

1 Computational_Geometry

1.1 Geometry.cpp

```

1 const double PI=atan2(0.0,-1.0);
2 template<typename T>
3 struct point{
4     T x,y;
5     point(){ }
6     point(const T&x,const T&y):x(x),y(y){ }
7     point operator+(const point &b)const{
8         return point(x+b.x,y+b.y); }
9     point operator-(const point &b)const{
10        return point(x-b.x,y-b.y); }
11     point operator*(const T &b)const{
12        return point(x*b,y*b); }
13     point operator/(const T &b)const{
14        return point(x/b,y/b); }
15     bool operator==(const point &b)const{
16        return x==b.x&&y==b.y; }
17     T dot(const point &b)const{
18        return x*b.x+y*b.y; }
19     T cross(const point &b)const{
20        return x*b.y-y*b.x; }
21     point normal()const{ //求法向量
22        return point(-y,x); }
23     T abs2()const{ //向量長度的平方
24        return dot(*this); }
25     T rad(const point &b)const{ //兩向量的弧度
26     return fabs(atan2(fabs(cross(b)),dot(b))); }
27     T getA()const{ //對x軸的弧度
28     T A=atan2(y,x); //超過180度會變負的
29     if(A<=-PI/2)A+=PI*2;
30     return A;
31 }
32 };
33 template<typename T>
34 struct line{
35     line(){ }
36     point<T> p1,p2;
37     T a,b,c; //ax+by+c=0
38     line(const point<T>&x,const point<T>&y):p1(x),p2(y){ }
39     void pton()const{ //轉成一般式
40         a=p1.y-p2.y;
41         b=p2.x-p1.x;
42         c=-a*p1.x-b*p1.y;
43     }
44     T ori(const point<T> &p)const{ //點和有向直
45         //線的關係 · >0左邊 · =0在線上<0右邊
46         return (p2-p1).cross(p-p1);
47     }
48     T btw(const point<T> &p)const{ //點投影落在
49         //線段上<=0
50         return (p1-p).dot(p2-p);
51     }
52     bool point_on_segment(const point<T>&p)
53         const{ //點是否在線段上
54         return ori(p)==0&&btw(p)<=0;
55     }
56     T dis2(const point<T> &p,bool is_segment
57         =0)const{ //點跟直線/線段的距離平方
58         point<T> v=p2-p1,v1=p-p1;
59         if(is_segment){
60             point<T> v2=p-p2;
61             if(v.dot(v1)<=0)return v1.abs2();
62             if(v.dot(v2)>=0)return v2.abs2();
63         }
64         T tmp=v.cross(v1);
65         return tmp*tmp/v.abs2();
66     }
67     T seg_dis2(const line<T> &l)const{ //兩線段
68         //距離平方
69         return min({dis2(l.p1,1),dis2(l.p2,1),l.
70             dis2(p1,1),l.dis2(p2,1)});
71     }
72     point<T> projection(const point<T> &p)
73         const{ //點對直線的投影
74         point<T> n=(p2-p1).normal();
75         return p-n*(p-p1).dot(n)/n.abs2();
76     }
77     point<T> mirror(const point<T> &p)const{
78         //點對直線的鏡射 · 要先呼叫pton轉成一般式
79         point<T> R;
80         T d=a*b+b*b;
81         R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
82         R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
83         return R;
84     }
85     bool equal(const line &l)const{ //直線相等
86         return ori(l.p1)==0&&ori(l.p2)==0;
87     }
88     bool parallel(const line &l)const{
89         return (p1-p2).cross(l.p1-l.p2)==0;
90     }
91     bool cross_seg(const line &l)const{
92         return (p2-p1).cross(l.p1-p1)*(p2-p1).
93             cross(l.p2-p1)<=0; //直線是否交線段
94     }
95     int line_intersect(const line &l)const{ //
96         //直線相交情況 · -1無限多點 · 1交於一點 · 0
97         //不相交
98         return parallel(l)?(ori(l.p1)==0?-1:0)
99             :1;
100     }
101     int seg_intersect(const line &l)const{
102         T c1=ori(l.p1), c2=ori(l.p2);
103         T c3=l.ori(p1), c4=l.ori(p2);
104         if(c1==0&&c2==0){ //共線
105             bool b1=btw(l.p1)>=0,b2=btw(l.p2)>=0;
106             T a3=l.btw(p1),a4=l.btw(p2);
107             if(b1&&b2&&a3==0&&a4==0) return 2;
108             if(b1&&b2&&a3>=0&&a4==0) return 3;
109             if(b1&&b2&&a3>=0&&a4>=0) return 0;
110             return -1; //無限交點
111         }else if(c1*c2<=0&&c3*c4<=0) return 1;
112         return 0; //不相交
113     }
114     point<T> line_intersection(const line &l)
115         const{ //直線交點
116         point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
117         //if(a.cross(b)==0) return INF;
118         return p1+a*(s.cross(b)/a.cross(b));
119     }
120     point<T> seg_intersection(const line &l)
121         const{ //線段交點
122         int res=seg_intersect(l);
123         if(res<=0) assert(0);
124         if(res==2) return p1;
125         if(res==3) return p2;
126         return line_intersection(l);
127     }
128     template<typename T>
129     struct polygon{
130         polygon(){ }
131         vector<point<T> > p; //逆時針順序
132         T area()const{ //面積
133             T ans=0;
134             for(int i=p.size()-1,j=0;j<(int)p.size()
135                 ;i=j++)
136                 ans+=p[i].cross(p[j]);
137             return ans/2;
138         }
139         point<T> center_of_mass()const{ //重心
140             T cx=0,cy=0,w=0;
141             for(int i=p.size()-1,j=0;j<(int)p.size()
142                 ;i=j++){
143                 T a=p[i].cross(p[j]);
144                 cx+=(p[i].x+p[j].x)*a;
145                 cy+=(p[i].y+p[j].y)*a;
146                 w+=a;
147             }
148             return point<T>(cx/3/w,cy/3/w);
149         }
150         char ahas(const point<T>&t)const{ //點是否
151             //在簡單多邊形內 · 是的話回傳1 · 在邊上回
152             //傳-1 · 否則回傳0
153         bool c=0;
154         for(int i=0,j=p.size()-1;i<p.size();j=i
155             ++){
156             if(line<T>(p[i],p[j]).point_on_segment
157                 (t))return -1;
158             else if((p[i].y>t.y)!(p[j].y>t.y)&&
159                 t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j]
160                     .y-p[i].y)+p[i].x)
161                 c=!c;
162             return c;
163         }
164         char point_in_convex(const point<T>&x)
165             const{
166             int l=1,r=(int)p.size()-2;
167             while(l<r){ //點是否在凸多邊形內 · 是的話
168                 //回傳1 · 在邊上回傳-1 · 否則回傳0
169                 int mid=(l+r)/2;
170                 T a1=(p[mid]-p[0]).cross(x-p[0]);
171                 T a2=(p[mid+1]-p[0]).cross(x-p[0]);
172                 if(a1>0&&a2<=0){
173                     T res=(p[mid+1]-p[mid]).cross(x-p[
174                         mid]);
175                     return res>0?1:(res==0?-1:0);
176                 }else if(a1<0)r=mid-1;
177                 else l=mid+1;
178             }
179             return 0;
180         }
181         vector<T> getA()const{ //凸包邊對x軸的夾角
182             vector<T>res; //一定是遞增的
183             for(size_t i=0;i<p.size();++i)
184                 res.push_back((p[(i+1)%p.size()]-p[i])
185                     .getA());
186             return res;
187         }
188         bool line_intersect(const vector<T>&A,
189             const line<T> &l)const{ //O(LogN)
190             int f1=upper_bound(A.begin(),A.end(),(l.
191                 p1-l.p2).getA())-A.begin();
192             int f2=upper_bound(A.begin(),A.end(),(l.
193                 p2-l.p1).getA())-A.begin();
194             return l.cross_seg(line<T>(p[f1],p[f2]))
195                 ;
196         }
197         polygon cut(const line<T> &l)const{ //凸包
198             //對直線切割 · 得到直線L左側的凸包
199             polygon ans;
200             for(int n=p.size(),i=n-1,j=0;j<n;i=j++){
201                 if(l.ori(p[i])>=0){
202                     ans.p.push_back(p[i]);
203                     if(l.ori(p[j])<0)
204                         ans.p.push_back(l.
205                             line_intersection(line<T>(p[i]
206                                 ,p[j])));
207                 }else if(l.ori(p[j])>0)
208                     ans.p.push_back(l.
209                         line_intersection(line<T>(p[i],p[j]
210                             )));
211             }
212             return ans;
213         }
214         static bool graham_cmp(const point<T>&a,
215             const point<T>&b){ //凸包排序函數
216             return (a.x<b.x)|| (a.x==b.x&&a.y<b.y);
217         }
218         void graham(vector<point<T> > &s){ //凸包
219             sort(s.begin(),s.end(),graham_cmp);
220             p.resize(s.size()+1);
221             int m=0;
222             for(size_t i=0;i<s.size();++i){
223                 while(m>=2&&(p[m-1]-p[m-2]).cross(s[i]
224                     -p[m-2])<=0)--m;
225                 p[m++]=s[i];
226             }
227             for(int i=s.size()-2,t=m+1;i>=0;--i){
228                 while(m>=t&&(p[m-1]-p[m-2]).cross(s[i]
229                     -p[m-2])<=0)--m;
230                 p[m++]=s[i];
231             }
232             if(s.size()>1)--m;
233             p.resize(m);
234         }
235         T diam()const{ //直徑
236             int n=p.size(),t=1;
237             T ans=0;
238             for(int i=0;i<n;i++){
239                 while((p[i]-p[t+1]).abs2()>(p[i]-p[t])
240                     .abs2())t=(t+1)%n;
241                 ans=max(ans,(p[i]-p[t]).abs2());
242             }
243             return p.pop_back(),ans;
244         }
245         T min_cover_rectangle()const{ //最小覆蓋矩形
246             int n=p.size(),t=1,r=1,l=1;
247             if(n<3)return 0; //也可以做最小周長矩形
248         }

```

```

212 T ans=1e99;p.push_back(p[0]);
213 for(int i=0;i<n;i++){
214     point<T> now=p[i+1]-p[i];
215     while(now.cross(p[t+1]-p[i])>now.cross
216           (p[t]-p[i]))t=(t+1)%n;
217     while(now.dot(p[r+1]-p[i])>now.dot(p[r
218           ]-p[i]))r=(r+1)%n;
219     if(!i)l=r;
220     while(now.dot(p[l+1]-p[i])<=now.dot(p[
221           l]-p[i]))l=(l+1)%n;
222     T d=now.abs2();
223     T tmp=now.cross(p[t]-p[i])*(now.dot(p[
224           r]-p[i])-now.dot(p[l]-p[i]))/d;
225     ans=min(ans,tmp);
226 }
227 return p.pop_back(),ans;
228
229 T max_triangle(){//最大內接三角形
230     int n=p.size(),a=1,b=2;
231     if(n<3)return 0;
232     T ans=0,tmp;p.push_back(p[0]);
233     for(int i=0;i<n;i++){
234         while((p[a]-p[i]).cross(p[b+1]-p[i])>(
235             tmp=(p[a]-p[i]).cross(p[b]-p[i]))
236             b=(b+1)%n;
237         ans=max(ans,tmp);
238         while((p[a+1]-p[i]).cross(p[b]-p[i])>(
239             tmp=(p[a]-p[i]).cross(p[b]-p[i]))
240             a=(a+1)%n;
241         ans=max(ans,tmp);
242     }
243     return p.pop_back(),ans/2;
244 }
245
246 T dis2(polygon &p1){//凸包最近距離平方
247     vector<point<T>> > &P=p,&Q=p1.p;
248     int n=P.size(),m=Q.size(),l=0,r=0;
249     for(int i=0;i<n;i++){if(P[i].y<P[l].y)l=i;
250     for(int i=0;i<m;i++){if(Q[i].y<Q[r].y)r=i;
251     P.push_back(P[0]);Q.push_back(Q[0]);
252     T ans=1e99;
253     for(int i=0;i<n;i++){
254         while((P[l]-P[l+1]).cross(Q[r+1]-Q[r])
255             <0)r=(r+1)%m;
256         ans=min(ans,line<T>(P[l],P[l+1]).
257             seg_dis2(line<T>(Q[r],Q[r+1])));
258         l=(l+1)%n;
259     }
260     return P.pop_back(),Q.pop_back(),ans;
261 }
262
263 static char sign(const point<T>&t){
264     return (t.y==0?t.x:t.y)<0;
265 }
266
267 static bool angle_cmp(const line<T>& A,
268     const line<T>& B){
269     point<T> a=A.p2-A.p1,b=B.p2-B.p1;
270     return sign(a)<sign(b)||((sign(a)==sign(b)
271         )&&a.cross(b)>0);
272 }
273
274 int halfplane_intersection(vector<line<T>
275     > &s){//半平面交
276     sort(s.begin(),s.end(),angle_cmp);//線段
277     //左側為該線段半平面
278     int L,R,n=s.size();
279     vector<point<T>> > px(n);
280     vector<line<T> > q(n);
281
282     q[L=R=0]=s[0];
283     for(int i=1;i<n;i++){
284         while(L<R&&s[i].ori(px[R-1])<=0)--R;
285         while(L<R&&s[i].ori(px[L])<=0)++L;
286         q[++R]=s[i];
287         if(q[R].parallel(q[R-1])){
288             --R;
289             if(q[R].ori(s[i].p1)>0)q[R]=s[i];
290         }
291         if(L<R)px[R-1]=q[R-1].
292             line_intersection(q[R]);
293     }
294     while(L<R&&q[L].ori(px[R-1])<=0)--R;
295     p.clear();
296     if(R-L==1)return 0;
297     px[R]=q[R].line_intersection(q[L]);
298     for(int i=L;i<=R;i++)p.push_back(px[i]);
299     return R-L+1;
300 }
301
302 template<typename T>
303 struct triangle{
304     point<T> a,b,c;
305     triangle(){
306         triangle(const point<T> &a,const point<T>
307             &b,const point<T> &c):a(a),b(b),c(c){
308     T area()const{
309         T t=(b-a).cross(c-a)/2;
310         return t>0?t:-t;
311     }
312     point<T> barycenter()const{//重心
313         return (a+b+c)/3;
314     }
315     point<T> circumcenter()const{//外心
316         static line<T> u,v;
317         u.p1=(a+b)/2;
318         u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-
319             b.x);
320         v.p1=(a+c)/2;
321         v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-
322             c.x);
323         return u.line_intersection(v);
324     }
325     point<T> incenter()const{//內心
326         T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2
327             ()),C=sqrt((a-b).abs2());
328         return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+
329             B*b.y+C*c.y)/(A+B+C);
330     }
331     point<T> perpencenter()const{//垂心
332         return barycenter()*3-circumcenter()*2;
333     }
334 };
335
336 template<typename T>
337 struct point3D{
338     T x,y,z;
339     point3D(){
340     point3D(const T&x,const T&y,const T&z):x(x
341         ),y(y),z(z){
342     point3D operator+(const point3D &b)const{
343         return point3D(x+b.x,y+b.y,z+b.z);
344     }
345     point3D operator-(const point3D &b)const{
346         return point3D(x-b.x,y-b.y,z-b.z);
347     }
348     point3D operator*(const T &b)const{
349         return point3D(x*b,y*b,z*b);
350     }
351     point3D operator/(const T &b)const{
352         return point3D(x/b,y/b,z/b);
353     }
354     bool operator==(const point3D &b)const{
355         return x==b.x&&y==b.y&&z==b.z;
356     }
357     T dot(const point3D &b)const{
358         return x*b.x+y*b.y+z*b.z;
359     }
360     point3D cross(const point3D &b)const{
361         return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x
362             *b.y-y*b.x);
363     }
364     T abs2()const{//向量長度的平方
365         return dot(*this);
366     }
367     T area2(const point3D &b)const{//和b、原點
368         //圍成面積的平方
369         return cross(b).abs2()/4;
370     };
371     template<typename T>
372     struct line3D{
373         point3D<T> p1,p2;
374         line3D(){
375         line3D(const point3D<T> &p1,const point3D<
376             T> &p2):p1(p1),p2(p2){
377     T dis2(const point3D<T> &p,bool is_segment
378         =0)const{//點跟直線/線段的距離平方
379         point3D<T> v=p2-p1,v1=p-p1;
380         if(is_segment){
381             point3D<T> v2=p-p2;
382             if(v.dot(v1)<=0)return v1.abs2();
383             if(v.dot(v2)>=0)return v2.abs2();
384         }
385         point3D<T> tmp=v.cross(v1);
386         return tmp.abs2()/v.abs2();
387     }
388     pair<point3D<T>,point3D<T>> > closest_pair(
389         const line3D<T> &l)const{
390         point3D<T> v1=(p1-p2),v2=(l.p1-l.p2);
391         point3D<T> N=v1.cross(v2),ab(p1-l.p1);
392         //if(N.abs2()==0)return NULL;平行或重合
393         T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//
394         //最近點距離
395         point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.
396             cross(d2),G=l.p1-p1;
397         T t1=(G.cross(d2)).dot(D)/D.abs2();
398         T t2=(G.cross(d1)).dot(D)/D.abs2();
399         return make_pair(p1+d1*t1,l.p1+d2*t2);
400     }
401     bool same_side(const point3D<T> &a,const
402         point3D<T> &b)const{
403         return (p2-p1).cross(a-p1).dot((p2-p1).
404             cross(b-p1))>0;
405     }
406 };
407
408 template<typename T>
409 struct plane{
410     point3D<T> p0,n;//平面上的點和法向量
411     plane(){
412     plane(const point3D<T> &p0,const point3D<T>
413         &n):p0(p0),n(n){
414     T dis2(const point3D<T> &p)const{//點到平
415         //面距離的平方
416         T tmp=(p-p0).dot(n);
417         return tmp*tmp/n.abs2();
418     }
419     point3D<T> projection(const point3D<T> &p)
420         const{
421         return p-n*(p-p0).dot(n)/n.abs2();
422     }
423     point3D<T> line_intersection(const line3D<
424         T> &l)const{
425         T tmp=n.dot(l.p2-l.p1);//等於0表示平行或
426         //重合該平面
427         return l.p1+(l.p2-l.p1)*(n.dot(p0-l.p1)/
428             tmp);
429     }
430     line3D<T> plane_intersection(const plane &
431         pl)const{
432         point3D<T> e=n.cross(pl.n),v=n.cross(e);
433         T tmp=pl.n.dot(v);//等於0表示平行或重合
434         //該平面
435         point3D<T> q=p0+(v*(pl.n.dot(pl.p0-p0))/
436             tmp);
437         return line3D<T>(q,q+e);
438     }
439 };
440
441 template<typename T>
442 struct triangle3D{
443     point3D<T> a,b,c;
444     triangle3D(){
445     triangle3D(const point3D<T> &a,const
446         point3D<T> &b,const point3D<T> &c):a(a
447         ),b(b),c(c){
448     bool point_in(const point3D<T> &p)const{//
449         //點在該平面上的投影在三角形中
450         return line3D<T>(b,c).same_side(p,a)&&
451             line3D<T>(a,c).same_side(p,b)&&
452             line3D<T>(a,b).same_side(p,c);
453     }
454 };
455
456 template<typename T>
457 struct tetrahedron{//四面體
458     point3D<T> a,b,c,d;
459     tetrahedron(){
460     tetrahedron(const point3D<T> &a,const
461         point3D<T> &b,const point3D<T> &c,
462         const point3D<T> &d):a(a),b(b),c(c),d(
463             d){
464     T volume6()const{//體積的六倍
465         return (d-a).dot((b-a).cross(c-a));
466     }
467     point3D<T> centroid()const{
468         return (a+b+c+d)/4;
469     }
470     bool point_in(const point3D<T> &p)const{
471         return triangle3D<T>(a,b,c).point_in(p)
472             &&triangle3D<T>(c,d,a).point_in(p);
473     }
474 };
475
476 template<typename T>
477 struct convexhull3D{
478     static const int MAXN=1005;
479     struct face{
480         int a,b,c;
481         face(int a,int b,int c):a(a),b(b),c(c){
482     };
483     vector<point3D<T>> pt;
484     vector<face> ans;
485     int fid[MAXN][MAXN];
486     void build(){
487         int n=pt.size();
488     }
489 };

