

1 Computational_Geometry

1.1 delaunay

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1 template<class T>
2 class Delaunay{
3     struct PT:public point<T>{
4         int g[2];
5         PT(const point<T> &p):
6             point<T>(p){ g[0]=g[1]=-1; }
7     };
8     static bool cmp(const PT &a,const PT &b){
9         return a.x<b.x||(a.x==b.x&&a.y<b.y);
10    }
11    struct edge{
12        int v,g[2];
13        edge(int v,int g0,int g1):
14            v(v){g[0]=g0,g[1]=g1;}
15    };
16    vector<PT> S;
17    vector<edge> E;
18    bool convex(int &from,int to,T LR){
19        for(int i=0;i<2;++i){
20            int c = E[S[from].g[i]].v;
21            auto A=S[from]-S[to], B=S[c]-S[to];
22            T v = A.cross(B)*LR;
23            if(v>0||(v==0&&B.abs2()<A.abs2()))
24                return from = c, true;
25        }
26        return false;
27    }
28    void addEdge(int v,int g0,int g1){
29        E.emplace_back(v,g0,g1);
30        E[E.back().g[0]].g[1] = E.size()-1;
31        E[E.back().g[1]].g[0] = E.size()-1;
32    }
33    void climb(int &p, int e, int n, int nl,
34              int nr, int LR){
35        for(int i=E[e].g[LR]; (S[nr]-S[nl]).
36            cross(S[E[i].v]-S[nl])>0;){
37            if(inCircle(S[E[i].v],S[nl],S[nr],S[E[
38                E[i].g[LR]].v])>0)
39                { p = i; break; }
40            for(int j=0;j<4;++j)
41                E[E[i^j/2].g[j%2^1]].g[j%2] = E[i^j
42                    /2].g[j%2];
43            int j=i; i=E[i].g[LR];
44            E[j].g[0]=E[i].g[1]=E[j^1].g[0]=E[j
45                ^1].g[1]=-1;
46        }
47    }
48    T det3(T a11,T a12,T a13,T a21,T a22,T a23
49        ,T a31,T a32,T a33){
50        return a11*(a22*a33-a32*a23)-a12*(a21*
51            a33-a31*a23)+a13*(a21*a32-a31*a22);
52    }
53    int inCircle(const PT &a, const PT &b,
54                const PT &c, const PT &p){
55        T as = a.abs2(), bs = b.abs2(), cs = c.abs2
56            (), ps = p.abs2();
57        T res = a.x * det3(b.y,bs,1,c.y,cs,1,p.y,ps
58            ,1)
59            -a.y * det3(b.x,bs,1,c.x,cs,1,p.x,ps,1)

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60        +as * det3(b.x,b.y,1,c.x,c.y,1,p.x,p.y,1)
61        -det3(b.x,b.y,bs,c.x,c.y,cs,p.x,p.y,ps);
62        return res<0 ? 1 : (res>0 ? -1 : 0);
63    }
64    void divide(int l, int r){
65        if(l==r)return;
66        if(l+1==r){
67            int A=S[l].g[0]=S[l].g[1]=E.size();
68            E.emplace_back(r,A,A);
69            int B=S[r].g[0]=S[r].g[1]=E.size();
70            E.emplace_back(l,B,B);
71            return;
72        }
73        int mid = (l+r)/2;
74        divide(l,mid), divide(mid+1, r);
75        int nl = mid, nr = mid+1;
76        for(;;){
77            if(convex(nl,nr,1)) continue;
78            if(S[nr].g[0]!=-1&&convex(nr,nl,-1))
79                continue;
80            break;
81        }
82        addEdge(nr,S[nl].g[0],S[nl].g[1]);
83        S[nl].g[1] = E.size()-1;
84        if(S[nr].g[0]==-1){
85            addEdge(nl,E.size(),E.size());
86            S[nr].g[1] = E.size()-1;
87        }else addEdge(nl,S[nr].g[0],S[nr].g[1]);
88        S[nr].g[0] = E.size()-1;
89        int cl = nl, cr = nr;
90        for(;;){
91            int pl=-1, pr=-1, side;
92            climb(pl,E.size()-2,nl,nl,nr,1);
93            climb(pr,E.size()-1,nr,nl,nr,0);
94            if(pl==-1&&pr==-1) break;
95            if(pl==-1||pr==-1) side = pl==-1;
96            else side=inCircle(S[E[pl].v],S[nl],S[
97                nr],S[E[pr].v])<=0;
98            if(side){
99                nr = E[pr].v;
100            }
101            addEdge(nr,E.size()-2,E[E.size()-2].g[1]);
102            addEdge(nl,E[pr^1].g[0],pr^1);
103        }else{
104            nl = E[pl].v;
105            addEdge(nr,pl^1,E[pl^1].g[1]);
106            addEdge(nl,E[E.size()-2].g[0],E.size()-2);
107        }
108    }
109    if(cl==nl&&cr==nr) return; //Collinearity
110    S[nl].g[0] = E.size()-2;
111    S[nr].g[1] = E.size()-1;
112    }
113    public:
114    void solve(const vector<point<T>> &P){
115        S.clear(), E.clear();
116        for(const auto &p:P) S.emplace_back(p);
117        sort(S.begin(),S.end(),cmp);
118        divide(0,int(S.size())-1);
119    }
120    vector<pair<int,int>> getEdge(){
121        vector<pair<int,int>> res;
122        for(size_t i=0;i<E.size();i+=2)
123            if(E[i].g[0]!=-1)
124                res.emplace_back(E[i].v,E[i^1].v);
125        return res;
126    }

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1.2 Geometry

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1 const double PI=atan2(0.0,-1.0);
2 template<typename T>
3 struct point{
4     T x,y;
5     point(){}
6     point(const T&x,const T&y):x(x),y(y){}
7     point operator+(const point &b)const{
8         return point(x+b.x,y+b.y); }
9     point operator-(const point &b)const{
10        return point(x-b.x,y-b.y); }
11     point operator*(const T &b)const{
12        return point(x*b,y*b); }
13     point operator/(const T &b)const{
14        return point(x/b,y/b); }
15     bool operator==(const point &b)const{
16        return x==b.x&&y==b.y; }
17     T dot(const point &b)const{
18        return x*b.x+y*b.y; }
19     T cross(const point &b)const{
20        return x*b.y-y*b.x; }
21     point normal()const{//求法向量
22        return point(-y,x); }
23     T abs2()const{//向量長度的平方
24        return dot(*this); }
25     T rad(const point &b)const{//兩向量的弧度
26        return fabs(atan2(fabs(cross(b)),dot(b))); }
27     T getA()const{//對x軸的弧度
28        T A=atan2(y,x); //超過180度會變負的
29        if(A<=-PI/2)A+=PI*2;
30        return A;
31    }
32 };
33 template<typename T>
34 struct line{
35     line(){}
36     point<T> p1,p2;
37     T a,b,c;//ax+by+c=0
38     line(const point<T>&x,const point<T>&y):p1
39         (x),p2(y){}
40     void pton(){} //轉成一般式
41     a=p1.y-p2.y;
42     b=p2.x-p1.x;
43     c=-a*p1.x-b*p1.y;
44 }
45 T ori(const point<T> &p)const{//點和有向直
46     線的關係，>0左邊、=0在線上、<0右邊
47     return (p2-p1).cross(p-p1);
48 }
49 T btw(const point<T> &p)const{//點投影落在
50     線段上<=0
51     return (p1-p).dot(p2-p);
52 }
53 bool point_on_segment(const point<T>&p)
54     const{//點是否在線段上
55     return ori(p)==0&&btw(p)<=0;
56 }

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57 T dis2(const point<T> &p,bool is_segment
58     =0)const{//點跟直線/線段的距離平方
59     point<T> v=p2-p1,v1=p-p1;
60     if(is_segment){
61         point<T> v2=p-p2;
62         if(v.dot(v1)<=0)return v1.abs2();
63         if(v.dot(v2)>=0)return v2.abs2();
64     }
65     T tmp=v.cross(v1);
66     return tmp*tmp/v.abs2();
67 }
68 T seg_dis2(const line<T> &l)const{//兩線段
69     距離平方
70     return min({dis2(l.p1,1),dis2(l.p2,1),l.
71         dis2(p1,1),l.dis2(p2,1)});
72 }
73 point<T> projection(const point<T> &p)
74     const{//點對直線的投影
75     point<T> n=(p2-p1).normal();
76     return p-n*(p-p1).dot(n)/n.abs2();
77 }
78 point<T> mirror(const point<T> &p)const{
79     //點對直線的鏡射，要先呼叫pton轉成一般式
80     point<T> R;
81     T d=a*a+b*b;
82     R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
83     R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
84     return R;
85 }
86 bool equal(const line &l)const{//直線相等
87     return ori(l.p1)==0&&ori(l.p2)==0;
88 }
89 bool parallel(const line &l)const{
90     return (p1-p2).cross(l.p1-l.p2)==0;
91 }
92 bool cross_seg(const line &l)const{
93     return (p2-p1).cross(l.p1-p1)*(p2-p1).
94         cross(l.p2-p1)<=0; //直線是否交線段
95 }
96 int line_intersect(const line &l)const{//
97     直線相交情況，-1無限多點、1交於一點、0
98     不相交
99     return parallel(l)?(ori(l.p1)==0?-1:0)
100         :1;
101 }
102 int seg_intersect(const line &l)const{
103     T c1=ori(l.p1), c2=ori(l.p2);
104     T c3=l.ori(p1), c4=l.ori(p2);
105     if(c1==0&&c2==0){ //共線
106         bool b1=btw(l.p1)>0,b2=btw(l.p2)>0;
107         T a3=l.btw(p1),a4=l.btw(p2);
108         if(b1&&b2&&a3==0&&a4==0) return 2;
109         if(b1&&b2&&a3>0&&a4==0) return 3;
110         if(b1&&b2&&a3>0&&a4>0) return 0;
111         return -1; //無限交點
112     }else if(c1*c2<=0&&c3*c4<=0)return 1;
113     return 0; //不相交
114 }
115 point<T> line_intersection(const line &l)
116     const{//直線交點*/
117     point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
118     //if(a.cross(b)==0)return INF;
119     return p1+a*(s.cross(b)/a.cross(b));
120 }

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108 point<T> seg_intersection(const line &l)
109     const{//線段交點
110     int res=seg_intersect(l);
111     if(res<=0) assert(0);
112     if(res==2) return p1;
113     if(res==3) return p2;
114     return line_intersection(l);
115 };
116 template<typename T>
117 struct polygon{
118     polygon(){
119     vector<point<T> > p;//逆時針順序
120     T area()const{//面積
121     T ans=0;
122     for(int i=p.size()-1,j=0;j<(int)p.size()
123     ;i=j++){
124     ans+=p[i].cross(p[j]);
125     return ans/2;
126 }
127 point<T> center_of_mass()const{//重心
128     T cx=0,cy=0,w=0;
129     for(int i=p.size()-1,j=0;j<(int)p.size()
130     ;i=j++){
131     T a=p[i].cross(p[j]);
132     cx+=(p[i].x+p[j].x)*a;
133     cy+=(p[i].y+p[j].y)*a;
134     w+=a;
135     }
136     return point<T>(cx/3/w,cy/3/w);
137 }
138 char ahas(const point<T>& t)const{//點是否
139     在簡單多邊形內、是的話回傳1、在邊上回
140     傳-1、否則回傳0
141     bool c=0;
142     for(int i=0,j=p.size()-1;i<p.size();j=i
143     ++){
144     if(line<T>(p[i],p[j]).point_on_segment
145     (t))return -1;
146     else if((p[i].y>t.y)!=p[j].y>t.y)&&
147     t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j
148     ].y-p[i].y)+p[i].x)
149     c=!c;
150     return c;
151 }
152 char point_in_convex(const point<T>&x)
153     const{
154     int l=1,r=(int)p.size()-2;
155     while(l<=r){//點是否在凸多邊形內、是的話
156     回傳1、在邊上回傳-1、否則回傳0
157     int mid=(l+r)/2;
158     T a1=(p[mid]-p[0]).cross(x-p[0]);
159     T a2=(p[mid+1]-p[0]).cross(x-p[0]);
160     if(a1>=0&&a2<=0){
161     T res=(p[mid+1]-p[mid]).cross(x-p[
162     mid]);
163     return res>0?1:(res>0?-1:0);
164     }else if(a1<0)r=mid-1;
165     else l=mid+1;
166 }
167 return 0;
168 }
169 vector<T> getA()const{//凸包邊對x軸的夾角
170     vector<T>res;//一定是遞增的
171     for(size_t i=0;i<p.size();++i)
172     res.push_back((p[(i+1)%p.size()]-p[i])
173     .getA());
174     return res;
175 }
176 bool line_intersect(const vector<T>&A,
177     const line<T> &l)const{//0(logN)
178     int f1=upper_bound(A.begin(),A.end(),(l.
179     p1-l.p2).getA())-A.begin();
180     int f2=upper_bound(A.begin(),A.end(),(l.
181     p2-l.p1).getA())-A.begin();
182     return l.cross_seg(line<T>(p[f1],p[f2]))
183     ;
184 }
185 polygon cut(const line<T> &l)const{//凸包
186     對直線切割、得到直線l左側的凸包
187     polygon ans;
188     for(int n=p.size(),i=n-1,j=0;j<n;i=j++){
189     if(l.ori(p[i])>=0){
190     ans.p.push_back(p[i]);
191     if(l.ori(p[j])<0)
192     ans.p.push_back(l.
193     line_intersection(line<T>(p[i
194     ],p[j])));
195     }else if(l.ori(p[j])>0)
196     ans.p.push_back(l.line_intersection(
197     line<T>(p[i],p[j])));
198     }
199     return ans;
200 }
201 static bool graham_cmp(const point<T>&a,
202     const point<T>&b){//凸包排序函數
203     return (a.x<b.x)||((a.x==b.x&&a.y<b.y));
204 }
205 void graham(vector<point<T> > &s){//凸包
206     sort(s.begin(),s.end(),graham_cmp);
207     p.resize(s.size()+1);
208     int m=0;
209     for(size_t i=0;i<s.size();++i){
210     while(m>=2&&(p[m-1]-p[m-2]).cross(s[i
211     ]-p[m-2])<=0)--m;
212     p[m++]=s[i];
213 }
214 for(int i=s.size()-2,t=m+1;i>=0;--i){
215     while(m>=t&&(p[m-1]-p[m-2]).cross(s[i
216     ]-p[m-2])<=0)--m;
217     p[m++]=s[i];
218 }
219 T diam(){//直徑
220     int n=p.size(),t=1;
221     T ans=0;p.push_back(p[0]);
222     for(int i=0;i<n;i++){
223     point<T> now=p[i+1]-p[i];
224     while(now.cross(p[t+1]-p[i])>now.cross
225     (p[t]-p[i]))t=(t+1)%n;
226     ans=max(ans,(p[i]-p[t]).abs2());
227 }
228 return p.pop_back(),ans;
229 }
230 T min_cover_rectangle(){//最小覆蓋矩形
231     int n=p.size(),t=1,r=1,l;
232     if(n<3)return 0;//也可以做最小周長矩形
233     T ans=1e99;p.push_back(p[0]);
234     for(int i=0;i<n;i++){
235     point<T> now=p[i+1]-p[i];
236     while(now.cross(p[t+1]-p[i])>now.cross
237     (p[t]-p[i]))t=(t+1)%n;
238     while(now.dot(p[r+1]-p[i])>now.dot(p[r
239     ]-p[i]))r=(r+1)%n;
240     if(!i)l=r;
241     while(now.dot(p[l+1]-p[i])<=now.dot(p[
242     l]-p[i]))l=(l+1)%n;
243     T d=now.abs2();
244     T tmp=now.cross(p[t]-p[i])*(now.dot(p[
245     r]-p[i])-now.dot(p[l]-p[i]))/d;
246     ans=min(ans,tmp);
247 }
248 return p.pop_back(),ans;
249 }
250 T dis2(polygon &p){//凸包最近距離平方
251     vector<point<T> > &P=p,&Q=p1.p;
252     int n=P.size(),m=Q.size(),l=0,r=0;
253     for(int i=0;i<n;++i)if(P[i].y<P[l].y)l=i;
254     for(int i=0;i<m;++i)if(Q[i].y<Q[r].y)r=i;
255     P.push_back(P[0]),Q.push_back(Q[0]);
256     T ans=1e99;
257     for(int i=0;i<n;++i){
258     while((P[l]-P[l+1]).cross(Q[r+1]-Q[r])
259     <0)r=(r+1)%m;
260     ans=min(ans,line<T>(P[l],P[l+1]).
261     seg_dis2(line<T>(Q[r],Q[r+1])));
262     l=(l+1)%n;
263 }
264 return P.pop_back(),Q.pop_back(),ans;
265 }
266 static char sign(const point<T>&t){
267     return (t.y==0?t.x:t.y)<0;
268 }
269 static bool angle_cmp(const line<T>&A,
270     const line<T>&B){
271     point<T> a=A.p2-A.p1,b=B.p2-B.p1;
272     return sign(a)<sign(b)||((sign(a)==sign(b
273     )&&a.cross(b)>0);
274 }
275 int halfplane_intersection(vector<line<T>
276     > &s){//半平面交
277     sort(s.begin(),s.end(),angle_cmp);//線段
278     左側為該線段半平面
279     int L,R,n=s.size();
280     vector<point<T> > px(n);
281     vector<line<T> > q(n);
282     q[L=R=0]=s[0];
283     for(int i=1;i<n;++i){
284     while(L<R&&s[i].ori(px[R-1])<=0)--R;
285     while(L<R&&s[i].ori(px[L])<=0)++L;
286     q[++R]=s[i];
287     if(q[R].parallel(q[R-1])){
288     --R;
289     if(q[R].ori(s[i].p1)>q[R]=s[i];
290     }
291     if(L<R)px[R-1]=q[R-1].
292     line_intersection(q[R]);
293 }
294 while(L<R&&q[L].ori(px[R-1])<=0)--R;
295 p.clear();
296 if(R-L<=1)return 0;
297 px[R]=q[R].line_intersection(q[L]);
298 for(int i=L;i<R;++i)p.push_back(px[i]);
299 return R-L+1;
300 }
301 template<typename T>
302 struct triangle{
303     point<T> a,b,c;
304     triangle(){
305     triangle(const point<T> &a,const point<T>
306     &b,const point<T> &c):a(a),b(b),c(c){}
307     T area()const{
308     T t=(b-a).cross(c-a)/2;
309     return t>0?t:-t;
310 }
311 point<T> barycenter()const{//重心
312     return (a+b+c)/3;
313 }
314 }
315 point<T> circumcenter()const{//外心
316     static line<T> u,v;
317     u.p1=(a+b)/2;
318     u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-
319     b.x);
320     v.p1=(a+c)/2;
321     v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-
322     c.x);
323     return u.line_intersection(v);
324 }
325 point<T> incenter()const{//內心
326     T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2
327     ()),C=sqrt((a-b).abs2());
328     return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+
329     B*b.y+C*c.y)/(A+B+C);
330 }
331 point<T> perpcenter()const{//垂心
332     return barycenter()*3-circumcenter()*2;
333 }
334 }
335 template<typename T>
336 struct point3D{
337     T x,y,z;
338     point3D(){
339     point3D(const T&x,const T&y,const T&z):x(x
340     ),y(y),z(z){}
341     point3D operator+(const point3D &b)const{
342     return point3D(x+b.x,y+b.y,z+b.z);}
343     point3D operator-(const point3D &b)const{
344     return point3D(x-b.x,y-b.y,z-b.z);}
345     point3D operator*(const T &b)const{
346     return point3D(x*b,y*b,z*b);}
347     point3D operator/(const T &b)const{
348     return point3D(x/b,y/b,z/b);}
349     bool operator==(const point3D &b)const{
350     return x==b.x&&y==b.y&&z==b.z;}
351     T dot(const point3D &b)const{
352     return x*b.x+y*b.y+z*b.z;}
353     point3D cross(const point3D &b)const{
354     return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x
355     *b.y-y*b.x);}
356     T abs2()const{//向量長度的平方
357     return dot(*this);}
358     T area2(const point3D &b)const{//和b、原點
359     圍成面積的平方
360     return cross(b).abs2()/4;}
361 }

```

