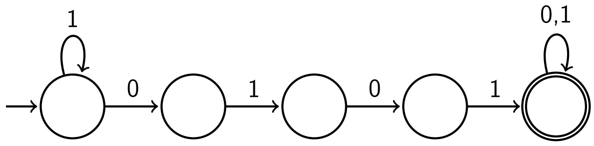
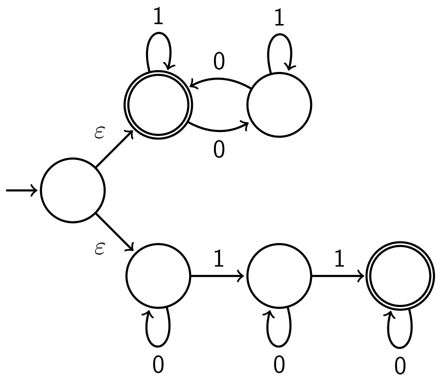
COT3210–Computability and Automata
Homework Problem Set 02 Solutions

Page 84: Exercise 1.7 The required NFAs are shown below.

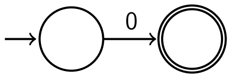
1.7b. $L(M) = \{w \mid w$ contains the substring 0101$\}$ with five states.



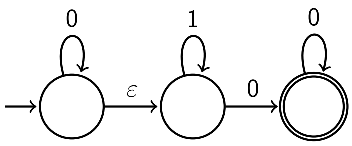
1.7c $L(M) = \{w \mid w$ contains an even number of 0s, or contains exactly two 1s$\}$ with six states.



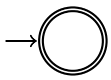
1.7d $L(M) = \{0\}$ with two states.



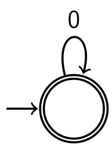
1.7e $L(M) = \{0^*1^*0^+\}$ with three states.



1.7g. $L(M) = \{\varepsilon\}$ with one state.



1.7h. $L(M) = 0^*$ with one state.

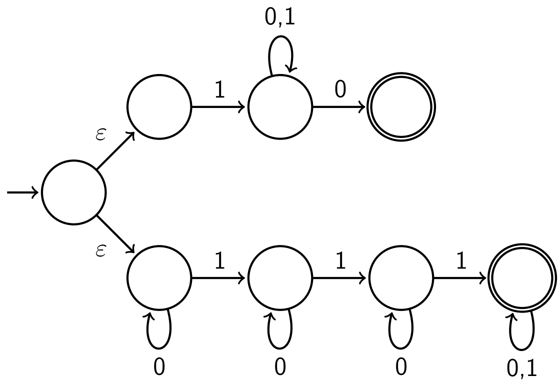


<u>Page 84: Exercise 1.8</u> The required NFAs are shown below.

1.8a. Let

$$A = \{w \,|\, w \text{ begins with a 1 and ends in a 0}\},$$
$$B = \{w \,|\, w \text{ has at least three 1s}\}.$$

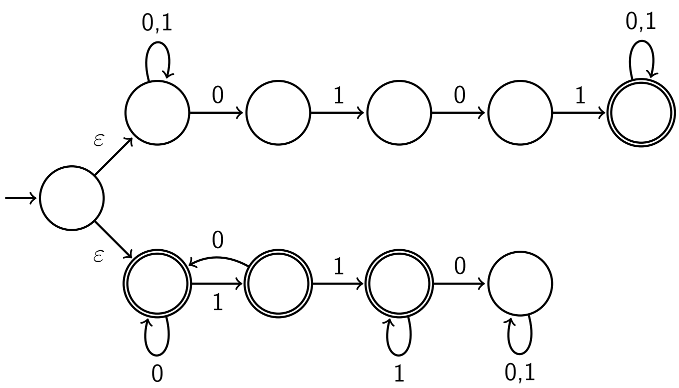
Construct the NFA M with $L(M) = A \cup B$.



1.8b. Let

$$A = \{w \,|\, w \text{ contains the substring 0101}\},$$
$$B = \{w \,|\, w \text{ doesn't contain the substring 110}\}.$$

Construct the NFA M with $L(M) = A \cup B$.

