SOLUTION

ISyE 3770 Statistics and Applications   Homework #7

1. Suppose that X and Y are random variables having the joint probability mass distribution

	f(x,y)	x = 2	x = 4
	1	0.10	0.15
y	3	0.20	0.3
	5	0.10	0.15

Find

a. E(2X - 3Y);
b. E(XY)

2. You are throwing two dice. Let X represent the number that occurs on a red die is tossed and Y the number that occurs on a green die. Find

a. E(X + Y);
b. E(X - Y);
c. E(XY)

3. . The joint probability density function for the continuous random variables X and Y is:

$h(x, y) = \begin{cases} \frac{2}{3}(x + 2y) & 0 \le x \le 1, 0 \le y \le 1 \end{cases}$ and 0, otherwise

a. Find P(Y> $\frac{1}{4}$ | X=$\frac{1}{10}$)
b. Find E(X)
c. Find E(Y | x=$\frac{1}{2}$)

4. 6.1.6 in textbook
5. 6.1.9 in textbook (no dot diagrams)
6. 6.S9 in textbook
7. 6.S10 in textbook

1A. $E(2X-3Y) = E(2X) - E(3Y) = 2E(X) - 3E(Y)$

$E(X) = 2(.40) + 4(.60) = 3.2$

$E(Y) = 1(.25) + 3(.5) + 5(.25) = 3.0$

Thus, $E(2X-3Y) = 6.4 - 9.0 = \underline{\underline{-2.6}}$

B. $E(XY) = \sum_x \sum_y xy \cdot f(x,y) = 2(.1) + 6(.2) + 10(.1) + 4(.15) + 12(.3) + 20(.15) = \underline{\underline{9.6}}$

NOTE: You cannot use E(X)*E(Y) unless you show independence.

2.

$f(x,y) =$

RED \ GR	1	2	3	4	5	6	f(RED)-X
1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	''	''	''	''	''	''	1/6
3	''	''	''	''	''	''	1/6
4	''	''	''	''	''	''	1/6
5	''	''	''	''	''	''	1/6
6	''	''	''	''	''	''	1/6
f(GR) Y	1/6	1/6	1/6	1/6	1/6	1/6	

a. $E(X+Y) = E(X) + E(Y)$ $\qquad E(X) = 21/6 \qquad E(Y) = 21/6$

$= 42/6 = \underline{\underline{7}}$

b. $E(X-Y) = \underline{\underline{0}}$

c. $E(XY) = \sum_x \sum_y xy \cdot f(x,y) = \frac{441}{36}$

$= \underline{\underline{12.25}}$

also $f(x) \cdot f(y) = f(x,y)$ are $x,y \Rightarrow$ INDEP

NOTE: You cannot use E(X)*E(Y) unless you show
independence.

3.

A. $f(y|x) = f(x,y)/f(x)$

$$f(x) = \int_0^1 \frac{2}{3}(x+2y)\,dy = 2/3\,(xy + y^2)\Big|_0^1 = 2/3\,(x+1) \quad 0 \le x \le 1$$

$$\frac{f(x,y)}{f(x)} = \frac{2/3\,(x+2y)}{2/3\,(x+1)} \;;\; \text{since } x = \frac{1}{10} \quad f(y|x) = \frac{1/10 + 2y}{11/10} = \frac{10}{11}\left(2y + \frac{1}{10}\right)$$

$$P(y > 1/4 \,|\, x = \tfrac{1}{10}) = \frac{10}{11}\int_{1/4}^{1} 2y + \frac{1}{10}\,dy = \frac{10}{11}\left(y^2 + \frac{y}{10}\right)\Big|_{1/4}^{1} = \frac{10}{11}\left[(11/10) - \left(\frac{1}{16} + \frac{1}{40}\right)\right]$$

$$= \frac{10}{11}\left(\frac{704 - 40 - 16}{640}\right) = \frac{10}{11}\left(\frac{648}{640}\right) = \frac{648}{704} = \frac{10.125}{11} = .920455 = \frac{81}{88}$$

