Deep Learning

Episode 4

Recurrent Neural Networks

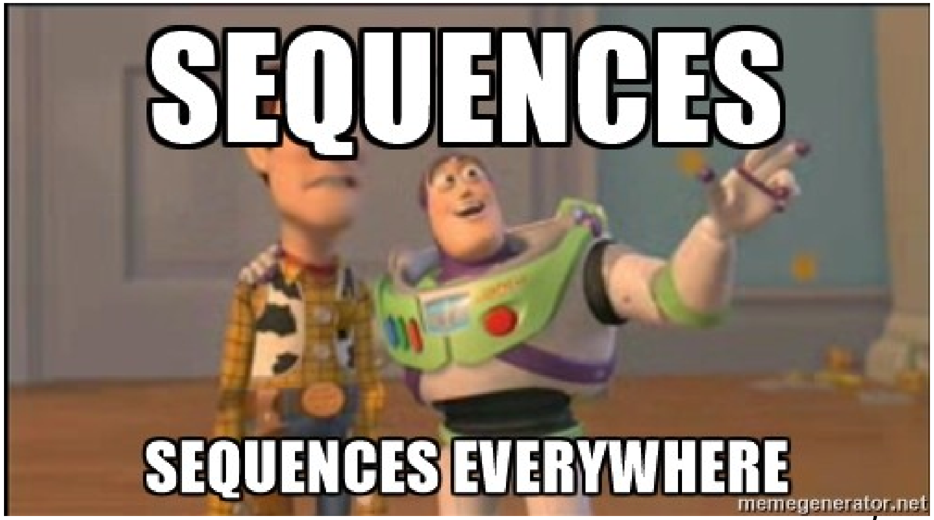
Yandex
Data Factory

LAMBDA

British Hedgehog
Preservation Society

Sequential data

- Time series
  - Financial data analysis
  - Demand prediction
  - Predict vehicle breakdown using sensor data
  - Medical sensors e.g. sugar level
- Sound
  - Speech recognition
  - Text to speech
  - Music generation
  - Music recommendation
  - …
- Text
  - Generating tweets, poetry
  - Sentiment analysis
  - See last lecture :)
- Spatio-temporal
  - Video
  - Precipitation maps
  - Ultrasonography



Could go on all day

Time series @finance

Data:
- Stock indices
- Commodities
- Forex

Objectives:
- Portfolio management
- Volatility targeting
- Estimating true value
- ...

Time series @finance

Data:
- Stock indices
- Commodities
- Forex

Objectives:
- ~~Portfolio management~~ ~ trading stuff
- ~~Volatility targeting~~ ~ evaluating risk
- Estimating true value
- ...

Natural language as time series

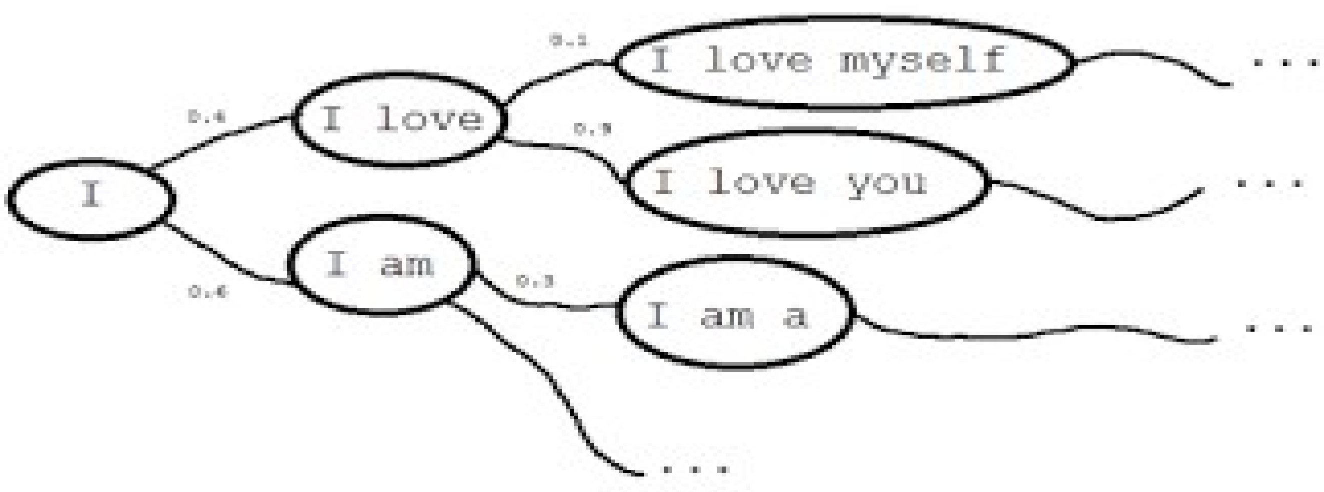
Data:
- Literature
- Conversation
- Tweets
- Book scans
- Speech

Language model

Objective:

- Learn P(text)

$$P(text)=P(w_{0,}w_{1,}\ldots,w_n)=P(w_0)\cdot P(w_1|w_0)\cdot P(w_2|w_1w_0)\cdot\ldots\cdot P(w_n|\ldots)$$



Language model

Why learning it?

- Detect languages as P(text|language)
- Sentiment analysis P(text|happy)
- Any text analysis you can imagine
- Generate texts!
  - Cool article http://bit.ly/1K610Ie
  - Generating clickbait: http://bit.ly/21cZM70

Language model

- Actual distribution

$$P(text) = P(w_{0,} w_{1,} \ldots, w_n) = P(w_0) \cdot P(w_1 | w_0) \cdot P(w_2 | w_1 w_0) \cdot \ldots \cdot P(w_n | \ldots)$$

- Bag of words assumption (independent words)

$$P(text) = P(w_{0,} w_{1,} \ldots, w_n) = P(w_0) \cdot P(w_1) \cdot P(w_2) \cdot \ldots \cdot P(w_n)$$

- **Anything better?**

Language model

- Actual distribution

$$P(text)=P(w_{0,}w_{1,}\dots,w_n)=P(w_0)\cdot P(w_1|w_0)\cdot P(w_2|w_1 w_0)\cdot\dots\cdot P(w_n|\dots)$$

- Bag of words assumption (independent words)

$$P(text)=P(w_{0,}w_{1,}\dots,w_n)=P(w_0)\cdot P(w_1)\cdot P(w_2)\cdot\dots\cdot P(w_n)$$

- Markov assumption

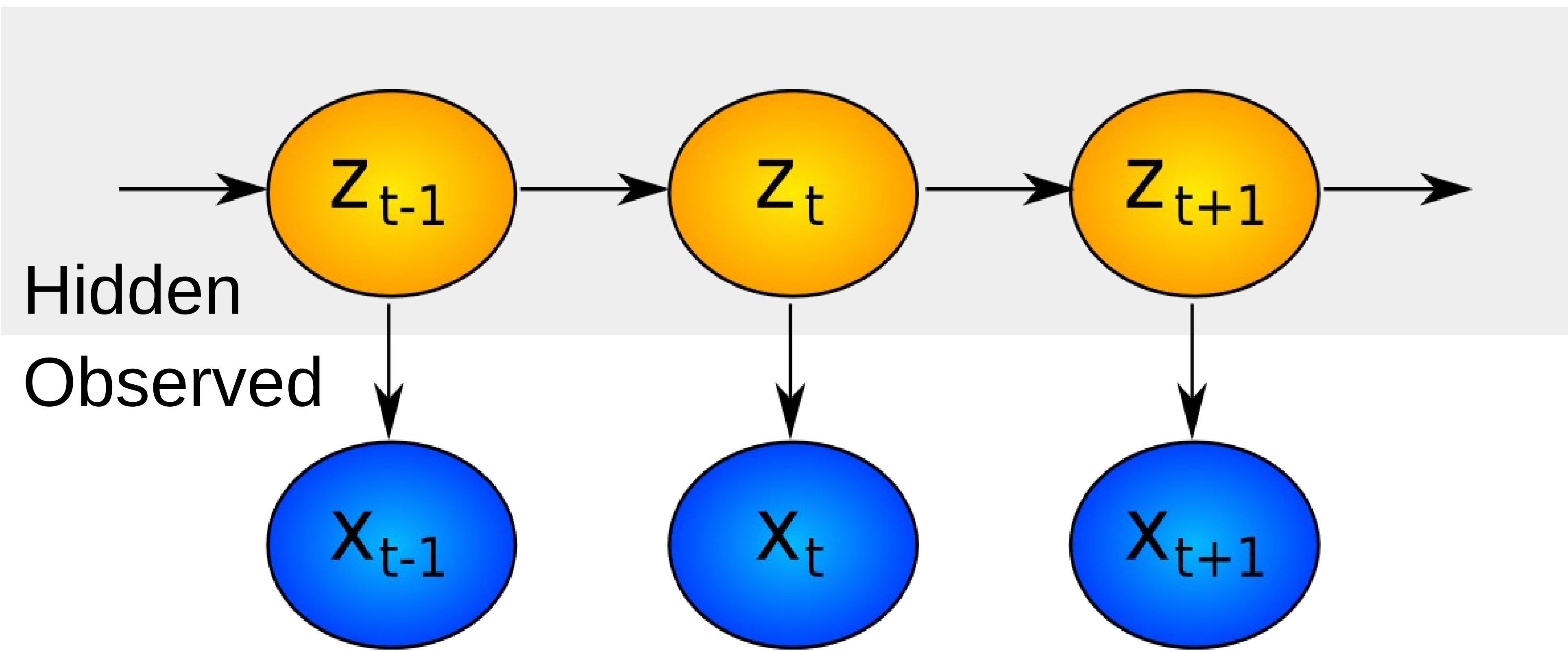
$$P(text)=P(w_{0,}w_{1,}\dots,w_n)=P(w_0)\cdot P(w_1|w\,0)\cdot P(w_2|w_1)\cdot\dots\cdot P(w_n|w_{n-1})$$

  - also 3-gram, 5-gram, 100-gram

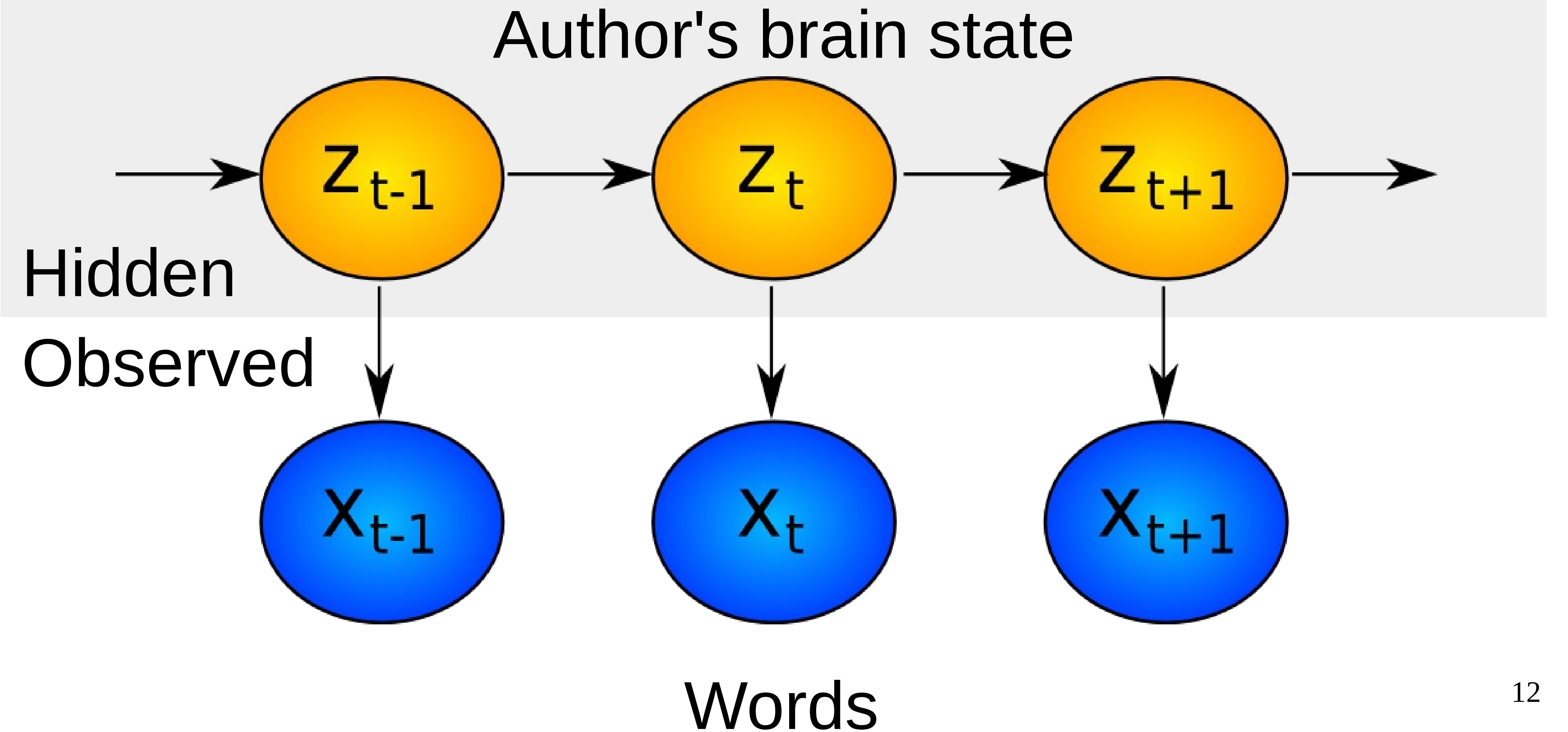
Can we learn* arbitrarily long dependencies?

