

1 Computational_Geometry

1.1 Geometry.cpp

```

1 const double PI=atan2(0.0,-1.0);
2 template<typename T>
3 struct point{
4     T x,y;
5     point(){
6     point(const T&x,const T&y):x(x),y(y){
7     point operator+(const point &b)const{
8         return point(x+b.x,y+b.y);
9     point operator-(const point &b)const{
10        return point(x-b.x,y-b.y);
11    point operator*(const T &b)const{
12        return point(x*b,y*b);
13    point operator/(const T &b)const{
14        return point(x/b,y/b);
15    bool operator==(const point &b)const{
16        return x==b.x&&y==b.y;
17    T dot(const point &b)const{
18        return x*b.x+y*b.y;
19    T cross(const point &b)const{
20        return x*b.y-y*b.x;
21    point normal()const{//求法向量
22        return point(-y,x);
23    T abs2()const{//向量長度的平方
24        return dot(*this);
25    }
26    T rad(const point &b)const{//兩向量的弧度
27        return fabs(atan2(fabs(cross(b)),dot(b)));
28    }
29    T getA()const{//對x軸的弧度
30        T A=atan2(y,x);{//超過180度會變負的
31        if(A<=-PI/2)A+=PI*2;
32        return A;
33    }
34};
35template<typename T>
36struct line{
37    line(){
38    point<T> p1,p2;
39    T a,b,c;//ax+by+c=0
40    line(const point<T>&x,const point<T>&y):p1(x),p2(y){
41    void pton()const{//轉成一般式
42        a=p1.y-p2.y;
43        b=p2.x-p1.x;
44        c=-a*p1.x-b*p1.y;
45    }
46    T cross(const point<T> &p)const{//點和有向
47        //直線的關係，>0左邊，=0在線上，<0右邊
48        return (p2-p1).cross(p-p1);
49    }
50    bool point_on_segment(const point<T>&p)
51        const{//點是否線段上
52        return cross(p)==0&&(p1-p).dot(p2-p)<=0;
53    }
54    T dis2(const point<T> &p,bool is_segment
55        =0)const{//點跟直線/線段的距離平方
56    point<T> v=p2-p1,v1=p-p1;
57    if(is_segment){
58        point<T> v2=p-p2;
59        if(v.dot(v1)<=0)return v1.abs2();
60        if(v.dot(v2)>=0)return v2.abs2();
61    }
62    T tmp=v.cross(v1);
63    return tmp*tmp/v.abs2();
64    }
65    T seg_dis2(const line<T> &l)const{//兩線段
66        //距離平方
67        return min({dis2(l.p1,1),dis2(l.p2,1),l.
68            dis2(p1,1),l.dis2(p2,1)});
69    }
70    point<T> projection(const point<T> &p)
71        const{//點對直線的投影
72    point<T> n=(p2-p1).normal();
73    return p-n*(p-p1).dot(n)/n.abs2();
74    }
75    point<T> mirror(const point<T> &p)const{//
76        //點對直線的鏡射
77        //要先呼叫pton轉成一般式
78    point<T> ans;
79    T d=a*p+b*b;
80    ans.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/
81        d;
82    ans.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/
83        d;
84    return ans;
85    }
86    bool equal(const line &l)const{//直線相等
87        return cross(l.p1)==0&&cross(l.p2)==0;
88    }
89    bool parallel(const line &l)const{//直線平
90        //行
91    return (p1-p2).cross(l.p1-l.p2)==0;
92    }
93    bool cross_seg(const line &l)const{//直線
94        //是否交線段
95    return (p2-p1).cross(l.p1-p1)*(p2-p1).
96        cross(l.p2-p1)<=0;
97    }
98    char line_intersect(const line &l)const{//
99        //直線相交情況，-1無限多點，1交於一點，0
100        //不相交
101    return parallel(l)?(cross(l.p1)==0?-1:0)
102        :1;
103    }
104    char seg_intersect(const line &l)const{//
105        //線段相交情況，-1無限多點，1交於一點，0
106        //不相交
107    T c1=(p2-p1).cross(l.p1-p1);
108    T c2=(p2-p1).cross(l.p2-p1);
109    T c3=(l.p2-l.p1).cross(p1-l.p1);
110    T c4=(l.p2-l.p1).cross(p2-l.p1);
111    if(c1==0&&c2==0){
112        if(p1==l.p1&&(p2-p1).dot(l.p2)<=0)
113            return 1;
114        if(p1==l.p2&&(p2-p1).dot(l.p1)<=0)
115            return 1;
116        if(p2==l.p1&&(p1-p2).dot(l.p2)<=0)
117            return 1;
118        if(p2==l.p2&&(p1-p2).dot(l.p1)<=0)
119            return 1;
120        return 1;
121    }
122    if(c1==0&&c2==0){
123        if(p1==l.p1&&(p2-p1).dot(l.p2)<=0)
124            return 1;
125        if(p1==l.p2&&(p2-p1).dot(l.p1)<=0)
126            return 1;
127        if(p2==l.p1&&(p1-p2).dot(l.p2)<=0)
128            return 1;
129        if(p2==l.p2&&(p1-p2).dot(l.p1)<=0)
130            return 1;
131        return 1;
132    }
133    return 0;
134    }
135    char line_in_convex(const point<T>&x)
136        const{
137    int l=1,r=(int)p.size()-2;
138    while(l<=r){
139        //點是否在凸多邊形內，是的話
140        //回傳1，在邊上回傳-1，否則回傳0
141        int mid=(l+r)/2;
142        T a1=(p[mid]-p[0]).cross(x-p[0]);
143        T a2=(p[mid+1]-p[0]).cross(x-p[0]);
144        if(a1>=0&&a2<=0){
145            T res=(p[mid+1]-p[mid]).cross(x-p[
146                mid]);
147            return res>0?1:(res>=0?-1:0);
148        }else if(a1<0)r=mid-1;
149        else l=mid+1;
150    }
151    return 0;
152    }
153    vector<T> getA()const{//凸包邊對x軸的夾角
154    vector<T> res;//一定是遞增的
155    for(size_t i=0;i<p.size();++i)
156        res.push_back((p[(i+1)%p.size()]-p[i])
157            .getA());
158    return res;
159    }
160    bool line_intersect(const vector<T>&A,
161        const line<T> &l)const{//O(LogN)
162    int f1=upper_bound(A.begin(),A.end(),(l.
163        p1-l.p2).getA())-A.begin();
164    int f2=upper_bound(A.begin(),A.end(),(l.
165        p2-l.p1).getA())-A.begin();
166    return l.cross_seg(line<T>(p[f1],p[f2]));
167    }
168    polygon cut(const line<T> &l)const{//凸包
169        //對直線切割，得到直線L左側的凸包
170    polygon ans;
171    for(int n=p.size(),i=n-1,j=0;j<n;i=j++){
172        if(l.cross(p[i])>=0){
173            ans.p.push_back(p[i]);
174            if(l.cross(p[j])<0)
175                ans.p.push_back(l.
176                    line_intersection(line<T>(p[i]
177                        ],p[j])));
178        }else if(l.cross(p[j])>=0)
179            ans.p.push_back(l.line_intersection(
180                line<T>(p[i],p[j]));
181        }
182    return ans;
183    }
184    static bool graham_cmp(const point<T>&a,
185        const point<T>&b){
186    return (a.x<b.x)||((a.x==b.x&&a.y<b.y));
187    }
188    void graham(vector<point<T>> &s){
189    sort(s.begin(),s.end(),graham_cmp);
190    p.resize(s.size()+1);
191    int m=0;
192    for(int i=0;i<(int)s.size();++i){
193        while(m>2&&(p[m-1]-p[m-2]).cross(s[i]
194            ]-p[m-2])<=0)-m;
195        p[m++]=s[i];
196    }
197    }
198    return c;
199    }else if(c1*c2<=0&&c3*c4<=0)return 1;
200    return 0;
201    }
202    point<T> line_intersection(const line &l)
203        const{//直線交點
204    point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
205    //if(a.cross(b)==0)return INF;
206    return p1+a*s.cross(b)/a.cross(b);
207    }
208    point<T> seg_intersection(const line &l)
209        const{//線段交點
210    T c1=(p2-p1).cross(l.p1-p1);
211    T c2=(p2-p1).cross(l.p2-p1);
212    T c3=(l.p2-l.p1).cross(p1-l.p1);
213    T c4=(l.p2-l.p1).cross(p2-l.p1);
214    if(c1==0&&c2==0){
215        if(p1==l.p1&&(p2-p1).dot(l.p2)<=0)
216            return p1;
217        if(p1==l.p2&&(p2-p1).dot(l.p1)<=0)
218            return p1;
219        if(p2==l.p1&&(p1-p2).dot(l.p2)<=0)
220            return p2;
221        if(p2==l.p2&&(p1-p2).dot(l.p1)<=0)
222            return p2;
223    }else if(c1*c2<=0&&c3*c4<=0)return
224        line_intersection(l);
225    //return INF;
226    }
227    }
228    template<typename T>
229    struct polygon{
230        polygon(){
231        vector<point<T>> > p;//逆時針順序
232        T area()const{//面積
233            T ans=0;
234            for(int i=p.size()-1,j=0;j<(int)p.size()
235                ;i=j++){
236                ans+=p[i].cross(p[j]);
237            }
238            return ans/2;
239        }
240    point<T> center_of_mass()const{//重心
241        T cx=0,cy=0,w=0;
242        for(int i=p.size()-1,j=0;j<(int)p.size()
243            ;i=j++){
244            T a=p[i].cross(p[j]);
245            cx+=(p[i].x+p[j].x)*a;
246            cy+=(p[i].y+p[j].y)*a;
247            w+=a;
248        }
249        return point<T>(cx/3/w,cy/3/w);
250    }
251    }
252    char ahas(const point<T>&t)const{//點是否
253        //在簡單多邊形內，是的話回傳1，在邊上回
254        //傳-1，否則回傳0
255    bool c=0;
256    for(int i=0,j=p.size()-1;i<p.size();i=i
257        ++){
258        if(line<T>(p[i],p[j]).point_on_segment
259            (t))return -1;
260        else if((p[i].y>t.y)!=p[j].y>t.y)&&
261            t.x<(p[j].x+p[i].x)*(t.y-p[i].y)/(p[j]
262                ].y-p[i].y)+p[i].x)
263            c=!c;
264    }
265    return c;
266    }

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198 }
199 for(int i=s.size()-2,t=m+1;i>=0;--i){
200     while(m>=t&&(p[m-1]-p[m-2]).cross(s[i]
201         ]-p[m-2])<=0)--m;
202     p[m++]=s[i];
203 }
204 if(s.size()>1)--m;
205 p.resize(m);
206 }
207 T diam(){//直徑
208     int n=p.size(),t=1;
209     T ans=0;p.push_back(p[0]);
210     for(int i=0;i<n;i++){
211         point<T> now=p[i+1]-p[i];
212         while(now.cross(p[t+1]-p[i])>now.cross
213             (p[t]-p[i]))t=(t+1)%n;
214         ans=max(ans,max((p[i]-p[t]).abs2(),(p[
215             i+1]-p[t+1]).abs2()));
216     }
217     return p.pop_back(),ans;
218 }
219 T min_cover_rectangle(){//最小覆蓋矩形
220     int n=p.size(),t=1,r=1,l;
221     if(n<3)return 0;//也可以做最小周長矩形
222     T ans=1e99;p.push_back(p[0]);
223     for(int i=0;i<n;i++){
224         point<T> now=p[i+1]-p[i];
225         while(now.cross(p[t+1]-p[i])>now.cross
226             (p[t]-p[i]))t=(t+1)%n;
227         while(now.dot(p[r+1]-p[i])>now.dot(p[r]
228             ]-p[i]))r=(r+1)%n;
229         if(l==r;
230         while(now.dot(p[l+1]-p[i])<=now.dot(p[
231             l]-p[i]))l=(l+1)%n;
232         T d=now.abs2();
233         T tmp=now.cross(p[t]-p[i])*(now.dot(p[
234             r]-p[i])-now.dot(p[l]-p[i]))/d;
235         ans=min(ans,tmp);
236     }
237     return p.pop_back(),ans;
238 }
239 T max_triangle(){//最大內接三角形
240     int n=p.size(),a=1,b=2;
241     if(n<3)return 0;
242     T ans=0,tmp;p.push_back(p[0]);
243     for(int i=0;i<n;i++){
244         while((p[a]-p[i]).cross(p[b+1]-p[i])>(
245             tmp=(p[a]-p[i]).cross(p[b]-p[i]))
246             b=(b+1)%n;
247         ans=max(ans,tmp);
248         while((p[a+1]-p[i]).cross(p[b]-p[i])>(
249             tmp=(p[a]-p[i]).cross(p[b]-p[i]))
250             a=(a+1)%n;
251         ans=max(ans,tmp);
252     }
253     return p.pop_back(),ans/2;
254 }
255 T dis2(polygon &p1){//凸包最近距離平方
256     vector<point<T> > &P=p,Q=p1.p;
257     int n=P.size(),m=Q.size(),l=0,r=0;
258     for(int i=0;i<n;i++){
259         if(P[i].y<P[l].y)l=i
260     }
261     for(int i=0;i<m;i++){
262         if(Q[i].y<Q[r].y)r=i
263     }
264     P.push_back(P[0]),Q.push_back(Q[0]);
265
266     T ans=1e99;
267     for(int i=0;i<n;i++){
268         while((P[i]-P[l+1]).cross(Q[r+1]-Q[r])
269             <0)r=(r+1)%m;
270         ans=min(ans,line<T>(P[l],P[l+1]).
271             seg_dis2(line<T>(Q[r],Q[r+1])));
272         l=(l+1)%n;
273     }
274     return P.pop_back(),Q.pop_back(),ans;
275 }
276 static char sign(const point<T>&t){
277     return (t.y==0?t.x:t.y)<0;
278 }
279 static bool angle_cmp(const line<T>& A,
280     const line<T>& B){
281     point<T> a=A.p2-A.p1,b=B.p2-B.p1;
282     return sign(a)<sign(b)||((sign(a)==sign(b)
283         )&&a.cross(b)>0);
284 }
285 int halfplane_intersection(vector<line<T>
286     > &s){//半平面交
287     sort(s.begin(),s.end(),angle_cmp);
288     //線段
289     //左側為該線段半平面
290     int L,R,n=s.size();
291     vector<point<T> > px(n);
292     vector<line<T> > q(n);
293     q[L=R=0]=s[0];
294     for(int i=1;i<n;i++){
295         while(L<R&&s[i].cross(px[R-1])<=0)--R;
296         while(L<R&&s[i].cross(px[L])<=0)+L;
297         q[++R]=s[i];
298         if(q[R].parallel(q[R-1])){
299             --R;
300             if(q[R].cross(s[i].p1)>0)q[R]=s[i];
301         }
302         if(L<R)px[R-1]=q[R-1].
303             line_intersection(q[R]);
304     }
305     while(L<R&&q[L].cross(px[R-1])<=0)--R;
306     p.clear();
307     if(R-L==1)return 0;
308     px[R]=q[R].line_intersection(q[L]);
309     for(int i=L;i<R;i++)p.push_back(px[i]);
310     return R-L+1;
311 }
312 template<typename T>
313 struct triangle{
314     point<T> a,b,c;
315     triangle(){
316     }
317     triangle(const point<T> &a,const point<T>
318         &b,const point<T> &c):a(a),b(b),c(c){
319     }
320     T area()const{
321         T t=(b-a).cross(c-a)/2;
322         return t>0?t:-t;
323     }
324     point<T> barycenter()const{//重心
325         return (a+b+c)/3;
326     }
327     point<T> circumcenter()const{//外心
328         static line<T> u,v;
329         u.p1=(a+b)/2;
330         u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-
331             b.x);
332         v.p1=(a+c)/2;
333         v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-
334             c.x);
335         return u.line_intersection(v);
336     }
337     point<T> incenter()const{//內心
338         T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2
339             ()),C=sqrt((a-b).abs2());
340         return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+
341             B*b.y+C*c.y)/(A+B+C);
342     }
343     point<T> perpercenter()const{//垂心
344         return barycenter()*3-circumcenter()*2;
345     }
346 };
347 template<typename T>
348 struct point3D{
349     T x,y,z;
350     point3D(){
351     }
352     point3D(const T&x,const T&y,const T&z):x(x
353         ),y(y),z(z){
354     }
355     point3D operator+(const point3D &b)const{
356         return point3D(x+b.x,y+b.y,z+b.z);
357     }
358     point3D operator-(const point3D &b)const{
359         return point3D(x-b.x,y-b.y,z-b.z);
360     }
361     point3D operator*(const T &b)const{
362         return point3D(x*b,y*b,z*b);
363     }
364     point3D operator/(const T &b)const{
365         return point3D(x/b,y/b,z/b);
366     }
367     bool operator==(const point3D &b)const{
368         return x==b.x&&y==b.y&&z==b.z;
369     }
370     T dot(const point3D &b)const{
371         return x*b.x+y*b.y+z*b.z;
372     }
373     point3D cross(const point3D &b)const{
374         return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x
375             *b.y-y*b.x);
376     }
377     T abs2()const{//向量長度的平方
378         return dot(*this);
379     }
380     T area2(const point3D &b)const{//和b、原點
381         //圍成面積的平方
382         return cross(b).abs2()/4;
383     }
384 };
385 template<typename T>
386 struct line3D{
387     point3D<T> p1,p2;
388     line3D(){
389     }
390     line3D(const point3D<T> &p1,const point3D<
391         T> &p2):p1(p1),p2(p2){
392     }
393     T dis2(const point3D<T> &p,bool is_segment
394         =0)const{//點跟直線/線段的距離平方
395         point3D<T> v=p2-p1,v1=p-p1;
396         if(is_segment){
397             point3D<T> v2=p-p2;
398             if(v.dot(v1)<=0)return v1.abs2();
399             if(v.dot(v2)>=0)return v2.abs2();
400         }
401         point3D<T> tmp=v.cross(v1);
402         return tmp.abs2()/v.abs2();
403     }
404     pair<point3D<T>,point3D<T> > closest_pair(
405         const line3D<T> &l1)const{
406         point3D<T> v1=(p1-p2),v2=(l1.p1-l.p2);
407         point3D<T> N=v1.cross(v2),ab(p1-l.p1);
408         //if(N.abs2()==0)return NULL;平行或重合
409     }
410 };
411 T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//
412     最近點對距離
413     point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.
414         cross(d2);
415     T t1=((l.p1-p1).cross(d2)).dot(D)/D.abs2
416         ();
417     T t2=((l.p1-p1).cross(d1)).dot(D)/D.abs2
418         ();
419     return make_pair(p1+d1*t1,l.p1+d2*t2);
420 }
421 bool same_side(const point3D<T> &a,const
422     point3D<T> &b)const{
423     return (p2-p1).cross(a-p1).dot((p2-p1).
424         cross(b-p1))>0;
425 }
426 template<typename T>
427 struct plane{
428     point3D<T> p0,n;//平面上的點和法向量
429     plane(){
430     }
431     plane(const point3D<T> &p0,const point3D<T>
432         &n):p0(p0),n(n){
433     }
434     T dis2(const point3D<T> &p)const{//點到平
435         //面距離的平方
436         T tmp=(p-p0).dot(n);
437         return tmp*tmp/n.abs2();
438     }
439     point3D<T> projection(const point3D<T> &p)
440         const{
441         return p-n*(p-p0).dot(n)/n.abs2();
442     }
443     point3D<T> line_intersection(const line3D<
444         T> &l1)const{
445         T tmp=n.dot(l1.p2-l.p1);//等於0表示平行或
446             重合該平面
447         return l1.p1+(l1.p2-l.p1)*(n.dot(p0-l.p1)/
448             tmp);
449     }
450     line3D<T> plane_intersection(const plane &
451         p1)const{
452         point3D<T> e=n.cross(p1.n),v=n.cross(e);
453         T tmp=p1.n.dot(v);//等於0表示平行或重合
454             該平面
455         point3D<T> q=p0+(v*(p1.n.dot(p1.p0-p0))/
456             tmp);
457         return line3D<T>(q,q+e);
458     }
459 };
460 template<typename T>
461 struct triangle3D{
462     point3D<T> a,b,c;
463     triangle3D(){
464     }
465     triangle3D(const point3D<T> &a,const
466         point3D<T> &b,const point3D<T> &c):a(a)
467         ,b(b),c(c){
468     }
469     bool point_in(const point3D<T> &p)const{//
470         //點在該平面上的投影在三角形中
471         return line3D<T>(b,c).same_side(p,a)&&
472             line3D<T>(a,c).same_side(p,b)&&
473             line3D<T>(a,b).same_side(p,c);
474     }
475 };
476 template<typename T>
477 struct tetrahedron{//四面體

