1 Computational Geometry

1.1 delaunay

```cpp
template<class T>
class Delaunay{
  struct PT:public point<T>{
    int g[2];
    PT(const point<T> &p):
      point<T>(p){ g[0]=g[1]=-1; }
  };
  static bool cmp(const PT &a,const PT &b){
    return a.x<b.x||(a.x==b.x&&a.y<b.y);
  }
  struct edge{
    int v,g[2];
    edge(int v,int g0,int g1):
      v(v){g[0]=g0,g[1]=g1;}
  };
  vector<PT> S;
  vector<edge> E;
  bool convex(int &from,int to,T LR){
    for(int i=0;i<2;++i){
      int c = E[S[from].g[i]].v;
      auto A=S[from]-S[to], B=S[c]-S[to];
      T v = A.cross(B)*LR;
      if(v>0||(v==0&&B.abs2()<A.abs2()))
        return from = c, true;
    }
    return false;
  }
  void addEdge(int v,int g0,int g1){
    E.emplace_back(v,g0,g1);
    E[E.back().g[0]].g[1] = E.size()-1;
    E[E.back().g[1]].g[0] = E.size()-1;
  }
  void climb(int &p, int e, int n, int nl, int nr, int LR){
    for(int i=E[e].g[LR]; (S[nr]-S[nl]).cross(S[E[i].v]-S[n])>0;){
      if(inCircle(S[E[i].v],S[nl],S[nr],S[E[E[i].g[LR]].v])>=0)
        { p = i; break; }
      for(int j=0;j<4;++j)
        E[E[i^j/2].g[j%2^1]].g[j%2] = E[i^j/2].g[j%2];
      int j=i; i=E[i].g[LR];
      E[j].g[0]=E[j].g[1]=E[j^1].g[0]=E[j^1].g[1]=-1;
    }
  }
  T det3(T a11,T a12,T a13,T a21,T a22,T a23,T a31,T a32,T a33){
    return a11*(a22*a33-a32*a23)-a12*(a21*a33-a31*a23)+a13*(a21*a32-a31*a22);
  }
  int inCircle(const PT &a, const PT &b, const PT &c, const PT &p){
T as = a.abs2(), bs = b.abs2(), cs = c.abs2(), ps = p.abs2();
T res = a.x * det3(b.y,bs,1,c.y,cs,1,p.y,ps,1)
-a.y * det3(b.x,bs,1,c.x,cs,1,p.x,ps,1)
+as * det3(b.x,b.y,1,c.x,c.y,1,p.x,p.y,1)
-det3(b.x,b.y,bs,c.x,c.y,cs,p.x,p.y,ps);
    return res<0 ? 1 : (res>0 ? -1 : 0);
  }
  void divide(int l, int r){
    if(l>=r)return;
    if(l+1==r){
      int A=S[l].g[0]=S[l].g[1]=E.size();
      E.emplace_back(r,A,A);
      int B=S[r].g[0]=S[r].g[1]=E.size();
      E.emplace_back(l,B,B);
      return;
    }
    int mid = (l+r)/2;
    divide(l,mid), divide(mid+1, r);
    int nl = mid, nr = mid+1;
    for(;;){
      if(convex(nl,nr,1)) continue;
      if(S[nr].g[0]!=-1&&convex(nr,nl,-1)) continue;
      break;
    }
    addEdge(nr,S[nl].g[0],S[nl].g[1]);
    S[nl].g[1] = E.size()-1;
    if(S[nr].g[0]==-1){
      addEdge(nl,E.size(),E.size());
      S[nr].g[1] = E.size()-1;
    }else addEdge(nl,S[nr].g[0],S[nr].g[1]);
    S[nr].g[0] = E.size()-1;
    int cl = nl, cr = nr;
    for(;;){
      int pl=-1, pr=-1, side;
      climb(pl,E.size()-2,nl,nl,nr,1);
      climb(pr,E.size()-1,nr,nl,nr,0);
      if(pl==-1&&pr==-1) break;
      if(pl==-1||pr==-1) side = pl==-1;
      else side=inCircle(S[E[pl].v],S[nl],S[nr],S[E[pr].v])<=0;
      if(side){
nr = E[pr].v;
addEdge(nr,E.size()-2,E[E.size()-2].g[1]);
addEdge(nl,E[pr^1].g[0],pr^1);
      }else{
nl = E[pl].v;
addEdge(nr,pl^1,E[pl^1].g[1]);
addEdge(nl,E[E.size()-2].g[0],E.size()-2);
      }
    }
    if(cl==nl&&cr==nr) return;//Collinearity
    S[nl].g[0] = E.size()-2;
    S[nr].g[1] = E.size()-1;
  }
public:
  void solve(const vector<point<T>> &P){
    S.clear(), E.clear();
    for(const auto &p:P) S.emplace_back(p);
    sort(S.begin(),S.end(),cmp);
    divide(0,int(S.size())-1);
  }
  vector<pair<int,int>> getEdge(){
    vector<pair<int,int>> res;
    for(size_t i=0;i<E.size();i+=2)
      if(E[i].g[0]!=-1)
        res.emplace_back(E[i].v,E[i^1].v);
    return res;
  }
};
```

1.2 Geometry

```cpp
const double PI=atan2(0.0,-1.0);
template<typename T>
struct point{
  T x,y;
  point(){}
  point(const T&x,const T&y):x(x),y(y){}
  point operator+(const point &b)const{
    return point(x+b.x,y+b.y); }
  point operator-(const point &b)const{
    return point(x-b.x,y-b.y); }
  point operator*(const T &b)const{
    return point(x*b,y*b); }
  point operator/(const T &b)const{
    return point(x/b,y/b); }
  bool operator==(const point &b)const{
    return x==b.x&&y==b.y; }
  T dot(const point &b)const{
    return x*b.x+y*b.y; }
  T cross(const point &b)const{
    return x*b.y-y*b.x; }
  point normal()const{//求法向量
    return point(-y,x); }
  T abs2()const{//向量長度的平方
    return dot(*this); }
  T rad(const point &b)const{//兩向量的弧度
return fabs(atan2(fabs(cross(b)),dot(b))); }
  T getA()const{//對x軸的弧度
    T A=atan2(y,x);//超過180度會變負的
    if(A<=-PI/2)A+=PI*2;
    return A;
  }
};
template<typename T>
struct line{
  line(){}
  point<T> p1,p2;
  T a,b,c;//ax+by+c=0
  line(const point<T>&x,const point<T>&y):p1(x),p2(y){}
  void pton(){//轉成一般式
    a=p1.y-p2.y;
    b=p2.x-p1.x;
    c=-a*p1.x-b*p1.y;
  }
  T ori(const point<T> &p)const{//點和有向直線的關係，>0左邊、=0在線上<0右邊
    return (p2-p1).cross(p-p1);
  }
  T btw(const point<T> &p)const{//點投影落在線段上<=0
    return (p1-p).dot(p2-p);
  }
  bool point_on_segment(const point<T>&p)const{//點是否在線段上
    return ori(p)==0&&btw(p)<=0;
  }
  T dis2(const point<T> &p,bool is_segment=0)const{//點跟直線/線段的距離平方
    point<T> v=p2-p1,v1=p-p1;
    if(is_segment){
      point<T> v2=p-p2;
      if(v.dot(v1)<=0)return v1.abs2();
      if(v.dot(v2)>=0)return v2.abs2();
    }
    T tmp=v.cross(v1);
    return tmp*tmp/v.abs2();
  }
  T seg_dis2(const line<T> &l)const{//兩線段距離平方
    return min({dis2(l.p1,1),dis2(l.p2,1),l.dis2(p1,1),l.dis2(p2,1)});
  }
  point<T> projection(const point<T> &p)const{//點對直線的投影
    point<T> n=(p2-p1).normal();
    return p-n*(p-p1).dot(n)/n.abs2();
  }
  point<T> mirror(const point<T> &p)const{
    //點對直線的鏡射，要先呼叫pton轉成一般式
    point<T> R;
    T d=a*a+b*b;
    R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
    R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
    return R;
  }
  bool equal(const line &l)const{//直線相等
    return ori(l.p1)==0&&ori(l.p2)==0;
  }
  bool parallel(const line &l)const{
    return (p1-p2).cross(l.p1-l.p2)==0;
  }
  bool cross_seg(const line &l)const{
    return (p2-p1).cross(l.p1-p1)*(p2-p1).cross(l.p2-p1)<=0;//直線是否交線段
  }
  int line_intersect(const line &l)const{//直線相交情況，-1無限多點、1交於一點、0不相交
    return parallel(l)?(ori(l.p1)==0?-1:0):1;
  }
  int seg_intersect(const line &l)const{
    T c1=ori(l.p1), c2=ori(l.p2);
    T c3=l.ori(p1), c4=l.ori(p2);
    if(c1==0&&c2==0){//共線
      bool b1=btw(l.p1)>=0,b2=btw(l.p2)>=0;
      T a3=l.btw(p1),a4=l.btw(p2);
      if(b1&&b2&&a3==0&&a4>=0) return 2;
      if(b1&&b2&&a3>=0&&a4==0) return 3;
      if(b1&&b2&&a3>=0&&a4>=0) return 0;
      return -1;//無限交點
    }else if(c1*c2<=0&&c3*c4<=0)return 1;
    return 0;//不相交
  }
  point<T> line_intersection(const line &l)const{/*直線交點*/
    point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
    //if(a.cross(b)==0)return INF;
    return p1+a*(s.cross(b)/a.cross(b));
  }
  point<T> seg_intersection(const line &l)const{//線段交點
    int res=seg_intersect(l);
    if(res<=0) assert(0);
    if(res==2) return p1;
    if(res==3) return p2;
    return line_intersection(l);
  }
};
template<typename T>
struct polygon{
  polygon(){}
  vector<point<T> > p;//逆時針順序
  T area()const{//面積
    T ans=0;
    for(int i=p.size()-1,j=0;j<(int)p.size();i=j++)
      ans+=p[i].cross(p[j]);
    return ans/2;
  }
  point<T> center_of_mass()const{//重心
    T cx=0,cy=0,w=0;
    for(int i=p.size()-1,j=0;j<(int)p.size();i=j++){
      T a=p[i].cross(p[j]);
      cx+=(p[i].x+p[j].x)*a;
      cy+=(p[i].y+p[j].y)*a;
      w+=a;
    }
    return point<T>(cx/3/w,cy/3/w);
  }
  char ahas(const point<T>& t)const{//點是否在簡單多邊形內，是的話回傳1、在邊上回傳-1、否則回傳0
    bool c=0;
    for(int i=0,j=p.size()-1;i<p.size();j=i++)
      if(line<T>(p[i],p[j]).point_on_segment(t))return -1;
      else if((p[i].y>t.y)!=(p[j].y>t.y)&&
      t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j].y-p[i].y)+p[i].x)
        c=!c;
    return c;
  }
  char point_in_convex(const point<T>&x)const{
    int l=1,r=(int)p.size()-2;
    while(l<=r){//點是否在凸多邊形內，是的話回傳1、在邊上回傳-1、否則回傳0
      int mid=(l+r)/2;
      T a1=(p[mid]-p[0]).cross(x-p[0]);
      T a2=(p[mid+1]-p[0]).cross(x-p[0]);
      if(a1>=0&&a2<=0){
        T res=(p[mid+1]-p[mid]).cross(x-p[mid]);
        return res>0?1:(res>=0?-1:0);
      }else if(a1<0)r=mid-1;
      else l=mid+1;
    }
    return 0;
  }
  vector<T> getA()const{//凸包邊對x軸的夾角
    vector<T>res;//一定是遞增的
    for(size_t i=0;i<p.size();++i)
      res.push_back((p[(i+1)%p.size()]-p[i]).getA());
    return res;
  }
  bool line_intersect(const vector<T>&A,const line<T> &l)const{//O(logN)
    int f1=upper_bound(A.begin(),A.end(),(l.p1-l.p2).getA())-A.begin();
    int f2=upper_bound(A.begin(),A.end(),(l.p2-l.p1).getA())-A.begin();
    return l.cross_seg(line<T>(p[f1],p[f2]));
  }
  polygon cut(const line<T> &l)const{//凸包對直線切割，得到直線l左側的凸包
    polygon ans;
    for(int n=p.size(),i=n-1,j=0;j<n;i=j++){
      if(l.ori(p[i])>=0){
        ans.p.push_back(p[i]);
        if(l.ori(p[j])<0)
          ans.p.push_back(l.line_intersection(line<T>(p[i],p[j])));
      }else if(l.ori(p[j])>0)
        ans.p.push_back(l.line_intersection(line<T>(p[i],p[j])));
    }
    return ans;
  }
  static bool monotone_chain_cmp(const point<T>& a,const point<T>& b){//凸包排序函數
    return (a.x<b.x)||(a.x==b.x&&a.y<b.y);
  }
  void monotone_chain(vector<point<T> > &s){//凸包
    sort(s.begin(),s.end(),monotone_chain_cmp);
    p.resize(s.size()+1);
    int m=0;
    for(size_t i=0;i<s.size();++i){
      while(m>=2&&(p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0)--m;
      p[m++]=s[i];
    }
    for(int i=s.size()-2,t=m+1;i>=0;--i){
      while(m>=t&&(p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0)--m;
      p[m++]=s[i];
    }
    if(s.size()>1)--m;
    p.resize(m);
  }
  T diam(){//直徑
    int n=p.size(),t=1;
    T ans=0;p.push_back(p[0]);
    for(int i=0;i<n;i++){
      point<T> now=p[i+1]-p[i];
      while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i]))t=(t+1)%n;
      ans=max(ans,(p[i]-p[t]).abs2());
    }
    return p.pop_back(),ans;
  }
  T min_cover_rectangle(){//最小覆蓋矩形
    int n=p.size(),t=1,r=1,l;
    if(n<3)return 0;//也可以做最小周長矩形
    T ans=1e99;p.push_back(p[0]);
    for(int i=0;i<n;i++){
      point<T> now=p[i+1]-p[i];
      while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i]))t=(t+1)%n;
      while(now.dot(p[r+1]-p[i])>now.dot(p[r]-p[i]))r=(r+1)%n;
      if(!i)l=r;
      while(now.dot(p[l+1]-p[i])<=now.dot(p[l]-p[i]))l=(l+1)%n;
      T d=now.abs2();
      T tmp=now.cross(p[t]-p[i])*(now.dot(p[r]-p[i])-now.dot(p[l]-p[i]))/d;
      ans=min(ans,tmp);
    }
    return p.pop_back(),ans;
  }
  T dis2(polygon &pl){//凸包最近距離平方
    vector<point<T> > &P=p,&Q=pl.p;
    int n=P.size(),m=Q.size(),l=0,r=0;
  for(int i=0;i<n;++i)if(P[i].y<P[l].y)l=i;
  for(int i=0;i<m;++i)if(Q[i].y<Q[r].y)r=i;
    P.push_back(P[0]),Q.push_back(Q[0]);
    T ans=1e99;
    for(int i=0;i<n;++i){
      while((P[l]-P[l+1]).cross(Q[r+1]-Q[r])<0)r=(r+1)%m;
      ans=min(ans,line<T>(P[l],P[l+1]).seg_dis2(line<T>(Q[r],Q[r+1])));
      l=(l+1)%n;
    }
    return P.pop_back(),Q.pop_back(),ans;
  }
  static char sign(const point<T>&t){
    return (t.y==0?t.x:t.y)<0;
  }
  static bool angle_cmp(const line<T>& A,const line<T>& B){
    point<T> a=A.p2-A.p1,b=B.p2-B.p1;
    return sign(a)<sign(b)||(sign(a)==sign(b)&&a.cross(b)>0);
  }
  int halfplane_intersection(vector<line<T> > &s){//半平面交
    sort(s.begin(),s.end(),angle_cmp);//線段左側為該線段半平面
    int L,R,n=s.size();
    vector<point<T> > px(n);
    vector<line<T> > q(n);
    q[L=R=0]=s[0];
    for(int i=1;i<n;++i){
      while(L<R&&s[i].ori(px[R-1])<=0)--R;
      while(L<R&&s[i].ori(px[L])<=0)++L;
      q[++R]=s[i];
      if(q[R].parallel(q[R-1])){
        --R;
        if(q[R].ori(s[i].p1)>0)q[R]=s[i];
      }
      if(L<R)px[R-1]=q[R-1].line_intersection(q[R]);
    }
    while(L<R&&q[L].ori(px[R-1])<=0)--R;
    p.clear();
    if(R-L<=1)return 0;
    px[R]=q[R].line_intersection(q[L]);
    for(int i=L;i<=R;++i)p.push_back(px[i]);
    return R-L+1;
  }
};
template<typename T>
struct triangle{
  point<T> a,b,c;
  triangle(){}
  triangle(const point<T> &a,const point<T> &b,const point<T> &c):a(a),b(b),c(c){}
  T area()const{
    T t=(b-a).cross(c-a)/2;
    return t>0?t:-t;
  }
  point<T> barycenter()const{//重心
    return (a+b+c)/3;
  }
  point<T> circumcenter()const{//外心
    static line<T> u,v;
    u.p1=(a+b)/2;
    u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-b.x);
    v.p1=(a+c)/2;
    v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-c.x);
    return u.line_intersection(v);
  }
  point<T> incenter()const{//內心
    T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2()),C=sqrt((a-b).abs2());
    return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+B*b.y+C*c.y)/(A+B+C);
  }
  point<T> perpencenter()const{//垂心
    return barycenter()*3-circumcenter()*2;
  }
};
template<typename T>
struct point3D{
  T x,y,z;
  point3D(){}
  point3D(const T&x,const T&y,const T&z):x(x),y(y),z(z){}
  point3D operator+(const point3D &b)const{
    return point3D(x+b.x,y+b.y,z+b.z);}
  point3D operator-(const point3D &b)const{
    return point3D(x-b.x,y-b.y,z-b.z);}
  point3D operator*(const T &b)const{
    return point3D(x*b,y*b,z*b);}
  point3D operator/(const T &b)const{
    return point3D(x/b,y/b,z/b);}
  bool operator==(const point3D &b)const{
    return x==b.x&&y==b.y&&z==b.z;}
  T dot(const point3D &b)const{
    return x*b.x+y*b.y+z*b.z;}
  point3D cross(const point3D &b)const{
    return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);}
  T abs2()const{//向量長度的平方
    return dot(*this);}
  T area2(const point3D &b)const{//和b、原點圍成面積的平方
    return cross(b).abs2()/4;}
};
template<typename T>
struct line3D{
  point3D<T> p1,p2;
  line3D(){}
  line3D(const point3D<T> &p1,const point3D<T> &p2):p1(p1),p2(p2){}
  T dis2(const point3D<T> &p,bool is_segment=0)const{//點跟直線/線段的距離平方
    point3D<T> v=p2-p1,v1=p-p1;
    if(is_segment){
      point3D<T> v2=p-p2;
      if(v.dot(v1)<=0)return v1.abs2();
      if(v.dot(v2)>=0)return v2.abs2();
    }
    point3D<T> tmp=v.cross(v1);
    return tmp.abs2()/v.abs2();
  }
  pair<point3D<T>,point3D<T> > closest_pair(const line3D<T> &l)const{
    point3D<T> v1=(p1-p2),v2=(l.p1-l.p2);
    point3D<T> N=v1.cross(v2),ab(p1-l.p1);
    //if(N.abs2()==0)return NULL;平行或重合
    T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//最近點對距離
    point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.cross(d2),G=l.p1-p1;
    T t1=(G.cross(d2)).dot(D)/D.abs2();
    T t2=(G.cross(d1)).dot(D)/D.abs2();
    return make_pair(p1+d1*t1,l.p1+d2*t2);
  }
  bool same_side(const point3D<T> &a,const point3D<T> &b)const{
    return (p2-p1).cross(a-p1).dot((p2-p1).cross(b-p1))>0;
  }
};
template<typename T>
struct plane{
  point3D<T> p0,n;//平面上的點和法向量
  plane(){}
  plane(const point3D<T> &p0,const point3D<T> &n):p0(p0),n(n){}
  T dis2(const point3D<T> &p)const{//點到平面距離的平方
    T tmp=(p-p0).dot(n);
    return tmp*tmp/n.abs2();
  }
  point3D<T> projection(const point3D<T> &p)const{
    return p-n*(p-p0).dot(n)/n.abs2();
  }
  point3D<T> line_intersection(const line3D<T> &l)const{
    T tmp=n.dot(l.p2-l.p1);//等於0表示平行或重合該平面
    return l.p1+(l.p2-l.p1)*(n.dot(p0-l.p1)/tmp);
  }
  line3D<T> plane_intersection(const plane &pl)const{
    point3D<T> e=n.cross(pl.n),v=n.cross(e);
    T tmp=pl.n.dot(v);//等於0表示平行或重合該平面
    point3D<T> q=p0+(v*(pl.n.dot(pl.p0-p0))/tmp);
    return line3D<T>(q,q+e);
  }
};
template<typename T>
struct triangle3D{
  point3D<T> a,b,c;
  triangle3D(){}
  triangle3D(const point3D<T> &a,const point3D<T> &b,const point3D<T> &c):a(a),b(b),c(c){}
  bool point_in(const point3D<T> &p)const{//點在該平面上的投影在三角形中
    return line3D<T>(b,c).same_side(p,a)&&line3D<T>(a,c).same_side(p,b)&&line3D<T>(a,b).same_side(p,c);
  }
};
template<typename T>
struct tetrahedron{//四面體
  point3D<T> a,b,c,d;
  tetrahedron(){}
  tetrahedron(const point3D<T> &a,const point3D<T> &b,const point3D<T> &c,const point3D<T> &d):a(a),b(b),c(c),d(d){}
  T volume6()const{//體積的六倍
    return (d-a).dot((b-a).cross(c-a));
  }
  point3D<T> centroid()const{
    return (a+b+c+d)/4;
  }
  bool point_in(const point3D<T> &p)const{
    return triangle3D<T>(a,b,c).point_in(p)&&triangle3D<T>(c,d,a).point_in(p);
  }
};
template<typename T>
struct convexhull3D{
  static const int MAXN=1005;
  struct face{
    int a,b,c;
    face(int a,int b,int c):a(a),b(b),c(c){}
  };
  vector<point3D<T>> pt;
  vector<face> ans;
  int fid[MAXN][MAXN];
  void build(){
    int n=pt.size();
    ans.clear();
    memset(fid,0,sizeof(fid));
    ans.emplace_back(0,1,2);//注意不能共線
    ans.emplace_back(2,1,0);
    int ftop = 0;
    for(int i=3, ftop=1; i<n; ++i,++ftop){
      vector<face> next;
      for(auto &f:ans){
        T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[f.a]).cross(pt[f.c]-pt[f.a]));
        if(d<=0) next.push_back(f);
        int ff=0;
        if(d>0) ff=ftop;
        else if(d<0) ff=-ftop;
        fid[f.a][f.b]=fid[f.b][f.c]=fid[f.c][f.a]=ff;
      }
      for(auto &f:ans){
        if(fid[f.a][f.b]>0 && fid[f.a][f.b]!=fid[f.b][f.a])
          next.emplace_back(f.a,f.b,i);
        if(fid[f.b][f.c]>0 && fid[f.b][f.c]!=fid[f.c][f.b])
          next.emplace_back(f.b,f.c,i);
        if(fid[f.c][f.a]>0 && fid[f.c][f.a]!=fid[f.a][f.c])
          next.emplace_back(f.c,f.a,i);
      }
      ans=next;
    }
  }
  point3D<T> centroid()const{
    point3D<T> res(0,0,0);
    T vol=0;
    for(auto &f:ans){
      T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c]));
      res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
      vol+=tmp;
    }
    return res/(vol*4);
  }
};
```

