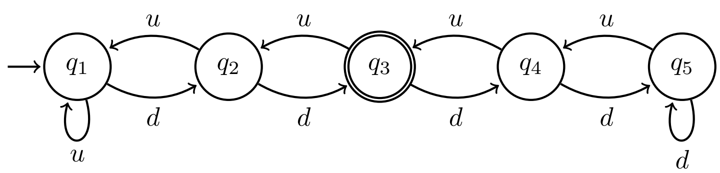
# COT3210–Computability and Automata

## Homework Problem Set 01 Solutions

Page 83: Exercise 1.3.

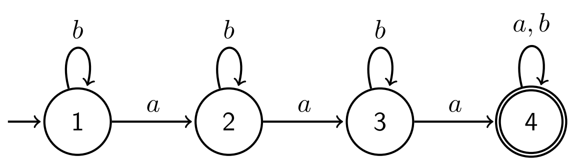



Page 83: Exercise 1.4a. We construct two machines $M_1$ and $M_2$ such that:

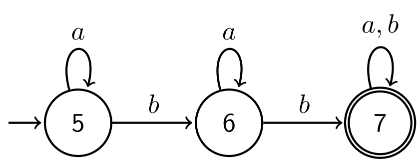
$$L(M_1) = \{w \,|\, w \text{ has at least three } a\text{'s}\},$$
$$L(M_2) = \{w \,|\, w \text{ has at least two } b\text{'s}\}.$$

Then we construct the machine $M$ such that $L(M) = L(M_1) \cap L(M_2)$.

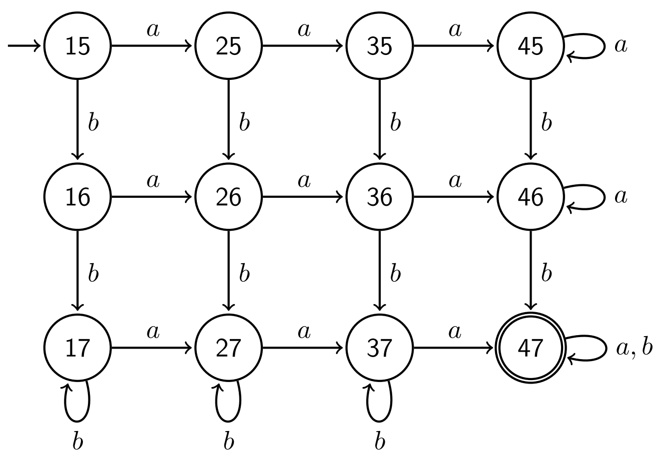
The state diagram for $M_1$ is given below:




For $M_2$ we have the following state diagram:




Finally, for $M$ with $L(M) = L(M_1) \cap L(M_2)$, we have the following state diagram:

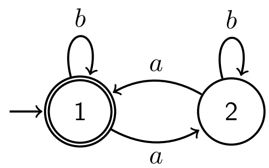


Page 83: Exercise 1.4c. We construct two machines $M_1$ and $M_2$ such that:

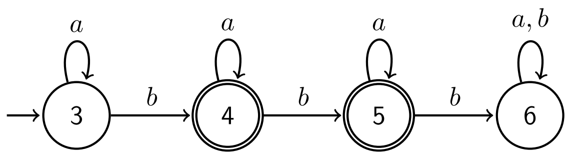
$$L(M_1) = \{w \mid w \text{ has an even number of } a\text{'s}\},$$
$$L(M_2) = \{w \mid w \text{ has one or two } b\text{'s}\}.$$

Then we construct the machine $M$ such that $L(M) = L(M_1) \cap L(M_2)$.