```

```

404 point3D<T> a,b,c,d;
405 tetrahedron({})
406 tetrahedron(const point3D<T> &a,const
    point3D<T> &b,const point3D<T> &c,
    const point3D<T> &d):a(a),b(b),c(c),d(
    d){}
407 T volume6()const{//體積的六倍
408     return (d-a).dot((b-a).cross(c-a));
409 }
410 point3D<T> centroid()const{
411     return (a+b+c+d)/4;
412 }
413 bool point_in(const point3D<T> &p)const{
414     return triangle3D<T>(a,b,c).point_in(p)
        &&triangle3D<T>(c,d,a).point_in(p);
415 }
416 };
417 template<typename T>
418 struct convexhull3D{
419     static const int MAXN=105;
420     struct face{
421         int a,b,c;
422         bool use;
423         face(){}
424         face(int a,int b,int c):a(a),b(b),c(c),
            use(1){}
425 };
426 vector<point3D<T> > pt;
427 vector<face> fc;
428 int fid[MAXN][MAXN];
429 static bool point_cmp(const point3D<T> &a,
    const point3D<T> &b){
430     return a.x<b.x|| (a.x==b.x&&(a.y<b.y|| (a.
        y==b.y&&a.z<b.z)));
431 }
432 bool outside(int p,int a,int b,int c)const
    {
433     return tetrahedron<T>(pt[a],pt[b],pt[c],
        pt[p]).volume6()<0;
434 }
435 bool outside(int p,int f)const{return
    outside(p,fc[f].a,fc[f].b,fc[f].c);}
436 void AddFace(int a,int b,int c,int p){
437     if(outside(p,a,b,c))fid[c][b]=fid[b][a]=
        fid[a][c]=fc.size(),fc.push_back(
        face(c,b,a));
438     else fid[a][b]=fid[b][c]=fid[c][a]=fc.
        size(),fc.push_back(face(a,b,c));
439 }
440 bool dfs(int p,int f){
441     if(!fc[f].use)return true;
442     if(outside(p,f)){
443         int a=fc[f].a,b=fc[f].b,c=fc[f].c;
444         fc[f].use=false;
445         if(!dfs(p,fid[b][a]))AddFace(p,a,b,c);
446         if(!dfs(p,fid[c][b]))AddFace(p,b,c,a);
447         if(!dfs(p,fid[a][c]))AddFace(p,c,a,b);
448         return true;
449     }else return false;
450 }
451 void build(){
452     bool ok=false;
453     fc.clear();
454     sort(pt.begin(),pt.end(),point_cmp);

```

```

455 pt.resize(unique(pt.begin(),pt.end())-pt
    .begin());
456 for(size_t i=2;i<pt.size();++i){
457     if((pt[0]-pt[i]).area2(pt[1]-pt[i])
        !=0){
458         ok=true;
459         swap(pt[i],pt[2]);
460         break;
461     }
462 }
463 if(!ok)return;
464 ok=false;
465 for(size_t i=3;i<pt.size();++i){
466     if(tetrahedron<T>(pt[0],pt[1],pt[2],pt
        [i]).volume6()!=0){
467         ok=true;
468         swap(pt[i],pt[3]);
469         break;
470     }
471 }
472 if(!ok)return;
473 for(int i=0;i<4;++i)AddFace(i,(i+1)%4,(i
    +2)%4,(i+3)%4);
474 for(size_t i=4;i<pt.size();++i){
475     for(int j=fc.size()-1;j>=0;--j){
476         if(outside(i,j)){
477             dfs(i,j);
478             break;
479         }
480     }
481 }
482 size_t sz=0;
483 for(size_t i=0;i<fc.size();++i)if(fc[i].
    use)fc[sz++]=fc[i];
484 fc.resize(sz);
485 }
486 point3D<T> centroid()const{
487     point3D<T> res(0,0,0);
488     T vol=0;
489     for(size_t i=0;i<fc.size();++i){
490         T tmp=pt[fc[i].a].dot(pt[fc[i].b].
            cross(pt[fc[i].c]));
491         res=res+(pt[fc[i].a]+pt[fc[i].b]+pt[fc
            [i].c])*tmp;
492         vol+=tmp;
493     }
494     return res/(vol*4);
495 }
496 };

```

1.2 SmallestCircle.cpp

```

1 #include "Geometry.cpp"
2 struct Circle{
3     typedef point<double> p;
4     typedef const point<double> cp;
5     p x;
6     double r2;
7     bool incircle(cp &c)const{return (x-c).
        abs2()<=r2;}
8 };
9

```

```

10 Circle TwoPointCircle(Circle::cp &a, Circle
    ::cp &b) {
11     Circle::p m=(a+b)/2;
12     return (Circle){m,(a-m).abs2()};
13 }
14
15 Circle outcircle(Circle::p a, Circle::p b,
    Circle::p c) {
16     if(TwoPointCircle(a,b).incircle(c))
        return TwoPointCircle(a,b);
17     if(TwoPointCircle(b,c).incircle(a))
        return TwoPointCircle(b,c);
18     if(TwoPointCircle(c,a).incircle(b))
        return TwoPointCircle(c,a);
19     Circle::p ret;
20     double a1=b.x-a.x, b1=b.y-a.y, c1=(a1*a1
        +b1*b1)/2;
21     double a2=c.x-a.x, b2=c.y-a.y, c2=(a2*a2
        +b2*b2)/2;
22     double d = a1*b2 - a2*b1;
23     ret.x=a.x+(c1*b2-c2*b1)/d;
24     ret.y=a.y+(a1*c2-a2*c1)/d;
25     return (Circle){ret,(ret-a).abs2()};
26 }
27 //rand required
28 Circle SmallestCircle(std::vector<Circle::p>
    &p){
29     int n=p.size();
30     if(n==1) return (Circle){p[0],0.0};
31     if(n==2) return TwoPointCircle(p[0],p
        [1]);
32     random_shuffle(p.begin(),p.end());
33     Circle c = {p[0],0.0};
34     for(int i=0;i<n;++i){
35         if(c.incircle(p[i])) continue;
36         c=Circle{p[i],0.0};
37         for(int j=0;j<i;++j){
38             if(c.incircle(p[j])) continue;
39             c=TwoPointCircle(p[i],p[j]);
40             for(int k=0;k<j;++k){
41                 if(c.incircle(p[k]))
                    continue;
42                 c=outcircle(p[i],p[j],p[k]);
43             }
44         }
45     }
46     return c;
47 }

```

```

10 for(int i=1;i<=r;++i)
11     if((v[i].x-v[mid].x)*(v[i].x-v[mid].x)<
        dis)t.push_back(v[i]);
12 sort(t.begin(),t.end(),point<T>::y_cmp);//
    如果用merge_sort的方式可以O(n)
13 for(int i=0;i<(int)t.size();++i)
14     for(int j=1;j<=3&&i+j<(int)t.size();++j)
15         if((tmd=(t[i]-t[i+j]).abs2())<dis)dis=
            tmd;
16     return dis;
17 }
18 template<typename T>
19 inline T closest_pair(vector<point<T> > &v){
20     vector<point<T> > t;
21     sort(v.begin(),v.end(),point<T>::x_cmp);
22     return closest_pair(v,t,0,v.size()-1);//最
    近點對距離
23 }

```

1.4 浮點數誤差模板.cpp

```

1 const double EPS=1e-9;
2 struct Double{
3     double d;
4     Double(double d=0):d(d){}
5     bool operator <(const Double &b)const{
6         return d-b.d<-EPS;}
7     bool operator >(const Double &b)const{
8         return d-b.d>EPS;}
9     bool operator ==(const Double &b)const{
10        return fabs(d-b.d)<=EPS;}
11     bool operator !=(const Double &b)const{
12        return fabs(d-b.d)>EPS;}
13     bool operator <=(const Double &b)const{
14        return d-b.d<=EPS;}
15     bool operator >=(const Double &b)const{
16        return d-b.d>=EPS;}
17     operator double()const{return d;}
18 };

```

2 Data_Structure

2.1 DLX.cpp

```

1 #define INF LLONG_MAX
2 #define MAXM 1030
3 #define MAXND 16390
4 struct DLX{
5     int n,m,sz,ansd;//高是n 寬是m的稀疏矩陣
6     int S[MAXM],H[MAXN];
7     int row[MAXND],col[MAXND];//每個節點代表的
        列行
8     int L[MAXND],R[MAXND],U[MAXND],D[MAXND];
9     vector<int> ans,ans1;
10    void init(int _n,int _m){
11        n=_n,m=_m;
12        for(int i=0;i<=m;++i){

```

1.3 最近點對.cpp

```

1 #define INF LLONG_MAX
2 template<typename T>
3 T closest_pair(vector<point<T> >&v,vector<
    point<T> >&t,int l,int r){
4     T dis=INF,tmd;
5     if(l==r)return dis;
6     int mid=(l+r)/2;
7     if((tmd=closest_pair(v,t,l,mid))<dis)dis=
        tmd;
8     if((tmd=closest_pair(v,t,mid+1,r))<dis)dis
        =tmd;
9     t.clear();

```

```

13     U[i]=D[i]=i,L[i]=i-1,R[i]=i+1;
14     S[i]=0;
15 }
16 R[m]=0,L[0]=m;
17 sz=m,ansd=INT_MAX;//ansd存最優解的個數
18 for(int i=1;i<=n;++i)H[i]=-1;
19 }
20 void add(int r,int c){
21     ++S[col[++sz]=c];
22     row[sz]=r;
23     D[sz]=D[c],U[D[c]]=sz,U[sz]=c,D[c]=sz;
24     if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
25     else R[sz]=R[H[r]],L[R[H[r]]]=sz,L[sz]=H[r],R[H[r]]=sz;
26 }
27 #define DFOR(i,A,s) for(int i=A[s];i!=s;i=
28     A[i])
29 void remove(int c){//刪除第c行和所有當前覆
30     蓋到第c行的列
31     L[R[c]]=L[c],R[L[c]]=R[c];//這裡刪除第c
32     行·若有些行不需要處理可以在開始時呼
33     叫他
34     DFOR(i,D,c)DFOR(j,R,i){U[D[j]]=U[j],D[U[
35     j]]=D[j],--S[col[j]]};
36 }
37 void restore(int c){//恢復第c行和所有當前
38     覆蓋到第c行的列·remove的逆操作
39     DFOR(i,U,c)DFOR(j,L,i){++S[col[j]],U[D[j]
40     ]=j,D[U[j]]=j};
41     L[R[c]]=c,R[L[c]]=c;
42 }
43 void remove2(int nd){//刪除nd所在的行當前
44     所有點(包括虛擬節點)·只保留nd
45     DFOR(i,D,nd)L[R[i]]=L[i],R[L[i]]=R[i];
46 }
47 void restore2(int nd){//刪除nd所在的行當前
48     所有點·為remove2的逆操作
49     DFOR(i,U,nd)L[R[i]]=R[L[i]]=i;
50 }
51 bool vis[MAXM];
52 int h()//估價函數 for IDA*
53 int res=0;
54 memset(vis,0,sizeof(vis));
55 DFOR(i,R,0)if(!vis[i]){
56     vis[i]=1;
57     ++res;
58     DFOR(j,D,i)DFOR(k,R,j)vis[col[k]]=1;
59 }
60 return res;
61 }
62 bool dfs(int d){//for精確覆蓋問題
63     if(d+h()>=ansd)return 0;//找最佳解用·找
64     任意解可以刪掉
65     if(!R[0]){ansd=d;return 1;}
66     int c=R[0];
67     DFOR(i,R,0)if(S[i]<S[c])c=i;
68     remove(c);
69     DFOR(i,D,c){
70         ans.push_back(row[i]);
71         DFOR(j,R,i)remove(col[j]);
72         if(dfs(d+1))return 1;
73         ans.pop_back();
74         DFOR(j,L,i)restore(col[j]);
75     }
76 }
77 void dfs2(int d){//for最小重複覆蓋問題
78     if(d+h()>=ansd)return 0;
79     if(!R[0]){ansd=d;ans=ansd;return;}
80     int c=R[0];
81     DFOR(i,R,0)if(S[i]<S[c])c=i;
82     DFOR(i,D,c){
83         anst.push_back(row[i]);
84         remove2(i);
85         DFOR(j,R,i)remove2(j),--S[col[j]];
86         dfs2(d+1);
87         anst.pop_back();
88         DFOR(j,L,i)restore2(j),++S[col[j]];
89         restore2(i);
90     }
91 }
92 bool exact_cover()//解精確覆蓋問題
93 ans.clear();//答案
94 return dfs(0);
95 }
96 void min_cover()//解最小重複覆蓋問題
97 ans.clear();//暫存用·答案還是存在ans裡
98 dfs2(0);
99 #undef DFOR
100 };

```

2.2 Dynamic_KD_tree.cpp

```

1 template<typename T,size_t kd>//有kd個維度
2 class kd_tree{
3 public:
4     struct point{
5         T d[kd];
6         T dist(const point &x)const{
7             T ret=0;
8             for(size_t i=0;i<kd;++i)ret+=std::
9                 abs(d[i]-x.d[i]);
10            return ret;
11        }
12        bool operator==(const point &p){
13            for(size_t i=0;i<kd;++i)
14                if(d[i]!=p.d[i])return 0;
15            return 1;
16        }
17        bool operator<(const point &b)const{
18            return d[0]<b.d[0];
19        }
20    };
21 private:
22     struct node{
23         node *l,*r;
24         point pid;
25         int s;
26         node(const point &p):l(0),r(0),pid(p),
27             s(1){}
28         ~node(){delete l;delete r;}
29         void up(){s=(l?l->s:0)+1+(r?r->s:0);}
30     };
31     *root;

```

```

32 const double alpha,loga;
33 const T INF;//記得要給INF·表示極大值
34 int maxn;
35 struct __cmp{
36     int sort_id;
37     bool operator()(const node*x,const
38         node*y)const{
39         return operator()(x->pid,y->pid);
40     }
41     bool operator()(const point &x,const
42         point &y)const{
43         if(x.d[sort_id]!=y.d[sort_id])
44             return x.d[sort_id]<y.d[sort_id];
45         for(size_t i=0;i<kd;++i)
46             if(x.d[i]!=y.d[i])return x.d[i]<y.
47                 d[i];
48         return 0;
49     }
50 }cmp;
51 int size(node *o){return o?o->s:0;}
52 std::vector<node*> A;
53 node* build(int k,int l,int r){
54     if(l>r)return 0;
55     if(k==kd)k=0;
56     int mid=(l+r)/2;
57     cmp.sort_id=k;
58     std::nth_element(A.begin()+l,A.begin()+
59         mid,A.begin()+r+1,cmp);
60     node *ret=A[mid];
61     ret->l=build(k+1,l,mid-1);
62     ret->r=build(k+1,mid+1,r);
63     ret->up();
64     return ret;
65 }
66 bool isbad(node*o){
67     return size(o->l)>alpha*o->s||size(o->
68         r)>alpha*o->s;
69 }
70 void flatten(node *u,typename std::
71     vector<node*>::iterator &it){
72     if(!u)return;
73     flatten(u->l,it);
74     *it=u;
75     flatten(u->r,++it);
76 }
77 void rebuild(node*&u,int k){
78     if((int)A.size()<u->s)A.resize(u->s);
79     typename std::vector<node*>::iterator
80         it=A.begin();
81     flatten(u,it);
82     u=build(k,0,u->s-1);
83 }
84 bool insert(node*&u,int k,const point &x
85     ,int dep){
86     if(!u){
87         u=new node(x);
88         return dep<=0;
89     }
90     ++u->s;
91     cmp.sort_id=k;
92     if(insert(cmp(x,u->pid)?u->l:u->r,(k
93         +1)%kd,x,dep-1)){
94         if(!isbad(u))return 1;
95         rebuild(u,k);
96     }
97 }

```

```

85     return 0;
86 }
87 node *findmin(node*o,int k){
88     if(!o)return 0;
89     if(cmp.sort_id==k)return o->l?findmin(
90         o->l,(k+1)%kd):o;
91     node *l=findmin(o->l,(k+1)%kd);
92     node *r=findmin(o->r,(k+1)%kd);
93     if(l&&!r)return cmp(l,o)?l:o;
94     if(!l&&r)return cmp(r,o)?r:o;
95     if(l&&r)return cmp(l,o)?l:o;
96     return cmp(r,o)?r:o;
97 }
98 bool erase(node *&u,int k,const point &x
99     ){
100     if(!u)return 0;
101     if(u->pid==x){
102         if(u->r);
103         else if(u->l){
104             u->r=u->l;
105             u->l=0;
106         }else{
107             delete u;
108             u=0;
109             return 1;
110         }
111         --u->s;
112         cmp.sort_id=k;
113         u->pid=findmin(u->r,(k+1)%kd)->pid;
114         return erase(u->r,(k+1)%kd,u->pid);
115     }
116     cmp.sort_id=k;
117     if(erase(cmp(x,u->pid)?u->l:u->r,(k+1)
118         %kd,x)){
119         --u->s;return 1;
120     }else return 0;
121 }
122 T heuristic(const T h[])const{
123     T ret=0;
124     for(size_t i=0;i<kd;++i)ret+=h[i];
125     return ret;
126 }
127 int qM;
128 std::priority_queue<std::pair<T,point >
129     > pQ;
130 void nearest(node *u,int k,const point &
131     x,T *h,T &mndist){
132     if(u==0||heuristic(h)>=mndist)return;
133     T dist=u->pid.dist(x),old=h[k];
134     /*mndist=std::min(mndist,dist);*/
135     if(dist<mndist){
136         pQ.push(std::make_pair(dist,u->pid))
137             ;
138         if((int)pQ.size()==qM+1)
139             mndist=pQ.top().first,pQ.pop();
140     }
141     if(x.d[k]<u->pid.d[k]){
142         nearest(u->l,(k+1)%kd,x,h,mndist);
143         h[k]=std::abs(x.d[k]-u->pid.d[k]);
144         nearest(u->r,(k+1)%kd,x,h,mndist);
145     }else{
146         nearest(u->r,(k+1)%kd,x,h,mndist);
147         h[k]=std::abs(x.d[k]-u->pid.d[k]);
148         nearest(u->l,(k+1)%kd,x,h,mndist);
149     }
150 }

