# 1 Computational Geometa T dis2(const point <T > &p, bool is\_segment

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# 1.1 Geometry

```
59
                                                60
1 const double PI=atan2(0.0,-1.0);
2 template<typename T>
                                                61
3 struct point{
                                                62
    T x,y;
    point(){}
    point(const T&x,const T&y):x(x),y(y){}
    point operator+(const point &b)const{
      return point(x+b.x,y+b.y); }
                                                65
    point operator-(const point &b)const{
      return point(x-b.x,y-b.y); }
    point operator*(const T &b)const{
                                                67
      return point(x*b,y*b); }
                                                68
    point operator/(const T &b)const{
                                                69
      return point(x/b,y/b); }
                                                70
    bool operator == (const point &b)const{
                                                71
      return x==b.x&&y==b.y; }
                                                72
    T dot(const point &b)const{
                                                73
      return x*b.x+v*b.y; }
                                                74
    T cross(const point &b)const{
      return x*b.y-y*b.x; }
                                                76
21
    point normal()const{//求法向量
                                                77
      return point(-y,x); }
22
                                                78
    T abs2()const{//向量長度的平方
                                                79
      return dot(*this); }
                                                80
    T rad(const point &b)const{//兩向量的弧度
                                                81
   return fabs(atan2(fabs(cross(b)),dot(b))); }
                                                82
27
    T getA()const{//對x軸的弧度
                                                83
      T A=atan2(y,x);//超過180度會變負的
                                                84
                                                85
      if(A<=-PI/2)A+=PI*2;</pre>
      return A:
31
32
   template<typename T>
   struct line{
    line(){}
                                                88
    point<T> p1,p2;
    T a,b,c;//ax+by+c=0
    line(const point<T>&x,const point<T>&y):p1
         (x),p2(y){}
    void pton(){//轉成一般式
40
      a=p1.y-p2.y;
      b=p2.x-p1.x;
41
      c=-a*p1.x-b*p1.v:
42
43
    T ori(const point<T> &p)const{//點和有向直
                                                97
          線的關係,>0左邊、=0在線上<0右邊
      return (p2-p1).cross(p-p1);
45
                                                99
46
                                               100
    T btw(const point<T> &p)const{//點投影落在 101
          線段 上 <=0
                                               102
48
      return (p1-p).dot(p2-p);
                                               103
49
    bool point_on_segment(const point<T>&p)
50
                                               104
         const{//點是否在線段上
                                               105
      return ori(p) == 0&&btw(p) <= 0;</pre>
                                               106
                                               107
```

```
=0) const { // 點 跟 直 線 / 線 段 的 距 離 平 方
  point<T> v=p2-p1.v1=p-p1:
                                           109
  if(is_segment){
                                           110
    point<T> v2=p-p2;
                                           111
    if(v.dot(v1)<=0)return v1.abs2();</pre>
                                           112
    if(v.dot(v2)>=0)return v2.abs2();
                                           113
                                           114
 T tmp=v.cross(v1);
  return tmp*tmp/v.abs2();
T seg dis2(const line<T> &1)const{//兩線段 118
  return min({dis2(1.p1,1),dis2(1.p2,1),1. 120
       dis2(p1,1),1.dis2(p2,1)});
                                           121
                                           122
point<T> projection(const point<T> &p)
     const { // 點對直線的投影
                                           123
                                           124
  point<T> n=(p2-p1).normal();
                                           125
 return p-n*(p-p1).dot(n)/n.abs2();
                                           126
point<T> mirror(const point<T> &p)const{
                                          127
  //點對直線的鏡射,要先呼叫pton轉成一般式 128
 point<T> R:
 T d=a*a+b*b:
 R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d; 130
  R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d; 131
  return R:
                                           133
                                          134
bool equal(const line &1)const{//直線相等
 return ori(1.p1)==0&&ori(1.p2)==0;
                                           136
bool parallel(const line &1)const{
 return (p1-p2).cross(l.p1-l.p2)==0;
                                           137
bool cross seg(const line &1)const{
                                           138
 return (p2-p1).cross(l.p1-p1)*(p2-p1).
       cross(1.p2-p1)<=0;//直線是否交線段
                                          139
int line intersect(const line &l)const{// 140
     直線相交情況,-1無限多點、1交於一點、0141
  return parallel(1)?(ori(1.p1)==0?-1:0)
                                           143
                                           144
                                           145
int seg intersect(const line &1)const{
 T c1=ori(l.p1), c2=ori(l.p2);
 T c3=1.ori(p1), c4=1.ori(p2);
                                           147
  if(c1==0&&c2==0){//共線
    bool b1=btw(1.p1)>=0,b2=btw(1.p2)>=0;
    T a3=1.btw(p1),a4=1.btw(p2);
                                           148
                                           149
    if(b1&&b2&&a3==0&&a4>=0) return 2;
                                           150
    if(b1&&b2&&a3>=0&&a4==0) return 3:
                                           151
   if(b1&&b2&&a3>=0&&a4>=0) return 0;
                                           152
    return -1://無限交點
  }else if(c1*c2<=0&&c3*c4<=0)return 1;</pre>
                                           153
 return 0;//不相交
                                           154
                                           155
point<T> line intersection(const line &1)
                                           156
     const{/*直線交點*/
                                           157
  point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
                                           158
  //if(a.cross(b)==0)return INF;
  return p1+a*(s.cross(b)/a.cross(b));
```

```
point<T> seg intersection(const line &1)
          const{//線段交點
                                                 162
       int res=seg intersect(1);
       if(res<=0) assert(0);</pre>
                                                 163
       if(res==2) return p1;
                                                 164
       if(res==3) return p2;
                                                 165
       return line intersection(1);
                                                 166
115 };
                                                 167
   template<typename T>
   struct polygon{
                                                 168
     polygon(){}
     vector<point<T> > p;//逆時針順序
                                                 169
     T area()const{//面積
                                                 170
       T ans=0;
       for(int i=p.size()-1,j=0;j<(int)p.size()</pre>
                                                 17
            ;i=j++)
                                                 172
          ans+=p[i].cross(p[i]);
                                                 173
       return ans/2;
                                                 174
                                                 175
     point<T> center of mass()const{//重心
                                                 176
       T cx=0, cy=0, w=0;
       for(int i=p.size()-1,j=0;j<(int)p.size()</pre>
             ;i=j++){
                                                 177
          T a=p[i].cross(p[j]);
                                                 178
          cx+=(p[i].x+p[j].x)*a;
          cy+=(p[i].y+p[j].y)*a;
                                                 179
                                                 180
                                                 181
       return point<T>(cx/3/w,cy/3/w);
     char ahas(const point<T>& t)const{//點是否
          在簡單多邊形內,是的話回傳1、在邊上回 183
                                                 184

值 - 1 、 否 則 回 值 a

       bool c=0;
                                                 186
       for(int i=0,j=p.size()-1;i<p.size();j=i</pre>
                                                 188
          if(line<T>(p[i],p[j]).point_on_segment
               (t))return -1;
                                                 190
          else if((p[i].y>t.y)!=(p[j].y>t.y)&&
          t.x<(p[i].x-p[i].x)*(t.y-p[i].y)/(p[j
                                                 191
              ].y-p[i].y)+p[i].x)
                                                 192
            c=!c;
                                                 193
       return c;
                                                 194
     char point_in_convex(const point<T>&x)
                                                 195
                                                 196
       int l=1,r=(int)p.size()-2;
                                                 197
       while(1<=r){//點是否在凸多邊形內,是的話
                                                 198
             回傳1、在邊上回傳-1、否則回傳0
                                                 199
          int mid=(1+r)/2;
                                                 200
         T a1=(p[mid]-p[0]).cross(x-p[0]);
         T a2=(p[mid+1]-p[0]).cross(x-p[0]);
                                                 202
          if(a1>=0&&a2<=0){
                                                 203
           T res=(p[mid+1]-p[mid]).cross(x-p[
                                                 204
                mid]);
                                                 205
           return res>0?1:(res>=0?-1:0);
          }else if(a1<0)r=mid-1;</pre>
                                                 206
          else l=mid+1:
                                                 207
                                                 208
       return 0;
                                                 209
     vector<T> getA()const{//凸包邊對x軸的夾角
       vector<T>res;//一定是遞增的
```

```
for(size t i=0;i<p.size();++i)</pre>
    res.push back((p[(i+1)\%p.size()]-p[i])
         .getA());
  return res:
bool line intersect(const vector<T>&A,
     const line<T> &1)const{//O(LoaN)
  int f1=upper bound(A.begin(), A.end(),(1.
       p1-1.p2).getA())-A.begin();
  int f2=upper bound(A.begin(), A.end(),(1.
       p2-1.p1).getA())-A.begin();
  return 1.cross seg(line<T>(p[f1],p[f2]))
polygon cut(const line<T> &l)const{//△包
     對 直 線 切 割 , 得 到 直 線 L 左 側 的 凸 包
  polygon ans;
  for(int n=p.size(),i=n-1,j=0;j<n;i=j++){</pre>
    if(l.ori(p[i])>=0){
      ans.p.push back(p[i]);
      if(1.ori(p[j])<0)</pre>
        ans.p.push_back(1.
             line intersection(line<T>(p[i
             ],p[j])));
    }else if(l.ori(p[j])>0)
      ans.p.push back(1.line intersection(
           line<T>(p[i],p[j])));
  return ans;
static bool graham cmp(const point<T>& a,
     const point<T>& b){//凸包排序函數
  return (a.x<b.x)||(a.x==b.x&&a.y<b.y);</pre>
void graham(vector<point<T> > &s){//凸包
  sort(s.begin(),s.end(),graham cmp);
  p.resize(s.size()+1);
  int m=0;
  for(size t i=0;i<s.size();++i){</pre>
    while (m \ge 2\& (p[m-1]-p[m-2]). cross (s[i
         ]-p[m-2])<=0)--m;
    p[m++]=s[i];
  for(int i=s.size()-2,t=m+1;i>=0;--i){
    while (m>=t&&(p[m-1]-p[m-2]).cross(s[i
         ]-p[m-2])<=0)--m;
    p[m++]=s[i];
  if(s.size()>1)--m;
  p.resize(m);
T diam(){//直徑
  int n=p.size(),t=1;
  T ans=0;p.push_back(p[0]);
  for(int i=0;i<n;i++){</pre>
    point<T> now=p[i+1]-p[i];
    while(now.cross(p[t+1]-p[i])>now.cross
         (p[t]-p[i]))t=(t+1)%n;
    ans=\max(ans,(p[i]-p[t]).abs2());
  return p.pop back(),ans;
T min_cover_rectangle(){//最小覆蓋矩形
  int n=p.size(),t=1,r=1,1;
```

```
if(n<3)return 0;//也可以做最小周長矩形
                                                           vector<line<T> > q(n);
213
        T ans=1e99; p. push back(p[0]);
                                                   264
                                                           q[L=R=0]=s[0];
        for(int i=0;i<n;i++){</pre>
                                                            for(int i=1;i<n;++i){</pre>
214
                                                   265
215
          point<T> now=p[i+1]-p[i];
                                                   266
                                                             while(L<R&&s[i].ori(px[R-1])<=0)--R;</pre>
         while(now.cross(p[t+1]-p[i])>now.cross 267
                                                             while(L<R&&s[i].ori(px[L])<=0)++L;</pre>
216
               (p[t]-p[i]))t=(t+1)%n;
                                                             q[++R]=s[i];
217
          while(now.dot(p[r+1]-p[i])>now.dot(p[r 269
                                                             if(q[R].parallel(q[R-1])){
               ]-p[i]))r=(r+1)%n;
                                                   270
                                                                --R;
218
          if(!i)l=r;
                                                   271
                                                                if(q[R].ori(s[i].p1)>0)q[R]=s[i];
          while (now.dot(p[l+1]-p[i]) < =now.dot(p[272])
219
               1]-p[i]))1=(1+1)%n;
                                                             if(L < R)px[R-1] = q[R-1].
                                                                  line intersection(q[R]);
220
         T d=now.abs2():
          T tmp=now.cross(p[t]-p[i])*(now.dot(p[274]
221
              r]-p[i])-now.dot(p[l]-p[i]))/d;
                                                           while(L<R&&q[L].ori(px[R-1])<=0)--R;</pre>
222
         ans=min(ans,tmp);
                                                   276
                                                           p.clear();
                                                   277
                                                           if(R-L<=1)return 0;</pre>
223
                                                           px[R]=q[R].line intersection(q[L]);
224
       return p.pop_back(),ans;
                                                   278
                                                           for(int i=L;i<=R;++i)p.push_back(px[i]);</pre>
225
                                                   279
                                                           return R-L+1;
     T max_triangle(){//最大內接三角形
                                                   280
226
                                                   281
227
        int n=p.size(),a=1,b=2;
                                                   282 };
228
       if(n<3)return 0;</pre>
                                                   283 template<typename T>
229
       T ans=0,tmp;p.push back(p[0]);
                                                   284 struct triangle{
        for(int i=0;i<n;++i){</pre>
230
                                                         point<T> a,b,c;
          while((p[a]-p[i]).cross(p[b+1]-p[i])>( 285
231
                                                         triangle(){}
               tmp=(p[a]-p[i]).cross(p[b]-p[i])))^{286}
                                                         triangle(const point<T> &a,const point<T>
              b=(b+1)%n;
                                                              &b, const point<T> &c):a(a),b(b),c(c){}^{342}
          ans=max(ans,tmp);
                                                         T area()const{
233
          while((p[a+1]-p[i]).cross(p[b]-p[i])>( 288
               tmp=(p[a]-p[i]).cross(p[b]-p[i])))<sup>289</sup>
                                                           T t=(b-a).cross(c-a)/2;
                                                           return t>0?t:-t;
              a=(a+1)%n;
                                                   291
         ans=max(ans,tmp);
234
235
                                                   292
                                                         point<T> barycenter()const{//重心
236
       return p.pop_back(),ans/2;
                                                   293
                                                           return (a+b+c)/3:
237
                                                   294
     T dis2(polygon &pl){//凸包最近距離平方
238
                                                   295
                                                         point<T> circumcenter()const{//外心
239
       vector<point<T> > &P=p,&Q=pl.p;
                                                   296
                                                           static line<T> u,v;
240
       int n=P.size(), m=Q.size(), l=0, r=0;
                                                   297
                                                           u.p1=(a+b)/2;
     for(int i=0;i<n;++i)if(P[i].y<P[1].y)l=i;</pre>
241
                                                   298
                                                           u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-
     for(int i=0;i<m;++i)if(Q[i].y<Q[r].y)r=i;</pre>
242
                                                                b.x);
243
       P.push back(P[0]), Q.push back(Q[0]);
                                                   299
                                                           v.p1=(a+c)/2;
244
       T ans=1e99;
                                                           v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-
245
       for(int i=0;i<n;++i){</pre>
         while((P[1]-P[1+1]).cross(Q[r+1]-Q[r]) 301
246
                                                           return u.line_intersection(v);
               <0)r=(r+1)%m;
          ans=min(ans,line\langle T \rangle (P[1],P[1+1]).
                                                         point<T> incenter()const{//內心
                                                   303
               seg_dis2(line<T>(Q[r],Q[r+1])));
                                                           T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2
                                                  304
         l=(1+1)%n;
                                                                ()),C=sqrt((a-b).abs2());
249
                                                           return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+
                                                   305
250
       return P.pop_back(),Q.pop_back(),ans;
                                                                B*b.y+C*c.y)/(A+B+C);
251
                                                   306
252
     static char sign(const point<T>&t){
                                                   307
                                                         point<T> perpencenter()const{//垂心
       return (t.y==0?t.x:t.y)<0;</pre>
253
                                                   308
                                                           return barycenter()*3-circumcenter()*2;
254
                                                   309
     static bool angle cmp(const line<T>& A,
255
                                                   310 };
           const line<T>& B){
                                                       template<typename T>
                                                   311
        point<T> a=A.p2-A.p1,b=B.p2-B.p1;
256
                                                   312 struct point3D{
       return sign(a)<sign(b)||(sign(a)==sign(b 313
257
                                                         T x,y,z;
             )&&a.cross(b)>0);
                                                         point3D(){}
258
                                                         point3D(const T&x,const T&y,const T&z):x(x
259
     int halfplane_intersection(vector<line<T>
                                                              ),y(y),z(z){}
          > &s){//半平面交
                                                         point3D operator+(const point3D &b)const{
       sort(s.begin(),s.end(),angle_cmp);//線段 317
                                                           return point3D(x+b.x,y+b.y,z+b.z);}
260
                                                         point3D operator-(const point3D &b)const{ 371
             左側為該線段半平面
                                                   318
                                                           return point3D(x-b.x,y-b.y,z-b.z);}
        int L.R.n=s.size():
                                                   319
261
                                                         point3D operator*(const T &b)const{
262
        vector<point<T> > px(n);
```

```
return point3D(x*b,y*b,z*b);}
     point3D operator/(const T &b)const{
       return point3D(x/b,y/b,z/b);}
                                                 374
     bool operator==(const point3D &b)const{
                                                 375
       return x==b.x&&y==b.y&&z==b.z;}
                                                 376
     T dot(const point3D &b)const{
       return x*b.x+v*b.v+z*b.z:}
                                                 377
     point3D cross(const point3D &b)const{
       return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x 378
            *b.y-y*b.x);}
     T abs2()const{//向量長度的平方
                                                 379
       return dot(*this);}
                                                 380
     T area2(const point3D &b)const{//和b、原點
          圍成面積的平方
                                                 381
       return cross(b).abs2()/4;}
                                                 382
334 };
335 template<typename T>
                                                 383
   struct line3D{
     point3D<T> p1,p2;
                                                 384
     line3D(){}
                                                 385
     line3D(const point3D<T> &p1,const point3D< 386
          T> &p2):p1(p1),p2(p2){}
     T dis2(const point3D<T> &p,bool is_segment 388
          =0) const { // 點跟直線/線段的距離平方
       point3D<T> v=p2-p1,v1=p-p1;
       if(is segment){
          point3D<T> v2=p-p2;
          if(v.dot(v1)<=0)return v1.abs2();</pre>
         if(v.dot(v2)>=0)return v2.abs2();
                                                 392
       point3D<T> tmp=v.cross(v1);
                                                 393
       return tmp.abs2()/v.abs2();
     pair<point3D<T>,point3D<T> > closest_pair( 394
          const line3D<T> &1)const{
                                                 395
       point3D < T > v1 = (p1 - p2), v2 = (1.p1 - 1.p2);
                                                 396
       point3D<T> N=v1.cross(v2),ab(p1-l.p1);
                                                 397
       //if(N.abs2()==0)return NULL;平行或重合
                                                 398
       T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//
                                                 399
             最近點對距離
       point3D<T> d1=p2-p1,d2=1.p2-1.p1,D=d1.
            cross(d2),G=1.p1-p1;
       T t1=(G.cross(d2)).dot(D)/D.abs2();
       T t2=(G.cross(d1)).dot(D)/D.abs2();
       return make_pair(p1+d1*t1,1.p1+d2*t2);
                                                 403
     bool same_side(const point3D<T> &a,const
                                                 404
          point3D<T> &b)const{
                                                 405
       return (p2-p1).cross(a-p1).dot((p2-p1).
                                                 406
            cross(b-p1))>0;
                                                 408
363 };
   template<typename T>
                                                 409
365 struct plane{
                                                 410
     point3D<T> p0,n;//平面上的點和法向量
                                                 411
     plane(){}
     plane(const point3D<T> &p0,const point3D<T 413
          > &n):p0(p0),n(n){}
                                                 414
     T dis2(const point3D<T> &p)const{//點到平
                                                415
                                                 416
          面距離的平方
                                                 417
       T tmp=(p-p0).dot(n);
                                                 418
       return tmp*tmp/n.abs2();
                                                 419
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```
point3D<T> projection(const point3D<T> &p)
   return p-n*(p-p0).dot(n)/n.abs2();
 point3D<T> line intersection(const line3D
      T> &1)const{
   T tmp=n.dot(1.p2-1.p1);//等於 Ø表示平行或
         重合該平面
    return 1.p1+(1.p2-1.p1)*(n.dot(p0-1.p1)/
        tmp):
 line3D<T> plane intersection(const plane &
      pl)const{
    point3D<T> e=n.cross(pl.n),v=n.cross(e);
   T tmp=pl.n.dot(v);//等於0表示平行或重合
    point3D<T> q=p0+(v*(pl.n.dot(pl.p0-p0))/
        tmp);
    return line3D<T>(q,q+e);
template<typename T>
struct triangle3D{
 point3D<T> a,b,c;
  triangle3D(){}
 triangle3D(const point3D<T> &a,const
      point3D<T> &b, const point3D<T> &c):a(a
      ),b(b),c(c){}
 bool point in(const point3D<T> &p)const{//
       點在該平面上的投影在三角形中
   return line3D<T>(b,c).same side(p,a)&&
        line3D<T>(a,c).same_side(p,b)&&
        line3D<T>(a,b).same_side(p,c);
template<typename T>
struct tetrahedron{//四面體
 point3D<T> a,b,c,d;
 tetrahedron(){}
 tetrahedron(const point3D<T> &a,const
      point3D<T> &b,const point3D<T> &c,
      const point3D<T> &d):a(a),b(b),c(c),d(
      d){}
 T volume6()const{//體積的六倍
   return (d-a).dot((b-a).cross(c-a));
 point3D<T> centroid()const{
   return (a+b+c+d)/4;
 bool point in(const point3D<T> &p)const{
   return triangle3D<T>(a,b,c).point in(p)
        &&triangle3D<T>(c,d,a).point_in(p);
};
template<typename T>
struct convexhull3D{
 static const int MAXN=1005;
 struct face{
   int a,b,c;
   face(int a,int b,int c):a(a),b(b),c(c){}
 vector<point3D<T>> pt;
 vector<face> ans;
 int fid[MAXN][MAXN];
```

