Computational Geometry

delaunav

```
i template < class T>
2 class Delaunay{
    struct PT:public point<T>{
      int g[2];
      PT(const point<T> &p):
        point<T>(p){ g[0]=g[1]=-1; }
    static bool cmp(const PT &a,const PT &b){
      return a.x<b.x||(a.x==b.x&&a.y<b.y);</pre>
    struct edge{
      int v,g[2];
      edge(int v,int g0,int g1):
        v(v){g[0]=g0,g[1]=g1;}
    vector<PT> S;
    vector<edge> E;
    bool convex(int &from,int to,T LR){
      for(int i=0;i<2;++i){</pre>
        int c = E[S[from].g[i]].v;
        auto A=S[from]-S[to], B=S[c]-S[to];
        T v = A.cross(B)*LR;
        if(v>0||(v==0&&B.abs2()<A.abs2()))
          return from = c, true;
      return false;
    void addEdge(int v,int g0,int g1){
      E.emplace back(v,g0,g1);
      E[E.back().g[0]].g[1] = E.size()-1;
      E[E.back().g[1]].g[0] = E.size()-1;
    void climb(int &p, int e, int n, int nl,
         int nr, int LR){
      for(int i=E[e].g[LR]; (S[nr]-S[nl]).
           cross(S[E[i].v]-S[n])>0;){
        if(inCircle(S[E[i].v],S[n1],S[nr],S[E[
             E[i].g[LR]].v])>=0)
          { p = i; break; }
        for(int j=0;j<4;++j)</pre>
          E[E[i^j/2].g[j\%2^1]].g[j\%2] = E[i^j
               /2].g[j%2];
        int j=i; i=E[i].g[LR];
        E[j].g[0]=E[j].g[1]=E[j^1].g[0]=E[j
             ^1].g[1]=-1;
    T det3(T a11,T a12,T a13,T a21,T a22,T a23 102
         ,T a31,T a32,T a33){
      return a11*(a22*a33-a32*a23)-a12*(a21*
           a33-a31*a23)+a13*(a21*a32-a31*a22);
    int inCircle(const PT &a, const PT &b,
         const PT &c, const PT &p){
|T| as = a.abs2(), bs = b.abs2(), cs = c.abs2
       (), ps = p.abs2();
48 T res = a.x * det3(b.y,bs,1,c.y,cs,1,p.y,ps
49 -a.y * det3(b.x,bs,1,c.x,cs,1,p.x,ps,1)
```

111

```
50 +as * det3(b.x,b.y,1,c.x,c.y,1,p.x,p.y,1)
  -det3(b.x,b.y,bs,c.x,c.y,cs,p.x,p.y,ps);
      return res<0 ? 1 : (res>0 ? -1 : 0);
    void divide(int 1, int r){
      if(1>=r)return;
      if(1+1==r){
        int A=S[1].g[0]=S[1].g[1]=E.size();
        E.emplace_back(r,A,A);
        int B=S[r].g[0]=S[r].g[1]=E.size();
        E.emplace back(1,B,B);
        return;
      int mid = (1+r)/2;
      divide(l,mid), divide(mid+1, r);
      int nl = mid, nr = mid+1:
        if(convex(nl,nr,1)) continue;
        if(S[nr].g[0]!=-1&&convex(nr,nl,-1))
             continue:
        break:
      addEdge(nr,S[nl].g[0],S[nl].g[1]);
      S[nl].g[1] = E.size()-1;
      if(S[nr].g[0]==-1){
        addEdge(nl,E.size(),E.size());
        S[nr].g[1] = E.size()-1;
      }else addEdge(nl,S[nr].g[0],S[nr].g[1]);
      S[nr].g[0] = E.size()-1;
      int cl = nl, cr = nr;
      for(;;){
        int pl=-1, pr=-1, side;
        climb(pl,E.size()-2,nl,nl,nr,1);
        climb(pr,E.size()-1,nr,nl,nr,0);
        if(pl==-1&&pr==-1) break;
        if(pl==-1||pr==-1) side = pl==-1;
        else side=inCircle(S[E[pl].v],S[nl],S[
             nr],S[E[pr].v])<=0;
        if(side){
  nr = E[pr].v;
  addEdge(nr,E.size()-2,E[E.size()-2].g[1]);
  addEdge(nl,E[pr^1].g[0],pr^1);
        }else{
  nl = E[pl].v;
  addEdge(nr,pl^1,E[pl^1].g[1]);
  addEdge(nl, E[E.size()-2].g[0], E.size()-2);
      if(cl==nl&&cr==nr) return;//Collinearity
      S[n1].g[0] = E.size()-2;
      S[nr].g[1] = E.size()-1;
    void solve(const vector<point<T>> &P){
      S.clear(), E.clear();
      for(const auto &p:P) S.emplace_back(p);
      sort(S.begin(),S.end(),cmp);
      divide(0,int(S.size())-1);
    vector<pair<int.int>> getEdge(){
      vector<pair<int,int>> res;
      for(size t i=0;i<E.size();i+=2)</pre>
        if(E[i].g[0]!=-1)
          res.emplace back(E[i].v,E[i^1].v);
      return res:
```

1.2 Geometry

114 };

```
const double PI=atan2(0.0,-1.0);
2 template<typename T>
                                                62
 3 struct point{
                                                63
    T x, y;
    point(){}
    point(const T&x,const T&y):x(x),y(y){}
                                                64
    point operator+(const point &b)const{
      return point(x+b.x,y+b.y); }
                                                65
    point operator-(const point &b)const{
                                                66
      return point(x-b.x,y-b.y); }
    point operator*(const T &b)const{
                                                67
      return point(x*b,y*b); }
                                                68
    point operator/(const T &b)const{
                                                69
      return point(x/b,y/b); }
    bool operator == (const point &b)const{
      return x==b.x&&y==b.y; }
    T dot(const point &b)const{
      return x*b.x+y*b.y; }
    T cross(const point &b)const{
      return x*b.y-y*b.x; }
    point normal()const{//求法向量
                                                77
      return point(-y,x); }
                                                78
    T abs2()const{//向量長度的平方
                                                79
      return dot(*this); }
    T rad(const point &b)const{//兩向量的弧度
return fabs(atan2(fabs(cross(b)),dot(b))); }
                                               82
    T getA()const{//對x軸的弧度
                                                83
                                                84
      T A=atan2(y,x);//超過180度會變負的
28
29
      if(A<=-PI/2)A+=PI*2;
30
      return A;
31
33 template<typename T>
34 struct line{
    line(){}
    point<T> p1.p2:
    T a,b,c;//ax+by+c=0
    line(const point<T>&x,const point<T>&y):p1
         (x),p2(y){}
    void pton(){//轉成一般式
      a=p1.v-p2.v:
      b=p2.x-p1.x;
      c=-a*p1.x-b*p1.v:
   T ori(const point<T> &p)const{//點和有向直
         線的關係, >0左邊、=0在線上<0右邊
      return (p2-p1).cross(p-p1);
                                                99
46
                                               100
    T btw(const point<T> &p)const{//點投影落在
                                               101
         線段 ト <= 0
                                               102
      return (p1-p).dot(p2-p);
                                               103
48
    bool point on segment(const point<T>&p)
                                               104
         const{//點是否在線段上
                                               105
      return ori(p) == 0&&btw(p) <= 0:
                                               106
```

```
T dis2(const point<T> &p,bool is segment
     =0) const { // 點 跟 直 線 / 線 段 的 距 離 平 方
  point<T> v=p2-p1,v1=p-p1;
  if(is_segment){
    point<T> v2=p-p2;
    if(v.dot(v1)<=0)return v1.abs2();</pre>
    if(v.dot(v2)>=0)return v2.abs2();
  T tmp=v.cross(v1);
  return tmp*tmp/v.abs2():
T seg dis2(const line<T> &1)const{//兩線段
  return min({dis2(l.p1,1),dis2(l.p2,1),l.
       dis2(p1,1),1.dis2(p2,1)});
point<T> projection(const point<T> &p)
     const { // 點 對 直 線 的 投 影
  point<T> n=(p2-p1).normal();
  return p-n*(p-p1).dot(n)/n.abs2();
point<T> mirror(const point<T> &p)const{
  //點對直線的鏡射·要先呼叫pton轉成一般式
  point<T> R;
  T d=a*a+b*b;
  R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
  R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
bool equal(const line &1)const{//直線相等
  return ori(1.p1)==0&&ori(1.p2)==0;
bool parallel(const line &1)const{
  return (p1-p2).cross(l.p1-l.p2)==0;
bool cross_seg(const line &1)const{
  return (p2-p1).cross(l.p1-p1)*(p2-p1).
       cross(1.p2-p1)<=0;//直線是否交線段
int line intersect(const line &l)const{//
     直線相交情況,-1無限多點、1交於一點、0
  return parallel(1)?(ori(1.p1)==0?-1:0)
       :1;
int seg_intersect(const line &1)const{
  T c1=ori(l.p1), c2=ori(l.p2);
  T c3=1.ori(p1), c4=1.ori(p2);
  if(c1==0&&c2==0){//共線
    bool b1=btw(1.p1)>=0,b2=btw(1.p2)>=0;
    T a3=1.btw(p1),a4=1.btw(p2);
    if(b1&&b2&&a3==0&&a4>=0) return 2;
    if(b1&&b2&&a3>=0&&a4==0) return 3;
    if(b1&&b2&&a3>=0&&a4>=0) return 0;
    return -1; //無限交點
  }else if(c1*c2<=0&&c3*c4<=0)return 1;</pre>
  return 0;//不相交
point<T> line_intersection(const line &l)
     const{/*直線交點*/
  point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
  //if(a.cross(b)==0)return INF;
  return p1+a*(s.cross(b)/a.cross(b));
```

```
point<T> seg intersection(const line &1)
          const{//線段交點
       int res=seg_intersect(1);
                                                  163
       if(res<=0) assert(0);</pre>
110
                                                  164
111
       if(res==2) return p1;
       if(res==3) return p2;
112
113
       return line intersection(1);
114
115
  | };
   template<typename T>
   struct polygon{
                                                  168
     polygon(){}
     vector<point<T> > p;//逆時針順序
                                                  169
     T area()const{//面積
120
121
       T ans=0;
       for(int i=p.size()-1,j=0;j<(int)p.size()</pre>
122
            ;i=j++)
                                                  172
123
         ans+=p[i].cross(p[j]);
                                                  173
124
       return ans/2;
                                                  174
125
                                                  175
     point<T> center of mass()const{//重心
127
       T cx=0, cy=0, w=0;
       for(int i=p.size()-1,j=0;j<(int)p.size()</pre>
            ;i=j++){
                                                  177
129
         T a=p[i].cross(p[j]);
                                                  178
130
         cx+=(p[i].x+p[j].x)*a;
         cy+=(p[i].y+p[j].y)*a;
131
                                                  179
132
                                                  180
133
       return point<T>(cx/3/w,cy/3/w);
134
135
136
     char ahas(const point<T>& t)const{//點是否
          在簡單多邊形內,是的話回傳1、在邊上回
          傳-1、否則回傳0
       bool c=0;
       for(int i=0,j=p.size()-1;i<p.size();j=i</pre>
         if(line<T>(p[i],p[j]).point_on_segment 186
              (t))return -1;
         else if((p[i].y>t.y)!=(p[j].y>t.y)&&
         t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j
                                                  189
              ].y-p[i].y)+p[i].x)
                                                  190
           c=!c;
142
143
       return c;
144
                                                  192
145
     char point_in_convex(const point<T>&x)
          const{
146
       int l=1,r=(int)p.size()-2;
       while(1<=r){//點是否在凸多邊形內,是的話
147
            回傳1、在邊上回傳-1、否則回傳0
148
         int mid=(1+r)/2;
149
         T a1=(p[mid]-p[0]).cross(x-p[0]);
150
         T a2=(p[mid+1]-p[0]).cross(x-p[0]);
151
         if(a1>=0&&a2<=0){
152
           T res=(p[mid+1]-p[mid]).cross(x-p[
                mid]);
                                                  202
           return res>0?1:(res>=0?-1:0);
153
                                                  203
         }else if(a1<0)r=mid-1;</pre>
154
                                                  204
155
         else l=mid+1;
                                                  205
156
157
       return 0;
                                                  206
158
     vector<T> getA()const{//凸包邊對x軸的夾角
159
                                                  208
       vector<T>res;//一定是遞增的
```

```
for(size t i=0;i<p.size();++i)</pre>
    res.push back((p[(i+1)%p.size()]-p[i]) 211
         .getA());
                                              212
  return res;
                                              213
                                              214
bool line intersect(const vector<T>&A.
                                              215
     const line<T> &1)const{//O(LogN)
                                              216
  int f1=upper bound(A.begin(),A.end(),(1.
       p1-l.p2).getA())-A.begin();
  int f2=upper_bound(A.begin(),A.end(),(1.
       p2-1.p1).getA())-A.begin();
  return 1.cross_seg(line<T>(p[f1],p[f2])) 219
                                              220
polygon cut(const line<T> &l)const{//△ 包
                                             221
     對 直線 切割,得到直線 L 左側的凸包
  polygon ans;
  for(int n=p.size(),i=n-1,j=0;j<n;i=j++){</pre>
                                              223
                                              224
    if(l.ori(p[i])>=0){
      ans.p.push back(p[i]);
                                              225
      if(1.ori(p[j])<0)
                                              226
        ans.p.push back(1.
                                              227
             line intersection(line<T>(p[i 228
             ],p[j])));
    }else if(l.ori(p[j])>0)
      ans.p.push_back(1.line_intersection( 231
           line<T>(p[i],p[j])));
                                              232
                                              233
  return ans;
                                              234
static bool monotone_chain_cmp(const point 235
     <T>& a, const point<T>& b){//凸包排序函
                                              236
                                              237
  return (a.x<b.x)||(a.x==b.x&&a.y<b.y);
                                              238
void monotone chain(vector<point<T> > &s){
     //凸包
                                              241
  sort(s.begin(),s.end(),
                                              242
       monotone_chain_cmp);
                                              243
  p.resize(s.size()+1);
  int m=0;
                                              244
  for(size_t i=0;i<s.size();++i){</pre>
                                              245
    while (m \ge 2\&\&(p[m-1]-p[m-2]).cross(s[i])
         ]-p[m-2])<=0)--m;
                                              246
    p[m++]=s[i];
                                              247
  for(int i=s.size()-2,t=m+1;i>=0;--i){
    while(m>=t&&(p[m-1]-p[m-2]).cross(s[i 248
         ]-p[m-2])<=0)--m;
    p[m++]=s[i];
                                              250
  if(s.size()>1)--m;
  p.resize(m);
                                              252
                                              253
                                              254
T diam(){//直徑
                                              255
  int n=p.size(),t=1;
                                              256
  T ans=0;p.push_back(p[0]);
                                              257
  for(int i=0;i<n;i++){</pre>
    point<T> now=p[i+1]-p[i];
                                              258
    while(now.cross(p[t+1]-p[i])>now.cross
                                             259
                                              260
         (p[t]-p[i]))t=(t+1)%n;
    ans=max(ans,(p[i]-p[t]).abs2());
                                              261
  return p.pop back(),ans;
                                              262
                                              263
```

```
T min cover rectangle(){//最小覆蓋矩形
                                              264
  int n=p.size(),t=1,r=1,l;
                                              265
                                              266
  if(n<3)return 0;//也可以做最小周長矩形
                                              267
  T ans=1e99;p.push_back(p[0]);
  for(int i=0;i<n;i++){</pre>
                                              268
                                              269
    point<T> now=p[i+1]-p[i];
    while(now.cross(p[t+1]-p[i])>now.cross 270| };
         (p[t]-p[i]))t=(t+1)%n;
    while (now.dot(p[r+1]-p[i]) > now.dot(p[r^{272}|struct triangle{}
                                              273
         ]-p[i]))r=(r+1)%n;
                                              274
    if(!i)l=r;
    while(now.dot(p[l+1]-p[i])<=now.dot(p[</pre>
         1]-p[i]))1=(1+1)%n;
    T d=now.abs2();
    T tmp=now.cross(p[t]-p[i])*(now.dot(p[
                                              277
                                              278
         r]-p[i]-now.dot(p[l]-p[i])/d;
                                              279
    ans=min(ans,tmp);
                                              280
  return p.pop back(),ans;
                                              281
                                              282
T dis2(polygon &pl){//凸包最近距離平方
                                              283
  vector<point<T> > &P=p,&Q=pl.p;
                                              284
  int n=P.size(),m=Q.size(),l=0,r=0;
                                              285
for(int i=0;i<n;++i)if(P[i].y<P[1].y)l=i;</pre>
                                              286
for(int i=0;i<m;++i)if(Q[i].y<Q[r].y)r=i;</pre>
 P.push_back(P[0]),Q.push_back(Q[0]);
                                              287
  T ans=1e99;
                                              288
  for(int i=0;i<n;++i){</pre>
    while((P[1]-P[1+1]).cross(Q[r+1]-Q[r]) 289
         <0)r=(r+1)%m;
                                              290
    ans=min(ans,line\langle T \rangle (P[1],P[1+1]).
                                              291
         seg_dis2(line<T>(Q[r],Q[r+1])));
                                              292
    l=(1+1)%n;
                                              293
  return P.pop_back(),Q.pop_back(),ans;
                                              294
static char sign(const point<T>&t){
                                              295
  return (t.y==0?t.x:t.y)<0;</pre>
                                              296
                                              297
static bool angle cmp(const line<T>& A,
     const line<T>& B){
  point<T> a=A.p2-A.p1,b=B.p2-B.p1;
  return sign(a)<sign(b)||(sign(a)==sign(b</pre>
                                              301
       )&&a.cross(b)>0);
                                              302
int halfplane_intersection(vector<line<T>
     > &s){//半平面交
                                              304
  sort(s.begin(),s.end(),angle_cmp);//線段
                                              305
       左側為該線段半平面
                                              306
                                              307
  int L,R,n=s.size();
                                              308
  vector<point<T> > px(n);
                                              309
  vector<line<T> > q(n);
                                              310
  q[L=R=0]=s[0];
                                              311
  for(int i=1;i<n;++i){</pre>
                                              312
    while(L<R&&s[i].ori(px[R-1])<=0)--R;</pre>
                                              313
    while(L<R&&s[i].ori(px[L])<=0)++L;</pre>
                                              314
    q[++R]=s[i];
                                              315
    if(q[R].parallel(q[R-1])){
                                              316
      if(q[R].ori(s[i].p1)>0)q[R]=s[i];
                                              317
    if(L < R)px[R-1] = q[R-1].
         line_intersection(q[R]);
                                              319
  while(L<R&&q[L].ori(px[R-1])<=0)--R;</pre>
```

```
p.clear();
       if(R-L<=1)return 0;</pre>
       px[R]=q[R].line intersection(q[L]);
       for(int i=L;i<=R;++i)p.push_back(px[i]);</pre>
       return R-L+1;
271 template<typename T>
    point<T> a,b,c;
    triangle(){}
     triangle(const point<T> &a,const point<T>
         &b, const point<T> &c):a(a),b(b),c(c){}
    T area()const{
      T t=(b-a).cross(c-a)/2;
       return t>0?t:-t;
    point<T> barycenter()const{//重心
      return (a+b+c)/3;
     point<T> circumcenter()const{//外心
      static line<T> u,v;
       u.p1=(a+b)/2;
       u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-
           b.x);
       v.p1=(a+c)/2;
       v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-
       return u.line_intersection(v);
     point<T> incenter()const{//内心
      T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2
           ()),C=sqrt((a-b).abs2());
       return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+
           B*b.y+C*c.y)/(A+B+C);
    point<T> perpencenter()const{//垂心
       return barycenter()*3-circumcenter()*2;
298 };
299 template<typename T>
300 struct point3D{
    T x,y,z;
    point3D(){}
     point3D(const T&x,const T&y,const T&z):x(x
         ),y(y),z(z){}
     point3D operator+(const point3D &b)const{
       return point3D(x+b.x,y+b.y,z+b.z);}
     point3D operator-(const point3D &b)const{
       return point3D(x-b.x,y-b.y,z-b.z);}
     point3D operator*(const T &b)const{
       return point3D(x*b,y*b,z*b);}
     point3D operator/(const T &b)const{
       return point3D(x/b,y/b,z/b);}
     bool operator==(const point3D &b)const{
       return x==b.x&&y==b.y&&z==b.z;}
     T dot(const point3D &b)const{
       return x*b.x+y*b.y+z*b.z;}
     point3D cross(const point3D &b)const{
       return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x
            *b.y-y*b.x);}
    T abs2()const{//向量長度的平方
       return dot(*this);}
    T area2(const point3D &b)const{//和b、原點
          圍成面積的平方
```

```
return cross(b).abs2()/4;}
322 };
323 template<typename T>
324 struct line3D{
    point3D<T> p1,p2;
     line3D(){}
    line3D(const point3D<T> &p1,const point3D< 374
         T> &p2):p1(p1),p2(p2){}
    T dis2(const point3D<T> &p,bool is_segment 376
          =0) const{//點 跟 直 線/線 段 的 距 離 平 方
                                                 377
       point3D<T> v=p2-p1,v1=p-p1;
330
       if(is segment){
331
         point3D<T> v2=p-p2;
         if(v.dot(v1)<=0)return v1.abs2();</pre>
332
         if(v.dot(v2)>=0)return v2.abs2();
333
                                                 380
334
335
       point3D<T> tmp=v.cross(v1);
336
       return tmp.abs2()/v.abs2();
337
    pair<point3D<T>,point3D<T> > closest pair(
338
          const line3D<T> &1)const{
       point3D < T > v1 = (p1-p2), v2 = (1.p1-1.p2);
339
340
       point3D<T> N=v1.cross(v2),ab(p1-l.p1);
341
       //if(N.abs2()==0)return NULL;平行或重合
      T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//
342
            最折點對距離
343
       point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.
            cross(d2),G=1.p1-p1;
      T t1=(G.cross(d2)).dot(D)/D.abs2();
      T t2=(G.cross(d1)).dot(D)/D.abs2();
345
       return make pair(p1+d1*t1,1.p1+d2*t2);
346
347
    bool same_side(const point3D<T> &a,const
          point3D<T> &b)const{
       return (p2-p1).cross(a-p1).dot((p2-p1).
            cross(b-p1))>0;
350
351
   };
   template<typename T>
  struct plane{
                                                    };
354
    point3D<T> p0,n;//平面上的點和法向量
     plane(){}
    plane(const point3D<T> &p0,const point3D<T
         > &n):p0(p0),n(n){}
    T dis2(const point3D<T> &p)const{//點到平
          面距離的平方
      T tmp=(p-p0).dot(n);
359
       return tmp*tmp/n.abs2();
360
    point3D<T> projection(const point3D<T> &p)
          const{
362
       return p-n*(p-p0).dot(n)/n.abs2();
363
    point3D<T> line intersection(const line3D
         T> &1)const{
      T tmp=n.dot(1.p2-1.p1);//等於 Ø表示平行或
            重合該平面
       return 1.p1+(1.p2-1.p1)*(n.dot(p0-1.p1)/
     line3D<T> plane intersection(const plane &
       point3D<T> e=n.cross(pl.n),v=n.cross(e);
```

```
return line3D<T>(q,q+e);
   template < typename T>
   struct triangle3D{
     point3D<T> a,b,c;
     triangle3D(){}
     triangle3D(const point3D<T> &a,const
          point3D<T> &b, const point3D<T> &c):a(a
          ),b(b),c(c){}
     bool point in(const point3D<T> &p)const{//
          點在該平面上的投影在三角形中
       return line3D<T>(b,c).same side(p,a)&&
           line3D<T>(a,c).same_side(p,b)&&
           line3D<T>(a,b).same_side(p,c);
384 template<typename T>
385 struct tetrahedron{//四面體
     point3D<T> a,b,c,d;
     tetrahedron(){}
     tetrahedron(const point3D<T> &a,const
          point3D<T> &b, const point3D<T> &c.
          const point3D<T> &d):a(a),b(b),c(c),d(
          d){}
     T volume6()const{//體積的六倍
       return (d-a).dot((b-a).cross(c-a));
     point3D<T> centroid()const{
       return (a+b+c+d)/4;
     bool point in(const point3D<T> &p)const{
       return triangle3D<T>(a,b,c).point in(p)
           &&triangle3D<T>(c,d,a).point_in(p);
   template<typename T>
   struct convexhull3D{
     static const int MAXN=1005;
     struct face{
       int a,b,c;
       face(int a,int b,int c):a(a),b(b),c(c){}
     vector<point3D<T>> pt;
     vector<face> ans;
     int fid[MAXN][MAXN];
     void build(){
       int n=pt.size();
       ans.clear();
       memset(fid,0,sizeof(fid));
       ans.emplace back(0,1,2)://注意不能共線
       ans.emplace_back(2,1,0);
       int ftop = 0;
       for(int i=3, ftop=1; i<n; ++i,++ftop){</pre>
         vector<face> next;
         for(auto &f:ans){
          T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[
               f.a]).cross(pt[f.c]-pt[f.a]));
           if(d<=0) next.push back(f);</pre>
           int ff=0;
           if(d>0) ff=ftop;
```

