Spring 2018: COT3210–Computability and Automata
Test 02: Answers

Instructions:

1. Put your Name and n# on your test.
2. Please write your answers legibly in the box provided under each question.
3. Answer all questions.

Name: _____ **n#:** _____

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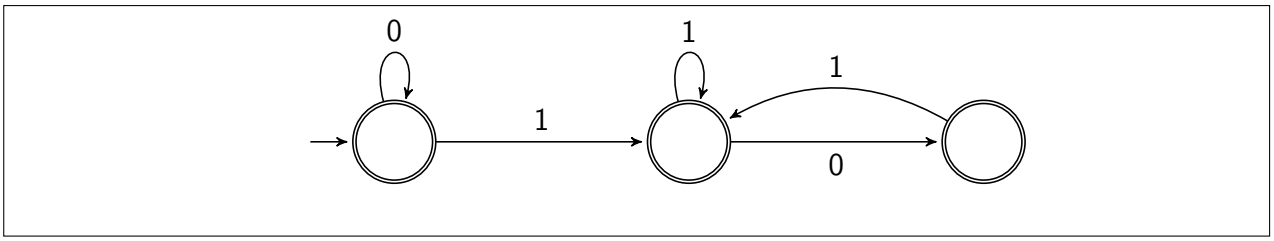
1. Shown below is the state diagram for an NFA that has no ε-transitions, 3 states, and will recognize the language

$$A = \{w \mid w \text{ does not contain the substring } 100\}.$$



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2. The required regular expressions are shown below:

   a. $\{w \mid w \text{ is of length at most } 3\} = (\varepsilon \cup \Sigma)^3$, or $\varepsilon \cup \Sigma \cup \Sigma\Sigma \cup \Sigma\Sigma\Sigma$.

   b. $A = \{w \mid w \text{ is of length at least 1 and begins and ends with the same symbol}\}$.

$$A = \Sigma \cup 0\,\Sigma^*\,0 \cup 1\,\Sigma^*\,1$$

   c. $\{w \mid \text{Every 0 is followed by at least one 1 in } w\} = 1^*(01^+)^*$.

