vector<int> F\_XOR\_T(vector<int> f, bool double sign lo, sign hi; m[j][k]=m[j][k]\*m[i][i]-m[i][k]\*mxinverse){ if( !(sign lo = sign(get(coef,lo))) ) 1 template<typename T> for(int i=0; (2<<i)<=f.size(); ++i)</pre> return lo; struct Matrix{ 67 for(int j=0; j<f.size(); j+=2<<i)</pre> if( !(sign\_hi = sign(get(coef,hi))) ) 18 using rt = std::vector<T>; 68 19 for(int k=0; k<(1<<i); ++k){</pre> return hi; using mt = std::vector<rt>; 69 T det=sign?-1:1; 20 int u=f[j+k], v=f[j+k+(1<<i)];</pre> if(sign lo \* sign hi > 0) return INF; using matrix = Matrix<T>; 70 for(int i=0;i<r;++i){</pre> 21 f[j+k+(1<< i)] = u-v, f[j+k] = u+v;for(int stp = 0; stp < 100 && hi - lo > int r.c: 71 det = det\*m[i][i]; eps; ++stp){ 22 72 det = det/lazy[i]; if(inverse) for(auto &a:f) a/=f.size(); double m = (lo+hi)/2.0; Matrix(int r,int c):r(r),c(c),m(r,rt(c)){} for(auto &j:m[i])j/=lazy[i]; return f; 24 19 int sign mid = sign(get(coef.m)); rt& operator[](int i){return m[i];} 74 25 } 20 if(!sign mid) return m; 10 matrix operator+(const matrix &a){ 75 return det; 21 if(sign lo\*sign mid < 0) hi = m;</pre> 11 matrix rev(r,c); 76 else lo = m: 77 }; 22 for(int i=0;i<r;++i)</pre> 23 13 for(int j=0;j<c;++j)</pre> 8.7 LinearCongruence.cpp 24 return (lo+hi)/2.0: rev[i][j]=m[i][j]+a.m[i][j]; 14 25 15 return rev; 8.10 MillerRobin.cpp 26 16 1 | pair<LL,LL> LinearCongruence(LL a[],LL b[], vector<double> cal(vector<double>coef, int n matrix operator-(const matrix &a){ LL m[], int n) { ) { matrix rev(r,c); $// a[i]*x = b[i] \pmod{m[i]}$ vector<double>res: for(int i=0;i<r;++i)</pre> 1 LL LLmul(LL a, LL b, const LL &mod) { for(int i=0;i<n;++i) {</pre> $if(n == 1){$ LL ans=0: 29 for(int j=0;j<c;++j)</pre> 20 LL x, y, d = extgcd(a[i],m[i],x,y);30 if(sign(coef[1])) res.pb(-coef[0]/coef while(b) { rev[i][j]=m[i][j]-a.m[i][j]; if(b[i]%d!=0) return make pair(-1LL,0LL) **if**(b&1) { [1]); 22 return rev: return res; 31 23 m[i] /= d; if(ans>=mod) ans-=mod; 32 matrix operator\*(const matrix &a){ b[i] = LLmul(b[i]/d,x,m[i]);vector<double>dcoef(n); 33 matrix rev(r,a.c); 34 for(int i = 0; i < n; ++i) dcoef[i] = coef matrix tmp(a.c,a.r); a<<=1, b>>=1: LL lastb = b[0], lastm = m[0]; [i+1]\*(i+1); if(a>=mod) a-=mod; for(int i=0;i<a.r;++i)</pre> for(int i=1;i<n;++i) {</pre> vector<double>droot = cal(dcoef, n-1); 10 for(int j=0;j<a.c;++j)</pre> LL x, y, d = extgcd(m[i],lastm,x,y); droot.insert(droot.begin(), -INF); 11 11 return ans; 36 tmp[j][i]=a.m[i][j]; if((lastb-b[i])%d!=0) return make pair 37 droot.pb(INF); 12 30 for(int i=0;i<r;++i)</pre> (-1LL,0LL); LL mod mul(LL a.LL b.LL m){ 38 for(int i = 0; i+1 < droot.size(); ++i){</pre> for(int j=0;j<a.c;++j)</pre> 13 lastb = LLmul((lastb-b[i])/d,x,(lastm/d) $a\%=m,b\%=m;/* fast for m < 2^58 */$ 39 double tmp = find(coef, n, droot[i], 14 32 for(int k=0;k<c;++k)</pre> )\*m[i]; droot[i+1]); 15 LL y=(LL)((double)a\*b/m+0.5);33 rev.m[i][j]+=m[i][k]\*tmp[j][k]; lastm = (lastm/d)\*m[i]; 14 LL r=(a\*b-v\*m)%m: 40 if(tmp < INF) res.pb(tmp);</pre> return rev; lastb = (lastb+b[i])%lastm; 15 17 return r<0?r+m:r;</pre> 41 16 18 42 return res; bool inverse(){ return make\_pair(lastb<0?lastb+lastm:lastb</pre> 17 43 template<typename T> Matrix t(r,r+c); ,lastm); T pow(T a,T b,T mod){//a^b%mod 44 for(int y=0;y<r;y++){</pre> 18 } int main () { t.m[y][c+y] = 1;for(;b;a=mod mul(a,a,mod),b>>=1) vector<double>ve; 40 for(int x=0;x<c;++x) vector<double>ans = cal(ve, n); 23 if(b&1)ans=mod\_mul(ans,a,mod); 41 t.m[y][x]=m[y][x]; 24 return ans: // 視情況把答案 +eps, 避免 -0 42 Lucas.cpp 25 **if**(!t.gas())

return false;

26 int sprp[3]={2,7,61};//int範圍可解

```
27 int llsprp
        [7] = \{2,325,9375,28178,450775,9780504,
28 1795265022};//至少unsigned Long Long範圍
  template<typename T>
  bool isprime(T n, int *sprp, int num){
    if(n==2)return 1;
    if(n<2||n%2==0)return 0;
33
    int t=0:
    T u=n-1;
34
     for(;u%2==0;++t)u>>=1;
     for(int i=0;i<num;++i){</pre>
       T a=sprp[i]%n;
38
       if(a==0||a==1||a==n-1)continue;
       T x=pow(a,u,n);
39
       if(x==1||x==n-1)continue;
       for(int j=0;j<t;++j){</pre>
         x=mod_mul(x,x,n);
42
         if(x==1)return 0;
         if(x==n-1)break;
       if(x==n-1)continue;
       return 0;
     return 1;
```