T tmp=pl.n.dot(v);//等於0表示平行或重合

point3D<T> q=p0+(v*(pl.n.dot(pl.p0-p0))/

tmp);

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```
else if(d<0) ff=-ftop;</pre>
           fid[f.a][f.b]=fid[f.b][f.c]=fid[f.c
                1[f.a]=ff:
         for(auto &f:ans){
           if(fid[f.a][f.b]>0 && fid[f.a][f.b
                ]!=fid[f.b][f.a])
             next.emplace back(f.a,f.b,i);
           if(fid[f.b][f.c]>0 && fid[f.b][f.c
                ]!=fid[f.c][f.b])
             next.emplace back(f.b,f.c,i);
           if(fid[f.c][f.a]>0 && fid[f.c][f.a
                                                  11
                ]!=fid[f.a][f.c])
                                                  12
             next.emplace back(f.c,f.a,i);
                                                  13
                                                  14
         ans=next;
                                                  15
                                                  16
                                                   17
     point3D<T> centroid()const{
                                                  18
       point3D<T> res(0,0,0);
                                                  19
       T vol=0;
                                                  20 T closest_pair(vector<point<T>> &v){
       for(auto &f:ans){
         T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c
                                                  23 }
         res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
         vol+=tmp;
       return res/(vol*4);
447 };
```

Data Structure

1 template < typename IT = point < T > * > 2 T cloest_pair(_IT L, _IT R){

inplace merge(L, mid, R, ycmp);

static vector<point> b; b.clear();

if((u->x-x)*(u->x-x)>=d) continue;

T dx=u->x-v->x, dy=u->y-v->y;

return closest_pair(v.begin(), v.end());

for(auto v=b.rbegin();v!=b.rend();++v){

T d = min(cloest pair(L,mid),cloest pair(

if(R-L <= 1) return INF;</pre>

IT mid = L+(R-L)/2;

for(auto u=L;u<R;++u){</pre>

b.push back(*u);

return d:

if(dy*dy>=d) break;

d=min(d,dx*dx+dy*dy);

sort(v.begin(),v.end(),xcmp);

mid,R));

T x = mid -> x;

```
using PT=point<T>; using CPT=const PT;
2 PT circumcenter(CPT &a, CPT &b, CPT &c){
   PT u=b-a, v=c-a;
   T c1=u.abs2()/2,c2=v.abs2()/2;
   T d=u.cross(v);
   return PT(a.x+(v.y*c1-u.y*c2)/d,a.y+(u.x*
        c2-v.x*c1)/d);
8 void solve(PT p[],int n,PT &c,T &r2){
   random shuffle(p,p+n);
   c=p[0]; r2=0; // c, r2 = 圓心, 半徑平方
 for(int i=1;i<n;i++)if((p[i]-c).abs2()>r2){
     c=p[i]; r2=0;
  for(int j=0;j<i;j++)if((p[j]-c).abs2()>r2){
       c.x=(p[i].x+p[j].x)/2;
       c.y=(p[i].y+p[j].y)/2;
       r2=(p[j]-c).abs2();
 for(int k=0;k<j;k++)if((p[k]-c).abs2()>r2){
          c=circumcenter(p[i],p[j],p[k]);
          r2=(p[i]-c).abs2();
```

1.4 最折點對

1.3 SmallestCircle

```
i #include < bits / stdc++.h>
using namespace std;
  const int MAXN = 100005;
4 struct node{
    double a,b,r,k,x,y;
    int id;
  } p[MAXN];
  double DP[MAXN];
  deque<int> q;
10 bool cmpK(const node &a,const node &b){
11
   return a.k>b.k;
12 }
13 bool cmpX(const node &a,const node &b){
   return a.x<b.x||(a.x==b.x&&a.y<b.y);
15
16 double Slope(int a,int b){
    if(!b) return -1e20:
    if(p[a].x==p[b].x) return 1e20;
18
    return (p[a].y-p[b].y)/(p[a].x-p[b].x);
19
20 }
void CDQ(int 1, int r){
    if(l==r){
22
23
      DP[1] = max(DP[1], DP[1-1]);
      p[1].y = DP[1]/(p[1].a*p[1].r+p[1].b);
24
25
      p[1].x = p[1].y*p[1].r;
26
      return;
    int mid = (1+r)/2:
    stable_partition(p+l,p+r+1,[&](const node
         &d){return d.id<=mid;});</pre>
    CDQ(1, mid); q.clear();
    for(int i=1, j; i<=mid; ++i){</pre>
```

2.1 CDQ DP

```
while((j=q.size())>1&&Slope(q[j-2],q[j
         -1])<Slope(q[j-1],i)) q.pop_back();
   a.push back(i):
 }q.push back(0);
 for(int i=mid+1; i<=r; ++i){</pre>
   while(q.size()>1&&Slope(q[0],q[1])>p[i].
        k) q.pop front();
   DP[p[i].id] = max(DP[p[i].id], p[i].a*p[
        q[0]].x+p[i].b*p[q[0]].y);
 CDO(mid+1,r):
 inplace merge(p+l,p+mid+1,p+r+1,cmpX);
double solve(int n,double S){
 DP[0] = S:
 sort(p+1,p+1+n,cmpK);
 CDQ(1,n);
 return DP[n];
int main(){
 int n; double S;
 scanf("%d%lf",&n,&S);
 for(int i=1; i<=n; ++i){</pre>
   scanf("%lf%lf%lf",&p[i].a,&p[i].b,&p[i].
   p[i].id = i, p[i].k = -p[i].a/p[i].b;
 printf("%.3lf\n", solve(n,S));
 return 0;
```

2.2 DLX

```
1 const int MAXN=4100, MAXM=1030, MAXND=16390;
2 struct DLX{
   int n,m,sz,ansd;//高是n, 寬是m的稀疏矩陣
   int S[MAXM],H[MAXN];
   int row[MAXND], col[MAXND]; //每個節點代表的
   int L[MAXND],R[MAXND],U[MAXND],D[MAXND];
   vector<int> ans,anst;
   void init(int n,int m){
     n=_n,m=_m;
     for(int i=0;i<=m;++i){</pre>
       U[i]=D[i]=i,L[i]=i-1,R[i]=i+1;
       S[i]=0;
     R[m]=0,L[0]=m;
     sz=m, ansd=INT MAX; //ansd存最優解的個數
     for(int i=1;i<=n;++i)H[i]=-1;</pre>
   void add(int r,int c){
     ++S[col[++sz]=c];
     row[sz]=r;
     D[sz]=D[c],U[D[c]]=sz,U[sz]=c,D[c]=sz;
     if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
     else R[sz]=R[H[r]],L[R[H[r]]]=sz,L[sz]=H
          [r],R[H[r]]=sz;
   #define DFOR(i,A,s) for(int i=A[s];i!=s;i=
```

```
DFOR(i,D,c)DFOR(j,R,i){U[D[j]]=U[j],D[U[
      j]]=D[j],--S[col[j]];}
                                          88
void restore(int c){//恢復第c行和所有當前
    覆蓋到第c行的列·remove的逆操作
                                          90 };
  DFOR(i,U,c)DFOR(j,L,i){++S[col[j]],U[D[j
      11=i,D[U[i]]=i;}
  L[R[c]]=c,R[L[c]]=c;
void remove2(int nd){//刪除nd所在的行當前
    所有點(包括虛擬節點),只保留nd
 DFOR(i,D,nd)L[R[i]]=L[i],R[L[i]]=R[i];
void restore2(int nd){//刪除nd所在的行當前
    所有點,為remove2的逆操作
  DFOR(i,U,nd)L[R[i]]=R[L[i]]=i;
bool vis[MAXM];
int h(){//估價函數 for IDA*
 int res=0:
  memset(vis,0,sizeof(vis));
  DFOR(i,R,0)if(!vis[i]){
   vis[i]=1;
   ++res;
   DFOR(j,D,i)DFOR(k,R,j)vis[col[k]]=1;
                                          13
  return res;
bool dfs(int d){//for精確覆蓋問題
 if(d+h()>=ansd)return 0;//找最佳解用,找
      任意解可以刪掉
  if(!R[0]){ansd=d;return 1;}
  int c=R[0];
 DFOR(i,R,0)if(S[i]<S[c])c=i;</pre>
  remove(c);
  DFOR(i,D,c){
   ans.push back(row[i]);
   DFOR(j,R,i)remove(col[j]);
   if(dfs(d+1))return 1;
   ans.pop back();
   DFOR(j,L,i)restore(col[j]);
  restore(c);
                                          30
  return 0;
                                          31
                                          32
void dfs2(int d){//for最小重複覆蓋問題
 if(d+h()>=ansd)return;
  if(!R[0]){ansd=d;ans=anst;return;}
                                          34
                                          35
  int c=R[0];
                                          36
  DFOR(i,R,0)if(S[i]<S[c])c=i;</pre>
  DFOR(i,D,c){
   anst.push_back(row[i]);
   remove2(i):
                                          39
   DFOR(j,R,i)remove2(j),--S[col[j]];
   dfs2(d+1):
   anst.pop back();
   DFOR(j,L,i)restore2(j),++S[col[j]];
```

void remove(int c){//刪除第c行和所有當前覆

L[R[c]]=L[c],R[L[c]]=R[c];//這裡刪除第c

蓋到第c行的列

restore2(i):

```
bool exact cover(){//解精確覆蓋問題
                                 return ans.clear(), dfs(0);
行,若有些行不需要處理可以在開始時呼 83
                                void min cover(){//解最小重複覆蓋問題
                                 anst.clear();//暫存用,答案還是存在ans裡
                               #undef DFOR
```

57

2.3 Dynamic KD tree

```
ı| template<typename T, size t kd>//有kd個維度
                                                    60
2 struct kd tree{
    struct point{
      T d[kd];
                                                    62
      T dist(const point &x)const{
                                                    63
        for(size t i=0:i<kd:++i)ret+=abs(d[i]-</pre>
                                                   65
              x.d[i]);
                                                    66
        return ret:
                                                    67
      bool operator == (const point &p){
                                                    69
        for(size t i=0:i<kd:++i)</pre>
           if(d[i]!=p.d[i])return 0;
                                                    71
        return 1:
                                                    72
      bool operator<(const point &b)const{</pre>
        return d[0]<b.d[0];</pre>
                                                    75
   };
19 private:
    struct node{
      node *1,*r;
      point pid:
                                                    79
      int s;
      node(const point &p):1(0),r(0),pid(p),s
                                                   81
            (1){}
                                                    82
      ~node(){delete l,delete r;}
                                                    83
      void up()\{s=(1?1->s:0)+1+(r?r->s:0);\}
                                                    84
    }*root:
                                                    85
    const double alpha,loga;
    const T INF;//記得要給INF,表示極大值
    int maxn;
                                                    87
    struct __cmp{
                                                    88
      int sort id;
                                                    89
      bool operator()(const node*x,const node*
                                                   90
        return operator()(x->pid,y->pid);
                                                    92
                                                    93
      bool operator()(const point &x,const
                                                    94
            point &v)const{
        if(x.d[sort id]!=y.d[sort id])
           return x.d[sort_id]<y.d[sort_id];</pre>
         for(size_t i=0;i<kd;++i)</pre>
           if(x.d[i]!=y.d[i])return x.d[i]<y.d[</pre>
                i];
                                                   100
41
        return 0:
                                                   101
                                                   102
    }cmp;
```

```
int size(node *o){return o?o->s:0;}
vector<node*> A;
node* build(int k,int l,int r){
  if(1>r) return 0;
  if(k==kd) k=0;
  int mid=(1+r)/2:
  cmp.sort id = k;
  nth element(A.begin()+1, A.begin()+mid, A.
       begin()+r+1,cmp);
  node *ret=A[mid];
  ret \rightarrow l = build(k+1,l,mid-1):
  ret->r = build(k+1,mid+1,r);
  ret->up();
  return ret;
bool isbad(node*o){
  return size(o->1)>alpha*o->s||size(o->r)
       >alpha*o->s;
void flatten(node *u, typename vector<node</pre>
     *>::iterator &it){
  if(!u)return:
  flatten(u->1,it);
  *it=u:
  flatten(u->r,++it);
void rebuild(node*&u,int k){
  if((int)A.size()<u->s)A.resize(u->s);
  auto it=A.begin();
  flatten(u,it);
  u=build(k,0,u->s-1);
bool insert(node*&u, int k, const point &x,
     int dep){
  if(!u) return u=new node(x), dep<=0;</pre>
  ++u->s:
  cmp.sort id=k;
  if(insert(cmp(x,u->pid)?u->1:u->r,(k+1)%
       kd,x,dep-1)){
    if(!isbad(u))return 1;
    rebuild(u,k);
  return 0;
node *findmin(node*o,int k){
  if(!o)return 0;
  if(cmp.sort id==k)return o->l?findmin(o
       ->1,(k+1)%kd):o;
  node *l=findmin(o->l,(k+1)%kd);
  node *r=findmin(o->r,(k+1)%kd);
  if(1&&!r)return cmp(1,0)?1:0;
  if(!1&&r)return cmp(r,o)?r:o;
  if(!1&&!r)return o;
  if(cmp(1,r))return cmp(1,o)?1:o;
  return cmp(r,o)?r:o;
bool erase(node *&u,int k,const point &x){
  if(!u)return 0;
  if(u->pid==x){
    if(u->r):
    else if(u->1) u->r=u->1, u->1=0;
    else return delete(u),u=0, 1;
    --u->s:
    cmp.sort id=k;
    u \rightarrow pid = findmin(u \rightarrow r, (k+1)\%kd) \rightarrow pid;
    return erase(u->r,(k+1)%kd,u->pid);
```

```
cmp.sort id=k;
       if(erase(cmp(x,u->pid)?u->1:u->r,(k+1)%)
            kd(x)
          return --u->s, 1;
108
       return 0:
109
     T heuristic(const T h[])const{
110
111
112
       for(size_t i=0;i<kd;++i)ret+=h[i];</pre>
113
       return ret;
114
     int qM;
115
     priority queue<pair<T,point>> pQ;
     void nearest(node *u,int k,const point &x,
          T *h,T &mndist){
       if(u==0||heuristic(h)>=mndist)return;
118
119
       T dist=u->pid.dist(x),old=h[k];
120
       /*mndist=std::min(mndist.dist):*/
121
       if(dist<mndist){</pre>
122
          pQ.push(std::make_pair(dist,u->pid));
123
          if((int)pQ.size()==qM+1)
           mndist=pQ.top().first,pQ.pop();
124
125
126
       if(x.d[k]<u->pid.d[k]){
127
         nearest(u->1,(k+1)%kd,x,h,mndist);
128
         h[k] = abs(x.d[k]-u->pid.d[k]);
129
         nearest(u->r,(k+1)%kd,x,h,mndist);
130
       }else{
131
          nearest(u->r,(k+1)%kd,x,h,mndist);
132
         h[k] = abs(x.d[k]-u->pid.d[k]);
         nearest(u->1,(k+1)%kd,x,h,mndist);
134
135
       h[k]=old;
136
     vector<point>in range;
     void range(node *u,int k,const point&mi,
          const point&ma){
       if(!u)return;
139
       bool is=1;
140
       for(int i=0;i<kd;++i)</pre>
         if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid
               .d[i])
            { is=0;break; }
       if(is) in_range.push_back(u->pid);
       if(mi.d[k] \le u - > pid.d[k]) range(u - > 1,(k+1))
            %kd,mi,ma);
       if(ma.d[k]>=u->pid.d[k])range(u->r,(k+1)
            %kd,mi,ma);
   public:
     kd tree(const T &INF, double a=0.75):
     root(0),alpha(a),loga(log2(1.0/a)),INF(INF
          ),maxn(1){}
151
     ~kd tree(){delete root;}
     void clear(){delete root,root=0,maxn=1;}
152
     void build(int n,const point *p){
154
       delete root, A. resize(maxn=n);
       for(int i=0;i<n;++i)A[i]=new node(p[i]);</pre>
155
156
       root=build(0,0,n-1);
157
158
     void insert(const point &x){
       insert(root,0,x,__lg(size(root))/loga);
159
       if(root->s>maxn)maxn=root->s;
160
161
     bool erase(const point &p){
```