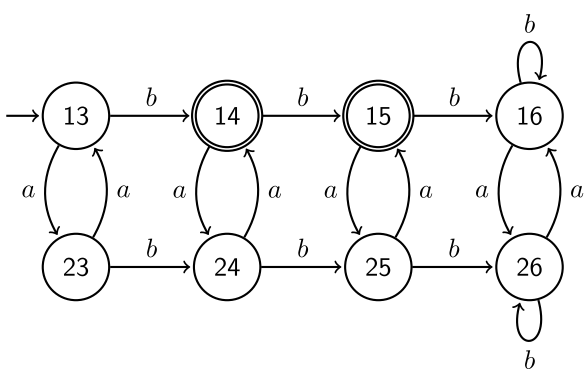
The state diagram for $M_1$ is given below:




For $M_2$ we have the following state diagram:




Finally, for $M$ with $L(M) = L(M_1) \cap L(M_2)$, we have the following state diagram:

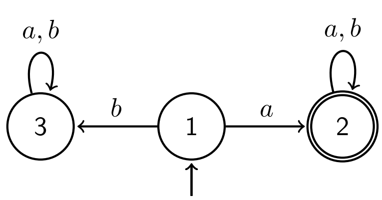



Page 83: Exercise 1.4e. We construct two machines $M_1$ and $M_2$ such that:

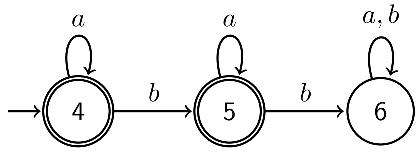
$$L(M_1) = \{w \mid w \text{ starts with an } a\},$$
$$L(M_2) = \{w \mid w \text{ has at most one } b\}.$$

Then we construct the machine $M$ such that $L(M) = L(M_1) \cap L(M_2)$.

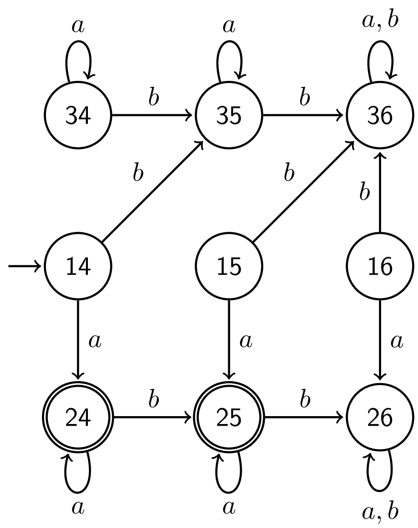
The state diagram for $M_1$ is given below:



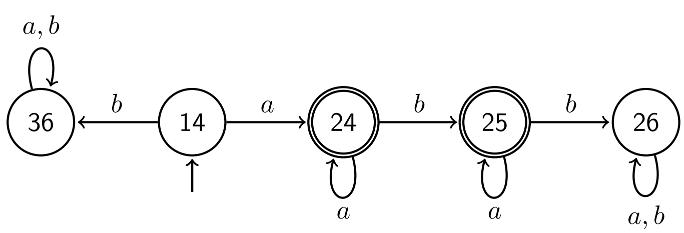
For $M_2$ we have the following state diagram:



Finally, for $M$ with $L(M) = L(M_1) \cap L(M_2)$, we have the following state diagram:



Note that in the state diagram we have obtained for the intersection machine, states labeled 34, 15, 16 may be eliminated as no transition reaches them. Also states 35 and 36 are both reject states, and it is sufficient to keep one of them. The following state diagram is obtained after removal of these states.

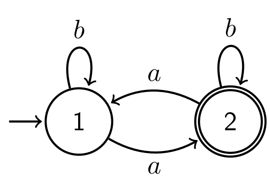


Page 83: Exercise 1.4f. We construct two machines $M_1$ and $M_2$ such that:

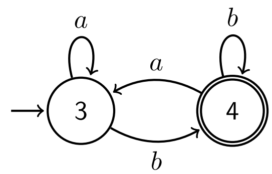
$$L(M_1) = \{w \mid w \text{ has an odd number of } a\text{'s}\},$$
$$L(M_2) = \{w \mid w \text{ ends with a } b\}.$$

Then we construct the machine $M$ such that $L(M) = L(M_1) \cap L(M_2)$.

