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Quiz 5

$$E(\Theta_1) = \frac{9\mu}{9} = \mu$$

$$E(\Upsilon_2) = \frac{3\mu - \mu + 2\mu}{4} = \frac{4\mu}{4} = \mu$$

- Because $E(\Theta_1) = \mu$ and $E(\Upsilon_2) = \mu$, both estimators Θ_1 and Υ_2 are unbiased

- We must then determine which estimator has the lower variance.

$$V(\Theta_1) = \frac{V(X_1) + V(X_2) + \ldots + V(X_9)}{81}$$

$$= \frac{9\sigma^2}{81} = \frac{\sigma^2}{9}$$

$$\underline{V(\Theta_1) = \frac{\sigma^2}{9}}$$

$$V(\Upsilon_2) = \frac{V(3X_1 - X_6 + 2X_4)}{16}$$

$$= \frac{V(3X_1) - V(X_6) + V(2X_4)}{16}$$

$$= \frac{9V(X_1) - V(X_6) + 4V(X_4)}{16}$$

$$= \frac{9\sigma^2 - \sigma^2 + 4\sigma^2}{16}$$

$$= \frac{12\sigma^2}{16}$$

$$= \frac{3\sigma^2}{4}$$

$$V(\Upsilon_2) = \frac{3\sigma^2}{4}$$

- $V(\Theta_1) = \frac{\sigma^2}{9}$ and $V(\Upsilon_2) = \frac{3\sigma^2}{4}$, and since $\frac{\sigma^2}{9} < \frac{3\sigma^2}{4}$, $\boxed{\Theta_1 \text{ is the better estimator}}$ because it has a smaller variance than Υ_2