Contents

1 Section 1.5: Quantifiers

Part 1: Evaluate the following. (Show your work to get full points).

(a) 7!

$$7 * 6 * 5 * 4 * 3 * 2 * 1 = \boxed{5040}$$

(b) $\sum_{x=1}^{20}[x]$

$$= 1 + 2 + 3 + ... + 18 + 19 + 20 = \boxed{210}$$

(c) $\sum_{i=1}^{20} w$

Sum formula, same as (b).

$$\sum_{k=1}^{n} k = \frac{1}{2}n(n+1)$$

$$= \frac{1}{2}(20)(21)$$

$$= \boxed{210}$$

(d) When c is a constant, $\sum_{x=1}^{3}\left[cx^3 + 1\right]$

$$= \left[c \cdot 1^3 + 1\right] + \left[c \cdot 2^3 + 1\right] + \left[c \cdot 3^3 + 1\right]$$

$$=18c + 3$$

$$= \boxed{3(6c+1)}$$

(e) Expand $(x-4)^2$

$$= x \cdot x + x \cdot (-4) + (-4) \cdot x + (-4) \cdot (-4)$$

$$= \boxed{x^2 + (-8)x + 16}$$

(f) For $\lambda > 0$, find $\sum_{x=0}^{\infty} \frac{\lambda^x}{x!}$

By Maclaurin series we have:

$$\sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = \boxed{e^\lambda}$$

(g) If $p(x) = \{ \ \frac{1}{8}; x = 0, 3\frac{3}{8}; x = 1, 20; \text{ otherwise} \quad ,$

then compute $\sum_{\text{all } x}[xp(x)]$

$$= (0 \cdot \frac{1}{8}) + (1 \cdot \frac{3}{8}) + (2 \cdot \frac{3}{8}) + (3 \cdot \frac{1}{8})$$

$$= (\frac{12}{8})$$

$$== \boxed{1\frac{1}{2}}$$

and compute $\sum_{\text{all } x}\left[(x - 1.5)^2 p(x)\right]$

$$= (-1.5^2 \cdot \frac{1}{8}) + (-0.5^2 \cdot \frac{3}{8}) + (0.5^2 \cdot \frac{3}{8}) + (1.5^2 \cdot \frac{1}{8})$$

$$= \boxed{\frac{3}{4}}$$

(h) $\int_1^3 x^2 dx$

$$= \left[\frac{x^3}{3} + c\right]\Big|_1^3$$

$$= \boxed{8.6667}$$

(i) $\int_0^1 \left(x^3 + 1\right) dx$

$$= \left[\frac{x^4}{4} + x + c\right]\Big|_0^1$$

$$= \frac{5}{4} = \boxed{1.25}$$

(j) When k is a constant, $\int_0^\infty \left[ke^{-\frac{x}{3}}\right] dx$

$$k \cdot \int_0^\infty \left[e^{-\frac{x}{3}}\right] dx$$

$$k \cdot \int_0^\infty \left[e^u\right] dx$$

$$u = -\frac{x}{3}$$

$$du = -\frac{1}{3}dx$$

$$dx = -3du$$

$$-3k \cdot \int_0^\infty \left[e^u\right] du$$

$$-3ke^u + C$$

$$= \boxed{-3ke^{-\frac{1}{3}} + C}$$

(k) If $f(x) = \begin{cases} 3x^{-4}; x > 1 \\ 0;\ \text{otherwise} \end{cases}$,

then compute $\int_{-\infty}^{\infty}[xf(x)]dx$

Since $(x \leq 1) \Rightarrow 0$,
I would change the bounds,
from 1 to $+\infty$ like so:

$\int_{1}^{\infty}[xf(x)]dx$

$\int_{1}^{\infty}[x \cdot 3x^{-4}]dx$

$3 \cdot \int_{1}^{\infty}[x^{-3}]dx$

$3 \cdot \int_{1}^{\infty}[\frac{1}{x^3}]dx$

$3 \cdot \int_{1}^{\infty}[\frac{1}{x^3}]dx$

$[-\frac{3}{2x^2}]\Big|_{0}^{\infty}$

$= \boxed{-\frac{3}{2x^2} + C}$

and compute:

$\int_{-\infty}^{\infty}[x^2 f(x)]dx$

Similar to first half,
restrict bounds of integration:

