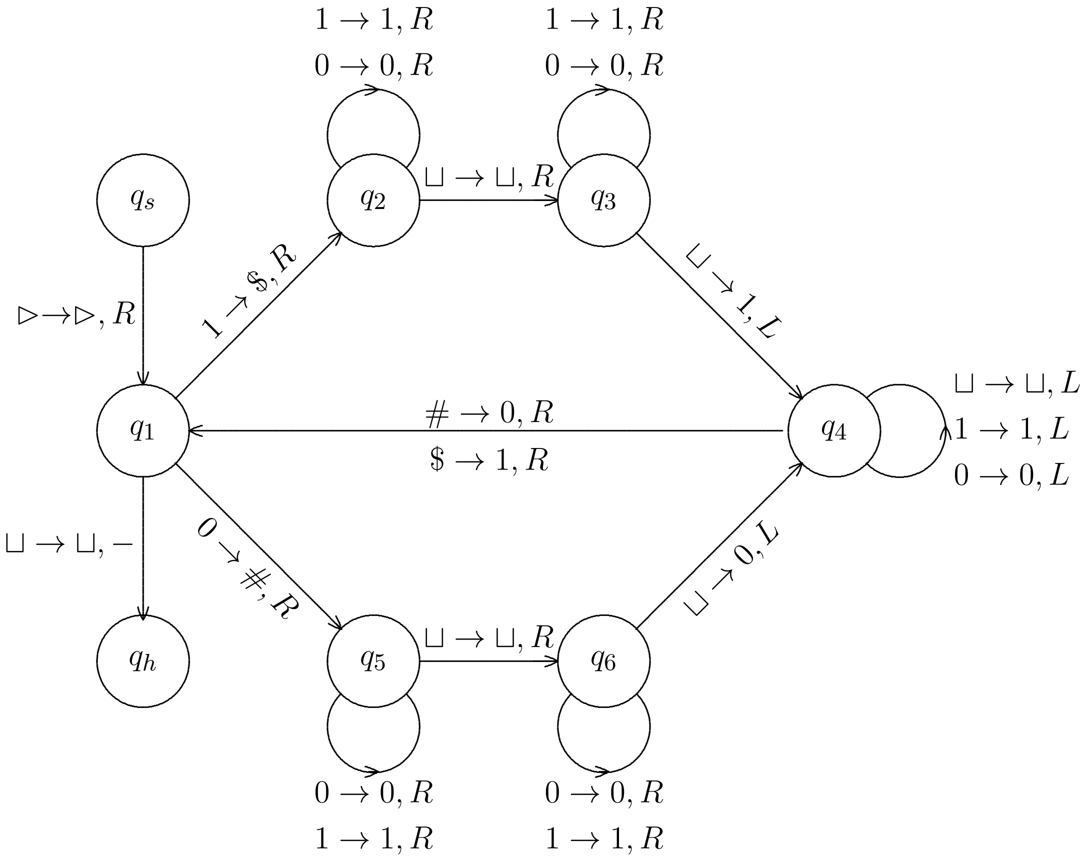
Fall 2016: COT3210–Computability and Automata
Answers to Supplementary Exercises II

1. Develop a Turing machine that will make a copy of the input right next to the input after leaving a blank space. Note that it does not matter what the alphabet contains, but to make things simple, we will assume that the input alphabet contains at least 0 and 1. The left end of the tape is marked by the tape symbol $\triangleright$, at which the machine starts. Tape alphabet contains at least the input alphabet, $\triangleright$, and the blank symbol ($\sqcup$). You may add symbols to the tape alphabet if needed for any other reason.

   The desired Turing Machine is formally defined as follows:

   $Q : \{q_s, q_1, q_2, q_3, q_4, q_5, q_6, q_h\}$; $\Sigma = \{0, 1\}$; $\Gamma = \{0, 1, \sqcup, \triangleright, \$, \#\}$
   Start state $= q_s$; Accept State $= q_h$; $\delta =$ Transition diagram as shown below.

