Computational Geometr

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1.1 Geometry

```
1 const double PI=atan2(0.0,-1.0);
   template<typename T>
   struct point{
    T x, y;
    point(){}
    point(const T&x,const T&y):x(x),y(y){}
    point operator+(const point &b)const{
      return point(x+b.x,y+b.y); }
    point operator-(const point &b)const{
      return point(x-b.x,y-b.y); }
    point operator*(const T &b)const{
      return point(x*b,v*b): }
    point operator/(const T &b)const{
13
      return point(x/b,v/b): }
14
15
    bool operator==(const point &b)const{
      return x==b.x&&y==b.y; }
16
17
    T dot(const point &b)const{
18
      return x*b.x+y*b.y; }
19
    T cross(const point &b)const{
20
      return x*b.y-y*b.x; }
    point normal()const{//求法向量
      return point(-y,x); }
23
    T abs2()const{//向量長度的平方
      return dot(*this); }
24
    T rad(const point &b)const{//兩向量的弧度
   return fabs(atan2(fabs(cross(b)),dot(b))); }
    T getA()const{//對x軸的弧度
      T A=atan2(y,x);//超過180度會變負的
29
      if(A<=-PI/2)A+=PI*2;
30
      return A;
   template<typename T>
   struct line{
    line(){}
    point<T> p1,p2;
    T a,b,c;//ax+by+c=0
    line(const point<T>&x,const point<T>&y):p1
         (x),p2(y){}
    void pton(){//轉成一般式
39
40
      a=p1.v-p2.v:
      b=p2.x-p1.x;
41
42
      c=-a*p1.x-b*p1.y;
43
                                               100
                                               101
    T ori(const point<T> &p)const{
   //點和有向直線的關係·>0左邊、=0在線上<0右邊
      return (p2-p1).cross(p-p1);
                                               103
                                               104
47
    T btw(const point<T> &p)const{
                                               105
      //點投影落在線段上<=0
                                               106
49
                                               107
50
      return (p1-p).dot(p2-p);
51
    bool point_on_segment(const point<T>&p)
52
                                               108
         const{//點是否在線段上
                                               109
                                               110
      return ori(p)==0&&btw(p)<=0;</pre>
53
                                               111
54
                                               112
    T dis2(const point<T> &p,bool is_segment
         =0) const { // 點 跟 直 線 / 線 段 的 距 離 平 方
```

```
point<T> v=p2-p1,v1=p-p1;
   if(is segment){
                                            114
     point<T> v2=p-p2;
                                            115
     if(v.dot(v1)<=0)return v1.abs2();</pre>
                                            116
     if(v.dot(v2)>=0)return v2.abs2();
                                            117
                                            118
   T tmp=v.cross(v1):
   return tmp*tmp/v.abs2();
                                            120
 T seg dis2(const line<T> &1)const{
   //兩線段距離平方
   return min({dis2(1.p1,1),dis2(1.p2,1),1. 124
        dis2(p1,1),1.dis2(p2,1)});
                                            125
                                            126
 point<T> projection(const point<T> &p)
      const { // 點 對 直 線 的 投 影
                                            127
   point<T> n=(p2-p1).normal();
                                            128
   return p-n*(p-p1).dot(n)/n.abs2();
                                            129
 point<T> mirror(const point<T> &p)const{
   //點對直線的鏡射·要先呼叫pton轉成一般式 132
   point<T> R:
   T d=a*a+b*b:
   R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d; 134
   R.v = (a*a*p.v-b*b*p.v-2*a*b*p.x-2*b*c)/d: 135
                                            137
                                           138
 bool equal(const line &1)const{//直線相等
   return ori(1.p1)==0&&ori(1.p2)==0;
                                            139
 bool parallel(const line &1)const{
   return (p1-p2).cross(l.p1-l.p2)==0;
                                            142
 bool cross seg(const line &1)const{
                                            143
   return (p2-p1).cross(l.p1-p1)*(p2-p1).
        cross(1.p2-p1)<=0;//直線是否交線段
 int line_intersect(const line &l)const{
                                           145
//直線相交情況·-1無限多點、1交於一點、0不相 146
   return parallel(1)?(ori(1.p1)==0?-1:0)
                                            148
                                            149
                                            150
 int seg intersect(const line &1)const{
   T c1=ori(l.p1), c2=ori(l.p2);
                                            151
   T c3=1.ori(p1), c4=1.ori(p2);
                                            152
   if(c1==0&&c2==0){//共線
     bool b1=btw(1.p1)>=0,b2=btw(1.p2)>=0;
     T a3=1.btw(p1),a4=1.btw(p2);
     if(b1&&b2&&a3==0&&a4>=0) return 2;
     if(b1&&b2&&a3>=0&&a4==0) return 3;
                                            155
     if(b1&&b2&&a3>=0&&a4>=0) return 0;
                                            156
                                            157
     return -1://無限交點
   }else if(c1*c2<=0&&c3*c4<=0)return 1;</pre>
                                            158
   return 0://不相交
                                            159
                                            160
 point<T> line intersection(const line &1)
                                            161
      162
   point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
                                            163
   //if(a.cross(b)==0)return INF;
                                            164
   return p1+a*(s.cross(b)/a.cross(b));
                                            165
                                            166
 point<T> seg intersection(const line &1)
      const{//線段交點
```

```
int res=seg intersect(1);
                                                 167
       if(res<=0) assert(0);</pre>
       if(res==2) return p1;
                                                 168
       if(res==3) return p2;
                                                 169
       return line intersection(1);
                                                 170
119 };
   template<typename T>
   struct polygon{
                                                 172
     polygon(){}
                                                 173
     vector<point<T> > p;//逆時針順序
     T area()const{//面積
                                                 174
       T ans=0:
       for(int i=p.size()-1,j=0;j<(int)p.size()</pre>
            :i=i++)
                                                 177
          ans+=p[i].cross(p[j]);
                                                 178
       return ans/2;
                                                 179
                                                 180
     point<T> center of mass()const{//重心
                                                 181
       T cx=0.cv=0.w=0:
       for(int i=p.size()-1,j=0;j<(int)p.size() ^{182}
            :i=i++){
         T a=p[i].cross(p[j]);
                                                 183
          cx+=(p[i].x+p[j].x)*a;
                                                 184
          cy+=(p[i].y+p[j].y)*a;
          w+=a;
                                                 185
                                                 186
       return point<T>(cx/3/w,cy/3/w);
                                                 187
                                                 188
     char ahas(const point<T>& t)const{
    //點是否在簡單多邊形內,是的話回傳1、在邊上
                                                 189
        回傳-1、否則回傳0
                                                 190
       bool c=0:
       for(int i=0,j=p.size()-1;i<p.size();j=i</pre>
         if(line<T>(p[i],p[j]).point_on_segment 193
              (t))return -1;
                                                 195
          else if((p[i].y>t.y)!=(p[j].y>t.y)&&
          t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j]
              ].y-p[i].y)+p[i].x)
                                                 197
           c=!c;
                                                 198
       return c;
                                                 199
                                                 200
     char point in convex(const point<T>&x)
          const{
                                                 201
       int l=1,r=(int)p.size()-2;
       while(l<=r){//點是否在凸多邊形內,是的話 ^{202}
                                                 203
             回傳1、在邊上回傳-1、否則回傳0
                                                 204
          int mid=(1+r)/2;
                                                 205
         T a1=(p[mid]-p[0]).cross(x-p[0]);
                                                 206
         T a2=(p[mid+1]-p[0]).cross(x-p[0]);
                                                 207
          if(a1>=0&&a2<=0){
           T res=(p[mid+1]-p[mid]).cross(x-p[
                mid]);
                                                 210
            return res>0?1:(res>=0?-1:0);
                                                 211
         }else if(a1<0)r=mid-1:</pre>
         else l=mid+1;
                                                 ^{212}
                                                 213
       return 0;
                                                 214
                                                 ^{215}
     vector<T> getA()const{//凸包邊對x軸的夾角
                                                 216
       vector<T>res://一定是遞增的
       for(size_t i=0;i<p.size();++i)</pre>
```

```
res.push back((p[(i+1)%p.size()]-p[i])
         .getA());
 return res;
bool line intersect(const vector<T>&A,
     const line<T> &1)const{//O(LogN)
  int f1=upper bound(A.begin(),A.end(),(1.
      p1-1.p2).getA())-A.begin();
  int f2=upper bound(A.begin(), A.end(),(1.
      p2-1.p1).getA())-A.begin();
 return 1.cross seg(line<T>(p[f1],p[f2]))
polvgon cut(const line<T> &1)const{
//凸包對直線切割,得到直線1左側的凸包
 polygon ans;
  for(int n=p.size(),i=n-1,j=0;j<n;i=j++){</pre>
   if(1.ori(p[i])>=0){
      ans.p.push back(p[i]);
     if(1.ori(p[j])<0)
        ans.p.push back(1.
            line intersection(line<T>(p[i
            1,p[i])));
   }else if(1.ori(p[j])>0)
      ans.p.push back(1.line intersection(
          line<T>(p[i],p[j])));
 return ans;
static bool graham cmp(const point<T>& a,
    const point<T>& b){//凸包排序函數
  return (a.x<b.x)||(a.x==b.x&&a.y<b.y);</pre>
void graham(vector<point<T> > &s){// □ 包
  sort(s.begin(),s.end(),graham_cmp);
 p.resize(s.size()+1);
  int m=0:
  for(size t i=0;i<s.size();++i){</pre>
   while (m>=2\&\&(p[m-1]-p[m-2]). cross(s[i
        ]-p[m-2])<=0)--m;
   p[m++]=s[i];
  for(int i=s.size()-2,t=m+1;i>=0;--i){
   while (m>=t&&(p[m-1]-p[m-2]).cross(s[i
        ]-p[m-2])<=0)--m;
   p[m++]=s[i];
 if(s.size()>1)--m;
 p.resize(m);
T diam(){//直徑
  int n=p.size(),t=1;
 T ans=0;p.push_back(p[0]);
  for(int i=0;i<n;i++){</pre>
   point<T> now=p[i+1]-p[i];
    while(now.cross(p[t+1]-p[i])>now.cross
         (p[t]-p[i]))t=(t+1)%n;
   ans=max(ans,(p[i]-p[t]).abs2());
 return p.pop_back(),ans;
T min cover rectangle(){//最小覆蓋矩形
 int n=p.size(),t=1,r=1,l;
  if(n<3)return 0;//也可以做最小周長矩形
 T ans=1e99; p.push back(p[0]);
```

```
for(int i=0;i<n;i++){</pre>
                                                   271
          point<T> now=p[i+1]-p[i];
221
                                                   272
          while(now.cross(p[t+1]-p[i])>now.cross 273
222
               (p[t]-p[i])) t=(t+1)%n;
          while(now.dot(p[r+1]-p[i])>now.dot(p[r 275
223
               ]-p[i])) r=(r+1)%n;
                                                   276
224
          if(!i)l=r:
                                                   277
          while (now.dot(p[l+1]-p[i]) < =now.dot(p[278])
225
               1]-p[i])) l=(l+1)%n;
          T d=now.abs2();
226
227
          T tmp=now.cross(p[t]-p[i])*(now.dot(p[280]
               r]-p[i])-now.dot(p[l]-p[i]))/d;
228
          ans=min(ans,tmp);
                                                   282
229
                                                   283
230
       return p.pop back(),ans;
                                                   284
231
                                                   285
                                                   286
     T max_triangle(){//最大內接三角形
232
                                                   287
        int n=p.size(),a=1,b=2;
233
                                                   288 };
       if(n<3)return 0;</pre>
234
235
       T ans=0,tmp;p.push_back(p[0]);
236
        for(int i=0;i<n;++i){</pre>
          while((p[a]-p[i]).cross(p[b+1]-p[i])>( 291
237
               tmp=(p[a]-p[i]).cross(p[b]-p[i])))^{292}
               b=(b+1)%n:
238
          ans=max(ans,tmp);
          while((p[a+1]-p[i]).cross(p[b]-p[i])>( 294
239
               tmp=(p[a]-p[i]).cross(p[b]-p[i])))^{295}
               a=(a+1)%n;
          ans=max(ans,tmp);
                                                   297
241
                                                   298
       return p.pop_back(),ans/2;
242
                                                   299
243
                                                   300
244
     T dis2(polygon &pl){//凸包最近距離平方
                                                   301
       vector<point<T> > &P=p,&Q=pl.p;
245
                                                   302
246
       int n=P.size(),m=Q.size(),l=0,r=0;
                                                   303
     for(int i=0;i<n;++i)if(P[i].y<P[1].y)l=i;</pre>
247
                                                   304
248
     for(int i=0;i<m;++i)if(Q[i].y<Q[r].y)r=i;</pre>
       P.push_back(P[0]),Q.push_back(Q[0]);
249
                                                   305
       T ans=1e99:
250
                                                   306
       for(int i=0;i<n;++i){</pre>
251
252
          while((P[1]-P[1+1]).cross(Q[r+1]-Q[r]) 307
               <0) r=(r+1)%m;
          ans=min(ans,line<T>(P[1],P[1+1]).
253
                                                   309
               seg dis2(line<T>(0[r],0[r+1])));
                                                  310
          l=(1+1)%n;
254
255
256
        return P.pop back(),Q.pop back(),ans;
257
                                                   312
258
     static char sign(const point<T>&t){
                                                   313
259
       return (t.y==0?t.x:t.y)<0;</pre>
                                                   314
260
                                                   315
     static bool angle cmp(const line<T>& A,
261
                                                   316 };
           const line<T>& B){
        point<T> a=A.p2-A.p1,b=B.p2-B.p1;
262
       return sign(a)<sign(b)||(sign(a)==sign(b 319
263
             )&&a.cross(b)>0):
264
265
     int halfplane intersection(vector<line<T>
          > &s){//半平面交
266
        sort(s.begin(),s.end(),angle_cmp);//線段
                                                   323
             左側為該線段半平面
                                                   325
        int L,R,n=s.size();
267
                                                   326
268
        vector<point<T> > px(n);
                                                   327
        vector<line<T> > a(n):
269
                                                   328
       q[L=R=0]=s[0];
```

```
for(int i=1;i<n;++i){</pre>
          while(L<R&&s[i].ori(px[R-1])<=0)--R;</pre>
                                                  330
          while(L<R&&s[i].ori(px[L])<=0)++L;</pre>
                                                   331
          q[++R]=s[i];
                                                   332
          if(q[R].parallel(q[R-1])){
                                                   333
                                                   334
            if(q[R].ori(s[i].p1)>0)q[R]=s[i];
                                                   335
          if(L<R)px[R-1]=q[R-1].
               line intersection(q[R]);
                                                   337
                                                   338
        while(L<R&&a[L].ori(px[R-1])<=0)--R:
                                                   339
        p.clear();
                                                   340
        if(R-L<=1)return 0:</pre>
                                                   341
        px[R]=q[R].line intersection(q[L]);
        for(int i=L;i<=R;++i)p.push_back(px[i]); 343 template<typename T>
        return R-L+1:
                                                   344
                                                   345
                                                   346
    template<typename T>
                                                   347
    struct triangle{
      point<T> a.b.c:
                                                   348
      triangle(){}
      triangle(const point<T> &a,const point<T>
          &b, const point<T> &c):a(a),b(b),c(c){}_{350}^{349}
     T area()const{
                                                   351
       T t=(b-a).cross(c-a)/2;
                                                   352
        return t>0?t:-t:
                                                   353
                                                   354
      point<T> barycenter()const{//重心
                                                   355
       return (a+b+c)/3;
                                                   356
                                                   357
      point<T> circumcenter()const{//外心
                                                   358
       static line<T> u,v;
       u.p1=(a+b)/2;
                                                   359
       u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x- 360
             b.x);
        v.p1=(a+c)/2;
                                                   362
       v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-
                                                   363
        return u.line_intersection(v);
                                                   364
      point<T> incenter()const{//內心
       T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2 366
             ()),C=sqrt((a-b).abs2());
        return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+ 368
             B*b.y+C*c.y)/(A+B+C);
                                                   369
      point<T> perpencenter()const{//垂心
        return barycenter()*3-circumcenter()*2; 370
317 template<typename T>
318 struct point3D{
     T x,y,z;
                                                   375
      point3D(){}
      point3D(const T&x,const T&y,const T&z):x(x
          ),y(y),z(z){}
                                                   377
                                                  378
      point3D operator+(const point3D &b)const{
        return point3D(x+b.x,y+b.y,z+b.z);}
                                                   380
      point3D operator-(const point3D &b)const{
                                                   381
        return point3D(x-b.x,y-b.y,z-b.z);}
                                                   382
      point3D operator*(const T &b)const{
       return point3D(x*b,y*b,z*b);}
```

```
point3D operator/(const T &b)const{
                                                 383
       return point3D(x/b,y/b,z/b);}
                                                 384
     bool operator==(const point3D &b)const{
                                                 385
       return x==b.x&&y==b.y&&z==b.z;}
     T dot(const point3D &b)const{
                                                 386
       return x*b.x+y*b.y+z*b.z;}
     point3D cross(const point3D &b)const{
                                                 387
       return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x
            *b.y-y*b.x);}
                                                 388
     T abs2()const{//向量長度的平方
                                                 389
       return dot(*this);}
     T area2(const point3D &b)const{
                                                 390
       //和b、原點圍成面積的平方
                                                 391
       return cross(b).abs2()/4;}
342 };
                                                 392
   struct line3D{
                                                 393
     point3D<T> p1,p2;
     line3D(){}
     line3D(const point3D<T> &p1,const point3D<</pre>
                                                 306
          T> &p2):p1(p1),p2(p2){}
     T dis2(const point3D<T> &p,bool is_segment 398
          =0) const { // 點 跟 直 線 / 線 段 的 距 離 平 方
       point3D<T> v=p2-p1,v1=p-p1;
       if(is segment){
         point3D<T> v2=p-p2;
          if(v.dot(v1)<=0)return v1.abs2();</pre>
         if(v.dot(v2)>=0)return v2.abs2();
                                                 402
                                                 403
       point3D<T> tmp=v.cross(v1):
       return tmp.abs2()/v.abs2();
                                                 404
     pair<point3D<T>,point3D<T> > closest pair(
                                                 405
          const line3D<T> &1)const{
       point3D<T> v1=(p1-p2), v2=(1.p1-1.p2);
       point3D<T> N=v1.cross(v2),ab(p1-l.p1);
       //if(N.abs2()==0)return NULL;平行或重合
       T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//
             最折點對距離
       point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.
            cross(d2),G=1.p1-p1;
       T t1=(G.cross(d2)).dot(D)/D.abs2();
                                                 411
       T t2=(G.cross(d1)).dot(D)/D.abs2();
       return make pair(p1+d1*t1,1.p1+d2*t2);
                                                 413
                                                 414
     bool same side(const point3D<T> &a,const
                                                 415
          point3D<T> &b)const{
                                                 416
       return (p2-p1).cross(a-p1).dot((p2-p1).
            cross(b-p1))>0;
                                                 418
                                                 419
   template<typename T>
                                                 420
   struct plane{
                                                 421
     point3D<T> p0,n;//平面上的點和法向量
     plane(){}
                                                 423
     plane(const point3D<T> &p0, const point3D<T 424
          > &n):p0(p0),n(n){}
                                                 425
     T dis2(const point3D<T> &p)const{
                                                 426
       //點到平面距離的平方
                                                 427
       T tmp=(p-p0).dot(n);
                                                 428
       return tmp*tmp/n.abs2():
                                                 430
     point3D<T> projection(const point3D<T> &p) 431
          const{
```

```
return p-n*(p-p0).dot(n)/n.abs2();
     point3D<T> line intersection(const line3D
         T> &1)const{
       T tmp=n.dot(1.p2-1.p1);//等於0表示平行或
            重合該平面
       return 1.p1+(1.p2-1.p1)*(n.dot(p0-1.p1)/
            tmp);
     line3D<T> plane intersection(const plane &
         pl)const{
       point3D<T> e=n.cross(pl.n),v=n.cross(e);
       T tmp=pl.n.dot(v);//等於0表示平行或重合
       point3D<T> q=p0+(v*(pl.n.dot(pl.p0-p0))/
           tmp);
       return line3D<T>(q,q+e);
394
   template<typename T>
397
   struct triangle3D{
     point3D<T> a,b,c;
     triangle3D(){}
     triangle3D(const point3D<T> &a,const
          point3D<T> &b, const point3D<T> &c):a(a
          ),b(b),c(c){}
     bool point in(const point3D<T> &p)const{
       //點在該平面上的投影在三角形中
       return line3D<T>(b,c).same_side(p,a)&&
           line3D<T>(a,c).same_side(p,b)&&
           line3D<T>(a,b).same side(p,c);
406
   template<typename T>
   struct tetrahedron{//四面體
     point3D<T> a,b,c,d;
     tetrahedron(){}
     tetrahedron(const point3D<T> &a.const
          point3D<T> &b, const point3D<T> &c,
          const point3D<T> &d):a(a),b(b),c(c),d(
         d){}
     T volume6()const{//體積的六倍
       return (d-a).dot((b-a).cross(c-a));
     point3D<T> centroid()const{
       return (a+b+c+d)/4;
     bool point in(const point3D<T> &p)const{
       return triangle3D<T>(a,b,c).point in(p)
           &&triangle3D<T>(c,d,a).point in(p);
   };
   template<typename T>
   struct convexhull3D{
     static const int MAXN=1005;
     struct face{
       int a,b,c;
       face(int a,int b,int c):a(a),b(b),c(c){}
     vector<point3D<T>> pt;
     vector<face> ans:
     int fid[MAXN][MAXN];
     void build(){
       int n=pt.size();
```

```
ans.clear();
434
        memset(fid,0,sizeof(fid));
                                                     17
        ans.emplace back(0,1,2);//注意不能共線
435
        ans.emplace_back(2,1,0);
                                                     19
436
        int ftop = \overline{0};
                                                     20
437
        for(int i=3, ftop=1; i<n; ++i,++ftop){</pre>
                                                     21
438
                                                     22
          vector<face> next;
439
          for(auto &f:ans){
                                                     23 }
440
            T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[
441
                 f.a]).cross(pt[f.c]-pt[f.a]));
            if(d<=0) next.push back(f);</pre>
442
443
            int ff=0;
            if(d>0) ff=ftop;
444
445
            else if(d<0) ff=-ftop;</pre>
            fid[f.a][f.b]=fid[f.b][f.c]=fid[f.c
446
                                                      1 template < typename IT = point < T > * >
                 ][f.a]=ff;
447
448
          for(auto &f:ans){
            if(fid[f.a][f.b]>0 && fid[f.a][f.b
449
                 ]!=fid[f.b][f.a])
              next.emplace back(f.a,f.b,i);
            if(fid[f.b][f.c]>0 && fid[f.b][f.c
                 ]!=fid[f.c][f.b])
              next.emplace back(f.b,f.c,i);
453
            if(fid[f.c][f.a]>0 && fid[f.c][f.a
                                                     10
                 ]!=fid[f.a][f.c])
                                                     11
              next.emplace back(f.c,f.a,i);
                                                     12
455
                                                     13
456
          ans=next;
                                                     14
457
                                                     15
458
                                                     16
459
     point3D<T> centroid()const{
                                                     17
        point3D<T> res(0,0,0);
460
                                                     18
        T vol=0;
461
                                                     19
462
        for(auto &f:ans){
          T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c 21
463
          res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
464
465
          vol+=tmp;
466
467
        return res/(vol*4);
468
469 };
```

