1 Computational_Geometr

1.1 Geometry.cpp

```cpp
const double PI=atan2(0.0,-1.0);
template<typename T>
struct point{
  T x,y;
  point(){}
  point(const T&x,const T&y):x(x),y(y){}
  point operator+(const point &b)const{
    return point(x+b.x,y+b.y);}
  point operator-(const point &b)const{
    return point(x-b.x,y-b.y);}
  point operator*(const T &b)const{
    return point(x*b,y*b);}
  point operator/(const T &b)const{
    return point(x/b,y/b);}
  bool operator==(const point &b)const{
    return x==b.x&&y==b.y;}
  T dot(const point &b)const{
    return x*b.x+y*b.y;}
  T cross(const point &b)const{
    return x*b.y-y*b.x;}
  point normal()const{//求法向量
    return point(-y,x);}
  T abs2()const{//向量長度的平方
    return dot(*this);
  }
  T rad(const point &b)const{//兩向量的弧度
    return fabs(atan2(fabs(cross(b)),dot(b)));
  }
  T getA()const{//對x軸的弧度
    T A=atan2(y,x);//超過180度會變負的
    if(A<=-PI/2)A+=PI*2;
    return A;
  }
};
template<typename T>
struct line{
  line(){}
  point<T> p1,p2;
  T a,b,c;//ax+by+c=0
  line(const point<T>&x,const point<T>&y):p1(x),p2(y){}
  void pton(){//轉成一般式
    a=p1.y-p2.y;
    b=p2.x-p1.x;
    c=-a*p1.x-b*p1.y;
  }
  T cross(const point<T> &p)const{//點和有向直線的關係，>0左邊、=0在線上<0右邊
    return (p2-p1).cross(p-p1);
  }
  bool point_on_segment(const point<T>&p)const{//點是否線段上
    return cross(p)==0&&(p1-p).dot(p2-p)<=0;
  }
  T dis2(const point<T> &p,bool is_segment=0)const{//點跟直線/線段的距離平方
    point<T> v=p2-p1,v1=p-p1;
    if(is_segment){
      point<T> v2=p-p2;
      if(v.dot(v1)<=0)return v1.abs2();
      if(v.dot(v2)>=0)return v2.abs2();
    }
    T tmp=v.cross(v1);
    return tmp*tmp/v.abs2();
  }
  T seg_dis2(const line<T> &l)const{//兩線段距離平方
    return min({dis2(l.p1,1),dis2(l.p2,1),l.dis2(p1,1),l.dis2(p2,1)});
  }
  point<T> projection(const point<T> &p)const{//點對直線的投影
    point<T> n=(p2-p1).normal();
    return p-n*(p-p1).dot(n)/n.abs2();
  }
  point<T> mirror(const point<T> &p)const{//點對直線的鏡射
    //要先呼叫pton轉成一般式
    point<T> ans;
    T d=a*a+b*b;
    ans.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
    ans.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
    return ans;
  }
  bool equal(const line &l)const{//直線相等
    return cross(l.p1)==0&&cross(l.p2)==0;
  }
  bool parallel(const line &l)const{//直線平行
    return (p1-p2).cross(l.p1-l.p2)==0;
  }
  bool cross_seg(const line &l)const{//直線是否交線段
    return (p2-p1).cross(l.p1-p1)*(p2-p1).cross(l.p2-p1)<=0;
  }
  char line_intersect(const line &l)const{//直線相交情況，-1無限多點、1交於一點、0不相交
    return parallel(l)?(cross(l.p1)==0?-1:0):1;
  }
  char seg_intersect(const line &l)const{//線段相交情況，-1無限多點、1交於一點、0不相交
    T c1=(p2-p1).cross(l.p1-p1);
    T c2=(p2-p1).cross(l.p2-p1);
    T c3=(l.p2-l.p1).cross(p1-l.p1);
    T c4=(l.p2-l.p1).cross(p2-l.p1);
    if(c1==0&&c2==0){
      if(p1==l.p1&&(p2-p1).dot(l.p2)<=0)return 1;
      if(p1==l.p2&&(p2-p1).dot(l.p1)<=0)return 1;
      if(p2==l.p1&&(p1-p2).dot(l.p2)<=0)return 1;
      if(p2==l.p2&&(p1-p2).dot(l.p1)<=0)return 1;
      return -1;
    }else if(c1*c2<=0&&c3*c4<=0)return 1;
    return 0;
  }
  point<T> line_intersection(const line &l)const{/*直線交點*/
    point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
    //if(a.cross(b)==0)return INF;
    return p1+a*s.cross(b)/a.cross(b);
  }
  point<T> seg_intersection(const line &l)const{//線段交點
    T c1=(p2-p1).cross(l.p1-p1);
    T c2=(p2-p1).cross(l.p2-p1);
    T c3=(l.p2-l.p1).cross(p1-l.p1);
    T c4=(l.p2-l.p1).cross(p2-l.p1);
    if(c1==0&&c2==0){
      if(p1==l.p1&&(p2-p1).dot(l.p2)<=0)return p1;
      if(p1==l.p2&&(p2-p1).dot(l.p1)<=0)return p1;
      if(p2==l.p1&&(p1-p2).dot(l.p2)<=0)return p2;
      if(p2==l.p2&&(p1-p2).dot(l.p1)<=0)return p2;
    }else if(c1*c2<=0&&c3*c4<=0)return line_intersection(l);
    //return INF;
  }
};
template<typename T>
struct polygon{
  polygon(){}
  vector<point<T> > p;//逆時針順序
  T area()const{//面積
    T ans=0;
    for(int i=p.size()-1,j=0;j<(int)p.size();i=j++)
      ans+=p[i].cross(p[j]);
    return ans/2;
  }
  point<T> center_of_mass()const{//重心
    T cx=0,cy=0,w=0;
    for(int i=p.size()-1,j=0;j<(int)p.size();i=j++){
      T a=p[i].cross(p[j]);
      cx+=(p[i].x+p[j].x)*a;
      cy+=(p[i].y+p[j].y)*a;
      w+=a;
    }
    return point<T>(cx/3/w,cy/3/w);
  }
  char ahas(const point<T>& t)const{//點是否在簡單多邊形內，是的話回傳1、在邊上回傳-1、否則回傳0
    bool c=0;
    for(int i=0,j=p.size()-1;i<p.size();j=i++)
      if(line<T>(p[i],p[j]).point_on_segment(t))return -1;
      else if((p[i].y>t.y)!=(p[j].y>t.y)&&
      t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j].y-p[i].y)+p[i].x)
        c=!c;
    return c;
  }
  char point_in_convex(const point<T>&x)const{
    int l=1,r=(int)p.size()-2;
    while(l<=r){//點是否在凸多邊形內，是的話回傳1、在邊上回傳-1、否則回傳0
      int mid=(l+r)/2;
      T a1=(p[mid]-p[0]).cross(x-p[0]);
      T a2=(p[mid+1]-p[0]).cross(x-p[0]);
      if(a1>=0&&a2<=0){
        T res=(p[mid+1]-p[mid]).cross(x-p[mid]);
        return res>0?1:(res>=0?-1:0);
      }else if(a1<0)r=mid-1;
      else l=mid+1;
    }
    return 0;
  }
  vector<T> getA()const{//凸包邊對x軸的夾角
    vector<T>res;//一定是遞增的
    for(size_t i=0;i<p.size();++i)
      res.push_back((p[(i+1)%p.size()]-p[i]).getA());
    return res;
  }
  bool line_intersect(const vector<T>&A,const line<T> &l)const{//O(LogN)
    int f1=upper_bound(A.begin(),A.end(),(l.p1-l.p2).getA())-A.begin();
    int f2=upper_bound(A.begin(),A.end(),(l.p2-l.p1).getA())-A.begin();
    return l.cross_seg(line<T>(p[f1],p[f2]));
  }
  polygon cut(const line<T> &l)const{//凸包對直線切割，得到直線l左側的凸包
    polygon ans;
    for(int n=p.size(),i=n-1,j=0;j<n;i=j++){
      if(l.cross(p[i])>=0){
        ans.p.push_back(p[i]);
        if(l.cross(p[j])<0)
          ans.p.push_back(l.line_intersection(line<T>(p[i],p[j])));
      }else if(l.cross(p[j])>0)
        ans.p.push_back(l.line_intersection(line<T>(p[i],p[j])));
    }
    return ans;
  }
  static bool graham_cmp(const point<T>& a,const point<T>& b){
    return (a.x<b.x)||(a.x==b.x&&a.y<b.y);//凸包排序函數
  }
  void graham(vector<point<T> > &s){//凸包
    sort(s.begin(),s.end(),graham_cmp);
    p.resize(s.size()+1);
    int m=0;
    for(int i=0;i<(int)s.size();++i){
      while(m>=2&&(p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0)--m;
      p[m++]=s[i];
    }
    for(int i=s.size()-2,t=m+1;i>=0;--i){
      while(m>=t&&(p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0)--m;
      p[m++]=s[i];
    }
    if(s.size()>1)--m;
    p.resize(m);
  }
  T diam(){//直徑
    int n=p.size(),t=1;
    T ans=0;p.push_back(p[0]);
    for(int i=0;i<n;i++){
      point<T> now=p[i+1]-p[i];
      while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i]))t=(t+1)%n;
      ans=max(ans,max((p[i]-p[t]).abs2(),(p[i+1]-p[t+1]).abs2()));
    }
    return p.pop_back(),ans;
  }
  T min_cover_rectangle(){//最小覆蓋矩形
    int n=p.size(),t=1,r=1,l;
    if(n<3)return 0;//也可以做最小周長矩形
    T ans=1e99;p.push_back(p[0]);
    for(int i=0;i<n;i++){
      point<T> now=p[i+1]-p[i];
      while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i]))t=(t+1)%n;
      while(now.dot(p[r+1]-p[i])>now.dot(p[r]-p[i]))r=(r+1)%n;
      if(!i)l=r;
      while(now.dot(p[l+1]-p[i])<=now.dot(p[l]-p[i]))l=(l+1)%n;
      T d=now.abs2();
      T tmp=now.cross(p[t]-p[i])*(now.dot(p[r]-p[i])-now.dot(p[l]-p[i]))/d;
      ans=min(ans,tmp);
    }
    return p.pop_back(),ans;
  }
  T max_triangle(){//最大內接三角形
    int n=p.size(),a=1,b=2;
    if(n<3)return 0;
    T ans=0,tmp;p.push_back(p[0]);
    for(int i=0;i<n;++i){
      while((p[a]-p[i]).cross(p[b+1]-p[i])>(tmp=(p[a]-p[i]).cross(p[b]-p[i])))b=(b+1)%n;
      ans=max(ans,tmp);
      while((p[a+1]-p[i]).cross(p[b]-p[i])>(tmp=(p[a]-p[i]).cross(p[b]-p[i])))a=(a+1)%n;
      ans=max(ans,tmp);
    }
    return p.pop_back(),ans/2;
  }
  T dis2(polygon &pl){//凸包最近距離平方
    vector<point<T> > &P=p,&Q=pl.p;
    int n=P.size(),m=Q.size(),l=0,r=0;
    for(int i=0;i<n;++i)if(P[i].y<P[l].y)l=i;
    for(int i=0;i<m;++i)if(Q[i].y<Q[r].y)r=i;
    P.push_back(P[0]),Q.push_back(Q[0]);
    T ans=1e99;
    for(int i=0;i<n;++i){
      while((P[l]-P[l+1]).cross(Q[r+1]-Q[r])<0)r=(r+1)%m;
      ans=min(ans,line<T>(P[l],P[l+1]).seg_dis2(line<T>(Q[r],Q[r+1])));
      l=(l+1)%n;
    }
    return P.pop_back(),Q.pop_back(),ans;
  }
  static char sign(const point<T>&t){
    return (t.y==0?t.x:t.y)<0;
  }
  static bool angle_cmp(const line<T>& A,const line<T>& B){
    point<T> a=A.p2-A.p1,b=B.p2-B.p1;
    return sign(a)<sign(b)||(sign(a)==sign(b)&&a.cross(b)>0);
  }
  int halfplane_intersection(vector<line<T> > &s){//半平面交
    sort(s.begin(),s.end(),angle_cmp);//線段左側為該線段半平面
    int L,R,n=s.size();
    vector<point<T> > px(n);
    vector<line<T> > q(n);
    q[L=R=0]=s[0];
    for(int i=1;i<n;++i){
      while(L<R&&s[i].cross(px[R-1])<=0)--R;
      while(L<R&&s[i].cross(px[L])<=0)++L;
      q[++R]=s[i];
      if(q[R].parallel(q[R-1])){
        --R;
        if(q[R].cross(s[i].p1)>0)q[R]=s[i];
      }
      if(L<R)px[R-1]=q[R-1].line_intersection(q[R]);
    }
    while(L<R&&q[L].cross(px[R-1])<=0)--R;
    p.clear();
    if(R-L<=1)return 0;
    px[R]=q[R].line_intersection(q[L]);
    for(int i=L;i<=R;++i)p.push_back(px[i]);
    return R-L+1;
  }
};
template<typename T>
struct triangle{
  point<T> a,b,c;
  triangle(){}
  triangle(const point<T> &a,const point<T> &b,const point<T> &c):a(a),b(b),c(c){}
  T area()const{
    T t=(b-a).cross(c-a)/2;
    return t>0?t:-t;
  }
  point<T> barycenter()const{//重心
    return (a+b+c)/3;
  }
  point<T> circumcenter()const{//外心
    static line<T> u,v;
    u.p1=(a+b)/2;
    u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-b.x);
    v.p1=(a+c)/2;
    v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-c.x);
    return u.line_intersection(v);
  }
  point<T> incenter()const{//內心
    T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2()),C=sqrt((a-b).abs2());
    return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+B*b.y+C*c.y)/(A+B+C);
  }
  point<T> perpencenter()const{//垂心
    return barycenter()*3-circumcenter()*2;
  }
};
template<typename T>
struct point3D{
  T x,y,z;
  point3D(){}
  point3D(const T&x,const T&y,const T&z):x(x),y(y),z(z){}
  point3D operator+(const point3D &b)const{
    return point3D(x+b.x,y+b.y,z+b.z);}
  point3D operator-(const point3D &b)const{
    return point3D(x-b.x,y-b.y,z-b.z);}
  point3D operator*(const T &b)const{
    return point3D(x*b,y*b,z*b);}
  point3D operator/(const T &b)const{
    return point3D(x/b,y/b,z/b);}
  bool operator==(const point3D &b)const{
    return x==b.x&&y==b.y&&z==b.z;}
  T dot(const point3D &b)const{
    return x*b.x+y*b.y+z*b.z;}
  point3D cross(const point3D &b)const{
    return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);}
  T abs2()const{//向量長度的平方
    return dot(*this);}
  T area2(const point3D &b)const{//和b、原點圍成面積的平方
    return cross(b).abs2()/4;}
};
template<typename T>
struct line3D{
  point3D<T> p1,p2;
  line3D(){}
  line3D(const point3D<T> &p1,const point3D<T> &p2):p1(p1),p2(p2){}
  T dis2(const point3D<T> &p,bool is_segment=0)const{//點跟直線/線段的距離平方
    point3D<T> v=p2-p1,v1=p-p1;
    if(is_segment){
      point3D<T> v2=p-p2;
      if(v.dot(v1)<=0)return v1.abs2();
      if(v.dot(v2)>=0)return v2.abs2();
    }
    point3D<T> tmp=v.cross(v1);
    return tmp.abs2()/v.abs2();
  }
  pair<point3D<T>,point3D<T> > closest_pair(const line3D<T> &l)const{
    point3D<T> v1=(p1-p2),v2=(l.p1-l.p2);
    point3D<T> N=v1.cross(v2),ab(p1-l.p1);
    //if(N.abs2()==0)return NULL;平行或重合
    T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//最近點對距離
    point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.cross(d2);
    T t1=((l.p1-p1).cross(d2)).dot(D)/D.abs2();
    T t2=((l.p1-p1).cross(d1)).dot(D)/D.abs2();
    return make_pair(p1+d1*t1,l.p1+d2*t2);
  }
  bool same_side(const point3D<T> &a,const point3D<T> &b)const{
    return (p2-p1).cross(a-p1).dot((p2-p1).cross(b-p1))>0;
  }
};
template<typename T>
struct plane{
  point3D<T> p0,n;//平面上的點和法向量
  plane(){}
  plane(const point3D<T> &p0,const point3D<T> &n):p0(p0),n(n){}
  T dis2(const point3D<T> &p)const{//點到平面距離的平方
    T tmp=(p-p0).dot(n);
    return tmp*tmp/n.abs2();
  }
  point3D<T> projection(const point3D<T> &p)const{
    return p-n*(p-p0).dot(n)/n.abs2();
  }
  point3D<T> line_intersection(const line3D<T> &l)const{
    T tmp=n.dot(l.p2-l.p1);//等於0表示平行或重合該平面
    return l.p1+(l.p2-l.p1)*(n.dot(p0-l.p1)/tmp);
  }
  line3D<T> plane_intersection(const plane &pl)const{
    point3D<T> e=n.cross(pl.n),v=n.cross(e);
    T tmp=pl.n.dot(v);//等於0表示平行或重合該平面
    point3D<T> q=p0+(v*(pl.n.dot(pl.p0-p0))/tmp);
    return line3D<T>(q,q+e);
  }
};
template<typename T>
struct triangle3D{
  point3D<T> a,b,c;
  triangle3D(){}
  triangle3D(const point3D<T> &a,const point3D<T> &b,const point3D<T> &c):a(a),b(b),c(c){}
  bool point_in(const point3D<T> &p)const{//點在該平面上的投影在三角形中
    return line3D<T>(b,c).same_side(p,a)&&line3D<T>(a,c).same_side(p,b)&&line3D<T>(a,b).same_side(p,c);
  }
};
template<typename T>
struct tetrahedron{//四面體
```

```cpp
  point3D<T> a,b,c,d;
  tetrahedron(){}
  tetrahedron(const point3D<T> &a,const point3D<T> &b,const point3D<T> &c,const point3D<T> &d):a(a),b(b),c(c),d(d){}
  T volume6()const{//體積的六倍
    return (d-a).dot((b-a).cross(c-a));
  }
  point3D<T> centroid()const{
    return (a+b+c+d)/4;
  }
  bool point_in(const point3D<T> &p)const{
    return triangle3D<T>(a,b,c).point_in(p)&&triangle3D<T>(c,d,a).point_in(p);
  }
};
template<typename T>
struct convexhull3D{
  static const int MAXN=105;
  struct face{
    int a,b,c;
    bool use;
    face(){}
    face(int a,int b,int c):a(a),b(b),c(c),use(1){}
  };
  vector<point3D<T> > pt;
  vector<face> fc;
  int fid[MAXN][MAXN];
  static bool point_cmp(const point3D<T> &a,const point3D<T> &b){
    return a.x<b.x||(a.x==b.x&&(a.y<b.y||(a.y==b.y&&a.z<b.z)));
  }
  bool outside(int p,int a,int b,int c)const{
    return tetrahedron<T>(pt[a],pt[b],pt[c],pt[p]).volume6()<0;
  }
  bool outside(int p,int f)const{return outside(p,fc[f].a,fc[f].b,fc[f].c);}
  void AddFace(int a,int b,int c,int p){
    if(outside(p,a,b,c))fid[c][b]=fid[b][a]=fid[a][c]=fc.size(),fc.push_back(face(c,b,a));
    else fid[a][b]=fid[b][c]=fid[c][a]=fc.size(),fc.push_back(face(a,b,c));
  }
  bool dfs(int p,int f){
    if(!fc[f].use)return true;
    if(outside(p,f)){
      int a=fc[f].a,b=fc[f].b,c=fc[f].c;
      fc[f].use=false;
      if(!dfs(p,fid[b][a]))AddFace(p,a,b,c);
      if(!dfs(p,fid[c][b]))AddFace(p,b,c,a);
      if(!dfs(p,fid[a][c]))AddFace(p,c,a,b);
      return true;
    }else return false;
  }
  void build(){
    bool ok=false;
    fc.clear();
    sort(pt.begin(),pt.end(),point_cmp);
    pt.resize(unique(pt.begin(),pt.end())-pt.begin());
    for(size_t i=2;i<pt.size();++i){
      if((pt[0]-pt[i]).area2(pt[1]-pt[i])!=0){
        ok=true;
        swap(pt[i],pt[2]);
        break;
      }
    }
    if(!ok)return;
    ok=false;
    for(size_t i=3;i<pt.size();++i){
      if(tetrahedron<T>(pt[0],pt[1],pt[2],pt[i]).volume6()!=0){
        ok=true;
        swap(pt[i],pt[3]);
        break;
      }
    }
    if(!ok)return;
    for(int i=0;i<4;++i)AddFace(i,(i+1)%4,(i+2)%4,(i+3)%4);
    for(size_t i=4;i<pt.size();++i){
      for(int j=fc.size()-1;j>=0;--j){
        if(outside(i,j)){
          dfs(i,j);
          break;
        }
      }
    }
    size_t sz=0;
    for(size_t i=0;i<fc.size();++i)if(fc[i].use)fc[sz++]=fc[i];
    fc.resize(sz);
  }
  point3D<T> centroid()const{
    point3D<T> res(0,0,0);
    T vol=0;
    for(size_t i=0;i<fc.size();++i){
      T tmp=pt[fc[i].a].dot(pt[fc[i].b].cross(pt[fc[i].c]));
      res=res+(pt[fc[i].a]+pt[fc[i].b]+pt[fc[i].c])*tmp;
      vol+=tmp;
    }
    return res/(vol*4);
  }
};
```