#### 8.11 NTT.cpp

1 2615053605667\*(2^18)+1,3

2 15\*(2^27)+1.31

```
3 479*(2^21)+1,3
4 7*17*(2^23)+1,3
5 3*3*211*(2^19)+1,5
6 25*(2^22)+1,3
  template<typename T, typename VT=vector<T> >
   struct NTT{
     const T P,G;
     NTT(T p=(1<<23)*7*17+1,T g=3):P(p),G(g){}
     unsigned bit reverse(unsigned a,int len){
       //Look FFT.cpp
12
13
     T pow_mod(T n,T k,T m){
14
15
       T ans=1;
       for (n=(n)=m?n\%m:n); k; k>>=1){
         if(k&1)ans=ans*n%m;
         n=n*n%m;
19
20
21
     void ntt(bool is_inv,VT &in,VT &out,int N)
23
       int bitlen= lg(N);
       for(int i=0;i<N;++i)out[bit_reverse(i,</pre>
24
            bitlen) | = in[i];
       for(int step=2,id=1;step<=N;step<<=1,++</pre>
26
         T wn=pow mod(G,(P-1)>>id,P),wi=1,u,t;
         const int mh=step>>1;
         for(int i=0;i<mh;++i){</pre>
           for(int j=i;j<N;j+=step){</pre>
30
             u=out[i],t=wi*out[i+mh]%P;
             out[i]=u+t;
31
             out[j+mh]=u-t;
```

```
if(out[j]>=P)out[j]-=P;
              if(out[j+mh]<0)out[j+mh]+=P;</pre>
34
35
36
            wi=wi*wn%P;
37
38
39
       if(is inv){
          for(int i=1;i<N/2;++i)swap(out[i],out[</pre>
          T invn=pow mod(N,P-2,P);
41
42
          for(int i=0;i<N;++i)out[i]=out[i]*invn</pre>
43
44
45 };
```

### 8.12 Simpson.cpp

double c=a+(b-a)/2;

1 | double simpson(double a, double b){

## 8.13 外星模運算.cpp

```
1 / a[0]^{a[1]^a[2]^{...}}
 2 #define maxn 1000000
 3 int euler[maxn+5];
 4 bool is prime[maxn+5];
 5 void init_euler(){
    is prime[1]=1;//一不是質數
     for(int i=1;i<=maxn;i++)euler[i]=i;</pre>
     for(int i=2;i<=maxn;i++){</pre>
       if(!is prime[i]){//是質數
         euler[i]--:
         for(int j=i<<1;j<=maxn;j+=i){</pre>
12
           is prime[j]=1;
           euler[j]=euler[j]/i*(i-1);
17 }
18 LL pow(LL a, LL b, LL mod) {//a^b%mod
    LL ans=1;
     for(;b;a=a*a%mod,b>>=1)
       if(b&1)ans=ans*a%mod;
     return ans;
22
```

```
24 bool isless(LL *a,int n,int k){
     if(*a==1)return k>1;
                                                    28
     if(--n==0)return *a<k;</pre>
26
                                                    29
27
     int next=0:
                                                    30
28
     for(LL b=1;b<k;++next)</pre>
       b*=*a:
30
     return isless(a+1,n,next);
31 }
   LL high pow(LL *a, int n, LL mod){
     if(*a==1||--n==0)return *a%mod;
34
     int k=0,r=euler[mod];
     for(LL tma=1:tma!=pow(*a,k+r,mod):++k)
        tma=tma*(*a)%mod;
     if(isless(a+1,n,k))return pow(*a,high pow(
          a+1.n.k).mod):
     int tmd=high_pow(a+1,n,r), t=(tmd-k+r)%r;
39
     return pow(*a,k+t,mod);
40
41 LL a[1000005];
42 int t.mod:
43
   int main(){
     init euler():
     scanf("%d",&t):
46
     #define n 4
47
     while(t--){
        for(int i=0;i<n;++i)scanf("%lld",&a[i]);</pre>
48
        scanf("%d",&mod);
49
50
        printf("%lld\n", high pow(a,n, mod));
51
52
     return 0;
```

## 8.14 質因數分解.cpp

if(n<MAXPRIME) {</pre>

```
1 LL func(const LL n,const LL mod,const int c)
    return (LLmul(n,n,mod)+c+mod)%mod;
5 LL pollorrho(const LL n, const int c) {//循
        環節長度
    LL a=1, b=1;
     a=func(a,n,c)%n;
     b=func(b,n,c)%n; b=func(b,n,c)%n;
     while(gcd(abs(a-b),n)==1) {
       a=func(a,n,c)%n;
11
       b=func(b,n,c)%n; b=func(b,n,c)%n;
12
13
     return gcd(abs(a-b),n);
14
15
   void prefactor(LL &n, vector<LL> &v) {
    for(int i=0;i<12;++i) {</pre>
       while(n%prime[i]==0) {
         v.push_back(prime[i]);
19
20
         n/=prime[i];
21
22
23
^{24}
  void smallfactor(LL n, vector<LL> &v) {
```

```
31
       v.push back(n);
     } else {
32
       for(int i=0:i<primecnt&&prime[i]*prime[i</pre>
            |<=n;++i) {</pre>
         while(n%prime[i]==0) {
34
35
           v.push back(prime[i]);
36
           n/=prime[i];
37
       if(n!=1) v.push back(n);
41
   void comfactor(const LL &n, vector<LL> &v) {
44
     if(n<1e9) {
       smallfactor(n,v);
45
46
       return:
47
48
     if(Isprime(n)) {
       v.push back(n);
       return;
51
     LL d;
     for(int c=3;;++c) {
53
54
       d = pollorrho(n,c);
55
       if(d!=n) break;
56
57
     comfactor(d,v);
58
     comfactor(n/d,v);
59
   void Factor(const LL &x, vector<LL> &v) {
     LL n = x;
     if(n==1) { puts("Factor 1"); return; }
     prefactor(n,v);
     if(n==1) return;
     comfactor(n,v);
66
67
     sort(v.begin(),v.end());
   void AllFactor(const LL &n, vector<LL> &v) {
     vector<LL> tmp;
     Factor(n,tmp);
     v.clear();
     v.push back(1);
     int len;
     LL now=1;
     for(int i=0;i<tmp.size();++i) {</pre>
       if(i==0 || tmp[i]!=tmp[i-1]) {
         len = v.size();
         now = 1;
       now*=tmp[i];
       for(int j=0;j<len;++j)</pre>
         v.push_back(v[j]*now);
85
```

while(isp[(int)n]) {

n/=isp[(int)n];

v.push back(isp[(int)n]);

#### other

## 9.1 WhatDay.cpp

```
int whatday(int y,int m,int d){
   if(m<=2)m+=12,--v:
   if(y<1752||y==1752&&m<9||y==1752&&m==9&&d
      return (d+2*m+3*(m+1)/5+y+y/4+5)%7;
   return (d+2*m+3*(m+1)/5+y+y/4-y/100+y/400)
```

### 9.2 上下最大正方形.cpp

```
1 void solve(int n,int a[],int b[]){// 1-base
     int ans=0:
     deque<int>da.db:
     for(int l=1,r=1;r<=n;++r){</pre>
       while(da.size()&&a[da.back()]>=a[r]){
         da.pop back();
       da.push back(r);
       while(db.size()&&b[db.back()]>=b[r]){
         db.pop_back();
12
       db.push_back(r);
       for(int d=a[da.front()]+b[db.front()];r-
            1+1>d;++1){
         if(da.front()==1)da.pop_front();
15
         if(db.front()==1)db.pop front();
         if(da.size()&&db.size()){
17
           d=a[da.front()]+b[db.front()];
18
19
20
       ans=max(ans,r-l+1);
21
    printf("%d\n",ans);
22
```

# 9.3 最大矩形.cpp

```
1 | LL max rectangle(vector<int> s){
   stack<pair<int,int > > st;
    st.push(make_pair(-1,0));
   s.push back(0);
   LL ans=0;
    for(size t i=0;i<s.size();++i){</pre>
      int h=s[i]:
      pair<int,int > now=make_pair(h,i);
      while(h<st.top().first){</pre>
        now=st.top();
        st.pop();
        ans=max(ans,(LL)(i-now.second)*now.
             first);
      if(h>st.top().first){
```

```
st.push(make pair(h,now.second));
16
17
18
    return ans;
```