```
bool d=erase(root,0,p);
       if(root&&root->s<alpha*maxn)rebuild();</pre>
     void rebuild(){
       if(root)rebuild(root,0);
       maxn=root->s;
    T nearest(const point &x,int k){
171
       gM=k;
      T mndist=INF,h[kd]={};
       nearest(root,0,x,h,mndist);
       mndist=pQ.top().first;
       pQ = priority_queue<pair<T,point>>();
       return mndist://回傳離x第k近的點的距離
     const vector<point> &range(const point&mi,
          const point&ma){
       in range.clear();
       range(root,0,mi,ma);
       return in_range;//回傳介於mi到ma之間的點
            vector
183
    int size(){return root?root->s:0;}
184
```

2.4 kd tree replace segment tree

```
il struct node{//kd樹代替高維線段樹
    node *1,*r;
    point pid, mi, ma;
    int s, data;
    node(const point &p,int d):1(0),r(0),pid(p
         ),mi(p),ma(p),s(1),data(d),dmin(d),
         dmax(d){}
    void up(){
      mi=ma=pid;
      s=1;
      if(1){
        for(int i=0;i<kd;++i){</pre>
          mi.d[i]=min(mi.d[i],l->mi.d[i]);
          ma.d[i]=max(ma.d[i],1->ma.d[i]);
        s+=1->s:
      if(r){
        for(int i=0;i<kd;++i){</pre>
          mi.d[i]=min(mi.d[i],r->mi.d[i]);
          ma.d[i]=max(ma.d[i],r->ma.d[i]);
        s+=r->s;
    void up2(){/*其他懶惰標記向上更新*/}
    void down(){/*其他懶惰標記下推*/}
  }*root;
  //檢查區間包含用的函數
28 bool range include(node *o, const point &L,
       const point &R){
    for(int i=0;i<kd;++i){</pre>
      if(L.d[i]>o->ma.d[i]||R.d[i]<o->mi.d[i])
           return 0;
```

```
}//(L,R)區間完全包含o的區間就回傳true
38
    return 1:
39
40 bool point in range(node *o, const point &L,
       const point &R){
    for(int i=0;i<kd;++i){</pre>
      if(L.d[i]>o->pid.d[i]||R.d[i]<o->pid.d[i
           ])return 0;
    }//(L,R)區間完全包含o->pid這個點就回傳true
    return 1;
45 }
46 | // 單點修改,以單點改值為例
47 void update(node *u, const point &x, int data,
       int k=0){
    if(!u)return;
    u->down();
    if(u->pid==x){
      u->data=data;
      u->up2();
      return;
53
    cmp.sort id=k;
    update(cmp(x,u->pid)?u->l:u->r,x,data,(k
        +1)%kd);
    u->up2();
58 }
60 void update(node *o,const point &L,const
      point &R.int data){
    if(!o)return;
    o->down():
63
    if(range in range(o,L,R)){
      //區間懶惰標記修改
      o->down();
      return;
    if(point_in_range(o,L,R)){
      //這個點在(L,R)區間,但是他的左右子樹不
          一定在區間中
      //單點懶惰標記修改
    if(o->1&&range_include(o->1,L,R))update(o
72
        ->1,L,R,data);
    if(o->r&&range include(o->r,L,R))update(o
         ->r,L,R,data);
    o->up2();
75 }
76 //區間查詢,以總和為例
int query(node *o,const point &L,const point
        &R){
    if(!o)return 0;
```

if(range in range(o,L,R))return o->sum;

if(point_in_range(o,L,R))ans+=o->data;

}//(L,R)區間有和o的區間有交集就回傳true

34 bool range_in_range(node *o,const point &L,

if(L.d[i]>o->mi.d[i]||o->ma.d[i]>R.d[i])

const point &R){

for(int i=0:i<kd:++i){</pre>

return 0:

32

33 }

return 1;

o->down();

int ans=0:

```
if(o->l&&range_include(o->l,L,R))ans+=
    query(o->l,L,R);
if(o->r&&range_include(o->r,L,R))ans+=
    query(o->r,L,R);
return ans;
}
```

2.5 reference point

```
1 template<typename T>
2 struct RefC{
    T data;
    int ref;
    _RefC(const T&d=0):data(d),ref(0){}
  template<typename T>
  struct _rp{
    RefC<T> *p;
    T *operator->(){return &p->data;}
    T &operator*(){return p->data;}
    operator RefC<T>*(){return p;}
    _rp &operator=(const _rp &t){
      if(p&&!--p->ref)delete p;
      p=t.p,p&&++p->ref;
      return *this;
17
    rp( RefC<T> *t=0):p(t){p&&++p->ref;}
    _rp(const _rp &t):p(t.p){p&&++p->ref;}
    ~ rp(){if(p&&!--p->ref)delete p;}
21 };
22 template<typename T>
23 inline rp<T> new rp(const T&nd){
   return _rp<T>(new _RefC<T>(nd));
```

2.6 skew heap

```
1    node *merge(node *a,node *b){
2        if(!a||!b) return a?a:b;
3        if(b->data<a->data) swap(a,b);
4        swap(a->l,a->r);
5        a->l=merge(b,a->l);
6        return a;
7    }
```

2.7 undo disjoint set

```
struct DisjointSet {
   // save() is like recursive
   // undo() is like return
   int n, fa[MXN], sz[MXN];
   vector<pair<int*,int>> h;
   vector<int> sp;
   void init(int tn) {
        n=tn;
        for (int i=0; i<n; i++) sz[fa[i]=i]=1;
}</pre>
```

```
sp.clear(); h.clear();
11
    void assign(int *k, int v) {
      h.PB({k, *k});
14
      *k=v;
    void save() { sp.PB(SZ(h)); }
    void undo() {
      assert(!sp.empty());
      int last=sp.back(); sp.pop_back();
      while (SZ(h)!=last) {
        auto x=h.back(); h.pop_back();
        *x.F=x.S;
    int f(int x) {
      while (fa[x]!=x) x=fa[x];
      return x;
    void uni(int x, int y) {
      x=f(x); y=f(y);
      if (x==y) return ;
      if (sz[x]<sz[y]) swap(x, y);</pre>
      assign(&sz[x], sz[x]+sz[y]);
      assign(&fa[y], x);
36 }djs;
```

2.8 整體二分

```
1 void totBS(int L, int R, vector<Item> M){
2    if(Q.empty()) return; //維護全域B陣列
3    if(L==R) 整個M的答案=r, return;
4    int mid = (L+R)/2;
5    vector<Item> mL, mR;
6    do_modify_B_with_divide(mid,M);
7    //讓B陣列在遞迴的時候只會保留[L~mid]的資訊
undo_modify_B(mid,M);
5    totBS(L,mid,mL);
10    totBS(mid+1,R,mR);
11 }
```

3 Flow

3.1 dinic

3.2 Gomory Hu

T ans=0, f=0;

return ans;

int g[MAXN];

vector<edge> e;

e.clear();

void init(int n){

g[u]=e.size()-1;

g[v]=e.size()-1;

int bfs(int s,int t){

while(q.size()){

queue<int> q;

q.push(s);

LV[s]=1;

}

T df:

return 0;

directed=false){

memset(g,-1,sizeof(int)*((n=_n)+1));

e.push back(edge(u,g[v],directed?0:cap))

void add edge(int u,int v,T cap,bool

e.push back(edge(v,g[u],cap));

memset(LV,0,sizeof(int)*(n+1));

int u=q.front();q.pop();

q.push(e[i].v);

T dfs(int u,int t,T CF=INF){

e[i].r-=df;

return df;

return LV[u]=0;

e[i].r=e[i].cap;

e[i^1].r+=df;

if(u==t)return CF;

memcpy(cur,g,sizeof(int)*(n+1));

for(int i=g[u];~i;i=e[i].pre){

if(e[i].v==t)return 1;

for(int &i=cur[u];~i;i=e[i].pre){

T dinic(int s,int t,bool clean=true){

if(clean)for(size t i=0;i<e.size();++i)</pre>

while(bfs(s,t))while(f=dfs(s,t))ans+=f;

if(LV[e[i].v]==LV[u]+1&&e[i].r){

if(df=dfs(e[i].v,t,min(CF,e[i].r))){

if(!LV[e[i].v]&&e[i].r){

LV[e[i].v]=LV[u]+1;

```
| //最小割樹+求任兩點間最小割
2 |//0-base, root=0
3 | LL e[MAXN][MAXN]; //任兩點間最小割
4 int p[MAXN]; //parent
5 ISAP D; // original graph
6 void gomory_hu(){
```

```
fill(e[0], e[n], INF);
for( int s = 1; s < n; ++s ) {
   int t = p[s];
   ISAP F = D;
   LL tmp = F.min_cut(s, t);
   for( int i = 1; i < s; ++i )
        e[s][i] = e[i][s] = min(tmp, e[t][i]);
   for( int i = s+1; i <= n; ++i )
        if( p[i] == t && F.vis[i] ) p[i] = s;
}
</pre>
```

3.3 ISAP with cut

static const int MAXN=105;

static const T INF=INT MAX;

int d[MAXN],gap[MAXN],cur[MAXN];

1 template < typename T>

int n;//點數

struct edge{

int v,pre;

2 struct ISAP{

20

22

23

24

25

26

28

32

33

34

35

37

38

39

41

42

43

fill(p, p+n, 0);

```
T cap,r:
  edge(int v,int pre,T cap):v(v),pre(pre),
      cap(cap),r(cap){}
int g[MAXN];
vector<edge> e;
void init(int _n){
 memset(g, -1, sizeof(int)*((n= n)+1));
  e.clear();
void add edge(int u,int v,T cap,bool
    directed=false){
  e.push_back(edge(v,g[u],cap));
  g[u]=e.size()-1;
  e.push_back(edge(u,g[v],directed?0:cap))
  g[v]=e.size()-1;
T dfs(int u,int s,int t,T CF=INF){
  if(u==t)return CF:
  T tf=CF,df;
  for(int &i=cur[u];~i;i=e[i].pre){
    if(e[i].r&&d[u]==d[e[i].v]+1){
      df=dfs(e[i].v,s,t,min(tf,e[i].r));
      e[i].r-=df:
      e[i^1].r+=df;
      if(!(tf-=df)||d[s]==n)return CF-tf;
  int mh=n:
  for(int i=cur[u]=g[u];~i;i=e[i].pre){
    if(e[i].r&&d[e[i].v]<mh)mh=d[e[i].v];</pre>
  if(!--gap[d[u]])d[s]=n;
  else ++gap[d[u]=++mh];
  return CF-tf:
T isap(int s,int t,bool clean=true){
  memset(d,0,sizeof(int)*(n+1));
  memset(gap,0,sizeof(int)*(n+1));
```

```
e[i].r=e[i].cap;
       for(gap[0]=n;d[s]<n;)MF+=dfs(s,s,t);</pre>
       return MF:
52
53
     vector<int> cut e;//最小割邊集
     bool vis[MAXN];
55
     void dfs cut(int u){
       vis[u]=1;//表示u屬於source的最小割集
57
       for(int i=g[u];~i;i=e[i].pre)
58
         if(e[i].r>0&&!vis[e[i].v])dfs_cut(e[i
              1.v);
59
     T min_cut(int s,int t){
60
61
       T ans=isap(s,t);
62
       memset(vis,0,sizeof(bool)*(n+1));
63
       dfs_cut(s), cut_e.clear();
       for(int u=0;u<=n;++u)if(vis[u])</pre>
64
65
         for(int i=g[u];~i;i=e[i].pre)
66
           if(!vis[e[i].v])cut_e.push_back(i);
67
       return ans;
68
```

memcpy(cur,g,sizeof(int)*(n+1));

if(clean) for(size t i=0;i<e.size();++i)</pre>

3.4 MinCostMaxFlow

```
i template<typename TP>
2 struct MCMF{
    static const int MAXN=440;
     static const TP INF=999999999;
    struct edge{
      int v,pre;
       edge(int v,int pre,TP r,TP cost):v(v),
           pre(pre),r(r),cost(cost){}
10
     int n,S,T;
    TP dis[MAXN],PIS,ans;
11
12
    bool vis[MAXN];
    vector<edge> e:
13
    int g[MAXN];
14
     void init(int n){
15
16
      memset(g,-1,sizeof(int)*((n=_n)+1));
17
      e.clear();
18
    void add edge(int u,int v,TP r,TP cost,
19
          bool directed=false){
       e.push_back(edge(v,g[u],r,cost));
20
21
      g[u]=e.size()-1;
22
      e.push back(
      edge(u,g[v],directed?0:r,-cost));
23
24
      g[v]=e.size()-1;
25
     TP augment(int u, TP CF){
26
27
      if(u==T||!CF)return ans+=PIS*CF,CF;
28
      vis[u]=1;
29
      TP r=CF,d;
      for(int i=g[u];~i;i=e[i].pre){
30
31
        if(e[i].r&&!e[i].cost&&!vis[e[i].v]){
          d=augment(e[i].v,min(r,e[i].r));
```

```
e[i].r-=d;
      e[i^1].r+=d;
      if(!(r-=d))break;
  return CF-r;
bool modlabel(){
  for(int u=0;u<=n;++u)dis[u]=INF;</pre>
  static deque<int>q;
  dis[T]=0,q.push_back(T);
  while(q.size()){
    int u=q.front();q.pop_front();
    for(int i=g[u];~i;i=e[i].pre){
      if(e[i^1].r&&(dt=dis[u]-e[i].cost)
           dis[e[i].v]){
        if((dis[e[i].v]=dt)<=dis[q.size()?</pre>
             q.front():S]){
          q.push_front(e[i].v);
        }else q.push_back(e[i].v);
  for(int u=0;u<=n;++u)</pre>
    for(int i=g[u];~i;i=e[i].pre)
      e[i].cost+=dis[e[i].v]-dis[u];
  return PIS+=dis[S], dis[S]<INF;</pre>
TP mincost(int s,int t){
 S=s,T=t;
  PIS=ans=0;
  while(modlabel()){
    do memset(vis,0,sizeof(bool)*(n+1));
    while(augment(S,INF));
  }return ans;
```

Graph

Augmenting Path

```
| #define MAXN1 505
2 #define MAXN2 505
3 int n1, n2; //n1 個 點 連 向 n2 個 點
4 int match [MAXN2]; // 屬於 n2 的 點 匹 配 了 哪 個 點
5 vector<int > g[MAXN1];//圖 0-base
6 bool vis[MAXN2];//是否走訪過
 bool dfs(int u){
   for(int v:g[u]){
      if(vis[v]) continue;
      vis[v]=1;
      if(match[v]==-1||dfs(match[v]))
        return match[v]=u, 1;
12
    return 0;
16 int max match(){
   int ans=0;
```

```
memset(match,-1,sizeof(int)*n2);
     for(int i=0;i<n1;++i){</pre>
       memset(vis,0,sizeof(bool)*n2);
       if(dfs(i)) ++ans;
22
    return ans;
```

Augmenting Path multiple

```
1 #define MAXN1 1005
 #define MAXN2 505
 int n1, n2;
 //n1個點連向n2個點,其中n2個點可以匹配很多邊
 vector<int> g[MAXN1];//圖 0-base
 size t c[MAXN2];
 //每個屬於n2點最多可以接受幾條匹配邊
 vector<int> matchs[MAXN2];
 //每個屬於n2的點匹配了那些點
 bool vis[MAXN2];
 bool dfs(int u){
   for(int v:g[u]){
     if(vis[v])continue;
     vis[v] = 1;
     if(matchs[v].size()<c[v]){</pre>
       return matchs[v].push_back(u), 1;
     }else for(size t j=0;j<matchs[v].size()</pre>
          ;++j){
       if(dfs(matchs[v][j]))
         return matchs[v][j]=u, 1;
   return 0;
 int max match(){
   for(int i=0;i<n2;++i) matchs[i].clear();</pre>
   for(int u=0;u<n1;++u){</pre>
     memset(vis,0,sizeof(bool)*n2);
     if(dfs(u))++cnt;
   return cnt;
```

blossom matching

```
1 #define MAXN 505
  int n; //1-base
  vector<int> g[MAXN];
  int MH[MAXN]; //output MH
  int pa[MAXN],st[MAXN],S[MAXN],v[MAXN],t;
  int lca(int x,int y){
    for(++t;;swap(x,y)){
      if(!x) continue;
      if(v[x]==t) return x;
      v[x] = t;
      x = st[pa[MH[x]]];
12
```

```
|A| #define gpush(x) g.push(x),S[x]=0
                                                          Set C(min(A.size(), B.size()));
15 void flower(int x,int y,int l,queue<int>&q){ 19
                                                          auto it = set difference(A.begin(), A.end
    while(st[x]!=1){
                                                               (),B.begin(),B.end(),C.begin());
                                                          C.erase(it, C.end());
      pa[x]=y;
                                                   20
      if(S[y=MH[x]]==1)qpush(y);
                                                  21
                                                          return C;
      st[x]=st[y]=1, x=pa[y];
                                                   22
                                                   23
                                                       void BronKerbosch1(Set R, Set P, Set X){
21 }
                                                         if(P.empty()&&X.empty()){
                                                           // R form an maximal clique
22 bool bfs(int x){
    iota(st+1, st+n+1, 1);
                                                   26
                                                            return;
    memset(S+1,-1,sizeof(int)*n);
                                                   27
    queue<int>q; qpush(x);
                                                         for(auto v: P){
                                                           BronKerbosch1(setUnion(R,{v}),
    while(q.size()){
      x=q.front(),q.pop();
                                                                 setIntersection(P,G[v]),
                                                                 setIntersection(X,G[v]));
      for(int y:g[x]){
        if(S[y]==-1){
                                                           P = setDifference(P,{v});
                                                           X = setUnion(X, \{v\});
           pa[y]=x,S[y]=1;
                                                   31
          if(!MH[y]){
                                                   32
32
             for(int lst;x;y=lst,x=pa[y])
                                                   33
               lst=MH[x],MH[x]=y,MH[y]=x;
                                                   34
                                                       void init(int _n){
33
             return 1;
                                                   35
                                                         G.clear();
34
                                                         G.resize((n = _n) + 1);
                                                   36
                                                   37
           qpush(MH[y]);
        }else if(!S[y]&&st[y]!=st[x]){
                                                       void addEdge(int u, int v){
           int l=lca(y,x);
                                                         G[u].emplace back(v);
                                                   39
           flower(y,x,1,q),flower(x,y,1,q);
                                                   40
                                                         G[v].emplace_back(u);
                                                   41
      }
                                                       void solve(int n){
                                                   42
41
                                                   43
                                                         Set P;
    return 0;
                                                         for(int i=1; i<=n; ++i){</pre>
                                                            sort(G[i].begin(), G[i].end());
44 }
                                                     G[i].erase(unique(G[i].begin(), G[i].end()),
  int blossom(){
45
    memset(MH+1,0,sizeof(int)*n);
                                                           G[i].end());
    int ans=0;
                                                           P.emplace_back(i);
    for(int i=1; i<=n; ++i)</pre>
      if(!MH[i]&&bfs(i)) ++ans;
                                                   49
                                                         BronKerbosch1({}, P, {});
    return ans;
                                                   50
                                                   51
51
```

4.4 BronKerbosch

using Set = vector<int>:

C.erase(it, C.end());

const Set &B){

return C;

1 struct maximalCliques{

13

15

16

size t n; //1-base vector<Set> G: static Set setUnion(const Set &A, const Set &B){ Set C(A.size() + B.size()); auto it = set_union(A.begin(),A.end(),B. begin(),B.end(),C.begin()); C.erase(it, C.end()); return C; 10 static Set setIntersection(const Set &A, const Set &B){ Set C(min(A.size(), B.size())); auto it = set_intersection(A.begin(),A.

static Set setDifference(const Set &A,

end(),B.begin(),B.end(),C.begin());

graphISO