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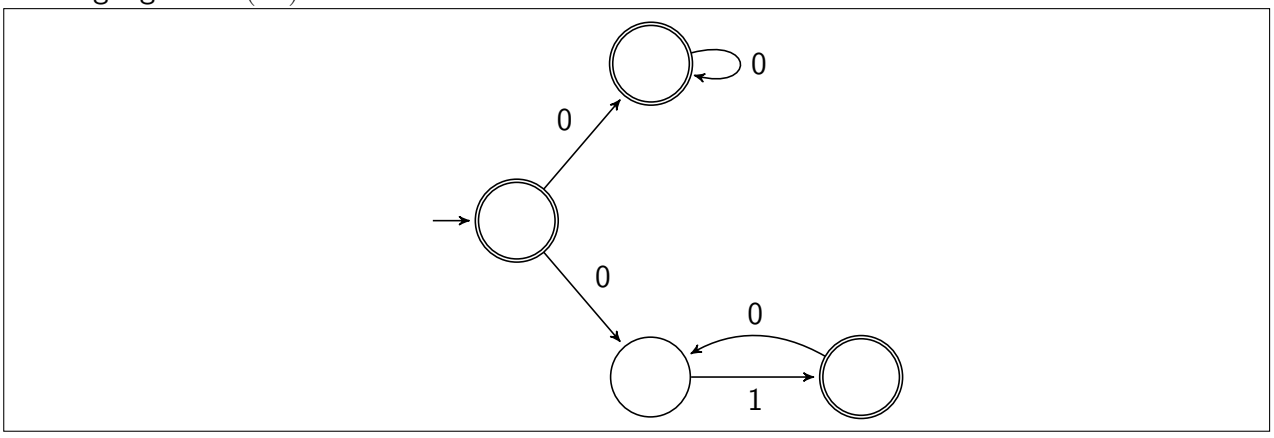
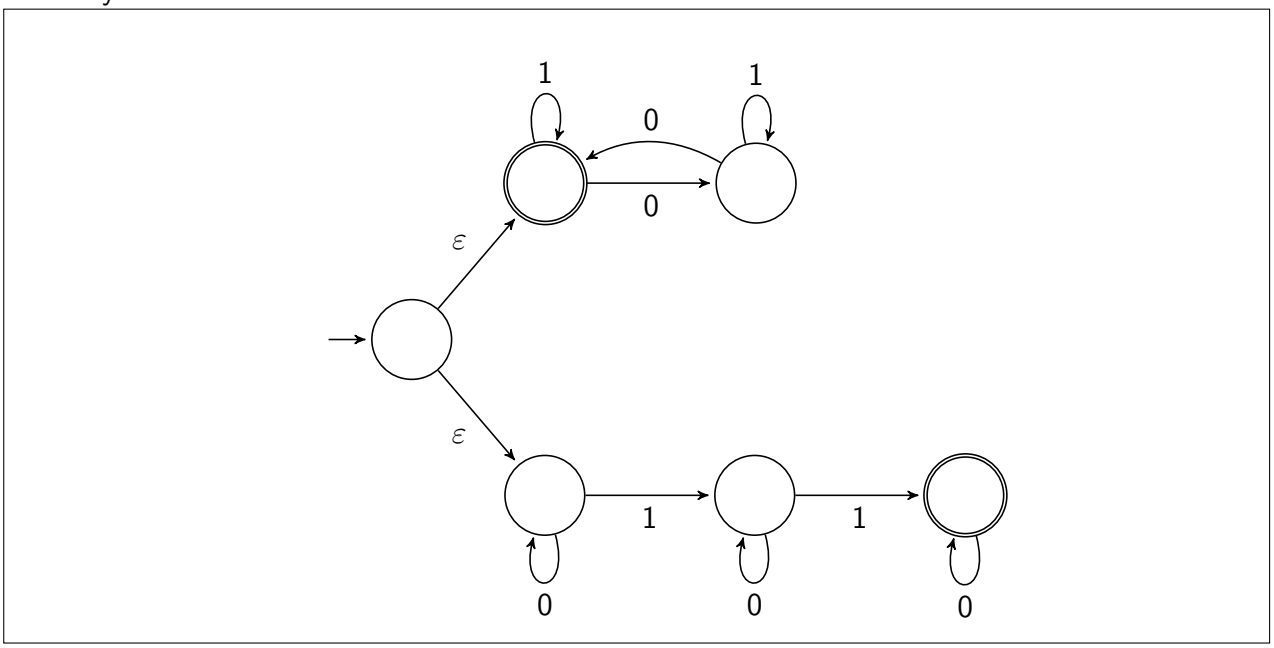
3. Shown below is the state diagram for an NFA that has no ε-transitions, 4 states, and will recognize the language $0^* \cup (01)^*$.



4. Shown below is the state diagram for an NFA that has six states and accepts the language

$$\{w \mid w \text{ contains an even number of 0s or contains exactly two 1s}\}.$$

You may use ε-transitions as needed.