53

54

55

56

57

58

59

60

61

62

64

66

75

77

84

85

87

88

89

95

96

97

# String

class ac automaton{

## 10.1 AC 自動機.cpp

1 template < char L='a', char R='z'>

```
struct joe{
       int next[R-L+1],fail,efl,ed,cnt_dp,vis;
                                                    65
       joe():ed(0),cnt dp(0),vis(0){
         for(int i=0;i<=R-L;++i)next[i]=0;</pre>
    };
   public:
     std::vector<joe> S;
                                                    69
     std::vector<int> a:
                                                    70
     int qs,qe,vt;
                                                    71
12
13
     ac automaton():S(1),qs(0),qe(0),vt(0){}
                                                    72
                                                    73
     void clear(){
14
                                                    74
15
      q.clear();
16
       S.resize(1):
17
       for(int i=0;i<=R-L;++i)S[0].next[i]=0;</pre>
                                                    76
       S[0].cnt dp=S[0].vis=qs=qe=vt=0;
19
     void insert(const char *s){
20
                                                    78
       int o=0:
21
                                                    79
22
       for(int i=0,id;s[i];++i){
                                                    80
23
         id=s[i]-L;
                                                    81
24
         if(!S[o].next[id]){
           S.push_back(joe());
25
26
            S[o].next[id]=S.size()-1;
27
28
         o=S[o].next[id];
29
30
       ++S[o].ed;
31
32
     void build fail(){
33
       S[0].fail=S[0].efl=-1;
34
       q.clear();
       q.push_back(0);
                                                    90
35
       ++ae;
                                                    91
36
       while(qs!=qe){
37
                                                    92
         int pa=q[qs++],id,t;
                                                    93
         for(int i=0;i<=R-L;++i){</pre>
                                                    94
39
40
           t=S[pa].next[i];
           if(!t)continue;
41
            id=S[pa].fail;
            while(~id&&!S[id].next[i])id=S[id].
43
            S[t].fail=~id?S[id].next[i]:0;
44
            S[t].efl=S[S[t].fail].ed?S[t].fail:S
45
                 [S[t].fail].efl;
                                                   100
            q.push back(t);
                                                   101
47
            ++ae;
                                                   102
48
                                                   103
```

```
/*DP出每個前綴在字串s出現的次數並傳回所有
                                       106
    字串被s匹配成功的次數O(N+M)*/
                                       107
int match 0(const char *s){
                                       108
 int ans=0,id,p=0,i;
                                       109
 for(i=0;s[i];++i){
   id=s[i]-L;
                                       110
   while(!S[p].next[id]&&p)p=S[p].fail;
                                       111
   if(!S[p].next[id])continue;
                                       112 };
   p=S[p].next[id];
   ++S[p].cnt dp;/*匹配成功則它所有後綴都
        可以被匹配(DP計算)*/
 for(i=qe-1;i>=0;--i){
   ans+=S[q[i]].cnt_dp*S[q[i]].ed;
   if(~S[q[i]].fail)S[S[q[i]].fail].
        cnt_dp+=S[q[i]].cnt_dp;
 return ans;
/*多串匹配走efL邊並傳回所有字串被s匹配成功
    的 次 數 O(N*M^1.5)*/
int match 1(const char *s)const{
 int ans=0,id,p=0,t;
 for(int i=0;s[i];++i){
   id=s[i]-L;
   while(!S[p].next[id]&&p)p=S[p].fail;
   if(!S[p].next[id])continue;
   p=S[p].next[id];
   if(S[p].ed)ans+=S[p].ed;
   for(t=S[p].efl;~t;t=S[t].efl){
     ans+=S[t].ed;/*因為都走efL邊所以保證
         匹配成功*/
 return ans;
/*枚舉(s的子字串nA)的所有相異字串各恰一次
    並傳回次數O(N*M^(1/3))*/
int match_2(const char *s){
 int ans=0,id,p=0,t;
 ++vt:
 /*把戳記vt+=1,只要vt沒溢位,所有S[p].
      vis==vt就會變成false
 這種利用vt的方法可以0(1)歸零vis陣列*/
 for(int i=0;s[i];++i){
   id=s[i]-L;
   while(!S[p].next[id]&&p)p=S[p].fail;
   if(!S[p].next[id])continue;
   p=S[p].next[id];
   if(S[p].ed&&S[p].vis!=vt){
     S[p].vis=vt;
     ans+=S[p].ed;
   for(t=S[p].efl;~t&&S[t].vis!=vt;t=S[t
        1.ef1){
     S[t].vis=vt;
     ans+=S[t].ed;/*因為都走efl邊所以保證
         匹配成功*/
 return ans;
```

## 10.2 hash.cpp

void evolution(){

for(qs=1;qs!=qe;){

int p=q[qs++];

/\*把AC自動機變成真的自動機\*/

for(int i=0;i<=R-L;++i)</pre>

pl.faill.next[i]:

if(S[p].next[i]==0)S[p].next[i]=S[S[

```
1 #define MAXN 1000000
2 #define mod 1073676287
3 /*mod 必須要是質數*/
 4 typedef long long T;
 5 char s[MAXN+5];
6 T h[MAXN+5]; /*hash 陣列*/
  T h base[MAXN+5];/*h base[n]=(prime^n)%mod*/
  void hash init(int len,T prime){
    h base[0]=1;
    for(int i=1;i<=len;++i){</pre>
      h[i]=(h[i-1]*prime+s[i-1])%mod;
11
      h base[i]=(h base[i-1]*prime)%mod;
13
14 }
  T get hash(int l,int r){/*閉區間寫法,設編號
       為0 ~ Len-1*/
     return (h[r+1]-(h[1]*h_base[r-1+1])%mod+
         mod)%mod:
```

## 10.3 KMP.cpp

```
1 /*產生fail function*/
  void kmp fail(char *s,int len,int *fail){
    int id=-1;
    for(int i=1:i<len:++i){</pre>
      while(~id&&s[id+1]!=s[i])id=fail[id];
      if(s[id+1]==s[i])++id;
      fail[i]=id;
11 /*以字串B匹配字串A·傳回匹配成功的數量(用B的
  int kmp_match(char *A,int lenA,char *B,int
       lenB,int *fail){
    int id=-1.ans=0:
    for(int i=0;i<lenA;++i){</pre>
      while(~id&&B[id+1]!=A[i])id=fail[id];
      if(B[id+1]==A[i])++id;
      if(id==lenB-1){/*匹配成功*/
        ++ans, id=fail[id];
18
19
20
21
    return ans;
```

}while( i != begin );