```
il const int MAXN=1005, K=30: //K要夠大
  const long long A=3,B=11,C=2,D=19,P=0
       xdefaced:
  long long f[K+1][MAXN];
  vector<int> g[MAXN],rg[MAXN];
  int n:
  void init(){
    for(int i=0;i<n;++i){</pre>
      f[0][i]=1;
      g[i].clear(), rg[i].clear();
10
11 }
12 void add_edge(int u,int v){
13
    g[u].push_back(v), rg[v].push_back(u);
14 }
15 long long point_hash(int u){//0(N)
    for(int t=1;t<=K;++t){</pre>
16
17
      for(int i=0;i<n;++i){</pre>
18
        f[t][i]=f[t-1][i]*A%P;
19
         for(int j:g[i])f[t][i]=(f[t][i]+f[t
              -1][j]*B%P)%P;
```

```
for(int j:rg[i])f[t][i]=(f[t][i]+f[t
             -1][j]*C%P)%P;
        if(i==u)f[t][i]+=D;//如果圖太大的話。
             把這行刪掉,執行一次後f[K]就會是所
             有點的答案
                                                     memset(My,0,sizeof(int)*(n+1));
        f[t][i]%=P;
                                                      memset(Mx,0,sizeof(int)*(n+1));
                                                      memset(ly,0,sizeof(LL)*(n+1));
    }
                                                      for(int x=1; x<=n; ++x){</pre>
    return f[K][u];
                                                       lx[x] = -INF;
                                                        for(int y=1; y<=n; ++y)</pre>
27 vector<long long> graph hash(){
                                                         lx[x] = max(lx[x],g[x][y]);
    vector<long long> ans;
    for(int i=0;i<n;++i)ans.push back(</pre>
                                                     for(int x=1; x<=n; ++x) bfs(x);</pre>
         point hash(i));//O(N^2)
                                                     LL ans = 0:
    sort(ans.begin(),ans.end());
                                                     for(int y=1; y<=n; ++y) ans+=g[My[y]][y];</pre>
    return ans:
                                                      return ans:
```

4.6 KM

```
1 #define MAXN 405
2 #define INF 0x3f3f3f3f3f3f3f3f3f
3 int n:// 1-base, 0表示沒有匹配
4 LL g[MAXN][MAXN]; //input graph
5 int My[MAXN], Mx[MAXN]; //output match
6 LL lx[MAXN],ly[MAXN],pa[MAXN],Sy[MAXN];
 bool vx[MAXN],vy[MAXN];
 void augment(int y){
   for(int x, z; y; y = z){
     x=pa[y],z=Mx[x];
     My[y]=x, Mx[x]=y;
 void bfs(int st){
   for(int i=1; i<=n; ++i)</pre>
     Sy[i] = INF, vx[i]=vy[i]=0;
   queue<int> q; q.push(st);
   for(;;){
      while(q.size()){
       int x=q.front(); q.pop();
        for(int y=1; y<=n; ++y) if(!vy[y]){</pre>
         LL t = 1x[x]+1y[y]-g[x][y];
         if(t==0){
           pa[y]=x;
           if(!My[y]){augment(y);return;}
            vy[y]=1,q.push(My[y]);
         }else if(Sy[y]>t) pa[y]=x,Sy[y]=t;
     LL cut = INF;
     for(int y=1; y<=n; ++y)</pre>
       if(!vy[y]&&cut>Sy[y]) cut=Sy[y];
     for(int j=1; j<=n; ++j){</pre>
       if(vx[j]) lx[j] -= cut;
       if(vy[j]) ly[j] += cut;
        else Sy[j] -= cut;
     for(int y=1; y<=n; ++y){</pre>
       if(!vy[y]&&Sy[y]==0){
         if(!My[y]){augment(y);return;}
         vy[y]=1, q.push(My[y]);
```

4.7 MaximumClique

static const int MAXN=105;

1 struct MaxClique{

```
int N.ans:
    int g[MAXN][MAXN],dp[MAXN],stk[MAXN][MAXN
    int sol[MAXN], tmp[MAXN]; //sol[0~ans-1]為答
    void init(int n){
      N=n;//0-base
       memset(g,0,sizeof(g));
    void add edge(int u,int v){
       g[u][v]=g[v][u]=1;
    int dfs(int ns,int dep){
      if(!ns){
         if(dep>ans){
           memcpy(sol,tmp,sizeof tmp);
           return 1;
         }else return 0:
       for(int i=0;i<ns;++i){</pre>
22
         if(dep+ns-i<=ans)return 0;</pre>
         int u=stk[dep][i],cnt=0;
         if(dep+dp[u]<=ans)return 0;</pre>
         for(int j=i+1; j<ns; ++j){</pre>
           int v=stk[dep][j];
           if(g[u][v])stk[dep+1][cnt++]=v;
         tmp[dep]=u;
         if(dfs(cnt,dep+1))return 1;
32
       return 0;
    int clique(){
      int u.v.ns:
       for(ans=0,u=N-1;u>=0;--u){
         for(ns=0,tmp[0]=u,v=u+1;v<N;++v)</pre>
           if(g[u][v])stk[1][ns++]=v;
         dfs(ns,1),dp[u]=ans;
```

```
42 }
43 };
```

1 #include < cfloat > //for DBL MAX

return ans;

4.8 MinimumMeanCycle

```
1 int dp[MAXN][MAXN]; // 1-base, O(NM)
3 vector<tuple<int,int,int>> edge;
4 double mmc(int n){//allow negative weight
    const int INF=0x3f3f3f3f;
    for(int t=0:t<n:++t){</pre>
      memset(dp[t+1],0x3f,sizeof(dp[t+1]));
      for(const auto &e:edge){
        int u,v,w;
        tie(u,v,w) = e;
        dp[t+1][v]=min(dp[t+1][v],dp[t][u]+w);
12
13
    double res = DBL MAX;
    for(int u=1;u<=n;++u){</pre>
      if(dp[n][u]==INF) continue;
      double val = -DBL MAX:
      for(int t=0;t<n;++t)</pre>
        val=max(val,(dp[n][u]-dp[t][u])*1.0/(n
              -t));
       res=min(res,val);
21
22
    return res;
```

4.9 Rectilinear MST

```
1 / / 平面曼哈頓最小生成樹構造圖(去除非必要邊)
2 #define T int
3 #define INF 0x3f3f3f3f
4 struct point{
   T x,y;
    int id;//從0開始編號
    point(){}
    T dist(const point &p)const{
      return abs(x-p.x)+abs(y-p.y);
11
12 bool cmpx(const point &a.const point &b){
    return a.x<b.x||(a.x==b.x&&a.y<b.y);
14 }
15 struct edge{
    int u,v;
    T cost;
    edge(int u,int v,T c):u(u),v(v),cost(c){}
    bool operator<(const edge&e)const{</pre>
      return cost<e.cost;</pre>
20
21
22 };
23 struct bit node{
24
   T mi:
   int id;
```

```
bit node(const T&mi=INF, int id=-1):mi(mi),
          id(id){}
27 };
28 vector<bit_node> bit;
29 void bit_update(int i,const T&data,int id){
    for(;i;i-=i&(-i)){
      if(data<bit[i].mi)bit[i]=bit node(data,</pre>
32
33
34 int bit find(int i,int m){
35
    bit node x;
    for(;i<=m;i+=i&(-i)) if(bit[i].mi<x.mi)x=</pre>
         bit[i];
    return x.id;
38 }
39 vector<edge> build_graph(int n,point p[]){
    vector<edge> e;//edge for MST
    for(int dir=0;dir<4;++dir){//4種座標變換
      if(dir%2) for(int i=0;i<n;++i) swap(p[i</pre>
42
           ].x,p[i].y);
       else if(dir==2) for(int i=0;i<n;++i) p[i
           ].x=-p[i].x;
      sort(p,p+n,cmpx);
      vector<T> ga(n), gb;
      for(int i=0;i<n;++i)ga[i]=p[i].y-p[i].x;</pre>
      gb=ga, sort(gb.begin(),gb.end());
      gb.erase(unique(gb.begin(),gb.end()),gb.
            end());
      int m=gb.size();
      bit=vector<bit node>(m+1);
51
      for(int i=n-1; i>=0; --i){
        int pos=lower_bound(gb.begin(),gb.end
              (),ga[i])-gb.begin()+1;
        int ans=bit find(pos,m);
53
        if(~ans)e.push_back(edge(p[i].id,p[ans
             ].id,p[i].dist(p[ans])));
55
        bit_update(pos,p[i].x+p[i].y,i);
56
57
    return e;
```

4.10 treeISO

```
1 const int MAXN=100005;
  const long long X=12327,P=0xdefaced;
  vector<int> g[MAXN];
  bool vis[MAXN];
5 long long dfs(int u){//hash ver
    vis[u]=1;
    vector<long long> tmp;
    for(auto v:g[u])if(!vis[v])tmp.PB(dfs(v));
    if(tmp.empty())return 177;
    long long ret=4931;
    sort(tmp.begin(),tmp.end());
11
12
    for(auto v:tmp)ret=((ret*X)^v)%P;
13
    return ret;
14 }
16 string dfs(int x,int p){
    vector<string> c;
    for(int y:g[x])
```

if (!onstk[i] && SPFA(i)){

found = 1;

```
if(y!=p)c.emplace back(dfs(y,x));
                                                              while (stk.size()>=2){
   sort(c.begin(),c.end());
                                                                int u = stk.back(); stk.pop back
   string ret("("):
                                                                     ();
   for(auto &s:c)ret+=s;
                                                                int v = stk.back(); stk.pop_back
   ret+=")";
                                                                     ();
   return ret;
                                                                match[u] = v:
                                                                match[v] = u;
 4.11 一般圖最小權完美匹配
                                                          if (!found) break;
                                                        int ret = 0;
                                                        for (int i=0; i<n; i++)</pre>
1 struct Graph {
   // Minimum General Weighted Matching (
                                                         ret += edge[i][match[i]];
        Perfect Match) 0-base
                                                        ret /= 2:
   static const int MXN = 105;
                                                        return ret:
   int n, edge[MXN][MXN];
   int match[MXN], dis[MXN], onstk[MXN];
                                                   }graph;
   vector<int> stk;
   void init(int _n) {
     n = _n;
                                                   4.12 全局最小割
     for (int i=0; i<n; i++)</pre>
       for (int j=0; j<n; j++)</pre>
         edge[i][j] = 0;
                                                    const int INF=0x3f3f3f3f;
                                                    template<typename T>
   void add edge(int u, int v, int w) {
                                                    struct stoer wagner{// 0-base
     edge[u][v] = edge[v][u] = w;
                                                      static const int MAXN=150;
                                                     T g[MAXN][MAXN], dis[MAXN];
   bool SPFA(int u){
                                                      int nd[MAXN],n,s,t;
     if (onstk[u]) return true;
                                                      void init(int _n){
     stk.push back(u);
                                                       n= n;
     onstk[u] = 1;
                                                        for(int i=0;i<n;++i)</pre>
     for (int v=0; v<n; v++){</pre>
                                                          for(int j=0;j<n;++j)g[i][j]=0;</pre>
       if (u != v && match[u] != v && !onstk[
            v1){
                                                      void add_edge(int u,int v,T w){
         int m = match[v];
                                                        g[u][v]=g[v][u]+=w;
                                                 13
         if (dis[m] > dis[u] - edge[v][m] +
                                                 14
               edge[u][v]){
                                                      T min_cut(){
           dis[m] = dis[u] - edge[v][m] +
                                                       T ans=INF:
                edge[u][v];
                                                        for(int i=0;i<n;++i)nd[i]=i;</pre>
           onstk[v] = 1;
                                                        for(int ind,tn=n;tn>1;--tn){
            stk.push back(v);
                                                          for(int i=1;i<tn;++i)dis[nd[i]]=0;</pre>
           if (SPFA(m)) return true;
                                                          for(int i=1;i<tn;++i){</pre>
           stk.pop back();
                                                            ind=i;
           onstk[v] = 0;
                                                            for(int j=i;j<tn;++j){</pre>
                                                              dis[nd[j]]+=g[nd[i-1]][nd[j]];
                                                              if(dis[nd[ind]]<dis[nd[j]])ind=j;</pre>
     onstk[u] = 0;
                                                            swap(nd[ind],nd[i]);
     stk.pop_back();
                                                 27
     return false:
                                                          if(ans>dis[nd[ind]])ans=dis[t=nd[ind
                                                               ]],s=nd[ind-1];
   int solve() {
                                                          for(int i=0;i<tn;++i)</pre>
     // find a match
                                                            g[nd[ind-1]][nd[i]]=g[nd[i]][nd[ind
     for (int i=0; i<n; i+=2){</pre>
                                                                 -1]]+=g[nd[i]][nd[ind]];
        match[i] = i+1, match[i+1] = i;
                                                        return ans;
                                                 32
     for(;;){
       int found = 0;
        for (int i=0; i<n; i++) dis[i] = onstk</pre>
            [i] = 0:
        for (int i=0; i<n; i++){</pre>
          stk.clear();
                                                   4.13 弦圖完美消除序列
```

```
1 struct chordal{
                                                  7 const int INF=0x3f3f3f3f;
    static const int MAXN=1005;
                                                  8 int dp[1<<MAXM][MAXN];</pre>
    int n:// 0-base
                                                  9 int g[MAXN][MAXN]:// 圖
    vector<int>G[MAXN];
                                                 void init(){memset(g,0x3f,sizeof(g));}
    int rank[MAXN],label[MAXN];
                                                 void add edge(int u,int v,int w){
    bool mark[MAXN];
    void init(int n){n= n;
                                                 13 }
      for(int i=0;i<n;++i)G[i].clear();</pre>
                                                 void steiner(int n,int r,int *p){
    void add_edge(int u,int v){
      G[u].push back(v);
                                                 17
12
      G[v].push back(u);
13
                                                 19
    vector<int> MCS(){
                                                 20
      memset(rank,-1,sizeof(int)*n);
                                                 21
      memset(label,0,sizeof(int)*n);
                                                 22
      priority queue<pair<int,int> > pq;
                                                 23
      for(int i=0;i<n;++i)pq.push(make_pair(0,</pre>
                                                 24
                                                 25
      for(int i=n-1;i>=0;--i)for(;;){
                                                 26
        int u=pq.top().second;pq.pop();
        if(~rank[u])continue;
                                                 27
        rank[u]=i;
                                                 28 29 }
        for(auto v:G[u])if(rank[v]==-1){
          pq.push(make pair(++label[v],v));
        break;
      vector<int> res(n);
28
      for(int i=0;i<n;++i)res[rank[i]]=i;</pre>
      return res;
30
31
32
    bool check(vector<int> ord){//弦圖判定
      for(int i=0;i<n;++i)rank[ord[i]]=i;</pre>
      memset(mark,0,sizeof(bool)*n);
34
      for(int i=0;i<n;++i){</pre>
35
        vector<pair<int,int> > tmp;
36
        for(auto u:G[ord[i]])if(!mark[u])
          tmp.push back(make pair(rank[u],u));
38
39
        sort(tmp.begin(),tmp.end());
                                                 10
40
        if(tmp.size()){
          int u=tmp[0].second;
          set<int> S:
42
          for(auto v:G[u])S.insert(v);
          for(size t j=1;j<tmp.size();++j)</pre>
44
            if(!S.count(tmp[j].second))return
        mark[ord[i]]=1;
48
49
      return 1;
                                                 21
  4.14 最小斯坦納樹 DP
11 //n個點,其中r個要構成斯坦納樹
2 //答案在max(dp[(1<<r)-1][k]) k=0~n-1
3 //p表示要構成斯坦納樹的點集
```

 $4 //0(n^3 + n*3^r + n^2*2^r)$

6 const int MAXN=30, MAXM=8;// 0-base

5 #define REP(i,n) for(int i=0;i<(int)n;++i)</pre>

4.15 最小樹形圖朱劉

g[u][v]=g[v][u]=min(g[v][u],w);

g[i][j]=min(g[i][j],g[i][k]+g[k][j]);

REP(i,r)REP(j,n)dp[1<<i][j]=g[p[i]][j];</pre>

for(int s=i&(i-1);s;s=i&(s-1)) tmp=min(tmp,dp[s][j]+dp[i^s][j]);

REP(k,n)dp[i][k]=min(dp[i][k],g[j][k]+

REP(k,n)REP(i,n)REP(j,n)

for(int i=1;i<(1<<r);++i){</pre>

if(!(i&(i-1)))continue;

REP(j,n)dp[i][j]=INF;

REP(i,n)g[i][i]=0;

int tmp=INF:

tmp);

REP(j,n){

```
1 template < typename T>
2 struct zhu liu{
    static const int MAXN=110, MAXM=10005;
    struct node{
       int u.v:
      T w,tag;
       node *1,*r;
       node(int u=0, int v=0, T w=0): u(u), v(v), w(
            w),tag(0),l(0),r(0){}
       void down(){
         w+=tag:
11
         if(1)1->tag+=tag;
12
         if(r)r->tag+=tag;
13
         tag=0;
14
15
     }mem[MAXM];//靜態記憶體
    node *pq[MAXN*2],*E[MAXN*2];
16
17
     int st[MAXN*2],id[MAXN*2],m;
18
     void init(int n){
19
       for(int i=1;i<=n;++i){</pre>
         pq[i]=E[i]=0, st[i]=id[i]=i;
20
      }m=0;
22
    node *merge(node *a, node *b){//skew heap
23
       if(!a||!b)return a?a:b;
25
       a->down(),b->down();
       if(b->w<a->w)return merge(b,a);
27
       swap(a->1,a->r);
       a \rightarrow 1 = merge(b, a \rightarrow 1);
28
29
      return a;
30
31
    void add edge(int u,int v,T w){
       if(u!=v)pq[v]=merge(pq[v],&(mem[m++]=
32
            node(u,v,w)));
33
    int find(int x,int *st){
```

```
return st[x]==x?x:st[x]=find(st[x],st);
    T build(int root,int n){
      T ans=0; int N=n, all=n;
      for(int i=1;i<=N;++i){</pre>
        if(i==root||!pq[i])continue;
        while(pq[i]){
          pq[i]->down(),E[i]=pq[i];
          pq[i]=merge(pq[i]->l,pq[i]->r);
          if(find(E[i]->u,id)!=find(i,id))
               break:
        if(find(E[i]->u,id)==find(i,id))
             continue:
        ans+=E[i]->w;
        if(find(E[i]->u,st)==find(i,st)){
          if(pq[i])pq[i]->tag-=E[i]->w;
          pq[++N]=pq[i];id[N]=N;
          for(int u=find(E[i]->u,id);u!=i;u=
               find(E[u]->u,id)){
            if(pq[u])pq[u]->tag-=E[u]->w;
            id[find(u,id)]=N;
            pq[N]=merge(pq[N],pq[u]);
          st[N]=find(i,st);
          id[find(i,id)]=N;
        }else st[find(i,st)]=find(E[i]->u,st)
             ,--all;
      return all==1?ans:-INT_MAX;//圖不連通就
           無解
62 };
```

4.16 穩定婚姻模板

```
1 | queue < int > 0;
2| for ( i : 所有考生 ) {
  設定在第0志願;
  Q.push(考生i);
6 while(Q.size()){
  當前考生=Q.front();Q.pop();
  while ( 此考生未分發 ) {
   指標移到下一志願;
   if ( 已經沒有志願 or 超出志願總數 )
       break:
    計算該考生在該科系加權後的總分;
   if (不符合科系需求) continue;
   if (目前科系有餘額) {
     依加權後分數高低順序將考生id加入科系錄
         取名單中;
     break;
   if (目前科系已額滿) {
     if ( 此考生成績比最低分數還高 ) {
       依加權後分數高低順序將考生id加入科系
          錄取名單:
      Q.push(被踢出的考生);
```

```
23 }
```

5 Language

int s,x,y;//s->xy | s->x, if y==-1

CNF(int s,int x,int y,int c):s(s),x(x),y(y

5.1 CNF

1 #define MAXN 55

int cost;

),cost(c){}

struct CNF{

CNF(){}