void build(){

```
int n=pt.size();
422
        ans.clear();
423
424
       memset(fid,0,sizeof(fid));
425
       ans.emplace back(0,1,2);//注意不能共線
        ans.emplace back(2,1,0);
426
        int ftop = \overline{0};
427
        for(int i=3, ftop=1; i<n; ++i,++ftop){</pre>
428
          vector<face> next;
429
          for(auto &f:ans){
430
            T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[
431
                 f.a]).cross(pt[f.c]-pt[f.a]));
            if(d<=0) next.push back(f);</pre>
432
433
            int ff=0:
            if(d>0) ff=ftop;
434
435
            else if(d<0) ff=-ftop;</pre>
            fid[f.a][f.b]=fid[f.c]=fid[f.c
436
                 ][f.a]=ff;
437
438
          for(auto &f:ans){
439
            if(fid[f.a][f.b]>0 && fid[f.a][f.b
                 ]!=fid[f.b][f.a])
              next.emplace back(f.a,f.b,i);
            if(fid[f.b][f.c]>0 && fid[f.b][f.c
                 ]!=fid[f.c][f.b])
              next.emplace back(f.b,f.c,i);
442
            if(fid[f.c][f.a]>0 && fid[f.c][f.a
                 ]!=fid[f.a][f.c])
              next.emplace_back(f.c,f.a,i);
444
445
446
          ans=next;
447
448
     point3D<T> centroid()const{
449
450
       point3D < T > res(0.0.0):
451
       T vol=0;
        for(auto &f:ans){
452
          T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c 21
453
          res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
454
455
          vol+=tmp;
456
       return res/(vol*4);
457
458
459 };
```

1.2 SmallestCircle

c.x=(p[i].x+p[j].x)/2;

#### .3 最近點對

```
1 template < typename _IT = point < T > * >
  T cloest pair( IT L, IT R){
     if(R-L <= 1) return INF;</pre>
     IT mid = L+(R-L)/2;
     T x = mid -> x:
     T d = min(cloest_pair(L,mid),cloest_pair(
          mid,R));
     inplace merge(L, mid, R, ycmp);
     static vector<point> b; b.clear();
     for(auto u=L;u<R;++u){</pre>
       if((u->x-x)*(u->x-x)>=d) continue;
10
       for(auto v=b.rbegin();v!=b.rend();++v){
         T dx=u\rightarrow x-v\rightarrow x, dy=u\rightarrow y-v\rightarrow y;
12
         if(dy*dy>=d) break;
13
14
         d=min(d,dx*dx+dy*dy);
15
16
       b.push_back(*u);
17
18
     return d;
19
20 T closest_pair(vector<point<T>> &v){
     sort(v.begin(),v.end(),xcmp);
     return closest pair(v.begin(), v.end());
```

# 2 Data Structure

#### 2.1 DLX

```
1 | const int MAXN=4100, MAXM=1030, MAXND=16390;
                                                 2 struct DLX{
1 using PT=point<T>: using CPT=const PT:
                                                     int n,m,sz,ansd;//高是n · 寬是m的稀疏矩陣
2 PT circumcenter(CPT &a,CPT &b,CPT &c){
                                                     int S[MAXM],H[MAXN];
    PT u=b-a, v=c-a;
    T c1=u.abs2()/2,c2=v.abs2()/2;
                                                     int row[MAXND], col[MAXND]; //每個節點代表的
    T d=u.cross(v);
    return PT(a.x+(v.y*c1-u.y*c2)/d,a.y+(u.x*
                                                     int L[MAXND],R[MAXND],U[MAXND],D[MAXND];
         c2-v.x*c1)/d);
                                                     vector<int> ans,anst;
                                                     void init(int n,int m){
  void solve(PT p[],int n,PT &c,T &r2){
                                                       n= n, m= m;
    random shuffle(p,p+n);
                                                       for(int i=0;i<=m;++i){</pre>
    c=p[0]; r2=0; // c, r2 = 圓心, 半徑平方
                                                        U[i]=D[i]=i,L[i]=i-1,R[i]=i+1;
   for(int i=1;i<n;i++)if((p[i]-c).abs2()>r2){
                                                12
                                                         S[i]=0;
      c=p[i]; r2=0;
13 for(int j=0;j<i;j++)if((p[j]-c).abs2()>r2){
                                                       R[m]=0,L[0]=m;
```

```
69
void add(int r,int c){
                                         70
  ++S[col[++sz]=c];
                                         71
  row[sz]=r;
                                         72
  D[sz]=D[c],U[D[c]]=sz,U[sz]=c,D[c]=sz;
                                         73
  if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
  else R[sz]=R[H[r]],L[R[H[r]]]=sz,L[sz]=H
                                         75
      [r],R[H[r]]=sz;
                                         76
#define DFOR(i,A,s) for(int i=A[s];i!=s;i= 78
void remove(int c){//刪除第c行和所有當前覆
                                         80
                                         81
     蓋到第c行的列
  L[R[c]]=L[c],R[L[c]]=R[c];//這裡刪除第c
      行,若有些行不需要處理可以在開始時呼
  DFOR(i,D,c)DFOR(j,R,i){U[D[j]]=U[j],D[U[
      j]]=D[j],--S[col[j]];}
void restore(int c){//恢復第c行和所有當前
    覆蓋到第c行的列,remove的逆操作
                                         90 };
  DFOR(i,U,c)DFOR(j,L,i){++S[col[j]],U[D[j
      ]]=j,D[U[j]]=j;}
  L[R[c]]=c,R[L[c]]=c;
void remove2(int nd){//刪除nd所在的行當前
    所有點(包括虛擬節點),只保留nd
  DFOR(i,D,nd)L[R[i]]=L[i],R[L[i]]=R[i];
void restore2(int nd){//刪除nd所在的行當前
    所有點,為remove2的逆操作
  DFOR(i,U,nd)L[R[i]]=R[L[i]]=i;
bool vis[MAXM];
int h(){//估價函數 for IDA*
  int res=0:
  memset(vis,0,sizeof(vis));
  DFOR(i,R,0)if(!vis[i]){
   vis[i]=1;
    ++res;
                                         11
    DFOR(j,D,i)DFOR(k,R,j)vis[col[k]]=1;
                                         12
                                         13
  return res;
                                         14
                                         15
bool dfs(int d){//for精確覆蓋問題
                                         16
  if(d+h()>=ansd)return 0;//找最佳解用,找
      任意解可以刪掉
                                         19
  if(!R[0]){ansd=d;return 1;}
  int c=R[0];
  DFOR(i,R,0)if(S[i]<S[c])c=i;</pre>
                                         22
  remove(c);
                                         23
  DFOR(i,D,c){
                                         24
   ans.push_back(row[i]);
   DFOR(i,R,i)remove(col[j]);
                                         25
   if(dfs(d+1))return 1;
                                         26
    ans.pop back();
   DFOR(j,L,i)restore(col[j]);
  restore(c);
  return 0:
```

sz=m, ansd=INT MAX; //ansd存最優解的個數

for(int i=1;i<=n;++i)H[i]=-1;</pre>

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66

```
void dfs2(int d){//for最小重複覆蓋問題
 if(d+h()>=ansd)return;
 if(!R[0]){ansd=d;ans=anst;return;}
 int c=R[0];
 DFOR(i,R,0)if(S[i]<S[c])c=i;</pre>
 DFOR(i,D,c){
   anst.push back(row[i]);
   remove2(i);
   DFOR(j,R,i)remove2(j),--S[col[j]];
   dfs2(d+1);
   anst.pop back();
   DFOR(j,L,i)restore2(j),++S[col[j]];
   restore2(i);
bool exact_cover(){//解精確覆蓋問題
 return ans.clear(), dfs(0);
void min_cover() { // 解最小重複覆蓋問題
 anst.clear();//暫存用,答案還是存在ans裡
  dfs2(0);
#undef DFOR
```

#### 2.2 Dynamic KD tree

```
1 template<typename T, size_t kd>//有kd個維度
 struct kd tree{
   struct point{
     T d[kd];
     T dist(const point &x)const{
       T ret=0;
        for(size t i=0;i<kd;++i)ret+=std::abs(</pre>
            d[i]-x.d[i]);
        return ret;
     bool operator==(const point &p){
        for(size t i=0;i<kd;++i)</pre>
         if(d[i]!=p.d[i])return 0;
        return 1:
     bool operator<(const point &b)const{</pre>
       return d[0]<b.d[0];</pre>
   };
 private:
   struct node{
     node *1.*r:
     point pid;
     int s:
     node(const point &p):1(0),r(0),pid(p),s
           (1)\{\}
     ~node(){delete l,delete r;}
     void up()\{s=(1?1->s:0)+1+(r?r->s:0);\}
    }*root:
   const double alpha,loga;
   const T INF;//記得要給INF,表示極大值
   int maxn;
   struct cmp{
     int sort id;
```

```
bool operator()(const node*x,const node*
                                                          if(!1&&r)return cmp(r,o)?r:o;
                                                                                                             if(mi.d[k]<=u->pid.d[k])range(u->1,(k+1) 11
                                                                                                     149
                                                          if(!1&&!r)return o;
                                                                                                                  %kd,mi,ma);
                                                          if(cmp(1,r))return cmp(1,0)?1:0;
                                                                                                             if(ma.d[k])=u-pid.d[k])range(u-pr,(k+1)
         return operator()(x->pid,y->pid);
34
                                                   91
                                                                                                     150
                                                          return cmp(r,o)?r:o;
                                                                                                                  %kd,mi,ma);
35
                                                   92
36
       bool operator()(const point &x,const
                                                   93
                                                                                                     151
            point &y)const{
                                                        bool erase(node *&u,int k,const point &x){ 152 public:
                                                   94
37
         if(x.d[sort id]!=v.d[sort id])
                                                   95
                                                          if(!u)return 0:
                                                                                                           kd tree(const T &INF, double a=0.75):root
38
           return x.d[sort_id]<y.d[sort_id];</pre>
                                                   96
                                                          if(u->pid==x){
                                                                                                                (0),alpha(a),loga(log2(1.0/a)),INF(INF 18
         for(size t i=0;i<kd;++i)</pre>
                                                            if(u->r);
                                                                                                                ),maxn(1){}
39
           if(x.d[i]!=y.d[i])return x.d[i]<y.d[</pre>
                                                            else if(u->1) u->r=u->1, u->1=0;
                                                                                                           ~kd tree(){delete root;}
                                                   98
                                                                                                           void clear(){delete root,root=0,maxn=1;}
                i];
                                                   99
                                                            else{
                                                                                                     155
         return 0:
                                                  100
                                                              delete u:
                                                                                                           void build(int n.const point *p){
41
                                                                                                     156
                                                              return u=0, 1;
                                                                                                     157
                                                                                                             delete root, A.resize(maxn=n);
42
                                                  101
                                                                                                             for(int i=0:i<n:++i)A[i]=new node(p[i]):</pre>
43
     }cmp;
                                                  102
                                                                                                     158
     int size(node *o){return o?o->s:0;}
                                                  103
                                                            --u->s:
                                                                                                     159
                                                                                                             root=build(0.0.n-1);
45
     std::vector<node*> A;
                                                  104
                                                            cmp.sort id=k;
                                                                                                     160
46
    node* build(int k,int l,int r){
                                                            u \rightarrow pid = findmin(u \rightarrow r, (k+1)%kd) \rightarrow pid:
                                                                                                           void insert(const point &x){
                                                  105
                                                                                                     161
47
       if(1>r) return 0;
                                                            return erase(u->r,(k+1)%kd,u->pid);
                                                                                                             insert(root,0,x,__lg(size(root))/loga);
                                                  106
                                                                                                     162
       if(k==kd) k=0:
                                                                                                             if(root->s>maxn)maxn=root->s:
48
                                                  107
                                                                                                     163
       int mid=(1+r)/2:
49
                                                  108
                                                          cmp.sort id=k;
                                                                                                     164
50
       cmp.sort id = k;
                                                  109
                                                          if(erase(cmp(x,u->pid)?u->1:u->r,(k+1)%
                                                                                                     165
                                                                                                           bool erase(const point &p){
       std::nth element(A.begin()+1,A.begin()+
                                                                                                             bool d=erase(root.0.p);
                                                               kd.x)
                                                                                                     166
            mid.A.begin()+r+1.cmp);
                                                  110
                                                            return --u->s, 1:
                                                                                                     167
                                                                                                             if(root&&root->s<alpha*maxn)rebuild();</pre>
       node *ret=A[mid];
                                                  111
                                                          return 0;
                                                                                                     168
                                                                                                             return d:
52
       ret \rightarrow l = build(k+1, l, mid-1);
53
                                                  112
                                                                                                     169
       ret->r = build(k+1,mid+1,r);
                                                                                                           void rebuild(){
54
                                                  113
                                                        T heuristic(const T h[])const{
                                                                                                     170
                                                                                                             if(root)rebuild(root,0);
55
       ret->up();
                                                  114
                                                                                                     171
56
       return ret:
                                                  115
                                                          for(size t i=0:i<kd:++i)ret+=h[i]:</pre>
                                                                                                     172
                                                                                                             maxn=root->s:
57
                                                  116
                                                          return ret:
                                                                                                     173
58
    bool isbad(node*o){
                                                  117
                                                                                                     174
                                                                                                           T nearest(const point &x,int k){
59
       return size(o->1)>alpha*o->s||size(o->r) 118
                                                        int aM:
                                                                                                     175
            >alpha*o->s;
                                                        std::priority queue<std::pair<T,point > >
                                                                                                     176
                                                                                                             T mndist=INF,h[kd]={};
                                                                                                             nearest(root,0,x,h,mndist);
60
     void flatten(node *u, typename std::vector< 120</pre>
                                                        void nearest(node *u,int k,const point &x, 178
                                                                                                             mndist=pQ.top().first;
61
                                                             T *h.T &mndist){
          node*>::iterator &it){
                                                                                                             pQ=std::priority_queue<std::pair<T,point
                                                          if(u==0||heuristic(h)>=mndist)return;
       if(!u)return:
62
                                                                                                                   > >();
                                                          T dist=u->pid.dist(x),old=h[k];
63
       flatten(u->1,it);
                                                  122
                                                                                                     180
                                                                                                             return mndist;//回傳離x第k近的點的距離
                                                          /*mndist=std::min(mndist,dist);*/
64
       *it=u;
                                                  123
                                                                                                     181
65
       flatten(u->r,++it);
                                                  124
                                                          if(dist<mndist){</pre>
                                                                                                           const std::vector<point> &range(const
                                                                                                     182
66
                                                  125
                                                            pQ.push(std::make_pair(dist,u->pid));
                                                                                                                point&mi, const point&ma){
     void rebuild(node*&u,int k){
                                                            if((int)pQ.size()==qM+1)
67
                                                  126
                                                                                                             in range.clear();
                                                                                                     183
       if((int)A.size()<u->s)A.resize(u->s);
                                                  127
                                                               mndist=pQ.top().first,pQ.pop();
                                                                                                     184
                                                                                                             range(root,0,mi,ma);
       typename std::vector<node*>::iterator it 128
                                                                                                             return in_range;//回傳介於mi到ma之間的點
                                                                                                     185
            =A.begin():
                                                          if(x.d[k]<u->pid.d[k]){
                                                  129
                                                                                                                  vector
                                                            nearest(u->1,(k+1)%kd,x,h,mndist);
       flatten(u,it);
                                                  130
                                                                                                     186
       u=build(k,0,u->s-1);
                                                  131
                                                            h[k]=std::abs(x.d[k]-u->pid.d[k]);
                                                                                                           int size(){return root?root->s:0;}
                                                                                                     187
72
                                                            nearest(u->r,(k+1)%kd,x,h,mndist);
                                                  132
73
     bool insert(node*&u,int k,const point &x,
                                                          }else{
          int dep){
                                                            nearest(u->r,(k+1)%kd,x,h,mndist);
                                                  134
       if(!u) return u=new node(x), dep<=0;</pre>
                                                            h[k]=std::abs(x.d[k]-u->pid.d[k]);
74
                                                  135
75
       ++u->s;
                                                            nearest(u->1,(k+1)%kd,x,h,mndist);
                                                  136
                                                                                                                kd tree replace segment 534
76
       cmp.sort id=k;
                                                  137
       if(insert(cmp(x,u->pid)?u->1:u->r,(k+1)% 138
                                                          h[k]=old;
            kd,x,dep-1)){
         if(!isbad(u))return 1;
                                                        std::vector<point>in range;
                                                                                                       1 | struct node { //kd 樹代 替高維線段樹
         rebuild(u,k);
                                                  141
                                                        void range(node *u,int k,const point&mi,
                                                                                                           node *1,*r;
79
80
                                                             const point&ma){
                                                                                                           point pid, mi, ma;
                                                          if(!u)return;
       return 0;
                                                  142
                                                                                                           int s. data:
                                                          bool is=1;
                                                                                                           node(const point &p,int d):1(0),r(0),pid(p
82
                                                  143
     node *findmin(node*o,int k){
                                                  144
                                                          for(int i=0:i<kd:++i)</pre>
                                                                                                                ),mi(p),ma(p),s(1),data(d),dmin(d),
                                                            if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid
       if(!o)return 0;
                                                                                                                dmax(d){}
       if(cmp.sort id==k)return o->l?findmin(o
                                                                 .d[i]){
                                                                                                           void up(){
            ->1,(k+1)%kd):o;
                                                               is=0:break:
                                                                                                             mi=ma=pid:
                                                  146
       node *l=findmin(o->l,(k+1)%kd);
                                                  147
                                                                                                             s=1;
       node *r=findmin(o->r,(k+1)%kd);
                                                          if(is)in range.push back(u->pid);
       if(1&&!r)return cmp(1,0)?1:0;
                                                                                                                for(int i=0;i<kd;++i){</pre>
```

```
mi.d[i]=min(mi.d[i],l->mi.d[i]);
          ma.d[i]=max(ma.d[i],1->ma.d[i]);
13
14
        s+=1->s;
15
       if(r){
16
17
         for(int i=0:i<kd:++i){</pre>
          mi.d[i]=min(mi.d[i],r->mi.d[i]);
          ma.d[i]=max(ma.d[i],r->ma.d[i]);
20
21
        s+=r->s;
22
23
     void up2(){/*其他懶惰標記向上更新*/}
     void down(){/*其他懶惰標記下推*/}
   }*root;
26
   //檢查區間包含用的函數
   bool range include(node *o,const point &L,
       const point &R){
     for(int i=0:i<kd:++i){</pre>
      if(L.d[i]>o->ma.d[i]||R.d[i]<o->mi.d[i])
     }//(L,R)區間有和o的區間有交集就回傳true
32
     return 1;
33
   bool range in range(node *o, const point &L,
        const point &R){
     for(int i=0;i<kd;++i){</pre>
      if(L.d[i]>o->mi.d[i]||o->ma.d[i]>R.d[i])
           return 0:
     }//(L,R)區間完全包含o的區間就回傳true
     return 1;
39
   bool point in range(node *o, const point &L,
        const point &R){
     for(int i=0;i<kd;++i){</pre>
      if(L.d[i]>o->pid.d[i]||R.d[i]<o->pid.d[i
           ])return 0;
     }//(L,R)區間完全包含o->pid這個點就回傳true
43
44
     return 1;
45
46 //單點修改,以單點改值為例
   void update(node *u,const point &x,int data,
       int k=0){
     if(!u)return:
     u->down();
     if(u->pid==x){
      u->data=data;
      u->up2();
      return:
     cmp.sort id=k;
     update(cmp(x,u->pid)?u->1:u->r,x,data,(k
         +1)%kd);
     u->up2();
59 //區間修改
   void update(node *o,const point &L,const
       point &R, int data){
     if(!o)return;
     o->down();
     if(range_in_range(o,L,R)){
      //區間懶惰標記修改
      o->down();
```

```
return;
    if(point_in_range(o,L,R)){
      //這個點在(L,R)區間,但是他的左右子樹不
           一定在區間中
      //單點懶惰標記修改
71
    if(o->l&&range include(o->l,L,R))update(o
         ->1,L,R,data);
    if(o->r&&range include(o->r,L,R))update(o
         ->r,L,R,data);
    o->up2();
75
  //區間查詢·以總和為例
  int query(node *o,const point &L,const point
        &R){
    if(!o)return 0;
    o->down();
    if(range in range(o,L,R))return o->sum;
    int ans=0;
    if(point_in_range(o,L,R))ans+=o->data;
    if(o->l&&range include(o->l,L,R))ans+=
         query(o->1,L,R);
    if(o->r&&range_include(o->r,L,R))ans+=
         query(o->r,L,R);
    return ans;
86
```

