2D Green function, modified bessel function

已知时空域解，作FFT求频空域解。

$$c^2\phi(x,t) = \frac{1}{2\pi}\frac{H(t\pm\frac{R}{c})}{\sqrt{t^2-\frac{R^2}{c^2}}}, \quad R^2 = x_1^2 + x_2^2$$

令 $\frac{R}{c} = a$. 对 $\frac{H(t-a)}{\sqrt{t^2-a^2}}$ 作FFT为.

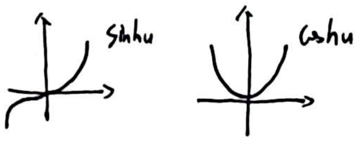
$$F(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t}\frac{H(t-a)}{\sqrt{t^2-a^2}}dt = \int_a^{\infty}\frac{e^{-i\omega t}}{\sqrt{t^2-a^2}}dt$$

令 $t = a\cosh u$. $u>0$. 则 $\int_a^{\infty}dt \to \int_0^{\infty}du$

则：$$F(\omega) = \int_0^{\infty}\frac{e^{-i\omega a\cosh u}}{a\sinh u}a\sinh u\, du = \int_0^{\infty}e^{-i\omega a\cosh u}du$$

注：$\cosh u = \frac{e^{u}+e^{-u}}{2}$, $\sinh u = \frac{e^{u}-e^{-u}}{2}$, hyperbolic function.

$$\begin{cases} \cosh u^2 + \sinh u^2 = 1 \\ \frac{d}{du}\cosh u = \sinh u \\ \frac{d}{du}\sinh u = \cosh u \end{cases}$$



$u=0$时, $\cosh u = 1$.

Modified Bessel function (wikipedia):

$$K_\alpha(x) = \int_0^{\infty}e^{-x\cosh t}\cosh(\alpha t)\,dt$$

order

则 $F(\omega) = K_0(i\omega a) = K_0(\frac{i\omega R}{c})$