```


2.3 kd_tree_replace_segment

```

145 h[k]=old;
146 }
147 std::vector<point> in_range;
148 void range(node *u, int k, const point &mi,
149         const point &ma){
150     if(!u) return;
151     bool is=1;
152     for(int i=0; i<kd; ++i)
153         if(u->pid.d[i]<mi.d[i] || ma.d[i]<u->
154             pid.d[i]){
155             is=0; break;
156         }
157     if(is) in_range.push_back(u->pid);
158     if(mi.d[k]<=u->pid.d[k]) range(u->l, (k
159         +1)%kd, mi, ma);
160     if(ma.d[k]>=u->pid.d[k]) range(u->r, (k
161         +1)%kd, mi, ma);
162 }
163 public:
164 kd_tree(const T &INF, double a=0.75): root(
165     (0), alpha(a), loga(log2(1.0/a)), INF(
166     INF), maxn(1)){
167     ~kd_tree(){ delete root; }
168 void clear(){ delete root; root=0; maxn=1; }
169 void build(int n, const point *p){
170     delete root; A.resize(maxn=n);
171     for(int i=0; i<n; ++i) A[i]=new node(p[i
172         ]);
173     root=build(0,0,n-1);
174 }
175 void insert(const point &x){
176     insert(root,0,x, __lg(size(root))/loga)
177     ;
178     if(root->s>maxn) maxn=root->s;
179 }
180 bool erase(const point &p){
181     bool d=erase(root,0,p);
182     if(root&&root->s<alpha*maxn) rebuild();
183     return d;
184 }
185 void rebuild(){
186     if(root) rebuild(root,0);
187     maxn=root->s;
188 }
189 T nearest(const point &x, int k){
190     qM=k;
191     T mndist=INF, h[kd]={};
192     nearest(root,0,x,h,mndist);
193     mndist=pQ.top().first;
194     pQ=std::priority_queue<std::pair<T,
195         point >>()>;
196     return mndist; // 回傳離x第k近的點的距離
197 }
198 const std::vector<point> &range(const
199     point &mi, const point &ma){
200     in_range.clear();
201     range(root,0,mi,ma);
202     return in_range; // 回傳介於mi到ma之間的
203     點vector
204 }
205 int size(){ return root?root->s:0; }
206 };

```

/*kd樹代替高維線段樹*/

```

256 struct node{
257     node *l,*r;
258     point pid,mi,ma;
259     int s;
260     int data;
261     node(const point &p, int d): l(0), r(0), pid(p
262         ), mi(p), ma(p), s(1), data(d), dmin(d),
263         dmax(d){}
264 void up(){
265     mi=ma=pid;
266     s=1;
267     if(l){
268         for(int i=0; i<kd; ++i){
269             mi.d[i]=min(mi.d[i], l->mi.d[i]);
270             ma.d[i]=max(ma.d[i], l->ma.d[i]);
271         }
272         s+=l->s;
273     }
274     if(r){
275         for(int i=0; i<kd; ++i){
276             mi.d[i]=min(mi.d[i], r->mi.d[i]);
277             ma.d[i]=max(ma.d[i], r->ma.d[i]);
278         }
279         s+=r->s;
280     }
281 void up2(){
282     // 其他懶惰標記向上更新
283 }
284 void down(){
285     // 其他懶惰標記下推
286 }
287 } *root;
288 /*檢查區間包含用的函數*/
289 inline bool range_include(node *o, const
290     point &L, const point &R){
291     for(int i=0; i<kd; ++i){
292         if(L.d[i]>o->ma.d[i] || R.d[i]<o->mi.d[i])
293             return 0;
294     }
295     // 只要(L,R)區間有和o的區間有交集就回傳
296     true
297     return 1;
298 }
299 inline bool range_in_range(node *o, const
300     point &L, const point &R){
301     for(int i=0; i<kd; ++i){
302         if(L.d[i]>o->mi.d[i] || o->ma.d[i]>R.d[i])
303             return 0;
304     }
305     // 如果(L,R)區間完全包含o的區間就回傳true
306     return 1;
307 }
308 inline bool point_in_range(node *o, const
309     point &L, const point &R){
310     for(int i=0; i<kd; ++i){
311         if(L.d[i]>o->pid.d[i] || R.d[i]<o->pid.d[i]
312             ) return 0;
313     }
314     // 如果(L,R)區間完全包含o->pid這個點就回傳
315     true
316     return 1;
317 }
318 }

```

2.3 kd_tree_replace_segment

```

544 /*單點修改 · 以單點改值為例*/
545 void update(node *u, const point &x, int data,
546     int k=0){
547     if(!u) return;
548     u->down();
549     if(u->pid==x){
550         u->data=data;
551         u->up2();
552         return;
553     }
554     cmp.sort_id=k;
555     update(cmp(x, u->pid)?u->l:u->r, x, data, (k
556         +1)%kd);
557     u->up2();
558 }
559 /*區間修改*/
560 void update(node *o, const point &L, const
561     point &R, int data){
562     if(!o) return;
563     o->down();
564     if(range_in_range(o, L, R)){
565         // 區間懶惰標記修改
566         o->down();
567         return;
568     }
569     if(point_in_range(o, L, R)){
570         // 這個點在(L,R)區間 · 但是他的左右子樹不
571         一定在區間中
572         // 單點懶惰標記修改
573         if(o->l && range_include(o->l, L, R)) update(o
574             ->l, L, R, data);
575         if(o->r && range_include(o->r, L, R)) update(o
576             ->r, L, R, data);
577         o->up2();
578     }
579     // 區間查詢 · 以總和為例*/
580 int query(node *o, const point &L, const point
581     &R){
582     if(!o) return 0;
583     o->down();
584     if(range_in_range(o, L, R)) return o->sum;
585     int ans=0;
586     if(point_in_range(o, L, R)) ans+=o->data;
587     if(o->l && range_include(o->l, L, R)) ans+=
588         query(o->l, L, R);
589     if(o->r && range_include(o->r, L, R)) ans+=
590         query(o->r, L, R);
591     return ans;
592 }

```

2.4 persistent_segment_tree.cpp

```

1 #include<bits/stdc++.h> //POJ 2104
2 using namespace std;
3 struct node{
4     int l,r;
5     int data;

```

```

6     node(int l, int r, int d): l(l), r(r), data(d)
7     {}
8 };
9 vector<node> nds;
10 inline void up(int o, int l, int r){
11     nds[o].data=nds[l].data+nds[r].data;
12 }
13 inline int new_node(int l, int r, int d){
14     nds.push_back(node(l, r, d));
15     return nds.size()-1;
16 }
17 inline int new_node(const node &nd){
18     nds.push_back(nd);
19     return nds.size()-1;
20 }
21 int build_tree(int l, int r){
22     int nd=new_node(-1, -1, 0);
23     if(l==r) return nd;
24     int mid=(l+r)/2;
25     int L=build_tree(l, mid); // 執行時vector會被
26     重構
27     int R=build_tree(mid+1, r); // 一定要這樣寫
28     nds[nd].l=L;
29     nds[nd].r=R;
30     // up(nd, L, R);
31     return nd;
32 }
33 int insert(int l, int r, int rt, int x, int d){
34     if(x<l || r<x) return rt;
35     int nd=new_node(nds[rt]);
36     if(l==r && l==x) nds[nd].data+=d;
37     else{
38         int mid=(l+r)/2;
39         int L=insert(l, mid, nds[nd].l, x, d);
40         int R=insert(mid+1, r, nds[nd].r, x, d);
41         nds[nd].l=L;
42         nds[nd].r=R;
43         up(nd, L, R);
44     }
45     return nd;
46 }
47 inline int cal(int L, int R){
48     return nds[R].data-nds[L].data;
49 }
50 int find(int l, int r, int L, int R, int k){
51     if(l==r) return l;
52     int mid=(l+r)/2;
53     int add=cal(nds[L].l, nds[R].l);
54     if(k<=add) return find(l, mid, nds[L].l, nds[R]
55         .l, k);
56     return find(mid+1, r, nds[L].r, nds[R].r, k-
57         add);
58 }
59 int n,m;
60 int s[100005];
61 int root[100005];
62 int main(){
63     while(~scanf("%d%d", &n, &m)){
64         nds.clear();
65         vector<int> lsh;
66         for(int i=1; i<=n; ++i){
67             scanf("%d", &s[i]);
68             lsh.push_back(s[i]);
69         }
70         sort(lsh.begin(), lsh.end());

```

```

67 lsh.resize(unique(lsh.begin(),lsh.end())
68           -lsh.begin());
69 int N=(int)lsh.size()-1;
70 root[0]=build_tree(0,N);
71 for(int i=1;i<=n;++i){
72     s[i]=lower_bound(lsh.begin(),lsh.end()
73                     ,s[i])-lsh.begin();
74     root[i]=insert(0,N,root[i-1],s[i],1);
75 }
76 while(m--){
77     int a,b,k;
78     scanf("%d%d%d",&a,&b,&k);
79     int res=find(0,N,root[a-1],root[b],k);
80     printf("%d\n",lsh[res]);
81 }
82 return 0;

```

2.7 操作分治.cpp

```

1 void dq(int l,int r){
2     if(l==r) return;
3     int mid=(l+r)/2;
4     dq(l,mid);
5     //處理[l,mid]的操作對[mid+1,r]的影響
6     dq(mid+1,r);
7 }

```

2.8 整體二分.cpp

```

1 void BS(int l,int r,vector<Item> &vs){
2     //答案該<L會有的已經做完了
3     if(l==r)整個vs的答案=l;////////
4     int mid=(l+r)/2;
5     do_thing(l,mid);//做答案<=mid會做的事
6     vector<Item> left=vs裡滿足的;
7     vector<Item> right=vs-left;
8     undo_thing(l,mid);
9     BS(l,mid,left);
10    do_thing(l,mid);
11    BS(mid+1,r,right);////////
12 }

```

3 default

3.1 debug.cpp

```

1 void split(node *o,node *&a,node *&b,int k){
2     if(!o)a=b=0;
3     else{
4         //o=new node(*o);
5         o->down();
6         if(k<=size(o->l)){
7             b=o;
8             split(o->l,a,b->l,k);
9         }else{
10            a=o;
11            split(o->r,a->r,b,k-size(o->l)-1);
12        }
13        o->up();
14    }
15 }
16 node *merge(node *a,node *b){
17     if(!a||!b) return a?a:b;
18     static int x;
19     if(x++%(a->s+b->s)<a->s){
20         //a=new node(*a);
21         a->down();
22         a->r=merge(a->r,b);
23         a->up();
24         return a;
25     }else{
26         //b=new node(*b);
27         b->down();
28         b->l=merge(a,b->l);
29         b->up();

```

```

1 #ifndef DEBUG
2 #define dbg(...) {\
3     fprintf(stderr,"%s - %d : (%s) = ",
4             __PRETTY_FUNCTION__,__LINE__,#
5             __VA_ARGS__);\
6     _DO(__VA_ARGS__);\
7 }
8 template<typename I> void _DO(I&&x){cerr<<x
9     <<endl;}
10 template<typename I,typename...T> void _DO(I
11     &&x,T&&...tail){cerr<<x<<" ";_DO(tail
12     ...);}
13 #else
14 #define dbg(...)
15 #endif

```

3.2 ext.cpp

```

1 #include<bits/extc++.h>
2 #include<ext/pd_ds/assoc_container.hpp>
3 #include<ext/pd_ds/tree_policy.hpp>
4 using namespace __gnu_cxx;
5 using namespace __gnu_pbds;
6 template<typename T>
7 using pbds_set = tree<T,null_type,less<T>,
8     rb_tree_tag,
9     tree_order_statistics_node_update>;
10 template<typename T,typename U>
11 using pbds_map = tree<T,U,less<T>,
12     rb_tree_tag,
13     tree_order_statistics_node_update>;
14 using heap = __gnu_pbds::priority_queue<int
15     >;
16 //s.find_by_order(1);//0 base
17 //s.order_of_key(1);

```

3.3 IncStack.cpp

```

1 //Magic
2 #pragma GCC optimize "Ofast"
3 //stack resize,change esp to rsp if 64-bit
4 //system
5 asm("mov %0,%esp\n" :: "g"(mem+1000000));
6 //linux stack resize
7 #include<sys/resource.h>
8 void increase_stack(){
9     const rlim_t ks=64*1024*1024;
10    struct rlimit rl;
11    int res=getrlimit(RLIMIT_STACK,&rl);
12    if(!res&&rl.rlim_cur<ks){
13        rl.rlim_cur=ks;
14        res=setrlimit(RLIMIT_STACK,&rl);
15    }

```

3.4 input.cpp

```

1 inline int read(){
2     int x=0; bool f=0; char c=getchar();
3     while(ch<'0'&&ch<'9')f|=ch=='-';ch=getchar();
4     while('0'<=ch&&ch<'9')x=x*10-'0'+ch,ch=
5         getchar();
6     return f?-x:x;
7 }
8 volatile
9 // #!/bin/bash
10 // g++ -std=c++11 -O2 -Wall -Wextra -Wno-
11 // unused-result -DDEBUG $1 && ./a.out
12 // -fsanitize=address -fsanitize=undefined
13 // -fsanitize=return

```

4 Flow

4.1 dinic.cpp

```

1 template<typename T>
2 struct DINIC{
3     static const int MAXN=105;
4     static const T INF=INT_MAX;
5     int n;//點數
6     int level[MAXN],cur[MAXN];
7     struct edge{
8         int v,pre;
9         T cap,flow,r;
10        edge(int v,int pre,T cap):v(v),pre(pre),
11            cap(cap),flow(0),r(cap){}
12    };
13    vector<edge> e;
14    void init(int _n){
15        memset(g,-1,sizeof(int)*((n=_n)+1));
16        e.clear();
17    }
18    void add_edge(int u,int v,T cap,bool
19        directed=false){
20        e.push_back(edge(v,g[u],cap));
21        g[u]=e.size()-1;
22        e.push_back(edge(u,g[v],directed?0:cap));
23        g[v]=e.size()-1;
24    }
25    int bfs(int s,int t){
26        memset(level,0,sizeof(int)*(n+1));
27        memcpy(cur,g,sizeof(int)*(n+1));
28        queue<int> q;
29        q.push(s);
30        level[s]=1;
31        while(q.size()){
32            int u=q.front();q.pop();
33            for(int i=g[u];~i;i=e[i].pre){
34                if(!level[e[i].v]&&e[i].r){
35                    level[e[i].v]=level[u]+1;
36                    q.push(e[i].v);
37                    if(e[i].v==t) return 1;
38                }
39            }
40        }
41        return 0;
42    }
43    T dfs(int u,int t,T cur_flow=INF){
44        if(u==t) return cur_flow;
45        T df;
46        for(int &i=cur[u];~i;i=e[i].pre){
47            if(!level[e[i].v]==level[u]+1&&e[i].r){
48                if(df=dfs(e[i].v,t,min(cur_flow,e[i]
49                    .r))){
50                    e[i].flow+=df;
51                    e[i^1].flow-=df;
52                    e[i].r-=df;
53                    e[i^1].r+=df;
54                    return df;
55                }
56            }
57        }
58        return 0;
59    }

```



```

22     }
23     }
24 }
25 }
26 return false;
27 }
28 inline int max_match(){
29     for(int i=0;i<n2;++i)match_list[i].clear()
30     ;
31     int cnt=0;
32     for(int u=0;u<n1;++u){
33         memset(vis,0,sizeof(bool)*n2);
34         if(dfs(u))++cnt;
35     }
36     return cnt;

```

5.3 blossom_matching.cpp

```

1 #define MAXN 505
2 vector<int>g[MAXN];
3 int pa[MAXN],match[MAXN],st[MAXN],S[MAXN],v[
4     MAXN];
5 int t,n;
6 inline int lca(int x,int y){
7     for(++t;swap(x,y)){
8         if(x==0)continue;
9         if(v[x]==t)return x;
10        v[x]=t;
11        x=st[pa[match[x]]];
12    }
13 }
14 #define qpush(x) q.push(x),S[x]=0
15 inline void flower(int x,int y,int l,queue<
16     int> &q){
17     while(st[x]!=1){
18         pa[x]=y;
19         if(S[y==match[x]]==1)qpush(y);
20         st[x]=st[y]=1,x=pa[y];
21     }
22 }
23 inline bool bfs(int x){
24     for(int i=1;i<n;++i)st[i]=i;
25     memset(S+1,-1,sizeof(int)*n);
26     queue<int>q;qpush(x);
27     while(q.size()){
28         x=q.front(),q.pop();
29         for(size_t i=0;i<g[x].size();++i){
30             int y=g[x][i];
31             if(S[y]==-1){
32                 pa[y]=x,S[y]=1;
33                 if(!match[y]){
34                     for(int lst=x;y=lst,x=pa[y])
35                         lst=match[x],match[x]=y,match[y]
36                             =x;
37                     return 1;
38                 }
39                 qpush(match[y]);
40             }
41             else if(!S[y]&&st[y]!=st[x]){
42                 int l=lca(y,x);
43                 flower(y,x,l,q),flower(x,y,l,q);
44             }
45         }
46     }

```

```

42     }
43     return 0;
44 }
45 inline int blossom(){
46     int ans=0;
47     for(int i=1;i<n;++i)
48         if(!match[i]&&bfs(i))++ans;
49     return ans;
50 }

```

5.4 graphISO.cpp

```

1 const int MAXN=1005,K=30;//K要夠大
2 const long long A=3,B=11,C=2,D=19,P=0
3     xdefaced;
4 long long f[K+1][MAXN];
5 vector<int> g[MAXN],rg[MAXN];
6 int n;
7 inline void init(){
8     for(int i=0;i<n;++i){
9         f[0][i]=1;
10        g[i].clear();
11        rg[i].clear();
12    }
13 }
14 inline void add_edge(int u,int v){
15     g[u].push_back(v);
16     rg[v].push_back(u);
17 }
18 inline long long point_hash(int u){//O(N)
19     for(int t=1;t<K;++t){
20         for(int i=0;i<n;++i){
21             f[t][i]=f[t-1][i]*A%P;
22             for(int j:g[i])f[t][i]=(f[t][i]+f[t-1][j]*B%P)%P;
23             for(int j:rg[i])f[t][i]=(f[t][i]+f[t-1][j]*C%P)%P;
24             if(i==u)f[t][i]+=D;//如果圖太大的話，
25             //把這行刪掉，執行一次後f[K]就會是所
26             //有點的答案
27             f[t][i]%=P;
28         }
29     }
30     return f[K][u];
31 }
32 inline vector<long long> graph_hash(){
33     vector<long long> ans;
34     for(int i=0;i<n;++i)ans.push_back(
35         point_hash(i));
36     sort(ans.begin(),ans.end());
37     return ans;
38 }