1.2 SmallestCircle

1.3 最近點對

r2=(p[j]-c).abs2();

r2=(p[i]-c).abs2();

for(int k=0;k<j;k++)if((p[k]-c).abs2()>r2){

c=circumcenter(p[i],p[j],p[k]);

```
2 T cloest_pair(_IT L, _IT R){
    if(R-L <= 1) return INF;</pre>
     IT mid = L+(R-L)/2;
     T x = mid -> x:
    T d = min(cloest_pair(L,mid),cloest_pair(
          mid.R)):
     inplace_merge(L, mid, R, ycmp);
     static vector<point> b; b.clear();
     for(auto u=L;u<R;++u){</pre>
      if((u->x-x)*(u->x-x)>=d) continue;
       for(auto v=b.rbegin();v!=b.rend();++v){
        T^{dx=u-x-v-x}, dy=u-y-v-y;
         if(dy*dy>=d) break;
         d=min(d,dx*dx+dy*dy);
      b.push_back(*u);
    return d;
20 T closest pair(vector<point<T>> &v){
    sort(v.begin(),v.end(),xcmp);
    return closest_pair(v.begin(), v.end());
```

Data Structure

DLX2.1

```
1 const int MAXN=4100, MAXM=1030, MAXND=16390;
                                                  2 struct DLX{
1 using PT=point<T>; using CPT=const PT;
                                                       int n,m,sz,ansd;// 高 是 n , 寬 是 m 的 稀 疏 矩 陣
2 PT circumcenter(CPT &a,CPT &b,CPT &c){
                                                       int S[MAXM],H[MAXN];
    PT u=b-a, v=c-a:
                                                      int row[MAXND], col[MAXND]; //每個節點代表的
    T c1=u.abs2()/2,c2=v.abs2()/2;
    T d=u.cross(v);
    return PT(a.x+(v.y*c1-u.y*c2)/d,a.y+(u.x*
                                                       int L[MAXND],R[MAXND],U[MAXND],D[MAXND];
         c2-v.x*c1)/d);
                                                       vector<int> ans,anst;
                                                       void init(int _n,int _m){
  void solve(PT p[],int n,PT &c,T &r2){
                                                        n= n,m= m;
    random_shuffle(p,p+n);
                                                         for(int i=0;i<=m;++i){</pre>
                                                  10
                                                          U[i]=D[i]=i,L[i]=i-1,R[i]=i+1;
                                                  11
    c=p[0]; r2=0; // c, r2 = 圓心, 半徑平方
                                                          S[i]=0;
   for(int i=1;i<n;i++)if((p[i]-c).abs2()>r2){
                                                 12
      c=p[i]; r2=0;
                                                  13
                                                 14
                                                        R[m]=0,L[0]=m;
   for(int j=0;j<i;j++)if((p[j]-c).abs2()>r2){
         c.x=(p[i].x+p[j].x)/2;
                                                         sz=m, ansd=INT MAX; //ansd 存 最 優 解 的 個 數
14
                                                 15
         c.y=(p[i].y+p[j].y)/2;
                                                         for(int i=1;i<=n;++i)H[i]=-1;</pre>
```

```
++S[col[++sz]=c];
                                         72
  row[sz]=r;
                                         73
  D[sz]=D[c],U[D[c]]=sz,U[sz]=c,D[c]=sz;
                                         74
  if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
  else R[sz]=R[H[r]],L[R[H[r]]]=sz,L[sz]=H 76
      [r],R[H[r]]=sz;
#define DFOR(i,A,s) for(int i=A[s];i!=s;i=
void remove(int c){
                                         81
  //刪除第c行和所有當前覆蓋到第c行的列
                                         82
  L[R[c]]=L[c],R[L[c]]=R[c];//這裡刪除第c
      行,若有些行不需要處理可以在開始時呼
  DFOR(i,D,c)DFOR(j,R,i){U[D[j]]=U[j],D[U[
      j]]=D[j],--S[col[j]];}
void restore(int c){//恢復第c行和所有當前
     覆蓋到第c行的列, remove的逆操作
  DFOR(i,U,c)DFOR(j,L,i){++S[col[j]],U[D[j
      ]]=j,D[U[j]]=j;}
  L[R[c]]=c,R[L[c]]=c;
void remove2(int nd){//刪除nd所在的行當前
    所有點(包括虛擬節點),只保留nd
  DFOR(i,D,nd)L[R[i]]=L[i],R[L[i]]=R[i];
void restore2(int nd){//刪除nd所在的行當前
    所有點,為remove2的逆操作
  DFOR(i,U,nd)L[R[i]]=R[L[i]]=i;
bool vis[MAXM];
int h(){//估價函數 for IDA*
  int res=0:
  memset(vis,0,sizeof(vis));
  DFOR(i,R,0)if(!vis[i]){
   vis[i]=1:
                                         11
   ++res:
    DFOR(j,D,i)DFOR(k,R,j)vis[col[k]]=1;
                                         13
                                         14
  return res;
                                         1.5
                                         16
bool dfs(int d){//for精確覆蓋問題
                                         17
  if(d+h()>=ansd)return 0;//找最佳解用,找
                                         18
      任意解可以刪掉
                                         20
  if(!R[0]){ansd=d; return 1;}
  int c=R[0];
                                         22
  DFOR(i,R,0)if(S[i]<S[c])c=i;</pre>
  remove(c);
  DFOR(i,D,c){
   ans.push_back(row[i]);
                                         25
   DFOR(j,R,i)remove(col[j]);
                                         26
   if(dfs(d+1))return 1;
                                         27
   ans.pop_back();
   DFOR(j,L,i)restore(col[j]);
  restore(c);
```

void add(int r,int c){

19

20

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66

67

return 0;

void dfs2(int d){//for最小重複覆蓋問題

if(d+h()>=ansd)return;

```
int c=R[0];
      DFOR(i,R,0)if(S[i]<S[c])c=i;</pre>
      DFOR(i,D,c){
        anst.push_back(row[i]);
        remove2(i);
        DFOR(j,R,i)remove2(j),--S[col[j]];
        dfs2(d+1);
        anst.pop back();
        DFOR(j,L,i)restore2(j),++S[col[j]];
        restore2(i);
    bool exact cover(){//解精確覆蓋問題
      return ans.clear(), dfs(0);
    void min_cover(){//解最小重複覆蓋問題
      anst.clear();//暫存用,答案還是存在ans裡
      dfs2(0);
    #undef DFOR
91 };
```

if(!R[0]){ansd=d;ans=anst;return;}

71

2.2 Dynamic KD tree

```
1| template<typename T,size_t kd>//有kd個維度
2 struct kd tree{
    struct point{
      T d[kd];
      T dist(const point &x)const{
        for(size t i=0;i<kd;++i)ret+=std::abs(</pre>
             d[i]-x.d[i]);
        return ret;
      bool operator == (const point &p){
        for(size t i=0;i<kd;++i)</pre>
          if(d[i]!=p.d[i])return 0;
      bool operator<(const point &b)const{</pre>
        return d[0]<b.d[0]:
   };
  private:
    struct node{
      node *1.*r:
      point pid;
      node(const point &p):1(0),r(0),pid(p),s
      ~node(){delete l,delete r;}
      void up()\{s=(1?1->s:0)+1+(r?r->s:0);\}
    }*root;
    const double alpha,loga;
    const T INF://記得要給INF,表示極大值
    int maxn;
    struct cmp{
      int sort id;
      bool operator()(const node*x,const node*
          v)const{
        return operator()(x->pid,y->pid);
```

return cmp(r,o)?r:o;

93

```
bool operator()(const point &x,const
                                                                                                   153 public:
                                                                                                                                                              if(r){
                                                  94
                                                                                                                                                                for(int i=0;i<kd;++i){</pre>
            point &y)const{
                                                       bool erase(node *&u,int k,const point &x){ 154
                                                                                                         kd tree(const T &INF, double a=0.75):root
                                                                                                                                                       17
                                                  95
         if(x.d[sort id]!=y.d[sort id])
                                                         if(!u)return 0;
                                                                                                                                                                  mi.d[i]=min(mi.d[i],r->mi.d[i]);
37
                                                  96
                                                                                                              (0),alpha(a),loga(log2(1.0/a)),INF(INF 18
           return x.d[sort id]<y.d[sort id];</pre>
                                                                                                                                                                  ma.d[i]=max(ma.d[i],r->ma.d[i]);
38
                                                  97
                                                         if(u->pid==x){
                                                                                                              ),maxn(1){}
         for(size t i=0;i<kd;++i)</pre>
                                                                                                         ~kd tree(){delete root;}
39
                                                  98
                                                           if(u->r);
                                                                                                                                                       20
                                                                                                         void clear(){delete root.root=0.maxn=1:}
           if(x.d[i]!=v.d[i])
                                                  99
                                                           else if(u->1) u->r=u->1, u->1=0:
                                                                                                   156
                                                                                                                                                       21
                                                                                                                                                                s+=r->s:
             return x.d[i]<y.d[i];</pre>
                                                 100
                                                           else{
                                                                                                   157
                                                                                                         void build(int n,const point *p){
41
                                                                                                                                                       22
                                                                                                           delete root, A.resize(maxn=n);
42
         return 0;
                                                 101
                                                             delete u;
                                                                                                   158
                                                                                                                                                       23
43
                                                 102
                                                             return u=0, 1;
                                                                                                   159
                                                                                                           for(int i=0;i<n;++i)A[i]=new node(p[i]);</pre>
                                                                                                                                                           void up2(){/*其他懒惰標記向上更新*/}
44
     }cmp;
                                                 103
                                                                                                   160
                                                                                                           root=build(0,0,n-1);
                                                                                                                                                           void down(){/*其他懶惰標記下推*/}
     int size(node *o){return o?o->s:0:}
                                                 104
                                                           --u->s:
                                                                                                   161
                                                                                                                                                         }*root:
                                                           cmp.sort id=k;
     std::vector<node*> A;
                                                                                                         void insert(const point &x){
                                                 105
                                                                                                   162
                                                                                                                                                         //檢查區間包含用的函數
    node* build(int k,int l,int r){
                                                           u \rightarrow pid = findmin(u \rightarrow r, (k+1)%kd) \rightarrow pid:
                                                 106
                                                                                                   163
                                                                                                           insert(root,0,x,__lg(size(root))/loga);
                                                                                                                                                         bool range include(node *o,const point &L,
48
       if(1>r) return 0:
                                                 107
                                                           return erase(u->r,(k+1)%kd,u->pid);
                                                                                                   164
                                                                                                           if(root->s>maxn)maxn=root->s:
                                                                                                                                                              const point &R){
49
       if(k==kd) k=0;
                                                 108
                                                                                                   165
                                                                                                                                                            for(int i=0;i<kd;++i){</pre>
       int mid=(1+r)/2:
                                                         cmp.sort id=k:
                                                                                                         bool erase(const point &p){
50
                                                 109
                                                                                                   166
                                                                                                                                                             if(L.d[i]>o->ma.d[i]||R.d[i]<o->mi.d[i])
       cmp.sort_id = k;
                                                         if(erase(cmp(x,u->pid)?u->1:u->r,(k+1)%
                                                                                                           bool d=erase(root,0,p);
51
                                                 110
                                                                                                                                                                  return 0;
       std::nth element(A.begin()+1,A.begin()+
                                                                                                           if(root&&root->s<alpha*maxn)rebuild();</pre>
                                                              kd,x))
                                                                                                   168
                                                                                                                                                           }//(L,R)區間有和o的區間有交集就回傳true
            mid,A.begin()+r+1,cmp);
                                                           return --u->s, 1;
                                                 111
                                                                                                   169
                                                                                                           return d:
                                                                                                                                                       32
                                                                                                                                                            return 1;
       node *ret=A[mid];
53
                                                 112
                                                         return 0;
                                                                                                   170
                                                                                                                                                       33
       ret \rightarrow l = build(k+1,l,mid-1):
                                                                                                         void rebuild(){
54
                                                 113
                                                                                                   171
                                                                                                                                                         bool range in range(node *o,const point &L,
                                                       T heuristic(const T h[])const{
55
       ret->r = build(k+1.mid+1.r):
                                                 114
                                                                                                   172
                                                                                                           if(root)rebuild(root.0);
                                                                                                                                                               const point &R){
56
       ret->up();
                                                 115
                                                                                                           maxn=root->s;
                                                                                                   173
                                                                                                                                                            for(int i=0;i<kd;++i){</pre>
                                                         for(size t i=0;i<kd;++i)ret+=h[i];</pre>
57
       return ret:
                                                 116
                                                                                                   174
                                                                                                                                                             if(L.d[i]>o->mi.d[i]||o->ma.d[i]>R.d[i])
58
                                                 117
                                                         return ret:
                                                                                                   175
                                                                                                         T nearest(const point &x,int k){
                                                                                                                                                                  return 0;
59
    bool isbad(node*o){
                                                 118
                                                                                                   176
                                                                                                                                                           }//(L,R)區間完全包含o的區間就回傳true
60
       return size(o->1)>alpha*o->s||size(o->r) 119
                                                       int aM:
                                                                                                   177
                                                                                                           T mndist=INF,h[kd]={};
                                                                                                                                                           return 1:
           >alpha*o->s;
                                                       std::priority_queue<std::pair<T,point > >
                                                                                                           nearest(root,0,x,h,mndist);
                                                                                                                                                       39
61
                                                                                                           mndist=pQ.top().first;
                                                                                                                                                         bool point_in_range(node *o,const point &L,
                                                       void nearest(node *u,int k,const point &x, 180
62
     void flatten(node *u, typename std::vector< 121</pre>
                                                                                                           pQ=std::priority queue<std::pair<T,point
                                                                                                                                                      40
                                                                                                                                                               const point &R){
                                                            T *h,T &mndist){
          node*>::iterator &it){
                                                                                                                 > >();
                                                                                                                                                            for(int i=0;i<kd;++i){</pre>
       if(!u)return:
                                                         if(u==0||heuristic(h)>=mndist)return;
63
                                                                                                           return mndist://回傳離x第k近的點的距離
                                                                                                   181
                                                                                                                                                             if(L.d[i]>o->pid.d[i]||R.d[i]<o->pid.d[i
                                                         T dist=u->pid.dist(x),old=h[k];
       flatten(u->1,it);
                                                 123
64
                                                                                                   182
                                                                                                                                                                  ])return 0;
       *it=u;
                                                         /*mndist=std::min(mndist,dist);*/
65
                                                 124
                                                                                                         const std::vector<point> &range(const
                                                                                                                                                           }//(L.R) 區間完全包含o->pid這個點就回傳true
       flatten(u->r,++it);
                                                 125
                                                         if(dist<mndist){</pre>
66
                                                                                                              point&mi,const point&ma){
67
                                                 126
                                                           pQ.push(std::make_pair(dist,u->pid));
                                                                                                                                                       44
                                                                                                                                                           return 1:
                                                                                                           in range.clear();
                                                           if((int)pQ.size()==qM+1)
                                                                                                                                                       45
68
     void rebuild(node*&u,int k){
                                                 127
                                                                                                           range(root,0,mi,ma);
69
       if((int)A.size()<u->s)A.resize(u->s);
                                                 128
                                                             mndist=pQ.top().first,pQ.pop();
                                                                                                                                                       46 //單點修改,以單點改值為例
                                                                                                           return in range;//回傳介於mi到ma之間的點
                                                                                                   186
       typename std::vector<node*>::iterator it 129
                                                                                                                                                         void update(node *u,const point &x,int data,
                                                                                                                vector
                                                         if(x.d[k]<u->pid.d[k]){
            =A.begin();
                                                 130
                                                                                                                                                              int k=0){
                                                                                                   187
       flatten(u,it);
                                                           nearest(u->1,(k+1)%kd,x,h,mndist);
                                                 131
                                                                                                                                                            if(!u)return;
71
                                                                                                   188
                                                                                                         int size(){return root?root->s:0;}
                                                           h[k]=std::abs(x.d[k]-u->pid.d[k]);
       u=build(k,0,u->s-1);
72
                                                 132
                                                                                                                                                           u->down();
73
                                                           nearest(u->r,(k+1)%kd,x,h,mndist);
                                                 133
                                                                                                                                                           if(u->pid==x){
    bool insert(node*&u,int k,const point &x,
                                                 134
                                                                                                                                                             u->data=data;
                                                           nearest(u->r,(k+1)%kd,x,h,mndist);
          int dep){
                                                 135
                                                                                                                                                             u->up2();
75
       if(!u) return u=new node(x), dep<=0;</pre>
                                                           h[k]=std::abs(x.d[k]-u->pid.d[k]);
                                                 136
                                                                                                                                                             return;
                                                                                                       2.3 kd tree replace segment 534
76
       ++u->s;
                                                           nearest(u->1,(k+1)%kd,x,h,mndist);
                                                 137
       cmp.sort id=k;
                                                                                                                                                            cmp.sort id=k;
       if(insert(cmp(x,u->pid)?u->1:u->r,(k+1)% 139
                                                         h[k]=old;
                                                                                                                                                            update(cmp(x,u->pid)?u->1:u->r,x,data,(k
            kd,x,dep-1)){
                                                                                                     1 struct node { //kd 樹代 替高維線段樹
                                                                                                                                                                +1)%kd);
         if(!isbad(u))return 1;
                                                       std::vector<point>in range;
                                                                                                         node *1,*r;
                                                                                                                                                           u->up2():
         rebuild(u,k);
                                                 142
                                                       void range(node *u,int k,const point&mi,
                                                                                                         point pid, mi, ma;
80
                                                                                                         int s, data;
                                                            const point&ma){
                                                                                                                                                         //區間修改
82
       return 0;
                                                 143
                                                         if(!u)return;
                                                                                                         node(const point &p,int d):1(0),r(0),pid(p
                                                                                                                                                         void update(node *o,const point &L,const
                                                         bool is=1;
                                                                                                              ),mi(p),ma(p),s(1),data(d),dmin(d),
                                                 144
                                                                                                                                                              point &R, int data){
     node *findmin(node*o,int k){
                                                 145
                                                         for(int i=0;i<kd;++i)</pre>
                                                                                                              dmax(d){}
                                                                                                                                                            if(!o)return;
       if(!o)return 0:
                                                           if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid
                                                                                                         void up(){
                                                 146
                                                                                                                                                           o->down();
       if(cmp.sort_id==k)return o->1?findmin(o
                                                                .d[i]){
                                                                                                           mi=ma=pid;
                                                                                                                                                           if(range in range(o,L,R)){
            ->1,(k+1)%kd):o;
                                                 147
                                                             is=0:break:
                                                                                                           s=1:
                                                                                                                                                             //區間懶惰標記修改
       node *l=findmin(o->l,(k+1)%kd);
                                                                                                           if(1){
                                                 148
                                                                                                                                                       65
                                                                                                                                                             o->down();
       node *r=findmin(o->r,(k+1)%kd);
                                                 149
                                                         if(is)in range.push back(u->pid);
                                                                                                              for(int i=0;i<kd;++i){</pre>
                                                                                                                                                       66
                                                                                                                                                             return:
       if(1&&!r)return cmp(1,o)?1:o;
                                                 150
                                                         if(mi.d[k] <= u -> pid.d[k]) range(u -> 1,(k+1))
                                                                                                               mi.d[i]=min(mi.d[i],1->mi.d[i]);
                                                                                                                                                       67
       if(!1&&r)return cmp(r,o)?r:o;
                                                                                                               ma.d[i]=max(ma.d[i],1->ma.d[i]);
                                                              %kd,mi,ma);
                                                                                                                                                           if(point_in_range(o,L,R)){
       if(!1&&!r)return o;
                                                         if(ma.d[k]>=u->pid.d[k])range(u->r,(k+1)
       if(cmp(1,r))return cmp(1,o)?1:o;
                                                              %kd,mi,ma);
                                                                                                              s+=1->s;
```

```
//這個點在(L,R)區間,但是他的左右子樹不
           一定在區間中
      //單點懶惰標記修改
71
    if(o->1&&range_include(o->1,L,R))update(o
72
         ->1,L,R,data);
    if(o->r&&range include(o->r,L,R))update(o
         ->r,L,R,data);
    o->up2();
75
   //區間查詢·以總和為例
  int query(node *o,const point &L,const point
        &R){
    if(!o)return 0;
    o->down();
    if(range_in_range(o,L,R))return o->sum;
    int ans=0;
    if(point_in_range(o,L,R))ans+=o->data;
    if(o->l&&range include(o->l,L,R))ans+=
         querv(o->1,L,R):
    if(o->r&&range_include(o->r,L,R))ans+=
         query(o->r,L,R);
    return ans;
86 }
```