```

```

422 ans.clear();
423 memset(fid,0,sizeof(fid));
424 ans.emplace_back(0,1,2); //注意不能共線
425 ans.emplace_back(2,1,0);
426 int ftop = 0;
427 for(int i=3, ftop=1; i<n; ++i, ++ftop){
428     vector<face> next;
429     for(auto &f:ans){
430         T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[
431             f.a]).cross(pt[f.c]-pt[f.a]));
432         if(d<=0) next.push_back(f);
433         int ff=0;
434         if(d>0) ff=ftop;
435         else if(d<0) ff=-ftop;
436         fid[f.a][f.b]=fid[f.b][f.c]=fid[f.c
437             ][f.a]=ff;
438     }
439     for(auto &f:ans){
440         if(fid[f.a][f.b]>0 && fid[f.a][f.b
441             ]!=fid[f.b][f.a])
442             next.emplace_back(f.a,f.b,i);
443         if(fid[f.b][f.c]>0 && fid[f.b][f.c
444             ]!=fid[f.c][f.b])
445             next.emplace_back(f.b,f.c,i);
446         if(fid[f.c][f.a]>0 && fid[f.c][f.a
447             ]!=fid[f.a][f.c])
448             next.emplace_back(f.c,f.a,i);
449     }
450     ans=next;
451 }
452 point3D<T> centroid(){const{
453     point3D<T> res(0,0,0);
454     T vol=0;
455     for(auto &f:ans){
456         T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c
457             ]));
458         res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
459         vol+=tmp;
460     }
461     return res/(vol*4);
462 }
463 }
464 }

```

1.2 SmallestCircle.cpp

```

1 #include "Geometry.cpp"
2 struct Circle{
3     typedef point<double> p;
4     typedef const point<double> cp;
5     p x;
6     double r2;
7     bool incircle(cp &c){const{return (x-c).
8         abs2())<=r2;}}
9 };
10 Circle TwoPointCircle(Circle::cp &a, Circle
11     ::cp &b) {
12     Circle::p m=(a+b)/2;
13     return (Circle){m,(a-m).abs2()};
14 }

```

```

15 Circle outcircle(Circle::p a, Circle::p b,
16     Circle::p c) {
17     if(TwoPointCircle(a,b).incircle(c))
18         return TwoPointCircle(a,b);
19     if(TwoPointCircle(b,c).incircle(a))
20         return TwoPointCircle(b,c);
21     if(TwoPointCircle(c,a).incircle(b))
22         return TwoPointCircle(c,a);
23     Circle::p ret;
24     double a1=b.x-a.x, b1=b.y-a.y, c1=(a1*a1
25         +b1*b1)/2;
26     double a2=c.x-a.x, b2=c.y-a.y, c2=(a2*a2
27         +b2*b2)/2;
28     double d = a1*b2 - a2*b1;
29     ret.x=a.x+(c1*b2-c2*b1)/d;
30     ret.y=a.y+(a1*c2-a2*c1)/d;
31     return (Circle){ret,(ret-a).abs2()};
32 }
33 //rand required
34 Circle SmallestCircle(std::vector<Circle::p>
35     &p){
36     int n=p.size();
37     if(n==1) return (Circle){p[0],0.0};
38     if(n==2) return TwoPointCircle(p[0],p
39         [1]);
40     random_shuffle(p.begin(),p.end());
41     Circle c = {p[0],0.0};
42     for(int i=0;i<n;++i){
43         if(c.incircle(p[i])) continue;
44         c=Circle{p[i],0.0};
45         for(int j=0;j<i;++j){
46             if(c.incircle(p[j])) continue;
47             c=TwoPointCircle(p[i],p[j]);
48             for(int k=0;k<j;++k){
49                 if(c.incircle(p[k]))
50                     continue;
51                 c=outcircle(p[i],p[j],p[k]);
52             }
53         }
54     }
55     return c;
56 }

```

1.3 最近點對.cpp

```

1 template<typename _IT=point<T>* >
2 T cloest_pair(_IT L, _IT R){
3     if(R-L <= 1) return INF;
4     _IT mid = L+(R-L)/2;
5     T x = mid->x;
6     T d = min(cloest_pair(L,mid),cloest_pair(
7         mid,R));
8     inplace_merge(L, mid, R, ycmp);
9     static vector<point> b; b.clear();
10     for(auto u=L;u<R;++u){
11         if((u->x-x)*(u->x-x)>=d) continue;
12         for(auto v=b.rbegin();v!=b.rend();++v){
13             T dx=u->x-v->x, dy=u->y-v->y;
14             if(dy*dy>=d) break;
15             d=min(d,dx*dx+dy*dy);
16         }
17     }
18     b.push_back(*u);
19 }

```

```

19 return d;
20 }
21 T closest_pair(vector<point<T>> &v){
22     sort(v.begin(),v.end(),xcmp);
23     return closest_pair(v.begin(),v.end());
24 }

```

2 Data_Structure

2.1 DLX.cpp

```

1 const int MAXN=4100, MAXM=1030, MAXND=16390;
2 struct DLX{
3     int n,m,sz,ansd; //高是n 寬是m的稀疏矩陣
4     int S[MAXN],H[MAXN];
5     int row[MAXN],col[MAXN]; //每個節點代表的
6         列跟行
7     int L[MAXN],R[MAXN],U[MAXN],D[MAXN];
8     vector<int> ans,anst;
9     void init(int _n,int _m){
10         n=_n,m=_m;
11         for(int i=0;i<=m;++i){
12             U[i]=D[i]=i,L[i]=i-1,R[i]=i+1;
13             S[i]=0;
14         }
15         R[m]=0,L[0]=m;
16         sz=m,ansd=INT_MAX; //ansd存最優解的個數
17         for(int i=1;i<=n;++i)H[i]=-1;
18     }
19     void add(int r,int c){
20         ++S[col[++sz]=c];
21         row[sz]=r;
22         D[sz]=D[c],U[D[c]]=sz,U[sz]=c,D[c]=sz;
23         if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
24         else R[sz]=R[H[r]],L[R[H[r]]]=sz,L[sz]=H
25             [r],R[H[r]]=sz;
26     }
27     #define DFOR(i,A,s) for(int i=A[s];i!=s;i=
28         A[i])
29     void remove(int c){ //刪除第c行和所有當前覆
30         蓋到第c行的列
31         L[R[c]]=L[c],R[L[c]]=R[c]; //這裡刪除第c
32         行 若有些行不需要處理可以在開始時呼
33         叫他
34         DFOR(i,D,c)DFOR(j,R,i){U[D[j]]=U[j],D[U
35             [j]]=D[j],--S[col[j]]};
36     }
37     void restore(int c){ //恢復第c行和所有當前
38         覆蓋到第c行的列 remove的逆操作
39         DFOR(i,U,c)DFOR(j,L,i){++S[col[j]],U[D[j
40             ]]=j,D[U[j]]=j;}}
41     void remove2(int nd){ //刪除nd所在的行當前
42         所有點 (包括虛擬節點) 只保留nd
43         DFOR(i,D,nd)L[R[i]]=L[i],R[L[i]]=R[i];
44     }
45     void restore2(int nd){ //刪除nd所在的行當前
46         所有點 為remove2的逆操作

```

```

38     DFOR(i,U,nd)L[R[i]]=R[L[i]]=i;
39 }
40 bool vis[MAXN];
41 int h(){ //估價函數 for IDA*
42     int res=0;
43     memset(vis,0,sizeof(vis));
44     DFOR(i,R,0)if(!vis[i]){
45         vis[i]=1;
46         ++res;
47         DFOR(j,D,i)DFOR(k,R,j)vis[col[k]]=1;
48     }
49     return res;
50 }
51 bool dfs(int d){ //for精確覆蓋問題
52     if(d+h()>=ansd) return 0; //找最佳解用 找
53     任意解可以刪掉
54     if(!R[0]){ansd=d; return 1;}
55     int c=R[0];
56     DFOR(i,R,0)if(S[i]<S[c])c=i;
57     remove(c);
58     DFOR(i,D,c){
59         ans.push_back(row[i]);
60         DFOR(j,R,i)remove(col[j]);
61         if(dfs(d+1)) return 1;
62         ans.pop_back();
63         DFOR(j,L,i)restore(col[j]);
64     }
65     restore(c);
66     return 0;
67 }
68 void dfs2(int d){ //for最小重複覆蓋問題
69     if(d+h()>=ansd) return;
70     if(!R[0]){ansd=d; ans=anst; return;}
71     int c=R[0];
72     DFOR(i,R,0)if(S[i]<S[c])c=i;
73     DFOR(i,D,c){
74         anst.push_back(row[i]);
75         remove2(i);
76         DFOR(j,R,i)remove2(col[j]);
77         dfs2(d+1);
78         anst.pop_back();
79         DFOR(j,L,i)restore2(col[j]);
80         restore2(i);
81     }
82 }
83 bool exact_cover(){ //解精確覆蓋問題
84     return ans.clear(), dfs(0);
85 }
86 void min_cover(){ //解最小重複覆蓋問題
87     anst.clear(); //暫存用 答案還是存在ans裡
88     dfs2(0);
89     #undef DFOR
90 }

```

2.2 Dynamic_KD_tree.cpp

```

1 template<typename T,size_t kd> //有kd個維度
2 struct kd_tree{
3     struct point{
4         T d[kd];

```

```

5 T dist(const point &x) const {
6     T ret=0;
7     for(size_t i=0; i<kd; ++i) ret+=std::abs(
8         d[i]-x.d[i]);
9     return ret;
10 }
11 bool operator==(const point &p) {
12     for(size_t i=0; i<kd; ++i)
13         if(d[i]!=p.d[i]) return 0;
14     return 1;
15 }
16 bool operator<(const point &b) const {
17     return d[0]<b.d[0];
18 }
19 private:
20 struct node {
21     node *l,*r;
22     point pid;
23     int s;
24     node(const point &p):l(0),r(0),pid(p),s
25         (1){}
26     ~node(){delete l; delete r;}
27     void up(){s=(l?l->s:0)+1+(r?r->s:0);}
28 } *root;
29 const double alpha, loga;
30 const T INF; //記得要給INF，表示極大值
31 int maxn;
32 struct __cmp {
33     int sort_id;
34     bool operator()(const node *x, const node *
35         y) const {
36         return operator()(x->pid, y->pid);
37     }
38     bool operator()(const point &x, const
39         point &y) const {
40         if(x.d[sort_id]!=y.d[sort_id])
41             return x.d[sort_id]<y.d[sort_id];
42         for(size_t i=0; i<kd; ++i)
43             if(x.d[i]!=y.d[i]) return x.d[i]<y.d[
44                 i];
45         return 0;
46     }
47 } cmp;
48 int size(node *o){return o?o->s:0;}
49 std::vector<node*> A;
50 node* build(int k, int l, int r){
51     if(l>r) return 0;
52     if(k==kd) k=0;
53     int mid=(l+r)/2;
54     cmp.sort_id = k;
55     std::nth_element(A.begin()+l, A.begin()+
56         mid, A.begin()+r+1, cmp);
57     node *ret=A[mid];
58     ret->l = build(k+1, l, mid-1);
59     ret->r = build(k+1, mid+1, r);
60     ret->up();
61     return ret;
62 }
63 bool isbad(node *o){
64     return size(o->l)>alpha*o->s || size(o->r)
65         >alpha*o->s;
66 }
67 void flatten(node *u, typename std::vector<
68     node*>::iterator &it){
69     if(!u) return;
70     flatten(u->l, it);
71     *it=u;
72     flatten(u->r, ++it);
73 }
74 void rebuild(node *u, int k){
75     if((int)A.size()<u->s) A.resize(u->s);
76     typename std::vector<node*>::iterator it
77         =A.begin();
78     flatten(u, it);
79     u=build(k, 0, u->s-1);
80 }
81 bool insert(node *u, int k, const point &x,
82     int dep){
83     if(!u) return u=new node(x), dep<=0;
84     ++u->s;
85     cmp.sort_id=k;
86     if(insert(cmp(x, u->pid)?u->l:u->r, (k+1)%
87         kd, x, dep-1)){
88         if(!isbad(u)) return 1;
89         rebuild(u, k);
90     }
91     return 0;
92 }
93 node *findmin(node *o, int k){
94     if(!o) return 0;
95     if(cmp.sort_id==k) return o->l?findmin(o
96         ->l, (k+1)%kd):o;
97     node *l=findmin(o->l, (k+1)%kd);
98     node *r=findmin(o->r, (k+1)%kd);
99     if(l&&!r) return cmp(l,o)?l:o;
100     if(!l&&r) return cmp(r,o)?r:o;
101     if(!l&&!r) return 0;
102     if(cmp(l,r)) return cmp(l,o)?l:o;
103     return cmp(r,o)?r:o;
104 }
105 void erase(node *u, int k, const point &x){
106     if(!u) return 0;
107     if(u->pid==x){
108         if(u->r)
109             else if(u->l) u->r=u->l, u->l=0;
110         else{
111             delete u;
112             return u=0, 1;
113         }
114     }
115     --u->s;
116     cmp.sort_id=k;
117     u->pid=findmin(u->r, (k+1)%kd->pid);
118     return erase(u->r, (k+1)%kd, u->pid);
119 }
120 cmp.sort_id=k;
121 if(erase(cmp(x, u->pid)?u->l:u->r, (k+1)%
122     kd, x)){
123     return --u->s, 1;
124     return 0;
125 }
126 T heuristic(const T h[]) const {
127     T ret=0;
128     for(size_t i=0; i<kd; ++i) ret+=h[i];
129     return ret;
130 }
131 int qM;
132 std::priority_queue<std::pair<T, point >>
133     pQ;
134 void nearest(node *u, int k, const point &x,
135     T *h, T &mndist){
136     if(u==0 || heuristic(h)>=mndist) return;
137     T dist=u->pid.dist(x), old=h[k];
138     /*mndist=std::min(mndist, dist);*/
139     if(dist<mndist){
140         pQ.push(std::make_pair(dist, u->pid));
141         if((int)pQ.size()==qM+1)
142             mndist=pQ.top().first, pQ.pop();
143     }
144     if(x.d[k]<u->pid.d[k]){
145         nearest(u->l, (k+1)%kd, x, h, mndist);
146         h[k]=std::abs(x.d[k]-u->pid.d[k]);
147         nearest(u->r, (k+1)%kd, x, h, mndist);
148     } else {
149         nearest(u->r, (k+1)%kd, x, h, mndist);
150         h[k]=std::abs(x.d[k]-u->pid.d[k]);
151         nearest(u->l, (k+1)%kd, x, h, mndist);
152     }
153     h[k]=old;
154 }
155 std::vector<point> in_range;
156 void range(node *u, int k, const point &mi,
157     const point &ma){
158     if(!u) return;
159     bool is=1;
160     for(int i=0; i<kd; ++i)
161         if(u->pid.d[i]<mi.d[i] || ma.d[i]<u->pid
162             .d[i]){
163             is=0; break;
164         }
165     if(is) in_range.push_back(u->pid);
166     if(mi.d[k]<u->pid.d[k]) range(u->l, (k+1)
167         %kd, mi, ma);
168     if(ma.d[k]>u->pid.d[k]) range(u->r, (k+1)
169         %kd, mi, ma);
170 }
171 public:
172 kd_tree(const T &INF, double a=0.75):root
173     (0), alpha(a), loga(log2(1.0/a)), INF(INF
174         ), maxn(1){}
175 ~kd_tree(){delete root;}
176 void clear(){delete root; root=0, maxn=1;}
177 void build(int n, const point *p){
178     delete root; A.resize(maxn=n);
179     for(int i=0; i<n; ++i) A[i]=new node(p[i]);
180     root=build(0, 0, n-1);
181 }
182 void insert(const point &x){
183     insert(root, 0, x, __lg(size(root))/loga);
184     if(root->s>maxn) maxn=root->s;
185 }
186 bool erase(const point &p){
187     bool d=erase(root, 0, p);
188     if(root&&root->s<alpha*maxn) rebuild();
189     return d;
190 }
191 void rebuild(){
192     if(root) rebuild(root, 0);
193     maxn=root->s;
194 }
195 T nearest(const point &x, int k){
196     qM=k;
197     T mndist=INF, h[kd]={};
198     nearest(root, 0, x, h, mndist);
199     mndist=pQ.top().first;
200     pQ=std::priority_queue<std::pair<T, point
201         >>());
202     return mndist; //回傳離x第k近的點的距離
203 }
204 const std::vector<point> &range(const
205     point &mi, const point &ma){
206     in_range.clear();
207     range(root, 0, mi, ma);
208     return in_range; //回傳介於mi到ma之間的點
209     vector
210 }
211 int size(){return root?root->s:0;}
212 };

```

2.3 kd_tree_replace_segment_tr

```

1 /*kd樹代替高維線段樹*/
2 struct node {
3     node *l,*r;
4     point pid, mi, ma;
5     int s;
6     int data;
7     node(const point &p, int d):l(0),r(0),pid(p
8         ), mi(p), ma(p), s(1), data(d), dmin(d),
9         dmax(d){}
10 void up(){
11     mi=ma=pid;
12     s=1;
13     if(l){
14         for(int i=0; i<kd; ++i){
15             mi.d[i]=min(mi.d[i], l->mi.d[i]);
16             ma.d[i]=max(ma.d[i], l->ma.d[i]);
17         }
18     }
19     if(r){
20         for(int i=0; i<kd; ++i){
21             mi.d[i]=min(mi.d[i], r->mi.d[i]);
22             ma.d[i]=max(ma.d[i], r->ma.d[i]);
23         }
24     }
25     s+=l->s;
26 }
27 void up2(){
28     //其他懶惰標記向上更新
29 }
30 void down(){
31     //其他懶惰標記下推
32 }
33 } *root;
34 /*檢查區間包含用的函數*/
35 inline bool range_include(node *o, const
36     point &L, const point &R){
37     for(int i=0; i<kd; ++i){
38         if(L.d[i]>o->ma.d[i] || R.d[i]<o->mi.d[i])
39             return 0;
40     }
41     return 1;
42 }
43 inline bool range_in_range(node *o, const
44     point &L, const point &R){
45     for(int i=0; i<kd; ++i){

```



```

43     if(L.d[i]>o->mi.d[i]||o->ma.d[i]>R.d[i])
44         return 0;
45 }//如果(L,R)區間完全包含o的區間就回傳true
46 return 1;
47 }
48 inline bool point_in_range(node *o,const
49     point &L,const point &R){
50     point i=0;i<kd;+i){
51         if(L.d[i]>o->pid.d[i]||R.d[i]<o->pid.d[i]
52             )return 0;
53 }//如果(L,R)區間完全包含o->pid這個點就回傳
54     true
55     return 1;
56 }
57 /*單點修改·以單點改值為例*/
58 void update(node *u,const point &x,int data,
59     int k=0){
60     if(!u)return;
61     u->down();
62     if(u->pid==x){
63         u->data=data;
64         u->up2();
65         return;
66     }
67     cmp.sort_id=k;
68     update(cmp(x,u->pid)?u->l:u->r,x,data,(k
69         +1)%kd);
70     u->up2();
71 }
72 /*區間修改*/
73 void update(node *o,const point &L,const
74     point &R,int data){
75     if(!o)return;
76     o->down();
77     if(range_in_range(o,L,R)){
78         //區間懶惰標記修改
79         o->down();
80         return;
81     }
82     if(point_in_range(o,L,R)){
83         //這個點在(L,R)區間·但是他的左右子樹不
84         //一定在區間中
85         //單點懶惰標記修改
86     }
87     if(o->l&&range_include(o->l,L,R))update(o
88         ->l,L,R,data);
89     if(o->r&&range_include(o->r,L,R))update(o
90         ->r,L,R,data);
91     o->up2();
92 }