```

5.5 KM.cpp

```

1 #define MAXN 405
2 #define INF 0x3f3f3f3f
3 int n;// 1-base · 0表示沒有匹配

```

```

4 int g[MAXN][MAXN],lx[MAXN],ly[MAXN],pa[MAXN
5     ],slack_y[MAXN];
6 int match_y[MAXN],match_x[MAXN];
7 bool vx[MAXN],vy[MAXN];
8 void augment(int y){
9     for(int x,z;y=y;z){
10        x=pa[y],z=match_x[x];
11        match_y[y]=x,match_x[x]=y;
12    }
13 }
14 void bfs(int st){
15     for(int i=1;i<n;++i)slack_y[i]=INF,vx[i]=
16         vy[i]=0;
17     queue<int> q;q.push(st);
18     for(;;){
19         while(q.size()){
20             int x=q.front();q.pop();
21             vx[x]=1;
22             for(int y=1;y<n;++y)if(!vy[y]){
23                 int t=lx[x]+ly[y]-g[x][y];
24                 if(t==0){
25                     pa[y]=x;
26                     if(!match_y[y]){augment(y);return
27                         ;}
28                     vy[y]=1,q.push(match_y[y]);
29                 }
30                 else if(slack_y[y]>t)pa[y]=x,
31                     slack_y[y]=t;
32             }
33         }
34         int cut=INF;
35         for(int y=1;y<n;++y){
36             if(!vy[y]&&cut>slack_y[y])cut=slack_y[
37                 y];
38         }
39         for(int j=1;j<n;++j){
40             if(vx[j])lx[j]-=cut;
41             if(vy[j])ly[j]+=cut;
42             else slack_y[j]-=cut;
43         }
44         for(int y=1;y<n;++y){
45             if(!vy[y]&&slack_y[y]==0){
46                 if(!match_y[y]){augment(y);return;}
47                 vy[y]=1,q.push(match_y[y]);
48             }
49         }
50     }
51 }
52 long long KM(){
53     memset(match_y,0,sizeof(int)*(n+1));
54     memset(ly,0,sizeof(int)*(n+1));
55     for(int x=1;x<n;++x){
56         lx[x]=-INF;
57         for(int y=1;y<n;++y)
58             lx[x]=max(lx[x],g[x][y]);
59     }
60     for(int x=1;x<n;++x)bfs(x);
61     long long ans=0;
62     for(int y=1;y<n;++y)ans+=g[match_x[y]][y];
63     return ans;
64 }

```

5.6 MaximumClique.cpp

```

1 struct MaxClique{
2     static const int MAXN=105;
3     int N,ans;
4     int g[MAXN][MAXN],dp[MAXN],stk[MAXN][MAXN
5         ];
6     int sol[MAXN],tmp[MAXN];//sol[0~ans-1]為答
7         案
8     void init(int n){
9         N=n;//0-base
10        memset(g,0,sizeof(g));
11    }
12    void add_edge(int u,int v){
13        g[u][v]=g[v][u]=1;
14    }
15    int dfs(int ns,int dep){
16        if(!ns){
17            if(dep>ans){
18                ans=dep;
19                memcpy(sol,tmp,sizeof tmp);
20                return 1;
21            }
22            else return 0;
23        }
24        for(int i=0;i<ns;++i){
25            if(dep+ns-i<ans)return 0;
26            int u=stk[dep][i],cnt=0;
27            if(dep+dp[u]<=ans)return 0;
28            for(int j=i+1;j<ns;++j){
29                int v=stk[dep][j];
30                if(g[u][v])stk[dep+1][cnt++]=v;
31            }
32            tmp[dep]=u;
33            if(dfs(cnt,dep+1))return 1;
34        }
35        return 0;
36    }
37    int clique(){
38        int u,v,ns;
39        for(ans=0,u=N-1;u>0;--u){
40            for(ns=0,tmp[0]=u,v=u+1;v<N;++v)
41                if(g[u][v])stk[1][ns++]=v;
42            dfs(ns,1),dp[u]=ans;
43        }
44        return ans;
45    }
46 };

```

5.7 MinimumMeanCycle.cpp

```

1 #include<cstdio>>//for DBL_MAX
2 int dp[maxN+1][maxN+1];
3 double mnc(int n){
4     int u,v,w;
5     const int inf=0x7f7f7f7f;
6     memset(dp,0x7f,sizeof(dp));
7     memset(dp[0],0,sizeof(dp[0]));
8     for(int i=0;i<n;++i){
9         for(auto e:E){//tuple<int,int,int>
10             of u,v,w
11             tie(u,v,w)=e;

```



```

11     if(dp[i][u]!=inf)
12         dp[i+1][v]=min(dp[i+1][v],dp
13             [i][u]+w);
14     }
15     double res = DBL_MAX;
16     for(int i=1;i<=n;++i){
17         double val = DBL_MIN;
18         for(int j=0;j<n;++j)
19             val=max(val,double(dp[n][i]-
20                 dp[i][j])/(n-j));
21         res=min(res,val);
22     }
23     return res;

```

5.8 Minimum_General_Weighte

```

1 struct Graph {
2     // Minimum General Weighted Matching (
3     // Perfect Match) 0-base
4     static const int MXN = 105;
5     int n, edge[MXN][MXN];
6     int match[MXN],dis[MXN],onstk[MXN];
7     vector<int> stk;
8
9     void init(int _n) {
10         n = _n;
11         for (int i=0; i<n; i++)
12             for (int j=0; j<n; j++)
13                 edge[i][j] = 0;
14     }
15     void add_edge(int u, int v, int w) {
16         edge[u][v] = edge[v][u] = w;
17     }
18     bool SPFA(int u){
19         if (onstk[u]) return true;
20         stk.push_back(u);
21         onstk[u] = 1;
22         for (int v=0; v<n; v++){
23             if (u != v && match[u] != v && !onstk[
24                 v]){
25                 int m = match[v];
26                 if (dis[m] > dis[u] - edge[v][m] +
27                     edge[u][v]){
28                     dis[m] = dis[u] - edge[v][m] +
29                         edge[u][v];
30                     onstk[v] = 1;
31                     stk.push_back(v);
32                     if (SPFA(m)) return true;
33                     stk.pop_back();
34                     onstk[v] = 0;
35                 }
36             }
37         }
38         onstk[u] = 0;
39         stk.pop_back();
40         return false;
41     }
42
43     int solve() {
44         // find a match

```

```

42     for (int i=0; i<n; i+=2){
43         match[i] = i+1;
44         match[i+1] = i;
45     }
46     for(;;){
47         int found = 0;
48         for (int i=0; i<n; i++)
49             dis[i] = onstk[i] = 0;
50         for (int i=0; i<n; i++){
51             stk.clear();
52             if (!onstk[i] && SPFA(i)){
53                 found = 1;
54                 while (stk.size()>=2){
55                     int u = stk.back(); stk.pop_back
56                         ();
57                     int v = stk.back(); stk.pop_back
58                         ();
59                     match[u] = v;
60                     match[v] = u;
61                 }
62                 if (!found) break;
63             }
64             int ret = 0;
65             for (int i=0; i<n; i++)
66                 ret += edge[i][match[i]];
67             ret /= 2;
68             return ret;
69         }
70     }graph;

```

5.9 Rectilinear_Steiner_tree.cpp

```

1 //平面曼哈頓最小生成樹構造圖(去除非必要邊)
2 #include<vector>
3 #include<algorithm>
4 #define T int
5 #define INF 0x3f3f3f3f
6 struct point{
7     T x,y;
8     int id;//每個點的編號都要不一樣，從0開始編
9     point(){}
10     T dist(const point &p)const{
11         return std::abs(x-p.x)+std::abs(y-p.y);
12     }
13 };
14 inline bool cmpx(const point &a,const point
15     &b){
16     return a.x<b.x||(a.x==b.x&&a.y<b.y);
17 }
18 struct edge{
19     int u,v;
20     T cost;
21     edge(int u,int v,const T&c):u(u),v(v),cost
22         (c){}
23     bool operator<(const edge&e)const{
24         return cost<e.cost;
25     }
26 };
27 struct bit_node{

```

```

26     T mi;
27     int id;
28     bit_node(const T&mi=INF,int id=-1):mi(mi),
29         id(id){}
30 };
31 std::vector<bit_node> bit;
32 inline void bit_update(int i,const T&data,
33     int id){
34     for(;;i=i&(-i)){
35         if(data<bit[i].mi)bit[i]=bit_node(data,
36             id);
37     }
38 }
39 inline int bit_find(int i,int m){
40     bit_node x;
41     for(;;i<=m;i=i&(-i)){
42         if(bit[i].mi<=m)x=bit[i];
43     }
44     return x.id;
45 }
46 inline std::vector<edge> build_graph(int n,
47     point p[]){
48     std::vector<edge> e;//回傳的邊就可以用來求
49     最小生成樹
50     for(int dir=0;dir<4;dir++){//4種座標變換
51         if(dir%2){
52             for(int i=0;i<n;i++)std::swap(p[i].x,p
53                 [i].y);
54         }else if(dir==2){
55             for(int i=0;i<n;i++)p[i].x=-p[i].x;
56         }
57         std::sort(p,p+n,cmpx);
58         std::vector<T>ga(n),gb;
59         for(int i=0;i<n;i++)ga[i]=p[i].y-p[i].x;
60         gb=ga;
61         std::sort(gb.begin(),gb.end());
62         gb.resize(std::unique(gb.begin(),gb.end
63             ())-gb.begin());
64         int m=gb.size();
65         bit=std::vector<bit_node>(m+1);
66         for(int i=n-1;i>=0;i--){
67             int pos=std::lower_bound(gb.begin(),gb
68                 .end(),ga[i])-gb.begin()+1;
69             int ans=bit_find(pos,m);
70             if(~ans)e.push_back(edge(p[i].id,p[ans
71                 ].id,p[i].dist(p[ans].id)));
72             bit_update(pos,p[i].x+p[i].y,i);
73         }
74     }
75     return e;
76 }

```

5.10 treeISO.cpp

```

1 const int MAXN=100005;
2 const long long X=12327,P=0xdefaced;
3 vector<int> g[MAXN];
4 bool vis[MAXN];
5 long long dfs(int u){//hash ver
6     vis[u]=1;
7     vector<long long> tmp;

```

```

8     for(auto v:g[u])if(!vis[v])tmp.push_back(
9         dfs(v));
10     if(tmp.empty())return 177;
11     long long ret=4931;
12     sort(tmp.begin(),tmp.end());
13     for(auto v:tmp)ret=((ret*X)^v)%P;
14     return ret;
15 }
16 //-----
17 string dfs(int x,int p){
18     vector<string> c;
19     for(int y:g[x])
20         if(y!=p)c.emplace_back(dfs(y,x));
21     sort(c.begin(),c.end());
22     string ret("(");
23     for(auto &s:c)ret+=s;
24     ret+=")";
25     return ret;
26 }

```

5.11 全局最小割.cpp

```

1 const int INF=0x3f3f3f3f;
2 template<typename T>
3 struct stoer_wagner{// 0-base
4     static const int MAXN=150;
5     T g[MAXN][MAXN],dis[MAXN];
6     int nd[MAXN],n,s,t;
7     void init(int _n){
8         n=_n;
9         for(int i=0;i<n;i++)
10             for(int j=0;j<n;j++)g[i][j]=0;
11     }
12     void add_edge(int u,int v,T w){
13         g[u][v]=g[v][u]+=w;
14     }
15     T min_cut(){
16         T ans=INF;
17         for(int i=0;i<n;i++)nd[i]=i;
18         for(int ind,tn=n-1;tn>1;--tn){
19             for(int i=1;i<tn;i++)dis[nd[i]]=0;
20             for(int i=1;i<tn;i++){
21                 ind=i;
22                 for(int j=i;j<tn;j++){
23                     dis[nd[j]]+=g[nd[i-1]][nd[j]];
24                     if(dis[nd[ind]]<dis[nd[j]])ind=j;
25                 }
26                 swap(nd[ind],nd[i]);
27             }
28             if(ans>dis[nd[ind]])ans=dis[t=nd[ind
29                 ],s=nd[ind-1]];
30             for(int i=0;i<tn;i++)
31                 g[nd[ind-1]][nd[i]]=g[nd[i]][nd[ind
32                     -1]]+=g[nd[i]][nd[ind]];
33         }
34         return ans;
35     }
36 };

```

5.12 平面圖判定.cpp

```

1 static const int MAXN = 20;
2 struct Edge{
3     int u, v;
4     Edge(int s, int d) : u(s), v(d) {}
5 };
6 bool isK33(int n, int degree[]){
7     int t = 0, z = 0;
8     for(int i=0; i<n; ++i){
9         if(degree[i] == 3)++t;
10        else if(degree[i] == 0)++z;
11        else return false;
12    }
13    return t == 6 && t + z == n;
14 }
15 bool isK5(int n, int degree[]){
16     int f = 0, z = 0;
17     for(int i=0; i<n; ++i){
18         if(degree[i] == 4)++f;
19         else if(degree[i] == 0)++z;
20         else return false;
21    }
22    return f == 5 && f + z == n;
23 }
24 // it judge a given graph is Homeomorphic
    with K33 or K5
25 bool isHomeomorphic(bool G[MAXN][MAXN],
    const int n){
26     for(;;){
27         int cnt = 0;
28         for(int i=0; i<n; ++i){
29             vector<Edge> E;
30             for(int j=0; j<n && E.size()<3; ++j)
31                 if(G[i][j] && i != j)
32                     E.push_back(Edge(i, j));
33             if(E.size() == 1){
34                 G[i][E[0].v] = G[E[0].v][i] = false;
35             } else if(E.size() == 2){
36                 G[i][E[0].v] = G[E[0].v][i] = false;
37                 G[i][E[1].v] = G[E[1].v][i] = false;
38                 G[E[0].v][E[1].v] = G[E[1].v][E[0].v]
39                     = true;
40                 ++cnt;
41             }
42             if(cnt == 0) break;
43         }
44         static int degree[MAXN];
45         fill(degree, degree + n, 0);
46         for(int i=0; i<n; ++i){
47             for(int j=i+1; j<n; ++j){
48                 if(!G[i][j]) continue;
49                 ++degree[i];
50                 ++degree[j];
51             }
52         }
53         return !(isK33(n, degree) || isK5(n,
54             degree));
55 }

```

5.13 弦圖完美消除序列.cpp

```

1 struct chordal{
2     static const int MAXN=1005;

```

```

3     int n; // 0-base
4     vector<int> G[MAXN];
5     int rank[MAXN], label[MAXN];
6     bool mark[MAXN];
7     void init(int _n){n=_n;
8         for(int i=0; i<n; ++i) G[i].clear();
9     }
10    void add_edge(int u, int v){
11        G[u].push_back(v);
12        G[v].push_back(u);
13    }
14    vector<int> MCS(){
15        memset(rank, -1, sizeof(int)*n);
16        memset(label, 0, sizeof(int)*n);
17        priority_queue<pair<int, int>> pq;
18        for(int i=0; i<n; ++i) pq.push(make_pair(0,
19            i));
20        for(int i=n-1; i>=0; --i) for(;;){
21            int u=pq.top().second; pq.pop();
22            if(~rank[u]) continue;
23            rank[u]=i;
24            for(auto v:G[u]) if(rank[v]==-1){
25                pq.push(make_pair(++label[v], v));
26            }
27            break;
28        }
29        vector<int> res(n);
30        for(int i=0; i<n; ++i) res[rank[i]]=i;
31        return res;
32    }
33    bool check(vector<int> ord){ //弦圖判定
34        for(int i=0; i<n; ++i) rank[ord[i]]=i;
35        memset(mark, 0, sizeof(bool)*n);
36        for(int i=0; i<n; ++i){
37            vector<pair<int, int>> tmp;
38            for(auto u:G[ord[i]]) if(!mark[u])
39                tmp.push_back(make_pair(rank[u], u));
40            sort(tmp.begin(), tmp.end());
41            if(tmp.size()){
42                int u=tmp[0].second;
43                set<int> S;
44                for(auto v:G[u]) S.insert(v);
45                for(size_t j=1; j<tmp.size(); ++j)
46                    if(!S.count(tmp[j].second)) return
47                        0;
48            }
49            mark[ord[i]]=1;
50        }
51        return 1;
52    }
53 }

```

5.14 最小斯坦納樹 DP.cpp

```

1 //n個點，其中r個要構成斯坦納樹
2 //答案在max(dp[(1<r)-1][k]) k=0~n-1
3 //p表示要構成斯坦納樹的點集
4 //O( n^3 + n^3*r + n^2*2^r )
5 #define REP(i,n) for(int i=0; i<(int)n; ++i)
6 const int MAXN=30, MAXM=8; // 0-base
7 const int INF=0x3f3f3f3f;
8 int dp[1<<MAXM][MAXN];

```

```

9 int g[MAXN][MAXN]; //圖
10 void init(){memset(g, 0x3f, sizeof(g));}
11 void add_edge(int u, int v, int w){
12     g[u][v]=g[v][u]=min(g[v][u], w);
13 }
14 void steiner(int n, int r, int *p){
15     REP(k,n) REP(i,n) REP(j,n)
16         g[i][j]=min(g[i][j], g[i][k]+g[k][j]);
17     REP(i,n) g[i][i]=0;
18     REP(i,r) REP(j,n) dp[1<<i][j]=g[p[i]][j];
19     for(int i=1; i<(1<<r); ++i){
20         if(!(i&(i-1))) continue;
21         REP(j,n) dp[i][j]=INF;
22         REP(j,n){
23             int tmp=INF;
24             for(int s=i&(i-1); s; s=s-1)
25                 tmp=min(tmp, dp[s][j]+dp[i^s][j]);
26             REP(k,n) dp[i][k]=min(dp[i][k], g[j][k]+
27                 tmp);
28         }
29     }
30 }

```

5.15 最小樹形圖 __ 朱劉.cpp

```

1 template<typename T>
2 struct zhu_liu{
3     static const int MAXN=110, MAXM=10005;
4     struct node{
5         int u, v;
6         T w, tag;
7         node *l, *r;
8         node(int u=0, int v=0, T w=0):u(u), v(v), w(
9             w), tag(0), l(0), r(0){}
10        void down(){
11            w+=tag;
12            if(l) l->tag+=tag;
13            if(r) r->tag+=tag;
14            tag=0;
15        }
16        mem[ MAXM ]; //靜態記憶體
17        node *pq[ MAXN*2 ], *E[ MAXN*2 ];
18        int st[ MAXN*2 ], id[ MAXN*2 ], m;
19        void init(int n){
20            for(int i=1; i<n; ++i){
21                pq[i]=E[i]=0;
22                st[i]=id[i]=i;
23            }
24            m=0;
25        }
26        node *merge(node *a, node *b){ //skew heap
27            if(!a || !b) return a?a:b;
28            a->down(), b->down();
29            if(b->w<a->w) return merge(b, a);
30            swap(a->l, a->r);
31            a->l=merge(b, a->l);
32            return a;
33        }
34        void add_edge(int u, int v, T w){
35            if(u!=v) pq[v]=merge(pq[v], &(mem[m++]=
36                node(u, v, w)));
37        }
38        int find(int x, int *st){
39            return st[x]==x?x:st[x]=find(st[x], st);
40        }
41    };