2.4 reference point

```
1 template < typename T>
2 struct _RefC{
    T data:
    int ref;
    RefC(const T&d=0):data(d),ref(0){}
7 template<typename T>
   struct _rp{
     RefC<T> *p;
    T *operator->(){return &p->data;}
    T & operator*() { return p->data; }
    operator _RefC<T>*(){return p;}
13
    rp &operator=(const rp &t){
      if(p&&!--p->ref)delete p;
14
      p=t.p,p&&++p->ref;
15
      return *this;
16
17
    _rp(_RefC<T> *t=0):p(t){p&&++p->ref;}
    _rp(const _rp &t):p(t.p){p&&++p->ref;}
20
    ~_rp(){if(p&&!--p->ref)delete p;}
21
   template<typename T>
  inline _rp<T> new_rp(const T&nd){
    return _rp<T>(new _RefC<T>(nd));
24
25
```

$2.5 ext{ skew_heap}$

```
node *merge(node *a,node *b){
    if(!a||!b) return a?a:b;
    if(b->data<a->data) swap(a,b);
    swap(a->1,a->r);
```

```
5     a->l=merge(b,a->l);
6     return a;
7 }
```

2.6 undo disjoint set

```
1 | struct DisjointSet {
    // save() is like recursive
     // undo() is like return
     int n, fa[MXN], sz[MXN];
     vector<pair<int*,int>> h;
     vector<int> sp;
     void init(int tn) {
       for (int i=0; i<n; i++) sz[fa[i]=i]=1;</pre>
       sp.clear(); h.clear();
11
     void assign(int *k, int v) {
      h.PB({k, *k});
       *k=v:
15
16
     void save() { sp.PB(SZ(h)); }
     void undo() {
       assert(!sp.empty());
       int last=sp.back(); sp.pop back();
       while (SZ(h)!=last) {
21
         auto x=h.back(); h.pop_back();
22
         *x.F=x.S;
23
      }
24
     int f(int x) {
       while (fa[x]!=x) x=fa[x];
27
       return x;
28
     void uni(int x, int y) {
       x=f(x); y=f(y);
       if (x==y) return ;
31
32
       if (sz[x]<sz[y]) swap(x, y);</pre>
       assign(&sz[x], sz[x]+sz[y]);
34
       assign(&fa[y], x);
35
36 }djs;
```

2.7 整體二分

```
1 | void totBS(int L, int R, vector<Item> M){
2 | if(Q.empty()) return; //維護全域B陣列
3 | if(L=R) 整個M的答案=r, return;
4 | int mid = (L+R)/2;
5 | vector<Item> mL, mR;
6 | do_modify_B_with_divide(mid,M);
7 | //讓B陣列在遞迴的時候只會保留[L~mid]的資訊
8 | undo_modify_B(mid,M);
9 | totBS(L,mid,mL);
10 | totBS(mid+1,R,mR);
11 | }
```

3 Flow

3.1 dinic

```
1 template<typename T>
 2 struct DINIC{
     static const int MAXN=105;
     static const T INF=INT MAX;
     int n, LV[MAXN], cur[MAXN];
     struct edge{
       int v,pre;
       T cap,r;
       edge(int v,int pre,T cap):v(v),pre(pre),
            cap(cap),r(cap){}
     int g[MAXN];
11
     vector<edge> e;
     void init(int _n){
13
       memset(g,-1,sizeof(int)*((n=n)+1));
14
15
       e.clear():
16
                                                   11
     void add edge(int u,int v,T cap,bool
17
                                                   12
          directed=false){
                                                   13
       e.push_back(edge(v,g[u],cap));
18
                                                   14
       g[u]=e.size()-1;
                                                   15
19
       e.push_back(edge(u,g[v],directed?0:cap))
20
                                                   16
                                                   17
       g[v]=e.size()-1;
21
22
23
     int bfs(int s,int t){
24
       memset(LV,0,sizeof(int)*(n+1));
                                                   20
25
       memcpy(cur,g,sizeof(int)*(n+1));
26
       queue<int> q;
27
       q.push(s);
                                                   22
28
       LV[s]=1;
                                                   23
29
       while(q.size()){
                                                   24
         int u=q.front();q.pop();
30
         for(int i=g[u];~i;i=e[i].pre){
                                                   26
31
32
           if(!LV[e[i].v]&&e[i].r){
                                                   27
             LV[e[i].v]=LV[u]+1;
                                                   28
33
                                                   29
34
             q.push(e[i].v);
35
             if(e[i].v==t)return 1;
                                                   30
36
                                                   31
37
        }
                                                   32
38
       }
                                                   33
39
       return 0;
                                                   34
40
                                                   35
     T dfs(int u,int t,T CF=INF){
41
       if(u==t)return CF;
                                                   37
43
                                                   38
       for(int &i=cur[u];~i;i=e[i].pre){
         if(LV[e[i].v]==LV[u]+1&&e[i].r){
           if(df=dfs(e[i].v,t,min(CF,e[i].r))){
46
47
             e[i].r-=df;
             e[i^1].r+=df;
             return df;
52
53
       return LV[u]=0;
54
     T dinic(int s,int t,bool clean=true){
       if(clean)for(size t i=0;i<e.size();++i)</pre>
```

3.2 ISAP with cut

e[i].r=e[i].cap;

while(bfs(s,t))while(f=dfs(s,t))ans+=f;

T ans=0, f=0;

return ans;

59

60

61

```
1 template<typename T>
 struct ISAP{
    static const int MAXN=105;
   static const T INF=INT MAX;
    int n://點數
    int d[MAXN],gap[MAXN],cur[MAXN];
    struct edge{
     int v,pre;
      edge(int v,int pre,T cap):v(v),pre(pre),
          cap(cap),r(cap){}
   int g[MAXN];
    vector<edge> e;
    void init(int _n){
     memset(g, -1, sizeof(int)*((n= n)+1));
     e.clear();
    void add edge(int u,int v,T cap,bool
         directed=false){
     e.push_back(edge(v,g[u],cap));
     g[u]=e.size()-1;
     e.push_back(edge(u,g[v],directed?0:cap))
     g[v]=e.size()-1;
   T dfs(int u, int s, int t, T CF=INF){
     if(u==t)return CF;
     T tf=CF,df;
      for(int &i=cur[u];~i;i=e[i].pre){
       if(e[i].r&&d[u]==d[e[i].v]+1){
          df=dfs(e[i].v,s,t,min(tf,e[i].r));
          e[i].r-=df;
          e[i^1].r+=df;
          if(!(tf-=df)||d[s]==n)return CF-tf;
      int mh=n;
      for(int i=cur[u]=g[u];~i;i=e[i].pre){
       if(e[i].r&&d[e[i].v]<mh)mh=d[e[i].v];</pre>
      if(!--gap[d[u]])d[s]=n;
     else ++gap[d[u]=++mh];
     return CF-tf;
   T isap(int s,int t,bool clean=true){
      memset(d,0,sizeof(int)*(n+1));
     memset(gap,0,sizeof(int)*(n+1));
      memcpy(cur,g,sizeof(int)*(n+1));
     if(clean) for(size t i=0;i<e.size();++i)</pre>
       e[i].r=e[i].cap;
      T MF=0;
      for(gap[0]=n;d[s]<n;)MF+=dfs(s,s,t);</pre>
```

39

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54

55

56

57

58

59

60

61

62

63

64

65

66

67

68 };

bool modlabel(){

static deque<int>q;

for(int u=0;u<=n;++u)</pre>

TP mincost(int s,int t){

while(modlabel()){

while(augment(S,INF));

while(q.size()){

dis[T]=0,q.push back(T);

for(int u=0;u<=n;++u)dis[u]=INF;</pre>

int u=q.front();q.pop front();

for(int i=g[u];~i;i=e[i].pre){

q.front():S]){

for(int i=g[u];~i;i=e[i].pre)

return PIS+=dis[S], dis[S]<INF;</pre>

q.push front(e[i].v);

}else q.push back(e[i].v);

e[i].cost+=dis[e[i].v]-dis[u];

do memset(vis,0,sizeof(bool)*(n+1));

dis[e[i].v]){

if(e[i^1].r&&(dt=dis[u]-e[i].cost)

if((dis[e[i].v]=dt)<=dis[q.size()?</pre>

```
vector<int> cut e;//最小割邊集
    bool vis[MAXN];
54
55
    void dfs_cut(int u){
       vis[u]=1;//表示u屬於source的最小割集
       for(int i=g[u];~i;i=e[i].pre)
         if(e[i].r>0&&!vis[e[i].v])dfs cut(e[i
             ].v);
60
    T min cut(int s,int t){
      T ans=isap(s,t);
61
      memset(vis,0,sizeof(bool)*(n+1));
62
      dfs_cut(s), cut_e.clear();
63
       for(int u=0;u<=n;++u)if(vis[u])</pre>
64
         for(int i=g[u];~i;i=e[i].pre)
          if(!vis[e[i].v])cut_e.push_back(i);
66
67
       return ans;
68
69 };
```

3.3 MinCostMaxFlow

static const int MAXN=440;

1 template<typename TP>

2 struct MCMF{

```
static const TP INF=999999999;
    struct edge{
       int v,pre;
       TP r, cost;
       edge(int v,int pre,TP r,TP cost):v(v),
            pre(pre),r(r),cost(cost){}
     int n,S,T;
     TP dis[MAXN],PIS,ans;
    bool vis[MAXN];
13
     vector<edge> e;
    int g[MAXN];
14
    void init(int n){
       memset(g,-1,sizeof(int)*((n=_n)+1));
16
       e.clear();
17
18
19
     void add_edge(int u,int v,TP r,TP cost,
          bool directed=false){
       e.push_back(edge(v,g[u],r,cost));
       g[u]=e.size()-1;
22
       e.push back(
       edge(u,g[v],directed?0:r,-cost));
24
       g[v]=e.size()-1;
25
26
     TP augment(int u, TP CF){
       if(u==T||!CF)return ans+=PIS*CF,CF;
       vis[u]=1;
29
       TP r=CF,d;
       for(int i=g[u];~i;i=e[i].pre){
         if(e[i].r&&!e[i].cost&&!vis[e[i].v]){
32
           d=augment(e[i].v,min(r,e[i].r));
33
           e[i].r-=d;
34
           e[i^1].r+=d;
35
           if(!(r-=d))break;
36
37
       return CF-r;
```

4 Graph

S=s,T=t;

PIS=ans=0:

}return ans;

4.1 Augmenting_Path

```
1 | #define MAXN1 505
 2 #define MAXN2 505
 3 int n1, n2; //n1 個點 連向 n2 個點
 4 | int match[MAXN2]; // 屬於 n2的點匹配了哪個點
 5 vector<int > g[MAXN1];//\bigseterms
 6 bool vis[MAXN2];//是否走訪過
   bool dfs(int u){
     for(size_t i=0;i<g[u].size();++i){</pre>
       int v=g[u][i];
       if(vis[v])continue;
11
       vis[v]=1;
       if(match[v]==-1||dfs(match[v]))
         return match[v]=u, 1;
13
14
     return 0;
16 }
   inline int max match(){
     int ans=0:
     memset(match,-1,sizeof(int)*n2);
     for(int i=0;i<n1;++i){</pre>
       memset(vis,0,sizeof(bool)*n2);
       if(dfs(i))++ans;
22
```

```
return ans;
                                                 12
                                                    #define qpush(x) q.push(x),S[x]=0
                                                    void flower(int x,int y,int l,queue<int> &q)
                                                      while(st[x]!=1){
         Augmenting Path multiple
                                                        pa[x]=y;
                                                        if(S[y=match[x]]==1)qpush(y);
                                                 18
                                                        st[x]=st[y]=1, x=pa[y];
1 #define MAXN1 1005
                                                 19
2 #define MAXN2 505
                                                 20
3 int n1, n2; //n1 個點連向n2個點,其中n2個點可以
                                                    bool bfs(int x){
       匹配很多邊
                                                      for(int i=1;i<=n;++i)st[i]=i;</pre>
                                                 22
4 vector<int> g[MAXN1];//圖
                                                      memset(S+1,-1,sizeof(int)*n);
5 | int c[MAXN2]; //每個屬於 n2 點最多可以接受幾條
                                                      queue<int>q; qpush(x);
                                                 25
                                                      while(q.size()){
                                                        x=q.front(),q.pop();
6 | vector<int> match_list[MAXN2];//每個屬於n2的
                                                        for(size_t i=0;i<g[x].size();++i){</pre>
       點匹配了那些點
                                                 28
                                                          int y=g[x][i];
7 bool vis[MAXN2];//是否走訪過
                                                          if(S[y]==-1){
                                                 29
  bool dfs(int u){
                                                 30
                                                            pa[y]=x,S[y]=1;
    for(size_t i=0;i<g[u].size();++i){</pre>
                                                 31
                                                            if(!match[y]){
      int v=g[u][i];
                                                 32
                                                               for(int lst;x;y=lst,x=pa[y])
11
      if(vis[v])continue;
                                                 33
                                                                 lst=match[x], match[x]=y, match[y
12
      vis[v]=true;
                                                                     ]=x;
      if((int)match_list[v].size()<c[v]){</pre>
13
                                                              return 1;
                                                 34
         return match_list[v].push_back(u),
14
                                                 35
             true:
                                                            qpush(match[y]);
                                                 36
      }else{
15
                                                          }else if(!S[y]&&st[y]!=st[x]){
16
         for(size_t j=0;j<match_list[v].size()</pre>
                                                            int l=lca(y,x);
             ;++j){
                                                 39
                                                            flower(y,x,l,q),flower(x,y,l,q);
           int next_u=match_list[v][j];
17
                                                 40
           if(dfs(next_u))
18
                                                 41
             return match_list[v][j]=u, true;
19
                                                 42
20
                                                 43
                                                      return 0;
21
      }
                                                 44
22
                                                 45
                                                    int blossom(){
23
    return false;
                                                      int ans=0;
24
                                                      for(int i=1;i<=n;++i)</pre>
  int max match(){
                                                        if(!match[i]&&bfs(i))++ans;
    for(int i=0;i<n2;++i)match list[i].clear()</pre>
                                                      return ans;
    int cnt=0;
    for(int u=0;u<n1;++u){</pre>
      memset(vis,0,sizeof(bool)*n2);
      if(dfs(u))++cnt;
                                                    4.4 graphISO
31
32
    return cnt;
```

4.3 blossom matching

```
1 const int MAXN=1005, K=30; // K要夠大
  const long long A=3,B=11,C=2,D=19,P=0
        xdefaced;
  long long f[K+1][MAXN];
  vector<int> g[MAXN],rg[MAXN];
  int n;
  void init(){
    for(int i=0;i<n;++i){</pre>
      f[0][i]=1;
       g[i].clear(), rg[i].clear();
11 }
  void add edge(int u,int v){
    g[u].push_back(v), rg[v].push_back(u);
  long long point hash(int u)\{//O(N)\}
    for(int t=1;t<=K;++t){</pre>
       for(int i=0;i<n;++i){</pre>
```

if(!vy[y]&&cut>slack_y[y])cut=slack_y[

у];

for(int j=1;j<=n;++j){</pre>

if(vx[j])lx[j]-=cut;

tmp[dep]=u;

return 0;

if(dfs(cnt,dep+1))return 1;

30

31

31

32

33

vector<long long> tmp;

if(tmp.empty())return 177;
long long ret=4931;

sort(tmp.begin(),tmp.end());