## 2.4 reference point

```
1 template < typename T>
2 struct _RefC{
    T data:
    int ref;
    RefC(const T&d=0):data(d),ref(0){}
  template<typename T>
   struct rp{
    RefC<T> *p;
    T *operator->(){return &p->data;}
    T & operator*() { return p->data; }
    operator _RefC<T>*(){return p;}
    rp &operator=(const rp &t){
13
14
      if(p&&!--p->ref)delete p;
15
      p=t.p,p&&++p->ref;
16
      return *this:
17
    _rp(_RefC<T> *t=0):p(t){p&&++p->ref;}
    _rp(const _rp &t):p(t.p){p&&++p->ref;}
20
    ~_rp(){if(p&&!--p->ref)delete p;}
21
  template<typename T>
  inline _rp<T> new_rp(const T&nd){
    return rp<T>(new RefC<T>(nd));
24
25
```

# 2.5 skew heap

```
1 | node *merge(node *a, node *b){
   if(!a||!b) return a?a:b;
   if(b->data<a->data) swap(a,b);
    swap(a->1,a->r);
   a->1=merge(b,a->1);
   return a;
```

#### 2.6 undo disjoint set

```
1 | struct DisjointSet {
    // save() is like recursive
     // undo() is like return
     int n, fa[MXN], sz[MXN];
     vector<pair<int*,int>> h;
     vector<int> sp:
     void init(int tn) {
       for (int i=0; i<n; i++) sz[fa[i]=i]=1;</pre>
       sp.clear(); h.clear();
10
11
     void assign(int *k, int v) {
12
13
       h.PB({k, *k});
14
       *k=v:
15
16
     void save() { sp.PB(SZ(h)); }
17
     void undo() {
       assert(!sp.empty());
18
       int last=sp.back(); sp.pop back();
19
       while (SZ(h)!=last) {
21
         auto x=h.back(); h.pop_back();
22
         *x.F=x.S;
23
24
25
     int f(int x) {
       while (fa[x]!=x) x=fa[x];
27
       return x:
28
29
     void uni(int x, int y) {
30
       x=f(x); y=f(y);
       if (x==y) return ;
31
       if (sz[x]<sz[y]) swap(x, y);</pre>
33
       assign(&sz[x], sz[x]+sz[y]);
       assign(&fa[y], x);
34
35
36 }djs;
```

# 整體 一分

```
1 void totBS(int L, int R, vector<Item> M){
   if(Q.empty()) return; //維護全域B陣列
   if(L==R) 整個M的答案=r, return;
   int mid = (L+R)/2:
   vector<Item> mL, mR;
   do_modify_B_with_divide(mid,M);
   //讓B陣列在遞迴的時候只會保留[L~mid]的資訊
   undo modify B(mid,M);
   totBS(L,mid,mL);
   totBS(mid+1,R,mR);
```

```
Flow
```

```
3.1 dinic
```

```
1 template<typename T>
 2 struct DINIC{
     static const int MAXN=105;
     static const T INF=INT MAX;
     int n, LV[MAXN], cur[MAXN];
     struct edge{
       int v,pre;
       T cap,r;
       edge(int v,int pre,T cap):v(v),pre(pre),
             cap(cap),r(cap){}
10
     int g[MAXN];
11
     vector<edge> e:
     void init(int _n){
13
       memset(g,-1,sizeof(int)*((n=n)+1));
15
       e.clear():
16
                                                   11
     void add edge(int u,int v,T cap,bool
                                                   12
          directed=false){
                                                   13
       e.push_back(edge(v,g[u],cap));
18
                                                   14
       g[u]=e.size()-1;
19
       e.push_back(edge(u,g[v],directed?0:cap))
                                                   16
                                                   17
       g[v]=e.size()-1;
21
22
23
     int bfs(int s,int t){
24
       memset(LV,0,sizeof(int)*(n+1));
                                                   20
25
       memcpy(cur,g,sizeof(int)*(n+1));
                                                   21
26
       queue<int> q;
27
       q.push(s);
                                                   22
28
       LV[s]=1;
                                                   23
29
       while(q.size()){
                                                   24
         int u=q.front();q.pop();
30
31
          for(int i=g[u];~i;i=e[i].pre){
                                                   26
32
           if(!LV[e[i].v]&&e[i].r){
                                                   27
33
             LV[e[i].v]=LV[u]+1;
                                                   28
                                                   29
34
             q.push(e[i].v);
35
             if(e[i].v==t)return 1;
                                                   30
36
                                                   31
37
         }
                                                   32
38
       }
                                                   33
39
       return 0;
                                                   34
40
                                                   35
     T dfs(int u,int t,T CF=INF){
41
       if(u==t)return CF;
42
                                                   37
       T df:
                                                   38
43
       for(int &i=cur[u];~i;i=e[i].pre){
         if(LV[e[i].v]==LV[u]+1&&e[i].r){
           if(df=dfs(e[i].v,t,min(CF,e[i].r))){
47
             e[i].r-=df;
             e[i^1].r+=df;
48
49
              return df;
52
       return LV[u]=0;
54
     T dinic(int s,int t,bool clean=true){
       if(clean)for(size t i=0;i<e.size();++i)</pre>
```

# 3.2 ISAP with cut

e[i].r=e[i].cap;

while(bfs(s,t))while(f=dfs(s,t))ans+=f;

T ans=0, f=0;

return ans:

```
1 template<typename T>
 struct ISAP{
    static const int MAXN=105;
   static const T INF=INT MAX;
    int n://點數
    int d[MAXN],gap[MAXN],cur[MAXN];
    struct edge{
     int v,pre;
      edge(int v,int pre,T cap):v(v),pre(pre),
          cap(cap),r(cap){}
   int g[MAXN];
   vector<edge> e;
    void init(int _n){
     memset(g, -1, sizeof(int)*((n= n)+1));
     e.clear();
    void add edge(int u,int v,T cap,bool
         directed=false){
     e.push_back(edge(v,g[u],cap));
     g[u]=e.size()-1;
     e.push_back(edge(u,g[v],directed?0:cap))
     g[v]=e.size()-1;
   T dfs(int u, int s, int t, T CF=INF){
     if(u==t)return CF;
     T tf=CF,df;
      for(int &i=cur[u];~i;i=e[i].pre){
       if(e[i].r&&d[u]==d[e[i].v]+1){
          df=dfs(e[i].v,s,t,min(tf,e[i].r));
          e[i].r-=df;
          e[i^1].r+=df;
          if(!(tf-=df)||d[s]==n)return CF-tf;
      int mh=n;
      for(int i=cur[u]=g[u];~i;i=e[i].pre){
       if(e[i].r&&d[e[i].v]<mh)mh=d[e[i].v];</pre>
      if(!--gap[d[u]])d[s]=n;
     else ++gap[d[u]=++mh];
     return CF-tf;
   T isap(int s,int t,bool clean=true){
      memset(d,0,sizeof(int)*(n+1));
     memset(gap,0,sizeof(int)*(n+1));
      memcpy(cur,g,sizeof(int)*(n+1));
     if(clean) for(size t i=0;i<e.size();++i)</pre>
       e[i].r=e[i].cap;
      T MF=0;
      for(gap[0]=n;d[s]<n;)MF+=dfs(s,s,t);</pre>
```

39

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59

60

61

62

63

64

65

66

67

68 };

bool modlabel(){

static deque<int>q;

for(int u=0;u<=n;++u)</pre>

TP mincost(int s,int t){

while(modlabel()){

while(augment(S,INF));

while(q.size()){

dis[T]=0,q.push back(T);

for(int u=0;u<=n;++u)dis[u]=INF;</pre>

int u=q.front();q.pop front();

for(int i=g[u];~i;i=e[i].pre){

q.front():S]){

for(int i=g[u];~i;i=e[i].pre)

return PIS+=dis[S], dis[S]<INF;</pre>

q.push front(e[i].v);

}else q.push back(e[i].v);

e[i].cost+=dis[e[i].v]-dis[u];

do memset(vis,0,sizeof(bool)\*(n+1));

dis[e[i].v]){

if(e[i^1].r&&(dt=dis[u]-e[i].cost)

if((dis[e[i].v]=dt)<=dis[q.size()?</pre>

```
vector<int> cut e;//最小割邊集
    bool vis[MAXN];
54
55
    void dfs_cut(int u){
       vis[u]=1;//表示u屬於source的最小割集
       for(int i=g[u];~i;i=e[i].pre)
         if(e[i].r>0&&!vis[e[i].v])dfs cut(e[i
             ].v);
60
    T min cut(int s,int t){
      T ans=isap(s,t);
61
      memset(vis,0,sizeof(bool)*(n+1));
62
      dfs_cut(s), cut_e.clear();
63
       for(int u=0;u<=n;++u)if(vis[u])</pre>
64
         for(int i=g[u];~i;i=e[i].pre)
          if(!vis[e[i].v])cut_e.push_back(i);
66
67
       return ans;
68
69 };
```

#### 3.3 MinCostMaxFlow

static const int MAXN=440;

1 template<typename TP>

2 struct MCMF{

```
static const TP INF=999999999;
    struct edge{
       int v,pre;
       TP r, cost;
       edge(int v,int pre,TP r,TP cost):v(v),
            pre(pre),r(r),cost(cost){}
     int n,S,T;
     TP dis[MAXN],PIS,ans;
    bool vis[MAXN];
13
     vector<edge> e;
    int g[MAXN];
14
    void init(int n){
       memset(g,-1,sizeof(int)*((n=_n)+1));
16
       e.clear();
17
18
19
     void add_edge(int u,int v,TP r,TP cost,
          bool directed=false){
       e.push_back(edge(v,g[u],r,cost));
       g[u]=e.size()-1;
22
       e.push back(
       edge(u,g[v],directed?0:r,-cost));
24
       g[v]=e.size()-1;
25
26
     TP augment(int u, TP CF){
       if(u==T||!CF)return ans+=PIS*CF,CF;
       vis[u]=1;
29
       TP r=CF,d;
       for(int i=g[u];~i;i=e[i].pre){
         if(e[i].r&&!e[i].cost&&!vis[e[i].v]){
32
           d=augment(e[i].v,min(r,e[i].r));
33
           e[i].r-=d;
34
           e[i^1].r+=d;
35
           if(!(r-=d))break;
36
37
       return CF-r;
```

# 4 Graph

S=s,T=t;

PIS=ans=0:

}return ans;

## 4.1 Augmenting\_Path

```
1 | #define MAXN1 505
 2 #define MAXN2 505
 3 int n1, n2; //n1 個點 連向 n2 個點
 4 | int match[MAXN2]; // 屬於 n2的點匹配了哪個點
 5 vector<int > g[MAXN1];//\bigseterms
 6 bool vis[MAXN2];//是否走訪過
   bool dfs(int u){
     for(size_t i=0;i<g[u].size();++i){</pre>
       int v=g[u][i];
       if(vis[v])continue;
11
       vis[v]=1;
       if(match[v]==-1||dfs(match[v]))
         return match[v]=u, 1;
13
14
     return 0;
16 }
   inline int max match(){
     int ans=0:
     memset(match,-1,sizeof(int)*n2);
     for(int i=0;i<n1;++i){</pre>
       memset(vis,0,sizeof(bool)*n2);
       if(dfs(i))++ans;
22
```

```
return ans;
                                                 12
                                                    #define qpush(x) q.push(x),S[x]=0
                                                    void flower(int x,int y,int l,queue<int> &q)
                                                      while(st[x]!=1){
         Augmenting Path multiple
                                                        pa[x]=y;
                                                        if(S[y=match[x]]==1)qpush(y);
                                                 18
                                                        st[x]=st[y]=1, x=pa[y];
1 #define MAXN1 1005
                                                 19
2 #define MAXN2 505
                                                 20
3 int n1, n2; //n1 個點連向n2個點,其中n2個點可以
                                                    bool bfs(int x){
       匹配很多邊
                                                      for(int i=1;i<=n;++i)st[i]=i;</pre>
                                                 22
4 vector<int> g[MAXN1];//圖
                                                      memset(S+1,-1,sizeof(int)*n);
5 | int c[MAXN2]; //每個屬於 n2 點最多可以接受幾條
                                                      queue<int>q; qpush(x);
                                                 25
                                                      while(q.size()){
                                                        x=q.front(),q.pop();
6 | vector<int> match_list[MAXN2];//每個屬於n2的
                                                        for(size_t i=0;i<g[x].size();++i){</pre>
       點匹配了那些點
                                                 28
                                                          int y=g[x][i];
7 bool vis[MAXN2];//是否走訪過
                                                          if(S[y]==-1){
                                                 29
  bool dfs(int u){
                                                 30
                                                            pa[y]=x,S[y]=1;
    for(size_t i=0;i<g[u].size();++i){</pre>
                                                 31
                                                            if(!match[y]){
      int v=g[u][i];
                                                 32
                                                               for(int lst;x;y=lst,x=pa[y])
11
      if(vis[v])continue;
                                                 33
                                                                 lst=match[x], match[x]=y, match[y
12
      vis[v]=true;
                                                                     ]=x;
      if((int)match_list[v].size()<c[v]){</pre>
13
                                                              return 1;
                                                 34
         return match_list[v].push_back(u),
14
                                                 35
             true:
                                                            qpush(match[y]);
                                                 36
      }else{
15
                                                          }else if(!S[y]&&st[y]!=st[x]){
16
         for(size_t j=0;j<match_list[v].size()</pre>
                                                            int l=lca(y,x);
             ;++j){
                                                 39
                                                            flower(y,x,l,q),flower(x,y,l,q);
           int next_u=match_list[v][j];
17
                                                 40
           if(dfs(next_u))
18
                                                 41
             return match_list[v][j]=u, true;
19
                                                 42
20
                                                 43
                                                      return 0;
21
      }
                                                 44
22
                                                 45
                                                    int blossom(){
23
    return false;
                                                      int ans=0;
24
                                                      for(int i=1;i<=n;++i)</pre>
  int max match(){
                                                        if(!match[i]&&bfs(i))++ans;
    for(int i=0;i<n2;++i)match list[i].clear()</pre>
                                                      return ans;
    int cnt=0;
    for(int u=0;u<n1;++u){</pre>
      memset(vis,0,sizeof(bool)*n2);
      if(dfs(u))++cnt;
                                                    4.4 graphISO
31
32
    return cnt;
```

# 4.3 blossom matching

```
1 const int MAXN=1005, K=30; // K要夠大
  const long long A=3,B=11,C=2,D=19,P=0
        xdefaced;
  long long f[K+1][MAXN];
  vector<int> g[MAXN],rg[MAXN];
  int n;
  void init(){
    for(int i=0;i<n;++i){</pre>
      f[0][i]=1;
       g[i].clear(), rg[i].clear();
11 }
  void add edge(int u,int v){
    g[u].push_back(v), rg[v].push_back(u);
  long long point hash(int u)\{//O(N)\}
    for(int t=1;t<=K;++t){</pre>
       for(int i=0;i<n;++i){</pre>
```

if(!vy[y]&&cut>slack\_y[y])cut=slack\_y[

у];

for(int j=1;j<=n;++j){</pre>

if(vx[j])lx[j]-=cut;

tmp[dep]=u;

return 0;

if(dfs(cnt,dep+1))return 1;

30

31

31

32

33

vector<long long> tmp;

if(tmp.empty())return 177;
long long ret=4931;

sort(tmp.begin(),tmp.end());