1.3 SmallestCircle

```
using PT=point<T>; using CPT=const PT;
PT circumcenter(CPT &a,CPT &b,CPT &c){
  PT u=b-a, v=c-a;
  T c1=u.abs2()/2,c2=v.abs2()/2;
  T d=u.cross(v);
  return PT(a.x+(v.y*c1-u.y*c2)/d,a.y+(u.x*c2-v.x*c1)/d);
}
void solve(PT p[],int n,PT &c,T &r2){
  random_shuffle(p,p+n);
  c=p[0]; r2=0; // c,r2 = 圓心,半徑平方
for(int i=1;i<n;i++)if((p[i]-c).abs2()>r2){
    c=p[i]; r2=0;
for(int j=0;j<i;j++)if((p[j]-c).abs2()>r2){
      c.x=(p[i].x+p[j].x)/2;
      c.y=(p[i].y+p[j].y)/2;
      r2=(p[j]-c).abs2();
for(int k=0;k<j;k++)if((p[k]-c).abs2()>r2){
        c=circumcenter(p[i],p[j],p[k]);
        r2=(p[i]-c).abs2();
      }
    }
  }
}
```

1.4 最近點對

```
template<typename _IT=point<T>* >
T cloest_pair(_IT L, _IT R){
  if(R-L <= 1) return INF;
  _IT mid = L+(R-L)/2;
  T x = mid->x;
  T d = min(cloest_pair(L,mid),cloest_pair(mid,R));
  inplace_merge(L, mid, R, ycmp);
  static vector<point> b; b.clear();
  for(auto u=L;u<R;++u){
    if((u->x-x)*(u->x-x)>=d) continue;
    for(auto v=b.rbegin();v!=b.rend();++v){
      T dx=u->x-v->x, dy=u->y-v->y;
      if(dy*dy>=d) break;
      d=min(d,dx*dx+dy*dy);
    }
    b.push_back(*u);
  }
  return d;
}
T closest_pair(vector<point<T>> &v){
  sort(v.begin(),v.end(),xcmp);
  return closest_pair(v.begin(),v.end());
}
```

2 Data Structure

2.1 CDQ DP

```
#include<bits/stdc++.h>
using namespace std;
const int MAXN = 100005;
struct node{
  double a,b,r,k,x,y;
  int id;
} p[MAXN];
double DP[MAXN];
deque<int> q;
bool cmpK(const node &a,const node &b){
  return a.k>b.k;
}
bool cmpX(const node &a,const node &b){
  return a.x<b.x||(a.x==b.x&&a.y<b.y);
}
double Slope(int a,int b){
  if(!b) return -1e20;
  if(p[a].x==p[b].x) return 1e20;
  return (p[a].y-p[b].y)/(p[a].x-p[b].x);
}
void CDQ(int l, int r){
  if(l==r){
    DP[l] = max(DP[l],DP[l-1]);
    p[l].y = DP[l]/(p[l].a*p[l].r+p[l].b);
    p[l].x = p[l].y*p[l].r;
    return;
  }
  int mid = (l+r)/2;
  stable_partition(p+l,p+r+1,[&](const node &d){return d.id<=mid;});
  CDQ(l, mid); q.clear();
  for(int i=l, j; i<=mid; ++i){
```

```cpp
    while((j=q.size())>1&&Slope(q[j-2],q[j-1])<Slope(q[j-1],i)) q.pop_back();
    q.push_back(i);
  }q.push_back(0);
  for(int i=mid+1; i<=r; ++i){
    while(q.size()>1&&Slope(q[0],q[1])>p[i].k) q.pop_front();
    DP[p[i].id] = max(DP[p[i].id], p[i].a*p[q[0]].x+p[i].b*p[q[0]].y);
  }
  CDQ(mid+1,r);
  inplace_merge(p+l,p+mid+1,p+r+1,cmpX);
}
double solve(int n,double S){
  DP[0] = S;
  sort(p+1,p+1+n,cmpK);
  CDQ(1,n);
  return DP[n];
}
int main(){
  int n; double S;
  scanf("%d%lf",&n,&S);
  for(int i=1; i<=n; ++i){
    scanf("%lf%lf%lf",&p[i].a,&p[i].b,&p[i].r);
    p[i].id = i, p[i].k = -p[i].a/p[i].b;
  }
  printf("%.3lf\n",solve(n,S));
  return 0;
}
```

2.2 DLX

```cpp
const int MAXN=4100, MAXM=1030, MAXND=16390;
struct DLX{
  int n,m,sz,ansd;//高是n，寬是m的稀疏矩陣
  int S[MAXM],H[MAXN];
  int row[MAXND],col[MAXND];//每個節點代表的列跟行
  int L[MAXND],R[MAXND],U[MAXND],D[MAXND];
  vector<int> ans,anst;
  void init(int _n,int _m){
    n=_n,m=_m;
    for(int i=0;i<=m;++i){
      U[i]=D[i]=i,L[i]=i-1,R[i]=i+1;
      S[i]=0;
    }
    R[m]=0,L[0]=m;
    sz=m,ansd=INT_MAX;//ansd存最優解的個數
    for(int i=1;i<=n;++i)H[i]=-1;
  }
  void add(int r,int c){
    ++S[col[++sz]=c];
    row[sz]=r;
    D[sz]=D[c],U[D[c]]=sz,U[sz]=c,D[c]=sz;
    if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
    else R[sz]=R[H[r]],L[R[H[r]]]=sz,L[sz]=H[r],R[H[r]]=sz;
  }
  #define DFOR(i,A,s) for(int i=A[s];i!=s;i=A[i])
  void remove(int c){//刪除第c行和所有當前覆蓋到第c行的列
    L[R[c]]=L[c],R[L[c]]=R[c];//這裡刪除第c行，若有些行不需要處理可以在開始時呼叫他
    DFOR(i,D,c)DFOR(j,R,i){U[D[j]]=U[j],D[U[j]]=D[j],--S[col[j]];}
  }
  void restore(int c){//恢復第c行和所有當前覆蓋到第c行的列，remove的逆操作
    DFOR(i,U,c)DFOR(j,L,i){++S[col[j]],U[D[j]]=j,D[U[j]]=j;}
    L[R[c]]=c,R[L[c]]=c;
  }
  void remove2(int nd){//刪除nd所在的行當前所有點(包括虛擬節點)，只保留nd
    DFOR(i,D,nd)L[R[i]]=L[i],R[L[i]]=R[i];
  }
  void restore2(int nd){//刪除nd所在的行當前所有點，為remove2的逆操作
    DFOR(i,U,nd)L[R[i]]=R[L[i]]=i;
  }
  bool vis[MAXM];
  int h(){//估價函數 for IDA*
    int res=0;
    memset(vis,0,sizeof(vis));
    DFOR(i,R,0)if(!vis[i]){
      vis[i]=1;
      ++res;
      DFOR(j,D,i)DFOR(k,R,j)vis[col[k]]=1;
    }
    return res;
  }
  bool dfs(int d){//for精確覆蓋問題
    if(d+h()>=ansd)return 0;//找最佳解用，找任意解可以刪掉
    if(!R[0]){ansd=d;return 1;}
    int c=R[0];
    DFOR(i,R,0)if(S[i]<S[c])c=i;
    remove(c);
    DFOR(i,D,c){
      ans.push_back(row[i]);
      DFOR(j,R,i)remove(col[j]);
      if(dfs(d+1))return 1;
      ans.pop_back();
      DFOR(j,L,i)restore(col[j]);
    }
    restore(c);
    return 0;
  }
  void dfs2(int d){//for最小重複覆蓋問題
    if(d+h()>=ansd)return;
    if(!R[0]){ansd=d;ans=anst;return;}
    int c=R[0];
    DFOR(i,R,0)if(S[i]<S[c])c=i;
    DFOR(i,D,c){
      anst.push_back(row[i]);
      remove2(i);
      DFOR(j,R,i)remove2(j),--S[col[j]];
      dfs2(d+1);
      anst.pop_back();
      DFOR(j,L,i)restore2(j),++S[col[j]];
      restore2(i);
    }
  }
  bool exact_cover(){//解精確覆蓋問題
    return ans.clear(), dfs(0);
  }
  void min_cover(){//解最小重複覆蓋問題
    anst.clear();//暫存用，答案還是存在ans裡
    dfs2(0);
  }
  #undef DFOR
};
```

2.3 Dynamic KD tree

```cpp
template<typename T,size_t kd>//有kd個維度
struct kd_tree{
  struct point{
    T d[kd];
    T dist(const point &x)const{
      T ret=0;
      for(size_t i=0;i<kd;++i)ret+=abs(d[i]-x.d[i]);
      return ret;
    }
    bool operator==(const point &p){
      for(size_t i=0;i<kd;++i)
        if(d[i]!=p.d[i])return 0;
      return 1;
    }
    bool operator<(const point &b)const{
      return d[0]<b.d[0];
    }
  };
private:
  struct node{
    node *l,*r;
    point pid;
    int s;
    node(const point &p):l(0),r(0),pid(p),s(1){}
    ~node(){delete l,delete r;}
    void up(){s=(l?l->s:0)+1+(r?r->s:0);}
  }*root;
  const double alpha,loga;
  const T INF;//記得要給INF，表示極大值
  int maxn;
  struct __cmp{
    int sort_id;
    bool operator()(const node*x,const node*y)const{
      return operator()(x->pid,y->pid);
    }
    bool operator()(const point &x,const point &y)const{
      if(x.d[sort_id]!=y.d[sort_id])
        return x.d[sort_id]<y.d[sort_id];
      for(size_t i=0;i<kd;++i)
        if(x.d[i]!=y.d[i])return x.d[i]<y.d[i];
      return 0;
    }
  }cmp;
  int size(node *o){return o?o->s:0;}
  vector<node*> A;
  node* build(int k,int l,int r){
    if(l>r) return 0;
    if(k==kd) k=0;
    int mid=(l+r)/2;
    cmp.sort_id = k;
    nth_element(A.begin()+l,A.begin()+mid,A.begin()+r+1,cmp);
    node *ret=A[mid];
    ret->l = build(k+1,l,mid-1);
    ret->r = build(k+1,mid+1,r);
    ret->up();
    return ret;
  }
  bool isbad(node*o){
    return size(o->l)>alpha*o->s||size(o->r)>alpha*o->s;
  }
  void flatten(node *u,typename vector<node*>::iterator &it){
    if(!u)return;
    flatten(u->l,it);
    *it=u;
    flatten(u->r,++it);
  }
  void rebuild(node*&u,int k){
    if((int)A.size()<u->s)A.resize(u->s);
    auto it=A.begin();
    flatten(u,it);
    u=build(k,0,u->s-1);
  }
  bool insert(node*&u,int k,const point &x,int dep){
    if(!u) return u=new node(x), dep<=0;
    ++u->s;
    cmp.sort_id=k;
    if(insert(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x,dep-1)){
      if(!isbad(u))return 1;
      rebuild(u,k);
    }
    return 0;
  }
  node *findmin(node*o,int k){
    if(!o)return 0;
    if(cmp.sort_id==k)return o->l?findmin(o->l,(k+1)%kd):o;
    node *l=findmin(o->l,(k+1)%kd);
    node *r=findmin(o->r,(k+1)%kd);
    if(l&&!r)return cmp(l,o)?l:o;
    if(!l&&r)return cmp(r,o)?r:o;
    if(!l&&!r)return o;
    if(cmp(l,r))return cmp(l,o)?l:o;
    return cmp(r,o)?r:o;
  }
  bool erase(node *&u,int k,const point &x){
    if(!u)return 0;
    if(u->pid==x){
      if(u->r);
      else if(u->l) u->r=u->l, u->l=0;
      else return delete(u),u=0, 1;
      --u->s;
      cmp.sort_id=k;
      u->pid=findmin(u->r,(k+1)%kd)->pid;
      return erase(u->r,(k+1)%kd,u->pid);
    }
    cmp.sort_id=k;
    if(erase(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x))
      return --u->s, 1;
    return 0;
  }
  T heuristic(const T h[])const{
    T ret=0;
    for(size_t i=0;i<kd;++i)ret+=h[i];
    return ret;
  }
  int qM;
  priority_queue<pair<T,point>> pQ;
  void nearest(node *u,int k,const point &x,T *h,T &mndist){
    if(u==0||heuristic(h)>=mndist)return;
    T dist=u->pid.dist(x),old=h[k];
    /*mndist=std::min(mndist,dist);*/
    if(dist<mndist){
      pQ.push(std::make_pair(dist,u->pid));
      if((int)pQ.size()==qM+1)
        mndist=pQ.top().first,pQ.pop();
    }
    if(x.d[k]<u->pid.d[k]){
      nearest(u->l,(k+1)%kd,x,h,mndist);
      h[k] = abs(x.d[k]-u->pid.d[k]);
      nearest(u->r,(k+1)%kd,x,h,mndist);
    }else{
      nearest(u->r,(k+1)%kd,x,h,mndist);
      h[k] = abs(x.d[k]-u->pid.d[k]);
      nearest(u->l,(k+1)%kd,x,h,mndist);
    }
    h[k]=old;
  }
  vector<point>in_range;
  void range(node *u,int k,const point&mi,const point&ma){
    if(!u)return;
    bool is=1;
    for(int i=0;i<kd;++i)
      if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid.d[i])
        { is=0;break; }
    if(is) in_range.push_back(u->pid);
    if(mi.d[k]<=u->pid.d[k])range(u->l,(k+1)%kd,mi,ma);
    if(ma.d[k]>=u->pid.d[k])range(u->r,(k+1)%kd,mi,ma);
  }
public:
  kd_tree(const T &INF,double a=0.75):
  root(0),alpha(a),loga(log2(1.0/a)),INF(INF),maxn(1){}
  ~kd_tree(){delete root;}
  void clear(){delete root,root=0,maxn=1;}
  void build(int n,const point *p){
    delete root,A.resize(maxn=n);
    for(int i=0;i<n;++i)A[i]=new node(p[i]);
    root=build(0,0,n-1);
  }
  void insert(const point &x){
    insert(root,0,x,__lg(size(root))/loga);
    if(root->s>maxn)maxn=root->s;
  }
  bool erase(const point &p){
    bool d=erase(root,0,p);
    if(root&&root->s<alpha*maxn)rebuild();
    return d;
  }
  void rebuild(){
    if(root)rebuild(root,0);
    maxn=root->s;
  }
  T nearest(const point &x,int k){
    qM=k;
    T mndist=INF,h[kd]={};
    nearest(root,0,x,h,mndist);
    mndist=pQ.top().first;
    pQ = priority_queue<pair<T,point>>();
    return mndist;//回傳離x第k近的點的距離
  }
  const vector<point> &range(const point&mi,const point&ma){
    in_range.clear();
    range(root,0,mi,ma);
    return in_range;//回傳介於mi到ma之間的點vector
  }
  int size(){return root?root->s:0;}
};
```