* without infinitely many parameters

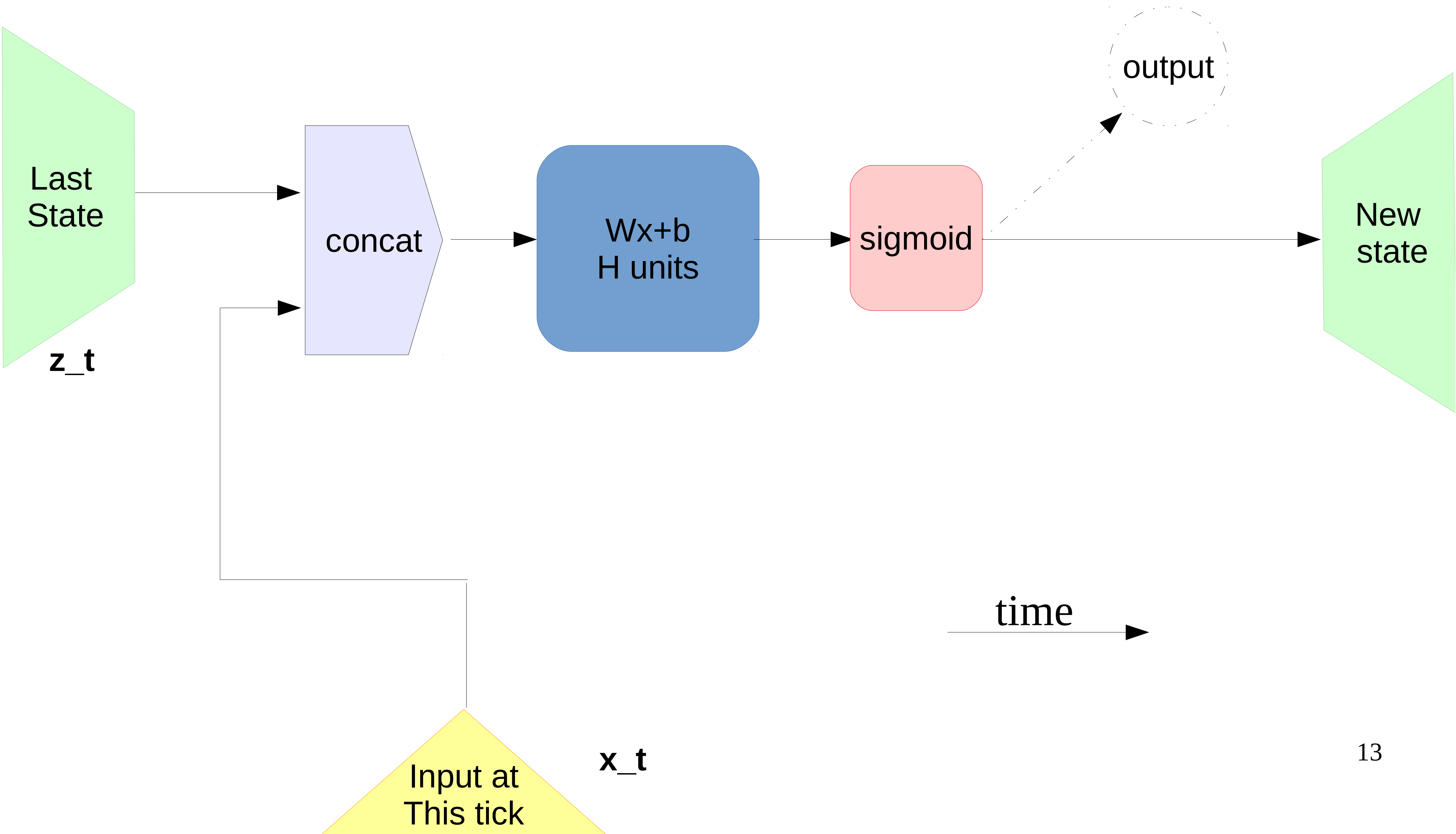
Hidden Markov Models: what's hidden



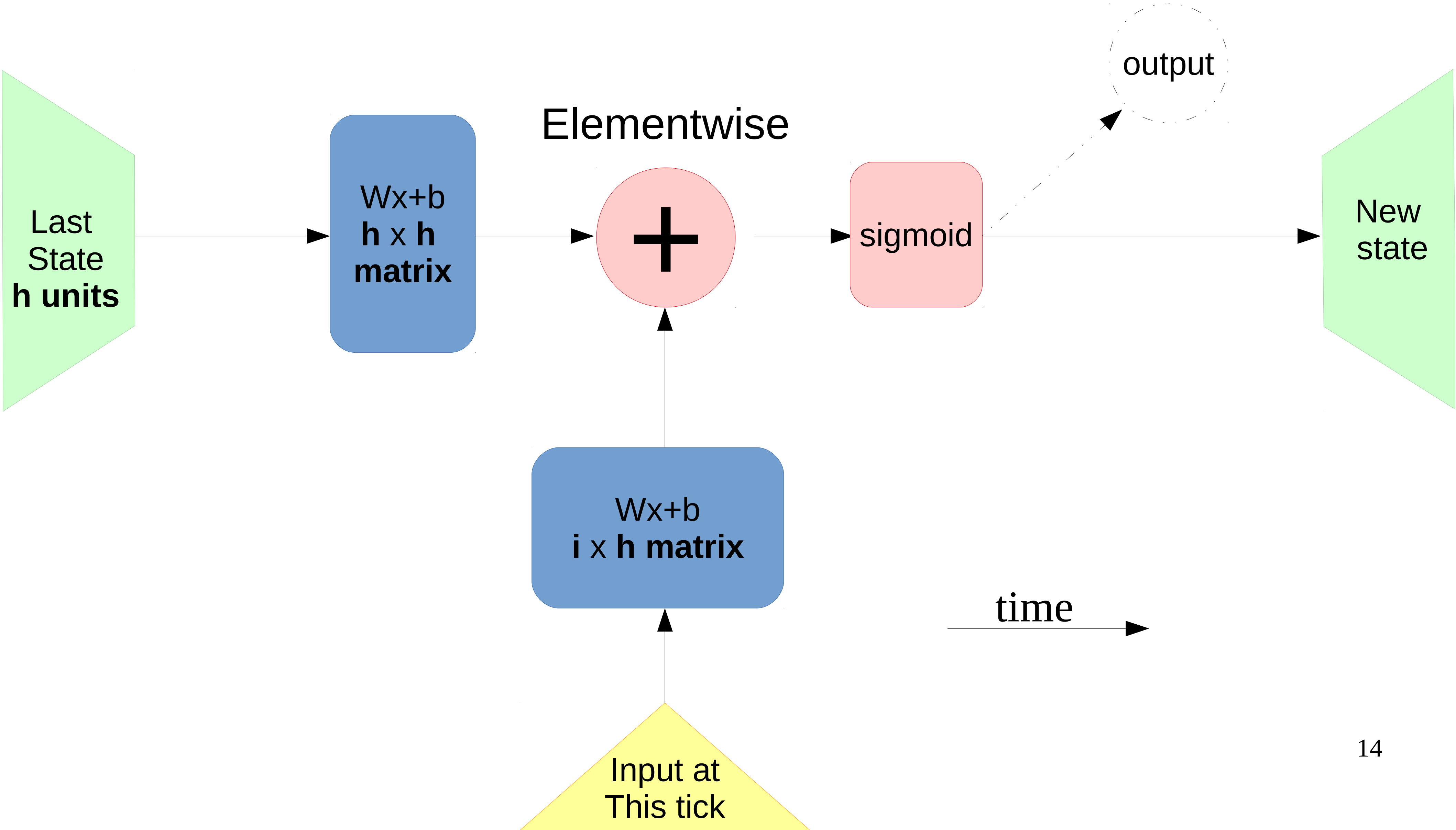
Hidden Markov Models: what is hidden



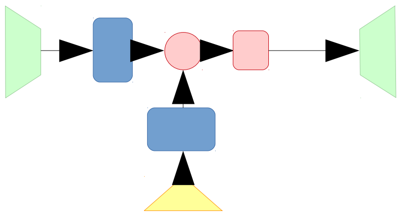
Recurrent neural network: one step



Recurrent neural network: one step

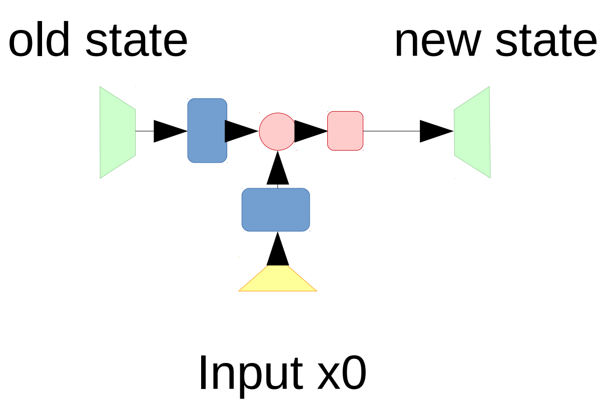


Recurrent neural network

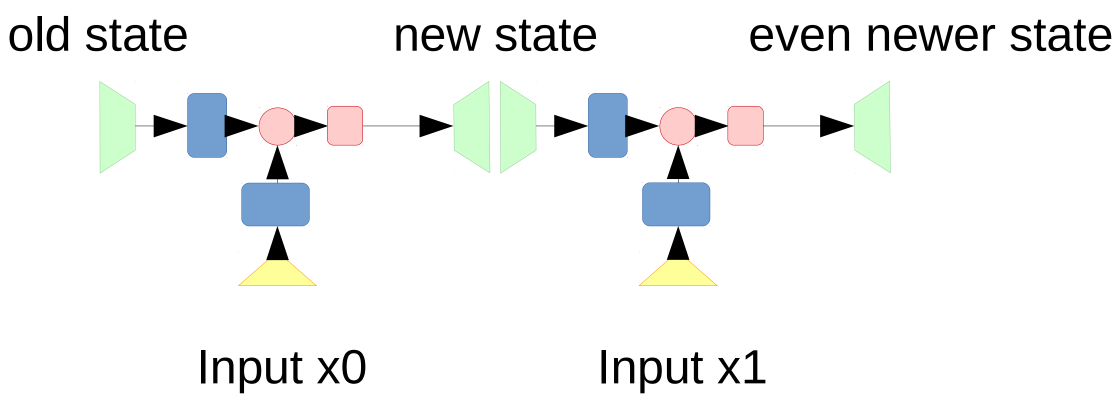


Zoom-out
of previous slide

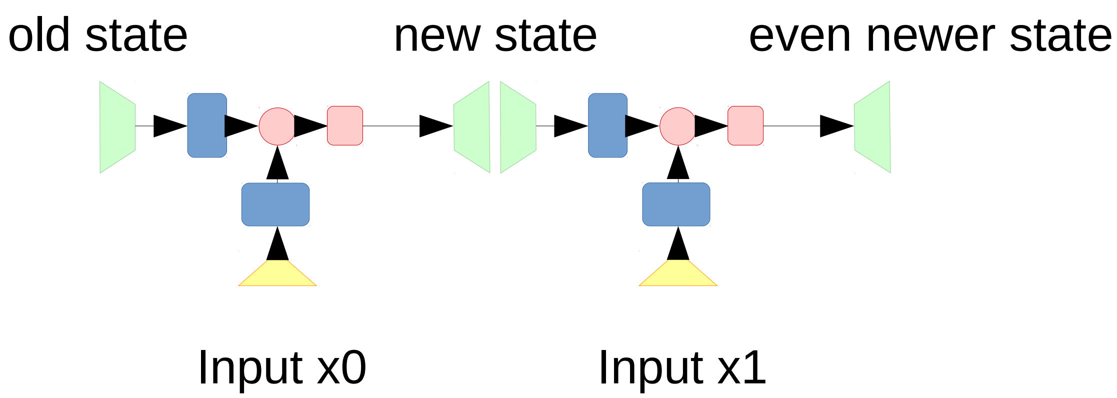
Recurrent neural network



Recurrent neural network

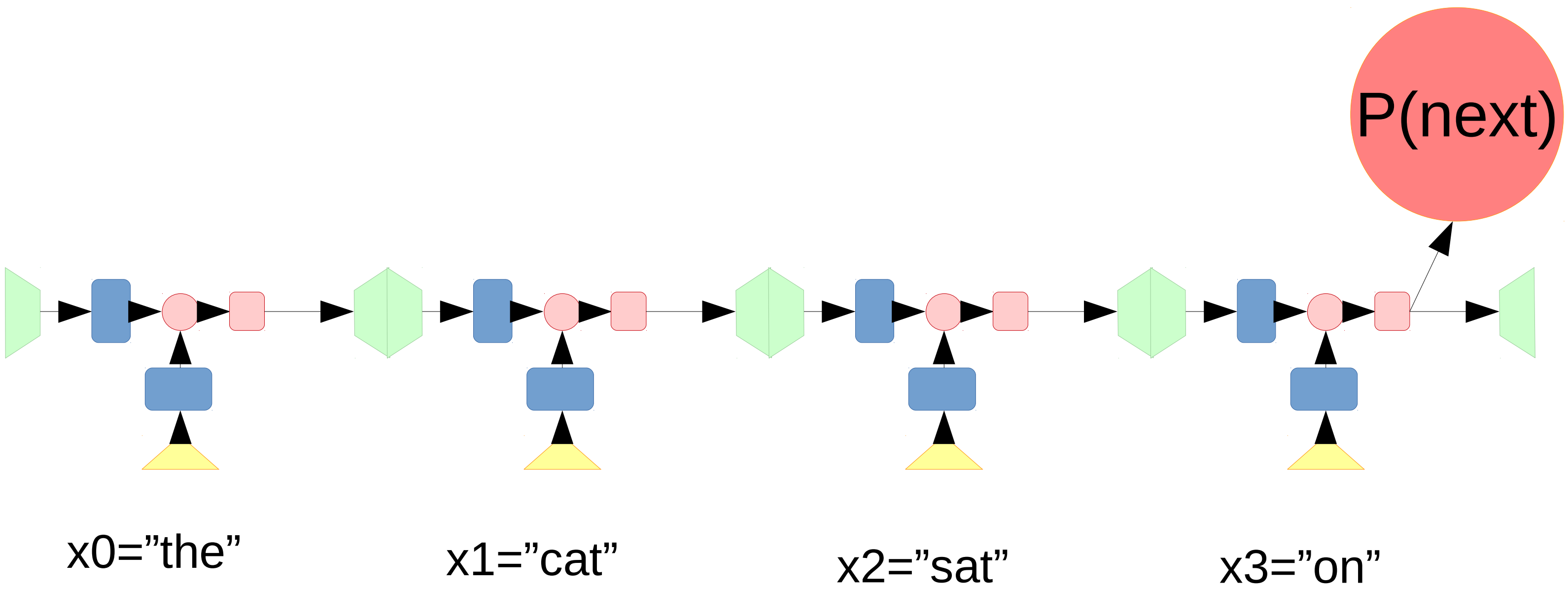


Recurrent neural network

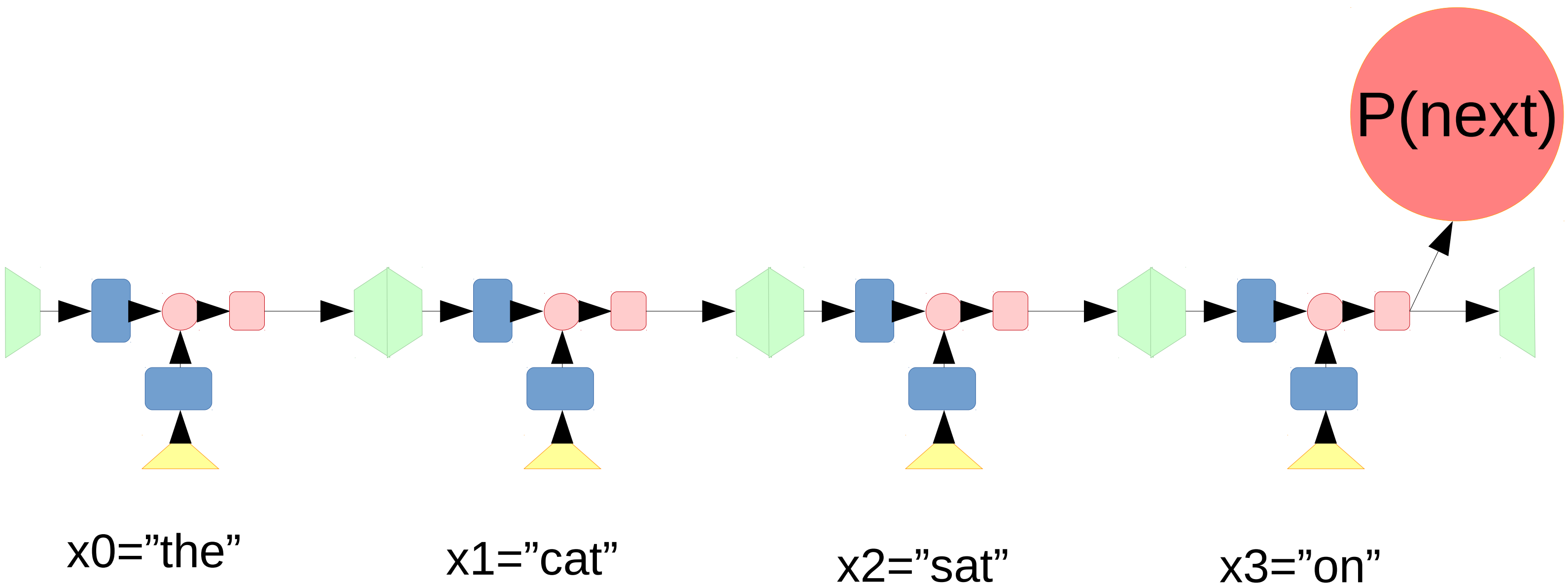


We use **same weight matrices** for all steps

Recurrent neural network

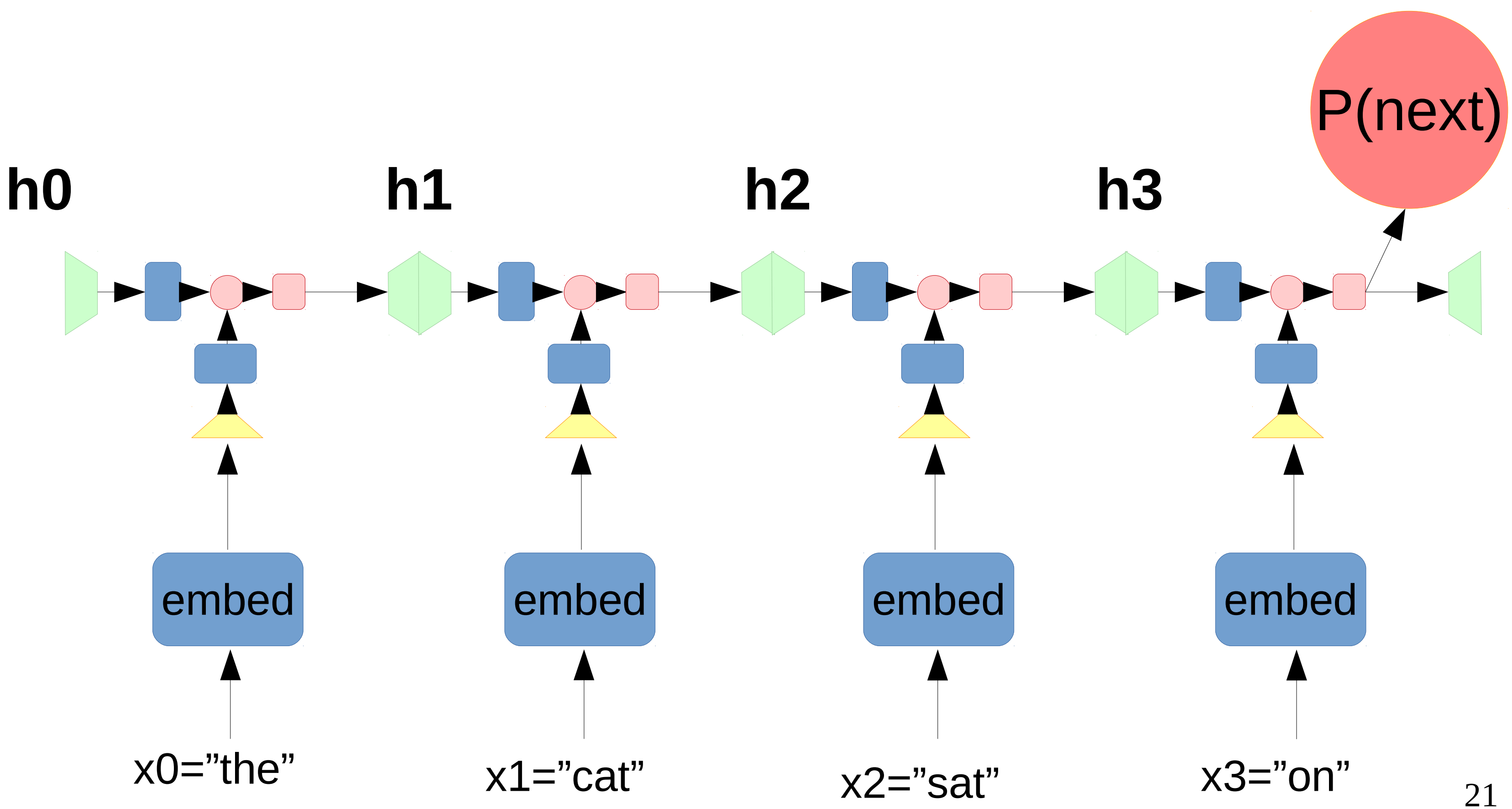


Recurrent neural network



How can we represent words?

Recurrent neural network

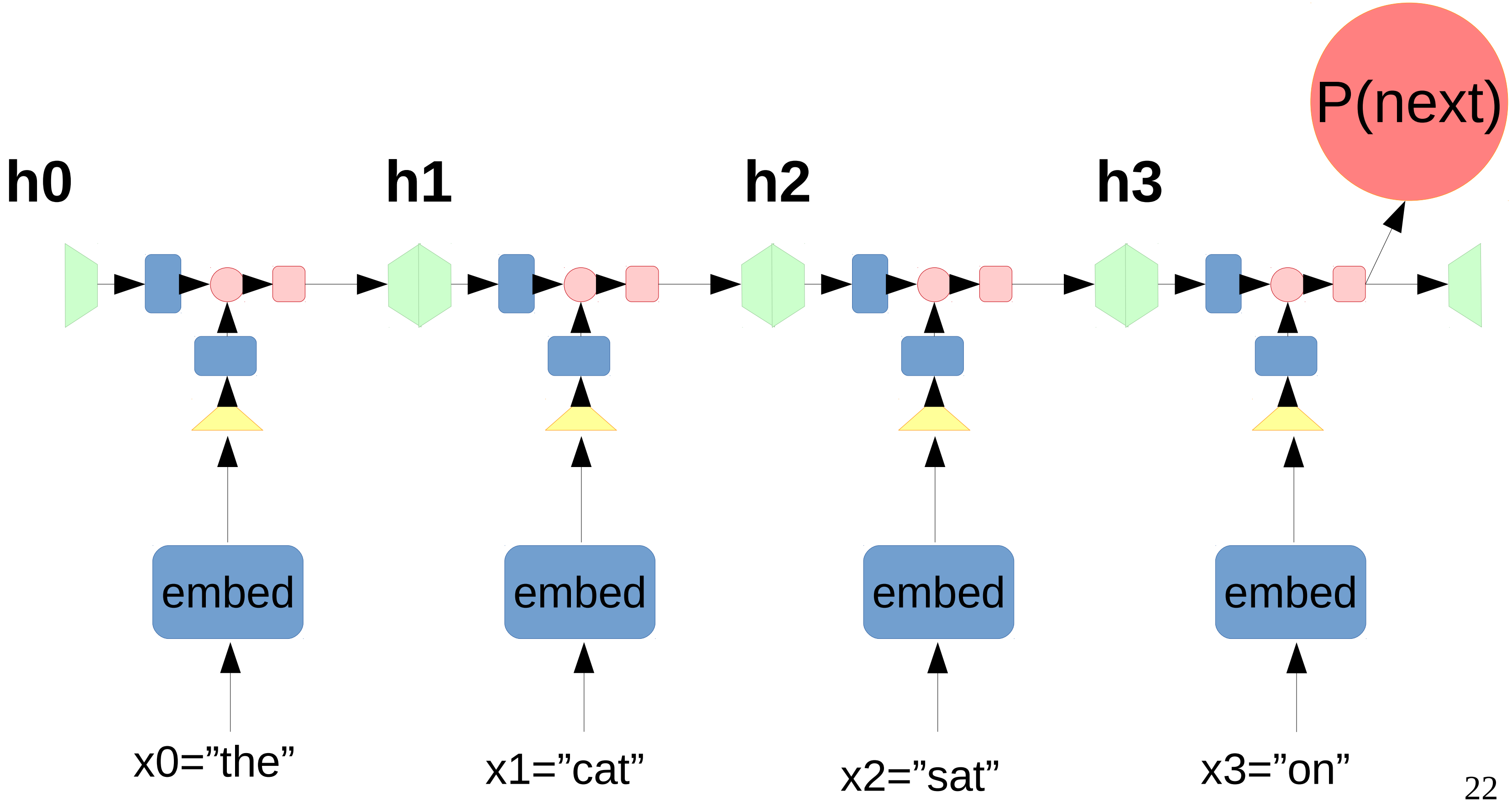


Recurrent neural network

$h_0 = \bar{0}$

$h_1 = \sigma(W_{hid} \cdot h_0 + W_{inp} \cdot x_0 + b)$

$h_2 = ?$



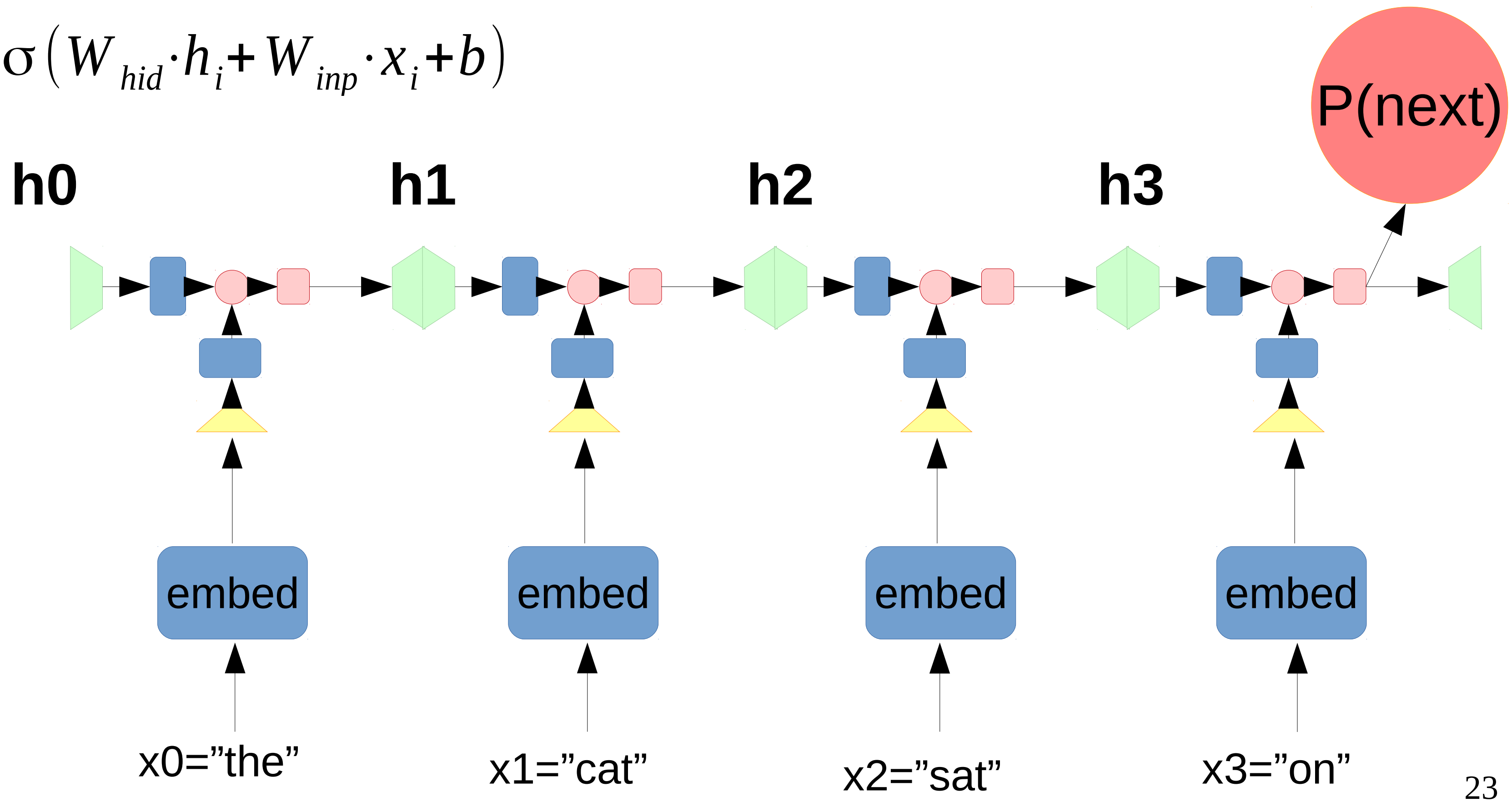
Recurrent neural network

$$h_0 = \bar{0}$$

$$h_1 = \sigma\left(W_{hid} \cdot h_0 + W_{inp} \cdot x_0 + b\right)$$

$$h_2 = \sigma\left(W_{hid} \cdot h_1 + W_{inp} \cdot x_1 + b\right) = \sigma\left(W_{hid} \cdot \sigma\left(W_{hid} \cdot h_0 + W_{inp} \cdot x_0 + b\right) + W_{inp} \cdot x_1 + b\right)$$