for(auto v:g[u])if(!vis[v])tmp.PB(dfs(v));

```
f[t][i]=f[t-1][i]*A%P;
                                                            if(vy[j])ly[j]+=cut;
                                                                                                                                                             bool operator<(const edge&e)const{</pre>
                                                  35
19
         for(int j:g[i])f[t][i]=(f[t][i]+f[t
                                                  36
                                                            else slack_y[j]-=cut;
                                                                                                          int clique(){
                                                                                                                                                        20
                                                                                                                                                               return cost<e.cost;</pre>
              -1][j]*B%P)%P;
                                                                                                                                                        21
                                                  37
                                                                                                     35
                                                                                                            int u,v,ns;
         for(int j:rg[i])f[t][i]=(f[t][i]+f[t
                                                  38
                                                          for(int y=1;y<=n;++y){</pre>
                                                                                                     36
                                                                                                            for(ans=0,u=N-1;u>=0;--u){
                                                                                                                                                        22
                                                                                                                                                           };
                                                           if(!vy[y]&&slack_y[y]==0){
                                                                                                              for(ns=0, tmp[0]=u, v=u+1; v<N;++v)</pre>
                                                                                                                                                           struct bit node{
              -1][j]*C%P)%P;
                                                  39
                                                                                                     37
                                                                                                                                                        23
         if(i==u)f[t][i]+=D;//如果圖太大的話
                                                  40
                                                              if(!match y[y]){augment(y);return;}
                                                                                                     38
                                                                                                                if(g[u][v])stk[1][ns++]=v;
                                                                                                                                                        24
                                                                                                                                                             T mi;
                                                              vy[y]=1,q.push(match_y[y]);
                                                                                                     39
                                                                                                              dfs(ns,1),dp[u]=ans;
                                                                                                                                                        25
                                                  41
              把這行刪掉,執行一次後f[K]就會是所
                                                                                                                                                             bit_node(const T&mi=INF,int id=-1):mi(mi),
                                                  42
                                                                                                     40
              有點的答案
                                                  43
                                                                                                     41
                                                                                                            return ans;
                                                                                                                                                                  id(id){}
         f[t][i]%=P;
                                                                                                     42
                                                  44
                                                                                                                                                        27
                                                                                                                                                           };
23
                                                                                                     43 };
                                                                                                                                                        28
                                                                                                                                                           vector<bit node> bit;
                                                  45
24
                                                                                                                                                           void bit update(int i,const T&data,int id){
                                                  46
                                                     long long KM(){
25
    return f[K][u];
                                                                                                                                                             for(;i;i-=i&(-i)){
                                                       memset(match_y,0,sizeof(int)*(n+1));
                                                  47
26
                                                  48
                                                        memset(ly,0,sizeof(int)*(n+1));
                                                                                                                                                               if(data<bit[i].mi)bit[i]=bit node(data,</pre>
   vector<long long> graph hash(){
                                                  49
                                                        for(int x=1;x<=n;++x){</pre>
                                                                                                               MinimumMeanCycle
                                                                                                                                                                    id);
     vector<long long> ans;
                                                  50
                                                         1x[x]=-INF;
                                                                                                                                                        32
    for(int i=0;i<n;++i)ans.push_back(</pre>
                                                          for(int y=1;y<=n;++y)</pre>
                                                  51
                                                                                                                                                        33
          point hash(i));//O(N^2)
                                                            lx[x]=max(lx[x],g[x][y]);
                                                  52
                                                                                                                                                        34
                                                                                                                                                           int bit_find(int i,int m){
     sort(ans.begin(),ans.end());
                                                                                                      1 #include < cfloat > //for DBL MAX
                                                  53
                                                                                                                                                        35
                                                                                                                                                             bit_node x;
                                                                                                      1 int dp[MAXN][MAXN]; // 1-base, O(NM)
31
    return ans;
                                                  54
                                                        for(int x=1;x<=n;++x)bfs(x);</pre>
                                                                                                                                                        36
                                                                                                                                                             for(;i<=m;i+=i&(-i)) if(bit[i].mi<x.mi)x=</pre>
                                                                                                        vector<tuple<int,int,int>> edge;
                                                  55
                                                        long long ans=0;
                                                                                                                                                                  bit[i];
                                                                                                        double mmc(int n){//allow negative weight
                                                        for(int y=1;y<=n;++y)ans+=g[match_y[y]][y</pre>
                                                                                                                                                        37
                                                                                                                                                             return x.id;
                                                                                                          const int INF=0x3f3f3f3f;
                                                                                                                                                        38
                                                                                                          for(int t=0;t<n;++t){</pre>
                                                                                                                                                           vector<edge> build_graph(int n,point p[]){
                                                  57
                                                       return ans;
        KM
                                                                                                            memset(dp[t+1],0x3f,sizeof(dp[t+1]));
                                                                                                                                                             vector<edge> e;//edge for MST
                                                                                                            for(const auto &e:edge){
                                                                                                                                                             for(int dir=0;dir<4;++dir){//4種座標變換
                                                                                                              int u,v,w;
                                                                                                                                                               if(dir%2) for(int i=0;i<n;++i) swap(p[i</pre>
                                                                                                              tie(u,v,w) = e;
1 #define MAXN 405
                                                                                                                                                                    ].x,p[i].y);
                                                                                                     11
                                                                                                              dp[t+1][v]=min(dp[t+1][v],dp[t][u]+w);
2 #define INF 0x3f3f3f3f
                                                                                                                                                               else if(dir==2) for(int i=0;i<n;++i) p[i</pre>
                                                     4.6 MaximumClique
                                                                                                     12
3 int n;// 1-base · 0表示沒有匹配
                                                                                                                                                                    ].x=-p[i].x;
                                                                                                     13
  int g[MAXN][MAXN],lx[MAXN],ly[MAXN],pa[MAXN
                                                                                                                                                               sort(p,p+n,cmpx);
                                                                                                     14
                                                                                                          double res = DBL_MAX;
        ],slack_y[MAXN];
                                                                                                                                                               vector<T> ga(n), gb;
                                                                                                     15
                                                                                                          for(int u=1;u<=n;++u){</pre>
                                                   1 | struct MaxClique{
  int match_y[MAXN], match_x[MAXN];
                                                                                                                                                               for(int i=0;i<n;++i)ga[i]=p[i].y-p[i].x;</pre>
                                                                                                            if(dp[n][u]==INF) continue;
                                                                                                     16
                                                       static const int MAXN=105;
  bool vx[MAXN],vy[MAXN];
                                                                                                            double val = -DBL_MAX;
                                                                                                                                                               gb=ga, sort(gb.begin(),gb.end());
                                                       int N.ans:
   void augment(int y){
                                                                                                                                                               gb.erase(unique(gb.begin(),gb.end()),gb.
                                                                                                            for(int t=0;t<n;++t)</pre>
                                                       int g[MAXN][MAXN], dp[MAXN], stk[MAXN][MAXN
    for(int x,z;y;y=z){
                                                                                                                                                                    end());
                                                                                                              val=max(val,(dp[n][u]-dp[t][u])*1.0/(n
       x=pa[y],z=match_x[x];
                                                                                                                                                               int m=gb.size();
                                                                                                                   -t));
                                                       int sol[MAXN],tmp[MAXN];//sol[0~ans-1]為答
       match_y[y]=x,match_x[x]=y;
                                                                                                                                                               bit=vector<bit_node>(m+1);
                                                                                                     20
                                                                                                            res=min(res, val);
11
                                                                                                                                                        51
                                                                                                                                                               for(int i=n-1;i>=0;--i){
                                                                                                     21
                                                        void init(int n){
12
                                                                                                                                                                 int pos=lower_bound(gb.begin(),gb.end
                                                                                                     22
                                                                                                          return res;
   void bfs(int st){
                                                         N=n;//0-base
                                                                                                                                                                      (),ga[i])-gb.begin()+1;
    for(int i=1;i<=n;++i)slack_y[i]=INF,vx[i]=</pre>
                                                         memset(g,0,sizeof(g));
                                                                                                                                                                 int ans=bit_find(pos,m);
          vy[i]=0;
                                                                                                                                                                 if(~ans)e.push_back(edge(p[i].id,p[ans
     queue<int> q;q.push(st);
                                                        void add_edge(int u,int v){
                                                                                                                                                                      ].id,p[i].dist(p[ans])));
     for(;;){
16
                                                  11
                                                         g[u][v]=g[v][u]=1;
                                                                                                                                                        55
                                                                                                                                                                 bit_update(pos,p[i].x+p[i].y,i);
                                                                                                        4.8 Rectilinear MST
       while(q.size()){
                                                   12
                                                                                                                                                        56
                                                        int dfs(int ns,int dep){
18
         int x=q.front();q.pop();
                                                                                                                                                             }
                                                                                                                                                        57
19
                                                   14
                                                         if(!ns){
                                                                                                                                                        58
                                                                                                                                                             return e;
                                                                                                      1 / / 平面曼哈頓最小生成樹構造圖(去除非必要邊)
20
         for(int y=1;y<=n;++y)if(!vy[y]){</pre>
                                                  15
                                                            if(dep>ans){
                                                                                                      2 #define T int
           int t=lx[x]+ly[y]-g[x][y];
                                                   16
                                                              ans=dep;
                                                                                                      3 #define INF 0x3f3f3f3f
           if(t==0){
                                                   17
                                                              memcpy(sol,tmp,sizeof tmp);
                                                                                                        struct point{
23
             pa[y]=x;
                                                   18
                                                              return 1;
                                                                                                         T x,y;
                                                                                                                                                                 treeISO
             if(!match_y[y]){augment(y);return
                                                            }else return 0;
                                                  20
                                                                                                          int id;//從0開始編號
             vy[y]=1,q.push(match y[y]);
                                                  21
                                                          for(int i=0;i<ns;++i){</pre>
                                                                                                          point(){}
                                                                                                                                                         1 const int MAXN=100005;
           }else if(slack_y[y]>t)pa[y]=x,
                                                  22
                                                           if(dep+ns-i<=ans)return 0;</pre>
                                                                                                          T dist(const point &p)const{
                slack_y[y]=t;
                                                  23
                                                            int u=stk[dep][i],cnt=0;
                                                                                                            return abs(x-p.x)+abs(y-p.y);
                                                                                                                                                           const long long X=12327,P=0xdefaced;
                                                            if(dep+dp[u]<=ans)return 0;</pre>
                                                                                                                                                           vector<int> g[MAXN];
                                                            for(int j=i+1; j<ns; ++j){</pre>
28
                                                  25
                                                                                                     11 };
                                                                                                                                                         4 bool vis[MAXN];
       int cut=INF:
                                                              int v=stk[dep][j];
                                                                                                        bool cmpx(const point &a,const point &b){
                                                                                                                                                           long long dfs(int u){//hash ver
30
       for(int y=1;y<=n;++y){</pre>
                                                              if(g[u][v])stk[dep+1][cnt++]=v;
                                                                                                          return a.x<b.x||(a.x==b.x&&a.y<b.y);
                                                                                                                                                             vis[u]=1;
```

14 }

15 struct edge{

int u,v;

edge(int u,int v,T c):u(u),v(v),cost(c){}

```
for(auto v:tmp)ret=((ret*X)^v)%P;
    return ret;
13
14
   ·
//-----
   string dfs(int x,int p){
    vector<string> c;
    for(int y:g[x])
18
19
     if(y!=p)c.emplace_back(dfs(y,x));
    sort(c.begin(),c.end());
    string ret("(");
22
    for(auto &s:c)ret+=s;
23
    ret+=")":
    return ret;
24
25
  4.10 一般圖最小權完美匹配
1 | struct Graph {
    // Minimum General Weighted Matching (
         Perfect Match) 0-base
    static const int MXN = 105;
    int n, edge[MXN][MXN];
    int match[MXN],dis[MXN],onstk[MXN];
    vector<int> stk;
    void init(int n) {
```

for (int i=0; i<n; i++)</pre>

edge[i][j] = 0;

bool SPFA(int u){

onstk[u] = 1;

stk.push back(u);

v1){

for (int j=0; j<n; j++)</pre>

edge[u][v] = edge[v][u] = w;

if (onstk[u]) return true;

for (int v=0; v<n; v++){</pre>

int m = match[v];

onstk[v] = 1;

edge[u][v]){

stk.push back(v);

stk.pop back();

onstk[v] = 0;

for (int i=0; i<n; i+=2){</pre>

match[i] = i+1, match[i+1] = i;

edge[u][v];

if (SPFA(m)) return true;

void add edge(int u, int v, int w) {

if (u != v && match[u] != v && !onstk[

if (dis[m] > dis[u] - edge[v][m] +

dis[m] = dis[u] - edge[v][m] +

10

11

12

13

14

15

16

17

18

19

20

22

23

26

28

29

30

31

32

36

41

}

onstk[u] = 0;

return false;

int solve() {

for(;;){

stk.pop back();

// find a match

# 4.11 全局最小割

}

**int** ret = 0:

return ret;

int found = 0:

stk.clear();

found = 1:

if (!found) break;

for (int i=0; i<n; i++)</pre>

ret += edge[i][match[i]];

47

48

49

51

52

53

54

55

56

57

58

59

60

61

62

63

64

65 }graph;

for (int i=0; i<n; i++){</pre>

match[u] = v;

match[v] = u;

if (!onstk[i] && SPFA(i)){

while (stk.size()>=2){

for (int i=0; i<n; i++) dis[i] = onstk 34 };</pre>

int u = stk.back(); stk.pop\_back

int v = stk.back(); stk.pop back

11

12

13

16

17

18

19

20

21

22

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32

33

34

35

36

37

38

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41

42

43

44

45

46

47

49

50

51

return !(isK33(n, degree) || isK5(n,

degree));

```
1 const int INF=0x3f3f3f3f3f;
2 template<typename T>
3 struct stoer_wagner{// 0-base
     static const int MAXN=150;
     T g[MAXN][MAXN], dis[MAXN];
     int nd[MAXN],n,s,t;
     void init(int n){
       for(int i=0:i<n:++i)</pre>
         for(int j=0;j<n;++j)g[i][j]=0;</pre>
11
     void add edge(int u,int v,T w){
12
       g[u][v]=g[v][u]+=w;
13
14
     T min cut(){
15
16
       T ans=INF;
       for(int i=0;i<n;++i)nd[i]=i;</pre>
17
       for(int ind,tn=n;tn>1;--tn){
18
         for(int i=1;i<tn;++i)dis[nd[i]]=0;</pre>
19
         for(int i=1;i<tn;++i){</pre>
20
21
            ind=i:
            for(int j=i;j<tn;++j){</pre>
22
23
              dis[nd[j]]+=g[nd[i-1]][nd[j]];
24
              if(dis[nd[ind]]<dis[nd[j]])ind=j;</pre>
25
            swap(nd[ind],nd[i]);
26
27
         if(ans>dis[nd[ind]])ans=dis[t=nd[ind
28
              ]],s=nd[ind-1];
          for(int i=0;i<tn;++i)</pre>
            g[nd[ind-1]][nd[i]]=g[nd[i]][nd[ind
                 -1]]+=g[nd[i]][nd[ind]];
31
       return ans;
```

# 4.13 弦圖完美消除序列

```
1 struct chordal{
  4.12 平面圖判定
                                                        static const int MAXN=1005;
                                                        int n:// 0-base
                                                        vector<int>G[MAXN];
1 static const int MAXN = 20;
                                                        int rank[MAXN],label[MAXN];
2 struct Edge{
                                                       bool mark[MAXN];
    int u, v;
                                                        void init(int n){n= n;
    Edge(int s, int d) : u(s), v(d) {}
                                                          for(int i=0;i<n;++i)G[i].clear();</pre>
  bool isK33(int n, int degree[]){
                                                   10
                                                       void add_edge(int u,int v){
    int t = 0, z = 0;
                                                   11
                                                         G[u].push back(v);
    for(int i=0;i<n;++i){</pre>
                                                   12
                                                          G[v].push back(u);
      if(degree[i] == 3)++t;
                                                   13
       else if(degree[i] == 0)++z;
                                                        vector<int> MCS(){
                                                   14
       else return false:
                                                          memset(rank,-1,sizeof(int)*n);
                                                   15
                                                          memset(label,0,sizeof(int)*n);
                                                   16
    return t == 6 && t + z == n;
                                                   17
                                                          priority queue<pair<int,int> > pq;
                                                          for(int i=0;i<n;++i)pq.push(make_pair(0,</pre>
14
  bool isK5(int n, int degree[]){
15
    int f = 0, z = 0;
                                                          for(int i=n-1;i>=0;--i)for(;;){
                                                   19
    for(int i=0;i<n;++i){</pre>
                                                            int u=pq.top().second;pq.pop();
                                                   20
      if(degree[i] == 4)++f;
                                                  21
                                                            if(~rank[u])continue;
       else if(degree[i] == 0)++z;
                                                   22
                                                            rank[u]=i:
       else return false:
                                                   23
                                                            for(auto v:G[u])if(rank[v]==-1){
                                                              pq.push(make pair(++label[v],v));
                                                   24
    return f == 5 \&\& f + z == n;
                                                   25
23
                                                           break;
                                                   26
24 // it judge a given graph is Homeomorphic
                                                   27
       with K33 or K5
                                                   28
                                                          vector<int> res(n);
25 bool isHomeomorphic(bool G[MAXN][MAXN],
                                                   29
                                                          for(int i=0;i<n;++i)res[rank[i]]=i;</pre>
       const int n){
                                                   30
                                                          return res:
     for(;;){
                                                   31
       int cnt = 0;
                                                        bool check(vector<int> ord){//弦圖判定
       for(int i=0;i<n;++i){</pre>
                                                          for(int i=0;i<n;++i)rank[ord[i]]=i;</pre>
         vector<Edge> E;
                                                          memset(mark.0.sizeof(bool)*n);
         for(int j=0;j<n&E.size()<3;++j)</pre>
                                                          for(int i=0;i<n;++i){</pre>
           if(G[i][j] && i != j)
                                                            vector<pair<int,int> > tmp;
             E.push back(Edge(i, j));
                                                            for(auto u:G[ord[i]])if(!mark[u])
         if(E.size() == 1){
                                                              tmp.push_back(make_pair(rank[u],u));
           G[i][E[0].v] = G[E[0].v][i] = false; 39
                                                            sort(tmp.begin(),tmp.end());
         }else if(E.size() == 2){
                                                            if(tmp.size()){
           G[i][E[0].v] = G[E[0].v][i] = false;
                                                              int u=tmp[0].second;
           G[i][E[1].v] = G[E[1].v][i] = false; 42
                                                              set<int> S;
           G[E[0].v][E[1].v] = G[E[1].v][E[0].v <sub>43</sub>
                                                              for(auto v:G[u])S.insert(v);
                ] = true;
                                                              for(size_t j=1;j<tmp.size();++j)</pre>
                                                   44
           ++cnt;
                                                                if(!S.count(tmp[j].second))return
                                                   45
      if(cnt == 0)break;
                                                            mark[ord[i]]=1;
                                                   47
                                                   48
     static int degree[MAXN];
                                                   49
                                                          return 1;
     fill(degree, degree + n, 0);
                                                   50
     for(int i=0;i<n;++i){</pre>
                                                   51 };
       for(int j=i+1; j<n; ++j){</pre>
         if(!G[i][j])continue;
         ++degree[i]:
         ++degree[j];
52
    }
```

## 4.14 最小斯坦納樹 DP

```
1 //n個點,其中r個要構成斯坦納樹
                                                  30
2 //答案在max(dp[(1<<r)-1][k]) k=0~n-1
                                                  31
3 | //p表示要構成斯坦納樹的點集
4 //0( n^3 + n*3^r + n^2*2^r )
5 #define REP(i,n) for(int i=0;i<(int)n;++i)</pre>
                                                  33
6 const int MAXN=30, MAXM=8; // 0-base
                                                  34
7 const int INF=0x3f3f3f3f3f;
                                                  35
8 int dp[1<<MAXM][MAXN];</pre>
                                                  36
9 int g[MAXN][MAXN];// 🗟
                                                  37
void init(){memset(g,0x3f,sizeof(g));}
                                                  38
  void add_edge(int u,int v,int w){
                                                  39
    g[u][v]=g[v][u]=min(g[v][u],w);
                                                  40
13
                                                  41
14
   void steiner(int n,int r,int *p){
                                                  42
    REP(k,n)REP(i,n)REP(j,n)
                                                  43
      g[i][j]=min(g[i][j],g[i][k]+g[k][j]);
                                                  44
     REP(i,n)g[i][i]=0;
    REP(i,r)REP(j,n)dp[1<<i][j]=g[p[i]][j];</pre>
                                                  45
     for(int i=1;i<(1<<r);++i){</pre>
                                                  46
20
       if(!(i&(i-1)))continue;
       REP(j,n)dp[i][j]=INF;
                                                  47
22
       REP(j,n){
                                                  48
23
         int tmp=INF:
                                                  49
         for(int s=i&(i-1);s;s=i&(s-1))
24
                                                  50
25
           tmp=min(tmp,dp[s][j]+dp[i^s][j]);
26
         REP(k,n)dp[i][k]=min(dp[i][k],g[j][k]+
              tmp);
                                                  52
27
                                                  53
28
                                                  54
                                                  55
```

# 4.15 最小樹形圖 朱劉

1 template<typename T>

```
struct zhu liu{
    static const int MAXN=110, MAXM=10005;
    struct node{
       int u,v;
       T w, tag;
       node(int u=0, int v=0, T w=0): u(u), v(v), w(
            w),tag(0),1(0),r(0){}
       void down(){
10
         w+=tag:
11
         if(1)1->tag+=tag;
12
         if(r)r->tag+=tag;
13
         tag=0;
14
     }mem[MAXM];//靜態記憶體
     node *pq[MAXN*2],*E[MAXN*2];
    int st[MAXN*2],id[MAXN*2],m;
    void init(int n){
       for(int i=1;i<=n;++i){</pre>
         pq[i]=E[i]=0, st[i]=id[i]=i;
^{21}
       }m=0;
22
     node *merge(node *a, node *b){//skew heap
       if(!a||!b)return a?a:b;
       a->down(),b->down();
       if(b->w<a->w)return merge(b,a);
       swap(a->1,a->r);
       a->1=merge(b,a->1);
```

# 4.16 穩定婚姻模板

return a;

void add edge(int u,int v,T w){

node(u,v,w)));

int find(int x,int \*st){

T build(int root, int n){

while(pa[i]){

ans+=E[i]->w:

56

57

58

59

60

61

62 };

T ans=0; int N=n, all=n;

for(int i=1:i<=N:++i){</pre>

break;

continue;

**if**(u!=v)pq[v]=merge(pq[v],&(mem[m++]=

return st[x]==x?x:st[x]=find(st[x],st);

pq[i]=merge(pq[i]->l,pq[i]->r);

if(find(E[i]->u,id)==find(i,id))

if(find(E[i]->u,st)==find(i,st)){

if(pq[i])pq[i]->tag-=E[i]->w;

find(E[u]->u,id)){

pq[N]=merge(pq[N],pq[u]);

for(int u=find(E[i]->u,id);u!=i;u=

if(pq[u])pq[u]->tag-=E[u]->w;

