

# 1 Computational\_Geometry

## 1.1 Geometry

```

1 const double PI=atan2(0.0,-1.0);
2 template<typename T>
3 struct point{
4     T x,y;
5     point(){ }
6     point(const T&x,const T&y):x(x),y(y){ }
7     point operator+(const point &b)const{
8         return point(x+b.x,y+b.y); }
9     point operator-(const point &b)const{
10        return point(x-b.x,y-b.y); }
11     point operator*(const T &b)const{
12        return point(x*b,y*b); }
13     point operator/(const T &b)const{
14        return point(x/b,y/b); }
15     bool operator==(const point &b)const{
16        return x==b.x&&y==b.y; }
17     T dot(const point &b)const{
18        return x*b.x+y*b.y; }
19     T cross(const point &b)const{
20        return x*b.y-y*b.x; }
21     point normal()const{ //求法向量
22        return point(-y,x); }
23     T abs2()const{ //向量長度的平方
24        return dot(*this); }
25     T rad(const point &b)const{ //兩向量的弧度
26     return fabs(atan2(fabs(cross(b)),dot(b))); }
27     T getA()const{ //對x軸的弧度
28     T A=atan2(y,x); //超過180度會變負的
29     if(A<=-PI/2)A+=PI*2;
30     return A;
31 }
32 };
33 template<typename T>
34 struct line{
35     line(){ }
36     point<T> p1,p2;
37     T a,b,c; //ax+by+c=0
38     line(const point<T>&x,const point<T>&y):p1
39         (x),p2(y){ }
40     void pton()const{ //轉成一般式
41         a=p1.y-p2.y;
42         b=p2.x-p1.x;
43         c=-a*p1.x-b*p1.y;
44     }
45     T ori(const point<T> &p)const{ //點和有向直
46         線的關係 · >0左邊 · =0在線上<0右邊
47         return (p2-p1).cross(p-p1);
48     }
49     T btw(const point<T> &p)const{ //點投影落在
50         線段上<=0
51         return (p1-p).dot(p2-p);
52     }
53     bool point_on_segment(const point<T>&p)
54         const{ //點是否在線段上
55         return ori(p)==0&&btw(p)<=0;
56     }
57     T dis2(const point<T> &p,bool is_segment
58         =0)const{ //點跟直線/線段的距離平方
59     point<T> v=p2-p1,v1=p-p1;
60     if(is_segment){
61         point<T> v2=p-p2;
62         if(v.dot(v1)<=0)return v1.abs2();
63         if(v.dot(v2)>=0)return v2.abs2();
64     }
65     T tmp=v.cross(v1);
66     return tmp*tmp/v.abs2();
67 }
68 T seg_dis2(const line<T> &l)const{ //兩線段
69     距離平方
70     return min({dis2(l.p1,1),dis2(l.p2,1),l.
71         dis2(p1,1),l.dis2(p2,1)});
72 }
73 point<T> projection(const point<T> &p)
74     const{ //點對直線的投影
75     point<T> n=(p2-p1).normal();
76     return p-n*(p-p1).dot(n)/n.abs2();
77 }
78 point<T> mirror(const point<T> &p)const{
79     //點對直線的鏡射 · 要先呼叫pton轉成一般式
80     point<T> R;
81     T d=a*b+b*b;
82     R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
83     R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
84     return R;
85 }
86 bool equal(const line &l1)const{ //直線相等
87     return ori(l1.p1)==0&&ori(l1.p2)==0;
88 }
89 bool parallel(const line &l1)const{
90     return (p1-p2).cross(l1.p1-l1.p2)==0;
91 }
92 bool cross_seg(const line &l1)const{
93     return (p2-p1).cross(l1.p1-p1)*(p2-p1).
94         cross(l1.p2-p1)<=0; //直線是否交線段
95 }
96 int line_intersect(const line &l1)const{ //
97     直線相交情況 · -1無限多點 · 1交於一點 · 0
98     不相交
99     return parallel(l1)?(ori(l1.p1)==0?-1:0)
100         :1;
101 }
102 int seg_intersect(const line &l1)const{
103     T c1=ori(l1.p1), c2=ori(l1.p2);
104     T c3=l1.ori(p1), c4=l1.ori(p2);
105     if(c1==0&&c2==0){ //共線
106         bool b1=btw(l1.p1)>=0,b2=btw(l1.p2)>=0;
107         T a3=l1.btw(p1),a4=l1.btw(p2);
108         if(b1&&b2&&a3==0&&a4==0) return 2;
109         if(b1&&b2&&a3>0&&a4==0) return 3;
110         if(b1&&b2&&a3>0&&a4>0) return 0;
111         return -1; //無限交點
112     }else if(c1*c2<0&&c3*c4<0)return 1;
113     return 0; //不相交
114 }
115 point<T> line_intersection(const line &l1)
116     const{ //直線交點*/
117     point<T> a=p2-p1,b=l1.p2-l1.p1,s=l1.p1-p1;
118     //if(a.cross(b)==0)return INF;
119     return p1+a*(s.cross(b)/a.cross(b));
120 }
121 point<T> seg_intersection(const line &l1)
122     const{ //線段交點
123     int res=seg_intersect(l1);
124     if(res<=0) assert(0);
125     if(res==2) return p1;
126     if(res==3) return p2;
127     return line_intersection(l1);
128 }
129 template<typename T>
130 struct polygon{
131     polygon(){ }
132     vector<point<T> > p; //逆時針順序
133     T area()const{ //面積
134     T ans=0;
135     for(int i=p.size()-1,j=0;j<(int)p.size()
136         ;i=j++)
137         ans+=p[i].cross(p[j]);
138     return ans/2;
139 }
140 point<T> center_of_mass()const{ //重心
141     T cx=0,cy=0,w=0;
142     for(int i=p.size()-1,j=0;j<(int)p.size()
143         ;i=j++){
144         T a=p[i].cross(p[j]);
145         cx+=(p[i].x+p[j].x)*a;
146         cy+=(p[i].y+p[j].y)*a;
147         w+=a;
148     }
149     return point<T>(cx/3/w,cy/3/w);
150 }
151 char ahas(const point<T>&t)const{ //點是否
152     在簡單多邊形內 · 是的話回傳1 · 在邊上回
153     傳-1 · 否則回傳0
154     bool c=0;
155     for(int i=0,j=p.size()-1;i<p.size();j=i
156         ++){
157         if(line<T>(p[i],p[j]).point_on_segment
158             (t))return -1;
159         else if((p[i].y>t.y)!(p[j].y>t.y)&&
160             t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j]
161                 .y-p[i].y)+p[i].x)
162             c=!c;
163     }
164     return c;
165 }
166 char point_in_convex(const point<T>&x)
167     const{
168     int l=1,r=(int)p.size()-2;
169     while(l<r){ //點是否在凸多邊形內 · 是的話
170         回傳1 · 在邊上回傳-1 · 否則回傳0
171         int mid=(l+r)/2;
172         T a1=(p[mid]-p[0]).cross(x-p[0]);
173         T a2=(p[mid+1]-p[0]).cross(x-p[0]);
174         if(a1>0&&a2<=0){
175             T res=(p[mid+1]-p[mid]).cross(x-p[
176                 mid]);
177             return res>0?1:(res>0?-1:0);
178         }else if(a1<0)r=mid-1;
179         else l=mid+1;
180     }
181     return 0;
182 }
183 vector<T> getA()const{ //凸包邊對x軸的夾角
184     vector<T>res; //一定是遞增的
185     for(size_t i=0;i<p.size();++i)
186         res.push_back((p[(i+1)%p.size()]-p[i])
187             .getA());
188     return res;
189 }
190 bool line_intersect(const vector<T>&A,
191     const line<T> &l1)const{ //O(LogN)
192     int f1=upper_bound(A.begin(),A.end(),(l.
193         p1-l.p2).getA())-A.begin();
194     int f2=upper_bound(A.begin(),A.end(),(l.
195         p2-l.p1).getA())-A.begin();
196     return l.cross_seg(line<T>(p[f1],p[f2]))
197         ;
198 }
199 polygon cut(const line<T> &l1)const{ //凸包
200     對直線切割 · 得到直線L左側的凸包
201     polygon ans;
202     for(int n=p.size(),i=n-1,j=0;j<n;i=j++){
203         if(l.ori(p[i])>=0){
204             ans.p.push_back(p[i]);
205             if(l.ori(p[j])<0)
206                 ans.p.push_back(l.
207                     line_intersection(line<T>(p[i]
208                         ],p[j])));
209         }else if(l.ori(p[j])>0)
210             ans.p.push_back(l.
211                 line_intersection(line<T>(p[i],p[j]
212                     )));
213     }
214     return ans;
215 }
216 static bool graham_cmp(const point<T>&a,
217     const point<T>&b){ //凸包排序函數
218     return (a.x<b.x)|| (a.x==b.x&&a.y<b.y);
219 }
220 void graham(vector<point<T> > &s){ //凸包
221     sort(s.begin(),s.end(),graham_cmp);
222     p.resize(s.size()+1);
223     int m=0;
224     for(size_t i=0;i<s.size();++i){
225         while(m>=2&&(p[m-1]-p[m-2]).cross(s[i]
226             -p[m-2])<=0)--m;
227         p[m++]=s[i];
228     }
229     for(int i=s.size()-2,t=m+1;i>=0;--i){
230         while(m>=t&&(p[m-1]-p[m-2]).cross(s[i]
231             -p[m-2])<=0)--m;
232         p[m++]=s[i];
233     }
234     if(s.size()>1)--m;
235     p.resize(m);
236 }
237 T diam(){ //直徑
238     int n=p.size(),t=1;
239     T ans=0;p.push_back(p[0]);
240     for(int i=0;i<n;i++){
241         point<T> now=p[i+1]-p[i];
242         while(now.cross(p[t+1]-p[i])>now.cross
243             (p[t]-p[i]))t=(t+1)%n;
244         ans=max(ans,(p[i]-p[t]).abs2());
245     }
246     return p.pop_back(),ans;
247 }
248 T min_cover_rectangle(){ //最小覆蓋矩形
249     int n=p.size(),t=1,r=1,l=1;

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212 if(n<3)return 0;//也可以做最小周長矩形
213 T ans=1e99;p.push_back(p[0]);
214 for(int i=0;i<n;i++){
215     point<T> now=p[i+1]-p[i];
216     while(now.cross(p[t+1]-p[i])>now.cross
217         (p[t]-p[i]))t=(t+1)%n;
218     while(now.dot(p[r+1]-p[i])>now.dot(p[
219         ]-p[i]))r=(r+1)%n;
218     if(!l)r=l;
219     while(now.dot(p[l+1]-p[i])<now.dot(p[
220         ]-p[i]))l=(l+1)%n;
220     T d=now.abs2();
221     T tmp=now.cross(p[t]-p[i])*(now.dot(p[
222         ]-p[i])-now.dot(p[l]-p[i]))/d;
222     ans=min(ans,tmp);
223 }
224 return p.pop_back(),ans;
225 }
226 T max_triangle(){//最大內接三角形
227     int n=p.size(),a=1,b=2;
228     if(n<3)return 0;
229     T ans=0,tmp;p.push_back(p[0]);
230     for(int i=0;i<n;i++){
231         while((p[a]-p[i]).cross(p[b+1]-p[i])>
232             tmp=(p[a]-p[i]).cross(p[b]-p[i]))
233             b=(b+1)%n;
232         ans=max(ans,tmp);
233         while((p[a+1]-p[i]).cross(p[b]-p[i])>
234             tmp=(p[a]-p[i]).cross(p[b]-p[i]))
235             a=(a+1)%n;
234         ans=max(ans,tmp);
235     }
236     return p.pop_back(),ans/2;
237 }
238 T dis2(polygon &p1){//凸包最近距離平方
239     vector<point<T> > &P=p,&Q=p1.p;
240     int n=P.size(),m=Q.size(),l=0,r=0;
241     for(int i=0;i<n;i++){
242         if(P[i].y<P[l].y)l=i;
243         if(Q[i].y<Q[r].y)r=i;
244     }
245     P.push_back(P[0]),Q.push_back(Q[0]);
246     T ans=1e99;
247     for(int i=0;i<n;i++){
248         while((P[l]-P[l+1]).cross(Q[r+1]-Q[r])
249             <0)r=(r+1)%m;
250         ans=min(ans,dis2(P[l],P[l+1]).
251             seg_dis2(line<T>(Q[r],Q[r+1])));
252         l=(l+1)%n;
253     }
254     return P.pop_back(),Q.pop_back(),ans;
255 }
256 static char sign(const point<T>&t){
257     return (t.y==0?t.x:t.y)<0;
258 }
259 static bool angle_cmp(const line<T>& A,
260     const line<T>& B){
261     point<T> a=A.p2-A.p1,b=B.p2-B.p1;
262     return sign(a)<sign(b)||((sign(a)==sign(b)
263         )&&a.cross(b)>0);
264 }
265 int halfplane_intersection(vector<line<T>
266     > &s){//半平面交
267     sort(s.begin(),s.end(),angle_cmp);//線段
268     左側為該線段半平面
269     int L,R,n=s.size();
270     vector<point<T> > px(n);
271     vector<line<T> > q(n);
272     q[L=R=0]=s[0];
273     for(int i=1;i<n;i++){
274         while(L<R&&s[i].ori(px[R-1])<=0)--R;
275         while(L<R&&s[i].ori(px[L])<=0)++L;
276         q[++R]=s[i];
277         if(q[R].parallel(q[R-1])){
278             --R;
279             if(q[R].ori(s[i].p1)>0)q[R]=s[i];
280         }
281         if(L<R)px[R-1]=q[R-1].
282             line_intersection(q[R]);
283     }
284     while(L<R&&q[L].ori(px[R-1])<=0)--R;
285     p.clear();
286     if(R-L<=1)return 0;
287     px[R]=q[R].line_intersection(q[L]);
288     for(int i=L;i<R;i++)p.push_back(px[i]);
289     return R-L+1;
290 }
291 template<typename T>
292 struct triangle{
293     point<T> a,b,c;
294     triangle(const point<T> &a,const point<T>
295         &b,const point<T> &c):a(a),b(b),c(c){}
296     T area()const{
297         T t=(b-a).cross(c-a)/2;
298         return t>0?t:-t;
299     }
300     point<T> barycenter()const{//重心
301         return (a+b+c)/3;
302     }
303     point<T> circumcenter()const{//外心
304         static line<T> u,v;
305         u.p1=(a+b)/2;
306         u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-
307             b.x);
308         v.p1=(a+c)/2;
309         v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-
310             c.x);
311         return u.line_intersection(v);
312     }
313     point<T> incenter()const{//內心
314         T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2
315             ()),C=sqrt((a-b).abs2());
316         return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+
317             B*b.y+C*c.y)/(A+B+C);
318     }
319     point<T> perpencenter()const{//垂心
320         return barycenter()*3-circumcenter()*2;
321     }
322 };
323 template<typename T>
324 struct point3D{
325     T x,y,z;
326     point3D(){}
327     point3D(const T&x,const T&y,const T&z):x(x)
328         ,y(y),z(z){}
329     point3D operator+(const point3D &b)const{
330         return point3D(x+b.x,y+b.y,z+b.z);
331     }
332     point3D operator-(const point3D &b)const{
333         return point3D(x-b.x,y-b.y,z-b.z);
334     }
335     point3D operator*(const T &b)const{
336         return point3D(x*b,y*b,z*b);
337     }
338     point3D operator/(const T &b)const{
339         return point3D(x/b,y/b,z/b);
340     }
341     bool operator==(const point3D &b)const{
342         return x==b.x&&y==b.y&&z==b.z;
343     }
344     T dot(const point3D &b)const{
345         return x*b.x+y*b.y+z*b.z;
346     }
347     point3D cross(const point3D &b)const{
348         return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x
349             *b.y-y*b.x);
350     }
351     T abs2()const{//向量長度的平方
352         return dot(*this);
353     }
354     T area2(const point3D &b)const{//和b、原點
355         圍成面積的平方
356         return cross(b).abs2()/4;
357     }
358 };
359 template<typename T>
360 struct line3D{
361     point3D<T> p1,p2;
362     line3D(){}
363     line3D(const point3D<T> &p1,const point3D<
364         T> &p2):p1(p1),p2(p2){}
365     T dis2(const point3D<T> &p,bool is_segment
366         =0)const{//點跟直線/線段的距離平方
367         point3D<T> v=p2-p1,v1=p-p1;
368         if(is_segment){
369             point3D<T> v2=p-p2;
370             if(v.dot(v1)<=0)return v1.abs2();
371             if(v.dot(v2)>=0)return v2.abs2();
372         }
373         point3D<T> tmp=v.cross(v1);
374         return tmp.abs2()/v.abs2();
375     }
376     pair<point3D<T>,point3D<T> > closest_pair(
377         const line3D<T> &l)const{
378         point3D<T> v1=(p1-p2),v2=(l.p1-l.p2);
379         point3D<T> N=v1.cross(v2),ab(p1-l.p1);
380         //if(N.abs2()==0)return NULL;平行或重合
381         T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//
382         最近點對距離
383         point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.
384             cross(d2),G=l.p1-p1;
385         T t1=(G.cross(d2)).dot(D)/D.abs2();
386         T t2=(G.cross(d1)).dot(D)/D.abs2();
387         return make_pair(p1+d1*t1,l.p1+d2*t2);
388     }
389     bool same_side(const point3D<T> &a,const
390         point3D<T> &b)const{
391         return (p2-p1).cross(a-p1).dot((p2-p1).
392             cross(b-p1))>0;
393     }
394 };
395 template<typename T>
396 struct plane{
397     point3D<T> p0,n;//平面上的點和法向量
398     plane(){}
399     plane(const point3D<T> &p0,const point3D<T>
400         &n):p0(p0),n(n){}
401     T dis2(const point3D<T> &p)const{//點到平
402         面距離的平方
403         T tmp=(p-p0).dot(n);
404         return tmp*tmp/n.abs2();
405     }
406 };
407 point3D<T> projection(const point3D<T> &p)
408     const{
409     return p-n*(p-p0).dot(n)/n.abs2();
410 }
411 point3D<T> line_intersection(const line3D<T>
412     &l)const{
413     T tmp=n.dot(l.p2-l.p1);//等於0表示平行或
414     重合該平面
415     return l.p1+(l.p2-l.p1)*(n.dot(p0-l.p1)/
416         tmp);
417 }
418 line3D<T> plane_intersection(const plane &
419     p1)const{
420     point3D<T> e=n.cross(p1.n),v=n.cross(e);
421     T tmp=p1.n.dot(v);//等於0表示平行或重合
422     該平面
423     point3D<T> q=p0+(v*(p1.n.dot(p1.p0-p0))/
424         tmp);
425     return line3D<T>(q,q+e);
426 }
427 template<typename T>
428 struct triangle3D{
429     point3D<T> a,b,c;
430     triangle3D(){}
431     triangle3D(const point3D<T> &a,const
432         point3D<T> &b,const point3D<T> &c):a(a)
433         ,b(b),c(c){}
434     bool point_in(const point3D<T> &p)const{//
435         點在該平面上的投影在三角形中
436         return line3D<T>(b,c).same_side(p,a)&&
437             line3D<T>(a,c).same_side(p,b)&&
438             line3D<T>(a,b).same_side(p,c);
439     }
440 };
441 template<typename T>
442 struct tetrahedron{//四面體
443     point3D<T> a,b,c,d;
444     tetrahedron(){}
445     tetrahedron(const point3D<T> &a,const
446         point3D<T> &b,const point3D<T> &c,
447         const point3D<T> &d):a(a),b(b),c(c),d(d)
448     {}
449     T volume6()const{//體積的六倍
450         return (d-a).dot((b-a).cross(c-a));
451     }
452     point3D<T> centroid()const{
453         return (a+b+c+d)/4;
454     }
455     bool point_in(const point3D<T> &p)const{
456         return triangle3D<T>(a,b,c).point_in(p)
457             &&triangle3D<T>(c,d,a).point_in(p);
458     }
459 };
460 template<typename T>
461 struct convexhull3D{
462     static const int MAXN=1005;
463     struct face{
464         int a,b,c;
465         face(int a,int b,int c):a(a),b(b),c(c){}
466     };
467     vector<point3D<T> > pt;
468     vector<face> ans;
469     int fid[MAXN][MAXN];

