Binomial

❖ If $X_1, X_2, \dots, X_k$ are **independent** random variables so that $X_i \sim B(n_i\,,\ p)$ for $i = 1, 2, \dots, k$, then the sum of those random variables

same

$$Y = X_1 + X_2 + \cdots + X_k \sim B\left(\sum_{i=1}^{k} n_i \quad , \quad p\right)$$

Binomial

Poisson

❖ If $X_1, X_2, \dots, X_k$ are **independent** random variables so that $X_i \sim P(\lambda_i)$ for $i = 1, 2, \dots, k$, then the sum of those random variables

mean

$$Y = X_1 + X_2 + \cdots + X_k \sim P\left(\sum_{i=1}^{k} \lambda_i\right)$$

Poisson

Ex 2: (Poisson channel)

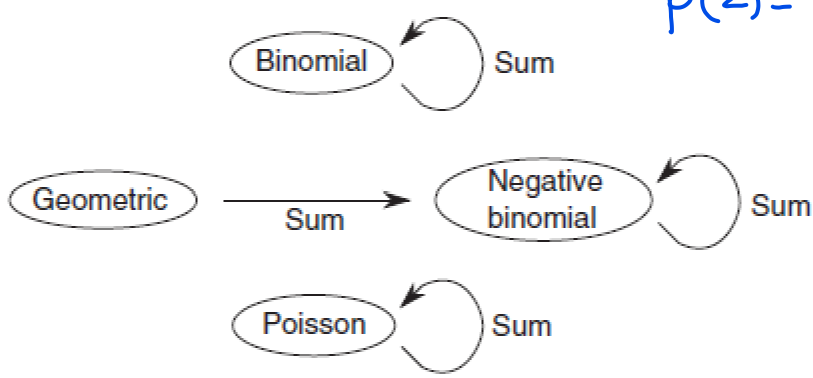
Average number of photoelectrons

The Poisson(λ) random variable is a good model for the number of photoelectrons generated in a photodetector when the incident light intensity is λ . Now suppose that an additional light source of intensity μ is also directed at the photodetector. Then we expect that the number of photoelectrons generated should be related to the total light intensity λ +μ.

If X and Y are independent Poisson random variables with respective parameters λ and μ, then Z = X +Y has a Poisson(λ + μ).

pmf of $Z = X+Y$ is

$$p(z) = \begin{cases} \dfrac{e^{-(\lambda+\mu)} (\lambda+\mu)^z}{z!} & ; z = 0, 1, 2, \dots \quad \mu > 0,\ \lambda > 0 \\ 0 & ; \text{otherwise} \end{cases}$$



Normal

❖ If $X_1, X_2, \dots, X_k$ are **independent** random variables so that $X_i \sim N(\mu_i\,,\ \sigma_i^2)$ for $i = 1, 2, \dots, k$, then the sum of those random variables

Normal

$$Y_1 = X_1 + X_2 + \cdots + X_k \sim N\left(\sum_{i=1}^{k} \mu_i \quad , \quad \sum_{i=1}^{k} \sigma_i^2\right)$$

$E(Y_1) = E(X_1 + X_2 + \cdots + X_k)$
$= E(X_1) + E(X_2) + \cdots + E(X_k)$
$\mu_1 + \mu_2 + \cdots + \mu_k$

$Var(Y_1) = Var(X_1 + X_2 + \cdots + X_k)$
$= Var(X_1) + Var(X_2) + \cdots + Var(X_k)$ all covariances are zero because of independence.

and the linear function of those random variables

Normal

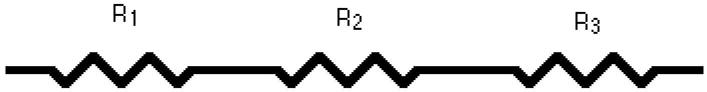
$$Y_2 = a_1 X_1 + a_2 X_2 + \cdots + a_k X_k \sim N\left(\sum_{i=1}^{k} a_i \mu_i \quad , \quad \sum_{i=1}^{k} a_i^2 \sigma_i^2\right)$$

$Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2ab\, Cov(X, Y)$

$\Rightarrow Var(X+Y) = Var(X) + Var(Y) + 2\, Cov(X, Y)$.

If X, Y are independent, $Cov(X,Y) = 0$.

Ex 3: Let R_1, R_2 and R_3 be resistance of randomly selected 3 resistors each with resistance independent from others.



The total resistance is $R = R_1 + R_2 + R_3$.

$\boxed{\mu = 10}$ $\qquad \sigma = 0.4 \Rightarrow \sigma^2 = 0.4^2$

Each resistance has a normal distribution with mean of 10 ohms and standard deviation of 0.4 ohms.

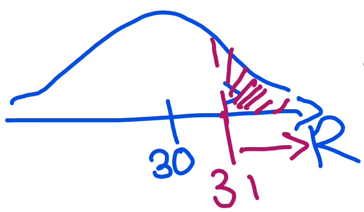
(a) Compute the probability that total resistance is more than 31 ohms. $P(R>31) = ?$

Since R is a sum of independent Normal (Gaussian) random variables, we know R has a Normal (Gaussian) distribution with

$$\text{Mean} = \sum_{i=1}^{3} \mu_i = \sum_{i=1}^{3} 10 = 30$$

$$\text{Variance} = \sum_{i=1}^{3} \sigma_i^2 = \sum_{i=1}^{3} (0.4)^2 = 0.48$$

Standard deviation of R



Then in MATLAB, use

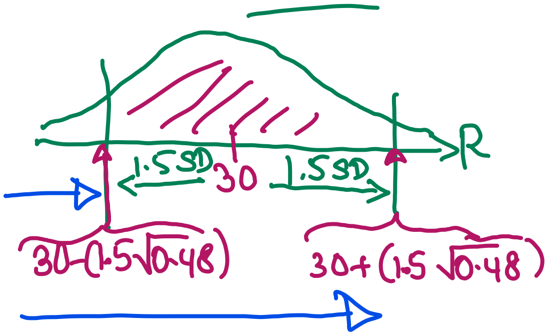
normcdf(31, 30, $\underbrace{\text{sqrt(0.48)}}_{}$, 'upper')

and get $P(R>31) = \underline{\underline{0.0745}}$ Rare

(b) Compute the probability that total resistance is within 1.5 standard deviations of the mean total resistance. 30

$$P\left(30 - 1.5\sqrt{0.48} < R < 30 + 1.5\sqrt{0.48}\right)$$

$$= P\left(R < 30 + 1.5\sqrt{0.48}\right) - P\left(R < 30 - 1.5\sqrt{0.48}\right)$$



In MATLAB, you can do

p = normcdf([30-1.5*sqrt(0.48), 30+1.5*sqrt(0.48)], 30, sqrt(0.48))

then do

p(2)-p(1)

and get $\underline{\underline{0.8664}}$ very likely