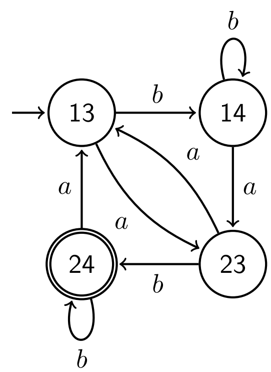
The state diagram for $M_1$ is given below:



For $M_2$ we have the following state diagram:




Finally, for $M$ with $L(M) = L(M_1) \cap L(M_2)$, we have the following state diagram:

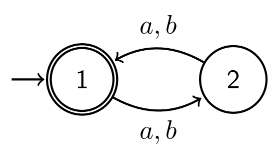



Page 83: Exercise 1.4g. We construct two machines $M_1$ and $M_2$ such that:

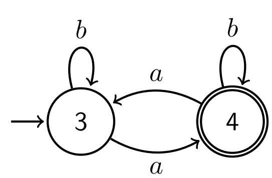
$$L(M_1) = \{w \mid w \text{ has even length}\},$$
$$L(M_2) = \{w \mid w \text{ has an odd number of } a\text{'s}\}.$$

Then we construct the machine $M$ such that $L(M) = L(M_1) \cap L(M_2)$.

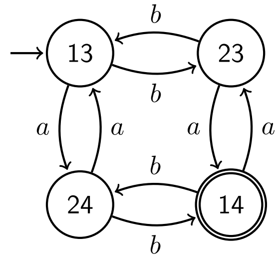
The state diagram for $M_1$ is given below:




For $M_2$ we have the following state diagram:




Finally, for $M$ with $L(M) = L(M_1) \cap L(M_2)$, we have the following state diagram:



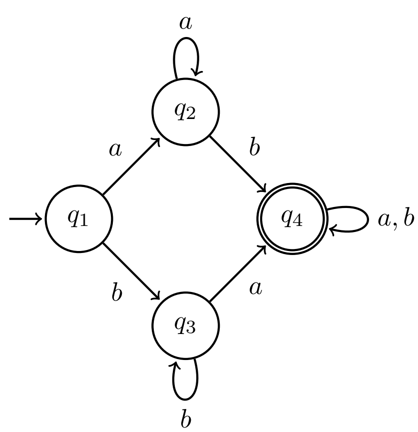
Page 84: Exercise 1.5c. To obtain the machine $M$ with

$$L(M) = \{w \mid w \text{ contains neither the substring } ab \text{ nor the substring } ba\},$$

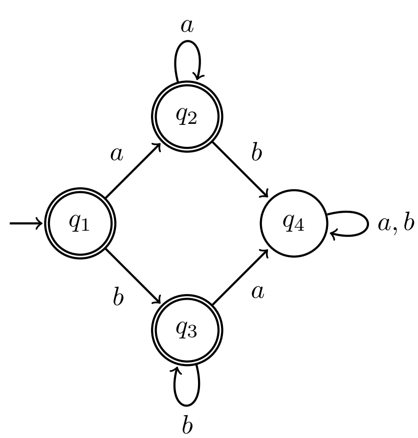
we construct the machine $M_1$ so that $L(M) = \overline{L(M_1)}$.
Thus, $M_1$ recognizes the language

$$L(M_1) = \{w \mid w \text{ contains the substring } ab \text{ or the substring } ba \text{ or both}\}.$$

$M_1$ has following state diagram.