```

```

421 void build(){
422     int n=pt.size();
423     ans.clear();
424     memset(fid,0,sizeof(fid));
425     ans.emplace_back(0,1,2); //注意不能共線
426     ans.emplace_back(2,1,0);
427     int ftop = 0;
428     for(int i=3, ftop=1; i<n; ++i, ++ftop){
429         vector<face> next;
430         for(auto &f:ans){
431             T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[
432                 f.a]).cross(pt[f.c]-pt[f.a]));
433             if(d<=0) next.push_back(f);
434             int ff=0;
435             if(d>0) ff=ftop;
436             else if(d<0) ff=-ftop;
437             fid[f.a][f.b]=fid[f.b][f.c]=fid[f.c
438                 ][f.a]=ff;
439         }
440         for(auto &f:ans){
441             if(fid[f.a][f.b]>0 && fid[f.a][f.b
442                 ]!=fid[f.b][f.a])
443                 next.emplace_back(f.a,f.b,i);
444             if(fid[f.b][f.c]>0 && fid[f.b][f.c
445                 ]!=fid[f.c][f.b])
446                 next.emplace_back(f.b,f.c,i);
447             if(fid[f.c][f.a]>0 && fid[f.c][f.a
448                 ]!=fid[f.a][f.c])
449                 next.emplace_back(f.c,f.a,i);
450         }
451         ans=next;
452     }
453     point3D<T> centroid()const{
454         point3D<T> res(0,0,0);
455         T vol=0;
456         for(auto &f:ans){
457             T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c
458                 ]));
459             res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
460             vol+=tmp;
461         }
462         return res/(vol*4);
463     }
464 }
465 };

```

## 1.2 SmallestCircle

```

1 #include "Geometry.cpp"
2 struct Circle{
3     typedef point<double> p;
4     typedef const point<double> cp;
5     p x;
6     double r2;
7     bool incircle(cp &c)const{return (x-c).
8         abs2()<=r2;}
9 };
10 Circle TwoPointCircle(Circle::cp &a, Circle
11     ::cp &b) {
12     Circle::p m=(a+b)/2;
13     return (Circle){m,(a-m).abs2()};

```

```

14 Circle outcircle(Circle::p a, Circle::p b,
15     Circle::p c) {
16     if(TwoPointCircle(a,b).incircle(c))
17         return TwoPointCircle(a,b);
18     if(TwoPointCircle(b,c).incircle(a))
19         return TwoPointCircle(b,c);
20     if(TwoPointCircle(c,a).incircle(b))
21         return TwoPointCircle(c,a);
22     Circle::p ret;
23     double a1=b.x-a.x, b1=b.y-a.y, c1=(a1*a1
24         +b1*b1)/2;
25     double a2=c.x-a.x, b2=c.y-a.y, c2=(a2*a2
26         +b2*b2)/2;
27     double d = a1*b2 - a2*b1;
28     ret.x=a.x+(c1*b2-c2*b1)/d;
29     ret.y=a.y+(a1*c2-a2*c1)/d;
30     return (Circle){ret,(ret-a).abs2()};
31 }
32 //rand required
33 Circle SmallestCircle(std::vector<Circle::p>
34     &p){
35     int n=p.size();
36     if(n==1) return (Circle){p[0],0.0};
37     if(n==2) return TwoPointCircle(p[0],p
38         [1]);
39     random_shuffle(p.begin(),p.end());
40     Circle c = {p[0],0.0};
41     for(int i=0;i<n;++i){
42         if(c.incircle(p[i])) continue;
43         c=Circle{p[i],0.0};
44         for(int j=0;j<i;++j){
45             if(c.incircle(p[j])) continue;
46             c=TwoPointCircle(p[i],p[j]);
47             for(int k=0;k<j;++k){
48                 if(c.incircle(p[k]))
49                     continue;
50                 c=outcircle(p[i],p[j],p[k]);
51             }
52         }
53     }
54     return c;
55 }

```

## 1.3 最近點對

```

1 template<typename _IT=point<T>* >
2 T closest_pair(_IT L, _IT R){
3     if(R-L <= 1) return INF;
4     _IT mid = L+(R-L)/2;
5     T x = mid->x;
6     T d = min(closest_pair(L,mid),closest_pair(
7         mid,R));
8     inplace_merge(L, mid, R, ycmp);
9     static vector<point> b; b.clear();
10    for(auto u=L;u<R;++u){
11        if((u->x-x)*(u->x-x)>=d) continue;
12        for(auto v=b.rbegin();v!=b.rend();++v){
13            T dx=u->x-v->x, dy=u->y-v->y;
14            if(dy*dy>=d) break;
15            d=min(d,dx*dx+dy*dy);
16        }
17        b.push_back(*u);

```

```

17 }
18 return d;
19 }
20 T closest_pair(vector<point<T>> &v){
21     sort(v.begin(),v.end(),xcmp);
22     return closest_pair(v.begin(),v.end());
23 }

```

## 2 Data\_Structure

### 2.1 DLX

```

1 const int MAXN=4100, MAXM=1030, MAXND=16390;
2 struct DLX{
3     int n,m,sz,ansd; //高是n 寬是m的稀疏矩陣
4     int S[MAXN],H[MAXN];
5     int row[MAXN],col[MAXN]; //每個節點代表的
6         列跟行
7     int L[MAXN],R[MAXN],U[MAXN],D[MAXN];
8     vector<int> ans,anst;
9     void init(int _n,int _m){
10         n=_n,m=_m;
11         for(int i=0;i<=m;++i){
12             U[i]=D[i]=i,L[i]=i-1,R[i]=i+1;
13             S[i]=0;
14         }
15         R[m]=0,L[0]=m;
16         sz=m,ansd=INT_MAX; //ansd存最優解的個數
17         for(int i=1;i<=n;++i)H[i]=-1;
18     }
19     void add(int r,int c){
20         ++S[col[++sz]=c];
21         row[sz]=r;
22         D[sz]=D[c],U[D[c]]=sz,U[sz]=c,D[c]=sz;
23         if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
24         else R[sz]=R[H[r]],L[R[H[r]]]=sz,L[sz]=H
25             [r],R[H[r]]=sz;
26     }
27     #define DFOR(i,A,s) for(int i=A[s];i!=s;i=
28         A[i])
29     void remove(int c){ //刪除第c行和所有當前覆
30         蓋到第c行的列
31         L[R[c]]=L[c],R[L[c]]=R[c]; //這裡刪除第c
32         行 若有些行不需要處理可以在開始時呼
33         叫他
34         DFOR(i,D,c)DFOR(j,R,i){U[D[j]]=U[j],D[U[
35             j]]=D[j],--S[col[j]]};
36     }
37     void restore(int c){ //恢復第c行和所有當前
38         覆蓋到第c行的列 remove的逆操作
39         DFOR(i,U,c)DFOR(j,L,i){++S[col[j]],U[D[j
40             ]]=j,D[U[j]]=j;}
41         L[R[c]]=c,R[L[c]]=c;
42     }
43     void remove2(int nd){ //刪除nd所在的行當前
44         所有點(包括虛擬節點) 只保留nd
45         DFOR(i,D,nd)L[R[i]]=L[i],R[L[i]]=R[i];
46     }

```

```

37 void restore2(int nd){ //刪除nd所在的行當前
38     所有點 為remove2的逆操作
39     DFOR(i,U,nd)L[R[i]]=R[L[i]]=i;
40 }
41 bool vis[MAXN];
42 int h(){ //估價函數 for IDA*
43     int res=0;
44     memset(vis,0,sizeof(vis));
45     DFOR(i,R,0)if(!vis[i]){
46         vis[i]=1;
47         ++res;
48         DFOR(j,D,i)DFOR(k,R,j)vis[col[k]]=1;
49     }
50     return res;
51 }
52 bool dfs(int d){ //for精確覆蓋問題
53     if(d+h())>=ansd return 0; //找最佳解用 找
54     任意解可以刪掉
55     if(!R[0]){ansd=d;return 1;}
56     int c=R[0];
57     DFOR(i,R,0)if(S[i]<S[c])c=i;
58     remove(c);
59     DFOR(i,D,c){
60         ans.push_back(row[i]);
61         DFOR(j,R,i)remove(col[j]);
62         if(dfs(d+1))return 1;
63         ans.pop_back();
64         DFOR(j,L,i)restore(col[j]);
65     }
66     restore(c);
67     return 0;
68 }
69 void dfs2(int d){ //for最小重複覆蓋問題
70     if(d+h())>=ansd return;
71     if(!R[0]){ansd=d;ans=anst;return;}
72     int c=R[0];
73     DFOR(i,R,0)if(S[i]<S[c])c=i;
74     DFOR(i,D,c){
75         anst.push_back(row[i]);
76         remove2(i);
77         DFOR(j,R,i)remove2(j),--S[col[j]];
78         dfs2(d+1);
79         anst.pop_back();
80         DFOR(j,L,i)restore2(j),++S[col[j]];
81         restore2(i);
82     }
83 }
84 bool exact_cover(){ //解精確覆蓋問題
85     return ans.clear(), dfs(0);
86 }
87 void min_cover(){ //解最小重複覆蓋問題
88     anst.clear(); //暫存用 答案還是存在ans裡
89     dfs2(0);
90     #undef DFOR
91 };

```

### 2.2 Dynamic\_KD\_tree

```

1 template<typename T,size_t kd> //有kd個維度
2 struct kd_tree{

```

```

3 struct point{
4     T d[kd];
5     T dist(const point &x) const{
6         T ret=0;
7         for(size_t i=0;i<kd;++i) ret+=std::abs(
8             d[i]-x.d[i]);
9         return ret;
10    }
11    bool operator==(const point &p){
12        for(size_t i=0;i<kd;++i)
13            if(d[i]!=p.d[i]) return 0;
14        return 1;
15    }
16    bool operator<(const point &b) const{
17        return d[0]<b.d[0];
18    }
19 private:
20    struct node{
21        node *l,*r;
22        point pid;
23        int s;
24        node(const point &p):l(0),r(0),pid(p),s
25            (1){}
26        ~node(){delete l;delete r;}
27        void up(){s=(l?l->s:0)+1+(r?r->s:0);}
28    }*root;
29    const double alpha,loga;
30    const T INF;//記得要給INF 表示極大值
31    int maxn;
32    struct __cmp{
33        int sort_id;
34        bool operator()(const node*x,const node*
35            y) const{
36            return operator()(x->pid,y->pid);
37        }
38        bool operator()(const point &x,const
39            point &y) const{
40            if(x.d[sort_id]!=y.d[sort_id])
41                return x.d[sort_id]<y.d[sort_id];
42            for(size_t i=0;i<kd;++i)
43                if(x.d[i]!=y.d[i]) return x.d[i]<y.d[
44                    i];
45            return 0;
46        }
47    }cmp;
48    int size(node *o){return o?o->s:0;}
49    std::vector<node*> A;
50    node* build(int k,int l,int r){
51        if(l>r) return 0;
52        if(k==kd) k=0;
53        int mid=(l+r)/2;
54        cmp.sort_id = k;
55        std::nth_element(A.begin()+l,A.begin()+
56            mid,A.begin()+r+1,cmp);
57        node *ret=A[mid];
58        ret->l = build(k+1,l,mid-1);
59        ret->r = build(k+1,mid+1,r);
60        ret->up();
61        return ret;
62    }
63    bool isbad(node*o){
64        return size(o->l)>alpha*o->s||size(o->r)
65            >alpha*o->s;
66    }
67    void flatten(node *u,typename std::vector<
68        node*>::iterator &it){
69        if(!u) return;
70        flatten(u->l,it);
71        *it=u;
72        flatten(u->r,++it);
73    }
74    void rebuild(node*&u,int k){
75        if((int)A.size()<u->s)A.resize(u->s);
76        typename std::vector<node*>::iterator it
77            =A.begin();
78        flatten(u,it);
79        u=build(k,0,u->s-1);
80    }
81    bool insert(node*&u,int k,const point &x,
82        int dep){
83        if(!u) return u=new node(x), dep<=0;
84        ++u->s;
85        cmp.sort_id=k;
86        if(insert(cmp(x,u->pid)?u->l:u->r,(k+1)%
87            kd,x,dep-1)){
88            if(!isbad(u)) return 1;
89            rebuild(u,k);
90        }
91        return 0;
92    }
93    node *findmin(node*o,int k){
94        if(!o) return 0;
95        if(cmp.sort_id==k) return o->l?findmin(o
96            ->l,(k+1)%kd):o;
97        node *l=findmin(o->l,(k+1)%kd);
98        node *r=findmin(o->r,(k+1)%kd);
99        if(l&&!r) return cmp(l,o)?l:o;
100        if(!l&&r) return cmp(r,o)?r:o;
101        if(!l&&!r) return 0;
102        if(cmp(l,r)) return cmp(l,o)?l:o;
103        return cmp(r,o)?r:o;
104    }
105    bool erase(node *&u,int k,const point &x){
106        if(!u) return 0;
107        if(u->pid==x){
108            if(u->r);
109            else if(u->l) u->r=u->l, u->l=0;
110            else{
111                delete u;
112                return u=0, 1;
113            }
114            --u->s;
115            cmp.sort_id=k;
116            u->pid=findmin(u->r,(k+1)%kd)->pid;
117            return erase(u->r,(k+1)%kd,u->pid);
118        }
119        cmp.sort_id=k;
120        if(erase(cmp(x,u->pid)?u->l:u->r,(k+1)%
121            kd,x))
122            return --u->s, 1;
123        return 0;
124    }
125    T heuristic(const T h[]) const{
126        T ret=0;
127        for(size_t i=0;i<kd;++i) ret+=h[i];
128        return ret;
129    }
130    int qM;
131    std::priority_queue<std::pair<T,point > >
132        pQ;
133    void nearest(node *u,int k,const point &x,
134        T *h,T &mndist){
135        if(u==0||heuristic(h)>=mndist) return;
136        T dist=u->pid.dist(x),old=h[k];
137        /*mndist=std::min(mndist,dist);*/
138        if(dist<mndist){
139            pQ.push(std::make_pair(dist,u->pid));
140            if((int)pQ.size()==qM+1)
141                mndist=pQ.top().first,pQ.pop();
142        }
143        if(x.d[k]<u->pid.d[k]){
144            nearest(u->l,(k+1)%kd,x,h,mndist);
145            h[k]=std::abs(x.d[k]-u->pid.d[k]);
146            nearest(u->r,(k+1)%kd,x,h,mndist);
147        } else{
148            nearest(u->r,(k+1)%kd,x,h,mndist);
149            h[k]=std::abs(x.d[k]-u->pid.d[k]);
150            nearest(u->l,(k+1)%kd,x,h,mndist);
151        }
152        h[k]=old;
153    }
154    std::vector<point> in_range;
155    void range(node *u,int k,const point &mi,
156        const point &ma){
157        if(!u) return;
158        bool is=1;
159        for(int i=0;i<kd;++i)
160            if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid
161                .d[i]){
162                is=0;break;
163            }
164        if(is) in_range.push_back(u->pid);
165        if(mi.d[k]<u->pid.d[k]) range(u->l,(k+1)
166            %kd,mi,ma);
167        if(ma.d[k]>u->pid.d[k]) range(u->r,(k+1)
168            %kd,mi,ma);
169    }
170    public:
171    kd_tree(const T &INF,double a=0.75):root
172        (0),alpha(a),loga(log2(1.0/a)),INF(INF
173            ),maxn(1){}
174    ~kd_tree(){delete root;}
175    void clear(){delete root;root=0,maxn=1;}
176    void build(int n,const point *p){
177        delete root,A.resize(maxn=n);
178        for(int i=0;i<n;++i) A[i]=new node(p[i]);
179        root=build(0,0,n-1);
180    }
181    void insert(const point &x){
182        insert(root,0,x,__lg(size(root))/loga);
183        if(root->s>maxn) maxn=root->s;
184    }
185    bool erase(const point &p){
186        bool d=erase(root,0,p);
187        if(root&&root->s<alpha*maxn) rebuild();
188        return d;
189    }
190    void rebuild(){
191        if(root) rebuild(root,0);
192        maxn=root->s;
193    }
194    T nearest(const point &x,int k){
195        qM=k;
196        T mndist=INF,h[kd]={};
197        nearest(root,0,x,h,mndist);
198        mndist=pQ.top().first;
199    }
200    pQ=std::priority_queue<std::pair<T,point
201        > >();
202    return mndist;//回傳離x第k近的點的距離
203    }
204    const std::vector<point> &range(const
205        point &mi,const point &ma){
206        in_range.clear();
207        range(root,0,mi,ma);
208        return in_range;//回傳介於mi到ma之間的點
209        vector
210    }
211    int size(){return root?root->s:0;}
212    };
213
214 2.3 kd_tree_replace_segment_tree
215
216 1 /*kd樹代替高維線段樹*/
217 2 struct node{
218 3     node *l,*r;
219 4     point pid,mi,ma;
220 5     int s;
221 6     int data;
222 7     node(const point &p,int d):l(0),r(0),pid(p
223         ),mi(p),ma(p),s(1),data(d),dmin(d),
224         dmax(d){}
225 8     void up(){
226 9         mi=ma=pid;
227 10        s=1;
228 11        if(l){
229 12            for(int i=0;i<kd;++i){
230 13                mi.d[i]=min(mi.d[i],l->mi.d[i]);
231 14                ma.d[i]=max(ma.d[i],l->ma.d[i]);
232 15            }
233 16            s+=l->s;
234 17        }
235 18        if(r){
236 19            for(int i=0;i<kd;++i){
237 20                mi.d[i]=min(mi.d[i],r->mi.d[i]);
238 21                ma.d[i]=max(ma.d[i],r->ma.d[i]);
239 22            }
240 23            s+=r->s;
241 24        }
242 25    }
243 26    void up2(){
244 27        //其他懶惰標記向上更新
245 28    }
246 29    void down(){
247 30        //其他懶惰標記下推
248 31    }
249 32    }*root;
250 33
251 34 /*檢查區間包含用的函數*/
252 35 inline bool range_include(node *o,const
253     point &L,const point &R){
254 36     for(int i=0;i<kd;++i){
255 37         if(L.d[i]>o->ma.d[i]||R.d[i]<o->mi.d[i])
256 38             return 0;
257 39     }//只要(L,R)區間有和o的區間有交集就回傳
258 40     true
259     return 1;
260 }

```



```

41 inline bool range_in_range(node *o, const
    point &L, const point &R){
42     for(int i=0; i<kd; ++i){
43         if(L.d[i]>o->mi.d[i] || o->ma.d[i]>R.d[i])
            return 0;
44     } //如果 (L,R) 區間完全包含 o 的區間就回傳 true
    return 1;
45 }
46 inline bool point_in_range(node *o, const
    point &L, const point &R){
47     for(int i=0; i<kd; ++i){
48         if(L.d[i]>o->pid.d[i] || R.d[i]<o->pid.d[i]
            )return 0;
49     } //如果 (L,R) 區間完全包含 o->pid 這個點就回傳
        true
    return 1;
50 }
51 }
52
53 /*單點修改 · 以單點改值為例*/
54 void update(node *u, const point &x, int data,
    int k=0){
55     if(!u)return;
56     u->down();
57     if(u->pid==x){
58         u->data=data;
59         u->up2();
60         return;
61     }
62     cmp.sort_id=k;
63     update(cmp(x, u->pid)?u->l:u->r, x, data, (k
        +1)%kd);
64     u->up2();
65 }
66
67 /*區間修改*/
68 void update(node *o, const point &L, const
    point &R, int data){
69     if(!o)return;
70     o->down();
71     if(range_in_range(o, L, R)){
72         //區間懶惰標記修改
73         o->down();
74         return;
75     }
76     if(point_in_range(o, L, R)){
77         //這個點在 (L, R) 區間 · 但是他的左右子樹不
            一定在區間中
78         //單點懶惰標記修改
79     }
80     if(o->l&&range_include(o->l, L, R))update(o
        ->l, L, R, data);
81     if(o->r&&range_include(o->r, L, R))update(o
        ->r, L, R, data);
82     o->up2();
83 }
84
85 /*區間查詢 · 以總和為例*/
86 int query(node *o, const point &L, const point
    &R){
87     if(!o)return 0;
88     o->down();
89     if(range_in_range(o, L, R))return o->sum;
90     int ans=0;
91     if(point_in_range(o, L, R))ans+=o->data;
92

```

```

93     if(o->l&&range_include(o->l, L, R))ans+=
        query(o->l, L, R);
94     if(o->r&&range_include(o->r, L, R))ans+=
        query(o->r, L, R);
95     return ans;
96 }