1.2 SmallestCircle.cpp

```cpp
#include"Geometry.cpp"
struct Circle{
    typedef point<double> p;
    typedef const point<double> cp;
    p x;
    double r2;
    bool incircle(cp &c)const{return (x-c).abs2()<=r2;}
};

Circle TwoPointCircle(Circle::cp &a, Circle::cp &b) {
    Circle::p m=(a+b)/2;
    return (Circle){m,(a-m).abs2()};
}

Circle outcircle(Circle::p a, Circle::p b, Circle::p c) {
    if(TwoPointCircle(a,b).incircle(c)) return TwoPointCircle(a,b);
    if(TwoPointCircle(b,c).incircle(a)) return TwoPointCircle(b,c);
    if(TwoPointCircle(c,a).incircle(b)) return TwoPointCircle(c,a);
    Circle::p ret;
    double a1=b.x-a.x, b1=b.y-a.y, c1=(a1*a1+b1*b1)/2;
    double a2=c.x-a.x, b2=c.y-a.y, c2=(a2*a2+b2*b2)/2;
    double d = a1*b2 - a2*b1;
    ret.x=a.x+(c1*b2-c2*b1)/d;
    ret.y=a.y+(a1*c2-a2*c1)/d;
    return (Circle){ret,(ret-a).abs2()};
}
//rand required
Circle SmallestCircle(std::vector<Circle::p> &p){
    int n=p.size();
    if(n==1) return (Circle){p[0],0.0};
    if(n==2) return TwoPointCircle(p[0],p[1]);
    random_shuffle(p.begin(),p.end());
    Circle c = {p[0],0.0};
    for(int i=0;i<n;++i){
        if(c.incircle(p[i])) continue;
        c=Circle{p[i],0.0};
        for(int j=0;j<i;++j){
            if(c.incircle(p[j])) continue;
            c=TwoPointCircle(p[i],p[j]);
            for(int k=0;k<j;++k){
                if(c.incircle(p[k])) continue;
                c=outcircle(p[i],p[j],p[k]);
            }
        }
    }
    return c;
}
```

1.3 最近點對.cpp

```cpp
#define INF LLONG_MAX/*預設是long long最大值*/
template<typename T>
T closest_pair(vector<point<T> >&v,vector<point<T> >&t,int l,int r){
  T dis=INF,tmd;
  if(l>=r)return dis;
  int mid=(l+r)/2;
  if((tmd=closest_pair(v,t,l,mid))<dis)dis=tmd;
  if((tmd=closest_pair(v,t,mid+1,r))<dis)dis=tmd;
  t.clear();
  for(int i=l;i<=r;++i)
    if((v[i].x-v[mid].x)*(v[i].x-v[mid].x)<dis)t.push_back(v[i]);
  sort(t.begin(),t.end(),point<T>::y_cmp);/*如果用merge_sort的方式可以O(n)*/
  for(int i=0;i<(int)t.size();++i)
    for(int j=1;j<=3&&i+j<(int)t.size();++j)
      if((tmd=(t[i]-t[i+j]).abs2())<dis)dis=tmd;
  return dis;
}
template<typename T>
inline T closest_pair(vector<point<T> > &v){
  vector<point<T> >t;
  sort(v.begin(),v.end(),point<T>::x_cmp);
  return closest_pair(v,t,0,v.size()-1);/*最近點對距離*/
}
```

1.4 浮點數誤差模板.cpp

```cpp
const double EPS=1e-9;
struct Double{
  double d;
  Double(double d=0):d(d){}
  bool operator <(const Double &b)const{return d-b.d<-EPS;}
  bool operator >(const Double &b)const{return d-b.d>EPS;}
  bool operator ==(const Double &b)const{return fabs(d-b.d)<=EPS;}
  bool operator !=(const Double &b)const{return fabs(d-b.d)>EPS;}
  bool operator <=(const Double &b)const{return d-b.d<=EPS;}
  bool operator >=(const Double &b)const{return d-b.d>=-EPS;}
  operator double()const{return d;}
};
```

2 Data_Structure

2.1 DLX.cpp

```cpp
#define MAXN 4100
#define MAXM 1030
#define MAXND 16390
struct DLX{
  int n,m,sz,ansd;//高是n，寬是m的稀疏矩陣
  int S[MAXM],H[MAXN];
  int row[MAXND],col[MAXND];//每個節點代表的列跟行
  int L[MAXND],R[MAXND],U[MAXND],D[MAXND];
  vector<int> ans,anst;
  void init(int _n,int _m){
    n=_n,m=_m;
    for(int i=0;i<=m;++i){
      U[i]=D[i]=i,L[i]=i-1,R[i]=i+1;
      S[i]=0;
    }
    R[m]=0,L[0]=m;
    sz=m,ansd=INT_MAX;//ansd存最優解的個數
    for(int i=1;i<=n;++i)H[i]=-1;
  }
  void add(int r,int c){
    ++S[col[++sz]=c];
    row[sz]=r;
    D[sz]=D[c],U[D[c]]=sz,U[sz]=c,D[c]=sz;
    if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
    else R[sz]=R[H[r]],L[R[H[r]]]=sz,L[sz]=H[r],R[H[r]]=sz;
  }
  #define DFOR(i,A,s) for(int i=A[s];i!=s;i=A[i])
  void remove(int c){//刪除第c行和所有當前覆蓋到第c行的列
    L[R[c]]=L[c],R[L[c]]=R[c];//這裡刪除第c行，若有些行不需要處理可以在開始時呼叫他
    DFOR(i,D,c)DFOR(j,R,i){U[D[j]]=U[j],D[U[j]]=D[j],--S[col[j]];}
  }
  void restore(int c){//恢復第c行和所有當前覆蓋到第c行的列，remove的逆操作
    DFOR(i,U,c)DFOR(j,L,i){++S[col[j]],U[D[j]]=j,D[U[j]]=j;}
    L[R[c]]=c,R[L[c]]=c;
  }
  void remove2(int nd){//刪除nd所在的行當前所有點(包括虛擬節點)，只保留nd
    DFOR(i,D,nd)L[R[i]]=L[i],R[L[i]]=R[i];
  }
  void restore2(int nd){//刪除nd所在的行當前所有點，為remove2的逆操作
    DFOR(i,U,nd)L[R[i]]=R[L[i]]=i;
  }
  bool vis[MAXM];
  int h(){//估價函數 for IDA*
    int res=0;
    memset(vis,0,sizeof(vis));
    DFOR(i,R,0)if(!vis[i]){
      vis[i]=1;
      ++res;
      DFOR(j,D,i)DFOR(k,R,j)vis[col[k]]=1;
    }
    return res;
  }
  bool dfs(int d){//for精確覆蓋問題
    if(d+h()>=ansd)return 0;//找最佳解用，找任意解可以刪掉
    if(!R[0]){ansd=d;return 1;}
    int c=R[0];
    DFOR(i,R,0)if(S[i]<S[c])c=i;
    remove(c);
    DFOR(i,D,c){
      ans.push_back(row[i]);
      DFOR(j,R,i)remove(col[j]);
      if(dfs(d+1))return 1;
      ans.pop_back();
      DFOR(j,L,i)restore(col[j]);
    }
    restore(c);
    return 0;
  }
  void dfs2(int d){//for最小重複覆蓋問題
    if(d+h()>=ansd)return;
    if(!R[0]){ansd=d;ans=anst;return;}
    int c=R[0];
    DFOR(i,R,0)if(S[i]<S[c])c=i;
    DFOR(i,D,c){
      anst.push_back(row[i]);
      remove2(i);
      DFOR(j,R,i)remove2(j),--S[col[j]];
      dfs2(d+1);
      anst.pop_back();
      DFOR(j,L,i)restore2(j),++S[col[j]];
      restore2(i);
    }
  }
  bool exact_cover(){//解精確覆蓋問題
    ans.clear();//答案
    return dfs(0);
  }
  void min_cover(){//解最小重複覆蓋問題
    anst.clear();//暫存用，答案還是存在ans裡
    dfs2(0);
  }
  #undef DFOR
};
```

2.2 Dynamic_KD_tree.cpp

```cpp
template<typename T,size_t kd>//有kd個維度
class kd_tree{
  public:
    struct point{
      T d[kd];
      T dist(const point &x)const{
        T ret=0;
        for(size_t i=0;i<kd;++i)ret+=std::abs(d[i]-x.d[i]);
        return ret;
      }
      bool operator==(const point &p){
        for(size_t i=0;i<kd;++i)
          if(d[i]!=p.d[i])return 0;
        return 1;
      }
      bool operator<(const point &b)const{
        return d[0]<b.d[0];
      }
    };
  private:
    struct node{
      node *l,*r;
      point pid;
      int s;
      node(const point &p):l(0),r(0),pid(p),s(1){}
      ~node(){delete l,delete r;}
      void up(){s=(l?l->s:0)+1+(r?r->s:0);}
    }*root;
    const double alpha,loga;
    const T INF;//記得要給INF，表示極大值
    int maxn;
    struct __cmp{
      int sort_id;
      bool operator()(const node*x,const node*y)const{
        return operator()(x->pid,y->pid);
      }
      bool operator()(const point &x,const point &y)const{
        if(x.d[sort_id]!=y.d[sort_id])
          return x.d[sort_id]<y.d[sort_id];
        for(size_t i=0;i<kd;++i)
          if(x.d[i]!=y.d[i])return x.d[i]<y.d[i];
        return 0;
      }
    }cmp;
    int size(node *o){return o?o->s:0;}
    std::vector<node*> A;
    node* build(int k,int l,int r){
      if(l>r)return 0;
      if(k==kd)k=0;
      int mid=(l+r)/2;
      cmp.sort_id=k;
      std::nth_element(A.begin()+l,A.begin()+mid,A.begin()+r+1,cmp);
      node *ret=A[mid];
      ret->l=build(k+1,l,mid-1);
      ret->r=build(k+1,mid+1,r);
      ret->up();
      return ret;
    }
    bool isbad(node*o){
      return size(o->l)>alpha*o->s||size(o->r)>alpha*o->s;
    }
    void flatten(node *u,typename std::vector<node*>::iterator &it){
      if(!u)return;
      flatten(u->l,it);
      *it=u;
      flatten(u->r,++it);
    }
    void rebuild(node*&u,int k){
      if((int)A.size()<u->s)A.resize(u->s);
      typename std::vector<node*>::iterator it=A.begin();
      flatten(u,it);
      u=build(k,0,u->s-1);
    }
    bool insert(node*&u,int k,const point &x,int dep){
      if(!u){
        u=new node(x);
        return dep<=0;
      }
      ++u->s;
      cmp.sort_id=k;
      if(insert(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x,dep-1)){
        if(!isbad(u))return 1;
        rebuild(u,k);
      }
      return 0;
    }
    node *findmin(node*o,int k){
      if(!o)return 0;
      if(cmp.sort_id==k)return o->l?findmin(o->l,(k+1)%kd):o;
      node *l=findmin(o->l,(k+1)%kd);
      node *r=findmin(o->r,(k+1)%kd);
      if(l&&!r)return cmp(l,o)?l:o;
      if(!l&&r)return cmp(r,o)?r:o;
      if(!l&&!r)return o;
      if(cmp(l,r))return cmp(l,o)?l:o;
      return cmp(r,o)?r:o;
    }
    bool erase(node *&u,int k,const point &x){
      if(!u)return 0;
      if(u->pid==x){
        if(u->r);
        else if(u->l){
          u->r=u->l;
          u->l=0;
        }else{
          delete u;
          u=0;
          return 1;
        }
        --u->s;
        cmp.sort_id=k;
        u->pid=findmin(u->r,(k+1)%kd)->pid;
        return erase(u->r,(k+1)%kd,u->pid);
      }
      cmp.sort_id=k;
      if(erase(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x)){
        --u->s;return 1;
      }else return 0;
    }
    T heuristic(const T h[])const{
      T ret=0;
      for(size_t i=0;i<kd;++i)ret+=h[i];
      return ret;
    }
    int qM;
    std::priority_queue<std::pair<T,point > >pQ;
    void nearest(node *u,int k,const point &x,T *h,T &mndist){
      if(u==0||heuristic(h)>=mndist)return;
      T dist=u->pid.dist(x),old=h[k];
      /*mndist=std::min(mndist,dist);*/
      if(dist<mndist){
        pQ.push(std::make_pair(dist,u->pid));
        if((int)pQ.size()==qM+1)
          mndist=pQ.top().first,pQ.pop();
      }
      if(x.d[k]<u->pid.d[k]){
        nearest(u->l,(k+1)%kd,x,h,mndist);
        h[k]=std::abs(x.d[k]-u->pid.d[k]);
        nearest(u->r,(k+1)%kd,x,h,mndist);
      }else{
        nearest(u->r,(k+1)%kd,x,h,mndist);
        h[k]=std::abs(x.d[k]-u->pid.d[k]);
        nearest(u->l,(k+1)%kd,x,h,mndist);
      }
      h[k]=old;
    }
    std::vector<point>in_range;
    void range(node *u,int k,const point&mi,const point&ma){
      if(!u)return;
      bool is=1;
      for(int i=0;i<kd;++i)
        if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid.d[i]){
          is=0;break;
        }
      if(is)in_range.push_back(u->pid);
      if(mi.d[k]<=u->pid.d[k])range(u->l,(k+1)%kd,mi,ma);
      if(ma.d[k]>=u->pid.d[k])range(u->r,(k+1)%kd,mi,ma);
    }
  public:
    kd_tree(const T &INF,double a=0.75):root(0),alpha(a),loga(log2(1.0/a)),INF(INF),maxn(1){}
    ~kd_tree(){delete root;}
    void clear(){delete root,root=0,maxn=1;}
    void build(int n,const point *p){
      delete root,A.resize(maxn=n);
      for(int i=0;i<n;++i)A[i]=new node(p[i]);
      root=build(0,0,n-1);
    }
    void insert(const point &x){
      insert(root,0,x,__lg(size(root))/loga);
      if(root->s>maxn)maxn=root->s;
    }
    bool erase(const point &p){
      bool d=erase(root,0,p);
      if(root&&root->s<alpha*maxn)rebuild();
      return d;
    }
    void rebuild(){
      if(root)rebuild(root,0);
      maxn=root->s;
    }
    T nearest(const point &x,int k){
      qM=k;
      T mndist=INF,h[kd]={};
      nearest(root,0,x,h,mndist);
      mndist=pQ.top().first;
      pQ=std::priority_queue<std::pair<T,point > >();
      return mndist;//回傳離x第k近的點的距離
    }
    const std::vector<point> &range(const point&mi,const point&ma){
      in_range.clear();
      range(root,0,mi,ma);
      return in_range;//回傳介於mi到ma之間的點vector
    }
    int size(){return root?root->s:0;}
};
```

