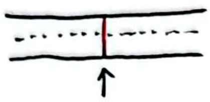
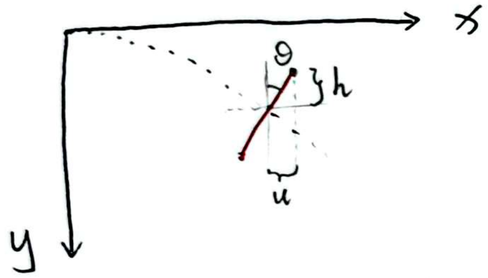
1. 形变前



→ 中性轴（无应变）此处近似为几何中心
neutral axis

截面
与中性轴垂直.

形变后:



① 截面依旧垂直于轴

② 截面旋转了θ

$\theta \approx \tan\theta = \frac{dy}{dx} = y'$

③ 截面上的点到轴线的y方向距离h.

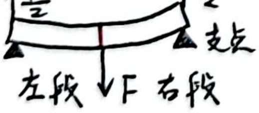
④ 截面点的水平（x方向）形变量u.

2. 小形变. 线弹性.

曲率: $K(x) = \frac{|y''|}{[1+(y')^2]^{\frac{3}{2}}}$. 小形变近似: $y' << 1$.

则 $K(x) \approx |y''|$

力矩:



考察左段: 其截面上的力矩由右段相邻面的应力产生,
而该应力由右侧支点上的力产生.

该截面上右段对左段的力矩 = 面上力矩的积分
$= \frac{F}{2} \cdot$ 力臂(即一半杆长)

$\sigma_{xx} = E\varepsilon_{xx}$, $\sigma_{xx}(x,h)$

where $\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$,

where $u_x = h\cdot\tan\theta$,

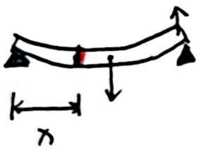
where, $\tan\theta = \frac{dy}{dx}$

则: $\sigma_{xx} = Eh\frac{d^2y}{dx^2}$

应力产生的力矩 $M(x) = \int_A \sigma_{xx}(x,h)\, h\, dA = E\frac{d^2y}{dx^2}\underbrace{\int_A h^2 dA}_{\text{记作}I}$

可见对实/空心杆同样适用 $\quad = EIy''$

对左段:



"两支点间隔"

$M(x) = F(\frac{L}{2}-x) - \frac{F}{2}(L-x) = -\frac{F}{2}x$, 杆长$L$.

则有方程

$EIy'' = -\frac{F}{2}x$, 解得: $y=\frac{1}{EI}\left(-\frac{F}{12}x^3 + C_1x + D_1\right)$

对右段同理. 解得: $y = \frac{1}{EI}\left(-\frac{F}{12}x^3 + \frac{F}{6}(x-\frac{L}{2})^3 + C_2x + D_2\right)$

边界条件: 左段 $x=0$, $y=0$; 右段 $x=L$, $y=0$. $x=\frac{L}{2}$时, 左右段 y, y' 相等.

得: $D_1=D_2=0$, $C_1=C_2=\cancel{\frac{F}{12}}\ \frac{1}{16}FL^2$

对左段:

$y=\frac{1}{EI}\left(-\frac{F}{12}x^3+\frac{1}{16}FL^2x\right)$, 当$x=\frac{L}{2}$时, $y=\frac{1}{48}\frac{FL^3}{I}\cdot\frac{1}{E}$

积分求I时忽略角度θ，对圆环：

$$I = \int_0^{r_1} 4\left(\sqrt{r_2^2 - h^2} - \sqrt{r_1^2 - h^2}\right) h^2 \, dh$$

$$+ \int_{r_1}^{r_2} 4\sqrt{r_2^2 - h^2} \, h^2 \, dh$$