

COL106

Data Structures and Algorithms

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COL106 Plagiarism Policy and Honor Code

- Copying or collaborating or use of any other unethical means in exams, quizzes is not permitted
- Every assignment will have its own honor code
 - Indicating what is permitted and what is not
- Read the honor code carefully, ask in case of doubts
- Violation of Honor code will lead to
 - D grade
 - Disciplinary committee (DISCO)
 - Any other penalty deemed fit by the instructors

Should we have Quiz
Today?

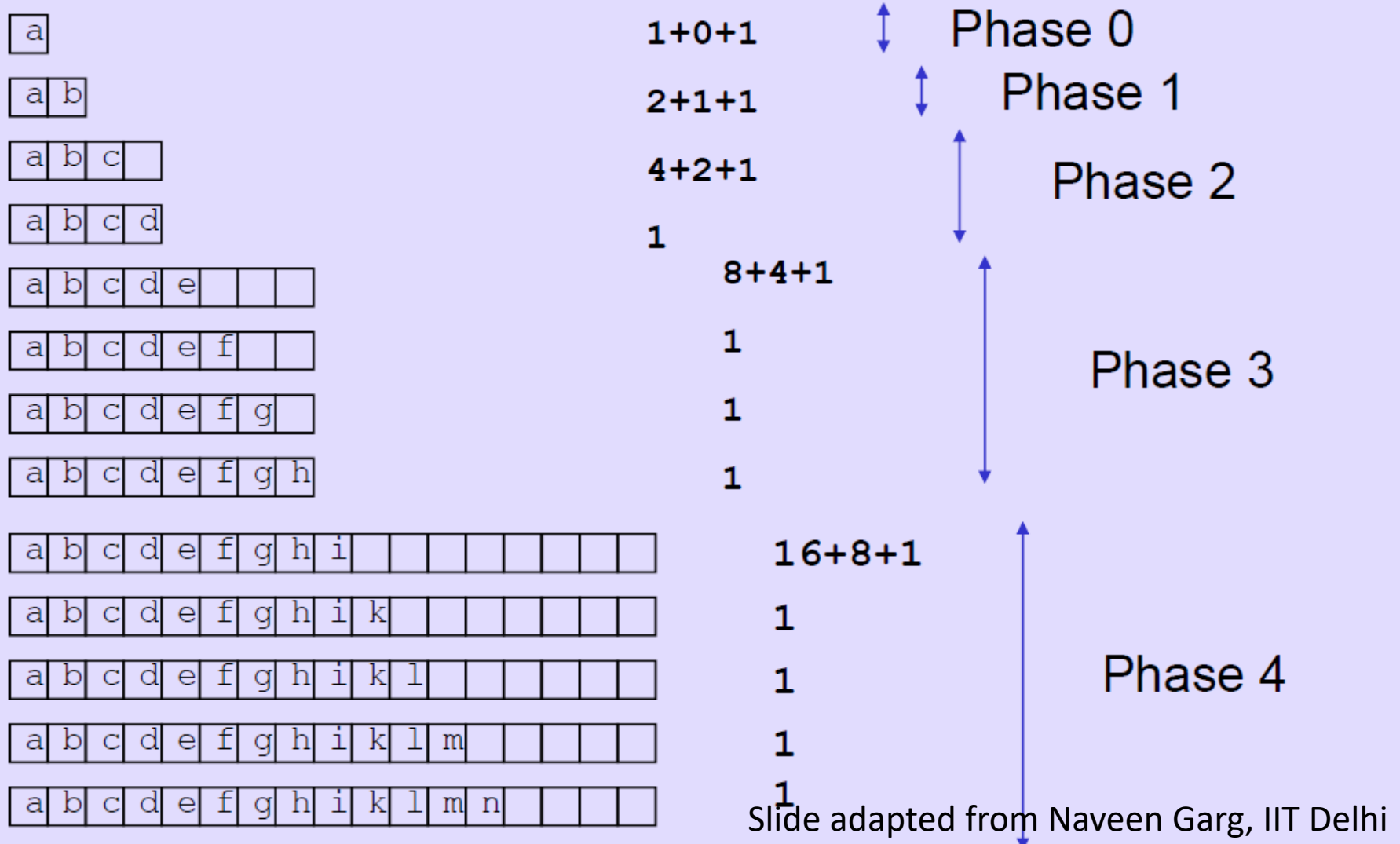
Dynamic Array-Based Stacks: Analysis

A Growable Stack: Multiplicative Increase

```
#define C 2
void Stack::push(int e) {
    if (t >= stackCapacity - 1) { // Reallocate space and copy
        int *temp = new int[stackCapacity * C]; // TBD: Check for error
        for (i = 0; i <= t; i++)
            temp[i] = S[i];
        delete S;
        S = temp;
        stackCapacity *= C;
    }
    S[++t] = e;
}
```

