COT3210–Computability and Automata
Practice Problems for Final

1. Let $\Sigma = \{0, 1\}$. Draw the state diagram for a deterministic finite automaton (DFA) which accepts the language consisting of strings w described in each of the following:

   i. The length of w is a multiple of 3.
   ii. w ends with 00.
   iii. w has an even number of 0s and an odd number of 1s.
   iv. w is any string not in L(R), with $R = (01^{+})^{*}$.
   v. w is any strong not in $0^{*} \cup 1^{*}$.
   vi. w contains an even number of 0s or contains exactly two 1s.

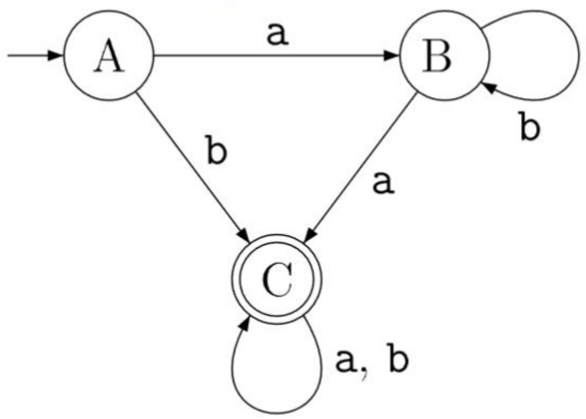
2. Let $\Sigma = \{0, 1\}$. Draw the state diagram for a non-deterministic finite automaton (NFA) which accepts the language consisting of strings w described in each of the following. Your NFA should also use the number of states where specified:

   i. w contains an even number of 0s or exactly two 1s (6 states).
   ii. w is given by the regular expression $0^{*}1^{*}0^{+}$ (3 states).
   iii. w is the language described by the regular expression $1^{*}(001^{+})^{*}$ (3 states).
   iv. w is the concatenation of strings containing at least three 1s and the empty string.
   v. w is the star of the language of strings containing at least two 0s and at most one 1.

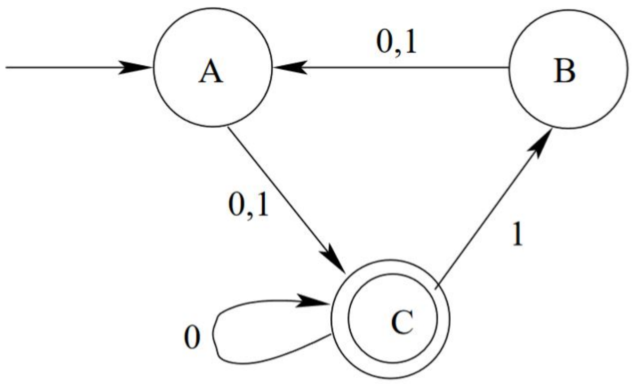
3. Let $\Sigma = \{0, 1\}$. Give regular expressions for languages of strings $w \in \Sigma^{*}$ described below:

   i. w has an even number of 0s and an odd number of 1s and does not contain the substring 01.
   ii. w is any string except 11 and 111.
   iii. w starts with a 0 and has odd length, or starts with a 1 and has even length.
   iv. w contains at least two 0s and at most one 1.

4. Let $\Sigma = \{\mathsf{a}, \mathsf{b}\}$. Convert the following DFA to its equivalent regular expression.



5. Let $\Sigma = \{0, 1\}$. Convert the following DFA to its equivalent regular expression.



6. Use the pumping lemma to prove that the following languages are not regular.

   i. $\{w\#x \,|\, w, x \in \{0,1\}$ and the number of 0s in w equals the number of 1s in $x\}$
   ii. $\{wwww \,|\, w \in \{0,1\}^*\}$
   iii. $\{0^{2n}1^{3n}0^{n} \,|\, n \geq 0\}$

7. Determine the language generated by each of the following grammars. Assume

   i. Assume $\Sigma = \{0, 1\}$.
   $S \to 0S0 \,|\, 0B0$
   $B \to 1B \,|\, 1$
   ii. Assume $\Sigma = \{a, b, c\}$.
   $S \to abScB \,|\, \varepsilon$
   $B \to bB \,|\, b$

8. Find a CFG G that generates each of the following languages.

   i. Assume $\Sigma = \{a, b, c, d\}$. $L(G) = \{a^n\, b^m\, c^m\, d^{2n} \,|\, n \geq 0, m > 0\}$.
   ii. Assume $\Sigma = \{a, b\}$. $L(G) = \{a^n\, b^m \,|\, 0 \leq n \leq m \leq 2n\}$.
   iii. Assume $\Sigma = \{a, b, c\}$. $L(G) = \{a^n b^m c^k \,|\, k = n + m\}$.

9. Transform the following grammar to Chomsky Normal Form.
   $S \to aXbY$
   $X \to aX \,|\, \varepsilon$
   $Y \to bY \,|\, \varepsilon$

10. Construct a NPDA that accepts the language specified in each of the following:

    i. The set of all palindromes over $\{a, b\}$.
    ii. $\{1^n 0^n \,|\, n > 0\} \cup \{1^n 0^{2n} \,|\, n > 0\}$

11. Construct a Turing Machine with $\Sigma = \{1\}$ which calculates the following function, assuming $x > 2$:

$$f(x) = \begin{cases} x + 2, & \text{if } x \text{ is even;} \\ x - 2, & \text{if } x \text{ is odd.} \end{cases}$$

12. Given $f(n) = n^{\log_2 n}$ and $g(n) = n^3$, determine whether $f(n) = O(g(n))$, or $g(n) = O(f(n))$.

13. Show that $RELPRIME \in P$.

14. Show that $CONNECTED \in P$.

15. Find $gcd(2831, 317)$. Are 2831 and 317 relatively prime?

16. Estimate the maximum number of division steps needed in order to compute $gcd(4713, k)$?

17. Is $(\overline{x_1} \vee x_2 \vee \overline{x_3}) \wedge (x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3})$ satisfiable?

18. Show that $HAMPATH \in NP$.

19. Show that $CLIQUE \in NP$.

20. Give three examples of NP-complete problems, with a clear statement of problem for each example.