}else st[find(i,st)]=find(E[i]->u,st)

return all==1?ans:-INT\_MAX;//圖不連通就

pq[++N]=pq[i];id[N]=N;

id[find(u,id)]=N;

st[N]=find(i,st);

id[find(i,id)]=N:

,--all;

if(find(E[i]->u,id)!=find(i,id))

if(i==root||!pq[i])continue;

pq[i]->down(),E[i]=pq[i];

```
1 | queue < int > Q;
2 for ( i: 所有考生 ) {
   設定在第0志願;
   0.push(考生i);
6 while(Q.size()){
   當前考生=Q.front();Q.pop();
   while (此考生未分發) {
     指標移到下一志願;
     if ( 已經沒有志願 or 超出志願總數 )
        break;
     計算該考生在該科系加權後的總分;
11
     if (不符合科系需求) continue:
12
     if (目前科系有餘額) {
13
      依加權後分數高低順序將考牛id加入科系錄
14
          取名單中;
15
      break:
```

```
if (目前科系已額滿) {
18
      if ( 此考生成績比最低分數還高 ) {
                                    46
19
        依加權後分數高低順序將考生id加入科系
           錄取名單;
                                    48
        Q.push(被踢出的考生);
20
21
22
23
                                    52
24 }
                                    53
```

# 5 Linear\_Programming

### 5.1 最大密度子圖

1 typedef double T;//POJ 3155

2 const int MAXN=105:

3 struct edge{

```
int u,v;
    edge(int u=0,int v=0,T w=0):u(u),v(v),w(w)
7 };
8 vector<edge> E;
9 int n,m;// 1-base
10 T de[MAXN], pv[MAXN]; // 每個點的邊權和和點權(
       有些題目會給)
11 void init(){
   E.clear();
12
    for(int i=1;i<=n;++i)de[i]=pv[i]=0;</pre>
13
void add_edge(int u,int v,T w){
    E.push back(edge(u,v,w));
    de[u]+=w.de[v]+=w:
18 }
19 T U;//二分搜的最大值
  void get_U(){
21
    U=0:
    for(int i=1;i<=n;++i)U+=2*pv[i];</pre>
    for(size_t i=0;i<E.size();++i)U+=E[i].w;</pre>
25 | ISAP<T> isap; //網路流
26 int s,t;//原匯點
  void build(T L){
    isap.init(n+2);
    for(size t i=0;i<E.size();++i)</pre>
      isap.add edge(E[i].u,E[i].v,E[i].w);
    for(int v=1; v<=n; ++v){</pre>
      isap.add edge(s,v,U);
       isap.add_edge(v,t,U+2*L-de[v]-2*pv[v]);
34
35 }
36
  int main(){
    while(~scanf("%d%d",&n,&m)){
       if(!m){
         puts("1\n1");
         continue;
42
       init();
       int u,v;
```

```
for(int i=0;i<m;++i){</pre>
   scanf("%d%d",&u,&v);
    add edge(u,v,1);
 get_U();
 s=n+1,t=n+2;
 T l=0,r=U,k=1.0/(n*n):
 while(r-1>k){//二分搜最大值
   T mid=(1+r)/2;
   build(mid);
   T res=(U*n-isap.isap(s,t))/2;
   if(res>0)l=mid;
   else r=mid:
  build(1);
 isap.min cut(s,t);
  vector<int> ans;
  for(int i=1;i<=n;++i)</pre>
   if(isap.vis[i])ans.push back(i);
  printf("%d\n",ans.size());
  for(size t i=0;i<ans.size();++i)</pre>
   printf("%d\n",ans[i]);
return 0;
```

# 6 Number Theory

#### 6.1 basic

```
1 template < typename T>
  void gcd(const T &a,const T &b,T &d,T &x,T &
     if(!b) d=a,x=1,y=0;
     else gcd(b,a%b,d,y,x), y-=x*(a/b);
  long long int phi[N+1];
   void phiTable(){
     for(int i=1;i<=N;i++)phi[i]=i;</pre>
     for(int i=1;i<=N;i++)for(x=i*2;x<=N;x+=i)</pre>
          phi[x]-=phi[i];
11
   void all divdown(const LL &n) {// all n/x
     for(LL a=1;a<=n;a=n/(n/(a+1))){</pre>
       // dosomething;
14
15 }
16 const int MAXPRIME = 1000000:
   int iscom[MAXPRIME], prime[MAXPRIME],
        primecnt;
   int phi[MAXPRIME], mu[MAXPRIME];
   void sieve(void){
     memset(iscom,0,sizeof(iscom));
     primecnt = 0;
     phi[1] = mu[1] = 1;
     for(int i=2;i<MAXPRIME;++i) {</pre>
       if(!iscom[i]) {
         prime[primecnt++] = i;
         mu[i] = -1;
27
         phi[i] = i-1;
```

```
for(int j=0;j<primecnt;++j) {</pre>
                                                        return -1;
                                                                                                                                                              int n=s.size(),res=0;
30
         int k = i * prime[i];
                                                   92
                                                                                                     152
                                                                                                                                                              for(int i=0;i<n;++i){</pre>
         if(k>=MAXPRIME) break;
                                                                                                                                                                int t=0;
31
                                                                                                     153 //java code
                                                      LL Tonelli_Shanks(const LL &n, const LL &p) 154 //求sqrt(N)的連分數
32
         iscom[k] = prime[j];
                                                                                                                                                                for(int j=i+1;j<n;++j)</pre>
33
         if(i%prime[j]==0) {
                                                                                                                                                                  if(s[i]<s[i])++t;</pre>
                                                                                                         public static void Pell(int n){
                                                                                                                                                         11
           mu[k] = 0;
                                                   95
                                                        // x^2 = n \pmod{p}
                                                                                                                                                                res+=t*factorial[n-i-1];
                                                                                                          BigInteger N,p1,p2,q1,q2,a0,a1,a2,g1,g2,h1
35
           phi[k] = phi[i] * prime[j];
                                                   96
                                                        if(n==0) return 0:
                                                                                                                                                         13
                                                                                                                ,h2,p,q;
           break:
                                                        if(Legendre(n,p)!=1) while(1) { puts("SQRT 157
36
                                                                                                                                                         14
                                                                                                                                                              return res;
                                                                                                           g1=q2=p1=BigInteger.ZERO;
37
         } else {
                                                              ROOT does not exist"); }
                                                                                                                                                         15
                                                                                                           h1=q1=p2=BigInteger.ONE;
                                                        int S = 0;
                                                                                                                                                            vector<int> decode(int a,int n){
           mu[k] = -mu[i];
                                                   98
                                                                                                           a0=a1=BigInteger.valueOf((int)Math.sqrt
39
           phi[k] = phi[i] * (prime[j]-1);
                                                   99
                                                        LL 0 = p-1;
                                                                                                                                                              vector<int> res;
                                                                                                                (1.0*n));
                                                        while( !(Q&1) ) { Q>>=1; ++S; }
                                                                                                                                                              vector<bool> vis(n.0):
40
                                                                                                                                                         18
                                                                                                     160
                                                                                                           BigInteger ans=a0.multiply(a0);
                                                        if(S==1) return modexp(n%p,(p+1)/4,p);
                                                                                                                                                              for(int i=n-1;i>=0;--i){
41
                                                  101
                                                                                                           if(ans.equals(BigInteger.valueOf(n))){
                                                                                                                                                         19
                                                                                                     161
                                                        LL z = 2;
42
                                                  102
                                                                                                                                                         20
                                                                                                                                                                int t=a/factorial[i],j;
                                                                                                             System.out.println("No solution!");
43
                                                  103
                                                        for(;Legendre(z,p)!=-1;++z)
                                                                                                                                                         21
                                                                                                                                                                for(j=0;j<n;++j)</pre>
                                                  104
                                                        LL c = modexp(z,Q,p);
                                                                                                                                                         22
                                                                                                                                                                  if(!vis[j]){
44
                                                                                                     164
   bool g_test(const LL &g, const LL &p, const
                                                        LL R = modexp(n\%p,(Q+1)/2,p), t = modexp(n_{165})
                                                                                                                                                         23
                                                                                                                                                                    if(t==0)break;
                                                                                                           while(true){
        vector<LL> &v) {
                                                                                                                                                         24
                                                                                                                                                                    --t;
                                                                                                             g2=a1.multiply(h1).substract(g1);
     for(int i=0;i<v.size();++i)</pre>
                                                        int M = S:
                                                                                                                                                         25
                                                  106
                                                                                                             h2=N.substract(g2.pow(2)).divide(h1);
                                                                                                                                                                res.push_back(j);
47
       if(modexp(g,(p-1)/v[i],p)==1)
                                                  107
                                                        while(1) {
                                                                                                                                                         26
                                                                                                             a2=g2.add(a0).divide(h2);
48
         return false;
                                                  108
                                                          if(t==1) return R;
                                                                                                                                                         27
                                                                                                                                                                vis[j]=1;
                                                                                                     169
                                                                                                             p=a1.multiply(p2).add(p1);
                                                          LL b = modexp(c,1L << (M-i-1),p);
49
    return true:
                                                  109
                                                                                                                                                         28
                                                                                                                                                                a%=factorial[i];
                                                                                                             q=a1.multiply(q2).add(q1);
                                                          R = LLmul(R,b,p);
50
                                                  110
                                                                                                                                                         29
                                                                                                             if(p.pow(2).substract(N.multiply(q.pow
                                                          t = LLmul(b,b,p), t, p);
   LL primitive_root(const LL &p) {
                                                  111
                                                                                                                  (2))).compareTo(BigInteger.ONE)==0)
                                                                                                                                                         30
                                                                                                                                                              return res;
52
    if(p==2) return 1;
                                                  112
                                                          c = LLmul(b,b,p);
    vector<LL> v:
                                                          M = i:
53
                                                  113
                                                                                                             g1=g2;h1=h2;a1=a2;
                                                                                                     172
54
     Factor(p-1,v);
                                                  114
                                                                                                     173
                                                                                                             p1=p2;p2=p;
55
    v.erase(unique(v.begin(), v.end()), v.end
                                                 115
                                                        return -1;
                                                                                                     174
                                                                                                             q1=q2;q2=q;
                                                                                                                                                            6.4 FFT
          ());
                                                  116 }
                                                                                                     175
     for(LL g=2;g<p;++g)</pre>
56
                                                                                                     176
                                                                                                           System.out.println(p+" "+q);
57
                                                      template<typename T>
      if(g test(g,p,v))
         return g;
                                                      T Euler(T n){
                                                                                                                                                          1 template<typename T, typename VT=vector<
    puts("primitive root NOT FOUND");
                                                                                                                                                                 complex<T> > >
59
                                                  120
                                                        T ans=n:
                                                        for(T i=2;i*i<=n;++i){</pre>
                                                                                                                                                            struct FFT{
60
    return -1;
                                                  121
                                                         if(n%i==0){
                                                                                                                                                              const T pi;
61
                                                  122
                                                                                                              bit set
   int Legendre(const LL &a, const LL &p) {
                                                            ans=ans/i*(i-1);
                                                                                                                                                              FFT(const T pi=acos((T)-1)):pi(pi){}
                                                  123
       return modexp(a%p,(p-1)/2,p); }
                                                  124
                                                            while(n%i==0)n/=i;
                                                                                                                                                              unsigned bit_reverse(unsigned a,int len){
                                                                                                                                                            a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)>>1);
                                                  125
                                                                                                       1 void sub set(int S){
   LL inv(const LL &a, const LL &n) {
                                                  126
                                                                                                                                                           a=((a&0x33333333U)<<2)|((a&0xCCCCCCCU)>>2);
                                                                                                           int sub=S;
                                                  127
                                                                                                                                                           a=((a&0x0F0F0F0FU)<<4)|((a&0xF0F0F0F0U)>>4);
    LL d,x,y;
                                                        if(n>1)ans=ans/n*(n-1);
                                                                                                           do{
                                                                                                                                                           a=((a&0x00FF00FFU)<<8)|((a&0xFF00FF00U)>>8);
    gcd(a,n,d,x,y);
                                                  128
                                                        return ans;
                                                                                                             //對某集合的子集合的處理
    return d==1 ? (x+n)%n : -1;
                                                                                                                                                           a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)
                                                  129
                                                                                                             sub=(sub-1)&S;
                                                  130
                                                                                                                                                                 >>16);
                                                                                                           }while(sub!=S);
                                                      //Chinese_remainder_theorem
                                                                                                                                                                return a>>(32-len);
69
                                                                                                                                                         12
   int inv[maxN];
                                                      template<typename T>
                                                                                                         void k sub set(int k,int n){
                                                      T pow_mod(T n,T k,T m){
   LL invtable(int n,LL P){
                                                                                                                                                              void fft(bool is_inv,VT &in,VT &out,int N)
                                                                                                           int comb=(1<<k)-1,S=1<<n;</pre>
                                                                                                           while(comb<S){</pre>
    for(int i=2;i<n;++i)</pre>
                                                        for(n=(n)=m?n\%m:n);k;k>>=1){
                                                                                                                                                                int bitlen=__lg(N),num=is_inv?-1:1;
                                                  135
                                                                                                                                                         14
                                                                                                             //對大小為k的子集合的處理
                                                         if(k&1)ans=ans*n%m;
                                                                                                                                                                for(int i=0;i<N;++i)out[bit_reverse(i,</pre>
       inv[i]=(P-(P/i))*inv[P%i]%P;
                                                  136
                                                                                                             int x=comb&-comb,y=comb+x;
                                                                                                      12
75
                                                          n=n*n%m;
                                                                                                                                                                     bitlen) | = in[i];
                                                                                                      13
                                                                                                             comb = ((comb\&\sim y)/x>>1)|y;
                                                                                                                                                                for(int step=2;step<=N;step<<=1){</pre>
                                                                                                      14
   LL log mod(const LL &a, const LL &b, const
                                                        return ans;
                                                                                                                                                         17
                                                                                                                                                                  const int mh=step>>1;
       LL &p) {
                                                                                                                                                                  for(int i=0;i<mh;++i){</pre>
    // a ^ x = b \pmod{p}
                                                      template<typename T>
                                                                                                                                                                    complex<T> wi=exp(complex<T>(0,i*num
    int m=sqrt(p+.5), e=1;
                                                      T crt(vector<T> &m, vector<T> &a){
                                                                                                                                                                         *pi/mh));
    LL v=inv(modexp(a,m,p), p);
                                                        T M=1,tM,ans=0;
                                                                                                                                                                    for(int j=i;j<N;j+=step){</pre>
                                                                                                         6.3 cantor expansion
     map<LL,int> x;
                                                        for(int i=0;i<(int)m.size();++i)M*=m[i];</pre>
                                                                                                                                                         21
                                                                                                                                                                      int k=j+mh;
                                                        for(int i=0;i<(int)a.size();++i){</pre>
    x[1]=0:
                                                  145
                                                                                                                                                         22
                                                                                                                                                                      complex<T> u=out[j],t=wi*out[k];
                                                          tM=M/m[i];
    for(int i=1;i<m;++i) {</pre>
                                                  146
                                                                                                                                                                      out[j]=u+t;
      e = LLmul(e,a,p);
                                                  147
                                                          ans=(ans+(a[i]*tM%M)*pow mod(tM,Euler(m[
                                                                                                       1 int factorial[MAXN];
                                                                                                                                                         24
                                                                                                                                                                      out[k]=u-t;
       if(!x.count(e)) x[e] = i;
                                                               i])-1,m[i])%M)%M;
                                                                                                       2 void init(){
                                                                                                                                                         25
86
                                                          /*如果m[i]是質數, Euler(m[i])-1=m[i]-2
                                                                                                           factorial[0]=1;
                                                                                                                                                         26
                                                  148
     for(int i=0;i<m;++i) {</pre>
                                                                                                           for(int i=1;i<=MAXN;++i)factorial[i]=</pre>
                                                               就不用算Euler了*/
       if(x.count(b)) return i*m + x[b];
                                                                                                                factorial[i-1]*i;
                                                                                                                                                                if(is inv)for(int i=0;i<N;++i)out[i]/=N;</pre>
                                                  149
       b = LLmul(b,v,p);
                                                                                                                                                         29
                                                       return ans;
                                                                                                       6 int encode(const vector(int> &s){
```

#### 6.5 find\_real\_root

```
1 / / an*x^n + ... + a1x + a0 = 0;
  int sign(double x){
    return x < -eps ? -1 : x > eps;
   double get(const vector<double>&coef, double
    double e = 1, s = 0;
    for(auto i : coef) s += i*e, e *= x;
    return s:
   double find(const vector<double>&coef, int n
       , double lo, double hi){
    double sign lo, sign hi;
    if( !(sign_lo = sign(get(coef,lo))) )
          return lo;
    if( !(sign hi = sign(get(coef,hi))) )
          return hi;
    if(sign lo * sign hi > 0) return INF;
    for(int stp = 0: stp < 100 && hi - lo >
          eps; ++stp){
       double m = (lo+hi)/2.0:
       int sign mid = sign(get(coef,m));
       if(!sign mid) return m;
20
       if(sign lo*sign mid < 0) hi = m;</pre>
       else lo = m;
22
23
    return (lo+hi)/2.0;
24
25
26
   vector<double> cal(vector<double>coef, int n
    vector<double>res:
    if(n == 1){
       if(sign(coef[1])) res.pb(-coef[0]/coef
            [1]);
       return res;
32
    vector<double>dcoef(n);
    for(int i = 0; i < n; ++i) dcoef[i] = coef</pre>
          [i+1]*(i+1);
    vector<double>droot = cal(dcoef, n-1);
    droot.insert(droot.begin(), -INF);
    droot.pb(INF);
    for(int i = 0; i+1 < droot.size(); ++i){</pre>
       double tmp = find(coef, n, droot[i],
            droot[i+1]):
       if(tmp < INF) res.pb(tmp);</pre>
42
    return res;
43
   int main () {
    vector<double>ve;
    vector<double>ans = cal(ve, n);
    // 視情況把答案 +eps,避免 -0
48
49 }
```

#### 6.6 FWT

```
1 | vector<int> F OR T(vector<int> f, bool
        inverse){
     for(int i=0; (2<<i)<=f.size(); ++i)</pre>
       for(int j=0; j<f.size(); j+=2<<i)</pre>
         for(int k=0; k<(1<<i); ++k)</pre>
            f[j+k+(1<<i)] += f[j+k]*(inverse)
                 ?-1:1);
     return f:
   vector<int> rev(vector<int> A) {
     for(int i=0; i<A.size(); i+=2)</pre>
       swap(A[i],A[i^(A.size()-1)]);
11
     return A:
12 }
vector<int> F_AND_T(vector<int> f, bool
        inverse){
    return rev(F_OR_T(rev(f), inverse));
15 }
vector<int> F_XOR_T(vector<int> f, bool
        inverse){
     for(int i=0; (2<<i)<=f.size(); ++i)</pre>
       for(int j=0; j<f.size(); j+=2<<i)</pre>
18
19
         for(int k=0; k<(1<<i); ++k){</pre>
20
            int u=f[j+k], v=f[j+k+(1<<i)];</pre>
21
            f[j+k+(1<< i)] = u-v, f[j+k] = u+v;
     if(inverse) for(auto &a:f) a/=f.size();
     return f;
```

## 6.7 LinearCongruence

```
1 pair<LL,LL> LinearCongruence(LL a[],LL b[],
        LL m[], int n) {
     // a[i]*x = b[i] \pmod{m[i]}
     for(int i=0;i<n;++i) {</pre>
      LL x, y, d = extgcd(a[i],m[i],x,y);
       if(b[i]%d!=0) return make pair(-1LL,0LL)
       m[i] /= d;
      b[i] = LLmul(b[i]/d,x,m[i]);
     LL lastb = b[0], lastm = m[0];
     for(int i=1;i<n;++i) {</pre>
      LL x, y, d = extgcd(m[i],lastm,x,y);
       if((lastb-b[i])%d!=0) return make pair
            (-1LL,0LL);
       lastb = LLmul((lastb-b[i])/d,x,(lastm/d)
13
            )*m[i];
       lastm = (lastm/d)*m[i];
14
       lastb = (lastb+b[i])%lastm;
15
16
    return make_pair(lastb<0?lastb+lastm:lastb</pre>
17
          .lastm):
18 }
```