```

```

37 }
38 T build(int root, int n){
39     T ans=0; int N=n, all=n;
40     for(int i=1; i<N; ++i){
41         if(i==root || !pq[i]) continue;
42         while(pq[i]){
43             pq[i]->down(), E[i]=pq[i];
44             pq[i]=merge(pq[i]->l, pq[i]->r);
45             if(find(E[i]->u, id)!=find(i, id))
46                 break;
47             if(find(E[i]->u, id)==find(i, id))
48                 continue;
49             ans+=E[i]->w;
50             if(find(E[i]->u, st)==find(i, st)){
51                 if(pq[i]) pq[i]->tag-=E[i]->w;
52                 pq[i]=merge(pq[i]->l, pq[i]->r);
53                 if(pq[i]) pq[i]->tag-=E[i]->w;
54                 id[find(u, id)]=N;
55                 pq[N]=merge(pq[N], pq[u]);
56             }
57             st[N]=find(i, st);
58             id[find(i, id)]=N;
59             } else st[find(i, st)]=find(E[i]->u, st)
60                 , --all;
61         }
62     }
63     return all==1?ans:-INT_MAX; //圖不連通就
    無解

```

5.16 穩定婚姻模板.cpp

```

1 queue<int> Q;
2 for ( i : 所有考生 ) {
3     設定在第0志願;
4     Q.push( 考生i );
5 }
6 while(Q.size()){
7     當前考生=Q.front(); Q.pop();
8     while ( 此考生未分發 ) {
9         指標移到下一志願;
10        if ( 已經沒有志願 or 超出志願總數 )
11            break;
12        計算該考生在該科系加權後的總分;
13        if ( 不符合科系需求 ) continue;
14        if ( 目前科系有餘額 ) {
15            依加權後分數高低順序將考生id加入科系錄
16            取名單中;
17            break;
18        }
19        if ( 目前科系已額滿 ) {
20            if ( 此考生成績比最低分數還高 ) {
21                依加權後分數高低順序將考生id加入科系
22                錄取名單;
23                Q.push( 被踢出的考生 );
24            }
25        }
26    }
27 }

```

6 language

6.1 CNF.cpp

```

23 }
24 }

#define MAXN 55
struct CNF{
    int s,x,y;//s->xy | s->x, if y== -1
    int cost;
    CNF(){}
    CNF(int s,int x,int y,int c):s(s),x(x),y(y),cost(c){}
};
int state;//規則數量
map<char,int> rule;//每個字元對應到的規則
vector<CNF> cnf;
inline void init(){
    state=0;
    rule.clear();
    cnf.clear();
}
inline void add_to_cnf(char s,const string &
    p,int cost){
    //加入一個s -> <p>的文法 代價為cost
    if(rule.find(s)==rule.end())rule[s]=state++;
    for(auto c:p)if(rule.find(c)==rule.end())
        rule[c]=state++;
    if(p.size()==1){
        cnf.push_back(CNF(rule[s],rule[p[0]],-1,
            cost));
    }else{
        int left=rule[s];
        int sz=p.size();
        for(int i=0;i<sz-2;++i){
            cnf.push_back(CNF(left,rule[p[i]],
                state,0));
            left=state++;
        }
        cnf.push_back(CNF(left,rule[p[sz-2]],
            rule[p[sz-1]],cost));
    }
}
vector<long long> dp[MAXN][MAXN];
vector<bool> neg_INF[MAXN][MAXN];//如果花費
    是真的可能會有無限小的情形
inline void relax(int l,int r,const CNF &c,
    long long cost,bool neg_c=0){
    if(!neg_INF[l][r][c.s]&&(neg_INF[l][r][c.s]
        )||cost<dp[l][r][c.s]){
        if(neg_c||neg_INF[l][r][c.s]){
            dp[l][r][c.s]=0;
            neg_INF[l][r][c.s]=true;
        }else dp[l][r][c.s]=cost;
    }
}
inline void bellman(int l,int r,int n){

```

```

43 for(int k=1;k<=state;++k)
44     for(auto c:cnf)
45         if(c.y!=-1)relax(l,r,c,dp[l][r][c.x]+
            .cost,k==n);
46 }
47 inline void cyk(const vector<int> &tok){
48     for(int i=0;i<(int)tok.size();++i){
49         for(int j=0;j<(int)tok.size();++j){
50             dp[i][j]=vector<long long>(state+1,
                INT_MAX);
51             neg_INF[i][j]=vector<bool>(state+1,
                false);
52         }
53         dp[i][i][tok[i]]=0;
54         bellman(i,i,tok.size());
55     }
56     for(int r=1;r<(int)tok.size();++r){
57         for(int l=r-1;l>=0;--l){
58             for(int k=1;k<r;++k)
59                 for(auto c:cnf)
60                     if(~c.y)relax(l,r,c,dp[l][k][c.x]+
                        dp[k+1][r][c.y]+c.cost);
61             bellman(l,r,tok.size());
62         }
63     }
64 }

```

7 Linear_Programming

7.1 最大密度子圖.cpp

```

1 typedef double T;//POJ 3155
2 const int MAXN=105;
3 struct edge{
4     int u,v;
5     T w;
6     edge(int u=0,int v=0,T w=0):u(u),v(v),w(w)
7     {}
8 };
9 vector<edge> E;
10 int n,m;// 1-base
11 T de[MAXN],pv[MAXN];//每個點的邊權和和點權(
    有些題目會給)
12 void init(){
13     E.clear();
14     for(int i=1;i<=n;++i)de[i]=pv[i]=0;
15 }
16 void add_edge(int u,int v,T w){
17     E.push_back(edge(u,v,w));
18     de[u]+=w,de[v]+=w;
19 }
20 T U;//二分搜的最大值
21 void get_U(){
22     U=0;
23     for(int i=1;i<=n;++i)U+=2*pv[i];
24     for(size_t i=0;i<E.size();++i)U+=E[i].w;
25 }
26 ISAP<T> isap;//網路流
27 int s,t;//原匯點
28 void build(T L){

```

```

28 isap.init(n+2);
29 for(size_t i=0;i<E.size();++i){
30     isap.add_edge(E[i].u,E[i].v,E[i].w);
31 }
32 for(int v=1;v<=n;++v){
33     isap.add_edge(s,v,U);
34     isap.add_edge(v,t,U+2*L-de[v]-2*pv[v]);
35 }
36 }
37 int main(){
38     while(~scanf("%d%d",&n,&m)){
39         if(!m){
40             puts("1\n1");
41             continue;
42         }
43         init();
44         int u,v;
45         for(int i=0;i<m;++i){
46             scanf("%d%d",&u,&v);
47             add_edge(u,v,1);
48         }
49         get_U();
50         s=n+1,t=n+2;
51         T l=0,r=U,k=1.0/(n*n);
52         while(r-l>k){//二分搜最大值
53             T mid=(l+r)/2;
54             build(mid);
55             T res=(U*n-isap.isap(s,t))/2;
56             if(res>0)l=mid;
57             else r=mid;
58         }
59         build(l);
60         isap.min_cut(s,t);
61         vector<int> ans;
62         for(int i=1;i<=n;++i){
63             if(isap.vis[i])ans.push_back(i);
64         }
65         printf("%d\n",ans.size());
66         for(size_t i=0;i<ans.size();++i){
67             printf("%d\n",ans[i]);
68         }
69     }
70     return 0;
71 }

```

8 Number_Theory

8.1 basic.cpp

```

1 template<typename T>
2 void gcd(const T &a,const T &b,T &d,T &x,T &
    y){
3     if(!b) d=a,x=1,y=0;
4     else gcd(b,a%b,d,y,x), y-=x*(a/b);
5 }
6 long long int phi[N+1];
7 void phiTable(){
8     for(int i=1;i<=N;++i)phi[i]=i;
9     for(int i=1;i<=N;++i)for(x=i*2;x<=N;x+=i)
        phi[x]-=phi[i];
10 }

```

```

11 void all_divdown(const LL &n){// all n/x
12     for(LL a=1;a<=n;a=n/(n/(a+1)))
13         // dosomething;
14     }
15 }
16 const int MAXPRIME = 1000000;
17 int iscom[MAXPRIME], prime[MAXPRIME],
    primecnt;
18 int phi[MAXPRIME], mu[MAXPRIME];
19 void sieve(void){
20     memset(iscom,0,sizeof(iscom));
21     primecnt = 0;
22     phi[1] = mu[1] = 1;
23     for(int i=2;i<MAXPRIME;++i) {
24         if(!iscom[i]) {
25             prime[primecnt++] = i;
26             mu[i] = -1;
27             phi[i] = i-1;
28         }
29         for(int j=0;j<primecnt;++j) {
30             int k = i * prime[j];
31             if(k>MAXPRIME) break;
32             iscom[k] = prime[j];
33             if(i%prime[j]==0) {
34                 mu[k] = 0;
35                 phi[k] = phi[i] * prime[j];
36                 break;
37             } else {
38                 mu[k] = -mu[i];
39                 phi[k] = phi[i] * (prime[j]-1);
40             }
41         }
42     }
43 }
44 }
45 bool g_test(const LL &g, const LL &p, const
    vector<LL> &v) {
46     for(int i=0;i<v.size();++i)
47         if(modexp(g,(p-1)/v[i],p)==1)
48             return false;
49     return true;
50 }
51 LL primitive_root(const LL &p) {
52     if(p==2) return 1;
53     vector<LL> v;
54     Factor(p-1,v);
55     v.erase(unique(v.begin(), v.end()), v.end()
        ());
56     for(LL g=2;g<p;++g)
57         if(g_test(g,p,v))
58             return g;
59     puts("primitive_root NOT FOUND");
60     return -1;
61 }
62 int Legendre(const LL &a, const LL &p) {
63     return modexp(a%p,(p-1)/2,p);
64 }
65 LL inv(const LL &a, const LL &n) {
66     LL d,x,y;
67     gcd(a,n,d,x,y);
68     return d==1 ? (x+n)%n : -1;
69 }
70 int inv[maxn];
71 LL invtable(int n,LL P){
72     inv[1]=1;

```

```

73 for(int i=2;i<n;++i)
74   inv[i]=(P-(P/i))*inv[P%i]%P;
75 }
76
77 LL log_mod(const LL &a, const LL &b, const
78   LL &p) {
79   //  $a^x = b \pmod p$ 
80   int m=sqrt(p+.5), e=1;
81   LL v=inv(modexp(a,m,p), p);
82   map<LL,int> x;
83   x[1]=0;
84   for(int i=1;i<m;++i) {
85     e = Llmul(e,a,p);
86     if(!x.count(e)) x[e] = i;
87   }
88   for(int i=0;i<m;++i) {
89     if(x.count(b)) return i*m + x[b];
90     b = Llmul(b,v,p);
91   }
92   return -1;
93 }
94
95 LL Tonelli_Shanks(const LL &n, const LL &p)
96 {
97   //  $x^2 = n \pmod p$ 
98   if(n==0) return 0;
99   if(Legendre(n,p)!=1) while(1) { puts("SQRT
100     ROOT does not exist"); }
101   int S = 0;
102   LL Q = p-1;
103   while( !(Q&1) ) { Q>>=1; ++S; }
104   if(S==1) return modexp(n%p,(p+1)/4,p);
105   LL z = 2;
106   for(; Legendre(z,p)!=-1; ++z)
107     LL c = modexp(z,Q,p);
108   LL R = modexp(n%p,(Q+1)/2,p), t = modexp(n
109     %p,Q,p);
110   int M = S;
111   while(1) {
112     if(t==1) return R;
113     LL b = modexp(c,1L<<(M-i-1),p);
114     R = Llmul(R,b,p);
115     t = Llmul(Llmul(b,b,p), t, p);
116     c = Llmul(b,b,p);
117     M = i;
118   }
119   return -1;
120 }
121
122 template<typename T>
123 T Euler(T n){
124   T ans=n;
125   for(T i=2;i*i<=n;++i){
126     if(n%i==0){
127       ans=ans/i*(i-1);
128       while(n%i==0)n/=i;
129     }
130   }
131   if(n>1)ans=ans*n*(n-1);
132   return ans;
133 }
134
135 //Chinese_remainder_theorem
136 template<typename T>
137 T pow_mod(T n, T k, T m){
138   T ans=1;

```

```

135 for(n=(n>m?n%m:n);k;k>>=1){
136   if(k&1)ans=ans*n%m;
137   n=n*n%m;
138 }
139 return ans;
140 }
141
142 template<typename T>
143 T crt(vector<T> &m,vector<T> &a){
144   T M=1,tM,ans=0;
145   for(int i=0;i<(int)m.size();++i)M*=m[i];
146   for(int i=0;i<(int)a.size();++i){
147     tM=M/m[i];
148     ans=(ans+(a[i]*tM%M)*pow_mod(tM,Euler(m[i]
149       i))-1,m[i])%M)%M;
150   }
151   //如果m[i]是質數 · Euler(m[i])-1=m[i]-2 · 就不用算Euler了*/
152   return ans;
153 }
154
155 //java code
156 //求sqrt(N)的連分數
157 public static void Pell(int n){
158   BigInteger N,p1,p2,q1,q2,a0,a1,a2,g1,g2,h1
159     ,h2,p,q;
160   g1=q2=p1=BigInteger.ZERO;
161   h1=q1=p2=BigInteger.ONE;
162   a0=a1=BigInteger.valueOf((int)Math.sqrt
163     (1.0*n));
164   BigInteger ans=a0.multiply(a0);
165   if(ans.equals(BigInteger.valueOf(n))){
166     System.out.println("No solution!");
167     return;
168   }
169   while(true){
170     g2=a1.multiply(h1).subtract(g1);
171     h2=N.subtract(g2.pow(2)).divide(h1);
172     a2=g2.add(a0).divide(h2);
173     p=a1.multiply(p2).add(p1);
174     q=a1.multiply(q2).add(q1);
175     if(p.pow(2).subtract(N.multiply(q.pow
176       (2))).compareTo(BigInteger.ONE)==0)
177       break;
178     g1=g2;h1=h2;a1=a2;
179     p1=p2;p2=p;
180     q1=q2;q2=q;
181   }
182   System.out.println(p+" "+q);
183 }

```

8.2 bit_set.cpp

```

1 void sub_set(int S){
2   int sub=S;
3   do{
4     //對某集合的子集合的處理
5     sub=(sub-1)&S;
6   }while(sub!=S);
7 }
8
9 void k_sub_set(int k,int n){
10  int comb=(1<<k)-1,S=1<<n;
11  while(comb<S){

```

```

11 //對大小為k的子集合的處理
12 int x=comb&-comb,y=comb+x;
13 comb=((comb&y)/x>>1)|y;
14 }
15 }

```

8.3 cantor_expansion.cpp

```

1 int factorial[MAXN];
2 void init(){
3   factorial[0]=1;
4   for(int i=1;i<=MAXN;++i)factorial[i]=
5     factorial[i-1]*i;
6 }
7
8 int encode(const vector<int> &s){
9   int n=s.size(),res=0;
10  for(int i=0;i<n;++i){
11    int t=0;
12    for(int j=i+1;j<n;++j)
13      if(s[j]<s[i])++t;
14    res+=t*factorial[n-i-1];
15  }
16  return res;
17 }
18
19 vector<int> decode(int a,int n){
20   vector<int> res;
21   vector<bool> vis(n,0);
22   for(int i=n-1;i>=0;--i){
23     int t=a/factorial[i];
24     for(j=0;j<n;++j)
25       if(!vis[j]){
26         if(t==0)break;
27         --t;
28       }
29     res.push_back(j);
30     vis[j]=1;
31     a%=factorial[i];
32   }
33   return res;
34 }

```

8.4 FFT.cpp

```

1 template<typename T,typename VT=std::vector<
2   std::complex<T> > >
3 struct FFT{
4   const T pi;
5   FFT(const T pi=acos((-1)):pi(pi){}
6   unsigned int bit_reverse(unsigned int a,
7     int len){
8     a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)
9       >>1);
10    a=((a&0x33333333U)<<2)|((a&0xCCCCCCCCU)
11      >>2);
12    a=((a&0x0F0F0F0FU)<<4)|((a&0xFF0F0F0FU)
13      >>4);
14    a=((a&0x00FF00FFU)<<8)|((a&0xFFFF00FFU)
15      >>8);
16    a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)
17      >>16);

```

```

11 return a>>(32-len);
12 }
13 void fft(bool is_inv,VT &in,VT &out,int N)
14 {
15   int bitlen=std::lg(N),num=is_inv?-1:1;
16   for(int i=0;i<N;++i)out[bit_reverse(i,
17     bitlen)]=in[i];
18   for(int step=2;step<=N;step<=1){
19     const int mh=step>>1;
20     for(int i=0;i<mh;++i){
21       std::complex<T> wi=exp(std::complex<
22         T>(0,i*num*pi/mh));
23       for(int j=i;j<N;j+=step){
24         int k=j+mh;
25         std::complex<T> u=out[j],t=wi*out[
26           k];
27         out[j]=u+t;
28         out[k]=u-t;
29       }
30     }
31   }
32 }

```

8.5 find_real_root.cpp

```

1 //  $an^x + \dots + a_1x + a_0 = 0$ ;
2 int sign(double x){
3   return x < -eps ? -1 : x > eps;
4 }
5
6 double get(const vector<double>&coef, double
7   x){
8   double e = 1, s = 0;
9   for(auto i : coef) s += i*e, e *= x;
10  return s;
11 }
12
13 double find(const vector<double>&coef, int n
14   , double lo, double hi){
15   double sign_lo, sign_hi;
16   if( !(sign_lo = sign(get(coef,lo))) )
17     return lo;
18   if( !(sign_hi = sign(get(coef,hi))) )
19     return hi;
20   if(sign_lo * sign_hi > 0) return INF;
21   for(int stp = 0; stp < 100 && hi - lo >
22     eps; ++stp){
23     double m = (lo+hi)/2.0;
24     int sign_mid = sign(get(coef,m));
25     if(!sign_mid) return m;
26     if(sign_lo*sign_mid < 0) hi = m;
27     else lo = m;
28   }
29   return (lo+hi)/2.0;
30 }
31
32 vector<double> cal(vector<double>coef, int n
33   ){
34   vector<double>res;
35   if(n == 1){

```



```

30 if(sign(coef[1])) res.pb(-coef[0]/coef
31 [1]);
32 return res;
33 }
34 vector<double>dcoef(n);
35 for(int i = 0; i < n; ++i) dcoef[i] = coef
36 [i+1]*(i+1);
37 vector<double>droot = cal(dcoef, n-1);
38 droot.insert(droot.begin(), -INF);
39 droot.pb(INF);
40 for(int i = 0; i+1 < droot.size(); ++i){
41 double tmp = find(coef, n, droot[i],
42 droot[i+1]);
43 if(tmp < INF) res.pb(tmp);
44 }
45 return res;
46 }
47 int main () {
48 vector<double>ve;
49 vector<double>ans = cal(ve, n);
50 // 視情況把答案 +eps · 避免 -0
51 }