```

## 2.4 reference\_point

```

1 template<typename T>
2 struct _RefC{
3     T data;
4     int ref;
5     _RefC(const T&d=0):data(d),ref(0){}
6 };
7 template<typename T>
8 struct _rp{
9     _RefC<T> *p;
10    T *operator->(){return &p->data;}
11    T &operator*(){return p->data;}
12    operator _RefC<T>*(){return p;}
13    _rp &operator=(const _rp &t){
14        if(p&&!--p->ref)delete p;
15        p=t.p, p&&+p->ref;
16        return *this;
17    }
18    _rp(_RefC<T> *t=0):p(t){p&&+p->ref;}
19    _rp(const _rp &t):p(t.p){p&&+p->ref;}
20    ~_rp(){if(p&&!--p->ref)delete p;}
21 };
22 template<typename T>
23 inline _rp<T> new_rp(const T&nd){
24     return _rp<T>(new _RefC<T>(nd));
25 }

```

## 2.5 skew\_heap

```

1 node *merge(node *a, node *b){
2     if(!a || !b) return a?a:b;
3     if(b->data<a->data) swap(a, b);
4     swap(a->l, a->r);
5     a->l=merge(b, a->l);
6     return a;
7 }

```

## 2.6 undo\_disjoint\_set

```

1 struct DisjointSet {
2     // save() is like recursive
3     // undo() is like return
4     int n, fa[MAXN], sz[MAXN];
5     vector<pair<int*, int*>> h;
6     vector<int> sp;
7     void init(int tn) {
8         n=tn;
9         for (int i=0; i<n; i++) sz[fa[i]=i]=1;

```

```

10         sp.clear(); h.clear();
11     }
12     void assign(int *k, int v) {
13         h.PB({k, *k});
14         *k=v;
15     }
16     void save() { sp.PB(SZ(h)); }
17     void undo() {
18         assert(!sp.empty());
19         int last=sp.back(); sp.pop_back();
20         while (SZ(h)!=last) {
21             auto x=h.back(); h.pop_back();
22             *x.F=x.S;
23         }
24     }
25     int f(int x) {
26         while (fa[x]!=x) x=fa[x];
27         return x;
28     }
29     void uni(int x, int y) {
30         x=f(x); y=f(y);
31         if (x==y) return ;
32         if (sz[x]<sz[y]) swap(x, y);
33         assign(&sz[x], sz[x]+sz[y]);
34         assign(&fa[y], x);
35     }
36 } djs;

```

## 2.7 整體二分

```

1 void totBS(int L, int R, vector<Item> M){
2     if(Q.empty()) return; //維護全域B陣列
3     if(L==R) 整個M的答案=r, return;
4     int mid = (L+R)/2;
5     vector<Item> mL, mR;
6     do_modify_B_with_divide(mid, M);
7     //讓B陣列在遞迴的時候只會保留 [L~mid] 的資訊
8     undo_modify_B(mid, M);
9     totBS(L, mid, mL);
10    totBS(mid+1, R, mR);
11 }

```

## 3 Flow

### 3.1 dinic

```

1 template<typename T>
2 struct DINIC{
3     static const int MAXN=105;
4     static const T INF=INT_MAX;
5     int n, LV[MAXN], cur[MAXN];
6     struct edge{
7         int v, pre;
8         T cap, r;
9         edge(int v, int pre, T cap):v(v), pre(pre),
            cap(cap), r(cap){}
10    };

```

```

11    int g[MAXN];
12    vector<edge> e;
13    void init(int _n){
14        memset(g, -1, sizeof(int)*((n=_n)+1));
15        e.clear();
16    }
17    void add_edge(int u, int v, T cap, bool
        directed=false){
18        e.push_back(edge(v, g[u], cap));
19        g[u]=e.size()-1;
20        e.push_back(edge(u, g[v], directed?0:cap))
            ;
21        g[v]=e.size()-1;
22    }
23    int bfs(int s, int t){
24        memset(LV, 0, sizeof(int)*(n+1));
25        memcpy(cur, g, sizeof(int)*(n+1));
26        queue<int> q;
27        q.push(s);
28        LV[s]=1;
29        while(q.size()){
30            int u=q.front(); q.pop();
31            for(int i=g[u]; ~i; i=e[i].pre){
32                if(!LV[e[i].v]&&e[i].r){
33                    LV[e[i].v]=LV[u]+1;
34                    q.push(e[i].v);
35                    if(e[i].v==t) return 1;
36                }
37            }
38        }
39        return 0;
40    }
41    T dfs(int u, int t, T CF=INF){
42        if(u==t) return CF;
43        T df;
44        for(int &i=cur[u]; ~i; i=e[i].pre){
45            if(LV[e[i].v]==LV[u]+1&&e[i].r){
46                if(df=dfs(e[i].v, t, min(CF, e[i].r))){
47                    e[i].r-=df;
48                    e[i^1].r+=df;
49                    return df;
50                }
51            }
52        }
53        return LV[u]=0;
54    }
55    T dinic(int s, int t, bool clean=true){
56        if(clean)for(size_t i=0; i<e.size(); ++i)
            e[i].r=e[i].cap;
57        T ans=0, f=0;
58        while(bfs(s, t))while(f=dfs(s, t))ans+=f;
59        return ans;
60    }
61 }
62 };

```

### 3.2 ISAP\_with\_cut

```

1 template<typename T>
2 struct ISAP{
3     static const int MAXN=105;
4     static const T INF=INT_MAX;
5     int n; //點數
6     int d[MAXN], gap[MAXN], cur[MAXN];

```

```

7 struct edge{
8     int v,pre;
9     T cap,r;
10    edge(int v,int pre,T cap):v(v),pre(pre),
        cap(cap),r(cap){}
11 };
12 int g[MAXN];
13 vector<edge> e;
14 void init(int _n){
15     memset(g,-1,sizeof(int)*((n=_n)+1));
16     e.clear();
17 }
18 void add_edge(int u,int v,T cap,bool
        directed=false){
19     e.push_back(edge(v,g[u],cap));
20     g[u]=e.size()-1;
21     e.push_back(edge(u,g[v],directed?0:cap))
        ;
22     g[v]=e.size()-1;
23 }
24 T dfs(int u,int s,int t,T CF=INF){
25     if(u==t)return CF;
26     T tf=CF,df;
27     for(int &i=cur[u];~i;i=e[i].pre){
28         if(e[i].r&&d[u]==d[e[i].v]+1){
29             df=dfs(e[i].v,s,t,min(tf,e[i].r));
30             e[i].r-=df;
31             e[i^1].r+=df;
32             if(!(tf==df)||d[s]==n)return CF-tf;
33         }
34     }
35     int mh=n;
36     for(int i=cur[u]=g[u];~i;i=e[i].pre){
37         if(e[i].r&&d[e[i].v]<mh)mh=d[e[i].v];
38     }
39     if(!--gap[d[u]])d[s]=n;
40     else ++gap[d[u]]=++mh;
41     return CF-tf;
42 }
43 T isap(int s,int t,bool clean=true){
44     memset(d,0,sizeof(int)*(n+1));
45     memset(gap,0,sizeof(int)*(n+1));
46     memcpy(cur,g,sizeof(int)*(n+1));
47     if(clean) for(size_t i=0;i<e.size();++i)
48         e[i].r=e[i].cap;
49     T MF=0;
50     for(gap[0]=n;d[s]<n;)MF+=dfs(s,s,t);
51     return MF;
52 }
53 vector<int> cut_e;//最小割邊集
54 bool vis[MAXN];
55 void dfs_cut(int u){
56     vis[u]=1;//表示u屬於source的最小割集
57     for(int i=g[u];~i;i=e[i].pre)
58         if(e[i].r>0&&vis[e[i].v])dfs_cut(e[i]
            .v);
59 }
60 T min_cut(int s,int t){
61     T ans=isap(s,t);
62     memset(vis,0,sizeof(bool)*(n+1));
63     dfs_cut(s); cut_e.clear();
64     for(int u=0;u<n;++u)if(vis[u])
65         for(int i=g[u];~i;i=e[i].pre)
66             if(!vis[e[i].v])cut_e.push_back(i);
67     return ans;

```

### 3.3 MinCostMaxFlow

```

1 template<typename TP>
2 struct MCMF{
3     static const int MAXN=440;
4     static const TP INF=999999999;
5     struct edge{
6         int v,pre;
7         TP r,cost;
8         edge(int v,int pre,TP r,TP cost):v(v),
            pre(pre),r(r),cost(cost){}
9     };
10    int n,S,T;
11    TP dis[MAXN],PIS,ans;
12    bool vis[MAXN];
13    vector<edge> e;
14    int g[MAXN];
15    void init(int _n){
16        memset(g,-1,sizeof(int)*((n=_n)+1));
17        e.clear();
18    }
19    void add_edge(int u,int v,TP r,TP cost,
        bool directed=false){
20        e.push_back(edge(v,g[u],r,cost));
21        g[u]=e.size()-1;
22        e.push_back(
23            edge(u,g[v],directed?0:r,-cost));
24        g[v]=e.size()-1;
25    }
26    TP augment(int u,TP CF){
27        if(u==T||!CF)return ans+=PIS*CF,CF;
28        vis[u]=1;
29        TP r=CF,d;
30        for(int i=g[u];~i;i=e[i].pre){
31            if(e[i].r&&!e[i].cost&&vis[e[i].v]){
32                d=augment(e[i].v,min(r,e[i].r));
33                e[i].r-=d;
34                e[i^1].r+=d;
35                if(!(r-=d))break;
36            }
37        }
38        return CF-r;
39    }
40    bool modlabel(){
41        for(int u=0;u<n;++u)dis[u]=INF;
42        static deque<int>q;
43        dis[T]=0,q.push_back(T);
44        while(q.size()){
45            int u=q.front();q.pop_front();
46            TP dt;
47            for(int i=g[u];~i;i=e[i].pre){
48                if(e[i^1].r&&(dt=dis[u]-e[i].cost)<
                    dis[e[i].v]){
49                    if((dis[e[i].v]=dt)<=dis[q.size()])
50                        q.front():S){}
51                    q.push_front(e[i].v);
52                }else q.push_back(e[i].v);
53            }
54        }

```

```

55     for(int u=0;u<n;++u)
56         for(int i=g[u];~i;i=e[i].pre)
57             e[i].cost+=dis[e[i].v]-dis[u];
58     return PIS+=dis[S], dis[S]<INF;
59 }
60 TP mincost(int s,int t){
61     S=s,T=t;
62     PIS=ans=0;
63     while(modlabel()){
64         do memset(vis,0,sizeof(bool)*(n+1));
65         while(augment(S,INF));
66     }return ans;
67 }
68 };

```

## 4 Graph

### 4.1 Augmenting\_Path

```

1 #define MAXN1 505
2 #define MAXN2 505
3 int n1,n2;//n1個點連向n2個點
4 int match[MAXN2];//屬於n2的點匹配了哪個點
5 vector<int> g[MAXN1];//圖
6 bool vis[MAXN2];//是否走訪過
7 bool dfs(int u){
8     for(size_t i=0;i<g[u].size();++i){
9         int v=g[u][i];
10        if(vis[v])continue;
11        vis[v]=1;
12        if(match[v]==-1||dfs(match[v]))
13            return match[v]=u, 1;
14    }
15    return 0;
16 }
17 inline int max_match(){
18     int ans=0;
19     memset(match,-1,sizeof(int)*n2);
20     for(int i=0;i<n1;++i){
21         memset(vis,0,sizeof(bool)*n2);
22         if(dfs(i))++ans;
23     }
24     return ans;
25 }

```

### 4.2 Augmenting\_Path\_multiple

```

1 #define MAXN1 1005
2 #define MAXN2 505
3 int n1,n2;//n1個點連向n2個點·其中n2個點可以
        匹配很多邊
4 vector<int> g[MAXN1];//圖
5 int c[MAXN2];//每個屬於n2點最多可以接受幾條
        匹配邊
6 vector<int> match_list[MAXN2];//每個屬於n2的
        點匹配了那些點

```

```

7 bool vis[MAXN2];//是否走訪過
8 bool dfs(int u){
9     for(size_t i=0;i<g[u].size();++i){
10        int v=g[u][i];
11        if(vis[v])continue;
12        vis[v]=true;
13        if((int)match_list[v].size()<c[v]){
14            return match_list[v].push_back(u),
                true;
15        }else{
16            for(size_t j=0;j<match_list[v].size()
                ;++j){
17                int next_u=match_list[v][j];
18                if(dfs(next_u))
19                    return match_list[v][j]=u, true;
20            }
21        }
22    }
23    return false;
24 }
25 int max_match(){
26     for(int i=0;i<n2;++i)match_list[i].clear()
        ;
27     int cnt=0;
28     for(int u=0;u<n1;++u){
29         memset(vis,0,sizeof(bool)*n2);
30         if(dfs(u))++cnt;
31     }
32     return cnt;
33 }

```

### 4.3 blossom\_matching

```

1 #define MAXN 505
2 vector<int>g[MAXN];
3 int pa[MAXN],match[MAXN],st[MAXN],S[MAXN],v[
        MAXN];
4 int t,n;
5 int lca(int x,int y){
6     for(++t;swap(x,y)){
7         if(x==0)continue;
8         if(v[x]==t)return x;
9         v[x]=t;
10        x=st[pa[match[x]]];
11    }
12 }
13 #define qpush(x) q.push(x),S[x]=0
14 void flower(int x,int y,int l,queue<int> &q)
        {
15     while(st[x]!=1){
16         pa[x]=y;
17         if(S[y==match[x]]==1)qpush(y);
18         st[x]=st[y]=1, x=pa[y];
19     }
20 }
21 bool bfs(int x){
22     for(int i=1;i<=n;++i)st[i]=i;
23     memset(S+1,-1,sizeof(int)*n);
24     queue<int>q; qpush(x);
25     while(q.size()){
26         x=q.front(),q.pop();
27         for(size_t i=0;i<g[x].size();++i){
28             int y=g[x][i];

```

```

29 if(S[y]==-1){
30   pa[y]=x,S[y]=1;
31   if(!match[y]){
32     for(int lst;x=y;lst,x=pa[y])
33       lst=match[x],match[x]=y,match[y]
34         =x;
35     return 1;
36   }
37   qpush(match[y]);
38   }else if(!S[y]&&st[y]!=st[x]){
39     int l=lc(y,x);
40     flower(y,x,l,q),flower(x,y,l,q);
41   }
42 }
43 return 0;
44 }
45 int blossom(){
46   int ans=0;
47   for(int i=1;i<n;++i)
48     if(!match[i]&&bfs(i))++ans;
49   return ans;
50 }

```

## 4.4 graphISO

```

1 const int MAXN=1005,K=30;///K要夠大
2 const long long A=3,B=11,C=2,D=19,P=0
3   xdefaced;
4 long long f[K+1][MAXN];
5 vector<int> g[MAXN],rg[MAXN];
6 int n;
7 void init(){
8   for(int i=0;i<n;++i){
9     f[0][i]=1;
10    g[i].clear(), rg[i].clear();
11  }
12 }
13 void add_edge(int u,int v){
14   g[u].push_back(v), rg[v].push_back(u);
15 }
16 long long point_hash(int u){///O(N)
17   for(int t=1;t<=K;++t){
18     for(int i=0;i<n;++i){
19       f[t][i]=f[t-1][i]*A%P;
20       for(int j:g[i])f[t][i]=(f[t][i]+f[t-1][j]*B%P)%P;
21       for(int j:rg[i])f[t][i]=(f[t][i]+f[t-1][j]*C%P)%P;
22       if(i==u)f[t][i]+=D;///如果圖太大的話，把這行刪掉，執行一次後f[K]就會是所
23         有點的答案
24       f[t][i]%=P;
25     }
26   }
27   return f[K][u];
28 }
29 vector<long long> graph_hash(){
30   vector<long long> ans;
31   for(int i=0;i<n;++i)ans.push_back(
32     point_hash(i));///O(N^2)
33   sort(ans.begin(),ans.end());

```

```

31 return ans;
32 }

```

## 4.5 KM

```

1 #define MAXN 405
2 #define INF 0x3f3f3f3f
3 int n;///1-base · 0表示沒有匹配
4 int g[MAXN][MAXN],lx[MAXN],ly[MAXN],pa[MAXN]
5   ],slack_y[MAXN];
6 int match_y[MAXN],match_x[MAXN];
7 bool vx[MAXN],vy[MAXN];
8 void augment(int y){
9   for(int x,z;y;y=z){
10    x=pa[y],z=match_x[x];
11    match_y[y]=x,match_x[x]=y;
12  }
13 }
14 void bfs(int st){
15   for(int i=1;i<n;++i)slack_y[i]=INF,vx[i]=
16     vy[i]=0;
17   queue<int> q;q.push(st);
18   for(;;){
19     while(q.size()){
20       int x=q.front();q.pop();
21       vx[x]=1;
22       for(int y=1;y<n;++y)if(!vy[y]){
23         int t=lx[x]+ly[y]-g[x][y];
24         if(t==0){
25           pa[y]=x;
26           if(!match_y[y]){augment(y);return}
27           ;
28         }
29         vy[y]=1,q.push(match_y[y]);
30       }else if(slack_y[y]>t)pa[y]=x,
31         slack_y[y]=t;
32     }
33   }
34   int cut=INF;
35   for(int y=1;y<n;++y){
36     if(!vy[y]&&cut>slack_y[y])cut=slack_y[
37       y];
38   }
39   for(int j=1;j<n;++j){
40     if(vx[j])lx[j]-=cut;
41     if(vy[j])ly[j]+=cut;
42     else slack_y[j]-=cut;
43   }
44   for(int y=1;y<n;++y){
45     if(!vy[y]&&slack_y[y]==0){
46       if(!match_y[y]){augment(y);return;}
47       vy[y]=1,q.push(match_y[y]);
48     }
49   }
50 }
51 long long KM(){
52   memset(match_y,0,sizeof(int)*(n+1));
53   memset(ly,0,sizeof(int)*(n+1));
54   for(int x=1;x<n;++x){
55     lx[x]=-INF;
56     for(int y=1;y<n;++y)
57       lx[x]=max(lx[x],g[x][y]);
58   }

```

```

54 for(int x=1;x<n;++x)bfs(x);
55 long long ans=0;
56 for(int y=1;y<n;++y)ans+=g[match_y[y]][y];
57 return ans;
58 }

```

## 4.6 MaximumClique

```

1 struct MaxClique{
2   static const int MAXN=105;
3   int N,ans;
4   int g[MAXN][MAXN],dp[MAXN],stk[MAXN][MAXN]
5     ];
6   int sol[MAXN],tmp[MAXN];///sol[0~ans-1]為答案
7   void init(int n){
8     N=n;///0-base
9     memset(g,0,sizeof(g));
10  }
11  void add_edge(int u,int v){
12    g[u][v]=g[v][u]=1;
13  }
14  int dfs(int ns,int dep){
15    if(!ns){
16      if(dep>ans){
17        ans=dep;
18        memcpy(sol,tmp,sizeof tmp);
19        return 1;
20      }else return 0;
21    }
22    for(int i=0;i<ns;++i){
23      if(dep+ns-i<=ans)return 0;
24      int u=stk[dep][i],cnt=0;
25      if(dep+dp[u]<=ans)return 0;
26      for(int j=i+1;j<ns;++j){
27        int v=stk[dep][j];
28        if(g[u][v])stk[dep+1][cnt++]=v;
29      }
30      tmp[dep]=u;
31      if(dfs(cnt,dep+1))return 1;
32    }
33    return 0;
34  }
35  int clique(){
36    int u,v,ns;
37    for(ans=0,u=N-1;u>0;--u){
38      for(ns=0,tmp[0]=u,v=u+1;v<N;++v)
39        if(g[u][v])stk[1][ns++]=v;
40      dfs(ns,1),dp[u]=ans;
41    }
42    return ans;
43  }

```

## 4.7 MinimumMeanCycle

```

1 #include<csdint>///for DBL_MAX
2 int dp[maxN+1][maxN+1];

```

```

3 double mnc(int n){
4   int u,v,w;
5   const int inf=0x7f7f7f7f;
6   memset(dp,0x7f,sizeof(dp));
7   memset(dp[0],0,sizeof(dp[0]));
8   for(int i=0;i<n;++i){
9     for(auto e:E){
10       tie(u,v,w)=e;
11       if(dp[i][u]!=inf)
12         dp[i+1][v]=min(dp[i+1][v],dp[i][u]+w);
13     }
14     double res = DBL_MAX;
15     for(int i=1;i<n;++i){
16       double val = DBL_MIN;
17       for(int j=0;j<n;++j)
18         val=max(val,double(dp[n][i]-dp[i][j]
19           ))/(n-j));
20       res=min(res,val);
21     }
22   }
23   return res;
24 }

