Discrete : PMF $p(x)=P(X=x)$

$-\infty$ ∞

Continuous Random Variables and Probability Density Functions

Recall: If the sample space of experiment or support of a random variable is uncountably infinite, then the random variable is a **continuous random variable.**

Probability density functions are associated with (absolutely) continuous random variables. For continuous random variables, the probability of any point value is zero (i.e., $P(X = a) = 0$). As a result, we define the probability density function (pdf) for a continuous random variable differently.

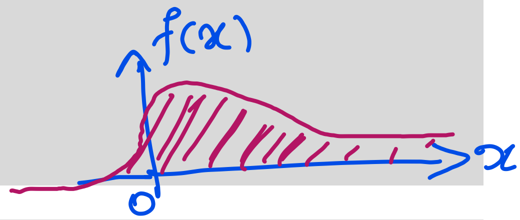
Definition: (Probability Density Function, pdf)

The **probability density function** or **pdf** of a continuous random variable X, denoted by $f(x)$ or $f_X(x)$, is such that

$f(x) \neq P(X=x)$

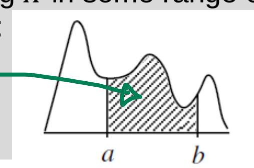
- $f(x) \geq 0$ for all x in X
- $\int_{-\infty}^{\infty} f(x)dx = 1$ $= P(-\infty < X < \infty)$

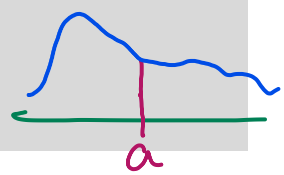
Total probability is 1 $= P(\Omega) = 1$



The **pdf** is a curve that describes the probability of observing X in some range of values, such as between a and b where $a < b$. The probability is defined as:

$$\boldsymbol{P(a < X < b) = \int_a^b f(x)\,dx}$$





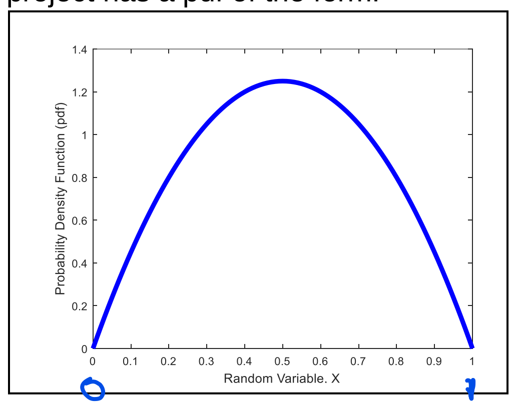
$P(X=a) = 0 = \int_a^a f(x)\,dx$

Ex 1: The time (measured in years), X, required to complete a software project has a pdf of the form:

$$f(x) = \begin{cases} kx(1-x)\,; & 0 \leq x \leq 1 \\ 0\,; & \text{Otherwise} \end{cases}$$

For a valid pdf

$\int_{-\infty}^{\infty} f(x)dx = 1$ and $f(x) \geq 0$ for all possible x



(a) Find the value of k so that $f(x)$ is a valid pdf.

Set $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{0} (0)\,dx + \int_0^1 kx(1-x)\,dx + \int_1^{\infty} (0)\,dx = 1$

$k\int_0^1 x(1-x)\,dx = 1 \Rightarrow k = 6.$

Then $f(x) = 6x(1-x)$ is non-negative when $0 \leq x \leq 1$.

Therefore, $k = 6$.

(b) Compute the probability that the project will be completed in less than 4 months.