```

```

86 /*區間查詢·以總和為例*/
87 int query(node *o,const point &L,const point
88     &R){
89     if(!o)return 0;
90     o->down();
91     if(range_in_range(o,L,R))return o->sum;
92     int ans=0;
93     if(point_in_range(o,L,R))ans+=o->data;
94     if(o->l&&range_include(o->l,L,R))ans+=
95         query(o->l,L,R);

```

```

94     if(o->r&&range_include(o->r,L,R))ans+=
95         query(o->r,L,R);
96     return ans;
97 }

```

2.4 reference_point.cpp

```

1 template<typename T>
2 struct _RefC{
3     T data;
4     int ref;
5     _RefC(const T&d=0):data(d),ref(0){}
6 };
7 template<typename T>
8 struct _rp{
9     _RefC<T> *p;
10    T *operator->(){return &p->data;}
11    T &operator*(){return p->data;}
12    operator _RefC<T>*(){return p;}
13    _rp &operator=(const _rp &t){
14        if(p&&!--p->ref)delete p;
15        p=t.p,p&&+p->ref;
16        return *this;
17    }
18    _rp(_RefC<T> *t=0):p(t){p&&+p->ref;}
19    _rp(const _rp &t):p(t.p){p&&+p->ref;}
20    ~_rp(){if(p&&!--p->ref)delete p;}
21 };
22 template<typename T>
23 inline _rp<T> new_rp(const T&nd){
24     return _rp<T>(new _RefC<T>(nd));
25 }

```

2.5 skew_heap.cpp

```

1 node *merge(node *a,node *b){
2     if(!a||!b) return a?a:b;
3     if(b->data<a->data) swap(a,b);
4     swap(a->l,a->r);
5     a->l=merge(b,a->l);
6     return a;
7 }

```

2.6 undo_disjoint_set.cpp

```

1 struct DisjointSet {
2     // save() is like recursive
3     // undo() is like return
4     int n, fa[MXN], sz[MXN];
5     vector<pair<int*,int>> h;
6     vector<int> sp;
7     void init(int tn) {
8         n=tn;
9         for (int i=0; i<n; i++) sz[fa[i]=i]=1;
10        sp.clear(); h.clear();
11    }

```

```

12 void assign(int *k, int v) {
13     h.PB({k, *k});
14     *k=v;
15 }
16 void save() { sp.PB(SZ(h)); }
17 void undo() {
18     assert(!sp.empty());
19     int last=sp.back(); sp.pop_back();
20     while (SZ(h)!=last) {
21         auto x=h.back(); h.pop_back();
22         *x.F=x.S;
23     }
24 }
25 int f(int x) {
26     while (fa[x]!=x) x=fa[x];
27     return x;
28 }
29 void uni(int x, int y) {
30     x=f(x); y=f(y);
31     if (x==y) return;
32     if (sz[x]<sz[y]) swap(x, y);
33     assign(&sz[x], sz[x]+sz[y]);
34     assign(&fa[y], x);
35 }
36 }djs;

```

2.7 整體二分.cpp

```

1 void totBS(int L, int R, vector<Item> M){
2     if(Q.empty()) return; //維護全域B陣列
3     if(L==R) 整個M的答案=r, return;
4     int mid = (L+R)/2;
5     vector<Item> mL, mR;
6     do_modify_B_with_divide(mid,M);
7     //讓B陣列在遞迴的時候只會保留[L~mid]的資訊
8     undo_modify_B(mid,M);
9     totBS(L,mid,mL);
10    totBS(mid+1,R,mR);
11 }

```

3 default

3.1 debug.cpp

```

1 //volatile
2 #ifndef DEBUG
3 #define dbg(...) {\
4     fprintf(stderr,"%s - %d : (%s) = ",
5         __PRETTY_FUNCTION__,__LINE__,#
6         __VA_ARGS__);\
7     _DO(__VA_ARGS__);\
8 }
9 template<typename I> void _DO(I&&x){cerr<<x
10     <<endl;}
11 template<typename I,typename...T> void _DO(I
12     &&x,T&&...tail){cerr<<x<<" ";_DO(tail
13     ...);}

```

```

9 #else
10 #define dbg(...)
11 #endif

```

3.2 ext.cpp

```

1 #include<bits/extc++.h>
2 #include<ext/pd_ds/assoc_container.hpp>
3 #include<ext/pd_ds/tree_policy.hpp>
4 using namespace __gnu_cxx;
5 using namespace __gnu_pbds;
6 template<typename T>
7 using pbds_set = tree<T,null_type,less<T>,
8     rb_tree_tag,
9     tree_order_statistics_node_update>;
10 template<typename T,typename U>
11 using pbds_map = tree<T,U,less<T>,
12     rb_tree_tag,
13     tree_order_statistics_node_update>;
14 using heap=__gnu_pbds::priority_queue<int>;
15 //s.find_by_order(1);//0 base
16 //s.order_of_key(1);

```

3.3 IncStack.cpp

```

1 //Magic
2 #pragma GCC optimize "Ofast"
3 //stack resize,change esp to rsp if 64-bit
4 //system
5 asm("mov %0,%%esp\n" :: "g"(mem+1000000));
6 -Wl,--stack,214748364 -trigraphs
7 //Linux stack resize
8 #include<sys/resource.h>
9 void increase_stack(){
10     const rlim_t ks=64*1024*1024;
11     struct rlimit rl;
12     int res=getrlimit(RLIMIT_STACK,&rl);
13     if(!res&&rl.rlim_cur<ks){
14         rl.rlim_cur=ks;
15         res=setrlimit(RLIMIT_STACK,&rl);
16     }

```

3.4 input.cpp

```

1 inline int read(){
2     int x=0; bool f=0; char c=getchar();
3     while (ch<'0'||'9'<ch)f|=ch=='-',ch=getchar
4         ();
5     while ('0'<=ch&&ch<='9')x=x*10-'0'+ch,ch=
6         getchar();
7     return f?-x:x;
8 }
9 // #!/bin/bash
10 // g++ -std=c++11 -O2 -Wall -Wextra -Wno-
11 // unused-result -DDEBUG $1 && ./a.out

```

```
9 // -fsanitize=address -fsanitize=undefined
   -fsanitize=return
```

4 Flow

4.1 dinic.cpp

```
1 template<typename T>
2 struct DINIC{
3     static const int MAXN=105;
4     static const T INF=INT_MAX;
5     int n, level[MAXN], cur[MAXN];
6     struct edge{
7         int v,pre;
8         T cap,flow,r;
9         edge(int v,int pre,T cap):v(v),pre(pre),
10             cap(cap),flow(0),r(cap){}
11 };
12 int g[MAXN];
13 vector<edge> e;
14 void init(int _n){
15     memset(g,-1,sizeof(int)*((n=_n)+1));
16     e.clear();
17 }
18 void add_edge(int u,int v,T cap,bool
19     directed=false){
20     e.push_back(edge(v,g[u],cap));
21     g[u]=e.size()-1;
22     e.push_back(edge(u,g[v],directed?0:cap));
23     g[v]=e.size()-1;
24 }
25 int bfs(int s,int t){
26     memset(level,0,sizeof(int)*(n+1));
27     memcpy(cur,g,sizeof(int)*(n+1));
28     queue<int> q;
29     q.push(s);
30     level[s]=1;
31     while(q.size()){
32         int u=q.front();q.pop();
33         for(int i=g[u];~i;i=e[i].pre){
34             if(!level[e[i].v]&&e[i].r){
35                 level[e[i].v]=level[u]+1;
36                 q.push(e[i].v);
37                 if(e[i].v==t)return 1;
38             }
39         }
40     }
41     return 0;
42 }
43 T dfs(int u,int t,T cur_flow=INF){
44     if(u==t)return cur_flow;
45     T df;
46     for(int &i=cur[u];~i;i=e[i].pre){
47         if(level[e[i].v]==level[u]+1&&e[i].r){
48             if(df=dfs(e[i].v,t,min(cur_flow,e[i].r))){
49                 e[i].flow+=df;
50                 e[i^1].flow-=df;
51                 e[i].r-=df;
52                 e[i^1].r+=df;
53             }
54         }
55     }
56     return df;
57 }
```

```
51     return df;
52 }
53 }
54 }
55 return level[u]==0;
56 }
57 T dinic(int s,int t,bool clean=true){
58     if(clean){
59         for(size_t i=0;i<e.size();++i){
60             e[i].flow=0;
61             e[i].r=e[i].cap;
62         }
63     }
64     T ans=0, mf=0;
65     while(bfs(s,t))while(mf=dfs(s,t))ans+=mf;
66     return ans;
67 }
68 }
```

4.2 ISAP_with_cut.cpp

```
1 template<typename T>
2 struct ISAP{
3     static const int MAXN=105;
4     static const T INF=INT_MAX;
5     int n; //點數
6     int d[MAXN], gap[MAXN], cur[MAXN];
7     struct edge{
8         int v,pre;
9         T cap,flow,r;
10        edge(int v,int pre,T cap):v(v),pre(pre),
11            cap(cap),flow(0),r(cap){}
12 };
13 int g[MAXN];
14 vector<edge> e;
15 void init(int _n){
16     memset(g,-1,sizeof(int)*((n=_n)+1));
17     e.clear();
18 }
19 void add_edge(int u,int v,T cap,bool
20     directed=false){
21     e.push_back(edge(v,g[u],cap));
22     g[u]=e.size()-1;
23     e.push_back(edge(u,g[v],directed?0:cap));
24     g[v]=e.size()-1;
25 }
26 T dfs(int u,int s,int t,T cur_flow=INF){
27     if(u==t)return cur_flow;
28     T tf=cur_flow,df;
29     for(int &i=cur[u];~i;i=e[i].pre){
30         if(e[i].r&&d[u]==d[e[i].v]+1){
31             df=dfs(e[i].v,s,t,min(tf,e[i].r));
32             e[i].flow+=df;
33             e[i^1].flow-=df;
34             e[i].r-=df;
35             e[i^1].r+=df;
36             if(!(tf==df)||d[s]==n)return
37                 cur_flow-tf;
38         }
39     }
40     return df;
41 }
```

```
38 for(int i=cur[u]=g[u];~i;i=e[i].pre){
39     if(e[i].r&&d[e[i].v]<mh)mh=d[e[i].v];
40 }
41 if(!--gap[d[u]])d[s]=n;
42 else ++gap[d[u]]+=mh;
43 return cur_flow-tf;
44 }
45 T isap(int s,int t,bool clean=true){
46     memset(d,0,sizeof(int)*(n+1));
47     memset(gap,0,sizeof(int)*(n+1));
48     memcpy(cur,g,sizeof(int)*(n+1));
49     if(clean) for(size_t i=0;i<e.size();++i){
50         {
51             e[i].flow=0;
52             e[i].r=e[i].cap;
53         }
54     }
55     T max_flow=0;
56     for(gap[0]=n;d[s]<n;)max_flow+=dfs(s,s,t);
57     return max_flow;
58 }
59 vector<int> cut_e; //最小割邊集
60 bool vis[MAXN];
61 void dfs_cut(int u){
62     vis[u]=1; //表示u屬於source的最小割集
63     for(int i=g[u];~i;i=e[i].pre){
64         if(e[i].flow<e[i].cap&&!vis[e[i].v])
65             dfs_cut(e[i].v);
66     }
67 }
68 T min_cut(int s,int t){
69     T ans=isap(s,t);
70     memset(vis,0,sizeof(bool)*(n+1));
71     dfs_cut(s);
72     for(int u=0;u<n;++u){
73         if(vis[u])for(int i=g[u];~i;i=e[i].pre){
74             if(!vis[e[i].v])cut_e.push_back(i);
75         }
76     }
77     return ans;
78 }
```

4.3 MinCostMaxFlow.cpp

```
1 template<typename _T>
2 struct MCMF{
3     static const int MAXN=440;
4     static const _T INF=999999999;
5     struct edge{
6         int v,pre;
7         _T cap,cost;
8         edge(int v,int pre,_T cap,_T cost):v(v),
9             pre(pre),cap(cap),cost(cost){}
10 };
11 int n,S,T;
12 _T dis[MAXN],piS,ans;
13 bool vis[MAXN];
14 vector<edge> e;
15 int g[MAXN];
16 void init(int _n){
17     memset(g,-1,sizeof(int)*((n=_n)+1));
18     e.clear();
19 }
```

```
19 void add_edge(int u,int v,_T cap,_T cost,
20     bool directed=false){
21     e.push_back(edge(v,g[u],cap,cost));
22     g[u]=e.size()-1;
23     e.push_back(edge(u,g[v],directed?0:cap,-
24         cost));
25     g[v]=e.size()-1;
26 }
27 _T augment(int u,_T cur_flow){
28     if(u==T||!cur_flow)return ans+=piS*
29         cur_flow,cur_flow;
30     vis[u]=1;
31     _T r=cur_flow,d;
32     for(int i=g[u];~i;i=e[i].pre){
33         if(e[i].cap&&!e[i].cost&&vis[e[i].v]){
34             {
35                 d=augment(e[i].v,min(r,e[i].cap));
36                 e[i].cap-=d;
37                 e[i^1].cap+=d;
38                 if(!(r-=d))break;
39             }
40         }
41     }
42     return cur_flow-r;
43 }
44 bool modlabel(){
45     for(int u=0;u<n;++u)dis[u]=INF;
46     static deque<int> q;
47     dis[T]=0,q.push_back(T);
48     while(q.size()){
49         int u=q.front();q.pop_front();
50         _T dt;
51         for(int i=g[u];~i;i=e[i].pre){
52             if(e[i^1].cap&&(dt=dis[u]-e[i].cost)
53                 <dis[e[i].v]){
54                 if((dis[e[i].v]=dt)<=dis[q.size()])
55                     q.push_front(e[i].v);
56                 else q.push_back(e[i].v);
57             }
58         }
59     }
60     for(int u=0;u<n;++u)
61         for(int i=g[u];~i;i=e[i].pre)
62             e[i].cost+=dis[e[i].v]-dis[u];
63     return piS+=dis[S], dis[S]<INF;
64 }
65 _T mincost(int s,int t){
66     S=s,T=t;
67     piS=ans=0;
68     while(modlabel()){
69         do memset(vis,0,sizeof(bool)*(n+1));
70         while(augment(S,INF));
71     }
72     return ans;
73 }
```

5 Graph

5.1 Augmenting_Path.cpp

```
1 #define MAXN 1 505
```

```

2 | #define MAXN2 505
3 | int n1,n2;//n1個點連向n2個點
4 | int match[MAXN2];//屬於n2的點匹配了哪個點
5 | vector<int> g[MAXN1];//圖
6 | bool vis[MAXN2];//是否走訪過
7 | bool dfs(int u){
8 |     for(size_t i=0;i<g[u].size();++i){
9 |         int v=g[u][i];
10 |         if(vis[v])continue;
11 |         vis[v]=1;
12 |         if(match[v]==-1||dfs(match[v]))
13 |             return match[v]=u, 1;
14 |     }
15 |     return 0;
16 | }
17 | inline int max_match(){
18 |     int ans=0;
19 |     memset(match,-1,sizeof(int)*n2);
20 |     for(int i=0;i<n1;++i){
21 |         memset(vis,0,sizeof(bool)*n2);
22 |         if(dfs(i))++ans;
23 |     }
24 |     return ans;
25 | }

```

5.2 Augmenting_Path_multiple.

```

1 | #define MAXN1 1005
2 | #define MAXN2 505
3 | int n1,n2;//n1個點連向n2個點，其中n2個點可以
4 |     匹配很多邊
5 | vector<int> g[MAXN1];//圖
6 | int c[MAXN2];//每個屬於n2點最多可以接受幾條
7 |     匹配邊
8 | vector<int> match_list[MAXN2];//每個屬於n2的
9 |     點匹配了那些點
10 | bool vis[MAXN2];//是否走訪過
11 | bool dfs(int u){
12 |     for(size_t i=0;i<g[u].size();++i){
13 |         int v=g[u][i];
14 |         if(vis[v])continue;
15 |         vis[v]=true;
16 |         if((int)match_list[v].size()<c[v]){
17 |             return match_list[v].push_back(u),
18 |                 true;
19 |         }else{
20 |             for(size_t j=0;j<match_list[v].size()
21 |                 ;++j){
22 |                 int next_u=match_list[v][j];
23 |                 if(dfs(next_u))
24 |                     return match_list[v][j]=u, true;
25 |             }
26 |         }
27 |     }
28 |     return false;
29 | }
30 | int max_match(){
31 |     for(int i=0;i<n2;++i)match_list[i].clear()
32 |     ;
33 |     int cnt=0;
34 |     for(int u=0;u<n1;++u){

```

```

29 |     memset(vis,0,sizeof(bool)*n2);
30 |     if(dfs(u))++cnt;
31 | }
32 | return cnt;
33 | }

```

5.3 blossom_matching.cpp

```

1 | #define MAXN 505
2 | vector<int>g[MAXN];
3 | int pa[MAXN],match[MAXN],st[MAXN],S[MAXN],v[
4 |     MAXN];
5 | int t,n;
6 | int lca(int x,int y){
7 |     for(++t;;swap(x,y)){
8 |         if(x==0)continue;
9 |         if(v[x]==t)return x;
10 |         v[x]=t;
11 |         x=st[pa[match[x]]];
12 |     }
13 | }
14 | #define qpush(x) q.push(x),S[x]=0
15 | void flower(int x,int y,int l,queue<int> &q){
16 |     {
17 |         while(st[x]!=1){
18 |             pa[x]=y;
19 |             if(S[y==match[x]]==1)qpush(y);
20 |             st[x]=st[y]=1, x=pa[y];
21 |         }
22 |     }
23 | bool bfs(int x){
24 |     for(int i=1;i<=n;++i)st[i]=i;
25 |     memset(S+1,-1,sizeof(int)*n);
26 |     queue<int>q; qpush(x);
27 |     while(q.size()){
28 |         x=q.front(),q.pop();
29 |         for(size_t i=0;i<g[x].size();++i){
30 |             int y=g[x][i];
31 |             if(S[y]==-1){
32 |                 pa[y]=x,S[y]=1;
33 |                 if(!match[y]){
34 |                     for(int lst;x=y,lst,x=pa[y])
35 |                         lst=match[x],match[x]=y,match[y]
36 |                             =x;
37 |                     return 1;
38 |                 }
39 |                 qpush(match[y]);
40 |             }else if(!S[y]&&st[y]!=st[x]){
41 |                 int l=lca(y,x);
42 |                 flower(y,x,l,q),flower(x,y,l,q);
43 |             }
44 |         }
45 |     }
46 |     return 0;
47 | }
48 | int blossom(){
49 |     int ans=0;
50 |     for(int i=1;i<=n;++i)
51 |         if(!match[i]&&bfs(i))++ans;
52 |     return ans;
53 | }