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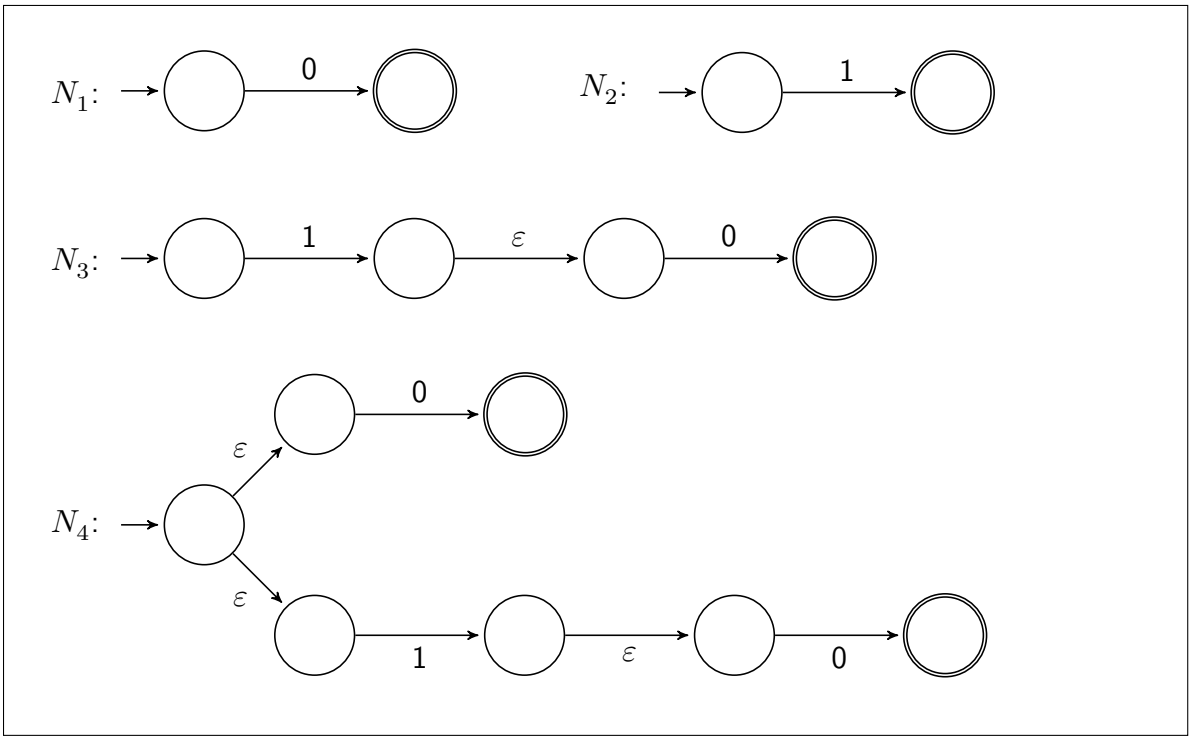
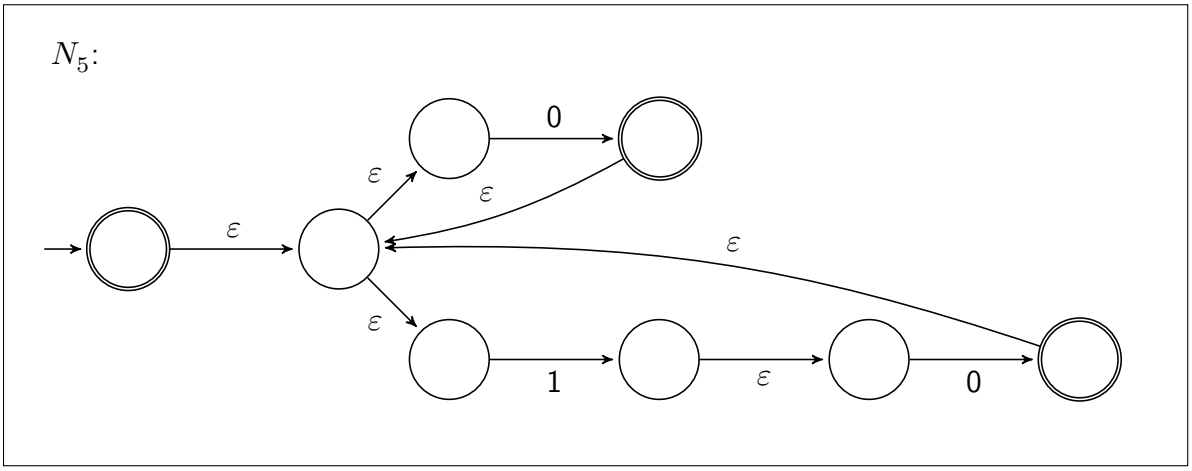
5. The state diagrams for the required machines are shown below:

   a. N_1, N_2, and N_3 with $L(N_1) = \{0\}$, $L(N_2) = \{1\}$, and

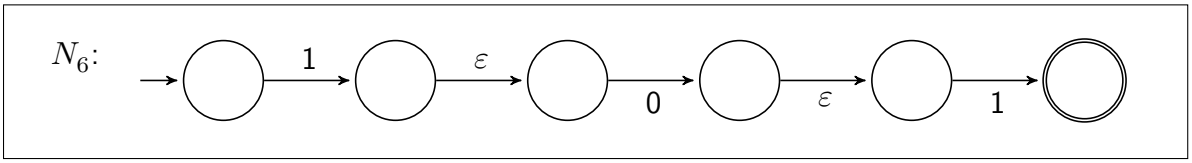
   b. The NFA N_4 with $L(N_4) = L(N_1) \cup L(N_3) = 0 \cup 10$. $L(N_3) = L(N_2) \circ L(N_1) = 10$.



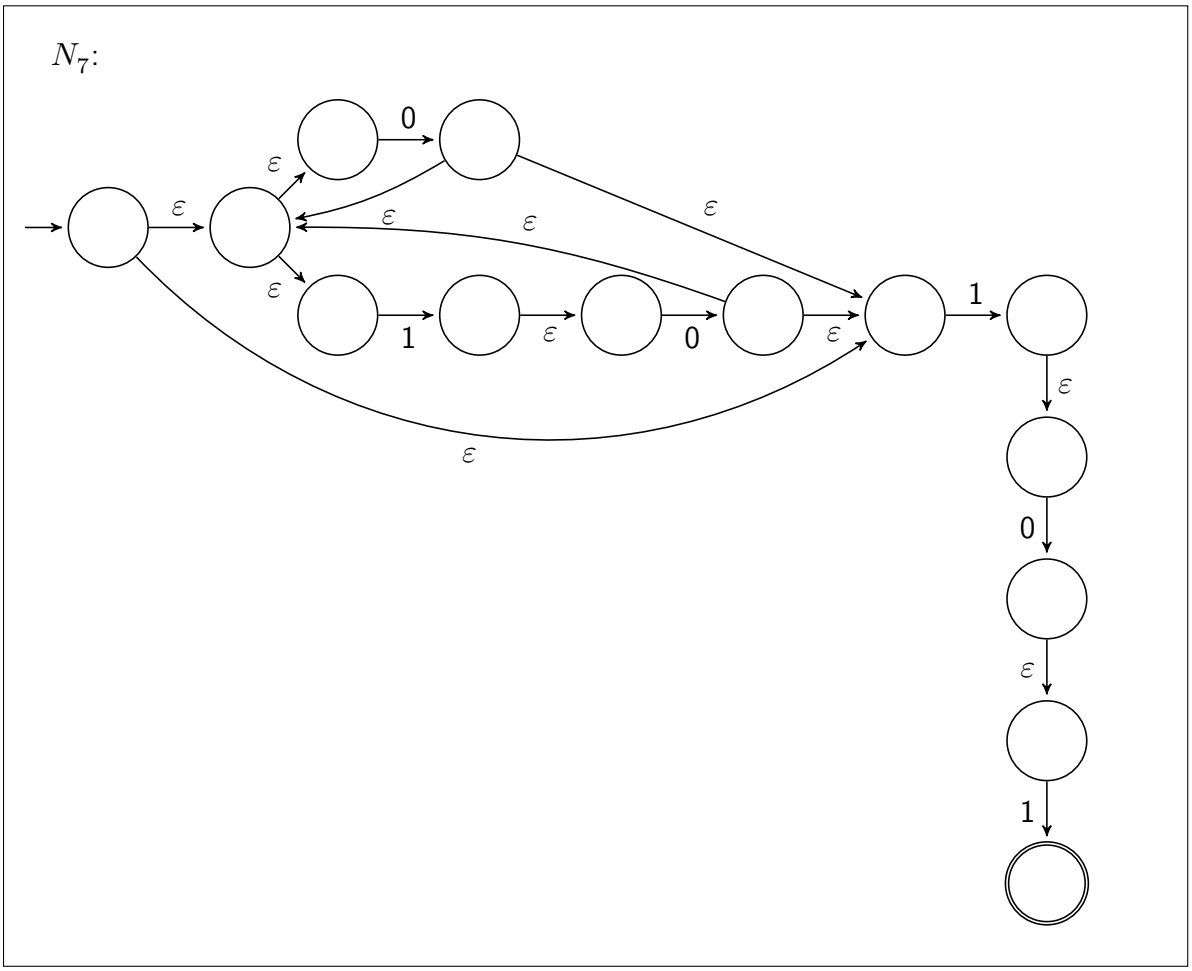
c. The NFA N_5 with $L(N_5) = [L(N_4)]^* = (0 \cup 10)^*$.