2.3 kd_tree_replace_segment_t

```cpp
/*kd樹代替高維線段樹*/
struct node{
  node *l,*r;
  point pid,mi,ma;
  int s;
  int data;
  node(const point &p,int d):l(0),r(0),pid(p),mi(p),ma(p),s(1),data(d),dmin(d),dmax(d){}
  void up(){
    mi=ma=pid;
    s=1;
    if(l){
      for(int i=0;i<kd;++i){
        mi.d[i]=min(mi.d[i],l->mi.d[i]);
        ma.d[i]=max(ma.d[i],l->ma.d[i]);
      }
      s+=l->s;
    }
    if(r){
      for(int i=0;i<kd;++i){
        mi.d[i]=min(mi.d[i],r->mi.d[i]);
        ma.d[i]=max(ma.d[i],r->ma.d[i]);
      }
      s+=r->s;
    }
  }
  void up2(){
    //其他懶惰標記向上更新
  }
  void down(){
    //其他懶惰標記下推
  }
}*root;

/*檢查區間包含用的函數*/
inline bool range_include(node *o,const point &L,const point &R){
  for(int i=0;i<kd;++i){
    if(L.d[i]>o->ma.d[i]||R.d[i]<o->mi.d[i])return 0;
  }//只要(L,R)區間有和o的區間有交集就回傳true
  return 1;
}
inline bool range_in_range(node *o,const point &L,const point &R){
  for(int i=0;i<kd;++i){
    if(L.d[i]>o->mi.d[i]||o->ma.d[i]>R.d[i])return 0;
  }//如果(L,R)區間完全包含o的區間就回傳true
  return 1;
}
inline bool point_in_range(node *o,const point &L,const point &R){
  for(int i=0;i<kd;++i){
    if(L.d[i]>o->pid.d[i]||R.d[i]<o->pid.d[i])return 0;
  }//如果(L,R)區間完全包含o->pid這個點就回傳true
  return 1;
}

/*單點修改,以單點改值為例*/
void update(node *u,const point &x,int data,int k=0){
  if(!u)return;
  u->down();
  if(u->pid==x){
    u->data=data;
    u->up2();
    return;
  }
  cmp.sort_id=k;
  update(cmp(x,u->pid)?u->l:u->r,x,data,(k+1)%kd);
  u->up2();
}

/*區間修改*/
void update(node *o,const point &L,const point &R,int data){
  if(!o)return;
  o->down();
  if(range_in_range(o,L,R)){
    //區間懶惰標記修改
    o->down();
    return;
  }
  if(point_in_range(o,L,R)){
    //這個點在(L,R)區間,但是他的左右子樹不一定在區間中
    //單點懶惰標記修改
  }
  if(o->l&&range_include(o->l,L,R))update(o->l,L,R,data);
  if(o->r&&range_include(o->r,L,R))update(o->r,L,R,data);
  o->up2();
}

/*區間查詢,以總和為例*/
int query(node *o,const point &L,const point &R){
  if(!o)return 0;
  o->down();
  if(range_in_range(o,L,R))return o->sum;
  int ans=0;
  if(point_in_range(o,L,R))ans+=o->data;
  if(o->l&&range_include(o->l,L,R))ans+=query(o->l,L,R);
  if(o->r&&range_include(o->r,L,R))ans+=query(o->r,L,R);
  return ans;
}
```

2.4 persistent_segment_tree.cpp

```cpp
#include<bits/stdc++.h>//POJ 2104
using namespace std;
struct node{
  int l,r;
  int data;
  node(int l,int r,int d):l(l),r(r),data(d){}
};
vector<node> nds;
inline void up(int o,int l,int r){
  nds[o].data=nds[l].data+nds[r].data;
}
inline int new_node(int l,int r,int d){
  nds.push_back(node(l,r,d));
  return nds.size()-1;
}
inline int new_node(const node &nd){
  nds.push_back(nd);
  return nds.size()-1;
}
int build_tree(int l,int r){
  int nd=new_node(-1,-1,0);
  if(l==r)return nd;
  int mid=(l+r)/2;
  int L=build_tree(l,mid);//執行時vector會被重構
  int R=build_tree(mid+1,r);//一定要這樣寫
  nds[nd].l=L;
  nds[nd].r=R;
  //up(nd,L,R);
  return nd;
}
int insert(int l,int r,int rt,int x,int d){
  if(x<l||r<x)return rt;
  int nd=new_node(nds[rt]);
  if(l==r&&l==x)nds[nd].data+=d;
  else{
    int mid=(l+r)/2;
    int L=insert(l,mid,nds[nd].l,x,d);
    int R=insert(mid+1,r,nds[nd].r,x,d);
    nds[nd].l=L;
    nds[nd].r=R;
    up(nd,L,R);
  }
  return nd;
}
inline int cal(int L,int R){
  return nds[R].data-nds[L].data;
}
int find(int l,int r,int L,int R,int k){
  if(l==r)return l;
  int mid=(l+r)/2;
  int add=cal(nds[L].l,nds[R].l);
  if(k<=add)return find(l,mid,nds[L].l,nds[R].l,k);
  return find(mid+1,r,nds[L].r,nds[R].r,k-add);
}
int n,m;
int s[100005];
int root[100005];
int main(){
  while(~scanf("%d%d",&n,&m)){
    nds.clear();
    vector<int> lsh;
    for(int i=1;i<=n;++i){
      scanf("%d",&s[i]);
      lsh.push_back(s[i]);
    }
    sort(lsh.begin(),lsh.end());
    lsh.resize(unique(lsh.begin(),lsh.end())
         -lsh.begin());
    int N=(int)lsh.size()-1;
    root[0]=build_tree(0,N);
    for(int i=1;i<=n;++i){
      s[i]=lower_bound(lsh.begin(),lsh.end()
          ,s[i])-lsh.begin();
      root[i]=insert(0,N,root[i-1],s[i],1);
    }
    while(m--){
      int a,b,k;
      scanf("%d%d%d",&a,&b,&k);
      int res=find(0,N,root[a-1],root[b],k);
      printf("%d\n",lsh[res]);
    }
  }
  return 0;
}
```

2.5 reference_point.cpp

```cpp
#include<bits/stdc++.h>
using namespace std;
template<typename T>
struct _RefCounter{
  T data;
  int ref;
  _RefCounter(const T&d=0):data(d),ref(0){}
};
template<typename T>
struct ref_pointer{
  _RefCounter<T> *p;
  T *operator->(){return &(*p).data;}
  T &operator*(){return p->data;}
  operator int(){return(int)(long long)p;}
  ref_pointer&operator=(const ref_pointer &t
      ){
    if(p&&--(*p).ref==0)delete p;
    p=t.p;
    p&&++(*p).ref;
    return*this;
  }
  ref_pointer(_RefCounter<T> *t=0):p(t){
    p&&++(*p).ref;
  }
  ref_pointer(const ref_pointer &t):p(t.p){
    p&&++(*p).ref;
  }
  ~ref_pointer(){
    if(p&&--(*p).ref==0)delete p;
  }
};
template<typename T>
inline const ref_pointer<T> new_ref(const T&
    nd){
  return ref_pointer<T>(new _RefCounter<T>(
      nd));
}
struct P{
  int a,b;
  P(int A,int B):a(A),b(B){}
}p(2,3);
int main(){
  ref_pointer<int>b=new_ref(int(5));
  ref_pointer<int>a=new_ref(*b);
  ref_pointer<P>c=new_ref(p);
  return 0;
}
```

2.6 skew_heap.cpp

```cpp
node *merge(node *a,node *b){
  if(!a||!b)return a?a:b;
  if(b->data<a->data)swap(a,b);
  swap(a->l,a->r);
  a->l=merge(b,a->l);
  return a;
}
```

2.7 split_merge.cpp

```cpp
void split(node *o,node *&a,node *&b,int k){
  if(!o)a=b=0;
  else{
    //o=new node(*o);
    o->down();
    if(k<=size(o->l)){
      b=o;
      split(o->l,a,b->l,k);
    }else{
      a=o;
      split(o->r,a->r,b,k-size(o->l)-1);
    }
    o->up();
  }
}
node *merge(node *a,node *b){
  if(!a||!b)return a?a:b;
  static int x;
  if(x++%(a->s+b->s)<a->s){
    //a=new node(*a);
    a->down();
    a->r=merge(a->r,b);
    a->up();
    return a;
  }else{
    //b=new node(*b);
    b->down();
    b->l=merge(a,b->l);
    b->up();
    return b;
  }
}
```

2.8 treap.cpp

```cpp
template<typename T>
class treap{
  private:
  struct node{
    T data;
    unsigned fix;
    int s;
    node *ch[2];
    node(const T&d):data(d),s(1){}
    node():s(0){ch[0]=ch[1]=this;}
  }*nil,*root;
  unsigned x;
  unsigned ran(){return x=x*0xdefaced+1;}
  void rotate(node *&a,bool d){
    node *b=a;
    a=a->ch[!d];
    a->s=b->s;
    b->ch[!d]=a->ch[d];
    a->ch[d]=b;
    b->s=b->ch[0]->s+b->ch[1]->s+1;
  }
  void insert(node *&o,const T &data){
    if(!o->s){
      o=new node(data),o->fix=ran();
      o->ch[0]=o->ch[1]=nil;
    }else{
      o->s++;
      bool d=o->data<data;
      insert(o->ch[d],data);
      if(o->ch[d]->fix>o->fix)rotate(o,!d)
           ;
    }
  }
  node *merge(node *a,node *b){
    if(!a->s||!b->s)return a->s?a:b;
    if(a->fix>b->fix){
      a->ch[1]=merge(a->ch[1],b);
      a->s=a->ch[0]->s+a->ch[1]->s+1;
      return a;
    }else{
      b->ch[0]=merge(a,b->ch[0]);
      b->s=b->ch[0]->s+b->ch[1]->s+1;
      return b;
    }
  }
  bool erase(node *&o,const T &data){
    if(!o->s)return 0;
    if(o->data==data){
      node *t=o;
      o=merge(o->ch[0],o->ch[1]);
      delete t;
      return 1;
    }
    if(erase(o->ch[o->data<data],data)){
      o->s--;return 1;
    }else return 0;
  }
  void clear(node *&o){
    if(o->s)clear(o->ch[0]),clear(o->ch
         [1]),delete o;
  }
public:
  treap(unsigned s=20150119):nil(new node)
       ,root(nil),x(s){}
  ~treap(){clear(root),delete nil;}
  void clear(){clear(root),root=nil;}
  void insert(const T &data){
    insert(root,data);
  }
    bool erase(const T &data){
      return erase(root,data);
    }
    bool find(const T&data){
      for(node *o=root;o->s;)
      if(o->data==data)return 1;
      else o=o->ch[o->data<data];
      return 0;
    }
    int rank(const T&data){
      int cnt=0;
      for(node *o=root;o->s;)
      if(o->data<data)cnt+=o->ch[0]->s+1,o=o
          ->ch[1];
      else o=o->ch[0];
      return cnt;
    }
    const T&kth(int k){
      for(node *o=root;;)
      if(k<=o->ch[0]->s)o=o->ch[0];
      else if(k==o->ch[0]->s+1)return o->
          data;
      else k-=o->ch[0]->s+1,o=o->ch[1];
    }
    const T&operator[](int k){
      return kth(k);
    }
    const T&preorder(const T&data){
      node *x=root,*y=0;
      while(x->s)
      if(x->data<data)y=x,x=x->ch[1];
      else x=x->ch[0];
      if(y)return y->data;
      return data;
    }
    const T&successor(const T&data){
      node *x=root,*y=0;
      while(x->s)
      if(data<x->data)y=x,x=x->ch[0];
      else x=x->ch[1];
      if(y)return y->data;
      return data;
    }
    int size(){return root->s;}
};
```

2.9 操作分治.cpp

```cpp
void dq(int l,int r){
  if(l==r)return;
  int mid=(l+r)/2;
  dq(l,mid);
  處理[l,mid]的操作對[mid+1,r]的影響
  dq(mid+1,r);
}
```

2.10 整體二分.cpp

```cpp
void BS(int l,int r,vector<Item> &vs){
  //答案該<l會有的已經做完了
  if(l==r)整個vs的答案=l;//??????
  int mid=(l+r)/2;
  do_thing(l,mid);//做答案<=mid會做的事
  vector<Item> left=vs裡滿足的;
  vector<Item> right=vs-left;
  undo_thing(l,mid);
  BS(l,mid,left);
  do_thing(l,mid);
  BS(mid+1,r,right);//??????
}
```

3 default

3.1 debug.cpp

```
#ifdef DEBUG
#define dbg(...) {\
  fprintf(stderr,"%s - %d : (%s) = ",__PRETTY_FUNCTION__,__LINE__,#__VA_ARGS__);\
  _DO(__VA_ARGS__);\
}
template<typename I> void _DO(I&&x){cerr<<x<<endl;}
template<typename I,typename...T> void _DO(I&&x,T&&...tail){cerr<<x<<", ";_DO(tail...);}
#else
#define dbg(...)
#endif
```

3.2 ext.cpp

```
#include<bits/extc++.h>
#include<ext/pd_ds/assoc_container.hpp>
#include<ext/pd_ds/tree_policy.hpp>
using namespace  __gnu_cxx;
using namespace  __gnu_pbds;
template<typename T>
using pbds_set = tree<T,null_type,less<T>,rb_tree_tag,tree_order_statistics_node_update>;
template<typename T,typename U>
using pbds_map = tree<T,U,less<T>,rb_tree_tag,tree_order_statistics_node_update>;
using heap = __gnu_pbds::priority_queue<int>;
//s.find_by_order(1);//0 base
//s.order_of_key(1);
```

3.3 IncStack.cpp

```
//Magic
#pragma GCC optimize "Ofast"
//stack resize,change esp to rsp if 64-bit system
asm("mov %0,%%esp\n" ::"g"(mem+10000000));
//linux stack resize
#include<sys/resource.h>
void increase_stack(){
  const rlim_t ks=64*1024*1024;
  struct rlimit rl;
  int res=getrlimit(RLIMIT_STACK,&rl);
  if(!res&&rl.rlim_cur<ks){
    rl.rlim_cur=ks;
    res=setrlimit(RLIMIT_STACK,&rl);
  }
}
```

3.4 input.cpp

```
inline int read(){
  int x=0; bool f=0; char c=getchar();
  while(ch<'0'||'9'<ch)f|=ch=='-',ch=getchar();
  while('0'<=ch&&ch<='9')x=x*10-'0'+ch,ch=getchar();
  return f?-x:x;
}

// #!/bin/bash
// g++ -std=c++11 -O2 -Wall -Wextra -Wno-unused-result -DDEBUG $1 && ./a.out
//  -fsanitize=address -fsanitize=undefined -fsanitize=return
```

4 Flow

4.1 dinic.cpp

```
template<typename T>
struct DINIC{
  static const int MAXN=105;
  static const T INF=INT_MAX;
  int n;//點數
  int level[MAXN],cur[MAXN];
  struct edge{
    int v,pre;
    T cap,flow,r;
    edge(int v,int pre,T cap):v(v),pre(pre),cap(cap),flow(0),r(cap){}
  };
  int g[MAXN];
  vector<edge> e;
  void init(int _n){
    memset(g,-1,sizeof(int)*((n=_n)+1));
    e.clear();
  }
  void add_edge(int u,int v,T cap,bool directed=false){
    e.push_back(edge(v,g[u],cap));
    g[u]=e.size()-1;
    e.push_back(edge(u,g[v],directed?0:cap));
    g[v]=e.size()-1;
  }
  int bfs(int s,int t){
    memset(level,0,sizeof(int)*(n+1));
    memcpy(cur,g,sizeof(int)*(n+1));
    queue<int >q;
    q.push(s);
    level[s]=1;
    while(q.size()){
      int u=q.front();q.pop();
      for(int i=g[u];~i;i=e[i].pre){
        if(!level[e[i].v]&&e[i].r){
          level[e[i].v]=level[u]+1;
          q.push(e[i].v);
          if(e[i].v==t)return 1;
        }
      }
    }
    return 0;
  }
  T dfs(int u,int t,T cur_flow=INF){
    if(u==t)return cur_flow;
    T df;
    for(int &i=cur[u];~i;i=e[i].pre){
      if(level[e[i].v]==level[u]+1&&e[i].r){
        if(df=dfs(e[i].v,t,min(cur_flow,e[i].r))){
          e[i].flow+=df;
          e[i^1].flow-=df;
          e[i].r-=df;
          e[i^1].r+=df;
          return df;
        }
      }
    }
    return level[u]=0;
  }
  T dinic(int s,int t,bool clean=true){
    if(clean){
      for(size_t i=0;i<e.size();++i){
        e[i].flow=0;
        e[i].r=e[i].cap;
      }
    }
    T ans=0,mf=0;
    while(bfs(s,t))while(mf=dfs(s,t))ans+=mf;
    return ans;
  }
};
```

4.2 ISAP_with_cut.cpp

```
template<typename T>
struct ISAP{
  static const int MAXN=105;
  static const T INF=INT_MAX;
  int n;//點數
  int d[MAXN],gap[MAXN],cur[MAXN];
  struct edge{
    int v,pre;
    T cap,flow,r;
    edge(int v,int pre,T cap):v(v),pre(pre),cap(cap),flow(0),r(cap){}
  };
  int g[MAXN];
  vector<edge> e;
  void init(int _n){
    memset(g,-1,sizeof(int)*((n=_n)+1));
    e.clear();
  }
  void add_edge(int u,int v,T cap,bool directed=false){
    e.push_back(edge(v,g[u],cap));
    g[u]=e.size()-1;
    e.push_back(edge(u,g[v],directed?0:cap));
    g[v]=e.size()-1;
  }
  T dfs(int u,int s,int t,T cur_flow=INF){
    if(u==t)return cur_flow;
    T tf=cur_flow,df;
    for(int &i=cur[u];~i;i=e[i].pre){
      if(e[i].r&&d[u]==d[e[i].v]+1){
        df=dfs(e[i].v,s,t,min(tf,e[i].r));
        e[i].flow+=df;
        e[i^1].flow-=df;
        e[i].r-=df;
        e[i^1].r+=df;
        if(!(tf-=df)||d[s]==n)return cur_flow-tf;
      }
    }
    int mh=n;
    for(int i=cur[u]=g[u];~i;i=e[i].pre){
      if(e[i].r&&d[e[i].v]<mh)mh=d[e[i].v];
    }
    if(!--gap[d[u]])d[s]=n;
    else ++gap[d[u]=++mh];
    return cur_flow-tf;
  }
  T isap(int s,int t,bool clean=true){
    memset(d,0,sizeof(int)*(n+1));
    memset(gap,0,sizeof(int)*(n+1));
    memcpy(cur,g,sizeof(int)*(n+1));
    if(clean){
      for(size_t i=0;i<e.size();++i){
        e[i].flow=0;
        e[i].r=e[i].cap;
      }
    }
    T max_flow=0;
    for(gap[0]=n;d[s]<n;)max_flow+=dfs(s,s,t);
    return max_flow;
  }
  vector<int> cut_e;//最小割邊集
  bool vis[MAXN];
  void dfs_cut(int u){
    vis[u]=1;//表示u屬於source的最小割集
    for(int i=g[u];~i;i=e[i].pre){
      if(e[i].flow<e[i].cap&&!vis[e[i].v])dfs_cut(e[i].v);
    }
```

```cpp
  }
  T min_cut(int s,int t){
    T ans=isap(s,t);
    memset(vis,0,sizeof(bool)*(n+1));
    dfs_cut(s),cut_e.clear();
    for(int u=0;u<=n;++u){
      if(vis[u])for(int i=g[u];~i;i=e[i].pre){
        if(!vis[e[i].v])cut_e.push_back(i);
      }
    }
    return ans;
  }
};
```