```

5.4 graphISO.cpp

```

1 | const int MAXN=1005,K=30;//K要夠大
2 | const long long A=3,B=11,C=2,D=19,P=0
3 |     xdefaced;
4 | long long f[K+1][MAXN];
5 | vector<int> g[MAXN],rg[MAXN];
6 | int n;
7 | void init(){
8 |     for(int i=0;i<n;++i){
9 |         f[0][i]=1;
10 |         g[i].clear(), rg[i].clear();
11 |     }
12 | }
13 | void add_edge(int u,int v){
14 |     g[u].push_back(v), rg[v].push_back(u);
15 | }
16 | long long point_hash(int u){//O(N)
17 |     for(int t=1;t<=K;++t){
18 |         for(int i=0;i<n;++i){
19 |             f[t][i]=f[t-1][i]*AP;
20 |             for(int j:g[i])f[t][i]=(f[t][i]+f[t-1][j]*BP)%P;
21 |             for(int j:rg[i])f[t][i]=(f[t][i]+f[t-1][j]*CP)%P;
22 |             if(i==u)f[t][i]+=D;//如果圖太大的話，
23 |                 把這行刪掉，執行一次後f[K]就會是所
24 |                 有點的答案
25 |             f[t][i]%=P;
26 |         }
27 |     }
28 |     return f[K][u];
29 | }
30 | vector<long long> graph_hash(){
31 |     vector<long long> ans;
32 |     for(int i=0;i<n;++i)ans.push_back(
33 |         point_hash(i));
34 |     sort(ans.begin(),ans.end());
35 |     return ans;
36 | }

```

5.5 KM.cpp

```

1 | #define MAXN 405
2 | #define INF 0x3f3f3f3f
3 | int n;// 1-base，0表示沒有匹配
4 | int g[MAXN][MAXN],lx[MAXN],ly[MAXN],pa[MAXN]
5 |     ,slack_y[MAXN];
6 | int match_y[MAXN],match_x[MAXN];
7 | bool vx[MAXN],vy[MAXN];
8 | void augment(int y){
9 |     for(int x,z;y=y;z){
10 |         x=pa[y],z=match_x[x];
11 |         match_y[y]=x,match_x[x]=y;
12 |     }
13 | }
14 | void bfs(int st){
15 |     for(int i=1;i<=n;++i)slack_y[i]=INF,vx[i]=
16 |         vy[i]=0;
17 |     queue<int> q;q.push(st);
18 |     for(;;){

```

```

17 | while(q.size()){
18 |     int x=q.front();q.pop();
19 |     vx[x]=1;
20 |     for(int y=1;y<=n;++y)if(!vy[y]){
21 |         int t=lx[x]+ly[y]-g[x][y];
22 |         if(t==0){
23 |             pa[y]=x;
24 |             if(!match_y[y]){augment(y);return
25 |                 ;}
26 |             vy[y]=1,q.push(match_y[y]);
27 |         }else if(slack_y[y]>t)pa[y]=x,
28 |             slack_y[y]=t;
29 |     }
30 |     int cut=INF;
31 |     for(int y=1;y<=n;++y){
32 |         if(!vy[y]&&cut>slack_y[y])cut=slack_y[
33 |             y];
34 |     }
35 |     for(int j=1;j<=n;++j){
36 |         if(vx[j])lx[j]-=cut;
37 |         if(vy[j])ly[j]+=cut;
38 |         else slack_y[j]-=cut;
39 |     }
40 |     for(int y=1;y<=n;++y){
41 |         if(!vy[y]&&slack_y[y]==0){
42 |             if(!match_y[y]){augment(y);return;}
43 |             vy[y]=1,q.push(match_y[y]);
44 |         }
45 |     }
46 | }
47 | long long KM(){
48 |     memset(match_y,0,sizeof(int)*(n+1));
49 |     memset(ly,0,sizeof(int)*(n+1));
50 |     for(int x=1;x<=n;++x){
51 |         lx[x]=-INF;
52 |         for(int y=1;y<=n;++y)
53 |             lx[x]=max(lx[x],g[x][y]);
54 |     }
55 |     for(int x=1;x<=n;++x)bfs(x);
56 |     long long ans=0;
57 |     for(int y=1;y<=n;++y)ans+=g[match_y[y]][y];
58 |     return ans;
59 | }

```

5.6 MaximumClique.cpp

```

1 | struct MaxClique{
2 |     static const int MAXN=105;
3 |     int N,ans;
4 |     int g[MAXN][MAXN],dp[MAXN],stk[MAXN][MAXN]
5 |         ;
6 |     int sol[MAXN],tmp[MAXN];
7 |     //sol[0~ans-1]為答案
8 |     void init(int n){
9 |         N=n;
10 |         //0-base
11 |         memset(g,0,sizeof(g));
12 |     }
13 |     void add_edge(int u,int v){
14 |         g[u][v]=g[v][u]=1;
15 |     }

```

5.8 Rectilinear_MST.cpp

```

12 }
13 int dfs(int ns,int dep){
14     if(!ns){
15         if(dep>ans){
16             ans=dep;
17             memcpy(sol,tmp,sizeof tmp);
18             return 1;
19         }else return 0;
20     }
21     for(int i=0;i<ns;++i){
22         if(dep+ns-i<=ans) return 0;
23         int u=stk[dep][i],cnt=0;
24         if(dep+dp[u]<=ans) return 0;
25         for(int j=i+1;j<ns;++j){
26             int v=stk[dep][j];
27             if(g[u][v])stk[dep+1][cnt++]=v;
28         }
29         tmp[dep]=u;
30         if(dfs(cnt,dep+1))return 1;
31     }
32     return 0;
33 }
34 int clique(){
35     int u,v,ns;
36     for(ans=0,u=N-1;u>=0;--u){
37         for(ns=0,tmp[0]=u,v=u+1;v<N;++v)
38             if(g[u][v])stk[1][ns++]=v;
39         dfs(ns,1),dp[u]=ans;
40     }
41     return ans;
42 }
43 };

```

5.7 MinimumMeanCycle.cpp

```

1 #include<cstdio> //for DBL_MAX
2 int dp[maxN+1][maxN+1];
3 double mnc(int n){
4     int u,v,w;
5     const int inf=0x7f7f7f7f;
6     memset(dp,0x7f,sizeof(dp));
7     memset(dp[0],0,sizeof(dp[0]));
8     for(int i=0;i<n;++i){
9         for(auto e:E){
10             tie(u,v,w)=e;
11             if(dp[i][u]!=inf)
12                 dp[i+1][v]=min(dp[i+1][v],dp[i][u]+w);
13         }
14         double res = DBL_MAX;
15         for(int i=1;i<n;++i){
16             double val = DBL_MIN;
17             for(int j=0;j<n;++j)
18                 val=max(val,double(dp[n][i]-dp[i][j]
19                                     ))/(n-j));
20             res=min(res,val);
21         }
22     }
23 }

```

```

1 //平面曼哈頓最小生成樹構造圖(去除非必要邊)
2 #define T int
3 #define INF 0x3f3f3f3f
4 struct point{
5     T x,y;
6     int id; //從0開始編號
7     point(){
8         T dist(const point &p)const{
9             return abs(x-p.x)+abs(y-p.y);
10        }
11    };
12    bool cmpx(const point &a,const point &b){
13        return a.x<b.x||(a.x==b.x&&a.y<b.y);
14    }
15    struct edge{
16        int u,v;
17        T cost;
18        edge(int u,int v,T c):u(u),v(v),cost(c){
19            bool operator<(const edge&e)const{
20                return cost<e.cost;
21            }
22        };
23    } struct bit_node{
24        T mi;
25        int id;
26        bit_node(const T&mi=INF,int id=-1):mi(mi),
27            id(id){
28        };
29    } vector<bit_node> bit;
30    void bit_update(int i,const T&data,int id){
31        for(;i=i&(-i)){
32            if(data<bit[i].mi)bit[i]=bit_node(data,
33                id);
34        }
35    } int bit_find(int i,int m){
36        bit_node x;
37        for(;i<m;i=i&(-i)) if(bit[i].mi<x.mi)x=
38            bit[i];
39        return x.id;
40    } vector<edge> build_graph(int n,point p[]){
41        vector<edge> e; //edge for MST
42        for(int dir=0;dir<4;dir++){ //4種座標變換
43            if(dir%2) for(int i=0;i<n;++i) swap(p[i]
44                ].x,p[i].y);
45            else if(dir==2) for(int i=0;i<n;++i) p[i]
46                ].x=-p[i].x;
47            sort(p,p+n,cmpx);
48            vector<T> ga(n), gb;
49            for(int i=0;i<n;++i)ga[i]=p[i].y-p[i].x;
50            gb=ga, sort(gb.begin(),gb.end());
51            gb.erase(unique(gb.begin(),gb.end()),gb.
52                end());
53            int m=gb.size();
54            bit=vector<bit_node>(m+1);
55            for(int i=n-1;i>=0;--i){
56                int pos=lower_bound(gb.begin(),gb.end
57                    (),ga[i])-gb.begin()+1;
58                int ans=bit_find(pos,m);
59                if(~ans)e.push_back(edge(p[i].id,p[ans
60                    ].id,p[i].dist(p[ans])));
61                bit_update(pos,p[i].x+p[i].y,i);
62            }
63        }
64    }

```

```

56 }
57 }
58 return e;
59 }

```

5.9 treeISO.cpp

```

1 const int MAXN=100005;
2 const long long X=12327,P=0xdefaced;
3 vector<int> g[MAXN];
4 bool vis[MAXN];
5 long long dfs(int u){//hash ver
6     vis[u]=1;
7     vector<long long> tmp;
8     for(auto v:g[u])if(!vis[v])tmp.pb(dfs(v));
9     if(tmp.empty())return 177;
10    long long ret=4931;
11    sort(tmp.begin(),tmp.end());
12    for(auto v:tmp)ret=((ret*X)^v)%P;
13    return ret;
14 }
15 //-----
16 string dfs(int x,int p){
17     vector<string> c;
18     for(int y:g[x])
19         if(y!=p)c.emplace_back(dfs(y,x));
20     sort(c.begin(),c.end());
21     string ret("(");
22     for(auto &s:c)ret+=s;
23     ret+=")";
24     return ret;
25 }

```

5.10 一般圖最小權完美匹配.cpp

```

1 struct Graph {
2     // Minimum General Weighted Matching (
3     Perfect Match) 0-base
4     static const int MXN = 105;
5     int n, edge[MXN][MXN];
6     int match[MXN],dis[MXN],onstk[MXN];
7     vector<int> stk;
8     void init(int _n) {
9         n = _n;
10        for (int i=0; i<n; i++)
11            for (int j=0; j<n; j++)
12                edge[i][j] = 0;
13    }
14    void add_edge(int u, int v, int w) {
15        edge[u][v] = edge[v][u] = w;
16    }
17    bool SPFA(int u){
18        if (onstk[u]) return true;
19        stk.push_back(u);
20        onstk[u] = 1;
21        for (int v=0; v<n; v++){
22            if (u != v && match[u] != v && !onstk[
23                v]){
24                int m = match[v];
25            }
26        }
27    }
28 }

```

```

23     if (dis[m] > dis[u] - edge[v][m] +
24         edge[u][v]){
25         dis[m] = dis[u] - edge[v][m] +
26             edge[u][v];
27         onstk[v] = 1;
28         stk.push_back(v);
29         if (SPFA(m)) return true;
30         stk.pop_back();
31         onstk[v] = 0;
32     }
33 }
34 onstk[u] = 0;
35 stk.pop_back();
36 return false;
37 }
38 int solve() {
39     // find a match
40     for (int i=0; i<n; i+=2){
41         match[i] = i+1, match[i+1] = i;
42     }
43     for(;;){
44         int found = 0;
45         for (int i=0; i<n; i++){
46             dis[i] = onstk
47                 [i] = 0;
48             for (int i=0; i<n; i++){
49                 stk.clear();
50                 if (!onstk[i] && SPFA(i)){
51                     found = 1;
52                     while (stk.size()>2){
53                         int u = stk.back(); stk.pop_back
54                             ();
55                         int v = stk.back(); stk.pop_back
56                             ();
57                         match[u] = v;
58                         match[v] = u;
59                     }
60                 }
61             }
62             if (!found) break;
63         }
64         int ret = 0;
65         for (int i=0; i<n; i++)
66             ret += edge[i][match[i]];
67         ret /= 2;
68         return ret;
69     }
70 } graph;

```

5.11 全局最小割.cpp

```

1 const int INF=0x3f3f3f3f;
2 template<typename T>
3 struct stoer_wagner{// 0-base
4     static const int MAXN=150;
5     T g[MAXN][MAXN],dis[MAXN];
6     int nd[MAXN],n,s,t;
7     void init(int _n){
8         n=_n;
9         for(int i=0;i<n;++i)
10             for(int j=0;j<n;++j)g[i][j]=0;
11     }
12     void add_edge(int u,int v,T w){
13     }
14 }

```



```

13     g[u][v]=g[v][u]+=w;
14 }
15 T min_cut(){
16     T ans=INF;
17     for(int i=0;i<n;++i)nd[i]=i;
18     for(int ind=tn;n>tn>1;--tn){
19         for(int i=1;i<tn;++i)dis[nd[i]]=0;
20         for(int i=1;i<tn;++i){
21             ind=i;
22             for(int j=i;j<tn;++j){
23                 dis[nd[j]]+=g[nd[i-1]][nd[j]];
24                 if(dis[nd[ind]]<dis[nd[j]])ind=j;
25             }
26             swap(nd[ind],nd[i]);
27         }
28         if(ans>dis[nd[ind]])ans=dis[t=nd[ind]];
29         for(int i=0;i<tn;++i)
30             g[nd[ind-1]][nd[i]]=g[nd[i]][nd[ind-1]]+=g[nd[i]][nd[ind]];
31     }
32     return ans;
33 }
34 };

```

5.12 平面圖判定.cpp

```

1 static const int MAXN = 20;
2 struct Edge{
3     int u, v;
4     Edge(int s, int d) : u(s), v(d) {}
5 };
6 bool isK33(int n, int degree[]){
7     int t = 0, z = 0;
8     for(int i=0;i<n;++i){
9         if(degree[i] == 3)++t;
10        else if(degree[i] == 0)++z;
11        else return false;
12    }
13    return t == 6 && t + z == n;
14 }
15 bool isK5(int n, int degree[]){
16     int f = 0, z = 0;
17     for(int i=0;i<n;++i){
18         if(degree[i] == 4)++f;
19         else if(degree[i] == 0)++z;
20         else return false;
21    }
22    return f == 5 && f + z == n;
23 }
24 // it judge a given graph is Homeomorphic
25 // with K33 or K5
26 bool isHomeomorphic(bool G[MAXN][MAXN],
27     const int n){
28     for(;;){
29         int cnt = 0;
30         for(int i=0;i<n;++i){
31             vector<Edge> E;
32             for(int j=0;j<n&&E.size()<3;++j)
33                 if(G[i][j] && i != j)
34                     E.push_back(Edge(i, j));
35             if(E.size() == 1){
36                 G[i][E[0].v] = G[E[0].v][i] = false;

```

```

35     }else if(E.size() == 2){
36         G[i][E[0].v] = G[E[0].v][i] = false;
37         G[i][E[1].v] = G[E[1].v][i] = false;
38         G[E[0].v][E[1].v] = G[E[1].v][E[0].v]
39             = true;
40         ++cnt;
41     }
42     if(cnt == 0)break;
43 }
44 static int degree[MAXN];
45 fill(degree, degree + n, 0);
46 for(int i=0;i<n;++i){
47     for(int j=i+1; j<n; ++j){
48         if(!G[i][j])continue;
49         ++degree[i];
50         ++degree[j];
51     }
52 }
53 return !(isK33(n, degree) || isK5(n,
54     degree));

```

5.13 弦圖完美消除序列.cpp

```

1 struct chordal{
2     static const int MAXN=1005;
3     int n;// 0-base
4     vector<int>G[MAXN];
5     int rank[MAXN],label[MAXN];
6     bool mark[MAXN];
7     void init(int _n){n=_n;
8         for(int i=0;i<n;++i)G[i].clear();
9     }
10    void add_edge(int u,int v){
11        G[u].push_back(v);
12        G[v].push_back(u);
13    }
14    vector<int> MCS(){
15        memset(rank,-1,sizeof(int)*n);
16        memset(label,0,sizeof(int)*n);
17        priority_queue<pair<int,int>> pq;
18        for(int i=0;i<n;++i)pq.push(make_pair(0,
19            i));
20        for(int i=n-1;i>=0;--i)for(;;){
21            int u=pq.top().second;pq.pop();
22            if(~rank[u])continue;
23            rank[u]=i;
24            for(auto v:G[u])if(rank[v]==-1){
25                pq.push(make_pair(++label[v],v));
26            }
27            break;
28        }
29        vector<int> res(n);
30        for(int i=0;i<n;++i)res[rank[i]]=i;
31        return res;
32    }
33    bool check(vector<int> ord){//弦圖判定
34        for(int i=0;i<n;++i)rank[ord[i]]=i;
35        memset(mark,0,sizeof(bool)*n);
36        for(int i=0;i<n;++i){
37            vector<pair<int,int>> tmp;
38            for(auto u:G[ord[i]])if(!mark[u])

```

```

38         tmp.push_back(make_pair(rank[u],u));
39         sort(tmp.begin(),tmp.end());
40         if(tmp.size()){
41             int u=tmp[0].second;
42             set<int> S;
43             for(auto v:G[u])S.insert(v);
44             for(size_t j=1;j<tmp.size();++j)
45                 if(!S.count(tmp[j].second))return
46                     0;
47             mark[ord[i]]=1;
48         }
49         return 1;
50     }
51 };

```

5.14 最小斯坦納樹 DP.cpp

```

1 //n個點，其中r個要構成斯坦納樹
2 //答案在max(dp[(1<r)-1][k]) k=0~n-1
3 //p表示要構成斯坦納樹的點集
4 //O( n^3 + n^3*r + n^2*2^r )
5 #define REP(i,n) for(int i=0;i<(int)n;++i)
6 const int MAXN=30,MAXM=8;// 0-base
7 const int INF=0x3f3f3f3f;
8 int dp[1<MAXN][MAXN];
9 int g[MAXN][MAXN];
10 void init(){memset(g,0,sizeof(g));}
11 void add_edge(int u,int v,int w){
12     g[u][v]=g[v][u]=min(g[v][u],w);
13 }
14 void steiner(int n,int r,int *p){
15     REP(k,n)REP(i,n)REP(j,n)
16         g[i][j]=min(g[i][j],g[i][k]+g[k][j]);
17     REP(i,n)g[i][i]=0;
18     REP(i,r)REP(j,n)dp[1<i][j]=g[p[i]][j];
19     for(int i=1;i<(1<r);++i){
20         if(!(i&(i-1)))continue;
21         REP(j,n)dp[i][j]=INF;
22         REP(j,n){
23             int tmp=INF;
24             for(int s=i&(i-1);s>=i&(s-1))
25                 tmp=min(tmp,dp[s][j]+dp[i^s][j]);
26             REP(k,n)dp[i][k]=min(dp[i][k],g[j][k]+
27                 tmp);
28         }
29     }

```

5.15 最小樹形圖 __ 朱劉.cpp

```

1 template<typename T>
2 struct zhu_liu{
3     static const int MAXN=110,MAXM=10005;
4     struct node{
5         int u,v;
6         T w,tag;
7         node *l,*r;
8         node(int u=0,int v=0,T w=0):u(u),v(v),w(
9             w),tag(0),l(0),r(0){}