```
8 int state; // 規則數量
9| map<char, int> rule; // 每個字元對應到的規則
       小寫字母為終端字符
  vector<CNF> cnf;
  void init(){
    state=0;
    rule.clear();
    cnf.clear();
  void add to cnf(char s,const string &p,int
    //加入一個s -> 的文法,代價為cost
    if(rule.find(s)==rule.end())rule[s]=state
    for(auto c:p)if(rule.find(c)==rule.end())
        rule[c]=state++;
    if(p.size()==1){
      cnf.push back(CNF(rule[s],rule[p[0]],-1,
          cost));
    }else{
      int left=rule[s];
      int sz=p.size();
      for(int i=0;i<sz-2;++i){</pre>
        cnf.push_back(CNF(left,rule[p[i]],
            state,0));
        left=state++;
      cnf.push back(CNF(left,rule[p[sz-2]],
           rule[p[sz-1]],cost));
32 vector<long long> dp[MAXN][MAXN];
33 vector (bool > neg_INF[MAXN][MAXN];//如果花費
       是負的可能會有無限小的情形
  void relax(int l,int r,const CNF &c,long
       long cost,bool neg_c=0){
    if(!neg_INF[1][r][c.s]&&(neg_INF[1][r][c.x
         ]||cost<dp[1][r][c.s])){
      if(neg_c||neg_INF[1][r][c.x]){
        dp[1][r][c.s]=0;
        neg_INF[1][r][c.s]=true;
      }else dp[l][r][c.s]=cost;
```

```
42 | void bellman(int l,int r,int n){
    for(int k=1;k<=state;++k)</pre>
      for(auto c:cnf)
         if(c.y==-1)relax(l,r,c,dp[l][r][c.x]+c 33
              .cost,k==n);
47 void cyk(const vector<int> &tok){
    for(int i=0;i<(int)tok.size();++i){</pre>
      for(int j=0;j<(int)tok.size();++j){</pre>
         dp[i][i]=vector<long long>(state+1,
              INT MAX):
         neg_INF[i][j]=vector<bool>(state+1,
              false);
52
53
      dp[i][i][tok[i]]=0;
      bellman(i,i,tok.size());
54
55
    for(int r=1;r<(int)tok.size();++r){</pre>
      for(int l=r-1;l>=0;--1){
         for(int k=1;k<r;++k)</pre>
           for(auto c:cnf)
             if(~c.y)relax(1,r,c,dp[1][k][c.x]+ 51 }
                  dp[k+1][r][c.y]+c.cost);
         bellman(l,r,tok.size());
62
63
64 }
```

```
31
        if(a[x][j] < a[x][y]) y = j;
32
      if(a[x][y]>=0) return VDB();//infeasible
      pivot(x, y);
34
35
     for(int x,y;;){
      for(int j=y=1; j <= n; ++j)</pre>
36
37
        if(a[0][j] > a[0][y]) y = j;
38
      if(a[0][y]<=0) break;
39
      x = -1:
      for(int i=1; i<=m; ++i) if(a[i][y] > 0)
        if(x == -1 || a[i][0]/a[i][y]
           < a[x][0]/a[x][y]) x = i;
      if(x == -1) return VDB();//unbounded
      pivot(x, y);
45
    VDB ans(n + 1);
    for(int i = 1; i <= m; ++i)</pre>
      if(left[i] <= n) ans[left[i]] = a[i][0];</pre>
    ans[0] = -a[0][0];
    return ans;
```

for(int j=y=1; j <= n; ++j)</pre>

7 Number Theory

7.1 basic

Linear Programming

6.1 simplex

```
1 /*target:
    max \setminus sum_{j=1}^n A_{0,j}*x_{j}
   condition:
    \sum_{j=1}^n A_{i,j}*x_j <= A_{i,0} | i=1\sim m
    x_j >= 0 \mid j=1\sim n
   VDB = vector<double>*/
   template < class VDB >
   VDB simplex(int m,int n,vector<VDB> a){
     vector<int> left(m+1), up(n+1);
     iota(left.begin(), left.end(), n);
     iota(up.begin(), up.end(), 0);
     auto pivot = [&](int x, int y){
       swap(left[x], up[y]);
       auto k = a[x][y]; a[x][y] = 1;
       vector<int> pos;
       for(int j = 0; j <= n; ++j){</pre>
         a[x][j] /= k;
         if(a[x][j] != 0) pos.push_back(j);
18
19
       for(int i = 0; i <= m; ++i){</pre>
20
         if(a[i][y]==0 || i == x) continue;
         k = a[i][y], a[i][y] = 0;
         for(int j : pos) a[i][j] -= k*a[x][j];
23
24
25
26
     for(int x,y;;){
       for(int i=x=1; i <= m; ++i)</pre>
28
         if(a[i][0]<a[x][0]) x = i;</pre>
       if(a[x][0]>=0) break;
```

```
i template<typename T>
 void gcd(const T &a,const T &b,T &d,T &x,T &
     if(!b) d=a, x=1, y=0;
     else gcd(b,a%b,d,y,x), y-=x*(a/b);
   long long int phi[N+1];
   void phiTable(){
     for(int i=1;i<=N;i++)phi[i]=i;</pre>
     for(int i=1;i<=N;i++)for(x=i*2;x<=N;x+=i)</pre>
          phi[x]-=phi[i];
void all divdown(const LL &n) {// all n/x
     for(LL a=1;a<=n;a=n/(n/(a+1))){</pre>
12
       // dosomething;
13
14
15 }
16 const int MAXPRIME = 1000000;
int iscom[MAXPRIME], prime[MAXPRIME],
        primecnt;
18 int phi[MAXPRIME], mu[MAXPRIME];
19 void sieve(void){
20
     memset(iscom,0,sizeof(iscom));
     primecnt = 0:
21
     phi[1] = mu[1] = 1;
22
     for(int i=2;i<MAXPRIME;++i) {</pre>
23
24
       if(!iscom[i]) {
25
         prime[primecnt++] = i;
         mu[i] = -1;
26
27
         phi[i] = i-1;
28
29
       for(int j=0;j<primecnt;++j) {</pre>
30
         int k = i * prime[j];
         if(k>=MAXPRIME) break;
```

```
iscom[k] = prime[i];
        if(i%prime[i]==0) {
          mu[k] = 0;
          phi[k] = phi[i] * prime[j];
          break;
        } else {
          mu[k] = -mu[i];
          phi[k] = phi[i] * (prime[j]-1);
42
  bool g test(const LL &g, const LL &p, const
       vector<LL> &v) {
    for(int i=0;i<v.size();++i)</pre>
      if(modexp(g,(p-1)/v[i],p)==1)
        return false;
    return true:
  LL primitive_root(const LL &p) {
    if(p==2) return 1;
    vector<LL> v;
    Factor(p-1,v):
    v.erase(unique(v.begin(), v.end()), v.end
    for(LL g=2;g<p;++g)</pre>
      if(g_test(g,p,v))
        return g;
    puts("primitive root NOT FOUND");
    return -1;
  int Legendre(const LL &a, const LL &p) {
       return modexp(a%p,(p-1)/2,p); }
  LL inv(const LL &a, const LL &n) {
   LL d,x,v;
    gcd(a,n,d,x,y);
    return d==1 ? (x+n)%n : -1;
  int inv[maxN];
  LL invtable(int n, LL P){
    inv[1]=1:
    for(int i=2;i<n;++i)</pre>
      inv[i]=(P-(P/i))*inv[P%i]%P;
  LL log mod(const LL &a, const LL &b, const
       LL &p) {
    // a ^ x = b \pmod{p}
    int m=sqrt(p+.5), e=1;
    LL v=inv(modexp(a,m,p), p);
    map<LL,int> x;
    x[1]=0;
    for(int i=1;i<m;++i) {</pre>
      e = LLmul(e,a,p);
      if(!x.count(e)) x[e] = i;
    for(int i=0:i<m:++i) {</pre>
      if(x.count(b)) return i*m + x[b];
      b = LLmul(b,v,p);
    return -1;
```

```
if(Legendre(n,p)!=1) while(1) { puts("SORT 157
           ROOT does not exist"); }
     LL 0 = p-1:
     while( !(Q&1) ) { Q>>=1; ++S; }
     if(S==1) return modexp(n\%p,(p+1)/4,p);
     for(;Legendre(z,p)!=-1;++z)
     LL c = modexp(z,Q,p);
     LL R = modexp(n\%p,(Q+1)/2,p), t = modexp(n
          %p,Q,p);
     int M = S:
     while(1) {
       if(t==1) return R;
       LL b = modexp(c,1L << (M-i-1),p);
       R = LLmul(R,b,p);
       t = LLmul(LLmul(b,b,p), t, p);
       c = LLmul(b,b,p);
       M = i;
    return -1;
   template<typename T>
   T Euler(T n){
    T ans=n:
     for(T i=2;i*i<=n;++i){</pre>
       if(n%i==0){
         ans=ans/i*(i-1):
         while(n%i==0)n/=i;
     if(n>1)ans=ans/n*(n-1);
     return ans:
   //Chinese remainder theorem
   template<typename T>
   T pow mod(T n,T k,T m){
    T ans=1:
     for(n=(n>=m?n%m:n);k;k>>=1){
       if(k&1)ans=ans*n%m;
       n=n*n%m;
     return ans;
   template<typename T>
   T crt(vector<T> &m, vector<T> &a){
    T M=1,tM,ans=0;
     for(int i=0;i<(int)m.size();++i)M*=m[i];</pre>
     for(int i=0;i<(int)a.size();++i){</pre>
       tM=M/m[i];
       ans=(ans+(a[i]*tM%M)*pow mod(tM,Euler(m[
            i])-1,m[i])%M)%M;
       /*如果m[i]是質數, Euler(m[i])-1=m[i]-2,
            就不用算Euler了*/
     return ans;
151
  //iava code
```

94 LL Tonelli Shanks(const LL &n, const LL &p)

 $// x^2 = n \pmod{p}$

if(n==0) return 0;

123

124

125

126

127

128

129

134

149

150

152

```
154 / / 求 sart (N) 的 連 分 數
public static void Pell(int n){
     BigInteger N,p1,p2,q1,q2,a0,a1,a2,g1,g2,h1
                                                  12
           ,h2,p,q;
     g1=q2=p1=BigInteger.ZERO;
     h1=q1=p2=BigInteger.ONE:
     a0=a1=BigInteger.valueOf((int)Math.sqrt
           (1.0*n));
     BigInteger ans=a0.multiply(a0);
     if(ans.equals(BigInteger.valueOf(n))){
       System.out.println("No solution!");
163
       return ;
 164
     while(true){
       g2=a1.multiply(h1).substract(g1);
                                                   24
       h2=N.substract(g2.pow(2)).divide(h1);
167
       a2=g2.add(a0).divide(h2);
169
       p=a1.multiply(p2).add(p1);
       q=a1.multiply(q2).add(q1);
170
       if(p.pow(2).substract(N.multiply(q.pow
                                                   29
            (2))).compareTo(BigInteger.ONE)==0)
                                                   30
            break:
       g1=g2;h1=h2;a1=a2;
173
       p1=p2;p2=p;
174
       q1=q2;q2=q;
175
176
     System.out.println(p+" "+q);
177
```

7.4 FFT

11

13

14

18

19

20

21

22

23

15 }

for(int j=i+1; j<n;++j)</pre>

16 vector<int> decode(int a,int n){

vector<bool> vis(n,0);

for(j=0;j<n;++j)</pre>

if(!vis[j]){

res.push_back(j);

a%=factorial[i]:

--t;

vis[j]=1;

return res;

for(int i=n-1;i>=0;--i){ int t=a/factorial[i],j;

if(t==0)break;

res+=t*factorial[n-i-1]:

if(s[i]<s[i])++t;

return res;

vector<int> res:

```
| template<typename T, typename VT=vector<</pre>
        complex<T>>>
  struct FFT{
    const T pi;
    FFT(const T pi=acos((T)-1)):pi(pi){}
    unsigned bit_reverse(unsigned a,int len){
  a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)>>1);
  a=((a&0x33333333U)<<2)|((a&0xCCCCCCCU)>>2);
 a=((a&0x0F0F0F0FU)<<4)|((a&0xF0F0F0F0U)>>4);
9 a=((a&0x00FF00FFU)<<8)|((a&0xFF00FF00U)>>8);
a = ((a\&0x0000FFFFU) << 16) | ((a\&0xFFFF0000U))
       >>16);
       return a>>(32-len);
12
     void fft(bool is inv,VT &in,VT &out,int N)
13
14
       int bitlen=__lg(N),num=is_inv?-1:1;
       for(int i=0;i<N;++i)out[bit_reverse(i,</pre>
15
            bitlen)]=in[i];
       for(int step=2;step<=N;step<<=1){</pre>
17
         const int mh=step>>1;
18
         for(int i=0:i<mh:++i){</pre>
           complex<T> wi=exp(complex<T>(0,i*num
19
                *pi/mh));
           for(int j=i;j<N;j+=step){</pre>
             int k=j+mh;
21
             complex<T> u=out[j],t=wi*out[k];
             out[j]=u+t;
             out[k]=u-t;
       if(is_inv)for(int i=0;i<N;++i)out[i]/=N;</pre>
29
30 };
```

7.2 bit set

160

168

172

```
void sub set(int S){
   int sub=S:
      //對某集合的子集合的處理
      sub=(sub-1)&S;
    }while(sub!=S);
  void k sub set(int k,int n){
    int comb=(1<<k)-1,S=1<<n;</pre>
    while(comb<S){</pre>
      //對大小為k的子集合的處理
      int x=comb&-comb,y=comb+x;
      comb = ((comb\&\sim y)/x>>1)|y;
14
15 }
```

7.3 cantor expansion

```
int factorial[MAXN];
void init(){
   factorial[0]=1:
    for(int i=1;i<=MAXN;++i)factorial[i]=</pre>
         factorial[i-1]*i;
6 int encode(const vector(int> &s){
   int n=s.size().res=0;
    for(int i=0;i<n;++i){</pre>
      int t=0;
```

7.5 find real root

```
1 | // an*x^n + ... + a1x + a0 = 0;
1 int sign(double x){
    return x < -eps ? -1 : x > eps;
 double get(const vector<double>&coef, double
    double e = 1, s = 0;
    for(auto i : coef) s += i*e, e *= x;
    return s:
  double find(const vector<double>&coef, int n
       , double lo, double hi){
    double sign lo, sign hi;
    if( !(sign_lo = sign(get(coef,lo))) )
         return lo;
    if( !(sign hi = sign(get(coef,hi))) )
         return hi;
    if(sign lo * sign hi > 0) return INF:
    for(int stp = 0; stp < 100 && hi - lo >
         eps; ++stp){
      double m = (lo+hi)/2.0;
      int sign mid = sign(get(coef,m));
      if(!sign mid) return m;
      if(sign lo*sign mid < 0) hi = m;</pre>
      else lo = m;
    return (lo+hi)/2.0;
  vector<double> cal(vector<double>coef, int n
    vector<double>res:
    if(n == 1){
      if(sign(coef[1])) res.pb(-coef[0]/coef
           [1]);
      return res;
    vector<double>dcoef(n);
    for(int i = 0; i < n; ++i) dcoef[i] = coef</pre>
         [i+1]*(i+1);
    vector<double>droot = cal(dcoef, n-1);
    droot.insert(droot.begin(), -INF);
    droot.pb(INF);
    for(int i = 0; i+1 < droot.size(); ++i){</pre>
      double tmp = find(coef, n, droot[i],
           droot[i+1]);
      if(tmp < INF) res.pb(tmp);</pre>
    return res;
45 int main () {
   vector<double>ve;
   vector<double>ans = cal(ve, n);
   // 視情況把答案 +eps,避免 -0
```

```
1 | vector<int> F OR T(vector<int> f, bool
    for(int i=0: (2<<i)<=f.size(): ++i)</pre>
      for(int j=0; j<f.size(); j+=2<<i)</pre>
        for(int k=0; k<(1<<i); ++k)</pre>
          f[j+k+(1<< i)] += f[j+k]*(inverse)
               ?-1:1);
    return f:
 vector<int> rev(vector<int> A) {
   for(int i=0; i<A.size(); i+=2)</pre>
      swap(A[i],A[i^(A.size()-1)]);
    return A;
 vector<int> F AND T(vector<int> f, bool
   return rev(F_OR_T(rev(f), inverse));
 vector<int> F XOR T(vector<int> f, bool
       inverse){
    for(int i=0; (2<<i)<=f.size(); ++i)</pre>
      for(int j=0; j<f.size(); j+=2<<i)</pre>
        for(int k=0; k<(1<<i); ++k){</pre>
          int u=f[j+k], v=f[j+k+(1<<i)];</pre>
          f[j+k+(1<<i)] = u-v, f[j+k] = u+v;
   if(inverse) for(auto &a:f) a/=f.size();
    return f:
```

7.7 LinearCongruence

```
| pair<LL,LL> LinearCongruence(LL a[],LL b[],
      LL m[], int n) {
    // a[i]*x = b[i] (mod m[i])
    for(int i=0;i<n;++i) {</pre>
      LL x, y, d = extgcd(a[i],m[i],x,y);
      if(b[i]%d!=0) return make pair(-1LL,0LL)
      b[i] = LLmul(b[i]/d,x,m[i]);
    LL lastb = b[0], lastm = m[0];
    for(int i=1;i<n;++i) {</pre>
      LL x, y, d = extgcd(m[i], lastm, x, y);
      if((lastb-b[i])%d!=0) return make_pair
           (-1LL,0LL);
      lastb = LLmul((lastb-b[i])/d,x,(lastm/d)
           )*m[i];
      lastm = (lastm/d)*m[i]:
      lastb = (lastb+b[i])%lastm;
   return make pair(lastb<0?lastb+lastm:lastb</pre>
         ,lastm);
```

7.8 Lucas

```
| 1 | 11 | C(11 | n, 11 | m, 11 | p) { // n! / m! / (n-m)! 
if(n<m) return 0;</pre>
```

```
3 return f[n]*inv(f[m],p)%p*inv(f[n-m],p)%p;
5 11 L(11 n, 11 m, 11 p){
   if(!m) return 1;
   return C(n%p,m%p,p)*L(n/p,m/p,p)%p;
  ll Wilson(ll n, ll p){ // n!%p
   if(!n)return 1;
   11 res=Wilson(n/p, p);
   if((n/p)%2) return res*(p-f[n%p])%p;
   return res*f[n%p]%p; //(p-1)!%p=-1
14 }
```

7.9 Matrix

2 struct Matrix{

int r,c;

mt m;

34

41

1 template<typename T>

using rt = std::vector<T>;

using matrix = Matrix<T>;

using mt = std::vector<rt>;

rt& operator[](int i){return m[i];}

```
matrix operator+(const matrix &a){
       matrix rev(r,c);
       for(int i=0;i<r;++i)</pre>
         for(int j=0;j<c;++j)</pre>
            rev[i][j]=m[i][j]+a.m[i][j];
       return rev;
     matrix operator-(const matrix &a){
       matrix rev(r,c);
       for(int i=0;i<r;++i)</pre>
         for(int j=0;j<c;++j)</pre>
            rev[i][j]=m[i][j]-a.m[i][j];
       return rev;
     matrix operator*(const matrix &a){
       matrix rev(r,a.c);
       matrix tmp(a.c,a.r);
       for(int i=0;i<a.r;++i)</pre>
         for(int j=0;j<a.c;++j)</pre>
            tmp[j][i]=a.m[i][j];
       for(int i=0:i<r:++i)</pre>
         for(int j=0;j<a.c;++j)</pre>
            for(int k=0;k<c;++k)</pre>
              rev.m[i][j]+=m[i][k]*tmp[j][k];
33
       return rev;
35
     bool inverse(){
       Matrix t(r,r+c);
       for(int y=0;y<r;y++){</pre>
         t.m[y][c+y] = 1;
         for(int x=0;x<c;++x)</pre>
           t.m[y][x]=m[y][x];
42
       if(!t.gas())
         return false:
       for(int y=0;y<r;y++)</pre>
         for(int x=0;x<c;++x)</pre>
            m[y][x]=t.m[y][c+x]/t.m[y][y];
       return true;
```

