機器人學 project1

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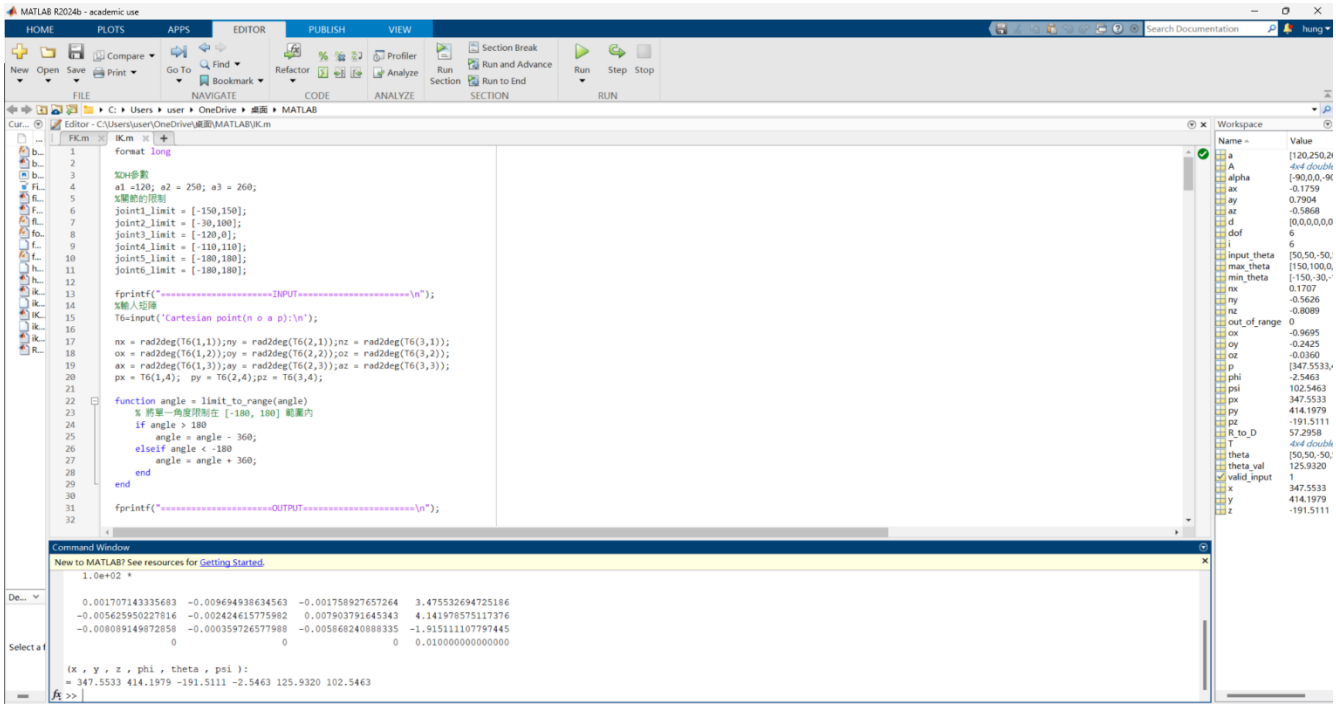
一、介面說明

開發平台 ： MATLAB



開啟程式碼後依照下列程序:

1. 程式編輯視窗
2. 按下 “Run” 執行程式碼
3. 在終端機中輸入要執行的程式
4. 依照指示輸入變數後按“Enter”

二、程式架構說明

Fk.m

```
% DH Model - Puma 560
d = [0, 0, 0, 0, 0, 0];
a = [120, 250, 260, 0, 0, 0];
alpha = [-90, 0, 0, -90, 90, 0];
theta = [0, 0, 0, 0, 0, 0];
max_theta = [150, 100, 0, 110, 180, 180];
min_theta = [-150, -30, -120, -110, -180, -180];
```

```
% calculate the transformation matrices
for i = 1:6
    A = [cosd(theta(i)), -sind(theta(i)) * cosd(alpha(i)), sind(theta(i)) * sind(alpha(i)), a(i) * cosd(theta(i));
         sind(theta(i)),  cosd(theta(i)) * cosd(alpha(i)), -cosd(theta(i)) * sind(alpha(i)), a(i) * sind(theta(i));
         0,                   sind(alpha(i)),                      cosd(alpha(i)),                     d(i);
         0,                   0,                                   0,                                  1];
    T = T * A;
end
```