Analysis: Multiplicative Increase

start with an array of size 0. cost of a special push is $3N + 1$



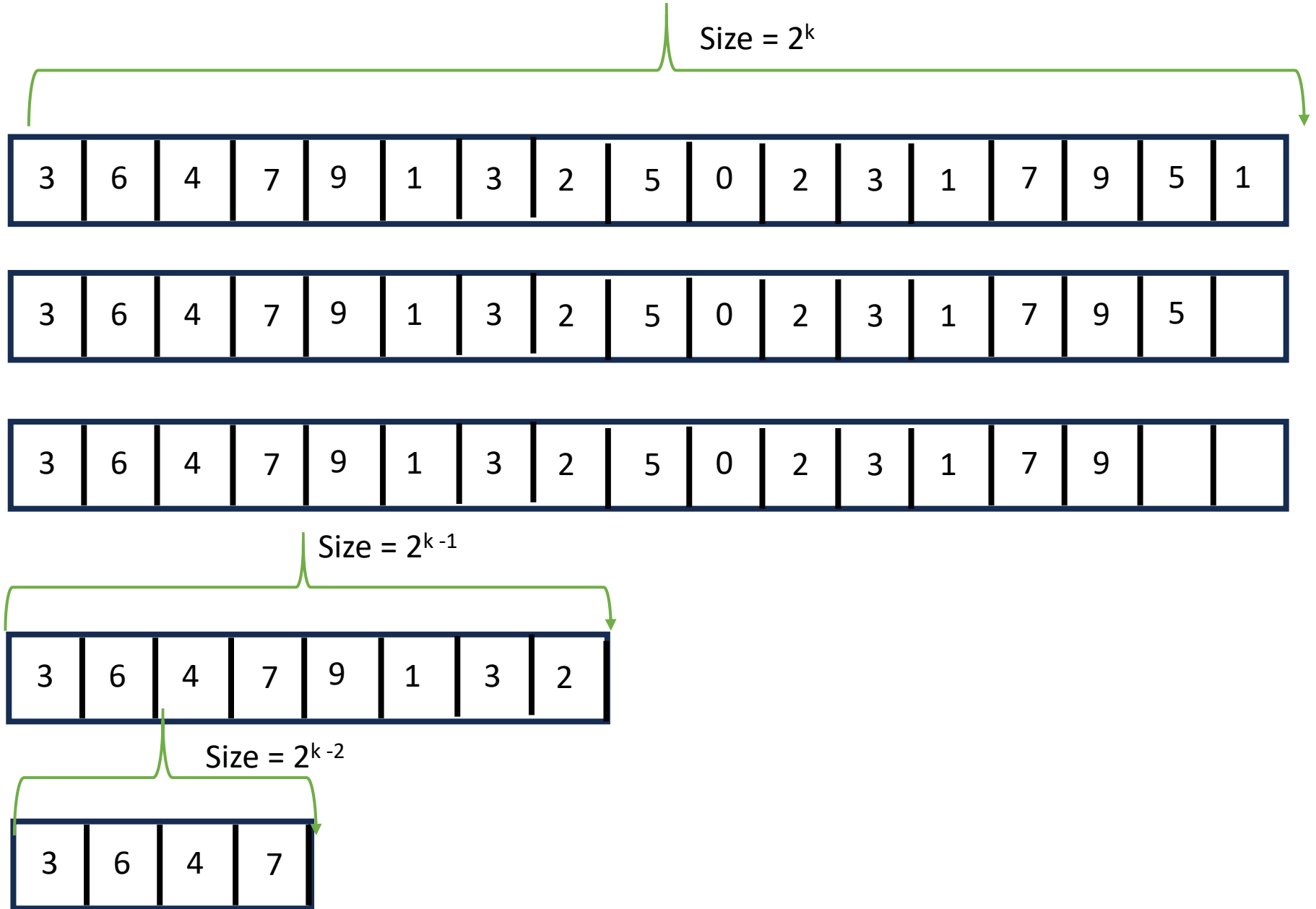
Analysis: Multiplicative Increase

- In phase i the array has size 2^i
- Total cost of phase i is
 - 2^i is the cost of creating the array
 - 2^{i-1} is the cost of copying elements into new array
 - 2^{i-1} is the cost of the 2^{i-1} pushes done in this phase
- Hence, cost of phase i is 2^{i+1}
- If we do n pushes, we will have $\log n$ phases.
- Total cost of n pushes
- $= 2 + 4 + 8 + \dots + 2^{\log n + 1} = 4n - 1$
- $T(n) = O(n)$

Shrinking: Multiplicative Decrease

```
#define C 2
int Stack::pop() {
    int result = S[t];
    if (t <= stackCapacity / C) { // Reallocate space and copy
        int *temp = new int[stackCapacity / C]; // TBD: Check error
        for (i = 0; i <= t; i++)
            temp[i] = S[i];
        delete S;
        S = temp;
        stackCapacity /= C;
    }
    t--;
    return result;
}
```


Analysis: Multiplicative Decrease



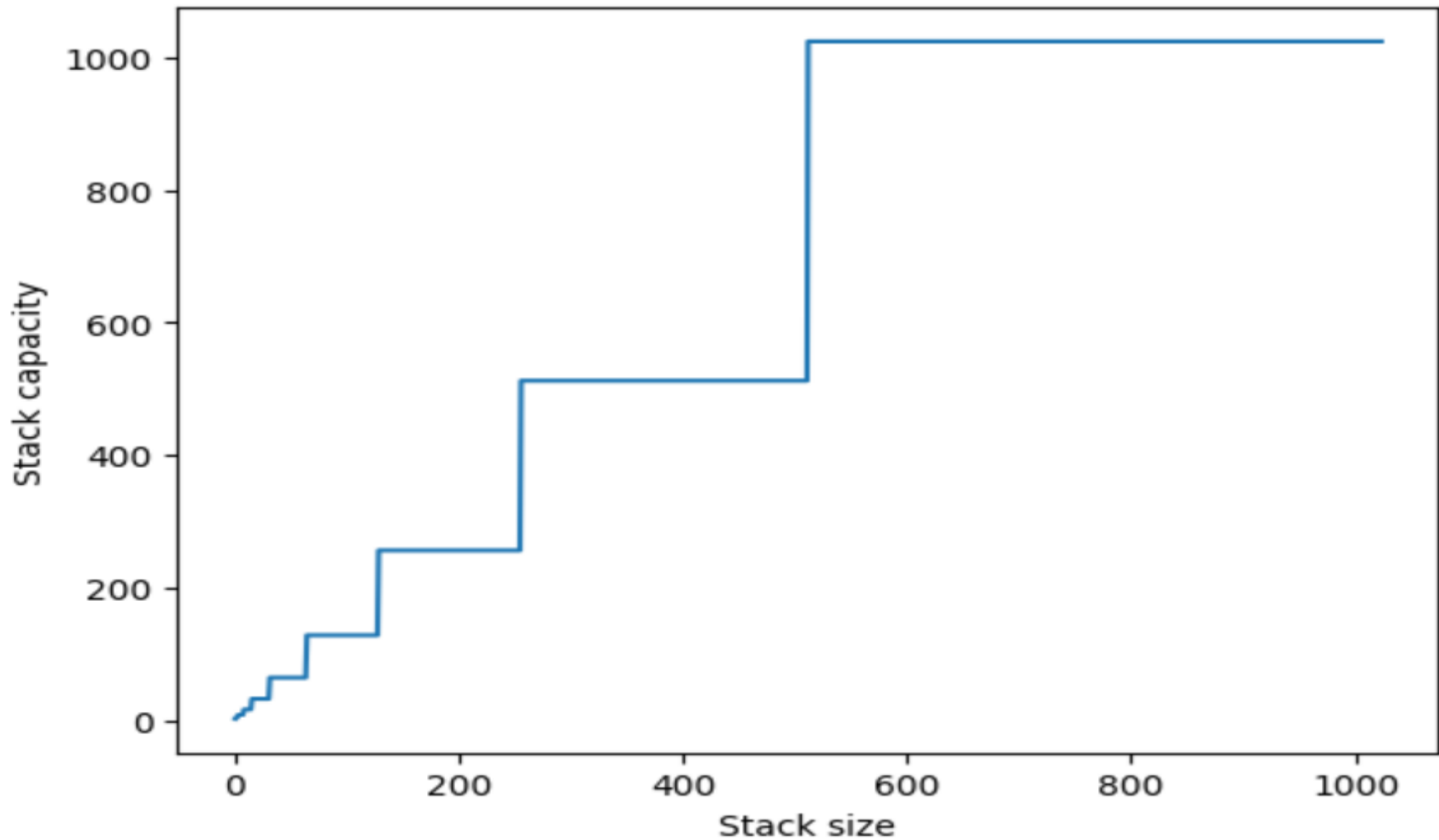
Analysis of Dynamic Stacks

- Assume get $N (= 2^k)$ push() followed by N pop()
- $T(2N) = \text{Time for } N \text{ push} + \text{Time for } N \text{ pop}$
- $= O(N) + O(N) = O(N)$
- $T(N) = O(N)$ – True or false?

Analysis of Dynamic Stacks

- Assume get $N = 2^k$ push() followed by N pop()
- $T(2N) = \text{Time for } N \text{ push} + \text{Time for } N \text{ pop}$
- $= O(N) + O(N) = O(N)$
- $T(N) = O(N)$ – **False.**
- Because $T(N)$ represents the time taken for the worst-case input

Analysis of Dynamic Stacks



Consider a sequence of 513 `push()` operations followed by 511 (`pop()` + `push()`) operation.