#### 10.4 manacher.cpp return res; 1 //原字串: asdsasdsa 2 // 先把字串變成這樣: @#a#s#d#s#a#s#d#s#a# void manacher(char \*s,int len,int \*z){ suffix array lcp.cpp int 1=0, r=0; for(int i=1;i<len;++i){</pre> z[i]=r>i?min(z[2\*l-i],r-i):1; while(s[i+z[i]]==s[i-z[i]])++z[i]; 1| #define radix\_sort(x,y){\ **if**(z[i]+i>r)r=z[i]+i,l=i; for(i=0;i<A;++i)c[i]=0;\</pre> $}//ans = max(z)-1$ for(i=0;i<n;++i)c[x[y[i]]]++;\</pre> for(i=1;i<A;++i)c[i]+=c[i-1];\</pre> for(i=n-1;~i;--i)sa[--c[x[y[i]]]]=y[i];\ #define sac(r,a,b) r[a]!=r[b]||a+k>=n||r[a+k]|10.5 minimal string\_rotation.cp ]!=r[b+k] void suffix\_array(const char \*s,int n,int \* sa,int \*rank,int \*tmp,int \*c){ int A='z'+1,i,k,id=0; int min\_string\_rotation(const string &s){ for(i=0;i<n;++i)rank[tmp[i]=i]=s[i];</pre> int n=s.size(),i=0,j=1,k=0; radix sort(rank,tmp); while(i<n&&j<n&&k<n){</pre> for(k=1;id<n-1;k<<=1){ int t=s[(i+k)%n]-s[(j+k)%n]; for(id=0,i=n-k;i<n;++i)tmp[id++]=i;</pre> for(i=0;i<n;++i)if(sa[i]>=k)tmp[id++]=sa if(t){ [i]-k; **if**(t>0)i+=k; else i+=k: 15 radix\_sort(rank,tmp); swap(rank,tmp); 16 if(i==j)++j; for(rank[sa[0]]=id=0,i=1;i<n;++i)</pre> 17 k=0; rank[sa[i]]=id+=sac(tmp,sa[i-1],sa[i]) 11 12 A=id+1; return min(i,j);//最小循環表示法起始位置 13 20 14 21 } 22 //h: 高度數組 sa:後綴數組 rank: 排名 void suffix array\_lcp(const char \*s,int len, int \*h,int \*sa,int \*rank){ 10.6 reverseBWT.cpp for(int i=0;i<len;++i)rank[sa[i]]=i;</pre> for(int i=0,k=0;i<len;++i){</pre> if(rank[i]==0)continue; 1 const int MAXN = 305, MAXC = 'Z'; 27 **if**(k)--k; int ranks[MAXN], tots[MAXC], first[MAXC]; while(s[i+k]==s[sa[rank[i]-1]+k])++k; 28 void rankBWT(const string &bw){ 29 h[rank[i]]=k; memset(ranks,0,sizeof(int)\*bw.size()); 30 memset(tots,0,sizeof(tots); h[0]=0; 31 for(size t i=0:i<bw.size():++i)</pre> ranks[i] = tots[int(bw[i])]++; void firstCol(){ memset(first,0,sizeof(first)); int totc = 0: 10.8 Z.cpp for(int c='A';c<='Z';++c){ 12 if(!tots[c]) continue; 13 14 first[c] = totc; 1 void z alg(char \*s,int len,int \*z){ 15 totc += tots[c]; int l=0.r=0: 16 17 string reverseBwt(string bw, int begin){ for(int i=1;i<len;++i){</pre> z[i]=i>r?0:(i-l+z[i-l]<z[l]?z[i-l]:r-i rankBWT(bw), firstCol(); **int** i = begin: //原字串最後一個元素的位置 while(i+z[i]<len&&s[i+z[i]]==s[z[i]])++z string res; 22 do{ if(i+z[i]-1>r)r=i+z[i]-1,l=i; char c = bw[i]; res = c + res;i = first[int(c)] + ranks[i];

```
Tarjan
```

12

13

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 $^{22}$ 

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dom[id[idom[u]]].push back(id[u]);

#### 11.1 dominator tree.cpp

```
11.2 tnfshb017 2 sat.cpp
1 struct dominator tree{
   static const int MAXN=5005;
   int n;// 1-base
                                                  1 | #include < bits / stdc++.h>
   vector<int> suc[MAXN],pre[MAXN];
                                                    using namespace std;
   int fa[MAXN],dfn[MAXN],id[MAXN],Time;
                                                    #define MAXN 8001
   int semi[MAXN],idom[MAXN];
                                                    #define MAXN2 MAXN*4
   int anc[MAXN], best[MAXN];//disjoint set
                                                    #define n(X)((X)+2*N)
   vector<int> dom[MAXN];//dominator tree
                                                    vector<int> v[MAXN2], rv[MAXN2], vis t;
   void init(int n){
                                                    int N,M;
                                                    void addedge(int s,int e){
     for(int i=1;i<=n;++i)suc[i].clear(),pre[</pre>
                                                      v[s].push_back(e);
          i].clear();
                                                     rv[e].push_back(s);
                                                 11
   void add_edge(int u,int v){
                                                    int scc[MAXN2];
     suc[u].push_back(v);
                                                    bool vis[MAXN2]={false};
     pre[v].push_back(u);
                                                    void dfs(vector<int> *uv,int n,int k=-1){
                                                      vis[n]=true;
   void dfs(int u){
                                                      for(int i=0;i<uv[n].size();++i)</pre>
     dfn[u]=++Time,id[Time]=u;
                                                        if(!vis[uv[n][i]])
     for(auto v:suc[u]){
                                                          dfs(uv,uv[n][i],k);
       if(dfn[v])continue;
                                                 19
                                                      if(uv==v)vis t.push back(n);
        dfs(v),fa[dfn[v]]=dfn[u];
                                                 20
                                                      scc[n]=k;
                                                 21
                                                    void solve(){
   int find(int x){
                                                 23
                                                      for(int i=1;i<=N;++i){</pre>
     if(x==anc[x])return x;
                                                        if(!vis[i])dfs(v,i);
     int y=find(anc[x]);
                                                        if(!vis[n(i)])dfs(v,n(i));
     if(semi[best[x]]>semi[best[anc[x]]])best
                                                26
          [x]=best[anc[x]];
                                                 27
                                                      memset(vis,0,sizeof(vis));
     return anc[x]=y;
                                                 28
                                                      int c=0;
                                                 29
                                                      for(int i=vis_t.size()-1;i>=0;--i)
   void tarjan(int r){
                                                        if(!vis[vis t[i]])
     Time=0:
                                                          dfs(rv,vis_t[i],c++);
     for(int t=1;t<=n;++t){</pre>
        dfn[t]=idom[t]=0;//u=r或是u無法到達r時
                                                 33 int main(){
             idom[id[u]]=0
                                                      int a,b;
        dom[t].clear();
                                                      scanf("%d%d",&N,&M);
        anc[t]=best[t]=semi[t]=t;
                                                      for(int i=1;i<=N;++i){</pre>
                                                        // (A or B)&(!A & !B) A^B
     dfs(r);
                                                        a=i*2-1;
     for(int y=Time;y>=2;--y){
                                                 39
                                                        b=i*2;
       int x=fa[y],idy=id[y];
                                                        addedge(n(a),b);
        for(auto z:pre[idy]){
                                                        addedge(n(b),a);
          if(!(z=dfn[z]))continue;
                                                        addedge(a,n(b));
          find(z);
                                                        addedge(b,n(a));
          semi[y]=min(semi[y],semi[best[z]]);
                                                      while(M--){
        dom[semi[y]].push back(y);
                                                        scanf("%d%d",&a,&b);
        anc[y]=x;
                                                        a = a>0?a*2-1:-a*2;
        for(auto z:dom[x]){
                                                        b = b>0?b*2-1:-b*2;
          find(z);
                                                        // A or B
          idom[z]=semi[best[z]]<x?best[z]:x;</pre>
                                                        addedge(n(a),b);
                                                        addedge(n(b),a);
        dom[x].clear();
                                                 52
                                                      solve();
     for(int u=2;u<=Time;++u){</pre>
                                                      bool check=true;
       if(idom[u]!=semi[u])idom[u]=idom[idom[
                                                      for(int i=1;i<=2*N;++i)</pre>
                                                        if(scc[i]==scc[n(i)])
```

check=false;

