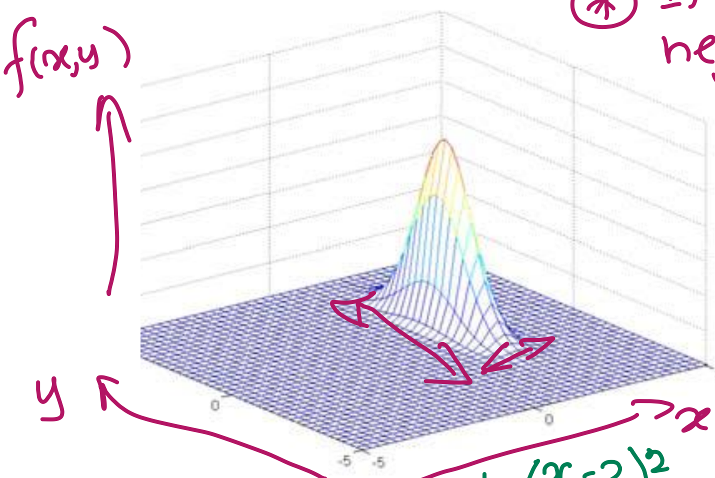
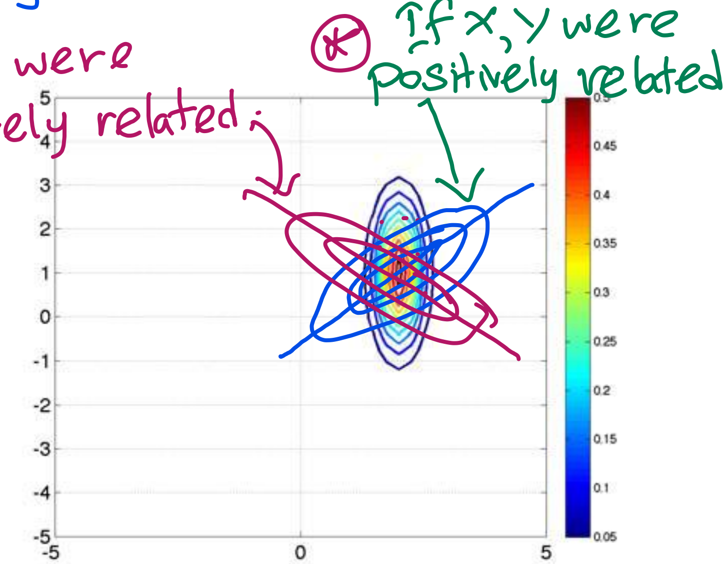
Here, X has Normal with $\mu = 2$, $\sigma = 0.3$

Y has Normal with $\mu = 1$, $\sigma = 1$

Here, measurement along x-axis has smaller uncertainty than the measurement along the y-axis.





$$f(x) = \frac{1}{\sqrt{2\pi}(0.3)} e^{-\frac{1}{2}\left(\frac{x-2}{0.3}\right)^2} ; -\infty < x < \infty$$

Since X, Y are independent, the joint pdf is

$$f(x,y) = f(x) * f(y) = \frac{1}{\sqrt{2\pi}(0.3)} e^{-\frac{1}{2}\left(\frac{x-2}{0.3}\right)^2} * \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y-1)^2}$$

$$= \frac{1}{0.6\pi} e^{-\frac{1}{2}\left[\left(\frac{x-2}{0.3}\right)^2 + (y-1)^2\right]} ; \begin{array}{l} -\infty < x < \infty \\ -\infty < y < \infty \end{array}$$

Ex 10: Let $X_1, X_2, \dots, X_n$ be independent and identically distributed normal random variables with mean μ and variance σ^2. Write the joint pdf of these variables.

Each X_i has a Normal (μ, σ^2). $f(x_i) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2}$

Since X_i are independent, the joint pdf is

$$f(x_1, x_2, \dots, x_n) = f(x_1) * f(x_2) * \dots * f(x_n) = \prod_{i=1}^{n} f(x_i)$$

$$= \prod_{i=1}^{n} \left[\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2}\right]$$

$$= \begin{cases} \frac{1}{(2\pi)^{\frac{n}{2}} \sigma^{-2}} e^{-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)^2} ; & -\infty < x_1, x_2, \dots, x_n < \infty, \ -\infty < \mu < \infty, \ \sigma > 0 \\ 0 , & \text{otherwise} \end{cases}$$