2.4 kd tree replace segment tree

```cpp
struct node{//kd樹代替高維線段樹
  node *l,*r;
  point pid,mi,ma;
  int s, data;
  node(const point &p,int d):l(0),r(0),pid(p),mi(p),ma(p),s(1),data(d),dmin(d),dmax(d){}
  void up(){
    mi=ma=pid;
    s=1;
    if(l){
      for(int i=0;i<kd;++i){
        mi.d[i]=min(mi.d[i],l->mi.d[i]);
        ma.d[i]=max(ma.d[i],l->ma.d[i]);
      }
      s+=l->s;
    }
    if(r){
      for(int i=0;i<kd;++i){
        mi.d[i]=min(mi.d[i],r->mi.d[i]);
        ma.d[i]=max(ma.d[i],r->ma.d[i]);
      }
      s+=r->s;
    }
  }
  void up2(){/*其他懶惰標記向上更新*/}
  void down(){/*其他懶惰標記下推*/}
}*root;
//檢查區間包含用的函數
bool range_include(node *o,const point &L,const point &R){
  for(int i=0;i<kd;++i){
    if(L.d[i]>o->ma.d[i]||R.d[i]<o->mi.d[i])return 0;
  }//(L,R)區間有和o的區間有交集就回傳true
  return 1;
}
bool range_in_range(node *o,const point &L,const point &R){
  for(int i=0;i<kd;++i){
    if(L.d[i]>o->mi.d[i]||o->ma.d[i]>R.d[i])return 0;
  }//(L,R)區間完全包含o的區間就回傳true
  return 1;
}
bool point_in_range(node *o,const point &L,const point &R){
  for(int i=0;i<kd;++i){
    if(L.d[i]>o->pid.d[i]||R.d[i]<o->pid.d[i])return 0;
  }//(L,R)區間完全包含o->pid這個點就回傳true
  return 1;
}
//單點修改，以單點改值為例
void update(node *u,const point &x,int data,int k=0){
  if(!u)return;
  u->down();
  if(u->pid==x){
    u->data=data;
    u->up2();
    return;
  }
  cmp.sort_id=k;
  update(cmp(x,u->pid)?u->l:u->r,x,data,(k+1)%kd);
  u->up2();
}
//區間修改
void update(node *o,const point &L,const point &R,int data){
  if(!o)return;
  o->down();
  if(range_in_range(o,L,R)){
    //區間懶惰標記修改
    o->down();
    return;
  }
  if(point_in_range(o,L,R)){
    //這個點在(L,R)區間，但是他的左右子樹不一定在區間中
    //單點懶惰標記修改
  }
  if(o->l&&range_include(o->l,L,R))update(o->l,L,R,data);
  if(o->r&&range_include(o->r,L,R))update(o->r,L,R,data);
  o->up2();
}
//區間查詢，以總和為例
int query(node *o,const point &L,const point &R){
  if(!o)return 0;
  o->down();
  if(range_in_range(o,L,R))return o->sum;
  int ans=0;
  if(point_in_range(o,L,R))ans+=o->data;
  if(o->l&&range_include(o->l,L,R))ans+=query(o->l,L,R);
  if(o->r&&range_include(o->r,L,R))ans+=query(o->r,L,R);
  return ans;
}
```

2.5 reference point

```cpp
template<typename T>
struct _RefC{
  T data;
  int ref;
  _RefC(const T&d=0):data(d),ref(0){}
};
template<typename T>
struct _rp{
  _RefC<T> *p;
  T *operator->(){return &p->data;}
  T &operator*(){return p->data;}
  operator _RefC<T>*(){return p;}
  _rp &operator=(const _rp &t){
    if(p&&!--p->ref)delete p;
    p=t.p,p&&++p->ref;
    return *this;
  }
  _rp(_RefC<T> *t=0):p(t){p&&++p->ref;}
  _rp(const _rp &t):p(t.p){p&&++p->ref;}
  ~_rp(){if(p&&!--p->ref)delete p;}
};
template<typename T>
inline _rp<T> new_rp(const T&nd){
  return _rp<T>(new _RefC<T>(nd));
}
```

2.6 skew heap

```cpp
node *merge(node *a,node *b){
  if(!a||!b) return a?a:b;
  if(b->data<a->data) swap(a,b);
  swap(a->l,a->r);
  a->l=merge(b,a->l);
  return a;
}
```

2.7 undo disjoint set

```cpp
struct DisjointSet {
  // save() is like recursive
  // undo() is like return
  int n, fa[MXN], sz[MXN];
  vector<pair<int*,int>> h;
  vector<int> sp;
  void init(int tn) {
    n=tn;
    for (int i=0; i<n; i++) sz[fa[i]=i]=1;
    sp.clear(); h.clear();
  }
  void assign(int *k, int v) {
    h.PB({k, *k});
    *k=v;
  }
  void save() { sp.PB(SZ(h)); }
  void undo() {
    assert(!sp.empty());
    int last=sp.back(); sp.pop_back();
    while (SZ(h)!=last) {
      auto x=h.back(); h.pop_back();
      *x.F=x.S;
    }
  }
  int f(int x) {
    while (fa[x]!=x) x=fa[x];
    return x;
  }
  void uni(int x, int y) {
    x=f(x); y=f(y);
    if (x==y) return ;
    if (sz[x]<sz[y]) swap(x, y);
    assign(&sz[x], sz[x]+sz[y]);
    assign(&fa[y], x);
  }
}djs;
```

2.8 整體二分

```cpp
void totBS(int L, int R, vector<Item> M){
  if(Q.empty()) return; //維護全域B陣列
  if(L==R) 整個M的答案=r, return;
  int mid = (L+R)/2;
  vector<Item> mL, mR;
  do_modify_B_with_divide(mid,M);
  //讓B陣列在遞迴的時候只會保留[L~mid]的資訊
  undo_modify_B(mid,M);
  totBS(L,mid,mL);
  totBS(mid+1,R,mR);
}
```

3 Flow

3.1 dinic

```cpp
template<typename T>
struct DINIC{
  static const int MAXN=105;
  static const T INF=INT_MAX;
  int n, LV[MAXN], cur[MAXN];
  struct edge{
    int v,pre;
    T cap,r;
    edge(int v,int pre,T cap):v(v),pre(pre),
         cap(cap),r(cap){}
  };
  int g[MAXN];
  vector<edge> e;
  void init(int _n){
    memset(g,-1,sizeof(int)*((n=_n)+1));
    e.clear();
  }
  void add_edge(int u,int v,T cap,bool
       directed=false){
    e.push_back(edge(v,g[u],cap));
    g[u]=e.size()-1;
    e.push_back(edge(u,g[v],directed?0:cap))
         ;
    g[v]=e.size()-1;
  }
  int bfs(int s,int t){
    memset(LV,0,sizeof(int)*(n+1));
    memcpy(cur,g,sizeof(int)*(n+1));
    queue<int> q;
    q.push(s);
    LV[s]=1;
    while(q.size()){
      int u=q.front();q.pop();
      for(int i=g[u];~i;i=e[i].pre){
        if(!LV[e[i].v]&&e[i].r){
          LV[e[i].v]=LV[u]+1;
          q.push(e[i].v);
          if(e[i].v==t)return 1;
        }
      }
    }
    return 0;
  }
  T dfs(int u,int t,T CF=INF){
    if(u==t)return CF;
    T df;
    for(int &i=cur[u];~i;i=e[i].pre){
      if(LV[e[i].v]==LV[u]+1&&e[i].r){
        if(df=dfs(e[i].v,t,min(CF,e[i].r))){
          e[i].r-=df;
          e[i^1].r+=df;
          return df;
        }
      }
    }
    return LV[u]=0;
  }
  T dinic(int s,int t,bool clean=true){
    if(clean)for(size_t i=0;i<e.size();++i)
      e[i].r=e[i].cap;
    T ans=0, f=0;
    while(bfs(s,t))while(f=dfs(s,t))ans+=f;
    return ans;
  }
};
```

3.2 Gomory Hu

```cpp
//最小割樹+求任兩點間最小割
//0-base, root=0
LL e[MAXN][MAXN]; //任兩點間最小割
int p[MAXN]; //parent
ISAP D; // original graph
void gomory_hu(){
  fill(p, p+n, 0);
  fill(e[0], e[n], INF);
  for( int s = 1; s < n; ++s ) {
    int t = p[s];
    ISAP F = D;
    LL tmp = F.min_cut(s, t);
    for( int i = 1; i < s; ++i )
      e[s][i] = e[i][s] = min(tmp, e[t][i]);
    for( int i = s+1; i <= n; ++i )
      if( p[i] == t && F.vis[i] ) p[i] = s;
  }
}
```

3.3 ISAP with cut

```cpp
template<typename T>
struct ISAP{
  static const int MAXN=105;
  static const T INF=INT_MAX;
  int n;//點數
  int d[MAXN],gap[MAXN],cur[MAXN];
  struct edge{
    int v,pre;
    T cap,r;
    edge(int v,int pre,T cap):v(v),pre(pre),
         cap(cap),r(cap){}
  };
  int g[MAXN];
  vector<edge> e;
  void init(int _n){
    memset(g,-1,sizeof(int)*((n=_n)+1));
    e.clear();
  }
  void add_edge(int u,int v,T cap,bool
       directed=false){
    e.push_back(edge(v,g[u],cap));
    g[u]=e.size()-1;
    e.push_back(edge(u,g[v],directed?0:cap))
         ;
    g[v]=e.size()-1;
  }
  T dfs(int u,int s,int t,T CF=INF){
    if(u==t)return CF;
    T tf=CF,df;
    for(int &i=cur[u];~i;i=e[i].pre){
      if(e[i].r&&d[u]==d[e[i].v]+1){
        df=dfs(e[i].v,s,t,min(tf,e[i].r));
        e[i].r-=df;
        e[i^1].r+=df;
        if(!(tf-=df)||d[s]==n)return CF-tf;
      }
    }
    int mh=n;
    for(int i=cur[u]=g[u];~i;i=e[i].pre){
      if(e[i].r&&d[e[i].v]<mh)mh=d[e[i].v];
    }
    if(!--gap[d[u]])d[s]=n;
    else ++gap[d[u]=++mh];
    return CF-tf;
  }
  T isap(int s,int t,bool clean=true){
    memset(d,0,sizeof(int)*(n+1));
    memset(gap,0,sizeof(int)*(n+1));
    memcpy(cur,g,sizeof(int)*(n+1));
    if(clean) for(size_t i=0;i<e.size();++i)
      e[i].r=e[i].cap;
    T MF=0;
    for(gap[0]=n;d[s]<n;)MF+=dfs(s,s,t);
    return MF;
  }
  vector<int> cut_e;//最小割邊集
  bool vis[MAXN];
  void dfs_cut(int u){
    vis[u]=1;//表示u屬於source的最小割集
    for(int i=g[u];~i;i=e[i].pre)
      if(e[i].r>0&&!vis[e[i].v])dfs_cut(e[i
           ].v);
  }
  T min_cut(int s,int t){
    T ans=isap(s,t);
    memset(vis,0,sizeof(bool)*(n+1));
    dfs_cut(s), cut_e.clear();
    for(int u=0;u<=n;++u)if(vis[u])
      for(int i=g[u];~i;i=e[i].pre)
        if(!vis[e[i].v])cut_e.push_back(i);
    return ans;
  }
};
```

3.4 MinCostMaxFlow

```cpp
template<typename TP>
struct MCMF{
  static const int MAXN=440;
  static const TP INF=999999999;
  struct edge{
    int v,pre;
    TP r,cost;
    edge(int v,int pre,TP r,TP cost):v(v),
         pre(pre),r(r),cost(cost){}
  };
  int n,S,T;
  TP dis[MAXN],PIS,ans;
  bool vis[MAXN];
  vector<edge> e;
  int g[MAXN];
  void init(int _n){
    memset(g,-1,sizeof(int)*((n=_n)+1));
    e.clear();
  }
  void add_edge(int u,int v,TP r,TP cost,
       bool directed=false){
    e.push_back(edge(v,g[u],r,cost));
    g[u]=e.size()-1;
    e.push_back(
    edge(u,g[v],directed?0:r,-cost));
    g[v]=e.size()-1;
  }
  TP augment(int u,TP CF){
    if(u==T||!CF)return ans+=PIS*CF,CF;
    vis[u]=1;
    TP r=CF,d;
    for(int i=g[u];~i;i=e[i].pre){
      if(e[i].r&&!e[i].cost&&!vis[e[i].v]){
        d=augment(e[i].v,min(r,e[i].r));
        e[i].r-=d;
        e[i^1].r+=d;
        if(!(r-=d))break;
      }
    }
    return CF-r;
  }
  bool modlabel(){
    for(int u=0;u<=n;++u)dis[u]=INF;
    static deque<int>q;
    dis[T]=0,q.push_back(T);
    while(q.size()){
      int u=q.front();q.pop_front();
      TP dt;
      for(int i=g[u];~i;i=e[i].pre){
        if(e[i^1].r&&(dt=dis[u]-e[i].cost)<
             dis[e[i].v]){
          if((dis[e[i].v]=dt)<=dis[q.size()?
               q.front():S]){
            q.push_front(e[i].v);
          }else q.push_back(e[i].v);
        }
      }
    }
    for(int u=0;u<=n;++u)
      for(int i=g[u];~i;i=e[i].pre)
        e[i].cost+=dis[e[i].v]-dis[u];
    return PIS+=dis[S], dis[S]<INF;
  }
  TP mincost(int s,int t){
    S=s,T=t;
    PIS=ans=0;
    while(modlabel()){
      do memset(vis,0,sizeof(bool)*(n+1));
      while(augment(S,INF));
    }return ans;
  }
};
```

4 Graph

4.1 Augmenting Path

```
#define MAXN1 505
#define MAXN2 505
int n1,n2;//n1個點連向n2個點
int match[MAXN2];//屬於n2的點匹配了哪個點
vector<int > g[MAXN1];//圖 0-base
bool vis[MAXN2];//是否走訪過
bool dfs(int u){
  for(int v:g[u]){
    if(vis[v]) continue;
    vis[v]=1;
    if(match[v]==-1||dfs(match[v]))
      return match[v]=u, 1;
  }
  return 0;
}
int max_match(){
  int ans=0;
  memset(match,-1,sizeof(int)*n2);
  for(int i=0;i<n1;++i){
    memset(vis,0,sizeof(bool)*n2);
    if(dfs(i)) ++ans;
  }
  return ans;
}
```

4.2 Augmenting Path multiple

```
#define MAXN1 1005
#define MAXN2 505
int n1,n2;
//n1個點連向n2個點，其中n2個點可以匹配很多邊
vector<int> g[MAXN1];//圖 0-base
size_t c[MAXN2];
//每個屬於n2點最多可以接受幾條匹配邊
vector<int> matchs[MAXN2];
//每個屬於n2的點匹配了那些點
bool vis[MAXN2];
bool dfs(int u){
  for(int v:g[u]){
    if(vis[v])continue;
    vis[v] = 1;
    if(matchs[v].size()<c[v]){
      return matchs[v].push_back(u), 1;
    }else for(size_t j=0;j<matchs[v].size()
         ;++j){
      if(dfs(matchs[v][j]))
        return matchs[v][j]=u, 1;
    }
  }
  return 0;
}
int max_match(){
  for(int i=0;i<n2;++i) matchs[i].clear();
  int cnt=0;
  for(int u=0;u<n1;++u){
    memset(vis,0,sizeof(bool)*n2);
    if(dfs(u))++cnt;
  }
  return cnt;
}
```

4.3 blossom matching

```
#define MAXN 505
int n; //1-base
vector<int> g[MAXN];
int MH[MAXN]; //output MH
int pa[MAXN],st[MAXN],S[MAXN],v[MAXN],t;
int lca(int x,int y){
  for(++t;;swap(x,y)){
    if(!x) continue;
    if(v[x]==t) return x;
    v[x] = t;
    x = st[pa[MH[x]]];
  }
}
#define qpush(x) q.push(x),S[x]=0
void flower(int x,int y,int l,queue<int>&q){
  while(st[x]!=l){
    pa[x]=y;
    if(S[y=MH[x]]==1)qpush(y);
    st[x]=st[y]=l, x=pa[y];
  }
}
bool bfs(int x){
  iota(st+1, st+n+1, 1);
  memset(S+1,-1,sizeof(int)*n);
  queue<int>q; qpush(x);
  while(q.size()){
    x=q.front(),q.pop();
    for(int y:g[x]){
      if(S[y]==-1){
        pa[y]=x,S[y]=1;
        if(!MH[y]){
          for(int lst;x;y=lst,x=pa[y])
            lst=MH[x],MH[x]=y,MH[y]=x;
          return 1;
        }
        qpush(MH[y]);
      }else if(!S[y]&&st[y]!=st[x]){
        int l=lca(y,x);
        flower(y,x,l,q),flower(x,y,l,q);
      }
    }
  }
  return 0;
}
int blossom(){
  memset(MH+1,0,sizeof(int)*n);
  int ans=0;
  for(int i=1; i<=n; ++i)
    if(!MH[i]&&bfs(i)) ++ans;
  return ans;
}
```

4.4 BronKerbosch

```
struct maximalCliques{
  using Set = vector<int>;
  size_t n; //1-base
  vector<Set> G;
  static Set setUnion(const Set &A, const
       Set &B){
    Set C(A.size() + B.size());
    auto it = set_union(A.begin(),A.end(),B.
         begin(),B.end(),C.begin());
    C.erase(it, C.end());
    return C;
  }
  static Set setIntersection(const Set &A,
       const Set &B){
    Set C(min(A.size(), B.size()));
    auto it = set_intersection(A.begin(),A.
         end(),B.begin(),B.end(),C.begin());
    C.erase(it, C.end());
    return C;
  }
  static Set setDifference(const Set &A,
       const Set &B){
    Set C(min(A.size(), B.size()));
    auto it = set_difference(A.begin(),A.end
         (),B.begin(),B.end(),C.begin());
    C.erase(it, C.end());
    return C;
  }
  void BronKerbosch1(Set R, Set P, Set X){
    if(P.empty()&&X.empty()){
      // R form an maximal clique
      return;
    }
    for(auto v: P){
      BronKerbosch1(setUnion(R,{v}),
           setIntersection(P,G[v]),
           setIntersection(X,G[v]));
      P = setDifference(P,{v});
      X = setUnion(X,{v});
    }
  }
  void init(int _n){
    G.clear();
    G.resize((n = _n) + 1);
  }
  void addEdge(int u, int v){
    G[u].emplace_back(v);
    G[v].emplace_back(u);
  }
  void solve(int n){
    Set P;
    for(int i=1; i<=n; ++i){
      sort(G[i].begin(), G[i].end());
G[i].erase(unique(G[i].begin(), G[i].end()),
       G[i].end());
      P.emplace_back(i);
    }
    BronKerbosch1({}, P, {});
  }
};
```