```

## 4.8 Rectilinear\_MST

```

1 ///平面曼哈頓最小生成樹構造圖(去除非必要邊)
2 #define T int
3 #define INF 0x3f3f3f3f
4 struct point{
5   T x,y;
6   int id;///從0開始編號
7   point(){}
8   T dist(const point &p)const{
9     return abs(x-p.x)+abs(y-p.y);
10  }
11 };
12 bool cmpx(const point &a,const point &b){
13   return a.x<b.x||((a.x==b.x&&a.y<b.y));
14 }
15 struct edge{
16   int u,v;
17   T cost;
18   edge(int u,int v,T c):u(u),v(v),cost(c){}
19   bool operator<(const edge&e)const{
20     return cost<e.cost;
21   }
22 };
23 struct bit_node{
24   T mi;
25   int id;
26   bit_node(const T&mi=INF,int id=-1):mi(mi),
27     id(id){}
28 };
29 vector<bit_node> bit;
30 void bit_update(int i,const T&data,int id){
31   for(;;i=i&(-i)){
32     if(data<bit[i].mi)bit[i]=bit_node(data,
33       id);
34   }
35 }
36 int bit_find(int i,int m){
37   bit_node x;

```

```

36 for(;i<=m;i+=i&(-i)) if(bit[i].mi<x.mi)x=
    bit[i];
37 return x.id;
38 }
vector<edge> build_graph(int n,point p[]){
39 vector<edge> e;//edge for MST
40 for(int dir=0;dir<4;+dir){//4種座標變換
41 if(dir%2) for(int i=0;i<n;+i) swap(p[i]
42 .x,p[i].y);
43 else if(dir==2) for(int i=0;i<n;+i) p[i]
44 .x=-p[i].x;
45 sort(p,p+n,cmpx);
46 vector<T> ga(n), gb;
47 for(int i=0;i<n;+i)ga[i]=p[i].y-p[i].x;
48 gb=ga, sort(gb.begin(),gb.end());
49 gb.erase(unique(gb.begin(),gb.end()),gb.
50 end());
51 int m=gb.size();
52 bit=vector<bit_node>(m+1);
53 for(int i=n-1;i>0;--i){
54 int pos=lower_bound(gb.begin(),gb.end
55 (),ga[i])-gb.begin()+1;
56 int ans=bit_find(pos,m);
57 if(~ans)e.push_back(edge(p[i].id,p[ans
58 ].id,p[i].dist(p[ans])));
59 bit_update(pos,p[i].x+p[i].y,i);
60 }
61 return e;
62 }

```

## 4.9 treeISO

```

1 const int MAXN=100005;
2 const long long X=12327,P=0xdefaced;
3 vector<int> g[MAXN];
4 bool vis[MAXN];
5 long long dfs(int u){//hash ver
6 vis[u]=1;
7 vector<long long> tmp;
8 for(auto v:g[u])if(!vis[v])tmp.pb(dfs(v));
9 if(tmp.empty())return 177;
10 long long ret=4931;
11 sort(tmp.begin(),tmp.end());
12 for(auto v:tmp)ret=((ret*X)^v)%P;
13 return ret;
14 }
15 //-----
16 string dfs(int x,int p){
17 vector<string> c;
18 for(int y:g[x])
19 if(y!=p)c.emplace_back(dfs(y,x));
20 sort(c.begin(),c.end());
21 string ret("(");
22 for(auto &s:c)ret+=s;
23 ret+=")";
24 return ret;
25 }

```

## 4.10 一般圖最小權完美匹配

```

1 struct Graph {
2 // Minimum General Weighted Matching (
3 // Perfect Match) 0-base
4 static const int MXN = 105;
5 int n, edge[MXN][MXN];
6 int match[MXN],dis[MXN],onstk[MXN];
7 vector<int> stk;
8 void init(int _n) {
9 n = _n;
10 for (int i=0; i<n; i++)
11 for (int j=0; j<n; j++)
12 edge[i][j] = 0;
13 }
14 void add_edge(int u, int v, int w) {
15 edge[u][v] = edge[v][u] = w;
16 }
17 bool SPFA(int u){
18 if (onstk[u]) return true;
19 stk.push_back(u);
20 onstk[u] = 1;
21 for (int v=0; v<n; v++){
22 if (u != v && match[u] != v && !onstk[
23 v]){
24 int m = match[v];
25 if (dis[m] > dis[u] - edge[v][m] +
26 edge[u][v]){
27 dis[m] = dis[u] - edge[v][m] +
28 edge[u][v];
29 onstk[v] = 1;
30 stk.push_back(v);
31 if (SPFA(m)) return true;
32 stk.pop_back();
33 onstk[v] = 0;
34 }
35 }
36 }
37 onstk[u] = 0;
38 stk.pop_back();
39 return false;
40 }
41 int solve() {
42 // find a match
43 for (int i=0; i<n; i+=2){
44 match[i] = i+1, match[i+1] = i;
45 }
46 for(;;){
47 int found = 0;
48 for (int i=0; i<n; i++) dis[i] = onstk
49 [i] = 0;
50 for (int i=0; i<n; i++){
51 stk.clear();
52 if (!onstk[i] && SPFA(i)){
53 found = 1;
54 while (stk.size())>=2){
55 int u = stk.back(); stk.pop_back
56 ();
57 int v = stk.back(); stk.pop_back
58 ();
59 match[u] = v;
60 match[v] = u;
61 }
62 }
63 if (!found) break;
64 }
65 int ret = 0;

```

```

60 for (int i=0; i<n; i++)
61 ret += edge[i][match[i]];
62 ret /= 2;
63 return ret;
64 }
65 }graph;

```

## 4.11 全局最小割

```

1 const int INF=0x3f3f3f3f;
2 template<typename T>
3 struct stoer_wagner{// 0-base
4 static const int MAXN=150;
5 T g[MAXN][MAXN],dis[MAXN];
6 int nd[MAXN],n,s,t;
7 void init(int _n){
8 n=_n;
9 for(int i=0;i<n;+i)
10 for(int j=0;j<n;+j)g[i][j]=0;
11 }
12 void add_edge(int u,int v,T w){
13 g[u][v]=g[v][u]+=w;
14 }
15 T min_cut(){
16 T ans=INF;
17 for(int i=0;i<n;+i)nd[i]=i;
18 for(int ind,tn=n;tn>1;--tn){
19 for(int i=1;i<tn;+i)dis[nd[i]]=0;
20 for(int i=1;i<tn;+i){
21 ind=i;
22 for(int j=i;j<tn;+j){
23 dis[nd[j]]+=g[nd[i-1]][nd[j]];
24 if(dis[nd[ind]]<dis[nd[j]])ind=j;
25 }
26 swap(nd[ind],nd[i]);
27 }
28 if(ans>dis[nd[ind]])ans=dis[t=nd[ind
29 ]],s=nd[ind-1];
30 for(int i=0;i<tn;+i)
31 g[nd[ind-1]][nd[i]]=g[nd[i]][nd[ind
32 -1]]+g[nd[i]][nd[ind]];
33 }
34 return ans;
35 }
36 }

```

## 4.12 平面圖判定

```

1 static const int MAXN = 20;
2 struct Edge{
3 int u, v;
4 Edge(int s, int d) : u(s), v(d) {}
5 };
6 bool isK33(int n, int degree[]){
7 int t = 0, z = 0;
8 for(int i=0;i<n;+i){
9 if(degree[i] == 3)++t;
10 else if(degree[i] == 0)++z;
11 else return false;
12 }

```

```

13 return t == 6 && t + z == n;
14 }
15 bool isK5(int n, int degree[]){
16 int f = 0, z = 0;
17 for(int i=0;i<n;+i){
18 if(degree[i] == 4)++f;
19 else if(degree[i] == 0)++z;
20 else return false;
21 }
22 return f == 5 && f + z == n;
23 }
24 // it judge a given graph is Homeomorphic
25 // with K33 or K5
26 bool isHomeomorphic(bool G[MAXN][MAXN],
27 const int n){
28 for(;;){
29 int cnt = 0;
30 for(int i=0;i<n;+i){
31 vector<Edge> E;
32 for(int j=0;j<n&&E.size()<3;+j)
33 if(G[i][j] && i != j)
34 E.push_back(Edge(i, j));
35 if(E.size() == 1){
36 G[i][E[0].v] = G[E[0].v][i] = false;
37 }else if(E.size() == 2){
38 G[i][E[0].v] = G[E[0].v][i] = false;
39 G[i][E[1].v] = G[E[1].v][i] = false;
40 G[E[0].v][E[1].v] = G[E[1].v][E[0].v]
41 = true;
42 ++cnt;
43 }
44 if(cnt == 0)break;
45 }
46 static int degree[MAXN];
47 fill(degree, degree + n, 0);
48 for(int i=0;i<n;+i){
49 for(int j=i+1; j<n; +j){
50 if(!G[i][j])continue;
51 ++degree[i];
52 ++degree[j];
53 }
54 }
55 return !(isK33(n, degree) || isK5(n,
56 degree));
57 }

```

## 4.13 弦圖完美消除序列

```

1 struct chordal{
2 static const int MAXN=1005;
3 int n;// 0-base
4 vector<int>G[MAXN];
5 int rank[MAXN],label[MAXN];
6 bool mark[MAXN];
7 void init(int _n){n=_n;
8 for(int i=0;i<n;+i)G[i].clear();
9 }
10 void add_edge(int u,int v){
11 G[u].push_back(v);
12 G[v].push_back(u);
13 }
14 vector<int> MCS(){

```



```

15  memset(rank, -1, sizeof(int)*n);
16  memset(label, 0, sizeof(int)*n);
17  priority_queue<pair<int, int> > pq;
18  for(int i=0; i<n; ++i) pq.push(make_pair(0,
19      i));
19  for(int i=n-1; i>=0; --i) for(;;){
20      int u=pq.top().second; pq.pop();
21      if(~rank[u]) continue;
22      rank[u]=i;
23      for(auto v:G[u]) if(rank[v]==-1){
24          pq.push(make_pair(++label[v], v));
25      }
26      break;
27  }
28  vector<int> res(n);
29  for(int i=0; i<n; ++i) res[rank[i]]=i;
30  return res;
31 }
32 bool check(vector<int> ord){ //弦圖判定
33     for(int i=0; i<n; ++i) rank[ord[i]]=i;
34     memset(mark, 0, sizeof(bool)*n);
35     for(int i=0; i<n; ++i){
36         vector<pair<int, int> > tmp;
37         for(auto u:G[ord[i]]) if(!mark[u])
38             tmp.push_back(make_pair(rank[u], u));
39         sort(tmp.begin(), tmp.end());
40         if(tmp.size()){
41             int u=tmp[0].second;
42             set<int> S;
43             for(auto v:G[u]) S.insert(v);
44             for(size_t j=1; j<tmp.size(); ++j)
45                 if(!S.count(tmp[j].second)) return
46                     0;
47             mark[ord[i]]=1;
48         }
49         return 1;
50     }
51 };

```

## 4.14 最小斯坦納樹 DP

```

1  //n個點，其中r個要構成斯坦納樹
2  //答案在max(dp[(1<<r)-1][k]) k=0~n-1
3  //p表示要構成斯坦納樹的點集
4  //O( n^3 + n*3^r + n^2*2^r )
5  #define REP(i,n) for(int i=0; i<(int)n; ++i)
6  const int MAXN=30, MAXM=8; // 0-base
7  const int INF=0x3f3f3f3f;
8  int dp[1<<MAXM][MAXN];
9  int g[MAXN][MAXN]; //圖
10 void init(){memset(g, 0x3f, sizeof(g));}
11 void add_edge(int u, int v, int w){
12     g[u][v]=g[v][u]=min(g[v][u], w);
13 }
14 void steiner(int n, int r, int *p){
15     REP(k,n) REP(i,n) REP(j,n)
16         g[i][j]=min(g[i][j], g[i][k]+g[k][j]);
17     REP(i,n) g[i][i]=0;
18     REP(i,r) REP(j,n) dp[1<<i][j]=g[p[i]][j];
19     for(int i=1; i<(1<<r); ++i){
20         if(!(i&(i-1))) continue;

```

```

21     REP(j,n) dp[i][j]=INF;
22     REP(j,n){
23         int tmp=INF;
24         for(int s=i&(i-1); s; s=i&(s-1))
25             tmp=min(tmp, dp[s][j]+dp[i^s][j]);
26         REP(k,n) dp[i][k]=min(dp[i][k], g[j][k]+
27             tmp);
28     }
29 }

```

## 4.15 最小樹形圖 — 朱劉

```

1  template<typename T>
2  struct zhu_liu{
3      static const int MAXN=110, MAXM=10005;
4      struct node{
5          int u, v;
6          T w, tag;
7          node *l, *r;
8          node(int u=0, int v=0, T w=0):u(u), v(v), w(
9              w), tag(0), l(0), r(0){}
10         void down(){
11             w+=tag;
12             if(l) l->tag+=tag;
13             if(r) r->tag+=tag;
14             tag=0;
15         }
16         mem[MAXN]; //靜態記憶體
17         node *pq[MAXN*2], *E[MAXN*2];
18         int st[MAXN*2], id[MAXN*2], m;
19         void init(int n){
20             for(int i=1; i<=n; ++i){
21                 pq[i]=E[i]=0, st[i]=id[i]=i;
22                 m=0;
23             }
24             *merge(node *a, node *b){ //skew heap
25                 if(!a||!b) return a?a:b;
26                 a->down(), b->down();
27                 if(b->w<a->w) return merge(b, a);
28                 swap(a->l, a->r);
29                 a->l=merge(b, a->l);
30                 return a;
31             }
32             void add_edge(int u, int v, T w){
33                 if(u!=v) pq[v]=merge(pq[v], &(mem[m++]=
34                     node(u, v, w)));
35             }
36             int find(int x, int *st){
37                 return st[x]=x?x:st[x]=find(st[x], st);
38             }
39             T build(int root, int n){
40                 T ans=0; int N=n, all=n;
41                 for(int i=1; i<=N; ++i){
42                     if(i==root||!pq[i]) continue;
43                     while(pq[i]){
44                         pq[i]->down(), E[i]=pq[i];
45                         pq[i]=merge(pq[i]->l, pq[i]->r);
46                         if(find(E[i]->u, id)!=find(i, id))
47                             break;
48                     }
49                     if(find(E[i]->u, id)==find(i, id))
50                         continue;

```

```

47     ans+=E[i]->w;
48     if(find(E[i]->u, st)==find(i, st)){
49         if(pq[i]) pq[i]->tag-=E[i]->w;
50         pq[++N]=pq[i]; id[N]=N;
51         for(int u=find(E[i]->u, id); u!=i; u=
52             find(E[u]->u, id)){
53             if(pq[u]) pq[u]->tag-=E[u]->w;
54             id[find(u, id)]=N;
55             pq[N]=merge(pq[N], pq[u]);
56         }
57         st[N]=find(i, st);
58         id[find(i, id)]=N;
59         pq[N]=merge(pq[N], pq[u]);
60     } else st[find(i, st)]=find(E[i]->u, st)
61         , --all;
62 }
63 }
64 }
65 }
66 }
67 }
68 }
69 }
70 }
71 }
72 }
73 }
74 }
75 }
76 }
77 }
78 }
79 }
80 }
81 }
82 }
83 }
84 }
85 }
86 }
87 }
88 }
89 }
90 }
91 }
92 }
93 }
94 }
95 }
96 }
97 }
98 }
99 }
100 }

```

## 4.16 穩定婚姻模板

```

1  queue<int> Q;
2  for ( i : 所有考生 ) {
3      設定在第0志願;
4      Q.push( 考生i );
5  }
6  while(Q.size()){
7      當前考生=Q.front(); Q.pop();
8      while ( 此考生未分發 ) {
9          指標移到下一志願;
10         if ( 已經沒有志願 or 超出志願總數 )
11             break;
12         計算該考生在該科系加權後的總分;
13         if ( 不符合科系需求 ) continue;
14         if ( 目前科系有餘額 ) {
15             依加權後分數高低順序將考生id加入科系錄
16                 取名單中;
17             break;
18         }
19         if ( 目前科系已額滿 ) {
20             if ( 此考生成績比最低分數還高 ) {
21                 依加權後分數高低順序將考生id加入科系
22                     錄取名單;
23                 Q.push( 被踢出的考生 );
24             }
25         }
26     }
27 }
28 }
29 }
30 }
31 }
32 }
33 }
34 }
35 }
36 }
37 }
38 }
39 }
40 }
41 }
42 }
43 }
44 }
45 }
46 }
47 }
48 }
49 }
50 }
51 }
52 }
53 }
54 }
55 }
56 }
57 }
58 }
59 }
60 }
61 }
62 }
63 }

```

## 5 Linear\_Programming

### 5.1 最大密度子圖

```

1  typedef double T; //POJ 3155
2  const int MAXN=105;
3  struct edge{
4      int u, v;
5      T w;
6      edge(int u=0, int v=0, T w=0):u(u), v(v), w(w)
7          {}
8  };
9  vector<edge> E;
10 int n, m; // 1-base
11 T de[MAXN], pv[MAXN]; //每個點的邊權和和點權(
12     有些題目會給)
13 void init(){
14     E.clear();
15     for(int i=1; i<=n; ++i) de[i]=pv[i]=0;
16 }
17 void add_edge(int u, int v, T w){
18     E.push_back(edge(u, v, w));
19     de[u]+=w, de[v]+=w;
20 }
21 T U; //二分搜的最大值
22 void get_U(){
23     U=0;
24     for(int i=1; i<=n; ++i) U+=2*pv[i];
25     for(size_t i=0; i<E.size(); ++i) U+=E[i].w;
26 }
27 ISAP<T> isap; //網路流
28 int s, t; //原匯點
29 void build(T L){
30     isap.init(n+2);
31     for(size_t i=0; i<E.size(); ++i)
32         isap.add_edge(E[i].u, E[i].v, E[i].w);
33     for(int v=1; v<=n; ++v){
34         isap.add_edge(s, v, U);
35         isap.add_edge(v, t, U+2*L-de[v]-2*pv[v]);
36     }
37 }
38 int main(){
39     while(~scanf("%d%d", &n, &m)){
40         if(!m){
41             puts("1\n1");
42             continue;
43         }
44         init();
45         int u, v;
46         for(int i=0; i<m; ++i){
47             scanf("%d%d", &u, &v);
48             add_edge(u, v, 1);
49         }
50         get_U();
51         s=n+1, t=n+2;
52         T l=0, r=U, k=1.0/(n*n);
53         while(r-l>k){ //二分搜最大值
54             T mid=(l+r)/2;
55             build(mid);
56             T res=(U*n-isap.isap(s, t))/2;
57             if(res>0) l=mid;
58             else r=mid;
59         }
60         build(l);
61         isap.min_cut(s, t);
62         vector<int> ans;
63         for(int i=1; i<=n; ++i)
64             if(isap.vis[i]) ans.push_back(i);
65         printf("%d\n", ans.size());
66     }
67 }

```

```

64   for(size_t i=0;i<ans.size();++i)
65       printf("%d\n",ans[i]);
66   }
67   return 0;
68 }

```

## 6 Number\_Theory

### 6.1 basic

```

1  template<typename T>
2  void gcd(const T &a,const T &b,T &d,T &x,T &y){
3      if(!b) d=a,x=1,y=0;
4      else gcd(b,a%b,d,y,x), y-=x*(a/b);
5  }
6  long long int phi[N+1];
7  void phiTable(){
8      for(int i=1;i<=N;i++)phi[i]=i;
9      for(int i=1;i<=N;i++)for(x=i*2;x<=N;x+=i)
10         phi[x]-=phi[i];
11 }
12 void all_divdown(const LL &n) { // all n/x
13     for(LL a=1;a<=n;a=n/(n/(a+1))) {
14         // dosomething;
15     }
16 }
17 const int MAXPRIME = 1000000;
18 int iscom[MAXPRIME], prime[MAXPRIME],
19     primecnt;
20 int phi[MAXPRIME], mu[MAXPRIME];
21 void sieve(void){
22     memset(iscom,0,sizeof(iscom));
23     primecnt = 0;
24     phi[1] = mu[1] = 1;
25     for(int i=2;i<MAXPRIME;++i) {
26         if(!iscom[i]) {
27             prime[primecnt++] = i;
28             mu[i] = -1;
29             phi[i] = i-1;
30         }
31         for(int j=0;j<primecnt;++j) {
32             int k = i * prime[j];
33             if(k>MAXPRIME) break;
34             iscom[k] = prime[j];
35             if(i%prime[j]==0) {
36                 mu[k] = 0;
37                 phi[k] = phi[i] * prime[j];
38                 break;
39             } else {
40                 mu[k] = -mu[i];
41                 phi[k] = phi[i] * (prime[j]-1);
42             }
43         }
44     }
45 }
46 bool g_test(const LL &g, const LL &p, const
47     vector<LL> &v) {
48     for(int i=0;i<v.size();++i)
49         if(modexp(g,(p-1)/v[i],p)==1)