4.3 MinCostMaxFlow.cpp

```cpp
template<typename _T>
struct MCMF{
  static const int MAXN=440;
  static const _T INF=999999999;
  struct edge{
    int v,pre;
    _T cap,cost;
    edge(int v,int pre,_T cap,_T cost):v(v),pre(pre),cap(cap),cost(cost){}
  };
  int n,S,T;
  _T dis[MAXN],piS,ans;
  bool vis[MAXN];
  vector<edge> e;
  int g[MAXN];
  void init(int _n){
    memset(g,-1,sizeof(int)*((n=_n)+1));
    e.clear();
  }
  void add_edge(int u,int v,_T cap,_T cost,bool directed=false){
    e.push_back(edge(v,g[u],cap,cost));
    g[u]=e.size()-1;
    e.push_back(edge(u,g[v],directed?0:cap,-cost));
    g[v]=e.size()-1;
  }
  _T augment(int u,_T cur_flow){
    if(u==T||!cur_flow)return ans+=piS*cur_flow,cur_flow;
    vis[u]=1;
    _T r=cur_flow,d;
    for(int i=g[u];~i;i=e[i].pre){
      if(e[i].cap&&!e[i].cost&&!vis[e[i].v]){
        d=augment(e[i].v,min(r,e[i].cap));
        e[i].cap-=d;
        e[i^1].cap+=d;
        if(!(r-=d))break;
      }
    }
    return cur_flow-r;
  }
  bool modlabel(){
    for(int u=0;u<=n;++u)dis[u]=INF;
    static deque<int>q;
    dis[T]=0,q.push_back(T);
    while(q.size()){
      int u=q.front();q.pop_front();
      _T dt;
      for(int i=g[u];~i;i=e[i].pre){
        if(e[i^1].cap&&(dt=dis[u]-e[i].cost)<dis[e[i].v]){
          if((dis[e[i].v]=dt)<=dis[q.size()?q.front():S]){
            q.push_front(e[i].v);
          }else q.push_back(e[i].v);
        }
      }
    }
    for(int u=0;u<=n;++u)
      for(int i=g[u];~i;i=e[i].pre)
        e[i].cost+=dis[e[i].v]-dis[u];
    piS+=dis[S];
    return dis[S]<INF;
  }
  _T mincost(int s,int t){
    S=s,T=t;
    piS=ans=0;
    while(modlabel()){
      do memset(vis,0,sizeof(bool)*(n+1));
      while(augment(S,INF));
    }
    return ans;
  }
};
```

5 Graph

5.1 Augmenting_Path.cpp

```cpp
#define MAXN1 505
#define MAXN2 505
int n1,n2;//n1個點連向n2個點
int match[MAXN2];//屬於n2的點匹配了哪個點
vector<int > g[MAXN1];//圖
bool vis[MAXN2];//是否走訪過
bool dfs(int u){
  for(size_t i=0;i<g[u].size();++i){
    int v=g[u][i];
    if(vis[v])continue;
    vis[v]=1;
    if(match[v]==-1||dfs(match[v])){
      match[v]=u;
      return 1;
    }
  }
  return 0;
}
inline int max_match(){
  int ans=0;
  memset(match,-1,sizeof(int)*n2);
  for(int i=0;i<n1;++i){
    memset(vis,0,sizeof(bool)*n2);
    if(dfs(i))++ans;
  }
  return ans;
}
```

5.2 Augmenting_Path_multiple.

```cpp
#define MAXN1 1005
#define MAXN2 505
int n1,n2;//n1個點連向n2個點，其中n2個點可以匹配很多邊
vector<int > g[MAXN1];//圖
int c[MAXN2];//每個屬於n2點最多可以接受幾條匹配邊
vector<int> match_list[MAXN2];//每個屬於n2的點匹配了那些點
bool vis[MAXN2];//是否走訪過
bool dfs(int u){
  for(size_t i=0;i<g[u].size();++i){
    int v=g[u][i];
    if(vis[v])continue;
    vis[v]=true;
    if((int)match_list[v].size()<c[v]){
      match_list[v].push_back(u);
      return true;
    }else{
      for(size_t j=0;j<match_list[v].size();++j){
        int next_u=match_list[v][j];
        if(dfs(next_u)){
          match_list[v][j]=u;
          return true;
        }
      }
    }
  }
  return false;
}
inline int max_match(){
  for(int i=0;i<n2;++i)match_list[i].clear();
  int cnt=0;
  for(int u=0;u<n1;++u){
    memset(vis,0,sizeof(bool)*n2);
    if(dfs(u))++cnt;
  }
  return cnt;
}
```

5.3 blossom_matching.cpp

```cpp
#define MAXN 505
vector<int>g[MAXN];
int pa[MAXN],match[MAXN],st[MAXN],S[MAXN],v[MAXN];
int t,n;
inline int lca(int x,int y){
  for(++t;;swap(x,y)){
    if(x==0)continue;
    if(v[x]==t)return x;
    v[x]=t;
    x=st[pa[match[x]]];
  }
}
#define qpush(x) q.push(x),S[x]=0
inline void flower(int x,int y,int l,queue<int> &q){
  while(st[x]!=l){
    pa[x]=y;
    if(S[y=match[x]]==1)qpush(y);
    st[x]=st[y]=l,x=pa[y];
  }
}
inline bool bfs(int x){
  for(int i=1;i<=n;++i)st[i]=i;
  memset(S+1,-1,sizeof(int)*n);
  queue<int>q;qpush(x);
  while(q.size()){
    x=q.front(),q.pop();
    for(size_t i=0;i<g[x].size();++i){
      int y=g[x][i];
      if(S[y]==-1){
        pa[y]=x,S[y]=1;
        if(!match[y]){
          for(int lst;x;y=lst,x=pa[y])
            lst=match[x],match[x]=y,match[y]=x;
          return 1;
        }
        qpush(match[y]);
      }else if(!S[y]&&st[y]!=st[x]){
        int l=lca(y,x);
        flower(y,x,l,q),flower(x,y,l,q);
      }
    }
  }
  return 0;
}
inline int blossom(){
  int ans=0;
  for(int i=1;i<=n;++i)
    if(!match[i]&&bfs(i))++ans;
  return ans;
}
```

5.4 graphISO.cpp

```cpp
const int MAXN=1005,K=30;//K要夠大
const long long A=3,B=11,C=2,D=19,P=0xdefaced;
long long f[K+1][MAXN];
vector<int> g[MAXN],rg[MAXN];
int n;
inline void init(){
  for(int i=0;i<n;++i){
    f[0][i]=1;
    g[i].clear();
    rg[i].clear();
  }
}
inline void add_edge(int u,int v){
  g[u].push_back(v);
  rg[v].push_back(u);
}
inline long long point_hash(int u){//O(N)
  for(int t=1;t<=K;++t){
    for(int i=0;i<n;++i){
      f[t][i]=f[t-1][i]*A%P;
      for(int j:g[i])f[t][i]=(f[t][i]+f[t-1][j]*B%P)%P;
      for(int j:rg[i])f[t][i]=(f[t][i]+f[t-1][j]*C%P)%P;
      if(i==u)f[t][i]+=D;//如果圖太大的話，把這行刪掉，執行一次後f[K]就會是所有點的答案
      f[t][i]%=P;
    }
  }
  return f[K][u];
}
inline vector<long long> graph_hash(){
  vector<long long> ans;
  for(int i=0;i<n;++i)ans.push_back(point_hash(i));//O(N^2)
  sort(ans.begin(),ans.end());
  return ans;
}
```

5.5 KM.cpp

```
#define MAXN 405
#define INF 0x3f3f3f3f
int n;// 1-base，0表示沒有匹配
int g[MAXN][MAXN],lx[MAXN],ly[MAXN],pa[MAXN],slack_y[MAXN];
int match_y[MAXN],match_x[MAXN];
bool vx[MAXN],vy[MAXN];
void augment(int y){
  for(int x,z;y;y=z){
    x=pa[y],z=match_x[x];
    match_y[y]=x,match_x[x]=y;
  }
}
void bfs(int st){
  for(int i=1;i<=n;++i)slack_y[i]=INF,vx[i]=vy[i]=0;
  queue<int> q;q.push(st);
  for(;;){
    while(q.size()){
      int x=q.front();q.pop();
      vx[x]=1;
      for(int y=1;y<=n;++y)if(!vy[y]){
        int t=lx[x]+ly[y]-g[x][y];
        if(t==0){
          pa[y]=x;
          if(!match_y[y]){augment(y);return;}
          vy[y]=1,q.push(match_y[y]);
        }else if(slack_y[y]>t)pa[y]=x,slack_y[y]=t;
      }
    }
    int cut=INF;
    for(int y=1;y<=n;++y){
      if(!vy[y]&&cut>slack_y[y])cut=slack_y[y];
    }
    for(int j=1;j<=n;++j){
      if(vx[j])lx[j]-=cut;
      if(vy[j])ly[j]+=cut;
      else slack_y[j]-=cut;
    }
    for(int y=1;y<=n;++y){
      if(!vy[y]&&slack_y[y]==0){
        if(!match_y[y]){augment(y);return;}
        vy[y]=1,q.push(match_y[y]);
      }
    }
  }
}
long long KM(){
  memset(match_y,0,sizeof(int)*(n+1));
  memset(ly,0,sizeof(int)*(n+1));
  for(int x=1;x<=n;++x){
    lx[x]=-INF;
    for(int y=1;y<=n;++y)
      lx[x]=max(lx[x],g[x][y]);
  }
  for(int x=1;x<=n;++x)bfs(x);
  long long ans=0;
  for(int y=1;y<=n;++y)ans+=g[match_y[y]][y];
  return ans;
}
```

5.6 MaximumClique.cpp

```
struct MaxClique{
  static const int MAXN=105;
  int N,ans;
  int g[MAXN][MAXN],dp[MAXN],stk[MAXN][MAXN];
  int sol[MAXN],tmp[MAXN];//sol[0~ans-1]為答案
  void init(int n){
    N=n;//0-base
    memset(g,0,sizeof(g));
  }
  void add_edge(int u,int v){
    g[u][v]=g[v][u]=1;
  }
  int dfs(int ns,int dep){
    if(!ns){
      if(dep>ans){
        ans=dep;
        memcpy(sol,tmp,sizeof tmp);
        return 1;
      }else return 0;
    }
    for(int i=0;i<ns;++i){
      if(dep+ns-i<=ans)return 0;
      int u=stk[dep][i],cnt=0;
      if(dep+dp[u]<=ans)return 0;
      for(int j=i+1;j<ns;++j){
        int v=stk[dep][j];
        if(g[u][v])stk[dep+1][cnt++]=v;
      }
      tmp[dep]=u;
      if(dfs(cnt,dep+1))return 1;
    }
    return 0;
  }
  int clique(){
    int u,v,ns;
    for(ans=0,u=N-1;u>=0;--u){
      for(ns=0,tmp[0]=u,v=u+1;v<N;++v)
        if(g[u][v])stk[1][ns++]=v;
      dfs(ns,1),dp[u]=ans;
    }
    return ans;
  }
};
```

5.7 MinimumMeanCycle.cpp

```
#include<cstdint>//for DBL_MAX
int dp[maxN+1][maxN+1];
double mnc(int n){
    int u,v,w;
    const int inf=0x7f7f7f7f;
    memset(dp,0x7f,sizeof(dp));
    memset(dp[0],0,sizeof(dp[0]));
    for(int i=0;i<n;++i){
        for(auto e:E){//tuple<int,int,int> of u,v,w
            tie(u,v,w)=e;
            if(dp[i][u]!=inf)
                dp[i+1][v]=min(dp[i+1][v],dp[i][u]+w);
        }
        double res = DBL_MAX;
        for(int i=1;i<=n;++i){
            double val = DBL_MIN;
            for(int j=0;j<n;++j)
                val=max(val,double(dp[n][i]-dp[i][j])/(n-j));
            res=min(res,val);
        }
    }
    return res;
}
```

5.8 Minimum_General_Weighte

```
struct Graph {
  // Minimum General Weighted Matching (Perfect Match) 0-base
  static const int MXN = 105;

  int n, edge[MXN][MXN];
  int match[MXN],dis[MXN],onstk[MXN];
  vector<int> stk;

  void init(int _n) {
    n = _n;
    for (int i=0; i<n; i++)
      for (int j=0; j<n; j++)
        edge[i][j] = 0;
  }
  void add_edge(int u, int v, int w) {
    edge[u][v] = edge[v][u] = w;
  }
  bool SPFA(int u){
    if (onstk[u]) return true;
    stk.push_back(u);
    onstk[u] = 1;
    for (int v=0; v<n; v++){
      if (u != v && match[u] != v && !onstk[v]){
        int m = match[v];
        if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
          dis[m] = dis[u] - edge[v][m] + edge[u][v];
          onstk[v] = 1;
          stk.push_back(v);
          if (SPFA(m)) return true;
          stk.pop_back();
          onstk[v] = 0;
        }
      }
    }
    onstk[u] = 0;
    stk.pop_back();
    return false;
  }

  int solve() {
    // find a match
    for (int i=0; i<n; i+=2){
      match[i] = i+1;
      match[i+1] = i;
    }
    for(;;){
      int found = 0;
      for (int i=0; i<n; i++)
        dis[i] = onstk[i] = 0;
      for (int i=0; i<n; i++){
        stk.clear();
        if (!onstk[i] && SPFA(i)){
          found = 1;
          while (stk.size()>=2){
            int u = stk.back(); stk.pop_back();
            int v = stk.back(); stk.pop_back();
            match[u] = v;
            match[v] = u;
          }
        }
      }
      if (!found) break;
    }
    int ret = 0;
    for (int i=0; i<n; i++)
      ret += edge[i][match[i]];
    ret /= 2;
    return ret;
  }
}graph;
```

5.9 Rectilinear_Steiner_tree.cpp

```cpp
//平面曼哈頓最小生成樹構造圖(去除非必要邊)
#include<vector>
#include<algorithm>
#define T int
#define INF 0x3f3f3f3f
struct point{
  T x,y;
  int id;//每個點的編號都要不一樣，從0開始編號
  point(){}
  T dist(const point &p)const{
    return std::abs(x-p.x)+std::abs(y-p.y);
  }
};
inline bool cmpx(const point &a,const point &b){
  return a.x<b.x||(a.x==b.x&&a.y<b.y);
}
struct edge{
  int u,v;
  T cost;
  edge(int u,int v,const T&c):u(u),v(v),cost(c){}
  bool operator<(const edge&e)const{
    return cost<e.cost;
  }
};
struct bit_node{
  T mi;
  int id;
  bit_node(const T&mi=INF,int id=-1):mi(mi),id(id){}
};
std::vector<bit_node> bit;
inline void bit_update(int i,const T&data,int id){
  for(;i;i-=i&(-i)){
    if(data<bit[i].mi)bit[i]=bit_node(data,id);
  }
}
inline int bit_find(int i,int m){
  bit_node x;
  for(;i<=m;i+=i&(-i)){
    if(bit[i].mi<x.mi)x=bit[i];
  }
  return x.id;
}
inline std::vector<edge> build_graph(int n,point p[]){
  std::vector<edge> e;//回傳的邊就可以用來求最小生成樹
  for(int dir=0;dir<4;++dir){//4種座標變換
    if(dir%2){
      for(int i=0;i<n;++i)std::swap(p[i].x,p[i].y);
    }else if(dir==2){
      for(int i=0;i<n;++i)p[i].x=-p[i].x;
    }
    std::sort(p,p+n,cmpx);
    std::vector<T>ga(n),gb;
    for(int i=0;i<n;++i)ga[i]=p[i].y-p[i].x;
    gb=ga;
    std::sort(gb.begin(),gb.end());
    gb.resize(std::unique(gb.begin(),gb.end())-gb.begin());
    int m=gb.size();
    bit=std::vector<bit_node>(m+1);
    for(int i=n-1;i>=0;--i){
      int pos=std::lower_bound(gb.begin(),gb.end(),ga[i])-gb.begin()+1;
      int ans=bit_find(pos,m);
      if(~ans)e.push_back(edge(p[i].id,p[ans].id,p[i].dist(p[ans])));
      bit_update(pos,p[i].x+p[i].y,i);
    }
  }
  return e;
}
```

5.10 treeISO.cpp

```cpp
const int MAXN=100005;
const long long X=12327,P=0xdefaced;
vector<int> g[MAXN];
bool vis[MAXN];
long long dfs(int u){//hash ver
  vis[u]=1;
  vector<long long> tmp;
  for(auto v:g[u])if(!vis[v])tmp.push_back(dfs(v));
  if(tmp.empty())return 177;
  long long ret=4931;
  sort(tmp.begin(),tmp.end());
  for(auto v:tmp)ret=((ret*X)^v)%P;
  return ret;
}
//-----------------------------
string dfs(int x,int p){
  vector<string> c;
  for(int y:g[x])
    if(y!=p)c.emplace_back(dfs(y,x));
  sort(c.begin(),c.end());
  string ret("(");
  for(auto &s:c)ret+=s;
  ret+=")";
  return ret;
}
```