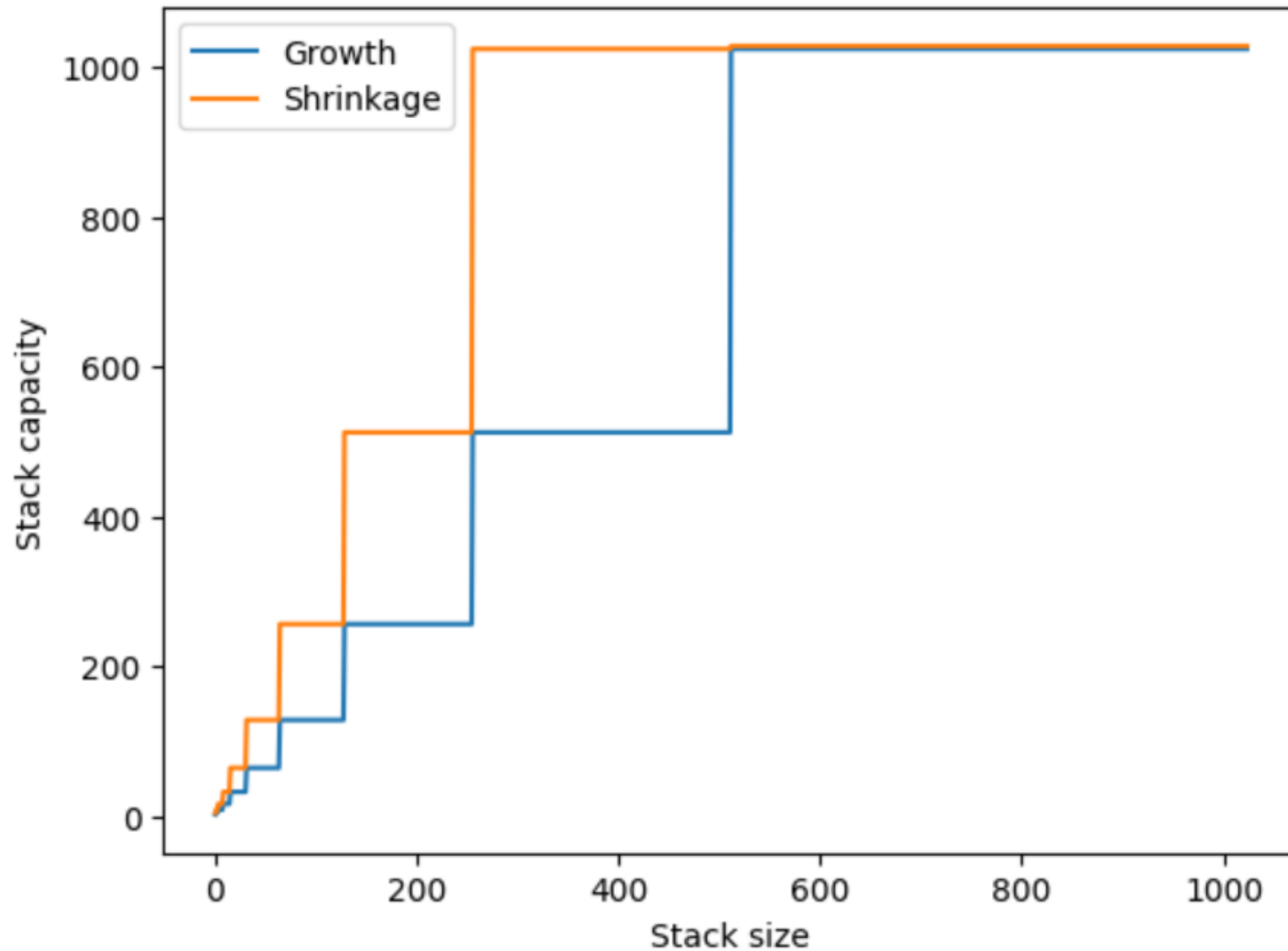
Dynamic Stacks: Worst case

- In general, consider 2^{k+1} push() operations
- Followed by 2^{k-1} push()+pop() operations
- Total number of operations
$$2^{k+1} + 2 * 2^{k-1} = 2^{k+1} + 2^k = 2^{k+1} + 2^k = N$$
- Time taken by 2^{k+1} push() operations
$$O(2^{k+1}) = O(N/2) = O(N)$$
- Time taken by each push()+pop() operation
$$2^{k+1} + 2^k + 2^k = 2^{k+2} = 2N - 2$$
- Total time taken by 2^{k-1} [= (N-1)/4] push()+pop() operations
$$((N-1)/4) * (2N - 2) = O(N^2)$$

Solution: Hysteresis

- **Grow** the stack as earlier
- While shrinking the stack, do not shrink when
 - $\text{size} \leq \text{capacity} / C$
- Shrink when
 - $\text{size} \leq \text{capacity} / C^2$

Dynamic Stack with Hysteresis





Queues and Lists

Images courtesy: www.123rf.com; www.livemint.com

Queues

- What is a Queue?
- Container of objects
 - Put an object (enqueue)
 - Take an object out (dequeue)
 - Order of objects: First-In First-Out (FIFO)
- Real life examples of a queue



Image credit: Jurgen Ziewe/Getty Images; Taken from: [Cue vs. Queue: How to Choose the Right Word \(thoughtco.com\)](https://www.thoughtco.com/cue-vs-queue-how-to-choose-the-right-word-2279478.html)



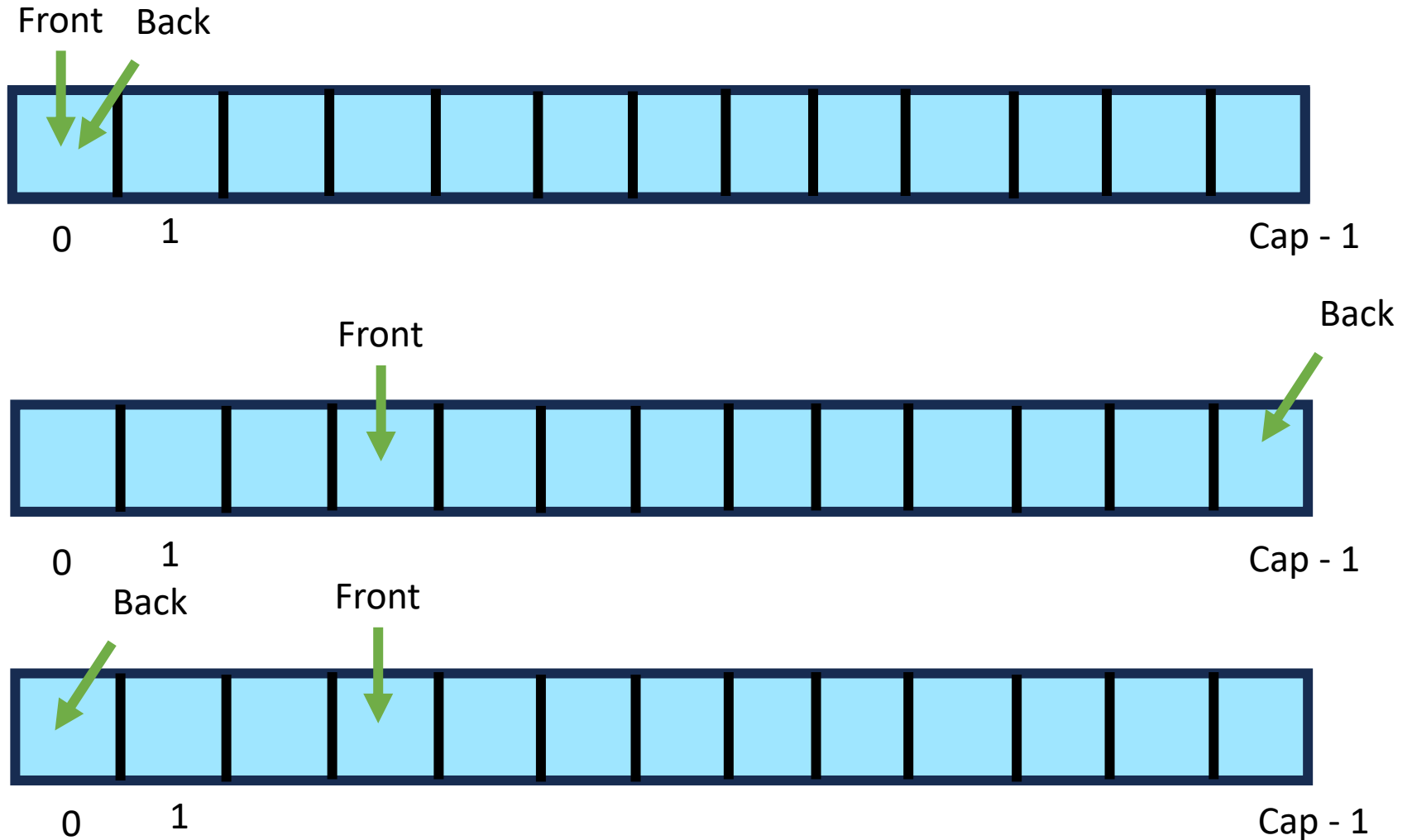
Female voters standing in a queue for casting their votes during the 3rd Phase of General Elections-2014, in New Delhi on April 10, 2014.
Image source: Wikipedia