```

322 };
323 template<typename T>
324 struct line3D{
325     point3D<T> p1,p2;
326     line3D(){}
327     line3D(const point3D<T> &p1,const point3D<
328         T> &p2):p1(p1),p2(p2){}
329     T dis2(const point3D<T> &p,bool is_segment
330         =0)const{//點跟直線/線段的距離平方
331         point3D<T> v=p2-p1,v1=p-p1;
332         if(is_segment){
333             point3D<T> v2=p-p2;
334             if(v.dot(v1)<=0)return v1.abs2();
335             if(v.dot(v2)>=0)return v2.abs2();
336         }
337         point3D<T> tmp=v.cross(v1);
338         return tmp.abs2()/v.abs2();
339     }
340     pair<point3D<T>,point3D<T>> closest_pair(
341         const line3D<T> &l)const{
342         point3D<T> v1=(p1-p2),v2=(l.p1-l.p2);
343         point3D<T> N=v1.cross(v2),ab(p1-l.p1);
344         //if(N.abs2()==0)return NULL;平行或重合
345         T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//
346         最近點對距離
347         point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.
348         cross(d2),G=l.p1-p1;
349         T t1=(G.cross(d2)).dot(D)/D.abs2();
350         T t2=(G.cross(d1)).dot(D)/D.abs2();
351         return make_pair(p1+d1*t1,l.p1+d2*t2);
352     }
353     bool same_side(const point3D<T> &a,const
354         point3D<T> &b)const{
355         return (p2-p1).cross(a-p1).dot((p2-p1).
356             cross(b-p1))>0;
357     }
358 };
359 template<typename T>
360 struct plane{
361     point3D<T> p0,n;//平面上的點和法向量
362     plane(){}
363     plane(const point3D<T> &p0,const point3D<T>
364         &n):p0(p0),n(n){}
365     T dis2(const point3D<T> &p)const{//點到平
366         面距離的平方
367         T tmp=(p-p0).dot(n);
368         return tmp*tmp/n.abs2();
369     }
370     point3D<T> projection(const point3D<T> &p)
371         const{
372         return p-n*(p-p0).dot(n)/n.abs2();
373     }
374     point3D<T> line_intersection(const line3D<
375         T> &l)const{
376         T tmp=n.dot(l.p2-l.p1);//等於0表示平行或
377         重合該平面
378         return l.p1+(l.p2-l.p1)*(n.dot(p0-l.p1)/
379             tmp);
380     }
381     line3D<T> plane_intersection(const plane &
382         pl)const{
383         point3D<T> e=n.cross(pl.n),v=n.cross(e);
384         T tmp=pl.n.dot(v);//等於0表示平行或重合
385         該平面
386         point3D<T> q=p0+(v*(pl.n.dot(pl.p0-p0))/
387             tmp);
388         return line3D<T>(q,q+e);
389     }
390 };
391 template<typename T>
392 struct triangle3D{
393     point3D<T> a,b,c;
394     triangle3D(){}
395     triangle3D(const point3D<T> &a,const
396         point3D<T> &b,const point3D<T> &c):a(a),b(b),c(c){}
397     bool point_in(const point3D<T> &p)const{//
398         點在該平面上的投影在三角形中
399         return line3D<T>(b,c).same_side(p,a)&&
400             line3D<T>(a,c).same_side(p,b)&&
401             line3D<T>(a,b).same_side(p,c);
402     }
403 };
404 template<typename T>
405 struct tetrahedron{//四面體
406     point3D<T> a,b,c,d;
407     tetrahedron(){}
408     tetrahedron(const point3D<T> &a,const
409         point3D<T> &b,const point3D<T> &c,
410         const point3D<T> &d):a(a),b(b),c(c),d(d){}
411     T volume6()const{//體積的六倍
412         return (d-a).dot((b-a).cross(c-a));
413     }
414     point3D<T> centroid()const{
415         return (a+b+c+d)/4;
416     }
417     bool point_in(const point3D<T> &p)const{
418         return triangle3D<T>(a,b,c).point_in(p)
419             &&triangle3D<T>(c,d,a).point_in(p);
420     }
421 };
422 template<typename T>
423 struct convexhull3D{
424     static const int MAXN=1005;
425     struct face{
426         int a,b,c;
427         face(int a,int b,int c):a(a),b(b),c(c){}
428     };
429     vector<point3D<T>> pt;
430     vector<face> ans;
431     int fid[MAXN][MAXN];
432     void build(){
433         int n=pt.size();
434         ans.clear();
435         memset(fid,0,sizeof(fid));
436         ans.emplace_back(0,1,2);//注意不能共線
437         ans.emplace_back(2,1,0);
438         int ftop = 0;
439         for(int i=3, ftop=1; i<n; ++i,++ftop){
440             vector<face> next;
441             for(auto &f:ans){
442                 T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[
443                     f.a]).cross(pt[f.c]-pt[f.a]));
444                 if(d<=0) next.push_back(f);
445                 int ff=0;
446                 if(d>0) ff=ftop;
447                 else if(d<0) ff=-ftop;
448                 fid[f.a][f.b]=fid[f.b][f.c]=fid[f.c]
449                     ][f.a]=ff;
450             }
451             for(auto &f:ans){
452                 if(fid[f.a][f.b]>0 && fid[f.a][f.b]
453                     !=fid[f.b][f.a])
454                     next.emplace_back(f.a,f.b,i);
455                 if(fid[f.b][f.c]>0 && fid[f.b][f.c]
456                     !=fid[f.c][f.b])
457                     next.emplace_back(f.b,f.c,i);
458                 if(fid[f.c][f.a]>0 && fid[f.c][f.a]
459                     !=fid[f.a][f.c])
460                     next.emplace_back(f.c,f.a,i);
461             }
462             ans=next;
463         }
464         point3D<T> centroid()const{
465             point3D<T> res(0,0,0);
466             T vol=0;
467             for(auto &f:ans){
468                 T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c]
469                     ));
470                 res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
471                 vol+=tmp;
472             }
473             return res/(vol*4);
474         }
475     };
476 };
477
478 template<typename _IT=point<T>* >
479 T closest_pair(_IT L, _IT R){
480     if(R-L <= 1) return INF;
481     _IT mid = L+(R-L)/2;
482     T x = mid->x;
483     T d = min(closest_pair(L,mid),closest_pair(
484         mid,R));
485     inplace_merge(L, mid, R, ycmp);
486     static vector<point> b; b.clear();
487     for(auto u=L;u<R;++u){
488         if((u->x-x)*(u->x-x)>=d) continue;
489         for(auto v=b.rbegin();v!=b.rend();++v){
490             T dx=u->x-v->x, dy=u->y-v->y;
491             if(dy*dy>=d) break;
492             d=min(d,dx*dx+dy*dy);
493         }
494         b.push_back(*u);
495     }
496     return d;
497 }
498 T closest_pair(vector<point<T>> &v){
499     sort(v.begin(),v.end(),xcmp);
500     return closest_pair(v.begin(),v.end());
501 }
502
503 #include<bits/stdc++.h>
504 using namespace std;
505 const int MAXN = 100005;
506 struct node{
507     double a,b,r,k,x,y;
508     int id;
509 } p[MAXN];
510 double DP[MAXN];
511 deque<int> q;
512 bool cmpK(const node &a,const node &b){
513     return a.k>b.k;
514 }
515 bool cmpX(const node &a,const node &b){
516     return a.x<b.x||(a.x==b.x&&a.y<b.y);
517 }
518 double Slope(int a,int b){
519     if(!b) return -1e20;
520     if(p[a].x==p[b].x) return 1e20;
521     return (p[a].y-p[b].y)/(p[a].x-p[b].x);
522 }
523 void CDQ(int l, int r){
524     if(l==r){
525         DP[l] = max(DP[l],DP[l-1]);
526         p[l].y = DP[l]/(p[l].a*p[l].r+p[l].b);
527         p[l].x = p[l].y*p[l].r;
528         return;
529     }
530     int mid = (l+r)/2;
531     stable_partition(p+l,p+r+1,[&](const node
532         &d){return d.id<=mid;});
533     CDQ(l, mid); q.clear();
534     for(int i=l, j; i<=mid; ++i){
535         using PT=point<T>; using CPT=const PT;
536         PT circumcenter(CPT &a,CPT &b,CPT &c){
537             PT u=b-a, v=c-a;
538             T c1=u.abs2()/2,c2=v.abs2()/2;
539             T d=u.cross(v);
540             return PT(a.x+(v.y*c1-u.y*c2)/d,a.y+(u.x*
541                 c2-v.x*c1)/d);
542         }
543         void solve(PT p[],int n,PT &c,T &r2){
544             random_shuffle(p,p+n);
545             c=p[0]; r2=0; // c,r2 = 圓心,半徑平方
546             for(int i=1;i<n;i++){if((p[i]-c).abs2())>r2){
547                 c=p[i]; r2=0;
548             }
549             for(int j=0;j<i;j++){if((p[j]-c).abs2())>r2){
550                 c.x=(p[i].x+p[j].x)/2;
551                 c.y=(p[i].y+p[j].y)/2;
552                 r2=(p[j]-c).abs2();
553             }
554             for(int k=0;k<j;k++){if((p[k]-c).abs2())>r2){
555                 c=circumcenter(p[i],p[j],p[k]);
556                 r2=(p[i]-c).abs2();
557             }
558         }
559     }
560 }
561
562 1.4 最近點對

```

2 Data_Structure

2.1 CDQ_DP

```

32 while((j=q.size())>1&&Slope(q[j-2],q[j-1])<Slope(q[j-1],i)) q.pop_back();
33 q.push_back(i);
34 q.push_back(0);
35 for(int i=mid+1; i<=r; ++i){
36 while(q.size())>1&&Slope(q[0],q[1])>p[i].k) q.pop_front();
37 DP[p[i].id] = max(DP[p[i].id], p[i].a*p[q[0]].x+p[i].b*p[q[0]].y);
38 }
39 CDQ(mid+1,r);
40 inplace_merge(p+1,p+mid+1,p+r+1,cmpX);
41 }
42 double solve(int n,double S){
43 DP[0] = S;
44 sort(p+1,p+1+n,cmpK);
45 CDQ(1,n);
46 return DP[n];
47 }
48 int main(){
49 int n; double S;
50 scanf("%d%lf",&n,&S);
51 for(int i=1; i<=n; ++i){
52 scanf("%lf%lf%lf",&p[i].a,&p[i].b,&p[i].r);
53 p[i].id = i, p[i].k = -p[i].a/p[i].b;
54 }
55 printf("%.3lf\n",solve(n,S));
56 return 0;
57 }

```

2.2 DLX

```

1 const int MAXN=4100, MAXM=1030, MAXND=16390;
2 struct DLX{
3 int n,m,sz,ansd; //高是n · 寬是m的稀疏矩陣
4 int S[MAXN],H[MAXN];
5 int row[MAXN],col[MAXN]; //每個節點代表的列與行
6 int L[MAXN],R[MAXN],U[MAXN],D[MAXN];
7 vector<int> ans,anst;
8 void init(int _n,int _m){
9 n=_n,m=_m;
10 for(int i=0;i<=m;++i){
11 U[i]=D[i]=i,L[i]=i-1,R[i]=i+1;
12 S[i]=0;
13 }
14 R[m]=0,L[0]=m;
15 sz=m,ansd=INT_MAX; //ansd存最優解的個數
16 for(int i=1;i<=n;++i)H[i]=-1;
17 }
18 void add(int r,int c){
19 ++S[col[++sz]=c];
20 row[sz]=r;
21 D[sz]=D[c],U[D[c]]=sz,U[sz]=c,D[c]=sz;
22 if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
23 else R[sz]=R[H[r]],L[R[H[r]]]=sz,L[sz]=H[r],R[H[r]]=sz;
24 }
25 #define DFOR(i,A,s) for(int i=A[s];i!=s;i=A[i])

```

```

26 void remove(int c){ //刪除第c行和所有當前覆蓋到第c行的列
27 L[R[c]]=L[c],R[L[c]]=R[c]; //這裡刪除第c行 · 若有些行不需要處理可以在開始時叫他
28 DFOR(i,D,c)DFOR(j,R,i){U[D[j]]=U[j],D[U[j]]=D[j],--S[col[j]];}
29 }
30 void restore(int c){ //恢復第c行和所有當前覆蓋到第c行的列 · remove的逆操作
31 DFOR(i,U,c)DFOR(j,L,i){++S[col[j]],U[D[j]]=j,D[U[j]]=j;}
32 L[R[c]]=c,R[L[c]]=c;
33 }
34 void remove2(int nd){ //刪除nd所在的行當前所有點(包括虛擬節點) · 只保留nd
35 DFOR(i,D,nd)L[R[i]]=L[i],R[L[i]]=R[i];
36 }
37 void restore2(int nd){ //刪除nd所在的行當前所有點 · 為remove2的逆操作
38 DFOR(i,U,nd)L[R[i]]=R[L[i]]=i;
39 }
40 bool vis[MAXN];
41 int h(){ //估價函數 for IDA*
42 int res=0;
43 memset(vis,0,sizeof(vis));
44 DFOR(i,R,0)if(!vis[i]){
45 vis[i]=1;
46 ++res;
47 DFOR(j,D,i)DFOR(k,R,j)vis[col[k]]=1;
48 }
49 return res;
50 }
51 bool dfs(int d){ //for精確覆蓋問題
52 if(d+h())>=ansd return 0; //找最佳解用 · 找任意解可以刪掉
53 if(!R[0]){ansd=d;return 1;}
54 int c=R[0];
55 DFOR(i,R,0)if(S[i]<S[c])c=i;
56 remove(c);
57 DFOR(i,D,c){
58 ans.push_back(row[i]);
59 DFOR(j,R,i)remove(col[j]);
60 if(dfs(d+1))return 1;
61 ans.pop_back();
62 DFOR(j,L,i)restore(col[j]);
63 }
64 restore(c);
65 return 0;
66 }
67 void dfs2(int d){ //for最小重複覆蓋問題
68 if(d+h())>=ansd return 0;
69 if(!R[0]){ansd=d;ans=anst;return 1;}
70 int c=R[0];
71 DFOR(i,R,0)if(S[i]<S[c])c=i;
72 DFOR(i,D,c){
73 anst.push_back(row[i]);
74 remove2(i);
75 DFOR(j,R,i)remove2(j),--S[col[j]];
76 dfs2(d+1);
77 anst.pop_back();
78 DFOR(j,L,i)restore2(j),++S[col[j]];
79 restore2(i);

```

```

80 }
81 }
82 bool exact_cover(){ //解精確覆蓋問題
83 return ans.clear(), dfs(0);
84 }
85 void min_cover(){ //解最小重複覆蓋問題
86 anst.clear(); //暫存用 · 答案還是存在ans裡
87 dfs2(0);
88 }
89 #undef DFOR
90 };

```

2.3 Dynamic_KD_tree

```

1 template<typename T,size_t kd> //有kd個維度
2 struct kd_tree{
3 struct point{
4 T d[kd];
5 T dist(const point &x)const{
6 T ret=0;
7 for(size_t i=0;i<kd;++i)ret+=abs(d[i]-x.d[i]);
8 return ret;
9 }
10 bool operator==(const point &p){
11 for(size_t i=0;i<kd;++i)
12 if(d[i]!=p.d[i])return 0;
13 return 1;
14 }
15 bool operator<(const point &b)const{
16 return d[0]<b.d[0];
17 }
18 };
19 private:
20 struct node{
21 node *l,*r;
22 point pid;
23 int s;
24 node(const point &p):l(0),r(0),pid(p),s(1){}
25 ~node(){delete l;delete r;}
26 void up(){s=(l?l->s:0)+1+(r?r->s:0);}
27 }*root;
28 const double alpha,loga;
29 const T INF; //記得要給INF · 表示極大值
30 int maxn;
31 struct __cmp{
32 int sort_id;
33 bool operator()(const node*x,const node*y)const{
34 return operator()(x->pid,y->pid);
35 }
36 bool operator()(const point &x,const point &y)const{
37 if(x.d[sort_id]!=y.d[sort_id])
38 return x.d[sort_id]<y.d[sort_id];
39 for(size_t i=0;i<kd;++i)
40 if(x.d[i]!=y.d[i])return x.d[i]<y.d[i];
41 return 0;
42 }
43 }cmp;

```

```

44 int size(node *o){return o?o->s:0;}
45 vector<node*> A;
46 node* build(int k,int l,int r){
47 if(l>r) return 0;
48 if(k==kd) k=0;
49 int mid=(l+r)/2;
50 cmp.sort_id = k;
51 nth_element(A.begin()+l,A.begin()+mid,A.begin()+r+1,cmp);
52 node *ret=A[mid];
53 ret->l = build(k+1,l,mid-1);
54 ret->r = build(k+1,mid+1,r);
55 ret->up();
56 return ret;
57 }
58 bool isbad(node*o){
59 return size(o->l)>alpha*o->s||size(o->r)>alpha*o->s;
60 }
61 void flatten(node *u,typename vector<node*>::iterator &it){
62 if(!u)return;
63 flatten(u->l,it);
64 *it=u;
65 flatten(u->r,++it);
66 }
67 void rebuild(node*&u,int k){
68 if((int)A.size()<u->s)A.resize(u->s);
69 auto it=A.begin();
70 flatten(u,it);
71 u=build(k,0,u->s-1);
72 }
73 bool insert(node*&u,int k,const point &x,int dep){
74 if(!u) return u=new node(x), dep<=0;
75 ++u->s;
76 cmp.sort_id=k;
77 if(insert(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x,dep-1)){
78 if(!isbad(u))return 1;
79 rebuild(u,k);
80 }
81 return 0;
82 }
83 node *findmin(node*o,int k){
84 if(!o)return 0;
85 if(cmp.sort_id==k)return o->l?findmin(o->l,(k+1)%kd):o;
86 node *l=findmin(o->l,(k+1)%kd);
87 node *r=findmin(o->r,(k+1)%kd);
88 if(l&&r)return cmp(l,o)?l:o;
89 if(!l&&r)return cmp(r,o)?r:o;
90 if(l&&r)return o;
91 if(cmp(l,r))return cmp(l,o)?l:o;
92 return cmp(r,o)?r:o;
93 }
94 bool erase(node *&u,int k,const point &x){
95 if(!u)return 0;
96 if(u->pid==x){
97 if(u->r);
98 else if(u->l) u->r=u->l, u->l=0;
99 else return delete(u),u=0, 1;
100 --u->s;
101 cmp.sort_id=k;
102 u->pid=findmin(u->r,(k+1)%kd)->pid;
103 return erase(u->r,(k+1)%kd,u->pid);

```



```

104 }
105 cmp.sort_id=k;
106 if(erase(cmp(x,u->pid)?u->l:u->r,(k+1)%
107     kd,x))
108     return --u->s, 1;
109 return 0;
110 }
111 T heuristic(const T h[])const{
112     T ret=0;
113     for(size_t i=0;i<kd;++i)ret+=h[i];
114     return ret;
115 }
116 int qM;
117 priority_queue<pair<T,point>> pQ;
118 void nearest(node *u,int k,const point &x,
119     T *h,T &mndist){
120     if(u==0||heuristic(h)>=mndist)return;
121     T dist=u->pid.dist(x),old=h[k];
122     /*mndist=std::min(mndist,dist);*/
123     if(dist<mndist){
124         pQ.push(std::make_pair(dist,u->pid));
125         if((int)pQ.size()==qM+1)
126             mndist=pQ.top().first,pQ.pop();
127     }
128     if(x.d[k]<u->pid.d[k]){
129         nearest(u->l,(k+1)%kd,x,h,mndist);
130         h[k] = abs(x.d[k]-u->pid.d[k]);
131         nearest(u->r,(k+1)%kd,x,h,mndist);
132     }else{
133         nearest(u->r,(k+1)%kd,x,h,mndist);
134         h[k] = abs(x.d[k]-u->pid.d[k]);
135         nearest(u->l,(k+1)%kd,x,h,mndist);
136     }
137     h[k]=old;
138 }
139 vector<point>in_range;
140 void range(node *u,int k,const point&mi,
141     const point&ma){
142     if(!u)return;
143     bool is=1;
144     for(int i=0;i<kd;++i)
145         if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid
146             .d[i])
147             { is=0;break; }
148     if(is) in_range.push_back(u->pid);
149     if(mi.d[k]<u->pid.d[k])range(u->l,(k+1)
150         %kd,mi,ma);
151     if(ma.d[k]>u->pid.d[k])range(u->r,(k+1)
152         %kd,mi,ma);
153 }
154 public:
155 kd_tree(const T &INF,double a=0.75):
156     root(0),alpha(a),loga(log2(1.0/a)),INF(INF
157         ),maxn(1){}
158 ~kd_tree(){delete root;}
159 void clear(){delete root,root=0,maxn=1;}
160 void build(int n,const point *p){
161     delete root,A.resize(maxn=n);
162     for(int i=0;i<n;++i)A[i]=new node(p[i]);
163     root=build(0,0,n-1);
164 }
165 void insert(const point &x){
166     insert(root,0,x,__lg(size(root))/loga);
167     if(root->s>maxn)maxn=root->s;
168 }
169 bool erase(const point &p){

```

```

163 bool d=erase(root,0,p);
164 if(root&&root->s<alpha*maxn)rebuild();
165 return d;
166 }
167 void rebuild(){
168     if(root)rebuild(root,0);
169     maxn=root->s;
170 }
171 T nearest(const point &x,int k){
172     qM=k;
173     T mndist=INF,h[kd]={};
174     nearest(root,0,x,h,mndist);
175     mndist=pQ.top().first;
176     pQ = priority_queue<pair<T,point>>();
177     return mndist;//回傳離x第k近的點的距離
178 }
179 const vector<point> &range(const point&mi,
180     const point&ma){
181     in_range.clear();
182     range(root,0,mi,ma);
183     return in_range;//回傳介於mi到ma之間的點
184     vector
185 }
186 int size(){return root?root->s:0;}
187 };

```

2.4 kd_tree_replace_segment

