Spring 2018: COT3210–Computability and Automata
Test 03–Answers

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1. A CFG for the language $\{w | w$ has odd length$\}$ is $S \to 00S \,|\, 01S \,|\, 10S \,|\, 11S \,|\, 0 \,|\, 1$.

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2. A CFG for the language $\{w |$ the first and the last symbol of w are different$\}$ is:

$$S \to 0A1 \mid 1A0$$
$$A \to 0A \mid 1A \mid \varepsilon$$

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3. A CFG for the language $\{0^n 1^m | m, n \geq 0$ and $2n \leq m \leq 3n\}$ is: $S \to 0S11 \,|\, 0S111 \,|\, \varepsilon$.

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4. Given the following grammar for the language $L(G)$:

$$S \to \mathtt{a}S\mathtt{a} \mid \mathtt{a}B\mathtt{a} \mid \varepsilon$$
$$B \to \mathtt{b}B \mid \mathtt{b}$$

The required answers are:

i. Three strings in $L(G)$: ε; aa; aba.

ii. Three strings not in $L(G)$: aaa; b; ba.

iii. $L(G) = \varepsilon \cup \{\mathtt{a}^n\mathtt{b}^m\mathtt{a}^n \,|\, n > 0,\, m \geq 0\}$.

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5. Given the grammar G shown below:

$$E \to E + T \mid T$$
$$T \to T \times F \mid F$$
$$F \to (E) \mid \mathtt{a} \mid \mathtt{b}$$

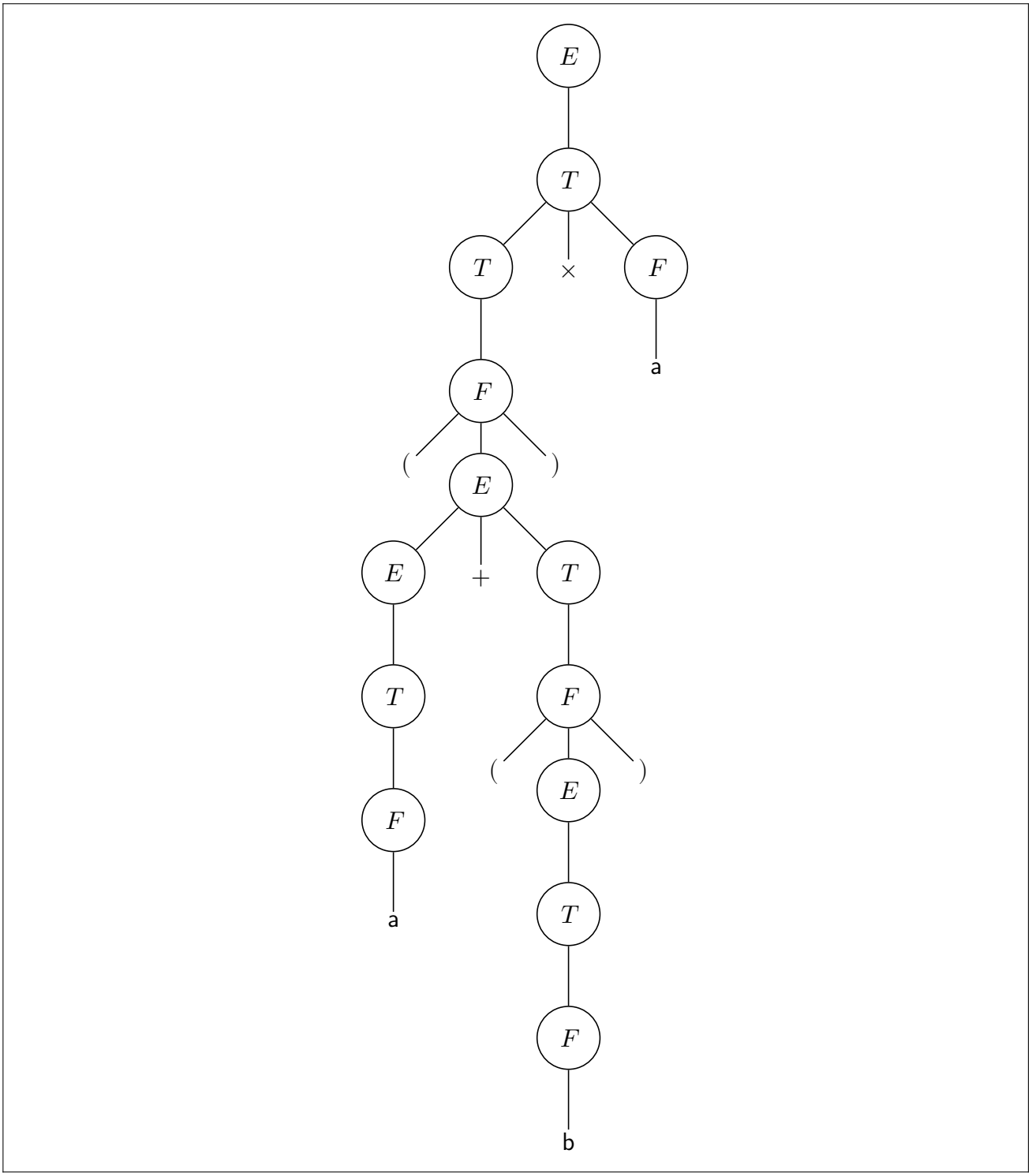
The leftmost derivation for the string $(\mathtt{a} + (\mathtt{b})) \times \mathtt{a}$ is:

$$E \Rightarrow T \Rightarrow T \times F \Rightarrow F \times F \Rightarrow (E) \times F \Rightarrow (E + T) \times F \Rightarrow (T + T) \times F \Rightarrow$$
$$(F + T) \times F \Rightarrow (\mathtt{a} + T) \times F \Rightarrow (\mathtt{a} + F) \times F \Rightarrow (a + (E)) \times F \Rightarrow (\mathtt{a} + (T)) \times F$$
$$\Rightarrow (\mathtt{a} + (F)) \times F \Rightarrow (\mathtt{a} + (\mathtt{b})) \times F \Rightarrow (\mathtt{a} + (\mathtt{b})) \times \mathtt{a}$$

6. Suppose $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, and the grammar G shown below are given.

$$E \to E + T \mid T$$
$$T \to T \times F \mid F$$
$$F \to (E) \mid \mathtt{a} \mid \mathtt{b}$$

Show the parse tree for the string (a+(b))×a.



