Probability Rules

Introduction

Experiment is an activity from which an outcome (or result) is obtained.

Outcome of an experiment is subjected to uncertainty (i.e. exact result is unknown until the experiment is performed). However, possible results are known before conducting the experiment. Such a study is called a **random experiment.**

- **Ex 1**: Toss a coin once and observe the top face. All possible outcomes are: H, T
- **Ex 2**: Roll a pair (2) of regular (6-side) dice. Suppose the total on the dice is of interest. All possible outcomes are: 2, 3, 4, 5, 6, . . . , 12
- **Ex 3 (Computer memories)**: Suppose we store an n-bit word consisting of all 0s at a particular location. When we read it back, we may not get all 0s. In fact, any n-bit word may be read out if the memory location is faulty. The set of all possible n-bit words are: $\{(b_1, \dots, b_n) \mid b_i = 0 \text{ or } 1\}$. If $n = 2$, then possible outcomes are: (0,0), (0,1), (1,0), (1,1) or 00, 10, 01, 11
- **Ex 4 (Optical communication systems)**: Since the output of a photodetector is a random number of photoelectrons. The logical possible outcomes are nonnegative integers. That is, {0,1,2, . . .}. Notice that we include 0 to account for the possibility that no photoelectrons are observed.
- **Ex 5 (Wireless communication systems)**: Noncoherent receivers measure the energy of the incoming waveform. Since energy is a nonnegative quantity, the possible outcomes are nonnegative real numbers. That is, $[0, \infty)$ Any value from 0 to ∞.

Sample Space is a set of all possible outcomes (or results) of a random experiment and denoted by **Ω**.

- **Ex 1**: Ω = { H , T }
- **Ex 2**: Ω = { 2 , 3 , . . . , 12 }
- **Ex 3**: Ω = { 00 , 01 , 10 , 11 }
- **Ex 4**: Ω = { 0 , 1 , 2 , ... }
- **Ex 5**: Ω = $\{x \mid x \in \mathbb{R}^+ \text{ or } x = 0\}$ $\mathbb{R}^+$ = Positive real numbers.

Mathematical Notation for listing possible outcomes:

{ x | membership rule}

Ex: $\left\{x \;\middle|\; \frac{x}{2} \in \mathbb{Z}\right\}$ conditioned $\{0, -1, 1, -2, 2, \dots\}$

which is read as "all the x values given (or knowing) that x/2 is an element of (∈) ℤ" where ℤ indicates integers. That is, { 0, −2, 2, −4, 4, −6, 6, . . . }

Event is a subset (small collection of outcomes) of sample space and denoted by a capitol letter.
Event and set are interchangeable.

$2 \times 2 \times 2 = 2^3 = 8$ outcomes in Ω

Ex 6: If the sample space is the set of all triples (b_1,b_2,b_3), where the b_i are 0 or 1, then any particular triple, say (0,0,0) or (1,0,1) would be an outcome.

Event W would be a subset such as the set of all triples with exactly one 1; that is, W = { (0,0,1), (0,1,0), (1,0,0) }

Ex 7: Which of following are possible events when you toss a coin once and observe the top face?

Sample space = Ω = { H , T }

✓ A = {H is observed } = { H }

✓ B = {either H or T is observed} = { H , T }

Null Set (Empty Event) is an event with no elements and denoted by Ø

In example 7, C = {neither H nor T is observed} = { } = $\emptyset$

Definitions of Set Operations:

Consider two events A, B. Then

- **Complement of** $\boldsymbol{A}$ is defined as a set of all elements in Ω that are not in A. It is denoted by $\boldsymbol{A'}$, $\boldsymbol{A^c}$, **or** $\boldsymbol{\bar{A}}$.

- **Intersection** of two events A, B is defined as a set of all elements common to events A, B. It is denoted by $\boldsymbol{A \cap B}$. It can also denoted by $\boldsymbol{A \text{ and } B}$.

- **Union** of events A, B is defined as a set of all elements in A or B or both. It is denoted by $\boldsymbol{A \cup B}$. It can also denoted by $\boldsymbol{A \text{ or } B}$.

- Event B is a **subset** of event A if every element in B is also in A. It is denoted by $\boldsymbol{B \subset A}$. This is also said as B is contained in A.

❖ **The null set is a subset of every event.**

Venn Diagrams are useful to see set operations. Events are represented by regions.