$$h_{i+1} = \sigma\left(W_{hid} \cdot h_i + W_{inp} \cdot x_i + b\right)$$



Recurrent neural network

$$h_0 = \bar{0}$$

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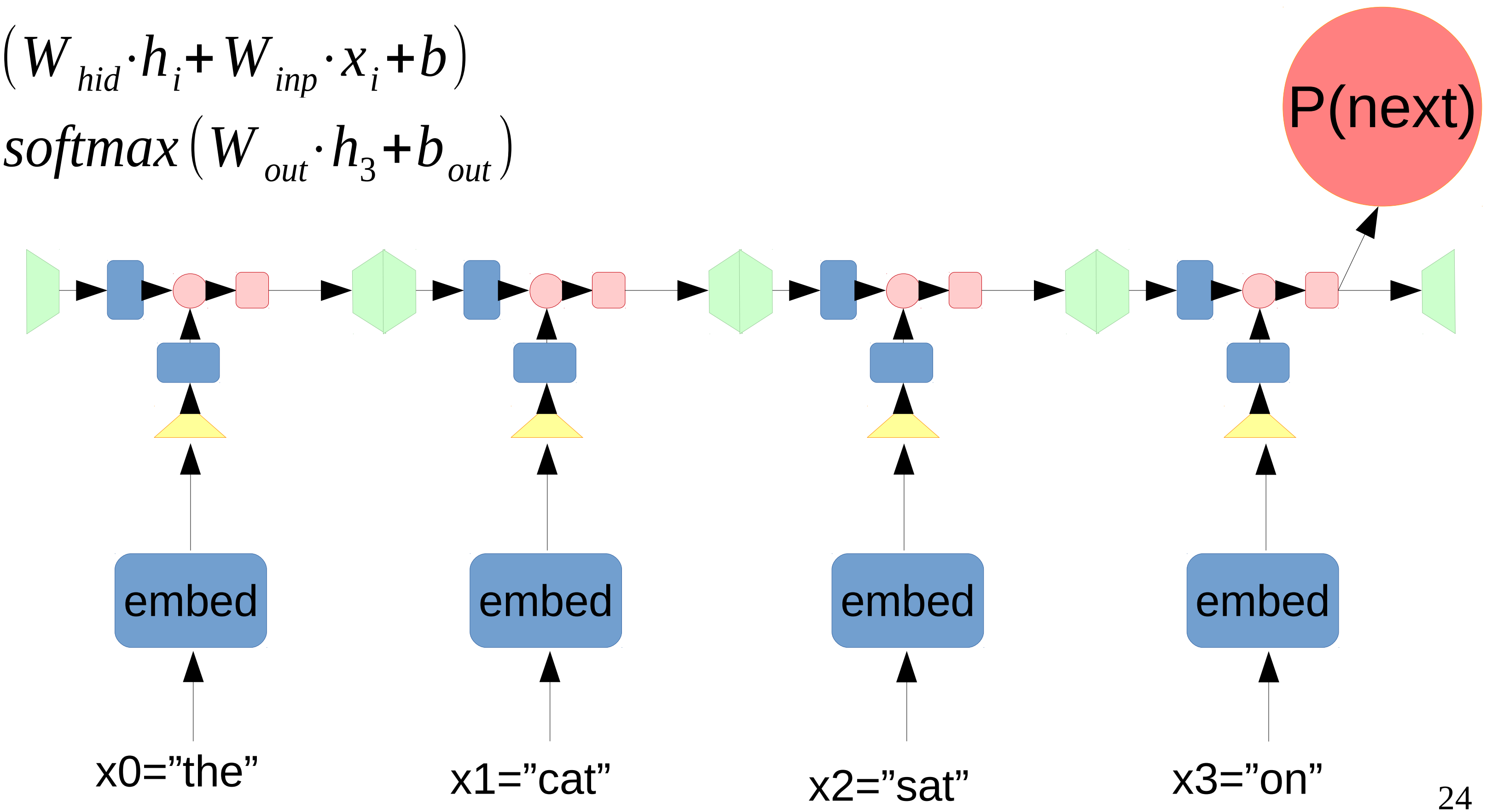
$$h_{i+1} = \sigma\left(W_{hid} \cdot h_i + W_{inp} \cdot x_i + b\right)$$

$$P\left(t_4\right) = softmax\left(W_{out} \cdot h_3 + b_{out}\right)$$

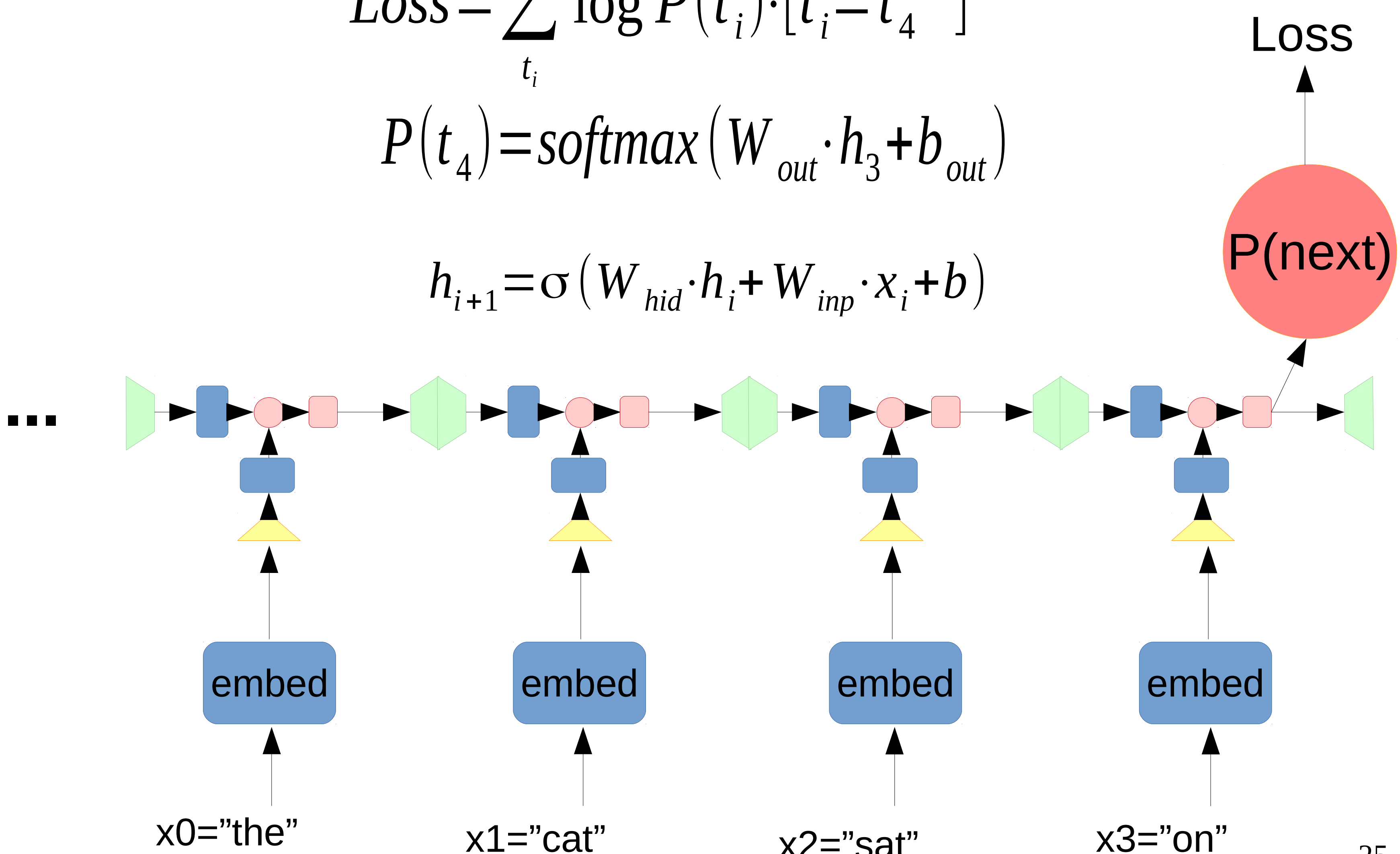


Recurrent neural network

$$Loss = \sum_{t_i} \log P(t_i) \cdot [t_i = t_4^{true}]$$

$$P(t_4) = softmax(W_{out} \cdot h_3 + b_{out})$$

$$h_{i+1} = \sigma(W_{hid} \cdot h_i + W_{inp} \cdot x_i + b)$$



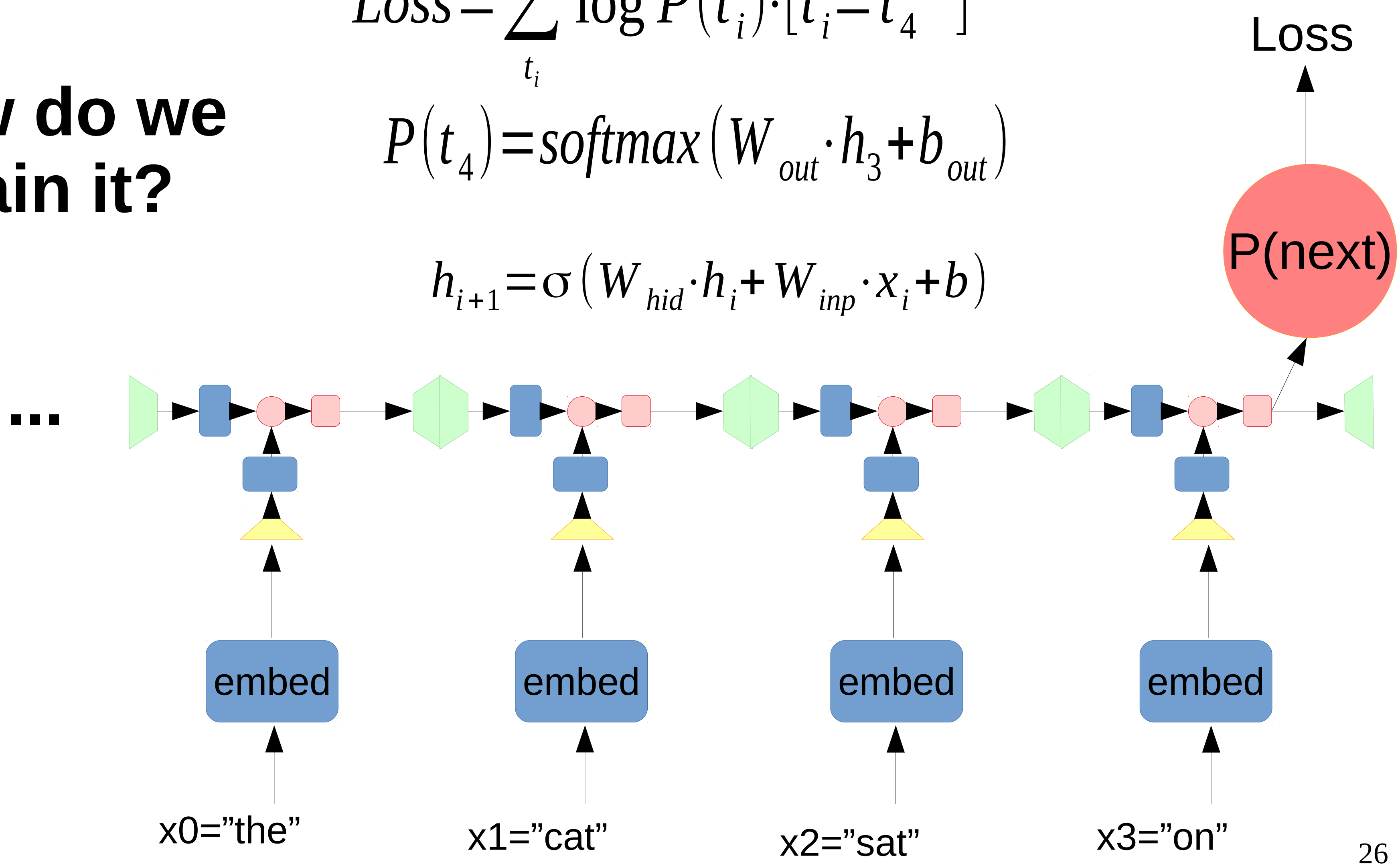
Recurrent neural network

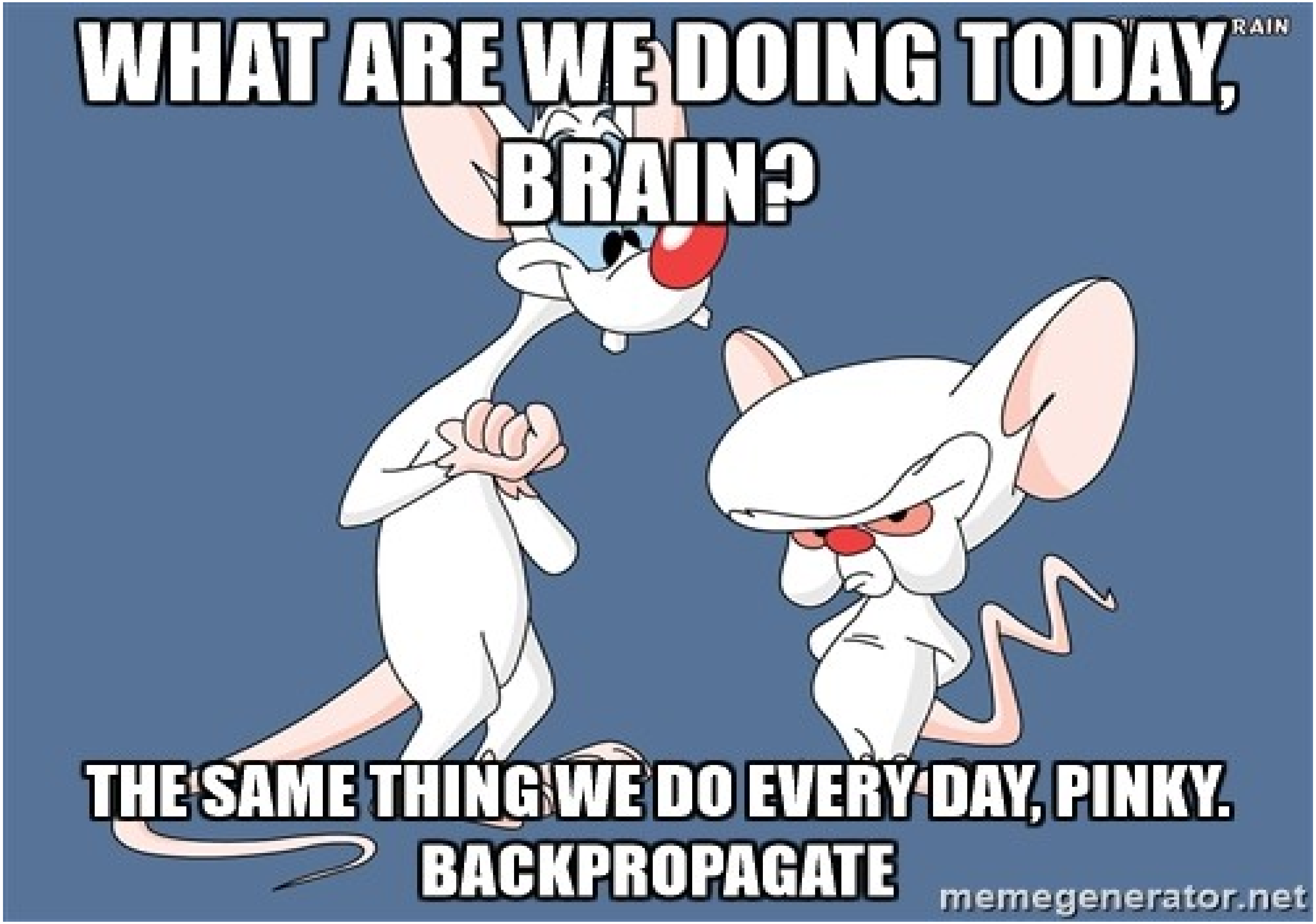
$$Loss = \sum_{t_i} \log P(t_i) \cdot [t_i = t_4^{true}]$$

How do we train it?

$$P(t_4) = softmax(W_{out} \cdot h_3 + b_{out})$$

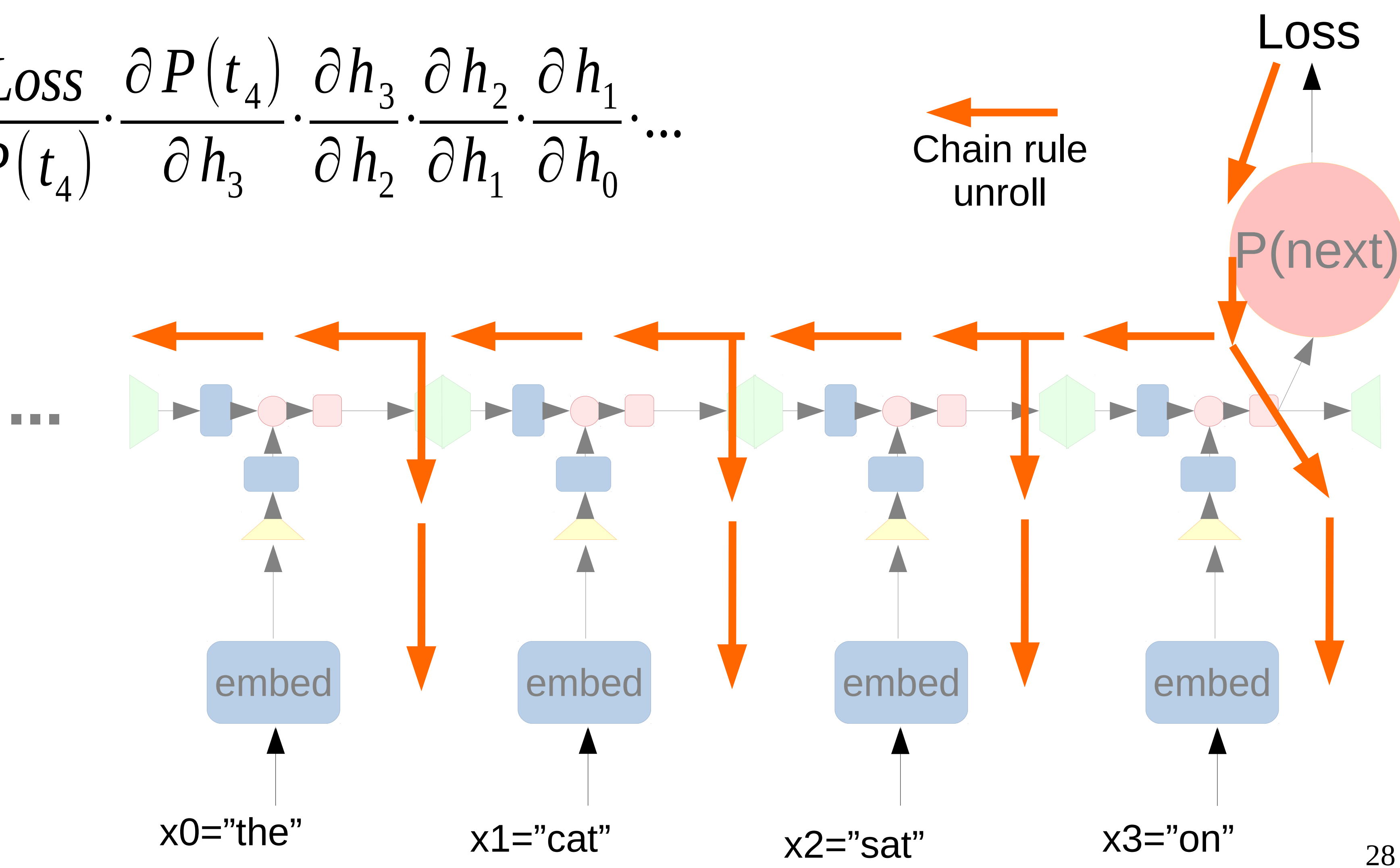
$$h_{i+1} = \sigma(W_{hid} \cdot h_i + W_{inp} \cdot x_i + b)$$