```

1 struct node{//kd樹代替高維線段樹
2     node *l,*r;
3     point pid,mi,ma;
4     int s, data;
5     node(const point &p,int d):l(0),r(0),pid(p
6         ),mi(p),ma(p),s(1),data(d),dmin(d),
7         dmax(d){}
8     void up(){
9         mi=ma=pid;
10        s=1;
11        if(l){
12            for(int i=0;i<kd;++i){
13                mi.d[i]=min(mi.d[i],l->mi.d[i]);
14                ma.d[i]=max(ma.d[i],l->ma.d[i]);
15            }
16            s+=l->s;
17        }
18        if(r){
19            for(int i=0;i<kd;++i){
20                mi.d[i]=min(mi.d[i],r->mi.d[i]);
21                ma.d[i]=max(ma.d[i],r->ma.d[i]);
22            }
23            s+=r->s;
24        }
25    }
26    void up2(){/*其他懶惰標記向上更新*/}
27    void down(){/*其他懶惰標記下推*/}
28 }*root;
29 //檢查區間包含用的函數
30 bool range_include(node *o,const point &L,
31     const point &R){
32     for(int i=0;i<kd;++i)
33         if(L.d[i]>o->ma.d[i]||R.d[i]<o->mi.d[i])
34             return 0;

```

```

31 }//(L,R)區間有和o的區間有交集就回傳true
32 return 1;
33 }
34 bool range_in_range(node *o,const point &L,
35     const point &R){
36     for(int i=0;i<kd;++i){
37         if(L.d[i]>o->mi.d[i]||o->ma.d[i]>R.d[i])
38             return 0;
39     }
40 }//(L,R)區間完全包含o的區間就回傳true
41 return 1;
42 }
43 bool point_in_range(node *o,const point &L,
44     const point &R){
45     for(int i=0;i<kd;++i){
46         if(L.d[i]>o->pid.d[i]||R.d[i]<o->pid.d[i]
47             )return 0;
48     }
49 }//(L,R)區間完全包含o->pid這個點就回傳true
50 return 1;
51 }
52 //單點修改 · 以單點改值為例
53 void update(node *u,const point &x,int data,
54     int k=0){
55     if(!u)return;
56     u->down();
57     if(u->pid==x){
58         u->data=data;
59         u->up2();
60         return;
61     }
62     cmp.sort_id=k;
63     update(cmp(x,u->pid)?u->l:u->r,x,data,(k
64         +1)%kd);
65     u->up2();
66 }
67 //區間修改
68 void update(node *o,const point &L,const
69     point &R,int data){
70     if(!o)return;
71     o->down();
72     if(range_in_range(o,L,R)){
73         //區間懶惰標記修改
74         o->down();
75         return;
76     }
77     if(point_in_range(o,L,R)){
78         //這個點在(L,R)區間 · 但是他的左右子樹不
79         一定在區間中
80         //單點懶惰標記修改
81     }
82     if(o->l&&range_include(o->l,L,R))update(o
83         ->l,L,R,data);
84     if(o->r&&range_include(o->r,L,R))update(o
85         ->r,L,R,data);
86     o->up2();
87 }
88 //區間查詢 · 以總和為例
89 int query(node *o,const point &L,const point
90     &R){
91     if(!o)return 0;
92     o->down();
93     if(range_in_range(o,L,R))return o->sum;
94     int ans=0;
95     if(point_in_range(o,L,R))ans+=o->data;

```

```

83 if(o->l&&range_include(o->l,L,R))ans+=
84     query(o->l,L,R);
85 if(o->r&&range_include(o->r,L,R))ans+=
86     query(o->r,L,R);
87 return ans;
88 }

```

2.5 reference_point

```

1 template<typename T>
2 struct _RefC{
3     T data;
4     int ref;
5     _RefC(const T&d=0):data(d),ref(0){}
6 };
7 template<typename T>
8 struct _rp{
9     _RefC<T> *p;
10    T *operator->(){return &p->data;}
11    T &operator*(){return p->data;}
12    operator _RefC<T>*(){return p;}
13    _rp &operator=(const _rp &t){
14        if(p&&!--p->ref)delete p;
15        p=t.p,p&&+p->ref;
16        return *this;
17    }
18    _rp(_RefC<T> *t=0):p(t){p&&+p->ref;}
19    _rp(const _rp &t):p(t.p){p&&+p->ref;}
20    ~_rp(){if(p&&!--p->ref)delete p;}
21 };
22 template<typename T>
23 inline _rp<T> new_rp(const T&d){
24     return _rp<T>(new _RefC<T>(d));
25 }

```

2.6 skew_heap

```

1 node *merge(node *a,node *b){
2     if(!a||!b) return a?a:b;
3     if(b->data<a->data) swap(a,b);
4     swap(a->l,a->r);
5     a->l=merge(b,a->l);
6     return a;
7 }

```

2.7 undo_disjoint_set

```

1 struct DisjointSet {
2     // save() is like recursive
3     // undo() is like return
4     int n, fa[MXN], sz[MXN];
5     vector<pair<int*,int>> h;
6     vector<int> sp;
7     void init(int tn) {
8         n=tn;
9         for (int i=0; i<n; i++) sz[fa[i]=i]=1;

```

```

10 sp.clear(); h.clear();
11 }
12 void assign(int *k, int v) {
13     h.PB({k, *k});
14     *k=v;
15 }
16 void save() { sp.PB(SZ(h)); }
17 void undo() {
18     assert(!sp.empty());
19     int last=sp.back(); sp.pop_back();
20     while (SZ(h)!=last) {
21         auto x=h.back(); h.pop_back();
22         *x.F=x.S;
23     }
24 }
25 int f(int x) {
26     while (fa[x]!=x) x=fa[x];
27     return x;
28 }
29 void uni(int x, int y) {
30     x=f(x); y=f(y);
31     if (x==y) return;
32     if (sz[x]<sz[y]) swap(x, y);
33     assign(&sz[x], sz[x]+sz[y]);
34     assign(&fa[y], x);
35 }
36 }djs;

```

2.8 整體二分

```

1 void totBS(int L, int R, vector<Item> M){
2     if(Q.empty()) return; //維護全域B陣列
3     if(L==R) 整個M的答案=r, return;
4     int mid = (L+R)/2;
5     vector<Item> mL, mR;
6     do_modify_B_with_divide(mid,M);
7     //讓B陣列在遞迴的時候只會保留[L~mid]的資訊
8     undo_modify_B(mid,M);
9     totBS(L,mid,mL);
10    totBS(mid+1,R,mR);
11 }

```

3 Flow

3.1 dinic

```

1 template<typename T>
2 struct DINIC{
3     static const int MAXN=105;
4     static const T INF=INT_MAX;
5     int n, LV[MAXN], cur[MAXN];
6     struct edge{
7         int v,pre;
8         T cap,r;
9         edge(int v,int pre,T cap):v(v),pre(pre),
10            cap(cap),r(cap){}
11 };

```

```

11 int g[MAXN];
12 vector<edge> e;
13 void init(int _n){
14     memset(g,-1,sizeof(int)*((n=_n)+1));
15     e.clear();
16 }
17 void add_edge(int u,int v,T cap,bool
18     directed=false){
19     e.push_back(edge(v,g[u],cap));
20     g[u]=e.size()-1;
21     e.push_back(edge(u,g[v],directed?0:cap));
22     g[v]=e.size()-1;
23 }
24 int bfs(int s,int t){
25     memset(LV,0,sizeof(int)*(n+1));
26     memcpy(cur,g,sizeof(int)*(n+1));
27     queue<int> q;
28     q.push(s);
29     LV[s]=1;
30     while(q.size()){
31         int u=q.front();q.pop();
32         for(int i=g[u];~i;i=e[i].pre){
33             if(!LV[e[i].v]&&e[i].r){
34                 LV[e[i].v]=LV[u]+1;
35                 q.push(e[i].v);
36                 if(e[i].v==t) return 1;
37             }
38         }
39     }
40     return 0;
41 }
42 T dfs(int u,int t,T CF=INF){
43     if(u==t) return CF;
44     T df;
45     for(int &i=cur[u];~i;i=e[i].pre){
46         if(LV[e[i].v]==LV[u]+1&&e[i].r){
47             if(df=dfs(e[i].v,t,min(CF,e[i].r))){
48                 e[i].r-=df;
49                 e[i^1].r+=df;
50                 return df;
51             }
52         }
53     }
54     return LV[u]=0;
55 }
56 T dinic(int s,int t,bool clean=true){
57     if(clean)for(size_t i=0;i<e.size();++i)
58         e[i].r=e[i].cap;
59     T ans=0, f=0;
60     while(bfs(s,t))while(f=dfs(s,t))ans+=f;
61     return ans;
62 };

```

3.2 Gomory_Hu

```

1 //最小割樹+求任兩點間最小割
2 //0-base, root=0
3 LL e[MAXN][MAXN]; //任兩點間最小割
4 int p[MAXN]; //parent
5 ISAP D; // original graph

```

```

6 void gomory_hu(){
7     fill(p, p+n, 0);
8     fill(e[0], e[n], INF);
9     for( int s = 1; s < n; ++s ) {
10         int t = p[s];
11         ISAP F = D;
12         LL tmp = F.min_cut(s, t);
13         for( int i = 1; i < s; ++i )
14             e[s][i] = e[i][s] = min(tmp, e[t][i]);
15         for( int i = s+1; i <= n; ++i )
16             if( p[i] == t && F.vis[i] ) p[i] = s;
17     }
18 }

```

3.3 ISAP_with_cut

```

1 template<typename T>
2 struct ISAP{
3     static const int MAXN=105;
4     static const T INF=INT_MAX;
5     int n;//點數
6     int d[MAXN],gap[MAXN],cur[MAXN];
7     struct edge{
8         int v,pre;
9         T cap,r;
10        edge(int v,int pre,T cap):v(v),pre(pre),
11           cap(cap),r(cap){}
12 };
13 int g[MAXN];
14 vector<edge> e;
15 void init(int _n){
16     memset(g,-1,sizeof(int)*((n=_n)+1));
17     e.clear();
18 }
19 void add_edge(int u,int v,T cap,bool
20     directed=false){
21     e.push_back(edge(v,g[u],cap));
22     g[u]=e.size()-1;
23     e.push_back(edge(u,g[v],directed?0:cap));
24     g[v]=e.size()-1;
25 }
26 T dfs(int u,int s,int t,T CF=INF){
27     if(u==t) return CF;
28     T tf=CF,df;
29     for(int &i=cur[u];~i;i=e[i].pre){
30         if(e[i].r&&d[u]==d[e[i].v]+1){
31             df=dfs(e[i].v,s,t,min(tf,e[i].r));
32             e[i].r-=df;
33             e[i^1].r+=df;
34             if(!(tf-=df)||d[s]==n) return CF-tf;
35         }
36     }
37     int mh=n;
38     for(int i=cur[u]=g[u];~i;i=e[i].pre){
39         if(e[i].r&&d[e[i].v]<mh)mh=d[e[i].v];
40     }
41     return CF-tf;
42 }
43 T isap(int s,int t,bool clean=true){
44     memset(d,0,sizeof(int)*(n+1));

```

```

45     memset(gap,0,sizeof(int)*(n+1));
46     memcpy(cur,g,sizeof(int)*(n+1));
47     if(clean) for(size_t i=0;i<e.size();++i)
48         e[i].r=e[i].cap;
49     T MF=0;
50     for(gap[0]=n;d[s]<n;)MF+=dfs(s,s,t);
51     return MF;
52 }
53 vector<int> cut_e;//最小割邊集
54 bool vis[MAXN];
55 void dfs_cut(int u){
56     vis[u]=1;//表示u屬於source的最小割集
57     for(int i=g[u];~i;i=e[i].pre)
58         if(e[i].r>0&&!vis[e[i].v])dfs_cut(e[i].v);
59 }
60 T min_cut(int s,int t){
61     T ans=isap(s,t);
62     memset(vis,0,sizeof(bool)*(n+1));
63     dfs_cut(s);
64     for(int u=0;u<n;++u)if(vis[u])
65         for(int i=g[u];~i;i=e[i].pre)
66             if(!vis[e[i].v])cut_e.push_back(i);
67     return ans;
68 }
69 };

```

3.4 MinCostMaxFlow

```

1 template<typename TP>
2 struct MCMF{
3     static const int MAXN=440;
4     static const TP INF=999999999;
5     struct edge{
6         int v,pre;
7         TP r,cost;
8         edge(int v,int pre,TP r,TP cost):v(v),
9            pre(pre),r(r),cost(cost){}
10 };
11 int n,S,T;
12 TP dis[MAXN],PIS,ans;
13 bool vis[MAXN];
14 vector<edge> e;
15 int g[MAXN];
16 void init(int _n){
17     memset(g,-1,sizeof(int)*((n=_n)+1));
18     e.clear();
19 }
20 void add_edge(int u,int v,TP r,TP cost,
21     bool directed=false){
22     e.push_back(edge(v,g[u],r,cost));
23     g[u]=e.size()-1;
24     e.push_back(
25         edge(u,g[v],directed?0:r,-cost));
26     g[v]=e.size()-1;
27 }
28 TP augment(int u,TP CF){
29     if(u==T||!CF) return ans+=PIS*CF,CF;
30     vis[u]=1;
31     TP r=CF,d;
32     for(int i=g[u];~i;i=e[i].pre){
33         if(e[i].r&&e[i].cost&&!vis[e[i].v]){

```

```

32     d=augment(e[i].v,min(r,e[i].r));
33     e[i].r=d;
34     e[i^1].r=d;
35     if(!(r==d))break;
36 }
37 return CF-r;
38 }
39 bool modlabel(){
40     for(int u=0;u<n;++u)dis[u]=INF;
41     static deque<int>q;
42     dis[T]=0,q.push_back(T);
43     while(q.size()){
44         int u=q.front();q.pop_front();
45         TP dt;
46         for(int i=g[u];~i;i=e[i].pre){
47             if(e[i^1].r&&(dt=dis[u]-e[i].cost)<
48                 dis[e[i].v]){
49                 if((dis[e[i].v]=dt)<=dis[q.size()])
50                     q.front():S){
51                         q.push_front(e[i].v);
52                     }else q.push_back(e[i].v);
53             }
54         }
55     }
56     for(int u=0;u<n;++u)
57         for(int i=g[u];~i;i=e[i].pre)
58             e[i].cost+=dis[e[i].v]-dis[u];
59     return PIS+=dis[S], dis[S]<INF;
60 }
61 TP mincost(int s,int t){
62     S=s,T=t;
63     PIS=ans=0;
64     while(modlabel()){
65         do memset(vis,0,sizeof(bool)*(n+1));
66         while(augment(S,INF));
67     }return ans;
68 }

```

4 Graph

4.1 Augmenting_Path

```

1 #define MAXN1 505
2 #define MAXN2 505
3 int n1,n2;//n1個點連向n2個點
4 int match[MAXN2];//屬於n2的點匹配了哪個點
5 vector<int> g[MAXN1];//圖 0-base
6 bool vis[MAXN2];//是否走訪過
7 bool dfs(int u){
8     for(int v:g[u]){
9         if(vis[v]) continue;
10        vis[v]=1;
11        if(match[v]==-1||dfs(match[v]))
12            return match[v]=u, 1;
13    }
14    return 0;
15 }
16 int max_match(){

```

```

17     int ans=0;
18     memset(match,-1,sizeof(int)*n2);
19     for(int i=0;i<n1;++i){
20         memset(vis,0,sizeof(bool)*n2);
21         if(dfs(i)) ++ans;
22     }
23     return ans;
24 }

```

4.2 Augmenting_Path_multiple

```

1 #define MAXN1 1005
2 #define MAXN2 505
3 int n1,n2;
4 //n1個點連向n2個點，其中n2個點可以匹配很多邊
5 vector<int> g[MAXN1];//圖 0-base
6 size_t c[MAXN2];
7 //每個屬於n2點最多可以接受幾條匹配邊
8 vector<int> matchs[MAXN2];
9 //每個屬於n2的點匹配了那些點
10 bool vis[MAXN2];
11 bool dfs(int u){
12     for(int v:g[u]){
13         if(vis[v])continue;
14         vis[v] = 1;
15         if(matchs[v].size()<c[v]){
16             return matchs[v].push_back(u), 1;
17         }else for(size_t j=0;j<matchs[v].size()
18             ;++j){
19             if(dfs(matchs[v][j]))
20                 return matchs[v][j]=u, 1;
21         }
22     }
23     return 0;
24 }
25 int max_match(){
26     for(int i=0;i<n2;++i) matchs[i].clear();
27     int cnt=0;
28     for(int u=0;u<n1;++u){
29         memset(vis,0,sizeof(bool)*n2);
30         if(dfs(u))++cnt;
31     }
32     return cnt;

```

4.3 blossom_matching

```

1 #define MAXN 505
2 int n; //1-base
3 vector<int> g[MAXN];
4 int MH[MAXN]; //output MH
5 int pa[MAXN],st[MAXN],S[MAXN],v[MAXN],t;
6 int lca(int x,int y){
7     for(++t;;swap(x,y)){
8         if(!x) continue;
9         if(v[x]==t) return x;
10        v[x] = t;
11        x = st[pa[MH[x]]];
12    }

```

```

13 }
14 #define qpush(x) q.push(x),S[x]=0
15 void flower(int x,int y,int l,queue<int>&q){
16     while(st[x]!=1){
17         pa[x]=y;
18         if(S[y==MH[x]]==1)qpush(y);
19         st[x]=st[y]=1, x=pa[y];
20     }
21 }
22 bool bfs(int x){
23     iota(st+1, st+n+1, 1);
24     memset(S+1,-1,sizeof(int)*n);
25     queue<int>q; qpush(x);
26     while(q.size()){
27         x=q.front(),q.pop();
28         for(int y:g[x]){
29             if(S[y]==-1){
30                 pa[y]=x,S[y]=1;
31                 if(!MH[y]){
32                     for(int lst;x=y=lst,x=pa[y])
33                         lst=MH[x],MH[x]=y,MH[y]=x;
34                     return 1;
35                 }
36                 qpush(MH[y]);
37             }else if(!S[y]&&st[y]!=st[x]){
38                 int l=lca(y,x);
39                 flower(y,x,l,q),flower(x,y,l,q);
40             }
41         }
42     }
43     return 0;
44 }
45 int blossom(){
46     memset(MH+1,0,sizeof(int)*n);
47     int ans=0;
48     for(int i=1;i<n;++i)
49         if(!MH[i]&&bfs(i)) ++ans;
50     return ans;
51 }

```

4.4 BronKerbosch

```

1 struct maximalCliques{
2     using Set = vector<int>;
3     size_t n; //1-base
4     vector<Set> G;
5     static Set setUnion(const Set &A, const
6         Set &B){
7         Set C(A.size() + B.size());
8         auto it = set_union(A.begin(),A.end(),B.
9             begin(),B.end(),C.begin());
10        C.erase(it, C.end());
11        return C;
12    }
13    static Set setIntersection(const Set &A,
14        const Set &B){
15        Set C(min(A.size(), B.size()));
16        auto it = set_intersection(A.begin(),A.
17            end(),B.begin(),B.end(),C.begin());
18        C.erase(it, C.end());
19        return C;
20    }

```

```

17     static Set setDifference(const Set &A,
18         const Set &B){
19         Set C(min(A.size(), B.size()));
20         auto it = set_difference(A.begin(),A.end
21             (),B.begin(),B.end(),C.begin());
22         C.erase(it, C.end());
23         return C;
24     }
25 void BronKerbosch1(Set R, Set P, Set X){
26     if(P.empty()&&X.empty()){
27         // R form an maximal clique
28         return;
29     }
30     for(auto v: P){
31         BronKerbosch1(setUnion(R,{v}),
32             setIntersection(P,G[v]),
33             setIntersection(X,G[v]));
34         P = setDifference(P,{v});
35         X = setUnion(X,{v});
36     }
37 }
38 void init(int _n){
39     G.clear();
40     G.resize((n = _n) + 1);
41 }
42 void addEdge(int u, int v){
43     G[u].emplace_back(v);
44     G[v].emplace_back(u);
45 }
46 void solve(int n){
47     Set P;
48     for(int i=1;i<n;++i){
49         sort(G[i].begin(), G[i].end());
50         G[i].erase(unique(G[i].begin(), G[i].end()),
51             G[i].end());
52         P.emplace_back(i);
53     }
54     BronKerbosch1({}, P, {});
55 }

```

4.5 graphISO