d. The NFA N_6 with $L(N_6) = L(N_3) \circ L(N_2) = 101$.

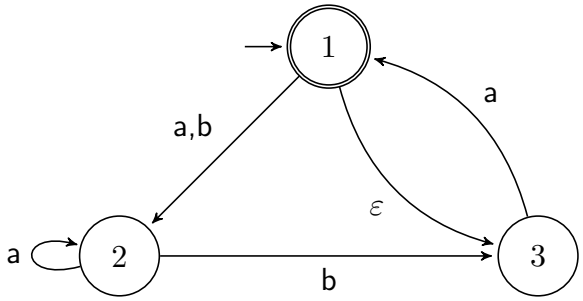


e. The NFA N_7 with $L(N_7) = L(N_5) \circ L(N_6) = (0 \cup 10)^*101$.



6. The state diagram for the given NFA and its equivalent DFA (along with the state transition table for the DFA) are shown below:

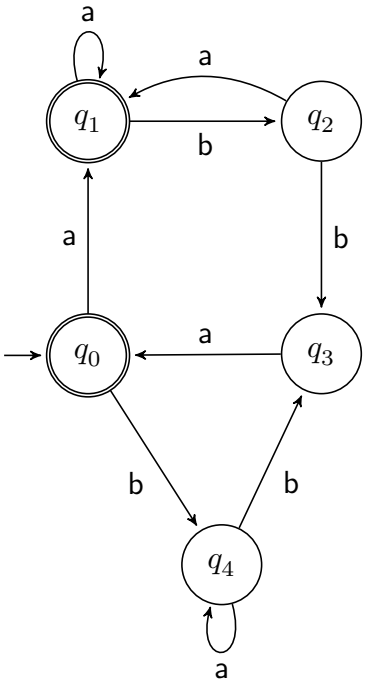
The given NFA:



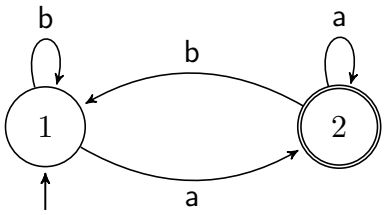
The state table for the equivalent DFA:

	a	b
$\{1\}^*$	$\{2\}$	$\{2\}$
$\{2\}$	$\{2\}$	$\{3\}$
$\{3\}$	$\{1,3\}$	$\emptyset$
$\{1,2\}^*$	$\{2\}$	$\{2,3\}$
$\rightarrow \{1,3\}^*$	$\{1,2,3\}$	$\{2\}$
$\{2,3\}$	$\{1,2,3\}$	$\{3\}$
$\{1,2,3\}^*$	$\{1,2,3\}$	$\{2,3\}1$
$\emptyset$	$\emptyset$	$\emptyset$