5.11 全局最小割.cpp

```cpp
const int INF=0x3f3f3f3f;
template<typename T>
struct stoer_wagner{// 0-base
  static const int MAXN=150;
  T g[MAXN][MAXN],dis[MAXN];
  int nd[MAXN],n,s,t;
  void init(int _n){
    n=_n;
    for(int i=0;i<n;++i)
      for(int j=0;j<n;++j)g[i][j]=0;
  }
  void add_edge(int u,int v,T w){
    g[u][v]=g[v][u]+=w;
  }
  T min_cut(){
    T ans=INF;
    for(int i=0;i<n;++i)nd[i]=i;
    for(int ind,tn=n;tn>1;--tn){
      for(int i=1;i<tn;++i)dis[nd[i]]=0;
      for(int i=1;i<tn;++i){
        ind=i;
        for(int j=i;j<tn;++j){
          dis[nd[j]]+=g[nd[i-1]][nd[j]];
          if(dis[nd[ind]]<dis[nd[j]])ind=j;
        }
        swap(nd[ind],nd[i]);
      }
      if(ans>dis[nd[ind]])ans=dis[t=nd[ind]],s=nd[ind-1];
      for(int i=0;i<tn;++i)
        g[nd[ind-1]][nd[i]]=g[nd[i]][nd[ind-1]]+=g[nd[i]][nd[ind]];
    }
    return ans;
  }
};
```

5.12 平面圖判定.cpp

```cpp
static const int MAXN = 20;
struct Edge{
  int u, v;
  Edge(int s, int d) : u(s), v(d) {}
};
bool isK33(int n, int degree[]){
  int t = 0, z = 0;
  for(int i=0;i<n;++i){
    if(degree[i] == 3)++t;
    else if(degree[i] == 0)++z;
    else return false;
  }
  return t == 6 && t + z == n;
}
bool isK5(int n, int degree[]){
  int f = 0, z = 0;
  for(int i=0;i<n;++i){
    if(degree[i] == 4)++f;
    else if(degree[i] == 0)++z;
    else return false;
  }
  return f == 5 && f + z == n;
}
// it judge a given graph is Homeomorphic with K33 or K5
bool isHomeomorphic(bool G[MAXN][MAXN], const int n){
  for(;;){
    int cnt = 0;
    for(int i=0;i<n;++i){
      vector<Edge> E;
      for(int j=0;j<n&&E.size()<3;++j)
        if(G[i][j] && i != j)
          E.push_back(Edge(i, j));
      if(E.size() == 1){
        G[i][E[0].v] = G[E[0].v][i] = false;
      }else if(E.size() == 2){
        G[i][E[0].v] = G[E[0].v][i] = false;
        G[i][E[1].v] = G[E[1].v][i] = false;
        G[E[0].v][E[1].v] = G[E[1].v][E[0].v] = true;
        ++cnt;
      }
    }
    if(cnt == 0)break;
  }
  static int degree[MAXN];
  fill(degree, degree + n, 0);
  for(int i=0;i<n;++i){
    for(int j=i+1; j<n; ++j){
      if(!G[i][j])continue;
      ++degree[i];
      ++degree[j];
    }
  }
  return  !(isK33(n, degree) || isK5(n, degree));
}
```

5.13 弦圖完美消除序列.cpp

```cpp
struct chordal{
  static const int MAXN=1005;
  int n;// 0-base
  vector<int>G[MAXN];
  int rank[MAXN],label[MAXN];
  bool mark[MAXN];
  void init(int _n){n=_n;
    for(int i=0;i<n;++i)G[i].clear();
  }
  void add_edge(int u,int v){
    G[u].push_back(v);
    G[v].push_back(u);
  }
  vector<int> MCS(){
    memset(rank,-1,sizeof(int)*n);
    memset(label,0,sizeof(int)*n);
    priority_queue<pair<int,int> > pq;
    for(int i=0;i<n;++i)pq.push(make_pair(0,i));
    for(int i=n-1;i>=0;--i)for(;;){
      int u=pq.top().second;pq.pop();
      if(~rank[u])continue;
      rank[u]=i;
      for(auto v:G[u])if(rank[v]==-1){
        pq.push(make_pair(++label[v],v));
      }
      break;
    }
    vector<int> res(n);
    for(int i=0;i<n;++i)res[rank[i]]=i;
    return res;
  }
  bool check(vector<int> ord){//弦圖判定
    for(int i=0;i<n;++i)rank[ord[i]]=i;
    memset(mark,0,sizeof(bool)*n);
    for(int i=0;i<n;++i){
      vector<pair<int,int> > tmp;
```

```cpp
        for(auto u:G[ord[i]])if(!mark[u])
          tmp.push_back(make_pair(rank[u],u));
        sort(tmp.begin(),tmp.end());
        if(tmp.size()){
          int u=tmp[0].second;
          set<int> S;
          for(auto v:G[u])S.insert(v);
          for(size_t j=1;j<tmp.size();++j)
            if(!S.count(tmp[j].second))return 0;
        }
        mark[ord[i]]=1;
      }
      return 1;
    }
};
```

5.14 最小斯坦納樹 DP.cpp

```cpp
//n個點，其中r個要構成斯坦納樹
//答案在max(dp[(1<<r)-1][k]) k=0~n-1
//p表示要構成斯坦納樹的點集
//O( n^3 + n*3^r + n^2*2^r )
#define REP(i,n) for(int i=0;i<(int)n;++i)
const int MAXN=30,MAXM=8;// 0-base
const int INF=0x3f3f3f3f;
int dp[1<<MAXM][MAXN];
int g[MAXN][MAXN];//圖
void init(){memset(g,0x3f,sizeof(g));}
void add_edge(int u,int v,int w){
  g[u][v]=g[v][u]=min(g[v][u],w);
}
void steiner(int n,int r,int *p){
  REP(k,n)REP(i,n)REP(j,n)
    g[i][j]=min(g[i][j],g[i][k]+g[k][j]);
  REP(i,n)g[i][i]=0;
  REP(i,r)REP(j,n)dp[1<<i][j]=g[p[i]][j];
  for(int i=1;i<(1<<r);++i){
    if(!(i&(i-1)))continue;
    REP(j,n)dp[i][j]=INF;
    REP(j,n){
      int tmp=INF;
      for(int s=i&(i-1);s;s=i&(s-1))
        tmp=min(tmp,dp[s][j]+dp[i^s][j]);
      REP(k,n)dp[i][k]=min(dp[i][k],g[j][k]+tmp);
    }
  }
}
```

5.15 最小樹形圖 _ 朱劉.cpp

```cpp
#define INF 0x3f3f3f3f
template<typename T>
struct zhu_liu{
  static const int MAXN=110;
  struct edge{
    int u,v;
    T w;
    edge(int u=0,int v=0,T w=0):u(u),v(v),w(w){}
  };
  vector<edge>E;// 0-base
  int pe[MAXN],id[MAXN],vis[MAXN];
  T in[MAXN];
  void init(){E.clear();}
  void add_edge(int u,int v,T w){
    if(u!=v)E.push_back(edge(u,v,w));
  }
  T build(int root,int n){
    T ans=0;int N=n;
    for(;;){
      for(int u=0;u<n;++u)in[u]=INF;
      for(size_t i=0;i<E.size();++i)
      if(E[i].u!=E[i].v&&E[i].w<in[E[i].v])
          pe[E[i].v]=i,in[E[i].v]=E[i].w;
      for(int u=0;u<n;++u)//無解
        if(u!=root&&in[u]==INF)return -INF;
      int cntnode=0;
      memset(id,-1,sizeof(int)*N);
      memset(vis,-1,sizeof(int)*N);
      for(int u=0;u<n;++u){
        if(u!=root)ans+=in[u];
        int v=u;
        for(;vis[v]!=u&&id[v]==-1&&v!=root;v=E[pe[v]].u)
          vis[v]=u;
        if(v!=root&&id[v]==-1){
          for(int x=E[pe[v]].u;x!=v;x=E[pe[x]].u)
            id[x]=cntnode;
          id[v]=cntnode++;
        }
      }
      if(!cntnode)break;//無環
      for(int u=0;u<n;++u)if(id[u]==-1)id[u]=cntnode++;
      for(size_t i=0;i<E.size();++i){
        int v=E[i].v;
        E[i].u=id[E[i].u];
        E[i].v=id[E[i].v];
        if(E[i].u!=E[i].v)E[i].w-=in[v];
      }
      n=cntnode;
      root=id[root];
    }
    return ans;
  }
};
```

5.16 穩定婚姻模板.cpp

```cpp
queue<int> Q;
for ( i : 所有考生 ) {
  設定在第0志願;
  Q.push(考生i);
}
while(Q.size()){
  當前考生=Q.front();Q.pop();
  while ( 此考生未分發 ) {
    指標移到下一志願;
    if ( 已經沒有志願 or 超出志願總數 ) break;
    計算該考生在該科系加權後的總分;
    if ( 不符合科系需求 ) continue;
    if ( 目前科系有餘額 ) {
      依加權後分數高低順序將考生id加入科系錄取名單中;
      break;
    }
    if ( 目前科系已額滿 ) {
      if ( 此考生成績比最低分數還高 ) {
        依加權後分數高低順序將考生id加入科系錄取名單;
        Q.push(被踢出的考生);
      }
    }
  }
}
```

6 language

6.1 CNF.cpp

```cpp
#define MAXN 55
struct CNF{
  int s,x,y;//s->xy | s->x, if y==-1
  int cost;
  CNF(){}
  CNF(int s,int x,int y,int c):s(s),x(x),y(y),cost(c){}
};
int state;//規則數量
map<char,int> rule;//每個字元對應到的規則，小寫字母為終端字符
vector<CNF> cnf;
inline void init(){
  state=0;
  rule.clear();
  cnf.clear();
}
inline void add_to_cnf(char s,const string &p,int cost){
  //加入一個s -> <p>的文法，代價為cost
  if(rule.find(s)==rule.end())rule[s]=state++;
  for(auto c:p)if(rule.find(c)==rule.end())rule[c]=state++;
  if(p.size()==1){
    cnf.push_back(CNF(rule[s],rule[p[0]],-1,cost));
  }else{
    int left=rule[s];
    int sz=p.size();
    for(int i=0;i<sz-2;++i){
      cnf.push_back(CNF(left,rule[p[i]],state,0));
      left=state++;
    }
    cnf.push_back(CNF(left,rule[p[sz-2]],rule[p[sz-1]],cost));
  }
}
vector<long long> dp[MAXN][MAXN];
vector<bool> neg_INF[MAXN][MAXN];//如果花費是負的可能會有無限小的情形
inline void relax(int l,int r,const CNF &c,long long cost,bool neg_c=0){
  if(!neg_INF[l][r][c.s]&&(neg_INF[l][r][c.x]||cost<dp[l][r][c.s])){
    if(neg_c||neg_INF[l][r][c.x]){
      dp[l][r][c.s]=0;
      neg_INF[l][r][c.s]=true;
    }else dp[l][r][c.s]=cost;
  }
}
inline void bellman(int l,int r,int n){
  for(int k=1;k<=state;++k)
    for(auto c:cnf)
      if(c.y==-1)relax(l,r,c,dp[l][r][c.x]+c.cost,k==n);
}
inline void cyk(const vector<int> &tok){
  for(int i=0;i<(int)tok.size();++i){
    for(int j=0;j<(int)tok.size();++j){
      dp[i][j]=vector<long long>(state+1,INT_MAX);
      neg_INF[i][j]=vector<bool>(state+1,false);
    }
    dp[i][i][tok[i]]=0;
    bellman(i,i,tok.size());
  }
  for(int r=1;r<(int)tok.size();++r){
    for(int l=r-1;l>=0;--l){
      for(int k=l;k<r;++k)
        for(auto c:cnf)
          if(~c.y)relax(l,r,c,dp[l][k][c.x]+dp[k+1][r][c.y]+c.cost);
      bellman(l,r,tok.size());
    }
  }
}
```

7 Linear_Programming

7.1 最大密度子圖.cpp

```cpp
typedef double T;//POJ 3155
const int MAXN=105;
struct edge{
  int u,v;
  T w;
  edge(int u=0,int v=0,T w=0):u(u),v(v),w(w){}
};
vector<edge> E;
int n,m;// 1-base
```

```cpp
T de[MAXN],pv[MAXN];//每個點的邊權和和點權(有些題目會給)
void init(){
  E.clear();
  for(int i=1;i<=n;++i)de[i]=pv[i]=0;
}
void add_edge(int u,int v,T w){
  E.push_back(edge(u,v,w));
  de[u]+=w,de[v]+=w;
}
T U;//二分搜的最大值
void get_U(){
  U=0;
  for(int i=1;i<=n;++i)U+=2*pv[i];
  for(size_t i=0;i<E.size();++i)U+=E[i].w;
}
ISAP<T> isap;//網路流
int s,t;//原匯點
void build(T L){
  isap.init(n+2);
  for(size_t i=0;i<E.size();++i){
    isap.add_edge(E[i].u,E[i].v,E[i].w);
  }
  for(int v=1;v<=n;++v){
    isap.add_edge(s,v,U);
    isap.add_edge(v,t,U+2*L-de[v]-2*pv[v]);
  }
}
int main(){
  while(~scanf("%d%d",&n,&m)){
    if(!m){
      puts("1\n1");
      continue;
    }
    init();
    int u,v;
    for(int i=0;i<m;++i){
      scanf("%d%d",&u,&v);
      add_edge(u,v,1);
    }
    get_U();
    s=n+1,t=n+2;
    T l=0,r=U,k=1.0/(n*n);
    while(r-l>k){//二分搜最大值
      T mid=(l+r)/2;
      build(mid);
      T res=(U*n-isap.isap(s,t))/2;
      if(res>0)l=mid;
      else r=mid;
    }
    build(l);
    isap.min_cut(s,t);
    vector<int> ans;
    for(int i=1;i<=n;++i){
      if(isap.vis[i])ans.push_back(i);
    }
    printf("%d\n",ans.size());
    for(size_t i=0;i<ans.size();++i){
      printf("%d\n",ans[i]);
    }
  }
  return 0;
}
```

8 Number_Theory

8.1 basic.cpp

```cpp
template<typename T>
void gcd(const T &a,const T &b,T &d,T &x,T &y){
  if(!b) d=a,x=1,y=0;
  else gcd(b,a%b,d,y,x), y-=x*(a/b);
}
long long int phi[N+1];
void phiTable(){
  for(int i=1;i<=N;i++)phi[i]=i;
  for(int i=1;i<=N;i++)for(x=i*2;x<=N;x+=i)phi[x]-=phi[i];
}
void all_divdown(const LL &n) {// all n/x
  for(LL a=1;a<=n;a=n/(n/(a+1))){
    // dosomething;
  }
}
const int MAXPRIME = 1000000;
int iscom[MAXPRIME], prime[MAXPRIME], primecnt;
int phi[MAXPRIME], mu[MAXPRIME];
void sieve(void){
  memset(iscom,0,sizeof(iscom));
  primecnt = 0;
  phi[1] = mu[1] = 1;
  for(int i=2;i<MAXPRIME;++i) {
    if(!iscom[i]) {
      prime[primecnt++] = i;
      mu[i] = -1;
      phi[i] = i-1;
    }
    for(int j=0;j<primecnt;++j) {
      int k = i * prime[j];
      if(k>=MAXPRIME) break;
      iscom[k] = prime[j];
      if(i%prime[j]==0) {
        mu[k] = 0;
        phi[k] = phi[i] * prime[j];
        break;
      } else {
        mu[k] = -mu[i];
        phi[k] = phi[i] * (prime[j]-1);
      }
    }
  }
}

bool g_test(const LL &g, const LL &p, const vector<LL> &v) {
  for(int i=0;i<v.size();++i)
    if(modexp(g,(p-1)/v[i],p)==1)
      return false;
  return true;
}
LL primitive_root(const LL &p) {
  if(p==2) return 1;
  vector<LL> v;
  Factor(p-1,v);
  v.erase(unique(v.begin(), v.end()), v.end());
  for(LL g=2;g<p;++g)
    if(g_test(g,p,v))
      return g;
  puts("primitive_root NOT FOUND");
  return -1;
}
int Legendre(const LL &a, const LL &p) { return modexp(a%p,(p-1)/2,p); }

LL inv(const LL &a, const LL &n) {
  LL d,x,y;
  gcd(a,n,d,x,y);
  return d==1 ? (x+n)%n : -1;
}

int inv[maxN];
LL invtable(int n,LL P){
  inv[1]=1;
  for(int i=2;i<n;++i)
    inv[i]=(P-(P/i))*inv[P%i]%P;
}

LL log_mod(const LL &a, const LL &b, const LL &p) {
  // a ^ x = b ( mod p )
  int m=sqrt(p+.5), e=1;
  LL v=inv(modexp(a,m,p), p);
  map<LL,int> x;
  x[1]=0;
  for(int i=1;i<m;++i) {
    e = LLmul(e,a,p);
    if(!x.count(e)) x[e] = i;
  }
  for(int i=0;i<m;++i) {
    if(x.count(b)) return i*m + x[b];
    b = LLmul(b,v,p);
  }
  return -1;
}

LL Tonelli_Shanks(const LL &n, const LL &p) {
  // x^2 = n ( mod p )
  if(n==0) return 0;
  if(Legendre(n,p)!=1) while(1) { puts("SQRT ROOT does not exist"); }
  int S = 0;
  LL Q = p-1;
  while( !(Q&1) ) { Q>>=1; ++S; }
  if(S==1) return modexp(n%p,(p+1)/4,p);
  LL z = 2;
  for(;Legendre(z,p)!=-1;++z)
  LL c = modexp(z,Q,p);
  LL R = modexp(n%p,(Q+1)/2,p), t = modexp(n%p,Q,p);
  int M = S;
  while(1) {
    if(t==1) return R;
    LL b = modexp(c,1L<<(M-i-1),p);
    R = LLmul(R,b,p);
    t = LLmul( LLmul(b,b,p), t, p);
    c = LLmul(b,b,p);
    M = i;
  }
  return -1;
}

template<typename T>
T Euler(T n){
  T ans=n;
  for(T i=2;i*i<=n;++i){
    if(n%i==0){
      ans=ans/i*(i-1);
      while(n%i==0)n/=i;
    }
  }
  if(n>1)ans=ans/n*(n-1);
  return ans;
}

//Chinese_remainder_theorem
template<typename T>
T pow_mod(T n,T k,T m){
  T ans=1;
  for(n=(n>=m?n%m:n);k;k>>=1){
    if(k&1)ans=ans*n%m;
    n=n*n%m;
  }
  return ans;
}
template<typename T>
T crt(vector<T> &m,vector<T> &a){
  T M=1,tM,ans=0;
  for(int i=0;i<(int)m.size();++i)M*=m[i];
  for(int i=0;i<(int)a.size();++i){
    tM=M/m[i];
    ans=(ans+(a[i]*tM%M)*pow_mod(tM,Euler(m[i])-1,m[i])%M)%M;
    /*如果m[i]是質數，Euler(m[i])-1=m[i]-2，就不用算Euler了*/
  }
  return ans;
}

//java code
//求sqrt(N)的連分數
public static void Pell(int n){
  BigInteger N,p1,p2,q1,q2,a0,a1,a2,g1,g2,h1,h2,p,q;
  g1=q2=p1=BigInteger.ZERO;
  h1=q1=p2=BigInteger.ONE;
  a0=a1=BigInteger.valueOf((int)Math.sqrt(1.0*n));
  BigInteger ans=a0.multiply(a0);
  if(ans.equals(BigInteger.valueOf(n))){
    System.out.println("No solution!");
    return ;
  }
  while(true){
    g2=a1.multiply(h1).substract(g1);
    h2=N.substract(g2.pow(2)).divide(h1);
    a2=g2.add(a0).divide(h2);
    p=a1.multiply(p2).add(p1);
    q=a1.multiply(q2).add(q1);
    if(p.pow(2).substract(N.multiply(q.pow(2))).compareTo(BigInteger.ONE)==0) break;
    g1=g2;h1=h2;a1=a2;
    p1=p2;p2=p;
    q1=q2;q2=q;
  }
  System.out.println(p+" "+q);
}
```