```

```

9     void down(){
10         w+=tag;
11         if(l)l->tag+=tag;
12         if(r)r->tag+=tag;
13         tag=0;
14     }
15     mem[MAXN];
16     node *pq[MAXN*2],*E[MAXN*2];
17     int st[MAXN*2],id[MAXN*2],m;
18     void init(int n){
19         for(int i=1;i<n;++i){
20             pq[i]=E[i]=0, st[i]=id[i]=i;
21             m=0;
22         }
23         node *merge(node *a,node *b){
24             if(!a||!b)return a?a:b;
25             a->down(),b->down();
26             if(b->w<a->w)return merge(b,a);
27             swap(a->l,a->r);
28             a->l=merge(b,a->l);
29             return a;
30         }
31         void add_edge(int u,int v,T w){
32             if(u!=v)pq[v]=merge(pq[v],&(mem[m++]=
33                 node(u,v,w)));
34         }
35         int find(int x,int *st){
36             return st[x]==x?x:st[x]=find(st[x],st);
37         }
38         T build(int root,int n){
39             T ans=0;int N=n,all=n;
40             for(int i=1;i<N;++i){
41                 if(i==root||!pq[i])continue;
42                 while(pq[i]){
43                     pq[i]->down(),E[i]=pq[i];
44                     pq[i]=merge(pq[i]->l,pq[i]->r);
45                     if(find(E[i]->u,id)!=find(i,id))
46                         break;
47                 }
48                 if(find(E[i]->u,id)==find(i,id))
49                     continue;
50                 ans+=E[i]->w;
51                 if(find(E[i]->u,st)==find(i,st)){
52                     if(pq[i])pq[i]->tag-=E[i]->w;
53                     pq[i]=pq[i];id[N]=N;
54                     for(int u=find(E[i]->u,id);u!=i;u=
55                         find(E[u]->u,id)){
56                         if(pq[u])pq[u]->tag-=E[u]->w;
57                         id[find(u,id)]=N;
58                         pq[N]=merge(pq[N],pq[u]);
59                     }
60                     st[N]=find(i,st);
61                     id[find(i,id)]=N;
62                     }else st[find(i,st)]=find(E[i]->u,st)
63                         ,--all;
64             }
65             return all==1?ans:-INT_MAX;
66         }
67     };
68 };

```

5.16 穩定婚姻模板.cpp

```

1 queue<int> Q;
2 for ( i : 所有考生 ) {
3     設定在第0志願;
4     Q.push(考生i);
5 }
6 while(Q.size()){
7     當前考生=Q.front();Q.pop();
8     while ( 此考生未分發 ) {
9         指標移到下一志願;
10        if ( 已經沒有志願 or 超出志願總數 )
11            break;
12        計算該考生在該科系加權後的總分;
13        if ( 不符合科系需求 ) continue;
14        if ( 目前科系有餘額 ) {
15            依加權後分數高低順序將考生id加入科系錄取名單中;
16            break;
17        }
18        if ( 目前科系已額滿 ) {
19            if ( 此考生成績比最低分數還高 ) {
20                依加權後分數高低順序將考生id加入科系錄取名單;
21                Q.push(被踢出的考生);
22            }
23        }
24    }
}

```

6 language

6.1 CNF.cpp

```

1 #define MAXN 55
2 struct CNF{
3     int s,x,y;//s->xy | s->x, if y==1
4     int cost;
5     CNF(){}
6     CNF(int s,int x,int y,int c):s(s),x(x),y(y),cost(c){}
7 };
8 int state;//規則數量
9 map<char,int> rule;//每個字元對應到的規則，小寫字母為終端字符
10 vector<CNF> cnf;
11 void init(){
12     state=0;
13     rule.clear();
14     cnf.clear();
15 }
16 void add_to_cnf(char s,const string &p,int cost){
17     //加入一個s -> <p>的文法，代價為cost
18     if(rule.find(s)==rule.end())rule[s]=state++;
19     for(auto c:p)if(rule.find(c)==rule.end())rule[c]=state++;
20     if(p.size()==1){

```

```

21     cnf.push_back(CNF(rule[s],rule[p[0]],-1,cost));
22 }else{
23     int left=rule[s];
24     int sz=p.size();
25     for(int i=0;i<sz-2;++i){
26         cnf.push_back(CNF(left,rule[p[i]],state,0));
27         left=state++;
28     }
29     cnf.push_back(CNF(left,rule[p[sz-2]],rule[p[sz-1]],cost));
30 }
31 }
32 vector<long long> dp[MAXN][MAXN];
33 vector<bool> neg_INF[MAXN][MAXN];//如果花費
34 //是真的可能會有無限小的情形
35 void relax(int l,int r,const CNF &c,long long cost,bool neg_c=0){
36     if(!neg_INF[l][r][c.s]&&(neg_INF[l][r][c.x]||cost<dp[l][r][c.s])){
37         if(neg_c|neg_INF[l][r][c.x]){
38             dp[l][r][c.s]=0;
39             neg_INF[l][r][c.s]=true;
40         }else dp[l][r][c.s]=cost;
41     }
42 }
43 void bellman(int l,int r,int n){
44     for(int k=1;k<=state;++k)
45         for(auto c:cnf)
46             if(c.y==1)relax(l,r,c,dp[l][r][c.x]+c.cost,k==n);
47 }
48 void cyk(const vector<int> &tok){
49     for(int i=0;i<(int)tok.size();++i){
50         for(int j=0;j<(int)tok.size();++j){
51             dp[i][j]=vector<long long>(state+1,INT_MAX);
52             neg_INF[i][j]=vector<bool>(state+1,false);
53         }
54         dp[i][i][tok[i]]=0;
55         bellman(i,i,tok.size());
56     }
57     for(int r=1;r<(int)tok.size();++r){
58         for(int l=r-1;l>=0;--l){
59             for(int k=l;k<r;++k)
60                 for(auto c:cnf)
61                     if(~c.y)relax(l,r,c,dp[l][k][c.x]+dp[k+1][r][c.y]+c.cost);
62             bellman(l,r,tok.size());
63         }
64     }
}

```

7 Linear_Programming

7.1 最大密度子圖.cpp

```

1 typedef double T;//POJ 3155

```

```

2 const int MAXN=105;
3 struct edge{
4     int u,v;
5     T w;
6     edge(int u=0,int v=0,T w=0):u(u),v(v),w(w){}
7 };
8 vector<edge> E;
9 int n,m;// 1-base
10 T de[MAXN],pv[MAXN];//每個點的邊權和和點權(有些題目會給)
11 void init(){
12     E.clear();
13     for(int i=1;i<=n;++i)de[i]=pv[i]=0;
14 }
15 void add_edge(int u,int v,T w){
16     E.push_back(edge(u,v,w));
17     de[u]+=w,de[v]+=w;
18 }
19 T U;//二分搜的最大值
20 void get_U(){
21     U=0;
22     for(int i=1;i<=n;++i)U+=2*pv[i];
23     for(size_t i=0;i<E.size();++i)U+=E[i].w;
24 }
25 ISAP<T> isap;//網路流
26 int s,t;//原匯點
27 void build(T L){
28     isap.init(n+2);
29     for(size_t i=0;i<E.size();++i)
30         isap.add_edge(E[i].u,E[i].v,E[i].w);
31     for(int v=1;v<=n;++v){
32         isap.add_edge(s,v,U);
33         isap.add_edge(v,t,U+2*L-de[v]-2*pv[v]);
34     }
35 }
36 int main(){
37     while(~scanf("%d%d",&n,&m)){
38         if(!m){
39             puts("1\n1");
40             continue;
41         }
42         init();
43         int u,v;
44         for(int i=0;i<m;++i){
45             scanf("%d%d",&u,&v);
46             add_edge(u,v,1);
47         }
48         get_U();
49         s=n+1,t=n+2;
50         T l=0,r=U,k=1.0/(n*n);
51         while(r-l>k){//二分搜最大值
52             T mid=(l+r)/2;
53             build(mid);
54             T res=(U*n-isap.isap(s,t))/2;
55             if(res>0)l=mid;
56             else r=mid;
57         }
58         build(l);
59         isap.min_cut(s,t);
60         vector<int> ans;
61         for(int i=1;i<=n;++i)
62             if(isap.vis[i])ans.push_back(i);
63         printf("%d\n",ans.size());
64         for(size_t i=0;i<ans.size();++i)

```

```

65         printf("%d\n",ans[i]);
66     }
67     return 0;
68 }

```

8 Number_Theory

8.1 basic.cpp

```

1 template<typename T>
2 void gcd(const T &a,const T &b,T &d,T &x,T &y){
3     if(!b) d=a,x=1,y=0;
4     else gcd(b,a%b,d,y,x), y-=x*(a/b);
5 }
6 long long int phi[N+1];
7 void phiTable(){
8     for(int i=1;i<=N;i++)phi[i]=i;
9     for(int i=1;i<=N;i++)for(x=i*2;x<=N;x+=i)phi[x]-=phi[i];
10 }
11 void all_divdown(const LL &n){// all n/x
12     for(LL a=1;a<=n;a=n/(n/(a+1)))// dosomething;
13 }
14 }
15 }
16 const int MAXPRIME = 1000000;
17 int iscom[MAXPRIME], prime[MAXPRIME], primecnt;
18 int phi[MAXPRIME], mu[MAXPRIME];
19 void sieve(void){
20     memset(iscom,0,sizeof(iscom));
21     primecnt = 0;
22     phi[1] = mu[1] = 1;
23     for(int i=2;i<MAXPRIME;++i) {
24         if(!iscom[i]) {
25             prime[primecnt++] = i;
26             mu[i] = -1;
27             phi[i] = i-1;
28         }
29         for(int j=0;j<primecnt;++j) {
30             int k = i * prime[j];
31             if(k>MAXPRIME) break;
32             iscom[k] = prime[j];
33             if(i%prime[j]==0) {
34                 mu[k] = 0;
35                 phi[k] = phi[i] * prime[j];
36                 break;
37             } else {
38                 mu[k] = -mu[i];
39                 phi[k] = phi[i] * (prime[j]-1);
40             }
41         }
42     }
43 }
44 }
45 bool g_test(const LL &g, const LL &p, const vector<LL> &v) {
46     for(int i=0;i<v.size();++i)
47         if(modexp(g,(p-1)/v[i],p)==1)
48             return false;

```

```

49     return true;
50 }
51 LL primitive_root(const LL &p) {
52     if(p==2) return 1;
53     vector<LL> v;
54     Factor(p-1,v);
55     v.erase(unique(v.begin(), v.end()), v.end());
56     for(LL g=2;g<p;++g)
57         if(g_test(g,p,v))
58             return g;
59     puts("primitive_root NOT FOUND");
60     return -1;
61 }
62 int Legendre(const LL &a, const LL &p) {
63     return modexp(a%p,(p-1)/2,p); }
64 LL inv(const LL &a, const LL &n) {
65     LL d,x,y;
66     gcd(a,n,d,x,y);
67     return d==1 ? (x+n)%n : -1;
68 }
69
70 int inv[maxN];
71 LL invtable(int n,LL P){
72     inv[1]=1;
73     for(int i=2;i<n;++i)
74         inv[i]=(P-(P/i))*inv[P%i]%P;
75 }
76
77 LL log_mod(const LL &a, const LL &b, const
78     LL &p) {
79     // a ^ x = b ( mod p )
80     int m=sqrt(p+.5), e=1;
81     LL v=inv(modexp(a,m,p), p);
82     map<LL,int> x;
83     x[1]=0;
84     for(int i=1;i<m;++i) {
85         e = LLMul(e,a,p);
86         if(!x.count(e)) x[e] = i;
87     }
88     for(int i=0;i<m;++i) {
89         if(x.count(b)) return i*m + x[b];
90         b = LLMul(b,v,p);
91     }
92     return -1;
93 }
94 LL Tonelli-Shanks(const LL &n, const LL &p)
95 {
96     // x^2 = n ( mod p )
97     if(n==0) return 0;
98     if(Legendre(n,p)!=1) while(1) { puts("SQRT
99         ROOT does not exist"); }
100     int S = 0;
101     LL Q = p-1;
102     while( !(Q&1) ) { Q>>=1; ++S; }
103     if(S==1) return modexp(n%p,(p+1)/4,p);
104     LL z = 2;
105     for(; Legendre(z,p)!=-1;++z)
106         LL c = modexp(z,Q,p);
107     LL R = modexp(n%p,(Q+1)/2,p), t = modexp(n
108         %p,Q,p);
109     LL b = modexp(c,1L<<(M-i-1),p);
110     R = LLMul(R,b,p);
111     t = LLMul( LLMul(b,b,p), t, p);
112     c = LLMul(b,b,p);
113     M = i;
114 }
115     return -1;
116 }
117
118 template<typename T>
119 T Euler(T n){
120     T ans=n;
121     for(T i=2;i*i<=n;++i){
122         if(n%i==0){
123             ans=ans/i*(i-1);
124             while(n%i==0)n/=i;
125         }
126     }
127     if(n>1)ans=ans/n*(n-1);
128     return ans;
129 }
130
131 //Chinese_remainder_theorem
132 template<typename T>
133 T pow_mod(T n,T k,T m){
134     T ans=1;
135     for(n=(n>m?n%m:n);k;k>>=1){
136         if(k&1)ans=ans*n%m;
137         n=n*n%m;
138     }
139     return ans;
140 }
141
142 template<typename T>
143 T crt(vector<T> &m,vector<T> &a){
144     T M=1,tM,ans=0;
145     for(int i=0;i<(int)m.size();++i)M*=m[i];
146     for(int i=0;i<(int)a.size();++i){
147         tM=M/m[i];
148         ans=(ans+(a[i]*tM%M)*pow_mod(tM,Euler(m[i]),M))%M;
149     }
150     return ans;
151 }
152
153 //java code
154 //求sqrt(N)的連分數
155 public static void Pell(int n){
156     BigInteger N,p1,p2,q1,q2,a0,a1,a2,g1,g2,h1
157         ,h2,p,q;
158     g1=q2=p1=BigInteger.ZERO;
159     h1=q1=p2=BigInteger.ONE;
160     a0=a1=BigInteger.valueOf((int)Math.sqrt
161         (1.0*n));
162     BigInteger ans=a0.multiply(a0);
163     if(ans.equals(BigInteger.valueOf(n))){
164         System.out.println("No solution!");
165         return ;
166     }
167     while(true){
168         g2=a1.multiply(h1).subtract(g1);
169         h2=N.subtract(g2.pow(2)).divide(h1);
170         a2=g2.add(a0).divide(h2);
171         p=a1.multiply(p2).add(p1);

```

```

172     q=a1.multiply(q2).add(q1);
173     if(p.pow(2).subtract(N.multiply(q.pow
174         (2))).compareTo(BigInteger.ONE)==0)
175         break;
176     g1=g2;h1=h2;a1=a2;
177     p1=p2;p2=p;
178     q1=q2;q2=q;
179 }
180 System.out.println(p+ " "+q);
181 }

```

8.2 bit_set.cpp

```

1 void sub_set(int S){
2     int sub=S;
3     do{
4         //對某集合的子集合的處理
5         sub=(sub-1)&S;
6     }while(sub!=S);
7 }
8 void k_sub_set(int k,int n){
9     int comb=(1<<k)-1,S=1<<n;
10    while(comb<S){
11        //對大小為k的子集合的處理
12        int x=comb&-comb,y=comb+x;
13        comb=((comb&~y)/x>>1)|y;
14    }
15 }

```

8.3 cantor_expansion.cpp

```

1 int factorial[MAXN];
2 void init(){
3     factorial[0]=1;
4     for(int i=1;i<=MAXN;++i)factorial[i]=
5         factorial[i-1]*i;
6 }
7 int encode(const vector<int> &s){
8     int n=s.size(),res=0;
9     for(int i=0;i<n;++i){
10        int t=0;
11        for(int j=i+1;j<n;++j)
12            if(s[j]<s[i])++t;
13        res+=t*factorial[n-i-1];
14    }
15    return res;
16 }
17 vector<int> decode(int a,int n){
18     vector<int> res;
19     vector<bool> vis(n,0);
20     for(int i=n-1;i>0;--i){
21         int t=a/factorial[i];j;
22         for(j=0;j<n;++j)
23             if(!vis[j]){
24                 if(t==0)break;
25                 --t;
26             }
27         res.push_back(j);
28         vis[j]=1;

```

```

28     a%=factorial[i];
29 }
30 return res;
31 }

```

8.4 FFT.cpp

```

1 template<typename T,typename VT=vector<
2     complex<T> > >
3 struct FFT{
4     const T pi;
5     FFT(const T pi=acos((T)-1)):pi(pi){}
6     unsigned bit_reverse(unsigned a,int len){
7         a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)
8             >>1);
9         a=((a&0x33333333U)<<2)|((a&0xCCCCCCCCU)
10             >>2);
11         a=((a&0x0F0F0F0FU)<<4)|((a&0xFF0FF0FFU)
12             >>4);
13         a=((a&0x00FF00FFU)<<8)|((a&0xFFFF0000U)
14             >>8);
15         a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)
16             >>16);
17         return a>>(32-len);
18 }
19 void fft(bool is_inv,VT &in,VT &out,int N)
20 {
21     int bitlen=__lg(N),num=is_inv?-1:1;
22     for(int i=0;i<N;++i)out[bit_reverse(i,
23         bitlen)]=in[i];
24     for(int step=2;step<=N;step<<=1){
25         const int mh=step>>1;
26         for(int i=0;i<N;i+=mh){
27             complex<T> wi=exp(complex<T>(0,i*num
28                 *pi/mh));
29             for(int j=i;j<N;j+=step){
30                 int k=j+mh;
31                 complex<T> u=out[j],t=out[k];
32                 out[j]=u+t;
33                 out[k]=u-t;
34             }
35         }
36     }
37     if(is_inv)for(int i=0;i<N;++i)out[i]/=N;
38 }

```

8.5 find_real_root.cpp

```

1 // a_n*x^n + ... + a_1x + a_0 = 0;
2 int sign(double x){
3     return x < -eps ? -1 : x > eps;
4 }
5
6 double get(const vector<double>&coef, double
7     x){
8     double e = 1, s = 0;
9     for(auto i : coef) s += i*e, e *= x;
10    return s;

```

```

11 double find(const vector<double>&coef, int n
12 , double lo, double hi){
13     double sign_lo, sign_hi;
14     if( !(sign_lo = sign(get(coef,lo))) )
15         return lo;
16     if( !(sign_hi = sign(get(coef,hi))) )
17         return hi;
18     if(sign_lo * sign_hi > 0) return INF;
19     for(int stp = 0; stp < 100 && hi - lo >
20         eps; ++stp){
21         double m = (lo+hi)/2.0;
22         int sign_mid = sign(get(coef,m));
23         if(!sign_mid) return m;
24         if(sign_lo*sign_mid < 0) hi = m;
25         else lo = m;
26     }
27     return (lo+hi)/2.0;
28 }
29 vector<double> cal(vector<double>coef, int n
30 ){
31     vector<double>res;
32     if(n == 1){
33         if(sign(coef[1])) res.pb(-coef[0]/coef
34             [1]);
35         return res;
36     }
37     vector<double>dcoef(n);
38     for(int i = 0; i < n; ++i) dcoef[i] = coef
39         [i+1]*(i+1);
40     vector<double>droot = cal(dcoef, n-1);
41     droot.insert(droot.begin(), -INF);
42     droot.pb(INF);
43     for(int i = 0; i+1 < droot.size(); ++i){
44         double tmp = find(coef, n, droot[i],
45             droot[i+1]);
46         if(tmp < INF) res.pb(tmp);
47     }
48     return res;
49 }
50 int main () {
51     vector<double>ve;
52     vector<double>ans = cal(ve, n);
53     // 視情況把答案 +eps 避免 -0
54 }

```

8.6 FWT.cpp

```

1 vector<int> F_OR_T(vector<int> f, bool
2     inverse){
3     for(int i=0; (2<<i)<=f.size(); ++i)
4         for(int j=0; j<f.size(); j+=2<<i)
5             for(int k=0; k<(1<<i); ++k)
6                 f[j+k+(1<<i)] += f[j+k]*(inverse
7                     ?-1:1);
8     return f;
9 }
10 vector<int> rev(vector<int> A) {
11     for(int i=0; i<A.size(); i+=2)
12         swap(A[i],A[i^(A.size()-1)]);
13     return A;
14 }