4.5 graphISO

```
const int MAXN=1005,K=30;//K要夠大
const long long A=3,B=11,C=2,D=19,P=0
     xdefaced;
long long f[K+1][MAXN];
vector<int> g[MAXN],rg[MAXN];
int n;
void init(){
  for(int i=0;i<n;++i){
    f[0][i]=1;
    g[i].clear(), rg[i].clear();
  }
}
void add_edge(int u,int v){
  g[u].push_back(v), rg[v].push_back(u);
}
long long point_hash(int u){//O(N)
  for(int t=1;t<=K;++t){
    for(int i=0;i<n;++i){
      f[t][i]=f[t-1][i]*A%P;
      for(int j:g[i])f[t][i]=(f[t][i]+f[t-1][j]*B%P)%P;
      for(int j:rg[i])f[t][i]=(f[t][i]+f[t-1][j]*C%P)%P;
      if(i==u)f[t][i]+=D;//如果圖太大的話，把這行刪掉，執行一次後f[K]就會是所有點的答案
      f[t][i]%=P;
    }
  }
  return f[K][u];
}
vector<long long> graph_hash(){
  vector<long long> ans;
  for(int i=0;i<n;++i)ans.push_back(point_hash(i));//O(N^2)
  sort(ans.begin(),ans.end());
  return ans;
}
```

4.6 KM

```
#define MAXN 405
#define INF 0x3f3f3f3f3f3f3f3f
int n;// 1-base，0表示沒有匹配
LL g[MAXN][MAXN]; //input graph
int My[MAXN],Mx[MAXN]; //output match
LL lx[MAXN],ly[MAXN],pa[MAXN],Sy[MAXN];
bool vx[MAXN],vy[MAXN];
void augment(int y){
  for(int x, z; y; y = z){
    x=pa[y],z=Mx[x];
    My[y]=x,Mx[x]=y;
  }
}
void bfs(int st){
  for(int i=1; i<=n; ++i)
    Sy[i] = INF, vx[i]=vy[i]=0;
  queue<int> q; q.push(st);
  for(;;){
    while(q.size()){
      int x=q.front(); q.pop();
      vx[x]=1;
      for(int y=1; y<=n; ++y) if(!vy[y]){
        LL t = lx[x]+ly[y]-g[x][y];
        if(t==0){
          pa[y]=x;
          if(!My[y]){augment(y);return;}
          vy[y]=1,q.push(My[y]);
        }else if(Sy[y]>t) pa[y]=x,Sy[y]=t;
      }
    }
    LL cut = INF;
    for(int y=1; y<=n; ++y)
      if(!vy[y]&&cut>Sy[y]) cut=Sy[y];
    for(int j=1; j<=n; ++j){
      if(vx[j]) lx[j] -= cut;
      if(vy[j]) ly[j] += cut;
      else Sy[j] -= cut;
    }
    for(int y=1; y<=n; ++y){
      if(!vy[y]&&Sy[y]==0){
        if(!My[y]){augment(y);return;}
        vy[y]=1, q.push(My[y]);
      }
    }
  }
}
LL KM(){
  memset(My,0,sizeof(int)*(n+1));
  memset(Mx,0,sizeof(int)*(n+1));
  memset(ly,0,sizeof(LL)*(n+1));
  for(int x=1; x<=n; ++x){
    lx[x] = -INF;
    for(int y=1; y<=n; ++y)
      lx[x] = max(lx[x],g[x][y]);
  }
  for(int x=1; x<=n; ++x) bfs(x);
  LL ans = 0;
  for(int y=1; y<=n; ++y) ans+=g[My[y]][y];
  return ans;
}
```

4.7 MaximumClique

```
struct MaxClique{
  static const int MAXN=105;
  int N,ans;
  int g[MAXN][MAXN],dp[MAXN],stk[MAXN][MAXN];
  int sol[MAXN],tmp[MAXN];//sol[0~ans-1]為答案
  void init(int n){
    N=n;//0-base
    memset(g,0,sizeof(g));
  }
  void add_edge(int u,int v){
    g[u][v]=g[v][u]=1;
  }
  int dfs(int ns,int dep){
    if(!ns){
      if(dep>ans){
        ans=dep;
        memcpy(sol,tmp,sizeof tmp);
        return 1;
      }else return 0;
    }
    for(int i=0;i<ns;++i){
      if(dep+ns-i<=ans)return 0;
      int u=stk[dep][i],cnt=0;
      if(dep+dp[u]<=ans)return 0;
      for(int j=i+1;j<ns;++j){
        int v=stk[dep][j];
        if(g[u][v])stk[dep+1][cnt++]=v;
      }
      tmp[dep]=u;
      if(dfs(cnt,dep+1))return 1;
    }
    return 0;
  }
  int clique(){
    int u,v,ns;
    for(ans=0,u=N-1;u>=0;--u){
      for(ns=0,tmp[0]=u,v=u+1;v<N;++v)
        if(g[u][v])stk[1][ns++]=v;
      dfs(ns,1),dp[u]=ans;
    }
    return ans;
  }
};
```

4.8 MinimumMeanCycle

```
#include<cfloat> //for DBL_MAX
int dp[MAXN][MAXN]; // 1-base,O(NM)
vector<tuple<int,int,int>> edge;
double mmc(int n){//allow negative weight
  const int INF=0x3f3f3f3f;
  for(int t=0;t<n;++t){
    memset(dp[t+1],0x3f,sizeof(dp[t+1]));
    for(const auto &e:edge){
      int u,v,w;
      tie(u,v,w) = e;
      dp[t+1][v]=min(dp[t+1][v],dp[t][u]+w);
    }
  }
  double res = DBL_MAX;
  for(int u=1;u<=n;++u){
    if(dp[n][u]==INF) continue;
    double val = -DBL_MAX;
    for(int t=0;t<n;++t)
      val=max(val,(dp[n][u]-dp[t][u])*1.0/(n-t));
    res=min(res,val);
  }
  return res;
}
```

4.9 Rectilinear MST

```
//平面曼哈頓最小生成樹構造圖(去除非必要邊)
#define T int
#define INF 0x3f3f3f3f
struct point{
  T x,y;
  int id;//從0開始編號
  point(){}
  T dist(const point &p)const{
    return abs(x-p.x)+abs(y-p.y);
  }
};
bool cmpx(const point &a,const point &b){
  return a.x<b.x||(a.x==b.x&&a.y<b.y);
}
struct edge{
  int u,v;
  T cost;
  edge(int u,int v,T c):u(u),v(v),cost(c){}
  bool operator<(const edge&e)const{
    return cost<e.cost;
  }
};
struct bit_node{
  T mi;
  int id;
  bit_node(const T&mi=INF,int id=-1):mi(mi),id(id){}
};
vector<bit_node> bit;
void bit_update(int i,const T&data,int id){
  for(;i;i-=i&(-i)){
    if(data<bit[i].mi)bit[i]=bit_node(data,id);
  }
}
int bit_find(int i,int m){
  bit_node x;
  for(;i<=m;i+=i&(-i)) if(bit[i].mi<x.mi)x=bit[i];
  return x.id;
}
vector<edge> build_graph(int n,point p[]){
  vector<edge> e;//edge for MST
  for(int dir=0;dir<4;++dir){//4種座標變換
    if(dir%2) for(int i=0;i<n;++i) swap(p[i].x,p[i].y);
    else if(dir==2) for(int i=0;i<n;++i) p[i].x=-p[i].x;
    sort(p,p+n,cmpx);
    vector<T> ga(n), gb;
    for(int i=0;i<n;++i)ga[i]=p[i].y-p[i].x;
    gb=ga, sort(gb.begin(),gb.end());
    gb.erase(unique(gb.begin(),gb.end()),gb.end());
    int m=gb.size();
    bit=vector<bit_node>(m+1);
    for(int i=n-1;i>=0;--i){
      int pos=lower_bound(gb.begin(),gb.end(),ga[i])-gb.begin()+1;
      int ans=bit_find(pos,m);
      if(~ans)e.push_back(edge(p[i].id,p[ans].id,p[i].dist(p[ans])));
      bit_update(pos,p[i].x+p[i].y,i);
    }
  }
  return e;
}
```

4.10 treeISO

```
const int MAXN=100005;
const long long X=12327,P=0xdefaced;
vector<int> g[MAXN];
bool vis[MAXN];
long long dfs(int u){//hash ver
  vis[u]=1;
  vector<long long> tmp;
  for(auto v:g[u])if(!vis[v])tmp.PB(dfs(v));
  if(tmp.empty())return 177;
  long long ret=4931;
  sort(tmp.begin(),tmp.end());
  for(auto v:tmp)ret=((ret*X)^v)%P;
  return ret;
}
//-----------------------------
string dfs(int x,int p){
  vector<string> c;
  for(int y:g[x])
    if(y!=p)c.emplace_back(dfs(y,x));
  sort(c.begin(),c.end());
  string ret("(");
  for(auto &s:c)ret+=s;
  ret+=")";
  return ret;
}
```

4.11 一般圖最小權完美匹配

```
struct Graph {
  // Minimum General Weighted Matching (Perfect Match) 0-base
  static const int MXN = 105;
  int n, edge[MXN][MXN];
  int match[MXN],dis[MXN],onstk[MXN];
  vector<int> stk;
  void init(int _n) {
    n = _n;
    for (int i=0; i<n; i++)
      for (int j=0; j<n; j++)
        edge[i][j] = 0;
  }
  void add_edge(int u, int v, int w) {
    edge[u][v] = edge[v][u] = w;
  }
  bool SPFA(int u){
    if (onstk[u]) return true;
    stk.push_back(u);
    onstk[u] = 1;
    for (int v=0; v<n; v++){
      if (u != v && match[u] != v && !onstk[v]){
        int m = match[v];
        if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
          dis[m] = dis[u] - edge[v][m] + edge[u][v];
          onstk[v] = 1;
          stk.push_back(v);
          if (SPFA(m)) return true;
          stk.pop_back();
          onstk[v] = 0;
        }
      }
    }
    onstk[u] = 0;
    stk.pop_back();
    return false;
  }
  int solve() {
    // find a match
    for (int i=0; i<n; i+=2){
      match[i] = i+1, match[i+1] = i;
    }
    for(;;){
      int found = 0;
      for (int i=0; i<n; i++) dis[i] = onstk[i] = 0;
      for (int i=0; i<n; i++){
        stk.clear();
        if (!onstk[i] && SPFA(i)){
          found = 1;
          while (stk.size()>=2){
            int u = stk.back(); stk.pop_back();
            int v = stk.back(); stk.pop_back();
            match[u] = v;
            match[v] = u;
          }
        }
      }
      if (!found) break;
    }
    int ret = 0;
    for (int i=0; i<n; i++)
      ret += edge[i][match[i]];
    ret /= 2;
    return ret;
  }
}graph;
```

4.12 全局最小割

```
const int INF=0x3f3f3f3f;
template<typename T>
struct stoer_wagner{// 0-base
  static const int MAXN=150;
  T g[MAXN][MAXN],dis[MAXN];
  int nd[MAXN],n,s,t;
  void init(int _n){
    n=_n;
    for(int i=0;i<n;++i)
      for(int j=0;j<n;++j)g[i][j]=0;
  }
  void add_edge(int u,int v,T w){
    g[u][v]=g[v][u]+=w;
  }
  T min_cut(){
    T ans=INF;
    for(int i=0;i<n;++i)nd[i]=i;
    for(int ind,tn=n;tn>1;--tn){
      for(int i=1;i<tn;++i)dis[nd[i]]=0;
      for(int i=1;i<tn;++i){
        ind=i;
        for(int j=i;j<tn;++j){
          dis[nd[j]]+=g[nd[i-1]][nd[j]];
          if(dis[nd[ind]]<dis[nd[j]])ind=j;
        }
        swap(nd[ind],nd[i]);
      }
      if(ans>dis[nd[ind]])ans=dis[t=nd[ind]],s=nd[ind-1];
      for(int i=0;i<tn;++i)
        g[nd[ind-1]][nd[i]]=g[nd[i]][nd[ind-1]]+=g[nd[i]][nd[ind]];
    }
    return ans;
  }
};
```

4.13 弦圖完美消除序列

```
struct chordal{
  static const int MAXN=1005;
  int n;// 0-base
  vector<int>G[MAXN];
  int rank[MAXN],label[MAXN];
  bool mark[MAXN];
  void init(int _n){n=_n;
    for(int i=0;i<n;++i)G[i].clear();
  }
  void add_edge(int u,int v){
    G[u].push_back(v);
    G[v].push_back(u);
  }
  vector<int> MCS(){
    memset(rank,-1,sizeof(int)*n);
    memset(label,0,sizeof(int)*n);
    priority_queue<pair<int,int> > pq;
    for(int i=0;i<n;++i)pq.push(make_pair(0,i));
    for(int i=n-1;i>=0;--i)for(;;){
      int u=pq.top().second;pq.pop();
      if(~rank[u])continue;
      rank[u]=i;
      for(auto v:G[u])if(rank[v]==-1){
        pq.push(make_pair(++label[v],v));
      }
      break;
    }
    vector<int> res(n);
    for(int i=0;i<n;++i)res[rank[i]]=i;
    return res;
  }
  bool check(vector<int> ord){//弦圖判定
    for(int i=0;i<n;++i)rank[ord[i]]=i;
    memset(mark,0,sizeof(bool)*n);
    for(int i=0;i<n;++i){
      vector<pair<int,int> > tmp;
      for(auto u:G[ord[i]])if(!mark[u])
        tmp.push_back(make_pair(rank[u],u));
      sort(tmp.begin(),tmp.end());
      if(tmp.size()){
        int u=tmp[0].second;
        set<int> S;
        for(auto v:G[u])S.insert(v);
        for(size_t j=1;j<tmp.size();++j)
          if(!S.count(tmp[j].second))return 0;
      }
      mark[ord[i]]=1;
    }
    return 1;
  }
};
```

4.14 最小斯坦納樹 DP

```
//n個點，其中r個要構成斯坦納樹
//答案在max(dp[(1<<r)-1][k]) k=0~n-1
//p表示要構成斯坦納樹的點集
//O( n^3 + n*3^r + n^2*2^r )
#define REP(i,n) for(int i=0;i<(int)n;++i)
const int MAXN=30,MAXM=8;// 0-base
const int INF=0x3f3f3f3f;
int dp[1<<MAXM][MAXN];
int g[MAXN][MAXN];//圖
void init(){memset(g,0x3f,sizeof(g));}
void add_edge(int u,int v,int w){
  g[u][v]=g[v][u]=min(g[v][u],w);
}
void steiner(int n,int r,int *p){
  REP(k,n)REP(i,n)REP(j,n)
    g[i][j]=min(g[i][j],g[i][k]+g[k][j]);
  REP(i,n)g[i][i]=0;
  REP(i,r)REP(j,n)dp[1<<i][j]=g[p[i]][j];
  for(int i=1;i<(1<<r);++i){
    if(!(i&(i-1)))continue;
    REP(j,n)dp[i][j]=INF;
    REP(j,n){
      int tmp=INF;
      for(int s=i&(i-1);s;s=i&(s-1))
        tmp=min(tmp,dp[s][j]+dp[i^s][j]);
      REP(k,n)dp[i][k]=min(dp[i][k],g[j][k]+tmp);
    }
  }
}
```

4.15 最小樹形圖朱劉

```
template<typename T>
struct zhu_liu{
  static const int MAXN=110,MAXM=10005;
  struct node{
    int u,v;
    T w,tag;
    node *l,*r;
    node(int u=0,int v=0,T w=0):u(u),v(v),w(w),tag(0),l(0),r(0){}
    void down(){
      w+=tag;
      if(l)l->tag+=tag;
      if(r)r->tag+=tag;
      tag=0;
    }
  }mem[MAXM];//靜態記憶體
  node *pq[MAXN*2],*E[MAXN*2];
  int st[MAXN*2],id[MAXN*2],m;
  void init(int n){
    for(int i=1;i<=n;++i){
      pq[i]=E[i]=0, st[i]=id[i]=i;
    }m=0;
  }
  node *merge(node *a,node *b){//skew heap
    if(!a||!b)return a?a:b;
    a->down(),b->down();
    if(b->w<a->w)return merge(b,a);
    swap(a->l,a->r);
    a->l=merge(b,a->l);
    return a;
  }
  void add_edge(int u,int v,T w){
```

```cpp
    if(u!=v)pq[v]=merge(pq[v],&(mem[m++]=node(u,v,w)));
  }
  int find(int x,int *st){
    return st[x]==x?x:st[x]=find(st[x],st);
  }
  T build(int root,int n){
    T ans=0;int N=n,all=n;
    for(int i=1;i<=N;++i){
      if(i==root||!pq[i])continue;
      while(pq[i]){
        pq[i]->down(),E[i]=pq[i];
        pq[i]=merge(pq[i]->l,pq[i]->r);
        if(find(E[i]->u,id)!=find(i,id))break;
      }
      if(find(E[i]->u,id)==find(i,id))continue;
      ans+=E[i]->w;
      if(find(E[i]->u,st)==find(i,st)){
        if(pq[i])pq[i]->tag-=E[i]->w;
        pq[++N]=pq[i];id[N]=N;
        for(int u=find(E[i]->u,id);u!=i;u=find(E[u]->u,id)){
          if(pq[u])pq[u]->tag-=E[u]->w;
          id[find(u,id)]=N;
          pq[N]=merge(pq[N],pq[u]);
        }
        st[N]=find(i,st);
        id[find(i,id)]=N;
      }else st[find(i,st)]=find(E[i]->u,st),--all;
    }
    return all==1?ans:-INT_MAX;//圖不連通就無解
  }
};
```

4.16 穩定婚姻模板

```cpp
queue<int> Q;
for ( i : 所有考生 ) {
  設定在第0志願;
  Q.push(考生i);
}
while(Q.size()){
  當前考生=Q.front();Q.pop();
  while ( 此考生未分發 ) {
    指標移到下一志願;
    if ( 已經沒有志願 or 超出志願總數 ) break;
    計算該考生在該科系加權後的總分;
    if ( 不符合科系需求 ) continue;
    if ( 目前科系有餘額 ) {
      依加權後分數高低順序將考生id加入科系錄取名單中;
      break;
    }
    if ( 目前科系已額滿 ) {
      if ( 此考生成績比最低分數還高 ) {
        依加權後分數高低順序將考生id加入科系錄取名單;
        Q.push(被踢出的考生);
      }
    }
  }
}
```