Operations of Queues

- Enqueue
- Dequeue
- Front
- Size
- IsEmpty

Queue Implementation using Array

Queue Implementation using Array



Queue Implementation using Array

```
class Queue {  
private:  
    int f, b;  
    int *Q;  
    int cap;  
  
    // First element is Q[f]  
    // Q[b] is empty space where a new element is to be inserted  
    // Last element is Q[(b - 1 + cap) mod cap]  
    // If f == b then Q is empty  
    // If (b + 1) mod cap == f then Q is full  
    // Else Q has space to insert elements  
    // Number of elements Q can have: cap - 1
```

Queue Implementation using Array

public:

```
Queue(int sz) {  
    Q = new int[sz]; // TBD Check for errors  
    cap = sz;  
}  
~Queue() {  
    delete Q;  
}
```


Queue Implementation using Array

```
void Enqueue(int element) {  
    // First check for Q full  
    if (((b + 1) % cap) == f) throw Q_FULL_EXCEPTION;  
    else {  
        Q[b] = element;  
        b = (b + 1) % cap;  
    }  
}
```

Queue Implementation using Array

```
int Dequeue() {  
    // First check for Q empty  
    if (f == b) throw Q_EMPTY_EXCEPTION;  
    else {  
        int result;  
        result = Q[f];  
        f = (f + 1) % cap;  
        return result;  
    }  
}
```

Queue Implementation using Array

```
int Front() {  
    return Q[f];  
}
```

```
int Size() {  
    return ((f - b + cap) % cap);  
}
```

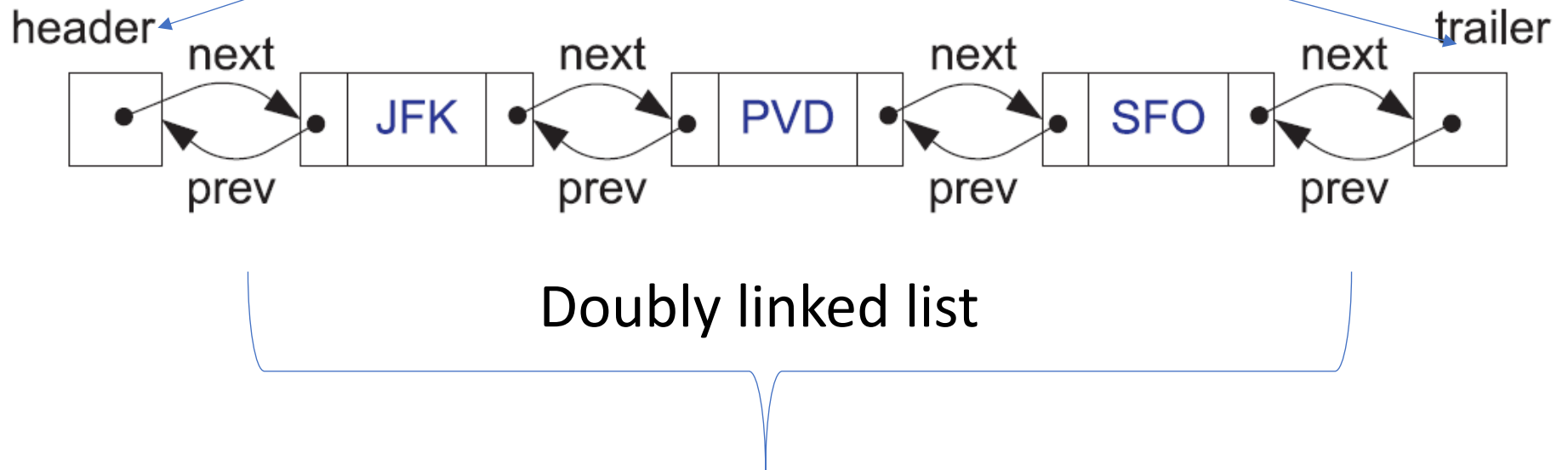
```
bool isEmpty() {  
    return (f == b);  
}
```

Double Ended Queues (Deque)

- A queue that supports **insertion** and **deletion** at **both** the **ends**
- Operations on Deque
 - insertFront(e)
 - insertBack(e)
 - eraseFront(e)
 - eraseBack(e)
 - front()
 - back()
 - size()
 - empty()

Data Structure for Deque

Sentinel nodes



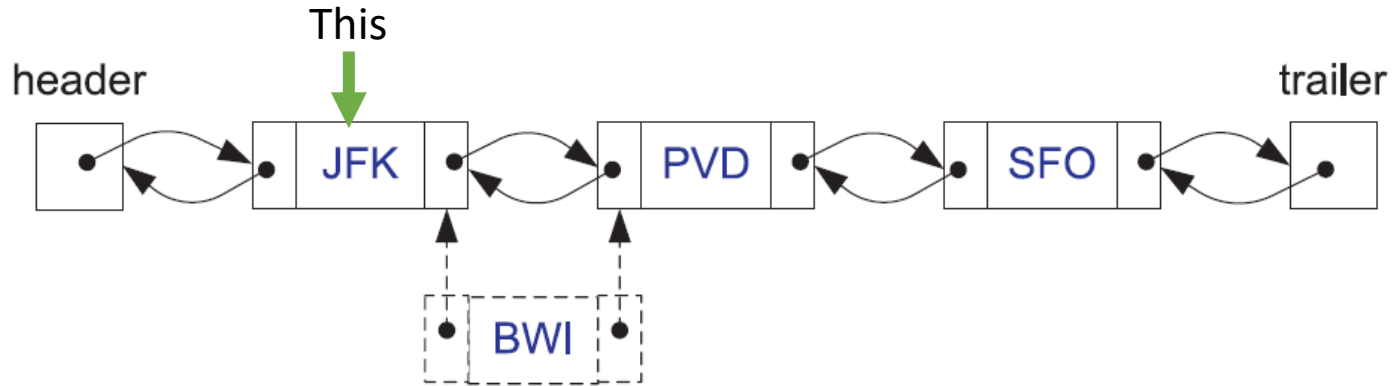
Basic Operations Needed

- insertBefore
- insertAfter
- DLLDelete
 - insertFront(): insertAfter(header)
 - insertBack(): insertBefore(trailer)
 - eraseFront(e): DLLDelete(header->next)
 - eraseBack(e): DLLDelete(trailer->prev)