WHAT ARE WE DOING TODAY,
BRAIN?
THE SAME THING WE DO EVERY DAY, PINKY.
BACKPROPAGATE
memegenerator.net

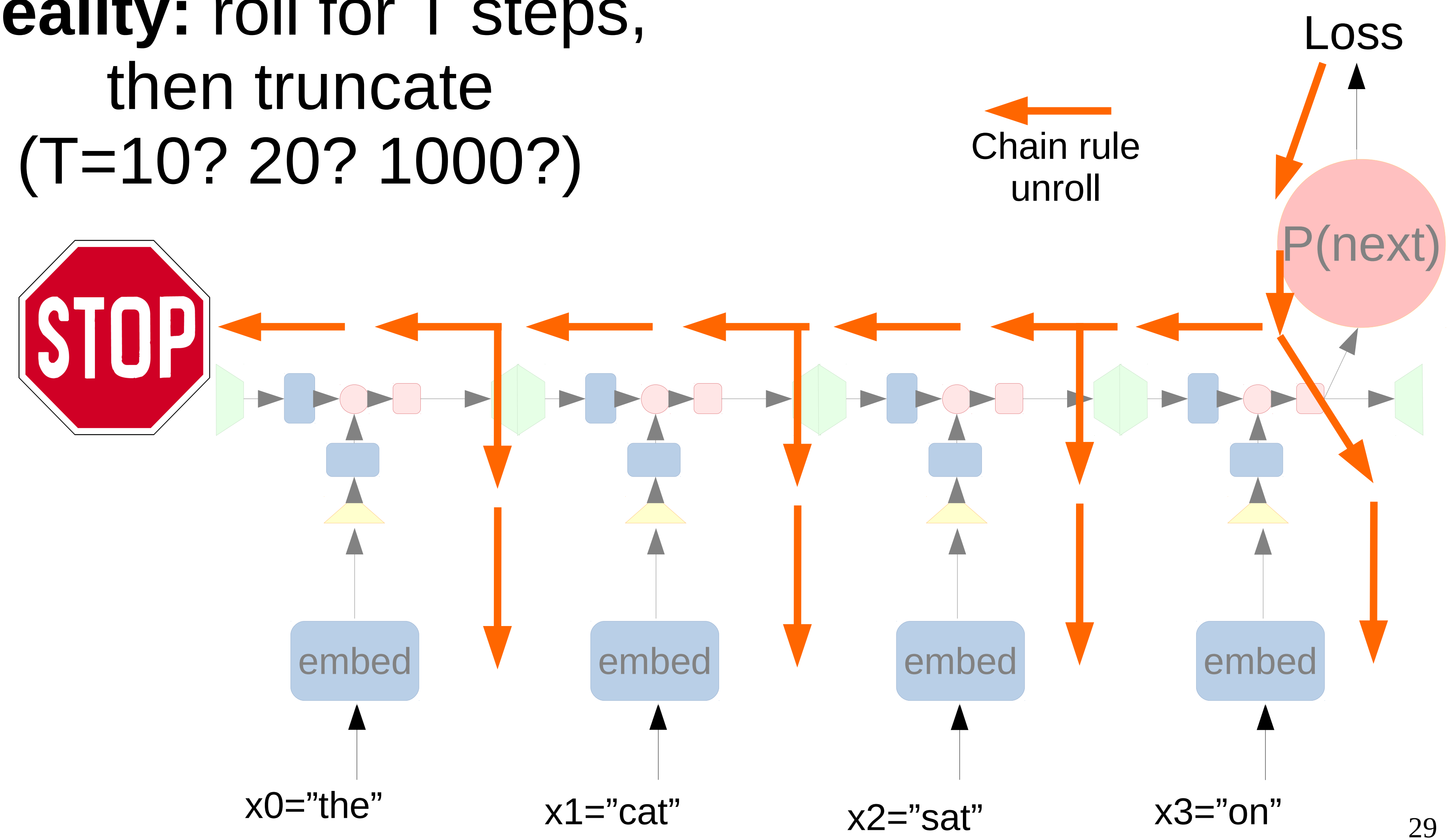
Backpropagation through time

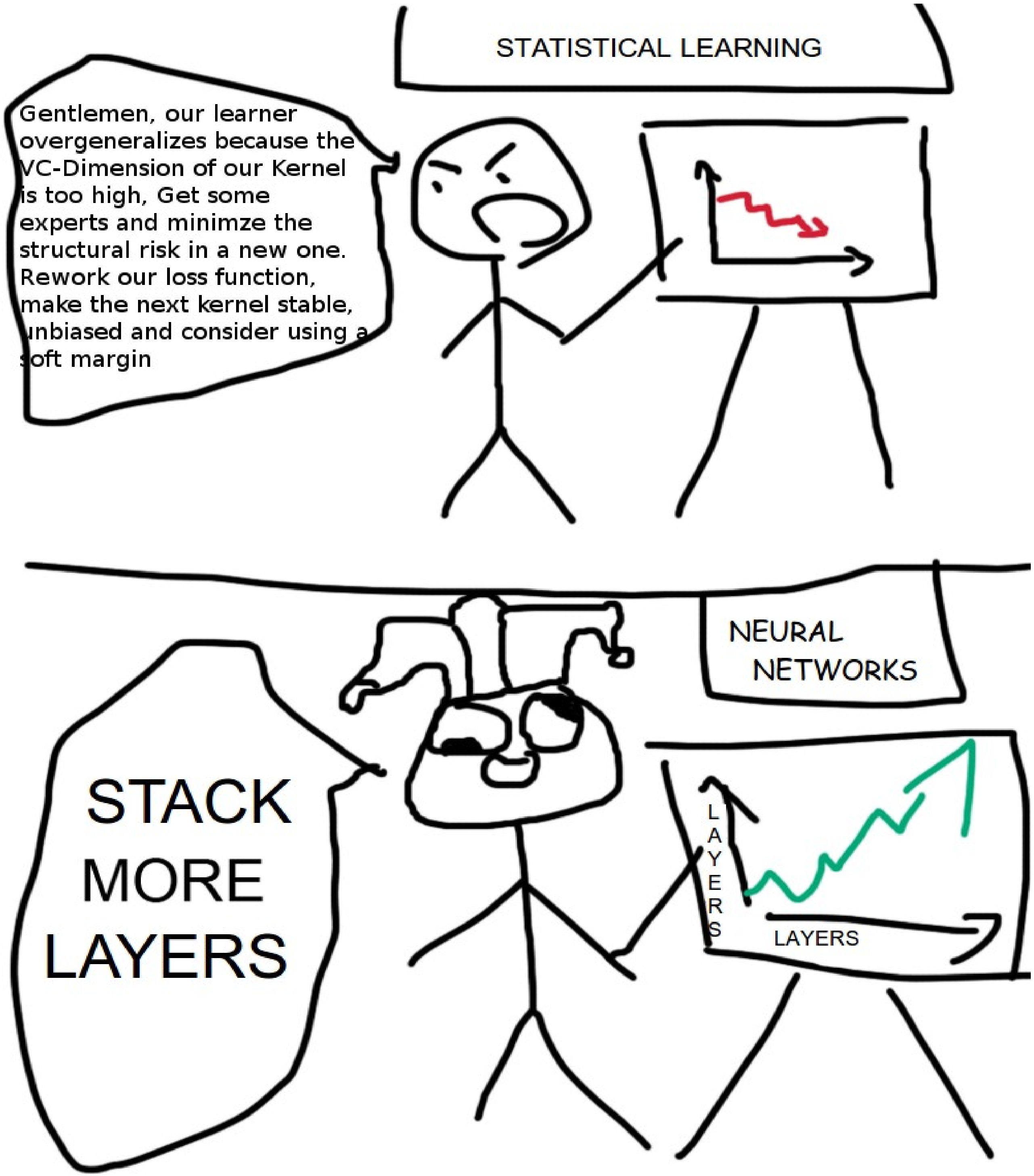
$$\frac{\partial Loss}{\partial P(t_4)} \cdot \frac{\partial P(t_4)}{\partial h_3} \cdot \frac{\partial h_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial h_1} \cdot \frac{\partial h_1}{\partial h_0} \cdot \ldots$$



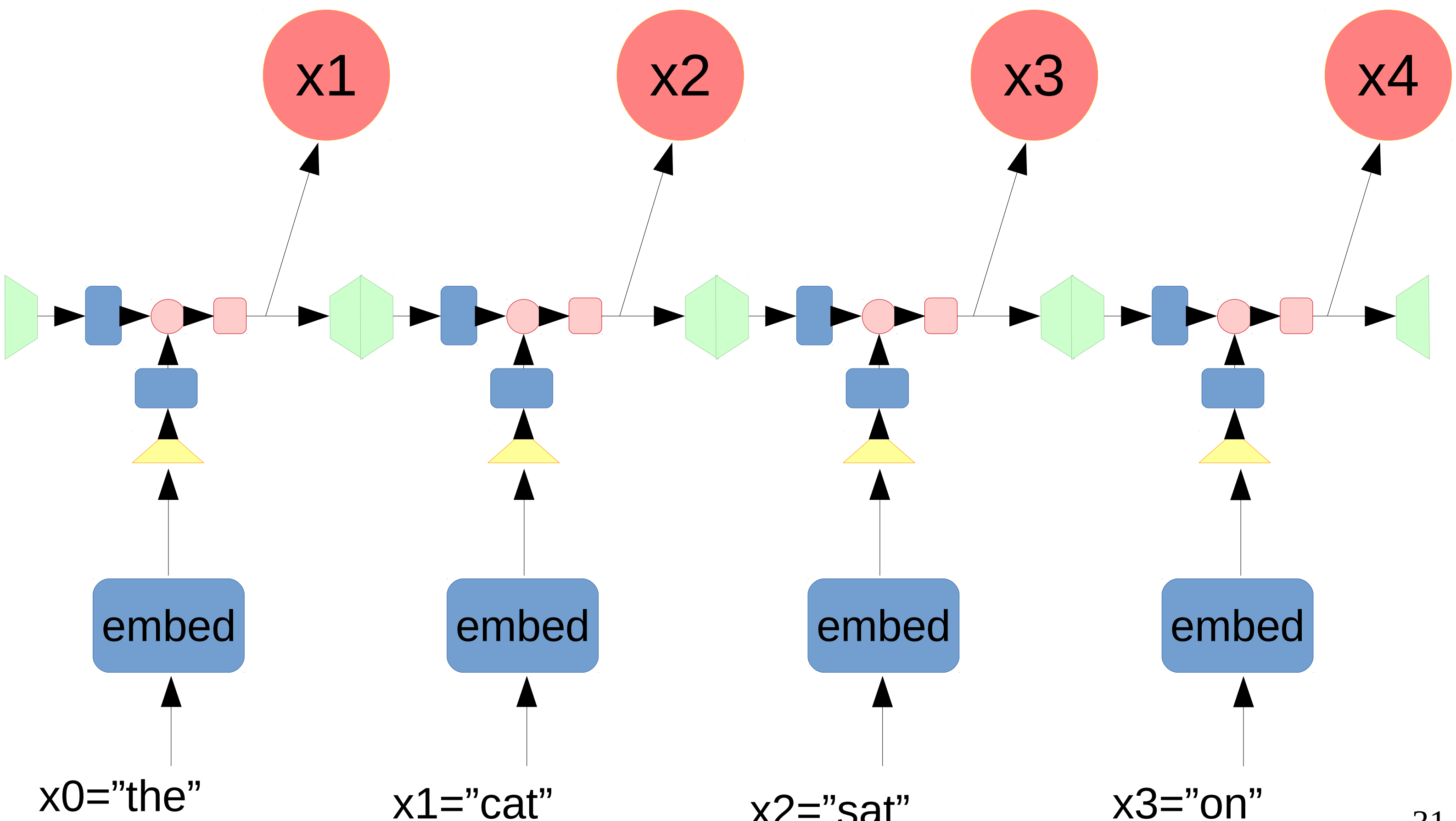
Truncated BPTT

Reality: roll for T steps,
then truncate
(T=10? 20? 1000?)



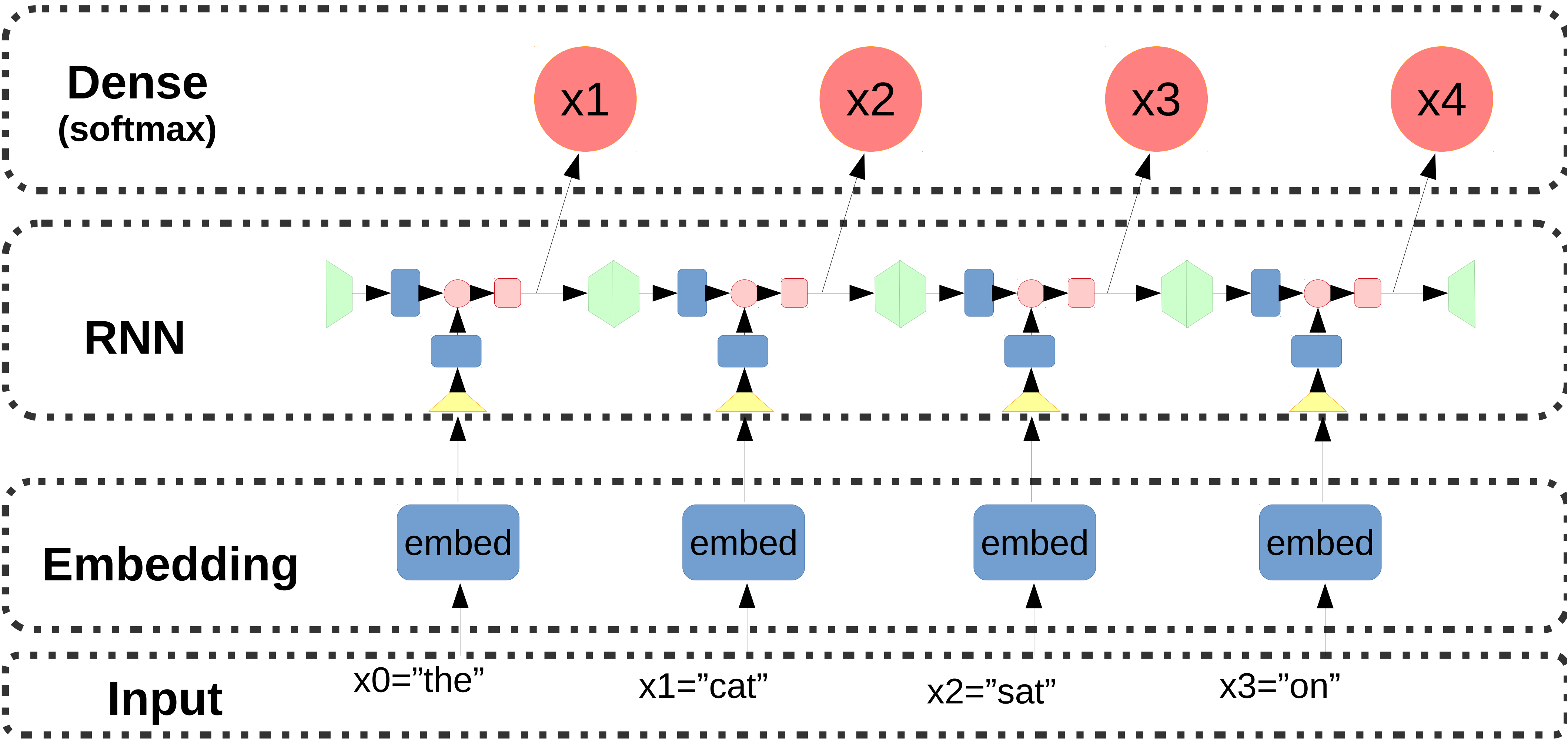

STATISTICAL LEARNING
Gentlemen, our learner overgeneralizes because the VC-Dimension of our Kernel is too high, Get some experts and minimze the structural risk in a new one. Rework our loss function, make the next kernel stable, unbiased and consider using a soft margin
NEURAL NETWORKS
STACK MORE LAYERS
LAYERS
LAYERS

What is layer, again?

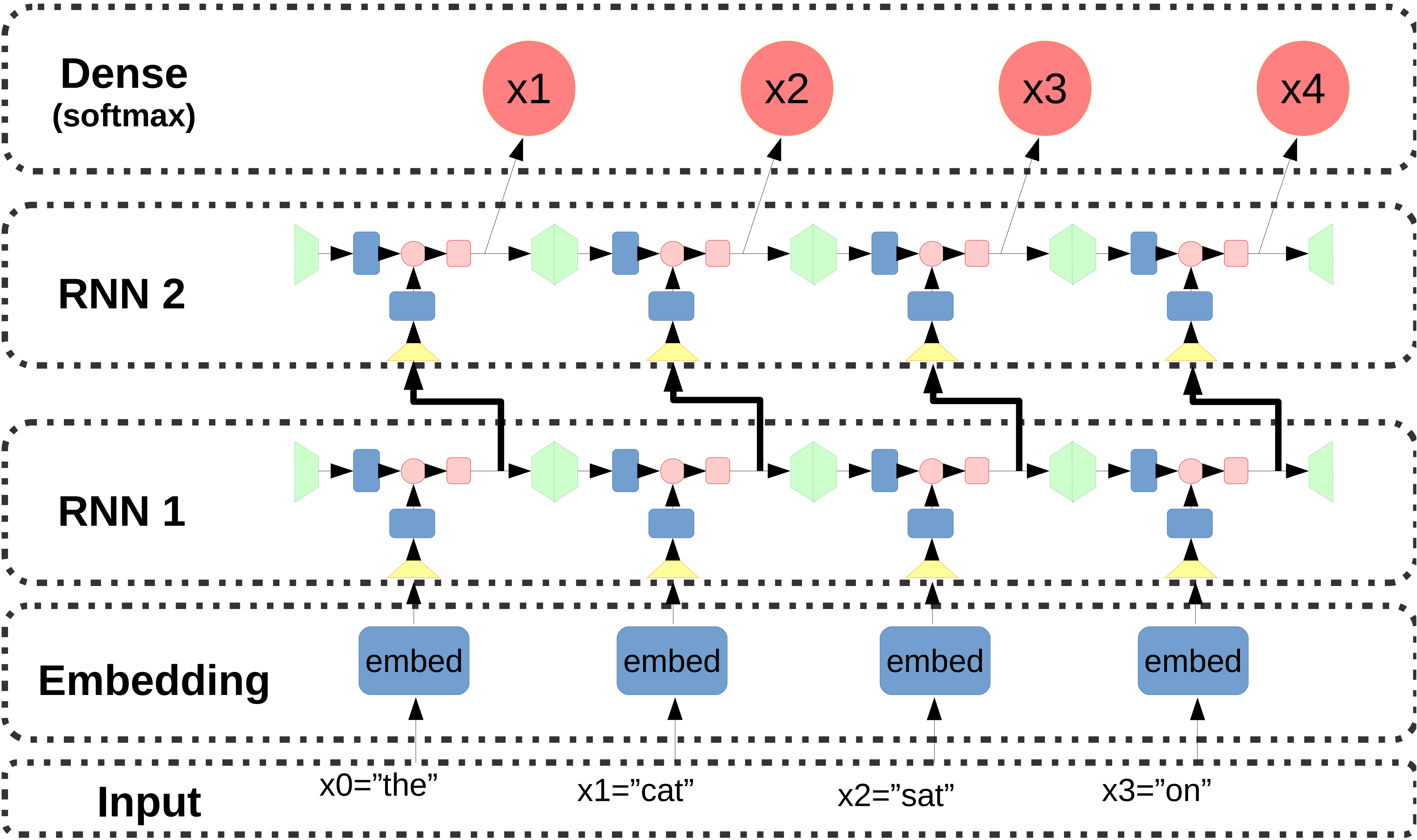


Layers

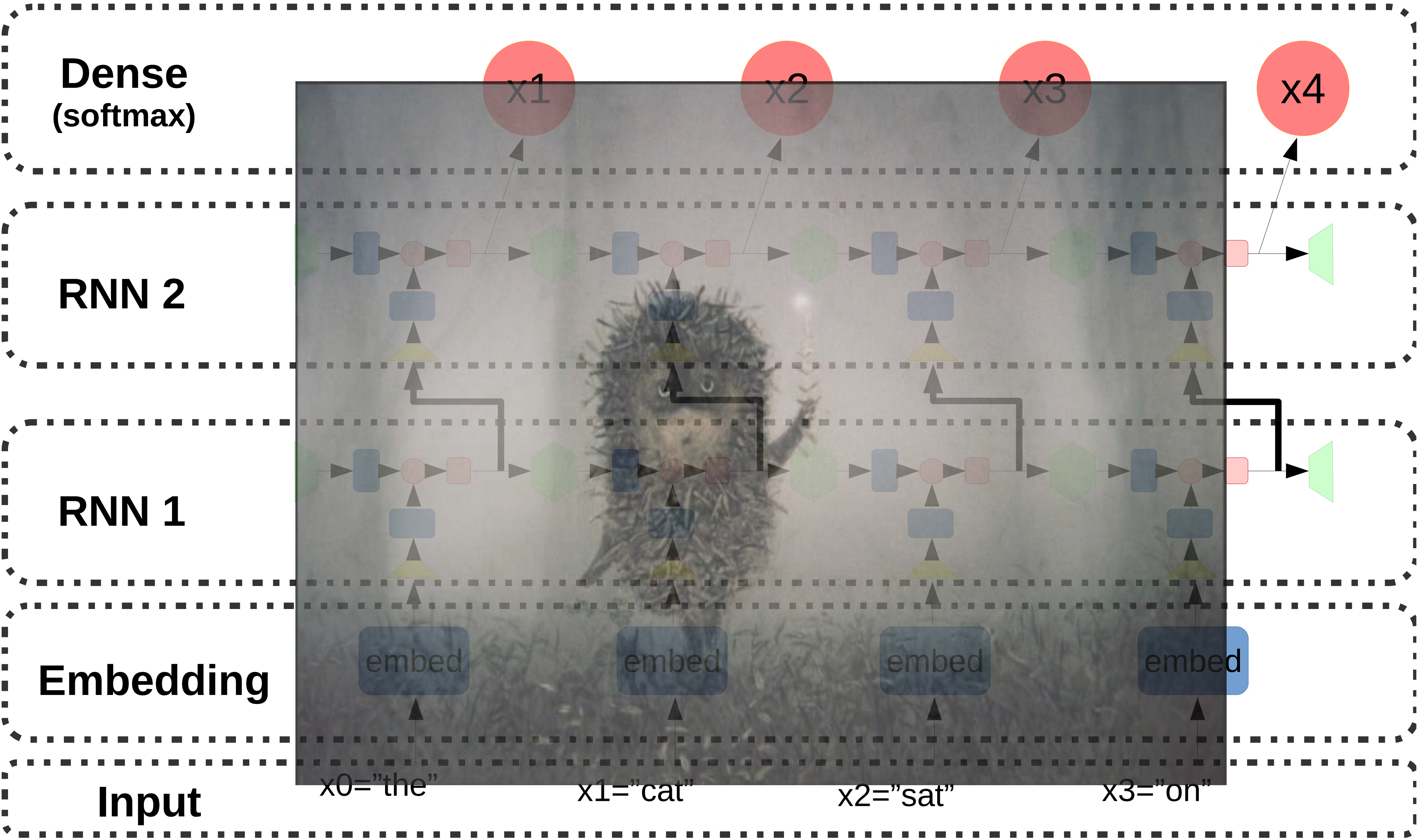
Where to stick more layers?



