Fall 2016: COT3210–Computability and Automata
Answers to Supplementary Exercises I

1. What is the language generated by the grammar given below?

$$S \to abB$$
$$A \to aaBb$$
$$B \to bbAa$$
$$A \to \varepsilon$$

We examine several productions and describe the language from the pattern that evolves:
For instance:

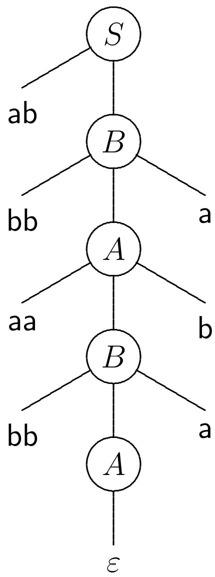
i. $S \Rightarrow abB \Rightarrow ab\ bb\ A\ a \Rightarrow ab\ bb\ a$

ii. $S \Rightarrow abB \Rightarrow ab\ bb\ A\ a \Rightarrow ab\ bb\ aa\ B\ b\ a \Rightarrow ab\ bbaa\ bb\ A\ aba \Rightarrow ab\ (bbaa)\ bb\ (ab)\ a$

iii. $S \Rightarrow abB \Rightarrow ab\ bb\ A\ a \Rightarrow ab\ bb\ aa\ B\ b\ a \Rightarrow ab\ bbaa\ bb\ A\ aba \Rightarrow ab\ bbaa\ bb\ aa\ B\ b\ aba$
$\Rightarrow ab\ bbaa\ bb\ aa\ bb\ A\ a\ b\ aba \Rightarrow ab\ bbaa\ bb\ aa\ bb\ a\ b\ aba$

The strings we obtained were $ab\ bb\ a$, $ab\ (bbaa)\ bb\ (ab)\ a$, and $ab\ (bbaa)^2\ bb\ (ab)^2\ a$.
We conclude the language generated by this grammar is $\{w | w = ab(bbaa)^n bb(ab)^n a\}$.

2. Show the parse tree for the string $abbbaabbaba$ for the grammar given above.



3. Transform the grammar given below into Chomsky normal form:

$$
\begin{aligned}
S &\to aSaaA \mid A \\
A &\to abA \mid bb
\end{aligned}
$$

After adding a new start variable S_0, the grammar becomes:

$$
\begin{aligned}
S_0 &\to S \\
S &\to aSaaA \mid A \\
A &\to abA \mid bb
\end{aligned}
$$

There are no ε-rules. Thus, we eliminate unit rules $S \to A$ and $S_0 \to S$. We obtain:

$$
\begin{aligned}
S_0 &\to aSaaA \mid abA \mid bb \\
S &\to aSaaA \mid abA \mid bb \\
A &\to abA \mid bb
\end{aligned}
$$

Now, we introduce new variables and new rules. Let $P \to a$, $Q \to b$.

Since $aSaaA$ now becomes $PSPPA$, let $S_0 \to PB$, $B \to SC$, $C \to PD$, and $D \to PA$.

Since abA becomes PQA, let $S \to PE$ and $E \to QA$. The grammar is now written in the form:

$$
\begin{aligned}
S_0 &\to PB \mid PE \mid QQ \\
S &\to PB \mid PE \mid QQ \\
A &\to PE \mid QQ \\
B &\to SC \\
C &\to PD \\
D &\to PA \\
E &\to QA \\
P &\to a \\
Q &\to b
\end{aligned}
$$

4. Transform the grammar given below into Chomsky normal form:

$$\begin{aligned} S &\to AB \mid aB \\ A &\to abb \mid \varepsilon \\ B &\to bbA \end{aligned}$$

After adding a new start variable S_0, the grammar becomes:

$$\begin{aligned} S_0 &\to S \\ S &\to AB \mid aB \\ A &\to abb \mid \varepsilon \\ B &\to bbA \end{aligned}$$

Removing $A \to \varepsilon$ yields:

$$\begin{aligned} S_0 &\to S \\ S &\to AB \mid aB \mid B \\ A &\to abb \\ B &\to bbA \mid bb \end{aligned}$$

Next, we remove the unit rules $S \to B$ and $S_0 \to S$ to obtain:

$$\begin{aligned} S_0 &\to AB \mid aB \mid bbA \mid bb \\ S &\to AB \mid aB \mid bbA \mid bb \\ A &\to abb \\ B &\to bbA \mid bb \end{aligned}$$

Now, we introduce new variables and new rules. Let $P \to a$, $Q \to b$.
Note that the rules for S_0 and S are the same and S does not appear on the right side.
Thus, we can drop the rule for S_0.
Since abb now becomes PQQ, bbA becomes QQA, we let $C \to QQ$. The grammar becomes:

$$\begin{aligned} S &\to AB \mid PB \mid CA \mid QQ \\ A &\to PC \\ B &\to CA \mid QQ \\ C &\to QQ \\ P &\to a \\ Q &\to b \end{aligned}$$