```

8.6 FWT.cpp

```

1 void XORtransform(LL *P, int k=log, bool inv
2 =0){
3 for(int len=1; 2*len<=(1<<k); len<=<1)
4 for(int i=0; i<(1<<k); i+=2*len)
5 for (int j=0; j<len; ++j){
6 LL u=P[i+j], v=P[i+len+j];
7 P[i+j]=u+v, P[i+len+j]=u-v;
8 }
9 if(inv)for(int i=0; i<(1<<k); ++i)P[i]/=(1<<
10 k);
11 void ANDtransform(LL *P, int k=log, bool inv
12 =0){
13 for(int len=1; 2*len<=(1<<k); len<=<1)
14 for(int i=0; i<(1<<k); i+=2*len)
15 for(int j=0; j<len; ++j){
16 LL u=P[i+j], v=P[i+len+j];
17 if(!inverse){
18 P[i+j]=v, P[i+len+j]=u+v;
19 //P[i+j]=u, P[i+len+j]=u+v; OR
20 version
21 }else{
22 P[i+j]=-u+v, P[i+len+j]=u;
23 //P[i+j]=u, P[i+len+j]=v-u; OR
24 version
25 }
26 }
27 }

```

8.7 LinearCongruence.cpp

```

1 pair<LL, LL> LinearCongruence(LL a[], LL b[],
2 LL m[], int n) {

```

```

2 // a[i]*x = b[i] ( mod m[i] )
3 for(int i=0; i<n; ++i) {
4 LL x, y, d = extgcd(a[i], m[i], x, y);
5 if(b[i]%d!=0) return make_pair(-1LL, 0LL);
6 ;
7 m[i] /= d;
8 b[i] = LLMul(b[i]/d, x, m[i]);
9 }
10 LL lastb = b[0], lastm = m[0];
11 for(int i=1; i<n; ++i) {
12 LL x, y, d = extgcd(m[i], lastm, x, y);
13 if((lastb-b[i])%d!=0) return make_pair
14 (-1LL, 0LL);
15 lastb = LLMul((lastb-b[i])/d, x, (lastm/d)
16 )*m[i];
17 lastm = (lastm/d)*m[i];
18 lastb = (lastb+b[i])%lastm;
19 }
20 return make_pair(lastb<0?lastb+lastm:lastb
21 , lastm);
22 }

```

8.8 Lucas.cpp

```

1 int mod_fact(int n, int &e){
2 e=0;
3 if(n==0)return 1;
4 int res=mod_fact(n/P, e);
5 e += n/P;
6 if((n/P)%2==0)return res*fact[n%P]%P;
7 return res*(P-fact[n%P])%P;
8 }
9 int Cmod(int n, int m){
10 int a1, a2, a3, e1, e2, e3;
11 a1=mod_fact(n, e1);
12 a2=mod_fact(m, e2);
13 a3=mod_fact(n-m, e3);
14 if(e1>e2+e3)return 0;
15 return a1*inv(a2*a3%P, P)%P;
16 }

```

8.9 Matrix.cpp

```

1 template<typename T>
2 struct Matrix{
3 using rt = std::vector<T>;
4 using mt = std::vector<rt>;
5 using matrix = Matrix<T>;
6 int r, c;
7 mt m;
8 Matrix(int r, int c):r(r), c(c), m(r, rt(c)){
9 rt& operator[](int i){return m[i];}
10 matrix operator+(const matrix &a){
11 matrix rev(r, c);
12 for(int i=0; i<r; ++i)
13 for(int j=0; j<c; ++j)
14 rev[i][j]=m[i][j]+a.m[i][j];
15 return rev;
16 }
17 matrix operator-(const matrix &a){

```

```

18 matrix rev(r, c);
19 for(int i=0; i<r; ++i)
20 for(int j=0; j<c; ++j)
21 rev[i][j]=m[i][j]-a.m[i][j];
22 return rev;
23 }
24 matrix operator*(const matrix &a){
25 matrix rev(r, a.c);
26 matrix tmp(a.c, a.r);
27 for(int i=0; i<a.r; ++i)
28 for(int j=0; j<a.c; ++j)
29 tmp[j][i]=a.m[i][j];
30 for(int i=0; i<r; ++i)
31 for(int j=0; j<a.c; ++j)
32 for(int k=0; k<c; ++k)
33 rev.m[i][j]+=m[i][k]*tmp[j][k];
34 return rev;
35 }
36 bool inverse(){
37 Matrix t(r, r+c);
38 for(int y=0; y<r; y++){
39 t.m[y][c+y] = 1;
40 for(int x=0; x<c; ++x)
41 t.m[y][x]=m[y][x];
42 }
43 if( !t.gas() )
44 return false;
45 for(int y=0; y<r; y++){
46 for(int x=0; x<c; ++x)
47 m[y][x]=t.m[y][c+x]/t.m[y][y];
48 return true;
49 }
50 T gas(){
51 vector<T> lazy(r, 1);
52 bool sign=false;
53 for(int i=0; i<r; ++i){
54 if( m[i][i]==0 ){
55 int j=i+1;
56 while(j<r&&!m[j][i])j++;
57 if(j==r)continue;
58 m[i].swap(m[j]);
59 sign=!sign;
60 }
61 for(int j=0; j<r; ++j){
62 if(i==j)continue;
63 lazy[j]=lazy[j]*m[i][i];
64 T mx=m[j][i];
65 for(int k=0; k<c; ++k)
66 m[j][k]=m[j][k]*m[i][i]-m[i][k]*mx;
67 }
68 }
69 T det=sign?-1:1;
70 for(int i=0; i<r; ++i){
71 det = det*m[i][i];
72 det = det/lazy[i];
73 for(auto &j:m[i])j/=lazy[i];
74 }
75 return det;
76 }
77 };

```

8.10 MillerRobin.cpp

```

1 LL LLMul(LL a, LL b, const LL &mod) {
2 LL ans=0;
3 while(b) {
4 if(b&1) {
5 ans+=a;
6 if(ans>=mod) ans-=mod;
7 }
8 a<<=1, b>>=1;
9 if(a==mod) a-=mod;
10 }
11 return ans;
12 }
13 LL mod_mul(LL a, LL b, LL m){
14 a%=m, b%=m; /* fast for m < 2^58 */
15 LL y=(LL)((double)a*b/m+0.5);
16 LL r=(a*b-y*m)%m;
17 return r<0?r+m:r;
18 }
19 template<typename T>
20 T pow(T a, T b, T mod){/*a^b%mod
21 T ans=1;
22 for(; b; a=mod_mul(a, a, mod), b>>=1)
23 if(b&1)ans=mod_mul(ans, a, mod);
24 return ans;
25 }
26 int sprp[3]={2, 7, 61}; /*int範圍可解
27 int llsprp
28 [7]={2, 325, 9375, 28178, 450775, 9780504, 17952650}
29 //至少unsigned long long範圍
30 template<typename T>
31 bool isprime(T n, int *sprp, int num){
32 if(n==2)return 1;
33 if(n<2||n%2==0)return 0;
34 int t=0;
35 T u=n-1;
36 for(; u%2==0; ++t)u>>=1;
37 for(int i=0; i<num; ++i){
38 T a=sprp[i]%n;
39 if(a==0||a==1||a==n-1)continue;
40 T x=pow(a, u, n);
41 if(x==1||x==n-1)continue;
42 for(int j=0; j<t; ++j){
43 x=mod_mul(x, x, n);
44 if(x==1)return 0;
45 if(x==n-1)break;
46 }
47 if(x==n-1)continue;
48 return 0;
49 }
50 return 1;
51 }

```

8.11 NTT.cpp

```

1 2615053605667*(2^18)+1, 3
2 15*(2^27)+1, 31
3 479*(2^21)+1, 3
4 7*17*(2^23)+1, 3
5 3*3*211*(2^19)+1, 5

```

```

6 25*(2^22)+1,3
7 template<typename T,typename VT=std::vector<
  T>>
8 struct NTT{
9   const T P,G;
10  NTT(T p=(1<<23)*7*17+1,T g=3):P(p),G(g){}
11  unsigned int bit_reverse(unsigned int a,
    int len){
12    a=((a&0x55555555U)<<1)|((a&0xAAAAAAAU)
      >>1);
13    a=((a&0x33333333U)<<2)|((a&0xCCCCCCU)
      >>2);
14    a=((a&0x0F0F0F0FU)<<4)|((a&0xF0F0F0FU)
      >>4);
15    a=((a&0x00FF00FFU)<<8)|((a&0xFF00FF00U)
      >>8);
16    a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)
      >>16);
17    return a>>(32-len);
18  }
19  T pow_mod(T n,T k,T m){
20    T ans=1;
21    for(n=(n>=m?n%m:n);k;>=1){
22      if(k&1)ans=ans*n%m;
23      n=n*n%m;
24    }
25    return ans;
26  }
27  void ntt(bool is_inv,VT &in,VT &out,int N)
    {
28    int bitlen=std::lg(N);
29    for(int i=0;i<N;++i)out[bit_reverse(i,
      bitlen)]=in[i];
30    for(int step=2,id=1;step<=N;step<=1,++
      id){
31      T wn=pow_mod(G,(P-1)>>id,P),wi=1,u,t;
32      const int mh=step>>1;
33      for(int i=0;i<mh;++i){
34        for(int j=i;j<N;j+=step){
35          u=out[j],t=wi*u*out[j+mh]%P;
36          out[j]=u+t;
37          out[j+mh]=u-t;
38          if(out[j]>=P)out[j]-=P;
39          if(out[j+mh]<0)out[j+mh]+=P;
40        }
41        wi=wi*wn%P;
42      }
43    }
44    if(is_inv){
45      for(int i=1;i<N/2;++i)std::swap(out[i],
        out[N-i]);
46      T invn=pow_mod(N,P-2,P);
47      for(int i=0;i<N;++i)out[i]=out[i]*invn
        %P;
48    }
49  }
50 };

```

8.12 Simpson.cpp

```

1 double simpson(double a,double b){
2   double c=a+(b-a)/2;
3   return (F(a)+4*F(c)+F(b))*(b-a)/6;

```

```

4 }
5 double asr(double a,double b,double eps,
  double A){
6   double c=a+(b-a)/2;
7   double L=simpson(a,c),R=simpson(c,b);
8   if( abs(L+R-A)<15*eps )
9     return L+R+(L+R-A)/15.0;
10  return asr(a,c,eps/2,L)+asr(c,b,eps/2,R)
    ;
11 }
12 double asr(double a,double b,double eps){
13   return asr(a,b,eps,simpson(a,b));
14 }

```

8.13 外星模運算.cpp

```

1 //a[0]^(a[1]^a[2]^...)
2 #include<bits/stdc++.h>
3 using namespace std;
4 #define maxn 1000000
5 int euler[maxn+5];
6 bool is_prime[maxn+5];
7 inline void init_euler(){
8   is_prime[1]=1; // - 不是質數
9   for(int i=1;i<=maxn;i++)euler[i]=i;
10  for(int i=2;i<=maxn;i++){
11    if(!is_prime[i]){//是質數
12      euler[i]--;
13      for(int j=i<1;j<=maxn;j+=i){
14        is_prime[j]=1;
15        euler[j]=euler[j]/i*(i-1);
16      }
17    }
18  }
19 }
20 inline long long pow(long long a,long long b
  ,long long mod){//a^b%mod
21   long long ans=1;
22   for(;b;a=a*a%mod,b>=1)
23     if(b&1)ans=ans*a%mod;
24   return ans;
25 }
26 bool isless(long long *a,int n,int k){
27   if(*a==1)return k>1;
28   if(--n==0)return *a<k;
29   int next=0;
30   for(long long b=1;b<k;++next)
31     b*=*a;
32   return isless(a+1,n,next);
33 }
34 long long high_pow(long long *a,int n,long
  long mod){
35   if(*a==1||--n==0)return *a%mod;
36   int k=0,r=euler[mod];
37   for(long long tma=1;tma!=pow(*a,k+r,mod)
    ;++k)
38     tma=tma*(a%mod);
39   if(isless(a+1,n,k))return pow(*a,high_pow(
    a+1,n,k),mod);
40   int tmd=high_pow(a+1,n,r);
41   int t=(tmd-k+r)%r;
42   return pow(*a,k+t,mod);

```

```

43 }
44 long long a[1000005];
45 int t,mod;
46 int main(){
47   init_euler();
48   scanf("%d",&t);
49   #define n 4
50   while(t--){
51     for(int i=0;i<n;++i)scanf("%lld",&a[i]);
52     scanf("%d",&mod);
53     printf("%lld\n",high_pow(a,n,mod));
54   }
55   return 0;
56 }

```

8.14 模運算模板.cpp

```

1 template<typename T,long long mod>
2 struct mod_t{//mod只能是質數
3   T data;
4   mod_t(){}
5   mod_t(const T &d):data((d%mod+mod)%mod){}
6   mod_t pow(T b)const{
7     mod_t ans(1);
8     for(mod_t now=*this;b;now=now*b,b/=2)
9       if(b%2)ans=ans*now;
10    return ans;
11  }
12  mod_t operator-(int)const{
13    return mod_t(mod-data);
14  }
15  mod_t operator+(const mod_t &b)const{
16    return mod_t((data+b.data)%mod);
17  }
18  mod_t operator-(const mod_t &b)const{
19    return mod_t((data-b.data+mod)%mod);
20  }
21  mod_t operator*(const mod_t &b)const{
22    return mod_t((data*b.data)%mod);
23  }
24  mod_t operator/(const mod_t &b)const{
25    return *this*b.pow(mod-2); // *this *
    Inverse(b)
26  }
27  operator T()const{return data;}
28  friend istream &operator>>(istream &i,
    mod_t &b){
29    T d;
30    i>>d;
31    b=mod_t(d);
32    return i;
33  }
34 };

```

8.15 質因數分解.cpp

```

1 LL func(const LL n,const LL mod,const int c)
  {
2   return (LLmul1(n,n,mod)+c+mod)%mod;

```

```

3 }
4
5 LL pollorro(const LL n, const int c) {//循
  環節長度
6   LL a=1, b=1;
7   a=func(a,n,c)%n;
8   b=func(b,n,c)%n; b=func(b,n,c)%n;
9   while(gcd(abs(a-b),n)==1) {
10    a=func(a,n,c)%n;
11    b=func(b,n,c)%n; b=func(b,n,c)%n;
12  }
13  return gcd(abs(a-b),n);
14 }
15
16 void prefactor(LL &n, vector<LL> &v) {
17   for(int i=0;i<12;++i) {
18     while(n%prime[i]==0) {
19       v.push_back(prime[i]);
20       n/=prime[i];
21     }
22   }
23 }
24
25 void smallfactor(LL n, vector<LL> &v) {
26   if(n<MAXPRIME) {
27     while(isp[(int)n]) {
28       v.push_back(isp[(int)n]);
29       n/=isp[(int)n];
30     }
31     v.push_back(n);
32   } else {
33     for(int i=0;i<primecnt&&prime[i]*prime[i]
      <=n;++i) {
34       while(n%prime[i]==0) {
35         v.push_back(prime[i]);
36         n/=prime[i];
37       }
38     }
39     if(n!=1) v.push_back(n);
40   }
41 }
42
43 void comfactor(const LL &n, vector<LL> &v) {
44   if(n<1e9) {
45     smallfactor(n,v);
46     return;
47   }
48   if(Isprime(n)) {
49     v.push_back(n);
50     return;
51   }
52   LL d;
53   for(int c=3;++c) {
54     d = pollorro(n,c);
55     if(d!=n) break;
56   }
57   comfactor(d,v);
58   comfactor(n/d,v);
59 }
60
61 void Factor(const LL &x, vector<LL> &v) {
62   LL n = x;
63   if(n==1) { puts("Factor 1"); return; }
64   prefactor(n,v);
65   if(n==1) return;

```

```

66 comfactor(n,v);
67 sort(v.begin(),v.end());
68 }
69
70 void AllFactor(const LL &n,vector<LL> &v) {
71     vector<LL> tmp;
72     Factor(n,tmp);
73     v.clear();
74     v.push_back(1);
75     int len;
76     LL now=1;
77     for(int i=0;i<tmp.size();++i) {
78         if(i==0 || tmp[i]!=tmp[i-1]) {
79             len = v.size();
80             now = 1;
81         }
82         now*=tmp[i];
83         for(int j=0;j<len;++j)
84             v.push_back(v[j]*now);
85     }
86 }

```

9 other

9.1 WhatDay.cpp

```

1 int whatday(int y,int m,int d){
2     if(m<=2)m+=12,--y;
3     if(y<1752||y==1752&&m<9||y==1752&&m==9&&
4         d<3)
5         return (d+2*m+3*(m+1)/5+y+y/4+5)%7;
6     return (d+2*m+3*(m+1)/5+y+y/4-y/100+y
7         /400)%7;
8 }

```

9.2 上下最大正方形.cpp

```

1 void solve(int n,int a[],int b[]){// 1-base
2     int ans=0;
3     deque<int>da,db;
4     for(int l=1,r=1;r<=n;++r){
5         while(da.size()&&a[da.back()]>=a[r]){
6             da.pop_back();
7         }
8         da.push_back(r);
9         while(db.size()&&b[db.back()]>=b[r]){
10            db.pop_back();
11        }
12        db.push_back(r);
13        for(int d=a[da.front()]+b[db.front()];r-
14            l+1>d;l++){
15            if(da.front()==l)da.pop_front();
16            if(db.front()==l)db.pop_front();
17            if(da.size()&&db.size()){
18                d=a[da.front()]+b[db.front()];
19            }
20        }
21        ans=max(ans,r-l+1);
22    }
23 }

```

```

21 }
22 printf("%d\n",ans);
23 }

```

9.3 最大矩形.cpp

```

1 long long max_rectangle(vector<int> s){
2     stack<pair<int,int> > st;
3     st.push(make_pair(-1,0));
4     s.push_back(0);
5     long long ans=0;
6     for(size_t i=0;i<s.size();++i){
7         int h=s[i];
8         pair<int,int> now=make_pair(h,i);
9         while(h<st.top().first){
10            now=st.top();
11            st.pop();
12            ans=max(ans,(long long)(i-now.second)*
13                now.first);
14        }
15        if(h>st.top().first){
16            st.push(make_pair(h,now.second));
17        }
18    }
19    return ans;
20 }