57 58 }dom;

```
if(check){
       printf("%d \ n", N);
       for(int i=1;i<=2*N;i+=2){</pre>
60
         if(scc[i]>scc[i+2*N]) putchar('+');
         else putchar('-');
62
64
       puts("");
65
     }else puts("0");
     return 0;
```

#### 11.3 橋連通分量.cpp

```
1 #define N 1005
2 struct edge{
    int u.v:
    bool is bridge;
    edge(int u=0,int v=0):u(u),v(v),is_bridge
  };
  vector<edge> E:
  vector<int> G[N];// 1-base
  int low[N], vis[N], Time;
int bcc_id[N],bridge_cnt,bcc_cnt;// 1-base
11 int st[N],top;//BCC用
  inline void add_edge(int u,int v){
    G[u].push_back(E.size());
    E.push back(edge(u,v));
    G[v].push_back(E.size());
    E.push back(edge(v,u));
17 }
18 void dfs(int u,int re=-1){//u當前點,re為u連
        接前一個點的邊
    int v:
    low[u]=vis[u]=++Time;
20
    st[top++]=u:
22
    for(size t i=0;i<G[u].size();++i){</pre>
23
      int e=G[u][i];v=E[e].v;
24
      if(!vis[v]){
25
         dfs(v,e^1);//e^1反向邊
        low[u]=min(low[u],low[v]);
26
        if(vis[u]<low[v]){</pre>
           E[e].is bridge=E[e^1].is bridge=1;
29
           ++bridge cnt;
       }else if(vis[v]<vis[u]&&e!=re)</pre>
32
        low[u]=min(low[u], vis[v]);
33
    if(vis[u]==low[u]){//處理BCC
34
35
      ++bcc cnt;// 1-base
      do bcc_id[v=st[--top]]=bcc_cnt;//每個點
            所 在 的 BCC
       while(v!=u);
38
   inline void bcc_init(int n){
    Time=bcc cnt=bridge cnt=top=0;
    E.clear();
    for(int i=1;i<=n;++i){</pre>
      G[i].clear();
       vis[i]=bcc_id[i]=0;
```

# 11.4 雙連通分量 & 割點.cpp

47 }

```
1 | #define N 1005
vector<int> G[N];// 1-base
 3 vector<int> bcc[N];//存每塊雙連通分量的點
 4 int low[N].vis[N].Time:
 5 int bcc_id[N],bcc_cnt;// 1-base
 6 bool is cut[N];//是否為割點
  int st[N],top;
   void dfs(int u,int pa=-1){//u當前點,pa父親
     int v, child=0;
     low[u]=vis[u]=++Time;
11
     st[top++]=u;
     for(size_t i=0;i<G[u].size();++i){</pre>
12
      if(!vis[v=G[u][i]]){
         dfs(v,u),++child;
         low[u]=min(low[u],low[v]);
15
16
         if(vis[u]<=low[v]){</pre>
           is_cut[u]=1;
17
           bcc[++bcc cnt].clear();
           int t;
19
             bcc id[t=st[--top]]=bcc cnt;
21
             bcc[bcc_cnt].push_back(t);
22
23
           }while(t!=v);
24
           bcc id[u]=bcc cnt;
25
           bcc[bcc cnt].push back(u);
26
27
      }else if(vis[v]<vis[u]&&v!=pa)//反向邊
         low[u]=min(low[u],vis[v]);
29
    if(pa==-1&&child<2)is_cut[u]=0;//u是dfs樹
          的根要特判
31 }
32 inline void bcc init(int n){
     Time=bcc cnt=top=0;
     for(int i=1;i<=n;++i){</pre>
35
      G[i].clear();
      is cut[i]=vis[i]=bcc id[i]=0;
36
```

# Tree problem

## 12.1 HeavyLight.cpp

```
1 | #include < vector >
2 #define MAXN 100005
int siz[MAXN], max son[MAXN], pa[MAXN], dep[
       MAXN];
4 int link top[MAXN],link[MAXN],cnt;
5 vector<int> G[MAXN];
6 void find max son(int u){
    siz[u]=1;
    max son[u]=-1;
```

```
for(auto v:G[u]){
      if(v==pa[u])continue;
10
                                               24
11
      pa[v]=u;
                                                25
      dep[v]=dep[u]+1;
12
                                                26
13
      find max son(v);
                                                27
      if(max son[u]==-1||siz[v]>siz[max son[u
                                                28
           ]])max son[u]=v;
                                                29
      siz[u]+=siz[v];
15
16
17
  void build link(int u,int top){
    link[u]=++cnt:
19
    link_top[u]=top;
20
    if(max son[u]==-1)return;
    build link(max_son[u],top);
    for(auto v:G[u]){
      if(v==max son[u]||v==pa[u])continue;
24
25
      build_link(v,v);
26
27
  int find_lca(int a,int b){
28
    //求LCA,可以在過程中對區間進行處理
    int ta=link top[a],tb=link top[b];
    while(ta!=tb){
      if(dep[ta]<dep[tb]){</pre>
        swap(ta,tb);
34
        swap(a,b);
35
                                                11 }
      //這裡可以對a所在的鏈做區間處理
36
37
      //區間為(Link[ta],Link[a])
38
      ta=link top[a=pa[ta]];
39
    //最後a,b會在同一條鏈,若a!=b還要在進行一
         次區間處理
    return dep[a]<dep[b]?a:b;</pre>
41
42 }
```

#### 12.2 LCA.cpp

```
1 #define MAXN 100000
2 #define MAX LOG 17
3 int pa[MAX LOG+1][MAXN+5];
4 int dep[MAXN+5];
5 vector<int> G[MAXN+5];
6 void dfs(int x, int p){//dfs(1,-1)};
    pa[0][x]=p;
    for(int i=0;i+1<MAX_LOG;++i)pa[i+1][x]=pa[</pre>
         i][pa[i][x]];
    for(auto &i:G[x]){
      if(i==p)continue;
11
       dep[i]=dep[x]+1;
12
      dfs(i,x);
13
14 }
inline int jump(int x,int d){
16 for(int i=0;i<d;++i)if((x>>i)&1)x=pa[k][x];
    return x;
  inline int find lca(int a,int b){
    if(dep[a]>dep[b])swap(a,b);
    b=jump(b,dep[b]-dep[a]);
    if(a==b)return a;
```

```
a=pa[i][a];
    b=pa[i][b];
return pa[0][a];
```

for(int i=MAX LOG;i>=0;--i){

if(pa[i][a]!=pa[i][b]){

#### 12.3 link\_cut\_tree.cpp

**int** ch[2],pa;//子節點跟父母

splay\_tree():pa(0),rev(0){ch[0]=ch[1]=0;}

return nd[nd[x].pa].ch[0]!=x&&nd[nd[x].pa

bool rev;//反轉的懶惰標記

7 // 有的時候用vector會TLE,要注意

9 bool isroot(int x){//判斷是否為這棵splay

8 | // 這邊以 node [0] 作為 null 節點

].ch[1]!=x;

void down(int x){// 懶惰標記下推

1 | struct splay\_tree{

6 vector<splay tree> nd;

