1至3D Green function for wave equation

$$\frac{\partial^2}{\partial t^2}\phi - \alpha^2\nabla^2\phi = f(\mathbf{x},t) \quad , \quad f(\mathbf{x},t) = \frac{\Phi}{\rho}$$

$$\begin{cases} \hat{\phi}(\mathbf{k},w) = \int_0^{\infty} dt \iiint_{-\infty}^{+\infty} d\mathbf{x}^3 \, \phi(\mathbf{x},t)\, e^{-i(\mathbf{k}\cdot\mathbf{x} - wt)} \\ \phi(\mathbf{x},t) = \int_{-\infty}^{+\infty} \frac{dw}{2\pi} \iiint_{-\infty}^{+\infty} \frac{d\mathbf{k}^3}{(2\pi)^3}\, \hat{\phi}(\mathbf{k},w)\, e^{i(\mathbf{k}\cdot\mathbf{x} - wt)} \end{cases}$$

similar for 1D FT.

$$f(\mathbf{x},t) \longleftrightarrow f(\mathbf{k},w) \quad \text{and} \quad u(\mathbf{x},t) \longleftrightarrow \hat{u}(\mathbf{k},w)$$

$$u_t \longleftrightarrow -iw\hat{u}$$

$$u_{tt} \longleftrightarrow -w^2\hat{u}$$

$$\nabla u(\mathbf{x},t) \longleftrightarrow i\mathbf{k}\hat{u}$$

$$\nabla^2 u(\mathbf{x},t) \longleftrightarrow -k^2\hat{u}$$

Then the scalar wave equation in the Fourier domain is.

$$-w^2\hat{\phi} + \alpha^2 k^2\hat{\phi} = \hat{f} \quad , \quad \text{if } f(\mathbf{x},t) = \delta(\mathbf{x})\,\delta(t)$$

$$\text{then } \hat{f} = \hat{f}(\mathbf{k},w) = 1$$

$$\hat{\phi}(\mathbf{k},w) = \frac{1}{\alpha^2\left(k^2 - \frac{w^2}{\alpha^2}\right)}$$

标量

then : $\phi(\mathbf{x},t) = \int_{-\infty}^{+\infty} \frac{dw}{2\pi} \iiint \frac{dk^3}{(2\pi)^3}\, \hat{\phi}(\mathbf{k},w)\, e^{i(\mathbf{k}\cdot\mathbf{x} - wt)}$