```

```

48     return false;
49     return true;
50 }
51 LL primitive_root(const LL &p) {
52     if(p==2) return 1;
53     vector<LL> v;
54     Factor(p-1,v);
55     v.erase(unique(v.begin(), v.end()), v.end());
56     for(LL g=2;g<p;++g)
57         if(g_test(g,p,v))
58             return g;
59     puts("primitive_root NOT FOUND");
60     return -1;
61 }
62 int Legendre(const LL &a, const LL &p) {
63     return modexp(a%p,(p-1)/2,p); }
64 LL inv(const LL &a, const LL &n) {
65     LL d,x,y;
66     gcd(a,n,d,x,y);
67     return d==1 ? (x+n)%n : -1;
68 }
69 int inv[maxN];
70 LL invtable(int n,LL P){
71     inv[1]=1;
72     for(int i=2;i<n;++i)
73         inv[i]=(P-(P/i))*inv[P%i]%P;
74 }
75 LL log_mod(const LL &a, const LL &b, const
76     LL &p) {
77     // a ^ x = b ( mod p )
78     int m=sqrt(p+.5), e=1;
79     LL v=inv(modexp(a,m,p), p);
80     map<LL,int> x;
81     x[1]=0;
82     for(int i=1;i<m;++i) {
83         e = LLMul(e,a,p);
84         if(!x.count(e)) x[e] = i;
85     }
86     for(int i=0;i<m;++i) {
87         if(x.count(b)) return i*m + x[b];
88         b = LLMul(b,v,p);
89     }
90     return -1;
91 }
92 LL Tonelli_Shanks(const LL &n, const LL &p)
93 {
94     // x^2 = n ( mod p )
95     if(n==0) return 0;
96     if(Legendre(n,p)!=1) while(1) { puts("SQRT
97         ROOT does not exist"); }
98     int S = 0;
99     LL Q = p-1;
100     while( !(Q&1) ) { Q>>=1; ++S; }
101     if(S==1) return modexp(n%p,(p+1)/4,p);
102     LL z = 2;
103     for(; Legendre(z,p)!=-1;++z)
104         LL c = modexp(z,Q,p);
105     LL R = modexp(n%p,(Q+1)/2,p), t = modexp(n
106         %p,Q,p);
107     int M = S;
108     while(1) {

```

```

108     if(t==1) return R;
109     LL b = modexp(c,1L<<(M-1),p);
110     R = LLMul(R,b,p);
111     t = LLMul( LLMul(b,b,p), t, p);
112     c = LLMul(b,b,p);
113     M = i;
114 }
115 return -1;
116 }
117 template<typename T>
118 T Euler(T n){
119     T ans=n;
120     for(T i=2;i*i<=n;++i){
121         if(n%i==0){
122             ans=ans/i*(i-1);
123             while(n%i==0)n/=i;
124         }
125     }
126     if(n>1)ans=ans/n*(n-1);
127     return ans;
128 }
129 //Chinese_remainder_theorem
130 template<typename T>
131 T pow_mod(T n,T k,T m){
132     T ans=1;
133     for(n=(n==m?n%m:n);k>=1){
134         if(k&1)ans=ans*n%m;
135         n=n*n%m;
136     }
137     return ans;
138 }
139 template<typename T>
140 T crt(vector<T> &m,vector<T> &a){
141     T M=1,tM,ans=0;
142     for(int i=0;i<(int)m.size();++i)M*=m[i];
143     for(int i=0;i<(int)a.size();++i){
144         tM=M/m[i];
145         ans=(ans+(a[i]*tM%M)*pow_mod(tM,Euler(m[i])-1,m[i])%M)%M;
146     }
147     /*如果m[i]是質數 · Euler(m[i])-1=m[i]-2 ·
148     就不用算Euler了*/
149     return ans;
150 }
151 //java code
152 //求sqrt(N)的連分數
153 public static void Pell(int n){
154     BigInteger N,p1,p2,q1,q2,a0,a1,a2,g1,g2,h1
155         ,h2,p,q;
156     g1=q2=p1=BigInteger.ZERO;
157     h1=q1=p2=BigInteger.ONE;
158     a0=a1=BigInteger.valueOf((int)Math.sqrt
159         (1.0*n));
160     BigInteger ans=a0.multiply(a0);
161     if(ans.equals(BigInteger.valueOf(n))){
162         System.out.println("No solution!");
163         return ;
164     }
165     while(true){
166         g2=a1.multiply(h1).subtract(g1);
167         h2=N.subtract(g2.pow(2)).divide(h1);
168         a2=g2.add(a0).divide(h2);

```

```

169     p=a1.multiply(p2).add(p1);
170     q=a1.multiply(q2).add(q1);
171     if(p.pow(2).subtract(N.multiply(q.pow
172         (2))).compareTo(BigInteger.ONE)==0)
173         break;
174     g1=g2;h1=h2;a1=a2;
175     p1=p2;p2=p;
176     q1=q2;q2=q;
177 }
178 System.out.println(p+" "+q);
179 }

```

### 6.2 bit\_set

```

1  void sub_set(int S){
2      int sub=S;
3      do{
4          //對某集合的子集合的處理
5          sub=(sub-1)&S;
6      }while(sub!=S);
7  }
8  void k_sub_set(int k,int n){
9      int comb=(1<<k)-1,S=1<<n;
10     while(comb<S){
11         //對大小為k的子集合的處理
12         int x=comb&-comb,y=comb+x;
13         comb=((comb&y)/x>>1)|y;
14     }
15 }

```

### 6.3 cantor\_expansion

```

1  int factorial[MAXN];
2  void init(){
3      factorial[0]=1;
4      for(int i=1;i<MAXN;++i)factorial[i]=
5          factorial[i-1]*i;
6  }
7  int encode(const vector<int> &s){
8      int n=s.size(),res=0;
9      for(int i=0;i<n;++i){
10         int t=0;
11         for(int j=i+1;j<n;++j)
12             if(s[j]<s[i])++t;
13         res+=t*factorial[n-i-1];
14     }
15     return res;
16 }
17 vector<int> decode(int a,int n){
18     vector<int> res;
19     vector<bool> vis(n,0);
20     for(int i=n-1;i>=0;--i){
21         int t=a/factorial[i],j;
22         for(j=0;j<n;++j)
23             if(!vis[j]){
24                 if(t==0)break;
25                 --t;
26             }
27         res.push_back(j);

```

## 6.4 FFT

```

27 vis[j]=1;
28 a%=factorial[i];
29 }
30 return res;
31 }

template<typename T, typename VT=vector<
    complex<T> > >
struct FFT{
    const T pi;
    FFT(const T pi=acos((T)-1)):pi(pi){}
    unsigned bit_reverse(unsigned a, int len){
        a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)>>1);
        a=((a&0x33333333U)<<2)|((a&0xCCCCCCCCU)>>2);
        a=((a&0x0F0F0F0FU)<<4)|((a&0xF0F0F0F0U)>>4);
        a=((a&0x00FF00FFU)<<8)|((a&0xFF00FF00U)>>8);
        a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)
            >>16);
        return a>>(32-len);
    }
    void fft(bool is_inv, VT &in, VT &out, int N)
    {
        int bitlen=__lg(N), num=is_inv?-1:1;
        for(int i=0; i<N; ++i) out[bit_reverse(i,
            bitlen)]=in[i];
        for(int step=2; step<=N; step<=1){
            const int mh=step>>1;
            for(int i=0; i<mh; ++i){
                complex<T> wi=exp(complex<T>(0, i*num
                    *pi/mh));
                for(int j=i; j<N; j+=step){
                    int k=j+mh;
                    complex<T> u=out[j], t=wi*out[k];
                    out[j]=u+t;
                    out[k]=u-t;
                }
            }
        }
        if(is_inv) for(int i=0; i<N; ++i) out[i]/=N;
    }
};

```

## 6.5 find\_real\_root

```

1 // an*x^n + ... + a1x + a0 = 0;
2 int sign(double x){
3     return x < -eps ? -1 : x > eps;
4 }
5
6 double get(const vector<double> &coef, double
    x){
7     double e = 1, s = 0;
8     for(auto i : coef) s += i*e, e *= x;
9     return s;
10 }
11
12 double find(const vector<double> &coef, int n
    , double lo, double hi){

```

```

13 double sign_lo, sign_hi;
14 if( !(sign_lo = sign(get(coef, lo))) )
15     return lo;
16 if( !(sign_hi = sign(get(coef, hi))) )
17     return hi;
18 if(sign_lo * sign_hi > 0) return INF;
19 for(int stp = 0; stp < 100 && hi - lo >
    eps; ++stp){
20     double m = (lo+hi)/2.0;
21     int sign_mid = sign(get(coef, m));
22     if(!sign_mid) return m;
23     if(sign_lo*sign_mid < 0) hi = m;
24     else lo = m;
25 }
26 return (lo+hi)/2.0;
27 }
28 vector<double> cal(vector<double> coef, int n
    ){
29     vector<double> res;
30     if(n == 1){
31         if(sign(coef[1])) res.pb(-coef[0]/coef
            [1]);
32         return res;
33     }
34     vector<double> dcoef(n);
35     for(int i = 0; i < n; ++i) dcoef[i] = coef
        [i+1]*(i+1);
36     vector<double> droot = cal(dcoef, n-1);
37     droot.insert(droot.begin(), -INF);
38     droot.pb(INF);
39     for(int i = 0; i+1 < droot.size(); ++i){
40         double tmp = find(coef, n, droot[i],
            droot[i+1]);
41         if(tmp < INF) res.pb(tmp);
42     }
43     return res;
44 }
45 int main() {
46     vector<double> ve;
47     vector<double> ans = cal(ve, n);
48     // 視情況把答案 +eps 避免 -0
49 }

```

## 6.6 FWT

```

1 vector<int> F_OR_T(vector<int> f, bool
    inverse){
2     for(int i=0; (2<<i)<=f.size(); ++i)
3         for(int j=0; j<f.size(); j+=2<<i)
4             for(int k=0; k<(1<<i); ++k)
5                 f[j+k+(1<<i)] += f[j+k]*(inverse
                    ?-1:1);
6     return f;
7 }
8 vector<int> rev(vector<int> A) {
9     for(int i=0; i<A.size(); i+=2)
10         swap(A[i], A[i^(A.size()-1)]);
11     return A;
12 }
13 vector<int> F_AND_T(vector<int> f, bool
    inverse){

```

```

14     return rev(F_OR_T(rev(f), inverse));
15 }
16 vector<int> F_XOR_T(vector<int> f, bool
    inverse){
17     for(int i=0; (2<<i)<=f.size(); ++i)
18         for(int j=0; j<f.size(); j+=2<<i)
19             for(int k=0; k<(1<<i); ++k){
20                 int u=f[j+k], v=f[j+k+(1<<i)];
21                 f[j+k+(1<<i)] = u-v, f[j+k] = u+v;
22             }
23     if(inverse) for(auto &a:f) a/=f.size();
24     return f;
25 }

```

## 6.7 LinearCongruence

```

1 pair<LL, LL> LinearCongruence(LL a[], LL b[],
    LL m[], int n) {
2     // a[i]*x = b[i] (mod m[i])
3     for(int i=0; i<n; ++i) {
4         LL x, y, d = extgcd(a[i], m[i], x, y);
5         if(b[i]%d!=0) return make_pair(-1LL, 0LL);
6         m[i] /= d;
7         b[i] = LLMul(b[i]/d, x, m[i]);
8     }
9     LL lastb = b[0], lastm = m[0];
10    for(int i=1; i<n; ++i) {
11        LL x, y, d = extgcd(m[i], lastm, x, y);
12        if((lastb-b[i])%d!=0) return make_pair
            (-1LL, 0LL);
13        lastb = LLMul((lastb-b[i])/d, x, (lastm/d)
            )*m[i];
14        lastm = (lastm/d)*m[i];
15        lastb = (lastb+b[i])%lastm;
16    }
17    return make_pair(lastb<0?lastb+lastm:lastb
        , lastm);
18 }

```

## 6.8 Lucas

```

1 int mod_fact(int n, int &e){
2     e=0;
3     if(n==0) return 1;
4     int res=mod_fact(n/P, e);
5     e += n/P;
6     if((n/P)%2==0) return res*fact[n%P]%P;
7     return res*(P-fact[n%P])%P;
8 }
9 int Cmod(int n, int m){
10    int a1, a2, a3, e1, e2, e3;
11    a1=mod_fact(n, e1);
12    a2=mod_fact(m, e2);
13    a3=mod_fact(n-m, e3);
14    if(e1>e2+e3) return 0;
15    return a1*inv(a2*a3%P, P)%P;
16 }

```

## 6.9 Matrix

```

1 template<typename T>
2 struct Matrix{
3     using rt = std::vector<T>;
4     using mt = std::vector<rt>;
5     using matrix = Matrix<T>;
6     int r, c;
7     mt m;
8     Matrix(int r, int c):r(r), c(c), m(r, rt(c)){}
9     rt& operator[](int i){return m[i];}
10    matrix operator+(const matrix &a){
11        matrix rev(r, c);
12        for(int i=0; i<r; ++i)
13            for(int j=0; j<c; ++j)
14                rev[i][j]=m[i][j]+a.m[i][j];
15        return rev;
16    }
17    matrix operator-(const matrix &a){
18        matrix rev(r, c);
19        for(int i=0; i<r; ++i)
20            for(int j=0; j<c; ++j)
21                rev[i][j]=m[i][j]-a.m[i][j];
22        return rev;
23    }
24    matrix operator*(const matrix &a){
25        matrix rev(r, a.c);
26        matrix tmp(a.c, a.r);
27        for(int i=0; i<a.r; ++i)
28            for(int j=0; j<a.c; ++j)
29                tmp[j][i]=a.m[j][i];
30        for(int i=0; i<r; ++i)
31            for(int j=0; j<a.c; ++j)
32                for(int k=0; k<c; ++k)
33                    rev.m[i][j]+=m[i][k]*tmp[j][k];
34        return rev;
35    }
36    bool inverse(){
37        Matrix t(r, r+c);
38        for(int y=0; y<r; ++y){
39            t.m[y][c+y] = 1;
40            for(int x=0; x<c; ++x)
41                t.m[y][x]=m[y][x];
42        }
43        if( !t.gas() )
44            return false;
45        for(int y=0; y<r; ++y)
46            for(int x=0; x<c; ++x)
47                m[y][x]=t.m[y][c+x]/t.m[y][y];
48        return true;
49    }
50    T gas(){
51        vector<T> lazy(r, 1);
52        bool sign=false;
53        for(int i=0; i<r; ++i){
54            if( m[i][i]==0 ){
55                int j=i+1;
56                while(j<r && !m[j][i]) j++;
57                if(j==r) continue;
58                m[i].swap(m[j]);
59                sign=!sign;
60            }
61            for(int j=0; j<r; ++j){
62                if(i==j) continue;
63                lazy[j]=lazy[j]*m[i][i];

```

```

64     T mx=m[j][i];
65     for(int k=0;k<c;++k)
66         m[j][k]=m[j][k]*m[i][i]-m[i][k]*mx;
67     }
68 }
69 T det=sign?-1:1;
70 for(int i=0;i<r;++i){
71     det = det*m[i][i];
72     det = det/lazy[i];
73     for(auto &j:m[i])j/=lazy[i];
74 }
75 return det;
76 }
77 };

```

## 6.10 MillerRobin

```

1 LL Llmul(LL a, LL b, const LL &mod) {
2     LL ans=0;
3     while(b) {
4         if(b&1) {
5             ans+=a;
6             if(ans>=mod) ans-=mod;
7         }
8         a<<=1, b>>=1;
9         if(a>=mod) a-=mod;
10    }
11    return ans;
12 }
13 LL mod_mul(LL a,LL b,LL m){
14     a%=m,b%=m; /* fast for m < 2^58 */
15     LL y=(LL)((double)a*b/m+0.5);
16     LL r=(a*b-y*m)%m;
17     return r<0?r+m:r;
18 }
19 template<typename T>
20 T pow(T a,T b,T mod){ //a^b%mod
21     T ans=1;
22     for(;b;a=mod_mul(a,a,mod),b>>=1)
23         if(b&1)ans=mod_mul(ans,a,mod);
24     return ans;
25 }
26 int sprp[3]={2,7,61}; //int範圍可解
27 int llsprp
    [7]={2,325,9375,28178,450775,9780504,
28 1795265022}; //至少unsigned long long範圍
29 template<typename T>
30 bool isprime(T n,int *sprp,int num){
31     if(n==2)return 1;
32     if(n<2||n%2==0)return 0;
33     int t=0;
34     T u=n-1;
35     for(;u%2==0;++t)u>>=1;
36     for(int i=0;i<num;++i){
37         T a=sprp[i]%n;
38         if(a==0||a==1||a==n-1)continue;
39         T x=pow(a,u,n);
40         if(x==1||x==n-1)continue;
41         for(int j=0;j<t;++j){
42             x=mod_mul(x,x,n);
43             if(x==1)return 0;

```

```

44         if(x==n-1)break;
45     }
46     if(x==n-1)continue;
47     return 0;
48 }
49 return 1;
50 }

```

## 6.11 NTT

```

1 2615053605667*(2^18)+1,3
2 15*(2^27)+1,31
3 479*(2^21)+1,3
4 7*17*(2^23)+1,3
5 3*3*211*(2^19)+1,5
6 25*(2^22)+1,3
7 template<typename T,typename VT=vector<T> >
8 struct NTT{
9     const T P,G;
10    NTT(T p=(1<<23)*7*17+1,T g=3):P(p),G(g){
11        unsigned bit_reverse(unsigned a,int len){
12            //Look FFT.cpp
13        }
14        T pow_mod(T n,T k,T m){
15            T ans=1;
16            for(n=(n>m?n%m:n);k;k>>=1){
17                if(k&1)ans=ans*n%m;
18                n=n*n%m;
19            }
20            return ans;
21        }
22        void ntt(bool is_inv,VT &in,VT &out,int N)
23        {
24            int bitlen=__lg(N);
25            for(int i=0;i<N;++i)out[bit_reverse(i,
26                bitlen)]=in[i];
27            for(int step=2,id=1;step<=N;step<<=1,++
28                id){
29                T wn=pow_mod(G,(P-1)>>id,P),wi=1,u,t;
30                const int mh=step>>1;
31                for(int i=0;i<mh;++i){
32                    for(int j=i;j<N;j+=step){
33                        u=out[j],t=wi*out[j+mh]%P;
34                        out[j]=u+t;
35                        out[j+mh]=u-t;
36                        if(out[j]>=P)out[j]-=P;
37                        if(out[j+mh]<0)out[j+mh]+=P;
38                    }
39                    wi=wi*wn%P;
40                }
41            }
42            if(is_inv){
43                for(int i=1;i<N/2;++i)swap(out[i],out[
44                    N-i]);
45                T invn=pow_mod(N,P-2,P);
46                for(int i=0;i<N;++i)out[i]=out[i]*invn
47                    %P;
48            }
49        }
50    };

```

## 6.12 Simpson

```

1 double simpson(double a,double b){
2     double c=a+(b-a)/2;
3     return (F(a)+4*F(c)+F(b))*(b-a)/6;
4 }
5 double asr(double a,double b,double eps,
6     double A){
7     double c=a+(b-a)/2;
8     double L=simpson(a,c),R=simpson(c,b);
9     if( abs(L+R-A)<15*eps )
10        return L+R+(L+R-A)/15.0;
11    return asr(a,c,eps/2,L)+asr(c,b,eps/2,R);
12 }
13 double asr(double a,double b,double eps){
14     return asr(a,b,eps,simpson(a,b));
15 }