執行程式後，可以看到

```
Forward kinematics
Please enter the joint variable (in rad):
theta1, theta2 , theta3 , theta4 , theta5, theta6 :
```

輸入 6 個(角度)來得到轉移矩陣 T6(我自行將弧度轉角度)

測資一:

[20 20 -20 20 20 20]

得到輸出:

```
[n o a p]:
   1.0e+02 *

   0.007797282437679  -0.006258180148738  -0.000193824180657   5.778387512885174
  -0.000582222717775  -0.001032939777916   0.009929453767560   2.103161056745715
  -0.006234051916208  -0.007730990663630  -0.001169777784405  -0.855050358314172
                   0                   0                   0   0.010000000000000

(x , y , z , phi , theta , psi ):
= 577.8388 210.3161 -85.5050 -51.1183 96.7177 91.1183
```

測資二:

[50 50 -50 50 50 50 50]

得到輸出:

```
[n o a p]:
   1.0e+02 *

    0.001707143335683  -0.009694938634563  -0.001758927657264   3.475532694725186
   -0.005625950227816  -0.002424615775982   0.007903791645343   4.141978575117376
   -0.008089149872858  -0.000359726577988  -0.005868240888335  -1.915111107797445
                    0                   0                   0   0.010000000000000

(x , y , z , phi , theta , psi ):
= 347.5533 414.1979 -191.5111 -2.5463 125.9320 102.5463
```

Ik.m

程式架構如程式中註解說明(由於程式過於冗長，故此不附上)

程式執行後，可輸入末端 Cartesian space 座標(n,o,a,p)

按下 enter 後得到 8 組 IK 的解，以及各軸是否超出工作限制的結果:

測資一:

```
Cartesian point(n o a p):
[0.5756 -0.2398 -0.7817 177.8;
0.7738 -0.1494 0.6156 308;
-0.2644 -0.9593 0.0996 -140.1
0 0 0 1.0000;]
```

測資二:

```
Cartesian point(n o a p):          ==========
[0.1736 0.0000 -0.9848 0.0000;
0.8529 0.5000 0.1504 325.2;
0.4924 -0.8660 0.0868 -158
0 0 0 1.0000;]
======================OUTPUT======================
your theta_list1 is:
joint3 is out of range!
joint4 is out of range!
60.003  -28.531  115.004  -121.456  10.005  -49.981
================================
your theta_list2 is:
joint3 is out of range!
60.003  -28.531  115.004  58.544  -10.005  130.019
================================
your theta_list3 is:
60.003  89.999  -115.004  -9.978  10.005  -49.981
================================
your theta_list4 is:
joint4 is out of range!
60.003  89.999  -115.004  170.022  -10.005  130.019
================================
your theta_list5 is:
joint2 is out of range!
joint3 is out of range!
-119.997  149.781  27.071  38.131  169.995  130.019
================================
your theta_list6 is:
joint2 is out of range!
joint3 is out of range!
joint4 is out of range!
-119.997  149.781  27.071  -141.869  -169.995  -49.981
================================
your theta_list7 is:
joint2 is out of range!
-119.997  177.394  -27.071  64.661  169.995  130.019
================================
your theta_list8 is:
joint2 is out of range!
joint4 is out of range!
-119.997  177.394  -27.071  -115.339  -169.995  -49.981
```

```
your theta_list1 is:
joint3 is out of range!
joint4 is out of range!
90.000  -23.812  119.002  -125.180  10.000  0.008
================================
your theta_list2 is:
joint3 is out of range!
90.000  -23.812  119.002  54.820  -10.000  -179.992
================================
your theta_list3 is:
90.000  99.003  -119.002  -9.992  10.000  0.008
================================
your theta_list4 is:
joint4 is out of range!
90.000  99.003  -119.002  170.008  -10.000  -179.992
================================
your theta_list5 is:
joint2 is out of range!
joint3 is out of range!
-90.000  137.862  44.283  27.846  170.000  -179.992
================================
your theta_list6 is:
joint2 is out of range!
joint3 is out of range!
joint4 is out of range!
-90.000  137.862  44.283  -152.154  -170.000  0.008
================================
your theta_list7 is:
joint2 is out of range!
-90.000  -176.941  -44.283  71.214  170.000  -179.992
================================
your theta_list8 is:
joint2 is out of range!
-90.000  -176.941  -44.283  -108.786  -170.000  0.008
```