8.2 bit_set.cpp

```cpp
void sub_set(int S){
  int sub=S;
  do{
    //對某集合的子集合的處理
    sub=(sub-1)&S;
  }while(sub!=S);
}
void k_sub_set(int k,int n){
  int comb=(1<<k)-1,S=1<<n;
  while(comb<S){
    //對大小為k的子集合的處理
    int x=comb&-comb,y=comb+x;
    comb=((comb&~y)/x>>1)|y;
  }
}
```

8.3 cantor_expansion.cpp

```cpp
int factorial[MAXN];
void init(){
  factorial[0]=1;
  for(int i=1;i<=MAXN;++i)factorial[i]=factorial[i-1]*i;
}
int encode(const vector<int> &s){
  int n=s.size(),res=0;
  for(int i=0;i<n;++i){
    int t=0;
    for(int j=i+1;j<n;++j)
      if(s[j]<s[i])++t;
    res+=t*factorial[n-i-1];
  }
  return res;
}
vector<int> decode(int a,int n){
  vector<int> res;
  vector<bool> vis(n,0);
  for(int i=n-1;i>=0;--i){
    int t=a/factorial[i],j;
    for(j=0;j<n;++j)
      if(!vis[j]){
        if(t==0)break;
        --t;
      }
    res.push_back(j);
    vis[j]=1;
    a%=factorial[i];
  }
  return res;
}
```

8.4 FFT.cpp

```cpp
template<typename T,typename VT=std::vector<std::complex<T> > >
struct FFT{
  const T pi;
  FFT(const T pi=acos((T)-1)):pi(pi){}
  unsigned int bit_reverse(unsigned int a,int len){
    a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)>>1);
    a=((a&0x33333333U)<<2)|((a&0xCCCCCCCCU)>>2);
    a=((a&0x0F0F0F0FU)<<4)|((a&0xF0F0F0F0U)>>4);
    a=((a&0x00FF00FFU)<<8)|((a&0xFF00FF00U)>>8);
    a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)>>16);
    return a>>(32-len);
  }
  void fft(bool is_inv,VT &in,VT &out,int N){
    int bitlen=std::__lg(N),num=is_inv?-1:1;
    for(int i=0;i<N;++i)out[bit_reverse(i,bitlen)]=in[i];
    for(int step=2;step<=N;step<<=1){
      const int mh=step>>1;
      for(int i=0;i<mh;++i){
        std::complex<T> wi=exp(std::complex<T>(0,i*num*pi/mh));
        for(int j=i;j<N;j+=step){
          int k=j+mh;
          std::complex<T> u=out[j],t=wi*out[k];
          out[j]=u+t;
          out[k]=u-t;
        }
      }
    }
    if(is_inv)for(int i=0;i<N;++i)out[i]/=N;
  }
};
```

8.5 find_real_root.cpp

```cpp
// an*x^n + ... + a1x + a0 = 0;
int sign(double x){
  return x < -eps ? -1 : x > eps;
}

double get(const vector<double>&coef, double x){
  double e = 1, s = 0;
  for(auto i : coef) s += i*e, e *= x;
  return s;
}

double find(const vector<double>&coef, int n, double lo, double hi){
  double sign_lo, sign_hi;
  if( !(sign_lo = sign(get(coef,lo))) ) return lo;
  if( !(sign_hi = sign(get(coef,hi))) ) return hi;
  if(sign_lo * sign_hi > 0) return INF;
  for(int stp = 0; stp < 100 && hi - lo > eps; ++stp){
    double m = (lo+hi)/2.0;
    int sign_mid = sign(get(coef,m));
    if(!sign_mid) return m;
    if(sign_lo*sign_mid < 0) hi = m;
    else lo = m;
  }
  return (lo+hi)/2.0;
}

vector<double> cal(vector<double>coef, int n){
  vector<double>res;
  if(n == 1){
    if(sign(coef[1])) res.pb(-coef[0]/coef[1]);
    return res;
  }
  vector<double>dcoef(n);
  for(int i = 0; i < n; ++i) dcoef[i] = coef[i+1]*(i+1);
  vector<double>droot = cal(dcoef, n-1);
  droot.insert(droot.begin(), -INF);
  droot.pb(INF);
  for(int i = 0; i+1 < droot.size(); ++i){
    double tmp = find(coef, n, droot[i], droot[i+1]);
    if(tmp < INF) res.pb(tmp);
  }
  return res;
}

int main () {
  vector<double>ve;
  vector<double>ans = cal(ve, n);
  // 視情況把答案 +eps，避免 -0
}
```

8.6 LinearCongruence.cpp

```cpp
pair<LL,LL> LinearCongruence(LL a[],LL b[],LL m[],int n) {
  // a[i]*x = b[i] ( mod m[i] )
  for(int i=0;i<n;++i) {
    LL x, y, d = extgcd(a[i],m[i],x,y);
    if(b[i]%d!=0) return make_pair(-1LL,0LL);
    m[i] /= d;
    b[i] = LLmul(b[i]/d,x,m[i]);
  }
  LL lastb = b[0], lastm = m[0];
  for(int i=1;i<n;++i) {
    LL x, y, d = extgcd(m[i],lastm,x,y);
    if((lastb-b[i])%d!=0) return make_pair(-1LL,0LL);
    lastb = LLmul((lastb-b[i])/d,x,(lastm/d))*m[i];
    lastm = (lastm/d)*m[i];
    lastb = (lastb+b[i])%lastm;
  }
  return make_pair(lastb<0?lastb+lastm:lastb,lastm);
}
```

8.7 Lucas.cpp

```cpp
int mod_fact(int n,int &e){
  e=0;
  if(n==0)return 1;
  int res=mod_fact(n/P,e);
  e += n/P;
  if((n/P)%2==0)return res*fact[n%P]%P;
  return res*(P-fact[n%P])%P;
}
int Cmod(int n,int m){
  int a1,a2,a3,e1,e2,e3;
  a1=mod_fact(n,e1);
  a2=mod_fact(m,e2);
  a3=mod_fact(n-m,e3);
  if(e1>e2+e3)return 0;
  return a1*inv(a2*a3%P,P)%P;
}
```

8.8 Matrix.cpp

```cpp
template<typename T>
struct Matrix{
  using rt = std::vector<T>;
  using mt = std::vector<rt>;
  using matrix = Matrix<T>;
  int r,c;
  mt m;
  Matrix(int r,int c):r(r),c(c),m(r,rt(c)){}
  rt& operator[](int i){return m[i];}
  matrix operator+(const matrix &a){
    matrix rev(r,c);
    for(int i=0;i<r;++i)
      for(int j=0;j<c;++j)
        rev[i][j]=m[i][j]+a.m[i][j];
    return rev;
  }
  matrix operator-(const matrix &a){
    matrix rev(r,c);
    for(int i=0;i<r;++i)
      for(int j=0;j<c;++j)
        rev[i][j]=m[i][j]-a.m[i][j];
    return rev;
  }
  matrix operator*(const matrix &a){
    matrix rev(r,a.c);
    matrix tmp(a.c,a.r);
    for(int i=0;i<a.r;++i)
      for(int j=0;j<a.c;++j)
        tmp[j][i]=a.m[i][j];
    for(int i=0;i<r;++i)
      for(int j=0;j<a.c;++j)
        for(int k=0;k<c;++k)
```

```cpp
          rev.m[i][j]+=m[i][k]*tmp[j][k];
    return rev;
  }
  bool inverse(){
    Matrix t(r,r+c);
    for(int y=0;y<r;y++){
      t.m[y][c+y] = 1;
      for(int x=0;x<c;++x)
        t.m[y][x]=m[y][x];
    }
    if( !t.gas() )
      return false;
    for(int y=0;y<r;y++)
      for(int x=0;x<c;++x)
        m[y][x]=t.m[y][c+x]/t.m[y][y];
    return true;
  }
  T gas(){
    vector<T> lazy(r,1);
    bool sign=false;
    for(int i=0;i<r;++i){
      if( m[i][i]==0 ){
        int j=i+1;
        while(j<r&&!m[j][i])j++;
        if(j==r)continue;
        m[i].swap(m[j]);
        sign=!sign;
      }
      for(int j=0;j<r;++j){
        if(i==j)continue;
        lazy[j]=lazy[j]*m[i][i];
        T mx=m[j][i];
        for(int k=0;k<c;++k)
          m[j][k]=m[j][k]*m[i][i]-m[i][k]*mx;
      }
    }
    T det=sign?-1:1;
    for(int i=0;i<r;++i){
      det = det*m[i][i];
      det = det/lazy[i];
      for(auto &j:m[i])j/=lazy[i];
    }
    return det;
  }
};
```

8.9 MillerRobin.cpp

```cpp
LL LLmul(LL a, LL b, const LL &mod) {
  LL ans=0;
  while(b) {
    if(b&1) {
      ans+=a;
      if(ans>=mod) ans-=mod;
    }
    a<<=1, b>>=1;
    if(a>=mod) a-=mod;
  }
  return ans;
}
LL mod_mul(LL a,LL b,LL m){
  a%=m,b%=m;/* fast for m < 2^58 */
  LL y=(LL)((double)a*b/m+0.5);
  LL r=(a*b-y*m)%m;
  return r<0?r+m:r;
}
template<typename T>
T pow(T a,T b,T mod){//a^b%mod
  T ans=1;
  for(;b;a=mod_mul(a,a,mod),b>>=1)
    if(b&1)ans=mod_mul(ans,a,mod);
  return ans;
}
int sprp[3]={2,7,61};//int範圍可解
int llsprp[7]={2,325,9375,28178,450775,9780504,179526
    //至少unsigned long long範圍
template<typename T>
bool isprime(T n,int *sprp,int num){
  if(n==2)return 1;
  if(n<2||n%2==0)return 0;
  int t=0;
  T u=n-1;
  for(;u%2==0;++t)u>>=1;
  for(int i=0;i<num;++i){
    T a=sprp[i]%n;
    if(a==0||a==1||a==n-1)continue;
    T x=pow(a,u,n);
    if(x==1||x==n-1)continue;
    for(int j=0;j<t;++j){
      x=mod_mul(x,x,n);
      if(x==1)return 0;
      if(x==n-1)break;
    }
    if(x==n-1)continue;
    return 0;
  }
  return 1;
}
```

8.10 NTT.cpp

```cpp
2615053605667*(2^18)+1,3
15*(2^27)+1,31
479*(2^21)+1,3
7*17*(2^23)+1,3
3*3*211*(2^19)+1,5
25*(2^22)+1,3
template<typename T,typename VT=std::vector<T> >
struct NTT{
  const T P,G;
  NTT(T p=(1<<23)*7*17+1,T g=3):P(p),G(g){}
  unsigned int bit_reverse(unsigned int a,int len){
    a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)>>1);
    a=((a&0x33333333U)<<2)|((a&0xCCCCCCCCU)>>2);
    a=((a&0x0F0F0F0FU)<<4)|((a&0xF0F0F0F0U)>>4);
    a=((a&0x00FF00FFU)<<8)|((a&0xFF00FF00U)>>8);
    a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)>>16);
    return a>>(32-len);
  }
  T pow_mod(T n,T k,T m){
    T ans=1;
    for(n=(n>=m?n%m:n);k;k>>=1){
      if(k&1)ans=ans*n%m;
      n=n*n%m;
    }
    return ans;
  }
  void ntt(bool is_inv,VT &in,VT &out,int N){
    int bitlen=std::__lg(N);
    for(int i=0;i<N;++i)out[bit_reverse(i,bitlen)]=in[i];
    for(int step=2,id=1;step<=N;step<<=1,++id){
      T wn=pow_mod(G,(P-1)>>id,P),wi=1,u,t;
      const int mh=step>>1;
      for(int i=0;i<mh;++i){
        for(int j=i;j<N;j+=step){
          u=out[j],t=wi*out[j+mh]%P;
          out[j]=u+t;
          out[j+mh]=u-t;
          if(out[j]>=P)out[j]-=P;
          if(out[j+mh]<0)out[j+mh]+=P;
        }
        wi=wi*wn%P;
      }
    }
    if(is_inv){
      for(int i=1;i<N/2;++i)std::swap(out[i],out[N-i]);
      T invn=pow_mod(N,P-2,P);
      for(int i=0;i<N;++i)out[i]=out[i]*invn%P;
    }
  }
};
```

8.11 Simpson.cpp

```cpp
double simpson(double a,double b){
    double c=a+(b-a)/2;
    return (F(a)+4*F(c)+F(b))*(b-a)/6;
}
double asr(double a,double b,double eps,double A){
    double c=a+(b-a)/2;
    double L=simpson(a,c),R=simpson(c,b);
    if( abs(L+R-A)<15*eps )
        return L+R+(L+R-A)/15.0;
    return asr(a,c,eps/2,L)+asr(c,b,eps/2,R);
}
double asr(double a,double b,double eps){
    return asr(a,b,eps,simpson(a,b));
}
```

8.12 外星模運算.cpp

```cpp
//a[0]^(a[1]^a[2]^...)
#include<bits/stdc++.h>
using namespace std;
#define maxn 1000000
int euler[maxn+5];
bool is_prime[maxn+5];
inline void init_euler(){
  is_prime[1]=1;//一不是質數
  for(int i=1;i<=maxn;i++)euler[i]=i;
  for(int i=2;i<=maxn;i++){
    if(!is_prime[i]){//是質數
      euler[i]--;
      for(int j=i<<1;j<=maxn;j+=i){
        is_prime[j]=1;
        euler[j]=euler[j]/i*(i-1);
      }
    }
  }
}
inline long long pow(long long a,long long b,long long mod){//a^b%mod
  long long ans=1;
  for(;b;a=a*a%mod,b>>=1)
    if(b&1)ans=ans*a%mod;
  return ans;
}
bool isless(long long *a,int n,int k){
  if(*a==1)return k>1;
  if(--n==0)return *a<k;
  int next=0;
  for(long long b=1;b<k;++next)
    b*=*a;
  return isless(a+1,n,next);
}
long long high_pow(long long *a,int n,long long mod){
  if(*a==1||--n==0)return *a%mod;
  int k=0,r=euler[mod];
  for(long long tma=1;tma!=pow(*a,k+r,mod);++k)
    tma=tma*(*a)%mod;
  if(isless(a+1,n,k))return pow(*a,high_pow(a+1,n,k),mod);
  int tmd=high_pow(a+1,n,r);
  int t=(tmd-k+r)%r;
  return pow(*a,k+t,mod);
}
long long a[1000005];
int t,mod;
int main(){
  init_euler();
  scanf("%d",&t);
  #define n 4
  while(t--){
    for(int i=0;i<n;++i)scanf("%lld",&a[i]);
    scanf("%d",&mod);
    printf("%lld\n",high_pow(a,n,mod));
  }
  return 0;
}
```

8.13 模運算模板.cpp

```cpp
template<typename T,long long mod>
struct mod_t{//mod只能是質數
  T data;
  mod_t(){}
  mod_t(const T &d):data((d%mod+mod)%mod){}
  mod_t pow(T b)const{
    mod_t ans(1);
    for(mod_t now=*this;b;now=now*now,b/=2)
      if(b%2)ans=ans*now;
    return ans;
  }
  mod_t operator-(int)const{
    return mod_t(mod-data);
  }
  mod_t operator+(const mod_t &b)const{
    return mod_t((data+b.data)%mod);
  }
  mod_t operator-(const mod_t &b)const{
    return mod_t((data-b.data+mod)%mod);
  }
  mod_t operator*(const mod_t &b)const{
    return mod_t((data*b.data)%mod);
  }
  mod_t operator/(const mod_t &b)const{
    return *this*b.pow(mod-2);//*this * Inverse(b)
  }
  operator T()const{return data;}
  friend istream &operator>>(istream &i, mod_t &b){
    T d;
    i>>d;
    b=mod_t(d);
    return i;
  }
};
```