```

1 const int MAXN=1005,K=30;//K要夠大
2 const long long A=3,B=11,C=2,D=19,P=0
3 xdefaced;
4 long long f[K+1][MAXN];
5 vector<int> g[MAXN],rg[MAXN];
6 int n;
7 void init(){
8     for(int i=0;i<n;++i){
9         f[0][i]=1;
10        g[i].clear(), rg[i].clear();
11    }
12 }
13 void add_edge(int u,int v){
14     g[u].push_back(v), rg[v].push_back(u);
15 }
16 long long point_hash(int u){//O(N)
17     for(int t=1;t<=K;++t){
18         for(int i=0;i<n;++i){
19             f[t][i]=f[t-1][i]*A%P;

```

```

19     for(int j:g[i])f[t][i]=(f[t][i]+f[t
20         -1][j]*B%P)%P;
21     for(int j:rg[i])f[t][i]=(f[t][i]+f[t
22         -1][j]*C%P)%P;
23     if(i==u)f[t][i]+=D; //如果圖太大的話，
24         把這行刪掉，執行一次後f[K]就會是所
25         有點的答案
26     f[t][i]%P;
27 }
28 vector<long long> graph_hash(){
29     vector<long long> ans;
30     for(int i=0;i<n;++i)ans.push_back(
31         point_hash(i)); //O(N^2)
32     sort(ans.begin(),ans.end());
33     return ans;
34 }

```

4.6 KM

```

1 #define MAXN 405
2 #define INF 0x3f3f3f3f3f3f3f3f
3 int n; // 1-base，0表示沒有匹配
4 LL g[MAXN][MAXN]; //input graph
5 int My[MAXN],Mx[MAXN]; //output match
6 LL lx[MAXN],ly[MAXN],pa[MAXN],Sy[MAXN];
7 bool vx[MAXN],vy[MAXN];
8 void augment(int y){
9     for(int x,z;y;z){
10         x=pa[y],z=Mx[x];
11         My[y]=x,Mx[x]=y;
12     }
13 }
14 void bfs(int st){
15     for(int i=1;i<n;++i)
16         Sy[i] = INF, vx[i]=vy[i]=0;
17     queue<int> q; q.push(st);
18     for(;;){
19         while(q.size()){
20             int x=q.front(); q.pop();
21             vx[x]=1;
22             for(int y=1;y<n;++y) if(!vy[y]){
23                 LL t = lx[x]+ly[y]-g[x][y];
24                 if(t==0){
25                     pa[y]=x;
26                     if(!My[y]){augment(y);return;}
27                     vy[y]=1,q.push(My[y]);
28                 }else if(Sy[y]>t) pa[y]=x,Sy[y]=t;
29             }
30         }
31         LL cut = INF;
32         for(int y=1;y<n;++y)
33             if(!vy[y]&&cut>Sy[y]) cut=Sy[y];
34         for(int j=1;j<n;++j){
35             if(vx[j]) lx[j] -= cut;
36             if(vy[j]) ly[j] += cut;
37             else Sy[j] -= cut;
38         }
39         for(int y=1;y<n;++y){
40             if(!vy[y]&&Sy[y]==0){

```

```

41         if(!My[y]){augment(y);return;}
42         vy[y]=1, q.push(My[y]);
43     }
44 }
45 }
46 }
47 LL KM(){
48     memset(My,0,sizeof(int)*(n+1));
49     memset(Mx,0,sizeof(int)*(n+1));
50     memset(ly,0,sizeof(LL)*(n+1));
51     for(int x=1;x<=n;++x){
52         lx[x] = -INF;
53         for(int y=1;y<=n;++y)
54             lx[x] = max(lx[x],g[x][y]);
55     }
56     for(int x=1;x<=n;++x) bfs(x);
57     LL ans = 0;
58     for(int y=1;y<=n;++y) ans+=g[My[y]][y];
59     return ans;
60 }

```

4.7 MaximumClique

```

1 struct MaxClique{
2     static const int MAXN=105;
3     int N,ans;
4     int g[MAXN][MAXN],dp[MAXN],stk[MAXN][MAXN]
5     ];
6     int sol[MAXN],tmp[MAXN]; //sol[0~ans-1]為答
7     案
8     void init(int n){
9         N=n; //0-base
10        memset(g,0,sizeof(g));
11    }
12    void add_edge(int u,int v){
13        g[u][v]=g[v][u]=1;
14    }
15    int dfs(int ns,int dep){
16        if(!ns){
17            if(dep>ans){
18                ans=dep;
19                memcpy(sol,tmp,sizeof tmp);
20                return 1;
21            }else return 0;
22        }
23        for(int i=0;i<n;++i){
24            if(dep+ns-i<=ans)return 0;
25            int u=stk[dep][i],cnt=0;
26            if(dep+dp[u]<=ans)return 0;
27            for(int j=i+1;j<n;++j){
28                int v=stk[dep][j];
29                if(g[u][v])stk[dep+1][cnt++]=v;
30            }
31            tmp[dep]=u;
32            if(dfs(cnt,dep+1))return 1;
33        }
34        return 0;
35    }
36    int clique(){
37        int u,v,ns;
38        for(ans=0,u=N-1;u>=0;--u){
39            for(ns=0,tmp[0]=u,v=u+1;v<N;++v)

```

```

38         if(g[u][v])stk[1][ns++]=v;
39         dfs(ns,1),dp[u]=ans;
40     }
41     return ans;
42 }
43 };

```

4.8 MinimumMeanCycle

```

1 #include<cstdio> //for DBL_MAX
2 int dp[MAXN][MAXN]; // 1-base, O(NM)
3 vector<tuple<int,int,int>> edge;
4 double mmc(int n){ //allow negative weight
5     const int INF=0x3f3f3f3f;
6     for(int t=0;t<n;++t){
7         memset(dp[t+1],0,sizeof(dp[t+1]));
8         for(const auto &e:edge){
9             int u,v,w;
10            tie(u,v,w) = e;
11            dp[t+1][v]=min(dp[t+1][v],dp[t][u]+w);
12        }
13    }
14    double res = DBL_MAX;
15    for(int u=1;u<=n;++u){
16        if(dp[n][u]==INF) continue;
17        double val = -DBL_MAX;
18        for(int t=0;t<n;++t)
19            val=max(val,(dp[n][u]-dp[t][u])*1.0/(n-t));
20        res=min(res,val);
21    }
22    return res;
23 }

```

4.9 Rectilinear_MST

```

1 //平面曼哈頓最小生成樹構造圖(去除非必要邊)
2 #define T int
3 #define INF 0x3f3f3f3f
4 struct point{
5     T x,y;
6     int id; //從0開始編號
7     point(){}
8     T dist(const point &p)const{
9         return abs(x-p.x)+abs(y-p.y);
10    }
11 };
12 bool cmpx(const point &a,const point &b){
13     return a.x<b.x|| (a.x==b.x&&a.y<b.y);
14 }
15 struct edge{
16     int u,v;
17     T cost;
18     edge(int u,int v,T c):u(u),v(v),cost(c){}
19     bool operator<(const edge&e)const{
20         return cost<e.cost;
21     }
22 };
23 struct bit_node{

```

```

24     T mi;
25     int id;
26     bit_node(const T&mi=INF,int id=-1):mi(mi),
27         id(id){}
28 };
29 vector<bit_node> bit;
30 void bit_update(int i,const T&data,int id){
31     for(;;i=i&(-i)){
32         if(data<bit[i].mi)bit[i]=bit_node(data,
33             id);
34     }
35 }
36 int bit_find(int i,int m){
37     bit_node x;
38     for(i<=m;i+=i&(-i)) if(bit[i].mi<x.mi)x=
39         bit[i];
40     return x.id;
41 }
42 vector<edge> build_graph(int n,point p[]){
43     vector<edge> e; //edge for MST
44     for(int dir=0;dir<4;++dir){ //4種座標變換
45         if(dir%2) for(int i=0;i<n;++i) swap(p[i]
46             .x,p[i].y);
47         else if(dir==2) for(int i=0;i<n;++i) p[i]
48             .x=-p[i].x;
49         sort(p,p+n,cmpx);
50         vector<T> ga(n), gb;
51         for(int i=0;i<n;++i)ga[i]=p[i].y-p[i].x;
52         gb=ga, sort(gb.begin(),gb.end());
53         gb.erase(unique(gb.begin(),gb.end()),gb.
54             end());
55         int m=gb.size();
56         bit=vector<bit_node>(m+1);
57         for(int i=n-1;i>=0;--i){
58             int pos=lower_bound(gb.begin(),gb.end
59                 (),ga[i])-gb.begin()+1;
60             int ans=bit_find(pos,m);
61             if(~ans)e.push_back(edge(p[i].id,p[ans]
62                 .id,p[i].dist(p[ans].id)));
63             bit_update(pos,p[i].x+p[i].y,i);
64         }
65     }
66     return e;
67 }

```

4.10 treeISO

```

1 const int MAXN=100005;
2 const long long X=12327,P=0xdefaced;
3 vector<int> g[MAXN];
4 bool vis[MAXN];
5 long long dfs(int u){ //hash ver
6     vis[u]=1;
7     vector<long long> tmp;
8     for(auto v:g[u])if(!vis[v])tmp.pb(dfs(v));
9     if(tmp.empty())return 177;
10    long long ret=4931;
11    sort(tmp.begin(),tmp.end());
12    for(auto v:tmp)ret=((ret*X)^v)%P;
13    return ret;
14 }
15 //-----
16 string dfs(int x,int p){

```



```

17 vector<string> c;
18 for(int y:g[x])
19     if(y!=p)c.emplace_back(dfs(y,x));
20 sort(c.begin(),c.end());
21 string ret("(");
22 for(auto &s:c)ret+=s;
23 ret+=")";
24 return ret;
25 }

```

4.11 一般圖最小權完美匹配

```

1 struct Graph {
2     // Minimum General Weighted Matching (
3     Perfect Match) 0-base
4     static const int MXN = 105;
5     int n, edge[MXN][MXN];
6     int match[MXN],dis[MXN],onstk[MXN];
7     vector<int> stk;
8     void init(int _n) {
9         n = _n;
10        for (int i=0; i<n; i++)
11            for (int j=0; j<n; j++)
12                edge[i][j] = 0;
13    }
14    void add_edge(int u, int v, int w) {
15        edge[u][v] = edge[v][u] = w;
16    }
17    bool SPFA(int u){
18        if (onstk[u]) return true;
19        stk.push_back(u);
20        onstk[u] = 1;
21        for (int v=0; v<n; v++){
22            if (u != v && match[u] != v && !onstk[
23                v]){
24                int m = match[v];
25                if (dis[m] > dis[u] - edge[v][m] +
26                    edge[u][v]){
27                    dis[m] = dis[u] - edge[v][m] +
28                        edge[u][v];
29                    onstk[v] = 1;
30                    stk.push_back(v);
31                    if (SPFA(m)) return true;
32                    stk.pop_back();
33                    onstk[v] = 0;
34                }
35            }
36        }
37        onstk[u] = 0;
38        stk.pop_back();
39        return false;
40    }
41    int solve() {
42        // find a match
43        for (int i=0; i<n; i+=2){
44            match[i] = i+1, match[i+1] = i;
45        }
46        for(;;){
47            int found = 0;
48            for (int i=0; i<n; i++) dis[i] = onstk
49                [i] = 0;
50            for (int i=0; i<n; i++){
51                stk.clear();

```

```

47         if (!onstk[i] && SPFA(i)){
48             found = 1;
49             while (stk.size()>2){
50                 int u = stk.back(); stk.pop_back
51                     ();
52                 int v = stk.back(); stk.pop_back
53                     ();
54                 match[u] = v;
55                 match[v] = u;
56             }
57             if (!found) break;
58         }
59         int ret = 0;
60         for (int i=0; i<n; i++)
61             ret += edge[i][match[i]];
62         ret /= 2;
63         return ret;
64     }
65 }graph;

```

4.12 全局最小割

```

1 const int INF=0x3f3f3f3f;
2 template<typename T>
3 struct stoer_wagner{// 0-base
4     static const int MAXN=150;
5     T g[MAXN][MAXN],dis[MAXN];
6     int nd[MAXN],n,s,t;
7     void init(int _n){
8         n=_n;
9         for(int i=0;i<n;++i)
10             for(int j=0;j<n;++j)g[i][j]=0;
11    }
12    void add_edge(int u,int v,T w){
13        g[u][v]=g[v][u]+=w;
14    }
15    T min_cut(){
16        T ans=INF;
17        for(int i=0;i<n;++i)nd[i]=i;
18        for(int tn=n;tn>1;--tn){
19            for(int i=1;i<tn;++i)dis[nd[i]]=0;
20            for(int i=1;i<tn;++i){
21                ind=i;
22                for(int j=i;j<tn;++j){
23                    dis[nd[j]]+=g[nd[i-1]][nd[j]];
24                    if(dis[nd[ind]]<dis[nd[j]])ind=j;
25                }
26                swap(nd[ind],nd[i]);
27            }
28            if(ans>dis[nd[ind]])ans=dis[t=nd[ind
29                ]],s=nd[ind-1];
30            for(int i=0;i<tn;++i)
31                g[nd[ind-1]][nd[i]]=g[nd[i]][nd[ind
32                    -1]]+=g[nd[i]][nd[ind]];
33        }
34        return ans;
35    }
36 }

```

4.13 弦圖完美消除序列

```

1 struct chordal{
2     static const int MAXN=1005;
3     int n;// 0-base
4     vector<int>G[MAXN];
5     int rank[MAXN],label[MAXN];
6     bool mark[MAXN];
7     void init(int _n){n=_n;
8         for(int i=0;i<n;++i)G[i].clear();
9     }
10    void add_edge(int u,int v){
11        G[u].push_back(v);
12        G[v].push_back(u);
13    }
14    vector<int> MCS(){
15        memset(rank,-1,sizeof(int)*n);
16        memset(label,0,sizeof(int)*n);
17        priority_queue<pair<int,int>> pq;
18        for(int i=0;i<n;++i)pq.push(make_pair(0,
19            i));
20        for(int i=n-1;i>=0;--i)for(;;){
21            int u=pq.top().second;pq.pop();
22            if(~rank[u])continue;
23            rank[u]=i;
24            for(auto v:G[u])if(rank[v]==-1){
25                pq.push(make_pair(++label[v],v));
26            }
27            break;
28        }
29        vector<int> res(n);
30        for(int i=0;i<n;++i)res[rank[i]]=i;
31        return res;
32    }
33    bool check(vector<int> ord){//弦圖判定
34        for(int i=0;i<n;++i)rank[ord[i]]=i;
35        memset(mark,0,sizeof(bool)*n);
36        for(int i=0;i<n;++i){
37            vector<pair<int,int>> tmp;
38            for(auto u:G[ord[i]])if(!mark[u])
39                tmp.push_back(make_pair(rank[u],u));
40            sort(tmp.begin(),tmp.end());
41            if(tmp.size()){
42                int u=tmp[0].second;
43                set<int> S;
44                for(auto v:G[u])S.insert(v);
45                for(size_t j=1;j<tmp.size();++j)
46                    if(!S.count(tmp[j].second))return
47                        0;
48            }
49            mark[ord[i]]=1;
50        }
51        return 1;
52    }
53 }

```

4.14 最小斯坦納樹 DP

```

1 //n個點，其中r個要構成斯坦納樹
2 //答案在max(dp[(1<r)-1][k]) k=0~n-1
3 //p表示要構成斯坦納樹的點集

```

```

4 //O( n^3 + n*3^r + n^2*2^r )
5 #define REP(i,n) for(int i=0;i<(int)n;++i)
6 const int MAXN=30,MAXM=8;// 0-base
7 const int INF=0x3f3f3f3f;
8 int dp[1<MAXM][MAXN];
9 int g[MAXN][MAXN];
10 void init(){memset(g,0,sizeof(g));}
11 void add_edge(int u,int v,int w){
12     g[u][v]=g[v][u]=min(g[v][u],w);
13 }
14 void steiner(int n,int r,int *p){
15     REP(k,n)REP(i,n)REP(j,n)
16         g[i][j]=min(g[i][j],g[i][k]+g[k][j]);
17     REP(i,n)g[i][i]=0;
18     REP(i,r)REP(j,n)dp[1<i][j]=g[p[i]][j];
19     for(int i=1;i<(1<r);++i){
20         if(!(i&(i-1)))continue;
21         REP(j,n)dp[i][j]=INF;
22         REP(j,n){
23             int tmp=INF;
24             for(int s=i&(i-1);s=s+(s-1))
25                 tmp=min(tmp,dp[s][j]+dp[i^s][j]);
26             REP(k,n)dp[i][k]=min(dp[i][k],g[j][k]+
27                 tmp);
28         }
29     }
30 }

```

4.15 最小樹形圖 — 朱劉

```

1 template<typename T>
2 struct zhu_liu{
3     static const int MAXN=110,MAXM=10005;
4     struct node{
5         int u,v;
6         T w>tag;
7         node *l,*r;
8         node(int u=0,int v=0,T w=0):u(u),v(v),w(
9             w),tag(0),l(0),r(0){}
10        void down(){
11            w+=tag;
12            if(l)l->tag+=tag;
13            if(r)r->tag+=tag;
14            tag=0;
15        }
16    }mem[MAXN];
17    node *pq[MAXN*2],*E[MAXN*2];
18    int st[MAXN*2],id[MAXN*2],m;
19    void init(int n){
20        for(int i=1;i<n;++i){
21            pq[i]=E[i]=0, st[i]=id[i]=i;
22            m=0;
23        }
24        node *merge(node *a,node *b){//skew heap
25            if(!a||!b)return a?b:
26                a->down(),b->down();
27            if(b->w<a->w)return merge(b,a);
28            swap(a->l,a->r);
29            a->l=merge(b,a->l);
30            return a;
31        }
32        void add_edge(int u,int v,T w){

```

```

32 if(u!=v)pq[v]=merge(pq[v],&mem[m++]=
33     node(u,v,w));
34 }
35 int find(int x,int *st){
36     return st[x]==x?x:st[x]=find(st[x],st);
37 }
38 T build(int root,int n){
39     T ans=0;int N=n,all=n;
40     for(int i=1;i<=N;++i){
41         if(i==root||!pq[i])continue;
42         while(pq[i]){
43             pq[i]->down(),E[i]=pq[i];
44             pq[i]=merge(pq[i]->l,pq[i]->r);
45             if(find(E[i]->u,id)!=find(i,id))
46                 break;
47         }
48         if(find(E[i]->u,id)==find(i,id))
49             continue;
50         ans+=E[i]->w;
51         if(find(E[i]->u,st)==find(i,st)){
52             if(pq[i]pq[i]->tag==E[i]->w;
53             pq[++N]=pq[i];id[N]=N;
54             for(int u=find(E[i]->u,id);u!=i;u=
55                 find(E[u]->u,id)){
56                 if(pq[u]pq[u]->tag==E[u]->w;
57                 id[find(u,id)]=N;
58                 pq[N]=merge(pq[N],pq[u]);
59             }
60             st[N]=find(i,st);
61             id[find(i,id)]=N;
62             }else st[find(i,st)]=find(E[i]->u,st)
63             ,--all;
64         }
65     }
66     return all==1?ans:-INT_MAX;//圖不連通就
67     無解
68 }
69 }

```

4.16 穩定婚姻模板

```

1 queue<int> Q;
2 for ( i : 所有考生 ) {
3     設定在第0志願;
4     Q.push(考生i);
5 }
6 while(Q.size()){
7     當前考生=Q.front();Q.pop();
8     while ( 此考生未分發 ) {
9         指標移到下一志願;
10        if ( 已經沒有志願 or 超出志願總數 )
11            break;
12        計算該考生在該科系加權後的總分;
13        if ( 不符合科系需求 ) continue;
14        if ( 目前科系有餘額 ) {
15            依加權後分數高低順序將考生id加入科系錄
16            取名單中;
17            break;
18        }
19        if ( 目前科系已額滿 ) {
20            if ( 此考生成績比最低分數還高 ) {

```

```

19        依加權後分數高低順序將考生id加入科系
20        錄取名單;
21        Q.push(被踢出的考生);
22    }
23 }
24 }

```

5 Linear_Programming

5.1 simplex