```
for(int j=0;j<r;++j){</pre>
                                                           if(i==j)continue;
                                                           lazy[j]=lazy[j]*m[i][i];
                                                          T mx=m[j][i];
                                                          for(int k=0;k<c;++k)</pre>
                                                             m[j][k]=m[j][k]*m[i][i]-m[i][k]*mx
                                               67
                                               68
                                                      T det=sign?-1:1;
                                                      for(int i=0;i<r;++i){</pre>
                                                        det = det*m[i][i];
                                                        det = det/lazy[i];
Matrix(int r,int c):r(r),c(c),m(r,rt(c)){}
                                                        for(auto &j:m[i])j/=lazy[i];
```

7.10 MillerRobin

return det;

T gas(){

vector<T> lazv(r,1):

for(int i=0;i<r;++i){</pre>

if(m[i][i]==0){

if(j==r)continue;

m[i].swap(m[j]);

while(j<r&&!m[j][i])j++;</pre>

int j=i+1;

sign=!sign;

bool sign=false;

51

52

75

76

77 };

```
| ULL LLmul(ULL a, ULL b, const ULL &mod) {
    LL ans=0;
    while(b) {
      if(b&1) {
        ans+=a:
        if(ans>=mod) ans-=mod;
      a<<=1. b>>=1:
      if(a>=mod) a-=mod;
    return ans;
12 }
13 ULL mod mul(ULL a, ULL b, ULL m){
    a\%=m, \overline{b}\%=m;/* fast for m < 2^58 */
    ULL y=(ULL)((double)a*b/m+0.5);
    ULL r=(a*b-y*m)%m;
    return r<0?r+m:r;</pre>
19 template<typename T>
20 T pow(T a,T b,T mod){//a^b mod}
    T ans=1:
    for(;b;a=mod mul(a,a,mod),b>>=1)
      if(b&1)ans=mod mul(ans,a,mod);
25 }
26 int sprp[3]={2,7,61};//int範圍可解
27 int llsprp
       [7] = \{2,325,9375,28178,450775,9780504,
28 1795265022};//至少unsigned Long Long範圍
```

7.6 FWT

```
29 template<typename T>
30 bool isprime(T n, int *sprp, int num){
    if(n==2)return 1:
    if(n<2||n%2==0)return 0;
    int t=0;
    T u=n-1:
    for(;u%2==0;++t)u>>=1;
    for(int i=0:i<num:++i){</pre>
      T a=sprp[i]%n;
      if(a==0||a==1||a==n-1)continue;
      T x=pow(a,u,n);
      if(x==1||x==n-1)continue;
      for(int j=0;j<t;++j){</pre>
        x=mod mul(x,x,n);
        if(x==1)return 0;
        if(x==n-1)break;
      if(x==n-1)continue;
      return 0:
    return 1;
```

7.11 NTT

 $1 \mid 2615053605667*(2^{18})+1,3$

```
15*(2^27)+1,31
  479*(2^21)+1,3
4 7*17*(2^23)+1,3
  3*3*211*(2^19)+1,5
  25*(2^22)+1,3
  template<typename T,typename VT=vector<T> >
  struct NTT{
    const T P.G:
    NTT(T p=(1<<23)*7*17+1,T g=3):P(p),G(g){}
    unsigned bit reverse(unsigned a,int len){
      //look FFT.cpp
    T pow_mod(T n,T k,T m){
      for(n=(n)=m?n\%m:n);k;k>>=1){
        if(k&1)ans=ans*n%m;
        n=n*n%m;
      return ans:
21
    void ntt(bool is inv,VT &in,VT &out,int N)
      int bitlen=__lg(N);
      for(int i=0;i<N;++i)out[bit reverse(i,</pre>
            bitlen) | = in[i];
      for(int step=2,id=1;step<=N;step<<=1,++</pre>
            id){
        T wn=pow_mod(G,(P-1)>>id,P),wi=1,u,t;
         const int mh=step>>1;
         for(int i=0;i<mh;++i){</pre>
           for(int j=i;j<N;j+=step){</pre>
            u=out[j],t=wi*out[j+mh]%P;
            out[j]=u+t;
             out[j+mh]=u-t;
            if(out[j]>=P)out[j]-=P;
            if(out[j+mh]<0)out[j+mh]+=P;</pre>
```

```
wi=wi*wn%P;
        }
       if(is inv){
         for(int i=1;i<N/2;++i)swap(out[i],out[</pre>
              N-il):
         T invn=pow mod(N,P-2,P);
         for(int i=0;i<N;++i)out[i]=out[i]*invn</pre>
45
  };
```

7.12 Simpson

43

```
i double simpson(double a, double b){
    double c=a+(b-a)/2;
    return (F(a)+4*F(c)+F(b))*(b-a)/6;
  double asr(double a, double b, double eps,
       double A){
    double c=a+(b-a)/2;
    double L=simpson(a,c),R=simpson(c,b);
    if( abs(L+R-A)<15*eps )</pre>
      return L+R+(L+R-A)/15.0:
    return asr(a,c,eps/2,L)+asr(c,b,eps/2,R);
12 double asr(double a, double b, double eps){
    return asr(a,b,eps,simpson(a,b));
```

7.13 外星模運算

```
|1|/a[0]^{a[1]^a[2]^{...}
  #define maxn 1000000
  int euler[maxn+5];
  bool is prime[maxn+5];
  void init euler(){
    is prime[1]=1://一不是質數
    for(int i=1;i<=maxn;i++)euler[i]=i;</pre>
    for(int i=2;i<=maxn;i++){</pre>
      if(!is_prime[i]){//是質數
         euler[i]--;
         for(int j=i<<1; j<=maxn; j+=i){</pre>
           is_prime[j]=1;
           euler[j]=euler[j]/i*(i-1);
  LL pow(LL a, LL b, LL mod) {//a^b%mod
    LL ans=1;
    for(;b;a=a*a%mod,b>>=1)
      if(b&1)ans=ans*a%mod;
    return ans:
24 bool isless(LL *a.int n.int k){
    if(*a==1)return k>1:
    if(--n==0)return *a<k;</pre>
```

```
int next=0;
     for(LL b=1;b<k;++next)</pre>
       b*=*a:
     return isless(a+1,n,next);
31 }
32 LL high pow(LL *a, int n, LL mod){
    if(*a==1||--n==0)return *a%mod;
     int k=0.r=euler[mod];
     for(LL tma=1; tma!=pow(*a,k+r,mod);++k)
       tma=tma*(*a)%mod;
     if(isless(a+1,n,k))return pow(*a,high pow(
          a+1,n,k),mod);
     int tmd=high_pow(a+1,n,r), t=(tmd-k+r)%r;
     return pow(*a,k+t,mod);
40
41 LL a[1000005];
42 int t.mod:
  int main(){
    init euler():
     scanf("%d",&t);
     #define n 4
     while(t--){
       for(int i=0;i<n;++i)scanf("%lld",&a[i]);</pre>
48
       scanf("%d",&mod);
       printf("%lld\n",high pow(a,n,mod));
51
52
    return 0;
53
```

7.14 數位統計

```
ı | 11 d[65], dp[65][2]; //up區間是不是完整
2 11 dfs(int p,bool is8,bool up){
   if(!p)return 1; // 回傳@是不是答案
   if(!up&&~dp[p][is8])return dp[p][is8];
   int mx = up?d[p]:9;//可以用的有那些
   11 ans=0:
   for(int i=0;i<=mx;++i){</pre>
     if( is8&&i==7 )continue;
     ans += dfs(p-1,i==8,up&ki==mx);
   if(!up)dp[p][is8]=ans;
   return ans;
14 11 f(11 N){
   int k=0;
   while(N){ // 把數字先分解到陣列
     d[++k] = N%10;
     N/=10;
   return dfs(k,false,true);
```

7.15 質因數分解

```
| LL func(const LL n, const LL mod, const int c)
   return (LLmul(n,n,mod)+c+mod)%mod;
```

```
5 LL pollorrho(const LL n, const int c) {//循
        環節長度
    LL a=1. b=1:
    a=func(a,n,c)%n;
    b=func(b,n,c)%n; b=func(b,n,c)%n;
     while(gcd(abs(a-b),n)==1) {
       a=func(a,n,c)%n;
      b=func(b,n,c)%n; b=func(b,n,c)%n;
11
12
13
    return gcd(abs(a-b),n);
14 }
15
   void prefactor(LL &n, vector<LL> &v) {
    for(int i=0;i<12;++i) {</pre>
       while(n%prime[i]==0) {
        v.push back(prime[i]);
19
20
        n/=prime[i];
21
22
23
25
  void smallfactor(LL n, vector<LL> &v) {
    if(n<MAXPRIME) {</pre>
27
       while(isp[(int)n]) {
        v.push_back(isp[(int)n]);
28
29
         n/=isp[(int)n];
30
31
      v.push back(n);
    } else {
32
       for(int i=0;i<primecnt&&prime[i]*prime[i</pre>
            ]<=n;++i) {</pre>
         while(n%prime[i]==0) {
           v.push_back(prime[i]);
           n/=prime[i];
37
       if(n!=1) v.push_back(n);
41
42
   void comfactor(const LL &n, vector<LL> &v) {
    if(n<1e9) {
       smallfactor(n,v);
       return;
     if(Isprime(n)) {
      v.push back(n);
       return;
51
     LL d;
     for(int c=3::++c) {
       d = pollorrho(n,c);
55
      if(d!=n) break;
56
57
    comfactor(d,v);
    comfactor(n/d,v);
58
59
61 void Factor(const LL &x, vector<LL> &v) {
    LL n = x:
    if(n==1) { puts("Factor 1"); return; }
    prefactor(n,v);
    if(n==1) return;
    comfactor(n,v);
```

```
sort(v.begin(),v.end());
68 }
  void AllFactor(const LL &n, vector<LL> &v) {
    vector<LL> tmp;
    Factor(n,tmp);
    v.clear();
    v.push back(1):
    int len:
    LL now=1;
    for(int i=0;i<tmp.size();++i) {</pre>
      if(i==0 || tmp[i]!=tmp[i-1]) {
        len = v.size();
        now = 1;
      now*=tmp[i];
      for(int j=0;j<len;++j)</pre>
        v.push_back(v[j]*now);
```

String

8.1 AC 自動機

```
1 template < char L='a', char R='z'>
2 class ac automaton{
   struct joe{
     int next[R-L+1],fail,efl,ed,cnt dp,vis;
     joe():ed(0),cnt dp(0),vis(0){
       for(int i=0;i<=R-L;++i)next[i]=0;</pre>
   };
  public:
   std::vector<joe> S;
   std::vector<int> q;
   int qs,qe,vt;
   ac automaton():S(1),qs(0),qe(0),vt(0){}
   void clear(){
     q.clear();
     S.resize(1):
     for(int i=0;i<=R-L;++i)S[0].next[i]=0;</pre>
     S[0].cnt dp=S[0].vis=qs=qe=vt=0;
   void insert(const char *s){
     for(int i=0,id;s[i];++i){
        id=s[i]-L;
       if(!S[o].next[id]){
         S.push back(joe());
         S[o].next[id]=S.size()-1;
       o=S[o].next[id];
     ++S[o].ed;
   void build fail(){
     S[0].fail=S[0].efl=-1;
     q.clear();
     q.push back(0);
     ++qe;
```

```
while(as!=ae){
   int pa=q[qs++],id,t;
   for(int i=0;i<=R-L;++i){</pre>
     t=S[pa].next[i];
     if(!t)continue;
     id=S[pa].fail;
     while(~id&&!S[id].next[i])id=S[id].
          fail:
     S[t].fail=~id?S[id].next[i]:0;
     S[t].efl=S[S[t].fail].ed?S[t].fail:S
          [S[t].fail].efl;
     q.push_back(t);
     ++qe;
/*DP出每個前綴在字串s出現的次數並傳回所有
    字串被s匹配成功的次數O(N+M)*/
int match 0(const char *s){
 int ans=0,id,p=0,i;
  for(i=0;s[i];++i){
   id=s[i]-L;
   while(!S[p].next[id]&&p)p=S[p].fail;
   if(!S[p].next[id])continue;
   p=S[p].next[id];
   ++S[p].cnt_dp;/*匹配成功則它所有後綴都
        可以被匹配(DP計算)*/
  for(i=qe-1;i>=0;--i){
   ans+=S[q[i]].cnt_dp*S[q[i]].ed;
   if(~S[q[i]].fail)S[S[q[i]].fail].
```

```
cnt_dp+=S[q[i]].cnt_dp;
 return ans;
/*多串匹配走efl邊並傳回所有字串被s匹配成功
    的 次 數 O(N*M^1.5)*/
int match 1(const char *s)const{
 int ans=0,id,p=0,t;
  for(int i=0;s[i];++i){
   id=s[i]-L;
   while(!S[p].next[id]&&p)p=S[p].fail;
   if(!S[p].next[id])continue;
   p=S[p].next[id];
   if(S[p].ed)ans+=S[p].ed;
   for(t=S[p].efl;~t;t=S[t].efl){
     ans+=S[t].ed;/*因為都走efl邊所以保證
         匹配成功*/
  return ans;
/*枚舉(s的子字串nA)的所有相異字串各恰一次
    並傳回次數O(N*M^(1/3))*/
int match 2(const char *s){
 int ans=0,id,p=0,t;
```

```
/*把戳記vt+=1,只要vt沒溢位,所有S[p].
    vis==vt就會變成false
這種利用vt的方法可以0(1)歸零vis陣列*/
for(int i=0;s[i];++i){
 id=s[i]-L;
  while(!S[p].next[id]&&p)p=S[p].fail:
 if(!S[p].next[id])continue;
```

```
p=S[p].next[id];
93
        if(S[p].ed&&S[p].vis!=vt){
           S[p].vis=vt;
           ans+=S[p].ed;
         for(t=S[p].efl;~t&&S[t].vis!=vt;t=S[t
              1.ef1){
           S[t].vis=vt;
           ans+=S[t].ed;/*因為都走efL邊所以保證
                匹配成功*/
100
101
102
      return ans;
103
104
    /*把AC自動機變成真的自動機*/
    void evolution(){
      for(qs=1;qs!=qe;){
107
        int p=q[qs++];
108
        for(int i=0;i<=R-L;++i)</pre>
109
           if(S[p].next[i]==0)S[p].next[i]=S[S[
               p].fail].next[i];
110
111
112 };
```

8.2 hash

```
| #define MAXN 1000000
2 #define mod 1073676287
3 /*mod 必須要是質數*/
4 typedef long long T;
5 char s[MAXN+5];
6 T h[MAXN+5]; /*hash 陣列*/
  T h base[MAXN+5];/*h_base[n]=(prime^n)%mod*/
8 void hash init(int len, T prime){
   h base[0]=1;
    for(int i=1;i<=len;++i){</pre>
     h[i]=(h[i-1]*prime+s[i-1])%mod;
      h_base[i]=(h_base[i-1]*prime)%mod;
13
14 }
15 | T get_hash(int 1, int r){/*閉區間寫法,設編號
       為0 ~ Len-1*/
    return (h[r+1]-(h[1]*h base[r-1+1])%mod+
         mod)%mod;
```

8.3 KMP

```
1 /*產生fail function*/
void kmp_fail(char *s,int len,int *fail){
   int id=-1:
    fail[0]=-1;
    for(int i=1;i<len;++i){</pre>
      while(~id&&s[id+1]!=s[i])id=fail[id];
     if(s[id+1]==s[i])++id;
      fail[i]=id;
```

```
|II| /*以字串B匹配字串A· 傳回匹配成功的數量(用B的
       fail)*/
int kmp match(char *A.int lenA.char *B.int
       lenB,int *fail){
    int id=-1,ans=0;
    for(int i=0;i<lenA;++i){</pre>
15
      while(~id&&B[id+1]!=A[i])id=fail[id];
      if(B[id+1]==A[i])++id;
16
      if(id==lenB-1){/*匹配成功*/
18
        ++ans, id=fail[id];
19
20
    return ans;
```

8.4 manacher

```
ı | //原字串: asdsasdsa
2 // 先把字串變成這樣: @#a#s#d#s#a#s#d#s#a#
void manacher(char *s,int len,int *z){
   int 1=0.r=0:
   for(int i=1;i<len;++i){</pre>
     z[i]=r>i?min(z[2*l-i],r-i):1;
     while(s[i+z[i]]==s[i-z[i]])++z[i];
     if(z[i]+i>r)r=z[i]+i,l=i;
   }//ans = max(z)-1
```

minimal string rotation

```
i| int min string rotation(const string &s){
    int n=s.size(),i=0,j=1,k=0;
    while(i<n&&j<n&&k<n){</pre>
      int t=s[(i+k)%n]-s[(j+k)%n];
      ++k;
      if(t){
        if(t>0)i+=k;
        else j+=k;
        if(i==j)++j;
        k=0;
12
   return min(i,j);//最小循環表示法起始位置
```

8.6 reverseBWT

```
1 int ranks[MAXN], tots[MAXC], first[MAXC];
3 void rankBWT(const string &bw){
   memset(ranks,0,sizeof(int)*bw.size());
   memset(tots,0,sizeof(tots);
   for(size_t i=0;i<bw.size();++i)</pre>
     ranks[i] = tots[int(bw[i])]++;
9 void firstCol(){
```

```
memset(first,0,sizeof(first));
    int totc = 0;
    for(int c='A':c<='Z':++c){
     if(!tots[c]) continue;
     first[c] = totc;
      totc += tots[c];
18 string reverseBwt(string bw,int begin){
   rankBWT(bw), firstCol();
    int i = begin; //原字串最後一個元素的位置
    string res;
    do{
     char c = bw[i];
     res = c + res;
     i = first[int(c)] + ranks[i];
    }while( i != begin );
    return res;
```

8.7 suffix array lcp

Tarjan

dominator tree

static const int MAXN=5005;

for(int i=1; i<=n; ++i)</pre>

void add edge(int u, int v){

G[u].push_back(v);

rG[v].push_back(u);

int find(int y,int x){

if(y <= x) return y;</pre>

return pa[y] = tmp;

void tarian(int root){

tree[i].clear();

int u = id[i];

find(v,i);

dfnCnt = 0:

dfs(root);

int tmp = find(pa[y],x);

best[y] = best[pa[y]];

for(int i=1; i<=n; ++i){</pre>

dfn[i] = idom[i] = 0;

best[i] = semi[i] = i;

for(int i=dfnCnt; i>1; --i){

for(auto v:rG[u]) if(v=dfn[v]){

id[dfn[u]=++dfnCnt]=u;

for(auto v:G[u]) if(!dfn[v])

dfs(v),pa[dfn[v]]=dfn[u];

if(semi[best[y]] > semi[best[pa[y]]])

void dfs(int u){

vector<int> G[MAXN], rG[MAXN];

int semi[MAXN], idom[MAXN], best[MAXN];

vector<int> tree[MAXN]; // tree here

G[i].clear(), rG[i].clear();

struct dominator_tree{

void init(int n){

int n:// 1-base

```
| #define radix_sort(x,y){\
    for(i=0;i<A;++i)c[i]=0;\</pre>
    for(i=0;i<n;++i)c[x[y[i]]]++;\</pre>
    for(i=1;i<A;++i)c[i]+=c[i-1];\</pre>
    for(i=n-1;~i;--i)sa[--c[x[y[i]]]]=y[i];\
  #define AC(r,a,b)\
   r[a]!=r[b]||a+k>=n||r[a+k]!=r[b+k]
  void suffix_array(const char *s,int n,int *
       sa,int *rank,int *tmp,int *c){
    int A='z'+1,i,k,id=0;
    for(i=0;i<n;++i)rank[tmp[i]=i]=s[i];</pre>
    radix sort(rank,tmp);
    for(k=1;id<n-1;k<<=1){</pre>
      for(id=0,i=n-k;i<n;++i)tmp[id++]=i;</pre>
      for(i=0;i<n;++i)</pre>
        if(sa[i]>=k)tmp[id++]=sa[i]-k;
      radix sort(rank,tmp);
      swap(rank,tmp);
      for(rank[sa[0]]=id=0,i=1;i<n;++i)</pre>
        rank[sa[i]]=id+=AC(tmp,sa[i-1],sa[i]);
      A=id+1;
                                                   27
24 //h:高度數組 sa:後綴數組 rank:排名
void suffix_array_lcp(const char *s,int len,
       int *h,int *sa,int *rank){
                                                   32
    for(int i=0;i<len;++i)rank[sa[i]]=i;</pre>
    for(int i=0,k=0;i<len;++i){</pre>
      if(rank[i]==0)continue;
      if(k)--k;
      while(s[i+k]==s[sa[rank[i]-1]+k])++k;
      h[rank[i]]=k;
    h[0]=0;// h[k]=lcp(sa[k],sa[k-1]);
```