8.14 質因數分解.cpp

```cpp
LL func(const LL n,const LL mod,const int c) {
  return (LLmul(n,n,mod)+c+mod)%mod;
}

LL pollorrho(const LL n, const int c) {//循環節長度
  LL a=1, b=1;
  a=func(a,n,c)%n;
  b=func(b,n,c)%n; b=func(b,n,c)%n;
  while(gcd(abs(a-b),n)==1) {
    a=func(a,n,c)%n;
    b=func(b,n,c)%n; b=func(b,n,c)%n;
  }
  return gcd(abs(a-b),n);
}

void prefactor(LL &n, vector<LL> &v) {
  for(int i=0;i<12;++i) {
    while(n%prime[i]==0) {
      v.push_back(prime[i]);
      n/=prime[i];
    }
  }
}

void smallfactor(LL n, vector<LL> &v) {
  if(n<MAXPRIME) {
    while(isp[(int)n]) {
      v.push_back(isp[(int)n]);
      n/=isp[(int)n];
    }
    v.push_back(n);
  } else {
    for(int i=0;i<primecnt&&prime[i]*prime[i]<=n;++i) {
      while(n%prime[i]==0) {
        v.push_back(prime[i]);
        n/=prime[i];
      }
    }
    if(n!=1) v.push_back(n);
  }
}

void comfactor(const LL &n, vector<LL> &v) {
  if(n<1e9) {
    smallfactor(n,v);
    return;
  }
  if(Isprime(n)) {
    v.push_back(n);
    return;
  }
  LL d;
  for(int c=3;;++c) {
    d = pollorrho(n,c);
    if(d!=n) break;
  }
  comfactor(d,v);
  comfactor(n/d,v);
}

void Factor(const LL &x, vector<LL> &v) {
  LL n = x;
  if(n==1) { puts("Factor 1"); return; }
  prefactor(n,v);
  if(n==1) return;
  comfactor(n,v);
  sort(v.begin(),v.end());
}

void AllFactor(const LL &n,vector<LL> &v) {
  vector<LL> tmp;
  Factor(n,tmp);
  v.clear();
  v.push_back(1);
  int len;
  LL now=1;
  for(int i=0;i<tmp.size();++i) {
    if(i==0 || tmp[i]!=tmp[i-1]) {
      len = v.size();
      now = 1;
    }
    now*=tmp[i];
    for(int j=0;j<len;++j)
      v.push_back(v[j]*now);
  }
}
```

9 other

9.1 WhatDay.cpp

```cpp
int whatday(int y,int m,int d){
  if(m<=2)m+=12,--y;
  if(y<1752||y==1752&&m<9||y==1752&&m==9&&d<3)
    return (d+2*m+3*(m+1)/5+y+y/4+5)%7;
  return (d+2*m+3*(m+1)/5+y+y/4-y/100+y/400)%7;
}
```

9.2 上下最大正方形.cpp

```cpp
void solve(int n,int a[],int b[]){// 1-base
  int ans=0;
  deque<int>da,db;
  for(int l=1,r=1;r<=n;++r){
    while(da.size()&&a[da.back()]>=a[r]){
      da.pop_back();
    }
    da.push_back(r);
    while(db.size()&&b[db.back()]>=b[r]){
      db.pop_back();
    }
    db.push_back(r);
    for(int d=a[da.front()]+b[db.front()];r-l+1>d;++l){
      if(da.front()==l)da.pop_front();
      if(db.front()==l)db.pop_front();
      if(da.size()&&db.size()){
        d=a[da.front()]+b[db.front()];
      }
    }
    ans=max(ans,r-l+1);
  }
  printf("%d\n",ans);
}
```

9.3 最大矩形.cpp

```cpp
long long max_rectangle(vector<int> s){
  stack<pair<int,int > > st;
  st.push(make_pair(-1,0));
  s.push_back(0);
  long long ans=0;
  for(size_t i=0;i<s.size();++i){
    int h=s[i];
    pair<int,int > now=make_pair(h,i);
    while(h<st.top().first){
      now=st.top();
      st.pop();
      ans=max(ans,(long long)(i-now.second)*now.first);
    }
    if(h>st.top().first){
      st.push(make_pair(h,now.second));
    }
  }
  return ans;
}
```

10 String

10.1 AC 自動機.cpp

```cpp
template<char L='a',char R='z'>
class ac_automaton{
  private:
    struct joe{
      int next[R-L+1],fail,efl,ed,cnt_dp,vis;
      joe():ed(0),cnt_dp(0),vis(0){
        for(int i=0;i<=R-L;++i)next[i]=0;
      }
    };
  public:
    std::vector<joe> S;
    std::vector<int> q;
    int qs,qe,vt;
    ac_automaton():S(1),qs(0),qe(0),vt(0){}
    void clear(){
      q.clear();
      S.resize(1);
      for(int i=0;i<=R-L;++i)S[0].next[i]=0;
      S[0].cnt_dp=S[0].vis=qs=qe=vt=0;
    }
    void insert(const char *s){
      int o=0;
      for(int i=0,id;s[i];++i){
        id=s[i]-L;
        if(!S[o].next[id]){
          S.push_back(joe());
          S[o].next[id]=S.size()-1;
        }
        o=S[o].next[id];
      }
      ++S[o].ed;
    }
    void build_fail(){
      S[0].fail=S[0].efl=-1;
      q.clear();
      q.push_back(0);
      ++qe;
      while(qs!=qe){
        int pa=q[qs++],id,t;
        for(int i=0;i<=R-L;++i){
          t=S[pa].next[i];
          if(!t)continue;
          id=S[pa].fail;
          while(~id&&!S[id].next[i])id=S[id].fail;
          S[t].fail=~id?S[id].next[i]:0;
          S[t].efl=S[S[t].fail].ed?S[t].fail:S[S[t].fail].efl;
          q.push_back(t);
          ++qe;
        }
      }
    }
    /*DP出每個前綴在字串s出現的次數並傳回所有字串被s匹配成功的次數O(N+M)*/
    int match_0(const char *s){
      int ans=0,id,p=0,i;
      for(i=0;s[i];++i){
        id=s[i]-L;
        while(!S[p].next[id]&&p)p=S[p].fail;
        if(!S[p].next[id])continue;
        p=S[p].next[id];
        ++S[p].cnt_dp;/*匹配成功則它所有後綴都可以被匹配(DP計算)*/
      }
      for(i=qe-1;i>=0;--i){
        ans+=S[q[i]].cnt_dp*S[q[i]].ed;
        if(~S[q[i]].fail)S[S[q[i]].fail].cnt_dp+=S[q[i]].cnt_dp;
      }
      return ans;
    }
    /*多串匹配走efl邊並傳回所有字串被s匹配成功的次數O(N*M^1.5)*/
    int match_1(const char *s)const{
      int ans=0,id,p=0,t;
      for(int i=0;s[i];++i){
        id=s[i]-L;
        while(!S[p].next[id]&&p)p=S[p].fail;
        if(!S[p].next[id])continue;
        p=S[p].next[id];
        if(S[p].ed)ans+=S[p].ed;
        for(t=S[p].efl;~t;t=S[t].efl){
          ans+=S[t].ed;/*因為都走efl邊所以保證匹配成功*/
        }
      }
      return ans;
    }
    /*枚舉(s的子字串∩A)的所有相異字串各恰一次並傳回次數O(N*M^(1/3))*/
    int match_2(const char *s){
      int ans=0,id,p=0,t;
      ++vt;
      /*把戳記vt+=1，只要vt沒溢位，所有S[p].vis==vt就會變成false
      這種利用vt的方法可以O(1)歸零vis陣列*/
      for(int i=0;s[i];++i){
        id=s[i]-L;
        while(!S[p].next[id]&&p)p=S[p].fail;
        if(!S[p].next[id])continue;
        p=S[p].next[id];
        if(S[p].ed&&S[p].vis!=vt){
          S[p].vis=vt;
          ans+=S[p].ed;
        }
        for(t=S[p].efl;~t&&S[t].vis!=vt;t=S[t].efl){
          S[t].vis=vt;
          ans+=S[t].ed;/*因為都走efl邊所以保證匹配成功*/
        }
      }
      return ans;
    }
    /*把AC自動機變成真的自動機*/
    void evolution(){
      for(qs=1;qs!=qe;){
        int p=q[qs++];
        for(int i=0;i<=R-L;++i)
          if(S[p].next[i]==0)S[p].next[i]=S[S[p].fail].next[i];
      }
    }
};
```

10.2 hash.cpp

```cpp
#define MAXN 1000000
#define prime_mod 1073676287
/*prime_mod 必須要是質數*/
typedef long long T;
char s[MAXN+5];
T h[MAXN+5];/*hash陣列*/
T h_base[MAXN+5];/*h_base[n]=(prime^n)%prime_mod*/
inline void hash_init(int len,T prime=0xdefaced){
  h_base[0]=1;
  for(int i=1;i<=len;++i){
    h[i]=(h[i-1]*prime+s[i-1])%prime_mod;
    h_base[i]=(h_base[i-1]*prime)%prime_mod;
  }
}
inline T get_hash(int l,int r){/*閉區間寫法，設編號為0 ~ len-1*/
  return (h[r+1]-(h[l]*h_base[r-l+1])%prime_mod+prime_mod)%prime_mod;
}
```

10.3 KMP.cpp

```cpp
/*產生fail function*/
inline void kmp_fail(char *s,int len,int *fail){
  int id=-1;
  fail[0]=-1;
  for(int i=1;i<len;++i){
    while(~id&&s[id+1]!=s[i])id=fail[id];
    if(s[id+1]==s[i])++id;
    fail[i]=id;
  }
}
/*以字串B匹配字串A，傳回匹配成功的數量(用B的fail)*/
inline int kmp_match(char *A,int lenA,char *B,int lenB,int *fail){
  int id=-1,ans=0;
  for(int i=0;i<lenA;++i){
    while(~id&&B[id+1]!=A[i])id=fail[id];
    if(B[id+1]==A[i])++id;
    if(id==lenB-1){/*匹配成功*/
      ++ans;
      id=fail[id];
    }
  }
  return ans;
}
```

10.4 manacher.cpp

```cpp
//原字串: asdsasdsa
//先把字串變成這樣: @#a#s#d#s#a#s#d#s#a#
inline void manacher(char *s,int len,int *z){
  int l=0,r=0;
  for(int i=1;i<len;++i){
    z[i]=r>i?min(z[2*l-i],r-i):1;
    while(s[i+z[i]]==s[i-z[i]])++z[i];
    if(z[i]+i>r)r=z[i]+i,l=i;
  }
}
```

10.5 minimal_string_rotation.cpp

```cpp
int min_string_rotation(const string &s){
  int n=s.size(),i=0,j=1,k=0;
  while(i<n&&j<n&&k<n){
    int t=s[(i+k)%n]-s[(j+k)%n];
    ++k;
    if(t){
      if(t>0)i+=k;
      else j+=k;
      if(i==j)++j;
      k=0;
    }
  }
  return min(i,j);//傳回最小循環表示法起始位置
}
```

10.6 suffix_array_lcp.cpp

```cpp
#define radix_sort(x,y){\
  for(i=0;i<A;++i)c[i]=0;\
  for(i=0;i<n;++i)c[x[y[i]]]++;\
  for(i=1;i<A;++i)c[i]+=c[i-1];\
  for(i=n-1;~i;--i)sa[--c[x[y[i]]]]=y[i];\
}
#define sac(r,a,b) r[a]!=r[b]||a+k>=n||r[a+k]!=r[b+k]
void suffix_array(const char *s,int n,int *sa,int *rank,int *tmp,int *c){
  int A='z'+1,i,k,id=0;
  for(i=0;i<n;++i)rank[tmp[i]=i]=s[i];
  radix_sort(rank,tmp);
  for(k=1;id<n-1;k<<=1){
    for(id=0,i=n-k;i<n;++i)tmp[id++]=i;
    for(i=0;i<n;++i)if(sa[i]>=k)tmp[id++]=sa[i]-k;
    radix_sort(rank,tmp);
    swap(rank,tmp);
    for(rank[sa[0]]=id=0,i=1;i<n;++i)
      rank[sa[i]]=id+=sac(tmp,sa[i-1],sa[i]);
    A=id+1;
  }
}
//h:高度數組 sa:後綴數組 rank:排名
void suffix_array_lcp(const char *s,int len,int *h,int *sa,int *rank){
  for(int i=0;i<len;++i)rank[sa[i]]=i;
  for(int i=0,k=0;i<len;++i){
    if(rank[i]==0)continue;
    if(k)--k;
    while(s[i+k]==s[sa[rank[i]-1]+k])++k;
    h[rank[i]]=k;
  }
  h[0]=0;
}
```

10.7 Z.cpp

```cpp
inline void z_alg(char *s,int len,int *z){
  int l=0,r=0;
  z[0]=len;
  for(int i=1;i<len;++i){
    z[i]=i>r?0:(i-l+z[i-l]<z[l]?z[i-l]:r-i+1);
    while(i+z[i]<len&&s[i+z[i]]==s[z[i]])++z[i];
    if(i+z[i]-1>r)r=i+z[i]-1,l=i;
  }
}
```

11 Tarjan

11.1 dominator_tree.cpp

```cpp
struct dominator_tree{
  static const int MAXN=5005;
  int n;// 1-base
  vector<int> suc[MAXN],pre[MAXN];
  int fa[MAXN],dfn[MAXN],id[MAXN],Time;
  int semi[MAXN],idom[MAXN];
  int anc[MAXN],best[MAXN];//disjoint set
  vector<int> dom[MAXN];//dominator_tree
  void init(int _n){
    n=_n;
    for(int i=1;i<=n;++i)suc[i].clear(),pre[i].clear();
  }
  void add_edge(int u,int v){
    suc[u].push_back(v);
    pre[v].push_back(u);
  }
  void dfs(int u){
    dfn[u]=++Time,id[Time]=u;
    for(auto v:suc[u]){
      if(dfn[v])continue;
      dfs(v),fa[dfn[v]]=dfn[u];
    }
  }
  int find(int x){
    if(x==anc[x])return x;
    int y=find(anc[x]);
    if(semi[best[x]]>semi[best[anc[x]]])best[x]=best[anc[x]];
    return anc[x]=y;
  }
  void tarjan(int r){
    Time=0;
    for(int t=1;t<=n;++t){
      dfn[t]=idom[t]=0;//u=r或是u無法到達r時idom[id[u]]=0
      dom[t].clear();
      anc[t]=best[t]=semi[t]=t;
    }
    dfs(r);
    for(int y=Time;y>=2;--y){
      int x=fa[y],idy=id[y];
      for(auto z:pre[idy]){
        if(!(z=dfn[z]))continue;
        find(z);
        semi[y]=min(semi[y],semi[best[z]]);
      }
      dom[semi[y]].push_back(y);
      anc[y]=x;
      for(auto z:dom[x]){
        find(z);
        idom[z]=semi[best[z]]<x?best[z]:x;
      }
      dom[x].clear();
    }
    for(int u=2;u<=Time;++u){
      if(idom[u]!=semi[u])idom[u]=idom[idom[u]];
      dom[id[idom[u]]].push_back(id[u]);
    }
  }
}dom;
```

11.2 tnfshb017_2_sat.cpp

```cpp
#include<bits/stdc++.h>
using namespace std;
#define MAXN 8001
#define MAXN2 MAXN*4
#define n(X) ((X)+2*N)
vector<int>  v[MAXN2];
vector<int> rv[MAXN2];
vector<int>  vis_t;
int N,M;
void addedge(int s,int e){
    v[s].push_back(e);
    rv[e].push_back(s);
}
int scc[MAXN2];
bool vis[MAXN2]={false};
void dfs(vector<int> *uv,int n,int k=-1){
    vis[n]=true;
    for(int i=0;i<uv[n].size();++i)
        if(!vis[uv[n][i]])
            dfs(uv,uv[n][i],k);
    if(uv==v)vis_t.push_back(n);
    scc[n]=k;
}
void solve(){
    for(int i=1;i<=N;++i){
        if(!vis[i])dfs(v,i);
        if(!vis[n(i)])dfs(v,n(i));
    }
    memset(vis,0,sizeof(vis));
    int c=0;
    for(int i=vis_t.size()-1;i>=0;--i)
        if(!vis[vis_t[i]])
            dfs(rv,vis_t[i],c++);
}
int main(){
    int a,b;
    scanf("%d%d",&N,&M);
    for(int i=1;i<=N;++i){
        // (A or B)&(!A & !B) A^B
        a=i*2-1;
        b=i*2;
        addedge(n(a),b);
        addedge(n(b),a);
        addedge(a,n(b));
        addedge(b,n(a));
    }
    while(M--){
        scanf("%d%d",&a,&b);
        a = a>0?a*2-1:-a*2;
        b = b>0?b*2-1:-b*2;
        // A or B
        addedge(n(a),b);
        addedge(n(b),a);
    }
    solve();
    bool check=true;
    for(int i=1;i<=2*N;++i)
        if(scc[i]==scc[n(i)])
            check=false;
    if(check){
        printf("%d\n",N);
        for(int i=1;i<=2*N;i+=2){
            if(scc[i]>scc[i+2*N])
                putchar('+');
            else
                putchar('-');
        }
        putchar('\n');
    }else puts("0");
    return 0;
}
```

11.3 橋連通分量.cpp

```cpp
#define N 1005
struct edge{
  int u,v;
  bool is_bridge;
  edge(int u=0,int v=0):u(u),v(v),is_bridge(0){}
};
vector<edge> E;
vector<int> G[N];// 1-base
int low[N],vis[N],Time;
int bcc_id[N],bridge_cnt,bcc_cnt;// 1-base
int st[N],top;//BCC用
inline void add_edge(int u,int v){
  G[u].push_back(E.size());
  E.push_back(edge(u,v));
  G[v].push_back(E.size());
  E.push_back(edge(v,u));
}
void dfs(int u,int re=-1){//u當前點，re為u連接前一個點的邊
  int v;
  low[u]=vis[u]=++Time;
  st[top++]=u;
  for(size_t i=0;i<G[u].size();++i){
    int e=G[u][i];v=E[e].v;
    if(!vis[v]){
      dfs(v,e^1);//e^1反向邊
      low[u]=min(low[u],low[v]);
      if(vis[u]<low[v]){
        E[e].is_bridge=E[e^1].is_bridge=1;
        ++bridge_cnt;
      }
    }else if(vis[v]<vis[u]&&e!=re)
      low[u]=min(low[u],vis[v]);
  }
  if(vis[u]==low[u]){//處理BCC
    ++bcc_cnt;// 1-base
    do bcc_id[v=st[--top]]=bcc_cnt;//每個點所在的BCC
    while(v!=u);
  }
}
inline void bcc_init(int n){
  Time=bcc_cnt=bridge_cnt=top=0;
  E.clear();
  for(int i=1;i<=n;++i){
    G[i].clear();
    vis[i]=bcc_id[i]=0;
  }
}
```

