Specific Probability Density Functions

Normal or Gaussian Random Variable and Distribution

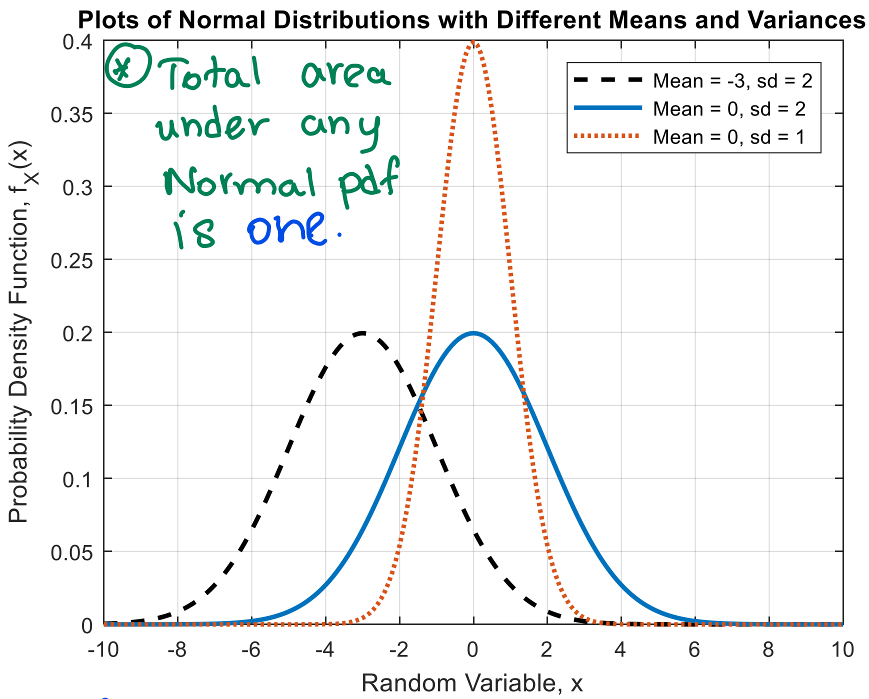
The normal distribution is the most widely used continuous probability distribution, mainly because it is tractable analytically, it follows the familiar bell shape which is consistent with a lot of population models, and the Central Limit Theorem says that, with a large enough sample, the normal distribution can be used to approximate a large variety of other distributions (e.g., Normal approximation to the Binomial). There are other continuous distributions such as chi-square, t and F distributions which are all by-products of the normal distribution.

$\mathbf{X} \sim \mathbf{N}(\mu, \sigma^2)$ μ, σ^2 are parameters.

$$\textbf{pdf}: \; f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} \quad ; \; -\infty < x < \infty, \qquad -\infty < \mu < \infty, \; \sigma > 0$$

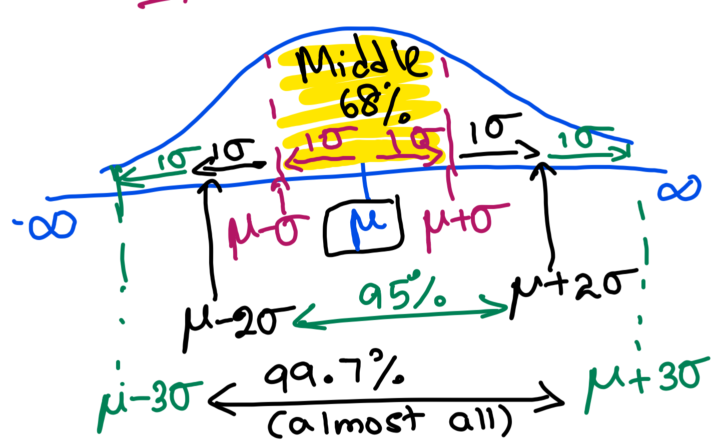
$\textbf{Mean}: E(X) = \mu$

$\textbf{Variance}: Var(X) = \sigma^2$



(*) Total area under any Normal pdf is one.

Empirical Rule



Six Sigma.

(*) If σ is smaller, Normal pdf is narrower and taller.

(*) If σ is higher, Normal pdf is wider and shorter.

