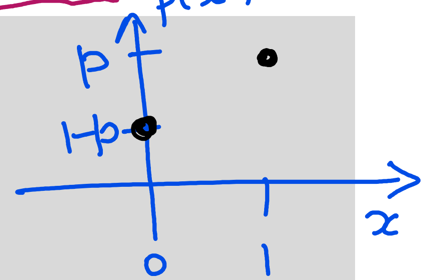
Specific Discrete Random Variables and Probability Models

Bernoulli Random Variable and Distribution

Binary

- The random variable is the number of successes in **one trial**
- There are only **2 disjoint and exhaustive outcomes of interest** in the trial (success, failure)
- The **probability of a "success"** is denoted by **p**

p is the parameter



X ~ Bernoulli (p) $\quad q = 1 - p$

sum is 1

$$\boxed{\textbf{pmf}}: \; p(x) = \begin{cases} \boxed{\begin{matrix} \mathbf{p} \\ \mathbf{1-p} \end{matrix}} & ; \; \mathbf{x = 1} \\ & ; \; \mathbf{x = 0} \\ \mathbf{0} & ; \; \textbf{otherwise} \end{cases} \qquad \text{and} \quad 0 < p < 1$$

$\mu =$ **Mean** : $E(X) = p = \sum [x\,p(x)] = (1 \times p) + (0 \times (1-p))$

$\sigma^2 =$ **Variance** : $Var(X) = p(1 - p)$

Ex 1: Roll a regular fair die once and let X be the number of times number 4 is observed. $\Omega = \{1, 2, 3, \boxed{4}, 5, 6\}$

Are there only 2 disjoint and exhaustive outcomes of interest for each trial? Yes

Success = Observing 4 , Failure = Observing other than 4 .

pmf of X is
$$p(x) = \begin{cases} 1/6 & ; x = 1 \\ 5/6 & ; x = 0 \\ 0 & ; \text{otherwise.} \end{cases}$$

Binomial Random Variable and Distribution

- The random variable is the number of successes in **n trials.**
- There is a **fixed number of identical Bernoulli trials**. The total number of trials is denoted by **n**
- There are only **2 disjoint and exhaustive outcomes of interest** for each trial (success, failure)
- The trials are **independent**
- The **probability of a "success" is a constant** across the trials. The probability of success is denoted by **p**

$1! = 1$
$0! = 1$

X ~ B (n, p) n, p are the parameters

$$\boxed{\textbf{pmf}}: \; p(x) = P(X = x) = \binom{\mathbf{n}}{\mathbf{x}} \mathbf{p^x (1-p)^{n-x}} \quad ; \quad x = 0, 1, 2, \ldots, n \quad \text{and} \quad 0 < p < 1$$

$\mu =$ **Mean** : $E(X) = np$

$\sigma^2 =$ **Variance** : $Var(X) = np(1 - p)$

$$\binom{\mathbf{n}}{\mathbf{x}} = {}^{\mathbf{n}}\mathbf{C_x} = \frac{\mathbf{n!}}{\mathbf{x!\,(n-x)!}} \quad \text{and} \quad q = 1 - p$$

$\binom{n}{x} = {}^nC_x = \begin{cases} \text{How many ways to select } x \text{ different items from } n \text{ different items.} \\ \text{How many ways to observed } x \text{ number of successes in } n \text{ number of trials.} \end{cases}$

For example: when you toss a coin 3 times, how many ways to observe 2 heads? HHT, HTH, THH

$n = 3, \; x = 2$

$$\binom{n}{x} = {}^nC_x = {}^3C_2 = \frac{3!}{2! \times (3-2)!} = \frac{3 \times 2 \times 1}{(2 \times 1) \times 1!} = \underline{\underline{3}}$$