2D和3D Laplace转极坐标.

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3D:

用数学技巧法则:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial}{\partial \theta}\frac{\partial \theta}{\partial x} + \frac{\partial}{\partial \varphi}\frac{\partial \varphi}{\partial x}$$

有:

$$\Delta u = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial \varphi^2}$$

当与θ,φ无关即对称时:

$$\Delta u = u_{rr} + \frac{2}{r}u_r \qquad \text{也可化为: } \Delta u = \frac{1}{r}\frac{\partial^2}{\partial r^2}(ru)$$

2D:

$$\Delta u = \frac{\partial^2}{\partial r^2}u + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

与θ无关时:

$$\Delta u = u_{rr} + \frac{1}{r}u_r$$