5 Language

5.1 CNF

```cpp
#define MAXN 55
struct CNF{
  int s,x,y;//s->xy | s->x, if y==-1
  int cost;
  CNF(){}
  CNF(int s,int x,int y,int c):s(s),x(x),y(y),cost(c){}
};
int state;//規則數量
map<char,int> rule;//每個字元對應到的規則，小寫字母為終端字符
vector<CNF> cnf;
void init(){
  state=0;
  rule.clear();
  cnf.clear();
}
void add_to_cnf(char s,const string &p,int cost){
  //加入一個s -> <p>的文法，代價為cost
  if(rule.find(s)==rule.end())rule[s]=state++;
  for(auto c:p)if(rule.find(c)==rule.end())rule[c]=state++;
  if(p.size()==1){
    cnf.push_back(CNF(rule[s],rule[p[0]],-1,cost));
  }else{
    int left=rule[s];
    int sz=p.size();
    for(int i=0;i<sz-2;++i){
      cnf.push_back(CNF(left,rule[p[i]],state,0));
      left=state++;
    }
    cnf.push_back(CNF(left,rule[p[sz-2]],rule[p[sz-1]],cost));
  }
}
vector<long long> dp[MAXN][MAXN];
vector<bool> neg_INF[MAXN][MAXN];//如果花費是負的可能會有無限小的情形
void relax(int l,int r,const CNF &c,long long cost,bool neg_c=0){
  if(!neg_INF[l][r][c.s]&&(neg_INF[l][r][c.x]||cost<dp[l][r][c.s])){
    if(neg_c||neg_INF[l][r][c.x]){
      dp[l][r][c.s]=0;
      neg_INF[l][r][c.s]=true;
    }else dp[l][r][c.s]=cost;
  }
}
void bellman(int l,int r,int n){
  for(int k=1;k<=state;++k)
    for(auto c:cnf)
      if(c.y==-1)relax(l,r,c,dp[l][r][c.x]+c.cost,k==n);
}
void cyk(const vector<int> &tok){
  for(int i=0;i<(int)tok.size();++i){
    for(int j=0;j<(int)tok.size();++j){
      dp[i][j]=vector<long long>(state+1,INT_MAX);
      neg_INF[i][j]=vector<bool>(state+1,false);
    }
    dp[i][i][tok[i]]=0;
    bellman(i,i,tok.size());
  }
  for(int r=1;r<(int)tok.size();++r){
    for(int l=r-1;l>=0;--l){
      for(int k=l;k<r;++k)
        for(auto c:cnf)
          if(~c.y)relax(l,r,c,dp[l][k][c.x]+dp[k+1][r][c.y]+c.cost);
      bellman(l,r,tok.size());
    }
  }
}
```

6 Linear Programming

6.1 simplex

```cpp
/*target:
  max \sum_{j=1}^n A_{0,j}*x_j
condition:
  \sum_{j=1}^n A_{i,j}*x_j <= A_{i,0} |i=1~m
  x_j >= 0 |j=1~n
VDB = vector<double>*/
template<class VDB>
VDB simplex(int m,int n,vector<VDB> a){
  vector<int> left(m+1), up(n+1);
  iota(left.begin(), left.end(), n);
  iota(up.begin(), up.end(), 0);
  auto pivot = [&](int x, int y){
    swap(left[x], up[y]);
    auto k = a[x][y]; a[x][y] = 1;
    vector<int> pos;
    for(int j = 0; j <= n; ++j){
      a[x][j] /= k;
      if(a[x][j] != 0) pos.push_back(j);
    }
    for(int i = 0; i <= m; ++i){
      if(a[i][y]==0 || i == x) continue;
      k = a[i][y], a[i][y] = 0;
      for(int j : pos) a[i][j] -= k*a[x][j];
    }
  };
  for(int x,y;;){
    for(int i=x=1; i <= m; ++i)
      if(a[i][0]<a[x][0]) x = i;
    if(a[x][0]>=0) break;
    for(int j=y=1; j <= n; ++j)
      if(a[x][j]<a[x][y]) y = j;
    if(a[x][y]>=0) return VDB();//infeasible
    pivot(x, y);
  }
  for(int x,y;;){
    for(int j=y=1; j <= n; ++j)
      if(a[0][j] > a[0][y]) y = j;
    if(a[0][y]<=0) break;
    x = -1;
    for(int i=1; i<=m; ++i) if(a[i][y] > 0)
      if(x == -1 || a[i][0]/a[i][y]
        < a[x][0]/a[x][y]) x = i;
    if(x == -1) return VDB();//unbounded
    pivot(x, y);
  }
  VDB ans(n + 1);
  for(int i = 1; i <= m; ++i)
    if(left[i] <= n) ans[left[i]] = a[i][0];
  ans[0] = -a[0][0];
  return ans;
}
```

7 Number Theory

7.1 basic

```cpp
template<typename T>
void gcd(const T &a,const T &b,T &d,T &x,T &y){
  if(!b) d=a,x=1,y=0;
  else gcd(b,a%b,d,y,x), y-=x*(a/b);
}
long long int phi[N+1];
void phiTable(){
  for(int i=1;i<=N;i++)phi[i]=i;
  for(int i=1;i<=N;i++)for(x=i*2;x<=N;x+=i)phi[x]-=phi[i];
}
void all_divdown(const LL &n) {// all n/x
  for(LL a=1;a<=n;a=n/(n/(a+1))){
    // dosomething;
  }
}
const int MAXPRIME = 1000000;
int iscom[MAXPRIME], prime[MAXPRIME], primecnt;
int phi[MAXPRIME], mu[MAXPRIME];
void sieve(void){
  memset(iscom,0,sizeof(iscom));
  primecnt = 0;
  phi[1] = mu[1] = 1;
  for(int i=2;i<MAXPRIME;++i) {
    if(!iscom[i]) {
      prime[primecnt++] = i;
      mu[i] = -1;
      phi[i] = i-1;
    }
    for(int j=0;j<primecnt;++j) {
      int k = i * prime[j];
      if(k>=MAXPRIME) break;
      iscom[k] = prime[j];
      if(i%prime[j]==0) {
        mu[k] = 0;
        phi[k] = phi[i] * prime[j];
        break;
      } else {
        mu[k] = -mu[i];
        phi[k] = phi[i] * (prime[j]-1);
      }
    }
  }
}

bool g_test(const LL &g, const LL &p, const
     vector<LL> &v) {
  for(int i=0;i<v.size();++i)
    if(modexp(g,(p-1)/v[i],p)==1)
      return false;
  return true;
}
LL primitive_root(const LL &p) {
  if(p==2) return 1;
  vector<LL> v;
  Factor(p-1,v);
  v.erase(unique(v.begin(), v.end()), v.end
       ());
  for(LL g=2;g<p;++g)
    if(g_test(g,p,v))
      return g;
  puts("primitive_root NOT FOUND");
  return -1;
}
int Legendre(const LL &a, const LL &p) {
     return modexp(a%p,(p-1)/2,p); }

LL inv(const LL &a, const LL &n) {
  LL d,x,y;
  gcd(a,n,d,x,y);
  return d==1 ? (x+n)%n : -1;
}

int inv[maxN];
LL invtable(int n,LL P){
  inv[1]=1;
  for(int i=2;i<n;++i)
    inv[i]=(P-(P/i))*inv[P%i]%P;
}

LL log_mod(const LL &a, const LL &b, const
     LL &p) {
  // a ^ x = b ( mod p )
  int m=sqrt(p+.5), e=1;
  LL v=inv(modexp(a,m,p), p);
  map<LL,int> x;
  x[1]=0;
  for(int i=1;i<m;++i) {
    e = LLmul(e,a,p);
    if(!x.count(e)) x[e] = i;
  }
  for(int i=0;i<m;++i) {
    if(x.count(b)) return i*m + x[b];
    b = LLmul(b,v,p);
  }
  return -1;
}

LL Tonelli_Shanks(const LL &n, const LL &p)
     {
  // x^2 = n ( mod p )
  if(n==0) return 0;
  if(Legendre(n,p)!=1) while(1) { puts("SQRT
       ROOT does not exist"); }
  int S = 0;
  LL Q = p-1;
  while( !(Q&1) ) { Q>>=1; ++S; }
  if(S==1) return modexp(n%p,(p+1)/4,p);
  LL z = 2;
  for(;Legendre(z,p)!=-1;++z)
  LL c = modexp(z,Q,p);
  LL R = modexp(n%p,(Q+1)/2,p), t = modexp(n
       %p,Q,p);
  int M = S;
  while(1) {
    if(t==1) return R;
    LL b = modexp(c,1L<<(M-i-1),p);
    R = LLmul(R,b,p);
    t = LLmul( LLmul(b,b,p), t, p);
    c = LLmul(b,b,p);
    M = i;
  }
  return -1;
}

template<typename T>
T Euler(T n){
  T ans=n;
  for(T i=2;i*i<=n;++i){
    if(n%i==0){
      ans=ans/i*(i-1);
      while(n%i==0)n/=i;
    }
  }
  if(n>1)ans=ans/n*(n-1);
  return ans;
}

//Chinese_remainder_theorem
template<typename T>
T pow_mod(T n,T k,T m){
  T ans=1;
  for(n=(n>=m?n%m:n);k;k>>=1){
    if(k&1)ans=ans*n%m;
    n=n*n%m;
  }
  return ans;
}
template<typename T>
T crt(vector<T> &m,vector<T> &a){
  T M=1,tM,ans=0;
  for(int i=0;i<(int)m.size();++i)M*=m[i];
  for(int i=0;i<(int)a.size();++i){
    tM=M/m[i];
    ans=(ans+(a[i]*tM%M)*pow_mod(tM,Euler(m[
         i])-1,m[i])%M)%M;
    /*如果m[i]是質數，Euler(m[i])-1=m[i]-2，
         就不用算Euler了*/
  }
  return ans;
}

//java code
//求sqrt(N)的連分數
public static void Pell(int n){
  BigInteger N,p1,p2,q1,q2,a0,a1,a2,g1,g2,h1
       ,h2,p,q;
  g1=q2=p1=BigInteger.ZERO;
  h1=q1=p2=BigInteger.ONE;
  a0=a1=BigInteger.valueOf((int)Math.sqrt
       (1.0*n));
  BigInteger ans=a0.multiply(a0);
  if(ans.equals(BigInteger.valueOf(n))){
    System.out.println("No solution!");
    return ;
  }
  while(true){
    g2=a1.multiply(h1).substract(g1);
    h2=N.substract(g2.pow(2)).divide(h1);
    a2=g2.add(a0).divide(h2);
    p=a1.multiply(p2).add(p1);
    q=a1.multiply(q2).add(q1);
    if(p.pow(2).substract(N.multiply(q.pow
         (2))).compareTo(BigInteger.ONE)==0)
         break;
    g1=g2;h1=h2;a1=a2;
    p1=p2;p2=p;
    q1=q2;q2=q;
  }
  System.out.println(p+" "+q);
}
```

7.2 bit set

```cpp
void sub_set(int S){
  int sub=S;
  do{
    //對某集合的子集合的處理
    sub=(sub-1)&S;
  }while(sub!=S);
}
void k_sub_set(int k,int n){
  int comb=(1<<k)-1,S=1<<n;
  while(comb<S){
    //對大小為k的子集合的處理
    int x=comb&-comb,y=comb+x;
    comb=((comb&~y)/x>>1)|y;
  }
}
```

7.3 cantor expansion

```cpp
int factorial[MAXN];
void init(){
  factorial[0]=1;
  for(int i=1;i<=MAXN;++i)factorial[i]=
       factorial[i-1]*i;
}
int encode(const vector<int> &s){
  int n=s.size(),res=0;
  for(int i=0;i<n;++i){
    int t=0;
    for(int j=i+1;j<n;++j)
      if(s[j]<s[i])++t;
    res+=t*factorial[n-i-1];
  }
  return res;
}
vector<int> decode(int a,int n){
  vector<int> res;
  vector<bool> vis(n,0);
  for(int i=n-1;i>=0;--i){
    int t=a/factorial[i],j;
    for(j=0;j<n;++j)
      if(!vis[j]){
        if(t==0)break;
        --t;
      }
    res.push_back(j);
    vis[j]=1;
    a%=factorial[i];
  }
  return res;
}
```

7.4 FFT

```cpp
template<typename T,typename VT=vector<
     complex<T> > >
struct FFT{
  const T pi;
  FFT(const T pi=acos((T)-1)):pi(pi){}
  unsigned bit_reverse(unsigned a,int len){
a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)>>1);
a=((a&0x33333333U)<<2)|((a&0xCCCCCCCCU)>>2);
a=((a&0x0F0F0F0FU)<<4)|((a&0xF0F0F0F0U)>>4);
a=((a&0x00FF00FFU)<<8)|((a&0xFF00FF00U)>>8);
a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)
     >>16);
    return a>>(32-len);
  }
  void fft(bool is_inv,VT &in,VT &out,int N)
       {
    int bitlen=__lg(N),num=is_inv?-1:1;
    for(int i=0;i<N;++i)out[bit_reverse(i,
         bitlen)]=in[i];
    for(int step=2;step<=N;step<<=1){
      const int mh=step>>1;
      for(int i=0;i<mh;++i){
        complex<T> wi=exp(complex<T>(0,i*num
             *pi/mh));
        for(int j=i;j<N;j+=step){
          int k=j+mh;
          complex<T> u=out[j],t=wi*out[k];
          out[j]=u+t;
          out[k]=u-t;
        }
      }
    }
    if(is_inv)for(int i=0;i<N;++i)out[i]/=N;
  }
};
```

7.5 find real root

```
// an*x^n + ... + a1x + a0 = 0;
int sign(double x){
  return x < -eps ? -1 : x > eps;
}

double get(const vector<double>&coef, double x){
  double e = 1, s = 0;
  for(auto i : coef) s += i*e, e *= x;
  return s;
}

double find(const vector<double>&coef, int n, double lo, double hi){
  double sign_lo, sign_hi;
  if( !(sign_lo = sign(get(coef,lo))) ) return lo;
  if( !(sign_hi = sign(get(coef,hi))) ) return hi;
  if(sign_lo * sign_hi > 0) return INF;
  for(int stp = 0; stp < 100 && hi - lo > eps; ++stp){
    double m = (lo+hi)/2.0;
    int sign_mid = sign(get(coef,m));
    if(!sign_mid) return m;
    if(sign_lo*sign_mid < 0) hi = m;
    else lo = m;
  }
  return (lo+hi)/2.0;
}

vector<double> cal(vector<double>coef, int n){
  vector<double>res;
  if(n == 1){
    if(sign(coef[1])) res.pb(-coef[0]/coef[1]);
    return res;
  }
  vector<double>dcoef(n);
  for(int i = 0; i < n; ++i) dcoef[i] = coef[i+1]*(i+1);
  vector<double>droot = cal(dcoef, n-1);
  droot.insert(droot.begin(), -INF);
  droot.pb(INF);
  for(int i = 0; i+1 < droot.size(); ++i){
    double tmp = find(coef, n, droot[i], droot[i+1]);
    if(tmp < INF) res.pb(tmp);
  }
  return res;
}

int main () {
  vector<double>ve;
  vector<double>ans = cal(ve, n);
  // 視情況把答案 +eps，避免 -0
}
```

7.6 FWT

```
vector<int> F_OR_T(vector<int> f, bool inverse){
  for(int i=0; (2<<i)<=f.size(); ++i)
    for(int j=0; j<f.size(); j+=2<<i)
      for(int k=0; k<(1<<i); ++k)
        f[j+k+(1<<i)] += f[j+k]*(inverse?-1:1);
  return f;
}
vector<int> rev(vector<int> A) {
  for(int i=0; i<A.size(); i+=2)
    swap(A[i],A[i^(A.size()-1)]);
  return A;
}
vector<int> F_AND_T(vector<int> f, bool inverse){
  return rev(F_OR_T(rev(f), inverse));
}
vector<int> F_XOR_T(vector<int> f, bool inverse){
  for(int i=0; (2<<i)<=f.size(); ++i)
    for(int j=0; j<f.size(); j+=2<<i)
      for(int k=0; k<(1<<i); ++k){
        int u=f[j+k], v=f[j+k+(1<<i)];
        f[j+k+(1<<i)] = u-v, f[j+k] = u+v;
      }
  if(inverse) for(auto &a:f) a/=f.size();
  return f;
}
```

7.7 LinearCongruence

```
pair<LL,LL> LinearCongruence(LL a[],LL b[],LL m[],int n) {
  // a[i]*x = b[i] ( mod m[i] )
  for(int i=0;i<n;++i) {
    LL x, y, d = extgcd(a[i],m[i],x,y);
    if(b[i]%d!=0) return make_pair(-1LL,0LL);
    m[i] /= d;
    b[i] = LLmul(b[i]/d,x,m[i]);
  }
  LL lastb = b[0], lastm = m[0];
  for(int i=1;i<n;++i) {
    LL x, y, d = extgcd(m[i],lastm,x,y);
    if((lastb-b[i])%d!=0) return make_pair(-1LL,0LL);
    lastb = LLmul((lastb-b[i])/d,x,(lastm/d))*m[i];
    lastm = (lastm/d)*m[i];
    lastb = (lastb+b[i])%lastm;
  }
  return make_pair(lastb<0?lastb+lastm:lastb,lastm);
}
```

7.8 Lucas

```
ll C(ll n, ll m, ll p){// n!/m!/(n-m)!
 if(n<m) return 0;
 return f[n]*inv(f[m],p)%p*inv(f[n-m],p)%p;
}
ll L(ll n, ll m, ll p){
 if(!m) return 1;
 return C(n%p,m%p,p)*L(n/p,m/p,p)%p;
}
ll Wilson(ll n, ll p){ // n!%p
  if(!n)return 1;
  ll res=Wilson(n/p, p);
  if((n/p)%2) return res*(p-f[n%p])%p;
  return  res*f[n%p]%p; //(p-1)!%p=-1
}
```