Then, the required machine $M$ is obtained by interchanging the accepting states of $M_1$ with the non-accepting states and vice versa. $M$ has the following state diagram:



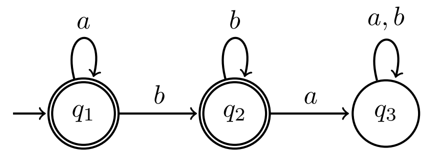

Page 84: Exercise 1.5d. To obtain the machine $M$ with

$$L(M) = \{w \mid w \text{ is any string not in } a^*b^*\},$$

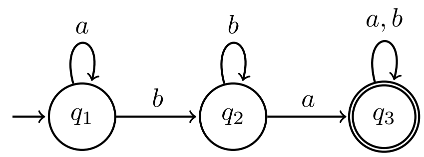
we construct the machine $M_1$ so that $L(M) = \overline{L(M_1)}$.
Thus, $M_1$ recognizes the language

$$L(M_1) = \{w \mid w \text{ is any string in } a^*b^*\}.$$

$M_1$ has following state diagram.



Then, the required machine $M$ is obtained by interchanging the accepting states of $M_1$ with the non-accepting states and vice versa. $M$ has the following state diagram:



<u>Page 84: Exercise 1.5e.</u> To obtain the machine $M$ with

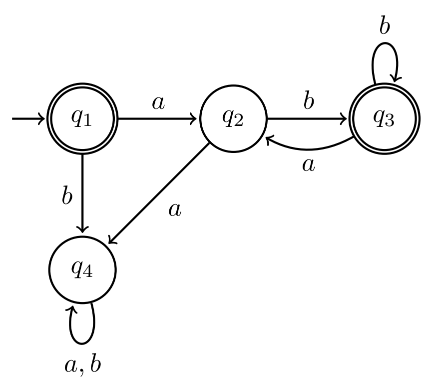
$$L(M) = \{w \,|\, w \text{ is any string not in } (ab^+)^*\},$$

we construct the machine $M_1$ so that $L(M) = \overline{L(M_1)}$.

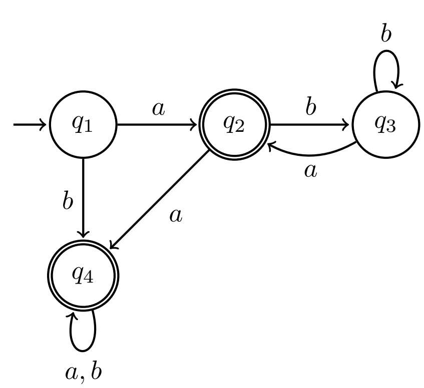
Thus, $M_1$ recognizes the language

$$L(M_1) = \{w \,|\, w \text{ is any string in } (ab^+)^*\}.$$

$M_1$ has following state diagram.



Then, the required machine $M$ is obtained by interchanging the accepting states of $M_1$ with the non-accepting states and vice versa. $M$ has the following state diagram:



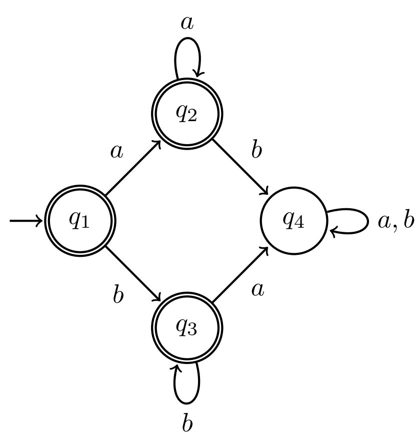
Page 84: Exercise 1.5f. To obtain the machine $M$ with

$$L(M) = \{w \,|\, w \text{ is any string not in } (a^* \cup b^*\},$$

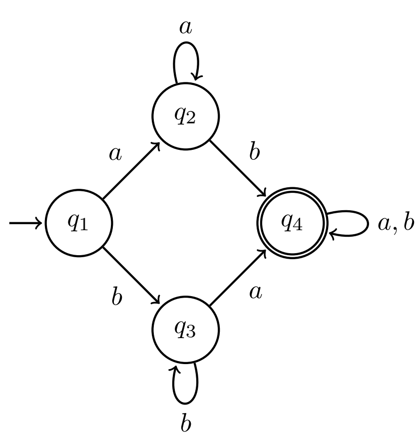
we construct the machine $M_1$ so that $L(M) = \overline{L(M_1)}$.
Thus, $M_1$ recognizes the language

$$L(M_1) = \{w \,|\, w \text{ is any string in } a^* \cup b^*\}.$$

$M_1$ has following state diagram.