```

## 6.13 外星模運算

```

1 //a[0]^(a[1]^a[2]^...)
2 #define maxn 1000000
3 int euler[maxn+5];
4 bool is_prime[maxn+5];
5 void init_euler(){
6     is_prime[1]=1; //不是質數
7     for(int i=1;i<=maxn;i++)euler[i]=i;
8     for(int i=2;i<=maxn;i++){
9         if(!is_prime[i]){ //是質數
10            euler[i]--;
11            for(int j=i<<1;j<=maxn;j+=i){
12                is_prime[j]=1;
13                euler[j]=euler[j]/i*(i-1);
14            }
15        }
16    }
17 }
18 LL pow(LL a,LL b,LL mod){ //a^b%mod
19     LL ans=1;
20     for(;b;a=a*a%mod,b>>=1)
21         if(b&1)ans=ans*a%mod;
22     return ans;
23 }
24 bool isless(LL *a,int n,int k){
25     if(*a==1||--n==0)return k>1;
26     if(--n==0)return *a<k;
27     int next=0;
28     for(LL b=1;b<k;++next)
29         b*=a;
30     return isless(a+1,n,next);
31 }
32 LL high_pow(LL *a,int n,LL mod){
33     if(*a==1||--n==0)return *a%mod;
34     int k=0,r=euler[mod];
35     for(LL tma=1;tma!=pow(*a,k+r,mod);++k)
36         tma=tma*(*a)%mod;
37     if(isless(a+1,n,k))return pow(*a,high_pow(
38         a+1,n,k),mod);
39     int tmd=high_pow(a+1,n,r), t=(tmd-k+r)%r;
40     return pow(*a,k+t,mod);
41 }

```

```

41 LL a[1000005];
42 int t,mod;
43 int main(){
44     init_euler();
45     scanf("%d",&t);
46     #define n 4
47     while(t--){
48         for(int i=0;i<n;++i)scanf("%lld",&a[i]);
49         scanf("%d",&mod);
50         printf("%lld\n",high_pow(a,n,mod));
51     }
52     return 0;
53 }

```

## 6.14 數位統計

```

1 ll d[65], dp[65][2]; //up區間是不是完整
2 ll dfs(int p,bool is8,bool up){
3     if(!p)return 1; // 回傳0是不是答案
4     if(!up&&dp[p][is8])return dp[p][is8];
5     int mx = up?d[p]:9; //可以用的有那些
6     ll ans=0;
7     for(int i=0;i<=mx;++i){
8         if( is8&&i==7 )continue;
9         ans += dfs(p-1,i==8,up&&i==mx);
10    }
11    if(!up)dp[p][is8]=ans;
12    return ans;
13 }
14 ll f(ll N){
15     int k=0;
16     while(N){ // 把數字先分解到陣列
17         d[++k] = N%10;
18         N/=10;
19     }
20     return dfs(k,false,true);
21 }

```

## 6.15 質因數分解

```

1 LL func(const LL n,const LL mod,const int c)
2 {
3     return (LLmul(n,n,mod)+c+mod)%mod;
4 }
5 LL pollorrho(const LL n, const int c) { //循
6     環節長度
7     LL a=1, b=1;
8     a=func(a,n,c)%n;
9     b=func(b,n,c)%n; b=func(b,n,c)%n;
10    while(gcd(abs(a-b),n)==1) {
11        a=func(a,n,c)%n;
12        b=func(b,n,c)%n; b=func(b,n,c)%n;
13    }
14    return gcd(abs(a-b),n);
15 }
16 void prefactor(LL &n, vector<LL> &v) {
17     for(int i=0;i<12;++i) {

```



```

18 while(n%prime[i]==0) {
19     v.push_back(prime[i]);
20     n/=prime[i];
21 }
22 }
23 }
24
25 void smallfactor(LL n, vector<LL> &v) {
26     if(n<MAXPRIME) {
27         while(isp[(int)n]) {
28             v.push_back(isp[(int)n]);
29             n/=isp[(int)n];
30         }
31         v.push_back(n);
32     } else {
33         for(int i=0; i<primecnt&&prime[i]*prime[i]
34             ]<=n; ++i) {
35             while(n%prime[i]==0) {
36                 v.push_back(prime[i]);
37                 n/=prime[i];
38             }
39             if(n!=1) v.push_back(n);
40         }
41     }
42
43 void comfactor(const LL &n, vector<LL> &v) {
44     if(n<le9) {
45         smallfactor(n,v);
46         return;
47     }
48     if(Isprime(n)) {
49         v.push_back(n);
50         return;
51     }
52     LL d;
53     for(int c=3; ++c) {
54         d = pollorrho(n,c);
55         if(d!=n) break;
56     }
57     comfactor(d,v);
58     comfactor(n/d,v);
59 }
60
61 void Factor(const LL &x, vector<LL> &v) {
62     LL n = x;
63     if(n==1) { puts("Factor 1"); return; }
64     prefactor(n,v);
65     if(n==1) return;
66     comfactor(n,v);
67     sort(v.begin(),v.end());
68 }
69
70 void AllFactor(const LL &n,vector<LL> &v) {
71     vector<LL> tmp;
72     Factor(n,tmp);
73     v.clear();
74     v.push_back(1);
75     int len;
76     LL now=1;
77     for(int i=0; i<tmp.size(); ++i) {
78         if(i==0 || tmp[i]!=tmp[i-1]) {
79             len = v.size();
80             now = 1;
81         }
82         now*=tmp[i];

```

```

83 for(int j=0; j<len; ++j)
84     v.push_back(v[j]*now);
85 }
86 }

```

## 7 String

### 7.1 AC 自動機

```

1 template<char L='a',char R='z'>
2 class ac_automaton{
3     struct joe{
4         int next[R-L+1],fail,efl,ed,cnt_dp,vis;
5         joe():ed(0),cnt_dp(0),vis(0){
6             for(int i=0; i<R-L; ++i)next[i]=0;
7         }
8     };
9     public:
10     std::vector<joe> S;
11     std::vector<int> q;
12     int qs,qe,vt;
13     ac_automaton():S(1),qs(0),qe(0),vt(0){
14         void clear(){
15             q.clear();
16             S.resize(1);
17             for(int i=0; i<R-L; ++i)S[0].next[i]=0;
18             S[0].cnt_dp=S[0].vis=qs=qe=vt=0;
19         }
20         void insert(const char *s){
21             int o=0;
22             for(int i=0; i<len; ++i){
23                 id=s[i]-L;
24                 if(!S[o].next[id]){
25                     S.push_back(joe());
26                     S[o].next[id]=S.size()-1;
27                 }
28                 o=S[o].next[id];
29             }
30             ++S[o].ed;
31         }
32         void build_fail(){
33             S[0].fail=S[0].efl=-1;
34             q.clear();
35             q.push_back(0);
36             ++qe;
37             while(qs!=qe){
38                 int pa=q[qs++],id,t;
39                 for(int i=0; i<R-L; ++i){
40                     t=S[pa].next[i];
41                     if(!t)continue;
42                     id=S[pa].fail;
43                     while(~id&&S[id].next[i])id=S[id].
44                         fail;
45                     S[t].fail=~id?S[id].next[i]:0;
46                     S[t].efl=S[t].fail?S[t].fail:S[
47                         t].fail].efl;
48                     q.push_back(t);
49                     ++qe;
50                 }
51             }
52         }

```

```

51 /*DP出每個前綴在字串s出現的次數並傳回所有
52 字串被s匹配成功的次數O(N*M)*/
53 int match_0(const char *s){
54     int ans=0,id,p=0,i;
55     for(i=0; s[i]; ++i){
56         id=s[i]-L;
57         while(!S[p].next[id]&&p=p=S[p].fail;
58             if(!S[p].next[id])continue;
59         p=S[p].next[id];
60         ++S[p].cnt_dp; /*匹配成功則它所有後綴都
61             可以被匹配(DP計算)*/
62     }
63     for(i=qe-1; i>=0; --i){
64         ans+=S[q[i]].cnt_dp*S[q[i]].ed;
65         if(~S[q[i]].fail)S[q[i]].fail].
66             cnt_dp+=S[q[i]].cnt_dp;
67     }
68     return ans;
69 }
70 /*多串匹配走efl邊並傳回所有字串被s匹配成功
71 的次數O(N*M^1.5)*/
72 int match_1(const char *s) const{
73     int ans=0,id,p=0,t;
74     for(int i=0; s[i]; ++i){
75         id=s[i]-L;
76         while(!S[p].next[id]&&p=p=S[p].fail;
77             if(!S[p].next[id])continue;
78         p=S[p].next[id];
79         if(S[p].ed)ans+=S[p].ed;
80         for(t=S[p].efl; ~t; t=S[t].efl){
81             ans+=S[t].ed; /*因為都走efl邊所以保證
82             匹配成功*/
83         }
84     }
85     return ans;
86 }
87 /*枚舉(s的子字串nA)的所有相異字串各恰一次
88 並傳回次數O(N*M^(1/3))*/
89 int match_2(const char *s){
90     int ans=0,id,p=0,t;
91     ++vt;
92     /*把載記vt+=1，只要vt沒溢位，所有S[p].
93     vis==vt就會變成false
94     這種利用vt的方法可以O(1)歸零vis陣列*/
95     for(int i=0; s[i]; ++i){
96         id=s[i]-L;
97         while(!S[p].next[id]&&p=p=S[p].fail;
98             if(!S[p].next[id])continue;
99         p=S[p].next[id];
100         if(S[p].ed&&S[p].vis!=vt){
101             S[p].vis=vt;
102             ans+=S[p].ed;
103         }
104         for(t=S[p].efl; ~t&&S[t].vis!=vt; t=S[t].
105             efl){
106             S[t].vis=vt;
107             ans+=S[t].ed; /*因為都走efl邊所以保證
108             匹配成功*/
109         }
110     }
111     return ans;
112 }
113 /*把AC自動機變成真的自動機*/

```

```

105 void evolution(){
106     for(qs=1; qs!=qe;){
107         int p=q[qs++];
108         for(int i=0; i<R-L; ++i)
109             if(S[p].next[i]==0)S[p].next[i]=S[
110                 p].fail].next[i];
111     }
112 }

```

### 7.2 hash

```

1 #define MAXN 1000000
2 #define mod 1073676287
3 /*mod 必須要是質數*/
4 typedef long long T;
5 char s[MAXN+5];
6 T h[MAXN+5]; /*hash陣列*/
7 T h_base[MAXN+5]; /*h_base[n]=(prime^n)%mod*/
8 void hash_init(int len,T prime){
9     h_base[0]=1;
10     for(int i=1; i<len; ++i){
11         h[i]=(h[i-1]*prime+s[i-1])%mod;
12         h_base[i]=(h_base[i-1]*prime)%mod;
13     }
14 }
15 T get_hash(int l,int r){ /*閉區間寫法，設編號
16     為0 ~ len-1*/
17     return (h[r+1]-(h[l]*h_base[r-l+1])%mod+
18         mod)%mod;

```

### 7.3 KMP

```

1 /*產生fail function*/
2 void kmp_fail(char *s,int len,int *fail){
3     int id=-1;
4     fail[0]=-1;
5     for(int i=1; i<len; ++i){
6         while(~id&&s[id+1]!=s[i])id=fail[id];
7         if(s[id+1]==s[i])++id;
8         fail[i]=id;
9     }
10 }
11 /*以字串B匹配字串A，傳回匹配成功的數量(用B的
12 fail)*/
13 int kmp_match(char *A,int lenA,char *B,int
14     lenB,int *fail){
15     int id=-1,ans=0;
16     for(int i=0; i<lenA; ++i){
17         while(~id&&B[id+1]!=A[i])id=fail[id];
18         if(B[id+1]==A[i])++id;
19         if(id==lenB-1){ /*匹配成功*/
20             ++ans, id=fail[id];
21         }
22     }
23     return ans;
24 }

```

## 7.4 manacher

```

1 //原字串: asdsasdsa
2 //先把字串變成這樣: @#a#s#d#s#a#s#d#s#a#
3 void manacher(char *s, int len, int *z){
4     int l=0, r=0;
5     for(int i=1; i<len; ++i){
6         z[i]=r>i?min(z[2*i-l], r-i):1;
7         while(s[i+z[i]]==s[i-z[i]])++z[i];
8         if(z[i]+i>r)r=z[i]+i, l=i;
9     } //ans = max(z)-1
10 }

```

## 7.5 minimal\_string\_rotation

```

1 int min_string_rotation(const string &s){
2     int n=s.size(), i=0, j=1, k=0;
3     while(i<n&&j<n&&k<n){
4         int t=s[(i+k)%n]-s[(j+k)%n];
5         ++k;
6         if(t){
7             if(t>0)i+=k;
8             else j+=k;
9             if(i==j)++j;
10            k=0;
11        }
12    }
13    return min(i, j); //最小循環表示法起始位置
14 }

```

## 7.6 reverseBWT

```

1 const int MAXN = 305, MAXC = 'Z';
2 int ranks[MAXN], tots[MAXC], first[MAXC];
3 void rankBWT(const string &bw){
4     memset(ranks, 0, sizeof(int)*bw.size());
5     memset(tots, 0, sizeof(int)*MAXC);
6     for(size_t i=0; i<bw.size(); ++i)
7         ranks[i] = tots[ bw[i] ]++;
8 }
9 void firstCol(){
10    memset(first, 0, sizeof(int)*MAXC);
11    int totc = 0;
12    for(int c='A'; c<='Z'; ++c){
13        if(!tots[c]) continue;
14        first[c] = totc;
15        totc += tots[c];
16    }
17 }
18 string reverseBwt(string bw, int begin){
19     rankBWT(bw, firstCol());
20     int i = begin; //原字串最後一個元素的位置
21     string res;
22     do{
23         char c = bw[i];
24         res = c + res;
25         i = first[ bw[i] ] + ranks[i];
26     } while( i != begin );

```

```

27     return res;
28 }

```

## 7.7 suffix\_array\_lcp

```

1 #define radix_sort(x, y){\
2     for(i=0; i<A; ++i)c[i]=0;\
3     for(i=0; i<n; ++i)c[x[y[i]]]++;\
4     for(i=1; i<A; ++i)c[i]+=c[i-1];\
5     for(i=n-1; ~i; --i)sa[--c[x[y[i]]]]=y[i];\
6 }
7 #define AC(r, a, b)\
8     r[a]!=r[b]||a+k>n||r[a+k]!=r[b+k]
9 void suffix_array(const char *s, int n, int *
10     sa, int *rank, int *tmp, int *c){
11     int A='z'+1, i, k, id=0;
12     for(i=0; i<n; ++i)rank[i]=s[i];
13     radix_sort(rank, tmp);
14     for(k=1; id<n-1; k<=1){
15         for(id=0, i=n-k; i<n; ++i)tmp[id++]=i;
16         for(i=0; i<n; ++i)
17             if(sa[i]>k)tmp[id++]=sa[i]-k;
18         radix_sort(rank, tmp);
19         swap(rank, tmp);
20         for(rank[sa[0]]=id=0, i=1; i<n; ++i)
21             rank[sa[i]]=id+=AC(tmp, sa[i-1], sa[i]);
22         A=id+1;
23     }
24     //h:高度數組 sa:後綴數組 rank:排名
25 void suffix_array_lcp(const char *s, int len,
26     int *h, int *sa, int *rank){
27     for(int i=0; i<len; ++i)rank[sa[i]]=i;
28     for(int i=0, k=0; i<len; ++i){
29         if(rank[i]==0)continue;
30         if(k--<0)
31             while(s[i+k]==s[sa[rank[i]-1]+k])++k;
32         h[rank[i]]=k;
33     }
34     h[0]=0; // h[k]=Lcp(sa[k], sa[k-1]);

```

## 7.8 Z

```

1 void z_alg(char *s, int len, int *z){
2     int l=0, r=0;
3     z[0]=len;
4     for(int i=1; i<len; ++i){
5         z[i]=i>r?0:(i-l+z[i-l]<z[l]?z[i-l]:r-i+1);
6         while(i+z[i]<len&&s[i+z[i]]==s[z[i]])++z[i];
7         if(i+z[i]-1>r)r=i+z[i]-1, l=i;
8     }
9 }

```

## 8 Tarjan

### 8.1 dominator\_tree

```

1 struct dominator_tree{
2     static const int MAXN=5005;
3     int n; // 1-base
4     vector<int> suc[MAXN], pre[MAXN];
5     int fa[MAXN], dfn[MAXN], id[MAXN], Time;
6     int semi[MAXN], idom[MAXN];
7     int anc[MAXN], best[MAXN]; //disjoint set
8     vector<int> dom[MAXN]; //dominator_tree
9     void init(int _n){
10         n=_n;
11         for(int i=1; i<=n; ++i)suc[i].clear(), pre[i].clear();
12     }
13     void add_edge(int u, int v){
14         suc[u].push_back(v);
15         pre[v].push_back(u);
16     }
17     void dfs(int u){
18         dfn[u]=++Time, id[Time]=u;
19         for(auto v:suc[u]){
20             if(dfn[v])continue;
21             dfs(v), fa[dfn[v]]=dfn[u];
22         }
23     }
24     int find(int x){
25         if(x==anc[x])return x;
26         int y=find(anc[x]);
27         if(semi[best[x]]>semi[best[anc[x]]])best[x]=best[anc[x]];
28         return anc[x]=y;
29     }
30     void tarjan(int r){
31         Time=0;
32         for(int t=1; t<=n; ++t){
33             dfn[t]=idom[t]=0; //u=r或是u無法到達r時
34             idom[id[t]]=0;
35             anc[t]=best[t]=semi[t]=t;
36         }
37         dfs(r);
38         for(int y=Time; y>=2; --y){
39             int x=fa[y], idy=id[y];
40             for(auto z:pre[idy]){
41                 if(!z)continue;
42                 find(z);
43                 semi[y]=min(semi[y], semi[best[z]]);
44             }
45             dom[semi[y]].push_back(y);
46             anc[y]=x;
47             for(auto z:dom[x]){
48                 find(z);
49                 idom[z]=semi[best[z]]<x?best[z]:x;
50             }
51             dom[x].clear();
52         }
53         for(int u=2; u<=Time; ++u){
54             if(idom[u]!=semi[u])idom[u]=idom[idom[u]];
55             dom[id[idom[u]]].push_back(id[u]);

```

```

56     }
57 }
58 }dom;

```

### 8.2 tnfsb017\_2\_sat

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 #define MAXN 8001
4 #define MAXN2 MAXN*4
5 #define n(X) ((X)+2*N)
6 vector<int> v[MAXN2], rv[MAXN2], vis_t;
7 int N, M;
8 void addedge(int s, int e){
9     v[s].push_back(e);
10    rv[e].push_back(s);
11 }
12 int scc[MAXN2];
13 bool vis[MAXN2]={false};
14 void dfs(vector<int> *uv, int n, int k=-1){
15     vis[n]=true;
16     for(int i=0; i<uv[n].size(); ++i)
17         if(!vis[uv[n][i]])
18             dfs(uv, uv[n][i], k);
19     if(uv==v)vis_t.push_back(n);
20     scc[n]=k;
21 }
22 void solve(){
23     for(int i=1; i<=N; ++i){
24         if(!vis[i])dfs(v, i);
25         if(!vis[n(i)])dfs(v, n(i));
26     }
27     memset(vis, 0, sizeof(vis));
28     int c=0;
29     for(int i=vis_t.size()-1; i>=0; --i)
30         if(!vis[vis_t[i]])
31             dfs(rv, vis_t[i], c++);
32 }
33 int main(){
34     int a, b;
35     scanf("%d%d", &N, &M);
36     for(int i=1; i<=N; ++i){
37         // (A or B) & (!A & !B) A^B
38         a=i*2-1;
39         b=i*2;
40         addedge(n(a), b);
41         addedge(n(b), a);
42         addedge(a, n(b));
43         addedge(b, n(a));
44     }
45     while(M--){
46         scanf("%d%d", &a, &b);
47         a=a>0?a*2-1:-a*2;
48         b=b>0?b*2-1:-b*2;
49         // A or B
50         addedge(n(a), b);
51         addedge(n(b), a);
52     }
53     solve();
54     bool check=true;
55     for(int i=1; i<=2*N; ++i)
56         if(scc[i]==scc[n(i)])
57             check=false;

```

```

58 if(check){
59     printf("%d\n",N);
60     for(int i=1;i<=2*N;i+=2){
61         if(scc[i]>scc[i+2*N]) putchar('+');
62         else putchar('-');
63     }
64     puts("");
65 }else puts("0");
66 return 0;
67 }

```

### 8.3 橋連通分量