7.9 Matrix

```
template<typename T>
struct Matrix{
  using rt = std::vector<T>;
  using mt = std::vector<rt>;
  using matrix = Matrix<T>;
  int r,c;
  mt m;
  Matrix(int r,int c):r(r),c(c),m(r,rt(c)){}
  rt& operator[](int i){return m[i];}
  matrix operator+(const matrix &a){
    matrix rev(r,c);
    for(int i=0;i<r;++i)
      for(int j=0;j<c;++j)
        rev[i][j]=m[i][j]+a.m[i][j];
    return rev;
  }
  matrix operator-(const matrix &a){
    matrix rev(r,c);
    for(int i=0;i<r;++i)
      for(int j=0;j<c;++j)
        rev[i][j]=m[i][j]-a.m[i][j];
    return rev;
  }
  matrix operator*(const matrix &a){
    matrix rev(r,a.c);
    matrix tmp(a.c,a.r);
    for(int i=0;i<a.r;++i)
      for(int j=0;j<a.c;++j)
        tmp[j][i]=a.m[i][j];
    for(int i=0;i<r;++i)
      for(int j=0;j<a.c;++j)
        for(int k=0;k<c;++k)
          rev.m[i][j]+=m[i][k]*tmp[j][k];
    return rev;
  }
  bool inverse(){
    Matrix t(r,r+c);
    for(int y=0;y<r;y++){
      t.m[y][c+y] = 1;
      for(int x=0;x<c;++x)
        t.m[y][x]=m[y][x];
    }
    if( !t.gas() )
      return false;
    for(int y=0;y<r;y++)
      for(int x=0;x<c;++x)
        m[y][x]=t.m[y][c+x]/t.m[y][y];
    return true;
  }
  T gas(){
    vector<T> lazy(r,1);
    bool sign=false;
    for(int i=0;i<r;++i){
      if( m[i][i]==0 ){
        int j=i+1;
        while(j<r&&!m[j][i])j++;
        if(j==r)continue;
        m[i].swap(m[j]);
        sign=!sign;
      }
      for(int j=0;j<r;++j){
        if(i==j)continue;
        lazy[j]=lazy[j]*m[i][i];
        T mx=m[j][i];
        for(int k=0;k<c;++k)
          m[j][k]=m[j][k]*m[i][i]-m[i][k]*mx;
      }
    }
    T det=sign?-1:1;
    for(int i=0;i<r;++i){
      det = det*m[i][i];
      det = det/lazy[i];
      for(auto &j:m[i])j/=lazy[i];
    }
    return det;
  }
};
```

7.10 MillerRobin

```
ULL LLmul(ULL a, ULL b, const ULL &mod) {
  LL ans=0;
  while(b) {
    if(b&1) {
      ans+=a;
      if(ans>=mod) ans-=mod;
    }
    a<<=1, b>>=1;
    if(a>=mod) a-=mod;
  }
  return ans;
}
ULL mod_mul(ULL a,ULL b,ULL m){
  a%=m,b%=m;/* fast for m < 2^58 */
  ULL y=(ULL)((double)a*b/m+0.5);
  ULL r=(a*b-y*m)%m;
  return r<0?r+m:r;
}
template<typename T>
T pow(T a,T b,T mod){//a^b%mod
  T ans=1;
  for(;b;a=mod_mul(a,a,mod),b>>=1)
    if(b&1)ans=mod_mul(ans,a,mod);
  return ans;
}
int sprp[3]={2,7,61};//int範圍可解
int llsprp[7]={2,325,9375,28178,450775,9780504,
```

```cpp
1795265022};//至少unsigned long long範圍
template<typename T>
bool isprime(T n,int *sprp,int num){
  if(n==2)return 1;
  if(n<2||n%2==0)return 0;
  int t=0;
  T u=n-1;
  for(;u%2==0;++t)u>>=1;
  for(int i=0;i<num;++i){
    T a=sprp[i]%n;
    if(a==0||a==1||a==n-1)continue;
    T x=pow(a,u,n);
    if(x==1||x==n-1)continue;
    for(int j=0;j<t;++j){
      x=mod_mul(x,x,n);
      if(x==1)return 0;
      if(x==n-1)break;
    }
    if(x==n-1)continue;
    return 0;
  }
  return 1;
}
```

7.11 NTT

```cpp
2615053605667*(2^18)+1,3
15*(2^27)+1,31
479*(2^21)+1,3
7*17*(2^23)+1,3
3*3*211*(2^19)+1,5
25*(2^22)+1,3
template<typename T,typename VT=vector<T> >
struct NTT{
  const T P,G;
  NTT(T p=(1<<23)*7*17+1,T g=3):P(p),G(g){}
  unsigned bit_reverse(unsigned a,int len){
    //look FFT.cpp
  }
  T pow_mod(T n,T k,T m){
    T ans=1;
    for(n=(n>=m?n%m:n);k;k>>=1){
      if(k&1)ans=ans*n%m;
      n=n*n%m;
    }
    return ans;
  }
  void ntt(bool is_inv,VT &in,VT &out,int N){
    int bitlen=__lg(N);
    for(int i=0;i<N;++i)out[bit_reverse(i,bitlen)]=in[i];
    for(int step=2,id=1;step<=N;step<<=1,++id){
      T wn=pow_mod(G,(P-1)>>id,P),wi=1,u,t;
      const int mh=step>>1;
      for(int i=0;i<mh;++i){
        for(int j=i;j<N;j+=step){
          u=out[j],t=wi*out[j+mh]%P;
          out[j]=u+t;
          out[j+mh]=u-t;
          if(out[j]>=P)out[j]-=P;
          if(out[j+mh]<0)out[j+mh]+=P;
        }
        wi=wi*wn%P;
      }
    }
    if(is_inv){
      for(int i=1;i<N/2;++i)swap(out[i],out[N-i]);
      T invn=pow_mod(N,P-2,P);
      for(int i=0;i<N;++i)out[i]=out[i]*invn%P;
    }
  }
};
```

7.12 Simpson

```cpp
double simpson(double a,double b){
  double c=a+(b-a)/2;
  return (F(a)+4*F(c)+F(b))*(b-a)/6;
}
double asr(double a,double b,double eps,double A){
  double c=a+(b-a)/2;
  double L=simpson(a,c),R=simpson(c,b);
  if( abs(L+R-A)<15*eps )
    return L+R+(L+R-A)/15.0;
  return asr(a,c,eps/2,L)+asr(c,b,eps/2,R);
}
double asr(double a,double b,double eps){
  return asr(a,b,eps,simpson(a,b));
}
```

7.13 外星模運算

```cpp
//a[0]^(a[1]^a[2]^...)
#define maxn 1000000
int euler[maxn+5];
bool is_prime[maxn+5];
void init_euler(){
  is_prime[1]=1;//一不是質數
  for(int i=1;i<=maxn;i++)euler[i]=i;
  for(int i=2;i<=maxn;i++){
    if(!is_prime[i]){//是質數
      euler[i]--;
      for(int j=i<<1;j<=maxn;j+=i){
        is_prime[j]=1;
        euler[j]=euler[j]/i*(i-1);
      }
    }
  }
}
LL pow(LL a,LL b,LL mod){//a^b%mod
  LL ans=1;
  for(;b;a=a*a%mod,b>>=1)
    if(b&1)ans=ans*a%mod;
  return ans;
}
bool isless(LL *a,int n,int k){
  if(*a==1)return k>1;
  if(--n==0)return *a<k;
  int next=0;
  for(LL b=1;b<k;++next)
    b*=*a;
  return isless(a+1,n,next);
}
LL high_pow(LL *a,int n,LL mod){
  if(*a==1||--n==0)return *a%mod;
  int k=0,r=euler[mod];
  for(LL tma=1;tma!=pow(*a,k+r,mod);++k)
    tma=tma*(*a)%mod;
  if(isless(a+1,n,k))return pow(*a,high_pow(a+1,n,k),mod);
  int tmd=high_pow(a+1,n,r), t=(tmd-k+r)%r;
  return pow(*a,k+t,mod);
}
LL a[1000005];
int t,mod;
int main(){
  init_euler();
  scanf("%d",&t);
  #define n 4
  while(t--){
    for(int i=0;i<n;++i)scanf("%lld",&a[i]);
    scanf("%d",&mod);
    printf("%lld\n",high_pow(a,n,mod));
  }
  return 0;
}
```

7.14 數位統計

```cpp
ll d[65], dp[65][2];//up區間是不是完整
ll dfs(int p,bool is8,bool up){
  if(!p)return 1; // 回傳0是不是答案
  if(!up&&~dp[p][is8])return dp[p][is8];
  int mx = up?d[p]:9;//可以用的有那些
  ll ans=0;
  for(int i=0;i<=mx;++i){
    if( is8&&i==7 )continue;
    ans += dfs(p-1,i==8,up&&i==mx);
  }
  if(!up)dp[p][is8]=ans;
  return ans;
}
ll f(ll N){
  int k=0;
  while(N){  // 把數字先分解到陣列
    d[++k] = N%10;
    N/=10;
  }
  return dfs(k,false,true);
}
```

7.15 質因數分解

```cpp
LL func(const LL n,const LL mod,const int c){
  return (LLmul(n,n,mod)+c+mod)%mod;
}

LL pollorrho(const LL n, const int c) {//循環節長度
  LL a=1, b=1;
  a=func(a,n,c)%n;
  b=func(b,n,c)%n; b=func(b,n,c)%n;
  while(gcd(abs(a-b),n)==1) {
    a=func(a,n,c)%n;
    b=func(b,n,c)%n; b=func(b,n,c)%n;
  }
  return gcd(abs(a-b),n);
}

void prefactor(LL &n, vector<LL> &v) {
  for(int i=0;i<12;++i) {
    while(n%prime[i]==0) {
      v.push_back(prime[i]);
      n/=prime[i];
    }
  }
}

void smallfactor(LL n, vector<LL> &v) {
  if(n<MAXPRIME) {
    while(isp[(int)n]) {
      v.push_back(isp[(int)n]);
      n/=isp[(int)n];
    }
    v.push_back(n);
  } else {
    for(int i=0;i<primecnt&&prime[i]*prime[i]<=n;++i) {
      while(n%prime[i]==0) {
        v.push_back(prime[i]);
        n/=prime[i];
      }
    }
    if(n!=1) v.push_back(n);
  }
}

void comfactor(const LL &n, vector<LL> &v) {
  if(n<1e9) {
    smallfactor(n,v);
    return;
  }
  if(Isprime(n)) {
    v.push_back(n);
    return;
  }
  LL d;
  for(int c=3;;++c) {
    d = pollorrho(n,c);
    if(d!=n) break;
  }
  comfactor(d,v);
  comfactor(n/d,v);
}

void Factor(const LL &x, vector<LL> &v) {
  LL n = x;
  if(n==1) { puts("Factor 1"); return; }
  prefactor(n,v);
  if(n==1) return;
  comfactor(n,v);
  sort(v.begin(),v.end());
}

void AllFactor(const LL &n,vector<LL> &v) {
  vector<LL> tmp;
  Factor(n,tmp);
  v.clear();
  v.push_back(1);
  int len;
  LL now=1;
  for(int i=0;i<tmp.size();++i) {
    if(i==0 || tmp[i]!=tmp[i-1]) {
      len = v.size();
      now = 1;
    }
    now*=tmp[i];
    for(int j=0;j<len;++j)
      v.push_back(v[j]*now);
  }
}
```

8 String

8.1 AC 自動機

```
template<char L='a',char R='z'>
class ac_automaton{
  struct joe{
    int next[R-L+1],fail,efl,ed,cnt_dp,vis;
    joe():ed(0),cnt_dp(0),vis(0){
      for(int i=0;i<=R-L;++i)next[i]=0;
    }
  };
public:
  std::vector<joe> S;
  std::vector<int> q;
  int qs,qe,vt;
  ac_automaton():S(1),qs(0),qe(0),vt(0){}
  void clear(){
    q.clear();
    S.resize(1);
    for(int i=0;i<=R-L;++i)S[0].next[i]=0;
    S[0].cnt_dp=S[0].vis=qs=qe=vt=0;
  }
  void insert(const char *s){
    int o=0;
    for(int i=0,id;s[i];++i){
      id=s[i]-L;
      if(!S[o].next[id]){
        S.push_back(joe());
        S[o].next[id]=S.size()-1;
      }
      o=S[o].next[id];
    }
    ++S[o].ed;
  }
  void build_fail(){
    S[0].fail=S[0].efl=-1;
    q.clear();
    q.push_back(0);
    ++qe;
    while(qs!=qe){
      int pa=q[qs++],id,t;
      for(int i=0;i<=R-L;++i){
        t=S[pa].next[i];
        if(!t)continue;
        id=S[pa].fail;
        while(~id&&!S[id].next[i])id=S[id].fail;
        S[t].fail=~id?S[id].next[i]:0;
        S[t].efl=S[S[t].fail].ed?S[t].fail:S[S[t].fail].efl;
        q.push_back(t);
        ++qe;
      }
    }
  }
  /*DP出每個前綴在字串s出現的次數並傳回所有字串被s匹配成功的次數O(N+M)*/
  int match_0(const char *s){
    int ans=0,id,p=0,i;
    for(i=0;s[i];++i){
      id=s[i]-L;
      while(!S[p].next[id]&&p)p=S[p].fail;
      if(!S[p].next[id])continue;
      p=S[p].next[id];
      ++S[p].cnt_dp;/*匹配成功則它所有後綴都可以被匹配(DP計算)*/
    }
    for(i=qe-1;i>=0;--i){
      ans+=S[q[i]].cnt_dp*S[q[i]].ed;
      if(~S[q[i]].fail)S[S[q[i]].fail].cnt_dp+=S[q[i]].cnt_dp;
    }
    return ans;
  }
  /*多串匹配走efl邊並傳回所有字串被s匹配成功的次數O(N*M^1.5)*/
  int match_1(const char *s)const{
    int ans=0,id,p=0,t;
    for(int i=0;s[i];++i){
      id=s[i]-L;
      while(!S[p].next[id]&&p)p=S[p].fail;
      if(!S[p].next[id])continue;
      p=S[p].next[id];
      if(S[p].ed)ans+=S[p].ed;
      for(t=S[p].efl;~t;t=S[t].efl){
        ans+=S[t].ed;/*因為都走efl邊所以保證匹配成功*/
      }
    }
    return ans;
  }
  /*枚舉(s的子字串∩A)的所有相異字串各恰一次並傳回次數O(N*M^(1/3))*/
  int match_2(const char *s){
    int ans=0,id,p=0,t;
    ++vt;
    /*把戳記vt+=1，只要vt沒溢位，所有S[p].vis==vt就會變成false
    這種利用vt的方法可以O(1)歸零vis陣列*/
    for(int i=0;s[i];++i){
      id=s[i]-L;
      while(!S[p].next[id]&&p)p=S[p].fail;
      if(!S[p].next[id])continue;
      p=S[p].next[id];
      if(S[p].ed&&S[p].vis!=vt){
        S[p].vis=vt;
        ans+=S[p].ed;
      }
      for(t=S[p].efl;~t&&S[t].vis!=vt;t=S[t].efl){
        S[t].vis=vt;
        ans+=S[t].ed;/*因為都走efl邊所以保證匹配成功*/
      }
    }
    return ans;
  }
  /*把AC自動機變成真的自動機*/
  void evolution(){
    for(qs=1;qs!=qe;){
      int p=q[qs++];
      for(int i=0;i<=R-L;++i)
        if(S[p].next[i]==0)S[p].next[i]=S[S[p].fail].next[i];
    }
  }
};
```

8.2 hash

```
#define MAXN 1000000
#define mod 1073676287
/*mod 必須要是質數*/
typedef long long T;
char s[MAXN+5];
T h[MAXN+5];/*hash陣列*/
T h_base[MAXN+5];/*h_base[n]=(prime^n)%mod*/
void hash_init(int len,T prime){
  h_base[0]=1;
  for(int i=1;i<=len;++i){
    h[i]=(h[i-1]*prime+s[i-1])%mod;
    h_base[i]=(h_base[i-1]*prime)%mod;
  }
}
T get_hash(int l,int r){/*閉區間寫法，設編號為0 ~ len-1*/
  return (h[r+1]-(h[l]*h_base[r-l+1])%mod+mod)%mod;
}
```

8.3 KMP

```
/*產生fail function*/
void kmp_fail(char *s,int len,int *fail){
  int id=-1;
  fail[0]=-1;
  for(int i=1;i<len;++i){
    while(~id&&s[id+1]!=s[i])id=fail[id];
    if(s[id+1]==s[i])++id;
    fail[i]=id;
  }
}
/*以字串B匹配字串A，傳回匹配成功的數量(用B的fail)*/
int kmp_match(char *A,int lenA,char *B,int lenB,int *fail){
  int id=-1,ans=0;
  for(int i=0;i<lenA;++i){
    while(~id&&B[id+1]!=A[i])id=fail[id];
    if(B[id+1]==A[i])++id;
    if(id==lenB-1){/*匹配成功*/
      ++ans, id=fail[id];
    }
  }
  return ans;
}
```

8.4 manacher

```
//原字串: asdsasdsa
//先把字串變成這樣: @#a#s#d#s#a#s#d#s#a#
void manacher(char *s,int len,int *z){
  int l=0,r=0;
  for(int i=1;i<len;++i){
    z[i]=r>i?min(z[2*l-i],r-i):1;
    while(s[i+z[i]]==s[i-z[i]])++z[i];
    if(z[i]+i>r)r=z[i]+i,l=i;
  }//ans = max(z)-1
}
```

8.5 minimal string rotation

```
int min_string_rotation(const string &s){
  int n=s.size(),i=0,j=1,k=0;
  while(i<n&&j<n&&k<n){
    int t=s[(i+k)%n]-s[(j+k)%n];
    ++k;
    if(t){
      if(t>0)i+=k;
      else j+=k;
      if(i==j)++j;
      k=0;
    }
  }
  return min(i,j);//最小循環表示法起始位置
}
```

8.6 reverseBWT

```
const int MAXN = 305, MAXC = 'Z';
int ranks[MAXN], tots[MAXC], first[MAXC];
void rankBWT(const string &bw){
  memset(ranks,0,sizeof(int)*bw.size());
  memset(tots,0,sizeof(tots));
  for(size_t i=0;i<bw.size();++i)
    ranks[i] = tots[int(bw[i])]++;
}
```

```cpp
void firstCol(){
  memset(first,0,sizeof(first));
  int totc = 0;
  for(int c='A';c<='Z';++c){
    if(!tots[c]) continue;
    first[c] = totc;
    totc += tots[c];
  }
}
string reverseBwt(string bw,int begin){
  rankBWT(bw), firstCol();
  int i = begin; //原字串最後一個元素的位置
  string res;
  do{
    char c = bw[i];
    res = c + res;
    i = first[int(c)] + ranks[i];
  }while( i != begin );
  return res;
}
```

8.7 suffix array lcp

```cpp
#define radix_sort(x,y){\
  for(i=0;i<A;++i)c[i]=0;\
  for(i=0;i<n;++i)c[x[y[i]]]++;\
  for(i=1;i<A;++i)c[i]+=c[i-1];\
  for(i=n-1;~i;--i)sa[--c[x[y[i]]]]=y[i];\
}
#define AC(r,a,b)\
  r[a]!=r[b]||a+k>=n||r[a+k]!=r[b+k]
void suffix_array(const char *s,int n,int *sa,int *rank,int *tmp,int *c){
  int A='z'+1,i,k,id=0;
  for(i=0;i<n;++i)rank[tmp[i]=i]=s[i];
  radix_sort(rank,tmp);
  for(k=1;id<n-1;k<<=1){
    for(id=0,i=n-k;i<n;++i)tmp[id++]=i;
    for(i=0;i<n;++i)
      if(sa[i]>=k)tmp[id++]=sa[i]-k;
    radix_sort(rank,tmp);
    swap(rank,tmp);
    for(rank[sa[0]]=id=0,i=1;i<n;++i)
      rank[sa[i]]=id+=AC(tmp,sa[i-1],sa[i]);
    A=id+1;
  }
}
//h:高度數組 sa:後綴數組 rank:排名
void suffix_array_lcp(const char *s,int len,int *h,int *sa,int *rank){
  for(int i=0;i<len;++i)rank[sa[i]]=i;
  for(int i=0,k=0;i<len;++i){
    if(rank[i]==0)continue;
    if(k)--k;
    while(s[i+k]==s[sa[rank[i]-1]+k])++k;
    h[rank[i]]=k;
  }
  h[0]=0;// h[k]=lcp(sa[k],sa[k-1]);
}
```