#### 6.8 Lucas

```
1 int mod fact(int n,int &e){
                                                         for(int y=0;y<r;y++)</pre>
   e=0;
                                                  46
    if(n==0)return 1;
                                                  47
   int res=mod fact(n/P,e);
                                                  48
                                                  49
   if((n/P)%2==0)return res*fact[n%P]%P;
    return res*(P-fact[n%P1)%P:
                                                  51
                                                  52
  int Cmod(int n,int m){
   int a1,a2,a3,e1,e2,e3;
                                                  54
    a1=mod fact(n,e1);
    a2=mod fact(m.e2):
    a3=mod fact(n-m,e3);
                                                  57
   if(e1>e2+e3)return 0:
   return a1*inv(a2*a3%P,P)%P:
                                                  59
                                                  60
                                                  61
                                                  62
                                                  63
  6.9 Matrix
                                                  64
                                                  65
1 template<typename T>
 struct Matrix{
```

```
matrix rev(r,c):
  for(int i=0;i<r;++i)</pre>
    for(int j=0;j<c;++j)</pre>
      rev[i][j]=m[i][j]+a.m[i][j];
  return rev;
matrix operator-(const matrix &a){
  matrix rev(r,c);
  for(int i=0;i<r;++i)</pre>
    for(int j=0;j<c;++j)</pre>
      rev[i][j]=m[i][j]-a.m[i][j];
  return rev:
matrix operator*(const matrix &a){
  matrix rev(r,a.c);
  matrix tmp(a.c,a.r);
  for(int i=0;i<a.r;++i)</pre>
    for(int j=0;j<a.c;++j)</pre>
      tmp[j][i]=a.m[i][j];
  for(int i=0;i<r;++i)</pre>
    for(int j=0;j<a.c;++j)</pre>
      for(int k=0;k<c;++k)</pre>
        rev.m[i][j]+=m[i][k]*tmp[j][k];
  return rev;
bool inverse(){
  Matrix t(r,r+c);
  for(int y=0;y<r;y++){</pre>
    t.m[y][c+y] = 1;
    for(int x=0;x<c;++x)</pre>
      t.m[y][x]=m[y][x];
  if(!t.gas())
```

Matrix(int r,int c):r(r),c(c),m(r,rt(c)){}

rt& operator[](int i){return m[i];}

matrix operator+(const matrix &a){

using rt = std::vector<T>:

using matrix = Matrix<T>;

int r.c:

10

11

12

13

14

15

20

22

23

30

32

33

40

41

42

return false;

using mt = std::vector<rt>:

```
for(int x=0;x<c;++x)
           m[y][x]=t.m[y][c+x]/t.m[y][y];
       return true;
     T gas(){
       vector<T> lazv(r,1);
       bool sign=false;
       for(int i=0;i<r;++i){</pre>
         if( m[i][i]==0 ){
           int j=i+1;
           while(j<r&&!m[j][i])j++;
           if(j==r)continue;
           m[i].swap(m[j]);
           sign=!sign;
         for(int j=0;j<r;++j){</pre>
           if(i==j)continue;
           lazy[j]=lazy[j]*m[i][i];
           T mx=m[j][i];
           for(int k=0;k<c;++k)</pre>
             m[j][k]=m[j][k]*m[i][i]-m[i][k]*mx
67
68
69
       T det=sign?-1:1:
       for(int i=0;i<r;++i){</pre>
70
71
         det = det*m[i][i];
72
         det = det/lazy[i];
         for(auto &j:m[i])j/=lazy[i];
74
75
       return det;
76
77 };
```

#### 6.10 MillerRobin

```
1 LL LLmul(LL a, LL b, const LL &mod) {
    LL ans=0:
    while(b) {
      if(b&1) {
        if(ans>=mod) ans-=mod;
      a<<=1. b>>=1:
      if(a>=mod) a-=mod;
11
    return ans;
12
  LL mod mul(LL a.LL b.LL m){
    a\%=m,b\%=m;/* fast for m < 2^58 */
14
15
    LL y=(LL)((double)a*b/m+0.5);
    LL r=(a*b-v*m)%m:
17
    return r<0?r+m:r;</pre>
18
  template<typename T>
   T pow(T a,T b,T mod){//a^b%mod
    for(:b:a=mod mul(a,a,mod),b>>=1)
23
      if(b&1)ans=mod_mul(ans,a,mod);
24
    return ans:
25
26 int sprp[3]={2,7,61};//int範圍可解
```

```
27 int llsprp
        [7] = \{2,325,9375,28178,450775,9780504,
28 1795265022};//至少unsigned Long Long範圍
  template<typename T>
  bool isprime(T n, int *sprp, int num){
    if(n==2)return 1;
    if(n<2||n%2==0)return 0;
33
    int t=0:
    T u=n-1;
34
     for(;u%2==0;++t)u>>=1;
     for(int i=0;i<num;++i){</pre>
      T a=sprp[i]%n;
38
       if(a==0||a==1||a==n-1)continue;
       T x=pow(a,u,n);
       if(x==1||x==n-1)continue;
       for(int j=0;j<t;++j){</pre>
         x=mod_mul(x,x,n);
42
         if(x==1)return 0;
         if(x==n-1)break;
45
       if(x==n-1)continue;
       return 0;
48
    return 1;
```

#### 6.11 NTT

1 2615053605667\*(2^18)+1,3

```
2 15*(2^27)+1.31
3 479*(2^21)+1,3
4 7*17*(2^23)+1,3
5 3*3*211*(2^19)+1,5
6 25*(2^22)+1,3
  template<typename T, typename VT=vector<T> >
   struct NTT{
    const T P,G;
    NTT(T p=(1<<23)*7*17+1,T g=3):P(p),G(g){}
    unsigned bit reverse(unsigned a,int len){
       //Look FFT.cpp
12
13
    T pow_mod(T n,T k,T m){
14
15
       T ans=1;
       for (n=(n)=m?n\%m:n); k; k>>=1){
         if(k&1)ans=ans*n%m;
         n=n*n%m;
19
20
21
     void ntt(bool is inv,VT &in,VT &out,int N)
23
       int bitlen= lg(N);
       for(int i=0;i<N;++i)out[bit_reverse(i,</pre>
24
            bitlen) | = in[i];
       for(int step=2,id=1;step<=N;step<<=1,++</pre>
26
         T wn=pow mod(G,(P-1)>>id,P),wi=1,u,t;
         const int mh=step>>1;
         for(int i=0;i<mh;++i){</pre>
           for(int j=i;j<N;j+=step){</pre>
30
             u=out[i],t=wi*out[i+mh]%P;
             out[i]=u+t;
31
             out[j+mh]=u-t;
```

```
if(out[j]>=P)out[j]-=P;
              if(out[j+mh]<0)out[j+mh]+=P;</pre>
34
35
36
            wi=wi*wn%P;
37
38
39
       if(is inv){
          for(int i=1;i<N/2;++i)swap(out[i],out[</pre>
               N-i]);
          T invn=pow mod(N,P-2,P);
41
42
          for(int i=0;i<N;++i)out[i]=out[i]*invn</pre>
43
44
45 };
```

# 6.12 Simpson

double c=a+(b-a)/2;

1 | double simpson(double a, double b){

# 6.13 外星模運算

```
1 / a[0]^{(a[1]^a[2]^{...})}
 2 #define maxn 1000000
 3 int euler[maxn+5];
 4 bool is prime[maxn+5];
 5 void init_euler(){
    is prime[1]=1;//一不是質數
     for(int i=1;i<=maxn;i++)euler[i]=i;</pre>
     for(int i=2;i<=maxn;i++){</pre>
       if(!is prime[i]){//是質數
         euler[i]--:
         for(int j=i<<1;j<=maxn;j+=i){</pre>
12
            is prime[j]=1;
            euler[j]=euler[j]/i*(i-1);
14
16
17 }
   LL pow(LL a, LL b, LL mod) { //a^b%mod
    LL ans=1;
     for(;b;a=a*a%mod,b>>=1)
       if(b&1)ans=ans*a%mod;
     return ans;
22
```

```
24 bool isless(LL *a,int n,int k){
     if(*a==1)return k>1;
     if(--n==0)return *a<k;</pre>
26
27
     int next=0:
28
     for(LL b=1;b<k;++next)</pre>
29
       b*=*a:
30
     return isless(a+1,n,next);
31 }
   LL high pow(LL *a, int n, LL mod){
     if(*a==1||--n==0)return *a%mod;
34
     int k=0,r=euler[mod];
     for(LL tma=1:tma!=pow(*a,k+r,mod):++k)
        tma=tma*(*a)%mod;
36
     if(isless(a+1,n,k))return pow(*a,high pow(
          a+1.n.k).mod):
     int tmd=high_pow(a+1,n,r), t=(tmd-k+r)%r;
     return pow(*a,k+t,mod);
39
40
41 LL a[1000005];
42 int t.mod:
43
   int main(){
     init euler():
     scanf("%d",&t):
46
     #define n 4
47
     while(t--){
        for(int i=0;i<n;++i)scanf("%lld",&a[i]);</pre>
48
49
        scanf("%d",&mod);
       printf("%lld\n",high_pow(a,n,mod));
50
51
52
     return 0;
```

## 6.14 數位統計

```
1 | 11 d[65], dp[65][2]; // up區間是不是完整
2 11 dfs(int p,bool is8,bool up){
    if(!p)return 1; // 回傳0是不是答案
    if(!up&&~dp[p][is8])return dp[p][is8];
    int mx = up?d[p]:9;//可以用的有那些
    11 ans=0:
    for(int i=0;i<=mx;++i){</pre>
      if( is8&&i==7 )continue;
      ans += dfs(p-1,i==8,up&&i==mx):
10
    if(!up)dp[p][is8]=ans;
    return ans;
12
13
  11 f(11 N){
14
    int k=0;
    while(N){ // 把數字先分解到陣列
      d[++k] = N%10;
17
18
      N/=10;
19
    return dfs(k,false,true);
20
```

# 6.15 質因數分解

```
return (LLmul(n,n,mod)+c+mod)%mod;
  LL pollorrho(const LL n, const int c) {//循
        環 箭 長 度
     LL a=1, b=1;
     a=func(a,n,c)%n:
     b=func(b,n,c)%n; b=func(b,n,c)%n;
     while(gcd(abs(a-b),n)==1) {
       a=func(a,n,c)%n;
       b=func(b,n,c)%n; b=func(b,n,c)%n;
     return gcd(abs(a-b),n);
14
   void prefactor(LL &n, vector<LL> &v) {
     for(int i=0;i<12;++i) {</pre>
       while(n%prime[i]==0) {
         v.push back(prime[i]);
20
         n/=prime[i];
21
^{22}
23
   void smallfactor(LL n, vector<LL> &v) {
     if(n<MAXPRIME) {</pre>
       while(isp[(int)n]) {
27
         v.push_back(isp[(int)n]);
28
29
         n/=isp[(int)n];
30
31
       v.push_back(n);
32
     } else {
       for(int i=0;i<primecnt&&prime[i]*prime[i</pre>
            1<=n;++i) {</pre>
         while(n%prime[i]==0) {
34
           v.push_back(prime[i]);
35
           n/=prime[i];
36
37
38
39
       if(n!=1) v.push back(n);
40
41
42
   void comfactor(const LL &n, vector<LL> &v) {
     if(n<1e9) {
       smallfactor(n,v);
46
       return:
47
     if(Isprime(n)) {
       v.push back(n);
49
50
       return;
51
52
     for(int c=3;;++c) {
54
       d = pollorrho(n,c);
       if(d!=n) break;
56
57
     comfactor(d,v);
     comfactor(n/d,v);
58
59
  void Factor(const LL &x, vector<LL> &v) {
    LL n = x;
```

1 | LL func(const LL n,const LL mod,const int c)

```
if(n==1) { puts("Factor 1"); return; }
     prefactor(n,v);
                                                     34
65
     if(n==1) return;
                                                     35
66
     comfactor(n,v);
                                                     36
67
     sort(v.begin(),v.end());
                                                     37
69
                                                      39
    /oid AllFactor(const LL &n, vector<LL> &v) {
71
     vector<LL> tmp;
                                                      41
72
     Factor(n,tmp);
                                                      42
     v.clear();
73
                                                      43
     v.push back(1):
74
75
     int len;
                                                     44
76
     LL now=1:
                                                      45
77
     for(int i=0;i<tmp.size();++i) {</pre>
78
       if(i==0 || tmp[i]!=tmp[i-1]) {
                                                     46
         len = v.size():
79
                                                     47
         now = 1;
80
                                                      48
                                                      49
81
                                                      50
82
       now*=tmp[i];
83
       for(int j=0;j<len;++j)</pre>
                                                     51
         v.push_back(v[j]*now);
84
85
                                                     52
86 }
                                                     53
                                                      54
                                                     55
                                                      56
```

# String

## 7.1 AC 自動機

```
1 template < char L='a', char R='z'>
2 class ac automaton{
    struct joe{
       int next[R-L+1],fail,efl,ed,cnt_dp,vis;
       joe():ed(0),cnt dp(0),vis(0){
         for(int i=0;i<=R-L;++i)next[i]=0;</pre>
    };
   public:
    std::vector<joe> S;
    std::vector<int> q;
    int qs,qe,vt;
    ac_automaton():S(1),qs(0),qe(0),vt(0){}
    void clear(){
       q.clear();
       S.resize(1);
       for(int i=0;i<=R-L;++i)S[0].next[i]=0;</pre>
       S[0].cnt dp=S[0].vis=qs=qe=vt=0;
19
20
     void insert(const char *s){
       int o=0;
       for(int i=0,id;s[i];++i){
23
         id=s[i]-L;
         if(!S[o].next[id]){
25
           S.push_back(joe());
26
           S[o].next[id]=S.size()-1;
27
28
         o=S[o].next[id];
       ++S[o].ed;
31
     void build fail(){
```

```
for(int i=0:i<=R-L:++i){</pre>
     t=S[pa].next[i];
     if(!t)continue;
     id=S[pa].fail;
     while(~id&&!S[id].next[i])id=S[id].
     S[t].fail=~id?S[id].next[i]:0;
     S[t].efl=S[S[t].fail].ed?S[t].fail:S
          [S[t].fail].efl;
                                       100
     q.push back(t);
                                       101
     ++qe;
                                       102
                                       103
                                       104
                                       105
/*DP出每個前綴在字串s出現的次數並傳回所有
                                       106
    字串被s匹配成功的次數O(N+M)*/
                                       107
int match 0(const char *s){
                                       108
 int ans=0,id,p=0,i;
                                       109
 for(i=0;s[i];++i){
   id=s[i]-L;
                                       110
   while(!S[p].next[id]&&p)p=S[p].fail;
                                       111
   if(!S[p].next[id])continue;
   p=S[p].next[id];
   ++S[p].cnt_dp;/*匹配成功則它所有後綴都
        可以被匹配(DP計算)*/
 for(i=qe-1;i>=0;--i){
   ans+=S[q[i]].cnt_dp*S[q[i]].ed;
   if(~S[q[i]].fail)S[S[q[i]].fail].
        cnt_dp+=S[q[i]].cnt_dp;
 return ans;
/*多串匹配走efL邊並傳回所有字串被s匹配成功
    的 次 數 O(N*M^1.5)*/
int match 1(const char *s)const{
 int ans=0,id,p=0,t;
 for(int i=0;s[i];++i){
   id=s[i]-L;
   while(!S[p].next[id]&&p)p=S[p].fail;
   if(!S[p].next[id])continue;
   p=S[p].next[id];
   if(S[p].ed)ans+=S[p].ed;
   for(t=S[p].efl;~t;t=S[t].efl){
     ans+=S[t].ed;/*因為都走efL邊所以保證
          匹配成功*/
 return ans;
/*枚舉(s的子字串nA)的所有相異字串各恰一次
    並傳回次數O(N*M^(1/3))*/
int match 2(const char *s){
 int ans=0,id,p=0,t;
 /*把戳記vt+=1,只要vt沒溢位,所有S[p].
      vis==vt就會變成false
  這種利用vt的方法可以0(1)歸零vis陣列*/
```

S[0].fail=S[0].efl=-1;

int pa=q[qs++],id,t;

q.clear();

++qe;

57

58

59

60

61

62

63

65

66

67

68

69

70

71

72

73

74

75

76

77

78

79

80

81

84

85

86

q.push\_back(0);

while(as!=ae){

```
for(int i=0;s[i];++i){
         id=s[i]-L;
         while(!S[p].next[id]&&p)p=S[p].fail;
         if(!S[p].next[id])continue;
         p=S[p].next[id];
         if(S[p].ed&&S[p].vis!=vt){
           S[p].vis=vt:
           ans+=S[p].ed;
         for(t=S[p].efl;~t&&S[t].vis!=vt;t=S[t
              1.ef1){
           S[t].vis=vt:
           ans+=S[t].ed;/*因為都走efL邊所以保證
                匹配成功*/
       }
       return ans;
     /*把AC自動機變成真的自動機*/
     void evolution(){
       for(qs=1;qs!=qe;){
         int p=a[as++];
         for(int i=0;i<=R-L;++i)</pre>
           if(S[p].next[i]==0)S[p].next[i]=S[S[
               p].fail].next[i];
     }
112 };
          hash
```

89

90

91

92

93

94

95

96

```
1 | #define MAXN 1000000
2 #define mod 1073676287
3 /*mod 必須要是質數*/
4 typedef long long T;
5 char s[MAXN+5];
6 T h[MAXN+5];/*hash陣列*/
7 T h_base[MAXN+5];/*h_base[n]=(prime^n)%mod*/
8 void hash init(int len,T prime){
    h base[0]=1;
    for(int i=1;i<=len;++i){</pre>
      h[i]=(h[i-1]*prime+s[i-1])%mod;
11
      h base[i]=(h base[i-1]*prime)%mod;
12
13
14 }
15 | T get hash(int l,int r){/*閉區間寫法,設編號
        為0 ~ Len-1*/
    return (h[r+1]-(h[1]*h_base[r-1+1])%mod+
         mod)%mod;
17 }
```

#### 7.3 KMP

```
1 /*產生fail function*/
void kmp fail(char *s,int len,int *fail){
  int id=-1;
   fail[0]=-1;
   for(int i=1;i<len;++i){</pre>
```

```
while(~id&&s[id+1]!=s[i])id=fail[id];
      if(s[id+1]==s[i])++id;
      fail[i]=id;
10 }
11 /*以字串B匹配字串A, 傳回匹配成功的數量(用B的
12 int kmp match(char *A,int lenA,char *B,int
       lenB, int *fail){
    int id=-1,ans=0;
    for(int i=0;i<lenA;++i){</pre>
      while(~id&&B[id+1]!=A[i])id=fail[id];
      if(B[id+1]==A[i])++id;
      if(id==lenB-1){/*匹配成功*/
17
        ++ans. id=fail[id]:
18
19
20
21
    return ans;
```

#### 7.4 manacher

```
1 //原字串: asdsasdsa
2 // 先 把 字 串 變 成 這 樣: @#a#s#d#s#a#s#d#s#a#
 void manacher(char *s,int len,int *z){
    int 1=0, r=0;
    for(int i=1;i<len;++i){</pre>
      z[i]=r>i?min(z[2*1-i],r-i):1;
      while(s[i+z[i]]==s[i-z[i]])++z[i];
      if(z[i]+i>r)r=z[i]+i,l=i;
    }//ans = max(z)-1
```

## 7.5 minimal string rotation

```
1 int min_string_rotation(const string &s){
    int n=s.size(),i=0,j=1,k=0;
    while(i<n&&i<n&&k<n){</pre>
      int t=s[(i+k)%n]-s[(j+k)%n];
      ++k;
      if(t){
        if(t>0)i+=k;
        else j+=k;
        if(i==j)++j;
        k=0;
12
    return min(i,j);//最小循環表示法起始位置
13
```