```

1 #define N 1005
2 struct edge{
3     int u,v;
4     bool is_bridge;
5     edge(int u=0,int v=0):u(u),v(v),is_bridge
6         (0){}
7 };
8 vector<edge> E;
9 vector<int> G[N]; // 1-base
10 int low[N],vis[N],Time;
11 int bcc_id[N],bridge_cnt,bcc_cnt; // 1-base
12 int st[N],top; // BCC用
13 inline void add_edge(int u,int v){
14     G[u].push_back(E.size());
15     E.push_back(edge(u,v));
16     G[v].push_back(E.size());
17     E.push_back(edge(v,u));
18 }
19 void dfs(int u,int re=-1){ // u當前點, re為u連
20     接前一個點的邊
21     int v;
22     low[u]=vis[u]=++Time;
23     st[top++]=u;
24     for(size_t i=0;i<G[u].size();++i){
25         int e=G[u][i];v=E[e].v;
26         if(!vis[v]){
27             dfs(v,e^1); // e^1 反向邊
28             low[u]=min(low[u],low[v]);
29             if(vis[u]<low[v]){
30                 E[e].is_bridge=E[e^1].is_bridge=1;
31                 ++bridge_cnt;
32             }
33         }else if(vis[v]<vis[u]&&e!=re){
34             low[u]=min(low[u],vis[v]);
35         }
36     }
37     if(vis[u]==low[u]){ // 處理BCC
38         ++bcc_cnt; // 1-base
39         do bcc_id[v=st[--top]]=bcc_cnt; // 每個點
40             所在的BCC
41         while(v!=u);
42     }
43 }
44 inline void bcc_init(int n){
45     Time=bcc_cnt=bridge_cnt=top=0;
46     E.clear();
47     for(int i=1;i<=n;++i){
48         G[i].clear();
49         vis[i]=bcc_id[i]=0;
50     }

```

```

47 }

```

### 8.4 雙連通分量 & 割點

```

1 #define N 1005
2 vector<int> G[N]; // 1-base
3 vector<int> bcc[N]; // 存每塊雙連通分量的點
4 int low[N],vis[N],Time;
5 int bcc_id[N],bcc_cnt; // 1-base
6 bool is_cut[N]; // 是否為割點
7 int st[N],top;
8 void dfs(int u,int pa=-1){ // u當前點, pa父親
9     int v,child=0;
10     low[u]=vis[u]=++Time;
11     st[top++]=u;
12     for(size_t i=0;i<G[u].size();++i){
13         if(!vis[v=G[u][i]]){
14             dfs(v,u),++child;
15             low[u]=min(low[u],low[v]);
16             if(vis[u]<=low[v]){
17                 is_cut[u]=1;
18                 bcc[++bcc_cnt].clear();
19                 int t;
20                 do{
21                     bcc_id[t=st[--top]]=bcc_cnt;
22                     bcc[bcc_cnt].push_back(t);
23                 }while(t!=v);
24                 bcc_id[u]=bcc_cnt;
25                 bcc[bcc_cnt].push_back(u);
26             }
27             }else if(vis[v]<vis[u]&&v!=pa){ // 反向邊
28                 low[u]=min(low[u],vis[v]);
29             }
30         }
31     }
32     if(pa!=-1&&child<2)is_cut[u]=0; // u是dfs樹
33     的根要特判
34 }
35 inline void bcc_init(int n){
36     Time=bcc_cnt=top=0;
37     for(int i=1;i<=n;++i){
38         G[i].clear();
39         is_cut[i]=vis[i]=bcc_id[i]=0;
40     }

```

## 9 Tree\_problem

### 9.1 HeavyLight

```

1 #include<vector>
2 #define MAXN 100005
3 int siz[MAXN],max_son[MAXN],pa[MAXN],dep[
4     MAXN];
5 int link_top[MAXN],link[MAXN],cnt;
6 vector<int> G[MAXN];
7 void find_max_son(int u){
8     siz[u]=1;
9     max_son[u]=-1;

```

```

9     for(auto v:G[u]){
10         if(v==pa[u])continue;
11         pa[v]=u;
12         dep[v]=dep[u]+1;
13         find_max_son(v);
14         if(max_son[u]==-1||siz[v]>siz[max_son[u]
15             ]){max_son[u]=v;
16             siz[u]+=siz[v];
17         }
18     }
19 void build_link(int u,int top){
20     link[u]=++cnt;
21     link_top[u]=top;
22     if(max_son[u]==-1)return;
23     build_link(max_son[u],top);
24     for(auto v:G[u]){
25         if(v==max_son[u]||v==pa[u])continue;
26         build_link(v,v);
27     }
28 }
29 int find_lca(int a,int b){
30     //求LCA, 可以在過程中對區間進行處理
31     int ta=link_top[a],tb=link_top[b];
32     while(ta!=tb){
33         if(dep[ta]<dep[tb]){
34             swap(ta,tb);
35             swap(a,b);
36         }
37         //這裡可以對a所在的鏈做區間處理
38         //區間為(Link[ta],Link[a])
39         ta=link_top[a=pa[ta]];
40     }
41     //最後a,b會在同一條鏈, 若a!=b還要在進行一
42     次區間處理
43     return dep[a]<dep[b]?a:b;

```

### 9.2 LCA

```

1 const int MAXN=100000; // 1-base
2 const int MLG=17; // Log2(MAXN)+1;
3 int pa[MLG+1][MAXN+5];
4 int dep[MAXN+5];
5 vector<int> G[MAXN+5];
6 void dfs(int x,int p=0){ // dfs(root);
7     pa[0][x]=p;
8     for(int i=0;i<=MLG;++i)
9         pa[i+1][x]=pa[i][pa[i][x]];
10     for(auto &i:G[x]){
11         if(i==p)continue;
12         dep[i]=dep[x]+1;
13         dfs(i,x);
14     }
15 }
16 inline int jump(int x,int d){
17     for(int i=0;i<=MLG;++i)
18         if((d>>i)&1) x=pa[i][x];
19     return x;
20 }
21 inline int find_lca(int a,int b){
22     if(dep[a]>dep[b])swap(a,b);
23     b=jump(b,dep[b]-dep[a]);

```

```

24 if(a==b)return a;
25 for(int i=MLG;i>=0;--i){
26     if(pa[i][a]!=pa[i][b]){
27         a=pa[i][a];
28         b=pa[i][b];
29     }
30 }
31 return pa[0][a];
32 }

```

### 9.3 link\_cut\_tree

```

1 struct splay_tree{
2     int ch[2],pa; // 子節點跟父母
3     bool rev; // 反轉的懶惰標記
4     splay_tree():pa(0),rev(0){ch[0]=ch[1]=0;}
5 };
6 vector<splay_tree> nd;
7 // 有的時候用vector會TLE, 要注意
8 // 這邊以node[0]作為null節點
9 bool isroot(int x){ // 判斷是否為這棵splay
10     tree的根
11     return nd[nd[x].pa].ch[0]!=x&&nd[nd[x].pa]
12         .ch[1]!=x;
13 }
14 void down(int x){ // 懶惰標記下推
15     if(nd[x].rev){
16         if(nd[x].ch[0]nd[nd[x].ch[0]].rev^=1;
17         if(nd[x].ch[1]nd[nd[x].ch[1]].rev^=1;
18         swap(nd[x].ch[0],nd[x].ch[1]);
19         nd[x].rev=0;
20     }
21 }
22 void push_down(int x){ // 所有祖先懶惰標記下推
23     if(!isroot(x))push_down(nd[x].pa);
24     down(x);
25 }
26 void up(int x){ // 將子節點的資訊向上更新
27 }
28 void rotate(int x){ // 旋轉, 會自行判斷轉的方
29     向
30     int y=nd[x].pa,z=nd[y].pa,d=(nd[y].ch[1]==
31         x);
32     nd[x].pa=z;
33     if(!isroot(y))nd[z].ch[nd[z].ch[1]==y]=x;
34     nd[y].ch[d]=nd[x].ch[d^1];
35     nd[nd[y].ch[d]].pa=y;
36     nd[y].pa=x,nd[x].ch[d^1]=y;
37     up(y),up(x);
38 }
39 }
40 void splay(int x){ // 將x伸展到splay tree的根
41     push_down(x);
42     while(!isroot(x)){
43         int y=nd[x].pa;
44         if(!isroot(y)){
45             int z=nd[y].pa;
46             if((nd[z].ch[0]==y)^(nd[y].ch[0]==x))
47                 rotate(y);
48             else rotate(x);
49         }
50         rotate(x);
51     }

```

```

45 }
46 int access(int x){
47     int last=0;
48     while(x){
49         splay(x);
50         nd[x].ch[1]=last;
51         up(x);
52         last=x;
53         x=nd[x].pa;
54     }
55     return last; //access後splay tree的根
56 }
57 void access(int x, bool is=0){ //is=0就是一般
    的access
58     int last=0;
59     while(x){
60         splay(x);
61         if(is&&!nd[x].pa){
62             //printf("%d\n", max(nd[last].ma, nd[nd[x].ch[1]].ma));
63         }
64         nd[x].ch[1]=last;
65         up(x);
66         last=x;
67         x=nd[x].pa;
68     }
69 }
70 void query_edge(int u, int v){
71     access(u);
72     access(v, 1);
73 }
74 void make_root(int x){
75     access(x), splay(x);
76     nd[x].rev ^= 1;
77 }
78 void make_root(int x){
79     nd[access(x)].rev ^= 1;
80     splay(x);
81 }
82 void cut(int x, int y){
83     make_root(x);
84     access(y);
85     splay(y);
86     nd[y].ch[0] = 0;
87     nd[x].pa = 0;
88 }
89 void cut_parents(int x){
90     access(x);
91     splay(x);
92     nd[nd[x].ch[0]].pa = 0;
93     nd[x].ch[0] = 0;
94 }
95 void link(int x, int y){
96     make_root(x);
97     nd[x].pa = y;
98 }
99 int find_root(int x){
100     x = access(x);
101     while(nd[x].ch[0]) x = nd[x].ch[0];
102     splay(x);
103     return x;
104 }
105 int query(int u, int v){
106     //傳回uv路徑splay tree的根結點
107     //這種寫法無法求LCA

```

```

108     make_root(u);
109     return access(v);
110 }
111 int query_lca(int u, int v){
112     //假設求鏈上點權的總和，sum是子樹的權重和，
    data是節點的權重
113     access(u);
114     int lca = access(v);
115     splay(u);
116     if(u == lca){
117         //return nd[lca].data + nd[nd[lca].ch[1]].sum
118     } else {
119         //return nd[lca].data + nd[nd[lca].ch[1]].sum + nd[u].sum
120     }
121 }
122 struct EDGE{
123     int a, b, w;
124 } e[10005];
125 int n;
126 vector<pair<int, int>> G[10005];
127 //first表示子節點，second表示邊的編號
128 int pa[10005], edge_node[10005];
129 //pa是父母節點，暫存用的，edge_node是每個編
    被存在哪個點裡面的陣列
130 void bfs(int root){
131     //在建構的時候把每個點都設成一個splay tree
132     queue<int> q;
133     for(int i=1; i<=n; ++i) pa[i] = 0;
134     q.push(root);
135     while(q.size()){
136         int u = q.front();
137         q.pop();
138         for(auto P: G[u]){
139             int v = P.first;
140             if(v != pa[u]){
141                 pa[v] = u;
142                 nd[v].pa = u;
143                 nd[v].data = e[P.second].w;
144                 edge_node[P.second] = v;
145                 up(v);
146                 q.push(v);
147             }
148         }
149     }
150 }
151 void change(int x, int b){
152     splay(x);
153     //nd[x].data = b;
154     up(x);
155 }

```

## 9.4 POJ\_tree

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 #define MAXN 10005
4 int n, k;
5 vector<pair<int, int>> g[MAXN];
6 int size[MAXN];
7 bool vis[MAXN];

```

```

8 inline void init(){
9     for(int i=0; i<=n; ++i){
10         g[i].clear();
11         vis[i] = 0;
12     }
13 }
14 void get_dis(vector<int> &dis, int u, int pa,
    int d){
15     dis.push_back(d);
16     for(size_t i=0; i<g[u].size(); ++i){
17         int v = g[u][i].first, w = g[u][i].second;
18         if(v != pa && !vis[v]) get_dis(dis, v, u, d+w);
19     }
20 }
21 vector<int> dis; //這東西如果放在函數裡會TLE
22 int cal(int u, int d){
23     dis.clear();
24     get_dis(dis, u, -1, d);
25     sort(dis.begin(), dis.end());
26     int l=0, r=dis.size()-1, res=0;
27     while(l<r){
28         while(l<r && dis[l]+dis[r]>k) --r;
29         res += r - l + 1;
30     }
31     return res;
32 }
33 pair<int, int> tree_centroid(int u, int pa,
    const int sz){
34     size[u] = 1; //找樹重心，second是重心
35     pair<int, int> res(INT_MAX, -1);
36     int ma = 0;
37     for(size_t i=0; i<g[u].size(); ++i){
38         int v = g[u][i].first;
39         if(v == pa || vis[v]) continue;
40         res = min(res, tree_centroid(v, u, sz));
41         size[u] += size[v];
42         ma = max(ma, size[v]);
43     }
44     ma = max(ma, sz - size[u]);
45     return min(res, make_pair(ma, u));
46 }
47 int tree_DC(int u, int sz){
48     int center = tree_centroid(u, -1, sz).second;
49     int ans = cal(center, 0);
50     vis[center] = 1;
51     for(size_t i=0; i<g[center].size(); ++i){
52         int v = g[center][i].first, w = g[center][i].second;
53         if(vis[v]) continue;
54         ans += cal(v, w);
55         ans += tree_DC(v, size[v]);
56     }
57     return ans;
58 }
59 int main(){
60     while(scanf("%d%d", &n, &k), n || k){
61         init();
62         for(int i=1; i<=n; ++i){
63             int u, v, w;
64             scanf("%d%d%d", &u, &v, &w);
65             g[u].push_back(make_pair(v, w));
66             g[v].push_back(make_pair(u, w));
67         }
68         printf("%d\n", tree_DC(1, n));
69     }

```

```

70     return 0;
71 }

```

## 10 default

### 10.1 debug

```

1 //volatile
2 #ifdef DEBUG
3 #define dbg(...) {\
4     fprintf(stderr, "%s - %d : (%s) = ",
        __PRETTY_FUNCTION__, __LINE__, #
        __VA_ARGS__); \
        _DO(__VA_ARGS__); \
5 }
6 }
7 template<typename I> void _DO(I&&x){ cerr<<x
    <<endl; }
8 template<typename I, typename...T> void _DO(I
    &&x, T&&...tail){ cerr<<x<<" "; _DO(tail
        ...); }
9 #else
10 #define dbg(...)
11 #endif

```

### 10.2 ext

```

1 #include<bits/extc++.h>
2 #include<ext/pd_ds/assoc_container.hpp>
3 #include<ext/pd_ds/tree_policy.hpp>
4 using namespace __gnu_cxx;
5 using namespace __gnu_pbds;
6 template<typename T>
7 using pbds_set = tree<T, null_type, less<T>,
    rb_tree_tag,
    tree_order_statistics_node_update>;
8 template<typename T, typename U>
9 using pbds_map = tree<T, U, less<T>,
    rb_tree_tag,
    tree_order_statistics_node_update>;
10 using heap = __gnu_pbds::priority_queue<int>;
11 //s.find_by_order(1); //0 base
12 //s.order_of_key(1);

```

### 10.3 IncStack

```

1 //Magic
2 #pragma GCC optimize "Ofast"
3 //stack resize, change esp to rsp if 64-bit
    system
4 asm("mov %0, %%esp\n" :: "g"(mem+1000000));
5 -Wl,--stack,214748364 -trigraphs
6 //Linux stack resize
7 #include<sys/resource.h>
8 void increase_stack(){

```



```

9  const rlim_t ks=64*1024*1024;
10 struct rlimit rl;
11 int res=getrlimit(RLIMIT_STACK,&rl);
12 if(!res&&rl.rlim_cur<ks){
13     rl.rlim_cur=ks;
14     res=setrlimit(RLIMIT_STACK,&rl);
15 }
16 }

```

## 10.4 input

```

1  inline int read(){
2      int x=0; bool f=0; char c=getchar();
3      while(ch<'0' || '9'<ch)f|=ch=='-' ,ch=getchar
4          ();
5      while('0'<=ch&&ch<='9')x=x*10-'0'+ch,ch=
6          getchar();
7      return f?-x:x;
8      // #!/bin/bash
9      // g++ -std=c++11 -O2 -Wall -Wextra -Wno-
10         unused-result -DDEBUG $1 && ./a.out
11 // -fsanitize=address -fsanitize=undefined
12 -fsanitize=return

```

## 11 language

### 11.1 CNF

```

1  #define MAXN 55
2  struct CNF{
3      int s,x,y;//s->xy | s->x, if y== -1
4      int cost;
5      CNF(){}
6      CNF(int s,int x,int y,int c):s(s),x(x),y(y)
7          ,cost(c){}
8  };
9  int state;//規則數量
10 map<char,int> rule;//每個字元對應到的規則
11 //小寫字母為終端字符
12 vector<CNF> cnf;
13 void init(){
14     state=0;
15     rule.clear();
16     cnf.clear();
17 }
18 void add_to_cnf(char s,const string &p,int
19     cost){
20     //加入一個s -> <p>的文法
21     //代價為cost
22     if(rule.find(s)==rule.end())rule[s]=state
23         ++;
24     for(auto c:p)if(rule.find(c)==rule.end())
25         rule[c]=state++;
26     if(p.size()==1){
27         cnf.push_back(CNF(rule[s],rule[p[0]],-1,
28             cost));
29     }else{

```

```

23 int left=rule[s];
24 int sz=p.size();
25 for(int i=0;i<sz-2;++i){
26     cnf.push_back(CNF(left,rule[p[i]],
27         state,0));
28     left=state++;
29     cnf.push_back(CNF(left,rule[p[sz-2]],
30         rule[p[sz-1]],cost));
31 }
32 vector<long long> dp[MAXN][MAXN];
33 vector<bool> neg_INF[MAXN][MAXN];//如果花費
34 //是真的可能會有無限小的情形
35 void relax(int l,int r,const CNF &c,long
36     long cost,bool neg_c=0){
37     if(!neg_INF[l][r][c.s]&&(neg_INF[l][r][c.x
38         ]||cost<dp[l][r][c.s])){
39         if(neg_c|!neg_INF[l][r][c.x]){
40             dp[l][r][c.s]=0;
41             neg_INF[l][r][c.s]=true;
42         }else dp[l][r][c.s]=cost;
43     }
44 }
45 void bellman(int l,int r,int n){
46     for(int k=1;k<=state;++k)
47         for(auto c:cnf)
48             if(c.y== -1)relax(l,r,c,dp[l][r][c.x]+
49                 c.cost,k==n);
50 }
51 void cyk(const vector<int> &tok){
52     for(int i=0;i<(int)tok.size();++i){
53         for(int j=0;j<(int)tok.size();++j){
54             dp[i][j]=vector<long long>(state+1,
55                 INT_MAX);
56             neg_INF[i][j]=vector<bool>(state+1,
57                 false);
58         }
59         dp[i][i][tok[i]]=0;
60         bellman(i,i,tok.size());
61     }
62     for(int r=1;r<(int)tok.size();++r){
63         for(int l=r-1;l>=0;--l){
64             for(int k=l;k<r;++k)
65                 for(auto c:cnf)
66                     if(~c.y)relax(l,r,c,dp[l][k][c.x]+
67                         dp[k+1][r][c.y]+c.cost);
68             bellman(l,r,tok.size());
69         }
70     }
71 }

```

## 12 other

### 12.1 WhatDay

```

1  int whatday(int y,int m,int d){
2      if(m<=2)m+=12,--y;
3      if(y<1752||y==1752&&m<9||y==1752&&m==9&&d
4          <3)

```

```

4      return (d+2*m+3*(m+1)/5+y+y/4+5)%7;
5      return (d+2*m+3*(m+1)/5+y+y/4-y/100+y/400)
6      %7;

```

### 12.2 上下最大正方形

```

1  void solve(int n,int a[],int b[]){// 1-base
2      int ans=0;
3      deque<int>da,db;
4      for(int l=1,r=1;r<=n;++r){
5          while(da.size()&&a[da.back()]>=a[r]){
6              da.pop_back();
7          }
8          da.push_back(r);
9          while(db.size()&&b[db.back()]>=b[r]){
10              db.pop_back();
11          }
12          db.push_back(r);
13          for(int d=a[da.front()]+b[db.front()];r-
14              l+1>d;++l){
15              if(da.front()==l)da.pop_front();
16              if(db.front()==l)db.pop_front();
17              if(da.size()&&db.size()){
18                  d=a[da.front()]+b[db.front()];
19              }
20          }
21          ans=max(ans,r-l+1);
22      }
23      printf("%d\n",ans);