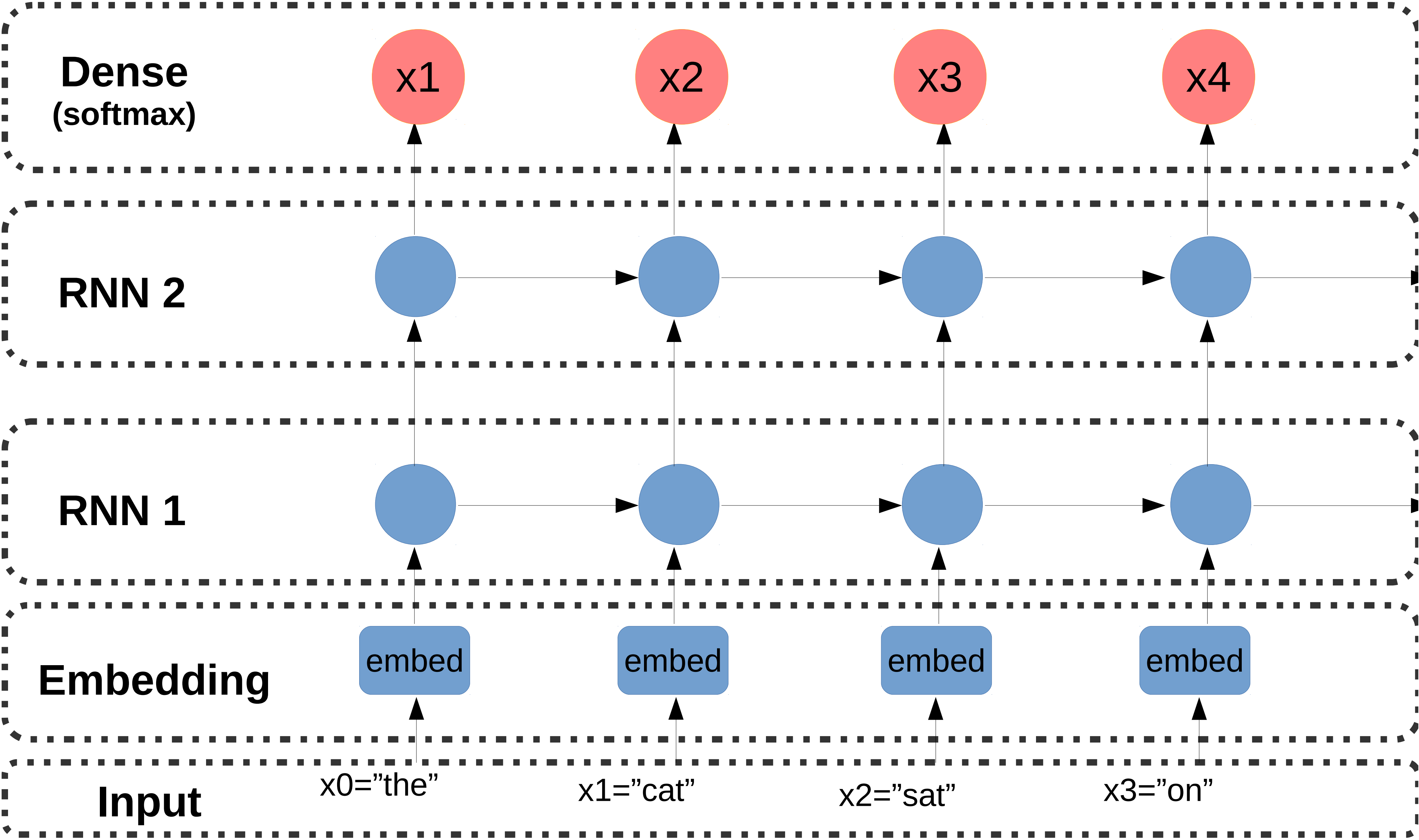
More layers


Dense
(softmax)
x1
x2
x3
x4
RNN 2
RNN 1
Embedding
embed
embed
embed
embed
Input
x0="the"
x1="cat"
x2="sat"
x3="on"

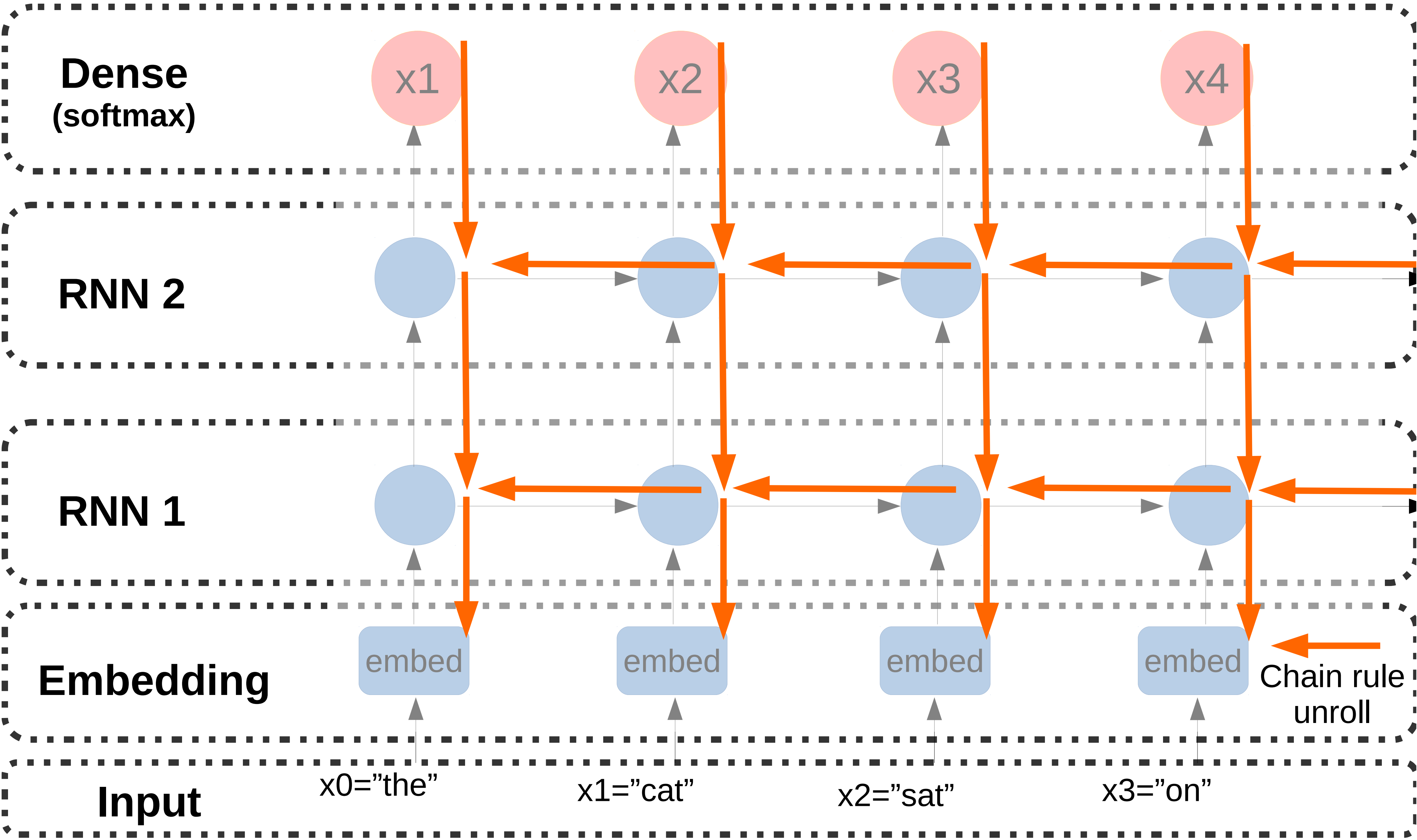
Too f**king complicated



2-layer RNN


Dense
(softmax)
x1
x2
x3
x4
RNN 2
RNN 1
Embedding
embed
embed
embed
embed
Input
x0="the"
x1="cat"
x2="sat"
x3="on"

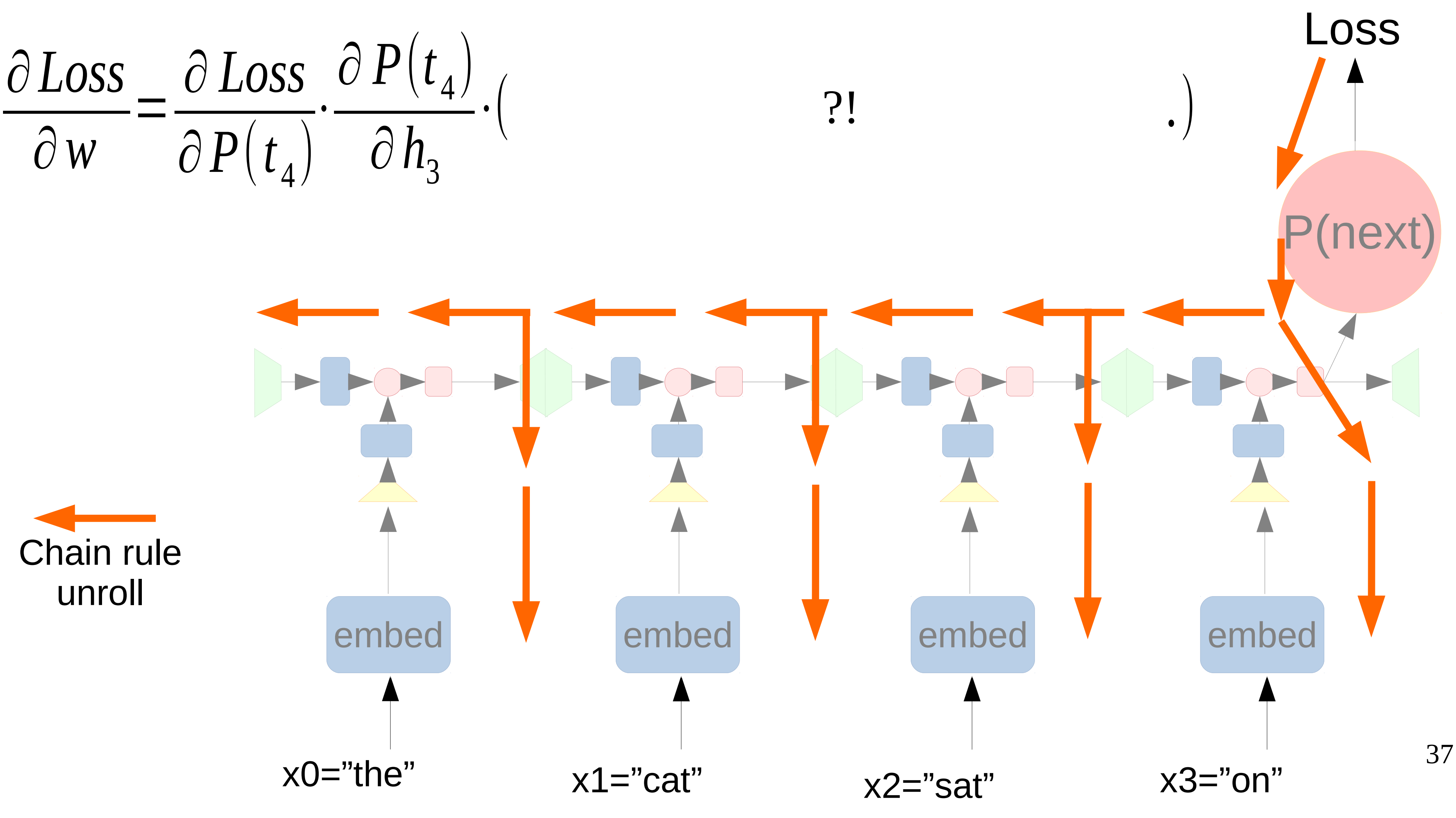
BPTT again


Dense
(softmax)
x1
x2
x3
x4
RNN 2
RNN 1
Embedding
embed
embed
embed
embed
Chain rule
unroll
Input
x0="the"
x1="cat"
x2="sat"
x3="on"

BPTT Again

$$h_{i+1} = \sigma\left(W_{hid} \cdot h_i + W_{inp} \cdot x_i + b\right)$$

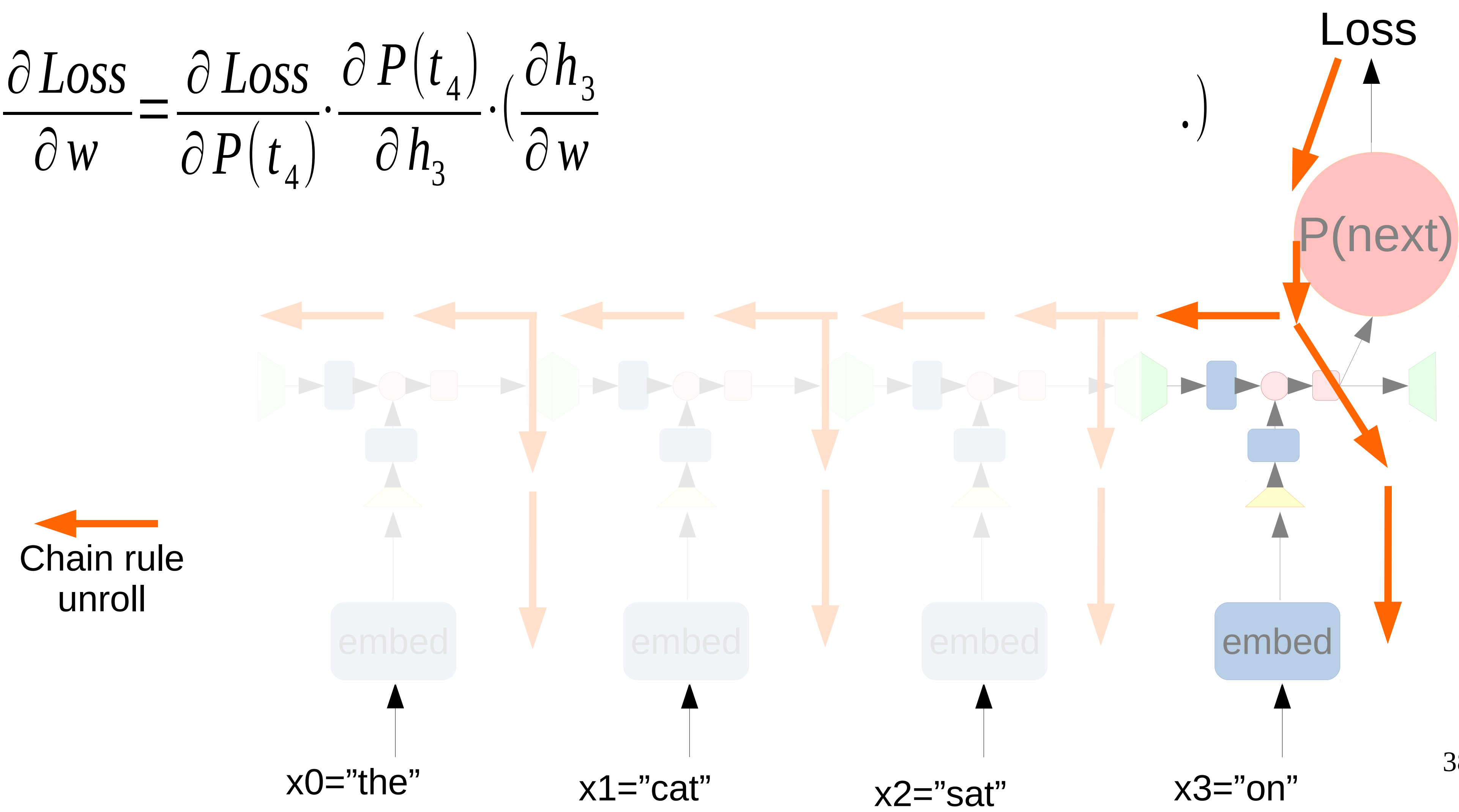
$$\frac{\partial Loss}{\partial w} = \frac{\partial Loss}{\partial P(t_4)} \cdot \frac{\partial P(t_4)}{\partial h_3} \cdot \Big(\qquad ?! \qquad . \Big)$$



BPTT Again

$$h_{i+1}=\sigma\left(W_{hid}\cdot h_i+W_{inp}\cdot x_i+b\right)$$

$$\frac{\partial Loss}{\partial w}=\frac{\partial Loss}{\partial P(t_4)}\cdot\frac{\partial P(t_4)}{\partial h_3}\cdot\Big(\frac{\partial h_3}{\partial w} \qquad\qquad\qquad\qquad .\Big)$$

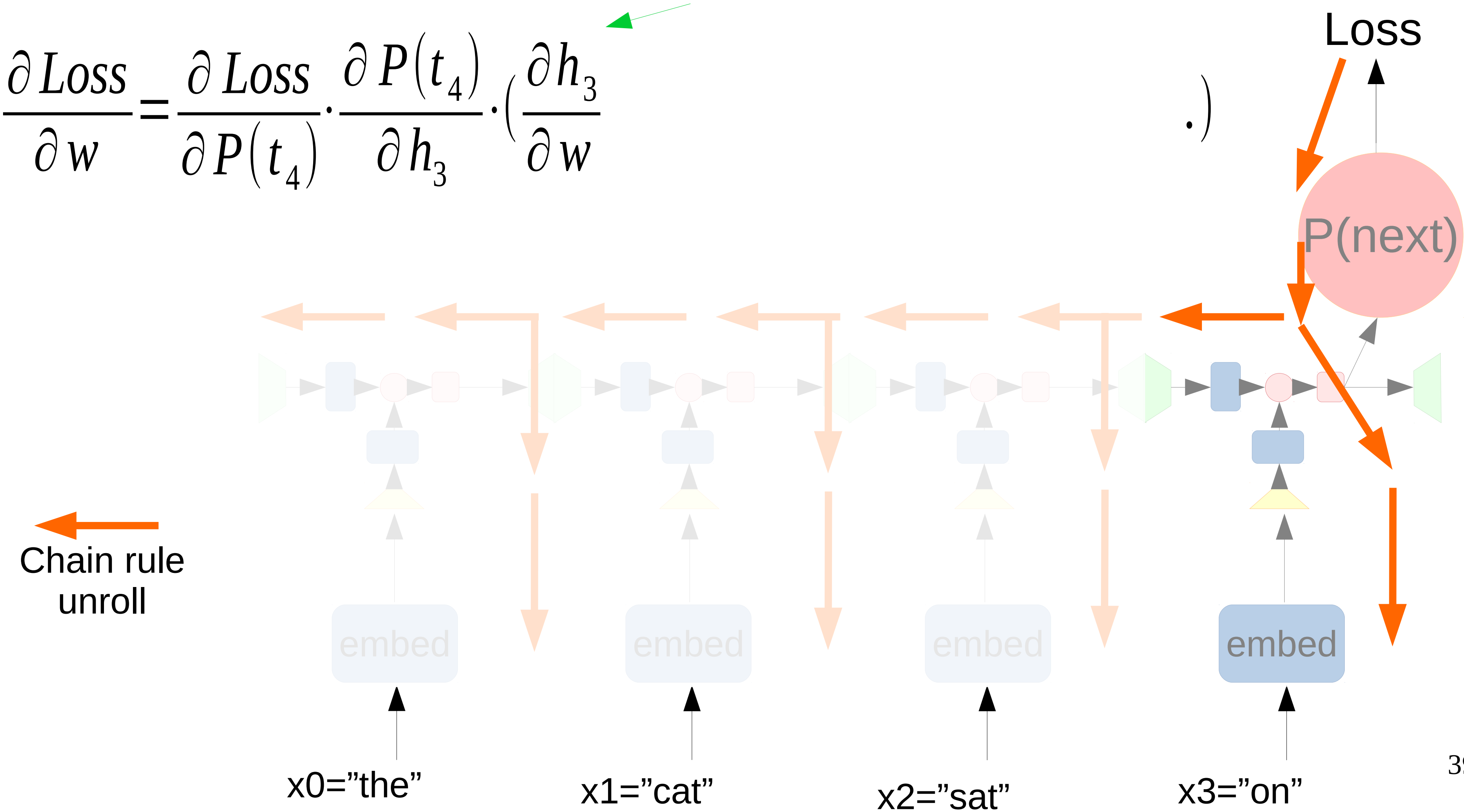


BPTT Again

$$h_{i+1} = \sigma\left(W_{hid} \cdot h_i + W_{inp} \cdot x_i + b\right)$$

consider h2 constant

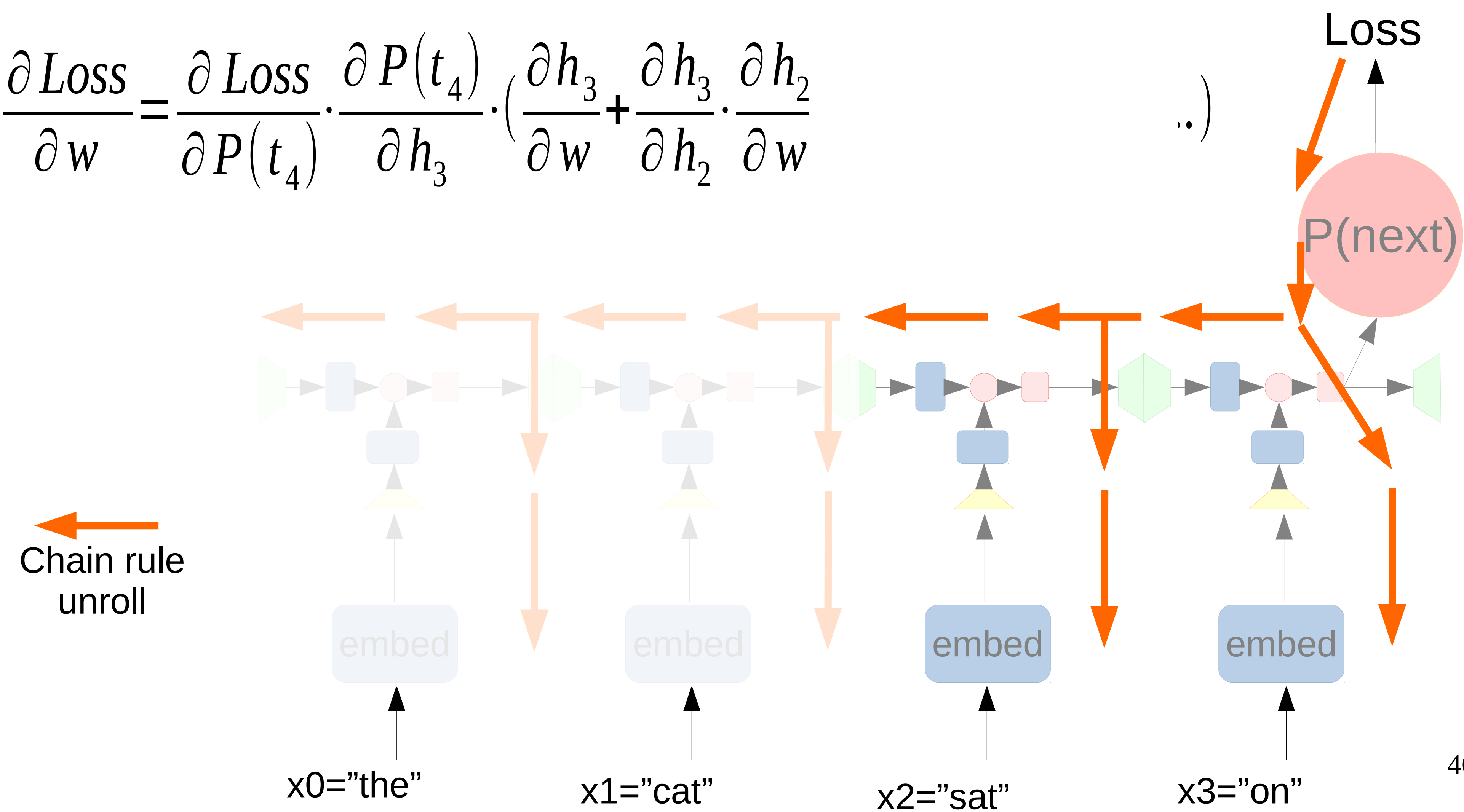
$$\frac{\partial Loss}{\partial w} = \frac{\partial Loss}{\partial P(t_4)} \cdot \frac{\partial P(t_4)}{\partial h_3} \cdot \left(\frac{\partial h_3}{\partial w} \qquad .\right)$$