```

1 /*target:
2 max \sum_{j=1}^n A_{0,j}*x_j
3 condition:
4 \sum_{j=1}^n A_{i,j}*x_j <= A_{i,0} | i=1~m
5 x_j >= 0 | j=1~n
6 VDB = vector<double>*/
7 template<class VDB>
8 VDB simplex(int m,int n,vector<VDB> a){
9     vector<int> left(m+1), up(n+1);
10    iota(left.begin(), left.end(), n);
11    iota(up.begin(), up.end(), 0);
12    auto pivot = [&](int x, int y){
13        swap(left[x], up[y]);
14        auto k = a[x][y]; a[x][y] = 1;
15        vector<int> pos;
16        for(int j = 0; j <= n; ++j){
17            a[x][j] /= k;
18            if(a[x][j] != 0) pos.push_back(j);
19        }
20        for(int i = 0; i <= m; ++i){
21            if(a[i][y]==0 || i == x) continue;
22            k = a[i][y], a[i][y] = 0;
23            for(int j : pos) a[i][j] -= k*a[x][j];
24        }
25    };
26    for(int x,y;){
27        for(int i=x+1; i <= m; ++i)
28            if(a[i][0]<a[x][0]) x = i;
29        if(a[x][0]>=0) break;
30        for(int j=y+1; j <= n; ++j)
31            if(a[x][j]<a[x][y]) y = j;
32        if(a[x][y]>=0) return VDB();//infeasible
33        pivot(x, y);
34    }
35    for(int x,y;){
36        for(int j=y+1; j <= n; ++j)
37            if(a[0][j] > a[0][y]) y = j;
38        if(a[0][y]<=0) break;
39        x = -1;
40        for(int i=1; i<=m; ++i) if(a[i][y] > 0)
41            if(x == -1 || a[i][0]/a[i][y]
42                < a[x][0]/a[x][y]) x = i;
43        if(x == -1) return VDB();//unbounded
44        pivot(x, y);
45    }
46    VDB ans(n + 1);
47    for(int i = 1; i <= m; ++i)
48        if(left[i] <= n) ans[left[i]] = a[i][0];

```

```

49 ans[0] = -a[0][0];
50 return ans;
51 }

```

6 Number_Theory

6.1 basic

```

1 template<typename T>
2 void gcd(const T &a,const T &b,T &d,T &x,T &
3     y){
4     if(!b) d=a,x=1,y=0;
5     else gcd(b,a%b,d,y,x), y-=x*(a/b);
6 }
7 long long int phi[N+1];
8 void phiTable(){
9     for(int i=1;i<=N;i++)phi[i]=i;
10    for(int i=1;i<=N;i++)for(x=i*2;x<=N;x+=i)
11        phi[x]-=phi[i];
12 }
13 void all_divdown(const LL &n){ // all n/x
14     for(LL a=1;a<=n;a=n/(n/(a+1))) {
15         // dosomething;
16     }
17 }
18 const int MAXPRIME = 1000000;
19 int iscom[MAXPRIME], prime[MAXPRIME],
20     primecnt;
21 int phi[MAXPRIME], mu[MAXPRIME];
22 void sieve(void){
23     memset(iscom,0,sizeof(iscom));
24     primecnt = 0;
25     phi[1] = mu[1] = 1;
26     for(int i=2;i<MAXPRIME;i++) {
27         if(!iscom[i]) {
28             prime[primecnt++] = i;
29             mu[i] = -1;
30             phi[i] = i-1;
31         }
32         for(int j=0;j<primecnt;j++) {
33             int k = i * prime[j];
34             if(k>MAXPRIME) break;
35             iscom[k] = prime[j];
36             if(i%prime[j]==0) {
37                 mu[k] = 0;
38                 phi[k] = phi[i] * prime[j];
39                 break;
40             } else {
41                 mu[k] = -mu[i];
42                 phi[k] = phi[i] * (prime[j]-1);
43             }
44         }
45     }
46 }
47 bool g_test(const LL &g, const LL &p, const
48     vector<LL> &v) {
49     for(int i=0;i<v.size();i++)
50         if(modexp(g,(p-1)/v[i],p)==1)
51             return false;
52     return true;

```

```

53 }
54 LL primitive_root(const LL &p) {
55     if(p==2) return 1;
56     vector<LL> v;
57     Factor(p-1,v);
58     v.erase(unique(v.begin(), v.end()), v.end()
59         ());
60     for(LL g=2;g<p;g++)
61         if(g_test(g,p,v))
62             return g;
63     puts("primitive_root NOT FOUND");
64     return -1;
65 }
66 int Legendre(const LL &a, const LL &p) {
67     return modexp(a%p,(p-1)/2,p); }
68 LL inv(const LL &a, const LL &n) {
69     LL d,x,y;
70     gcd(a,n,d,x,y);
71     return d==1 ? (x+n)%n : -1;
72 }
73 int inv[maxn];
74 LL invtable(int n,LL P){
75     inv[1]=1;
76     for(int i=2;i<n;i++)
77         inv[i]=(P-(P/i))*inv[P%i]%P;
78 }
79 LL log_mod(const LL &a, const LL &b, const
80     LL &p) {
81     // a ^ x = b ( mod p )
82     int m=sqrt(p+.5), e=1;
83     LL v=inv(modexp(a,m,p), p);
84     map<LL,int> x;
85     x[1]=0;
86     for(int i=1;i<m;i++) {
87         e = LLMul(e,a,p);
88         if(!x.count(e)) x[e] = i;
89     }
90     for(int i=0;i<m;i++) {
91         if(x.count(b)) return i*m + x[b];
92         b = LLMul(b,v,p);
93     }
94     return -1;
95 }
96 LL Tonelli_Shanks(const LL &n, const LL &p)
97 {
98     // x^2 = n ( mod p )
99     if(n==0) return 0;
100    if(Legendre(n,p)!=1) while(1) { puts("SQRT
101        ROOT does not exist"); }
102    int S = 0;
103    LL Q = p-1;
104    while( !(Q&1) ) { Q>>=1; ++S; }
105    if(S==1) return modexp(n%p,(p+1)/4,p);
106    LL z = 2;
107    for(;Legendre(z,p)!=-1; z++)
108        LL c = modexp(z,Q,p);
109    LL R = modexp(n%p,(Q+1)/2,p), t = modexp(n
110        %p,Q,p);
111    int M = S;
112    while(1) {
113        if(t==1) return R;
114        LL b = modexp(c,1L<<(M-i-1),p);

```

```

110 R = LLMul(R,b,p);
111 t = LLMul( LLMul(b,b,p), t, p);
112 c = LLMul(b,b,p);
113 M = i;
114 }
115 return -1;
116 }
117
118 template<typename T>
119 T Euler(T n){
120     T ans=n;
121     for(T i=2;i<=n;++i){
122         if(n%i==0){
123             ans=ans/i*(i-1);
124             while(n%i==0)n/=i;
125         }
126     }
127     if(n>1)ans=ans/n*(n-1);
128     return ans;
129 }
130
131 //Chinese_remainder_theorem
132 template<typename T>
133 T pow_mod(T n,T k,T m){
134     T ans=1;
135     for(n=(n>m?n%m:n);k;k>>=1){
136         if(k&1)ans=ans*n%m;
137         n=n*n%m;
138     }
139     return ans;
140 }
141
142 template<typename T>
143 T crt(vector<T> &m,vector<T> &a){
144     T M=1,tM,ans=0;
145     for(int i=0;i<(int)m.size();++i)M*=m[i];
146     for(int i=0;i<(int)a.size();++i){
147         tM=M/m[i];
148         ans=(ans+(a[i]*tM%M)*pow_mod(tM,Euler(m[i])-1,m[i])%M)%M;
149         /*如果m[i]是質數·Euler(m[i])-1=m[i]-2·
150         就不用算Euler了*/
151     }
152     return ans;
153 }
154
155 //java code
156 //求sqrt(N)的連分數
157 public static void Pell(int n){
158     BigInteger N,p1,p2,q1,q2,a0,a1,a2,g1,g2,h1
159         ,h2,p,q;
160     g1=q2=p1=BigInteger.ZERO;
161     h1=q1=p2=BigInteger.ONE;
162     a0=a1=BigInteger.valueOf((int)Math.sqrt
163         (1.0*n));
164     BigInteger ans=a0.multiply(a0);
165     if(ans.equals(BigInteger.valueOf(n))){
166         System.out.println("No solution!");
167         return ;
168     }
169     while(true){
170         g2=a1.multiply(h1).subtract(g1);
171         h2=N.subtract(g2.pow(2)).divide(h1);
172         a2=g2.add(a0).divide(h2);
173         p=a1.multiply(p2).add(p1);
174         q=a1.multiply(q2).add(q1);

```

```

171     if(p.pow(2).subtract(N.multiply(q.pow
172         (2))).compareTo(BigInteger.ONE)==0)
173         break;
174     g1=g2;h1=h2;a1=a2;
175     p1=p2;p2=p;
176     q1=q2;q2=q;
177 }
178
179 System.out.println(p+" "+q);
180 }

```

6.2 bit_set

```

1 void sub_set(int S){
2     int sub=S;
3     do{
4         //對某集合的子集合的處理
5         sub=(sub-1)&S;
6     }while(sub!=S);
7 }
8
9 void k_sub_set(int k,int n){
10    int comb=(1<<k)-1,S=1<<n;
11    while(comb<S){
12        //對大小為k的子集合的處理
13        int x=comb&-comb,y=comb+x;
14        comb=((comb&y)/x>>1)|y;
15    }
16 }

```

6.3 cantor_expansion

```

1 int factorial[MAXN];
2 void init(){
3     factorial[0]=1;
4     for(int i=1;i<=MAXN;++i)factorial[i]=
5         factorial[i-1]*i;
6 }
7
8 int encode(const vector<int> &s){
9     int n=s.size(),res=0;
10    for(int i=0;i<n;++i){
11        int t=0;
12        for(int j=i+1;j<n;++j)
13            if(s[j]<s[i])++t;
14        res+=t*factorial[n-i-1];
15    }
16    return res;
17 }
18
19 vector<int> decode(int a,int n){
20     vector<int> res;
21     vector<bool> vis(n,0);
22     for(int i=n-1;i>=0;--i){
23         int t=a/factorial[i];j;
24         for(j=0;j<n;++j)
25             if(!vis[j]){
26                 if(t==0)break;
27                 --t;
28             }
29         res.push_back(j);
30         vis[j]=1;
31         a%=factorial[i];

```

```

29 }
30 return res;
31 }

```

6.4 FFT

```

1 template<typename T,typename VT=vector<
2     complex<T> >>
3 struct FFT{
4     const T pi;
5     FFT(const T pi=acos((T)-1)):pi(pi){}
6     unsigned bit_reverse(unsigned a,int len){
7         a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)>>1);
8         a=((a&0x33333333U)<<2)|((a&0xCCCCCCCCU)>>2);
9         a=((a&0x0F0F0F0FU)<<4)|((a&0xFF0F0F0FU)>>4);
10        a=((a&0x00FF00FFU)<<8)|((a&0xFFFF0000U)>>8);
11        a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)>>16);
12        return a>>(32-len);
13    }
14    void fft(bool is_inv,VT &in,VT &out,int N)
15    {
16        int bitlen=__lg(N),num=is_inv?-1:1;
17        for(int i=0;i<N;++i)out[bit_reverse(i,
18            bitlen)]=in[i];
19        for(int step=2;step<=N;step<<=1){
20            const int mh=step>>1;
21            for(int i=0;i<mh;++i){
22                complex<T> wi=exp(complex<T>(0,i*num
23                    *pi/mh));
24                for(int j=i;j<N;j+=step){
25                    int k=j+mh;
26                    complex<T> u=out[j],t=wi*out[k];
27                    out[j]=u+t;
28                    out[k]=u-t;
29                }
30            }
31        }
32        if(is_inv)for(int i=0;i<N;++i)out[i]/=N;
33    }
34 }

```

6.5 find_real_root

```

1 // a*x^n + ... + a1x + a0 = 0;
2 int sign(double x){
3     return x < -eps ? -1 : x > eps;
4 }
5
6 double get(const vector<double>&coef, double
7     x){
8     double e = 1, s = 0;
9     for(auto i : coef) s += i*e, e *= x;
10    return s;
11 }
12
13 double find(const vector<double>&coef, int n
14     , double lo, double hi){
15     double sign_lo, sign_hi;

```

```

14 if( !(sign_lo = sign(get(coef,lo))) )
15     return lo;
16 if( !(sign_hi = sign(get(coef,hi))) )
17     return hi;
18 if(sign_lo * sign_hi > 0) return INF;
19 for(int stp = 0; stp < 100 && hi - lo >
20     eps; ++stp){
21     double m = (lo+hi)/2.0;
22     int sign_mid = sign(get(coef,m));
23     if(!sign_mid) return m;
24     if(sign_lo*sign_mid < 0) hi = m;
25     else lo = m;
26 }
27 return (lo+hi)/2.0;
28 }
29
30 vector<double> cal(vector<double>coef, int n
31 )){
32     vector<double>res;
33     if(n == 1){
34         if(sign(coef[1])) res.pb(-coef[0]/coef
35             [1]);
36         return res;
37     }
38     vector<double>dcoef(n);
39     for(int i = 0; i < n; ++i) dcoef[i] = coef
40         [i+1]*(i+1);
41     vector<double>droot = cal(dcoef, n-1);
42     droot.insert(droot.begin(), -INF);
43     droot.pb(INF);
44     for(int i = 0; i+1 < droot.size(); ++i){
45         double tmp = find(coef, n, droot[i],
46             droot[i+1]);
47         if(tmp < INF) res.pb(tmp);
48     }
49     return res;
50 }
51
52 int main () {
53     vector<double>ve;
54     vector<double>ans = cal(ve, n);
55     // 視情況把答案 +eps · 避免 -0
56 }

```

6.6 FWT

```

1 vector<int> F_OR_T(vector<int> f, bool
2     inverse){
3     for(int i=0; (2<<i)<=f.size(); ++i)
4         for(int j=0; j<f.size(); j+=2<<i)
5             f[j+k*(1<<i)] += f[j+k]*(inverse
6                 ?-1:1);
7     return f;
8 }
9
10 vector<int> rev(vector<int> A) {
11     for(int i=0; i<A.size(); i+=2)
12         swap(A[i],A[i^(A.size()-1)]);
13     return A;
14 }
15
16 vector<int> F_AND_T(vector<int> f, bool
17     inverse){
18     return rev(F_OR_T(rev(f), inverse));
19 }

```

```

15 }
16 vector<int> F_XOR_T(vector<int> f, bool
    inverse){
17     for(int i=0; (2<<i)<=f.size(); ++i)
18         for(int j=0; j<f.size(); j+=2<<i)
19             for(int k=0; k<(1<<i); ++k){
20                 int u=f[j+k], v=f[j+k+(1<<i)];
21                 f[j+k+(1<<i)] = u-v, f[j+k] = u+v;
22             }
23     if(inverse) for(auto &a:f) a/=f.size();
24     return f;
25 }

```

6.9 Matrix

6.7 LinearCongruence

```

1 pair<LL,LL> LinearCongruence(LL a[],LL b[],
    LL m[],int n) {
2     // a[i]*x = b[i] ( mod m[i] )
3     for(int i=0;i<n;++i) {
4         LL x, y, d = extgcd(a[i],m[i],x,y);
5         if(b[i]%d!=0) return make_pair(-1LL,0LL);
6         m[i] /= d;
7         b[i] = LLmul(b[i]/d,x,m[i]);
8     }
9     LL lastb = b[0], lastm = m[0];
10    for(int i=1;i<n;++i) {
11        LL x, y, d = extgcd(m[i],lastm,x,y);
12        if((lastb-b[i])%d!=0) return make_pair
            (-1LL,0LL);
13        lastb = LLmul((lastb-b[i])/d,x,(lastm/d)
            )*m[i];
14        lastm = (lastm/d)*m[i];
15        lastb = (lastb+b[i])%lastm;
16    }
17    return make_pair(lastb<0?lastb+lastm:lastb
        ,lastm);
18 }

```

6.8 Lucas

```

1 ll C(ll n, ll m, ll p){// n!/m!/(n-m)!
2     if(n<m) return 0;
3     return f[n]*inv(f[m],p)%p*inv(f[n-m],p)%p;
4 }
5 ll L(ll n, ll m, ll p){
6     if(!m) return 1;
7     return C(n%p,m%p,p)*L(n/p,m/p,p)%p;
8 }
9 ll Wilson(ll n, ll p){ // n!%p
10    if(!n)return 1;
11    ll res=Wilson(n/p, p);
12    if((n/p)%2) return res*(p-f[n%p])%p;
13    return res*f[n%p]%p; //(p-1)!%p=-1
14 }

```

```

1 template<typename T>
2 struct Matrix{
3     using rt = std::vector<T>;
4     using mt = std::vector<rt>;
5     using matrix = Matrix<T>;
6     int r,c;
7     mt m;
8     Matrix(int r,int c):r(r),c(c),m(r,rt(c)){
9         rt& operator[](int i){return m[i];}
10    matrix operator+(const matrix &a){
11        matrix rev(r,c);
12        for(int i=0;i<r;++i)
13            for(int j=0;j<c;++j)
14                rev[i][j]=m[i][j]+a.m[i][j];
15        return rev;
16    }
17    matrix operator-(const matrix &a){
18        matrix rev(r,c);
19        for(int i=0;i<r;++i)
20            for(int j=0;j<c;++j)
21                rev[i][j]=m[i][j]-a.m[i][j];
22        return rev;
23    }
24    matrix operator*(const matrix &a){
25        matrix rev(r,a.c);
26        matrix tmp(a.c,a.r);
27        for(int i=0;i<a.r;++i)
28            for(int j=0;j<a.c;++j)
29                tmp[j][i]=a.m[i][j];
30        for(int i=0;i<r;++i)
31            for(int j=0;j<a.c;++j)
32                for(int k=0;k<c;++k)
33                    rev.m[i][j]+=m[i][k]*tmp[j][k];
34        return rev;
35    }
36    bool inverse(){
37        Matrix t(r,r+c);
38        for(int y=0;y<r;y++){
39            t.m[y][c+y] = 1;
40            for(int x=0;x<c;++x)
41                t.m[y][x]=m[y][x];
42        }
43        if(!t.gas())
44            return false;
45        for(int y=0;y<r;y++){
46            for(int x=0;x<c;++x)
47                m[y][x]=t.m[y][c+x]/t.m[y][y];
48        return true;
49    }
50    T gas(){
51        vector<T> lazy(r,1);
52        bool sign=false;
53        for(int i=0;i<r;++i){
54            if(m[i][i]==0){
55                int j=i+1;
56                while(j<r&&!m[j][i])j++;
57                if(j==r)continue;
58                m[i].swap(m[j]);
59                sign=!sign;
60            }
61            for(int j=0;j<r;++j){
62                if(i==j)continue;
63                lazy[j]=lazy[j]*m[i][i];

```

```

64        T mx=m[j][i];
65        for(int k=0;k<c;++k)
66            m[j][k]=m[j][k]*m[i][i]-m[i][k]*mx;
67    }
68    T det=sign?-1:1;
69    for(int i=0;i<r;++i){
70        det = det*m[i][i];
71        det = det/lazy[i];
72        for(auto &j:m[i])j/=lazy[i];
73    }
74    return det;
75 }
76 }
77 };

```

6.10 MillerRobin

```

1 LL LLMul(LL a, LL b, const LL &mod) {
2     LL ans=0;
3     while(b) {
4         if(b&1) {
5             ans+=a;
6             if(ans>=mod) ans-=mod;
7         }
8         a<<=1, b>>=1;
9         if(a>=mod) a-=mod;
10    }
11    return ans;
12 }
13 LL mod_mul(LL a,LL b,LL m){
14    a%=m,b%=m;/* fast for m < 2^58 */
15    LL y=(LL)((double)a*b/m+0.5);
16    LL r=(a*b-y*m)%m;
17    return r<0?r+m:r;
18 }
19 template<typename T>
20 T pow(T a,T b,T mod){//a^b%mod
21    T ans=1;
22    for(;b;a=mod_mul(a,a,mod),b>>=1)
23        if(b&1)ans=mod_mul(ans,a,mod);
24    return ans;
25 }
26 int sprp[3]={2,7,61};//int範圍可解
27 int llsprp
    [7]={2,325,9375,28178,450775,9780504,
28    1795265022};//至少 unsigned long long範圍
29 template<typename T>
30 bool isprime(T n,int *sprp,int num){
31    if(n==2)return 1;
32    if(n<2||n%2==0)return 0;
33    int t=0;
34    T u=n-1;
35    for(;u%2==0;++t)u>>=1;
36    for(int i=0;i<num;++i){
37        T a=sprp[i]%n;
38        if(a==0||a==1||a==n-1)continue;
39        T x=pow(a,u,n);
40        if(x==1||x==n-1)continue;
41        for(int j=0;j<t;++j){
42            x=mod_mul(x,x,n);
43            if(x==1)return 0;

```

```

44        if(x==n-1)break;
45    }
46    if(x==n-1)continue;
47    return 0;
48 }
49 return 1;
50 }

```

6.11 NTT