```

```

12 }
13 vector<int> F_AND_T(vector<int> f, bool
14     inverse){
15     return rev(F_OR_T(rev(f), inverse));
16 }
17 vector<int> F_XOR_T(vector<int> f, bool
18     inverse){
19     for(int i=0; (2<<i)<=f.size(); ++i)
20         for(int j=0; j<f.size(); j+=2<<i)
21             for(int k=0; k<(1<<i); ++k){
22                 int u=f[j+k], v=f[j+k+(1<<i)];
23                 f[j+k+(1<<i)] = u-v, f[j+k] = u+v;
24             }
25     if(inverse) for(auto &a:f) a/=f.size();
26     return f;
27 }

```

8.7 LinearCongruence.cpp

```

1 pair<LL,LL> LinearCongruence(LL a[],LL b[],
2     LL m[],int n) {
3     // a[i]*x = b[i] ( mod m[i] )
4     for(int i=0;i<n;++i) {
5         LL x, y, d = extgcd(a[i],m[i],x,y);
6         if(b[i]%d!=0) return make_pair(-1LL,0LL);
7         m[i] /= d;
8         b[i] = LLmul(b[i]/d,x,m[i]);
9     }
10    LL lastb = b[0], lastm = m[0];
11    for(int i=1;i<n;++i) {
12        LL x, y, d = extgcd(m[i],lastm,x,y);
13        if((lastb-b[i])%d!=0) return make_pair
14            (-1LL,0LL);
15        lastb = LLmul((lastb-b[i])/d,x,(lastm/d)
16            )*m[i];
17        lastm = (lastm/d)*m[i];
18        lastb = (lastb+b[i])%lastm;
19    }
20    return make_pair(lastb<0?lastb+lastm:lastb
21        ,lastm);
22 }

```

8.8 Lucas.cpp

```

1 int mod_fact(int n,int &e){
2     e=0;
3     if(n==0)return 1;
4     int res=mod_fact(n/P,e);
5     e += n/P;
6     if((n/P)%2==0)return res*fact[n%P]%P;
7     return res*(P-fact[n%P])%P;
8 }
9 int Cmod(int n,int m){
10    int a1,a2,a3,e1,e2,e3;
11    a1=mod_fact(n,e1);
12    a2=mod_fact(m,e2);
13    a3=mod_fact(n-m,e3);
14    if(e1>e2+e3)return 0;
15    return a1*inv(a2*a3%P,P)%P;
16 }

```

```
16 }
```

8.9 Matrix.cpp

```

1 template<typename T>
2 struct Matrix{
3     using rt = std::vector<T>;
4     using mt = std::vector<vector<T>>;
5     using matrix = Matrix<T>;
6     int r,c;
7     mt m;
8     Matrix(int r,int c):r(r),c(c),m(r,rt(c)){
9         rt& operator[](int i){return m[i];}
10        matrix operator+(const matrix &a){
11            matrix rev(r,c);
12            for(int i=0;i<r;++i)
13                for(int j=0;j<c;++j)
14                    rev[i][j]=m[i][j]+a.m[i][j];
15            return rev;
16        }
17        matrix operator-(const matrix &a){
18            matrix rev(r,c);
19            for(int i=0;i<r;++i)
20                for(int j=0;j<c;++j)
21                    rev[i][j]=m[i][j]-a.m[i][j];
22            return rev;
23        }
24        matrix operator*(const matrix &a){
25            matrix rev(r,a.c);
26            matrix tmp(a.c,a.r);
27            for(int i=0;i<a.r;++i)
28                for(int j=0;j<a.c;++j)
29                    tmp[j][i]=a.m[i][j];
30            for(int i=0;i<r;++i)
31                for(int j=0;j<a.c;++j)
32                    for(int k=0;k<c;++k)
33                        rev.m[i][j]+=m[i][k]*tmp[j][k];
34            return rev;
35        }
36        bool inverse(){
37            Matrix t(r,r+c);
38            for(int y=0;y<r;y++){
39                t.m[y][c+y] = 1;
40                for(int x=0;x<c;++x)
41                    t.m[y][x]=m[y][x];
42            }
43            if( !t.gas() )
44                return false;
45            for(int y=0;y<r;y++){
46                for(int x=0;x<c;++x)
47                    m[y][x]=t.m[y][c+x]/t.m[y][y];
48            }
49            return true;
50        }
51        T gas(){
52            vector<T> lazy(r,1);
53            bool sign=false;
54            for(int i=0;i<r;++i){
55                if( m[i][i]==0 ){
56                    int j=i+1;
57                    while(j<r&&!m[j][i])j++;
58                    if(j==r)continue;
59                    if(j!=i)swap(m[j][i],m[i][j]);
60                    sign=!sign;
61                }
62            }
63            for(int i=0;i<r;++i){
64                if( m[i][i]==0 )continue;
65                lazy[i]=lazy[i]*m[i][i];
66                T mx=m[j][i];
67                for(int k=0;k<c;++k)
68                    m[j][k]=m[j][k]*m[i][i]-m[i][k]*mx;
69            }
70            T det=sign?-1:1;
71            for(int i=0;i<r;++i){
72                det = det*m[i][i];
73                for(auto &j:m[i])j/=lazy[i];
74            }
75            return det;
76        }
77    };

```

8.10 MillerRobin.cpp

```

1 LL LLmul(LL a, LL b, const LL &mod) {
2     LL ans=0;
3     while(b) {
4         if(b&1) {
5             ans+=a;
6             if(ans>=mod) ans-=mod;
7         }
8         a<<=1, b>>=1;
9         if(a>=mod) a-=mod;
10    }
11    return ans;
12 }
13 LL mod_mul(LL a,LL b,LL m){
14     a%=m,b%=m; /* fast for m < 2^58 */
15     LL y=(LL)((double)a*b/m+0.5);
16     LL r=(a*b-y*m)%m;
17     return r<0?r+m:r;
18 }
19 template<typename T>
20 T pow(T a,T b,T mod){ //a^b%mod
21     T ans=1;
22     for(;b;a=mod_mul(a,a,mod),b>>=1)
23         if(b&1)ans=mod_mul(ans,a,mod);
24     return ans;
25 }
26 int sprp[3]={2,7,61}; //int範圍可解
27 int llsp[3]={2,325,9375,28178,450775,9780504,
28     1795265022}; //至少 unsigned long long範圍
29 template<typename T>
30 bool isprime(T n,int *sprp,int num){
31     if(n==2)return 1;
32     if(n<2||n%2==0)return 0;
33     int t=0;
34     T u=n-1;
35     for(;u%2==0;++t)u>>=1;
36     for(int i=0;i<num;++i){
37         T a=sprp[i]%n;
38         if(a==0||a==1||a==n-1)continue;
39         T x=pow(a,u,n);
40         for(int j=1;j<t;++j){
41             if(x==1)continue;
42             x=x*x%n;
43         }
44         if(x!=n-1)return 0;
45     }
46     return 1;
47 }

```


8.11 NTT.cpp

```

40 if(x==1||x==n-1)continue;
41 for(int j=0;j<t;++j){
42     x=mod_mul(x,x,n);
43     if(x==1)return 0;
44     if(x==n-1)break;
45 }
46 if(x==n-1)continue;
47 return 0;
48 }
49 return 1;
50 }

2615053605667*(2^18)+1,3
15*(2^27)+1,31
479*(2^21)+1,3
7*17*(2^23)+1,3
3*3*211*(2^19)+1,5
25*(2^22)+1,3
template<typename T,typename VT=vector<T> >
struct NTT{
8     const T P,G;
9     NTT(T p=(1<<23)*7*17+1,T g=3):P(p),G(g){}
10     unsigned bit_reverse(unsigned a,int len){
11         //Look FFT.cpp
12     }
13     T pow_mod(T n,T k,T m){
14         T ans=1;
15         for(n=(n>=m?n%m:n);k>=>1){
16             if(k&1)ans=ans*n%m;
17             n=n*n%m;
18         }
19         return ans;
20     }
21     void ntt(bool is_inv,VT &in,VT &out,int N)
22     {
23         int bitlen=__lg(N);
24         for(int i=0;i<N;++i)out[bit_reverse(i,
25             bitlen)]=in[i];
26         for(int step=2,id=1;step<=N;step<=1,++
27             id){
28             T wn=pow_mod(G,(P-1)>>id,P),wi=1,u,t;
29             const int mh=step>>1;
30             for(int i=0;i<mh;++i){
31                 for(int j=i;j<N;j+=step){
32                     u=out[j],t=wi*out[j+mh]%P;
33                     out[j]=u+t;
34                     out[j+mh]=u-t;
35                     if(out[j]>=P)out[j]-=P;
36                     if(out[j+mh]<0)out[j+mh]+=P;
37                 }
38                 wi=wi*wn%P;
39             }
40             if(is_inv){
41                 for(int i=1;i<N/2;++i)swap(out[i],out[
42                     N-i]);
43                 T invn=pow_mod(N,P-2,P);
44                 for(int i=0;i<N;++i)out[i]=out[i]*invn
45                     %P;
46             }
47         }
48     }
49 }

```

8.12 Simpson.cpp

```

45 | };

1 double simpson(double a,double b){
2     double c=a+(b-a)/2;
3     return (F(a)+4*F(c)+F(b))*(b-a)/6;
4 }
5 double asr(double a,double b,double eps,
6     double A){
7     double c=a+(b-a)/2;
8     double L=simpson(a,c),R=simpson(c,b);
9     if( abs(L+R-A)<15*eps )
10         return L+R+(L+R-A)/15.0;
11     return asr(a,c,eps/2,L)+asr(c,b,eps/2,R);
12 }
13 double asr(double a,double b,double eps){
14     return asr(a,b,eps,simpson(a,b));
15 }

```

8.13 外星模運算.cpp

```

1 //a[0]^a[1]^a[2]^...
2 #define maxn 100000
3 int euler[maxn+5];
4 bool is_prime[maxn+5];
5 void init_euler(){
6     is_prime[1]=1; //不是質數
7     for(int i=1;i<=maxn;i++)euler[i]=i;
8     for(int i=2;i<=maxn;i++){
9         if(!is_prime[i]){//是質數
10             euler[i]--;
11             for(int j=i<1;j<=maxn;j+=i){
12                 is_prime[j]=1;
13                 euler[j]=euler[j]/i*(i-1);
14             }
15         }
16     }
17 }
18 LL pow(LL a,LL b,LL mod){ //a^b%mod
19     LL ans=1;
20     for(;b;a=a*a%mod,b>>=1)
21         if(b&1)ans=ans*a%mod;
22     return ans;
23 }
24 bool isless(LL *a,int n,int k){
25     if(*a==1)return k>1;
26     if(--n==0)return *a<k;
27     int next=0;
28     for(LL b=1;b<k;next)
29         b*=*a;
30     return isless(a+1,n,next);
31 }
32 LL high_pow(LL *a,int n,LL mod){
33     if(*a==1||--n==0)return *a%mod;
34     int k=0,r=euler[mod];
35     for(LL tma=1;tma!=pow(*a,k+r,mod);++k)
36         tma=tma*(*a)%mod;
37     if(isless(a+1,n,k))return pow(*a,high_pow(
38         a+1,n,k),mod);
39 }

```

```

38 int tmd=high_pow(a+1,n,r), t=(tmd-k+r)%r;
39 return pow(*a,k+t,mod);
40 }
41 LL a[1000005];
42 int t,mod;
43 int main(){
44     init_euler();
45     scanf("%d",&t);
46     #define n 4
47     while(t--){
48         for(int i=0;i<n;++i)scanf("%lld",&a[i]);
49         scanf("%d",&mod);
50         printf("%lld\n",high_pow(a,n,mod));
51     }
52     return 0;
53 }

```

8.14 質因數分解.cpp

```

1 LL func(const LL n,const LL mod,const int c)
2 {
3     return (LLmul(n,n,mod)+c+mod)%mod;
4 }
5 LL pollorro(const LL n, const int c) { //循環
6     環節長度
7     LL a=1, b=1;
8     a=func(a,n,c)%n;
9     b=func(b,n,c)%n; b=func(b,n,c)%n;
10    while(gcd(abs(a-b),n)==1) {
11        a=func(a,n,c)%n;
12        b=func(b,n,c)%n; b=func(b,n,c)%n;
13    }
14    return gcd(abs(a-b),n);
15 }
16 void prefactor(LL &n, vector<LL> &v) {
17     for(int i=0;i<12;++i) {
18         while(n%prime[i]==0) {
19             v.push_back(prime[i]);
20             n/=prime[i];
21         }
22     }
23 }
24 void smallfactor(LL n, vector<LL> &v) {
25     if(n<MAXPRIME) {
26         while(isp[(int)n]) {
27             v.push_back(isp[(int)n]);
28             n/=isp[(int)n];
29         }
30         v.push_back(n);
31     } else {
32         for(int i=0;i<primecnt&&prime[i]*prime[i]
33             ]<=n;++i) {
34             while(n%prime[i]==0) {
35                 v.push_back(prime[i]);
36                 n/=prime[i];
37             }
38         }
39         if(n!=1) v.push_back(n);
40     }
41 }
42 void comfactor(const LL &n, vector<LL> &v) {
43     if(n<1e9) {
44         smallfactor(n,v);
45         return;
46     }
47     if(Isprime(n)) {
48         v.push_back(n);
49         return;
50     }
51     LL d;
52     for(int c=3; c<=n; ++c) {
53         d = pollorro(n,c);
54         if(d!=n) break;
55     }
56     comfactor(d,v);
57     comfactor(n/d,v);
58 }
59 }
60 void Factor(const LL &x, vector<LL> &v) {
61     LL n = x;
62     if(n==1) { puts("Factor 1"); return; }
63     prefactor(n,v);
64     if(n==1) return;
65     comfactor(n,v);
66     sort(v.begin(),v.end());
67 }
68 }
69 void AllFactor(const LL &n,vector<LL> &v) {
70     vector<LL> tmp;
71     Factor(n,tmp);
72     v.clear();
73     v.push_back(1);
74     int len;
75     LL now=1;
76     for(int i=0;i<tmp.size();++i) {
77         if(i==0 || tmp[i]!=tmp[i-1]) {
78             len = v.size();
79             now = 1;
80         }
81         now*=tmp[i];
82         for(int j=0;j<len;++j)
83             v.push_back(v[j]*now);
84     }
85 }
86 }

```

9 other

9.1 WhatDay.cpp

```

1 int whatday(int y,int m,int d){
2     if(m<=2)m+=12,--y;
3     if(y<1752||y==1752&&m<9||y==1752&&m==9&&d
4         <3)
5         return (d+2*m+3*(m+1)/5+y+y/4+y/100+y/400)
6             %7;
7 }

```

9.2 上下最大正方形.cpp

```

1 void solve(int n,int a[],int b[]){// 1-base
2     int ans=0;
3     deque<int>da,db;
4     for(int l=1,r=1;r<=n;++r){
5         while(da.size()&&a[da.back()]>=a[r]){
6             da.pop_back();
7         }
8         da.push_back(r);
9         while(db.size()&&b[db.back()]>=b[r]){
10            db.pop_back();
11        }
12        db.push_back(r);
13        for(int d=a[da.front()]+b[db.front()];r-
14            1>d;d--){
15            if(da.front()==l)da.pop_front();
16            if(db.front()==l)db.pop_front();
17            if(da.size()&&db.size()){
18                d=a[da.front()]+b[db.front()];
19            }
20        }
21        ans=max(ans,r-l+1);
22    }
23    printf("%d\n",ans);
}

```

9.3 最大矩形.cpp

```

1 LL max_rectangle(vector<int> s){
2     stack<pair<int,int> > > st;
3     st.push(make_pair(-1,0));
4     s.push_back(0);
5     LL ans=0;
6     for(size_t i=0;i<s.size();++i){
7         int h=s[i];
8         pair<int,int> now=make_pair(h,i);
9         while(h<st.top().first){
10            now=st.top();
11            st.pop();
12            ans=max(ans,(LL)(i-now.second)*now.first);
13        }
14        if(h>st.top().first){
15            st.push(make_pair(h,now.second));
16        }
17    }
18    return ans;
19 }

```

10 String

10.1 AC 自動機.cpp

```

1 template<char L='a',char R='z'>
2 class ac_automaton{
3     struct joe{

```

```

4         int next[R-L+1],fail,efl,ed,cnt_dp,vis;
5         joe():ed(0),cnt_dp(0),vis(0){
6             for(int i=0;i<=R-L;++i)next[i]=0;
7         }
8     };
9     public:
10        std::vector<joe> S;
11        std::vector<int> q;
12        int qs,qe,vt;
13        ac_automaton():S(1),qs(0),qe(0),vt(0){
14            void clear(){
15                q.clear();
16                S.resize(1);
17                for(int i=0;i<=R-L;++i)S[0].next[i]=0;
18                S[0].cnt_dp=S[0].vis=qs=qe=vt=0;
19            }
20            void insert(const char *s){
21                int o=0;
22                for(int i=0,id;s[i];++i){
23                    id=s[i]-L;
24                    if(!S[o].next[id]){
25                        S.push_back(joe());
26                        S[o].next[id]=S.size()-1;
27                    }
28                    o=S[o].next[id];
29                }
30                ++S[o].ed;
31            }
32            void build_fail(){
33                S[0].fail=S[0].efl=-1;
34                q.clear();
35                q.push_back(0);
36                ++qe;
37                while(qs!=qe){
38                    int pa=q[qs++],id,t;
39                    for(int i=0;i<=R-L;++i){
40                        t=S[pa].next[i];
41                        if(!t)continue;
42                        id=S[pa].fail;
43                        while(~id&&!S[id].next[i])id=S[id].fail;
44                        S[t].fail=~id?S[id].next[i]:0;
45                        S[t].efl=S[S[t].fail].ed?S[t].fail:S[t].fail;efl;
46                        q.push_back(t);
47                        ++qe;
48                    }
49                }
50            }
51            /*DP出每個前綴在字串s出現的次數並傳回所有
52                字串被s匹配成功的次數O(N*M)*/
53            int match_0(const char *s){
54                int ans=0,id,p=0,t;
55                for(i=0;s[i];++i){
56                    id=s[i]-L;
57                    while(!S[p].next[id]&&p=S[p].fail;
58                        if(!S[p].next[id])continue;
59                    p=S[p].next[id];
60                    ++S[p].cnt_dp; /*匹配成功則它所有後綴都
61                        可以被匹配(DP計算)*/
62                }
63                for(i=qe-1;i>=0;--i){
64                    ans+=S[q[i]].cnt_dp*S[q[i]].ed;
65                    if(~S[q[i]].fail)S[q[i]].fail.
66                        cnt_dp+=S[q[i]].cnt_dp;

```

```

64                }
65                return ans;
66            }
67            /*多串匹配走efl邊並傳回所有字串被s匹配成功
68                的次數O(N*M^1.5)*/
69            int match_1(const char *s)const{
70                int ans=0,id,p=0,t;
71                for(int i=0;s[i];++i){
72                    id=s[i]-L;
73                    while(!S[p].next[id]&&p=S[p].fail;
74                        if(!S[p].next[id])continue;
75                    p=S[p].next[id];
76                    if(S[p].ed)ans+=S[p].ed;
77                    for(t=S[p].efl;~t;t=S[t].efl){
78                        ans+=S[t].ed; /*因為都走efl邊所以保證
79                            匹配成功*/
80                    }
81                }
82                return ans;
83            }
84            /*枚舉(s的子串nA)的所有相異字串各恰一次
85                並傳回次數O(N*M^(1/3))*/
86            int match_2(const char *s){
87                int ans=0,id,p=0,t;
88                ++vt;
89                /*把截記vt+=1，只要vt沒溢位，所有S[p].
90                    vis=vt就會變成false
91                這種利用vt的方法可以O(1)歸零vis陣列*/
92                for(int i=0;s[i];++i){
93                    id=s[i]-L;
94                    while(!S[p].next[id]&&p=S[p].fail;
95                        if(!S[p].next[id])continue;
96                    p=S[p].next[id];
97                    if(S[p].ed&&S[p].vis!=vt){
98                        S[p].vis=vt;
99                        ans+=S[p].ed;
100                    }
101                }
102                return ans;
103            }
104            /*把AC自動機變成真的自動機*/
105            void evolution(){
106                for(qs=1;qs!=qe;){
107                    int p=q[qs++];
108                    for(int i=0;i<=R-L;++i)
109                        if(S[p].next[i]==0)S[p].next[i]=S[S[p].fail].next[i];
110                }
111            }
112        };