BPTT Again

$$h_{i+1} = \sigma\left(W_{hid} \cdot h_i + W_{inp} \cdot x_i + b\right)$$

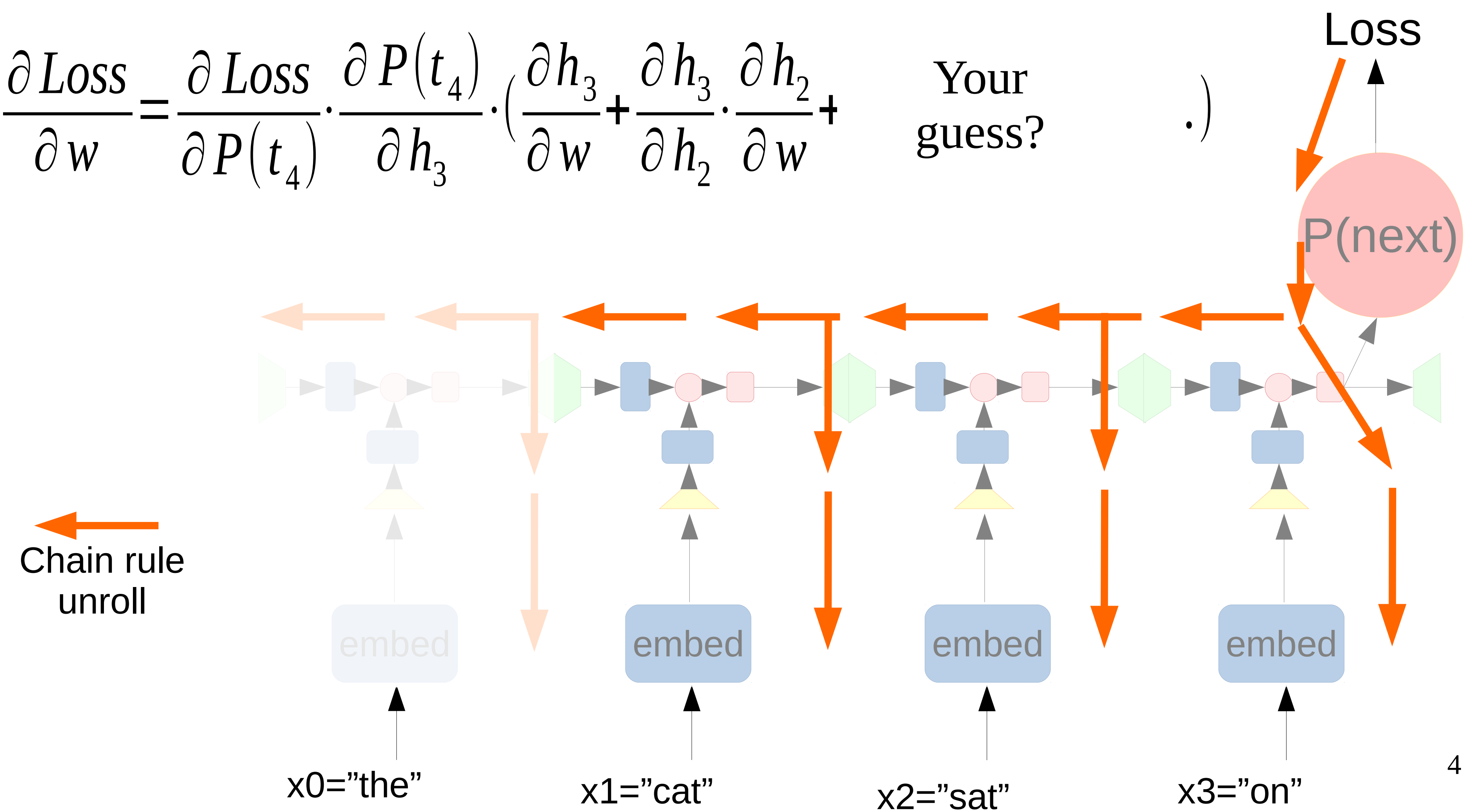
$$\frac{\partial Loss}{\partial w} = \frac{\partial Loss}{\partial P(t_4)} \cdot \frac{\partial P(t_4)}{\partial h_3} \cdot \left(\frac{\partial h_3}{\partial w} + \frac{\partial h_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial w} \quad \ldots\right)$$



BPTT Again

$$h_{i+1} = \sigma\left(W_{hid} \cdot h_i + W_{inp} \cdot x_i + b\right)$$

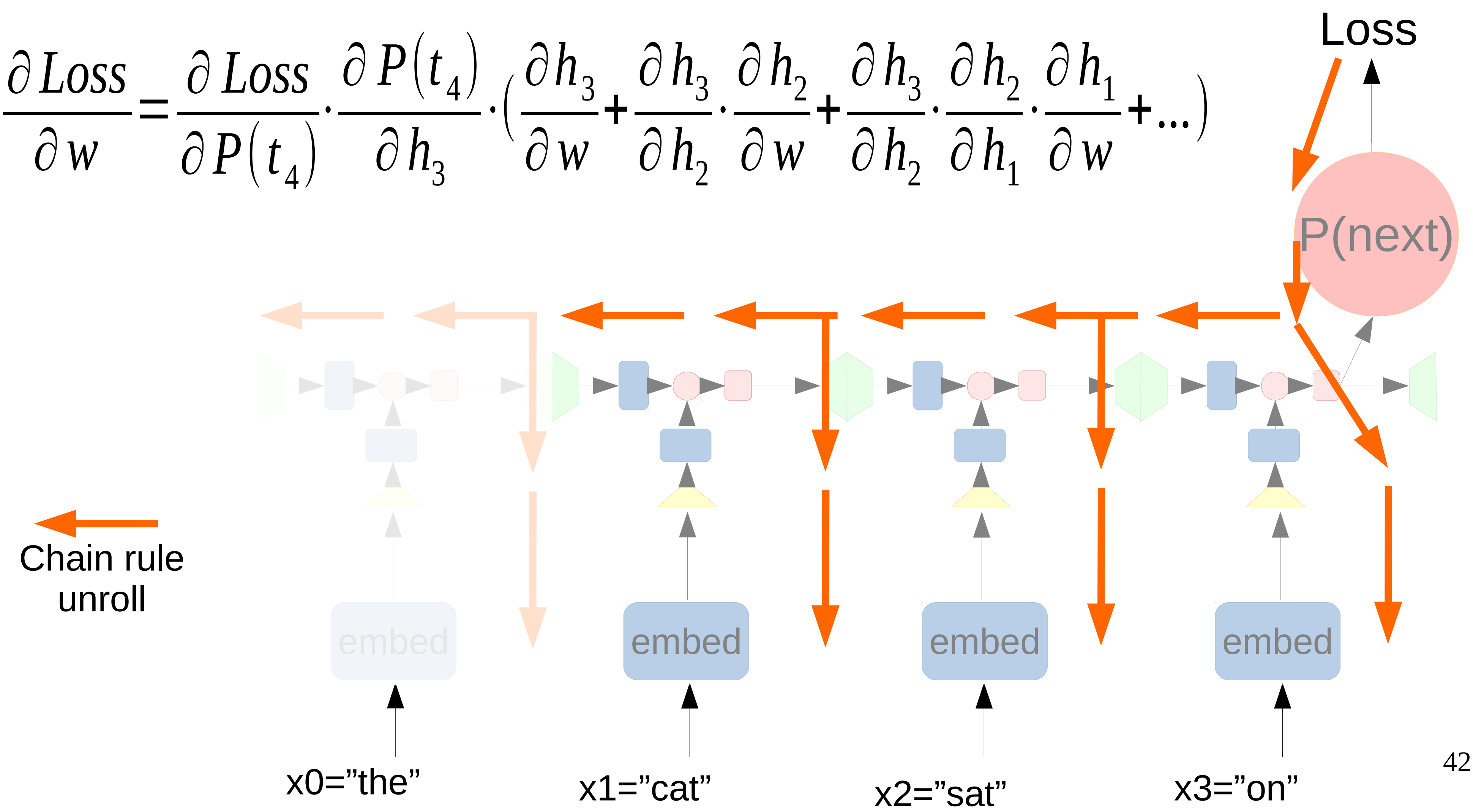
$$\frac{\partial Loss}{\partial w} = \frac{\partial Loss}{\partial P(t_4)} \cdot \frac{\partial P(t_4)}{\partial h_3} \cdot \left(\frac{\partial h_3}{\partial w} + \frac{\partial h_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial w} + \text{Your guess?} \; . \right)$$



BPTT Again

$$h_{i+1} = \sigma \left(W_{hid} \cdot h_i + W_{inp} \cdot x_i + b \right)$$

$$\frac{\partial Loss}{\partial w} = \frac{\partial Loss}{\partial P(t_4)} \cdot \frac{\partial P(t_4)}{\partial h_3} \cdot \left(\frac{\partial h_3}{\partial w} + \frac{\partial h_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial w} + \frac{\partial h_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial h_1} \cdot \frac{\partial h_1}{\partial w} + \dots \right)$$



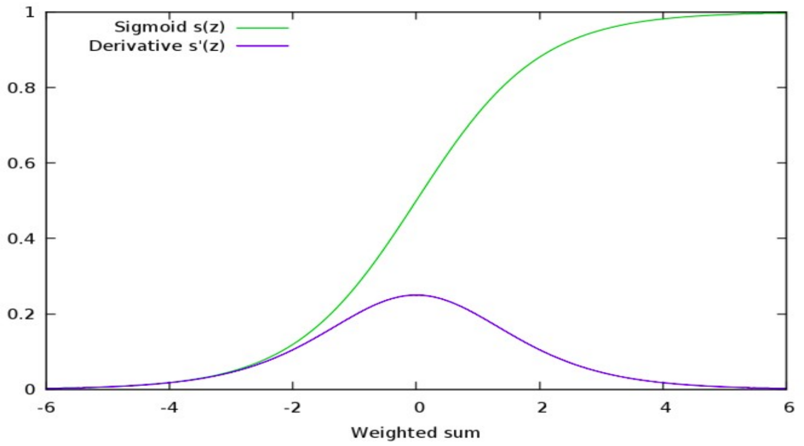
Gradient explosion and vanishing

$$h_{i+1} = \sigma\left(W_{hid} \cdot h_i + W_{inp} \cdot x_i + b\right)$$

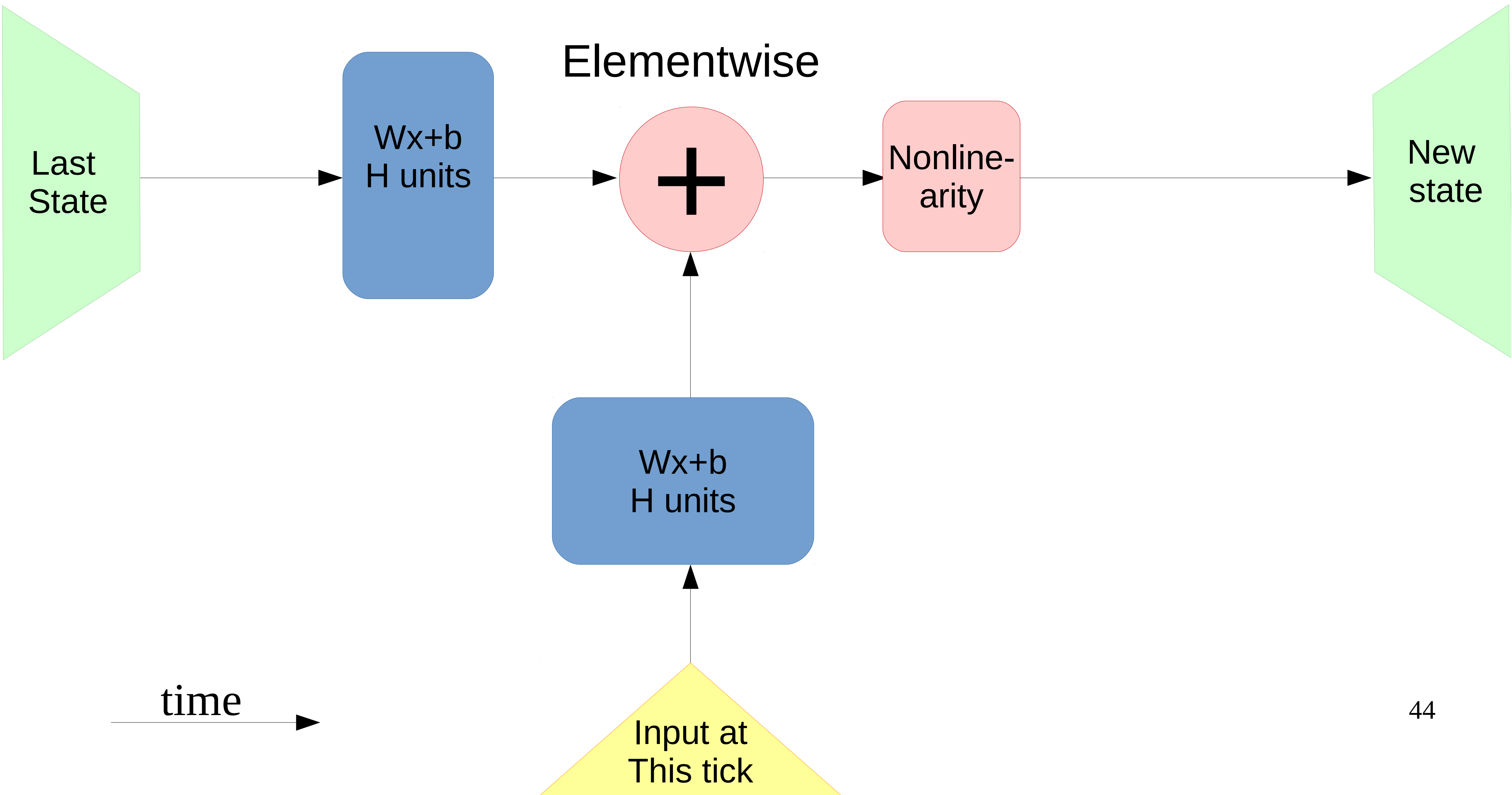
$$\frac{\partial Loss}{\partial w} = \frac{\partial Loss}{\partial P(t_4)} \cdot \frac{\partial P(t_4)}{\partial h_3} \cdot \left(\frac{\partial h_3}{\partial w} + \frac{\partial h_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial w} + \frac{\partial h_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial h_1} \cdot \frac{\partial h_1}{\partial w} + \ldots \right)$$

- Many sigmoids near 0 or 1
  - Gradients → 0
  - Not training for long-term dependencies

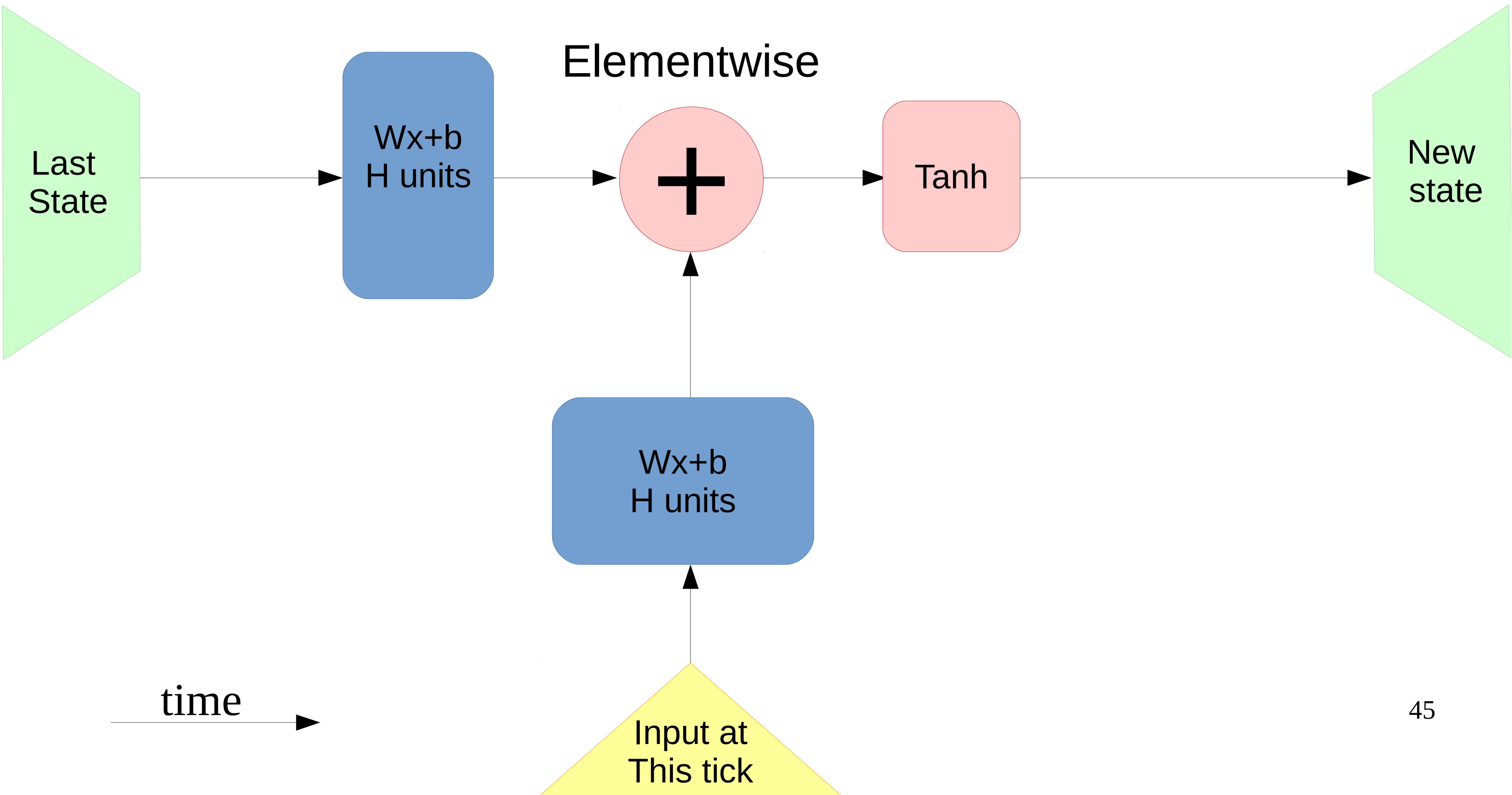
- Many nonzero values
  - Derivative stacks to >1
  - Gradients → inf
  - Weights → shit



RNN step

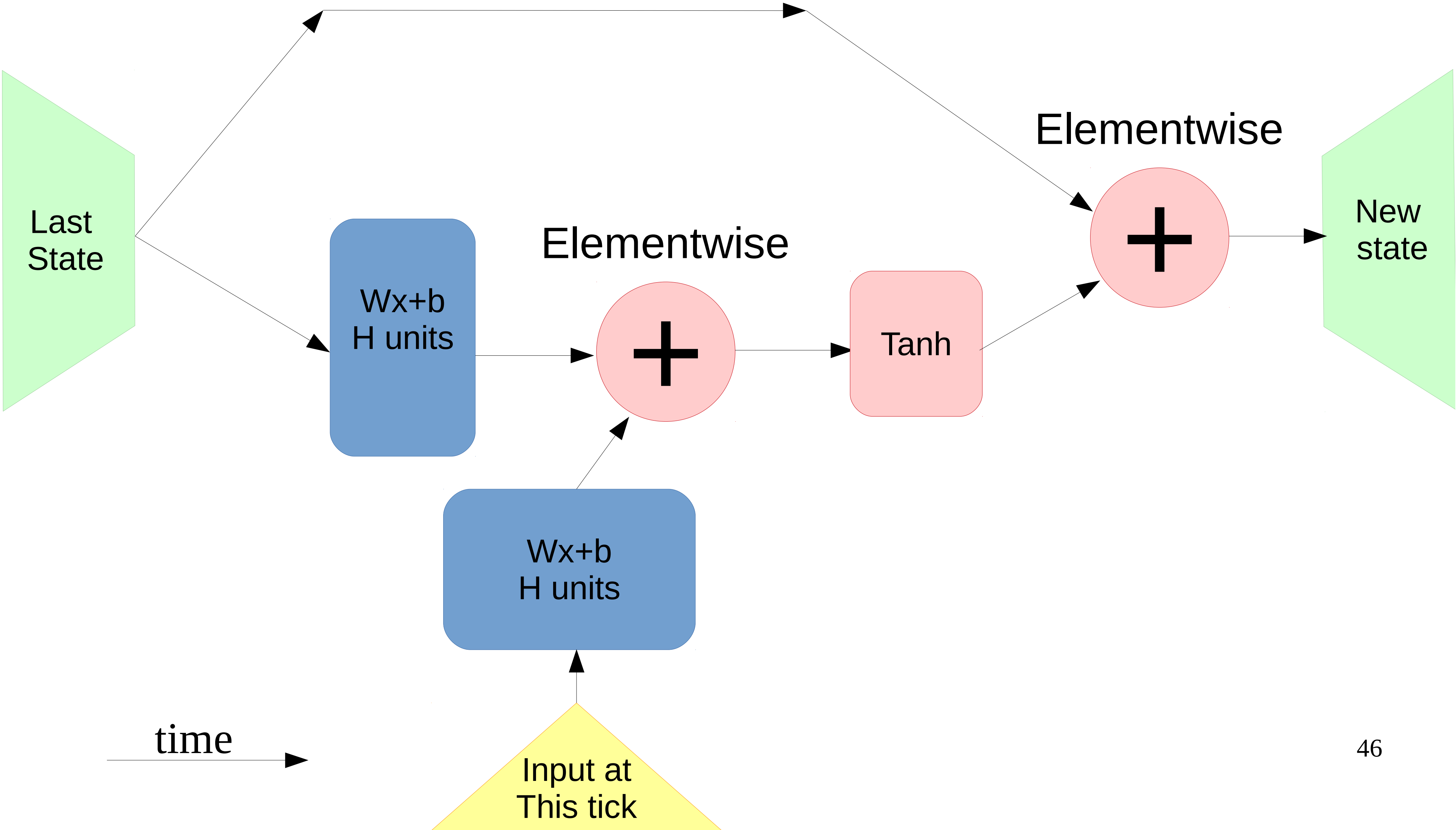

Last State
Wx+b H units
Elementwise
+
Nonline-arity
New state
Wx+b H units
Input at This tick
time