```

10 String

10.1 AC 自動機.cpp

```

1 template<char L='a',char R='z'>
2 class ac_automaton{
3 private:
4     struct joe{
5         int next[R-L+1],fail,efl,ed,cnt_dp,vis;
6     };
7     joe():ed(0),cnt_dp(0),vis(0){
8         for(int i=0;i<=R-L;++i)next[i]=0;
9     };
10    public:
11        std::vector<joe> S;
12        std::vector<int> q;
13        int qs,qe,vt;
14        ac_automaton():S(1),qs(0),qe(0),vt(0){
15            void clear(){
16                q.clear();
17                S.resize(1);
18                for(int i=0;i<=R-L;++i)S[0].next[i]=0;
19                S[0].cnt_dp=S[0].vis=qs=qe=vt=0;
20            }
21            void insert(const char *s){
22                int o=0;
23                for(int i=0,id;s[i];++i){
24                    id=s[i]-L;
25                    if(!S[o].next[id]){
26                        S.push_back(joe());
27                    }
28                }
29            }
30        }
31    };

```

```

32 S[o].next[id]=S.size()-1;
33 }
34 o=S[o].next[id];
35 }
36 ++S[o].ed;
37 }
38 void build_fail(){
39     S[0].fail=S[0].efl=-1;
40     q.clear();
41     q.push_back(0);
42     ++qe;
43     while(qs!=qe){
44         int pa=q[qs++],id,t;
45         for(int i=0;i<=R-L;++i){
46             t=S[pa].next[i];
47             if(!t)continue;
48             id=S[pa].fail;
49             while(~id&&!S[id].next[i])id=S[id]
50                 .fail;
51             S[t].fail=~id?S[id].next[i]:0;
52             S[t].efl=S[S[t].fail].ed?S[t].fail
53                 :S[S[t].fail].efl;
54             q.push_back(t);
55             ++qe;
56         }
57     }
58     /*DP出每個前綴在字串s出現的次數並傳回所
59     有字串被s匹配成功的次數O(N*M)*/
60     int match_0(const char *s){
61         int ans=0,id,p=0,i;
62         for(int i=0;s[i];++i){
63             id=s[i]-L;
64             while(!S[p].next[id]&&p) p=S[p].fail;
65             if(!S[p].next[id])continue;
66             p=S[p].next[id];
67             ++S[p].cnt_dp; /*匹配成功則它所有後綴
68                 都可以被匹配(DP計算)*/
69         }
70         for(i=qe-1;i>=0;--i){
71             ans+=S[q[i]].cnt_dp*S[q[i]].ed;
72             if(~S[q[i]].fail)S[q[i]].fail.
73                 cnt_dp+=S[q[i]].cnt_dp;
74         }
75         return ans;
76     }
77     /*多串匹配走efl邊並傳回所有字串被s匹配成
78     功的次數O(N*M^1.5)*/
79     int match_1(const char *s)const{
80         int ans=0,id,p=0,t;
81         for(int i=0;s[i];++i){
82             id=s[i]-L;
83             while(!S[p].next[id]&&p) p=S[p].fail;
84             if(!S[p].next[id])continue;
85             p=S[p].next[id];
86             if(S[p].ed)ans+=S[p].ed;
87             for(t=S[p].efl;~t;t=S[t].efl){
88                 ans+=S[t].ed; /*因為都走efl邊所以保
89                     證匹配成功*/
90             }
91         }
92         return ans;
93     }
94 }

```

```

95 /*枚舉(s的子字串na)的所有相異字串各恰一
96 次並傳回次數O(N*M^(1/3))*/
97 int match_2(const char *s){
98     int ans=0,id,p=0,t;
99     ++vt;
100     /*把戳記vt+=1，只要vt沒溢位，所有S[p].
101     vis=vt就會變成false
102     這種利用vt的方法可以O(1)歸零vis陣列*/
103     for(int i=0;s[i];++i){
104         id=s[i]-L;
105         while(!S[p].next[id]&&p) p=S[p].fail;
106         if(!S[p].next[id])continue;
107         p=S[p].next[id];
108         if(S[p].ed&&S[p].vis!=vt){
109             S[p].vis=vt;
110             ans+=S[p].ed;
111         }
112         for(t=S[p].efl;~t&&S[t].vis!=vt;t=S[
113             t].efl){
114             S[t].vis=vt;
115             ans+=S[t].ed; /*因為都走efl邊所以保
116                 證匹配成功*/
117         }
118     }
119     return ans;
120 }
121 /*把AC自動機變成真的自動機*/
122 void evolution(){
123     for(qs=1;qs!=qe;){
124         int p=q[qs++];
125         for(int i=0;i<=R-L;++i)
126             if(S[p].next[i]==0)S[p].next[i]=S[
127                 S[p].fail].next[i];
128     }
129 }
130 }

```

10.2 hash.cpp

```

1 #define MAXN 1000000
2 #define prime_mod 1073676287
3 /*prime_mod 必須要要是質數*/
4 typedef long long T;
5 char s[MAXN+5];
6 T h[MAXN+5]; /*hash陣列*/
7 T h_base[MAXN+5]; /*h_base[n]=(prime^n)%
8     prime_mod*/
9 inline void hash_init(int len,T prime=0
10     xdefaced){
11     h_base[0]=1;
12     for(int i=1;i<=len;++i){
13         h[i]=(h[i-1]*prime+s[i-1])%prime_mod;
14         h_base[i]=(h_base[i-1]*prime)%prime_mod;
15     }
16 }
17 inline T get_hash(int l,int r){/*閉區間寫
18     法，設編號為0 ~ Len-1*/
19     return (h[r+1]-(h[l]*h_base[r-l+1])%
20         prime_mod+prime_mod)%prime_mod;
21 }

```

10.3 KMP.cpp

```

1  /*產生fail function*/
2  inline void kmp_fail(char *s,int len,int *
3      fail){
4      int id=-1;
5      fail[0]=-1;
6      for(int i=1;i<len;++i){
7          while(~id&&s[id+1]!=s[i])id=fail[id];
8          if(s[id+1]==s[i])++id;
9          fail[i]=id;
10     }
11     /*以字串B匹配字串A，傳回匹配成功的數量(用B的
12     fail)*/
13     inline int kmp_match(char *A,int lenA,char *
14         B,int lenB,int *fail){
15         int id=-1,ans=0;
16         for(int i=0;i<lenA;++i){
17             while(~id&&B[id+1]!=A[i])id=fail[id];
18             if(B[id+1]==A[i])++id;
19             if(id==lenB-1){/*匹配成功*/
20                 ++ans;
21                 id=fail[id];
22             }
23         }
24         return ans;

```

10.4 manacher.cpp

```

1  //原字串: asdsasdsa
2  //先把字串變成這樣: @#a#s#d#s#a#s#d#s#a#
3  inline void manacher(char *s,int len,int *z)
4  {
5      int l=0,r=0;
6      for(int i=1;i<len;++i){
7          z[i]=r>i?min(z[2*i-l],r-i):1;
8          while(s[i+z[i]]==s[i-z[i]])++z[i];
9          if(z[i]+i>r)r=z[i]+i,l=i;
10     }

```

10.5 minimal_string_rotation.cpp

```

1  int min_string_rotation(const string &s){
2      int n=s.size(),i=0,j=1,k=0;
3      while(i<n&&j<n&&k<n){
4          int t=s[(i+k)%n]-s[(j+k)%n];
5          ++k;
6          if(t){
7              if(t>0)i+=k;
8              else j+=k;
9              if(i==j)++j;
10             k=0;
11         }
12     }

```

```

13     return min(i,j);/*傳回最小循環表示法起始位
14     置

```

10.6 suffix_array_lcp.cpp

```

1  #define radix_sort(x,y){\
2      for(i=0;i<A;++i)c[i]=0;\
3      for(i=0;i<n;++i)c[x[y[i]]]++;\
4      for(i=1;i<A;++i)c[i]+=c[i-1];\
5      for(i=n-1;~i;--i)sa[--c[x[y[i]]]]=y[i];\
6  }
7  #define sac(r,a,b) r[a]!=(r[b]||a+k>n||r[a+k]
8      !=r[b+k])
9  void suffix_array(const char *s,int n,int *
10     sa,int *rank,int *tmp,int *c){
11     int A='z'+1,i,k,id=0;
12     for(i=0;i<n;++i)rank[i]=s[i];
13     radix_sort(rank,tmp);
14     for(k=1;id<n-1;k<=1){
15         for(id=0,i=n-k;i<n;++i)tmp[id++]=i;
16         for(i=0;i<n;++i)if(sa[i]>=k)tmp[id++]=(sa[i]-k);
17         radix_sort(rank,tmp);
18         swap(rank,tmp);
19         for(rank[sa[0]]=id=0,i=1;i<n;++i)
20             rank[sa[i]]=id+=sac(tmp,sa[i-1],sa[i]);
21         A=id+1;
22     }
23     //h:高度數組 sa:後綴數組 rank:排名
24     void suffix_array_lcp(const char *s,int len,
25         int *h,int *sa,int *rank){
26         for(int i=0;i<len;++i)rank[sa[i]]=i;
27         for(int i=0,k=0;i<len;++i){
28             if(rank[i]==0)continue;
29             if(k--<0)continue;
30             while(s[i+k]==s[sa[rank[i]-1]+k])++k;
31             h[rank[i]]=k;
32         }
33     }

```

10.7 Z.cpp

```

1  inline void z_alg(char *s,int len,int *z){
2      int l=0,r=0;
3      z[0]=len;
4      for(int i=1;i<len;++i){
5          z[i]=i>r?0:(i-l+z[i-l]<z[l]?z[i-l]:r-i+1);
6          while(i+z[i]<len&&s[i+z[i]]==s[z[i]])++z[i];
7          if(i+z[i]-1>r)r=i+z[i]-1,l=i;
8      }
9  }

```

11 Tarjan

11.1 dominator_tree.cpp

```

1  struct dominator_tree{
2      static const int MAXN=5005;
3      int n;// 1-base
4      vector<int> suc[MAXN],pre[MAXN];
5      int fa[MAXN],dfn[MAXN],id[MAXN],Time;
6      int semi[MAXN],idom[MAXN];
7      int anc[MAXN],best[MAXN];/*disjoint set
8      vector<int> dom[MAXN];/*dominator_tree
9      void init(int _n){
10         n=_n;
11         for(int i=1;i<=n;++i)suc[i].clear(),pre[
12             i].clear();
13     }
14     void add_edge(int u,int v){
15         suc[u].push_back(v);
16         pre[v].push_back(u);
17     }
18     void dfs(int u){
19         dfn[u]=++Time,id[Time]=u;
20         for(auto v:suc[u]){
21             if(dfn[v])continue;
22             dfs(v),fa[dfn[v]]=dfn[u];
23         }
24     }
25     int find(int x){
26         if(x==anc[x])return x;
27         int y=find(anc[x]);
28         if(semi[best[x]]>semi[best[anc[x]]])best
29             [x]=best[anc[x]];
30         return anc[x]=y;
31     }
32     void tarjan(int r){
33         Time=0;
34         for(int t=1;t<=n;++t){
35             dfn[t]=idom[t]=0;/*u=r或是u無法到達r時
36             idom[id[u]]=0
37             dom[t].clear();
38             anc[t]=best[t]=semi[t]=t;
39         }
40         dfs(r);
41         for(int y=Time;y>=2;--y){
42             int x=fa[y],idy=id[y];
43             for(auto z:pre[idy]){
44                 if(!(z=dfn[z]))continue;
45                 find(z);
46                 semi[y]=min(semi[y],semi[best[z]]);
47             }
48             dom[semi[y]].push_back(y);
49             anc[y]=x;
50             for(auto z:dom[x]){
51                 find(z);
52                 idom[z]=semi[best[z]]<x?best[z]:x;
53             }
54             dom[x].clear();
55         }
56         for(int u=2;u<=Time;++u){
57             if(idom[u]!=semi[u])idom[u]=idom[idom[
58                 u]];
59             dom[id[idom[u]]].push_back(id[u]);

```

```

56     }
57     }
58 }dom;

```

11.2 tnfsb017_2_sat.cpp

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  #define MAXN 8001
4  #define MAXN2 MAXN*4
5  #define n(X) ((X)+2*N)
6  vector<int> v[MAXN2];
7  vector<int> rv[MAXN2];
8  vector<int> vis_t;
9  int N,M;
10 void addedge(int s,int e){
11     v[s].push_back(e);
12     rv[e].push_back(s);
13 }
14 int scc[MAXN2];
15 bool vis[MAXN2]={false};
16 void dfs(vector<int> *uv,int n,int k=-1){
17     vis[n]=true;
18     for(int i=0;i<uv[n].size();++i)
19         if(!vis[uv[n][i]])
20             dfs(uv,uv[n][i],k);
21     if(uv==v)vis_t.push_back(n);
22     scc[n]=k;
23 }
24 void solve(){
25     for(int i=1;i<=N;++i){
26         if(!vis[i])dfs(v,i);
27         if(!vis[n(i)])dfs(v,n(i));
28     }
29     memset(vis,0,sizeof(vis));
30     int c=0;
31     for(int i=vis_t.size()-1;i>=0;--i)
32         if(!vis[vis_t[i]])
33             dfs(rv,vis_t[i],c++);
34 }
35 int main(){
36     int a,b;
37     scanf("%d%d",&N,&M);
38     for(int i=1;i<=N;++i){
39         // (A or B)&(!A & !B) A^B
40         a=i*2-1;
41         b=i*2;
42         addedge(n(a),b);
43         addedge(n(b),a);
44         addedge(a,n(b));
45         addedge(b,n(a));
46     }
47     while(M--){
48         scanf("%d%d",&a,&b);
49         a = a>0?a*2-1:-a*2;
50         b = b>0?b*2-1:-b*2;
51         // A or B
52         addedge(n(a),b);
53         addedge(n(b),a);
54     }
55     solve();
56     bool check=true;
57     for(int i=1;i<=2*N;++i)

```



```

58     if(scc[i]==scc[n(i)])
59         check=false;
60     if(check){
61         printf("%d\n",N);
62         for(int i=1;i<=2*N;i+=2){
63             if(scc[i]>scc[i+2*N])
64                 putchar('+');
65             else
66                 putchar('-');
67         }
68         putchar('\n');
69     }else puts("0");
70     return 0;
71 }

```

11.3 橋連通分量.cpp

```

1  #define N 1005
2  struct edge{
3      int u,v;
4      bool is_bridge;
5      edge(int u=0,int v=0):u(u),v(v),is_bridge
6          (0){}
7  };
8  vector<edge> E;
9  vector<int> G[N]; // 1-base
10 int low[N],vis[N],Time;
11 int bcc_id[N],bridge_cnt,bcc_cnt; // 1-base
12 int st[N],top; // BCC用
13 inline void add_edge(int u,int v){
14     G[u].push_back(E.size());
15     E.push_back(edge(u,v));
16     G[v].push_back(E.size());
17     E.push_back(edge(v,u));
18 }
19 void dfs(int u,int re=-1){ // u當前點, re為u連
20     接前一個點的邊
21     int v;
22     low[u]=vis[u]=++Time;
23     st[top++]=u;
24     for(size_t i=0;i<G[u].size();++i){
25         int e=G[u][i];v=E[e].v;
26         if(!vis[v]){
27             dfs(v,e^1); // e^1 反向邊
28             low[u]=min(low[u],low[v]);
29             if(vis[u]<low[v]){
30                 E[e].is_bridge=E[e^1].is_bridge=1;
31                 ++bridge_cnt;
32             }
33         }else if(vis[v]<vis[u]&&e!=re){
34             low[u]=min(low[u],vis[v]);
35         }
36     }
37     if(vis[u]==low[u]){ // 處理BCC
38         ++bcc_cnt; // 1-base
39         do bcc_id[v=st[--top]]=bcc_cnt; // 每個點
40             所在的BCC
41         while(v!=u);
42     }
43 }
44 inline void bcc_init(int n){
45     Time=bcc_cnt=bridge_cnt=top=0;
46     E.clear();

```

```

43     for(int i=1;i<=n;++i){
44         G[i].clear();
45         vis[i]=bcc_id[i]=0;
46     }
47 }

```

11.4 雙連通分量 & 割點.cpp

```

1  #define N 1005
2  vector<int> G[N]; // 1-base
3  vector<int> bcc[N]; // 存每塊雙連通分量的點
4  int low[N],vis[N],Time;
5  int bcc_id[N],bcc_cnt; // 1-base
6  bool is_cut[N]; // 是否為割點
7  int st[N],top;
8  void dfs(int u,int pa=-1){ // u當前點, pa父親
9      int v,child=0;
10     low[u]=vis[u]=++Time;
11     st[top++]=u;
12     for(size_t i=0;i<G[u].size();++i){
13         int v=G[u][i];
14         if(!vis[v]){
15             dfs(v,u),++child;
16             low[u]=min(low[u],low[v]);
17             if(vis[u]<low[v]){
18                 is_cut[u]=1;
19                 bcc[++bcc_cnt].clear();
20                 int t;
21                 do{
22                     bcc_id[t=st[--top]]=bcc_cnt;
23                     bcc[bcc_cnt].push_back(t);
24                 }while(t!=v);
25                 bcc_id[u]=bcc_cnt;
26                 bcc[bcc_cnt].push_back(u);
27             }else if(vis[v]<vis[u]&&v!=pa){ // 反向邊
28                 low[u]=min(low[u],vis[v]);
29             }
30         }
31         if(pa!=-1&&child<2)is_cut[u]=0; // u是dfs樹
32             的根要特判
33     }
34     inline void bcc_init(int n){
35         Time=bcc_cnt=top=0;
36         for(int i=1;i<=n;++i){
37             G[i].clear();
38             is_cut[i]=vis[i]=bcc_id[i]=0;
39         }
40     }

```

12 Tree_problem

12.1 HeavyLight.cpp

```

1  #include<vector>
2  #define MAXN 100005
3  typedef std::vector<int>::iterator VIT;
4  int siz[MAXN],max_son[MAXN],pa[MAXN],dep[
5      MAXN];

```

```

5  int link_top[MAXN],link[MAXN],cnt;
6  std::vector<int> >G[MAXN];
7  void find_max_son(int x){
8      siz[x]=1;
9      max_son[x]=-1;
10     for(VIT i=G[x].begin();i!=G[x].end();++i){
11         if(*i==pa[x])continue;
12         pa[*i]=x;
13         dep[*i]=dep[x]+1;
14         find_max_son(*i);
15         if(max_son[x]==-1||siz[*i]>siz[max_son[x]
16             ])max_son[x]=*i;
17         siz[x]+=siz[*i];
18     }
19 }
20 void build_link(int x,int top){
21     link[x]=++cnt;
22     link_top[x]=top;
23     if(max_son[x]==-1)return;
24     build_link(max_son[x],top);
25     for(VIT i=G[x].begin();i!=G[x].end();++i){
26         if(*i==max_son[x]||*i==pa[x])continue;
27         build_link(*i,*i);
28     }
29 }
30 inline int find_lca(int a,int b){
31     //求LCA, 可以在過程中對區間進行處理
32     int ta=link_top[a],tb=link_top[b];
33     while(ta!=tb){
34         if(dep[ta]<dep[tb]){
35             std::swap(ta,tb);
36             std::swap(a,b);
37         }
38         //這裡可以對a所在的鏈做區間處理
39         //區間為(Link[ta],Link[a])
40         ta=link_top[a=pa[ta]];
41     }
42     //最後a,b會在同一條鏈上, 若a!=b還要在進行一
43     次區間處理
44     return dep[a]<dep[b]?a:b;

```

12.2 LCA.cpp

