$$E(aX+bY+c) = aE(X)+bE(Y)+c$$
$$Var(aX+bY+c) = Var(aX+bY) = a^2 Var(X) + b^2 Var(Y) + 2ab\,Cov(X,Y)$$

Ex 4: If X, Y are independent random variables such that $E(X) = E(Y) = \mu$, $Var(X) = Var(Y) = \sigma^2$,

find

$$E\left(\frac{X+Y}{2}\right) = E\left[\overset{a}{\tfrac{1}{2}}X + \overset{b}{\tfrac{1}{2}}Y\right] = \frac{1}{2}\underbrace{E(X)}_{\mu} + \frac{1}{2}\underbrace{E(Y)}_{\mu} = \underline{\underline{\mu}} = E(X)$$

$$Var\left(\frac{X+Y}{2}\right) = Var\left[\tfrac{1}{2}X + \tfrac{1}{2}Y\right] = \left(\tfrac{1}{2}\right)^2 \underbrace{Var(X)}_{\sigma^2} + \left(\tfrac{1}{2}\right)^2 \underbrace{Var(Y)}_{\sigma^2} + 2\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)\underbrace{Cov(X,Y)}_{\text{zero because } X, Y \text{ are independent}}$$

$$= \frac{1}{4}\sigma^2 + \frac{1}{4}\sigma^2 = \underline{\underline{\frac{\sigma^2}{2}}} = \frac{Var(X)}{2}$$

The above results can be extended to n independent random variables:

In general, if $X_1, X_2, \dots, X_n$ are independent random variables with $E(X_i) = \mu$, and $Var(X_i) = \sigma^2$, then the **mean (or average)** of these random variables is a random variable and given by

"X bar"

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{\sum_{i=1}^{n} X_i}{n} = \overbrace{\frac{1}{n}X_1 + \frac{1}{n}X_2 + \cdots + \frac{1}{n}X_n}^{n \text{ terms}}$$

and

$$* \quad E(\bar{X}) = \frac{1}{n}\overbrace{E(X_1)}^{\mu} + \frac{1}{n}\overbrace{E(X_2)}^{\mu} + \cdots + \frac{1}{n}\overbrace{E(X_n)}^{\mu}$$
$$= \frac{n\mu}{n}$$
$$= \mu$$

$\Rightarrow$ The distribution of $\bar{X}$ is centered at μ

$$* \quad Var(\bar{X}) = \left(\frac{1}{n}\right)^2 \underbrace{Var(X_1)}_{\sigma^2} + \left(\frac{1}{n}\right)^2 \underbrace{Var(X_2)}_{\sigma^2} + \cdots + \left(\frac{1}{n}\right)^2 \underbrace{Var(X_n)}_{\sigma^2}$$

All cov are zero

$$= \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

(*) Standard Deviation of $\bar{X}$ is $\sqrt{Var\,\bar{X}} = \frac{\sigma}{\sqrt{n}}$

This is the typical (average) difference between $\bar{X}$ and μ.

(*) Central Limit Theorem (CLT) (*)

The distribution of $\bar{X}$ is approximately Normal (Gaussian) when n is large. $\Rightarrow$