```
8.8 Z
                                                 43
                                                          tree[semi[i]].push_back(i);
                                                 44
                                                          for(auto v:tree[pa[i]]){
                                                 45
                                                            find(v, pa[i]);
void z alg(char *s,int len,int *z){
                                                            idom[v] = semi[best[v]]==pa[i]
  int 1=0, r=0;
                                                                 ? pa[i] : best[v];
  z[0]=len:
  for(int i=1;i<len;++i){</pre>
                                                 49
                                                          tree[pa[i]].clear();
    z[i]=i>r?0:(i-1+z[i-1]< z[1]?z[i-1]:r-i
                                                        for(int i=2; i<=dfnCnt; ++i){</pre>
                                                 51
    while(i+z[i]<len&&s[i+z[i]]==s[z[i]])++z</pre>
                                                          if(idom[i] != semi[i])
                                                            idom[i] = idom[idom[i]];
    if(i+z[i]-1>r)r=i+z[i]-1,l=i;
                                                 54
                                                          tree[id[idom[i]]].push_back(id[i]);
                                                 55
                                                 56
                                                 57 } dom;
```

tnfshb017 2 sat

| #include < bits / stdc++.h>

```
2 using namespace std;
                                               3 #define MAXN 8001
                                               4 #define MAXN2 MAXN*4
                                               5 #define n(X) ((X)+2*N)
                                                vector<int> v[MAXN2], rv[MAXN2], vis t;
                                                int N,M;
int pa[MAXN], dfn[MAXN], id[MAXN], dfnCnt;
                                                void addedge(int s,int e){
                                                  v[s].push_back(e);
                                                  rv[e].push back(s);
                                              11
                                                int scc[MAXN2];
                                                bool vis[MAXN2]={false};
                                              void dfs(vector<int> *uv,int n,int k=-1){
                                                  vis[n]=true;
                                                  for(int i=0;i<uv[n].size();++i)</pre>
                                                    if(!vis[uv[n][i]])
                                                       dfs(uv,uv[n][i],k);
                                                  if(uv==v)vis t.push back(n);
                                                  scc[n]=k;
                                              21
                                                void solve(){
                                                  for(int i=1;i<=N;++i){</pre>
                                                    if(!vis[i])dfs(v,i);
                                              25
                                                     if(!vis[n(i)])dfs(v,n(i));
                                              26
                                              27
                                                  memset(vis,0,sizeof(vis));
                                              28
                                                  int c=0;
                                              29
                                                  for(int i=vis_t.size()-1;i>=0;--i)
                                                    if(!vis[vis t[i]])
                                              30
                                                       dfs(rv,vis_t[i],c++);
                                              31
                                              32
                                              33 int main(){
                                                  int a,b;
                                                  scanf("%d%d",&N,&M);
                                                   for(int i=1;i<=N;++i){</pre>
                                                    // (A or B)&(!A & !B) A^B
                                                     a=i*2-1;
                                                     b=i*2;
                                                     addedge(n(a),b);
                                                     addedge(n(b),a);
                                                     addedge(a,n(b));
                                              42
                                              43
                                                     addedge(b,n(a));
      semi[i]=min(semi[i],semi[best[v]]);
                                                  while(M--){
```

```
b = b>0?b*2-1:-b*2:
       // A or B
       addedge(n(a),b);
       addedge(n(b),a);
51
52
53
     solve();
     bool check=true;
     for(int i=1;i<=2*N;++i)</pre>
       if(scc[i]==scc[n(i)])
57
         check=false;
     if(check){
       printf("%d \setminus n",N);
       for(int i=1;i<=2*N;i+=2){</pre>
         if(scc[i]>scc[i+2*N]) putchar('+');
62
         else putchar('-');
63
64
       puts("");
    }else puts("0");
    return 0;
```

scanf("%d%d",&a,&b);

a = a>0?a*2-1:-a*2;

9.3 橋連涌分量

```
1 #define N 1005
2 struct edge{
    int u,v;
    bool is bridge;
    edge(int u=0,int v=0):u(u),v(v),is_bridge
7 vector<edge> E;
8 vector<int> G[N];// 1-base
9 int low[N], vis[N], Time;
int bcc_id[N],bridge_cnt,bcc_cnt;// 1-base
int st[N],top;//BCC用
12 void add edge(int u,int v){
    G[u].push_back(E.size());
    E.emplace back(u,v);
    G[v].push back(E.size());
    E.emplace_back(v,u);
16
17 }
| void dfs(int u,int re=-1){//u當前點,re為u連
       接前一個點的邊
    int v:
    low[u]=vis[u]=++Time;
21
    st[top++]=u;
    for(int e:G[u]){
      v=E[e].v;
      if(!vis[v]){
        dfs(v,e^1);//e^1反向邊
        low[u]=min(low[u],low[v]);
        if(vis[u]<low[v]){</pre>
          E[e].is_bridge=E[e^1].is_bridge=1;
          ++bridge cnt;
      }else if(vis[v]<vis[u]&&e!=re)</pre>
32
        low[u]=min(low[u], vis[v]);
33
    if(vis[u]==low[u]){//處理BCC
      ++bcc_cnt;// 1-base
```

9.4 雙連通分量 & 割點

```
1 #define N 1005
vector<int> G[N];// 1-base
3 | vector<int> bcc[N];//存每塊雙連通分量的點
4 int low[N], vis[N], Time;
int bcc_id[N],bcc_cnt;// 1-base
6|bool is cut[N];//是否為割點
7 int st[N],top;
8 void dfs(int u,int pa=-1){//u當前點,pa父親
    int t, child=0;
    low[u]=vis[u]=++Time;
    st[top++]=u;
    for(int v:G[u]){
      if(!vis[v]){
        dfs(v,u),++child;
        low[u]=min(low[u],low[v]);
        if(vis[u]<=low[v]){</pre>
          is_cut[u]=1;
          bcc[++bcc_cnt].clear();
            bcc_id[t=st[--top]]=bcc_cnt;
            bcc[bcc cnt].push back(t);
          }while(t!=v);
          bcc_id[u]=bcc_cnt;
          bcc[bcc cnt].push back(u);
      }else if(vis[v]<vis[u]&&v!=pa)//反向邊
        low[u] = min(low[u], vis[v]);
    }//u是dfs樹的根要特判
    if(pa==-1&&child<2)is cut[u]=0;</pre>
31 void bcc init(int n){
    Time=bcc cnt=top=0;
    for(int i=1;i<=n;++i){</pre>
      G[i].clear();
      is_cut[i]=vis[i]=bcc_id[i]=0;
36
```

10 Tree Problem

10.1 HeavyLight

```
#include<vector>
  #define MAXN 100005
  int siz[MAXN], max son[MAXN], pa[MAXN], dep[
  int link top[MAXN],link[MAXN],cnt;
  vector<int> G[MAXN];
  void find max son(int u){
    siz[u]=\overline{1}:
    max son[u]=-1;
    for(auto v:G[u]){
      if(v==pa[u])continue;
      pa[v]=u;
      dep[v]=dep[u]+1;
      find max son(v);
      if(max_son[u]==-1||siz[v]>siz[max_son[u]
           ]])max son[u]=v;
      siz[u]+=siz[v];
  void build_link(int u,int top){
    link[u]=++cnt;
    link_top[u]=top;
    if(max_son[u]==-1)return;
    build_link(max_son[u],top);
    for(auto v:G[u]){
      if(v==max_son[u]||v==pa[u])continue;
      build link(v,v);
  int find lca(int a,int b){
    //求LCA, 可以在過程中對區間進行處理
    int ta=link top[a],tb=link top[b];
    while(ta!=tb){
      if(dep[ta]<dep[tb]){</pre>
        swap(ta,tb);
        swap(a,b);
      //這裡可以對a所在的鏈做區間處理
      //區間為(link[ta],link[a])
37
      ta=link top[a=pa[ta]];
    //最後a,b會在同一條鏈,若a!=b還要在進行一
         次區間處理
    return dep[a]<dep[b]?a:b;</pre>
```

10.2 LCA

```
const int MAXN=100000; // 1-base
const int MLG=17; //Log2(MAXN)+1;
int pa[MLG+2][MAXN+5];
int dep[MAXN+5];
vector<int> G[MAXN+5];
void dfs(int x,int p=0){//dfs(root);
pa[0][x]=p;
for(int i=0;i<=MLG;++i)
pa[i+1][x]=pa[i][pa[i][x]];
for(auto &i:G[x]){
    if(i==p)continue;
    dep[i]=dep[x]+1;
    dfs(i,x);
}</pre>
```

```
inline int jump(int x,int d){
    for(int i=0:i<=MLG:++i)</pre>
      if((d>>i)&1) x=pa[i][x];
18
    return x;
20
  inline int find lca(int a,int b){
    if(dep[a]>dep[b])swap(a,b);
    b=jump(b,dep[b]-dep[a]);
    if(a==b)return a;
    for(int i=MLG;i>=0;--i){
      if(pa[i][a]!=pa[i][b]){
        a=pa[i][a];
        b=pa[i][b];
29
31
    return pa[0][a];
32
```

10.3 link cut tree

```
struct splay_tree{
   int ch[2],pa;//子節點跟父母
   bool rev;//反轉的懶惰標記
   splay_tree():pa(0),rev(0){ch[0]=ch[1]=0;}
6 vector<splay_tree> nd;
8 / / 這邊以node [0] 作為null 節點
9| bool isroot(int x){//判斷是否為這棵splay
   return nd[nd[x].pa].ch[0]!=x&&nd[nd[x].pa
        ].ch[1]!=x;
11 | }
12 | void down(int x){// 懶惰標記下推
   if(nd[x].rev){
     if(nd[x].ch[0])nd[nd[x].ch[0]].rev^=1;
     if(nd[x].ch[1])nd[nd[x].ch[1]].rev^=1;
     swap(nd[x].ch[0],nd[x].ch[1]);
     nd[x].rev=0;
18
19 }
20 | void push_down(int x){//所有祖先懶惰標記下推
   if(!isroot(x))push_down(nd[x].pa);
   down(x);
23 }
24 | void up(int x){}//將子節點的資訊向上更新
25 | void rotate(int x){//旋轉,會自行判斷轉的方
    int y=nd[x].pa,z=nd[y].pa,d=(nd[y].ch[1]==
    nd[x].pa=z;
    if(!isroot(y))nd[z].ch[nd[z].ch[1]==y]=x;
    nd[y].ch[d]=nd[x].ch[d^1];
    nd[nd[y].ch[d]].pa=y;
    nd[y].pa=x,nd[x].ch[d^1]=y;
   up(y),up(x);
33 }
34 void splay(int x){//將x伸展到splay tree的根
   push down(x);
   while(!isroot(x)){
```

```
if(!isroot(y)){
        int z=nd[y].pa;
        if((nd[z].ch[0]==y)^(nd[y].ch[0]==x))
             rotate(v);
        else rotate(x);
42
      rotate(x);
  int access(int x){
    int last=0;
     while(x){
      splay(x);
      nd[x].ch[1]=last;
      up(x);
52
      last=x;
53
      x=nd[x].pa;
54
    return last;//access後splay tree的根
57 | void access(int x, bool is=0){//is=0就是一般
        的access
     int last=0;
     while(x){
60
       splay(x);
      if(is&&!nd[x].pa){
        //printf("%d\n", max(nd[last].ma,nd[nd[
62
             x].ch[1]].ma));
63
64
      nd[x].ch[1]=last;
65
      up(x);
      last=x;
67
      x=nd[x].pa;
68
69 }
70 void query_edge(int u,int v){
71
    access(u);
    access(v,1);
73
74 void make root(int x){
    access(x),splay(x);
76
    nd[x].rev^=1;
77 }
78 void make_root(int x){
    nd[access(x)].rev^=1;
    splay(x);
81 }
82 void cut(int x,int y){
    make_root(x);
    access(y);
    splay(y);
    nd[y].ch[0]=0;
    nd[x].pa=0;
88
89 void cut_parents(int x){
    access(x);
    splay(x);
    nd[nd[x].ch[0]].pa=0;
93
    nd[x].ch[0]=0;
95 void link(int x,int y){
    make_root(x);
    nd[x].pa=y;
98 }
```

int y=nd[x].pa;

```
99 int find root(int x){
    x=access(x);
    while(nd[x].ch[0])x=nd[x].ch[0];
    splay(x);
    return x;
103
104 }
int query(int u,int v){
106 // 傳回uv路徑splay tree的根結點
107 // 這種寫法無法求LCA
    make root(u);
    return access(v);
110 }
int query_lca(int u,int v){
112 //假設求鏈上點權的總和·sum是子樹的權重和
       data 是節點的權重
     access(u);
    int lca=access(v);
     splay(u);
    if(u==lca){
      //return nd[lca].data+nd[nd[lca].ch[1]].
118
    }else{
      //return nd[lca].data+nd[nd[lca].ch[1]].
           sum+nd[u].sum
120
121
  struct EDGE{
    int a,b,w;
124 }e[10005];
  int n;
126 vector<pair<int,int>> G[10005];
127 | //first表示子節點, second表示邊的編號
128 int pa[10005],edge_node[10005];
| 129 | //pa 是父母節點,暫存用的,edge_node 是每個編
        被存在哪個點裡面的陣列
130 void bfs(int root){
131 //在建構的時候把每個點都設成一個splay tree
    queue<int > q;
    for(int i=1;i<=n;++i)pa[i]=0;</pre>
133
134
    q.push(root);
135
    while(q.size()){
136
      int u=q.front();
137
      q.pop();
138
      for(auto P:G[u]){
139
        int v=P.first;
140
        if(v!=pa[u]){
141
          pa[v]=u;
142
          nd[v].pa=u;
143
          nd[v].data=e[P.second].w;
144
          edge_node[P.second]=v;
145
          up(v);
          q.push(v);
147
148
149
    }
  void change(int x,int b){
    splay(x);
    //nd[x].data=b;
    up(x);
```

10.4 POJ tree

#include < bits / stdc++.h>

using namespace std;

```
#define MAXN 10005
  int n.k:
  vector<pair<int,int> >g[MAXN];
  int size[MAXN];
  bool vis[MAXN];
  inline void init(){
    for(int i=0;i<=n;++i){</pre>
      g[i].clear();
      vis[i]=0;
  void get_dis(vector<int> &dis,int u,int pa,
       int d){
    dis.push_back(d);
    for(size_t i=0;i<g[u].size();++i){</pre>
      int v=g[u][i].first,w=g[u][i].second;
      if(v!=pa&&!vis[v])get_dis(dis,v,u,d+w);
  vector<int> dis://這東西如果放在函數裡會TLE
  int cal(int u,int d){
    dis.clear():
    get_dis(dis,u,-1,d);
    sort(dis.begin(),dis.end());
    int l=0, r=dis.size()-1, res=0;
    while(l<r){</pre>
      while(l<r&&dis[l]+dis[r]>k)--r;
      res+=r-(1++);
    return res;
pair<int,int> tree_centroid(int u,int pa,
       const int sz){
    size[u]=1;//找樹重心, second是重心
    pair<int,int> res(INT MAX,-1);
    for(size_t i=0;i<g[u].size();++i){</pre>
      int v=g[u][i].first;
      if(v==pa||vis[v])continue;
      res=min(res,tree centroid(v,u,sz));
      size[u]+=size[v]:
42
      ma=max(ma,size[v]);
    ma=max(ma,sz-size[u]);
    return min(res,make_pair(ma,u));
  int tree DC(int u,int sz){
    int center=tree centroid(u,-1,sz).second;
    int ans=cal(center,0);
    vis[center]=1;
    for(size_t i=0;i<g[center].size();++i){</pre>
      int v=g[center][i].first,w=g[center][i].
           second;
      if(vis[v])continue;
      ans-=cal(v,w);
      ans+=tree_DC(v,size[v]);
    return ans;
  int main(){
    while(scanf("%d%d",&n,&k),n||k){
```

11 default

11.1 debug

11.2 ext

```
| #include < bits / extc++.h>
2 #include<ext/pd_ds/assoc_container.hpp>
3 #include < ext/pd ds/tree policy.hpp>
4 using namespace __gnu_cxx;
s using namespace __gnu_pbds;
6 template<typename T>
7 using pbds set = tree<T,null type,less<T>,
       rb_tree_tag,
       tree order statistics node update>;
  template<typename T,typename U>
9 using pbds_map = tree<T,U,less<T>,
       rb tree tag,
       tree_order_statistics_node_update>;
using heap=__gnu_pbds::priority_queue<int>;
11 //s.find by order(1);//0 base
12 //s.order_of_key(1);
```

11.3 IncStack

```
2 #pragma GCC optimize "Ofast"
3 //stack resize.change esp to rsp if 64-bit
4 asm("mov %0, %%esp\n" :: "g"(mem+10000000));
  -Wl,--stack,214748364 -trigraphs
 6 #pragma comment(linker, "/STACK
       :1024000000.1024000000")
7 //linux stack resize
8 #include<sys/resource.h>
9 void increase stack(){
    const rlim_t ks=64*1024*1024;
    struct rlimit rl;
    int res=getrlimit(RLIMIT_STACK,&rl);
    if(!res&&rl.rlim_cur<ks){</pre>
      rl.rlim cur=ks;
15
      res=setrlimit(RLIMIT STACK,&rl);
16
17 }
```

11.4 input

```
inline int read(){
  int x=0; bool f=0; char c=getchar();
  while(ch<'0'||'9'<ch)f|=ch=='-',ch=getchar();

while('0'<=ch&&ch<='9')x=x*10-'0'+ch,ch=getchar();

return f?-x:x;

// #!/bin/bash
// g++ -std=c++11 -02 -Wall -Wextra -Wno-unused-result -DDEBUG $1 && ./a.out
// -fsanitize=address -fsanitize=undefined -fsanitize=return</pre>
```

12 other

12.1 WhatDay

12.2 上下最大正方形

```
void solve(int n,int a[],int b[]){// 1-base
int ans=0;
deque<int>da,db;
for(int l=1,r=1;r<=n;++r){</pre>
```

12.3 最大矩形

```
1 | LL max_rectangle(vector<int> s){
    stack<pair<int,int > > st;
    st.push(make pair(-1,0));
    s.push_back(0);
    LL ans=0:
    for(size t i=0;i<s.size();++i){</pre>
      int h=s[i];
      pair<int,int > now=make pair(h,i);
      while(h<st.top().first){</pre>
        now=st.top();
        st.pop();
        ans=max(ans,(LL)(i-now.second)*now.
             first);
      if(h>st.top().first){
        st.push(make pair(h,now.second));
17
   }
   return ans;
```

13 other language

13.1 java

13.1.1 文件操作

```
import java.io.*;
import java.util.*;
import java.math.*;
import java.text.*;
public class Main{
```

```
public static void main(String args[]){
    throws FileNotFoundException,
    IOException
  Scanner sc = new Scanner(new FileReader(
      "a.in"));
  PrintWriter pw = new PrintWriter(new
      FileWriter("a.out"));
 int n.m:
 n=sc.nextInt();//读入下一个INT
 m=sc.nextInt():
 for(ci=1; ci<=c; ++ci){</pre>
   pw.println("Case #"+ci+": easy for
        output");
 pw.close();// 关闭流并释放,这个很重要,
      否则是没有输出的
 sc.close();// 关闭流并释放
```

13.1.2 优先队列

13.1.3 Map

```
1  Map map = new HashMap();
2  map.put("sa","dd");
3  String str = map.get("sa").toString;
4  
5  for(Object obj : map.keySet()){
6    Object value = map.get(obj);
7  }
```

13.1.4 sort

```
static class cmp implements Comparator{
public int compare(Object o1,Object o2){
BigInteger b1=(BigInteger)o1;
BigInteger b2=(BigInteger)o2;
return b1.compareTo(b2);
}
public static void main(String[] args)
throws IOException{
Scanner cin = new Scanner(System.in);
```

```
int n;
n=cin.nextInt();
BigInteger[] seg = new BigInteger[n];
for (int i=0;i<n;i++)
seg[i]=cin.nextBigInteger();
Arrays.sort(seg,new cmp());
}</pre>
```

13.2 python heap