```

1  #define MAXN 100000
2  #define MAX_LOG 17
3  int pa[MAX_LOG+1][MAXN+5];
4  int dep[MAXN+5];
5  vector<int> G[MAXN+5];
6  void dfs(int x,int p){ //dfs(1,-1);
7      pa[0][x]=p;
8      for(int i=0;i+1<MAX_LOG;++i)pa[i+1][x]=pa[
9          i][pa[i][x]];
10     for(auto &i:G[x]){
11         if(i==p)continue;
12         dep[i]=dep[x]+1;
13         dfs(i,x);
14     }
15 }
16 inline int jump(int x,int d){
17     for(int i=0;i<d;++i)if((x>>i)&1)x=pa[i][x];
18     return x;

```

```

18 }
19 inline int find_lca(int a,int b){
20     if(dep[a]>dep[b])swap(a,b);
21     b=jump(b,dep[b]-dep[a]);
22     if(a==b)return a;
23     for(int i=MAX_LOG;i>=0;--i){
24         if(pa[i][a]!=pa[i][b]){
25             a=pa[i][a];
26             b=pa[i][b];
27         }
28     }
29     return pa[0][a];
30 }

```

12.3 link_cut_tree.cpp

```

1  struct splay_tree{
2      int ch[2],pa; //子節點跟父母
3      bool rev; //反轉的懶惰標記
4      splay_tree():pa(0),rev(0){ch[0]=ch[1]=0;}
5  };
6  vector<splay_tree> node;
7  //有的時候用vector會TLE, 要注意
8  //這邊以node[0]作為null節點
9  bool isroot(int x){ //判斷是否為這棵splay
10     tree的根
11     return node[node[x].pa].ch[0]!=x&&node[
12         node[x].pa].ch[1]!=x;
13 }
14 void down(int x){ //懶惰標記下推
15     if(node[x].rev){
16         if(node[x].ch[0])node[node[x].ch[0]].rev
17             ^=1;
18         if(node[x].ch[1])node[node[x].ch[1]].rev
19             ^=1;
20         std::swap(node[x].ch[0],node[x].ch[1]);
21         node[x].rev^=1;
22     }
23 }
24 void push_down(int x){ //將所有祖先的懶惰標記
25     下推
26     if(!isroot(x))push_down(node[x].pa);
27     down(x);
28 }
29 void up(int x){ //將子節點的資訊向上更新
30 }
31 void rotate(int x){ //旋轉, 會自行判斷轉的方
32     向
33     int y=node[x].pa,z=node[y].pa,d=(node[y].
34         ch[1]==x);
35     node[x].pa=z;
36     if(!isroot(y))node[z].ch[node[z].ch[1]==y
37         ]=x;
38     node[y].ch[d]=node[x].ch[d^1];
39     node[node[y].ch[d]].pa=y;
40     node[y].pa=x,node[x].ch[d^1]=y;
41     up(y),up(x);
42 }
43 void splay(int x){ //將節點x伸展到所在splay
44     tree的根
45     push_down(x);
46     while(!isroot(x)){

```

```

37 int y=node[x].pa;
38 if(!isroot(y)){
39     int z=node[y].pa;
40     if((node[z].ch[0]==y)^(node[y].ch[0]==
41         x))rotate(y);
42     else rotate(x);
43 }
44 rotate(x);
45 }
46 int access(int x){
47     int last=0;
48     while(x){
49         splay(x);
50         node[x].ch[1]=last;
51         up(x);
52         last=x;
53         x=node[x].pa;
54     }
55     return last; // 傳回access後splay tree的根
56 }
57 void access(int x, bool is=0){ // is=0就是一般
    的access
58     int last=0;
59     while(x){
60         splay(x);
61         if(is&&!node[x].pa){
62             //printf("%d\n", max(node[last].ma, node
63                 [node[x].ch[1]].ma));
64         }
65         node[x].ch[1]=last;
66         up(x);
67         last=x;
68         x=node[x].pa;
69     }
70 void query_edge(int u, int v){
71     access(u);
72     access(v, 1);
73 }
74 void make_root(int x){
75     access(x), splay(x);
76     node[x].rev^=1;
77 }
78 void make_root(int x){
79     node[access(x)].rev^=1;
80     splay(x);
81 }
82 void cut(int x, int y){
83     make_root(x);
84     access(y);
85     splay(y);
86     node[y].ch[0]=0;
87     node[x].pa=0;
88 }
89 void cut_parents(int x){
90     access(x);
91     splay(x);
92     node[node[x].ch[0]].pa=0;
93     node[x].ch[0]=0;
94 }
95 void link(int x, int y){
96     make_root(x);
97     node[x].pa=y;
98 }
99 int find_root(int x){
100     x=access(x);
101     while(node[x].ch[0])x=node[x].ch[0];
102     splay(x);
103     return x;
104 }
105 int query(int u, int v){
106     // 傳回uv路徑splay tree的根結點
107     // 這種寫法無法求LCA
108     make_root(u);
109     return access(v);
110 }
111 int query_lca(int u, int v){
112     // 假設求鏈上點權的總和，sum是子樹的權重和，
113     // data是節點的權重
114     access(u);
115     int lca=access(v);
116     splay(u);
117     if(u==lca){
118         // return node[lca].data+node[node[lca].
119             ch[1]].sum
120     }
121     else{
122         // return node[lca].data+node[node[lca].
123             ch[1]].sum+node[u].sum
124     }
125 }
126 struct EDGE{
127     int a, b, w;
128 }e[10005];
129 int n;
130 vector<pair<int, int>> >G[10005];
131 // first表示子節點，second表示邊的編號
132 int pa[10005], edge_node[10005];
133 // pa是父母節點，暫存用的，edge_node是每個編
134 // 被存在哪個點裡面的陣列
135 void bfs(int root){
136     // 在建構的時候把每個點都設成一個splay tree，
137     // 不會壞掉
138     queue<int> q;
139     for(int i=1; i<=n; ++i)pa[i]=0;
140     q.push(root);
141     while(q.size()){
142         int u=q.front();
143         q.pop();
144         for(int i=0; i<(int)G[u].size(); ++i){
145             int v=G[u][i].first;
146             if(v!=pa[u]){
147                 pa[v]=u;
148                 node[v].pa=u;
149                 node[v].data=e[G[u][i].second].w;
150                 edge_node[G[u][i].second]=v;
151                 up(v);
152                 q.push(v);
153             }
154         }
155     }
156 }
157 void change(int x, int b){
158     splay(x);
159     // node[x].data=b;
160     up(x);
161 }

```

12.4 POJ_tree.cpp

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 #define MAXN 10005
4 int n, k;
5 vector<pair<int, int>> >g[MAXN];
6 int size[MAXN];
7 bool vis[MAXN];
8 inline void init(){
9     for(int i=0; i<=n; ++i){
10         g[i].clear();
11         vis[i]=0;
12     }
13 }
14 void get_dis(vector<int> &dis, int u, int pa,
    int d){
15     dis.push_back(d);
16     for(size_t i=0; i<g[u].size(); ++i){
17         int v=g[u][i].first, w=g[u][i].second;
18         if(v!=pa&&!vis[v])get_dis(dis, v, u, d+w);
19     }
20 }
21 vector<int> dis; // 這東西如果放在函數裡會TLE
22 int cal(int u, int d){
23     dis.clear();
24     get_dis(dis, u, -1, d);
25     sort(dis.begin(), dis.end());
26     int l=0, r=dis.size()-1, res=0;
27     while(l<r){
28         while(l<r&&dis[l]+dis[r]>k)--r;
29         res+=r-(l+1);
30     }
31     return res;
32 }
33 pair<int, int> tree_centroid(int u, int pa,
    const int sz){
34     size[u]=1; // 找樹重心，second是重心
35     pair<int, int> res(INT_MAX, -1);
36     int ma=0;
37     for(size_t i=0; i<g[u].size(); ++i){
38         int v=g[u][i].first;
39         if(v==pa||vis[v])continue;
40         res=min(res, tree_centroid(v, u, sz));
41         size[u]+=size[v];
42         ma=max(ma, size[v]);
43     }
44     ma=max(ma, sz-size[u]);
45     return min(res, make_pair(ma, u));
46 }
47 int tree_DC(int u, int sz){
48     int center=tree_centroid(u, -1, sz).second;
49     int ans=cal(center, 0);
50     vis[center]=1;
51     for(size_t i=0; i<g[center].size(); ++i){
52         int v=g[center][i].first, w=g[center][i].
53             second;
54         if(vis[v])continue;
55         ans+=cal(v, w);
56         ans+=tree_DC(v, size[v]);
57     }
58     return ans;
59 }
60 int main(){
61     while(scanf("%d%d", &n, &k), n||k){

```

```

61     init();
62     for(int i=1; i<=n; ++i){
63         int u, v, w;
64         scanf("%d%d%d", &u, &v, &w);
65         g[u].push_back(make_pair(v, w));
66         g[v].push_back(make_pair(u, w));
67     }
68     printf("%d\n", tree_DC(1, n));
69 }
70 return 0;
71 }

```

13 zformula

13.1 formula.tex

13.1.1 Pick 公式

給定頂點坐標均是整點的簡單多邊形，面積 = 內部格點數 + 邊上格點數/2-1

13.1.2 圖論

- $V - E + F = 2$
- 對於平面圖， $F = E - V + n + 1$ ， n 是連通分量
- 對於平面圖， $E < 3V - 6$
- 對於連通圖 G ，最大獨立點集的大小設為 $I(G)$ ，最大匹配大小設為 $M(G)$ ，最小點覆蓋設為 $C_v(G)$ ，最小邊覆蓋設為 $C_e(G)$ 。對於任意連通圖：

$$(a) \quad I(G) + C_v(G) = |V|$$

$$(b) \quad M(G) + C_e(G) = |V|$$

- 對於連通二分圖：

$$(a) \quad I(G) = C_v(G)$$

$$(b) \quad M(G) = C_e(G)$$

- 最大權閉合圖：

$$(a) \quad C(u, V) = \infty, (u, v) \in E$$

$$(b) \quad C(S, v) = W_v, W_v > 0$$

$$(c) \quad C(v, T) = -W_v, W_v < 0$$

- 最大密度子圖：

$$(a) \quad C(u, v) = 1, (u, v) \in E$$

$$(b) \quad C(S, v) = U_v, v \in V$$

$$(c) \quad C(v, T) = U + 2g - d_v, v \in V$$

- 弦圖：

- 完美消除序列從後往前依次給每個點染色，給每個點染上可以染的最小顏色
- 最大團大小 = 色數
- 最大獨立集：完美消除序列從前往後能選就選
- 最小團覆蓋：最大獨立集的點和他延伸的邊構成
- 區間圖是弦圖
- 區間圖的完美消除序列：將區間按造又端點由小到大排序
- 區間圖染色：用線段樹做

```

1 double l=0,=m,stop=1.0/n/n;
2 while(r-l>=stop){
3     double(mid);
4     if((n*m-sol.maxFlow(s,t))/2>eps)l=mid;
5     else r=mid;
6 }
7 build(1);
8 sol.maxFlow(s,t);
9 vector<int> ans;
10 for(int i=1;i<=n;++i)
11     if(sol.vis[i])ans.push_back(i);

```

13.1.3 學長公式

- $\sum_{d|n} \phi(n) = n$
- $g(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} \mu(d) \times g(n/d)$
- Harmonic series $H_n = \ln(n) + \gamma + 1/(2n) - 1/(12n^2) + 1/(120n^4)$
- $\gamma = 0.57721566490153286060651209008240243104215$
- 格雷碼 $= n \oplus (n >> 1)$
- $SG(A+B) = SG(A) \oplus SG(B)$
- 選轉矩陣 $M(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

13.1.4 基本數論

- $\sum_{d|n} \mu(n) = [n == 1]$
- $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) \times g(m/d)$
- $\sum_{i=1}^n \sum_{j=1}^m$ 互質數量 $= \sum \mu(d) \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$
- $\sum_{i=1}^n \sum_{j=1}^m lcm(i, j) = n \sum_{d|n} d \times \phi(d)$

13.1.5 排組公式

- k 卡特蘭 $\frac{C_k^{2n}}{n(k-1)+1} \cdot C_m^n = \frac{n!}{m!(n-m)!}$
- $H(n, m) \cong x_1 + x_2 \dots + x_n = k, num = C_k^{n+k-1}$
- Stirling number of 2^{nd} , n 人分 k 組方法數目
 - $S(0, 0) = S(n, n) = 1$
 - $S(n, 0) = 0$
 - $S(n, k) = kS(n-1, k) + S(n-1, k-1)$
- Bell number, n 人分任意多組方法數目
 - $B_0 = 1$
 - $B_n = \sum_{i=0}^n S(n, i)$
 - $B_{n+1} = \sum_{k=0}^n C_k^n B_k$
 - $B_{p+n} \equiv B_n + B_{n+1} \pmod{p}$, p is prime
 - $B_{p^m+n} \equiv mB_n + B_{n+1} \pmod{p}$, p is prime
 - From $B_0 : 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975$
- Derangement, 錯排, 沒有人在自己位置上
 - $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \dots + (-1)^n \frac{1}{n!})$
 - $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 = 1, D_1 = 0$

(c) From $D_0 : 1, 0, 1, 2, 9, 44, 265, 1854, 14833, 133496$

6. Binomial Equality

- $\sum_k \binom{r}{m+k} \binom{s}{n-k} = \binom{r+s}{m+n}$
- $\sum_k \binom{l}{m+k} \binom{s}{n-k} = \binom{l+s}{l-m+n}$
- $\sum_k \binom{l}{m+k} \binom{s+k}{n} (-1)^k = \frac{(-1)^{l+m} \binom{s-m}{n-l}}{(-1)^{l+m} \binom{s-m}{l-n-m}}$
- $\sum_{k \leq l} \binom{l-k}{m} \binom{s-n}{l-n-m} (-1)^k = \frac{(-1)^{l+m} \binom{s-m-1}{l-n-m-1}}{(-1)^{l+m} \binom{s-m-1}{l-n-m-1}}$
- $\sum_{0 \leq k \leq l} \binom{l-k}{m} \binom{q+k}{n} = \binom{l+q+1}{m+n+1}$
- $\binom{r}{k} = (-1)^k \binom{k-r-1}{k}$
- $\binom{r}{m} \binom{m}{k} = \binom{r}{m-k} \binom{r-k}{m-k}$
- $\sum_{k \leq n} \binom{r+k}{k} = \binom{r+n+1}{n}$
- $\sum_{0 \leq k \leq n} \binom{m}{k} = \binom{n+1}{m+1}$
- $\sum_{k \leq m} \binom{m+r}{k} x^k y^{m-k} = \sum_{k \leq m} \binom{r}{k} (-x)^k (x+y)^{m-k}$

13.1.6 幕次, 幕次和

- $a^b \% P = a^{b \% \varphi(P) + \varphi(P)} \cdot b \geq \varphi(P)$
- $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
- $1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$
- $1^5 + 2^5 + 3^5 + \dots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12}$
- $0^k + 1^k + 2^k + \dots + n^k = P(k), P(k) = \frac{(n+1)^{k+1} - \sum_{i=0}^{k-1} C_{i+1}^{k+1} P(i)}{k+1}, P(0) = n+1$
- $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^n C_k^{n+1} B_k m^{n+1-k}$
- $\sum_{j=0}^m C_j^{m+1} B_j = 0, B_0 = 1$
- 除了 $B_1 = -1/2$ · 剩下的奇數項都是 0
- $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 = -1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 7/6, B_{16} = -3617/510, B_{18} = 43867/798, B_{20} = -174611/330,$

13.1.7 Burnside's lemma

- $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- $X^g = t^{c(g)}$
- G 表示有幾種轉法 · X^g 表示在那種轉法下 · 有幾種是會保持對稱的 · t 是顏色數 · $c(g)$ 是循環環不動的面數。
- 正立方體塗三顏色 · 轉 0 有 3^6 個元素不變 · 轉 90 有 6 種 · 每種有 3^3 不變 · 180 有 3×3^4 · 120(角) 有 8×3^2 · 180(邊) 有 6×3^3 · 全部 $\frac{1}{24} (3^6 + 6 \times 3^3 + 3 \times 3^4 + 8 \times 3^2 + 6 \times 3^3) = \frac{24}{57}$

13.1.8 Count on a tree

- Rooted tree: $s_{n+1} = \frac{1}{n} \sum_{i=1}^n (i \times a_i \times \sum_{j=1}^{\lfloor n/i \rfloor} a_{n+1-i \times j})$
- Unrooted tree:
 - Odd: $a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$
 - Even: $Odd + \frac{1}{2} a_{n/2} (a_{n/2} + 1)$
- Spanning Tree

- 完全圖 $n^n - 2$
- 一般圖 (Kirchhoff's theorem) $M[i][i] = \text{degree}(V_i), M[i][j] = -1, \text{if have } E(i, j), 0 \text{ if no edge. delete any one row and col in } A, \text{ans} = \det(A)$

13.2 java.tex

13.2.1 文件操作

```

1 import java.io.*;
2 import java.util.*;
3 import java.math.*;
4 import java.text.*;
5
6 public class Main
7 {
8
9     public static void main(String args[])
10         throws FileNotFoundException,
11         IOException
12     {
13         Scanner sc = new Scanner(new FileReader(
14             "a.in"));
15         PrintWriter pw = new PrintWriter(new
16             FileWriter("a.out"));
17         int n,m;
18         n=sc.nextInt();//读入下一个INT
19         m=sc.nextInt();
20
21         for(ci=1; ci<=c; ++ci)
22         {
23             pw.println("Case #"+ci+": easy for
24                 output");
25         }
26
27         pw.close();//关闭流并释放 · 这个很重要 ·
28             否则是没有输出的
29         sc.close();//关闭流并释放
30     }
31 }

```

13.2.2 优先队列

```

1 PriorityQueue queue = new PriorityQueue( 1,
2     new Comparator()
3 {
4     public int compare( Point a, Point b )

```

```

4 {
5     if( a.x < b.x || a.x == b.x && a.y < b.y )
6         return -1;
7     else if( a.x == b.x && a.y == b.y )
8         return 0;
9     else
10         return 1;
11 }
12 });

```

13.2.3 Map

```

1 Map map = new HashMap();
2 map.put("sa", "dd");
3 String str = map.get("sa").toString();
4
5 for(Object obj : map.keySet()){
6     Object value = map.get(obj);
7 }

```

13.2.4 sort

```

1 static class cmp implements Comparator
2 {
3     public int compare(Object o1, Object o2)
4     {
5         BigInteger b1=(BigInteger)o1;
6         BigInteger b2=(BigInteger)o2;
7         return b1.compareTo(b2);
8     }
9 }
10 public static void main(String[] args)
11     throws IOException
12 {
13     Scanner cin = new Scanner(System.in);
14     int n;
15     n=cin.nextInt();
16     BigInteger[] seg = new BigInteger[n];
17     for (int i=0; i<n; i++)
18         seg[i]=cin.nextBigInteger();
19     Arrays.sort(seg, new cmp());
20 }

```

ACM ICPC TEAM REFERENCE - NTHU JINKELA

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