for(auto v:g[u])if(!vis[v])tmp.PB(dfs(v));

```
f[t][i]=f[t-1][i]*A%P;
                                                            if(vy[j])ly[j]+=cut;
                                                                                                                                                             bool operator<(const edge&e)const{</pre>
                                                  35
19
         for(int j:g[i])f[t][i]=(f[t][i]+f[t
                                                   36
                                                            else slack_y[j]-=cut;
                                                                                                          int clique(){
                                                                                                                                                        20
                                                                                                                                                               return cost<e.cost;</pre>
              -1][j]*B%P)%P;
                                                                                                                                                        21
                                                   37
                                                                                                     35
                                                                                                            int u,v,ns;
         for(int j:rg[i])f[t][i]=(f[t][i]+f[t
                                                   38
                                                          for(int y=1;y<=n;++y){</pre>
                                                                                                     36
                                                                                                            for(ans=0,u=N-1;u>=0;--u){
                                                                                                                                                        22
                                                                                                                                                           };
                                                            if(!vy[y]&&slack_y[y]==0){
                                                                                                               for(ns=0, tmp[0]=u, v=u+1; v<N;++v)</pre>
                                                                                                                                                           struct bit node{
              -1][j]*C%P)%P;
                                                   39
                                                                                                     37
                                                                                                                                                        23
         if(i==u)f[t][i]+=D;//如果圖太大的話
                                                  40
                                                              if(!match y[y]){augment(y);return;}
                                                                                                     38
                                                                                                                if(g[u][v])stk[1][ns++]=v;
                                                                                                                                                        24
                                                                                                                                                             T mi;
                                                              vy[y]=1,q.push(match_y[y]);
                                                                                                     39
                                                                                                               dfs(ns,1),dp[u]=ans;
                                                                                                                                                        25
                                                  41
              把這行刪掉,執行一次後f[K]就會是所
                                                                                                                                                             bit_node(const T&mi=INF,int id=-1):mi(mi),
                                                   42
                                                                                                     40
              有點的答案
                                                   43
                                                                                                     41
                                                                                                            return ans;
                                                                                                                                                                  id(id){}
         f[t][i]%=P;
                                                                                                     42
                                                  44
                                                                                                                                                        27
                                                                                                                                                           };
23
                                                                                                     43 };
                                                                                                                                                        28
                                                                                                                                                           vector<bit node> bit;
                                                  45
24
                                                                                                                                                           void bit update(int i,const T&data,int id){
                                                   46
                                                      long long KM(){
25
    return f[K][u];
                                                                                                                                                             for(;i;i-=i&(-i)){
                                                       memset(match_y,0,sizeof(int)*(n+1));
                                                   47
26
                                                  48
                                                        memset(ly,0,sizeof(int)*(n+1));
                                                                                                                                                               if(data<bit[i].mi)bit[i]=bit node(data,</pre>
   vector<long long> graph hash(){
                                                   49
                                                        for(int x=1;x<=n;++x){</pre>
                                                                                                               MinimumMeanCycle
                                                                                                                                                                    id);
     vector<long long> ans;
                                                   50
                                                         1x[x]=-INF;
                                                                                                                                                        32
    for(int i=0;i<n;++i)ans.push_back(</pre>
                                                          for(int y=1;y<=n;++y)</pre>
                                                  51
                                                                                                                                                        33
          point hash(i));//O(N^2)
                                                            lx[x]=max(lx[x],g[x][y]);
                                                  52
                                                                                                                                                        34
                                                                                                                                                           int bit_find(int i,int m){
     sort(ans.begin(),ans.end());
                                                                                                      1 #include < cfloat > //for DBL MAX
                                                  53
                                                                                                                                                        35
                                                                                                                                                             bit_node x;
                                                                                                      1 int dp[MAXN][MAXN]; // 1-base, O(NM)
31
    return ans;
                                                  54
                                                        for(int x=1;x<=n;++x)bfs(x);</pre>
                                                                                                                                                        36
                                                                                                                                                             for(;i<=m;i+=i&(-i)) if(bit[i].mi<x.mi)x=</pre>
                                                                                                        vector<tuple<int,int,int>> edge;
                                                  55
                                                        long long ans=0;
                                                                                                                                                                  bit[i];
                                                                                                        double mmc(int n){//allow negative weight
                                                        for(int y=1;y<=n;++y)ans+=g[match_y[y]][y</pre>
                                                                                                                                                        37
                                                                                                                                                             return x.id;
                                                                                                          const int INF=0x3f3f3f3f;
                                                                                                                                                        38
                                                                                                          for(int t=0;t<n;++t){</pre>
                                                                                                                                                           vector<edge> build_graph(int n,point p[]){
                                                  57
                                                       return ans;
         KM
                                                                                                            memset(dp[t+1],0x3f,sizeof(dp[t+1]));
                                                                                                                                                             vector<edge> e;//edge for MST
                                                                                                            for(const auto &e:edge){
                                                                                                                                                             for(int dir=0;dir<4;++dir){//4種座標變換
                                                                                                              int u,v,w;
                                                                                                                                                               if(dir%2) for(int i=0;i<n;++i) swap(p[i</pre>
                                                                                                              tie(u,v,w) = e;
1 #define MAXN 405
                                                                                                                                                                    ].x,p[i].y);
                                                                                                     11
                                                                                                               dp[t+1][v]=min(dp[t+1][v],dp[t][u]+w);
2 #define INF 0x3f3f3f3f
                                                                                                                                                               else if(dir==2) for(int i=0;i<n;++i) p[i</pre>
                                                     4.6 MaximumClique
                                                                                                     12
3 int n;// 1-base · 0表示沒有匹配
                                                                                                                                                                    ].x=-p[i].x;
                                                                                                     13
  int g[MAXN][MAXN], lx[MAXN], ly[MAXN], pa[MAXN
                                                                                                                                                               sort(p,p+n,cmpx);
                                                                                                     14
                                                                                                          double res = DBL_MAX;
        ],slack_y[MAXN];
                                                                                                                                                               vector<T> ga(n), gb;
                                                                                                     15
                                                                                                          for(int u=1;u<=n;++u){</pre>
                                                   1 | struct MaxClique{
  int match_y[MAXN], match_x[MAXN];
                                                                                                                                                               for(int i=0;i<n;++i)ga[i]=p[i].y-p[i].x;</pre>
                                                                                                            if(dp[n][u]==INF) continue;
                                                                                                     16
                                                       static const int MAXN=105;
  bool vx[MAXN],vy[MAXN];
                                                                                                            double val = -DBL_MAX;
                                                                                                                                                               gb=ga, sort(gb.begin(),gb.end());
                                                       int N.ans:
   void augment(int y){
                                                                                                                                                               gb.erase(unique(gb.begin(),gb.end()),gb.
                                                                                                            for(int t=0;t<n;++t)</pre>
                                                       int g[MAXN][MAXN], dp[MAXN], stk[MAXN][MAXN
    for(int x,z;y;y=z){
                                                                                                                                                                    end());
                                                                                                              val=max(val,(dp[n][u]-dp[t][u])*1.0/(n
       x=pa[y],z=match_x[x];
                                                                                                                                                               int m=gb.size();
                                                                                                                   -t));
                                                       int sol[MAXN],tmp[MAXN];//sol[0~ans-1]為答
       match_y[y]=x,match_x[x]=y;
                                                                                                                                                               bit=vector<bit_node>(m+1);
                                                                                                     20
                                                                                                            res=min(res,val);
11
                                                                                                                                                        51
                                                                                                                                                               for(int i=n-1;i>=0;--i){
                                                                                                     21
                                                        void init(int n){
12
                                                                                                                                                                 int pos=lower_bound(gb.begin(),gb.end
                                                                                                     22
                                                                                                          return res;
   void bfs(int st){
                                                         N=n;//0-base
                                                                                                                                                                      (),ga[i])-gb.begin()+1;
    for(int i=1;i<=n;++i)slack_y[i]=INF,vx[i]=</pre>
                                                         memset(g,0,sizeof(g));
                                                                                                                                                                 int ans=bit_find(pos,m);
          vy[i]=0;
                                                                                                                                                                 if(~ans)e.push_back(edge(p[i].id,p[ans
     queue<int> q;q.push(st);
                                                        void add_edge(int u,int v){
                                                                                                                                                                      ].id,p[i].dist(p[ans])));
     for(;;){
16
                                                   11
                                                         g[u][v]=g[v][u]=1;
                                                                                                                                                        55
                                                                                                                                                                 bit_update(pos,p[i].x+p[i].y,i);
                                                                                                        4.8 Rectilinear MST
       while(q.size()){
                                                   12
                                                                                                                                                        56
                                                        int dfs(int ns,int dep){
18
         int x=q.front();q.pop();
                                                                                                                                                             }
                                                                                                                                                        57
19
                                                   14
                                                         if(!ns){
                                                                                                                                                        58
                                                                                                                                                             return e;
                                                                                                      1 / / 平面曼哈頓最小生成樹構造圖(去除非必要邊)
20
         for(int y=1;y<=n;++y)if(!vy[y]){</pre>
                                                   15
                                                            if(dep>ans){
                                                                                                      2 #define T int
           int t=lx[x]+ly[y]-g[x][y];
                                                   16
                                                              ans=dep;
                                                                                                      3 #define INF 0x3f3f3f3f
           if(t==0){
                                                   17
                                                              memcpy(sol,tmp,sizeof tmp);
                                                                                                        struct point{
23
             pa[y]=x;
                                                   18
                                                              return 1;
                                                                                                         T x,y;
                                                                                                                                                                 treeISO
             if(!match_y[y]){augment(y);return
                                                            }else return 0;
                                                   20
                                                                                                          int id;//從0開始編號
             vy[y]=1,q.push(match y[y]);
                                                  21
                                                          for(int i=0;i<ns;++i){</pre>
                                                                                                          point(){}
                                                                                                                                                         1 const int MAXN=100005;
           }else if(slack_y[y]>t)pa[y]=x,
                                                   22
                                                            if(dep+ns-i<=ans)return 0;</pre>
                                                                                                          T dist(const point &p)const{
                slack_y[y]=t;
                                                   23
                                                            int u=stk[dep][i],cnt=0;
                                                                                                            return abs(x-p.x)+abs(y-p.y);
                                                                                                                                                           const long long X=12327,P=0xdefaced;
                                                            if(dep+dp[u]<=ans)return 0;</pre>
                                                                                                                                                           vector<int> g[MAXN];
                                                            for(int j=i+1; j<ns; ++j){</pre>
28
                                                   25
                                                                                                     11 };
                                                                                                                                                         4 bool vis[MAXN];
       int cut=INF:
                                                              int v=stk[dep][j];
                                                                                                        bool cmpx(const point &a,const point &b){
                                                                                                                                                           long long dfs(int u){//hash ver
30
       for(int y=1;y<=n;++y){</pre>
                                                              if(g[u][v])stk[dep+1][cnt++]=v;
                                                                                                          return a.x<b.x||(a.x==b.x&&a.y<b.y);
                                                                                                                                                             vis[u]=1;
```

14 }

15 struct edge{

int u,v;

edge(int u,int v,T c):u(u),v(v),cost(c){}

```
for(auto v:tmp)ret=((ret*X)^v)%P;
13
    return ret;
14
   ·
//-----
   string dfs(int x,int p){
    vector<string> c;
18
    for(int y:g[x])
     if(y!=p)c.emplace_back(dfs(y,x));
19
    sort(c.begin(),c.end());
    string ret("(");
22
    for(auto &s:c)ret+=s;
23
    ret+=")":
    return ret;
24
25
```

4.10 一般圖最小權完美匹配

```
1 | struct Graph {
    // Minimum General Weighted Matching (
          Perfect Match) 0-base
     static const int MXN = 105;
     int n, edge[MXN][MXN];
     int match[MXN], dis[MXN], onstk[MXN];
     vector<int> stk;
     void init(int n) {
       for (int i=0; i<n; i++)</pre>
10
         for (int j=0; j<n; j++)</pre>
11
           edge[i][j] = 0;
12
     void add edge(int u, int v, int w) {
13
       edge[u][v] = edge[v][u] = w;
14
15
16
     bool SPFA(int u){
       if(onstk[u]) return true;
17
18
       stk.push back(u);
19
       onstk[u] = 1;
       for(int v=0; v<n; v++){</pre>
20
         if(u!=v && match[u]!=v && !onstk[v]){
21
           int m = match[v];
22
           if(dis[m]>dis[u]-edge[v][m]+edge[u][
23
                v]){
24
             dis[m] = dis[u]-edge[v][m]+edge[u
                  ][v];
             onstk[v] = 1;
25
             stk.push_back(v);
26
             if (SPFA(m)) return true;
27
             stk.pop_back();
29
             onstk[v] = 0;
30
31
        }
32
       onstk[u] = 0;
       stk.pop back();
       return false:
36
     int solve() {
       // find a match
       for (int i=0; i<n; i+=2){</pre>
         match[i] = i+1, match[i+1] = i;
41
42
       for(;;){
         int found = 0;
```

```
for(int i=0;i<n;i++) dis[i]=onstk[i</pre>
          for (int i=0; i<n; i++){</pre>
45
46
            stk.clear();
47
            if (!onstk[i] && SPFA(i)){
              found = 1:
49
              while (stk.size()>=2){
                int u=stk.back();stk.pop back();
50
                int v=stk.back();stk.pop back();
51
52
                match[u] = v;
53
                match[v] = u;
54
55
56
57
         if (!found) break:
58
       int ret = 0:
59
       for (int i=0; i<n; i++)</pre>
60
         ret += edge[i][match[i]];
61
       ret /= 2:
62
63
       return ret;
64
65 }graph;
```

4.11 全局最小割

```
1 const int INF=0x3f3f3f3f;
 2 template<typename T>
3 struct stoer_wagner{// 0-base
     static const int MAXN=150;
     T g[MAXN][MAXN], dis[MAXN];
     int nd[MAXN],n,s,t;
     void init(int n){
        for(int i=0;i<n;++i)</pre>
10
          for(int j=0;j<n;++j)g[i][j]=0;</pre>
11
     void add_edge(int u,int v,T w){
12
       g[u][v]=g[v][u]+=w;
14
     T min cut(){
15
       T ans=INF;
        for(int i=0;i<n;++i)nd[i]=i;</pre>
17
        for(int ind,tn=n;tn>1;--tn){
18
          for(int i=1;i<tn;++i)dis[nd[i]]=0;</pre>
19
          for(int i=1;i<tn;++i){</pre>
20
21
            ind=i:
22
            for(int j=i;j<tn;++j){</pre>
23
              dis[nd[i]]+=g[nd[i-1]][nd[i]];
              if(dis[nd[ind]]<dis[nd[j]])ind=j;</pre>
24
25
26
            swap(nd[ind],nd[i]);
27
28
          if(ans>dis[nd[ind]])
            ans=dis[t=nd[ind]],s=nd[ind-1];
29
          for(int i=0;i<tn;++i)</pre>
30
            g[nd[ind-1]][nd[i]]=g[nd[i]][nd[ind
31
                 -1]]+=g[nd[i]][nd[ind]];
       return ans;
34
35 };
```

4.12 平面圖判定

1 static const int MAXN = 20;

Edge(int s, int d): u(s), v(d) {}

2 struct Edge{

int u, v;

```
6 bool isK33(int n, int degree[]){
    int t = 0, z = 0;
     for(int i=0;i<n;++i){</pre>
       if(degree[i] == 3)++t;
       else if(degree[i] == 0)++z;
10
11
       else return false:
12
13
    return t == 6 && t + z == n:
14
  bool isK5(int n, int degree[]){
15
16
    int f = 0, z = 0;
    for(int i=0;i<n;++i){</pre>
17
18
       if(degree[i] == 4)++f;
       else if(degree[i] == 0)++z;
19
20
       else return false:
21
22
    return f == 5 \&\& f + z == n;
23
24 // it judge a given graph is Homeomorphic
        with K33 or K5
25 bool isHomeomorphic(bool G[MAXN][MAXN],
        const int n){
26
     for(;;){
27
       int cnt = 0:
       for(int i=0;i<n;++i){</pre>
28
29
         vector<Edge> E;
30
         for(int j=0;j<n&E.size()<3;++j)</pre>
31
           if(G[i][j] && i != j)
32
             E.push back(Edge(i, j));
33
         if(E.size() == 1){
34
           G[i][E[0].v] = G[E[0].v][i] = false; 38
         }else if(E.size() == 2){
35
36
           G[i][E[0].v] = G[E[0].v][i] = false; 40
37
           G[i][E[1].v] = G[E[1].v][i] = false; 41
38
           G[E[0].v][E[1].v] = G[E[1].v][E[0].v 42
                ] = true;
39
           ++cnt;
40
41
       if(cnt == 0)break;
42
43
     static int degree[MAXN];
     fill(degree, degree + n, 0);
     for(int i=0;i<n;++i){</pre>
       for(int j=i+1; j<n; ++j){</pre>
48
         if(!G[i][j])continue;
49
         ++degree[i];
50
         ++degree[j];
51
    }
52
    return !(isK33(n,degree)||isK5(n,degree));
53
```

4.14 最小斯坦納樹 DP

1 struct chordal{

11

12

13

14

15

19

20

21

22

23

24

25

26

27

28

29

30

31

43

44

45

46

47

48

49

50

int n;// 0-base

bool mark[MAXN];

vector<int>G[MAXN]; int rank[MAXN],label[MAXN];

void init(int n){n= n:

G[u].push back(v);

G[v].push back(u);

vector<int> MCS(){

rank[u]=i:

vector<int> res(n);

for(int i=0;i<n;++i){</pre>

if(tmp.size()){

set<int> S:

mark[ord[i]]=1;

return 1;

break;

return res:

static const int MAXN=1005;

void add_edge(int u,int v){

for(int i=0; i<n;++i)G[i].clear();</pre>

memset(rank,-1,sizeof(int)*n);

memset(label,0,sizeof(int)*n);

priority queue<pair<int.int> > pq;

int u=pq.top().second;pq.pop();

for(auto v:G[u])if(rank[v]==-1){

for(int i=0;i<n;++i)res[rank[i]]=i;</pre>

bool check(vector<int> ord){//弦圖判定

memset(mark,0,sizeof(bool)*n);

vector<pair<int,int> > tmp;

sort(tmp.begin(),tmp.end());

int u=tmp[0].second;

for(int i=0;i<n;++i)rank[ord[i]]=i;</pre>

for(auto u:G[ord[i]])if(!mark[u])

for(auto v:G[u])S.insert(v);

for(size_t j=1;j<tmp.size();++j)</pre>

if(!S.count(tmp[j].second))return

tmp.push back(make pair(rank[u],u));

pq.push(make pair(++label[v],v));

for(int i=n-1;i>=0;--i)for(;;){

if(~rank[u])continue;

for(int i=0;i<n;++i)pq.push(make_pair(0,</pre>

```
1 | //n 個 點 · 其中r 個 要 構 成 斯 坦 納 樹
2 //答案在max(dp[(1<<r)-1][k]) k=0~n-1
3 //p表示要構成斯坦納樹的點集
4 //0 (n^3 + n*3^r + n^2*2^r)
5 #define REP(i,n) for(int i=0;i<(int)n;++i)</pre>
6 const int MAXN=30, MAXM=8;// 0-base
```

4.13 弦圖完美消除序列

```
7 const int INF=0x3f3f3f3f;
  int dp[1<<MAXM][MAXN];</pre>
                                                    36
  int g[MAXN][MAXN];// 🗟
                                                    37
  void init(){memset(g,0x3f,sizeof(g));}
                                                    38
   void add edge(int u,int v,int w){
                                                    39
    g[u][v]=g[v][u]=min(g[v][u],w);
                                                    40
13
                                                    41
   void steiner(int n,int r,int *p){
14
                                                    42
15
     REP(k,n)REP(i,n)REP(j,n)
                                                    43
       g[i][j]=min(g[i][j],g[i][k]+g[k][j]);
                                                    44
17
     REP(i,n)g[i][i]=0;
     REP(i,r)REP(j,n)dp[1<<i][j]=g[p[i]][j];</pre>
18
                                                    45
19
     for(int i=1;i<(1<<r);++i){</pre>
                                                    46
20
       if(!(i&(i-1)))continue;
21
       REP(j,n)dp[i][j]=INF;
                                                    47
22
       REP(i,n){
                                                    48
         int tmp=INF:
23
                                                    49
         for(int s=i&(i-1);s;s=i&(s-1))
24
                                                    50
           tmp=min(tmp,dp[s][j]+dp[i^s][j]);
25
                                                    51
         REP(k,n)dp[i][k]=min(dp[i][k],g[j][k]+
26
              tmp);
                                                    52
27
                                                    53
28
                                                    54
29
                                                    55
                                                    56
```

4.15 最小樹形圖 朱劉

1 template < typename T>

```
struct zhu liu{
     static const int MAXN=110, MAXM=10005;
    struct node{
       int u,v;
       T w,tag;
       node *1.*r:
       node(int u=0, int v=0, T w=0):u(u), v(v), w(v)
            w),tag(0),1(0),r(0){}
       void down(){
         if(1)1->tag+=tag;
12
         if(r)r->tag+=tag;
13
         tag=0;
14
     }mem[MAXM];//靜態記憶體
     node *pq[MAXN*2],*E[MAXN*2];
     int st[MAXN*2],id[MAXN*2],m;
     void init(int n){
       for(int i=1;i<=n;++i){</pre>
         pq[i]=E[i]=0, st[i]=id[i]=i;
20
21
       }m=0;
22
     node *merge(node *a, node *b){//skew heap
       if(!a||!b)return a?a:b;
       a->down(),b->down();
       if(b->w<a->w)return merge(b,a);
       swap(a->1,a->r);
       a->l=merge(b,a->l);
29
       return a:
30
     void add edge(int u,int v,T w){
       if(u!=v)pq[v]=merge(pq[v],&(mem[m++]=
            node(u,v,w));
     int find(int x,int *st){
```

4.16 穩定婚姻模板

```
1 | queue < int > Q;
2| for ( i : 所有考生 ) {
   設定在第0志願:
   Q.push(考生i);
6 while(Q.size()){
   當前考生=Q.front();Q.pop();
   while (此考生未分發) {
     指標移到下一志願;
     if (已經沒有志願 or 超出志願總數)
        break;
11
     計算該考生在該科系加權後的總分:
     if (不符合科系需求) continue;
     if (目前科系有餘額) {
13
      依加權後分數高低順序將考生id加入科系錄
14
          取名單中:
15
      break;
16
    if (目前科系已額滿) {
17
      if ( 此考生成績比最低分數還高 ) {
18
        依加權後分數高低順序將考生id加入科系
19
           錄取名單:
        0.push(被踢出的考生);
20
```

return st[x]==x?x:st[x]=find(st[x],st);

pq[i]=merge(pq[i]->1,pq[i]->r);

if(find(E[i]->u,id)!=find(i,id))

if(find(E[i]->u,id)==find(i,id))

if(find(E[i]->u,st)==find(i,st)){

if(pq[i])pq[i]->tag-=E[i]->w;

 $find(E[u]->u,id)){$

pq[N]=merge(pq[N],pq[u]);

for(int u=find(E[i]->u,id);u!=i;u=

if(pq[u])pq[u]->tag-=E[u]->w;

}else st[find(i,st)]=find(E[i]->u,st)

return all==1?ans:-INT MAX;//圖不連通就

pq[++N]=pq[i];id[N]=N;

id[find(u,id)]=N;

st[N]=find(i,st);

id[find(i,id)]=N;

T build(int root, int n){

while(pa[i]){

T ans=0; int N=n, all=n;

for(int i=1;i<=N;++i){</pre>

continue:

ans+=E[i]->w:

57

58

59

60

61

62 };

if(i==root||!pq[i])continue;

pq[i]->down(),E[i]=pq[i];