11.4 雙連通分量 & 割點.cpp

```cpp
#define N 1005
vector<int> G[N];// 1-base
vector<int> bcc[N];//存每塊雙連通分量的點
int low[N],vis[N],Time;
int bcc_id[N],bcc_cnt;// 1-base
bool is_cut[N];//是否為割點
int st[N],top;
void dfs(int u,int pa=-1){//u當前點，pa父親
  int v,child=0;
  low[u]=vis[u]=++Time;
  st[top++]=u;
  for(size_t i=0;i<G[u].size();++i){
    if(!vis[v=G[u][i]]){
      dfs(v,u),++child;
      low[u]=min(low[u],low[v]);
      if(vis[u]<=low[v]){
        is_cut[u]=1;
        bcc[++bcc_cnt].clear();
        int t;
        do{
          bcc_id[t=st[--top]]=bcc_cnt;
          bcc[bcc_cnt].push_back(t);
        }while(t!=v);
        bcc_id[u]=bcc_cnt;
        bcc[bcc_cnt].push_back(u);
      }
    }else if(vis[v]<vis[u]&&v!=pa)//反向邊
      low[u]=min(low[u],vis[v]);
  }
  if(pa==-1&&child<2)is_cut[u]=0;//u是dfs樹的根要特判
}
inline void bcc_init(int n){
  Time=bcc_cnt=top=0;
  for(int i=1;i<=n;++i){
    G[i].clear();
    is_cut[i]=vis[i]=bcc_id[i]=0;
  }
}
```

12 Tree_problem

12.1 HeavyLight.cpp

```cpp
#include<vector>
#define MAXN 100005
typedef std::vector<int >::iterator VIT;
int siz[MAXN],max_son[MAXN],pa[MAXN],dep[MAXN];
int link_top[MAXN],link[MAXN],cnt;
std::vector<int >G[MAXN];
void find_max_son(int x){
  siz[x]=1;
  max_son[x]=-1;
  for(VIT i=G[x].begin();i!=G[x].end();++i){
    if(*i==pa[x])continue;
    pa[*i]=x;
    dep[*i]=dep[x]+1;
    find_max_son(*i);
    if(max_son[x]==-1||siz[*i]>siz[max_son[x]])max_son[x]=*i;
    siz[x]+=siz[*i];
  }
}
void build_link(int x,int top){
  link[x]=++cnt;
  link_top[x]=top;
  if(max_son[x]==-1)return;
  build_link(max_son[x],top);
  for(VIT i=G[x].begin();i!=G[x].end();++i){
    if(*i==max_son[x]||*i==pa[x])continue;
    build_link(*i,*i);
  }
}
inline int find_lca(int a,int b){
  //求LCA，可以在過程中對區間進行處理
  int ta=link_top[a],tb=link_top[b];
  while(ta!=tb){
    if(dep[ta]<dep[tb]){
      std::swap(ta,tb);
      std::swap(a,b);
    }
    //這裡可以對a所在的鏈做區間處理
    //區間為(link[ta],link[a])
    ta=link_top[a=pa[ta]];
  }
  //最後a,b會在同一條鏈，若a!=b還要在進行一次區間處理
  return dep[a]<dep[b]?a:b;
}
```

12.2 LCA.cpp

```cpp
#define MAXN 100000
#define MAX_LOG 17
int pa[MAX_LOG+1][MAXN+5];
int dep[MAXN+5];
vector<int>G[MAXN+5];
void dfs(int x,int p){//dfs(1,-1);
  pa[0][x]=p;
  for(int i=0;i+1<MAX_LOG;++i)pa[i+1][x]=pa[i][pa[i][x]];
  for(auto &i:G[x]){
    if(i==p)continue;
    dep[i]=dep[x]+1;
    dfs(i,x);
  }
}
inline int jump(int x,int d){
for(int i=0;i<d;++i)if((x>>i)&1)x=pa[k][x];
  return x;
}
inline int find_lca(int a,int b){
  if(dep[a]>dep[b])swap(a,b);
  b=jump(b,dep[b]-dep[a]);
  if(a==b)return a;
  for(int i=MAX_LOG;i>=0;--i){
    if(pa[i][a]!=pa[i][b]){
      a=pa[i][a];
      b=pa[i][b];
    }
  }
  return pa[0][a];
}
```

12.3 link_cut_tree.cpp

```cpp
struct splay_tree{
  int ch[2],pa;//子節點跟父母
  bool rev;//反轉的懶惰標記
  splay_tree():pa(0),rev(0){ch[0]=ch[1]=0;}
};
vector<splay_tree> node;
//有的時候用vector會TLE，要注意
//這邊以node[0]作為null節點
bool isroot(int x){//判斷是否為這棵splay tree的根
  return node[node[x].pa].ch[0]!=x&&node[node[x].pa].ch[1]!=x;
}
void down(int x){//懶惰標記下推
  if(node[x].rev){
    if(node[x].ch[0])node[node[x].ch[0]].rev^=1;
    if(node[x].ch[1])node[node[x].ch[1]].rev^=1;
    std::swap(node[x].ch[0],node[x].ch[1]);
    node[x].rev^=1;
  }
}
void push_down(int x){//將所有祖先的懶惰標記下推
  if(!isroot(x))push_down(node[x].pa);
  down(x);
}
void up(int x){}//將子節點的資訊向上更新
void rotate(int x){//旋轉，會自行判斷轉的方向
  int y=node[x].pa,z=node[y].pa,d=(node[y].ch[1]==x);
  node[x].pa=z;
  if(!isroot(y))node[z].ch[node[z].ch[1]==y]=x;
  node[y].ch[d]=node[x].ch[d^1];
  node[node[y].ch[d]].pa=y;
  node[y].pa=x,node[x].ch[d^1]=y;
  up(y),up(x);
}
void splay(int x){//將節點x伸展到所在splay tree的根
  push_down(x);
  while(!isroot(x)){
    int y=node[x].pa;
    if(!isroot(y)){
      int z=node[y].pa;
      if((node[z].ch[0]==y)^(node[y].ch[0]==x))rotate(y);
      else rotate(x);
    }
    rotate(x);
  }
}
int access(int x){
  int last=0;
  while(x){
    splay(x);
    node[x].ch[1]=last;
    up(x);
    last=x;
    x=node[x].pa;
  }
  return last;//回傳access後splay tree的根
}
void access(int x,bool is=0){//is=0就是一般的access
  int last=0;
  while(x){
    splay(x);
    if(is&&!node[x].pa){
      //printf("%d\n",max(node[last].ma,node[node[x].ch[1]].ma));
    }
    node[x].ch[1]=last;
    up(x);
    last=x;
    x=node[x].pa;
  }
}
void query_edge(int u,int v){
  access(u);
  access(v,1);
}
void make_root(int x){
  access(x),splay(x);
  node[x].rev^=1;
}
void make_root(int x){
  node[access(x)].rev^=1;
  splay(x);
}
void cut(int x,int y){
  make_root(x);
  access(y);
  splay(y);
  node[y].ch[0]=0;
  node[x].pa=0;
}
void cut_parents(int x){
  access(x);
  splay(x);
  node[node[x].ch[0]].pa=0;
  node[x].ch[0]=0;
}
void link(int x,int y){
  make_root(x);
  node[x].pa=y;
}
int find_root(int x){
  x=access(x);
  while(node[x].ch[0])x=node[x].ch[0];
  splay(x);
  return x;
}
int query(int u,int v){
//傳回uv路徑splay tree的根結點
//這種寫法無法求LCA
  make_root(u);
  return access(v);
}
int query_lca(int u,int v){
//假設求鏈上點權的總和，sum是子樹的權重和，data是節點的權重
  access(u);
  int lca=access(v);
  splay(u);
  if(u==lca){
    //return node[lca].data+node[node[lca].ch[1]].sum
  }else{
    //return node[lca].data+node[node[lca].ch[1]].sum+node[u].sum
  }
}
struct EDGE{
  int a,b,w;
}e[10005];
int n;
vector<pair<int ,int > >G[10005];
//first表示子節點，second表示邊的編號
int pa[10005],edge_node[10005];
//pa是父母節點，暫存用的，edge_node是每個編被存在哪個點裡面的陣列
void bfs(int root){
//在建構的時候把每個點都設成一個splay tree，不會壞掉
  queue<int > q;
  for(int i=1;i<=n;++i)pa[i]=0;
  q.push(root);
  while(q.size()){
    int u=q.front();
    q.pop();
    for(int i=0;i<(int)G[u].size();++i){
      int v=G[u][i].first;
      if(v!=pa[u]){
        pa[v]=u;
        node[v].pa=u;
        node[v].data=e[G[u][i].second].w;
        edge_node[G[u][i].second]=v;
        up(v);
        q.push(v);
      }
    }
  }
}
void change(int x,int b){
  splay(x);
  //node[x].data=b;
  up(x);
}
```

12.4 POJ_tree.cpp

```cpp
#include<bits/stdc++.h>
using namespace std;
#define MAXN 10005
int n,k;
vector<pair<int,int> >g[MAXN];
int size[MAXN];
bool vis[MAXN];
inline void init(){
  for(int i=0;i<=n;++i){
    g[i].clear();
    vis[i]=0;
  }
}
void get_dis(vector<int> &dis,int u,int pa,int d){
  dis.push_back(d);
  for(size_t i=0;i<g[u].size();++i){
    int v=g[u][i].first,w=g[u][i].second;
    if(v!=pa&&!vis[v])get_dis(dis,v,u,d+w);
  }
}
vector<int> dis;//這東西如果放在函數裡會TLE
int cal(int u,int d){
  dis.clear();
  get_dis(dis,u,-1,d);
  sort(dis.begin(),dis.end());
  int l=0,r=dis.size()-1,res=0;
  while(l<r){
    while(l<r&&dis[l]+dis[r]>k)--r;
    res+=r-(l++);
  }
  return res;
}
pair<int,int> tree_centroid(int u,int pa,const int sz){
  size[u]=1;//找樹重心，second是重心
  pair<int,int> res(INT_MAX,-1);
  int ma=0;
  for(size_t i=0;i<g[u].size();++i){
    int v=g[u][i].first;
    if(v==pa||vis[v])continue;
    res=min(res,tree_centroid(v,u,sz));
    size[u]+=size[v];
    ma=max(ma,size[v]);
  }
  ma=max(ma,sz-size[u]);
  return min(res,make_pair(ma,u));
}
int tree_DC(int u,int sz){
  int center=tree_centroid(u,-1,sz).second;
  int ans=cal(center,0);
  vis[center]=1;
  for(size_t i=0;i<g[center].size();++i){
    int v=g[center][i].first,w=g[center][i].second;
    if(vis[v])continue;
    ans-=cal(v,w);
    ans+=tree_DC(v,size[v]);
  }
  return ans;
}
int main(){
  while(scanf("%d%d",&n,&k),n||k){
    init();
    for(int i=1;i<n;++i){
      int u,v,w;
      scanf("%d%d%d",&u,&v,&w);
      g[u].push_back(make_pair(v,w));
      g[v].push_back(make_pair(u,w));
    }
    printf("%d\n",tree_DC(1,n));
  }
  return 0;
}
```

13 zformula

13.1 formula.tex

13.1.1 Pick 公式

給定頂點坐標均是整點的簡單多邊形，面積 = 內部格點數 + 邊上格點數/2-1

13.1.2 圖論

1. $V - E + F = 2$
2. 對於平面圖，$F = E - V + n + 1$，n 是連通分量
3. 對於平面圖，$E \le 3V - 6$
4. 對於連通圖 G，最大獨立點集的大小設為 I(G)，最大匹配大小設為 M(G)，最小點覆蓋設為 Cv(G)，最小邊覆蓋設為 Ce(G)。對於任意連通圖：
   (a) $I(G) + Cv(G) = |V|$
   (b) $M(G) + Ce(G) = |V|$
5. 對於連通二分圖：
   (a) $I(G) = Cv(G)$
   (b) $M(G) = Ce(G)$
6. 最大權閉合圖：
   (a) $C(u, V) = \infty, (u, v) \in E$
   (b) $C(S, v) = W_v, W_v > 0$
   (c) $C(v, T) = -W_v, W_v < 0$
7. 最大密度子圖：
   (a) $C(u, v) = 1, (u, v) \in E$
   (b) $C(S, v) = U_v, v \in V$
   (c) $C(v, T) = U + 2g - d_v, v \in V$
8. 弦圖：
   (a) 完美消除序列從後往前依次給每個點染色，給每個點染上可以染的最小顏色
   (b) 最大團大小 = 色數
   (c) 最大獨立集: 完美消除序列從前往後能選就選
   (d) 最小團覆蓋: 最大獨立集的點和他延伸的邊構成
   (e) 區間圖是弦圖
   (f) 區間圖的完美消除序列: 將區間按造又端點由小到大排序
   (g) 區間圖染色: 用線段樹做

```cpp
double l=0,=m,stop=1.0/n/n;
while(r-l>=stop){
  double(mid);
  if((n*m-sol.maxFlow(s,t))/2>eps)l=mid;
  else r=mid;
}
build(l);
sol.maxFlow(s,t);
vector<int> ans;
for(int i=1;i<=n;++i)
  if(sol.vis[i])ans.push_back(i);
```

13.1.3 學長公式

1. $\sum_{d|n} phi(n) = n$
2. $g(n) = \sum_{d|n} f(d) => f(n) = \sum_{d|n} mu(d) * g(n/d)$
3. $Harmonicseries H_n = ln(n) + \gamma + 1/(2n) - 1/(12n^2) + 1/(120n^4)$
4. $\gamma = 0.57721566490153286060651209008240243104215$
5. 格雷碼 $= n \oplus (n >> 1)$
6. $SG(A + B) = SG(A) \oplus SG(B)$
7. 選轉矩陣 $M(\theta) = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$

13.1.4 基本數論

1. $\sum_{d|n} \mu(n) = (n == 1)$
2. $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) * g(m/d)$
3. $\sum_{i=1}^{n} \sum_{j=1}^{m}$ 互質數量 $= \sum \mu(d) \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$
4. $\sum_{i=1}^{n} \sum_{j=1}^{n} lcm(i, j) = n \sum_{d|n} d\phi(d)$

13.1.5 排組公式

1. k 卡特蘭 $\frac{C_n^{kn}}{n(k-1)+1}$，$C_m^n = \frac{n!}{m!(n-m)!}$
2. $H(n, m) \cong x_1 + x_2 \ldots + x_n = k, num = C_k^{n+k-1}$
3. Stirling number of 2^{nd},n 人分 k 組方法數目
   (a) $S(0, 0) = S(n, n) = 1$
   (b) $S(n, 0) = 0$
   (c) $S(n, k) = kS(n - 1, k) + S(n - 1, k - 1)$
4. Bell number,n 人分任意多組方法數目
   (a) $B_0 = 1$
   (b) $B_n = \sum_{i=0}^{n} S(n, i)$
   (c) $B_{n+1} = \sum_{k=0}^{n} C_k^n B_k$
   (d) $B_{p+n} \equiv B_n + B_{n+1} mod p$, p is prime
   (e) $B_{p^m+n} \equiv mB_n + B_{n+1} mod p$, p is prime
   (f) From B0:$1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975$
5. Derangement, 錯排, 沒有人在自己位置上
   (a) $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \ldots + (-1)^n \frac{1}{n!})$
   (b) $D_n = (n - 1)(D_{n-1} + D_{n-2}), D_0 = 1, D_1 = 0$
   (c) From D0:$1, 0, 1, 2, 9, 44, 265, 1854, 14833, 133496$

13.1.6 冪次, 冪次和

1. $a^b \% P = a^{b\%\varphi(p)+\varphi(p)}, b \ge \varphi(p)$
2. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
3. $1^4 + 2^4 + 3^4 + \ldots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$
4. $1^5 + 2^5 + 3^5 + \ldots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12}$
5. $0^k + 1^k + 2^k + \ldots + n^k = P(k), P(k) = \frac{(n+1)^{k+1} - \sum_{i=0}^{k-1} C_i^{k+1} P(i)}{k+1}, P(0) = n + 1$
6. $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^{n} C_k^{n+1} B_k m^{n+1-k}$
7. $\sum_{j=0}^{m} C_j^{m+1} B_j = 0, B_0 = 1$
8. 除了 $B_1 = -1/2$，剩下的奇數項都是 0
9. $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 = -1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 7/6, B_{16} = -3617/510, B_{18} = 43867/798, B_{20} = -174611/330,$

13.1.7 Burnside's lemma

1. $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
2. $X^g = t^{c(g)}$
3. G 表示有幾種轉法，X^g 表示在那種轉法下，有幾種是會保持對稱的，t 是顏色數，$c(g)$ 是循環節不動的面數。
4. 正立方體塗三顏色，轉 0 有 3^6 個元素不變，轉 90 有 6 種，每種有 3^3 不變，180 有 3×3^4，120(角) 有 8×3^2，180(邊) 有 6×3^3，全部 $\frac{1}{24}\left(3^6 + 6 \times 3^3 + 3 \times 3^4 + 8 \times 3^2 + 6 \times 3^3\right) = 57$

13.1.8 Count on a tree

1. Rooted tree: $s_{n+1} = \frac{1}{n} \sum_{i=1}^{n} (i \times a_i \times \sum_{j=1}^{\lfloor n/i \rfloor} a_{n+1-i\times j})$
2. Unrooted tree:
   (a) Odd:$a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$
   (b) Even:$Odd + \frac{1}{2} a_{n/2}(a_{n/2} + 1)$
3. Spanning Tree
   (a) 完全圖 $n^n - 2$
   (b) 一般圖 (Kirchhoff's theorem)$M[i][i] = degree(V_i)$,$M[i][j] = -1$,if have $E(i, j)$,0 if no edge. delete any one row and col in A, $ans = det(A)$

ACM ICPC Team Reference - NTHU Jinkela

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