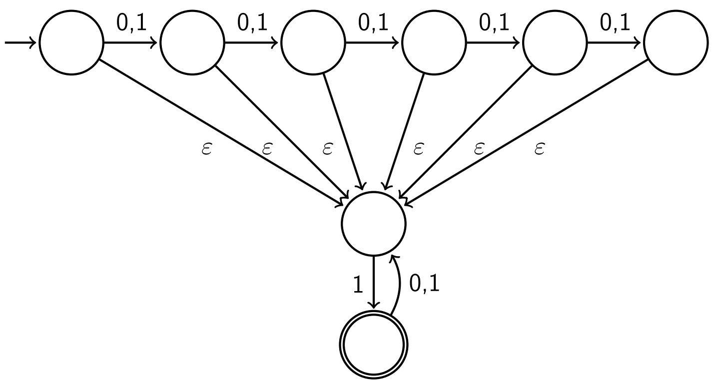


Page 85: Exercise 1.9 The answers are given below:

1.9a. Let

$$A = \{w \mid w \text{ has length at most } 5\},$$
$$B = \{w \mid w \text{ every odd position of } w \text{ is a } 1\}.$$

Construct M with $L(M) = A \circ B$.

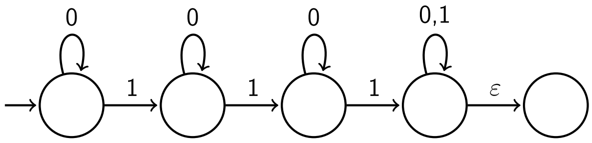


1.9b. Let

$$A = \{w \mid w \text{ has lat least three 1s}\},$$
$$B = \text{The empty set}.$$

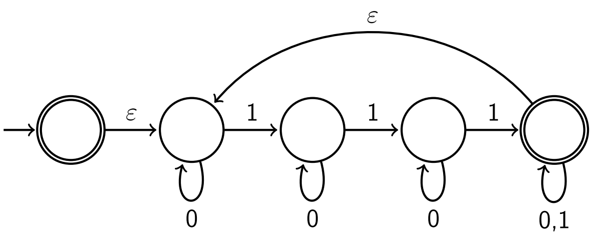
Construct M with $L(M) = A \circ B$.

M is as shown below.

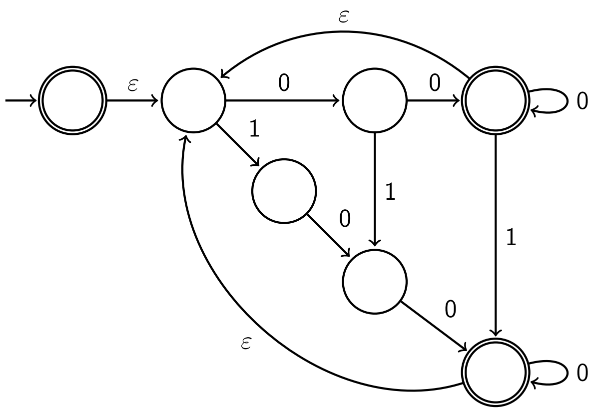


Page 85: Exercise 1.10 The answers are given below:

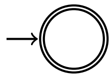
1.10a. Let $A = \{w \mid w \text{ has at least three 1s}\}$. Construct M with $L(M) = A^*$.



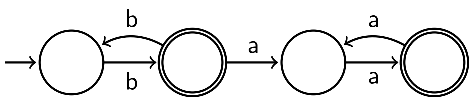
1.10b. Let $A = \{w \mid w$ has at least two 0s and at most one $1\}$. Construct M with $L(M) = A^*$.



1.10c. Let $A =$ The empty set. Construct M with $L(M) = A^*$.



Page 85: Exercise 1.12 D does not contain the substring ab. Therefore, D must contain strings with an odd number of b's followed by an even number of a's. These strings are represented by $b(bb)^*(aa)^*$.



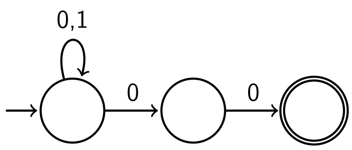
Page 85: Exercise 1.14 The answers are given below:

1.14a. Proving regular languages are closed under the complement.