5 Linear Programming

edge(int u=0, int v=0, T w=0):u(u),v(v),w(w)

5.1 最大密度子圖

1 typedef double T://POJ 3155

2 const int MAXN=105;

3 struct edge{

int u,v;

}

23

24

```
7 };
8 vector<edge> E:
9 int n,m;// 1-base
10 | T de[MAXN], pv[MAXN]; // 每個點的邊權和和點權(
        有些題目會給)
  void init(){
    E.clear();
    for(int i=1;i<=n;++i)de[i]=pv[i]=0;</pre>
13
15 void add edge(int u,int v,T w){
    E.push back(edge(u,v,w));
    de[u]+=w,de[v]+=w;
18
19 T U; // 二分搜的最大值
  void get U(){
    for(int i=1;i<=n;++i)U+=2*pv[i];</pre>
    for(size t i=0;i<E.size();++i)U+=E[i].w;</pre>
24
25 | ISAP<T> isap;//網路流
26 int s,t;//原匯點
  void build(T L){
27
    isap.init(n+2);
     for(size_t i=0;i<E.size();++i)</pre>
       isap.add_edge(E[i].u,E[i].v,E[i].w);
     for(int v=1; v<=n; ++v){</pre>
       isap.add edge(s,v,U);
       isap.add_edge(v,t,U+2*L-de[v]-2*pv[v]);
34
35
36
   int main(){
     while(~scanf("%d%d",&n,&m)){
       if(!m){
         puts("1\n1");
         continue:
41
       init();
42
43
       int u,v;
       for(int i=0;i<m;++i){</pre>
         scanf("%d%d",&u,&v);
         add_edge(u,v,1);
47
       get U();
       s=n+1, t=n+2;
       T = 0, r = U, k = 1.0/(n*n);
```

```
while(r-1>k){//二分搜最大值
         T mid=(1+r)/2;
52
         build(mid);
53
         T res=(U*n-isap.isap(s,t))/2;
         if(res>0)l=mid;
         else r=mid:
       build(1);
       isap.min cut(s,t);
      vector<int> ans;
61
       for(int i=1;i<=n;++i)</pre>
        if(isap.vis[i])ans.push back(i);
      printf("%d\n",ans.size());
       for(size t i=0:i<ans.size():++i)</pre>
65
         printf("%d\n",ans[i]);
66
    return 0:
67
```

6 Number_Theory

6.1 basic

```
1 | template < typename T>
  void gcd(const T &a,const T &b,T &d,T &x,T &
     if(!b) d=a,x=1,y=0;
    else gcd(b,a%b,d,y,x), y-=x*(a/b);
  long long int phi[N+1];
  void phiTable(){
    for(int i=1;i<=N;i++)phi[i]=i;</pre>
     for(int i=1;i<=N;i++)for(x=i*2;x<=N;x+=i)</pre>
         phi[x]-=phi[i];
  void all divdown(const LL &n) {// all n/x
11
    for(LL a=1;a<=n;a=n/(n/(a+1))){</pre>
      // dosomething;
14
15 }
16 const int MAXPRIME = 1000000;
  int iscom[MAXPRIME], prime[MAXPRIME],
       primecnt;
  int phi[MAXPRIME], mu[MAXPRIME];
  void sieve(void){
    memset(iscom,0,sizeof(iscom));
    primecnt = 0;
     phi[1] = mu[1] = 1;
     for(int i=2;i<MAXPRIME;++i) {</pre>
       if(!iscom[i]) {
         prime[primecnt++] = i;
         mu[i] = -1;
         phi[i] = i-1;
27
28
29
       for(int j=0;j<primecnt;++j) {</pre>
         int k = i * prime[j];
         if(k>=MAXPRIME) break;
32
         iscom[k] = prime[j];
         if(i%prime[i]==0) {
           mu[k] = 0;
           phi[k] = phi[i] * prime[j];
```

```
if(Legendre(n,p)!=1) while(1) { puts("SQRT 157)
           break:
                                                                                                           g1=q2=p1=BigInteger.ZERO;
                                                                                                                                                               return res;
         } else {
                                                              ROOT does not exist"); }
37
                                                                                                           h1=q1=p2=BigInteger.ONE;
                                                                                                                                                          15
           mu[k] = -mu[i];
                                                        int S = 0;
                                                                                                            a0=a1=BigInteger.valueOf((int)Math.sqrt
                                                                                                                                                             vector<int> decode(int a,int n){
                                                   98
                                                                                                     159
                                                                                                                                                          16
           phi[k] = phi[i] * (prime[j]-1);
39
                                                   99
                                                        LL Q = p-1;
                                                                                                                 (1.0*n));
                                                                                                                                                          17
                                                                                                                                                               vector<int> res;
40
                                                        while( !(0&1) ) { 0>>=1; ++S; }
                                                                                                            BigInteger ans=a0.multiply(a0);
                                                                                                                                                               vector<bool> vis(n,0);
                                                  100
                                                                                                     160
                                                                                                                                                          18
                                                                                                            if(ans.equals(BigInteger.valueOf(n))){
                                                                                                                                                               for(int i=n-1;i>=0;--i){
41
                                                  101
                                                        if(S==1) return modexp(n\%p,(p+1)/4,p);
42
                                                  102
                                                                                                     162
                                                                                                              System.out.println("No solution!"):
                                                                                                                                                          20
                                                                                                                                                                 int t=a/factorial[i],j;
43
                                                        for(;Legendre(z,p)!=-1;++z)
                                                                                                                                                                 for(i=0;i<n;++i)</pre>
                                                  103
                                                                                                     163
                                                                                                              return :
                                                                                                                                                          21
44
                                                  104
                                                        LL c = modexp(z,Q,p);
                                                                                                      164
                                                                                                                                                          22
                                                                                                                                                                   if(!vis[j]){
   bool g_test(const LL &g, const LL &p, const
                                                        LL R = modexp(n\%p,(Q+1)/2,p), t = modexp(n_{165})
                                                                                                            while(true){
                                                                                                                                                          23
                                                                                                                                                                     if(t==0)break;
        vector<LL> &v) {
                                                             %p,Q,p);
                                                                                                      166
                                                                                                              g2=a1.multiply(h1).substract(g1);
                                                                                                                                                          24
                                                                                                                                                                     --t;
     for(int i=0:i<v.size():++i)</pre>
                                                        int M = S:
                                                                                                              h2=N.substract(g2.pow(2)).divide(h1):
                                                  106
                                                                                                     167
                                                                                                                                                          25
       if(modexp(g,(p-1)/v[i],p)==1)
                                                        while(1) {
                                                                                                              a2=g2.add(a0).divide(h2);
47
                                                  107
                                                                                                     168
                                                                                                                                                          26
                                                                                                                                                                 res.push_back(j);
48
         return false:
                                                  108
                                                          if(t==1) return R:
                                                                                                      169
                                                                                                              p=a1.multiply(p2).add(p1);
                                                                                                                                                          27
                                                                                                                                                                 vis[j]=1;
                                                          LL b = modexp(c,1L<<(M-i-1),p);
49
    return true:
                                                  109
                                                                                                      170
                                                                                                              q=a1.multiply(q2).add(q1);
                                                                                                                                                          28
                                                                                                                                                                 a%=factorial[i];
50
                                                  110
                                                          R = LLmul(R,b,p);
                                                                                                     171
                                                                                                              if(p.pow(2).substract(N.multiply(q.pow
                                                                                                                                                          29
   LL primitive root(const LL &p) {
                                                          t = LLmul( LLmul(b,b,p), t, p);
                                                                                                                   (2))).compareTo(BigInteger.ONE)==0)
51
                                                  111
                                                                                                                                                          30
                                                                                                                                                               return res;
                                                          c = LLmul(b,b,p);
    if(p==2) return 1;
52
                                                  112
                                                                                                                   break;
53
     vector<LL> v:
                                                  113
                                                          M = i;
                                                                                                              g1=g2;h1=h2;a1=a2;
                                                                                                     172
54
     Factor(p-1,v);
                                                  114
                                                                                                     173
                                                                                                              p1=p2;p2=p;
55
    v.erase(unique(v.begin(), v.end()), v.end
                                                  115
                                                        return -1;
                                                                                                     174
                                                                                                              q1=q2;q2=q;
          ());
                                                  116
                                                                                                     175
                                                                                                                                                             6.4 FFT
56
    for(LL g=2;g<p;++g)</pre>
                                                  117
                                                                                                      176
                                                                                                           System.out.println(p+" "+q);
57
       if(g_test(g,p,v))
                                                  118
                                                      template<typename T>
                                                                                                      177 }
58
         return g;
                                                  119 T Euler(T n){
                                                                                                                                                           1 | template < typename T, typename VT = vector <
59
    puts("primitive root NOT FOUND");
                                                  120
                                                        T ans=n:
                                                                                                                                                                  complex<T>>>
                                                        for(T i=2;i*i<=n;++i){</pre>
60
    return -1;
                                                  121
                                                          if(n%i==0){
                                                                                                                                                             struct FFT{
61
                                                  122
                                                                                                                bit set
                                                                                                                                                               const T pi;
   int Legendre(const LL &a, const LL &p) {
                                                            ans=ans/i*(i-1);
                                                  123
                                                                                                                                                               FFT(const T pi=acos((T)-1)):pi(pi){}
       return modexp(a%p,(p-1)/2,p); }
                                                  124
                                                            while(n%i==0)n/=i;
                                                                                                                                                               unsigned bit reverse(unsigned a,int len){
                                                  125
                                                                                                        1 void sub_set(int S){
                                                                                                                                                             a = ((a\&0x55555555U) << 1) | ((a\&0xAAAAAAAAU) >> 1);
   LL inv(const LL &a, const LL &n) {
                                                  126
                                                                                                           int sub=S;
    LL d,x,y;
                                                  127
                                                        if(n>1) ans=ans/n*(n-1):
                                                                                                                                                             a=((a&0x33333333U)<<2)|((a&0xCCCCCCCU)>>2);
65
                                                                                                            do{
                                                        return ans;
                                                                                                                                                             a=((a\&0x0F0F0F0FU)<<4)|((a\&0xF0F0F0F0U)>>4);
    gcd(a,n,d,x,y);
                                                  128
                                                                                                             //對某集合的子集合的處理
                                                                                                                                                             a=((a&0x00FF00FFU)<<8)|((a&0xFF00FF00U)>>8);
    return d==1 ? (x+n)%n : -1;
                                                  129 }
                                                                                                              sub=(sub-1)&S;
                                                                                                                                                             a=((a\&0x0000FFFFU)<<16)|((a\&0xFFFF0000U)
68
                                                  130
                                                                                                            }while(sub!=S);
                                                  131
                                                      //Chinese remainder theorem
69
                                                                                                                                                          11
                                                                                                                                                                 return a>>(32-len);
   int inv[maxN];
                                                      template<typename T>
                                                                                                          void k_sub_set(int k,int n){
   LL invtable(int n,LL P){
                                                      T pow mod(T n,T k,T m){
                                                                                                                                                          12
                                                                                                           int comb=(1<<k)-1,S=1<<n;</pre>
    inv[1]=1;
                                                  134
                                                                                                                                                          1.3
                                                                                                                                                               void fft(bool is inv,VT &in,VT &out,int N)
72
                                                        T ans=1;
                                                                                                            while(comb<S){</pre>
                                                        for (n=(n)=m?n\%m:n);k;k>>=1)
73
    for(int i=2;i<n;++i)</pre>
                                                  135
                                                                                                              //對大小為k的子集合的處理
                                                                                                                                                                 int bitlen=__lg(N),num=is_inv?-1:1;
       inv[i]=(P-(P/i))*inv[P%i]%P;
                                                  136
                                                          if(k&1)ans=ans*n%m;
                                                                                                                                                          14
74
                                                                                                              int x=comb&-comb,y=comb+x;
                                                                                                                                                          15
                                                                                                                                                                 for(int i=0;i<N;++i)out[bit reverse(i,</pre>
                                                          n=n*n%m;
75
                                                  137
                                                                                                      13
                                                                                                              comb = ((comb\&\sim y)/x>>1)|y;
                                                                                                                                                                      bitlen) | = in[i];
76
                                                  138
                                                                                                      14
                                                                                                                                                                 for(int step=2;step<=N;step<<=1){</pre>
   LL log_mod(const LL &a, const LL &b, const
                                                  139
                                                        return ans;
                                                                                                      15 }
                                                                                                                                                                   const int mh=step>>1:
                                                                                                                                                          17
       LL &p) {
                                                  140 }
                                                                                                                                                                   for(int i=0;i<mh;++i){</pre>
     // a ^ x = b \pmod{p}
                                                      template<typename T>
                                                                                                                                                          18
    int m=sqrt(p+.5), e=1;
                                                      T crt(vector<T> &m, vector<T> &a){
                                                                                                                                                                     complex<T> wi=exp(complex<T>(0.i*num
    LL v=inv(modexp(a,m,p), p);
                                                                                                                                                                           *pi/mh));
                                                        T M=1,tM,ans=0;
                                                                                                         6.3 cantor expansion
                                                                                                                                                                     for(int j=i;j<N;j+=step){</pre>
                                                                                                                                                          20
    map<LL,int> x;
                                                  144
                                                        for(int i=0;i<(int)m.size();++i)M*=m[i];</pre>
                                                                                                                                                          21
                                                                                                                                                                       int k=i+mh:
                                                        for(int i=0;i<(int)a.size();++i){</pre>
82
    x[1]=0;
                                                  145
                                                                                                                                                                        complex<T> u=out[j],t=wi*out[k];
                                                                                                                                                          22
83
     for(int i=1;i<m;++i) {</pre>
                                                  146
                                                          tM=M/m[i];
                                                                                                                                                          23
                                                                                                                                                                       out[j]=u+t;
                                                          ans=(ans+(a[i]*tM%M)*pow mod(tM,Euler(m[
                                                                                                       1 int factorial[MAXN];
       e = LLmul(e,a,p);
                                                  147
                                                                                                                                                                       out[k]=u-t;
                                                                                                                                                          24
       if(!x.count(e)) x[e] = i;
                                                               i])-1,m[i])%M)%M;
                                                                                                       2 void init(){
                                                                                                                                                          25
86
                                                                                                           factorial[0]=1;
                                                          /*如果m[i]是質數·Euler(m[i])-1=m[i]-2·
                                                  148
                                                                                                            for(int i=1;i<=MAXN;++i)factorial[i]=</pre>
                                                                                                                                                          26
     for(int i=0;i<m;++i) {</pre>
                                                               就不用算Euler了*/
                                                                                                                                                          27
       if(x.count(b)) return i*m + x[b];
                                                                                                                factorial[i-1]*i;
                                                  149
                                                                                                                                                          28
                                                                                                                                                                 if(is_inv)for(int i=0;i<N;++i)out[i]/=N;</pre>
       b = LLmul(b,v,p);
                                                  150
                                                        return ans;
                                                                                                                                                          29
90
    }
                                                                                                         int encode(const vector<int> &s){
                                                  151 }
                                                                                                                                                          30 };
     return -1:
                                                                                                           int n=s.size(),res=0;
                                                  152
92
                                                                                                            for(int i=0;i<n;++i){</pre>
                                                  153 //iava code
93
                                                                                                             int t=0;
                                                  154 //求 sqrt (N)的 連分 數
   LL Tonelli Shanks(const LL &n, const LL &p)
                                                                                                              for(int j=i+1; j<n;++j)</pre>
                                                  155 public static void Pell(int n){
                                                                                                                if(s[i]<s[i])++t;
                                                                                                                                                             6.5 find real root
                                                        BigInteger N,p1,p2,q1,q2,a0,a1,a2,g1,g2,h1
    // x^2 = n \pmod{p}
                                                                                                              res+=t*factorial[n-i-1];
                                                                                                       12
    if(n==0) return 0;
```