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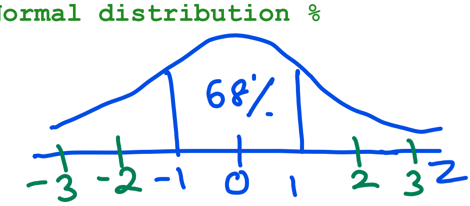
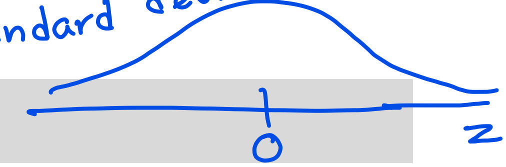
figure(1)
x = -10:0.1:10; % Set the range of values for Normal random variable%
mean1 = -3; std1 = 2; %Set the mean and standard deviation of 1st Normal distribution %
mean2 = 0; std2 = 2; %Set the mean and standard deviation of 2nd Normal distribution %
mean3 = 0; std3 = 1; %Set the mean and standard deviation of 3rd Normal distribution %
y1 = normpdf(x, mean1, std1); % Normal with mean1 and std1 %
y2 = normpdf(x, mean2, std2); % Normal with mean2 and std2 %
y3 = normpdf(x, mean3, std3); % Normal with mean3 and std3 %
plot(x, y1, '--k', x, y2, '-', x, y3, ':','LineWidth',2)
xlabel('Random Variable, x')
ylabel('Probability Density Function, f_X(x)')
title(['Plots of Normal Distributions with Different Means and Variances'])
legend([' Mean = ' num2str(mean1) , ', sd = ' num2str(std1)] , [' Mean = '
num2str(mean2), ', sd = '  num2str(std2)]  , [' Mean = ' num2str(mean3), ', sd = '
num2str(std3)])
grid on; % Adds grid on the plot %
```



Z-Score Standardizing Standard deviation

If X has a $N(\mu, \sigma^2)$, then $\boxed{Z = \dfrac{X-\mu}{\sigma}}$ has a $\boxed{N(0,1)}$



The disribution of Z is called the **standard normal distribution**

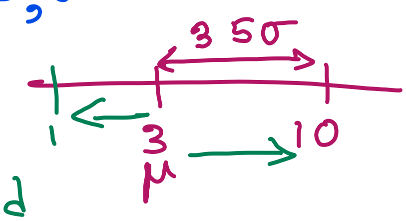
This is a special normal distribution.

Each value of Z describes the number of standard deviations each value of X is away from mean of X.

⊛ Z-score can be negative, 0, or positive.

For example, if a data value is 10, $\mu = 3$, $\sigma = 2$, then Z value for 10 is

$$Z = \frac{X-\mu}{\sigma} = \frac{10-3}{2} = 3.5$$



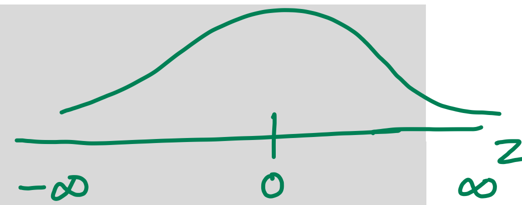
Therefore, data value 10 is 3.5 standard deviations higher than the mean.

Z ~ **N**(0, 1)

pdf : $f(z) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$; $-\infty < z < \infty$

Mean : $E(Z) = 0$

Variance : $Var(Z) = 1$



Use MATLAB to compute probability from Normal (Gaussian) Density:

Let random variable X has a normal distribution with mean, $\mu = 5$ and standard deviation, $\sigma = 2$.

(a) Find $P(X < 2.85)$ using **normcdf(2.85, 5, 2)** and get **0.1412**

(b) Find $P(X > 2.85)$ using **normcdf(2.85, 5, 2, 'upper')** and get **0.8588**

(c) Find $P(1 < X < 2.85)$ using **p = normcdf([1, 2.85], 5, 2)** and then using **p(2)-p(1)** to get **0.1184**

Let random variable Z has a normal distribution with mean, $\mu = 0$ and standard deviation, $\sigma = 1$. Find $P(Z < 2.85)$ using **normcdf(2.85)** and get **0.9978**

Use MATLAB to get the value of variable for a given probability from Normal Density:

Let random variable X has a normal distribution with mean, $\mu = 5$ and standard deviation, $\sigma = 2$, then

a) to find k so that $P(X < k) = 0.548$, use **norminv(0.548, 5, 2)** and get **5.2412**

b) to find s so that $P(X > s) = 0.7251$, use **norminv(1- 0.7251, 5, 2)** and get **3.8039**

Let random variable Z has a normal distribution with mean, $\mu = 0$ and standard deviation, $\sigma = 1$. Find d such that $P(Z < d) = 0.548$ using **norminv(0.548)** and get **0.1206**