7. Transform the grammar given below into Chomsky normal form:

$$S \to ASA \mid \mathtt{a}B$$
$$A \to B \mid S$$
$$B \to \mathtt{b} \mid \varepsilon$$

Step-by-step conversion of the given CFG to CNF:

1. Add a new start symbol S_0; and eliminate the rule $B \to \varepsilon$.

$$S_0 \to S$$
$$S \to ASA \mid \mathtt{a}B \mid \mathtt{a}$$
$$A \to B \mid S \mid \varepsilon$$
$$B \to \mathtt{b}$$

2. Eliminate the rule $A \to \varepsilon$;

$$S_0 \to S$$
$$S \to ASA \mid AS \mid SA \mid S \mid \mathtt{a}B \mid \mathtt{a}$$
$$A \to B \mid S$$
$$B \to \mathtt{b}$$

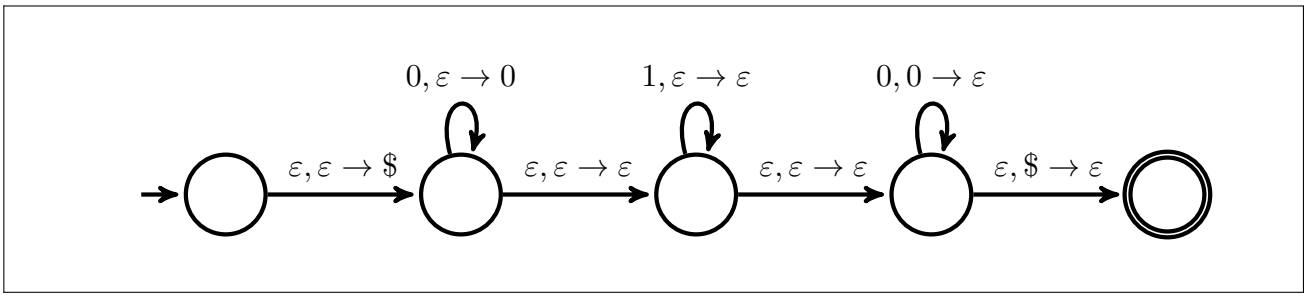
3. Eliminate unit rules $A \to B$, $S \to S$, and $S_0 \to S$

$$S_0 \to ASA \mid AS \mid SA \mid \mathtt{a}B \mid \mathtt{a}$$
$$S \to ASA \mid AS \mid SA \mid \mathtt{a}B \mid \mathtt{a}$$
$$A \to ASA \mid AS \mid SA \mid \mathtt{a}B \mid \mathtt{a} \mid \mathtt{b}$$
$$B \to \mathtt{b}$$

4. Make up new variables to obtain CNF:

$$S_0 \to VA \mid AS \mid SA \mid UB \mid \mathtt{a}$$
$$S \to VA \mid AS \mid SA \mid UB \mid \mathtt{a}$$
$$A \to ASA \mid AS \mid SA \mid UB \mid \mathtt{a} \mid \mathtt{b}$$
$$V \to AS$$
$$U \to \mathtt{a}$$
$$B \to \mathtt{b}$$

8. Draw the state diagram for the PDA that accepts the language
$$\{w = 0^n 1^m 0^n \,|\, m, n \geq 0\}.$$



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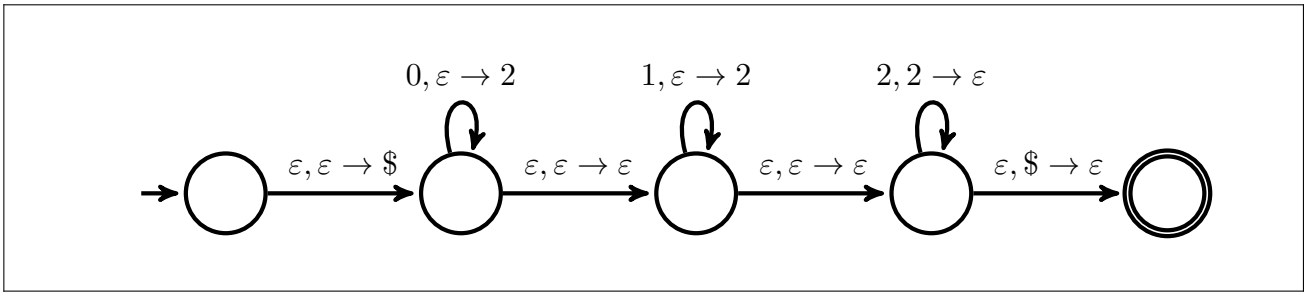
9. Draw the state diagram for the PDA that accepts the language
$$\{w = 0^m 1^n 2^p \,|\, m, n \geq 0, m + n = p\}.$$



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10. Draw the state diagram for the PDA that accepts the language
$$\{1^m 0^m 1^n 0^n \,|\, m, n \geq 0\}.$$