Given the DFA $D = (Q, \Sigma, \delta, q_0, F)$, define $D' = (Q, \Sigma, \delta, q_0, F')$ where $F' = Q - F$. In other words, the accept states are swapped with the nonaccept states.

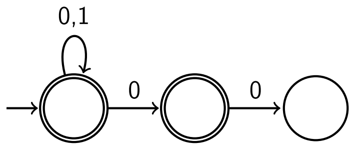
Suppose $w \in L(D)$, then the computation ends in state $q \in F$, so $q \notin F'$ and so $w \notin L(D')$. Similarly, suppose $w \in L(D')$. Then the computation ends in state $q \in F'$, so $q \notin F$ and so $w \notin L(D)$. Therefore $L(D') = \overline{L(D)}$.

1.14b. Languages accepted by NFAs are not closed under complementation.
Let M be the machine with $L(M) = \{w \mid w$ ends in $00\}$.



Then, $\overline{L(M)} = \{w \mid w$does not end in $00\}$. Now, let M' represent the machine obtained by swapping the accept states with the non-accept states, as shown in the state diagram below:

However, note that the string 100, which ends in 00, is accepted by M'.

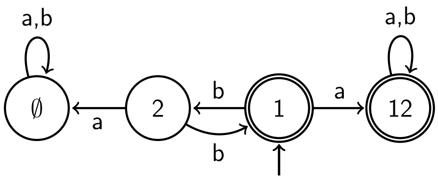


Thus, $L(M') \neq \overline{L(M)}$.

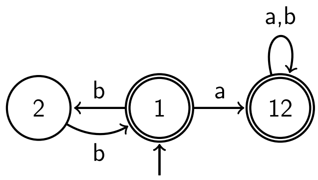
Page 86: Exercise 1.16 The answers are given below:

1.16a. Let $1 \equiv \{1\}$; $12 \equiv \{1,2\}$; $2 \equiv \{2\}$.

The state diagram for the desired DFA is as shown below:



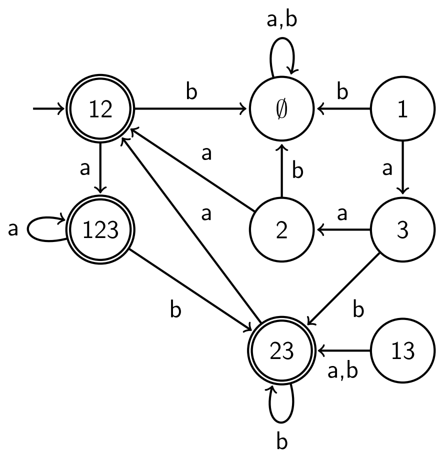
It is customary to not show the $\emptyset$ state. Thus, not every state will necessarily have a transition for each symbol in the alphabet. The revised state diagram with this modification of the above DFA is shown below:



1.16b. Let

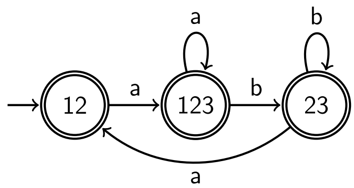
$$12 \equiv \{1,2\}, 123 \equiv \{1,2,3\}, 23 \equiv \{2,3\}, 13 \equiv \{1,3\},$$
$$3 \equiv \{3\}, \quad 1 \quad \equiv \{1\}, \qquad 2 \quad \equiv \{2\}.$$

The state diagram for the desired DFA is as shown below:



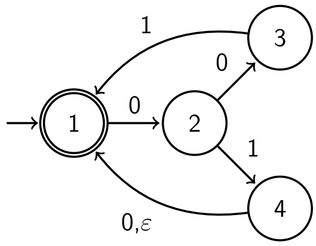
Note that states 13, 3, 1, 2, and $\emptyset$ can be removed.