```

### 12.3 最大矩形

```

1  LL max_rectangle(vector<int> s){
2      stack<pair<int,int>> st;
3      st.push(make_pair(-1,0));
4      s.push_back(0);
5      LL ans=0;
6      for(size_t i=0;i<s.size();++i){
7          int h=s[i];
8          pair<int,int> now=make_pair(h,i);
9          while(h<st.top().first){
10              now=st.top();
11              st.pop();
12              ans=max(ans,(LL)(i-now.second)*now.
13                  first);
14          }
15          if(h>st.top().first){
16              st.push(make_pair(h,now.second));
17          }
18      }
19      return ans;

```

## 13 zformula

### 13.1 formula

#### 13.1.1 Pick 公式

給定頂點坐標均是整點的簡單多邊形，面積 = 內部格點數 + 邊上格點數/2-1

#### 13.1.2 圖論

- $V - E + F = 2$
- 對於平面圖， $F = E - V + n + 1$ ， $n$  是連通分量
- 對於平面圖， $E \leq 3V - 6$
- 對於連通圖  $G$ ，最大獨立點集的大小設為  $I(G)$ ，最大匹配大小設為  $M(G)$ ，最小點覆蓋設為  $C_v(G)$ ，最小邊覆蓋設為  $C_e(G)$ 。對於任意連通圖：

$$(a) \quad I(G) + C_v(G) = |V|$$

$$(b) \quad M(G) + C_e(G) = |V|$$

- 對於連通二分圖：

$$(a) \quad I(G) = C_v(G)$$

$$(b) \quad M(G) = C_e(G)$$

- 最大權閉合圖：

$$(a) \quad C(u, V) = \infty, (u, v) \in E$$

$$(b) \quad C(S, v) = W_v, W_v > 0$$

$$(c) \quad C(v, T) = -W_v, W_v < 0$$

- 最大密度子圖：

$$(a) \quad C(u, v) = 1, (u, v) \in E$$

$$(b) \quad C(S, v) = U_v, v \in V$$

$$(c) \quad C(v, T) = U + 2g - d_v, v \in V$$

- 弦圖：

- 完美消除序列從後往前依次給每個點染色，給每個點染上可以染的最小顏色
- 最大團大小 = 色數
- 最大獨立集：完美消除序列從前往後能選就選
- 最小團覆蓋：最大獨立集的點和他延伸的邊構成
- 區間圖是弦圖
- 區間圖的完美消除序列：將區間按造又端點由小到大排序
- 區間圖染色：用線段樹做

```

1  double l=0,m,stop=1.0/n/n;
2  while(r-l>=stop){
3      double mid;
4      if((n*m-sol.maxFlow(s,t))/2>eps)l=mid;
5      else r=mid;
6  }
7  build(1);
8  sol.maxFlow(s,t);
9  vector<int> ans;
10 for(int i=1;i<=n;++i)
11     if(sol.vis[i])ans.push_back(i);

```

### 13.1.3 學長公式

- $\sum_{d|n} \phi(n) = n$
- $g(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} \mu(d) \times g(n/d)$
- Harmonic series  $H_n = \ln(n) + \gamma + 1/(2n) - 1/(12n^2) + 1/(120n^4)$
- $\gamma = 0.57721566490153286060651209008240243104215$
- 格雷碼  $= n \oplus (n >> 1)$
- $SG(A+B) = SG(A) \oplus SG(B)$
- 選轉矩陣  $M(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

### 13.1.4 基本數論

- $\sum_{d|n} \mu(n) = [n == 1]$
- $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) \times g(m/d)$
- $\sum_{i=1}^n \sum_{j=1}^m \text{互質數量} = \sum \mu(d) \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$
- $\sum_{i=1}^n \sum_{j=1}^m \text{lcm}(i, j) = n \sum_{d|n} d \times \phi(d)$

### 13.1.5 排組公式

- k 卡特蘭  $\frac{C_n^{kn}}{n(k-1)+1} \cdot C_m^n = \frac{n!}{m!(n-m)!}$
- $H(n, m) \cong x_1 + x_2 \dots + x_n = k, num = C_k^{n+k-1}$
- Stirling number of  $2^{nd}$ ,  $n$  人分  $k$  組方法數目
  - $S(0, 0) = S(n, n) = 1$
  - $S(n, 0) = 0$
  - $S(n, k) = kS(n-1, k) + S(n-1, k-1)$
- Bell number,  $n$  人分任意多組方法數目
  - $B_0 = 1$
  - $B_n = \sum_{i=0}^n S(n, i)$
  - $B_{n+1} = \sum_{k=0}^n C_k^n B_k$
  - $B_{p+n} \equiv B_n + B_{n+1} \pmod{p}$ ,  $p$  is prime
  - $B_p^m \equiv mB_n + B_{n+1} \pmod{p}$ ,  $p$  is prime
  - From  $B_0 : 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975$
- Derangement, 錯排, 沒有人在自己位置上
  - $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \dots + (-1)^n \frac{1}{n!})$
  - $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 = 1, D_1 = 0$
  - From  $D_0 : 1, 0, 1, 2, 9, 44, 265, 1854, 14833, 133496$
- Binomial Equality
  - $\sum_k \binom{r}{m+k} \binom{s}{n-k} = \binom{r+s}{m+n}$
  - $\sum_k \binom{l}{m+k} \binom{s}{n-k} = \binom{l+s}{l-m+n}$
  - $\sum_k \binom{l}{m+k} \binom{s+k}{n} (-1)^k = (-1)^{l+m} \binom{s-m}{n-l}$
  - $\sum_{k \leq l} \binom{l-k}{m} \binom{s}{k-n} (-1)^k = (-1)^{l+m} \binom{s-m-1}{l-n-m}$
  - $\sum_{0 \leq k \leq l} \binom{l-k}{m} \binom{q+k}{n} = \binom{l+q+1}{m+n+1}$
  - $\binom{r}{k} = (-1)^k \binom{k-r-1}{k}$

- $\binom{r}{m} \binom{m}{k} = \binom{r}{k} \binom{r-k}{m-k}$
- $\sum_{k \leq n} \binom{r+k}{k} = \binom{r+n+1}{n}$
- $\sum_{0 \leq k \leq n} \binom{k}{m} = \binom{n+1}{m+1}$
- $\sum_{k \leq m} \binom{m+r}{k} x^k y^{m-k} = \sum_{k \leq m} \binom{-r}{k} (-x)^k (x+y)^{m-k}$

### 13.1.6 幕次, 幕次和

- $a^b \% P = a^{b \% \varphi(P) + \varphi(P)}, b \geq \varphi(P)$
- $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
- $1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$
- $1^5 + 2^5 + 3^5 + \dots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12}$
- $0^k + 1^k + 2^k + \dots + n^k = P(k), P(k) = \frac{(n+1)^{k+1} - \sum_{i=0}^{k-1} C_i^{k+1} P(i)}{k+1}, P(0) = n+1$
- $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^n C_k^{n+1} B_k m^{n+1-k}$
- $\sum_{j=0}^m C_j^{m+1} B_j = 0, B_0 = 1$
- 除了  $B_1 = -1/2$ , 剩下的奇數項都是 0
- $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 = -1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 7/6, B_{16} = -3617/510, B_{18} = 43867/798, B_{20} = -174611/330,$

### 13.1.7 Burnside's lemma

- $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- $X^g = t^{c(g)}$
- $G$  表示有幾種轉法,  $X^g$  表示在那種轉法下, 有幾種是會保持對稱的,  $t$  是顏色數,  $c(g)$  是循環節不動的面數。
- 正立方體塗三顏色, 轉 0 有  $3^6$  個元素不變, 轉 90 有 6 種, 每種有  $3^3$  不變, 180 有  $3 \times 3^4$ , 120(角) 有  $8 \times 3^2$ , 180(邊) 有  $6 \times 3^3$ , 全部  $\frac{1}{24}(3^6 + 6 \times 3^3 + 3 \times 3^4 + 8 \times 3^2 + 6 \times 3^3) = \frac{57}{57}$

### 13.1.8 Count on a tree

- Rooted tree:  $s_{n+1} = \frac{1}{n} \sum_{i=1}^n (i \times a_i \times \sum_{j=1}^{\lfloor n/i \rfloor} a_{n+1-i \times j})$
- Unrooted tree:
  - Odd:  $a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$
  - Even:  $Odd + \frac{1}{2} a_{n/2} (a_{n/2} + 1)$
- Spanning Tree
  - 完全圖  $n^n - 2$
  - 一般圖 (Kirchhoff's theorem)  $M[i][i] = \text{degree}(V_i), M[i][j] = -1, \text{if have } E(i, j), 0 \text{ if no edge. delete any one row and col in } A, \text{ans} = \det(A)$

## 13.2 java

### 13.2.1 文件操作

```

1 import java.io.*;
2 import java.util.*;
3 import java.math.*;
4 import java.text.*;
5
6 public class Main{
7
8     public static void main(String args[]){
9         throws FileNotFoundException,
10         IOException
11         Scanner sc = new Scanner(new FileReader(
12             "a.in"));
13         PrintWriter pw = new PrintWriter(new
14             FileWriter("a.out"));
15         int n,m;
16         n=sc.nextInt();//读入下一个INT
17         m=sc.nextInt();
18
19         for(ci=1; ci<=c; ++ci){
20             pw.println("Case #"+ci+": easy for
21                 output");
22         }
23
24         pw.close();//关闭流并释放, 这个很重要,
25         否则是没有输出的
26         sc.close();//关闭流并释放
27     }
28 }
```

### 13.2.2 优先队列

```

1 PriorityQueue queue = new PriorityQueue( 1,
2     new Comparator(){
3         public int compare( Point a, Point b ){
4             if( a.x < b.x || a.x == b.x && a.y < b.y )
5                 return -1;
6             else if( a.x == b.x && a.y == b.y )
7                 return 0;
8             else return 1;
9         }
10    });
```

### 13.2.3 Map

```

1 Map map = new HashMap();
2 map.put("sa", "dd");
3 String str = map.get("sa").toString();
4
5 for(Object obj : map.keySet()){
6     Object value = map.get(obj);
7 }
```

### 13.2.4 sort

```

1 static class cmp implements Comparator{
2     public int compare(Object o1, Object o2){
3         BigInteger b1=(BigInteger)o1;
4         BigInteger b2=(BigInteger)o2;
5         return b1.compareTo(b2);
6     }
7 }
8 public static void main(String[] args)
9     throws IOException{
10     Scanner cin = new Scanner(System.in);
11     int n;
12     n=cin.nextInt();
13     BigInteger[] seg = new BigInteger[n];
14     for (int i=0; i<n; i++)
15         seg[i]=cin.nextBigInteger();
16     Arrays.sort(seg, new cmp());
17 }
```

## 14

### 14.1 ganadoQuote

```

1 ¡Allí estás!
2 ¡Un forastero!
3 ¡Agarrenlo!
4 ¡Os voy a romper a pedazos!
5 ¡Cógelo!
6 ¡Te voy a hacer picadillo!
7 ¡Te voy a matar!
8 ¡Míralo, está herido!
9 ¡Sos cerdo!
10 ¿Dónde estás?
11 ¡Detrás de tí, imbécil!
12 ¡No dejes que se escape!
13 ¡Basta, hijo de puta!
14 Lord Saddler...
15
16 ¡Mátalo!
17 ¡Allí estás!
18 Morir es vivir.
19 ¡Ííííí, ¡Quiero matar!
20 Muere, muere, muere...
21 Cerebros, cerebros, cerebros...
22 Cógedlo, cógedlo, cógedlo...
23 Lord Saddler...
24 Dieciséis.
25
26 ¡Va por él!
27 ¡Muérete!
28 ¡Cógelo!
29 ¡Te voy a matar!
30 ¡Bloqueale el paso!
31 ¡Te cogí!
32 ¡No dejes que se escape!
33
34 ¿Qué carajo estás haciendo aquí? ¡Lárgate,
    cabrón!
```

```
35 Hay un rumor de que hay un extranjero entre
    nosotros.
36 Nuestro jefe se encargará de la rata.
37 Su "Las Plagas" es mucho mejor que la
    nuestra.
38 Tienes razón, es un hombre.
39 Usa los músculos.
40 Se vuelve loco!
41 ¡Hey, acá!
42 ¡Por aquí!
43 ¡El Gigante!
44 ¡Del Lago!
45 ¡Cógelo!
46 ¡Cógenlo!
47 ¡Allí!
48 ¡Rápido!
49 ¡Empieza a rezar!
50 ¡Mátenlos!
51 ¡Te voy a romper en pedazos!
52 ¡La campana!
53 Ya es hora de rezar.
54 Tenemos que irnos.
55 ¡Maldita sea, mierda!
56 ¡Ya es hora de aplastar!
57 ¡Mierda!
58 ¡Puedes correr, pero no te puedes esconder!
59 ¡Sos cerdo!
60 ¡Está en la trampa!
61 ¡Ah, que madre!
62 ¡Vámonos!
63 ¡Ándale!
64 ¡Cabrón!
65 ¡Coño!
66 ¡Agárrenlo!
67 Cógerlo, Cógerlo...
68 ¡Allí está, mátaló!
69 ¡No dejas que se escape de la isla vivo!
70 ¡Hasta luego!
71 ¡Rápido, es un intruso!
```

14.2

```
1 /*****
2  L'Internationale,
3      Sera le genre humain.
4
5
6
7
8
9
10
11
12
13
14
15
16 *****/
17 Вставай, проклятьем заклеймённый,
18 Весь мир голодных и рабов!
19 Кипит наш разум возмущённый
20 И в смертный бой вести готов.
21 Весь мир наси́лья мы разрушим
```

```
22 До основанья, а затем
23 Мы наш, мы новый мир построим, –
24 Кто был ничем, тот станет всем.
25
26 Chorus
27 Это есть наш последний
28 И решительный бой;
29 С Интернационалом
30 Воспрянет род людской!
31
32 Никто не даст нам избавленья:
33 Ни бог, ни царь и не герой!
34 Добьёмся мы освобожденья
35 Своею собственной рукой.
36 Чтоб свергнуть гнёт рукой умелой,
37 Отвоевать своё добро, –
38 Вдуйте горн и куйте смело,
39 Пока железо горячо!
40
41 Chorus
42
43 Довольно кровь сосать, вампиры,
44 Тюрьмой, налогом, нищетой!
45 У вас — вся власть, все блага мира,
46 А наше право — звук пустой !
47 Мы жизнь построим по-иному –
48 И вот наш лозунг боевой:
49 Вся власть народу трудовому!
50 А дармоедов всех долой!
51
52 Chorus
53
54 Презренны вы в своём богатстве,
55 Угля и стали короли!
56 Вы ваши троны, тунейдцы,
57 На наших спинах возвели.
58 Заводы, фабрики, палаты –
59 Всё нашим создано трудом.
60 Пора! Мы требуем возврата
61 Того, что взято грабежом.
62
63 Chorus
64
65 Довольно королям в угоду
66 Дурманить нас в чаду войны!
67 Война тиранам! Мир Народу!
68 Бастуйте, армии сыны!
69 Когда ж тираны нас заставят
70 В бою героически пасть за них –
71 Убийцы, в вас тогда направим
72 Мы жерла пушек боевых!
73
74 Chorus
75
76 Лишь мы, работники всемирной
77 Великой армии труда,
78 Владеть землёй имеем право,
79 Но паразиты — никогда!
80 И если гром великий грянет
81 Над сворой псов и палачей, –
82 Для нас всё так же солнце станет
83 Сиять огнём своих лучей.
84
85 Chorus
```

14.3 保佑

```
1 //
2 //
3 //
4 //
5 //
6 //
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0\ = /0

佛祖保佑 永無BUG

神獸保佑 永無BUG!

```
59 // ##
60 // ##
61 // ##
62 // ##
63 // ##
64 // ##
65 // ##
66 // ##
67 // #####
68 // ##
69 // ##
70 // ##
71 // ##
72 // ##
73 // ##
74 // ##
75 // #####
76 //
77 // 元首保佑 永無BUG
78
79 //
80 //
81 //
82 //
83 //
84 //
85 //
86 //
87 //
88 //
89 //
90 //
91 //
92 //
```

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神獸保佑 永無BUG

# ACM ICPC TEAM REFERENCE - MADE IN ABYSS

## Contents

<b>1</b>	<b>Computational_Geometry</b>	<b>1</b>	<b>4</b>	<b>Graph</b>	<b>6</b>	6.11 NTT . . . . .	12	10.4 input . . . . .	17
1.1	Geometry . . . . .	1	4.1	Augmenting_Path . . . . .	6	6.12 Simpson . . . . .	12	<b>11 language</b>	<b>17</b>
1.2	SmallestCircle . . . . .	3	4.2	Augmenting_Path_multiple . . . . .	6	6.13 外星模運算 . . . . .	12	11.1 CNF . . . . .	17
1.3	最近點對 . . . . .	3	4.3	blossom_matching . . . . .	6	6.14 數位統計 . . . . .	12	<b>12 other</b>	<b>17</b>
<b>2</b>	<b>Data_Structure</b>	<b>3</b>	4.4	graphISO . . . . .	7	6.15 質因數分解 . . . . .	12	12.1 WhatDay . . . . .	17
2.1	DLX . . . . .	3	4.5	KM . . . . .	7	<b>7 String</b>	<b>13</b>	12.2 上下最大正方形 . . . . .	17
2.2	Dynamic_KD_tree . . . . .	3	4.6	MaximumClique . . . . .	7	7.1 AC 自動機 . . . . .	13	12.3 最大矩形 . . . . .	17
2.3	kd_tree_replace_segment_tree . . . . .	4	4.7	MinimumMeanCycle . . . . .	7	7.2 hash . . . . .	13	<b>13 zformula</b>	<b>17</b>
2.4	reference_point . . . . .	5	4.8	Rectilinear_MST . . . . .	7	7.3 KMP . . . . .	13	13.1 formula . . . . .	17
2.5	skew_heap . . . . .	5	4.9	treeISO . . . . .	8	7.4 manacher . . . . .	14	13.1.1 Pick 公式 . . . . .	17
2.6	undo_disjoint_set . . . . .	5	4.10	一般圖最小權完美匹配 . . . . .	8	7.5 minimal_string_rotation . . . . .	14	13.1.2 圖論 . . . . .	17
2.7	整體二分 . . . . .	5	4.11	全局最小割 . . . . .	8	7.6 reverseBWT . . . . .	14	13.1.3 學長公式 . . . . .	18
<b>3</b>	<b>Flow</b>	<b>5</b>	4.12	平面圖判定 . . . . .	8	7.7 suffix_array_lcp . . . . .	14	13.1.4 基本數論 . . . . .	18
3.1	dinic . . . . .	5	4.13	弦圖完美消除序列 . . . . .	8	7.8 Z . . . . .	14	13.1.5 排組公式 . . . . .	18
3.2	ISAP_with_cut . . . . .	5	4.14	最小斯坦納樹 DP . . . . .	9	<b>8 Tarjan</b>	<b>14</b>	13.1.6 冪次, 冪次和 . . . . .	18
3.3	MinCostMaxFlow . . . . .	6	4.15	最小樹形圖 _ 朱劉 . . . . .	9	8.1 dominator_tree . . . . .	14	13.1.7 Burnside's lemma . . . . .	18
			4.16	穩定婚姻模板 . . . . .	9	8.2 tnfsb017_2_sat . . . . .	14	13.1.8 Count on a tree . . . . .	18
			<b>5</b>	<b>Linear_Programming</b>	<b>9</b>	8.3 橋連通分量 . . . . .	15	13.2 java . . . . .	18
			5.1	最大密度子圖 . . . . .	9	8.4 雙連通分量 & 割點 . . . . .	15	13.2.1 文件操作 . . . . .	18
			<b>6</b>	<b>Number_Theory</b>	<b>10</b>	<b>9 Tree_problem</b>	<b>15</b>	13.2.2 优先队列 . . . . .	18
			6.1	basic . . . . .	10	9.1 HeavyLight . . . . .	15	13.2.3 Map . . . . .	18
			6.2	bit_set . . . . .	10	9.2 LCA . . . . .	15	13.2.4 sort . . . . .	18
			6.3	cantor_expansion . . . . .	10	9.3 link_cut_tree . . . . .	15	<b>14</b>	<b>18</b>
			6.4	FFT . . . . .	11	9.4 POJ_tree . . . . .	16	14.1 ganadoQuote . . . . .	18
			6.5	find_real_root . . . . .	11	<b>10 default</b>	<b>16</b>	14.2 . . . . .	19
			6.6	FWT . . . . .	11	10.1 debug . . . . .	16	14.3 保佑 . . . . .	19
			6.7	LinearCongruence . . . . .	11	10.2 ext . . . . .	16		
			6.8	Lucas . . . . .	11	10.3 IncStack . . . . .	16		
			6.9	Matrix . . . . .	11				
			6.10	MillerRobin . . . . .	12				