RNN step

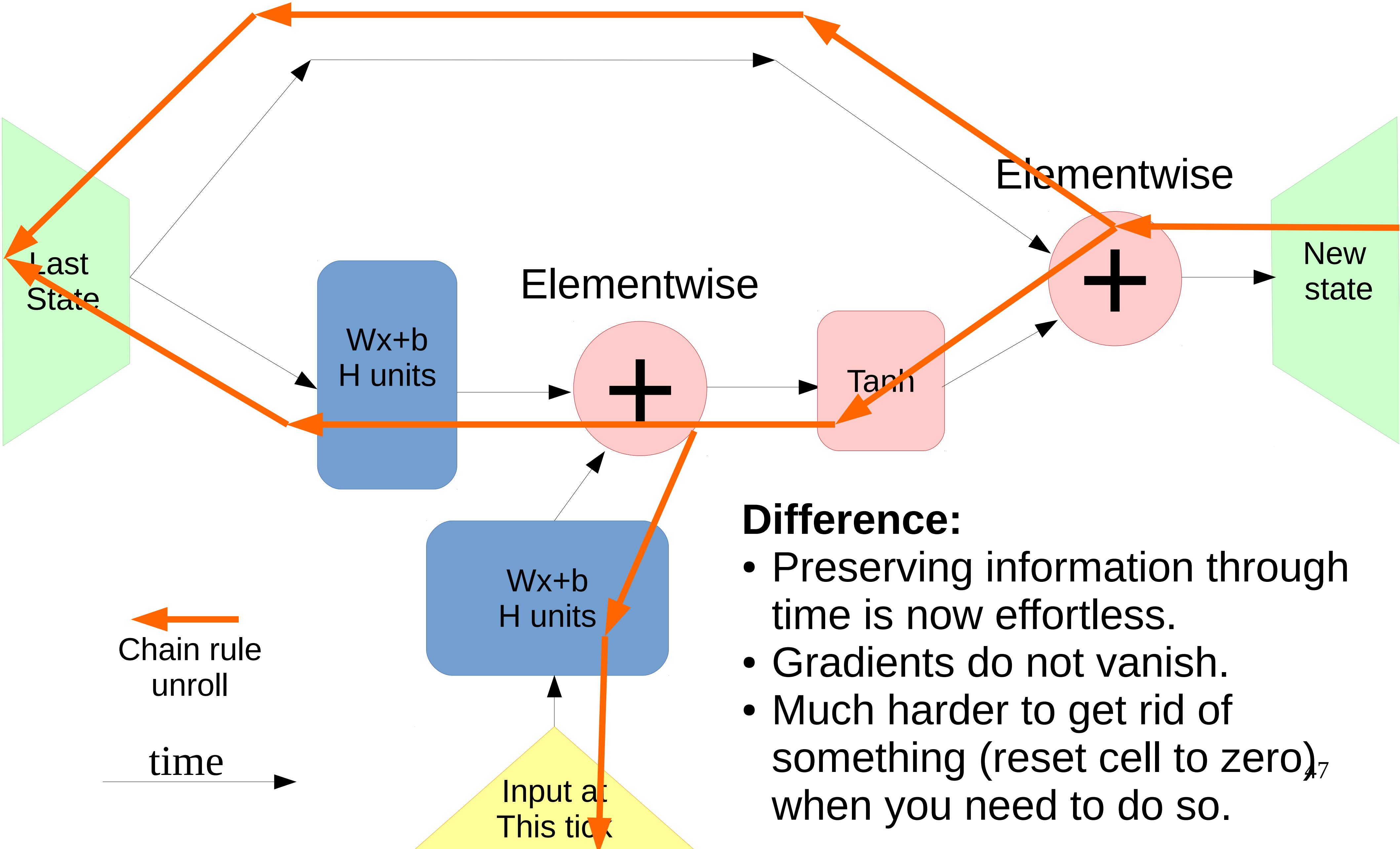


Last State
Wx+b
H units
Elementwise
+
Tanh
New state
Wx+b
H units
Input at This tick
time

Residual RNN step



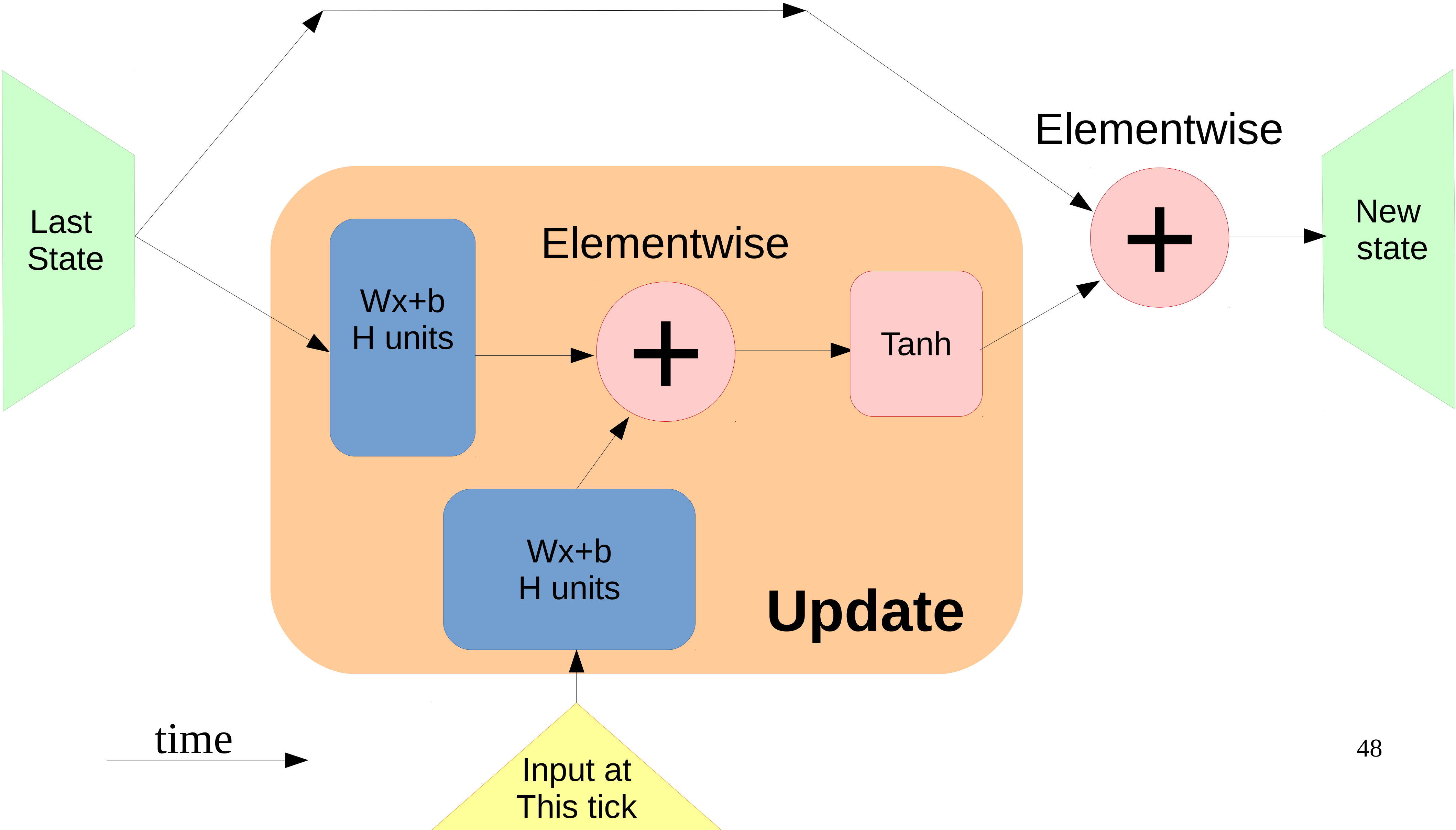
Residual RNN step



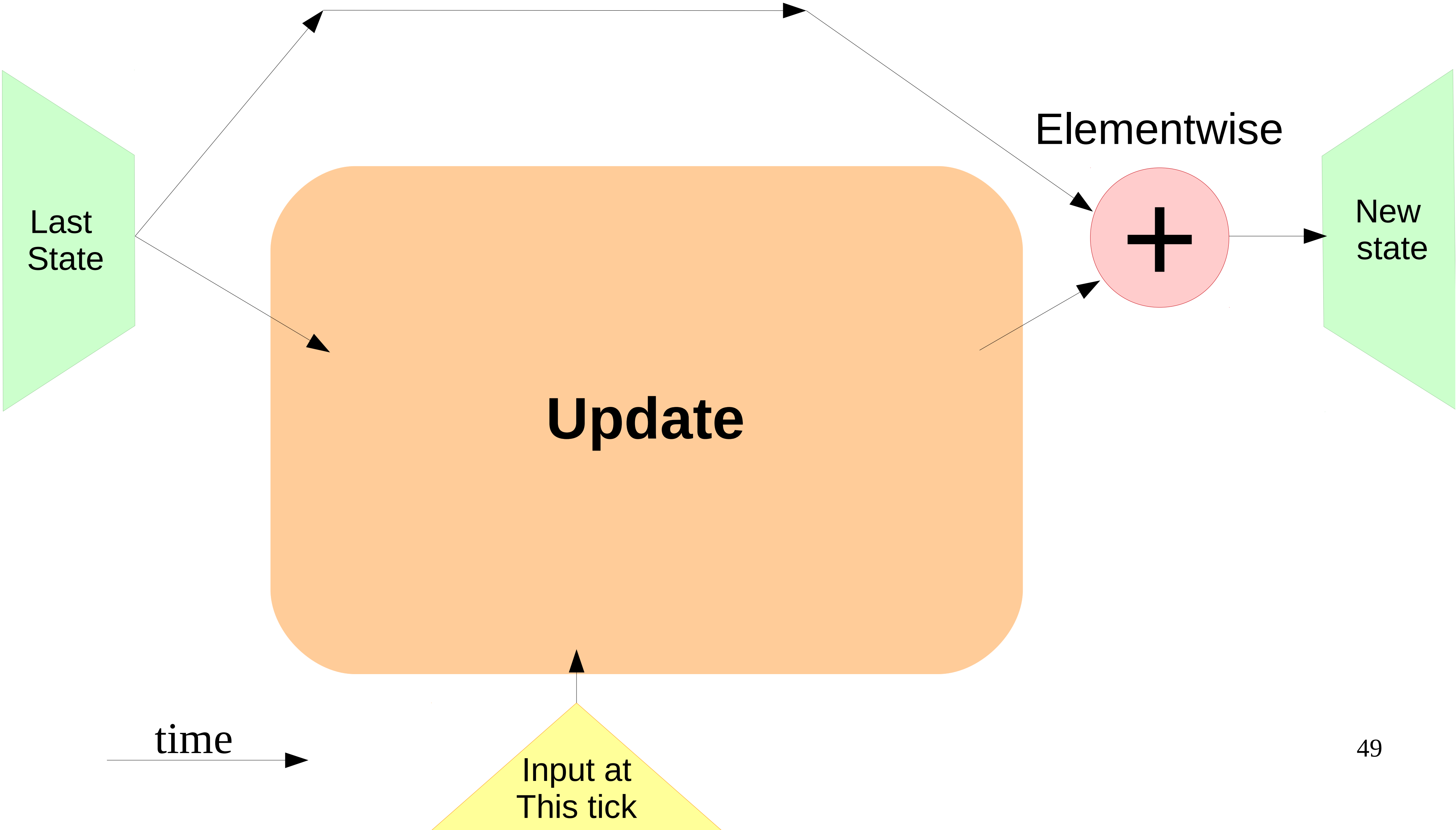
Difference:

- Preserving information through time is now effortless.
- Gradients do not vanish.
- Much harder to get rid of something (reset cell to zero) when you need to do so.

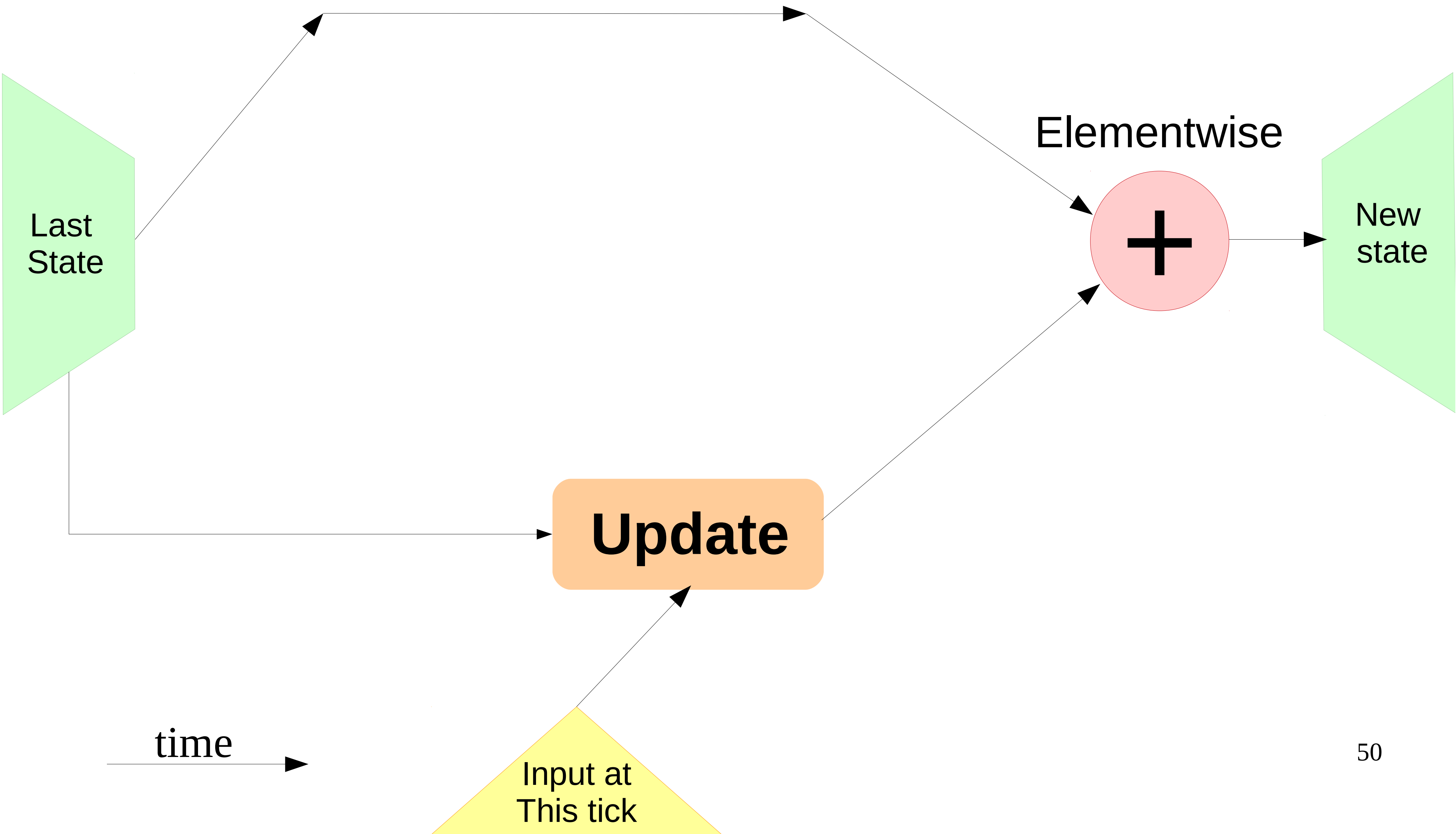
Residual RNN step



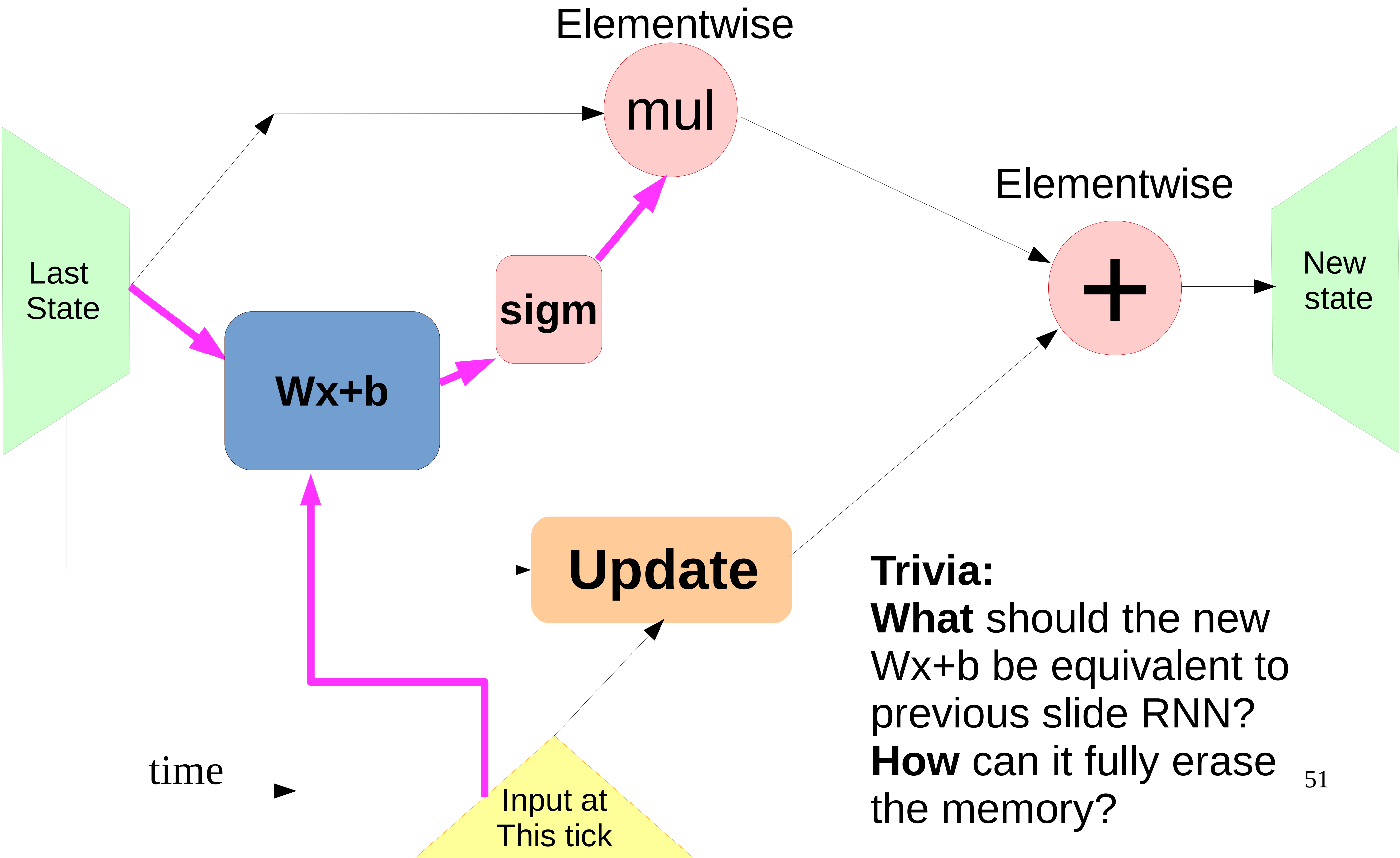
Residual RNN step



Residual RNN step

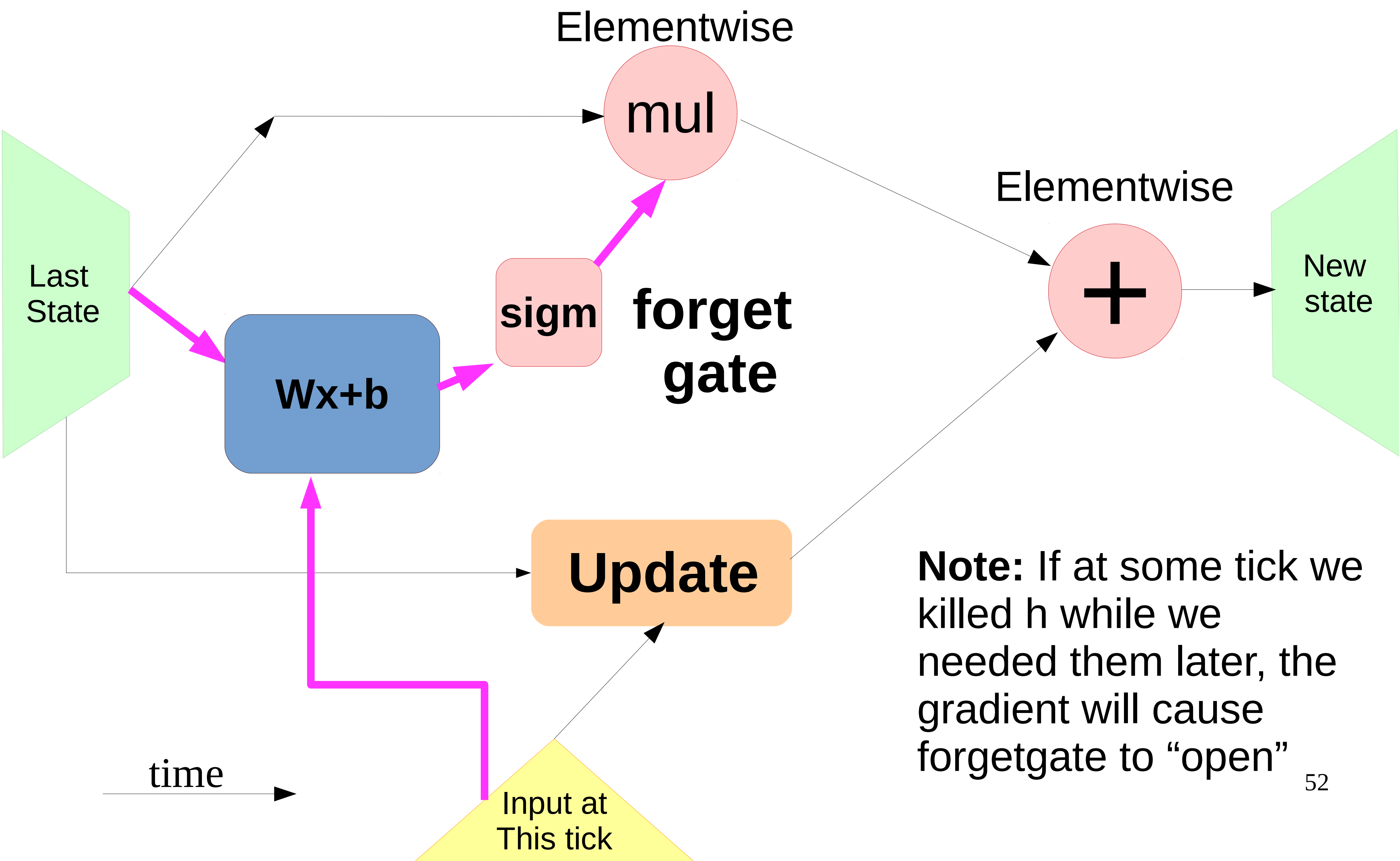


Residual RNN step



Trivia:
What should the new Wx+b be equivalent to previous slide RNN?
How can it fully erase the memory?