tree的 根

};

```
if(nd[x].rev){
      if(nd[x].ch[0])nd[nd[x].ch[0]].rev^=1;
      if(nd[x].ch[1])nd[nd[x].ch[1]].rev^=1;
      swap(nd[x].ch[0],nd[x].ch[1]);
17
      nd[x].rev=0;
18
19
  void push down(int x){//所有祖先懶惰標記下推
    if(!isroot(x))push_down(nd[x].pa);
22
23
24 | void up(int x){}//將子節點的資訊向上更新
  void rotate(int x){//旋轉,會自行判斷轉的方
    int y=nd[x].pa,z=nd[y].pa,d=(nd[y].ch[1]==
         x);
    nd[x].pa=z;
27
    if(!isroot(y))nd[z].ch[nd[z].ch[1]==y]=x;
    nd[y].ch[d]=nd[x].ch[d^1];
    nd[nd[y].ch[d]].pa=y;
    nd[y].pa=x,nd[x].ch[d^1]=y;
31
    up(y),up(x);
33
  void splay(int x){//將x伸展到splay tree的根
    push down(x);
    while(!isroot(x)){
      int y=nd[x].pa;
      if(!isroot(y)){
        int z=nd[y].pa;
        if((nd[z].ch[0]==y)^(nd[y].ch[0]==x))
            rotate(y);
41
        else rotate(x);
42
43
      rotate(x);
44
```

make root(u);

if(sol.vis[i])ans.push\_back(i);

```
46 int access(int x){
                                                                                                        for(int i=0;i<=n;++i){</pre>
                                                                                                                                                    71 }
                                                      return access(v);
     int last=0;
                                                110 }
                                                                                                   10
                                                                                                          g[i].clear();
     while(x){
                                                int query_lca(int u,int v){
                                                                                                          vis[i]=0;
                                                                                                   11
49
       splay(x);
                                                112 //假設求鏈上點權的總和·sum是子樹的權重和
                                                                                                   12
       nd[x].ch[1]=last;
                                                                                                   13
50
                                                                                                                                                              zformula
                                                         data是節點的權重
                                                                                                      void get dis(vector<int> &dis,int u,int pa,
51
       up(x);
                                                      access(u);
                                                113
52
       last=x:
                                                                                                          int d){
                                                      int lca=access(v);
                                                114
       x=nd[x].pa;
                                                                                                        dis.push_back(d);
53
                                                                                                   15
                                                      splay(u);
                                                115
                                                                                                                                                       13.1 formula.tex
                                                                                                        for(size t i=0;i<g[u].size();++i){</pre>
54
                                                                                                   16
                                                116
                                                      if(u==lca){
                                                                                                         int v=g[u][i].first,w=g[u][i].second;
     return last;//access後splay tree的根
                                                117
                                                        //return nd[lca].data+nd[nd[lca].ch[1]].
                                                                                                   18
                                                                                                          if(v!=pa&&!vis[v])get_dis(dis,v,u,d+w);
56
                                                                                                                                                       13.1.1 Pick 公式
                                                                                                   19
57 void access(int x,bool is=0){//is=0就是一般
                                                118
                                                      }else{
                                                                                                  20 }
                                                        //return nd[lca].data+nd[nd[lca].ch[1]].
        的access
                                                119
                                                                                                                                                       給定頂點坐標均是整點的簡單多邊形,面積 = 內部格點數 +
                                                                                                   21 | vector<int> dis;//這東西如果放在函數裡會TLE
                                                             sum+nd[u].sum
     int last=0:
                                                                                                                                                       邊上格點數/2-1
                                                                                                   22 int cal(int u,int d){
59
     while(x){
                                                120
                                                                                                       dis.clear();
       splay(x);
                                                121 }
60
                                                                                                       get_dis(dis,u,-1,d);
                                                122 struct EDGE{
       if(is&&!nd[x].pa){
                                                                                                                                                       13.1.2 圖論
                                                                                                        sort(dis.begin(),dis.end());
         //printf("%d\n",max(nd[last].ma,nd[nd[ 123
                                                      int a,b,w;
62
                                                                                                        int l=0,r=dis.size()-1,res=0;
                                                124 }e[10005];
              x].ch[1]].ma));
                                                                                                                                                          1. V - E + F = 2
                                                125 int n:
                                                                                                        while(l<r){
63
                                                                                                                                                          2. 對於平面圖 F = E - V + n + 1 \cdot n 是連通分量
                                                                                                   28
                                                                                                          while(l<r&&dis[l]+dis[r]>k)--r;
                                                126 vector<pair<int,int>> G[10005];
                                                                                                                                                          3. 對於平面圖 E \leq 3V - 6
64
       nd[x].ch[1]=last;
                                                                                                          res+=r-(1++);
                                                                                                                                                          4. 對於連通圖 G·最大獨立點集的大小設為 I(G),最
                                                127 | //first表示子節點, second表示邊的編號
65
       up(x);
                                                                                                   30
                                                                                                                                                             大匹配大小設為 M(G),最小點覆蓋設為 Cv(G)
66
       last=x;
                                                128 int pa[10005], edge_node[10005];
                                                                                                       return res;
                                                                                                   31
                                                                                                                                                             最小邊覆蓋設為 Ce(G)。對於任意連通圖:
       x=nd[x].pa;
                                                129 //pa是父母節點,暫存用的,edge node是每個編
68
                                                         被存在哪個點裡面的陣列
                                                                                                   33 pair<int,int> tree centroid(int u,int pa,
                                                                                                                                                              (a) I(G) + Cv(G) = |V|

(b) M(G) + Ce(G) = |V|
69
                                                130 void bfs(int root){
                                                                                                          const int sz){
    void query edge(int u,int v){
                                                131 //在建構的時候把每個點都設成一個splay tree
                                                                                                        size[u]=1;//找樹重心, second是重心
     access(u);
                                                                                                                                                          5. 對於連通二分圖:
71
                                                      queue<int > a:
                                                132
                                                                                                        pair<int,int> res(INT MAX,-1);
72
     access(v,1);
                                                      for(int i=1;i<=n;++i)pa[i]=0;</pre>
                                                133
                                                                                                        int ma=0;
                                                                                                                                                              (a) I(G) = Cv(G)
(b) M(G) = Ce(G)
73
                                                134
                                                      a.push(root);
                                                                                                        for(size_t i=0;i<g[u].size();++i){</pre>
                                                                                                   37
   void make root(int x){
                                                135
                                                      while(q.size()){
                                                                                                          int v=g[u][i].first;
     access(x),splay(x);

 最大權閉合圖:

                                                136
                                                        int u=q.front();
                                                                                                   39
                                                                                                          if(v==pa||vis[v])continue;
76
     nd[x].rev^=1;
                                                137
                                                        q.pop();
                                                                                                          res=min(res,tree centroid(v,u,sz));
77
                                                                                                   40
                                                                                                                                                              (a) C(u, V) = \infty, (u, v) \in E
(b) C(S, v) = W_v, W_v > 0
                                                        for(auto P:G[u]){
                                                138
   void make_root(int x){
                                                                                                   41
                                                                                                          size[u]+=size[v];
                                                          int v=P.first;
                                                139
                                                                                                                                                               (c) C(v,T) = -W_v, W_v < 0
                                                                                                          ma=max(ma,size[v]);
     nd[access(x)].rev^=1;
                                                                                                   ^{42}
79
                                                          if(v!=pa[u]){
                                                140
                                                                                                   43
                                                                                                                                                          7. 最大密度子圖:
     splay(x);
80
                                                141
                                                            pa[v]=u;
                                                                                                   44
                                                                                                        ma=max(ma,sz-size[u]);
81
                                                            nd[v].pa=u:
                                                142
                                                                                                       return min(res, make pair(ma,u));
   void cut(int x,int y){
                                                                                                   45
                                                                                                                                                               (a) C(u,v) = 1, (u,v) \in E
                                                143
                                                            nd[v].data=e[P.second].w;
                                                                                                                                                              (a) C(u, v) = 1, (u, v) \in E

(b) C(S, v) = U_v, v \in V

(c) C(v, T) = U + 2g - d_v, v \in V
     make_root(x);
                                                                                                   46
                                                144
                                                             edge_node[P.second]=v;
     access(y);
                                                                                                      int tree_DC(int u,int sz){
                                                145
                                                             up(v);
                                                                                                       int center=tree centroid(u,-1,sz).second;
                                                                                                                                                          8. 弦圖:
85
     splay(y);
                                                146
                                                            q.push(v);
                                                                                                       int ans=cal(center,0);
     nd[y].ch[0]=0;
                                                147
                                                                                                                                                               (a) 完美消除序列從後往前依次給每個點染色,給
                                                                                                   50
                                                                                                       vis[center]=1;
87
     nd[x].pa=0;
                                                148
                                                                                                                                                                  每個點染上可以染的最小顏色
                                                                                                        for(size t i=0;i<g[center].size();++i){</pre>
88
                                                149
                                                                                                                                                                  最大團大小 = 色數
最大獨立集: 完美消除序列從前往後能選就選
                                                                                                          int v=g[center][i].first,w=g[center][i].
   void cut parents(int x){
                                                150 }
                                                                                                              second:
     access(x):
                                                                                                                                                                  最小團覆蓋: 最大獨立集的點和他延伸的邊構
                                                151
                                                    void change(int x,int b){
     splay(x);
                                                                                                   53
                                                                                                          if(vis[v])continue;
                                                152
                                                      splay(x);
     nd[nd[x].ch[0]].pa=0;
                                                                                                   54
                                                                                                          ans-=cal(v,w);
                                                153
                                                      //nd[x].data=b;
                                                                                                   55
                                                                                                          ans+=tree DC(v,size[v]);
                                                                                                                                                                  區間圖的完美消除序列: 將區間按造又端點由
93
     nd[x].ch[0]=0;
                                                      up(x);
                                                154
                                                                                                   56
                                                                                                                                                                  小到大排序
94
                                                155 }
                                                                                                                                                               (g) 區間圖染色: 用線段樹做
                                                                                                   57
   void link(int x,int y){
                                                                                                        return ans;
                                                                                                   58
     make_root(x);
                                                                                                   59
                                                                                                      int main(){
97
     nd[x].pa=y;
                                                                                                                                                     1 double 1=0,=m,stop=1.0/n/n;
                                                                                                        while(scanf("%d%d",&n,&k),n||k){
98
                                                    12.4 POJ tree.cpp
                                                                                                                                                      while(r-l>=stop){
                                                                                                          init():
   int find root(int x){
                                                                                                   61
                                                                                                                                                         double(mid);
                                                                                                   62
                                                                                                          for(int i=1;i<n;++i){</pre>
     x=access(x);
                                                                                                                                                         if((n*m-sol.maxFlow(s,t))/2>eps)l=mid;
                                                                                                   63
                                                                                                           int u,v,w;
     while(nd[x].ch[0])x=nd[x].ch[0];
                                                  1 | #include < bits / stdc++.h>
                                                                                                                                                         else r=mid;
                                                                                                            scanf("%d%d%d",&u,&v,&w);
                                                                                                   64
102
     splay(x);
                                                  2 using namespace std:
                                                                                                            g[u].push_back(make_pair(v,w));
                                                                                                   65
103
     return x;
                                                  3 #define MAXN 10005
                                                                                                                                                      build(1);
                                                                                                            g[v].push back(make pair(u,w));
                                                                                                   66
104
                                                  4 int n,k;
                                                                                                                                                      sol.maxFlow(s,t);
                                                                                                   67
int query(int u,int v){
                                                  5 vector<pair<int,int> >g[MAXN];
                                                                                                                                                      vector<int> ans;
                                                                                                   68
                                                                                                          printf("%d\n",tree_DC(1,n));
106 | // 傳回uv路徑splay tree的根結點
                                                  6 int size[MAXN];
                                                                                                                                                    10 for(int i=1:i<=n:++i)
                                                                                                   69
107 // 這種寫法無法求LCA
```