Now the following FA is obtained:

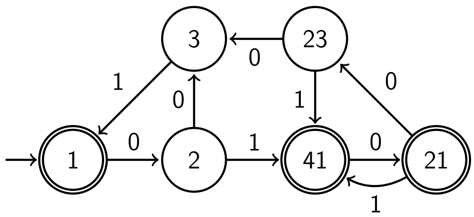


Page 86: Exercise 1.17 The answers are given below:

1.17a. A NFA that recognizes the language $(01 \cup 010 \cup 001)^*$:



1.17b. The equivalent DFA:

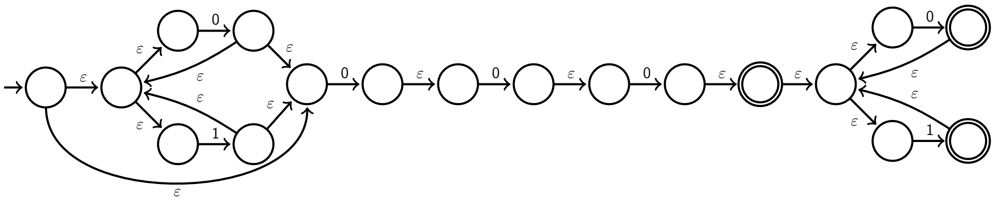


Page 86: Exercise 1.18 Note that any string of 0s and 1s is denoted by Σ^*.

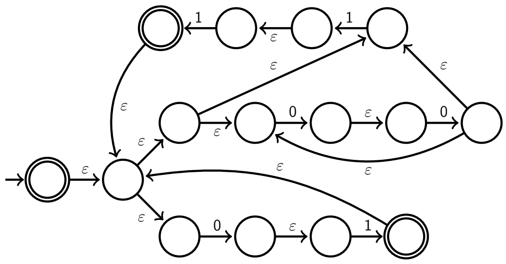
a. $\{w \,|\, w$ begins with a 1 and ends with a 0$\} \equiv 1\Sigma^*0$
b. $\{w \,|\, w$ contains at least three 1s$\} \equiv \Sigma^*1\Sigma^*1\Sigma^*1\Sigma^*$
c. $\{w \,|\, w$ contains the substring 0101$\} \equiv \Sigma^*0101\Sigma^*$
d. $\{w \,|\, w$ has length at least 3 and its third symbol is a 0$\} \equiv \Sigma\Sigma0\Sigma^*$
e. $\{w \,|\, w$ starts with 0 and has odd length,
or starts with 1 and has even length$\} \equiv 0(\Sigma\Sigma)^* \cup 1\Sigma(\Sigma\Sigma)^*$
f. $\{w \,|\, w$ doesn't contain the substring 110 $\} \equiv (0 \cup 10)^*1^*$
g. $\{w \,|\, w$ the length of w is at most 5 $\} \equiv (\varepsilon \cup \Sigma)^5$
h. $\{w|$ is any string except 11 and 111$\} \equiv \varepsilon \cup 1 \cup (0 \cup 10 \cup 110 \cup 111\Sigma)\Sigma^*$
i. $\{w|$ every odd position of w is a 1$\} \equiv (1\Sigma)^*(1 \cup \varepsilon)$
j. $\{w|$ contains at least two 0s and at most one 1$\} \equiv 000^* \cup (000^*1 \cup 100 \cup 010)0^*$
k. $\{\varepsilon, 0\} = \varepsilon \cup 0$
l. $\{w|w$ contains an even number of 0s or exactly two 1s$\} \equiv (1^*01^*01^*)^* \cup (0^*10^*10^*)$
m. $\emptyset$
n. $\{w|w$ is any string except the empty string$\} \equiv \Sigma^+$

Page 86: Exercise 1.19 The answers are given below:

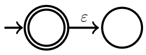
a. The NFA for $(0 \cup 1)^*000(0 \cup 1)^*$



b. The NFA for $(((00)^*(11)) \cup 01)^*$



c. The NFA for $\emptyset^*$ is show below:

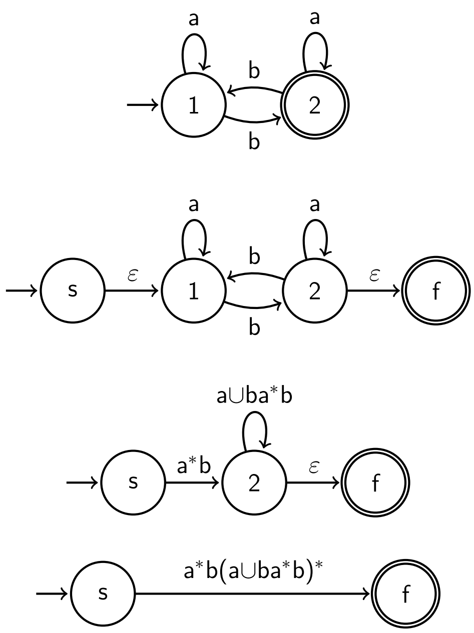


Page 87: Exercise 1.20 The answers are given below: (Assume $\Sigma = \{a, b\}$)

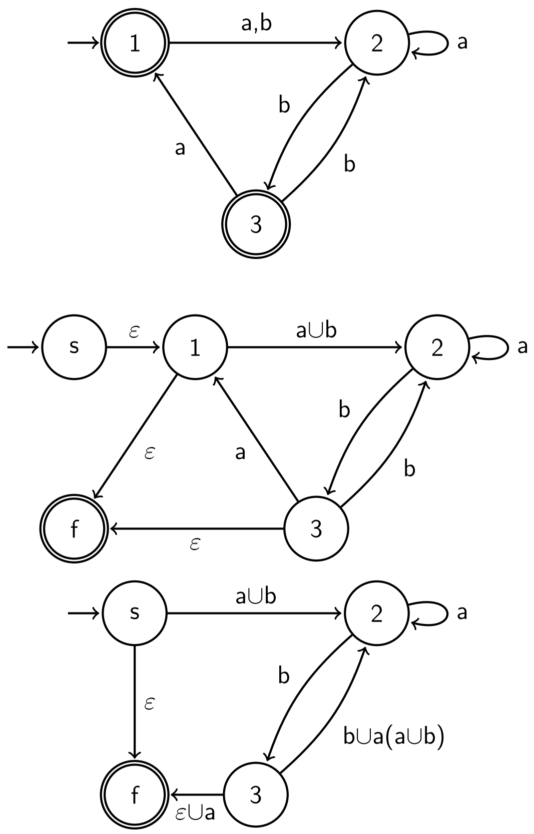
	RE	Members	Non-members
a.	a^*b^*	ε, a, b, aa, aab, bb	ba, aba
b.	$a(ba)^*b$	$abab$, $ababab$	ε, abb, ba
c.	$a^* \cup b^*$	a, aa, b, bb	ab, ba
d.	$(aaa)^*$	ε, aaa, $aaaaaa$	a, aa, $aaaa$
e.	$\Sigma^*a\Sigma^*b\Sigma^*a\Sigma^*$	aba, $baba$	ε, a, b ab
f.	$aba \cup bab$	aba, bab	ε, aa
g.	$(\varepsilon \cup a)b$	b, ab	ε, a, aa
h.	$(a \cup ba \cup bb)\Sigma^*$	a, ba, bb	ε, b

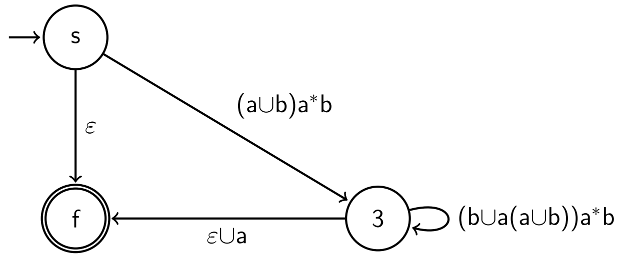
Page 86: Exercise 1.21 The answers are give below:

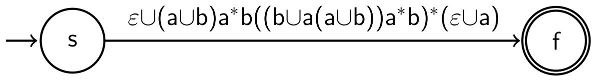
a. $a^*b(a \cup ba^*b)^*$



b. $\varepsilon \cup (a \cup b)a^*b((b \cup a(a \cup b))\, a^*b)^*(\varepsilon \cup a)$




s
(a∪b)a*b
ε
f
3
(b∪a(a∪b))a*b
ε∪a


s
ε∪(a∪b)a*b((b∪a(a∪b))a*b)*(ε∪a)
f