inverse){

29 template<typename T>

```
for(int i=0; (2<<i)<=f.size(); ++i)</pre>
                                                                                                            if((n/P)%2==0)return res*fact[n%P]%P;
1 // an*x^n + ... + a1x + a0 = 0;
                                                                                                                                                                T gas(){
                                                                                                            return res*(P-fact[n%P])%P;
2 int sign(double x){
                                                           for(int j=0; j<f.size(); j+=2<<i)</pre>
                                                                                                                                                           51
                                                                                                                                                                  vector<T> lazy(r,1);
    return x < -eps ? -1 : x > eps;
                                                             for(int k=0; k<(1<<i); ++k)</pre>
                                                                                                                                                                  bool sign=false;
                                                                                                                                                           52
                                                               f[j+k+(1<< i)] += f[j+k]*(inverse)
                                                                                                        9 int Cmod(int n,int m){
                                                                                                                                                           53
                                                                                                                                                                  for(int i=0;i<r;++i){</pre>
                                                                    ?-1:1);
                                                                                                            int a1,a2,a3,e1,e2,e3;
                                                                                                                                                           54
                                                                                                                                                                    if( m[i][i]==0 ){
                                                                                                       10
   double get(const vector<double>&coef, double
                                                                                                            a1=mod fact(n,e1);
                                                                                                                                                                       int j=i+1;
                                                                                                            a2=mod fact(m.e2):
                                                                                                                                                           56
                                                                                                                                                                       while(j<r&&!m[j][i])j++;</pre>
     double e = 1, s = 0;
                                                       vector<int> rev(vector<int> A) {
                                                                                                            a3=mod fact(n-m,e3);
                                                                                                                                                                       if(j==r)continue;
                                                                                                                                                           57
     for(auto i : coef) s += i*e, e *= x;
                                                         for(int i=0; i<A.size(); i+=2)</pre>
                                                                                                            if(e1>e2+e3)return 0;
                                                                                                                                                           58
                                                                                                                                                                       m[i].swap(m[j]);
    return s;
                                                          swap(A[i],A[i^(A.size()-1)]);
                                                                                                            return a1*inv(a2*a3%P,P)%P;
                                                                                                                                                                       sign=!sign;
                                                                                                                                                           59
10
                                                    11
                                                        return A;
                                                                                                                                                           60
                                                    12 }
                                                                                                                                                                     for(int j=0;j<r;++j){</pre>
11
                                                                                                                                                           61
   double find(const vector<double>&coef, int n
                                                   13 vector<int> F_AND_T(vector<int> f, bool
                                                                                                                                                                       if(i==j)continue;
                                                                                                                                                           62
        , double lo, double hi){
                                                            inverse){
                                                                                                                                                           63
                                                                                                                                                                       lazy[j]=lazy[j]*m[i][i];
    double sign_lo, sign_hi;
                                                                                                                Matrix
                                                        return rev(F OR T(rev(f), inverse));
                                                                                                                                                           64
                                                                                                                                                                       T mx=m[j][i];
14
    if( !(sign_lo = sign(get(coef,lo))) )
                                                   15 }
                                                                                                                                                           65
                                                                                                                                                                       for(int k=0;k<c;++k)</pre>
                                                    16 vector<int> F XOR T(vector<int> f, bool
          return lo;
                                                                                                                                                                         m[j][k]=m[j][k]*m[i][i]-m[i][k]*mx
                                                                                                        1 | template<typename T>
     if( !(sign_hi = sign(get(coef,hi))) )
                                                            inverse){
          return hi;
                                                         for(int i=0; (2<<i)<=f.size(); ++i)</pre>
                                                                                                          struct Matrix{
                                                   17
                                                                                                                                                           67
                                                                                                            using rt = std::vector<T>;
                                                           for(int j=0; j<f.size(); j+=2<<i)</pre>
     if(sign lo * sign hi > 0) return INF;
                                                   18
                                                                                                                                                           68
                                                             for(int k=0; k<(1<<i); ++k){</pre>
    for(int stp = 0; stp < 100 && hi - lo >
                                                   19
                                                                                                            using mt = std::vector<rt>;
                                                                                                                                                           69
                                                                                                                                                                  T det=sign?-1:1;
17
                                                               int u=f[j+k], v=f[j+k+(1<<i)];</pre>
                                                                                                            using matrix = Matrix<T>;
                                                                                                                                                                  for(int i=0;i<r;++i){</pre>
          eps; ++stp){
                                                   20
                                                                                                                                                           70
       double m = (lo+hi)/2.0:
                                                   21
                                                               f[j+k+(1<<i)] = u-v, f[j+k] = u+v;
                                                                                                            int r.c:
                                                                                                                                                           71
                                                                                                                                                                    det = det*m[i][i]:
       int sign_mid = sign(get(coef,m));
                                                                                                            mt m;
                                                                                                                                                                    det = det/lazy[i];
19
                                                   22
                                                                                                            Matrix(int r,int c):r(r),c(c),m(r,rt(c)){}
20
       if(!sign mid) return m;
                                                   23
                                                        if(inverse) for(auto &a:f) a/=f.size();
                                                                                                                                                           73
                                                                                                                                                                    for(auto &j:m[i])j/=lazy[i];
       if(sign lo*sign mid < 0) hi = m;</pre>
                                                        return f:
                                                                                                            rt& operator[](int i){return m[i];}
                                                                                                                                                           74
21
       else lo = m;
                                                                                                            matrix operator+(const matrix &a){
                                                                                                                                                           75
22
                                                                                                       10
                                                                                                                                                                  return det;
                                                                                                       11
                                                                                                               matrix rev(r,c);
                                                                                                                                                           76
23
                                                                                                                                                           77 };
    return (lo+hi)/2.0;
                                                                                                       12
                                                                                                               for(int i=0;i<r;++i)</pre>
24
25
                                                                                                       13
                                                                                                                 for(int j=0;j<c;++j)</pre>
                                                      6.7 LinearCongruence
                                                                                                       14
                                                                                                                   rev[i][j]=m[i][j]+a.m[i][j];
26
   vector<double> cal(vector<double>coef, int n
                                                                                                       15
                                                                                                               return rev;
                                                                                                       16
       ){
                                                                                                                                                              6.10 MillerRobin
                                                                                                            matrix operator-(const matrix &a){
     vector<double>res;
                                                     1 | pair<LL,LL> LinearCongruence(LL a[],LL b[],
    if(n == 1){
                                                            LL m[], int n) {
                                                                                                               matrix rev(r,c);
29
                                                                                                       18
       if(sign(coef[1])) res.pb(-coef[0]/coef
                                                         // a[i]*x = b[i] (mod m[i])
                                                                                                               for(int i=0;i<r;++i)</pre>
30
                                                                                                       19
                                                                                                                                                            1 LL LLmul(LL a, LL b, const LL &mod) {
                                                         for(int i=0;i<n;++i) {</pre>
                                                                                                                 for(int j=0;j<c;++j)</pre>
            [1]);
                                                                                                       20
                                                                                                                                                                LL ans=0:
31
       return res;
                                                           LL x, y, d = extgcd(a[i],m[i],x,y);
                                                                                                       ^{21}
                                                                                                                   rev[i][j]=m[i][j]-a.m[i][j];
                                                                                                                                                                while(b) {
32
                                                           if(b[i]%d!=0)return make pair(-1LL,0LL);
                                                                                                       22
                                                                                                               return rev;
                                                                                                                                                                  if(b&1) {
33
     vector<double>dcoef(n);
                                                           m[i] /= d;
                                                                                                       23
                                                                                                                                                                    ans+=a;
     for(int i = 0; i < n; ++i) dcoef[i] = coef</pre>
                                                          b[i] = LLmul(b[i]/d,x,m[i]);
                                                                                                            matrix operator*(const matrix &a){
                                                                                                       24
                                                                                                                                                                    if(ans>=mod) ans-=mod;
          [i+1]*(i+1);
                                                                                                               matrix rev(r,a.c);
     vector<double>droot = cal(dcoef, n-1);
                                                                                                               matrix tmp(a.c,a.r);
                                                         LL lastb = b[0], lastm = m[0];
                                                                                                                                                                  a<<=1, b>>=1;
    droot.insert(droot.begin(), -INF);
                                                         for(int i=1;i<n;++i) {</pre>
                                                                                                               for(int i=0;i<a.r;++i)</pre>
36
                                                                                                       27
                                                                                                                                                                  if(a>=mod) a-=mod;
                                                          LL x, y, d = extgcd(m[i],lastm,x,y);
                                                                                                                 for(int j=0;j<a.c;++j)</pre>
37
    droot.pb(INF);
                                                    11
                                                                                                       28
    for(int i = 0; i+1 < droot.size(); ++i){</pre>
                                                   12
                                                           if((lastb-b[i])%d!=0) return make_pair
                                                                                                       29
                                                                                                                   tmp[j][i]=a.m[i][j];
                                                                                                                                                           11
                                                                                                                                                                return ans;
       double tmp = find(coef, n, droot[i],
                                                                                                               for(int i=0;i<r;++i)</pre>
39
                                                                (-1LL,0LL);
                                                                                                       30
                                                                                                                                                           12
                                                           lastb = LLmul((lastb-b[i])/d,x,(lastm/d)
                                                                                                                 for(int j=0;j<a.c;++j)</pre>
            droot[i+1]);
                                                    13
                                                                                                       31
                                                                                                                                                              LL mod mul(LL a,LL b,LL m){
       if(tmp < INF) res.pb(tmp);</pre>
                                                                )*m[i];
                                                                                                       32
                                                                                                                   for(int k=0;k<c;++k)</pre>
                                                                                                                                                                a\%=m,b\%=m;/* fast for m < 2^58 */
    }
                                                           lastm = (lastm/d)*m[i];
                                                                                                       33
                                                                                                                     rev.m[i][j]+=m[i][k]*tmp[j][k];
                                                                                                                                                                LL y=(LL)((double)a*b/m+0.5);
                                                          lastb = (lastb+b[i])%lastm;
42
     return res;
                                                    15
                                                                                                       34
                                                                                                               return rev;
                                                                                                                                                                LL r=(a*b-y*m)%m;
43
                                                    16
                                                                                                       35
                                                                                                                                                                return r<0?r+m:r;</pre>
                                                         return make pair(lastb<0?lastb+lastm:lastb</pre>
                                                    17
                                                                                                            bool inverse(){
                                                                                                                                                           18
   int main () {
                                                              ,lastm);
                                                                                                               Matrix t(r,r+c);
                                                                                                                                                              template<typename T>
    vector<double>ve;
                                                                                                               for(int y=0;y<r;y++){</pre>
                                                                                                                                                              T pow(T a, T b, T mod){//a^b\%mod}
                                                                                                                                                           20
    vector<double>ans = cal(ve, n);
                                                                                                       39
                                                                                                                 t.m[y][c+y] = 1;
    // 視情況把答案 +eps, 避免 -0
                                                                                                       40
                                                                                                                 for(int x=0;x<c;++x)</pre>
48
                                                                                                                                                                for(;b;a=mod_mul(a,a,mod),b>>=1)
                                                                                                                   t.m[y][x]=m[y][x];
                                                      6.8 Lucas
                                                                                                                                                                  if(b&1)ans=mod_mul(ans,a,mod);
                                                                                                       ^{42}
                                                                                                                                                                return ans;
                                                                                                                                                           ^{24}
                                                                                                               if(!t.gas())
                                                                                                       43
                                                                                                                                                           25
                                                                                                                 return false;
                                                                                                                                                           26 int sprp[3]={2,7,61};//int範圍可解
        \mathbf{FWT}
                                                    1 | int mod fact(int n,int &e){
                                                                                                               for(int y=0;y<r;y++)</pre>
                                                                                                                                                              int llsprp
                                                        e=0:
                                                                                                                 for(int x=0;x<c;++x)
                                                                                                                                                                    [7] = \{2,325,9375,28178,450775,9780504,
                                                         if(n==0)return 1;
                                                                                                                   m[y][x]=t.m[y][c+x]/t.m[y][y];
1 vector<int> F OR T(vector<int> f, bool
                                                        int res=mod_fact(n/P,e);
                                                                                                                                                           28 1795265022};//至少unsigned Long Long範圍
                                                                                                       48
                                                                                                               return true;
```

```
30 bool isprime(T n, int *sprp, int num){
     if(n==2)return 1;
     if(n<2||n%2==0)return 0;
33
     int t=0;
     T u=n-1;
34
     for(;u%2==0;++t)u>>=1;
35
36
     for(int i=0:i<num:++i){</pre>
37
       T a=sprp[i]%n;
       if(a==0||a==1||a==n-1)continue;
38
39
       T x=pow(a,u,n);
40
       if(x==1||x==n-1)continue;
       for(int i=0:i<t:++i){</pre>
41
         x = mod mul(x,x,n);
42
43
         if(x==1)return 0:
44
         if(x==n-1)break;
45
       if(x==n-1)continue:
46
       return 0;
47
48
49
     return 1;
```

6.11 NTT

1 2615053605667*(2^18)+1,3

```
2 15*(2^27)+1,31
3 479*(2^21)+1,3
4 7*17*(2^23)+1,3
5 3*3*211*(2^19)+1,5
6 25*(2^22)+1.3
  template<typename T,typename VT=vector<T> >
   struct NTT{
    const T P.G:
    NTT(T p=(1<<23)*7*17+1,T g=3):P(p),G(g){}
    unsigned bit_reverse(unsigned a, int len){
12
       //Look FFT.cpp
13
14
    T pow mod(T n,T k,T m){
15
       T ans=1:
       for(n=(n)=m?n\%m:n);k;k>>=1){
16
         if(k&1)ans=ans*n%m;
17
         n=n*n%m;
18
19
20
       return ans;
     void ntt(bool is_inv,VT &in,VT &out,int N)
       int bitlen=__lg(N);
       for(int i=0;i<N;++i)out[bit reverse(i,</pre>
24
            bitlen)]=in[i];
       for(int step=2,id=1;step<=N;step<<=1,++</pre>
            id){
         T wn=pow_mod(G,(P-1)>>id,P),wi=1,u,t;
         const int mh=step>>1;
         for(int i=0:i<mh:++i){</pre>
           for(int j=i;j<N;j+=step){</pre>
             u=out[j],t=wi*out[j+mh]%P;
30
             out[j]=u+t;
32
             out[i+mh]=u-t;
             if(out[j]>=P)out[j]-=P;
             if(out[i+mh]<0)out[i+mh]+=P;</pre>
35
           wi=wi*wn%P;
```

6.12 Simpson

1 | double simpson(double a, double b){

6.13 外星模運算

```
1 / a[0]^{a[1]^{a[2]^{...}}
2 #define maxn 1000000
 3 int euler[maxn+5];
  bool is prime[maxn+5];
   void init euler(){
    is prime[1]=1;//一不是質數
     for(int i=1:i<=maxn:i++)euler[i]=i:</pre>
     for(int i=2;i<=maxn;i++){</pre>
       if(!is prime[i]){//是質數
         euler[i]--;
         for(int j=i<<1;j<=maxn;j+=i){</pre>
12
           is_prime[j]=1;
           euler[j]=euler[j]/i*(i-1);
17
   LL pow(LL a, LL b, LL mod){//a^b\%mod}
    LL ans=1:
     for(;b;a=a*a%mod,b>>=1)
      if(b&1)ans=ans*a%mod:
     return ans;
   bool isless(LL *a,int n,int k){
    if(*a==1)return k>1;
     if(--n==0)return *a<k;</pre>
    int next=0;
```

```
for(LL b=1;b<k;++next)</pre>
29
       b*=*a;
     return isless(a+1,n,next);
30
  LL high pow(LL *a, int n, LL mod){
    if(*a==1||--n==0)return *a%mod;
     int k=0.r=euler[mod]:
     for(LL tma=1;tma!=pow(*a,k+r,mod);++k)
       tma=tma*(*a)%mod;
     if(isless(a+1,n,k))return pow(*a,high pow(
          a+1,n,k),mod);
     int tmd=high pow(a+1,n,r), t=(tmd-k+r)%r;
    return pow(*a,k+t,mod);
39
40
41 LL a[1000005]:
42 int t, mod;
  int main(){
43
    init euler();
     scanf("%d",&t);
45
46
     #define n 4
47
     while(t--){
       for(int i=0;i<n;++i)scanf("%lld",&a[i]);</pre>
48
49
       scanf("%d",&mod):
50
       printf("%lld\n",high_pow(a,n,mod));
51
52
    return 0;
```

6.14 數位統計

```
1 11 d[65], dp[65][2];//up區間是不是完整
2 11 dfs(int p,bool is8,bool up){
    if(!p)return 1; // 回傳@是不是答案
    if(!up&&~dp[p][is8])return dp[p][is8];
    int mx = up?d[p]:9;//可以用的有那些
    11 ans=0:
    for(int i=0;i<=mx;++i){</pre>
      if( is8&&i==7 )continue;
      ans += dfs(p-1, i==8, up&&i==mx);
11
    if(!up)dp[p][is8]=ans;
12
    return ans;
13
  11 f(11 N){
14
    while(N){ // 把數字先分解到陣列
17
      d[++k] = N%10;
18
      N/=10;
19
    return dfs(k,false,true);
20
```

6.15 質因數分解

```
1 LL func(const LL n,const LL mod,const int c)
2 return (LLmul(n,n,mod)+c+mod)%mod;
3 }
4
```

```
5 LL pollorrho(const LL n, const int c) {//循
        環 箭 長 度
    LL a=1. b=1:
    a=func(a,n,c)%n;
    b=func(b,n,c)%n; b=func(b,n,c)%n;
    while(gcd(abs(a-b),n)==1) {
       a=func(a,n,c)%n;
       b=func(b,n,c)%n; b=func(b,n,c)%n;
11
12
     return gcd(abs(a-b),n);
   void prefactor(LL &n, vector<LL> &v) {
    for(int i=0:i<12:++i) {</pre>
       while(n%prime[i]==0) {
         v.push back(prime[i]);
20
         n/=prime[i];
21
22
23
   void smallfactor(LL n, vector<LL> &v) {
    if(n<MAXPRIME) {</pre>
       while(isp[(int)n]) {
28
         v.push_back(isp[(int)n]);
29
         n/=isp[(int)n];
31
       v.push back(n);
32
    } else {
       for(int i=0;i<primecnt&&prime[i]*prime[i</pre>
            1<=n;++i) {</pre>
34
         while(n%prime[i]==0) {
           v.push back(prime[i]);
35
           n/=prime[i];
36
37
38
39
       if(n!=1) v.push_back(n);
40
41
42
   void comfactor(const LL &n, vector<LL> &v) {
     if(n<1e9) {
       smallfactor(n,v);
46
       return;
47
     if(Isprime(n)) {
       v.push back(n);
50
       return:
51
52
     for(int c=3;;++c) {
       d = pollorrho(n,c);
55
       if(d!=n) break;
56
    comfactor(d,v);
     comfactor(n/d,v);
   void Factor(const LL &x, vector<LL> &v) {
    if(n==1) { puts("Factor 1"); return; }
    prefactor(n,v);
    if(n==1) return;
     comfactor(n,v);
    sort(v.begin(),v.end());
```

```
38
                                                       39
    void AllFactor(const LL &n,vector<LL> &v) {
                                                      40
71
     vector<LL> tmp;
                                                      41
     Factor(n,tmp);
72
                                                      42
73
     v.clear();
                                                      43
74
     v.push back(1):
75
     int len;
                                                      44
76
     LL now=1;
                                                      45
77
     for(int i=0;i<tmp.size();++i) {</pre>
78
       if(i==0 || tmp[i]!=tmp[i-1]) {
                                                      46
         len = v.size():
79
                                                      47
         now = 1;
80
                                                      48
81
                                                      49
82
       now*=tmp[i];
                                                      50
83
       for(int j=0;j<len;++j)</pre>
          v.push_back(v[j]*now);
84
85
                                                      52
                                                      53
                                                      54
                                                      55
                                                      56
                                                      57
```

7 String

7.1 AC 自動機

```
1 template < char L='a', char R='z'>
  class ac automaton{
    struct joe{
       int next[R-L+1], fail, efl, ed, cnt dp, vis;
       joe():ed(0),cnt dp(0),vis(0){
         for(int i=0;i<=R-L;++i)next[i]=0;</pre>
    };
   public:
    std::vector<joe> S;
    std::vector<int> q;
    int qs,qe,vt;
    ac_automaton():S(1),qs(0),qe(0),vt(0){}
14
    void clear(){
15
       q.clear();
16
       S.resize(1);
       for(int i=0;i<=R-L;++i)S[0].next[i]=0;</pre>
       S[0].cnt dp=S[0].vis=qs=qe=vt=0;
19
     void insert(const char *s){
20
       int o=0;
       for(int i=0,id;s[i];++i){
22
23
         id=s[i]-L;
         if(!S[o].next[id]){
           S.push_back(joe());
26
           S[o].next[id]=S.size()-1;
27
         o=S[o].next[id];
28
30
       ++S[o].ed;
     void build fail(){
       S[0].fail=S[0].efl=-1;
       q.clear();
       q.push back(0);
       ++ae;
       while(qs!=qe){
```

```
t=S[pa].next[i];
     if(!t)continue;
     id=S[pa].fail;
     while(~id&&!S[id].next[i])id=S[id].
     S[t].fail=~id?S[id].next[i]:0;
     S[t].efl=S[S[t].fail].ed?S[t].fail:S
          [S[t].fail].efl;
     q.push back(t);
                                       101
     ++qe;
 }
/*DP出每個前綴在字串s出現的次數並傳回所有
    字串被s匹配成功的次數O(N+M)*/
int match 0(const char *s){
                                       108
 int ans=0,id,p=0,i;
 for(i=0;s[i];++i){
   id=s[i]-L;
   while(!S[p].next[id]&&p)p=S[p].fail;
   if(!S[p].next[id])continue;
   p=S[p].next[id];
   ++S[p].cnt_dp;/*匹配成功則它所有後綴都
        可以被匹配(DP計算)*/
 for(i=qe-1;i>=0;--i){
   ans+=S[q[i]].cnt dp*S[q[i]].ed;
   if(~S[q[i]].fail)S[S[q[i]].fail].
        cnt_dp+=S[q[i]].cnt_dp;
 return ans;
/*多串匹配走efL邊並傳回所有字串被s匹配成功
    的 次 數 O(N*M^1.5)*/
int match 1(const char *s)const{
 int ans=0,id,p=0,t;
 for(int i=0;s[i];++i){
   id=s[i]-L;
   while(!S[p].next[id]&&p)p=S[p].fail;
   if(!S[p].next[id])continue;
   p=S[p].next[id];
   if(S[p].ed)ans+=S[p].ed;
   for(t=S[p].efl;~t;t=S[t].efl){
     ans+=S[t].ed;/*因為都走efl邊所以保證
          匹配成功*/
 return ans;
/*枚舉(s的子字串nA)的所有相異字串各恰一次
    並傳回次數O(N*M^(1/3))*/
int match_2(const char *s){
 int ans=0,id,p=0,t;
 /*把戳記vt+=1,只要vt沒溢位,所有S[p].
      vis==vt就會變成false
  這種利用vt的方法可以0(1)歸零vis陣列*/
  for(int i=0;s[i];++i){
   while(!S[p].next[id]&&p)p=S[p].fail;
   if(!S[p].next[id])continue;
```

int pa=q[qs++],id,t;

58

59

60

61

62

65

66

67

68

69

70

71

72

73

74

75

76

78

79

80

81

85

90

91

p=S[p].next[id];

for(int i=0;i<=R-L;++i){</pre>

```
if(S[p].ed&&S[p].vis!=vt){
94
           S[p].vis=vt;
95
           ans+=S[p].ed;
96
         for(t=S[p].efl;~t&&S[t].vis!=vt;t=S[t
97
              ].efl){
           S[t].vis=vt;
           ans+=S[t].ed;/*因為都走efL邊所以保證
                匹配成功*/
100
102
       return ans;
103
     /*把AC自動機變成真的自動機*/
104
105
     void evolution(){
       for(qs=1;qs!=qe;){
106
107
         int p=q[qs++];
         for(int i=0;i<=R-L;++i)</pre>
109
           if(S[p].next[i]==0)S[p].next[i]=S[S[
                p].fail].next[i];
110
111
     }
112 };
```

7.2 hash

```
1 #define MAXN 1000000
2 #define mod 1073676287
3 /*mod 必須要是質數*/
4 typedef long long T;
 5 char s[MAXN+5];
6 T h[MAXN+5];/*hash陣列*/
7 T h_base[MAXN+5];/*h_base[n]=(prime^n)%mod*/
8 void hash_init(int len,T prime){
    h base[0]=1:
    for(int i=1;i<=len;++i){</pre>
      h[i]=(h[i-1]*prime+s[i-1])%mod;
      h_base[i]=(h_base[i-1]*prime)%mod;
12
13
14 }
15 | T get_hash(int l,int r){/*閉區間寫法,設編號
       為0 ~ Len-1*/
    return (h[r+1]-(h[1]*h_base[r-1+1])%mod+
         mod)%mod;
17 }
```

7.3 KMP

```
1  /*產生fail function*/
2  void kmp_fail(char *s,int len,int *fail){
3    int id=-1;
4    fail[0]=-1;
5    for(int i=1;i<len;++i){
6       while(~id&&s[id+1]!=s[i])id=fail[id];
7       if(s[id+1]==s[i])++id;
8       fail[i]=id;
9    }
10 }</pre>
```

```
11 | /*以字串B匹配字串A, 傳回匹配成功的數量(用B的
       fail)*/
int kmp match(char *A,int lenA,char *B,int
       lenB, int *fail){
     int id=-1,ans=0;
    for(int i=0;i<lenA;++i){</pre>
      while(~id&&B[id+1]!=A[i])id=fail[id];
      if(B[id+1]==A[i])++id;
      if(id==lenB-1){/*匹配成功*/
        ++ans, id=fail[id];
18
19
20
    }
21
    return ans;
```