return 0;

7 bool vis[MAXN];

8 inline void init(){

#### 13.1.3 學長公式

- 1.  $\sum_{d|n} \phi(n) = n$
- 2.  $g(n) = \sum_{d|n} f(d) = \sum_{d|n} \mu(d) \times$
- 3. Harmonic series  $H_n = \ln(n) + \gamma + 1/(2n)$   $1/(12n^2) + 1/(120n^4)$
- 4.  $\gamma = 0.57721566490153286060651209008240243104215$
- 5. 格雷碼 =  $n \oplus (n >> 1)$
- 6.  $SG(A+B) = SG(A) \oplus SG(B)$
- 7. 選轉矩陣  $M(\theta) = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$

#### 13.1.4 基本數論

- 1.  $\sum_{d|n} \mu(n) = [n == 1]$
- 2.  $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) \times$
- 4.  $\sum_{i=1}^{n} \sum_{j=1}^{n} lcm(i,j) = n \sum_{d|n} d \times \phi(d)$

#### 13.1.5 排組公式

- 1. k 卡特蘭  $\frac{C_n^{kn}}{n(k-1)+1} \cdot C_m^n = \frac{n!}{m!(n-m)!}$
- 2.  $H(n,m) \cong x_1 + x_2 \dots + x_n = k, num = C_k^{n+k-1}$
- 3. Stirling number of  $2^{nd}$ ,n 人分 k 組方法數目
  - (a) S(0,0) = S(n,n) = 1
  - (b) S(n,0) = 0
- (c) S(n,k) = kS(n-1,k) + S(n-1,k-1)
- 4. Bell number,n 人分任意多組方法數目
  - (a)  $B_0 = 1$

  - (a)  $B_0 = \sum_{i=0}^{n} S(n, i)$ (b)  $B_n = \sum_{i=0}^{n} S(n, i)$ (c)  $B_{n+1} = \sum_{k=0}^{n} C_n^k B_k$ (d)  $B_{p+n} \equiv B_n + B_{n+1} mod p$ , p is prime (e)  $B_p m_{+n} \equiv m B_n + B_{n+1} mod p$ , p is prime (f)  $E_n = B_n + B_{n+1} mod p$ , p is prime
  - (f) From  $B_0: 1, 1, 2, 5, 15, 52$ , 203, 877, 4140, 21147, 115975
- 5. Derangement, 錯排, 沒有人在自己位置上
  - (a)  $D_n = n!(1 \frac{1}{1!} + \frac{1}{2!} \frac{1}{3!} \dots + (-1)^n \frac{1}{n!})$ (b)  $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 = 0$  $1, D_1 = 0$
  - (c) From  $D_0: 1, 0, 1, 2, 9, 44$ , 265, 1854, 14833, 133496
- 6. Binomial Equality
  - (a)  $\sum_{k} {r \choose m+k} {s \choose n-k} = {r+s \choose m+n}$
  - (b)  $\sum_{k} {l \choose m+k} {s \choose n+k} = {l+s \choose l-m+n}$