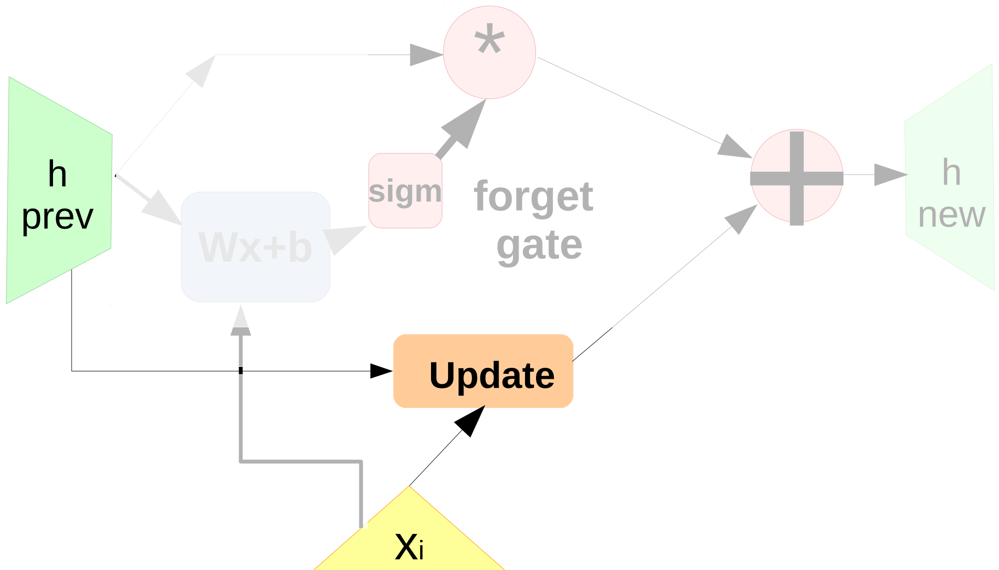
Residual RNN step



Note: If at some tick we killed h while we needed them later, the gradient will cause forgetgate to "open"

What we drew

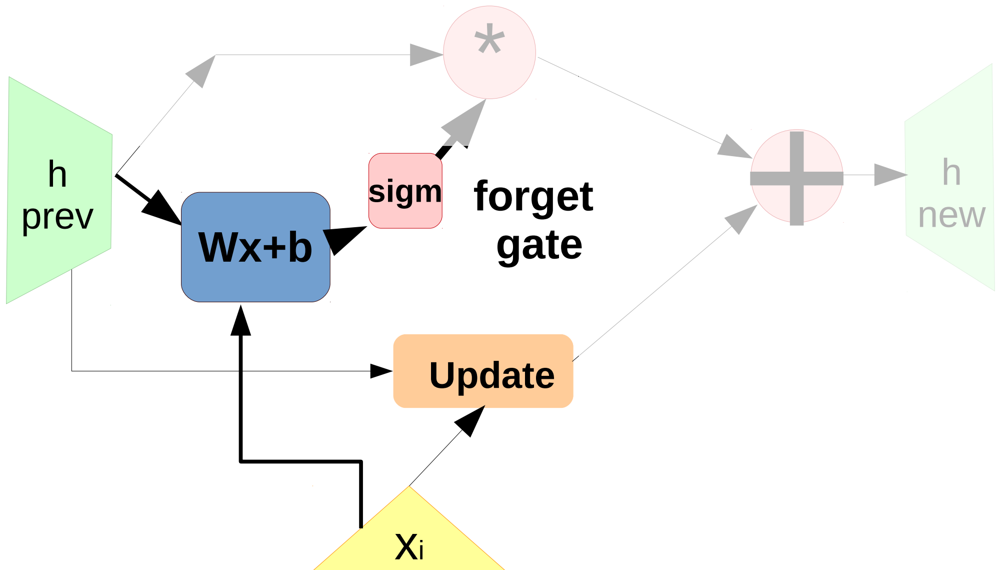
$$update(x_i, h_{i-1}) = \tanh\left(W_{hid}^{update} \cdot h_{i-1} + W_{inp}^{update} \cdot x_i + b^{update}\right)$$



What we drew

$$update(x_i, h_{i-1}) = \tanh\left(W_{hid}^{update} \cdot h_{i-1} + W_{inp}^{update} \cdot x_i + b^{update}\right)$$

$$forget(x_i, h_{i-1}) = \sigma\left(W_{hid}^{forget} \cdot h_{i-1} + W_{inp}^{forget} \cdot x_i + b^{forget}\right)$$



What we drew

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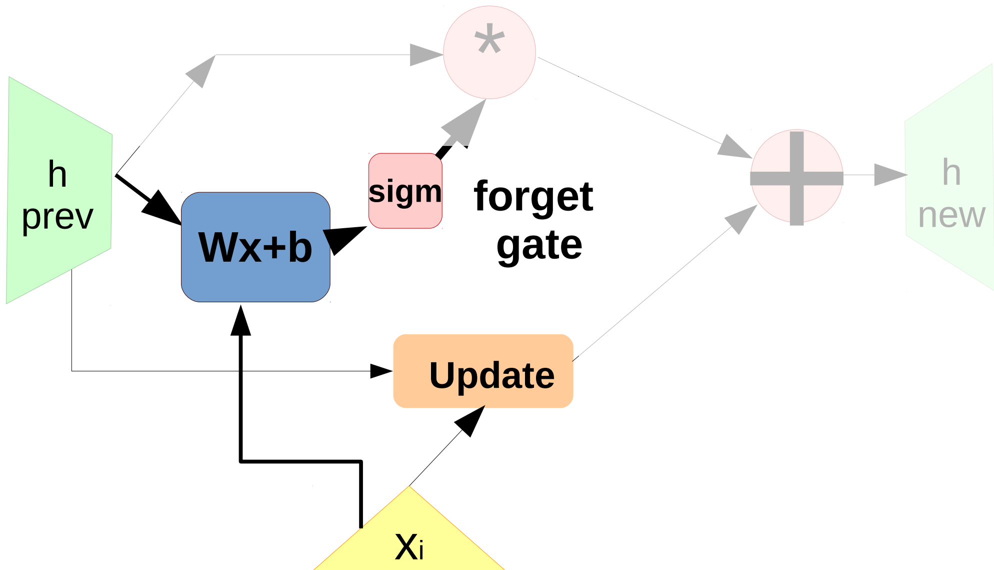


How to compute h_new?

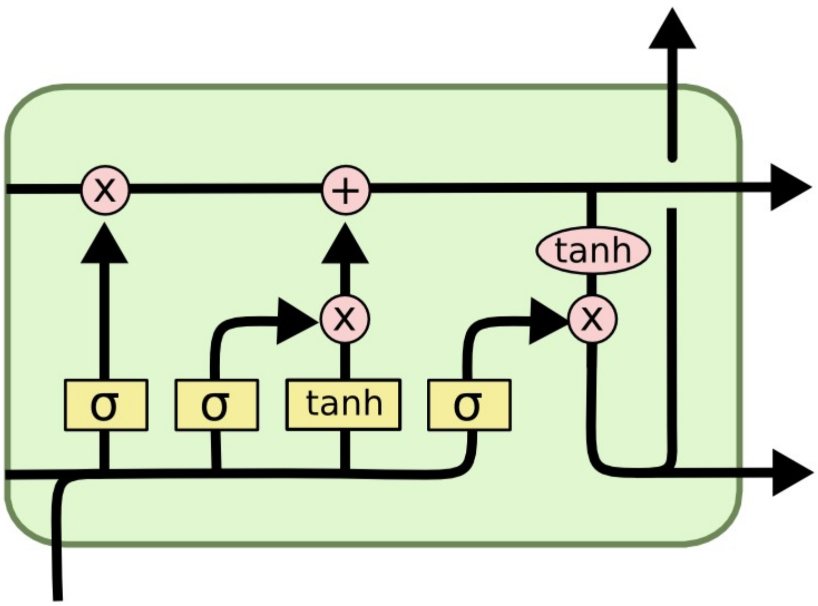
What we drew

$$update(x_i, h_{i-1}) = \tanh\left(W_{hid}^{update} \cdot h_{i-1} + W_{inp}^{update} \cdot x_i + b^{update}\right)$$

$$forget(x_i, h_{i-1}) = \sigma\left(W_{hid}^{forget} \cdot h_{i-1} + W_{inp}^{forget} \cdot x_i + b^{forget}\right)$$

$$h_i(x_i, h_{i-1}) = forget(x_i, h_{i-1}) \cdot h_{i-1} + update(x_i, h_{i-1})$$

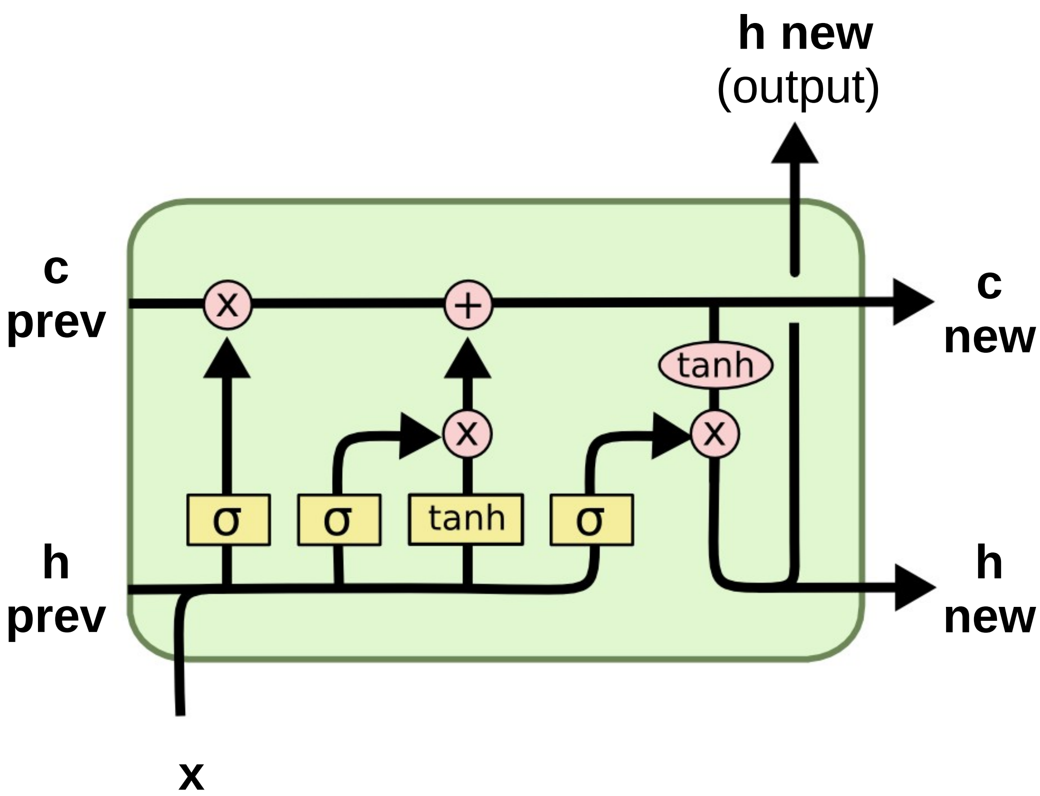
LSTM



2 hidden states:
- Cell (“private” state)
- Output (“public” state)

4 blocks:
- Update
- Forget gate
- Input gate
- Output gate

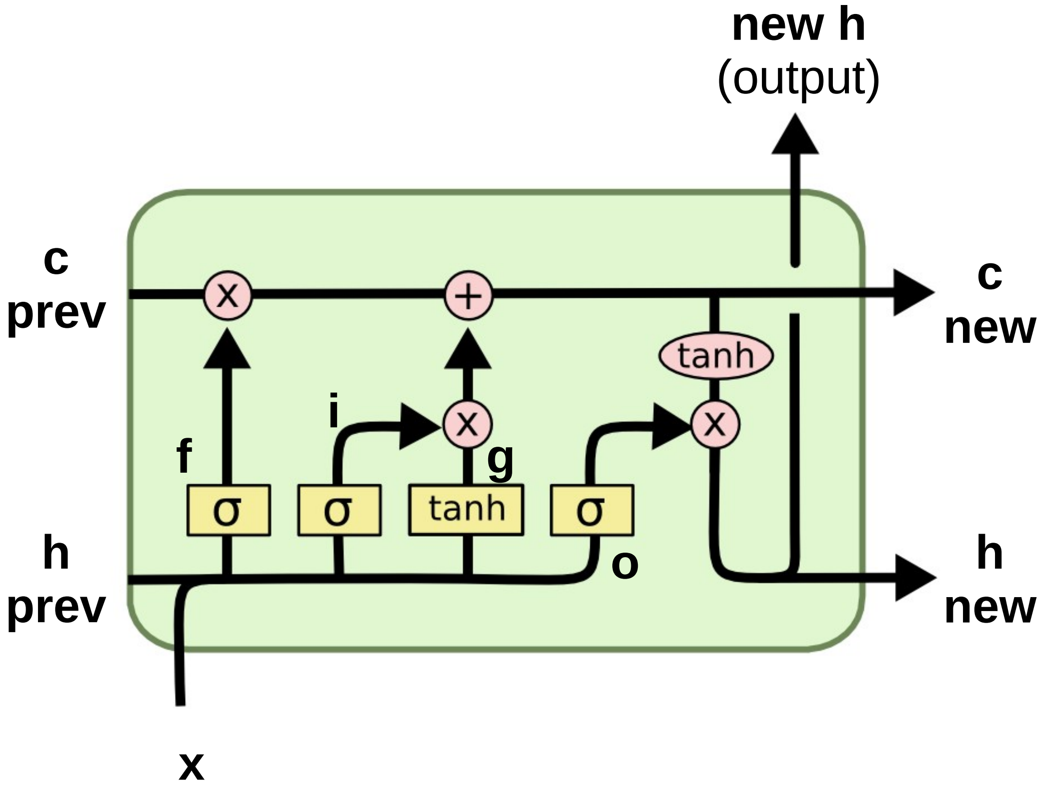
LSTM



$$i_t = Sigm(\theta_{xi} x_t + \theta_{hi} h_{t-1} + b_i)$$
$$f_t = Sigm(\theta_{xf} x_t + \theta_{hf} h_{t-1} + b_f)$$
$$o_t = Sigm(\theta_{xo} x_t + \theta_{ho} h_{t-1} + b_o)$$
$$g_t = Tanh(\theta_{xg} x_t + \theta_{hg} h_{t-1} + b_g)$$
$$c_t = f_t \otimes c_{t-1} + i_t \otimes g_t$$
$$h_t = o_t \otimes Tanh(c_t)$$

Where are the gates?

LSTM



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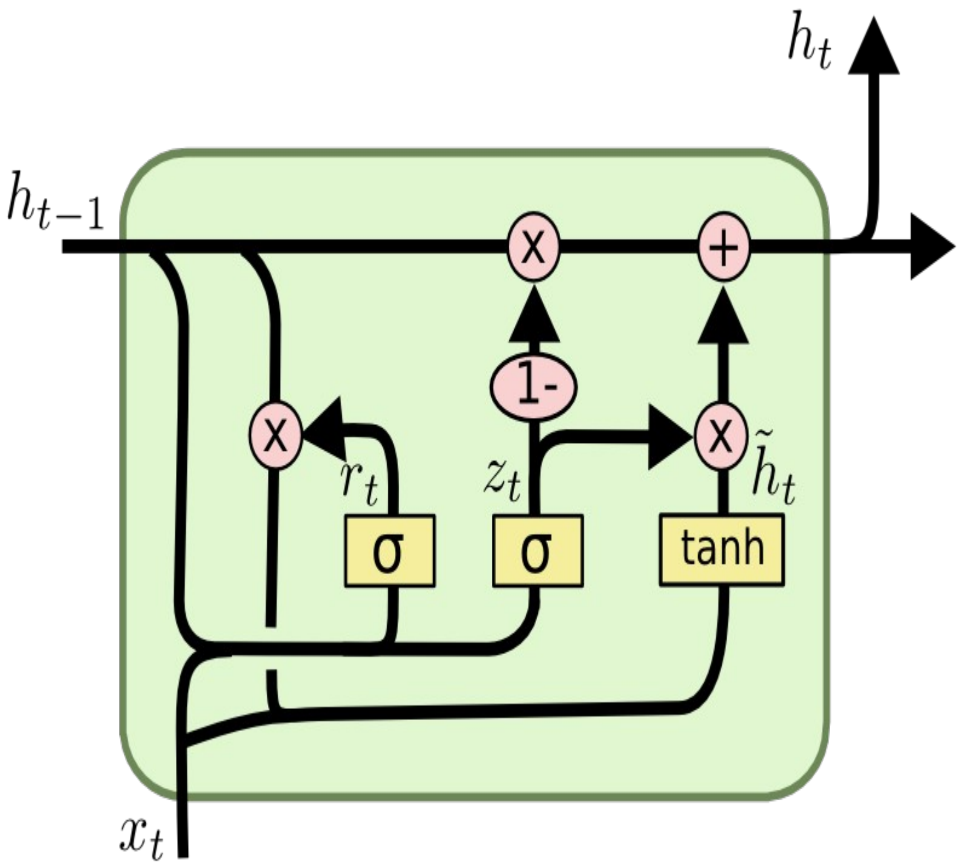
$$h_t = o_t \otimes Tanh(c_t)$$

GRU



$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$

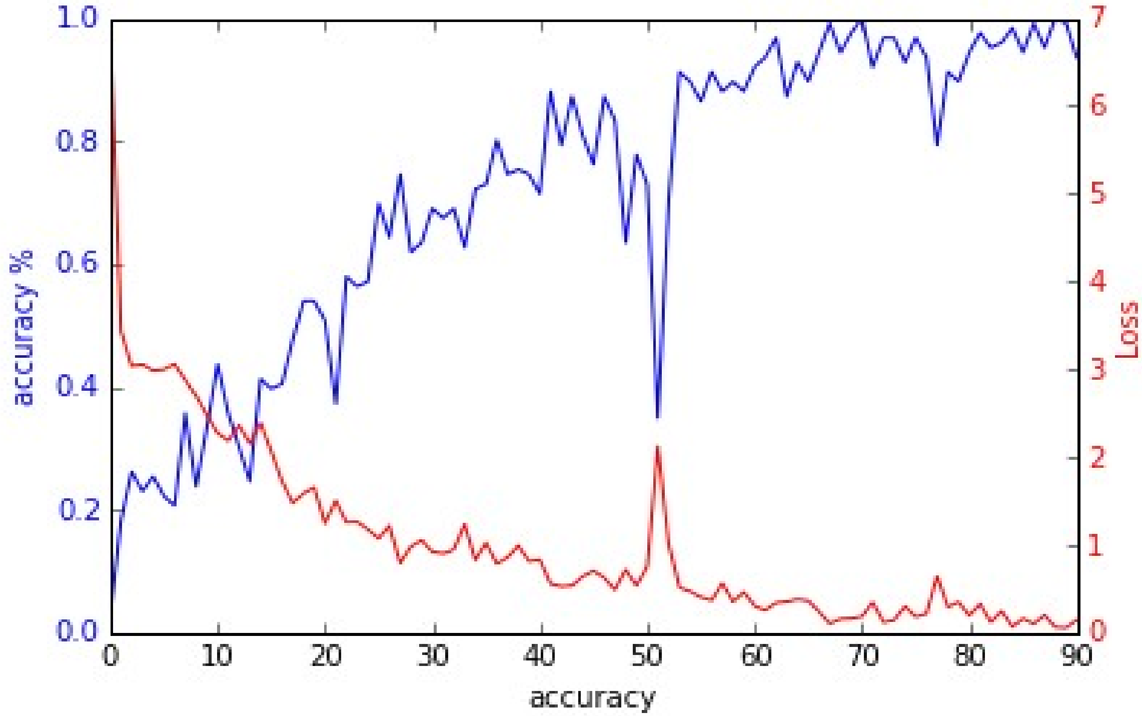
$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$

$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

Okay, the gradients no longer vanish
except they still do, if only slower

But how do we deal with exploding grads?



Ideas?

Gradient clipping

At each time tick,

- check if grad abs value is more than … 5?
- If so, clip it
  - large positive is now 5,
  - large negative is now -5

- How large is too large?
  - Reduce clipping threshold until explosions disappear

Gradient clipping

Where do I clip?

- Clip each element of $\delta L / \delta w$
- Clip each element of $\delta h_{i+1} / \delta h_i$
- Clip whole $\delta L / \delta w$ by norm
  - If $\left\| \frac{\delta L}{\delta w} \right\| > 5$, scale $\frac{\delta L}{\delta w} / \left\| \frac{\delta L}{\delta w} \right\| \cdot 5$

Generating stuff

Easy:

- Names, small phrases
- Orthographically correct delirium

Medium:

- Grammatically coherent text
- Resembling particular author

Hard:

- C/C++ source code
- Music
- Organic molecules
- LaTex articles
- Your course projects

Nuff

Coding time!