```

1 2615053605667*(2^18)+1,3
2 15*(2^27)+1,31
3 479*(2^21)+1,3
4 7*17*(2^23)+1,3
5 3*3*211*(2^19)+1,5
6 25*(2^22)+1,3
7 template<typename T,typename VT=vector<T> >
8 struct NTT{
9     const T P,G;
10    NTT(T p=(1<<23)*7*17+1,T g=3):P(p),G(g){}
11    unsigned bit_reverse(unsigned a,int len){
12        //look FFT.cpp
13    }
14    T pow_mod(T n,T k,T m){
15        T ans=1;
16        for(n=(n>=m?n%m:n);k>>=1){
17            if(k&1)ans=ans*n%m;
18            n=n*n%m;
19        }
20        return ans;
21 }
22 void ntt(bool is_inv,VT &in,VT &out,int N)
    {
23     int bitlen=__lg(N);
24     for(int i=0;i<N;++i)out[bit_reverse(i,
        bitlen)]=in[i];
25     for(int step=2,id=1;step<=N;step<=1,++
        id){
26         T wn=pow_mod(G,(P-1)>>id,P),wi=1,u,t;
27         const int mh=step>>1;
28         for(int i=0;i<N;i++){
29             for(int j=i;j<N;j+=step){
30                 u=out[j],t=wi*out[j+mh]%P;
31                 out[j]=u+t;
32                 out[j+mh]=u-t;
33                 if(out[j]>=P)out[j]-=P;
34                 if(out[j+mh]<0)out[j+mh]+=P;
35             }
36             wi=wi*wn%P;
37         }
38     }
39     if(is_inv){
40         for(int i=1;i<N/2;++i)swap(out[i],out[
            N-i]);
41         T invn=pow_mod(N,P-2,P);
42         for(int i=0;i<N;++i)out[i]=out[i]*invn
            %P;
43     }
44 }
45 };

```


6.12 Simpson

```

1 double simpson(double a,double b){
2     double c=a+(b-a)/2;
3     return (F(a)+4*F(c)+F(b))*(b-a)/6;
4 }
5 double asr(double a,double b,double eps,
6     double A){
7     double c=a+(b-a)/2;
8     double L=simpson(a,c),R=simpson(c,b);
9     if( abs(L+R-A)<15*eps )
10        return L+R+(L+R-A)/15.0;
11    return asr(a,c,eps/2,L)+asr(c,b,eps/2,R);
12 }
13 double asr(double a,double b,double eps){
14    return asr(a,b,eps,simpson(a,b));
15 }

```

6.13 外星模運算

```

1 //a[0]^a[1]^a[2]^...
2 #define maxn 1000000
3 int euler[maxn+5];
4 bool is_prime[maxn+5];
5 void init_euler(){
6     is_prime[1]=1; // --不是質數
7     for(int i=1;i<=maxn;i++)euler[i]=i;
8     for(int i=2;i<=maxn;i++){
9         if(!is_prime[i]){ //是質數
10             euler[i]--;
11             for(int j=i<<1;j<=maxn;j+=i){
12                 is_prime[j]=1;
13                 euler[j]=euler[j]/i*(i-1);
14             }
15         }
16     }
17 }
18 LL pow(LL a,LL b,LL mod){ //a^b%mod
19     LL ans=1;
20     for(;b;a=a%mod,b>>=1)
21         if(b&1)ans=ans*a%mod;
22     return ans;
23 }
24 bool isless(LL *a,int n,int k){
25     if(*a==1)return k<1;
26     if(--n==0)return *a<k;
27     int next=0;
28     for(LL b=1;b<k;next++)
29         b*=*a;
30     return isless(a+1,n,next);
31 }
32 LL high_pow(LL *a,int n,LL mod){
33     if(*a==1||--n==0)return *a%mod;
34     int k=0,r=euler[mod];
35     for(LL tma=1;tma!=pow(*a,k+r,mod);++k)
36         tma=tma*(*a)%mod;
37     if(isless(a+1,n,k))return pow(*a,high_pow(
38         a+1,n,k),mod);
39     int tmd=high_pow(a+1,n,r), t=(tmd-k+r)%r;
40     return pow(*a,k+t,mod);
41 }

```

```

41 LL a[1000005];
42 int t,mod;
43 int main(){
44     init_euler();
45     scanf("%d",&t);
46     #define n 4
47     while(t--){
48         for(int i=0;i<n;++i)scanf("%lld",&a[i]);
49         scanf("%d",&mod);
50         printf("%lld\n",high_pow(a,n,mod));
51     }
52     return 0;
53 }

```

6.14 數位統計

```

1 ll d[65], dp[65][2]; //up區間是不是完整
2 ll dfs(int p,bool is8,bool up){
3     if(!p)return 1; // 回傳0是不是答案
4     if(!up&&dp[p][is8])return dp[p][is8];
5     int mx = up?d[p]:9; //可以用的有那些
6     ll ans=0;
7     for(int i=0;i<=mx;++i){
8         if( is8&&i==7 )continue;
9         ans += dfs(p-1,i==8,up&&i==mx);
10    }
11    if(!up)dp[p][is8]=ans;
12    return ans;
13 }
14 ll f(ll N){
15     int k=0;
16     while(N){ // 把數字先分解到陣列
17         d[++k] = N%10;
18         N/=10;
19     }
20     return dfs(k,false,true);
21 }

```

6.15 質因數分解

```

1 LL func(const LL n,const LL mod,const int c)
2     {
3         return (LLmul(n,n,mod)+c+mod)%mod;
4     }
5 LL pollorro(const LL n, const int c) { //循
6     環節長度
7     LL a=1, b=1;
8     a=func(a,n,c)%n;
9     b=func(b,n,c)%n; b=func(b,n,c)%n;
10    while(gcd(abs(a-b),n)==1) {
11        a=func(a,n,c)%n;
12        b=func(b,n,c)%n; b=func(b,n,c)%n;
13    }
14    return gcd(abs(a-b),n);
15 }
16 void prefactor(LL &n, vector<LL> &v) {
17     for(int i=0;i<12;++i) {

```

```

18     while(n%prime[i]==0) {
19         v.push_back(prime[i]);
20         n/=prime[i];
21     }
22 }
23 void smallfactor(LL n, vector<LL> &v) {
24     if(n<MAXPRIME) {
25         while(isp[(int)n]) {
26             v.push_back(isp[(int)n]);
27             n/=isp[(int)n];
28         }
29         v.push_back(n);
30     } else {
31         for(int i=0;i<primecnt&&prime[i]*prime[i]
32             ]<=n;++i) {
33             while(n%prime[i]==0) {
34                 v.push_back(prime[i]);
35                 n/=prime[i];
36             }
37         }
38         if(n!=1) v.push_back(n);
39     }
40 }
41 void comfactor(const LL &n, vector<LL> &v) {
42     if(n<1e9) {
43         smallfactor(n,v);
44         return;
45     }
46     if(Isprime(n)) {
47         v.push_back(n);
48         return;
49     }
50     LL d;
51     for(int c=3;++c) {
52         d = pollorro(n,c);
53         if(d!=n) break;
54     }
55     comfactor(d,v);
56     comfactor(n/d,v);
57 }
58 void Factor(const LL &x, vector<LL> &v) {
59     LL n = x;
60     if(n==1) { puts("Factor 1"); return; }
61     prefactor(n,v);
62     if(n==1) return;
63     comfactor(n,v);
64     sort(v.begin(),v.end());
65 }
66 void AllFactor(const LL &n,vector<LL> &v) {
67     vector<LL> tmp;
68     Factor(n,tmp);
69     v.clear();
70     v.push_back(1);
71     int len;
72     LL now=1;
73     for(int i=0;i<tmp.size();++i) {
74         if(i==0 || tmp[i]!=tmp[i-1]) {
75             len = v.size();
76             now = 1;
77         }
78         for(int j=0;j<len;++j) {
79             v[j]*=tmp[i];
80         }
81     }
82     now*=tmp[i];

```

```

83     for(int j=0;j<len;++j)
84         v.push_back(v[j]*now);
85 }
86 }

```

7 String

7.1 AC 自動機

```

1 template<char L='a',char R='z'>
2 class ac_automaton{
3     struct joe{
4         int next[R-L+1],fail,efl,ed,cnt_dp,vis;
5         joe():ed(0),cnt_dp(0),vis(0){
6             for(int i=0;i<=R-L;++i)next[i]=0;
7         }
8     };
9     public:
10        std::vector<joe> S;
11        std::vector<int> q;
12        int qs,qe,vt;
13        ac_automaton():S(1),qs(0),qe(0),vt(0){
14            void clear(){
15                q.clear();
16                S.resize(1);
17                for(int i=0;i<=R-L;++i)S[0].next[i]=0;
18                S[0].cnt_dp=S[0].vis=qs=qe=vt=0;
19            }
20            void insert(const char *s){
21                int o=0;
22                for(int i=0,id;s[i];++i){
23                    id=s[i]-L;
24                    if(!S[o].next[id]){
25                        S.push_back(joe());
26                        S[o].next[id]=S.size()-1;
27                    }
28                    o=S[o].next[id];
29                }
30                ++S[o].ed;
31            }
32            void build_fail(){
33                S[0].fail=S[0].efl=-1;
34                q.clear();
35                q.push_back(0);
36                ++qe;
37                while(qs!=qe){
38                    int pa=q[qs++],id,t;
39                    for(int i=0;i<=R-L;++i){
40                        t=S[pa].next[i];
41                        if(!t)continue;
42                        id=S[pa].fail;
43                        while(~id&&S[id].next[i])id=S[id].fail;
44                        S[t].fail=~id?S[id].next[i]:0;
45                        S[t].efl=S[S[t].fail].ed+S[t].fail:S[t].fail;efl;
46                        q.push_back(t);
47                        ++qe;
48                    }
49                }
50            }

```

```

51 /*DP出每個前綴在字串s出現的次數並傳回所有
   字串被s匹配成功的次數O(N*M)*/
52 int match_0(const char *s){
53     int ans=0,id,p=0,i;
54     for(i=0;s[i];++i){
55         id=s[i]-L;
56         while(!S[p].next[id]&&p=S[p].fail;
57             if(!S[p].next[id])continue;
58             p=S[p].next[id];
59         ++S[p].cnt_dp; /*匹配成功則它所有後綴都
           可以被匹配(DP計算)*/
60     }
61     for(i=qe-1;i>=0;--i){
62         ans+=S[q[i]].cnt_dp*S[q[i]].ed;
63         if(~S[q[i]].fail)S[q[i]].fail.
           cnt_dp+=S[q[i]].cnt_dp;
64     }
65     return ans;
66 }
67 /*多串匹配走efl邊並傳回所有字串被s匹配成功
   的次數O(N*M^1.5)*/
68 int match_1(const char *s)const{
69     int ans=0,id,p=0,t;
70     for(int i=0;s[i];++i){
71         id=s[i]-L;
72         while(!S[p].next[id]&&p=S[p].fail;
73             if(!S[p].next[id])continue;
74             p=S[p].next[id];
75             if(S[p].ed)ans+=S[p].ed;
76             for(t=S[p].efl;~t;t=S[t].efl){
77                 ans+=S[t].ed; /*因為都走efl邊所以保證
                   匹配成功*/
78             }
79     }
80     return ans;
81 }
82 /*枚舉(s的子字串nA)的所有相異字串各恰一次
   並傳回次數O(N*M*(1/3))*/
83 int match_2(const char *s){
84     int ans=0,id,p=0,t;
85     ++vt;
86     /*把戳記vt+=1. 只要vt沒溢位. 所有S[p].
       vis==vt就會變成false
87     這種利用vt的方法可以O(1)歸零vis陣列*/
88     for(int i=0;s[i];++i){
89         id=s[i]-L;
90         while(!S[p].next[id]&&p=S[p].fail;
91             if(!S[p].next[id])continue;
92             p=S[p].next[id];
93             if(S[p].ed&&S[p].vis!=vt){
94                 S[p].vis=vt;
95                 ans+=S[p].ed;
96             }
97             for(t=S[p].efl;~t&&S[t].vis!=vt;t=S[t]
               .efl){
98                 S[t].vis=vt;
99                 ans+=S[t].ed; /*因為都走efl邊所以保證
                   匹配成功*/
100             }
101         }
102     }
103     return ans;
104 }
/*把AC自動機變成真的自動機*/

```

```

105 void evolution(){
106     for(qs=1;qs!=qe;){
107         int p=q[qs++];
108         for(int i=0;i<=R-L;++){
109             if(S[p].next[i]==0)S[p].next[i]=S[p]
               .fail.next[i];
110         }
111     }
112 }

```

7.2 hash

```

1 #define MAXN 1000000
2 #define mod 1073676287
3 /*mod 必須要是質數*/
4 typedef long long T;
5 char s[MAXN+5];
6 T h[MAXN+5]; /*hash陣列*/
7 T h_base[MAXN+5]; /*h_base[n]=(prime^n)%mod*/
8 void hash_init(int len,T prime){
9     h_base[0]=1;
10    for(int i=1;i<=len;++){
11        h[i]=(h[i-1]*prime+s[i-1])%mod;
12        h_base[i]=(h_base[i-1]*prime)%mod;
13    }
14 }
15 T get_hash(int l,int r){ /*閉區間寫法. 設編號
   為0 ~ len-1*/
16     return (h[r+1]-(h[l]*h_base[r-l+1])%mod+
       mod)%mod;
17 }

```

7.3 KMP

```

1 /*產生fail function*/
2 void kmp_fail(char *s,int len,int *fail){
3     int id=-1;
4     fail[0]=-1;
5     for(int i=1;i<len;++){
6         while(~id&&s[id+1]!=s[i])id=fail[id];
7         if(s[id+1]==s[i])++id;
8         fail[i]=id;
9     }
10 }
11 /*以字串B匹配字串A. 傳回匹配成功的數量(用B的
   fail)*/
12 int kmp_match(char *A,int lenA,char *B,int
   lenB,int *fail){
13     int id=-1,ans=0;
14     for(int i=0;i<lenA;++){
15         while(~id&&B[id+1]!=A[i])id=fail[id];
16         if(B[id+1]==A[i])++id;
17         if(id==lenB-1){ /*匹配成功*/
18             ++ans, id=fail[id];
19         }
20     }
21     return ans;
22 }

```

7.4 manacher

```

1 //原字串: asdsasdsa
2 //先把字串變成這樣: @#a#s#d#s#a#s#d#s#a#
3 void manacher(char *s,int len,int *z){
4     int l=0,r=0;
5     for(int i=1;i<len;++){
6         z[i]=r>i?min(z[2*i-l],r-i):1;
7         while(s[i+z[i]]==s[i-z[i]])++z[i];
8         if(z[i]+i>r)r=z[i]+i,l=i;
9     } //ans = max(z)-1
10 }

```

7.5 minimal_string_rotation

```

1 int min_string_rotation(const string &s){
2     int n=s.size(),i=0,j=1,k=0;
3     while(i<n&&j<n&&k<n){
4         int t=s[(i+k)%n]-s[(j+k)%n];
5         ++k;
6         if(t){
7             if(t>0)i+=k;
8             else j+=k;
9             if(i==j)++j;
10            k=0;
11        }
12    }
13    return min(i,j); //最小循環表示法起始位置
14 }

```

7.6 reverseBWT

```

1 const int MAXN = 305, MAXC = 'Z';
2 int ranks[MAXN], tots[MAXC], first[MAXC];
3 void rankBWT(const string &bw){
4     memset(ranks,0,sizeof(int)*bw.size());
5     memset(tots,0,sizeof(tots));
6     for(size_t i=0;i<bw.size();++){
7         ranks[i] = tots[int(bw[i])]++;
8     }
9     void firstCol(){
10        memset(first,0,sizeof(first));
11        int totc = 0;
12        for(int c='A';c<='Z';++c){
13            if(!tots[c]) continue;
14            first[c] = totc;
15            totc += tots[c];
16        }
17    }
18 string reverseBwt(string bw,int begin){
19     rankBWT(bw, firstCol());
20     int i = begin; //原字串最後一個元素的位置
21     string res;
22     do{
23         char c = bw[i];
24         res = c + res;
25         i = first[int(c)] + ranks[i];
26     }while (i != begin);

```

```

27     return res;
28 }

```

7.7 suffix_array_lcp

```

1 #define radix_sort(x,y){\
2     for(i=0;i<A;++){c[i]=0;\
3     for(i=0;i<n;++){c[x[y[i]]]++; \
4     for(i=1;i<A;++){c[i]+=c[i-1]; \
5     for(i=n-1;~i;--i)sa[--c[x[y[i]]]]=y[i]; \
6 }
7 #define AC(r,a,b){\
8     r[a]=r[b]|(a+k>n)?r[a+k]:r[b+k]
9 void suffix_array(const char *s,int n,int *
   sa,int *rank,int *tmp,int *c){
10     int A='z'+1,i,k,id=0;
11     for(i=0;i<n;++){rank[tmp[i]]=i;s[i];
12     radix_sort(rank,tmp);
13     for(k=1;id<n-1;k<=1){
14         for(id=0,i=n-k;i<n;++){tmp[id++]=i;
15         for(i=0;i<n;++){
16             if(sa[i]>=k)tmp[id++]=sa[i]-k;
17         radix_sort(rank,tmp);
18         swap(rank,tmp);
19         for(rank[sa[0]]=id=0,i=1;i<n;++){
20             rank[sa[i]]=id+=AC(tmp,sa[i-1],sa[i]);
21         A=id+1;
22     }
23 }
24 //h:高度數組 sa:後綴數組 rank:排名
25 void suffix_array_lcp(const char *s,int len,
   int *h,int *sa,int *rank){
26     for(int i=0;i<len;++){rank[sa[i]]=i;
27     for(int i=0,k=0;i<len;++){
28         if(rank[i]==0)continue;
29         if(k--<0;
30         while(s[i+k]==s[sa[rank[i]-1]+k])++k;
31         h[rank[i]]=k;
32     }
33     h[0]=0; // h[k]=lcp(sa[k],sa[k-1]);
34 }

```

7.8 Z

```

1 void z_alg(char *s,int len,int *z){
2     int l=0,r=0;
3     z[0]=len;
4     for(int i=1;i<len;++){
5         z[i]=i>r?0:(i-l+z[i-l]<z[i-l]?z[i-l]:r-i
           +1);
6         while(i+z[i]<len&&s[i+z[i]]==s[z[i]])++z
           [i];
7         if(i+z[i]-1>r)r=i+z[i]-1,l=i;
8     }
9 }

```

8 Tarjan

8.1 dominator_tree

```

1 struct dominator_tree{
2     static const int MAXN=5005;
3     int n; // 1-base
4     vector<int> G[MAXN], rG[MAXN];
5     int pa[MAXN], dfn[MAXN], id[MAXN], dfnCnt;
6     int semi[MAXN], idom[MAXN], best[MAXN];
7     vector<int> tree[MAXN]; // tree here
8     void init(int _n){
9         n = _n;
10        for(int i=1; i<=n; ++i)
11            G[i].clear(), rG[i].clear();
12    }
13    void add_edge(int u, int v){
14        G[u].push_back(v);
15        rG[v].push_back(u);
16    }
17    void dfs(int u){
18        id[dfn[u]=++dfnCnt]=u;
19        for(auto v:G[u]) if(!dfn[v])
20            dfs(v), pa[dfn[v]]=dfn[u];
21    }
22    int find(int y, int x){
23        if(y <= x) return y;
24        int tmp = find(pa[y], x);
25        if(semi[best[y]] > semi[best[pa[y]]])
26            best[y] = best[pa[y]];
27        return pa[y] = tmp;
28    }
29    void tarjan(int root){
30        dfnCnt = 0;
31        for(int i=1; i<=n; ++i){
32            dfn[i] = idom[i] = 0;
33            tree[i].clear();
34            best[i] = semi[i] = i;
35        }
36        dfs(root);
37        for(int i=dfnCnt; i>1; --i){
38            int u = id[i];
39            for(auto v:rG[u]) if(v=dfn[v]){
40                find(v, i);
41                semi[i]=min(semi[i], semi[best[v]]);
42            }
43            tree[semi[i]].push_back(i);
44            for(auto v:tree[pa[i]]){
45                find(v, pa[i]);
46                idom[v] = semi[best[v]]==pa[i]
47                    ? pa[i] : best[v];
48            }
49            tree[pa[i]].clear();
50        }
51        for(int i=2; i<=dfnCnt; ++i){
52            if(idom[i] != semi[i])
53                idom[i] = idom[idom[i]];
54            tree[id[idom[i]]].push_back(id[i]);
55        }
56    }
57 } dom;

```

8.2 tnfsb017_2_sat

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 #define MAXN 8001
4 #define MAXN2 MAXN*4
5 #define n(X) ((X)+2*N)
6 vector<int> v[MAXN2], rv[MAXN2], vis_t;
7 int N,M;
8 void addedge(int s,int e){
9     v[s].push_back(e);
10    rv[e].push_back(s);
11 }
12 int scc[MAXN2];
13 bool vis[MAXN2]={false};
14 void dfs(vector<int> *uv,int n,int k=-1){
15     vis[n]=true;
16     for(int i=0;i<uv[n].size();++i)
17         if(!vis[uv[n][i]])
18             dfs(uv,uv[n][i],k);
19     if(uv==v)vis_t.push_back(n);
20     scc[n]=k;
21 }
22 void solve(){
23     for(int i=1;i<=N;++i){
24         if(!vis[i])dfs(v,i);
25         if(!vis[n(i)])dfs(v,n(i));
26     }
27     memset(vis,0,sizeof(vis));
28     int c=0;
29     for(int i=vis_t.size()-1;i>=0;--i)
30         if(!vis[vis_t[i]])
31             dfs(rv,vis_t[i],c++);
32 }
33 int main(){
34     int a,b;
35     scanf("%d%d",&N,&M);
36     for(int i=1;i<=N;++i){
37         // (A or B)&(!A & !B) A^B
38         a=i*2-1;
39         b=i*2;
40         addedge(n(a),b);
41         addedge(n(b),a);
42         addedge(a,n(b));
43         addedge(b,n(a));
44     }
45     while(M--){
46         scanf("%d%d",&a,&b);
47         a = a>0?a*2-1:-a*2;
48         b = b>0?b*2-1:-b*2;
49         // A or B
50         addedge(n(a),b);
51         addedge(n(b),a);
52     }
53     solve();
54     bool check=true;
55     for(int i=1;i<=2*N;++i)
56         if(scc[i]==scc[n(i)])
57             check=false;
58     if(check){
59         printf("%d\n",N);
60         for(int i=1;i<=2*N;i+=2){
61             if(scc[i]>scc[i+2*N]) putchar('+');
62             else putchar('-');
63         }
64     }