三、數學運算說明、推導

$$A_{0n}=\begin{bmatrix} c\theta_n & -s\theta_n c\alpha_n & s\theta_n s\alpha_n & a_n c\theta_n \\ s\theta_n & c\theta_n c\alpha_n & -c\theta_n s\alpha_n & a_n s\theta_n \\ 0 & s\alpha_n & c\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1=\begin{bmatrix} c_1 & 0 & -s_1 & 120c_1 \\ s_1 & 0 & c_1 & 120s_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_4=\begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2=\begin{bmatrix} c_2 & -s_2 & 0 & 250c_2 \\ s_2 & c_2 & 0 & 250s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_5=\begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3=\begin{bmatrix} c_3 & -s_3 & 0 & 260c_3 \\ s_3 & c_3 & 0 & 260s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_6=\begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5A_6=\begin{bmatrix} c_5c_6 & -s_6c_5 & s_5 & 0 \\ s_5c_6 & -s_5s_6 & -c_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4A_5A_6=\begin{bmatrix} c_5c_6c_4-s_4s_6 & -c_4s_6c_5-s_4c_6 & s_5c_4 & 0 \\ s_4c_5c_6+c_4s_6 & -s_4s_6c_5+c_4c_6 & s_5s_4 & 0 \\ -s_5c_6 & s_5s_6 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3A_4A_5A_6=\begin{bmatrix} c_3c_4c_5c_6-s_3s_4s_6 & -c_3c_4c_5s_6-s_3s_4c_6 & c_3c_4s_5 & a_3c_3 \\ s_3c_4c_5c_6+c_3c_4s_6 & -s_3c_4c_5s_6+c_3c_4c_6 & s_3c_4s_5 & a_3s_3 \\ -s_5c_6 & s_5s_6 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2A_3A_4A_5A_6=\begin{bmatrix} c_{23}c_4c_5c_6-s_{23}s_4s_6 & -c_{23}c_4c_5s_6-s_{23}s_4c_6 & c_{23}c_4s_5 & 250c_2+260c_{23} & a_3c_2c_3-a_3s_2s_3+a_2c_2 \\ s_{23}c_4c_5c_6+c_{23}s_4s_6 & -s_{23}c_4c_5s_6+c_{23}s_4c_6 & s_{23}c_4s_5 & 250s_2+260s_{23} & a_3s_2c_3+a_3c_2s_3+a_2s_2 \\ -s_5c_6 & s_5s_6 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$T_6 = [n \; o \; a \; p]$

$n_x = C_1(C_{234}C_5C_6 - S_{234}S_6) + S_1S_5C_6$

$n_y = S_1(C_{234}C_5C_6 - S_{234}S_6) - C_1S_5C_6$

$n_z = -S_{234}C_5C_6 - C_{234}S_6$

$o_x = -C_1(C_{234}C_5S_6 + S_{234}C_6) - S_1S_5S_6$

$o_y = -S_1(C_{234}C_5S_6 + S_{234}C_6) + C_1S_5S_6$

$o_z = S_{234}C_5S_6 - C_{234}C_6$

$a_x = C_1C_{234}S_5 - S_1C_5$

$a_y = S_1C_{234}S_5 + C_1C_5$

$a_z = -S_{234}S_5$

$P_x = 120C_1 + 250C_1C_2 + 260C_1C_{23}$

$P_y = 120S_1 + 250S_1C_2 + 260S_1C_{23}$

$P_z = -250S_2 - 260S_{23}$ *

Inverse kinematic:

$$A_1^{-1} = \begin{bmatrix} C_1 & S_1 & 0 & -120 \\ 0 & 0 & -1 & 0 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2^{-1} = \begin{bmatrix} C_2 & S_2 & 0 & -250 \\ -S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3^{-1} = \begin{bmatrix} C_3 & S_3 & 0 & -260 \\ -S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4^{-1} = \begin{bmatrix} C_4 & S_4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S_4 & C_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5^{-1} = \begin{bmatrix} C_5 & S_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ S_5 & -C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6^{-1} = \begin{bmatrix} C_6 & S_6 & 0 & 0 \\ -S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^{-1}T_6 = \begin{bmatrix} C_1 & S_1 & 0 & -120 \\ 0 & 0 & -1 & 0 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot & a_3C_2C_3 - a_3S_2S_3 + a_2C_2 \\ \cdot & \cdot & \cdot & a_3S_2C_3 + a_3C_2S_3 + a_2S_2 \\ \cdot & \cdot & \cdot & 0 \\ & & & 1 \end{bmatrix}$$

$$\begin{cases} P_xC_1 + P_yS_1 - 120 = a_3C_2C_3 - a_3S_2S_3 + a_2C_2 \quad ① \\ -P_z = a_3S_2C_3 + a_3C_2S_3 + a_2S_2 \quad ② \\ -P_xS_1 + P_yC_1 = 0 \quad ③ \end{cases}$$