Then, the required machine $M$ is obtained by interchanging the accepting states of $M_1$ with the non-accepting states and vice versa. $M$ has the following state diagram:



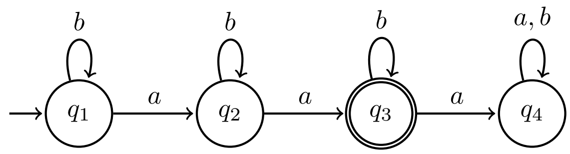

Page 84: Exercise 1.5g. To obtain the machine $M$ with

$$L(M) = \{w \,|\, w \text{ is any string that doesn't contain exactly two } a\text{'s}\},$$

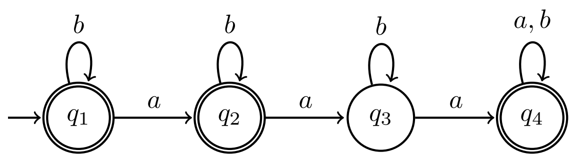
we construct the machine $M_1$ so that $L(M) = \overline{L(M_1)}$.
Thus, $M_1$ recognizes the language

$$L(M_1) = \{w \,|\, w \text{ contains exactly two } a\text{'s}\}.$$

$M_1$ has following state diagram.




Then, the required machine $M$ is obtained by interchanging the accepting states of $M_1$ with the non-accepting states and vice versa. $M$ has the following state diagram:



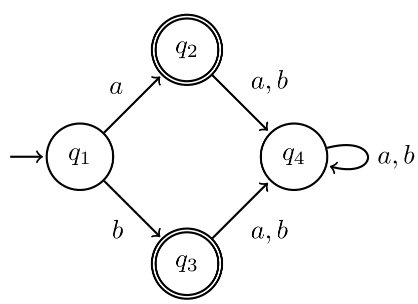

Page 84: Exercise 1.5h. To obtain the machine $M$ with

$$L(M) = \{w \,|\, w \text{ is any string except } a \text{ and } b\},$$

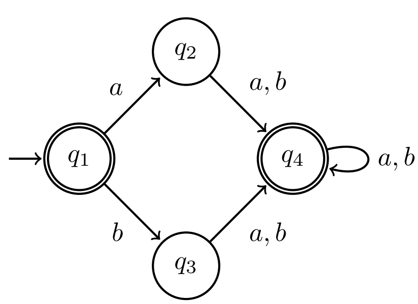
we construct the machine $M_1$ so that $L(M) = \overline{L(M_1)}$.
Thus, $M_1$ recognizes the language

$$L(M_1) = \{w \,|\, w \text{ is either } a \text{ or } b\}.$$

$M_1$ has following state diagram.



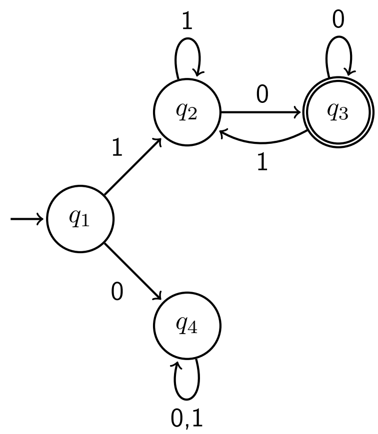

Then, the required machine $M$ is obtained by interchanging the accepting states of $M_1$ with the non-accepting states and vice versa. $M$ has the following state diagram:



Page 84: Exercise 1.6a. The state diagram for the machine $M$ with

$$L(M) = \{w \,|\, w \text{ begins with a 1 and ends with a 0}\}$$

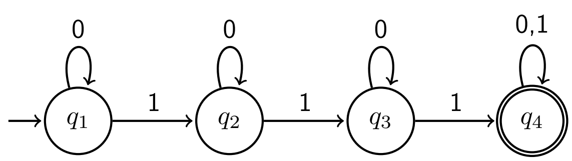
is shown below.