```

```

64     puts("");
65 }else puts("0");
66 return 0;
67 }

```

8.3 橋連通分量

```

1 #define N 1005
2 struct edge{
3     int u,v;
4     bool is_bridge;
5     edge(int u=0,int v=0):u(u),v(v),is_bridge
6         (0){}
7 };
8 vector<edge> E;
9 vector<int> G[N]; // 1-base
10 int low[N],vis[N],Time;
11 int bcc_id[N],bridge_cnt,bcc_cnt; // 1-base
12 int st[N],top; // BCC用
13 void add_edge(int u,int v){
14     G[u].push_back(E.size());
15     E.emplace_back(u,v);
16     G[v].push_back(E.size());
17     E.emplace_back(v,u);
18 }
19 void dfs(int u,int re=-1){ // u當前點, re為u連
20     接前一個點的邊
21     int v;
22     low[u]=vis[u]=++Time;
23     st[top++]=u;
24     for(int e:G[u]){
25         v=E[e].v;
26         if(!vis[v]){
27             dfs(v,e^1); // e^1反向邊
28             low[u]=min(low[u],low[v]);
29             if(vis[u]<low[v]){
30                 E[e].is_bridge=E[e^1].is_bridge=1;
31                 ++bridge_cnt;
32             }else if(vis[v]<vis[u]&&e!=re)
33                 low[u]=min(low[u],vis[v]);
34         }
35     }
36     if(vis[u]==low[u]){ // 處理BCC
37         ++bcc_cnt; // 1-base
38         do bcc_id[v=st[--top]]=bcc_cnt; // 每個點
39         所在的BCC
40         while(v!=u);
41     }
42 }
43 void bcc_init(int n){
44     Time=bcc_cnt=bridge_cnt=top=0;
45     E.clear();
46     for(int i=1;i<=n;++i){
47         G[i].clear();
48         vis[i]=bcc_id[i]=0;
49     }

```

8.4 雙連通分量 & 割點

```

1 #define N 1005
2 vector<int> G[N]; // 1-base
3 vector<int> bcc[N]; // 存每塊雙連通分量的點
4 int low[N],vis[N],Time;
5 int bcc_id[N],bcc_cnt; // 1-base
6 bool is_cut[N]; // 是否為割點
7 int st[N],top;
8 void dfs(int u,int pa=-1){ // u當前點, pa父親
9     int t, child=0;
10    low[u]=vis[u]=++Time;
11    st[top++]=u;
12    for(int v:G[u]){
13        if(!vis[v]){
14            dfs(v,u), ++child;
15            low[u]=min(low[u],low[v]);
16            if(vis[u]<=low[v]){
17                is_cut[u]=1;
18                bcc[++bcc_cnt].clear();
19                do{
20                    bcc_id[t=st[--top]]=bcc_cnt;
21                    bcc[bcc_cnt].push_back(t);
22                }while(t!=v);
23                bcc_id[u]=bcc_cnt;
24                bcc[bcc_cnt].push_back(u);
25            }
26        }else if(vis[v]<vis[u]&&v!=pa) // 反向邊
27            low[u] = min(low[u],vis[v]);
28        // u是dfs樹的根要特判
29        if(pa==-1&&child<2)is_cut[u]=0;
30    }
31 }
32 void bcc_init(int n){
33     Time=bcc_cnt=top=0;
34     for(int i=1;i<=n;++i){
35         G[i].clear();
36         is_cut[i]=vis[i]=bcc_id[i]=0;
37     }
38 }

```

9 Tree_problem

9.1 HeavyLight

```

1 #include<vector>
2 #define MAXN 100005
3 int siz[MAXN],max_son[MAXN],pa[MAXN],dep[
4     MAXN];
5 int link_top[MAXN],link[MAXN],cnt;
6 vector<int> G[MAXN];
7 void find_max_son(int u){
8     siz[u]=1;
9     max_son[u]=-1;
10    for(auto v:G[u]){
11        if(v==pa[u])continue;
12        pa[v]=u;
13        dep[v]=dep[u]+1;
14        find_max_son(v);
15        if(max_son[u]==-1||siz[v]>siz[max_son[u]
16            ])max_son[u]=v;
17        siz[u]+=siz[v];
18    }

```

9.3 link_cut_tree

```

17 }
18 void build_link(int u,int top){
19     link[u]=++cnt;
20     link_top[u]=top;
21     if(max_son[u]==-1)return;
22     build_link(max_son[u],top);
23     for(auto v:G[u]){
24         if(v==max_son[u]||v==pa[u])continue;
25         build_link(v,v);
26     }
27 }
28 int find_lca(int a,int b){
29     //求LCA，可以在過程中對區間進行處理
30     int ta=link_top[a],tb=link_top[b];
31     while(ta!=tb){
32         if(dep[ta]<dep[tb]){
33             swap(ta,tb);
34             swap(a,b);
35         }
36         //這裡可以對a所在的鏈做區間處理
37         //區間為(link[ta],link[a])
38         ta=link_top[a=pa[ta]];
39     }
40     //最後a,b會在同一條鏈，若a!=b還要在進行一次區間處理
41     return dep[a]<dep[b]?a:b;
42 }

```

9.2 LCA

```

1 const int MAXN=100000; // 1-base
2 const int MLG=17; //log2(MAXN)+1;
3 int pa[MLG+2][MAXN+5];
4 int dep[MAXN+5];
5 vector<int> G[MAXN+5];
6 void dfs(int x,int p=0){//dfs(root);
7     pa[0][x]=p;
8     for(int i=0;i<=MLG;++i)
9         pa[i+1][x]=pa[i][pa[i][x]];
10    for(auto &i:G[x]){
11        if(i==p)continue;
12        dep[i]=dep[x]+1;
13        dfs(i,x);
14    }
15 }
16 inline int jump(int x,int d){
17     for(int i=0;i<=MLG;++i)
18         if((d>>i)&1) x=pa[i][x];
19     return x;
20 }
21 inline int find_lca(int a,int b){
22     if(dep[a]>dep[b])swap(a,b);
23     b=jump(b,dep[b]-dep[a]);
24     if(a==b)return a;
25     for(int i=MLG;i>=0;--i){
26         if(pa[i][a]!=pa[i][b]){
27             a=pa[i][a];
28             b=pa[i][b];
29         }
30     }
31     return pa[0][a];
32 }

```

```

1 struct splay_tree{
2     int ch[2],pa; //子節點跟父母
3     bool rev; //反轉的懶惰標記
4     splay_tree():pa(0),rev(0){ch[0]=ch[1]=0;}
5 };
6 vector<splay_tree> nd;
7 //有的時候用vector會TLE，要注意
8 //這邊以node[0]作為null節點
9 bool isroot(int x){//判斷是否為這棵splay
10    tree的根
11    return nd[nd[x].pa].ch[0]!=x&&nd[nd[x].pa].ch[1]!=x;
12 }
13 void down(int x){//懶惰標記下推
14     if(nd[x].rev){
15         if(nd[x].ch[0])nd[nd[x].ch[0]].rev^=1;
16         if(nd[x].ch[1])nd[nd[x].ch[1]].rev^=1;
17         swap(nd[x].ch[0],nd[x].ch[1]);
18         nd[x].rev=0;
19     }
20 }
21 void push_down(int x){//所有祖先懶惰標記下推
22     if(!isroot(x))push_down(nd[x].pa);
23     down(x);
24 }
25 void up(int x){//將子節點的資訊向上更新
26     void rotate(int x){//旋轉，會自行判斷轉的方向
27         int y=nd[x].pa,z=nd[y].pa,d=(nd[y].ch[1]==x);
28         nd[x].pa=z;
29         if(!isroot(y))nd[z].ch[nd[z].ch[1]==y]=x;
30         nd[y].ch[d]=nd[x].ch[d^1];
31         nd[nd[y].ch[d]].pa=y;
32         nd[y].pa=x,nd[x].ch[d^1]=y;
33         up(y),up(x);
34     }
35 }
36 void splay(int x){//將x伸展到splay tree的根
37     push_down(x);
38     while(!isroot(x)){
39         int y=nd[x].pa;
40         if(!isroot(y)){
41             int z=nd[y].pa;
42             if((nd[z].ch[0]==y)^(nd[y].ch[0]==x))
43                 rotate(y);
44             else rotate(x);
45         }
46         rotate(x);
47     }
48 }
49 int access(int x){
50     int last=0;
51     while(x){
52         splay(x);
53         nd[x].ch[1]=last;
54         up(x);
55         last=x;
56         x=nd[x].pa;
57     }
58     return last; //access後splay tree的根
59 }

```

```

57 void access(int x,bool is=0){//is=0就是一般的access
58     int last=0;
59     while(x){
60         splay(x);
61         if(is&&!nd[x].pa){
62             //printf("%d\n",max(nd[last].ma,nd[nd[x].ch[1]].ma));
63         }
64         nd[x].ch[1]=last;
65         up(x);
66         last=x;
67         x=nd[x].pa;
68     }
69 }
70 void query_edge(int u,int v){
71     access(u);
72     access(v,1);
73 }
74 void make_root(int x){
75     access(x),splay(x);
76     nd[x].rev^=1;
77 }
78 void make_root(int x){
79     nd[access(x)].rev^=1;
80     splay(x);
81 }
82 void cut(int x,int y){
83     make_root(x);
84     access(y);
85     splay(y);
86     nd[y].ch[0]=0;
87     nd[x].pa=0;
88 }
89 void cut_parents(int x){
90     access(x);
91     splay(x);
92     nd[nd[x].ch[0]].pa=0;
93     nd[x].ch[0]=0;
94 }
95 void link(int x,int y){
96     make_root(x);
97     nd[x].pa=y;
98 }
99 int find_root(int x){
100    x=access(x);
101    while(nd[x].ch[0])x=nd[x].ch[0];
102    splay(x);
103    return x;
104 }
105 int query(int u,int v){
106     //傳回uv路徑splay tree的根結點
107     //這種寫法無法求LCA
108     make_root(u);
109     return access(v);
110 }
111 int query_lca(int u,int v){
112     //假設求鏈上點權的總和，sum是子樹的權重和，data是節點的權重
113     access(u);
114     int lca=access(v);
115     splay(u);
116     if(u==lca){
117         //return nd[lca].data+nd[nd[lca].ch[1]].sum
118     }
119 }

```

```

118 }else{
119     //return nd[lca].data+nd[nd[lca].ch[1]].sum+nd[u].sum
120 }
121 }
122 struct EDGE{
123     int a,b,w;
124 }e[10005];
125 int n;
126 vector<pair<int,int>> G[10005];
127 //first表示子節點，second表示邊的編號
128 int pa[10005],edge_node[10005];
129 //pa是父母節點，暫存用的，edge_node是每個編
130 //被存在哪個點裡面的陣列
131 void bfs(int root){
132     //在建構的時候把每個點都設成一個splay tree
133     queue<int> q;
134     for(int i=1;i<=n;++i)pa[i]=0;
135     q.push(root);
136     while(q.size()){
137         int u=q.front();
138         q.pop();
139         for(auto P:G[u]){
140             int v=P.first;
141             if(v!=pa[u]){
142                 pa[v]=u;
143                 nd[v].pa=u;
144                 nd[v].data=e[P.second].w;
145                 edge_node[P.second]=v;
146                 up(v);
147                 q.push(v);
148             }
149         }
150     }
151 }
152 void change(int x,int b){
153     splay(x);
154     //nd[x].data=b;
155     up(x);
156 }

```

9.4 POJ_tree

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 #define MAXN 10005
4 int n,k;
5 vector<pair<int,int>> g[MAXN];
6 int size[MAXN];
7 bool vis[MAXN];
8 inline void init(){
9     for(int i=0;i<=n;++i){
10         g[i].clear();
11         vis[i]=0;
12     }
13 }
14 void get_dis(vector<int> &dis,int u,int pa,int d){
15     dis.push_back(d);
16     for(size_t i=0;i<g[u].size();++i){
17         int v=g[u][i].first,w=g[u][i].second;
18         if(v!=pa&&!vis[v])get_dis(dis,v,u,d+w);
19     }
20 }

```



```

19 }
20 }
21 vector<int> dis;//這東西如果放在函數裡會TLE
22 int cal(int u,int d){
23     dis.clear();
24     get_dis(dis,u,-1,d);
25     sort(dis.begin(),dis.end());
26     int l=0,r=dis.size()-1,res=0;
27     while(l<r){
28         while(l<r&&dis[l]+dis[r]>k)--r;
29         res+=r-(l++);
30     }
31     return res;
32 }
33 pair<int,int> tree_centroid(int u,int pa,
    const int sz){
34     size[u]=1;//找樹重心，second是重心
35     pair<int,int> res(INT_MAX,-1);
36     int ma=0;
37     for(size_t i=0;i<g[u].size();++i){
38         int v=g[u][i].first;
39         if(v==pa||vis[v])continue;
40         res=min(res,tree_centroid(v,u,sz));
41         size[u]+=size[v];
42         ma=max(ma,size[v]);
43     }
44     ma=max(ma,sz-size[u]);
45     return min(res,make_pair(ma,u));
46 }
47 int tree_DC(int u,int sz){
48     int center=tree_centroid(u,-1,sz).second;
49     int ans=cal(center,0);
50     vis[center]=1;
51     for(size_t i=0;i<g[center].size();++i){
52         int v=g[center][i].first,w=g[center][i].second;
53         if(vis[v])continue;
54         ans+=cal(v,w);
55         ans+=tree_DC(v,size[v]);
56     }
57     return ans;
58 }
59 int main(){
60     while(scanf("%d%d",&n,&k),n||k){
61         init();
62         for(int i=1;i<n;++i){
63             int u,v,w;
64             scanf("%d%d%d",&u,&v,&w);
65             g[u].push_back(make_pair(v,w));
66             g[v].push_back(make_pair(u,w));
67         }
68         printf("%d\n",tree_DC(1,n));
69     }
70     return 0;
71 }

```

10 default

10.1 debug

```
1 //volatile
```

```

2 #ifndef DEBUG
3 #define dbg(...) {\
4     fprintf(stderr,"%s - %d : (%s) = ",
5         __PRETTY_FUNCTION__,__LINE__,#
6         __VA_ARGS__);\
7     _DO(__VA_ARGS__); \
8 }
9 template<typename I> void _DO(I&&x){cerr<<x
10     <<endl;}
11 template<typename I,typename...T> void _DO(I
12     &&x,T&&...tail){cerr<<x<<" ";_DO(tail
13     ...);}
14 #else
15 #define dbg(...)
16 #endif

```

10.2 ext

```

1 #include<bits/extc++.h>
2 #include<ext/pd_ds/assoc_container.hpp>
3 #include<ext/pd_ds/tree_policy.hpp>
4 using namespace __gnu_cxx;
5 using namespace __gnu_pbds;
6 template<typename T>
7 using pbds_set = tree<T,null_type,less<T>,
8     rb_tree_tag,
9     tree_order_statistics_node_update>;
10 template<typename T,typename U>
11 using pbds_map = tree<T,U,less<T>,
12     rb_tree_tag,
13     tree_order_statistics_node_update>;
14 using heap=__gnu_pbds::priority_queue<int>;
15 //s.find_by_order(1);//0 base
16 //s.order_of_key(1);

```

10.3 IncStack

```

1 //Magic
2 #pragma GCC optimize "Ofast"
3 //stack resize,change esp to rsp if 64-bit
4 system
5 asm("mov %0,%esp\n" ::"g"(mem+10000000));
6 -WL,--stack,214748364 -trigraphs
7 #pragma comment(linker, "/STACK
8 :1024000000,1024000000")
9 //linux stack resize
10 #include<sys/resource.h>
11 void increase_stack(){
12     const rlim_t ks=64*1024*1024;
13     struct rlimit rl;
14     int res=getrlimit(RLIMIT_STACK,&rl);
15     if(!res&&rl.rlim_cur<ks){
16         rl.rlim_cur=ks;
17         res=setrlimit(RLIMIT_STACK,&rl);
18     }
19 }

```

10.4 input

```

1 inline int read(){
2     int x=0; bool f=0; char c=getchar();
3     while(ch<'0' || '9'<ch)f|=ch=='-',ch=getchar
4     ();
5     while('0'<=ch&&ch<='9')x=x*10-'0'+ch,ch=
6     getchar();
7     return f?-x:x;
8 }
9 // #!/bin/bash
10 // g++ -std=c++11 -O2 -Wall -Wextra -Wno-
11 // unused-result -DDEBUG $1 && ./a.out
12 // -fsanitize=address -fsanitize=undefined
13 // -fsanitize=return

```

11 language

11.1 CNF

```

1 #define MAXN 55
2 struct CNF{
3     int s,x,y;//s->xy | s->x, if y==1
4     int cost;
5     CNF(){}
6     CNF(int s,int x,int y,int c):s(s),x(x),y(y)
7     ,cost(c){}
8 };
9 map<char,int> rule;//每個字元對應到的規則，
10 小寫字母為終端字符
11 vector<CNF> cnf;
12 void init(){
13     state=0;
14     rule.clear();
15     cnf.clear();
16 }
17 void add_to_cnf(char s,const string &p,int
18     cost){
19     //加入一個s -> <p>的文法，代價為cost
20     if(rule.find(s)==rule.end())rule[s]=state
21     ++;
22     for(auto c:p)if(rule.find(c)==rule.end())
23     rule[c]=state++;
24     if(p.size()==1){
25         cnf.push_back(CNF(rule[s],rule[p[0]],-1,
26             cost));
27     }else{
28         int left=rule[s];
29         int sz=p.size();
30         for(int i=0;i<sz-2;++i){
31             cnf.push_back(CNF(left,rule[p[i]],
32                 state,0));
33             left=state++;
34         }
35         cnf.push_back(CNF(left,rule[p[sz-2]],
36             rule[p[sz-1]],cost));
37     }
38 }
39 }
40 }

```

```

32 vector<long long> dp[MAXN][MAXN];
33 vector<bool> neg_INF[MAXN][MAXN];//如果花費
34 是負的可能會有無限小的情形
35 void relax(int l,int r,const CNF &c,long
36     long cost,bool neg_c=0){
37     if(!neg_INF[l][r][c.s]&&(neg_INF[l][r][c.x
38     ][cost<dp[l][r][c.s]])){
39         if(neg_c|neg_INF[l][r][c.x]){
40             dp[l][r][c.s]=0;
41             neg_INF[l][r][c.s]=true;
42         }else dp[l][r][c.s]=cost;
43     }
44 }
45 void bellman(int l,int r,int n){
46     for(int k=1;k<=state;++k)
47     for(auto c:cnf)
48     if(c.y==1)relax(l,r,c,dp[l][r][c.x]+c
49     .cost,k==n);
50 }
51 void cyk(const vector<int> &tok){
52     for(int i=0;i<(int)tok.size();++i){
53         for(int j=0;j<(int)tok.size();++j){
54             dp[i][j]=vector<long long>(state+1,
55                 INT_MAX);
56             neg_INF[i][j]=vector<bool>(state+1,
57                 false);
58         }
59         dp[i][i][tok[i]]=0;
60         bellman(i,i,tok.size());
61     }
62     for(int r=1;r<(int)tok.size();++r){
63         for(int l=r-1;l>=0;--l){
64             for(int k=1;k<r;++k)
65             for(auto c:cnf)
66             if(~c.y)relax(l,r,c,dp[l][k][c.x]+
67                 dp[k+1][r][c.y]+c.cost);
68             bellman(l,r,tok.size());
69         }
70     }
71 }

```

12 other

12.1 WhatDay

```

1 int whatday(int y,int m,int d){
2     if(m<=2)m+=12,--y;
3     if(y<1752||y==1752&&m<9||y==1752&&m==9&&d
4     <3)
5         return (d+2*m+3*(m+1)/5+y+y/4+y/100+y/400)
6         %7;
7 }

```

12.2 上下最大正方形