$\int_{1}^{\infty}[x^2 f(x)]dx$

$\int_{1}^{\infty}[x^2 \cdot 3x^{-4}]dx$

$3 \cdot \int_{1}^{\infty}[x^{-2}]dx$

$3 \cdot \int_{1}^{\infty}[\frac{1}{x^2}]dx$

$[-\frac{3}{x}]\Big|_{0}^{\infty}$

$= \boxed{-\frac{3}{x} + C}$

(l) $\int_0^y ye^{-y}e^{-x}dx$

$\int_0^y ye^{-y-x}dx$

$\int_0^y ye^{u}dx$

$u = -x - y$

$du = -1dx$

$dx = -du$

$-y \cdot \int_0^y e^{u}dx$

$-y \cdot e^{u}$

$= \boxed{-y \cdot e^{-y-x} + C}$

(m) $\int_0^1 \int_0^x \left(1 + x^2y\right) dydx$

$\int_0^x \left(1 + x^2y\right) dy$

$y + x^2 \cdot \frac{y^2}{2}\Big|_0^x$

$x + \frac{x^4}{2} + C$

$\int_0^1 [x + \frac{x^4}{2}]dx$

$\frac{x^2}{2} + \frac{x^5}{(2)(4)}\Big|_0^1$

$= \boxed{\frac{5}{8}}$

(n) If $p(x,y) = \begin{cases} \frac{1}{3}; (x,y) = (-1,1), (0,0), (1,1), \\ 0;\ \text{otherwise} \end{cases}$

then compute $\sum_{\text{all } x} \sum_{\text{all } y}[xp(x,y)]$

-1$\cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 0$

$= \boxed{0}$

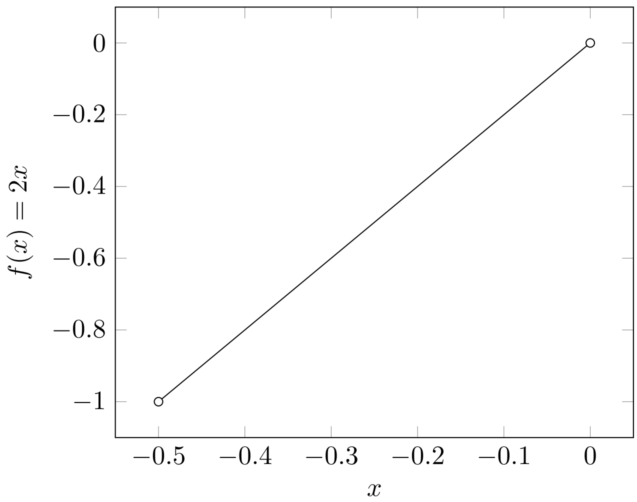
and compute $\sum_{\text{all } x} \sum_{\text{all } y}[xyp(x,y)]$

(-1)(1)$\cdot \frac{1}{3} + (0)(0) \cdot \frac{1}{3} + (1)(1) \cdot \frac{1}{3} + 0$

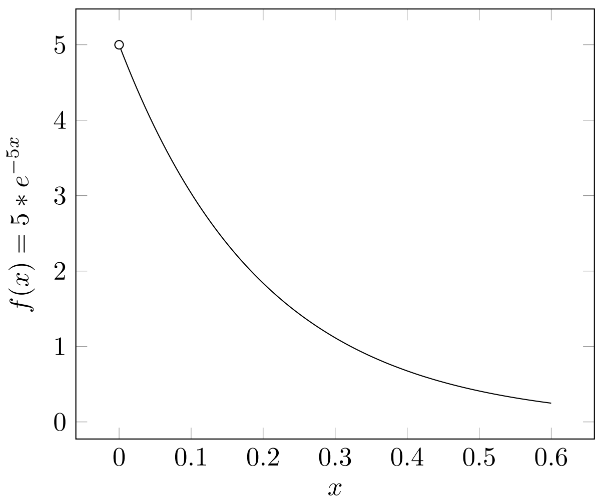
$= \boxed{0}$

Part 2: Sketch each of the following functions on separate x -y planes

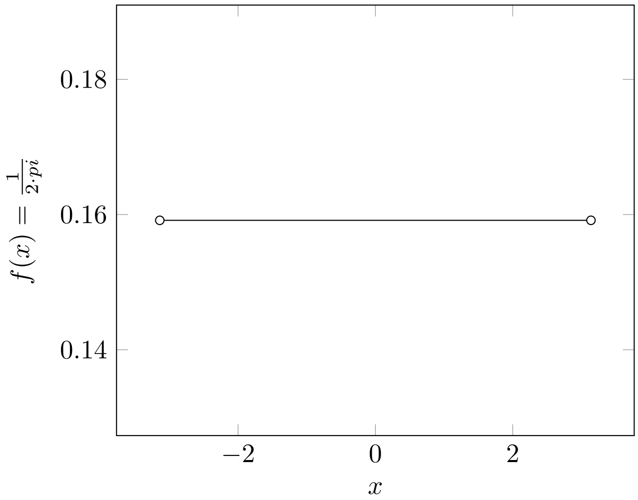
(a) $f(x) = \begin{cases} 2x; -0.5 < x < 0 \\ 0; \text{otherwise} \end{cases}$



(b) $f(x) = \begin{cases} 5e^{-5x}; x > 0 \\ 0; \text{otherwise} \end{cases}$



(c) $f(x) = \begin{cases} \frac{1}{2\pi} & ; -\pi < x < \pi \\ 0; & \text{otherwise} \end{cases}$



(d) $f(x) = \begin{cases} 0 & ; x < 0 \\ \frac{x^2}{4} & ; 0 \leq x < 1 \\ \frac{x+1}{4}; & 1 \leq x < 2 \\ 1; & x \geq 2 \end{cases}$