Page 84: Exercise 1.6b. The state diagram for the machine $M$ with

$$L(M) = \{w \,|\, w \text{ contains at least three 1s}\}$$

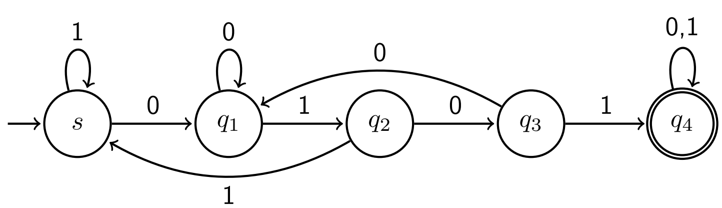
is shown below.




Page 84: Exercise 1.6c. The state diagram for the machine $M$ with

$$L(M) = \{w \,|\, w \text{ contains the substring 0101}\}$$

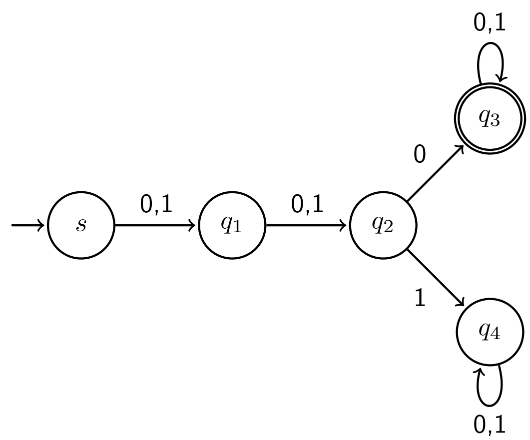
is shown below.



Page 84: Exercise 1.6d. The state diagram for the machine $M$ with

$$L(M) = \{w \mid w \text{ has length at least 3 and the third symbol is 0}\}$$

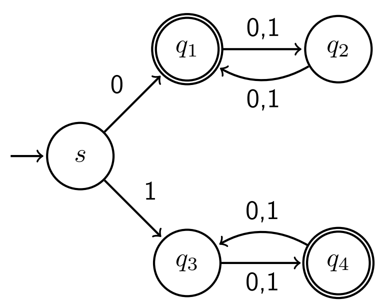
is shown below.




Page 84: Exercise 1.6e. The state diagram for the machine $M$ with

$$L(M) = \{w \mid w \text{ starts with a 0 and has odd length, or starts with a 1 and has even length}\}$$

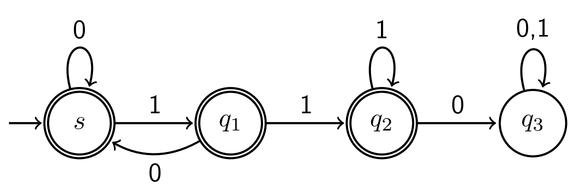
is shown below.




Page 84: Exercise 1.6f. The state diagram for the machine $M$ with

$$L(M) = \{w \mid w \text{ doesn't contain the substring 110}\}$$

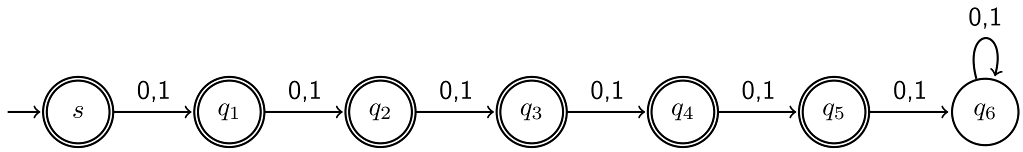
is shown below.