```

1 void solve(int n,int a[],int b[]){// 1-base
2   int ans=0;
3   deque<int>da,db;
4   for(int l=1,r=1;r<=n;++r){
5     while(da.size()&&a[da.back()]>=a[r]){
6       da.pop_back();
7     }
8     da.push_back(r);
9     while(db.size()&&b[db.back()]>=b[r]){
10      db.pop_back();
11    }
12    db.push_back(r);
13    for(int d=a[da.front()]+b[db.front()];r-
14      1+l>d;++l){
15      if(da.front()==l)da.pop_front();
16      if(db.front()==l)db.pop_front();
17      if(da.size()&&db.size()){
18        d=a[da.front()]+b[db.front()];
19      }
20    }
21    ans=max(ans,r-l+1);
22  }
23  printf("%d\n",ans);
}

```

12.3 最大矩形

```

1 LL max_rectangle(vector<int> s){
2   stack<pair<int,int> > st;
3   st.push(make_pair(-1,0));
4   s.push_back(0);
5   LL ans=0;
6   for(size_t i=0;i<s.size();++i){
7     int h=s[i];
8     pair<int,int> now=make_pair(h,i);
9     while(h<st.top().first){
10      now=st.top();
11      st.pop();
12      ans=max(ans,(LL)(i-now.second)*now.first);
13    }
14    if(h>st.top().first){
15      st.push(make_pair(h,now.second));
16    }
17  }
18  return ans;
19 }

```

13 zformula

13.1 formula

13.1.1 Pick 公式

給定頂點坐標均是整點的簡單多邊形，面積 = 內部格點數 + 邊上格點數/2-1

13.1.2 圖論

- 對於平面圖 $\cdot F = E - V + C + 1 \cdot C$ 是連通分量數
- 對於平面圖 $\cdot E \leq 3V - 6$
- 對於連通圖 G ，最大獨立點集的大小設為 $I(G)$ ，最大匹配大小設為 $M(G)$ ，最小點覆蓋設為 $C_v(G)$ ，最小邊覆蓋設為 $C_e(G)$ ，對於任意連通圖：

- $I(G) + C_v(G) = |V|$
- $M(G) + C_e(G) = |V|$

- 對於連通二分圖：

- $I(G) = C_v(G)$
- $M(G) = C_e(G)$

- 最大權閉合圖：

- $C(u, v) = \infty, (u, v) \in E$
- $C(S, v) = W_v, W_v > 0$
- $C(v, T) = -W_v, W_v < 0$
- $ans = \sum_{W_v > 0} W_v - flow(S, T)$

- 最大密度子圖：

- 求 $max\left(\frac{W_e + W_v}{|V|}\right), e \in E', v \in V'$
- $U = \sum_{v \in V} 2W_v + \sum_{e \in E} W_e$
- $C(u, v) = W_{(u,v)}, (u, v) \in E$ ，雙向邊
- $C(S, v) = U, v \in V$
- $D_u = \sum_{(u,v) \in E} W_{(u,v)}$
- $C(v, T) = U + 2g - D_v - 2W_v, v \in V$
- 二分搜 g ：
 $l = 0, r = U, eps = 1/n^2$
 if $((U \times |V| - flow(S, T))/2 > 0)$ $l = mid$
 else $r = mid$
- $ans = min_cut(S, T)$
- $|E| = 0$ 要特殊判斷

- 弦圖：

- 點數大於 3 的環都要有一條弦
- 完美消除序列從後往前依次給每個點染色，給每個點染上可以染的最小顏色
- 最大團大小 = 色數
- 最大獨立集：完美消除序列從前往後能選就選
- 最小團覆蓋：最大獨立集的點和他延伸的邊構成
- 區間圖是弦圖
- 區間圖的完美消除序列：將區間按造又端點由小到大排序
- 區間圖染色：用線段樹做

13.1.3 dinic 特殊圖複雜度

- 單位流： $O\left(\min\left(V^{3/2}, E^{1/2}\right)E\right)$
- 二分圖： $O\left(V^{1/2}E\right)$

13.1.4 0-1 分數規劃

$x_i \in \{0, 1\} \cdot x_i$ 可能會有其他限制，求 $max\left(\frac{\sum B_i x_i}{\sum C_i x_i}\right)$

- $D(i, g) = B_i - g \times C_i$
- $f(g) = \sum D(i, g) x_i$
- $f(g) = 0$ 時 g 為最佳解， $f(g) < 0$ 沒有意義
- 因為 $f(g)$ 單調可以二分搜 g
- 或用 Dinkelbach 通常比較快

```

1 binary_search(){
2   while(r-l>eps){
3     g=(l+r)/2;
4     for(i:所有元素)D[i]=B[i]-g*C[i];//D(i,g)
5     找出一組合法x[i]使f(g)最大;
6     if(f(g)>0) l=g;
7     else r=g;
8   }
9   Ans = r;
10 }
11 Dinkelbach(){
12   g=任意狀態(通常設為0);
13   do{
14     Ans=g;
15     for(i:所有元素)D[i]=B[i]-g*C[i];//D(i,g)
16     找出一組合法x[i]使f(g)最大;
17     p=0,q=0;
18     for(i:所有元素)
19       if(x[i])p+=B[i],q+=C[i];
20     g=p/q;//更新解，注意q=0的情況
21   }while(abs(Ans-g)>EPS);
22   return Ans;
23 }

```

13.1.5 學長公式

- $\sum_{d|n} \phi(n) = n$
- $g(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} \mu(d) \times g(n/d)$
- Harmonic series $H_n = \ln(n) + \gamma + 1/(2n) - 1/(12n^2) + 1/(120n^4)$
- $\gamma = 0.57721566490153286060651209008240243104215$
- 格雷碼： $n \oplus (n >> 1)$
- $SG(A + B) = SG(A) \oplus SG(B)$
- 選轉矩陣 $M(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

13.1.6 基本數論

- $\sum_{d|n} \mu(n) = [n == 1]$
- $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) \times g(m/d)$
- $\sum_{i=1}^n \sum_{j=1}^m \text{互質數量} = \sum \mu(d) \left\lfloor \frac{n}{d} \right\rfloor \left\lfloor \frac{m}{d} \right\rfloor$
- $\sum_{i=1}^n \sum_{j=1}^n lcm(i, j) = n \sum_{d|n} d \times \phi(d)$

13.1.7 排組公式

- k 卡特蘭 $\frac{C_n^{kn}}{n(k-1)+1} \cdot C_m^n = \frac{n!}{m!(n-m)!}$
- $H(n, m) \cong x_1 + x_2 + \dots + x_n = k, num = C_k^{n+k-1}$
- Stirling number of 2^{nd} , n 人分 k 組方法數目
 - $S(0, 0) = S(n, n) = 1$
 - $S(n, 0) = 0$
 - $S(n, k) = kS(n-1, k) + S(n-1, k-1)$
- Bell number, n 人分任意多組方法數目
 - $B_0 = 1$
 - $B_n = \sum_{i=0}^n S(n, i)$
 - $B_{n+1} = \sum_{k=0}^n C_k^n B_k$
 - $B_{p+n} \equiv B_n + B_{n+1} \pmod{p}$, p is prime
 - $B_{p^m+n} \equiv mB_n + B_{n+1} \pmod{p}$, p is prime
 - From $B_0 : 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975$
- Derangement, 錯排，沒有人在自己位置上
 - $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \dots + (-1)^n \frac{1}{n!})$
 - $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 = 1, D_1 = 0$
 - From $D_0 : 1, 0, 1, 2, 9, 44, 265, 1854, 14833, 133496$

- Binomial Equality

- $\sum_k \binom{r}{m+k} \binom{s}{n-k} = \binom{r+s}{m+n}$
- $\sum_k \binom{r}{m+k} \binom{s}{n+k} = \binom{l+s}{l-m+n}$
- $\sum_k \binom{r}{m+k} \binom{s+k}{n} (-1)^k = (-1)^{l+m} \binom{s-m}{n-l}$
- $\sum_{k \leq l} \binom{l-k}{m} \binom{s}{n-k} (-1)^k = \frac{(-1)^{l+m} \binom{s-m-1}{l-n-m-1}}{(l-n-m-1)}$
- $\sum_{0 \leq k \leq l} \binom{l-k}{m} \binom{q+k}{n} = \binom{l+q+1}{m+n+1}$
- $\binom{r}{k} = (-1)^k \binom{k-r-1}{k}$
- $\binom{r}{m} \binom{m}{k} = \binom{r}{k} \binom{r-k}{m-k}$
- $\sum_{k \leq n} \binom{r+k}{k} = \binom{r+n+1}{n}$
- $\sum_{0 \leq k \leq n} \binom{k}{m} = \binom{n+1}{m+1}$
- $\sum_{k \leq m} \binom{m+r}{k} x^k y^k = \sum_{k \leq m} \binom{-r}{k} (-x)^k (x+y)^{m-k} =$

13.1.8 冪次，冪次和

- $a^{b\%P} = a^{b\% \varphi(P) + \varphi(P)}, b \geq \varphi(P)$
- $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
- $1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$
- $1^5 + 2^5 + 3^5 + \dots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12}$
- $0^k + 1^k + \dots + n^k = P(k), P(k) = \frac{(n+1)^{k+1} - \sum_{i=0}^{k-1} C_i^{k+1} P(i)}{k+1}, P(0) = n+1$
- $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^{m-1} C_k^{n+1} B_k m^{n+1-k}$
- $\sum_{j=0}^m C_j^{m+1} B_j = 0, B_0 = 1$
- 除了 $B_1 = -1/2$ ，剩下的奇數項都是 0
- $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 = -1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 7/6, B_{16} = -3617/510, B_{18} = 43867/798, B_{20} = -174611/330,$

13.1.9 Burnside's lemma

- $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- $X^g = t^{c(g)}$
- G 表示有幾種轉法, X^g 表示在那種轉法下, 有幾種是會保持對稱的, t 是顏色數, $c(g)$ 是循環節不動的面數。
- 正立方體塗三顏色, 轉 0 有 3^6 個元素不變, 轉 90 有 6 種, 每種有 3^3 不變, 180 有 3×3^4 , 120(角) 有 8×3^2 , 180(邊) 有 6×3^3 , 全部 $\frac{1}{54}(3^6 + 6 \times 3^3 + 3 \times 3^4 + 8 \times 3^2 + 6 \times 3^3) = \frac{24}{54}$

13.1.10 Count on a tree

- Rooted tree: $s_{n+1} = \frac{1}{n} \sum_{i=1}^n (i \times a_i \times \sum_{j=1}^{\lfloor n/i \rfloor} a_{n+1-i \times j})$
- Unrooted tree:
 - Odd: $a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$
 - Even: $Odd + \frac{1}{2} a_{n/2} (a_{n/2} + 1)$
- Spanning Tree
 - 完全圖 $n^n - 2$
 - 一般圖 (Kirchhoff's theorem) $M[i][i] = \text{degree}(V_i), M[i][j] = -1, \text{if have } E(i, j), 0 \text{ if no edge. delete any one row and col in } A, \text{ans} = \det(A)$

13.2 java

13.2.1 文件操作

```
1 import java.io.*;
2 import java.util.*;
3 import java.math.*;
4 import java.text.*;
5
6 public class Main{
7
8     public static void main(String args[]){
9         throws FileNotFoundException,
10         IOException
11         Scanner sc = new Scanner(new FileReader(
12             "a.in"));
13         PrintWriter pw = new PrintWriter(new
14             FileWriter("a.out"));
15         int n,m;
16         n=sc.nextInt();//读入下一个INT
17         m=sc.nextInt();
18
19         for(ci=1; ci<=c; ++ci){
20             pw.println("Case #"+ci+": easy for
21                 output");
22         }
23
24         pw.close();//关闭流并释放, 这个很重要,
25             否则是没有输出的
26         sc.close();//关闭流并释放
```

```
21 | }
22 | }
```

13.2.2 优先队列

```
1 PriorityQueue queue = new PriorityQueue( 1,
2     new Comparator(){
3     public int compare( Point a, Point b ){
4         if( a.x < b.x || a.x == b.x && a.y < b.y )
5             return -1;
6         else if( a.x == b.x && a.y == b.y )
7             return 0;
8         else return 1;
9     });
```

13.2.3 Map

```
1 Map map = new HashMap();
2 map.put("sa","dd");
3 String str = map.get("sa").toString();
4
5 for(Object obj : map.keySet()){
6     Object value = map.get(obj );
7 }
```

13.2.4 sort

```
1 static class cmp implements Comparator{
2     public int compare(Object o1,Object o2){
3         BigInteger b1=(BigInteger)o1;
4         BigInteger b2=(BigInteger)o2;
5         return b1.compareTo(b2);
6     }
7 }
8 public static void main(String[] args)
9     throws IOException{
10     Scanner cin = new Scanner(System.in);
11     int n;
12     n=cin.nextInt();
13     BigInteger[] seg = new BigInteger[n];
14     for (int i=0;i<n;i++)
15         seg[i]=cin.nextBigInteger();
16     Arrays.sort(seg,new cmp());
```

14

14.1 ganadoQuote

```
1 ¡Allí está!
2 ¡Un forastero!
3 ¡Agarrenlo!
4 ¡Os voy a romper a pedazos!
5 ¡Cógelo!
6 ¡Te voy a hacer picadillo!
7 ¡Te voy a matar!
8 ¡Míralo, está herido!
9 ¡Sos cerdo!
10 ¿Dónde estás?
11 ¡Detrás de tí, imbécil!
12 ¡No dejes que se escape!
13 ¡Basta, hijo de puta!
14 Lord Saddler...
15
16 ¡Mátalo!
17 ¡Allí está!
18 Morir es vivir.
19 ¡Síííí, ¡Quiero matar!
20 Muere, muere, muere....
21 Cerebros,cerebros,cerebros...
22 Cógedlo, cógedlo, cógedlo...
23 Lord Saddler...
24 Dieciséis.
25
26 ¡Va por él!
27 ¡Muérete!
28 ¡Cógelo!
29 ¡Te voy a matar!
30 ¡Bloqueale el paso!
31 ¡Te cogí!
32 ¡No dejes que se escape!
33
34 ¿Qué carajo estás haciendo aquí? ¡Lárgate,
35     cabrón!
36 Hay un rumor de que hay un extranjero entre
37     nosotros.
38 Nuestro jefe se encargará de la rata.
39 Su "Las Plagas" es mucho mejor que la
40     nuestra.
41 Tienes razón, es un hombre.
42 Usa los músculos.
43 Se vuelve loco!
44 ¡Hey, acá!
45 ¡Por aquí!
46 ¡El Gigante!
47 ¡Del Lago!
48 ¡Cógelo!
49 ¡Cógenlo!
50 ¡Allí!
51 ¡Rápido!
52 ¡Empieza a rezar!
53 ¡Mátenlos!
54 ¡Te voy a romper en pedazos!
55 ¡La campana!
56 Ya es hora de rezar.
57 Tenemos que irnos.
58 ¡Maldita sea, mierda!
59 ¡Ya es hora de aplastar!
60 ¡Mierda!
61 ¡Puedes correr, pero no te puedes esconder!
62 ¡Sos cerdo!
63 ¡Está en la trampa!
64 ¡Ah, que madre!
65 ¡Vámonos!
```

```
63 ¡Ándale!
64 ¡Cabrón!
65 ¡Coño!
66 ¡Agárrenlo!
67 Cógerlo, Cógerlo...
68 ¡Allí está, máta!
69 ¡No dejas que se escape de la isla vivo!
70 ¡Hasta luego!
71 ¡Rápido, es un intruso!
```

14.2

```
1 /*****
2 L'Internationale,
3     Sera le genre humain.
4
5
6
7
8
9
10
11
12
13
14
15 \./
16 *****/
17 Вставай, проклятым заклеимённый,
18 Весь мир голодных и рабов!
19 Кипит наш разум возмущённый
20 И в смертный бой вести готов.
21 Весь мир насилия мы разрушим
22 До основания, а затем
23 Мы наш, мы новый мир построим, —
24 Кто был ничем, тот станет всем.
25
26 Chorus
27 Это есть наш последний
28 И решительный бой;
29 С Интернационалом
30 Воспрянет род людской!
31
32 Никто не даст нам избавленья:
33 Ни бог, ни царь и не герой!
34 Добьёмся мы освобожденья
35 Своею собственной рукой.
36 Чтоб свергнуть гнёт рукой умелой,
37 Отвоевать своё добро, —
38 Вдуйте горн и куйте смело,
39 Пока железо горячо!
40
41 Chorus
42
43 Довольно кровь сосать, вампиры,
44 Тьмой, налогом, нищетой!
45 У вас — вся власть, все блага мира,
46 А наше право — звук пустой !
47 Мы жизнь построим по-иному —
48 И вот наш лозунг боевой:
49 Вся власть народу трудовому!
50 А дармоедов всех долой!
51
```

```

88 //
89 //
90 //      < @\
91 //      \(\)\*\**\*/(\)/
92 //      \(\)\*\**\*/(\)/
93 //      u/u/u/****\u\u\u
94 //      u/u/u/****\u\u
95 //      |*|*|
96 //      |
97 //      /+--+\\
98 //      || 卐 ||
99 //      \\==//
100 //
101 //      神獸保佑 永無BUG

```

68	//	##	#####
69	//	##	##
70	//	##	##
71	//	##	##
72	//	##	##
73	//	##	##
74	//	##	##
75	//	##	##
76	//	#####	#####
77	//		##
78	//		##
79	//		##
80	//		##
81	//		##
82	//		##
83	//		##
84	//	#####	##
85	//		
86	//	元首保佑 永無BUG	

ACM ICPC TEAM REFERENCE - ANGRY CROW TAKES FLIGHT!

Contents

1 Computational_Geometry	1	3 Flow	6	6.8 Lucas	12	10.4 input	17
1.1 delaunay	1	3.1 dinic	6	6.9 Matrix	12	11 language	17
1.2 Geometry	1	3.2 Gomory_Hu	6	6.10 MillerRobin	12	11.1 CNF	17
1.3 SmallestCircle	3	3.3 ISAP_with_cut	6	6.11 NTT	12	12 other	17
1.4 最近點對	3	3.4 MinCostMaxFlow	6	6.12 Simpson	13	12.1 WhatDay	17
2 Data_Structure	3	4 Graph	7	6.13 外星模運算	13	12.2 上下最大正方形	18
2.1 CDQ_DP	3	4.1 Augmenting_Path	7	6.14 數位統計	13	12.3 最大矩形	18
2.2 DLX	4	4.2 Augmenting_Path_multiple	7	7 String	13	13 zformula	18
2.3 Dynamic_KD_tree	4	4.3 blossom_matching	7	7.1 AC 自動機	13	13.1 formula	18
2.4 kd_tree_replace_segment_tree	5	4.4 BronKerbosch	7	7.2 hash	14	13.1.1 Pick 公式	18
2.5 reference_point	5	4.5 graphISO	7	7.3 KMP	14	13.1.2 圖論	18
2.6 skew_heap	5	4.6 KM	8	7.4 manacher	14	13.1.3 dinic 特殊圖複雜度	18
2.7 undo_disjoint_set	5	4.7 MaximumClique	8	7.5 minimal_string_rotation	14	13.1.4 0-1 分數規劃	18
2.8 整體二分	6	4.8 MinimumMeanCycle	8	7.6 reverseBWT	14	13.1.5 學長公式	18
		4.9 Rectilinear_MST	8	7.7 suffix_array_lcp	14	13.1.6 基本數論	18
		4.10 treeISO	8	7.8 Z	14	13.1.7 排組公式	18
		4.11 一般圖最小權完美匹配	9	8 Tarjan	15	13.1.8 冪次, 冪次和	18
		4.12 全局最小割	9	8.1 dominator_tree	15	13.1.9 Burnside's lemma	19
		4.13 弦圖完美消除序列	9	8.2 tnfsb017_2_sat	15	13.1.10 Count on a tree	19
		4.14 最小斯坦納樹 DP	9	8.3 橋連通分量	15	13.2 java	19
		4.15 最小樹形圖_朱劉	9	8.4 雙連通分量 & 割點	15	13.2.1 文件操作	19
		4.16 穩定婚姻模板	10	9 Tree_problem	15	13.2.2 優先队列	19
		5 Linear_Programming	10	9.1 HeavyLight	15	13.2.3 Map	19
		5.1 simplex	10	9.2 LCA	16	13.2.4 sort	19
		6 Number_Theory	10	9.3 link_cut_tree	16	14	19
		6.1 basic	10	9.4 POJ_tree	16	14.1 ganadoQuote	19
		6.2 bit_set	11	10 default	17	14.2	19
		6.3 cantor_expansion	11	10.1 debug	17	14.3 保佑	20
		6.4 FFT	11	10.2 ext	17		
		6.5 find_real_root	11	10.3 IncStack	17		
		6.6 FWT	11				
		6.7 LinearCongruence	12				