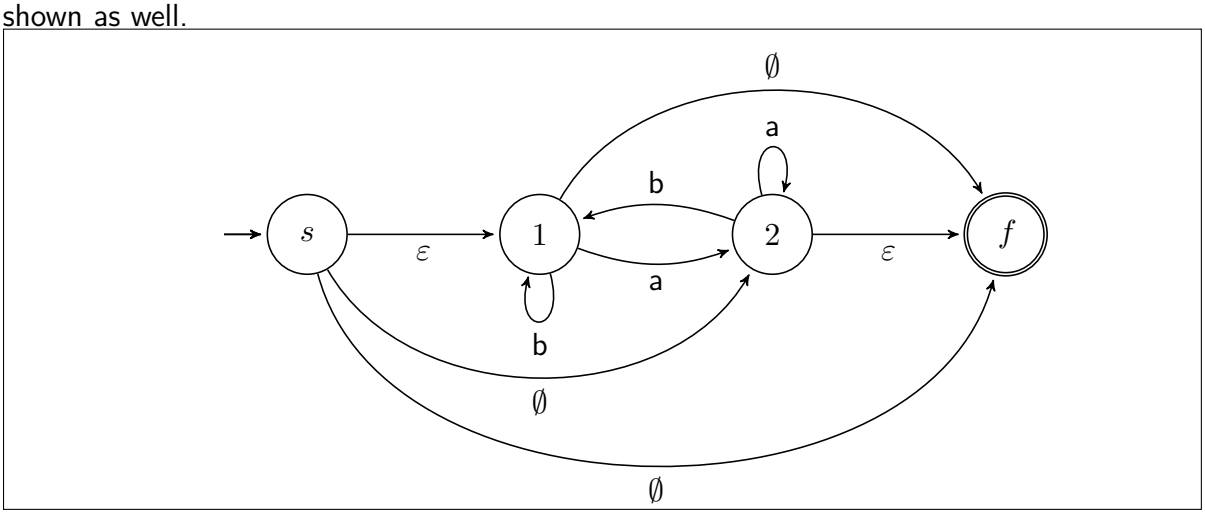
The state diagram for the DFA:
Let $q_0 = \{1,3\}$; $q_1 = \{1,2,3\}$; $q_2 = \{2,3\}$; $q_3 = \{3\}$; and $q_4 = \{2\}$.



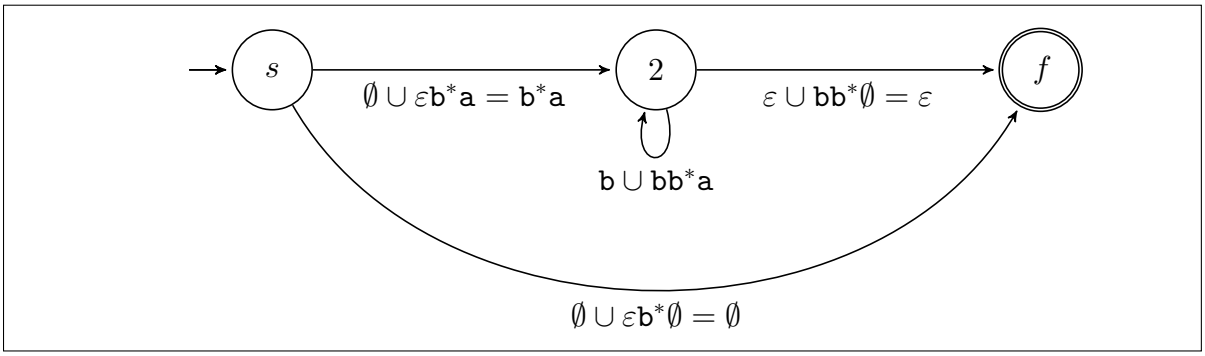
7. We convert the following DFA to its equivalent regular expression:



a. Shown below is the GNFA for the above diagram that we will need in order to obtain the regular expression. The start state is denoted by s and the final state by f. All the $\emptyset$ transitions are shown as well.



b. Shown below is the GNFA obtained after ripping state 1.



c. Shown below is the GNFA after ripping state 2.