Page 84: Exercise 1.6g. The state diagram for the machine $M$ with

$$L(M) = \{w \,|\, w \text{ has length at most 5}\}$$

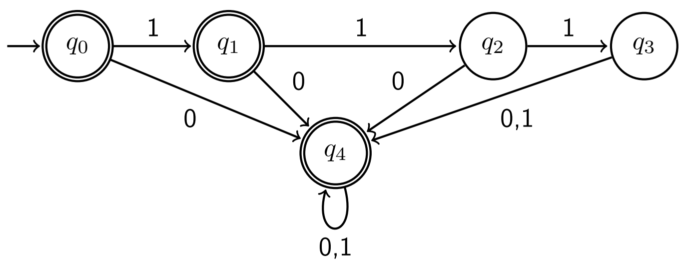
is shown below.




Page 84: Exercise 1.6h. The state diagram for the machine $M$ with

$$L(M) = \{w \,|\, w \text{ is any string except 11 and 111}\}$$

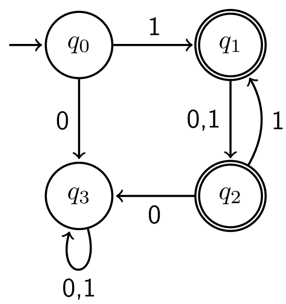
is shown below.




Page 84: Exercise 1.6i. The state diagram for the machine $M$ with

$$L(M) = \{w \,|\, \text{every odd position of } w \text{ is a 1}\}$$

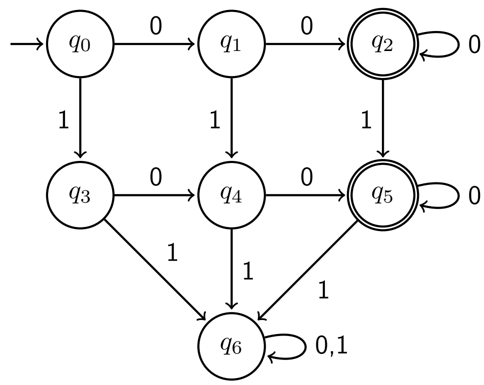
is shown below.



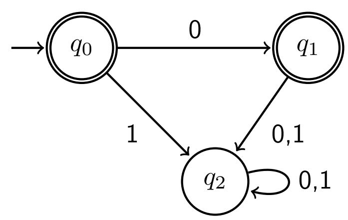

Page 84: Exercise 1.6j. The state diagram for the machine $M$ with

$$L(M) = \{w \,|\, w \text{ contains at least two 0s and at most one 1}\}$$

is shown below.



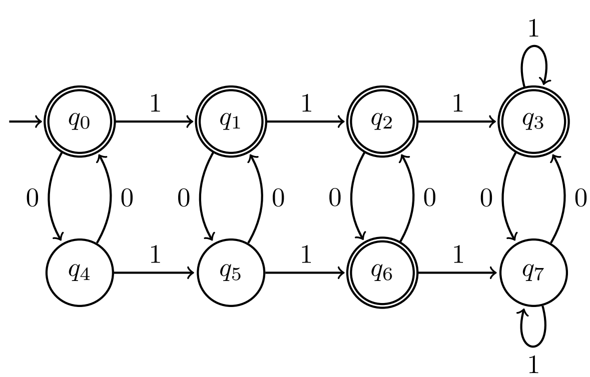

Page 84: Exercise 1.6k. The state diagram for the machine $M$ with $L(M) = \{\varepsilon, 0\}$ is shown below.



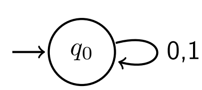

Page 84: Exercise 1.6l. The state diagram for the machine $M$ with

$$L(M) = \{w \mid w \text{ contains an even number of 0s, or contains exactly two 1s}\}$$

is shown below.




Page 84: Exercise 1.6m. The state diagram for the machine $M$ with $L(M) = \emptyset$ is shown below.



<u>Page 84: Exercise 1.6n.</u> The state diagram for the machine $M$ with

$$L(M) = \{w \mid w \text{ is any string except the empty string}\}$$

is shown below.