Scattering Attenuation of 2D elastic waves: Theory and numerical Modeling Using a Wavelet-Based Method

可能对我算真空中的一小球有帮助，估且学一下它的证明思路，后期可能借鉴得上。

~~k~~ k：波数

a：correlation distance

ka：normalized wavenumber.

u_j. $j=x, z$ 由 u_j^0 主波 u_j^s 散波 compose

背景场：

$$\rho_0 \frac{\partial^2 u_x^0}{\partial t^2} = \frac{\partial \sigma_{xx}^0}{\partial x} + \frac{\partial \sigma_{xz}^0}{\partial z}$$

$$\rho_0 \frac{\partial u_z^0}{\partial t^2} = \frac{\partial \sigma_{xz}^0}{\partial x} + \frac{\partial \sigma_{zz}^0}{\partial z}.$$

即：$\rho_0 u^0_{i,tt} = \sigma^0_{ij,j}$

$$\sigma_{xx}^0 = (\lambda_0 + 2\mu_0)\frac{\partial u_x^0}{\partial x} + \lambda_0 \frac{\partial u_z^0}{\partial z}$$

$$\sigma_{zz}^0 = \lambda_0 \frac{\partial u_x^0}{\partial x} + (\lambda_0 + 2\mu_0)\frac{\partial u_z^0}{\partial z}$$

$$\sigma_{xz}^0 = \mu_0 \left(\frac{\partial u_x^0}{\partial z} + \frac{\partial u_z^0}{\partial x}\right)$$

即：$\sigma_{ij}^0 = \lambda \delta_{ij} e_{kk}^0 + 2\mu e_{ij}^0$

垂直P波入射

$$u_x^0 = 0 \quad u_z^0 = e^{i(k_\alpha z - \omega t)}$$

$k_\alpha = \frac{\omega}{\alpha_0}$，$\alpha_0$ 是背景P波速度。

$\rho_0 u_{i,tt}^s - \sigma_{ij,j}^s = f_i^s$

~~fa~~ $f_x^s = -ik_\alpha \frac{\partial}{\partial x}(\delta\lambda) U_z^0$, $f_z^s = -\left[k_\alpha^2(\alpha_0^2\delta\rho - \delta\lambda) - 2\delta\mu\right)$

$+ ik_\alpha \frac{\partial}{\partial z}(\delta\lambda + 2\delta\mu)\Big] U_z^0$

①

然后用散射 f_x^s 结合均质Green得到散射波场 u.

$u_j^s(x) = \int_s f_k^s(x') G_{jk}(x, x') ds(x')$

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指引：学会"散射源"方法.

下面再简化 $\frac{\delta\alpha}{\alpha_0}$ $\frac{\delta\beta}{\beta_0}$ $\frac{\delta\rho}{\rho_0}$ 的关系(用经验公式)

再分别给出 P.S 波格林函数

然后代进去积分.

然后做 far-field approximation

由于本文是随机介质散射.所以积分中取了平均以表达"总体效应"

然后进一步将面积积分简化成和θ有关的表达式.

可推出了散射"辐射花样"