B. $$E(x) = \int_0^1 x \cdot 2/3\,(x+1)\,dx = 2/3\int_0^1 x^2 + x\,dx = 2/3\left(\frac{x^3}{3} + \frac{x^2}{2}\right)\Big|_0^1 = 2/3\left(\frac{1}{3} + \frac{1}{2}\right)$$

$$= 2/3\left(\frac{5}{6}\right) = \frac{5}{9} = .555\overline{5}$$

C. $$E(y|x=1/2) = \int_0^1 y \cdot f(y|x=1/2)$$

; $f(y|x=1/2)$ FROM ABOVE

$$= \frac{1/2 + 2y}{3/2} = 2/3\,(2y + 1/2)$$

$$E(y|x=1/2) = 2/3\int_0^1 y\cdot(2y + 1/2)\,dy = 2/3\int_0^1 2y^2 + 1/2\,y\,dy = 2/3\left(\frac{2y^3}{3} + \frac{y^2}{4}\right)\Big|_0^1$$

$$= 2/3\left[(2/3 + 1/4)\right] = 2/3\left(\frac{11}{12}\right) = 11/18 = .6\overline{111}$$

4 (6.1.6)

a. $\bar{x} = \frac{\sum_{i=1}^{19} x_i}{n}$

$\bar{x} = 14.34994737$

b. $s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$

$s^2 = 356.7423256$

5 (6.1.9) $\bar{X}_{EX}$ $\bar{X}_{NO\,EX}$ S_{EX} $S_{NO\,EX}$

$n_{EX} = 12$ $n_{NOEX} = 8$

$\bar{X}_{EX} = 287.8896667$ $S_{EX} = 106.1542685$

$\bar{X}_{NOEX} = 325.0097500$ $S_{NO\,EX} = 121.1980211$

6 (6.5.9

$n_1 = 8$ $n_2 = 8$

a. $R_1 = 4$ $R_2 = 4$ yes; only in terms of the range and $n=8$

b. $S_1 = 1.60356745$ $S_2 = 1.85164020$

NO ; sample 2 has a higher variance

c. s^2 considers all data, and range ignores all data except the max + min

7 (6.S10

$\bar{x}, q_2, s^2, s \qquad n=24$

43 44 44 45 45 | 46 46 46 47 48 | 48
49 49 49 49 50 | 50 50 50 51 51 | 51
52 52

A $\quad \bar{x} = 48.125 \qquad s^2 = 7.24456522 \qquad s = 2.69157300$

$q_2 \equiv$ median LOCATED AT INDEX $\frac{n+1}{2} = \frac{25}{2} = 12.5$

So, IT IS HALF-WAY BETWEEN DATA POINT
12 + 13

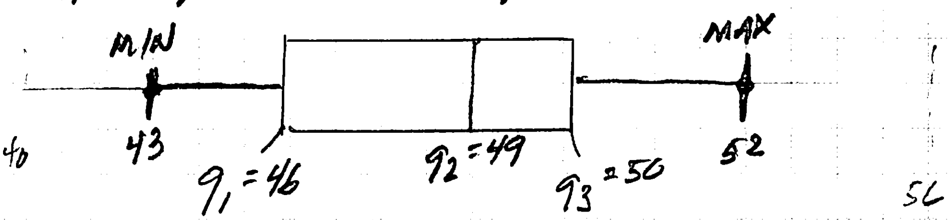
$x_{12} = 49 \qquad x_{13} = 49 \implies q_2 = 49$

B BOX PLOT.

$q_1 @ \frac{n+1}{4} @ \frac{25}{4} = 6.25 \qquad q_3 @ 3\left(\frac{n+1}{4}\right) = 18.75$

$q_1 = 46 \qquad q_3 = 50$

$IQR = q_3 - q_1 = 4 \qquad 1.5\, IQR = 6$



NO OUTLIERS