Hesse matrix

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$f: \mathbb{R}^n \to \mathbb{R}$ 即 f 是一个把 $x \in \mathbb{R}^n$ 的向量转成标量 $f(x) \in \mathbb{R}$ 的函数.

要所有2阶偏导均存在.

$$\mathbb{H}_f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ 21 & 22 & \cdots & 2n \\ \vdots & \vdots & \ddots & \vdots \\ n1 & n2 & & nn \end{bmatrix}$$

也即 $(\mathbb{H}_f)_{i,j} = \frac{\partial^2 f}{\partial x_i \partial x_j}$

若所有二阶导连续, 则 $\frac{\partial^2 f}{\partial x_i \partial x_j} = \frac{\partial^2 f}{\partial x_j \partial x_i}$, $\mathbb{H}_f$ 成为对称矩阵.

$\mathbb{H}(f(x)) = \mathbb{J}(\nabla f(x))^T$

「证明: $\mathbb{J} = \frac{d\mathbf{f}(x)}{dx} = \left[\frac{\partial \mathbf{f}(x)}{\partial x_1} \cdots \frac{\partial \mathbf{f}(x)}{\partial x_n}\right] = \begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_v}{\partial x_1} & & \frac{\partial f_v}{\partial x_n} \end{bmatrix}$

where $\mathbf{f} \in \mathbb{R}^v$, $x \in \mathbb{R}^n$

当 $v=1$, 即 $\mathbf{f} \in \mathbb{R}^1$ 即 $\mathbf{f}$ 是标量, 退化成 f, $\nabla f(x)$ 变成一个矢量,

$\mathbb{J}$ 作用于 $\nabla f(x)$ 成为一个矩阵,

即把 $\mathbb{J}$ 里的 $\mathbf{f}(x)$ 换成 $\frac{\partial f}{\partial x_1}\hat{e}_1 + \frac{\partial f}{\partial x_2}\hat{e}_2 + \cdots \frac{\partial f}{\partial x_n}\hat{e}_n$

则 $\mathbb{J}(\nabla f(x)) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1 \partial x_1} & \cdots & \frac{\partial f}{\partial x_n \partial x_1} \\ \vdots & \vdots & \vdots \\ \underbrace{\frac{\partial f}{\partial x_1 \partial x_n}} & \cdots & \end{bmatrix} = \mathbb{H}_f^T$

和 $\mathbb{H}_f$ 的 $\frac{\partial f}{\partial x_n \partial x_1}$ 相同.

□ 」