#### 7.6 reverseBWT

```
1 const int MAXN = 305, MAXC = 'Z';
int ranks[MAXN], tots[MAXC], first[MAXC];
3 void rankBWT(const string &bw){
   memset(ranks,0,sizeof(int)*bw.size());
```

```
memset(tots,0,sizeof(tots);
    for(size t i=0;i<bw.size();++i)</pre>
       ranks[i] = tots[int(bw[i])]++;
   void firstCol(){
    memset(first,0,sizeof(first));
    int totc = 0:
12
    for(int c='A';c<='Z';++c){</pre>
       if(!tots[c]) continue;
13
14
       first[c] = totc;
15
       totc += tots[c];
16
17
   string reverseBwt(string bw.int begin){
    rankBWT(bw), firstCol();
    int i = begin; //原字串最後一個元素的位置
    string res;
22
    do{
       char c = bw[i]:
23
       res = c + res;
      i = first[int(c)] + ranks[i];
    }while( i != begin );
    return res;
```

# 7.7 suffix array lcp

```
1 #define radix sort(x,y){\
     for(i=0;i<A;++i)c[i]=0;\</pre>
     for(i=0;i<n;++i)c[x[y[i]]]++;\</pre>
     for(i=1;i<A;++i)c[i]+=c[i-1];\</pre>
     for(i=n-1;~i;--i)sa[--c[x[y[i]]]]=y[i];\
6
   #define AC(r,a,b)\
     r[a]!=r[b]||a+k>=n||r[a+k]!=r[b+k]
                                                     13
   void suffix array(const char *s,int n,int *
                                                     14
        sa,int *rank,int *tmp,int *c){
                                                     15
     int A = 'z' + 1, i, k, id = 0;
                                                     16
     for(i=0;i<n;++i)rank[tmp[i]=i]=s[i];</pre>
                                                     17
12
     radix sort(rank,tmp);
                                                     18
13
     for(k=1;id<n-1;k<<=1){</pre>
                                                     19
       for(id=0,i=n-k;i<n;++i)tmp[id++]=i;</pre>
14
                                                     20
15
       for(i=0:i<n:++i)</pre>
                                                     21
         if(sa[i]>=k)tmp[id++]=sa[i]-k;
16
                                                     22
17
       radix sort(rank,tmp);
                                                     23
       swap(rank,tmp);
                                                     24
19
       for(rank[sa[0]]=id=0,i=1;i<n;++i)</pre>
                                                     25
         rank[sa[i]]=id+=AC(tmp,sa[i-1],sa[i]);
20
21
       A=id+1;
22
23 }
                                                     28
                                                     29
24 //h:高度數組 sa:後綴數組 rank:排名
   void suffix_array_lcp(const char *s,int len,
        int *h,int *sa,int *rank){
                                                     31
     for(int i=0;i<len;++i)rank[sa[i]]=i;</pre>
                                                     32
     for(int i=0,k=0;i<len;++i){</pre>
       if(rank[i]==0)continue;
       if(k)--k:
                                                     34
       while(s[i+k]==s[sa[rank[i]-1]+k])++k;
                                                     35
       h[rank[i]]=k;
                                                     36
32
     h[0]=0;// h[k]=lcp(sa[k],sa[k-1]);
```

# 7.8 Z

# 8 Tarjan

1 | struct dominator tree{

#### 8.1 dominator tree

```
static const int MAXN=5005;
int n:// 1-base
vector<int> suc[MAXN],pre[MAXN];
int fa[MAXN],dfn[MAXN],id[MAXN],Time;
int semi[MAXN],idom[MAXN];
int anc[MAXN], best[MAXN]; // disjoint set
vector<int> dom[MAXN];//dominator tree
void init(int _n){
 n= n;
  for(int i=1;i<=n;++i)suc[i].clear(),pre[</pre>
      il.clear();
void add_edge(int u,int v){
 suc[u].push back(v);
 pre[v].push back(u);
void dfs(int u){
 dfn[u]=++Time,id[Time]=u;
 for(auto v:suc[u]){
   if(dfn[v])continue;
    dfs(v),fa[dfn[v]]=dfn[u];
int find(int x){
 if(x==anc[x])return x;
  int y=find(anc[x]);
  if(semi[best[x]]>semi[best[anc[x]]])best 26
       [x]=best[anc[x]];
 return anc[x]=y;
void tarjan(int r){
  for(int t=1;t<=n;++t){</pre>
    dfn[t]=idom[t]=0;//u=r或是u無法到達r時
         idom[id[u]]=0
    dom[t].clear();
    anc[t]=best[t]=semi[t]=t;
  for(int y=Time;y>=2;--y){
    int x=fa[y],idy=id[y];
```

## 8.2 tnfshb017\_2\_sat

1 #include < bits / stdc++.h>

for(auto z:pre[idy]){

for(auto z:dom[x]){

for(int u=2;u<=Time;++u){</pre>

dom[x].clear();

anc[v]=x:

41

42

43

44

45

46

50

51

52

53

54

55

56

57

58 } dom;

if(!(z=dfn[z]))continue;

dom[semi[y]].push back(y);

semi[y]=min(semi[y],semi[best[z]]);

idom[z]=semi[best[z]]<x?best[z]:x;</pre>

if(idom[u]!=semi[u])idom[u]=idom[idom[

dom[id[idom[u]]].push back(id[u]);

```
using namespace std;
 3 #define MAXN 8001
 4 #define MAXN2 MAXN*4
 5 #define n(X) ((X)+2*N)
 6 vector<int> v[MAXN2], rv[MAXN2], vis_t;
 7 int N,M;
   void addedge(int s,int e){
     v[s].push_back(e);
     rv[e].push back(s);
11 }
12 int scc[MAXN2];
  bool vis[MAXN2]={false};
   void dfs(vector<int> *uv,int n,int k=-1){
     vis[n]=true;
     for(int i=0;i<uv[n].size();++i)</pre>
       if(!vis[uv[n][i]])
         dfs(uv,uv[n][i],k);
     if(uv==v)vis_t.push_back(n);
19
20
     scc[n]=k;
21
22
   void solve(){
     for(int i=1;i<=N;++i){</pre>
       if(!vis[i])dfs(v,i);
       if(!vis[n(i)])dfs(v,n(i));
     memset(vis,0,sizeof(vis));
     int c=0;
     for(int i=vis_t.size()-1;i>=0;--i)
       if(!vis[vis_t[i]])
         dfs(rv,vis_t[i],c++);
32 }
33 int main(){
     int a.b:
     scanf("%d%d",&N,&M);
     for(int i=1;i<=N;++i){</pre>
       // (A or B)&(!A & !B) A^B
       a=i*2-1;
       b=i*2;
       addedge(n(a),b);
```

```
addedge(n(b),a);
42
       addedge(a,n(b));
       addedge(b,n(a));
43
44
     while(M--){
       scanf("%d%d",&a,&b);
47
       a = a>0?a*2-1:-a*2:
       b = b>0?b*2-1:-b*2;
48
       // A or B
       addedge(n(a),b);
51
       addedge(n(b),a);
52
53
     solve();
     bool check=true:
     for(int i=1;i<=2*N;++i)</pre>
       if(scc[i]==scc[n(i)])
57
         check=false:
     if(check){
58
       printf("%d\n",N);
59
       for(int i=1;i<=2*N;i+=2){</pre>
61
         if(scc[i]>scc[i+2*N]) putchar('+');
         else putchar('-');
62
63
64
       puts("");
     }else puts("0");
65
     return 0:
```

#### 8.3 橋連涌分量

```
1 | #define N 1005
  struct edge{
    int u,v;
    bool is bridge:
    edge(int u=0, int v=0):u(u),v(v), is bridge
  vector<edge> E;
  vector<int> G[N];// 1-base
  int low[N], vis[N], Time;
  int bcc_id[N],bridge_cnt,bcc_cnt;// 1-base
  int st[N],top;//BCC用
  inline void add edge(int u,int v){
    G[u].push back(E.size());
    E.push back(edge(u,v));
    G[v].push back(E.size());
    E.push_back(edge(v,u));
17 }
18 | void dfs(int u,int re=-1){//u當前點,re為u連
        接前一個點的邊
    int v;
    low[u]=vis[u]=++Time;
21
    st[top++]=u;
    for(size_t i=0;i<G[u].size();++i){</pre>
      int e=G[u][i];v=E[e].v;
      if(!vis[v]){
         dfs(v,e^1);//e^1反向邊
        low[u]=min(low[u],low[v]);
        if(vis[u]<low[v]){</pre>
          E[e].is bridge=E[e^1].is bridge=1;
           ++bridge_cnt;
```

```
}else if(vis[v]<vis[u]&&e!=re)</pre>
         low[u]=min(low[u], vis[v]);
32
33
    if(vis[u]==low[u]){//處理BCC
34
35
       ++bcc cnt;// 1-base
       do bcc_id[v=st[--top]]=bcc_cnt;//每個點
36
            所在的BCC
       while(v!=u);
38
39
   inline void bcc init(int n){
    Time=bcc_cnt=bridge_cnt=top=0;
    E.clear();
42
    for(int i=1;i<=n;++i){</pre>
43
      G[i].clear();
       vis[i]=bcc id[i]=0;
46
```

### 8.4 雙連通分量 & 割點

```
1 #define N 1005
vector<int> G[N];// 1-base
3 vector < int > bcc[N]; // 存每塊雙連通分量的點
4 int low[N], vis[N], Time;
5 int bcc_id[N],bcc_cnt;// 1-base
6 bool is cut[N];//是否為割點
7 int st[N],top;
   void dfs(int u,int pa=-1){//u當前點,pa父親
    int v,child=0;
    low[u]=vis[u]=++Time;
11
    st[top++]=u;
    for(size_t i=0;i<G[u].size();++i){</pre>
12
      if(!vis[v=G[u][i]]){
13
14
        dfs(v,u),++child;
15
         low[u]=min(low[u],low[v]);
16
         if(vis[u]<=low[v]){</pre>
           is cut[u]=1;
17
           bcc[++bcc_cnt].clear();
18
19
           int t;
20
21
             bcc id[t=st[--top]]=bcc cnt;
             bcc[bcc cnt].push back(t);
22
23
           }while(t!=v);
           bcc id[u]=bcc cnt;
24
25
           bcc[bcc_cnt].push_back(u);
26
27
      }else if(vis[v]<vis[u]&&v!=pa)//反向邊
28
        low[u]=min(low[u], vis[v]);
29
    if(pa==-1&&child<2)is cut[u]=0;//u是dfs樹
          的根要特判
31
   inline void bcc_init(int n){
    Time=bcc cnt=top=0;
34
    for(int i=1;i<=n;++i){</pre>
35
      G[i].clear();
      is cut[i]=vis[i]=bcc id[i]=0;
36
37
38 }
```

# 9 Tree\_problem

## 9.1 HeavyLight

1 | #include < vector >

```
2 #define MAXN 100005
 int siz[MAXN], max_son[MAXN], pa[MAXN], dep[
       MAXN1:
   int link_top[MAXN],link[MAXN],cnt;
   vector<int> G[MAXN];
   void find max son(int u){
    siz[u]=1;
     max son[u]=-1;
     for(auto v:G[u]){
      if(v==pa[u])continue;
      pa[v]=u;
12
       dep[v]=dep[u]+1;
       find max son(v);
       if(max son[u]==-1||siz[v]>siz[max son[u
           ]])max_son[u]=v;
15
       siz[u]+=siz[v];
16
    }
17 }
   void build link(int u.int top){
    link[u]=++cnt;
20
    link top[u]=top;
     if(max_son[u]==-1)return;
21
     build link(max son[u],top);
22
     for(auto v:G[u]){
      if(v==max son[u]||v==pa[u])continue;
24
      build link(v,v);
26
27 }
   int find_lca(int a,int b){
    //求LCA, 可以在過程中對區間進行處理
    int ta=link top[a],tb=link top[b];
     while(ta!=tb){
31
32
      if(dep[ta]<dep[tb]){</pre>
33
        swap(ta,tb);
34
        swap(a,b);
35
      //這裡可以對a所在的鏈做區間處理
36
      //區間為(link[ta],link[a])
37
      ta=link_top[a=pa[ta]];
38
39
    // 最後a,b會在同一條鏈,若a!=b還要在進行一
         次區間處理
    return dep[a]<dep[b]?a:b;</pre>
41
```

## 9.2 LCA

```
const int MAXN=100000; // 1-base
const int MLG=17; //Log2(MAXN)+1;
int pa[MLG+1][MAXN+5];
int dep[MAXN+5];
vector<int> G[MAXN+5];
void dfs(int x,int p=0){//dfs(root);
pa[0][x]=p;
```

```
for(int i=0;i<=MLG;++i)</pre>
       pa[i+1][x]=pa[i][pa[i][x]];
     for(auto &i:G[x]){
10
11
       if(i==p)continue;
12
       dep[i]=dep[x]+1;
13
       dfs(i,x);
14
15
16
   inline int jump(int x,int d){
17
     for(int i=0;i<=MLG;++i)</pre>
18
       if((d>>i)&1) x=pa[i][x];
     return x:
19
20
   inline int find_lca(int a,int b){
     if(dep[a]>dep[b])swap(a,b);
23
     b=jump(b,dep[b]-dep[a]);
     if(a==b)return a:
24
     for(int i=MLG;i>=0;--i){
25
       if(pa[i][a]!=pa[i][b]){
26
         a=pa[i][a];
27
28
         b=pa[i][b];
29
30
     return pa[0][a];
31
```

#### 9.3 link cut tree

```
1 struct splay tree{
    int ch[2],pa;//子節點跟父母
    bool rev;//反轉的懶惰標記
    splay tree():pa(0),rev(0){ch[0]=ch[1]=0;}
5 };
6 vector<splay_tree> nd;
7 // 有的時候用vector會TLE,要注意
8 / / 這邊以node [ 0 ] 作為nul L 節點
9 bool isroot(int x){//判斷是否為這棵splay
       tree的根
    return nd[nd[x].pa].ch[0]!=x&&nd[nd[x].pa
        ].ch[1]!=x;
  void down(int x){// 懶 惰 標 記 下 推
    if(nd[x].rev){
      if(nd[x].ch[0])nd[nd[x].ch[0]].rev^=1;
15
      if(nd[x].ch[1])nd[nd[x].ch[1]].rev^=1;
      swap(nd[x].ch[0],nd[x].ch[1]);
16
17
      nd[x].rev=0;
18
19
  void push down(int x){//所有祖先懶惰標記下推
    if(!isroot(x))push_down(nd[x].pa);
22
23 }
24 | void up(int x){}//將子節點的資訊向上更新
  void rotate(int x){//旋轉·會自行判斷轉的方
    int y=nd[x].pa,z=nd[y].pa,d=(nd[y].ch[1]==
        x);
    nd[x].pa=z;
    if(!isroot(y))nd[z].ch[nd[z].ch[1]==y]=x;
    nd[y].ch[d]=nd[x].ch[d^1];
```

```
nd[nd[y].ch[d]].pa=y;
31
    nd[y].pa=x,nd[x].ch[d^1]=y;
32
    up(y),up(x);
33
34 void splay(int x){//將x伸展到splay tree的根
35
    push down(x);
    while(!isroot(x)){
      int y=nd[x].pa;
      if(!isroot(y)){
39
         int z=nd[y].pa;
        if((nd[z].ch[0]==y)^(nd[y].ch[0]==x))
40
             rotate(v);
        else rotate(x);
41
42
43
      rotate(x);
44
45
  int access(int x){
    int last=0:
    while(x){
48
      splay(x);
      nd[x].ch[1]=last;
      up(x);
52
      last=x:
      x=nd[x].pa;
54
55
    return last;//access後splay tree的根
56
  void access(int x,bool is=0){//is=0就是一般
        的access
     int last=0;
    while(x){
      splay(x);
      if(is\&\&!nd[x].pa){
        //printf("%d\n",max(nd[last].ma,nd[nd[
             x1.ch[1]1.ma));
64
      nd[x].ch[1]=last;
      up(x);
65
      last=x;
      x=nd[x].pa;
  void query_edge(int u,int v){
    access(u);
    access(v.1):
  void make root(int x){
    access(x),splay(x);
76
    nd[x].rev^=1;
77
  void make_root(int x){
    nd[access(x)].rev^=1;
    splay(x);
81
  void cut(int x,int y){
    make root(x);
    access(y);
    splay(y);
    nd[y].ch[0]=0;
    nd[x].pa=0;
  void cut_parents(int x){
89
    access(x);
    splay(x);
```

```
nd[nd[x].ch[0]].pa=0;
93
     nd[x].ch[0]=0;
                                               153
                                               154
94
    void link(int x,int y){
                                               155 }
     make root(x);
97
     nd[x].pa=y;
98
    int find_root(int x){
     x=access(x);
     while(nd[x].ch[0])x=nd[x].ch[0];
102
     splay(x);
103
     return x:
104
int query(int u,int v){
   //傳回uv路徑splay tree的根結點
   // 這種寫法無法求LCA
     make root(u);
     return access(v);
109
110
int query_lca(int u,int v){
112 //假設求鏈上點權的總和, sum是子樹的權重和
        data是節點的權重
     access(u);
     int lca=access(v);
115
     splay(u);
116
     if(u==lca){
       //return nd[lca].data+nd[nd[lca].ch[1]].
     }else{
       //return nd[lca].data+nd[nd[lca].ch[1]].
119
            sum+nd[u].sum
   struct EDGE{
     int a,b,w;
124 }e[10005];
126 vector<pair<int,int>> G[10005];
127 //first表示子節點, second表示邊的編號
128 int pa[10005],edge_node[10005];
129 //pa是父母節點,暫存用的,edge node是每個編
        被存在哪個點裡面的陣列
130 void bfs(int root){
131 //在建構的時候把每個點都設成一個splay tree
     queue<int > q;
     for(int i=1;i<=n;++i)pa[i]=0;</pre>
133
     q.push(root);
134
135
     while(q.size()){
136
       int u=q.front();
137
       q.pop();
       for(auto P:G[u]){
138
         int v=P.first;
139
140
         if(v!=pa[u]){
           pa[v]=u;
141
142
           nd[v].pa=u;
           nd[v].data=e[P.second].w;
143
144
           edge_node[P.second]=v;
145
           up(v);
146
           q.push(v);
147
148
149
150
void change(int x,int b){
```

# 9.4 POJ\_tree

1 #include < bits / stdc++.h>

2 using namespace std;

3 #define MAXN 10005

splay(x);

up(x);

//nd[x].data=b;

```
4 int n,k;
 5 vector<pair<int,int> >g[MAXN];
 6 int size[MAXN];
  bool vis[MAXN];
  inline void init(){
     for(int i=0:i<=n:++i){</pre>
       g[i].clear();
       vis[i]=0;
12
    }
13 }
   void get dis(vector<int> &dis.int u.int pa.
        int d){
     dis.push back(d):
     for(size_t i=0;i<g[u].size();++i){</pre>
       int v=g[u][i].first,w=g[u][i].second;
17
       if(v!=pa&&!vis[v])get dis(dis,v,u,d+w);
19
20 }
   vector<int> dis;//這東西如果放在函數裡會TLE
   int cal(int u,int d){
     dis.clear();
     get dis(dis,u,-1,d);
25
     sort(dis.begin(),dis.end());
     int l=0,r=dis.size()-1,res=0;
27
       while(1<r&&dis[1]+dis[r]>k)--r;
29
       res+=r-(1++);
30
31
     return res;
32 }
   pair<int,int> tree centroid(int u,int pa,
        const int sz){
     size[u]=1;//找樹重心, second是重心
     pair<int,int> res(INT MAX,-1);
     int ma=0;
     for(size_t i=0;i<g[u].size();++i){</pre>
37
       int v=g[u][i].first;
38
39
       if(v==pa||vis[v])continue;
       res=min(res,tree centroid(v,u,sz));
40
41
       size[u]+=size[v];
42
       ma=max(ma,size[v]);
43
     ma=max(ma,sz-size[u]);
     return min(res, make pair(ma, u));
46 }
47
   int tree DC(int u,int sz){
     int center=tree centroid(u,-1,sz).second;
     int ans=cal(center,0);
     vis[center]=1;
     for(size t i=0;i<g[center].size();++i){</pre>
       int v=g[center][i].first,w=g[center][i].
```

```
if(vis[v])continue;
54
       ans-=cal(v,w);
55
       ans+=tree_DC(v,size[v]);
56
57
     return ans;
58
59
   int main(){
     while(scanf("%d%d",&n,&k),n||k){
60
61
       init();
       for(int i=1;i<n;++i){</pre>
62
63
         int u,v,w;
         scanf("%d%d%d",&u,&v,&w);
64
         g[u].push_back(make_pair(v,w));
65
66
         g[v].push back(make pair(u,w));
67
68
       printf("%d\n",tree_DC(1,n));
69
70
     return 0;
```