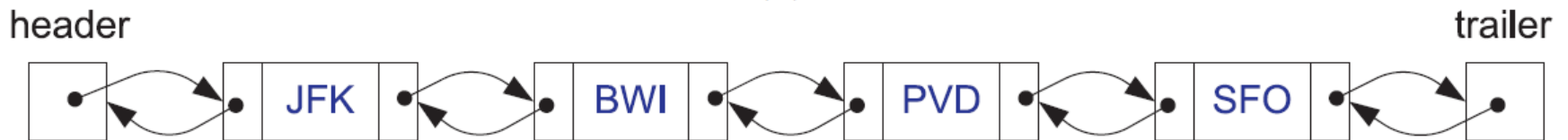
Deque using Doubly Linked Lists

```
class DLLNode {  
    // The Doubly linked list will have two sentinel nodes header and trailer.  
    // No data is kept in sentinel nodes  
    // Sentinel node invariants: (i) header->prev == NULL (ii) trailer->next == NULL  
    // The above invariants give a method to identify header and trailer  
    // Non-sentinel node invariants:  
    // (iii) n->next->prev == n (iv) n->prev->next == n  
  
private:  
    int data;  
  
public:  
    DLLNode *next, *prev;  
    void insertAfter(DLLNode *n); // Insert node n after this node  
    void insertBefore(DLLNode *n); // Insert node n before this node  
    void DLLdelete(); // Delete the this node  
};
```

Insertion in a Doubly Linked List

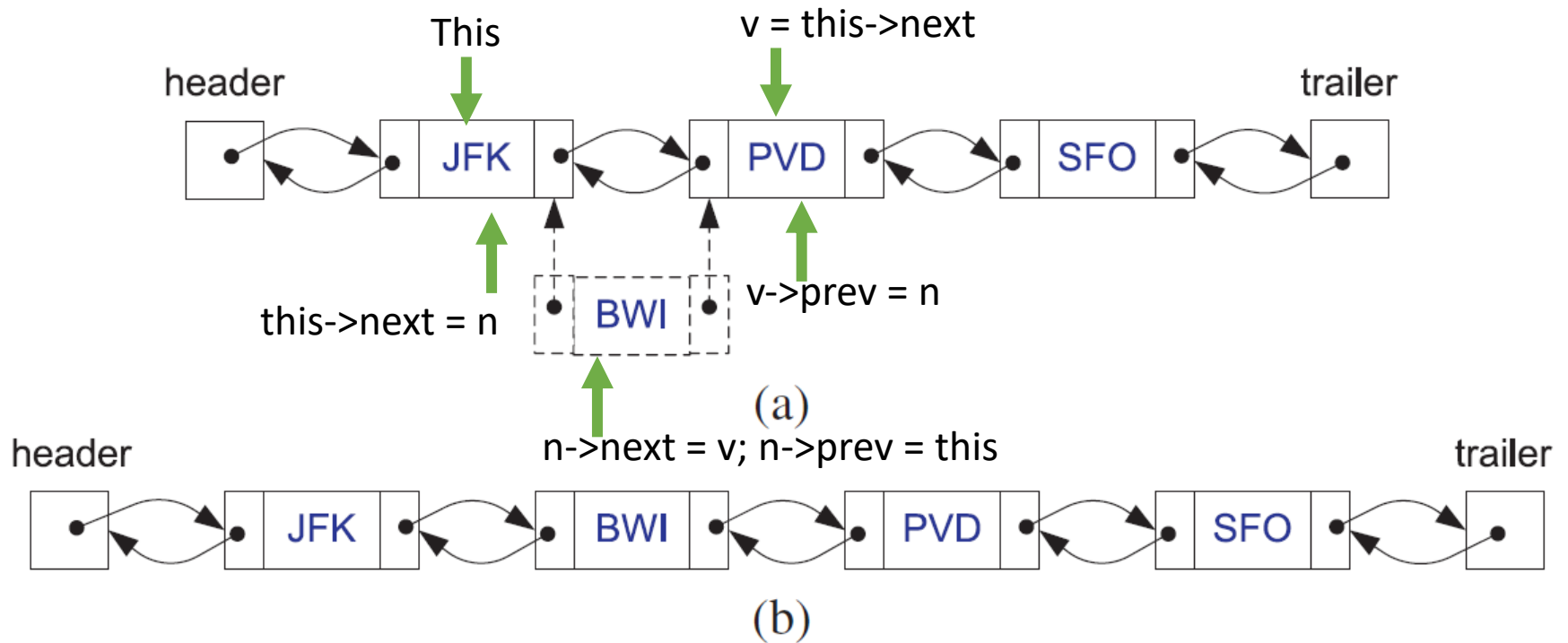


(a)



(b)

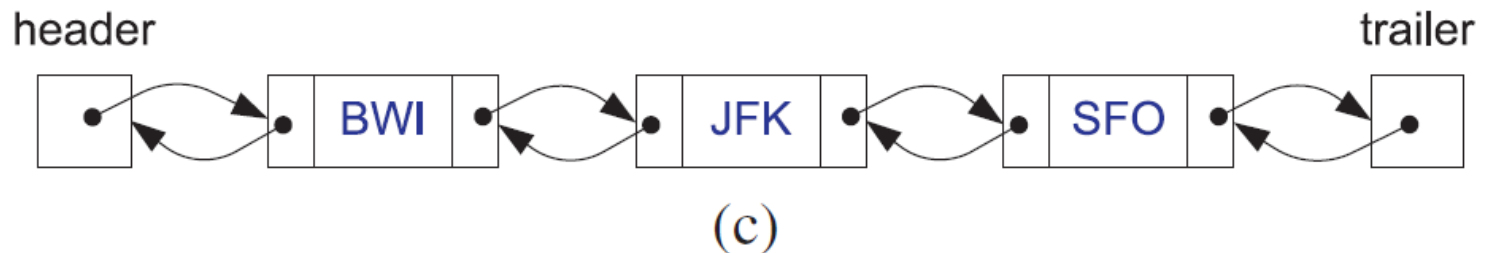
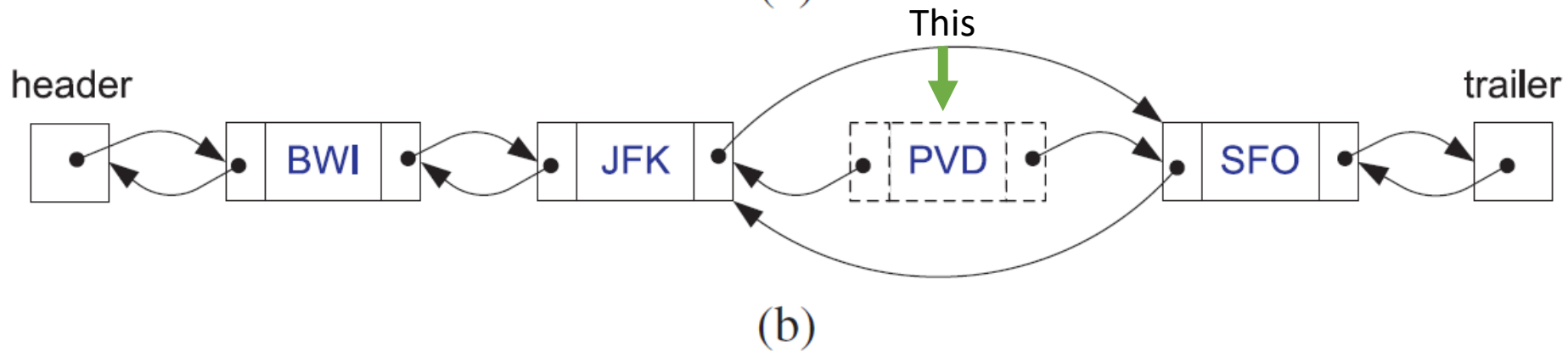
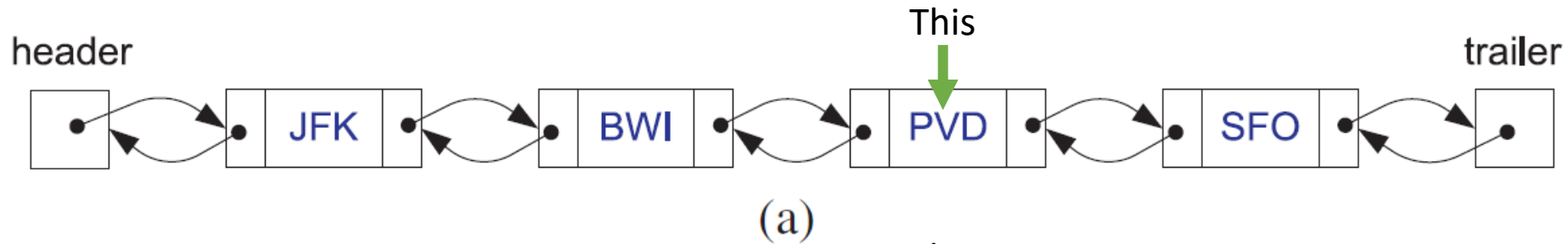
Insertion in a Doubly Linked List