7.4 manacher

7.5 minimal_string_rotation

```
int min_string_rotation(const string &s){
    int n=s.size(),i=0,j=1,k=0;
    while(i<n&&j<n&&k<n){
        int t=s[(i+k)%n]-s[(j+k)%n];
        ++k;
        if(t){
            if(t>0)i+=k;
            else j+=k;
            if(i==j)++j;
            k=0;
    }
    return min(i,j);//最小循環表示法起始位置
```

7.6 reverseBWT

```
const int MAXN = 305, MAXC = 'Z';
int ranks[MAXN], tots[MAXC], first[MAXC];
void rankBWT(const string &bw){
   memset(ranks,0,sizeof(int)*bw.size());
   memset(tots,0,sizeof(tots);
   for(size_t i=0;i<bw.size();++i)
      ranks[i] = tots[int(bw[i])]++;
}
youd firstCol(){</pre>
```

```
memset(first,0,sizeof(first));
    int totc = 0;
    for(int c='A';c<='Z';++c){</pre>
12
      if(!tots[c]) continue;
13
14
      first[c] = totc;
      totc += tots[c];
15
16
17
   string reverseBwt(string bw,int begin){
    rankBWT(bw), firstCol();
    int i = begin; //原字串最後一個元素的位置
    string res;
22
    do{
23
      char c = bw[i]:
       res = c + res;
      i = first[int(c)] + ranks[i];
    }while( i != begin );
    return res;
```

7.8 Z

```
void z_alg(char *s,int len,int *z){
    int l=0,r=0;
    z[0]=len;
    for(int i=1;i<len;++i){
        z[i]=i>r?0:(i-1+z[i-1]<z[l]?z[i-1]:r-i
        +1);
    while(i+z[i]<len&&s[i+z[i]]==s[z[i]])++z
    [i];
    if(i+z[i]-1>r)r=i+z[i]-1,l=i;
}

9 }
```

8 Tarjan

1 | struct dominator tree{

int n:// 1-base

36

37

dfs(r);

for(int y=Time;y>=2;--y){

int x=fa[y],idy=id[y];

8.1 dominator tree

static const int MAXN=5005;

7.7 suffix_array_lcp

```
1 #define radix sort(x,y){\
    for(i=0;i<A;++i)c[i]=0;\</pre>
    for(i=0;i<n;++i)c[x[y[i]]]++;\</pre>
    for(i=1;i<A;++i)c[i]+=c[i-1];\</pre>
    for(i=n-1;~i;--i)sa[--c[x[y[i]]]]=y[i];\
7 #define AC(r,a,b)\
    r[a]!=r[b]||a+k>=n||r[a+k]!=r[b+k]
   void suffix_array(const char *s,int n,int *
        sa,int *rank,int *tmp,int *c){
     int A='z'+1,i,k,id=0;
    for(i=0;i<n;++i)rank[tmp[i]=i]=s[i];</pre>
    radix sort(rank,tmp);
13
     for(k=1;id<n-1;k<<=1){
14
       for(id=0,i=n-k;i<n;++i)tmp[id++]=i;</pre>
15
       for(i=0;i<n;++i)</pre>
         if(sa[i]>=k)tmp[id++]=sa[i]-k;
16
       radix sort(rank,tmp);
       swap(rank,tmp);
       for(rank[sa[0]]=id=0,i=1;i<n;++i)</pre>
         rank[sa[i]]=id+=AC(tmp,sa[i-1],sa[i]);
       A=id+1;
22
23
24 //h: 高度數組 sa:後綴數組 rank:排名
   void suffix_array_lcp(const char *s,int len,
        int *h,int *sa,int *rank){
     for(int i=0;i<len;++i)rank[sa[i]]=i;</pre>
27
     for(int i=0,k=0;i<len;++i){</pre>
28
       if(rank[i]==0)continue;
29
       if(k)--k;
30
       while(s[i+k]==s[sa[rank[i]-1]+k])++k;
31
       h[rank[i]]=k;
32
33
    h[0]=0;// h[k]=lcp(sa[k],sa[k-1]);
```

vector<int> suc[MAXN],pre[MAXN]; int fa[MAXN],dfn[MAXN],id[MAXN],Time; int semi[MAXN],idom[MAXN]; int anc[MAXN],best[MAXN];//disjoint set vector<int> dom[MAXN];//dominator_tree void init(int _n){ n=_n; for(int i=1;i<=n;++i)suc[i].clear(),pre[i].clear(); } void add_edge(int u,int v){ susing namespace std; #define MAXN 8001 #define MAXN 2 MAXN*4 #define mAXN 2 MAXN*2 *vector<int> v[MAXN2] int N,M; void addedge(int s,i v[s].push_back(e); rv[e].push_back(s) } int scc[MAXN];

```
10
11
       suc[u].push back(v);
14
       pre[v].push back(u);
15
16
     void dfs(int u){
17
       dfn[u]=++Time,id[Time]=u;
18
19
       for(auto v:suc[u]){
20
         if(dfn[v])continue;
21
         dfs(v),fa[dfn[v]]=dfn[u];
22
23
24
     int find(int x){
       if(x==anc[x])return x;
       int y=find(anc[x]);
       if(semi[best[x]]>semi[best[anc[x]]])best 26
            [x]=best[anc[x]];
       return anc[x]=y;
29
     void tarjan(int r){
30
31
       for(int t=1;t<=n;++t){</pre>
32
         dfn[t]=idom[t]=0;//u=r或是u無法到達r時
33
              idom[id[u]]=0
34
         dom[t].clear();
35
         anc[t]=best[t]=semi[t]=t;
```

8.2 tnfshb017 2 sat

1 #include < bits / stdc++.h>

for(auto z:pre[idy]){

for(auto z:dom[x]){

for(int u=2:u<=Time:++u){</pre>

dom[x].clear();

anc[v]=x:

41

42

43

44

45

46

50

51

52

53

54

55

56

57

58 }dom;

if(!(z=dfn[z]))continue;

dom[semi[y]].push back(y);

semi[y]=min(semi[y],semi[best[z]]);

idom[z]=semi[best[z]]<x?best[z]:x;</pre>

if(idom[u]!=semi[u])idom[u]=idom[idom[

dom[id[idom[u]]].push back(id[u]);

```
3 #define MAXN 8001
 4 #define MAXN2 MAXN*4
 5 #define n(X) ((X)+2*N)
 6 vector<int> v[MAXN2], rv[MAXN2], vis_t;
 7 int N.M:
   void addedge(int s,int e){
     v[s].push_back(e);
     rv[e].push back(s);
11 }
12 int scc[MAXN2];
13 bool vis[MAXN2]={false};
   void dfs(vector<int> *uv,int n,int k=-1){
     vis[n]=true;
     for(int i=0;i<uv[n].size();++i)</pre>
       if(!vis[uv[n][i]])
         dfs(uv,uv[n][i],k);
     if(uv==v)vis_t.push_back(n);
19
20
     scc[n]=k;
21 }
22
   void solve(){
     for(int i=1;i<=N;++i){</pre>
       if(!vis[i])dfs(v,i);
       if(!vis[n(i)])dfs(v,n(i));
27
     memset(vis,0,sizeof(vis));
     int c=0;
     for(int i=vis t.size()-1;i>=0;--i)
       if(!vis[vis_t[i]])
         dfs(rv,vis_t[i],c++);
32 }
33 int main(){
34
     int a.b:
     scanf("%d%d",&N,&M);
     for(int i=1;i<=N;++i){</pre>
       // (A or B)&(!A & !B) A^B
       a=i*2-1;
       b=i*2;
       addedge(n(a),b);
```

```
addedge(n(b),a);
42
       addedge(a,n(b));
       addedge(b,n(a));
43
44
     while(M--){
       scanf("%d%d",&a,&b);
47
       a = a>0?a*2-1:-a*2:
       b = b>0?b*2-1:-b*2;
48
       // A or B
       addedge(n(a),b);
51
       addedge(n(b),a);
52
53
     solve();
     bool check=true:
     for(int i=1:i<=2*N:++i)</pre>
       if(scc[i]==scc[n(i)])
57
         check=false:
     if(check){
58
       printf("%d \setminus n",N);
59
       for(int i=1;i<=2*N;i+=2){</pre>
         if(scc[i]>scc[i+2*N]) putchar('+');
61
         else putchar('-');
62
63
64
       puts("");
     }else puts("0");
65
     return 0:
```

8.3 橋連涌分量

```
1 | #define N 1005
  struct edge{
    int u,v;
    bool is bridge:
    edge(int u=0,int v=0):u(u),v(v),is_bridge
  vector<edge> E;
  vector<int> G[N];// 1-base
  int low[N], vis[N], Time;
  int bcc_id[N],bridge_cnt,bcc_cnt;// 1-base
  int st[N],top;//BCC用
  inline void add edge(int u,int v){
    G[u].push back(E.size());
    E.push back(edge(u,v));
    G[v].push back(E.size());
    E.push_back(edge(v,u));
17 }
18 void dfs(int u,int re=-1){//u當前點,re為u連
        接前一個點的邊
     int v;
    low[u]=vis[u]=++Time:
21
    st[top++]=u;
    for(size_t i=0;i<G[u].size();++i){</pre>
      int e=G[u][i];v=E[e].v;
24
      if(!vis[v]){
25
        dfs(v,e^1);//e^1反向邊
        low[u]=min(low[u],low[v]);
        if(vis[u]<low[v]){</pre>
          E[e].is bridge=E[e^1].is bridge=1;
          ++bridge_cnt;
```

```
}else if(vis[v]<vis[u]&&e!=re)</pre>
         low[u]=min(low[u], vis[v]);
32
33
    if(vis[u]==low[u]){//處理BCC
34
35
       ++bcc cnt;// 1-base
       do bcc_id[v=st[--top]]=bcc_cnt;//每個點
36
            所在的BCC
       while(v!=u);
38
39
   inline void bcc init(int n){
    Time=bcc_cnt=bridge_cnt=top=0;
    E.clear();
42
    for(int i=1;i<=n;++i){</pre>
43
      G[i].clear();
       vis[i]=bcc id[i]=0;
46
```

8.4 雙連通分量 & 割點

```
1 #define N 1005
vector<int> G[N];// 1-base
3 vector < int > bcc[N]; // 存每塊雙連通分量的點
4 int low[N], vis[N], Time;
5 int bcc_id[N],bcc_cnt;// 1-base
6 bool is cut[N];//是否為割點
7 int st[N],top;
   void dfs(int u,int pa=-1){//u當前點,pa父親
    int v,child=0;
    low[u]=vis[u]=++Time;
11
    st[top++]=u;
    for(size_t i=0;i<G[u].size();++i){</pre>
12
      if(!vis[v=G[u][i]]){
13
14
        dfs(v,u),++child;
15
         low[u]=min(low[u],low[v]);
16
         if(vis[u]<=low[v]){</pre>
           is cut[u]=1;
17
           bcc[++bcc_cnt].clear();
18
19
           int t;
20
21
             bcc id[t=st[--top]]=bcc cnt;
             bcc[bcc cnt].push back(t);
22
23
           }while(t!=v);
           bcc id[u]=bcc cnt;
24
25
           bcc[bcc_cnt].push_back(u);
26
27
      }else if(vis[v]<vis[u]&&v!=pa)//反向邊
28
        low[u]=min(low[u], vis[v]);
29
    if(pa==-1&&child<2)is cut[u]=0;//u是dfs樹
          的根要特判
31
   inline void bcc_init(int n){
    Time=bcc cnt=top=0;
34
    for(int i=1;i<=n;++i){</pre>
35
      G[i].clear();
      is cut[i]=vis[i]=bcc id[i]=0;
36
37
38 }
```

9 Tree_problem

9.1 HeavyLight

1 | #include < vector >

```
2 #define MAXN 100005
 int siz[MAXN], max_son[MAXN], pa[MAXN], dep[
       MAXN1:
   int link_top[MAXN],link[MAXN],cnt;
   vector<int> G[MAXN];
   void find max son(int u){
    siz[u]=1;
     max son[u]=-1;
     for(auto v:G[u]){
      if(v==pa[u])continue;
      pa[v]=u;
12
       dep[v]=dep[u]+1;
       find max son(v);
       if(max son[u]==-1||siz[v]>siz[max son[u
           ]])max_son[u]=v;
15
       siz[u]+=siz[v];
16
    }
17 }
   void build link(int u.int top){
    link[u]=++cnt;
20
    link top[u]=top;
     if(max_son[u]==-1)return;
21
     build link(max son[u],top);
22
     for(auto v:G[u]){
      if(v==max son[u]||v==pa[u])continue;
24
      build link(v,v);
26
27 }
   int find_lca(int a,int b){
    //求LCA, 可以在過程中對區間進行處理
    int ta=link top[a],tb=link top[b];
     while(ta!=tb){
31
32
      if(dep[ta]<dep[tb]){</pre>
33
        swap(ta,tb);
34
        swap(a,b);
35
      //這裡可以對a所在的鏈做區間處理
36
      //區間為(link[ta],link[a])
37
      ta=link_top[a=pa[ta]];
38
39
    // 最後a,b會在同一條鏈,若a!=b還要在進行一
         次區間處理
    return dep[a]<dep[b]?a:b;</pre>
41
```

9.2 LCA

```
const int MAXN=100000; // 1-base
const int MLG=17; //Log2(MAXN)+1;
int pa[MLG+1][MAXN+5];
int dep[MAXN+5];
vector<int> G[MAXN+5];
void dfs(int x,int p=0){//dfs(root);
pa[0][x]=p;
```

```
for(int i=0;i<=MLG;++i)</pre>
       pa[i+1][x]=pa[i][pa[i][x]];
     for(auto &i:G[x]){
10
11
       if(i==p)continue;
12
       dep[i]=dep[x]+1;
13
       dfs(i,x);
14
15
16
   inline int jump(int x,int d){
17
     for(int i=0;i<=MLG;++i)</pre>
18
       if((d>>i)&1) x=pa[i][x];
     return x:
19
20
   inline int find_lca(int a,int b){
     if(dep[a]>dep[b])swap(a,b);
23
     b=jump(b,dep[b]-dep[a]);
     if(a==b)return a:
24
     for(int i=MLG;i>=0;--i){
25
       if(pa[i][a]!=pa[i][b]){
26
         a=pa[i][a];
27
28
         b=pa[i][b];
29
30
     return pa[0][a];
31
```

9.3 link cut tree

```
1 struct splay tree{
    int ch[2],pa;//子節點跟父母
    bool rev;//反轉的懶惰標記
    splay tree():pa(0),rev(0){ch[0]=ch[1]=0;}
5 };
6 vector<splay_tree> nd;
7 // 有的時候用vector會TLE,要注意
8 / / 這邊以node [ 0 ] 作為nul L 節點
9 bool isroot(int x){//判斷是否為這棵splay
       tree的根
    return nd[nd[x].pa].ch[0]!=x&&nd[nd[x].pa
        ].ch[1]!=x;
  void down(int x){// 懶 惰 標 記 下 推
    if(nd[x].rev){
      if(nd[x].ch[0])nd[nd[x].ch[0]].rev^=1;
15
      if(nd[x].ch[1])nd[nd[x].ch[1]].rev^=1;
      swap(nd[x].ch[0],nd[x].ch[1]);
16
17
      nd[x].rev=0;
18
19
  void push down(int x){//所有祖先懶惰標記下推
    if(!isroot(x))push_down(nd[x].pa);
22
23 }
24 | void up(int x){}//將子節點的資訊向上更新
  void rotate(int x){//旋轉·會自行判斷轉的方
    int y=nd[x].pa,z=nd[y].pa,d=(nd[y].ch[1]==
        x);
    nd[x].pa=z;
    if(!isroot(y))nd[z].ch[nd[z].ch[1]==y]=x;
    nd[y].ch[d]=nd[x].ch[d^1];
```

```
nd[nd[y].ch[d]].pa=y;
31
    nd[y].pa=x,nd[x].ch[d^1]=y;
32
    up(y),up(x);
33
34 void splay(int x){//將x伸展到splay tree的根
35
    push down(x);
    while(!isroot(x)){
      int y=nd[x].pa;
      if(!isroot(y)){
39
         int z=nd[y].pa;
        if((nd[z].ch[0]==y)^(nd[y].ch[0]==x))
40
             rotate(v);
        else rotate(x);
41
42
43
      rotate(x);
44
45
  int access(int x){
    int last=0:
    while(x){
48
      splay(x);
      nd[x].ch[1]=last;
      up(x);
52
      last=x:
      x=nd[x].pa;
54
55
    return last;//access後splay tree的根
56
  void access(int x,bool is=0){//is=0就是一般
        的access
     int last=0;
    while(x){
      splay(x);
      if(is&&!nd[x].pa){
        //printf("%d\n",max(nd[last].ma,nd[nd[
             x1.ch[1]1.ma));
64
      nd[x].ch[1]=last;
      up(x);
65
      last=x;
      x=nd[x].pa;
  void query_edge(int u,int v){
    access(u);
    access(v.1):
  void make root(int x){
    access(x),splay(x);
76
    nd[x].rev^=1;
77
  void make_root(int x){
    nd[access(x)].rev^=1;
    splay(x);
81
  void cut(int x,int y){
    make root(x);
    access(y);
    splay(y);
    nd[y].ch[0]=0;
    nd[x].pa=0;
  void cut_parents(int x){
89
    access(x);
    splay(x);
```

```
nd[nd[x].ch[0]].pa=0;
93
     nd[x].ch[0]=0;
                                               153
                                               154
94
    void link(int x,int y){
                                               155 }
     make root(x);
97
     nd[x].pa=y;
98
    int find_root(int x){
     x=access(x);
     while(nd[x].ch[0])x=nd[x].ch[0];
102
     splay(x);
103
     return x:
104
int query(int u,int v){
   //傳回uv路徑splay tree的根結點
   // 這種寫法無法求LCA
     make root(u);
     return access(v);
109
110
int query_lca(int u,int v){
112 //假設求鏈上點權的總和, sum是子樹的權重和
        data是節點的權重
     access(u);
     int lca=access(v);
115
     splay(u);
116
     if(u==lca){
       //return nd[lca].data+nd[nd[lca].ch[1]].
     }else{
       //return nd[lca].data+nd[nd[lca].ch[1]].
119
            sum+nd[u].sum
   struct EDGE{
     int a,b,w;
124 }e[10005];
126 vector<pair<int,int>> G[10005];
127 //first表示子節點, second表示邊的編號
128 int pa[10005],edge_node[10005];
129 //pa是父母節點,暫存用的,edge node是每個編
        被存在哪個點裡面的陣列
130 void bfs(int root){
131 //在建構的時候把每個點都設成一個splay tree
     queue<int > q;
     for(int i=1;i<=n;++i)pa[i]=0;</pre>
133
     q.push(root);
134
135
     while(q.size()){
136
       int u=q.front();
137
       q.pop();
       for(auto P:G[u]){
138
         int v=P.first;
139
140
         if(v!=pa[u]){
           pa[v]=u;
141
142
           nd[v].pa=u;
           nd[v].data=e[P.second].w;
143
144
           edge_node[P.second]=v;
145
           up(v);
146
           q.push(v);
147
148
149
150
void change(int x,int b){
```

9.4 POJ_tree

1 #include < bits / stdc++.h>

2 using namespace std;

3 #define MAXN 10005

splay(x);

up(x);

//nd[x].data=b;

```
4 int n,k;
 5 vector<pair<int,int> >g[MAXN];
 6 int size[MAXN];
  bool vis[MAXN];
  inline void init(){
     for(int i=0:i<=n:++i){</pre>
       g[i].clear();
       vis[i]=0;
12
    }
13 }
   void get dis(vector<int> &dis.int u.int pa.
        int d){
     dis.push back(d):
     for(size_t i=0;i<g[u].size();++i){</pre>
       int v=g[u][i].first,w=g[u][i].second;
17
       if(v!=pa&&!vis[v])get dis(dis,v,u,d+w);
19
20 }
   vector<int> dis;//這東西如果放在函數裡會TLE
   int cal(int u,int d){
     dis.clear();
     get dis(dis,u,-1,d);
25
     sort(dis.begin(),dis.end());
     int l=0,r=dis.size()-1,res=0;
27
       while(1<r&&dis[1]+dis[r]>k)--r;
29
       res+=r-(1++);
30
31
     return res;
32 }
   pair<int,int> tree centroid(int u,int pa,
        const int sz){
     size[u]=1;//找樹重心, second是重心
     pair<int,int> res(INT MAX,-1);
     int ma=0;
     for(size_t i=0;i<g[u].size();++i){</pre>
37
       int v=g[u][i].first;
38
39
       if(v==pa||vis[v])continue;
       res=min(res,tree centroid(v,u,sz));
40
41
       size[u]+=size[v];
42
       ma=max(ma,size[v]);
43
     ma=max(ma,sz-size[u]);
     return min(res, make pair(ma, u));
46 }
47
   int tree DC(int u,int sz){
     int center=tree centroid(u,-1,sz).second;
     int ans=cal(center,0);
     vis[center]=1;
     for(size t i=0;i<g[center].size();++i){</pre>
       int v=g[center][i].first,w=g[center][i].
```

```
if(vis[v])continue;
54
       ans-=cal(v,w);
55
       ans+=tree_DC(v,size[v]);
56
57
     return ans;
58
59
   int main(){
     while(scanf("%d%d",&n,&k),n||k){
60
61
       init();
       for(int i=1;i<n;++i){</pre>
62
63
         int u,v,w;
         scanf("%d%d%d",&u,&v,&w);
64
         g[u].push_back(make_pair(v,w));
65
66
         g[v].push back(make pair(u,w));
67
68
       printf("%d\n",tree_DC(1,n));
69
70
     return 0;
```