③

$$\therefore \tan\theta_1 = \frac{P_y}{P_x} \quad \therefore \theta_1 = \tan^{-1}\left(\frac{P_y}{P_x}\right) \text{ or } \tan^{-1}\left(\frac{P_y}{P_x}\right) + 180^\circ \ \#$$

$①^2 + ②^2$

$$\therefore (P_x C_1 + P_y S_1 - 120)^2 + (P_z)^2 = 260^2 C_3^2 + 260^2 S_3^2 + 250^2 + 2\left(260 \times 250 C_2^2 C_3 + 260 \times 250 S_2^2 C_3\right)$$

$$= 260^2 + 250^2 + 2(250 \times 260 \times C_3)$$

$$\therefore \theta_3 = \pm\cos^{-1}\left(\frac{(C_1 P_x + S_1 P_y - 120)^2 + P_z^2 - 250^2 - 260^2}{2 \times 250 \times 260}\right) \ \#$$

$$A_3^{-1} A_2^{-1} A_1^{-1} \times T = A_4 A_5 A_6$$

$$\therefore \begin{bmatrix} C_1C_{23} & S_1C_{23} & -S_{23} & -120C_{23} - 250C_3 - 260 \\ -C_1S_{23} & -S_1S_{23} & -C_{23} & 120S_{23} + 250S_3 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times T = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & C_4S_5 & 0 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & 0 \\ -S_5C_6 & S_5S_6 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore C_1C_{23}P_x + S_1C_{23}P_y - S_{23}P_z - 120C_{23} - 250C_3 - 260 = 0$$

$$-C_1S_{23}P_x - S_1S_{23}P_y - C_{23}P_z + 120S_{23} + 250S_3 = 0$$

$$C_{23}(C_1P_x + S_1P_y - 120) + S_{23}(-P_z) = 250C_3 + 260 \quad — ①$$

$$-S_{23}(C_1P_x + S_1P_y - 120) + C_{23}(-P_z) = -250S_3 \quad — ②$$

$$\sqrt{①^2 + ②^2} = \sqrt{(C_1P_x + S_1P_y - 120)^2 + (-P_z)^2} = \sqrt{(250C_3 + 260)^2 + (-250S_3)^2} = t$$

$$\sin\phi = \frac{C_1P_x + S_1P_y - 120}{t}, \quad \cos\phi = \frac{-P_z}{t}$$

$$\Rightarrow \tan\phi = \frac{C_1P_x + S_1P_y - 120}{-P_z} \quad \Rightarrow \phi = \tan^{-1}\left(\frac{C_1P_x + S_1P_y - 120}{-P_z}\right)$$

$$\cos(\theta_2+\theta_3)\sin\phi + \sin(\theta_2+\theta_3)\cos\phi = \sin(\theta_2+\theta_3+\phi) = \frac{250C_3+260}{t}$$

$$-\sin(\theta_2+\theta_3)\sin\phi + \cos(\theta_2+\theta_3)\cos\phi = \cos(\theta_2+\theta_3+\phi) = \frac{-250S_3}{t}$$

$$\Rightarrow \tan(\theta_2+\theta_3+\phi) = \frac{250C_3+260}{-250S_3}$$

$$\Rightarrow \theta_2+\theta_3+\phi = \tan^{-1}\left(\frac{250C_3+260}{-250S_3}\right)$$

$$\Rightarrow \theta_2 = \tan^{-1}\left(\frac{250C_3+260}{-250S_3}\right) - \tan^{-1}\left(\frac{C_1P_x+S_1P_y-120}{-P_z}\right) - \theta_3 \quad \text{※}$$