8.8 Z

```cpp
void z_alg(char *s,int len,int *z){
  int l=0,r=0;
  z[0]=len;
  for(int i=1;i<len;++i){
    z[i]=i>r?0:(i-l+z[i-l]<z[l]?z[i-l]:r-i+1);
    while(i+z[i]<len&&s[i+z[i]]==s[z[i]])++z[i];
    if(i+z[i]-1>r)r=i+z[i]-1,l=i;
  }
}
```

9 Tarjan

9.1 dominator tree

```cpp
struct dominator_tree{
  static const int MAXN=5005;
  int n;// 1-base
  vector<int> G[MAXN], rG[MAXN];
  int pa[MAXN], dfn[MAXN], id[MAXN], dfnCnt;
  int semi[MAXN], idom[MAXN], best[MAXN];
  vector<int> tree[MAXN]; // tree here
  void init(int _n){
    n = _n;
    for(int i=1; i<=n; ++i)
      G[i].clear(), rG[i].clear();
  }
  void add_edge(int u, int v){
    G[u].push_back(v);
    rG[v].push_back(u);
  }
  void dfs(int u){
    id[dfn[u]=++dfnCnt]=u;
    for(auto v:G[u]) if(!dfn[v])
      dfs(v),pa[dfn[v]]=dfn[u];
  }
  int find(int y,int x){
    if(y <= x) return y;
    int tmp = find(pa[y],x);
    if(semi[best[y]] > semi[best[pa[y]]])
      best[y] = best[pa[y]];
    return pa[y] = tmp;
  }
  void tarjan(int root){
    dfnCnt = 0;
    for(int i=1; i<=n; ++i){
      dfn[i] = idom[i] = 0;
      tree[i].clear();
      best[i] = semi[i] = i;
    }
    dfs(root);
    for(int i=dfnCnt; i>1; --i){
      int u = id[i];
      for(auto v:rG[u]) if(v=dfn[v]){
        find(v,i);
        semi[i]=min(semi[i],semi[best[v]]);
      }
      tree[semi[i]].push_back(i);
      for(auto v:tree[pa[i]]){
        find(v, pa[i]);
        idom[v] = semi[best[v]]==pa[i]
            ? pa[i] : best[v];
      }
      tree[pa[i]].clear();
    }
    for(int i=2; i<=dfnCnt; ++i){
      if(idom[i] != semi[i])
        idom[i] = idom[idom[i]];
      tree[id[idom[i]]].push_back(id[i]);
    }
  }
}dom;
```

9.2 tnfshb017 2 sat

```cpp
#include<bits/stdc++.h>
using namespace std;
#define MAXN 8001
#define MAXN2 MAXN*4
#define n(X) ((X)+2*N)
vector<int>  v[MAXN2], rv[MAXN2], vis_t;
int N,M;
void addedge(int s,int e){
  v[s].push_back(e);
  rv[e].push_back(s);
}
int scc[MAXN2];
bool vis[MAXN2]={false};
void dfs(vector<int> *uv,int n,int k=-1){
  vis[n]=true;
  for(int i=0;i<uv[n].size();++i)
    if(!vis[uv[n][i]])
      dfs(uv,uv[n][i],k);
  if(uv==v)vis_t.push_back(n);
  scc[n]=k;
}
void solve(){
  for(int i=1;i<=N;++i){
    if(!vis[i])dfs(v,i);
    if(!vis[n(i)])dfs(v,n(i));
  }
  memset(vis,0,sizeof(vis));
  int c=0;
  for(int i=vis_t.size()-1;i>=0;--i)
    if(!vis[vis_t[i]])
      dfs(rv,vis_t[i],c++);
}
int main(){
  int a,b;
  scanf("%d%d",&N,&M);
  for(int i=1;i<=N;++i){
    // (A or B)&(!A & !B) A^B
    a=i*2-1;
    b=i*2;
    addedge(n(a),b);
    addedge(n(b),a);
    addedge(a,n(b));
    addedge(b,n(a));
  }
  while(M--){
    scanf("%d%d",&a,&b);
    a = a>0?a*2-1:-a*2;
    b = b>0?b*2-1:-b*2;
    // A or B
    addedge(n(a),b);
    addedge(n(b),a);
  }
  solve();
  bool check=true;
  for(int i=1;i<=2*N;++i)
    if(scc[i]==scc[n(i)])
      check=false;
  if(check){
    printf("%d\n",N);
    for(int i=1;i<=2*N;i+=2){
      if(scc[i]>scc[i+2*N]) putchar('+');
      else putchar('-');
    }
    puts("");
  }else puts("0");
  return 0;
}
```

9.3 橋連通分量

```cpp
#define N 1005
struct edge{
  int u,v;
  bool is_bridge;
  edge(int u=0,int v=0):u(u),v(v),is_bridge(0){}
};
vector<edge> E;
vector<int> G[N];// 1-base
int low[N],vis[N],Time;
int bcc_id[N],bridge_cnt,bcc_cnt;// 1-base
int st[N],top;//BCC用
void add_edge(int u,int v){
  G[u].push_back(E.size());
  E.emplace_back(u,v);
  G[v].push_back(E.size());
  E.emplace_back(v,u);
}
void dfs(int u,int re=-1){//u當前點，re為u連接前一個點的邊
  int v;
  low[u]=vis[u]=++Time;
  st[top++]=u;
  for(int e:G[u]){
    v=E[e].v;
    if(!vis[v]){
      dfs(v,e^1);//e^1反向邊
      low[u]=min(low[u],low[v]);
      if(vis[u]<low[v]){
        E[e].is_bridge=E[e^1].is_bridge=1;
        ++bridge_cnt;
      }
    }else if(vis[v]<vis[u]&&e!=re)
      low[u]=min(low[u],vis[v]);
  }
  if(vis[u]==low[u]){//處理BCC
    ++bcc_cnt;// 1-base
    do bcc_id[v=st[--top]]=bcc_cnt;//每個點所在的BCC
    while(v!=u);
  }
}
void bcc_init(int n){
  Time=bcc_cnt=bridge_cnt=top=0;
  E.clear();
  for(int i=1;i<=n;++i){
    G[i].clear();
    vis[i]=bcc_id[i]=0;
  }
}
```

9.4 雙連通分量 & 割點

```cpp
#define N 1005
vector<int> G[N];// 1-base
vector<int> bcc[N];//存每塊雙連通分量的點
int low[N],vis[N],Time;
int bcc_id[N],bcc_cnt;// 1-base
bool is_cut[N];//是否為割點
int st[N],top;
void dfs(int u,int pa=-1){//u當前點，pa父親
  int t, child=0;
  low[u]=vis[u]=++Time;
  st[top++]=u;
  for(int v:G[u]){
    if(!vis[v]){
      dfs(v,u),++child;
      low[u]=min(low[u],low[v]);
      if(vis[u]<=low[v]){
        is_cut[u]=1;
        bcc[++bcc_cnt].clear();
        do{
          bcc_id[t=st[--top]]=bcc_cnt;
          bcc[bcc_cnt].push_back(t);
        }while(t!=v);
        bcc_id[u]=bcc_cnt;
        bcc[bcc_cnt].push_back(u);
      }
    }else if(vis[v]<vis[u]&&v!=pa)//反向邊
      low[u] = min(low[u],vis[v]);
  }//u是dfs樹的根要特判
  if(pa==-1&&child<2)is_cut[u]=0;
}
void bcc_init(int n){
  Time=bcc_cnt=top=0;
  for(int i=1;i<=n;++i){
    G[i].clear();
    is_cut[i]=vis[i]=bcc_id[i]=0;
  }
}
```

10 Tree Problem

10.1 HeavyLight

```cpp
#include<vector>
#define MAXN 100005
int siz[MAXN],max_son[MAXN],pa[MAXN],dep[MAXN];
int link_top[MAXN],link[MAXN],cnt;
vector<int> G[MAXN];
void find_max_son(int u){
  siz[u]=1;
  max_son[u]=-1;
  for(auto v:G[u]){
    if(v==pa[u])continue;
    pa[v]=u;
    dep[v]=dep[u]+1;
    find_max_son(v);
    if(max_son[u]==-1||siz[v]>siz[max_son[u]])max_son[u]=v;
    siz[u]+=siz[v];
  }
}
void build_link(int u,int top){
  link[u]=++cnt;
  link_top[u]=top;
  if(max_son[u]==-1)return;
  build_link(max_son[u],top);
  for(auto v:G[u]){
    if(v==max_son[u]||v==pa[u])continue;
    build_link(v,v);
  }
}
int find_lca(int a,int b){
  //求LCA，可以在過程中對區間進行處理
  int ta=link_top[a],tb=link_top[b];
  while(ta!=tb){
    if(dep[ta]<dep[tb]){
      swap(ta,tb);
      swap(a,b);
    }
    //這裡可以對a所在的鏈做區間處理
    //區間為(link[ta],link[a])
    ta=link_top[a=pa[ta]];
  }
  //最後a,b會在同一條鏈，若a!=b還要在進行一次區間處理
  return dep[a]<dep[b]?a:b;
}
```

10.2 LCA

```cpp
const int MAXN=100000; // 1-base
const int MLG=17; //log2(MAXN)+1;
int pa[MLG+2][MAXN+5];
int dep[MAXN+5];
vector<int> G[MAXN+5];
void dfs(int x,int p=0){//dfs(root);
  pa[0][x]=p;
  for(int i=0;i<=MLG;++i)
    pa[i+1][x]=pa[i][pa[i][x]];
  for(auto &i:G[x]){
    if(i==p)continue;
    dep[i]=dep[x]+1;
    dfs(i,x);
  }
}
inline int jump(int x,int d){
  for(int i=0;i<=MLG;++i)
    if((d>>i)&1) x=pa[i][x];
  return x;
}
inline int find_lca(int a,int b){
  if(dep[a]>dep[b])swap(a,b);
  b=jump(b,dep[b]-dep[a]);
  if(a==b)return a;
  for(int i=MLG;i>=0;--i){
    if(pa[i][a]!=pa[i][b]){
      a=pa[i][a];
      b=pa[i][b];
    }
  }
  return pa[0][a];
}
```

10.3 link cut tree

```cpp
struct splay_tree{
  int ch[2],pa;//子節點跟父母
  bool rev;//反轉的懶惰標記
  splay_tree():pa(0),rev(0){ch[0]=ch[1]=0;}
};
vector<splay_tree> nd;
//有的時候用vector會TLE，要注意
//這邊以node[0]作為null節點
bool isroot(int x){//判斷是否為這棵splay tree的根
  return nd[nd[x].pa].ch[0]!=x&&nd[nd[x].pa].ch[1]!=x;
}
void down(int x){//懶惰標記下推
  if(nd[x].rev){
    if(nd[x].ch[0])nd[nd[x].ch[0]].rev^=1;
    if(nd[x].ch[1])nd[nd[x].ch[1]].rev^=1;
    swap(nd[x].ch[0],nd[x].ch[1]);
    nd[x].rev=0;
  }
}
void push_down(int x){//所有祖先懶惰標記下推
  if(!isroot(x))push_down(nd[x].pa);
  down(x);
}
void up(int x){}//將子節點的資訊向上更新
void rotate(int x){//旋轉，會自行判斷轉的方向
  int y=nd[x].pa,z=nd[y].pa,d=(nd[y].ch[1]==x);
  nd[x].pa=z;
  if(!isroot(y))nd[z].ch[nd[z].ch[1]==y]=x;
  nd[y].ch[d]=nd[x].ch[d^1];
  nd[nd[y].ch[d]].pa=y;
  nd[y].pa=x,nd[x].ch[d^1]=y;
  up(y),up(x);
}
void splay(int x){//將x伸展到splay tree的根
  push_down(x);
  while(!isroot(x)){
    int y=nd[x].pa;
    if(!isroot(y)){
      int z=nd[y].pa;
      if((nd[z].ch[0]==y)^(nd[y].ch[0]==x))rotate(y);
      else rotate(x);
    }
    rotate(x);
  }
}
int access(int x){
  int last=0;
  while(x){
    splay(x);
    nd[x].ch[1]=last;
    up(x);
    last=x;
    x=nd[x].pa;
  }
  return last;//access後splay tree的根
}
void access(int x,bool is=0){//is=0就是一般的access
  int last=0;
  while(x){
    splay(x);
    if(is&&!nd[x].pa){
      //printf("%d\n",max(nd[last].ma,nd[nd[x].ch[1]].ma));
    }
    nd[x].ch[1]=last;
    up(x);
    last=x;
    x=nd[x].pa;
  }
}
void query_edge(int u,int v){
  access(u);
  access(v,1);
}
void make_root(int x){
  access(x),splay(x);
  nd[x].rev^=1;
}
void make_root(int x){
  nd[access(x)].rev^=1;
  splay(x);
}
void cut(int x,int y){
  make_root(x);
  access(y);
  splay(y);
  nd[y].ch[0]=0;
  nd[x].pa=0;
}
void cut_parents(int x){
  access(x);
  splay(x);
  nd[nd[x].ch[0]].pa=0;
  nd[x].ch[0]=0;
}
void link(int x,int y){
  make_root(x);
  nd[x].pa=y;
}
```

```cpp
int find_root(int x){
  x=access(x);
  while(nd[x].ch[0])x=nd[x].ch[0];
  splay(x);
  return x;
}
int query(int u,int v){
//傳回uv路徑splay tree的根結點
//這種寫法無法求LCA
  make_root(u);
  return access(v);
}
int query_lca(int u,int v){
//假設求鏈上點權的總和，sum是子樹的權重和，
     data是節點的權重
  access(u);
  int lca=access(v);
  splay(u);
  if(u==lca){
    //return nd[lca].data+nd[nd[lca].ch[1]].
        sum
  }else{
    //return nd[lca].data+nd[nd[lca].ch[1]].
        sum+nd[u].sum
  }
}
struct EDGE{
  int a,b,w;
}e[10005];
int n;
vector<pair<int,int>> G[10005];
//first表示子節點，second表示邊的編號
int pa[10005],edge_node[10005];
//pa是父母節點，暫存用的，edge_node是每個編
     被存在哪個點裡面的陣列
void bfs(int root){
//在建構的時候把每個點都設成一個splay tree
  queue<int > q;
  for(int i=1;i<=n;++i)pa[i]=0;
  q.push(root);
  while(q.size()){
    int u=q.front();
    q.pop();
    for(auto P:G[u]){
      int v=P.first;
      if(v!=pa[u]){
        pa[v]=u;
        nd[v].pa=u;
        nd[v].data=e[P.second].w;
        edge_node[P.second]=v;
        up(v);
        q.push(v);
      }
    }
  }
}
void change(int x,int b){
  splay(x);
  //nd[x].data=b;
  up(x);
}
```

10.4 POJ tree

```cpp
#include<bits/stdc++.h>
using namespace std;
#define MAXN 10005
int n,k;
vector<pair<int,int> >g[MAXN];
int size[MAXN];
bool vis[MAXN];
inline void init(){
  for(int i=0;i<=n;++i){
    g[i].clear();
    vis[i]=0;
  }
}
void get_dis(vector<int> &dis,int u,int pa,
     int d){
  dis.push_back(d);
  for(size_t i=0;i<g[u].size();++i){
    int v=g[u][i].first,w=g[u][i].second;
    if(v!=pa&&!vis[v])get_dis(dis,v,u,d+w);
  }
}
vector<int> dis;//這東西如果放在函數裡會TLE
int cal(int u,int d){
  dis.clear();
  get_dis(dis,u,-1,d);
  sort(dis.begin(),dis.end());
  int l=0,r=dis.size()-1,res=0;
  while(l<r){
    while(l<r&&dis[l]+dis[r]>k)--r;
    res+=r-(l++);
  }
  return res;
}
pair<int,int> tree_centroid(int u,int pa,
     const int sz){
  size[u]=1;//找樹重心，second是重心
  pair<int,int> res(INT_MAX,-1);
  int ma=0;
  for(size_t i=0;i<g[u].size();++i){
    int v=g[u][i].first;
    if(v==pa||vis[v])continue;
    res=min(res,tree_centroid(v,u,sz));
    size[u]+=size[v];
    ma=max(ma,size[v]);
  }
  ma=max(ma,sz-size[u]);
  return min(res,make_pair(ma,u));
}
int tree_DC(int u,int sz){
  int center=tree_centroid(u,-1,sz).second;
  int ans=cal(center,0);
  vis[center]=1;
  for(size_t i=0;i<g[center].size();++i){
    int v=g[center][i].first,w=g[center][i].
        second;
    if(vis[v])continue;
    ans-=cal(v,w);
    ans+=tree_DC(v,size[v]);
  }
  return ans;
}
int main(){
  while(scanf("%d%d",&n,&k),n||k){
    init();
    for(int i=1;i<n;++i){
      int u,v,w;
      scanf("%d%d%d",&u,&v,&w);
      g[u].push_back(make_pair(v,w));
      g[v].push_back(make_pair(u,w));
    }
    printf("%d\n",tree_DC(1,n));
  }
  return 0;
}
```

11 default

11.1 debug

```cpp
//volatile
#ifdef DEBUG
#define dbg(...) {\
  fprintf(stderr,"%s - %d : (%s) = ",
       __PRETTY_FUNCTION__,__LINE__,#
       __VA_ARGS__);\
  _DO(__VA_ARGS__);\
}
template<typename I> void _DO(I&&x){cerr<<x
     <<endl;}
template<typename I,typename...T> void _DO(I
     &&x,T&&...tail){cerr<<x<<", ";_DO(tail
     ...);}
#else
#define dbg(...)
#endif
```

11.2 ext

```cpp
#include<bits/extc++.h>
#include<ext/pd_ds/assoc_container.hpp>
#include<ext/pd_ds/tree_policy.hpp>
using namespace  __gnu_cxx;
using namespace  __gnu_pbds;
template<typename T>
using pbds_set = tree<T,null_type,less<T>,
     rb_tree_tag,
     tree_order_statistics_node_update>;
template<typename T,typename U>
using pbds_map = tree<T,U,less<T>,
     rb_tree_tag,
     tree_order_statistics_node_update>;
using heap=__gnu_pbds::priority_queue<int>;
//s.find_by_order(1);//0 base
//s.order_of_key(1);
```

11.3 IncStack

```cpp
//Magic
#pragma GCC optimize "Ofast"
//stack resize,change esp to rsp if 64-bit
     system
asm("mov %0,%%esp\n" ::"g"(mem+10000000));
-Wl,--stack,214748364 -trigraphs
#pragma comment(linker, "/STACK
     :1024000000,1024000000")
//linux stack resize
#include<sys/resource.h>
void increase_stack(){
  const rlim_t ks=64*1024*1024;
  struct rlimit rl;
  int res=getrlimit(RLIMIT_STACK,&rl);
  if(!res&&rl.rlim_cur<ks){
    rl.rlim_cur=ks;
    res=setrlimit(RLIMIT_STACK,&rl);
  }
}
```