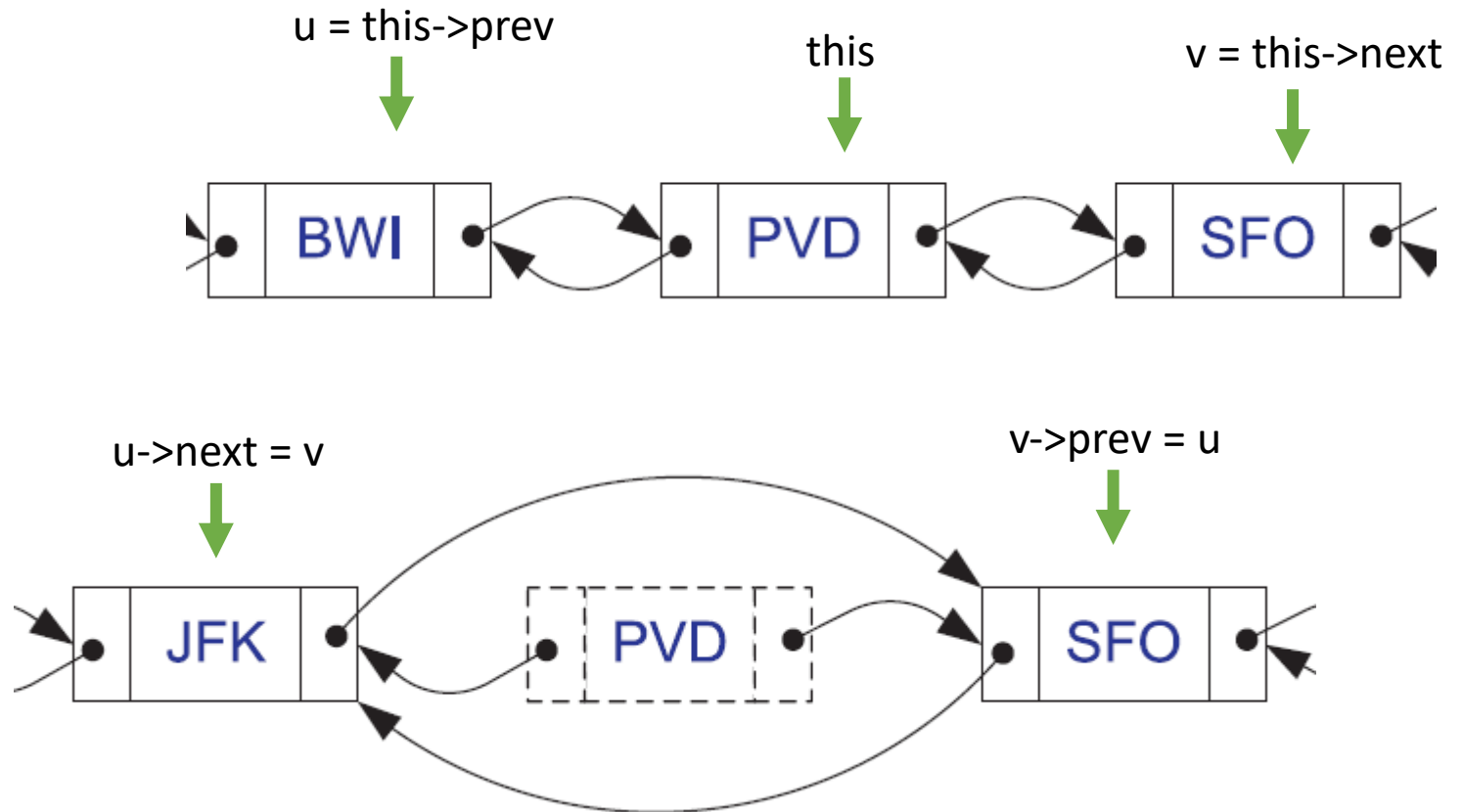
InsertAfter in a Doubly Linked List

```
void DLLNode::insertAfter(DLLNode *n) {  
  
    n->next = next;  
    n->prev = this;  
    next = n;  
    n->next->prev = n;  
  
}
```

Deletion in a Doubly Linked List



DLLNode::Delete()



DLLNode::Delete()

```
void DLLNode::DLLdelete() {  
  
    prev->next = next;  
    next->prev = prev;  
  
    delete this;  
  
}
```

Stacks using Deque

Stack Method	Deque Implementation
push(o)	insertFront(o)
pop()	eraseFront()
top()	front()
size()	size()
empty()	empty()