$$C_1C_{23}a_x + S_1C_{23}a_y - S_{23}a_z = C_4S_5$$

$$-C_1S_{23}a_x - S_1S_{23}a_y - C_{23}a_z = S_4S_5$$

$$C_{23}(C_1a_x+S_1a_y) - S_{23}a_z = C_4S_5$$

$$-S_{23}(C_1a_x+S_1a_y) - C_{23}a_z = S_4S_5$$

$$\tan\theta_4 = \frac{S_4S_5}{C_4S_5} = \frac{-S_{23}(C_1a_x+S_1a_y)-C_{23}a_z}{C_{23}(C_1a_x+S_1a_y)-S_{23}a_z}$$

$$\Rightarrow \theta_4 = \tan^{-1}\left(\frac{-S_{23}(C_1a_x+S_1a_y)-C_{23}a_z}{C_{23}(C_1a_x+S_1a_y)-S_{23}a_z}\right) \quad \text{or} \quad \tan^{-1}\left(\frac{-S_{23}(C_1a_x+S_1a_y)-C_{23}a_z}{C_{23}(C_1a_x+S_1a_y)-S_{23}a_z}\right) + 180^\circ \quad \text{※}$$

$$A_4^{-1}A_5^{-1}A_2^{-1}A_1^{-1}T = A_5 \cdot A_1$$

$$= \begin{bmatrix} C_{234} & S_1C_{234} & -S_{234} & -120C_{234} - 250C_{34} - 260C_4 \\ S_1 & -C_1 & 0 & 0 \\ -C_1S_{234} & -S_1S_{234} & -C_{234} & 120S_{234} + 250S_{34} + 260S_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} T = \begin{bmatrix} C_5C_6 & -C_5S_6 & S_5 & 0 \\ S_5C_6 & -S_5S_6 & -C_5 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_1C_{234}a_x + S_1C_{234}a_y - S_{234}a_z = S_5$$

$$S_1a_x - C_1a_y = -C_5$$

$$\tan\theta_5 = \frac{S_5}{C_5} = \frac{C_1C_{234}a_x + S_1C_{234}a_y - S_{234}a_z}{-S_1a_x + C_1a_y}$$

$$\theta_5 = \tan^{-1}\left(\frac{C_1C_{234}a_x + S_1C_{234}a_y - S_{234}a_z}{-S_1a_x + C_1a_y}\right) \quad \text{※}$$

$$-C_1 S_{234} n_x - S_1 S_{234} n_y - C_{234} n_z = S_6$$

$$-C_1 S_{234} o_x - S_1 S_{234} o_y - C_{234} o_z = C_6$$

$$\tan\theta_6 = \frac{C_1 S_{234} n_x + S_1 S_{234} n_y + C_{234} n_z}{C_1 S_{234} o_x + S_1 S_{234} o_y + C_{234} o_z}$$

$$\therefore \theta_6 = \tan^{-1}\left(\frac{C_1 S_{234} n_x + S_1 S_{234} n_y + C_{234} n_z}{C_1 S_{234} o_x + S_1 S_{234} o_y + C_{234} o_z}\right) \text{※}$$

四、加分題：討論兩種逆向運動學(代數法，幾何法)的優缺點

代數法（Algebraic Method）

優點：

1.精確性：代數法通常能夠提供較為準確的解，特別是對於複雜的多關節系統。

2.通用性：代數法的應用不受機械結構的特定形狀限制，可以用於各種機械系統。

3.高效性： 在某些情況下，代數法的計算速度可能比幾何法更快，尤其是當機械結構複雜且需要高精度時。

缺點：

1.複雜性：代數法的數學運算可能變得非常複雜，特別是對於具有大量關節的系統，導致解的公式難以推導和實現。

2.數學求解困難： 對於一些非線性或特殊機構，代數法的求解可能變得非常困難，並且可能需要使用數值方法。

幾何法（Geometric Method）

優點：

1.簡單直觀：幾何法基於三角學和幾何關係，通常比代數法更容易理解和實現。

2.即時反饋： 幾何法通常可以提供即時的解，這對於需要快速反應的應用（如機器人的運動控制）是有利的。

3.易於調整： 對於一些簡單的機械結構，可以較容易地進行手動調整以滿足特定的需求。

缺點：

1.精確度受限： 幾何法的精確度通常比代數法低，特別是在機械結構複雜且需要高精度的情況下。

2.特定機構： 幾何法的適用性可能受到機械結構形狀的限制，對於某些特定機構可能不夠通用。