## 10 default

## 10.1 debug

#### 10.2 ext

```
12 //s.order_of_key(1);
```

#### 10.3 IncStack

#### 10.4 input

```
inline int read(){
  int x=0; bool f=0; char c=getchar();
  while(ch<'0'||'9'<ch)f|=ch=='-',ch=getchar();

while('0'<=ch&&ch<='9')x=x*10-'0'+ch,ch=getchar();
  return f?-x:x;
}

// #!/bin/bash
// g++ -std=c++11 -02 -Wall -Wextra -Wno-unused-result -DDEBUG $1 && ./a.out
// -fsanitize=address -fsanitize=undefined-fsanitize=return</pre>
```

# 11 language

#### 11.1 CNF

```
1 #define MAXN 55
2 struct CNF{
3 int s,x,y;//s->xy | s->x, if y==-1
4 int cost;
5 CNF(){}
6 CNF(int s,int x,int y,int c):s(s),x(x),y(y),cost(c){}
7 };
8 int state;//規則數量
9 map<char,int> rule;//每個字元對應到的規則·
小寫字母為終端字符
10 vector<CNF> cnf;
```

```
11 void init(){
    state=0;
    rule.clear();
13
14
    cnf.clear();
15
   void add to cnf(char s,const string &p,int
    //加入一個s -> 的文法,代價為cost
    if(rule.find(s)==rule.end())rule[s]=state
    for(auto c:p)if(rule.find(c)==rule.end())
         rule[c]=state++;
    if(p.size()==1){
       cnf.push back(CNF(rule[s],rule[p[0]],-1,
22
    }else{
      int left=rule[s];
      int sz=p.size();
       for(int i=0;i<sz-2;++i){</pre>
         cnf.push_back(CNF(left,rule[p[i]],
             state,0));
        left=state++;
       cnf.push back(CNF(left,rule[p[sz-2]],
            rule[p[sz-1]],cost));
30
32 vector<long long> dp[MAXN][MAXN];
33 | vector<bool> neg INF[MAXN][MAXN];//如果花費
        是負的可能會有無限小的情形
  void relax(int 1,int r,const CNF &c,long
       long cost,bool neg_c=0){
    if(!neg_INF[1][r][c.s]&&(neg_INF[1][r][c.x
         ]||cost<dp[1][r][c.s])){
      if(neg_c||neg_INF[1][r][c.x]){
37
        dp[1][r][c.s]=0;
        neg_INF[1][r][c.s]=true;
39
      }else dp[l][r][c.s]=cost;
40
41
   void bellman(int l,int r,int n){
    for(int k=1;k<=state;++k)</pre>
      for(auto c:cnf)
        if(c.y==-1)relax(1,r,c,dp[1][r][c.x]+c
45
             .cost,k==n);
46
   void cyk(const vector<int> &tok){
    for(int i=0;i<(int)tok.size();++i){</pre>
      for(int j=0;j<(int)tok.size();++j){</pre>
49
        dp[i][j]=vector<long long>(state+1,
             INT MAX);
        neg_INF[i][j]=vector<bool>(state+1,
             false):
      dp[i][i][tok[i]]=0;
      bellman(i,i,tok.size());
55
    for(int r=1;r<(int)tok.size();++r){</pre>
      for(int l=r-1;l>=0;--1){
        for(int k=1;k<r;++k)</pre>
          for(auto c:cnf)
            if(~c.y)relax(1,r,c,dp[1][k][c.x]+
                 dp[k+1][r][c.y]+c.cost);
         bellman(l,r,tok.size());
62
```

# other

## 12.1 WhatDay

```
1 int whatday(int y,int m,int d){
  if(m < = 2)m + = 12, -- y;
   if(y<1752||y==1752&m<9||y==1752&m==9&d
     return (d+2*m+3*(m+1)/5+y+y/4+5)%7;
   return (d+2*m+3*(m+1)/5+y+y/4-y/100+y/400)
```

#### 12.2 上下最大正方形

```
1 void solve(int n,int a[],int b[]){// 1-base
    int ans=0:
    deque<int>da,db;
     for(int l=1,r=1;r<=n;++r){</pre>
       while(da.size()&&a[da.back()]>=a[r]){
         da.pop_back();
       da.push back(r);
       while(db.size()&&b[db.back()]>=b[r]){
         db.pop back();
11
12
       db.push back(r);
       for(int d=a[da.front()]+b[db.front()];r-
            1+1>d;++1){
         if(da.front()==1)da.pop_front();
         if(db.front()==1)db.pop_front();
         if(da.size()&&db.size()){
           d=a[da.front()]+b[db.front()];
18
19
20
      ans=max(ans,r-l+1);
21
    printf("%d\n",ans);
```

# 12.3 最大矩形

```
1 | LL max rectangle(vector<int> s){
    stack<pair<int,int > > st;
    st.push(make pair(-1,0));
    s.push back(0);
    LL ans=0;
    for(size t i=0;i<s.size();++i){</pre>
      int h=s[i];
      pair<int,int > now=make_pair(h,i);
      while(h<st.top().first){</pre>
```

```
now=st.top();
11
         st.pop();
         ans=max(ans,(LL)(i-now.second)*now.
12
              first);
13
       if(h>st.top().first){
14
15
         st.push(make pair(h,now.second));
16
17
18
    return ans;
```

# zformula

#### 13.1 formula

#### 13.1.1 Pick 公式

給定頂點坐標均是整點的簡單多邊形,面積 = 內部格點數 + 邊上格點數/2-1

#### 13.1.2 圖論

```
1. V - E + F = 2
```

- 2. 對於平面圖  $F = E V + n + 1 \cdot n$  是連通分量
- 3. 對於平面圖  $\cdot E \leq 3V 6$
- 4. 對於連通圖 G·最大獨立點集的大小設為 I(G)·最 大匹配大小設為 M(G), 最小點覆蓋設為 Cv(G), 最小邊覆蓋設為 Ce(G)。對於任意連通圖:
  - (a) I(G) + Cv(G) = |V|(b) M(G) + Ce(G) = |V|
- 5. 對於連通二分圖:
  - (a) I(G) = Cv(G)(b) M(G) = Ce(G)
- 6. 最大權閉合圖:
  - (a)  $C(u, V) = \infty, (u, v) \in E$ (b)  $C(S, v) = W_v, W_v > 0$ (c)  $C(v, T) = -W_v, W_v < 0$
- 7. 最大密度子圖:
  - (a)  $C(u, v) = 1, (u, v) \in E$ (b)  $C(S, v) = U_v, v \in V$ (c)  $C(v,T) = U + 2g - d_v, v \in V$
- - (a) 完美消除序列從後往前依次給每個點染色,給 每個點染上可以染的最小顏色
  - 最大團大小 = 色數
  - (c) 最大獨立集: 完美消除序列從前往後能選就選
  - (d) 最小團覆蓋: 最大獨立集的點和他延伸的邊構

  - (f) 區間圖的完美消除序列: 將區間按造又端點由 小到大排序
  - (g) 區間圖染色: 用線段樹做

```
1 double 1=0,=m, stop=1.0/n/n;
  while(r-1>=stop){
    double(mid);
    if((n*m-sol.maxFlow(s,t))/2>eps)l=mid;
    else r=mid;
  build(1):
  sol.maxFlow(s,t);
  vector<int> ans;
10 for(int i=1;i<=n;++i)
if(sol.vis[i])ans.push back(i);
```

#### 13.1.3 學長公式

```
1. \sum_{d|n} \phi(n) = n
```

- 2.  $g(n) = \sum_{d|n} f(d) = \sum_{d|n} \mu(d) \times$
- 3. Harmonic series  $H_n = \ln(n) + \gamma + 1/(2n)$  $1/(12n^2) + 1/(120n^4)$
- 4.  $\gamma = 0.5772156649015328606065120900824024310421$
- 5. 格雷碼 =  $n \oplus (n >> 1)$
- 6.  $SG(A+B) = SG(A) \oplus SG(B)$
- 7. 選轉矩陣  $M(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

#### 13.1.4 基本數論

- 1.  $\sum_{d|n} \mu(n) = [n == 1]$
- 2.  $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) \times$
- 3.  $\sum_{i=1}^{n} \sum_{j=1}^{m}$  互質數量 =  $\sum \mu(d) \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$
- 4.  $\sum_{i=1}^{n} \sum_{j=1}^{n} lcm(i,j) = n \sum_{d|n} d \times \phi(d)$

#### 13.1.5 排組公式

- 1. k 卡特蘭  $\frac{C_n^{kn}}{n(k-1)+1} \cdot C_m^n = \frac{n!}{m!(n-m)!}$
- 2.  $H(n,m) \cong x_1 + x_2 \dots + x_n = k, num = k$
- 3. Stirling number of  $2^{nd}$ , n 人分 k 組方法數目
  - (a) S(0,0) = S(n,n) = 1
  - (b) S(n,0) = 0
  - (c) S(n,k) = kS(n-1,k) + S(n-1,k-1)
- 4. Bell number, n 人分任意多組方法數目
  - (a)  $B_0 = 1$

  - (a)  $B_0 = 1$ (b)  $B_n = \sum_{i=0}^n S(n, i)$ (c)  $B_{n+1} = \sum_{k=0}^n C_k^k B_k$ (d)  $B_{p+n} \equiv B_n + B_{n+1} mod p$ , p is prime (e)  $B_{p^m+n} \equiv m B_n + B_{n+1} mod p$ , p is prime (f) From  $B_0 : 1, 1, 2, 5, 15, 52$ ,
  - 203, 877, 4140, 21147, 115975
- 5. Derangement, 錯排, 沒有人在自己位置上

  - (a)  $D_n = n!(1 \frac{1}{1!} + \frac{1}{2!} \frac{1}{3!} \dots + (-1)^n \frac{1}{n!})$ (b)  $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 =$  $1, D_1 = 0$

- (c) From  $D_0: 1, 0, 1, 2, 9, 44$ , 265, 1854, 14833, 133496
- 6. Binomial Equality

```
(a) \sum_{k} {r \choose m+k} {s \choose n-k} = {r+s \choose m+n}
```

- (b)  $\sum_{k} {l \choose m+k} {s \choose n+k} = {l+s \choose l-m+n}$
- (c)  $\sum_{k} {l \choose m+k} {s+k \choose n} (-1)^k = (-1)^{l+m} {s-m \choose n-l}$
- (d)  $\sum_{k \le l} {l \choose m} {s \choose k-n} (-1)^k$  $(-1)^{l+m} {s-m-1 \choose l-n-m}$
- (e)  $\sum_{0 \le k \le l} {l-k \choose m} {q+k \choose n} = {l+q+1 \choose m+n+1}$
- (f)  $\binom{r}{k} = (-1)^k \binom{k-r-1}{k}$
- (g)  $\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}$
- (h)  $\sum_{k \le n} {r+k \choose k} = {r+n+1 \choose n}$
- (i)  $\sum_{0 \le k \le n} {k \choose m} = {n+1 \choose m+1}$
- (j)  $\sum_{k \le m}^{-} {m+r \choose k} x^k y^k$  $\sum_{k \le m} {\binom{-r}{k}} (-x)^k (x+y)^{m-k}$

#### 13.1.6 冪次、冪次和

- 1.  $a^{b}\%P = a^{b\%\varphi(p) + \varphi(p)}, b > \varphi(p)$
- 2.  $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
- 3.  $1^4 + 2^4 + 3^4 + \ldots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{2} \frac{n}{20}$
- 4.  $1^5 + 2^5 + 3^5 + \ldots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} \frac{n^2}{12}$  9
- 5.  $0^k + 1^k + 2^k + \dots + n^k = P(k), P(k) = {}_{10}$   $\frac{(n+1)^{k+1} \sum_{i=0}^{k-1} C_i^{k+1} P(i)}{k+1}, P(0) = n+1$
- 6.  $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^n C_k^{n+1} B_k m^{n+1-k}$
- 7.  $\sum_{j=0}^{m} C_{j}^{m+1} B_{j} = 0, B_{0} = 1$
- 8. 除了  $B_1 = -1/2$ ,剩下的奇數項都是 0
- 9.  $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 =$  $-1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 17$  $7/6, B_{16} = -3617/510, B_{18}$  $43867/798, B_{20} = -174611/330,$

#### 13.1.7 Burnside's lemma

- 1.  $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- 2.  $X^g = t^{c(g)}$
- G 表示有幾種轉法, X<sup>g</sup> 表示在那種轉法下, 有幾種 是會保持對稱的 $\cdot t$  是顏色數 $\cdot c(g)$  是循環節不動的
- 4. 正立方體塗三顏色,轉 0 有 36 個元素不變,轉 90 有 6 種, 每種有 3<sup>3</sup> 不變, 180 有 3 × 3<sup>4</sup>, 3 120(角) 有 8 ×  $3^2$  · 180(邊) 有 6 ×  $3^3$  · 全部 4  $\frac{1}{24} \left( 3^6 + 6 \times 3^3 + 3 \times 3^4 + 8 \times 3^2 + 6 \times 3^3 \right) = 5$

#### 13.1.8 Count on a tree

- 1. Rooted tree:  $s_{n+1} = \frac{1}{n} \sum_{i=1}^{n} (i \times a_i \times 9)$  });
- 2. Unrooted tree:
  - (a) Odd: $a_n \sum_{i=1}^{n/2} a_i a_{n-i}$ (b) Even: $Odd + \frac{1}{2} a_{n/2} (a_{n/2} + 1)$
- 3. Spanning Tree
  - (a) 完全圖  $n^n 2$
  - (b) 一般圖 (Kirchhoff's theorem)M[i][i] = $degree(V_i), M[i][j] = -1, \text{if have } E(i, j), 0$  0 Object value = map.get(obj); if no edge. delete any one row and col in 7 3  $A. \ ans = det(A)$

#### 13.2 java

#### 13.2.1 文件操作

```
1 import java.io.*;
  import iava.util.*:
  import java.math.*;
  import java.text.*;
   public class Main{
    public static void main(String args[]){
         throws FileNotFoundException,
         IOException
      Scanner sc = new Scanner(new FileReader(
           "a.in"));
       PrintWriter pw = new PrintWriter(new
           FileWriter("a.out"));
      int n,m;
      n=sc.nextInt();//读入下一个INT
12
      m=sc.nextInt():
       for(ci=1; ci<=c; ++ci){</pre>
        pw.println("Case #"+ci+": easy for
             output");
       pw.close();// 关闭流并释放,这个很重要
           否则是没有输出的
      sc.close();// 关闭流并释放
20
21
```

#### 13.2.2 优先队列

```
1 | PriorityQueue queue = new PriorityQueue( 1,
      new Comparator(){
    public int compare( Point a, Point b ){
   if(a.x < b.x | | a.x == b.x && a.y < b.y)
     return -1;
   else if( a.x == b.x && a.v == b.v )
     return 0;
```

# 13.2.4 sort

7 else return 1;

13.2.3 Map

2 map.put("sa","dd");

1 | Map map = new HashMap();

3 String str = map.get("sa").toString;

5 for(Object obj : map.keySet()){

```
1 static class cmp implements Comparator{
    public int compare(Object o1,Object o2){
    BigInteger b1=(BigInteger)o1;
    BigInteger b2=(BigInteger)o2;
    return b1.compareTo(b2);
  public static void main(String[] args)
       throws IOException{
    Scanner cin = new Scanner(System.in);
11
    n=cin.nextInt();
    BigInteger[] seg = new BigInteger[n];
    for (int i=0;i<n;i++)</pre>
    seg[i]=cin.nextBigInteger();
    Arrays.sort(seg, new cmp());
```

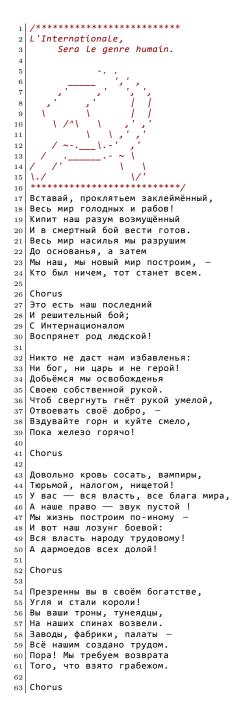
## 14

# 14.1 ganadoQuote

```
1 ¡Allí está!
 2 ¡Un forastero!
 3 ¡Agarrenlo!
 4 jOs voy a romper a pedazos!
 5 ¡Cógelo!
 6 ¡Te voy a hacer picadillo!
 7 | ¡Te voy a matar!
 8 ¡Míralo, está herido!
 9 iSos cerdo!
10 ¿Dónde estás?
11 ¡Detrás de tí, imbécil!
12 ¡No dejes que se escape!
13 ¡Basta, hijo de puta!
14 Lord Saddler...
16 ¡Mátalo!
17 ¡Allí está!
```

```
18 Morir es vivir.
  Sííííí, ¡Quiero matar!
  Muere, muere, muere....
  Cerebros, cerebros, cerebros...
22 Cógedlo, cógedlo, cógedlo...
23 Lord Saddler...
24 Dieciséis.
  ¡Va por él!
  ¡Muérete!
  ¡Cógelo!
  ¡Te vov a matar!
  ¡Bloqueale el paso!
  ¡Te cogí!
  ¡No dejes que se escape!
  ¿Qué carajo estás haciendo aquí? ¡Lárgate
  Hay un rumor de que hay un extranjero entre
       nosotros.
  Nuestro jefe se encargará de la rata.
  Su "Las Plagas" es mucho mejor que la
       nuestra.
  Tienes razón, es un hombre.
  Usa los músculos.
40 Se vuelve loco!
41 ¡Hey, acá!
42 ¡Por aquí!
43 ¡El Gigante!
44 ¡Del Lago!
45 ¡Cógelo!
46 ¡Cógenlo!
  ¡Allí!
47
  ¡Rápido!
  ¡Empieza a rezar!
  :Mátenlos!
  ¡Te voy a romper en pedazos!
  ¡La campana!
  Ya es hora de rezar.
  Tenemos que irnos.
  ¡Maldita sea, mierda!
  ¡Ya es hora de aplastar!
  ¡Puedes correr, pero no te puedes esconder!
  ¡Sos cerdo!
  ¡Está en la trampa!
  ¡Ah, que madre!
62 ¡Vámonos!
  ¡Ándale!
  ¡Cabrón!
  ¡Coño!
  ¡Agárrenlo!
  Cógerlo, Cógerlo...
  ¡Allí está, mátalo!
  ¡No dejas que se escape de la isla vivo!
70 ¡Hasta luego!
71 ¡Rápido, es un intruso!
```

## 14.2



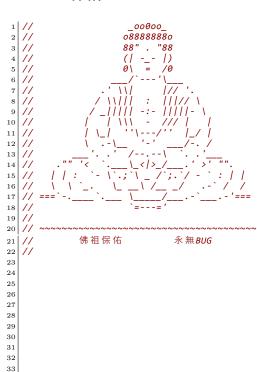
```
65 Довольно королям в угоду
66 Дурманить нас в чаду войны!
67 Война тиранам! Мир Народу!
68 Бастуйте, армии сыны!
69 Когда ж тираны нас заставят
70 В бою геройски пасть за них —
71 Убийцы, в вас тогда направим
72 Мы жерла пушек боевых!
73
74
   Chorus
75
   Лишь мы, работники всемирной
76
77 Великой армии труда,
78 Владеть землёй имеем право,
79 Но паразиты — никогда!
80 И если гром великий грянет
81 Над сворой псов и палачей, -
82 Для нас всё так же солнце станет
83 Сиять огнём своих лучей.
84
85 Chorus
```

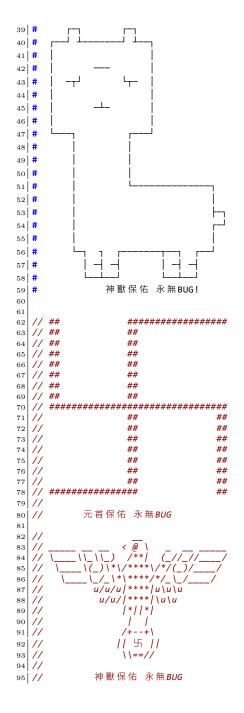
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34

35

36





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