```
import heapq

heap = [7,1,2,2]
heapq.heapify(heap)
print(heap) # [1, 2, 2, 7]
heapq.heappush(heap, 5)
print(heap) # [1, 2, 2, 7, 5]
print(heap, # [1, 2, 2, 7, 5]
print(heap, # [2, 2, 5, 7]
```

13.3 python input

```
1  ans = sum(map(float, input().split()))
2  # input: 1.1 2.2 3.3 4.4 5.5
3  print(ans) # 16.5
4
5  (n, m) = map(int, input().split()) # 300 200
6  print(n * m) # 60000
7
8  Arr = list(map(int, input().split()))
9  # input: 1 2 3 4 5
10  print(Arr) # [1, 2, 3, 4, 5]
```

14 zformula

14.1 formula

14.1.1 Pick 公式

給定頂點坐標均是整點的簡單多邊形·面積 = 內部格點數 + 邊上格點數/2-1

14.1.2 圖論

- 1. 對於平面圖 $\cdot F = E V + C + 1 \cdot C$ 是連通分量數 2. 對於平面圖 $\cdot E < 3V 6$
- 3. 對於連通圖 G·最大獨立點集的大小設為 I(G)·最大 匹配大小設為 M(G)·最小點覆蓋設為 Cv(G)·最小 邊覆蓋設為 Ce(G)·對於任意連通圖:
 - (a) I(G) + Cv(G) = |V|(b) M(G) + Ce(G) = |V|
- 4. 對於連通二分圖:

- (a) I(G) = Cv(G)
- (b) M(G) = Ce(G)

5. 最大權閉合圖:

- (a) $C(u,v) = \infty, (u,v) \in E$
- (b) $C(S, v) = W_v, W_v > 0$
- (c) $C(v,T) = -W_v, W_v < 0$
- (d) ans= $\sum_{W_v>0} W_v flow(S,T)$

6. 最大密度子圖:

- (b) $U = \sum_{v \in V} 2W_v + \sum_{e \in E} W_e$
- (c) $C(u,v) = W_{(u,v)}, (u,v) \in E$, 雙向邊
- (d) $C(S, v) = U, v \in V$
- (e) $D_u = \sum_{(u,v) \in E} W_{(u,v)}$
- (f) $C(v,T) = U + 2g D_v 2W_v, v \in V$
- (g) 二分搜 g: $l=0, r=U, eps=1/n^2$ if $((U\times |V|-flow(S,T))/2>0)$ l=mid else r=mid
- (h) ans= $min_cut(S,T)$
- (i) |E| = 0 要特殊判斷
- 7. 弦圖:
 - (a) 點數大於 3 的環都要有一條弦
 - (b) 完美消除序列從後往前依次給每個點染色,給 每個點染上可以染的最小顏色
 - (c) 最大團大小 = 色數
 - (d) 最大獨立集: 完美消除序列從前往後能選就選
 - (e) 最小團覆蓋: 最大獨立集的點和他延伸的邊構成
 - (f) 區間圖是弦圖
 - (g) 區間圖的完美消除序列: 將區間按造又端點由 小到大排序
 - (h) 區間圖染色: 用線段樹做

14.1.3 dinic 特殊圖複雜度

- 1. 單位流: $O\left(min\left(V^{3/2},E^{1/2}\right)E\right)$
- 2. 二分圖: $O(V^{1/2}E)$

14.1.4 0-1 分數規劃

```
x_i = \{0,1\} \cdot x_i 可能會有其他限制 · 求 max\left(\frac{\sum B_i x_i}{\sum C_i x_i}\right)
     1. D(i,g) = B_i - g \times C_i
     2. f(q) = \sum D(i, q)x_i
     3. f(g) = 0 時 g 為最佳解 f(g) < 0 沒有意義
     4. 因為 f(g) 單調可以二分搜 g
     5. 或用 Dinkelbach 通常比較快
i| binary_search(){
   while(r-l>eps){
     g=(1+r)/2;
      for(i:所有元素)D[i]=B[i]-g*C[i];//D(i,g)
      找出一組合法x[i]使f(g)最大;
     if(f(g)>0) l=g;
      else r=g;
   Ans = r;
 Dinkelbach(){
```

14.1.5 學長公式

```
1. \sum_{d|n} \phi(n) = n
```

g=任意狀態(通常設為0);

找出一組合法x[i]使f(g)最大;

if(x[i])p+=B[i],q+=C[i];

g=p/q;//更新解,注意q=0的情況

do{

Ans=g;

p=0,q=0;

return Ans;

for(i:所有元素)

}while(abs(Ans-g)>EPS);

2.
$$g(n) = \sum_{d|n} f(d) = f(n) = \sum_{d|n} \mu(d) \times g(n/d)$$

for(i:所有元素)D[i]=B[i]-g*C[i];//D(i,g)

- 3. Harmonic series $H_n = \ln(n) + \gamma + 1/(2n)$ $1/(12n^2) + 1/(120n^4)$
- 4. $\gamma = 0.57721566490153286060651209008240243104215$ 1. $a^b\%P = a^{b\%\varphi(p) + \varphi(p)}, b > \varphi(p)$
- 5. 格雷碼 = $n \oplus (n >> 1)$
- 6. $SG(A+B) = SG(A) \oplus SG(B)$
- 7. 選轉矩陣 $M(\theta) = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$

14.1.6 基本數論

- 1. $\sum_{d|n} \mu(n) = [n == 1]$
- 2. $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) \times$
- 4. $\sum_{i=1}^{n} \sum_{j=1}^{n} lcm(i,j) = n \sum_{d|n} d \times \phi(d)$

14.1.7 排組公式

- 1. k 卡特蘭 $\frac{C_n^{kn}}{n(k-1)+1} \cdot C_m^n = \frac{n!}{m!(n-m)!}$ 2. $H(n,m) \cong x_1 + x_2 \dots + x_n = k, num = C_k^{n+k-1}$
- 3. Stirling number of 2^{nd} ,n 人分 k 組方法數目
 - (a) S(0,0) = S(n,n) = 1
 - (b) S(n,0) = 0
 - (c) S(n,k) = kS(n-1,k) + S(n-1,k-1)
- 4. Bell number, n 人分任意多組方法數目
 - (a) $B_0 = 1$

 - (a) $B_0 = 1$ (b) $B_n = \sum_{i=0}^n S(n, i)$ (c) $B_{n+1} = \sum_{k=0}^n C_n^k B_k$ (d) $B_{p+n} \equiv B_n + B_{n+1} mod p$, p is prime (e) $B_p m_+ \equiv m B_n + B_{n+1} mod p$, p is prime (f) From $B_0 : 1, 1, 2, 5, 15, 52$, 203, 877, 4140, 21147, 115975
- 5. Derangement, 錯排, 沒有人在自己位置上
 - (a) $D_n = n!(1 \frac{1}{1!} + \frac{1}{2!} \frac{1}{3!} \dots + (-1)^n \frac{1}{n!})$ (b) $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 =$ $1, D_1 = 0$
 - (c) From $D_0: 1, 0, 1, 2, 9, 44$ 265, 1854, 14833, 133496
- 6. Binomial Equality
 - (a) $\sum_{k} {r \choose m+k} {s \choose n-k} = {r+s \choose m+n}$
 - (b) $\sum_{k} {i \choose m+k} {s \choose n+k} = {i+s \choose l-m+n}$
 - (c) $\sum_{k} {m+k \choose m+k} {s+k \choose n} (-1)^{k} = (-1)^{l+m} {s-m \choose n-l}$ (d) $\sum_{k \le l} {m \choose l} {s \choose k-n} (-1)^{k} = (-1)^{l+m} {s-m \choose n-l}$

 - $(-1)^{l+m} {n \choose l-n-m} (-1)^{l+m} {n \choose l-n-m} (-1)^{l+m} {n \choose l-n-m} (-1)^{l+m} {n \choose n} = {n+q+1 \choose m+n+1} (-1)^{m+1} {n \choose l-m} {n \choose m-1} (-1)^{m+1} {n \choose m$

 - (g) $\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}$

 - (h) $\sum_{k \le n} {r+k \choose k} = {r+n+1 \choose n}$ (i) $\sum_{0 \le k \le n} {k \choose k} = {n+1 \choose n}$ (j) $\sum_{k \le m} {m+r \choose k} x^k y^k$ $\sum_{k \le m} {r \choose k} (-x)^k (x+y)^{m-k}$

14.1.8 冪次, 冪次和

- 2. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$

- 6. $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^n C_k^{n+1} B_k m^{n+1-k}$
- 7. $\sum_{j=0}^{m} C_j^{m+1} B_j = 0, B_0 = 1$
- 8. 除了 $B_1 = -1/2$ · 剩下的奇數項都是 0
- 9. $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 = 22$ Cógedlo, cógedlo, cógedlo... $-1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 23$ Lord Saddler... $7/6, B_{16} = -3617/510, B_{18}$ $43867/798, B_{20} = -174611/330,$

14.1.9 Burnside's lemma

- 1. $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- 3. G 表示有幾種轉法, X^g 表示在那種轉法下, 有幾種 是會保持對稱的 $\cdot t$ 是顏色數 $\cdot c(g)$ 是循環節不動的 $_{32}$ | | No dejes que se escape!
- 4. 正立方體塗三顏色,轉0有36個元素不變 轉 90 有 6 種, 每 種 有 33 不 變, 180 有 3 × $\frac{1}{24}$ $(3^6+6\times3^3+3\times3^4+8\times3^2+6\times3^3)=57$

14.1.10 Count on a tree

- 1. Rooted tree: $s_{n+1} = \frac{1}{n} \sum_{i=1}^{n} (i \times a_i \times \sum_{j=1}^{\lfloor n/i \rfloor} a_{n+1-i \times j})$
- 2. Unrooted tree:

 - (a) $\operatorname{Odd}: a_n \sum_{i=1}^{n/2} a_i a_{n-i}$ (b) $\operatorname{Even}: Odd + \frac{1}{2} a_{n/2} (a_{n/2} + 1)$
- Spanning Tree
 - (a) 完全圖 $n^n 2$
 - (b) 般 圖(Kirchhoff's theorem) $M[i][i]=\frac{1}{51}$ i Te voy a romper en pedazos! $degree(V_i), M[i][j] = -1, \text{if have } E(i, j), 0$ 52 | ¡La campana! if no edge. delete any one row and col in A, $_{53}$ | Ya es hora de rezar. ans = det(A)

Интернационал

15.1 ganadoQuote

```
ı ¡Allí está!
                                                                                         2 ¡Un forastero!
                                                                                   = 3 | ¡Agarrenlo!
                                                                                         4 ¡Os voy a romper a pedazos!
                                                                                             ¡Cógelo!
                                                                                          6 ¡Te voy a hacer picadillo!
                                                                                         7 | ¡Te voy a matar!
                                                                                         8 ¡Míralo, está herido!
                                                                                         9 ¡Sos cerdo!
                                                                                         10 ¿Dónde estás?
2. 1 + 2 + 3 + ... + n^2 = \frac{n}{4} + \frac{n}{2} + \frac{n}{4} II | Detrás de tí, 3. 1^4 + 2^4 + 3^4 + ... + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30} | 12 | No dejes que so de ti, 15 + 2^5 + 3^5 + ... + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12} | 14 | Lord Saddler... 5. 0^k + 1^k + 2^k + ... + n^k = P(k), P(k) = \frac{(n+1)^{k+1} - \sum_{i=0}^{k-1} C_i^{k+1} P(i)}{\sum_{i=0}^{k-1} C_i^{k-1}} P(0) = n+1 | 16 | Mátalo! | Allí está!
                                                                                             ¡Detrás de tí, imbécil!
                                                                                             ¡No dejes que se escape!
                                                                                              ¡Basta, hijo de puta!
                                                                                        18 Morir es vivir.
                                                                                        19 Sííííí, ¡Quiero matar!
                                                                                        20 Muere, muere, muere....
                                                                                       21 Cerebros, cerebros, cerebros...
                                                                                   = 24 Dieciséis.
```

```
26 ¡Va por él!
                                            27 | ¡Muérete!
                                            28 ¡Cógelo!
                                            29 ¡Te voy a matar!
                                            30 ¡Bloqueale el paso!
                                           31 ¡Te cogí!
                                               ¿Qué carajo estás haciendo aquí? ¡Lárgate,
                                                    cabrón!
3^4 \cdot 120(角) 有 8 \times 3^2 \cdot 180(邊) 有 6 \times 3^3 \cdot 全部 35 Hay un rumor de que hay un extranjero entre
                                            36 Nuestro jefe se encargará de la rata.
                                            37 Su "Las Plagas" es mucho mejor que la
                                            38 Tienes razón, es un hombre.
                                            39 Usa los músculos.
                                            40 Se vuelve loco!
                                            41 ¡Hey, acá!
                                            42 ¡Por aquí!
                                            43 ¡El Gigante!
                                            44 ¡Del Lago!
                                            45 ¡Cógelo!
                                            46 ¡Cógenlo!
                                            47 ¡Allí!
                                            48 ¡Rápido!
                                            49 ¡Empieza a rezar!
                                            50 ¡Mátenlos!
                                            54 Tenemos que irnos.
                                               ¡Maldita sea, mierda!
                                               ¡Ya es hora de aplastar!
                                               ¡Puedes correr, pero no te puedes esconder!
                                               ¡Sos cerdo!
                                              ¡Está en la trampa!
                                            61 ¡Ah, que madre!
                                            62 ¡Vámonos!
                                            63 ¡Ándale!
                                              ¡Cabrón!
                                            65 | ¡Coño!
                                            66 ¡Agárrenlo!
                                            67 Cógerlo, Cógerlo...
                                            68 ¡Allí está, mátalo!
                                            69 ¡No dejas que se escape de la isla vivo!
                                            70 ¡Hasta luego!
```

71 ¡Rápido, es un intruso!

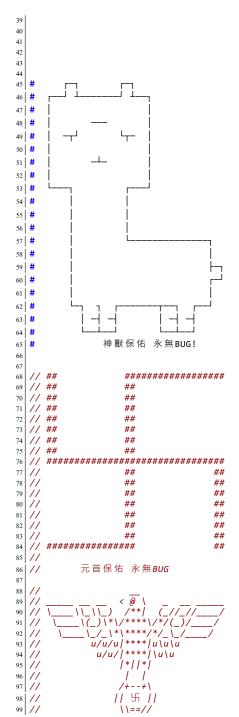
15.2 Интернационал

/*************** L'Internationale, Sera le genre humain. Вставай, проклятьем заклеймённый, Весь мир голодных и рабов! Кипит наш разум возмущённый 20 И в смертный бой вести готов. Весь мир насилья мы разрушим До основанья, а затем 23 Мы наш, мы новый мир построим, -Кто был ничем, тот станет всем. Chorus Это есть наш последний 28 И решительный бой; 29 С Интернационалом Воспрянет род людской! Никто не даст нам избавленья: Ни бог, ни царь и не герой! Добьёмся мы освобожденья Своею собственной рукой. Чтоб свергнуть гнёт рукой умелой, Отвоевать своё добро, -Вздувайте горн и куйте смело, Пока железо горячо! Chorus 43 Довольно кровь сосать, вампиры, 44 Тюрьмой, налогом, нищетой! 45 У вас — вся власть, все блага мира, 46 А наше право — звук пустой ! 47 Мы жизнь построим по-иному -48 И вот наш лозунг боевой: 49 Вся власть народу трудовому! 50 А дармоедов всех долой! Chorus 54 Презренны вы в своём богатстве, 55 Угля и стали короли! 56 Вы ваши троны, тунеядцы, 57 На наших спинах возвели. 58 Заводы, фабрики, палаты — 59 Всё нашим создано трудом. 60 Пора! Мы требуем возврата 61 Того, что взято грабежом. 63 Chorus

Довольно королям в угоду Дурманить нас в чаду войны! Война тиранам! Мир Народу! Бастуйте, армии сыны! Когда ж тираны нас заставят В бою геройски пасть за них -Убийцы, в вас тогда направим Мы жерла пушек боевых! Лишь мы, работники всемирной Великой армии труда, Владеть землёй имеем право, Но паразиты — никогда! И если гром великий грянет Над сворой псов и палачей, -Для нас всё так же солнце станет Сиять огнём своих лучей. Chorus

15.3 保佑





100 | // 101 | // 神獸保佑 永無*BUG*

ACM ICPC		3.4 MinCostMaxFlow	6		7.8 Lucas		11.3 IncStack	
Team Reference - Angry Crow	-	4.1 Augmenting Path	7 7 7 7		7.10 MillerRobin	13 13 13	12 other 12.1 WhatDay	17 17 17
Takes Flight!		4.4 BronKerbosch4.5 graphISO4.6 KM4.7 MaximumClique	7 7 8 8	8	7.14 數位統計	13	12.3 最大矩形	18
Contents		4.7 MaximumClique	8 8 8 9	ð	8.1 AC 自動機		13.1.1 文件操作	18 18 18
1 Computational Geometry 1.1 delaunay	1 1 1 3	4.12 全局最小割	9 9 9 9		8.5 minimal string rotation 8.6 reverseBWT 8.7 suffix array lcp	15	13.2 python heap	18 18
1.4 最近點對	3 3 5	5 Language	10	9	Tarjan 9.1 dominator tree		14.1.1 Pick 公式	18 18
2.1 CDQ DP	3 4	5.1 CNF	10 10		9.2 tnfshb017 2 sat	15 15	14.1.3 dinic 特殊圖複雜度 14.1.4 0-1 分數規劃 14.1.5 學長公式	19
•	4 (5	6. Linear Programming 6.1 simplex	10		9.4 雙連通分量 & 割點	16	14.1.5 學長公式	
2.4 kd tree replace segment tree	4 6 5 5 5 5 7	6.1 simplex	10 10 10 11	10	Tree Problem 10.1 HeavyLight	16 16 16 16		19 19 19 19
2.4 kd tree replace segment tree	4 6 5 7 5 7 5 6 6 6 6 6	6.1 simplex	10 10 10 11 11 11 12		Tree Problem 10.1 HeavyLight	16 16 16 16 17	14.1.6 基本數論	19 19 19 19 19

ACM ICPC Judge Test Angry Crow Takes Flight!

C++ Resource Test

```
#include <bits/stdc++.h>
using namespace std;

namespace system_test {

const size_t KB = 1024;
const size_t MB = KB * 1024;
const size_t GB = MB * 1024;
```

```
chrono::duration<double> diff = end -
10 size t block size, bound;
                                                          begin;
  void stack size dfs(size t depth = 1) {
                                                     return diff.count():
   if (depth >= bound)
                                                   void runtime error 1() {
    int8_t ptr[block_size]; // 若無法編譯將
                                                     // Segmentation fault
         block size 改成常數
                                                     int *ptr = nullptr;
    memset(ptr, 'a', block_size);
                                                     *(ptr + 7122) = 7122;
    cout << depth << endl;</pre>
                                                 42 }
    stack_size_dfs(depth + 1);
                                                   void runtime_error_2() {
                                                     // Segmentation fault
  void stack_size_and_runtime_error(size_t
                                                     int *ptr = (int *)memset;
       block size, size t bound = 1024) {
                                                     *ptr = 7122;
    system test::block size = block size;
                                                 48 }
    system_test::bound = bound;
    stack size dfs();
                                                   void runtime_error_3() {
                                                     // munmap_chunk(): invalid pointer
                                                     int *ptr = (int *)memset;
  double speed(int iter num) {
                                                     delete ptr;
    const int block_size = 1024;
    volatile int A[block size];
    auto begin = chrono::high resolution clock
                                                   void runtime_error_4() {
         ::now();
                                                     // free(): invalid pointer
    while (iter_num--)
                                                     int *ptr = new int[7122];
      for (int j = 0; j < block_size; ++j)</pre>
                                                     ptr += 1;
                                                     delete[] ptr;
    auto end = chrono::high resolution clock::
```

```
63 void runtime error 5() {
    // maybe illegal instruction
    int a = 7122, b = 0;
    cout << (a / b) << endl;</pre>
  void runtime error 6() {
    // floating point exception
    volatile int a = 7122, b = 0;
    cout << (a / b) << endl;
73 }
  void runtime error 7() {
    // call to abort.
    assert(false);
78 }
  } // namespace system test
82 #include <sys/resource.h>
void print_stack_limit() { // only work in
       Linux
    struct rlimit 1;
    getrlimit(RLIMIT STACK, &1);
    cout << "stack_size = " << l.rlim_cur << "</pre>
          byte" << endl;</pre>
87 }
```