  - (c)  $\sum_{k} {m+k \choose n+k} {s+k \choose n} {(-1)^k} = (-1)^{l+m} {s-m \choose n-l}$ (d)  $\sum_{k \le l} {l \choose m+k} {s \choose k-n} (-1)^k = (-1)^{l+m} {s-m \choose n-l}$
  - (e)  $\sum_{0 \le k \le l} {l-k \choose m} {q+k \choose n} = {l+q+1 \choose m+n+1}$
  - (f)  $\binom{r}{b} = (-1)^k \binom{k-r-1}{b}$

- (g)  $\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}$
- (h)  $\sum_{k \le n} {r+k \choose k} = {r+n+1 \choose n}$
- (i)  $\sum_{0 \le k \le n} {k \choose m} = {n+1 \choose m+1}$
- (j)  $\sum_{k \le m} {m+r \choose k} x^k y^k$  $\sum_{k \le m} {\binom{-r}{k}} (-x)^k (x+y)^{m-k}$

#### 13.1.6 冪次, 冪次和

- 1.  $a^b \% P = a^{b \% \varphi(p) + \varphi(p)}, b > \varphi(p)$
- 2.  $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
- 3.  $1^4 + 2^4 + 3^4 + \ldots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} \frac{n}{30}$
- 4.  $1^5 + 2^5 + 3^5 + \ldots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} \frac{n^2}{12}$
- 5.  $0^k + 1^k + 2^k + \dots + n^k = P(k), P(k) = {}^{10}$   $\frac{(n+1)^{k+1} \sum_{i=0}^{k-1} C_i^{k+1} P(i)}{\sum_{i=0}^{k-1} C_i^{k}}, P(0) = n+1$  11
- 6.  $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^n C_k^{n+1} B_k m^{n+1-k}$
- 7.  $\sum_{i=0}^{m} C_i^{m+1} B_i = 0, B_0 = 1$
- 8. 除了  $B_1 = -1/2$ ,剩下的奇數項都是 0
- 9.  $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 =$  $-1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = {}_{17}$  $7/6, B_{16} = -3617/510, B_{18}$  $43867/798, B_{20} = -174611/330,$

#### 13.1.7 Burnside's lemma

- 1.  $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- 2.  $X^g = t^{c(g)}$
- 3. G 表示有幾種轉法, $X^g$  表示在那種轉法下,有幾種 是會保持對稱的 $\cdot t$  是顏色數 $\cdot c(q)$  是循環節不動的
- 4. 正立方體塗三顏色,轉 0 有 36 個元素不變,轉 90 有 6 種, 每種有  $3^3$  不變, 180 有  $3\times3^4$ 120(角) 有  $8 \times 3^2 \cdot 180(邊)$  有  $6 \times 3^3 \cdot$  全部  $\frac{1}{24} \left( 3^{6} + 6 \times 3^{3} + 3 \times 3^{4} + 8 \times 3^{2} + 6 \times 3^{3} \right) =$

#### 13.1.8 Count on a tree

- 1. Rooted tree:  $s_{n+1} = \frac{1}{n} \sum_{i=1}^{n} (i \times a_i \times a_i)$  $\sum_{i=1}^{\lfloor n/i \rfloor} a_{n+1-i \times i})$
- 2. Unrooted tree:
  - (a) Odd: $a_n \sum_{i=1}^{n/2} a_i a_{n-i}$
  - (b) Even: $Odd + \frac{1}{2}a_{n/2}(a_{n/2} + 1)$
- 3. Spanning Tree
  - (a) 完全圖  $n^n 2$
  - (b) 一般圖 (Kirchhoff's theorem)M[i][i] = 5 for(Object obj: map.keySet()){ if no edge. delete any one row and col in 7 } A, ans = det(A)

#### 13.2 java.tex

#### 13.2.1 文件操作

```
1 import java.io.*;
 2 import java.util.*;
 3 import java.math.*;
 4 import java.text.*;
  public class Main{
    public static void main(String args[]){
         throws FileNotFoundException.
         IOException
      Scanner sc = new Scanner(new FileReader(
      PrintWriter pw = new PrintWriter(new
           FileWriter("a.out"));
      n=sc.nextInt();//读入下一个INT
      m=sc.nextInt();
      for(ci=1; ci<=c; ++ci){</pre>
        pw.println("Case #"+ci+": easy for
             output");
      pw.close();//矣闭流并释放,这个很重要
           否则是没有输出的
20
      sc.close();// 关闭流并释放
21
```

#### 13.2.2 优先队列

```
1 | PriorityQueue queue = new PriorityQueue( 1,
      new Comparator(){
   public int compare( Point a, Point b ){
   if(a.x < b.x | | a.x == b.x && a.y < b.y)
    return -1;
   else if( a.x == b.x && a.y == b.y )
     return 0;
   else return 1;
```

#### 13.2.3 Map

```
1 | Map map = new HashMap();
                                         2 map.put("sa","dd");
                                         3 String str = map.get("sa").toString;
degree(V_i), M[i][j] = -1, \text{if have } E(i, j), 0 \quad 6 | Object value = map.get(obj);
```

#### 13.2.4 sort

```
1 | static class cmp implements Comparator{
    public int compare(Object o1,Object o2){
    BigInteger b1=(BigInteger)o1;
    BigInteger b2=(BigInteger)o2;
    return b1.compareTo(b2);
  public static void main(String[] args)
       throws IOException{
    Scanner cin = new Scanner(System.in);
    int n;
    n=cin.nextInt();
     BigInteger[] seg = new BigInteger[n];
    for (int i=0;i<n;i++)</pre>
    seg[i]=cin.nextBigInteger();
    Arrays.sort(seg, new cmp());
15
```

	ACM ICPC		3.2 ext.cpp	5 5	7	Linear_Programming 7.1 最大密度子圖.cpp		10.7 suffix_array_lcp.cpp 10.8 Z.cpp	
	Team Reference -	4	3.4 input.cpp	6	8	Number_Theory           8.1 basic.cpp            8.2 bit_set.cpp	10	11 Tarjan 11.1 dominator_tree.cpp 11.2 tnfshb017 2 sat.cpp	
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$\mathbf{C}$	Contents	5	<ul> <li>6 Graph</li> <li>5.1 Augmenting_Path.cpp</li> <li>5.2 Augmenting_Path_multiple.cp</li> <li>5.3 blossom_matching.cpp</li> <li>5.4 graphISO.cpp</li> </ul>	p 7 7		8.7 LinearCongruence.cpp	12 12 12 12	12.1 HeavyLight.cpp	16 16 16
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2	1.3 最近點對.cpp       3         Data_Structure       3         2.1 DLX.cpp       3         2.2 Dynamic_KD_tree.cpp       4         2.3 kd_tree_replace_segment_tree.cp	3 4	5.8 Rectilinear_MST.cpp	8 8	9	other         9.1 WhatDay.cpp         9.2 上下最大正方形.cpp         9.3 最大矩形.cpp	14	13.1.2 圖論	18 18 18
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3	default         5           3.1 debug.cpp         5		6.1 CNF.cpp	<b>10</b> 10		10.4 manacher.cpp	15	13.2.2 优先队列	18