```

10.2 hash.cpp

```

1 #define MAXN 1000000
2 #define mod 1073676287

```

```

3 /*mod 必須要是質數*/
4 typedef long long T;
5 char s[MAXN+5];
6 T h[MAXN+5]; /*hash陣列*/
7 T h_base[MAXN+5]; /*h_base[n]=(prime^n)%mod*/
8 void hash_init(int len,T prime){
9     h_base[0]=1;
10    for(int i=1;i<=len;++i){
11        h[i]=(h[i-1]*prime+s[i-1])%mod;
12        h_base[i]=(h_base[i-1]*prime)%mod;
13    }
14 }
15 T get_hash(int l,int r){/*閉區間寫法，設編號
16     為0 ~ Len-1*/
17    return (h[r+1]-(h[l]*h_base[r-l+1])%mod+
18        mod)%mod;
19 }

```

10.3 KMP.cpp

```

1 /*產生fail function*/
2 void kmp_fail(char *s,int len,int *fail){
3     int id=-1;
4     fail[0]=-1;
5     for(int i=1;i<len;++i){
6         while(~id&&s[id+1]!=s[i])id=fail[id];
7         if(s[id+1]==s[i])++id;
8         fail[i]=id;
9     }
10 }
11 /*以字串B匹配字串A，傳回匹配成功的數量(用B的
12     fail)*/
13 int kmp_match(char *A,int lenA,char *B,int
14     lenB,int *fail){
15     int id=-1,ans=0;
16     for(int i=0;i<lenA;++i){
17         while(~id&&B[id+1]!=A[i])id=fail[id];
18         if(B[id+1]==A[i])++id;
19         if(id==lenB-1){/*匹配成功*/
20             ++ans, id=fail[id];
21         }
22     }
23     return ans;
24 }

```

10.4 manacher.cpp

```

1 //原字串: asdsasdsa
2 //先把字串變成這樣: @#a#s#d#s#a#s#d#s#a#
3 void manacher(char *s,int len,int *z){
4     int l=0,r=0;
5     for(int i=1;i<len;++i){
6         z[i]=r>i?min(z[2*i-l],r-i):1;
7         while(s[i+z[i]]==s[i-z[i]])++z[i];
8         if(z[i]>r)r=z[i]+i,l=i;
9     }
10    //ans = max(z)-1
11 }

```

10.5 minimal_string_rotation.cpp

```

1 int min_string_rotation(const string &s){
2     int n=s.size(),i=0,j=1,k=0;
3     while(i<n&&j<n&&k<n){
4         int t=s[(i+k)%n]-s[(j+k)%n];
5         ++k;
6         if(t){
7             if(t>0)i+=k;
8             else j+=k;
9             if(i==j)++j;
10            k=0;
11        }
12    }
13    return min(i,j); //最小循環表示法起始位置
14 }

```

10.6 reverseBWT.cpp

```

1 const int MAXN = 305, MAXC = 'Z';
2 int ranks[MAXN], tots[MAXC], first[MAXC];
3 void rankBWT(const string &bw){
4     memset(ranks,0,sizeof(int)*bw.size());
5     memset(tots,0,sizeof(tots));
6     for(size_t i=0;i<bw.size();++i)
7         ranks[i] = tots[bw[i]]++;
8 }
9 void firstCol(){
10    memset(first,0,sizeof(first));
11    int totc = 0;
12    for(int c='A';c<='Z';++c){
13        if(!tots[c]) continue;
14        first[c] = totc;
15        totc += tots[c];
16    }
17 }
18 string reverseBwt(string bw,int begin){
19     rankBWT(bw), firstCol();
20     int i = begin; //原字串最後一個元素的位置
21     string res;
22     do{
23         char c = bw[i];
24         res = c + res;
25         i = first[int(c)] + ranks[i];
26     }while( i != begin );
27     return res;
28 }

```

10.7 suffix_array_lcp.cpp

```

1 #define radix_sort(x,y){\
2     for(i=0;i<A;++i)c[i]=0;\
3     for(i=0;i<n;++i)c[x[y[i]]]++;\
4     for(i=1;i<A;++i)c[i]+=c[i-1];\
5     for(i=n-1;~i;--i)sa[--c[x[y[i]]]]=y[i];\
6 }
7 #define AC(r,a,b)\
8     r[a]!=(r[b]||a+k>n)||r[a+k]!=r[b+k]

```

```

1 void suffix_array(const char *s,int n,int *
2     sa,int *rank,int *tmp,int *c){
3     int A='Z'+1,i,k,id=0;
4     for(i=0;i<n;++i)rank[tmp[i]]=s[i];
5     radix_sort(rank,tmp);
6     for(k=1;id<n-1;k<=1){
7         for(id=0,i=n-k;i<n;++i)tmp[id++]=i;
8         for(i=0;i<n;++i)
9             if(sa[i]>k)tmp[id++]=sa[i]-k;
10        radix_sort(rank,tmp);
11        swap(rank,tmp);
12        for(rank[sa[0]]=id=0,i=1;i<n;++i)
13            rank[sa[i]]=id+AC(tmp,sa[i-1],sa[i]);
14        A=id+1;
15    }
16 }
17 //h:高度數組 sa:後綴數組 rank:排名
18 void suffix_array_lcp(const char *s,int len,
19     int *h,int *sa,int *rank){
20     for(int i=0;i<len;++i)rank[sa[i]]=i;
21     for(int i=0,k=0;i<len;++i){
22         if(rank[i]==0)continue;
23         if(k--<0)
24             while(s[i+k]==s[sa[rank[i]-1]+k])++k;
25         h[rank[i]]=k;
26     }
27     h[0]=0; // h[k]=Lcp(sa[k],sa[k-1]);
28 }

```

10.8 Z.cpp

```

1 void z_alg(char *s,int len,int *z){
2     int l=0,r=0;
3     z[0]=len;
4     for(int i=1;i<len;++i){
5         z[i]=i>r?0:(i-l+z[i-l]<z[l]?z[i-l]:r-i
6             +1);
7         while(i+z[i]<len&&s[i+z[i]]==s[z[i]])++z[i];
8         if(i+z[i]-1>r)r=i+z[i]-1,l=i;
9     }
10 }

```

11 Tarjan

11.1 dominator_tree.cpp

```

1 struct dominator_tree{
2     static const int MAXN=5005;
3     int n; // 1-base
4     vector<int> suc[MAXN],pre[MAXN];
5     int fa[MAXN],dfn[MAXN],id[MAXN],Time;
6     int semi[MAXN],idom[MAXN];
7     int anc[MAXN],best[MAXN]; //disjoint set
8     vector<int> dom[MAXN]; //dominator_tree
9     void init(int _n){
10         n=_n;

```

```

11     for(int i=1;i<=n;++i)suc[i].clear(),pre[
12         i].clear();
13     }
14     void add_edge(int u,int v){
15         suc[u].push_back(v);
16         pre[v].push_back(u);
17     }
18     void dfs(int u){
19         dfn[u]=++Time,id[Time]=u;
20         for(auto v:suc[u]){
21             if(dfn[v])continue;
22             dfs(v),fa[dfn[v]]=dfn[u];
23         }
24     }
25     int find(int x){
26         if(x==anc[x])return x;
27         int y=find(anc[x]);
28         if(semi[best[x]]>semi[best[anc[x]]])best
29             [x]=best[anc[x]];
30         return anc[x]=y;
31     }
32     void tarjan(int r){
33         Time=0;
34         for(int t=1;t<=n;++t){
35             dfn[t]=idom[t]=0; //u=r或是u無法到達r時
36             idom[id[u]]=0
37             dom[t].clear();
38             anc[t]=best[t]=semi[t]=t;
39         }
40         dfs(r);
41         for(int y=Time;y>=2;--y){
42             int x=fa[y],idy=id[y];
43             for(auto z:pre[idy]){
44                 if(!z=dfn[z])continue;
45                 find(z);
46                 semi[y]=min(semi[y],semi[best[z]]);
47                 dom[semi[y]].push_back(y);
48                 anc[y]=x;
49                 for(auto z:dom[x]){
50                     find(z);
51                     idom[z]=semi[best[z]]<x?best[z]:x;
52                 }
53                 dom[x].clear();
54             }
55             for(int u=2;u<=Time;++u){
56                 if(idom[u]!=semi[u])idom[u]=idom[idom[
57                     u]];
58                 dom[id[idom[u]]].push_back(id[u]);
59             }
60         }
61     }
62     }
63     }
64     }
65     }
66     }
67     }
68     }
69     }
70     }
71     }
72     }
73     }
74     }
75     }
76     }
77     }
78     }
79     }
80     }
81     }
82     }
83     }
84     }
85     }
86     }
87     }
88     }
89     }
90     }
91     }
92     }
93     }
94     }
95     }
96     }
97     }
98     }
99     }
100    }

```

11.2 tnfsb017_2_sat.cpp

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 #define MAXN 8001
4 #define MAXN2 MAXN*4
5 #define n(X) ((X)+2*N)
6 vector<int> v[MAXN2], rv[MAXN2], vis_t;
7 int N,M;
8 void addedge(int s,int e){

```

```

9     v[s].push_back(e);
10    rv[e].push_back(s);
11 }
12 int scc[MAXN2];
13 bool vis[MAXN2]={false};
14 void dfs(vector<int> *uv,int n,int k=-1){
15     vis[n]=true;
16     for(int i=0;i<uv[n].size();++i)
17         if(!vis[uv[n][i]])
18             dfs(uv,uv[n][i],k);
19     if(uv==v)vis_t.push_back(n);
20     scc[n]=k;
21 }
22 void solve(){
23     for(int i=1;i<=N;++i){
24         if(!vis[i])dfs(v,i);
25         if(!vis[n(i)])dfs(v,n(i));
26     }
27     memset(vis,0,sizeof(vis));
28     int c=0;
29     for(int i=vis_t.size()-1;i>=0;--i)
30         if(!vis[vis_t[i]])
31             dfs(rv,vis_t[i],c++);
32 }
33 int main(){
34     int a,b;
35     scanf("%d%d",&N,&M);
36     for(int i=1;i<=N;++i){
37         // (A or B)&(A & B) A^B
38         a=i*2-1;
39         b=i*2;
40         addedge(n(a),b);
41         addedge(n(b),a);
42         addedge(a,n(b));
43         addedge(b,n(a));
44     }
45     while(M--){
46         scanf("%d%d",&a,&b);
47         a = a>0?a*2-1:-a*2;
48         b = b>0?b*2-1:-b*2;
49         // A or B
50         addedge(n(a),b);
51         addedge(n(b),a);
52     }
53     solve();
54     bool check=true;
55     for(int i=1;i<=2*N;++i)
56         if(scc[i]==scc[n(i)])
57             check=false;
58     if(check){
59         printf("%d\n",N);
60         for(int i=1;i<=2*N;i+=2){
61             if(scc[i]>scc[i+2*N]) putchar('+');
62             else putchar('-');
63         }
64         puts("");
65     }else puts("0");
66     return 0;
67 }

```

11.3 橋連通分量.cpp

```

1 #define N 1005
2 struct edge{
3     int u,v;
4     bool is_bridge;
5     edge(int u,int v=0):u(u),v(v),is_bridge
6         (0){}
7 };
8 vector<edge> E;
9 vector<int> G[N]; // 1-base
10 int low[N],vis[N],Time;
11 int bcc_id[N],bridge_cnt,bcc_cnt; // 1-base
12 int st[N],top; // BCC用
13 inline void add_edge(int u,int v){
14     G[u].push_back(E.size());
15     E.push_back(edge(u,v));
16     G[v].push_back(E.size());
17     E.push_back(edge(v,u));
18 }
19 void dfs(int u,int re=-1){ // u當前點, re為u連
20     接前一個點的邊
21     int v;
22     low[u]=vis[u]=++Time;
23     st[top++]=u;
24     for(size_t i=0;i<G[u].size();++i){
25         int e=G[u][i];v=E[e].v;
26         if(!vis[v]){
27             dfs(v,e^1); // e^1 反向邊
28             low[u]=min(low[u],low[v]);
29             if(vis[u]<low[v]){
30                 E[e].is_bridge=E[e^1].is_bridge=1;
31                 ++bridge_cnt;
32             }
33         }else if(vis[v]<vis[u]&&v!=re){
34             low[u]=min(low[u],vis[v]);
35         }
36     }
37     if(vis[u]==low[u]){ // 處理BCC
38         ++bcc_cnt; // 1-base
39         do bcc_id[v=st[--top]]=bcc_cnt; // 每個點
40             所在的BCC
41         while(v!=u);
42     }
43 }
44 inline void bcc_init(int n){
45     Time=bcc_cnt=bridge_cnt=top=0;
46     E.clear();
47     for(int i=1;i<=n;++i){
48         G[i].clear();
49         vis[i]=bcc_id[i]=0;
50     }
51 }

```

11.4 雙連通分量 & 割點.cpp

```

1 #define N 1005
2 vector<int> G[N]; // 1-base
3 vector<int> bcc[N]; // 存每塊雙連通分量的點
4 int low[N],vis[N],Time;
5 int bcc_id[N],bcc_cnt; // 1-base
6 bool is_cut[N]; // 是否為割點
7 int st[N],top;
8 void dfs(int u,int pa=-1){ // u當前點, pa父親

```

```

9     int v,child=0;
10     low[u]=vis[u]=++Time;
11     st[top++]=u;
12     for(size_t i=0;i<G[u].size();++i){
13         if(!vis[v=G[u][i]]){
14             dfs(v,u,++child;
15             low[u]=min(low[u],low[v]);
16             if(vis[u]<=low[v]){
17                 is_cut[u]=1;
18                 bcc[++bcc_cnt].clear();
19                 int t;
20                 do{
21                     bcc_id[t=st[--top]]=bcc_cnt;
22                     bcc[bcc_cnt].push_back(t);
23                 }while(t!=v);
24                 bcc_id[u]=bcc_cnt;
25                 bcc[bcc_cnt].push_back(u);
26             }
27             else if(vis[v]<vis[u]&&v!=pa){ // 反向邊
28                 low[u]=min(low[u],vis[v]);
29             }
30         }
31         if(pa!=-1&&child<2)is_cut[u]=0; // u是dfs樹
32             的根要特判
33     }
34 }
35 inline void bcc_init(int n){
36     Time=bcc_cnt=top=0;
37     for(int i=1;i<=n;++i){
38         G[i].clear();
39         is_cut[i]=vis[i]=bcc_id[i]=0;
40     }
41 }

```

12 Tree_problem

12.1 HeavyLight.cpp

```

1 #include<vector>
2 #define MAXN 100005
3 int siz[MAXN],max_son[MAXN],pa[MAXN],dep[
4     MAXN];
5 int link_top[MAXN],link[MAXN],cnt;
6 vector<int> G[MAXN];
7 void find_max_son(int u){
8     siz[u]=1;
9     max_son[u]=-1;
10     for(auto v:G[u]){
11         if(v==pa[u])continue;
12         pa[v]=u;
13         dep[v]=dep[u]+1;
14         find_max_son(v);
15         if(max_son[u]==-1||siz[v]>siz[max_son[u]]
16             )max_son[u]=v;
17         siz[u]+=siz[v];
18     }
19 }
20 void build_link(int u,int top){
21     link[u]=++cnt;
22     link_top[u]=top;
23     if(max_son[u]==-1)return;
24     build_link(max_son[u],top);

```

```

23     for(auto v:G[u]){
24         if(v==max_son[u]||v==pa[u])continue;
25         build_link(v,v);
26     }
27 }
28 int find_lca(int a,int b){
29     // 求LCA, 可以在過程中對區間進行處理
30     int ta=link_top[a],tb=link_top[b];
31     while(ta!=tb){
32         if(dep[ta]>dep[tb]){
33             swap(ta,tb);
34             swap(a,b);
35         }
36         // 這裡可以對a所在的鏈做區間處理
37         // 區間為(Link[ta],Link[a])
38         ta=link_top[a=pa[ta]];
39     }
40     // 最後a,b會在同一條鏈, 若a!=b還要在進行一
41     次區間處理
42     return dep[a]<dep[b]?a:b;

```

12.2 LCA.cpp

```

1 #define MAXN 100000
2 #define MAX_LOG 17
3 int pa[MAX_LOG+1][MAXN+5];
4 int dep[MAXN+5];
5 vector<int> G[MAXN+5];
6 void dfs(int x,int p){ // dfs(1,-1);
7     pa[0][x]=p;
8     for(int i=0;i+1<MAX_LOG;++i)pa[i+1][x]=pa[
9         i][pa[i][x]];
10     for(auto &i:G[x]){
11         if(i==p)continue;
12         dep[i]=dep[x]+1;
13         dfs(i,x);
14     }
15 }
16 inline int jump(int x,int d){
17     for(int i=0;i<d;++i)if((x>>i)&1)x=pa[i][x];
18     return x;
19 }
20 inline int find_lca(int a,int b){
21     if(dep[a]>dep[b])swap(a,b);
22     b=jump(b,dep[b]-dep[a]);
23     if(a==b)return a;
24     for(int i=MAX_LOG;i>0;--i){
25         if(pa[i][a]!=pa[i][b]){
26             a=pa[i][a];
27             b=pa[i][b];
28         }
29     }
30     return pa[0][a];

```

12.3 link_cut_tree.cpp

```

1 struct splay_tree{
2     int ch[2],pa; // 子節點跟父母
3     bool rev; // 反轉的懶惰標記
4     splay_tree():pa(0),rev(0){ch[0]=ch[1]=0;}
5 };
6 vector<splay_tree> nd;
7 // 有的時候用vector會TLE, 要注意
8 // 這邊以node[0]作為null節點
9 bool isroot(int x){ // 判斷是否為這棵splay
10     tree的根
11     return nd[x].pa.ch[0]!=x&&nd[x].pa
12         .ch[1]!=x;
13 }
14 void down(int x){ // 懶惰標記下推
15     if(nd[x].rev){
16         if(nd[x].ch[0])nd[x].ch[0].rev^=1;
17         if(nd[x].ch[1])nd[x].ch[1].rev^=1;
18         swap(nd[x].ch[0],nd[x].ch[1]);
19         nd[x].rev=0;
20     }
21 }
22 void push_down(int x){ // 所有祖先懶惰標記下推
23     if(!isroot(x))push_down(nd[x].pa);
24     down(x);
25 }
26 void up(int x){ // 將子節點的資訊向上更新
27     rotate(int x){ // 旋轉, 會自行判斷轉的方
28         向
29         int y=nd[x].pa,z=nd[y].pa,d=(nd[y].ch[1]==
30             x);
31         nd[x].pa=z;
32         if(!isroot(y))nd[z].ch[nd[z].ch[1]==y]=x;
33         nd[y].ch[d]=nd[x].ch[d^1];
34         nd[nd[y].ch[d]].pa=y;
35         nd[y].pa=x,nd[x].ch[d^1]=y;
36         up(y),up(x);
37     }
38 }
39 void splay(int x){ // 將x伸展到splay tree的根
40     push_down(x);
41     while(!isroot(x)){
42         int y=nd[x].pa;
43         if(!isroot(y)){
44             int z=nd[y].pa;
45             if((nd[z].ch[0]==y)^(nd[y].ch[0]==x))
46                 rotate(y);
47             else rotate(x);
48         }
49         rotate(x);
50     }
51 }
52 int access(int x){
53     int last=0;
54     while(x){
55         splay(x);
56         nd[x].ch[1]=last;
57         up(x);
58         last=x;
59         x=nd[x].pa;
60     }
61     return last; // access後splay tree的根
62 }
63 void access(int x,bool is=0){ // is=0就是一般
64     的access

```