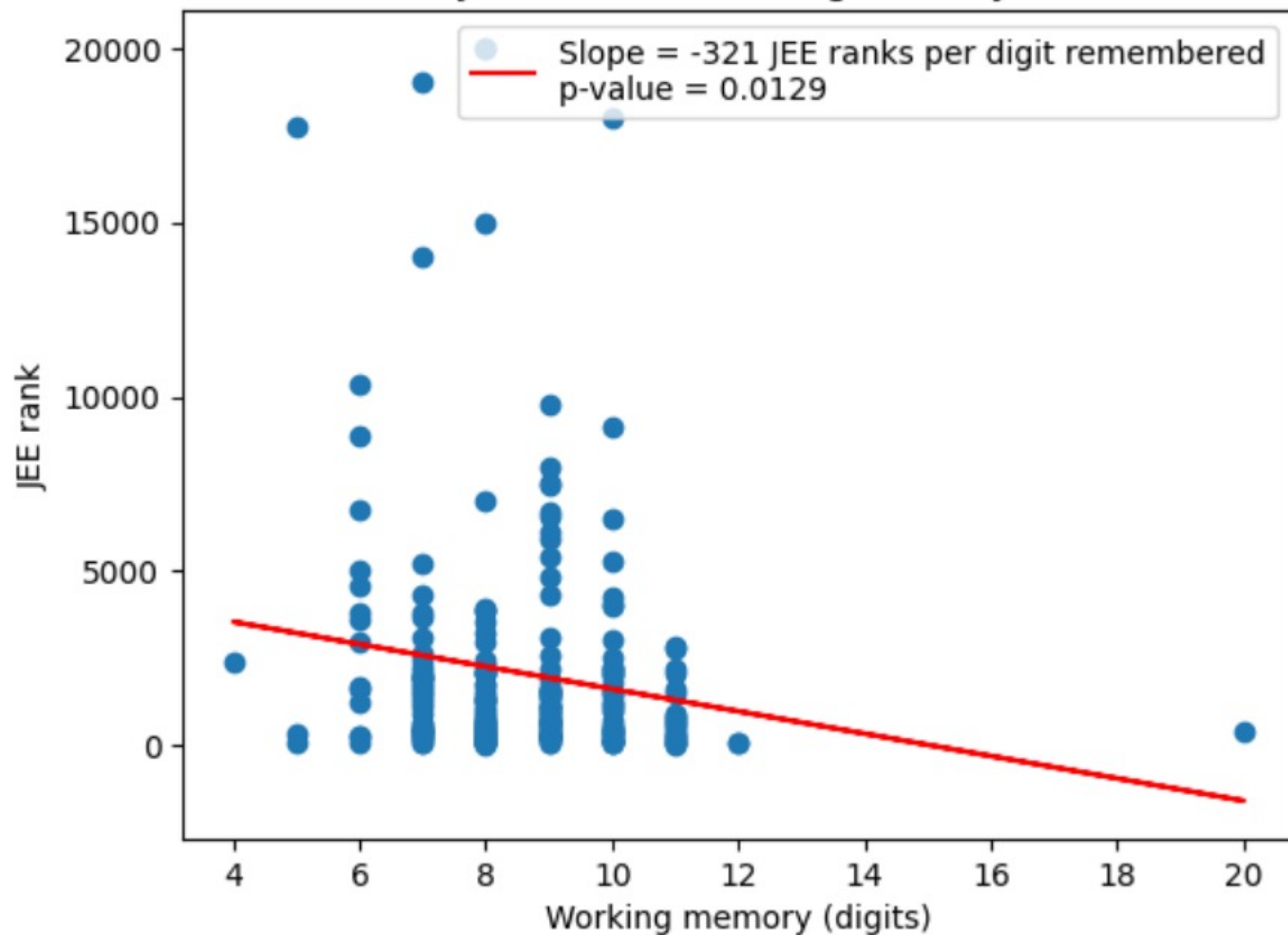
Queues using Deque

Queue Method	Deque Implementation
enqueue(o)	insertBack(o)
dequeue()	eraseFront()
front()	front()
size()	size()
empty()	empty()

