Spring 2018: COT3210–Computability and Automata
Homework Problem Set 05 Solutions

7.1 The answers are given below:

a. True
b. False
c. False
d. True
e. True
f. True

7.3 The answers are given below:

a. The remainder sequence produced by the Euclidean algorithm when started with the numbers 10505 and 1274 is shown below:

10505, 1274, 313, 22, 5, 2, 1, 0

Thus, $\gcd(1050, 1274) = 1$, which allows us to conclude that 10505 and 1274 are relatively prime.

b. The remainder sequence produced by the Euclidean algorithm when started with the numbers 8029 and 7289 is shown below:

8029, 7289, 740, 629, 111, 74, 37, 0

Thus, $\gcd(8029, 7289) = 37$, which allows us to conclude that 10505 and 1274 are not relatively prime.

7.5 The answer is given below:

The formula is not satisfiable. For any assignment of the boolean values for x and y, it always makes one of the four clauses false. Therefore, the formula, which is a conjunction of the four clauses, is always false for any assignment of x and y.

7.8 The answer is shown below:

The algorithm given in page 185 runs in $O(n^3)$ time. Stage 1 takes at most $O(n)$ steps to locate and mark the start node. Stage 2 causes at most $n+1$ repetitions, because each repetition except the last marks at least one additional node. Each execution of stage 3 uses at most $O(n^3)$ steps because G contains at most n to be checked and for each checked node, examining all adjacent nodes to see whether any have been marked uses at most $O(n^2)$ steps. Therefore in total, stages 2 and 3 take $O(n^4)$ time. Stage 4 uses $O(n)$ steps to scan all nodes. Therefore, the algorithm runs in $O(n^4)$ time and *CONNECTED* is in P.

7.9 The answer is shown below:

We construct a TM M that decides *TRIANGLE* in polynomial time.

M = "On input $\langle G \rangle$ where G is a graph:

1. For each triple of vertices v_1, v_2, v_3 in G:
2. If edges (v_1, v_2), (v_1, v_3), and (v_2, v_3), are all edges of G, *accept*.
3. No triangle has been found in G, so *reject*."

A graph with m vertices has $\binom{m}{3} = \frac{m!}{3!(m-3)!} = O(m^3)$ triples of vertices. Therefore, stage 2 will be repeated at most $O(m^3)$ times. In addition, each stage can be implemented to run in polynomial time. Therefore, *TRIANGLE* $\in$ P.

7.12 The answer is shown below:

A nondeterministic polynomial time algorithm for *ISO* operates as follows:

"On input $\langle G, H \rangle$ where G and H are undirected graphs:

1. Let m be the number of nodes of G and H. If they don't have the same number of nodes, *reject*.
2. Nondeterministically select a permutation π of m elements.
3. For each pair of nodes x and y of G check that (x, y) is an edge of G iff $(\pi(x), \pi(y))$ is an edge of H. If all agree, *accept*. If any differ, *reject*."

Stage 2 can be implemented in polynomial time nondeterministically. Stage 3 takes polynomial time. Therefore *ISO* $\in$ NP.