10 default

10.1 debug

10.2 ext

```
12 //s.order_of_key(1);
```

10.3 IncStack

10.4 input

```
inline int read(){
  int x=0; bool f=0; char c=getchar();
  while(ch<'0'||'9'<ch)f|=ch=='-',ch=getchar();

while('0'<=ch&&ch<='9')x=x*10-'0'+ch,ch=getchar();
  return f?-x:x;
}

// #!/bin/bash
// g++ -std=c++11 -02 -Wall -Wextra -Wno-unused-result -DDEBUG $1 && ./a.out
// -fsanitize=address -fsanitize=undefined-fsanitize=return</pre>
```

11 language

11.1 CNF

```
1 #define MAXN 55
2 struct CNF{
3 int s,x,y;//s->xy | s->x, if y==-1
4 int cost;
5 CNF(){}
6 CNF(int s,int x,int y,int c):s(s),x(x),y(y),cost(c){}
7 };
8 int state;//規則數量
9 map<char,int> rule;//每個字元對應到的規則·
小寫字母為終端字符
10 vector<CNF> cnf;
```

```
11 void init(){
    state=0;
    rule.clear();
13
14
    cnf.clear();
15
   void add to cnf(char s,const string &p,int
    //加入一個s -> 的文法,代價為cost
    if(rule.find(s)==rule.end())rule[s]=state
    for(auto c:p)if(rule.find(c)==rule.end())
         rule[c]=state++;
    if(p.size()==1){
       cnf.push back(CNF(rule[s],rule[p[0]],-1,
22
    }else{
      int left=rule[s];
      int sz=p.size();
       for(int i=0;i<sz-2;++i){</pre>
         cnf.push_back(CNF(left,rule[p[i]],
             state,0));
        left=state++;
       cnf.push back(CNF(left,rule[p[sz-2]],
            rule[p[sz-1]],cost));
30
32 vector<long long> dp[MAXN][MAXN];
33 | vector<bool> neg INF[MAXN][MAXN];//如果花費
        是負的可能會有無限小的情形
  void relax(int 1,int r,const CNF &c,long
       long cost,bool neg_c=0){
    if(!neg_INF[1][r][c.s]&&(neg_INF[1][r][c.x
         ]||cost<dp[1][r][c.s])){
      if(neg_c||neg_INF[1][r][c.x]){
37
        dp[1][r][c.s]=0;
        neg_INF[1][r][c.s]=true;
39
      }else dp[l][r][c.s]=cost;
40
41
   void bellman(int l,int r,int n){
    for(int k=1;k<=state;++k)</pre>
      for(auto c:cnf)
        if(c.y==-1)relax(1,r,c,dp[1][r][c.x]+c
45
             .cost,k==n);
46
   void cyk(const vector<int> &tok){
    for(int i=0;i<(int)tok.size();++i){</pre>
      for(int j=0;j<(int)tok.size();++j){</pre>
49
        dp[i][j]=vector<long long>(state+1,
             INT MAX);
        neg_INF[i][j]=vector<bool>(state+1,
             false):
      dp[i][i][tok[i]]=0;
      bellman(i,i,tok.size());
55
    for(int r=1;r<(int)tok.size();++r){</pre>
      for(int l=r-1;l>=0;--1){
        for(int k=1;k<r;++k)</pre>
          for(auto c:cnf)
            if(~c.y)relax(1,r,c,dp[1][k][c.x]+
                 dp[k+1][r][c.y]+c.cost);
         bellman(l,r,tok.size());
62
```

other

12.1 WhatDay

```
1 int whatday(int y,int m,int d){
  if(m < = 2)m + = 12, -- y;
   if(y<1752||y==1752&m<9||y==1752&m==9&d
     return (d+2*m+3*(m+1)/5+y+y/4+5)%7;
   return (d+2*m+3*(m+1)/5+y+y/4-y/100+y/400)
```

12.2 上下最大正方形

```
1 void solve(int n,int a[],int b[]){// 1-base
    int ans=0:
    deque<int>da,db;
     for(int l=1,r=1;r<=n;++r){</pre>
       while(da.size()&&a[da.back()]>=a[r]){
         da.pop_back();
       da.push back(r);
       while(db.size()&&b[db.back()]>=b[r]){
         db.pop back();
11
12
       db.push back(r);
       for(int d=a[da.front()]+b[db.front()];r-
            1+1>d;++1){
         if(da.front()==1)da.pop_front();
         if(db.front()==1)db.pop_front();
         if(da.size()&&db.size()){
           d=a[da.front()]+b[db.front()];
18
19
20
      ans=max(ans,r-l+1);
21
    printf("%d\n",ans);
```

12.3 最大矩形

```
1 | LL max rectangle(vector<int> s){
    stack<pair<int,int > > st;
    st.push(make pair(-1,0));
    s.push back(0);
    LL ans=0;
    for(size t i=0;i<s.size();++i){</pre>
      int h=s[i];
      pair<int,int > now=make_pair(h,i);
      while(h<st.top().first){</pre>
```

```
now=st.top();
11
         st.pop();
         ans=max(ans,(LL)(i-now.second)*now.
12
              first);
13
       if(h>st.top().first){
14
15
         st.push(make pair(h,now.second));
16
17
18
    return ans;
```

zformula

13.1 formula

13.1.1 Pick 公式

給定頂點坐標均是整點的簡單多邊形,面積 = 內部格點數 + 邊上格點數/2-1

13.1.2 圖論

```
1. V - E + F = 2
```

- 2. 對於平面圖 $F = E V + n + 1 \cdot n$ 是連通分量
- 3. 對於平面圖 $\cdot E \leq 3V 6$
- 4. 對於連通圖 G·最大獨立點集的大小設為 I(G)·最 大匹配大小設為 M(G), 最小點覆蓋設為 Cv(G), 最小邊覆蓋設為 Ce(G)。對於任意連通圖:
 - (a) I(G) + Cv(G) = |V|(b) M(G) + Ce(G) = |V|
- 5. 對於連通二分圖:
 - (a) I(G) = Cv(G)(b) M(G) = Ce(G)
- 6. 最大權閉合圖:
 - (a) $C(u, V) = \infty, (u, v) \in E$ (b) $C(S, v) = W_v, W_v > 0$ (c) $C(v, T) = -W_v, W_v < 0$
- 7. 最大密度子圖:
 - (a) $C(u, v) = 1, (u, v) \in E$ (b) $C(S, v) = U_v, v \in V$ (c) $C(v,T) = U + 2g - d_v, v \in V$
- - (a) 完美消除序列從後往前依次給每個點染色,給 每個點染上可以染的最小顏色
 - 最大團大小 = 色數
 - (c) 最大獨立集: 完美消除序列從前往後能選就選
 - (d) 最小團覆蓋: 最大獨立集的點和他延伸的邊構

 - (f) 區間圖的完美消除序列: 將區間按造又端點由 小到大排序
 - (g) 區間圖染色: 用線段樹做

```
1 double 1=0,=m, stop=1.0/n/n;
  while(r-1>=stop){
    double(mid);
    if((n*m-sol.maxFlow(s,t))/2>eps)l=mid;
    else r=mid;
  build(1):
  sol.maxFlow(s,t);
  vector<int> ans;
10 for(int i=1;i<=n;++i)
if(sol.vis[i])ans.push back(i);
```

13.1.3 學長公式

```
1. \sum_{d|n} \phi(n) = n
```

- 2. $g(n) = \sum_{d|n} f(d) = \sum_{d|n} \mu(d) \times$
- 3. Harmonic series $H_n = \ln(n) + \gamma + 1/(2n)$ $1/(12n^2) + 1/(120n^4)$
- 4. $\gamma = 0.5772156649015328606065120900824024310421$
- 5. 格雷碼 = $n \oplus (n >> 1)$
- 6. $SG(A+B) = SG(A) \oplus SG(B)$
- 7. 選轉矩陣 $M(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

13.1.4 基本數論

- 1. $\sum_{d|n} \mu(n) = [n == 1]$
- 2. $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) \times$
- 3. $\sum_{i=1}^{n} \sum_{j=1}^{m}$ 互質數量 = $\sum \mu(d) \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$
- 4. $\sum_{i=1}^{n} \sum_{j=1}^{n} lcm(i,j) = n \sum_{d|n} d \times \phi(d)$

13.1.5 排組公式

- 1. k 卡特蘭 $\frac{C_n^{kn}}{n(k-1)+1} \cdot C_m^n = \frac{n!}{m!(n-m)!}$
- 2. $H(n,m) \cong x_1 + x_2 \dots + x_n = k, num = k$
- 3. Stirling number of 2^{nd} , n 人分 k 組方法數目
 - (a) S(0,0) = S(n,n) = 1
 - (b) S(n,0) = 0
 - (c) S(n,k) = kS(n-1,k) + S(n-1,k-1)
- 4. Bell number, n 人分任意多組方法數目
 - (a) $B_0 = 1$

 - (a) $B_0 = 1$ (b) $B_n = \sum_{i=0}^n S(n, i)$ (c) $B_{n+1} = \sum_{k=0}^n C_k^k B_k$ (d) $B_{p+n} \equiv B_n + B_{n+1} mod p$, p is prime (e) $B_{p^m+n} \equiv m B_n + B_{n+1} mod p$, p is prime (f) From $B_0 : 1, 1, 2, 5, 15, 52$,
 - 203, 877, 4140, 21147, 115975
- 5. Derangement, 錯排, 沒有人在自己位置上
 - (a) $D_n = n!(1 \frac{1}{1!} + \frac{1}{2!} \frac{1}{3!} \dots + (-1)^n \frac{1}{n!})$ (b) $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 =$
 - $1, D_1 = 0$

- (c) From $D_0: 1, 0, 1, 2, 9, 44$, 265, 1854, 14833, 133496
- 6. Binomial Equality

```
(a) \sum_{k} {r \choose m+k} {s \choose n-k} = {r+s \choose m+n}
```

- (b) $\sum_{k} {l \choose m+k} {s \choose n+k} = {l+s \choose l-m+n}$
- (c) $\sum_{k} {l \choose m+k} {s+k \choose n} (-1)^k = (-1)^{l+m} {s-m \choose n-l}$
- (d) $\sum_{k \le l} {l \choose m} {s \choose k-n} (-1)^k$ $(-1)^{l+m} {s-m-1 \choose l-n-m}$
- (e) $\sum_{0 \le k \le l} {l-k \choose m} {q+k \choose n} = {l+q+1 \choose m+n+1}$
- (f) $\binom{r}{k} = (-1)^k \binom{k-r-1}{k}$
- (g) $\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}$
- (h) $\sum_{k \le n} {r+k \choose k} = {r+n+1 \choose n}$
- (i) $\sum_{0 \le k \le n} {k \choose m} = {n+1 \choose m+1}$
- (j) $\sum_{k \le m}^{-} {m+r \choose k} x^k y^k$ $\sum_{k \le m} {\binom{-r}{k}} (-x)^k (x+y)^{m-k}$

13.1.6 冪次、冪次和

- 1. $a^{b}\%P = a^{b\%\varphi(p) + \varphi(p)}, b > \varphi(p)$
- 2. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
- 3. $1^4 + 2^4 + 3^4 + \ldots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{2} \frac{n}{20}$
- 4. $1^5 + 2^5 + 3^5 + \ldots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} \frac{n^2}{12}$ 9
- 5. $0^k + 1^k + 2^k + \dots + n^k = P(k), P(k) = {}_{10}$ $\frac{(n+1)^{k+1} \sum_{i=0}^{k-1} C_i^{k+1} P(i)}{k+1}, P(0) = n+1$
- 6. $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^n C_k^{n+1} B_k m^{n+1-k}$
- 7. $\sum_{j=0}^{m} C_{j}^{m+1} B_{j} = 0, B_{0} = 1$
- 8. 除了 $B_1 = -1/2$,剩下的奇數項都是 0
- 9. $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 =$ $-1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 17$ $7/6, B_{16} = -3617/510, B_{18}$ $43867/798, B_{20} = -174611/330,$

13.1.7 Burnside's lemma

- 1. $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- 2. $X^g = t^{c(g)}$
- G 表示有幾種轉法, X^g 表示在那種轉法下, 有幾種 是會保持對稱的 $\cdot t$ 是顏色數 $\cdot c(g)$ 是循環節不動的
- 4. 正立方體塗三顏色,轉 0 有 36 個元素不變,轉 90 有 6 種, 每種有 3³ 不變, 180 有 3 × 3⁴, 3 120(角) 有 8 × 3^2 · 180(邊) 有 6 × 3^3 · 全部 4 $\frac{1}{24} \left(3^6 + 6 \times 3^3 + 3 \times 3^4 + 8 \times 3^2 + 6 \times 3^3 \right) = 5$

13.1.8 Count on a tree

- 1. Rooted tree: $s_{n+1} = \frac{1}{n} \sum_{i=1}^{n} (i \times a_i \times 9)$ });
- 2. Unrooted tree:
 - (a) Odd: $a_n \sum_{i=1}^{n/2} a_i a_{n-i}$ (b) Even: $Odd + \frac{1}{2} a_{n/2} (a_{n/2} + 1)$
- 3. Spanning Tree
 - (a) 完全圖 $n^n 2$
 - (b) 一般圖 (Kirchhoff's theorem)M[i][i] = $degree(V_i), M[i][j] = -1, \text{if have } E(i, j), 0$ 0 Object value = map.get(obj); if no edge. delete any one row and col in 7 3 $A. \ ans = det(A)$

13.2 java

13.2.1 文件操作

```
1 import java.io.*;
  import iava.util.*:
  import java.math.*;
  import java.text.*;
   public class Main{
    public static void main(String args[]){
         throws FileNotFoundException,
         IOException
      Scanner sc = new Scanner(new FileReader(
           "a.in"));
       PrintWriter pw = new PrintWriter(new
           FileWriter("a.out"));
      int n,m;
      n=sc.nextInt();//读入下一个INT
12
      m=sc.nextInt():
       for(ci=1; ci<=c; ++ci){</pre>
        pw.println("Case #"+ci+": easy for
             output");
       pw.close();// 关闭流并释放,这个很重要
           否则是没有输出的
      sc.close();// 关闭流并释放
20
21
```

13.2.2 优先队列

```
1 | PriorityQueue queue = new PriorityQueue( 1,
      new Comparator(){
    public int compare( Point a, Point b ){
   if(a.x < b.x | | a.x == b.x && a.y < b.y)
     return -1;
   else if( a.x == b.x && a.v == b.v )
     return 0;
```

13.2.4 sort

7 else return 1;

13.2.3 Map

2 map.put("sa","dd");

1 | Map map = new HashMap();

3 String str = map.get("sa").toString;

5 for(Object obj : map.keySet()){

```
1 static class cmp implements Comparator{
    public int compare(Object o1,Object o2){
    BigInteger b1=(BigInteger)o1;
    BigInteger b2=(BigInteger)o2;
    return b1.compareTo(b2);
  public static void main(String[] args)
       throws IOException{
    Scanner cin = new Scanner(System.in);
11
    n=cin.nextInt();
    BigInteger[] seg = new BigInteger[n];
    for (int i=0;i<n;i++)</pre>
    seg[i]=cin.nextBigInteger();
    Arrays.sort(seg, new cmp());
```

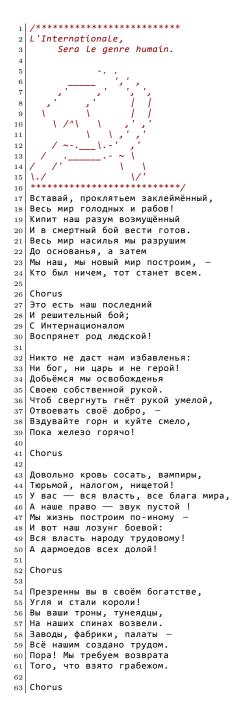
14

14.1 ganadoQuote

```
1 ¡Allí está!
 2 ¡Un forastero!
 3 ¡Agarrenlo!
 4 jOs voy a romper a pedazos!
 5 ¡Cógelo!
 6 ¡Te voy a hacer picadillo!
 7 | ¡Te voy a matar!
 8 ¡Míralo, está herido!
 9 iSos cerdo!
10 ¿Dónde estás?
11 ¡Detrás de tí, imbécil!
12 ¡No dejes que se escape!
13 ¡Basta, hijo de puta!
14 Lord Saddler...
16 ¡Mátalo!
17 ¡Allí está!
```

```
18 Morir es vivir.
  Sííííí, ¡Quiero matar!
  Muere, muere, muere....
  Cerebros, cerebros, cerebros...
22 Cógedlo, cógedlo, cógedlo...
23 Lord Saddler...
24 Dieciséis.
  ¡Va por él!
  ¡Muérete!
  ¡Cógelo!
  ¡Te vov a matar!
  ¡Bloqueale el paso!
  ¡Te cogí!
  ¡No dejes que se escape!
  ¿Qué carajo estás haciendo aquí? ¡Lárgate
  Hay un rumor de que hay un extranjero entre
       nosotros.
  Nuestro jefe se encargará de la rata.
  Su "Las Plagas" es mucho mejor que la
       nuestra.
  Tienes razón, es un hombre.
  Usa los músculos.
40 Se vuelve loco!
41 ¡Hey, acá!
42 ¡Por aquí!
43 ¡El Gigante!
44 ¡Del Lago!
45 ¡Cógelo!
46 ¡Cógenlo!
  ¡Allí!
47
  ¡Rápido!
  ¡Empieza a rezar!
  :Mátenlos!
  ¡Te voy a romper en pedazos!
  ¡La campana!
  Ya es hora de rezar.
  Tenemos que irnos.
  ¡Maldita sea, mierda!
  ¡Ya es hora de aplastar!
  ¡Puedes correr, pero no te puedes esconder!
  ¡Sos cerdo!
  ¡Está en la trampa!
  ¡Ah, que madre!
62 ¡Vámonos!
  ¡Ándale!
  ¡Cabrón!
  ¡Coño!
  ¡Agárrenlo!
  Cógerlo, Cógerlo...
  ¡Allí está, mátalo!
  ¡No dejas que se escape de la isla vivo!
70 ¡Hasta luego!
71 ¡Rápido, es un intruso!
```

14.2

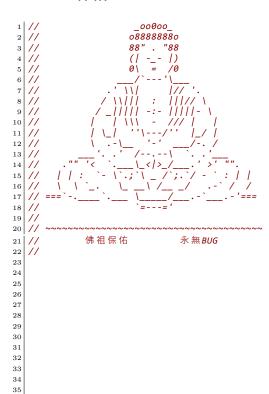


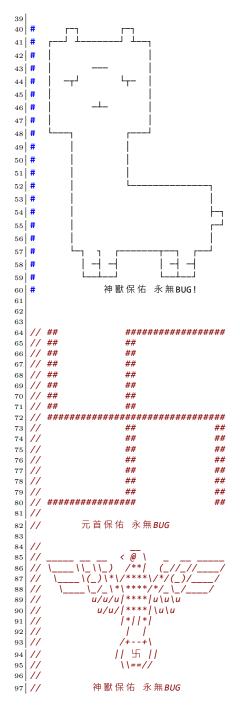
```
65 Довольно королям в угоду
66 Дурманить нас в чаду войны!
67 Война тиранам! Мир Народу!
68 Бастуйте, армии сыны!
69 Когда ж тираны нас заставят
70 В бою геройски пасть за них —
71 Убийцы, в вас тогда направим
72 Мы жерла пушек боевых!
73
74
   Chorus
75
   Лишь мы, работники всемирной
76
77 Великой армии труда,
78 Владеть землёй имеем право,
79 Но паразиты — никогда!
80 И если гром великий грянет
81 Над сворой псов и палачей, -
82 Для нас всё так же солнце станет
83 Сиять огнём своих лучей.
84
85 Chorus
```

14.3 保佑

36

37





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