```

58 int last=0;
59 while(x){
60     splay(x);
61     if(is&&!nd[x].pa){
62         //printf("%d\n",max(nd[Last].ma,nd[nd[
63             x].ch[1]].ma));
64     }
65     nd[x].ch[1]=last;
66     up(x);
67     last=x;
68     x=nd[x].pa;
69 }
70 void query_edge(int u,int v){
71     access(u);
72     access(v,1);
73 }
74 void make_root(int x){
75     access(x),splay(x);
76     nd[x].rev^=1;
77 }
78 void make_root(int x){
79     nd[access(x)].rev^=1;
80     splay(x);
81 }
82 void cut(int x,int y){
83     make_root(x);
84     access(y);
85     splay(y);
86     nd[y].ch[0]=0;
87     nd[x].pa=0;
88 }
89 void cut_parents(int x){
90     access(x);
91     splay(x);
92     nd[nd[x].ch[0]].pa=0;
93     nd[x].ch[0]=0;
94 }
95 void link(int x,int y){
96     make_root(x);
97     nd[x].pa=y;
98 }
99 int find_root(int x){
100     x=access(x);
101     while(nd[x].ch[0])x=nd[x].ch[0];
102     splay(x);
103     return x;
104 }
105 int query(int u,int v){
106     //傳回uv路徑splay tree的根結點
107     //這種寫法無法求LCA
108     make_root(u);
109     return access(v);
110 }
111 int query_lca(int u,int v){
112     //假設求鏈上點權的總和，sum是子樹的權重和，
113     data是節點的權重
114     access(u);
115     int lca=access(v);
116     splay(u);
117     if(u==lca){
118         //return nd[lca].data+nd[nd[lca].ch[1]].
119         sum
120     }else{

```

```

119         //return nd[lca].data+nd[nd[lca].ch[1]].
120         sum+nd[u].sum
121     }
122 }
123 struct EDGE{
124     int a,b,w;
125 }e[10005];
126 int n;
127 vector<pair<int,int>> G[10005];
128 //first表示子節點，second表示邊的編號
129 int pa[10005],edge_node[10005];
130 //pa是父母節點，暫存用的，edge_node是每個編
131     被存在哪個點裡面的陣列
132 void bfs(int root){
133     //在建構的時候把每個點都設成一個splay tree
134     queue<int> q;
135     for(int i=1;i<=n;++i)pa[i]=0;
136     q.push(root);
137     while(q.size()){
138         int u=q.front();
139         q.pop();
140         for(auto P:G[u]){
141             int v=P.first;
142             if(v!=pa[u]){
143                 pa[v]=u;
144                 nd[v].pa=u;
145                 nd[v].data=e[P.second].w;
146                 edge_node[P.second]=v;
147                 up(v);
148                 q.push(v);
149             }
150         }
151     }
152 }
153 void change(int x,int b){
154     splay(x);
155     //nd[x].data=b;
156     up(x);
157 }

```

12.4 POJ_tree.cpp

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 #define MAXN 10005
4 int n,k;
5 vector<pair<int,int>> >g[MAXN];
6 int size[MAXN];
7 bool vis[MAXN];
8 inline void init(){
9     for(int i=0;i<=n;++i){
10         g[i].clear();
11         vis[i]=0;
12     }
13 }
14 void get_dis(vector<int> &dis,int u,int pa,
15     int d){
16     dis.push_back(d);
17     for(size_t i=0;i<g[u].size();++i){
18         int v=g[u][i].first,w=g[u][i].second;
19         if(v!=pa&&!vis[v])get_dis(dis,v,u,d+w);
20     }

```

```

20 }
21 vector<int> dis;//這東西如果放在函數裡會TLE
22 int cal(int u,int d){
23     dis.clear();
24     get_dis(dis,u,-1,d);
25     sort(dis.begin(),dis.end());
26     int l=0,r=dis.size()-1,res=0;
27     while(l<r){
28         while(l<r&&dis[l]+dis[r]>k)--r;
29         res+=r-(l+1);
30     }
31     return res;
32 }
33 pair<int,int> tree_centroid(int u,int pa,
34     const int sz){
35     size[u]=1;//找樹重心，second是重心
36     pair<int,int> res(INT_MAX,-1);
37     int ma=0;
38     for(size_t i=0;i<g[u].size();++i){
39         int v=g[u][i].first;
40         if(v==pa||vis[v])continue;
41         res=min(res,tree_centroid(v,u,sz));
42         size[u]+=size[v];
43         ma=max(ma,size[v]);
44     }
45     ma=max(ma,sz-size[u]);
46     return min(res,make_pair(ma,u));
47 }
48 int tree_DC(int u,int sz){
49     int center=tree_centroid(u,-1,sz).second;
50     int ans=cal(center,0);
51     vis[center]=1;
52     for(size_t i=0;i<g[center].size();++i){
53         int v=g[center][i].first,w=g[center][i].
54             second;
55         if(vis[v])continue;
56         ans-=cal(v,w);
57         ans+=tree_DC(v,size[v]);
58     }
59     return ans;
60 }
61 int main(){
62     while(scanf("%d%d",&n,&k),n||k){
63         init();
64         for(int i=1;i<=n;++i){
65             int u,v,w;
66             scanf("%d%d%d",&u,&v,&w);
67             g[u].push_back(make_pair(v,w));
68             g[v].push_back(make_pair(u,w));
69         }
70         printf("%d\n",tree_DC(1,n));
71     }

```

13 zformula

13.1 formula.tex

13.1.1 Pick 公式

給定頂點坐標均是整點的簡單多邊形，面積 = 內部格點數 + 邊上格點數/2-1

13.1.2 圖論

1. $V - E + F = 2$
2. 對於平面圖， $F = E - V + n + 1$ ， n 是連通分量
3. 對於平面圖， $E \leq 3V - 6$
4. 對於連通圖 G ，最大獨立點集的大小設為 $I(G)$ ，最大匹配大小設為 $M(G)$ ，最小點覆蓋設為 $C_v(G)$ ，最小邊覆蓋設為 $C_e(G)$ 。對於任意連通圖：

$$\begin{aligned} (a) \quad & I(G) + C_v(G) = |V| \\ (b) \quad & M(G) + C_e(G) = |V| \end{aligned}$$

5. 對於連通二分圖：

$$\begin{aligned} (a) \quad & I(G) = C_v(G) \\ (b) \quad & M(G) = C_e(G) \end{aligned}$$

6. 最大權閉合圖：

$$\begin{aligned} (a) \quad & C(u, V) = \infty, (u, v) \in E \\ (b) \quad & C(S, v) = W_v, W_v > 0 \\ (c) \quad & C(v, T) = -W_v, W_v < 0 \end{aligned}$$

7. 最大密度子圖：

$$\begin{aligned} (a) \quad & C(u, v) = 1, (u, v) \in E \\ (b) \quad & C(S, v) = U_v, v \in V \\ (c) \quad & C(v, T) = U + 2g - d_v, v \in V \end{aligned}$$

8. 弦圖：

- (a) 完美消除序列從後往前依次給每個點染色，給每個點染上可以染的最小顏色
- (b) 最大團大小 = 色數
- (c) 最大獨立集：完美消除序列從前往後能選就選
- (d) 最小圈覆蓋：最大獨立集的點和他延伸的邊構成
- (e) 區間圖是弦圖
- (f) 區間圖的完美消除序列：將區間按造又端點由小到大排序
- (g) 區間圖染色：用線段樹做

```

1 double l=0,m,stop=1.0/n/n;
2 while(r-l>stop){
3     double mid;
4     if((n*m-sol.maxFlow(s,t))/2>eps)l=mid;
5     else r=mid;
6 }
7 build(1);
8 sol.maxFlow(s,t);
9 vector<int> ans;
10 for(int i=1;i<=n;++i)
11     if(sol.vis[i])ans.push_back(i);

```

13.1.3 學長公式

- $\sum_{d|n} \phi(n) = n$
- $g(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} \mu(d) \times g(n/d)$
- Harmonic series $H_n = \ln(n) + \gamma + 1/(2n) - 1/(12n^2) + 1/(120n^4)$
- $\gamma = 0.57721566490153286060651209008240243104215$
- 格雷碼 $= n \oplus (n >> 1)$
- $SG(A+B) = SG(A) \oplus SG(B)$
- 選轉矩陣 $M(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

13.1.4 基本數論

- $\sum_{d|n} \mu(n) = [n == 1]$
- $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) \times g(m/d)$
- $\sum_{i=1}^n \sum_{j=1}^m \text{互質數量} = \sum \mu(d) \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$
- $\sum_{i=1}^n \sum_{j=1}^m \text{lcm}(i, j) = n \sum_{d|n} d \times \phi(d)$

13.1.5 排組公式

- k 卡特蘭 $\frac{C_n^{kn}}{n(k-1)+1} \cdot C_m^n = \frac{n!}{m!(n-m)!}$
- $H(n, m) \cong x_1 + x_2 \dots + x_n = k, num = C_k^{n+k-1}$
- Stirling number of 2^{nd} , n 人分 k 組方法數目
 - $S(0, 0) = S(n, n) = 1$
 - $S(n, 0) = 0$
 - $S(n, k) = kS(n-1, k) + S(n-1, k-1)$
- Bell number, n 人分任意多組方法數目
 - $B_0 = 1$
 - $B_n = \sum_{i=0}^n S(n, i)$
 - $B_{n+1} = \sum_{k=0}^n C_k^n B_k$
 - $B_{p+n} \equiv B_n + B_{n+1} \pmod{p}$, p is prime
 - $B_p^{m+n} \equiv mB_n + B_{n+1} \pmod{p}$, p is prime
 - From $B_0 : 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975$
- Derangement, 錯排, 沒有人在自己位置上
 - $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \dots + (-1)^n \frac{1}{n!})$
 - $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 = 1, D_1 = 0$
 - From $D_0 : 1, 0, 1, 2, 9, 44, 265, 1854, 14833, 133496$
- Binomial Equality
 - $\sum_k \binom{r}{m+k} \binom{s}{n-k} = \binom{r+s}{m+n}$
 - $\sum_k \binom{l}{m+k} \binom{s}{n-k} = \binom{l+s}{l+m+n}$
 - $\sum_k \binom{l}{m+k} \binom{s+k}{n} (-1)^k = (-1)^{l+m} \binom{s-m}{n-l}$
 - $\sum_{k \leq l} \binom{l-k}{m} \binom{s}{k-n} (-1)^k = (-1)^{l+m} \binom{s-m-1}{l-n-m}$
 - $\sum_{0 \leq k \leq l} \binom{l-k}{m} \binom{q+k}{n} = \binom{l+q+1}{m+n+1}$
 - $\binom{r}{k} = (-1)^k \binom{k-r-1}{k}$

- $\binom{r}{m} \binom{m}{k} = \binom{r}{m-k} \binom{r-k}{m-k}$
- $\sum_{k \leq n} \binom{r+k}{k} = \binom{r+n+1}{n}$
- $\sum_{0 \leq k \leq n} \binom{k}{m} = \binom{n+1}{m+1}$
- $\sum_{k \leq m} \binom{m+r}{k} x^k y^{m-k} = \sum_{k \leq m} \binom{-r}{k} (-x)^k (x+y)^{m-k}$

13.1.6 幕次, 幕次和

- $a^b \% P = a^{b \% \varphi(P) + \varphi(P)} \cdot b \geq \varphi(P)$
- $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
- $1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$
- $1^5 + 2^5 + 3^5 + \dots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12}$
- $0^k + 1^k + 2^k + \dots + n^k = P(k), P(k) = \frac{(n+1)^{k+1} - \sum_{i=0}^{k-1} C_i^{k+1} P(i)}{k+1}, P(0) = n+1$
- $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^n C_k^{n+1} B_k m^{n+1-k}$
- $\sum_{j=0}^m C_j^{m+1} B_j = 0, B_0 = 1$
- 除了 $B_1 = -1/2$, 剩下的奇數項都是 0
- $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 = -1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 7/6, B_{16} = -3617/510, B_{18} = 43867/798, B_{20} = -174611/330,$

13.1.7 Burnside's lemma

- $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- $X^g = t^{c(g)}$
- G 表示有幾種轉法, X^g 表示在那種轉法下, 有幾種是會保持對稱的, t 是顏色數, $c(g)$ 是循環節不動的面數。
- 正立方體塗三顏色, 轉 0 有 3^6 個元素不變, 轉 90 有 6 種, 每種有 3^3 不變, 180 有 3×3^4 , 120(角) 有 8×3^2 , 180(邊) 有 6×3^3 , 全部 $\frac{1}{24}(3^6 + 6 \times 3^3 + 3 \times 3^4 + 8 \times 3^2 + 6 \times 3^3) = \frac{57}{24}$

13.1.8 Count on a tree

- Rooted tree: $s_{n+1} = \frac{1}{n} \sum_{i=1}^n (i \times a_i \times \sum_{j=1}^{\lfloor n/i \rfloor} a_{n+1-i \times j})$
- Unrooted tree:
 - Odd: $a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$
 - Even: $Odd + \frac{1}{2} a_{n/2} (a_{n/2} + 1)$
- Spanning Tree
 - 完全圖 $n^n - 2$
 - 一般圖 (Kirchhoff's theorem) $M[i][i] = \text{degree}(V_i), M[i][j] = -1, \text{if have } E(i, j), 0 \text{ if no edge. delete any one row and col in } A, \text{ans} = \det(A)$

13.2 java.tex

13.2.1 文件操作

```

1 import java.io.*;
2 import java.util.*;
3 import java.math.*;
4 import java.text.*;
5
6 public class Main{
7
8     public static void main(String args[]){
9         throws FileNotFoundException,
10            IOException
11         Scanner sc = new Scanner(new FileReader(
12             "a.in"));
13         PrintWriter pw = new PrintWriter(new
14             FileWriter("a.out"));
15         int n,m;
16         n=sc.nextInt();//读入下一个INT
17         m=sc.nextInt();
18
19         for(ci=1; ci<=c; ++ci){
20             pw.println("Case #"+ci+": easy for
21                 output");
22         }
23
24         pw.close();//关闭流并释放, 这个很重要,
25             否则是没有输出的
26         sc.close();//关闭流并释放
27     }
28 }

```

13.2.2 优先队列

```

1 PriorityQueue queue = new PriorityQueue( 1,
2     new Comparator(){
3         public int compare( Point a, Point b ){
4             if( a.x < b.x || a.x == b.x && a.y < b.y )
5                 return -1;
6             else if( a.x == b.x && a.y == b.y )
7                 return 0;
8             else return 1;
9         }
10    });

```

13.2.3 Map

```

1 Map map = new HashMap();
2 map.put("sa", "dd");
3 String str = map.get("sa").toString();
4
5 for(Object obj : map.keySet()){
6     Object value = map.get(obj);
7 }

```

13.2.4 sort

```

1 static class cmp implements Comparator{
2     public int compare(Object o1, Object o2){
3         BigInteger b1=(BigInteger)o1;
4         BigInteger b2=(BigInteger)o2;
5         return b1.compareTo(b2);
6     }
7 }
8 public static void main(String[] args)
9     throws IOException{
10     Scanner cin = new Scanner(System.in);
11     int n;
12     n=cin.nextInt();
13     BigInteger[] seg = new BigInteger[n];
14     for (int i=0;i<n;i++)
15         seg[i]=cin.nextBigInteger();
16     Arrays.sort(seg, new cmp());
17 }

```

ACM ICPC TEAM REFERENCE - MADE IN ABYSS

Contents

1 Computational_Geometry	1	3.2 ext.cpp	5	7 Linear_Programming	10	10.7 suffix_array_lcp.cpp	15
1.1 Geometry.cpp	1	3.3 IncStack.cpp	5	7.1 最大密度子圖.cpp	10	10.8 Z.cpp	15
1.2 SmallestCircle.cpp	3	3.4 input.cpp	5	8 Number_Theory	10	11 Tarjan	15
1.3 最近點對.cpp	3	4 Flow	6	8.1 basic.cpp	10	11.1 dominator_tree.cpp	15
2 Data_Structure	3	4.1 dinic.cpp	6	8.2 bit_set.cpp	11	11.2 tnfsb017_2_sat.cpp	15
2.1 DLX.cpp	3	4.2 ISAP_with_cut.cpp	6	8.3 cantor_expansion.cpp	11	11.3 橋連通分量.cpp	16
2.2 Dynamic_KD_tree.cpp	3	4.3 MinCostMaxFlow.cpp	6	8.4 FFT.cpp	11	11.4 雙連通分量 & 割點.cpp	16
2.3 kd_tree_replace_segment_tree.cpp	4	5 Graph	7	8.5 find_real_root.cpp	11	12 Tree_problem	16
2.4 reference_point.cpp	5	5.1 Augmenting_Path.cpp	7	8.6 FWT.cpp	12	12.1 HeavyLight.cpp	16
2.5 skew_heap.cpp	5	5.2 Augmenting_Path_multiple.cpp	7	8.7 LinearCongruence.cpp	12	12.2 LCA.cpp	16
2.6 undo_disjoint_set.cpp	5	5.3 blossom_matching.cpp	7	8.8 Lucas.cpp	12	12.3 link_cut_tree.cpp	16
2.7 整體二分.cpp	5	5.4 graphISO.cpp	7	8.9 Matrix.cpp	12	12.4 POJ_tree.cpp	17
3 default	5	5.5 KM.cpp	7	8.10 MillerRobin.cpp	12	13 zformula	17
3.1 debug.cpp	5	5.6 MaximumClique.cpp	7	8.11 NTT.cpp	13	13.1 formula.tex	17
6 language	10	5.7 MinimumMeanCycle.cpp	8	8.12 Simpson.cpp	13	13.1.1 Pick 公式	17
6.1 CNF.cpp	10	5.8 Rectilinear_MST.cpp	8	8.13 外星模運算.cpp	13	13.1.2 圖論	17
		5.9 treeISO.cpp	8	8.14 質因數分解.cpp	13	13.1.3 學長公式	18
		5.10 一般圖最小權完美匹配.cpp	8	9 other	13	13.1.4 基本數論	18
		5.11 全局最小割.cpp	8	9.1 WhatDay.cpp	13	13.1.5 排組公式	18
		5.12 平面圖判定.cpp	9	9.2 上下最大正方形.cpp	14	13.1.6 冪次, 冪次和	18
		5.13 弦圖完美消除序列.cpp	9	9.3 最大矩形.cpp	14	13.1.7 Burnside's lemma	18
		5.14 最小斯坦納樹 DP.cpp	9	10 String	14	13.1.8 Count on a tree	18
		5.15 最小樹形圖 _ 朱劉.cpp	9	10.1 AC 自動機.cpp	14	13.2 java.tex	18
		5.16 穩定婚姻模板.cpp	10	10.2 hash.cpp	14	13.2.1 文件操作	18
				10.3 KMP.cpp	14	13.2.2 優先隊列	18
				10.4 manacher.cpp	14	13.2.3 Map	18
				10.5 minimal_string_rotation.cpp	15	13.2.4 sort	18
				10.6 reverseBWT.cpp	15		