11.4 input

```cpp
inline int read(){
  int x=0; bool f=0; char c=getchar();
  while(ch<'0'||'9'<ch)f|=ch=='-',ch=getchar
       ();
  while('0'<=ch&&ch<='9')x=x*10-'0'+ch,ch=
       getchar();
  return f?-x:x;
}
// #!/bin/bash
// g++ -std=c++11 -O2 -Wall -Wextra -Wno-
     unused-result -DDEBUG $1 && ./a.out
//  -fsanitize=address -fsanitize=undefined
     -fsanitize=return
```

12 other

12.1 WhatDay

```cpp
int whatday(int y,int m,int d){
  if(m<=2)m+=12,--y;
  if(y<1752||y==1752&&m<9||y==1752&&m==9&&d
       <3)
    return (d+2*m+3*(m+1)/5+y+y/4+5)%7;
  return (d+2*m+3*(m+1)/5+y+y/4-y/100+y/400)
       %7;
}
```

12.2 上下最大正方形

```cpp
void solve(int n,int a[],int b[]){// 1-base
  int ans=0;
  deque<int>da,db;
  for(int l=1,r=1;r<=n;++r){
    while(da.size()&&a[da.back()]>=a[r]){
      da.pop_back();
    }
    da.push_back(r);
    while(db.size()&&b[db.back()]>=b[r]){
      db.pop_back();
    }
    db.push_back(r);
    for(int d=a[da.front()]+b[db.front()];r-l+1>d;++l){
      if(da.front()==l)da.pop_front();
      if(db.front()==l)db.pop_front();
      if(da.size()&&db.size()){
        d=a[da.front()]+b[db.front()];
      }
    }
    ans=max(ans,r-l+1);
  }
  printf("%d\n",ans);
}
```

12.3 最大矩形

```
LL max_rectangle(vector<int> s){
  stack<pair<int,int > > st;
  st.push(make_pair(-1,0));
  s.push_back(0);
  LL ans=0;
  for(size_t i=0;i<s.size();++i){
    int h=s[i];
    pair<int,int > now=make_pair(h,i);
    while(h<st.top().first){
      now=st.top();
      st.pop();
      ans=max(ans,(LL)(i-now.second)*now.first);
    }
    if(h>st.top().first){
      st.push(make_pair(h,now.second));
    }
  }
  return ans;
}
```

13 other language

13.1 java

13.1.1 文件操作

```
import java.io.*;
import java.util.*;
import java.math.*;
import java.text.*;

public class Main{

  public static void main(String args[]) throws FileNotFoundException, IOException
    Scanner sc = new Scanner(new FileReader("a.in"));
    PrintWriter pw = new PrintWriter(new FileWriter("a.out"));
    int n,m;
    n=sc.nextInt();//读入下一个INT
    m=sc.nextInt();

    for(ci=1; ci<=c; ++ci){
      pw.println("Case #"+ci+": easy for output");
    }

    pw.close();//关闭流并释放，这个很重要，否则是没有输出的
    sc.close();//关闭流并释放
  }
}
```

13.1.2 优先队列

```
PriorityQueue queue = new PriorityQueue( 1, new Comparator(){
  public int compare( Point a, Point b ){
  if( a.x < b.x || a.x == b.x && a.y < b.y )
    return -1;
  else if( a.x == b.x && a.y == b.y )
    return 0;
  else return 1;
  }
});
```

13.1.3 Map

```
Map map = new HashMap();
map.put("sa","dd");
String str = map.get("sa").toString;

for(Object obj : map.keySet()){
  Object value = map.get(obj );
}
```

13.1.4 sort

```
static class cmp implements Comparator{
  public int compare(Object o1,Object o2){
  BigInteger b1=(BigInteger)o1;
  BigInteger b2=(BigInteger)o2;
  return b1.compareTo(b2);
  }
}
public static void main(String[] args) throws IOException{
  Scanner cin = new Scanner(System.in);
  int n;
  n=cin.nextInt();
  BigInteger[] seg = new BigInteger[n];
  for (int i=0;i<n;i++)
  seg[i]=cin.nextBigInteger();
  Arrays.sort(seg,new cmp());
}
```

13.2 python heap

```
import heapq

heap = [7,1,2,2]
heapq.heapify(heap)
print(heap) # [1, 2, 2, 7]
heapq.heappush(heap, 5)
print(heap) # [1, 2, 2, 7, 5]
print(heapq.heappop(heap)) # 1
print(heap) # [2, 2, 5, 7]
```

13.3 python input

```
ans = sum(map(float, input().split()))
# input: 1.1 2.2 3.3 4.4 5.5
print(ans) # 16.5

(n, m) = map(int, input().split()) # 300 200
print(n * m) # 60000

Arr = list(map(int, input().split()))
# input: 1 2 3 4 5
print(Arr) # [1, 2, 3, 4, 5]
```

14 zformula

14.1 formula

14.1.1 Pick 公式

給定頂點坐標均是整點的簡單多邊形，面積 = 內部格點數 + 邊上格點數/2-1

14.1.2 圖論

1. 對於平面圖，$F = E - V + C + 1$，C 是連通分量數
2. 對於平面圖，$E \le 3V - 6$
3. 對於連通圖 G，最大獨立點集的大小設為 I(G)，最大匹配大小設為 M(G)，最小點覆蓋設為 Cv(G)，最小邊覆蓋設為 Ce(G)。對於任意連通圖：
   (a) $I(G) + Cv(G) = |V|$
   (b) $M(G) + Ce(G) = |V|$
4. 對於連通二分圖：
   (a) $I(G) = Cv(G)$
   (b) $M(G) = Ce(G)$
5. 最大權閉合圖：
   (a) $C(u, v) = \infty, (u, v) \in E$
   (b) $C(S, v) = W_v, W_v > 0$
   (c) $C(v, T) = -W_v, W_v < 0$
   (d) ans=$\sum_{W_v>0} W_v - flow(S, T)$
6. 最大密度子圖：
   (a) 求 $max\left(\frac{W_e+W_v}{|V'|}\right), e \in E', v \in V'$
   (b) $U = \sum_{v\in V} 2W_v + \sum_{e\in E} W_e$
   (c) $C(u, v) = W_{(u,v)}, (u, v) \in E$，雙向邊
   (d) $C(S, v) = U, v \in V$
   (e) $D_u = \sum_{(u,v)\in E} W_{(u,v)}$
   (f) $C(v, T) = U + 2g - D_v - 2W_v, v \in V$
   (g) 二分搜 g：
       $l = 0, r = U, eps = 1/n^2$
       if$((U \times |V| - flow(S, T))/2 > 0)$ $l = mid$
       else $r = mid$
   (h) ans=$min\_cut(S, T)$
   (i) $|E| = 0$ 要特殊判斷
7. 弦圖：
   (a) 點數大於 3 的環都要有一條弦
   (b) 完美消除序列從後往前依次給每個點染色，給每個點染上可以染的最小顏色
   (c) 最大團大小 = 色數
   (d) 最大獨立集: 完美消除序列從前往後能選就選
   (e) 最小團覆蓋: 最大獨立集的點和他延伸的邊構成
   (f) 區間圖是弦圖
   (g) 區間圖的完美消除序列: 將區間按造又端點由小到大排序
   (h) 區間圖染色: 用線段樹做

14.1.3 dinic 特殊圖複雜度

1. 單位流：$O\left(min\left(V^{3/2}, E^{1/2}\right)E\right)$
2. 二分圖：$O\left(V^{1/2}E\right)$

14.1.4 0-1 分數規劃

$x_i = \{0,1\}$，x_i 可能會有其他限制，求 $max\left(\frac{\sum B_i x_i}{\sum C_i x_i}\right)$

1. $D(i,g) = B_i - g \times C_i$
2. $f(g) = \sum D(i,g)x_i$
3. $f(g) = 0$ 時 g 為最佳解，$f(g) < 0$ 沒有意義
4. 因為 $f(g)$ 單調可以二分搜 g
5. 或用 Dinkelbach 通常比較快

```
binary_search(){
  while(r-l>eps){
    g=(l+r)/2;
    for(i:所有元素)D[i]=B[i]-g*C[i];//D(i,g)
    找出一組合法x[i]使f(g)最大;
    if(f(g)>0) l=g;
    else r=g;
  }
  Ans = r;
}
Dinkelbach(){
  g=任意狀態(通常設為0);
  do{
    Ans=g;
    for(i:所有元素)D[i]=B[i]-g*C[i];//D(i,g)
    找出一組合法x[i]使f(g)最大;
    p=0,q=0;
    for(i:所有元素)
      if(x[i])p+=B[i],q+=C[i];
    g=p/q;//更新解，注意q=0的情況
  }while(abs(Ans-g)>EPS);
  return Ans;
}
```

14.1.5 學長公式

1. $\sum_{d|n}\phi(n) = n$
2. $g(n) = \sum_{d|n} f(d) => f(n) = \sum_{d|n}\mu(d) \times g(n/d)$
3. Harmonic series $H_n = \ln(n) + \gamma + 1/(2n) - 1/(12n^2) + 1/(120n^4)$
4. $\gamma = 0.57721566490153286060651209008240243104215$
5. 格雷碼 $= n \oplus (n >> 1)$
6. $SG(A+B) = SG(A) \oplus SG(B)$
7. 選轉矩陣 $M(\theta) = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$

14.1.6 基本數論

1. $\sum_{d|n}\mu(n) = [n == 1]$
2. $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m}\mu(d) \times g(m/d)$
3. $\sum_{i=1}^{n}\sum_{j=1}^{m}$ 互質數量 $= \sum \mu(d)\left\lfloor\frac{n}{d}\right\rfloor\left\lfloor\frac{m}{d}\right\rfloor$
4. $\sum_{i=1}^{n}\sum_{j=1}^{n} lcm(i,j) = n\sum_{d|n} d \times \phi(d)$

14.1.7 排組公式

1. k 卡特蘭 $\frac{C_n^{kn}}{n(k-1)+1}$，$C_m^n = \frac{n!}{m!(n-m)!}$
2. $H(n,m) \cong x_1 + x_2 \ldots + x_n = k, num = C_k^{n+k-1}$
3. Stirling number of 2^{nd},n 人分 k 組方法數目
   (a) $S(0,0) = S(n,n) = 1$
   (b) $S(n,0) = 0$
   (c) $S(n,k) = kS(n-1,k) + S(n-1,k-1)$
4. Bell number,n 人分任意多組方法數目
   (a) $B_0 = 1$
   (b) $B_n = \sum_{i=0}^{n} S(n,i)$
   (c) $B_{n+1} = \sum_{k=0}^{n} C_k^n B_k$
   (d) $B_{p+n} \equiv B_n + B_{n+1} modp$, p is prime
   (e) $B_{p^m+n} \equiv mB_n + B_{n+1} modp$, p is prime
   (f) From $B_0 : 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975$
5. Derangement, 錯排, 沒有人在自己位置上
   (a) $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \ldots + (-1)^n\frac{1}{n!})$
   (b) $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 = 1, D_1 = 0$
   (c) From $D_0 : 1, 0, 1, 2, 9, 44, 265, 1854, 14833, 133496$
6. Binomial Equality
   (a) $\sum_k \binom{r}{m+k}\binom{s}{n-k} = \binom{r+s}{m+n}$
   (b) $\sum_k \binom{l}{m+k}\binom{s}{n+k} = \binom{l+s}{l-m+n}$
   (c) $\sum_k \binom{l}{m+k}\binom{s+k}{n}(-1)^k = (-1)^{l+m}\binom{s-m}{n-l}$
   (d) $\sum_{k\le l} \binom{l-k}{m}\binom{s}{k-n}(-1)^k = (-1)^{l+m}\binom{s-m-1}{l-n-m}$
   (e) $\sum_{0\le k\le l} \binom{l-k}{m}\binom{q+k}{n} = \binom{l+q+1}{m+n+1}$
   (f) $\binom{r}{k} = (-1)^k\binom{k-r-1}{k}$
   (g) $\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}$
   (h) $\sum_{k\le n} \binom{r+k}{k} = \binom{r+n+1}{n}$
   (i) $\sum_{0\le k\le n} \binom{k}{m} = \binom{n+1}{m+1}$
   (j) $\sum_{k\le m} \binom{m+r}{k}x^k y^k = \sum_{k\le m} \binom{-r}{k}(-x)^k(x+y)^{m-k}$

14.1.8 冪次, 冪次和

1. $a^b\%P = a^{b\%\varphi(p)+\varphi(p)}, b \ge \varphi(p)$
2. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
3. $1^4 + 2^4 + 3^4 + \ldots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$
4. $1^5 + 2^5 + 3^5 + \ldots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12}$
5. $0^k + 1^k + 2^k + \ldots + n^k = P(k), P(k) = \frac{(n+1)^{k+1} - \sum_{i=0}^{k-1} C_i^{k+1}P(i)}{k+1}, P(0) = n+1$
6. $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1}\sum_{k=0}^{n} C_k^{n+1}B_k m^{n+1-k}$
7. $\sum_{j=0}^{m} C_j^{m+1}B_j = 0, B_0 = 1$
8. 除了 $B_1 = -1/2$，剩下的奇數項都是 0
9. $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 = -1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 7/6, B_{16} = -3617/510, B_{18} = 43867/798, B_{20} = -174611/330,$

14.1.9 Burnside's lemma

1. $|X/G| = \frac{1}{|G|}\sum_{g\in G}|X^g|$
2. $X^g = t^{c(g)}$
3. G 表示有幾種轉法，X^g 表示在那種轉法下，有幾種是會保持對稱的，t 是顏色數，$c(g)$ 是循環節不動的面數。
4. 正立方體塗三顏色，轉 0 有 3^6 個元素不變，轉 90 有 6 種，每種有 3^3 不變，180 有 3×3^4，120(角) 有 8×3^2，180(邊) 有 6×3^3，全部 $\frac{1}{24}\left(3^6 + 6\times 3^3 + 3\times 3^4 + 8\times 3^2 + 6\times 3^3\right) = 57$

14.1.10 Count on a tree

1. Rooted tree: $s_{n+1} = \frac{1}{n}\sum_{i=1}^{n}(i \times a_i \times \sum_{j=1}^{\lfloor n/i\rfloor} a_{n+1-i\times j})$
2. Unrooted tree:
   (a) Odd:$a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$
   (b) Even:$Odd + \frac{1}{2}a_{n/2}(a_{n/2}+1)$
3. Spanning Tree
   (a) 完全圖 $n^n - 2$
   (b) 一般圖 (Kirchhoff's theorem)$M[i][i] = degree(V_i)$,$M[i][j] = -1$,if have $E(i,j)$,0 if no edge. delete any one row and col in A, $ans = det(A)$

15

15.1 ganadoQuote

```
¡Allí está!
¡Un forastero!
¡Agarrenlo!
¡Os voy a romper a pedazos!
¡Cógelo!
¡Te voy a hacer picadillo!
¡Te voy a matar!
¡Míralo, está herido!
¡Sos cerdo!
¿Dónde estás?
¡Detrás de tí, imbécil!
¡No dejes que se escape!
¡Basta, hijo de puta!
Lord Saddler...

¡Mátalo!
¡Allí está!
Morir es vivir.
Sííííí, ¡Quiero matar!
Muere, muere, muere....
Cerebros,cerebros,cerebros...
Cógedlo, cógedlo, cógedlo...
Lord Saddler...
Dieciséis.

¡Va por él!
¡Muérete!
¡Cógelo!
¡Te voy a matar!
¡Bloqueale el paso!
¡Te cogí!
¡No dejes que se escape!

¿Qué carajo estás haciendo aquí? ¡Lárgate,
    cabrón!
Hay un rumor de que hay un extranjero entre
    nosotros.
Nuestro jefe se encargará de la rata.
Su "Las Plagas" es mucho mejor que la
    nuestra.
Tienes razón, es un hombre.
Usa los músculos.
Se vuelve loco!
¡Hey, acá!
¡Por aquí!
¡El Gigante!
¡Del Lago!
¡Cógelo!
¡Cógenlo!
¡Allí!
¡Rápido!
¡Empieza a rezar!
¡Mátenlos!
¡Te voy a romper en pedazos!
¡La campana!
Ya es hora de rezar.
Tenemos que irnos.
¡Maldita sea, mierda!
¡Ya es hora de aplastar!
¡Mierda!
¡Puedes correr, pero no te puedes esconder!
¡Sos cerdo!
¡Está en la trampa!
¡Ah, que madre!
¡Vámonos!
¡Ándale!
¡Cabrón!
¡Coño!
¡Agárrenlo!
Cógerlo, Cógerlo...
¡Allí está, mátalo!
¡No dejas que se escape de la isla vivo!
¡Hasta luego!
¡Rápido, es un intruso!
```

15.2

```
/**************************
L'Internationale,
     Sera le genre humain.

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***************************/
Вставай, проклятьем заклеймённый,
Весь мир голодных и рабов!
Кипит наш разум возмущённый
И в смертный бой вести готов.
Весь мир насилья мы разрушим
До основанья, а затем
Мы наш, мы новый мир построим, –
Кто был ничем, тот станет всем.

Chorus
Это есть наш последний
И решительный бой;
С Интернационалом
Воспрянет род людской!

Никто не даст нам избавленья:
Ни бог, ни царь и не герой!
Добьёмся мы освобожденья
Своею собственной рукой.
Чтоб свергнуть гнёт рукой умелой,
Отвоевать своё добро, –
Вздувайте горн и куйте смело,
Пока железо горячо!

Chorus

Довольно кровь сосать, вампиры,
Тюрьмой, налогом, нищетой!
У вас — вся власть, все блага мира,
А наше право — звук пустой !
Мы жизнь построим по-иному –
И вот наш лозунг боевой:
Вся власть народу трудовому!
А дармоедов всех долой!

Chorus

Презренны вы в своём богатстве,
Угля и стали короли!
Вы ваши троны, тунеядцы,
На наших спинах возвели.
Заводы, фабрики, палаты –
Всё нашим создано трудом.
Пора! Мы требуем возврата
Того, что взято грабежом.

Chorus

Довольно королям в угоду
Дурманить нас в чаду войны!
Война тиранам! Мир Народу!
Бастуйте, армии сыны!
Когда ж тираны нас заставят
В бою геройски пасть за них –
Убийцы, в вас тогда направим
Мы жерла пушек боевых!

Chorus

Лишь мы, работники всемирной
Великой армии труда,
Владеть землёй имеем право,
Но паразиты — никогда!
И если гром великий грянет
Над сворой псов и палачей, –
Для нас всё так же солнце станет
Сиять огнём своих лучей.

Chorus
```

15.3 保佑

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//         神獸保佑 永無BUG
```

ACM ICPC Team Reference - Angry Crow Takes Flight!

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