$\langle\, q_s,\ \triangleright,\ q_1,\ \triangleright,\ R \,\rangle$	$\langle\, q_3,\ \sqcup,\ q_4,\ 1,\ L \,\rangle$	$\langle\, q_6,\ \sqcup,\ q_4,\ 0,\ L \,\rangle$
$\langle\, q_1,\ 1,\ q_2,\ \$,\ R \,\rangle$	$\langle\, q_1,\ 0,\ q_5,\ \#,\ R \,\rangle$	$\langle\, q_4,\ 1,\ q_4,\ 1,\ L \,\rangle$
$\langle\, q_2,\ 1,\ q_2,\ 1,\ R \,\rangle$	$\langle\, q_5,\ 1,\ q_5,\ 1,\ R \,\rangle$	$\langle\, q_4,\ 0,\ q_4,\ 0,\ L \,\rangle$
$\langle\, q_2,\ 0,\ q_2,\ 0,\ R \,\rangle$	$\langle\, q_5,\ 0,\ q_5,\ 0,\ R \,\rangle$	$\langle\, q_4,\ \sqcup,\ q_4,\ \sqcup,\ L \,\rangle$
$\langle\, q_2,\ \sqcup,\ q_3,\ \sqcup,\ R \,\rangle$	$\langle\, q_5,\ \sqcup,\ q_6,\ \sqcup,\ R \,\rangle$	$\langle\, q_4,\ \$,\ q_1,\ 1,\ R \,\rangle$
$\langle\, q_3,\ 1,\ q_3,\ 1,\ R \,\rangle$	$\langle\, q_6,\ 1,\ q_6,\ 1,\ R \,\rangle$	$\langle\, q_4,\ \#,\ q_1,\ 0,\ R \,\rangle$
$\langle\, q_3,\ 0,\ q_3,\ 0,\ R \,\rangle$	$\langle\, q_6,\ 0,\ q_6,\ 0,\ R \,\rangle$	$\langle\, q_1,\ \sqcup,\ q_h,\ \sqcup,\ - \,\rangle$



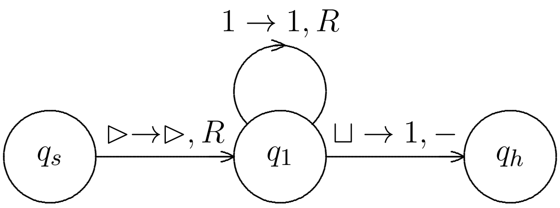
2. Develop a Turing machine to compute $f(x) = x+1$, assuming that x is a natural number represented in “unary” form. The unary representation of a natural number k is a sequence of k 1s. There is no 0 in this representation. The input alphabet accordingly contains just 1. The left end of the tape is marked by the tape symbol $\rhd$, at which the machine starts. The tape alphabet contains at least the input alphabet, $\rhd$, and the blank symbol ($\sqcup$). You may add symbols to the tape alphabet if needed for any other reason. The Turing machine will begin with a tape that contains a sequence of 1s and produce an output consisting of a 1 appended to the input sequence.

   The desired Turing Machine is formally defined as follows:

   $Q : \{q_s, q_1, q_h\}$; $\Sigma = \{1\}$; $\Gamma = \{1, \sqcup, \rhd\}$

   Start state $= q_s$; Accept State $= q_h$; $\delta =$ Transition diagram as shown below.

   $\langle\ q_s,\ \rhd,\ q_1,\ \rhd,\ R\ \rangle$
   $\langle\ q_1,\ 1,\ q_1,\ 1,\ R\ \rangle$
   $\langle\ q_1,\ \sqcup,\ q_h,\ 1,\ -\ \rangle$



3. Write a Turing machine to compute f(x) = x mod 2, assuming that x is a natural number represented in unary form. The Turing machine should begin with a tape that contains a finite sequence of 1s followed by an infinite number of blanks and rewrite the tape to contain a 1 followed by an infinite number of blanks if the initial number of 1s on the tape were odd, or to contain a 0 followed by an infinite number of blanks. Note that 0 is used only to represent the answer. There are no 0s in the input. The input alphabet contains 1, and the tape alphabet contains at the minimum the symbols, 1, 0, $\sqcup$, and $\triangleright$. You may add symbols to the tape alphabet if needed for any other reason. The left end of the tape is marked by the tape symbol $\triangleright$, at which the machine starts. The machine will halt at the cell following the $\triangleright$ where the machine writes a 1 or a 0. The rest of the tape should be all blanks when the machine halts.

   The desired Turing Machine is formally defined as follows:

   $Q : \{q_s, q_1, q_2, q_3, q_4, q_5, q_6, q_h\}$; $\Sigma = \{1\}$; $\Gamma = \{0, 1, \sqcup, \triangleright\}$

   Start state = q_s; Accept State = q_h; δ = Transition diagram as shown below.

$\langle\ q_s,\ \triangleright,\ q_1,\ \triangleright,\ R\ \rangle$	$\langle\ q_3,\ \triangleright,\ q_4,\ \triangleright,\ R\ \rangle$	$\langle\ q_5,\ \sqcup,\ q_5,\ \sqcup,\ L\ \rangle$
$\langle\ q_1,\ 1,\ q_2,\ \sqcup,\ R\ \rangle$	$\langle\ q_4,\ \sqcup,\ q_h,\ 1,\ -\ \rangle$	$\langle\ q_5,\ \triangleright,\ q_6,\ \triangleright,\ R\ \rangle$
$\langle\ q_2,\ \sqcup,\ q_3,\ \sqcup,\ L\ \rangle$	$\langle\ q_2,\ 1,\ q_1,\ \sqcup,\ R\ \rangle$	$\langle\ q_6,\ \sqcup,\ q_h,\ 0,\ -\ \rangle$
$\langle\ q_3,\ \sqcup,\ q_3,\ \sqcup,\ L\ \rangle$	$\langle\ q_1,\ \sqcup,\ q_5,\ \sqcup,\ L\ \rangle$