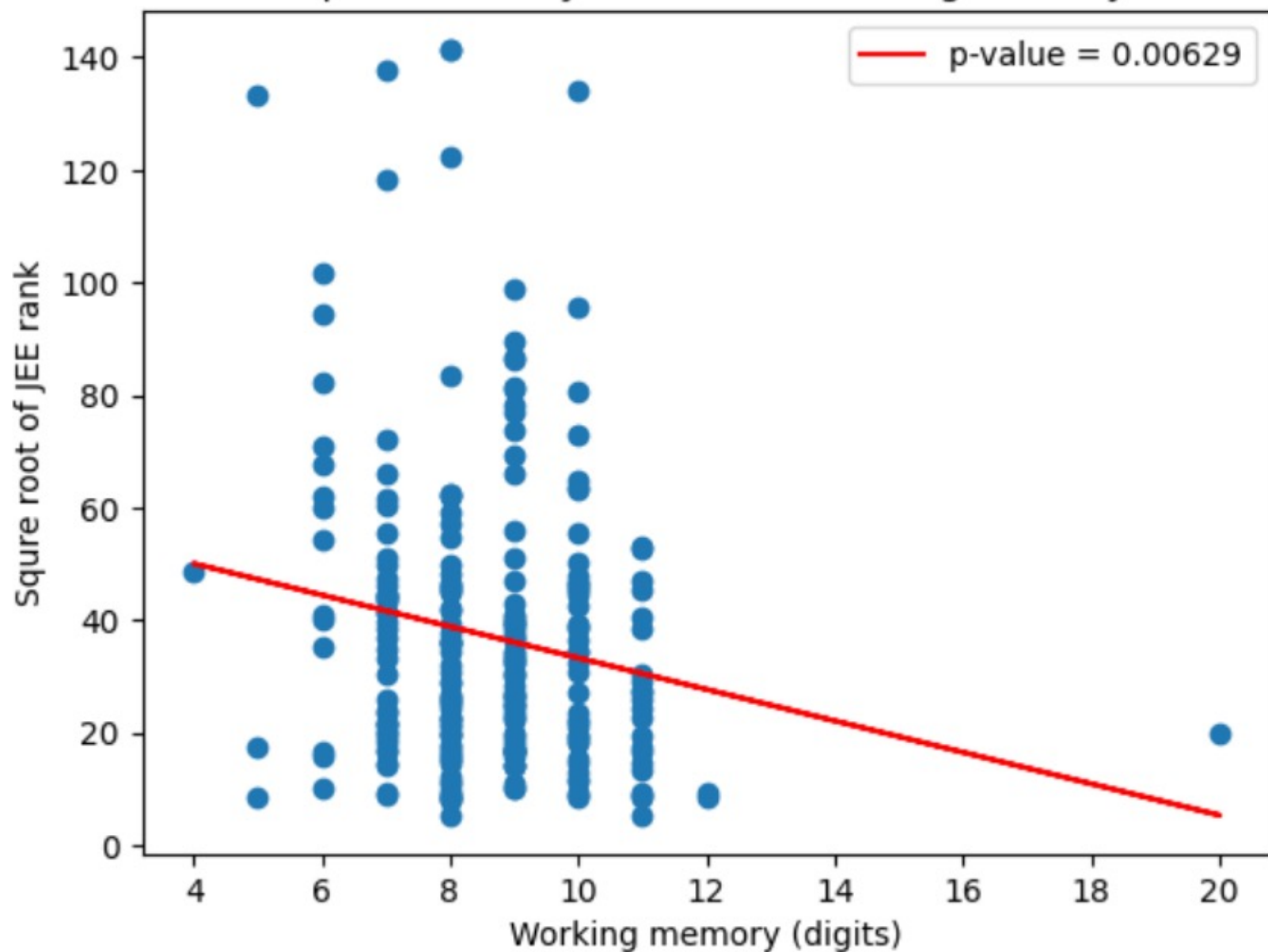
WM Experiment Results

248 responses

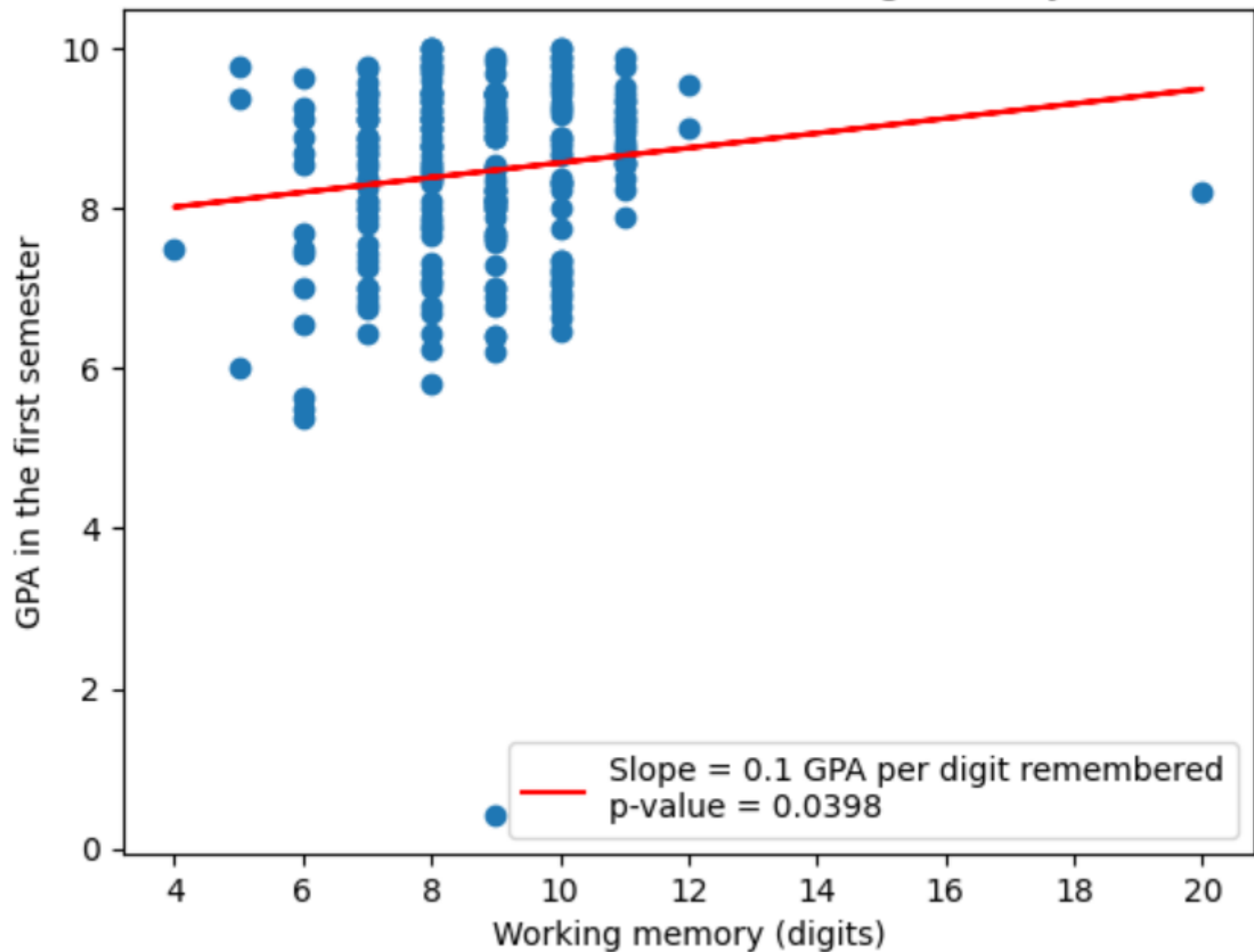
JEE Rank vs. Working Memory



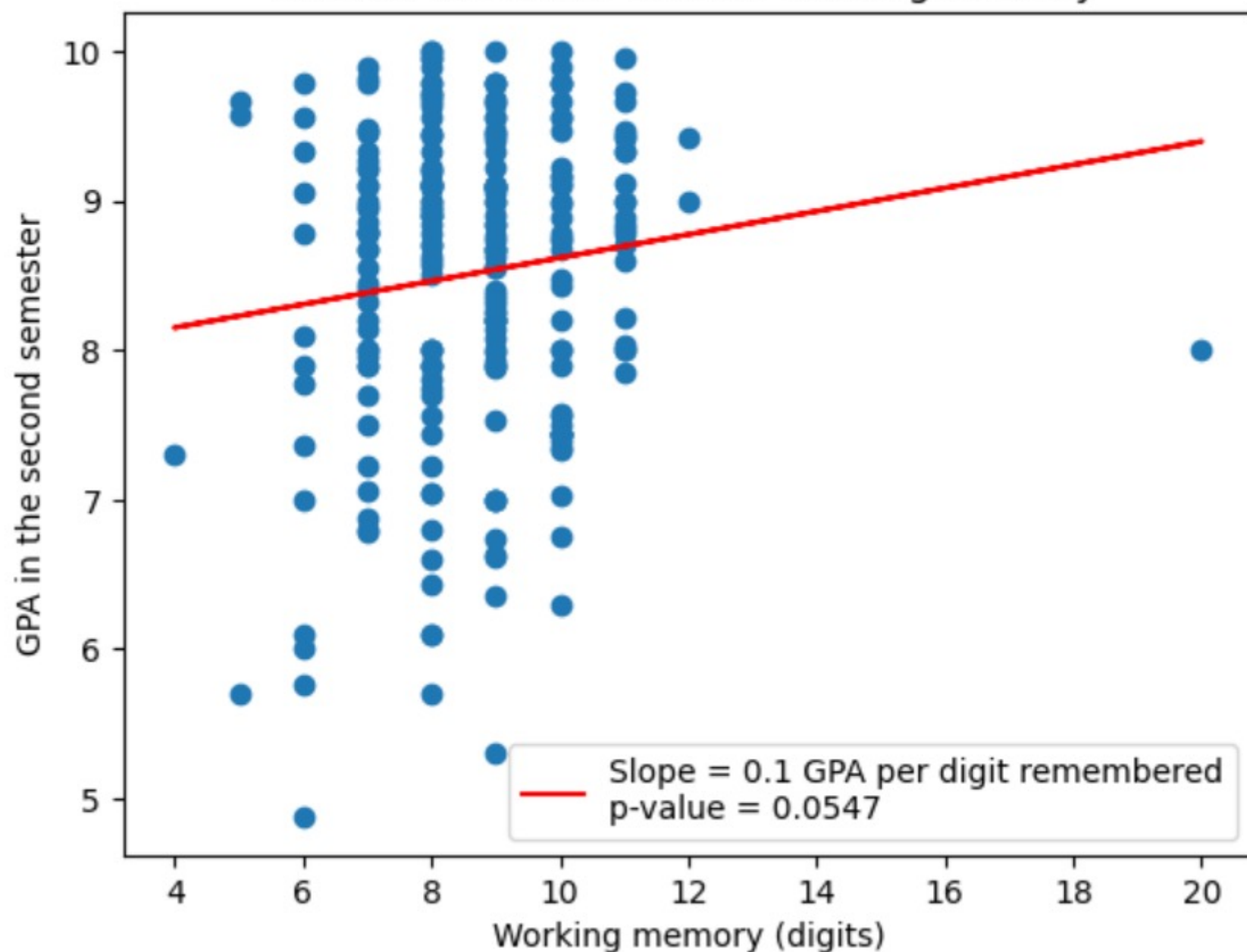
Square root of JEE Rank vs. Working Memory



First Semester GPA vs. Working Memory

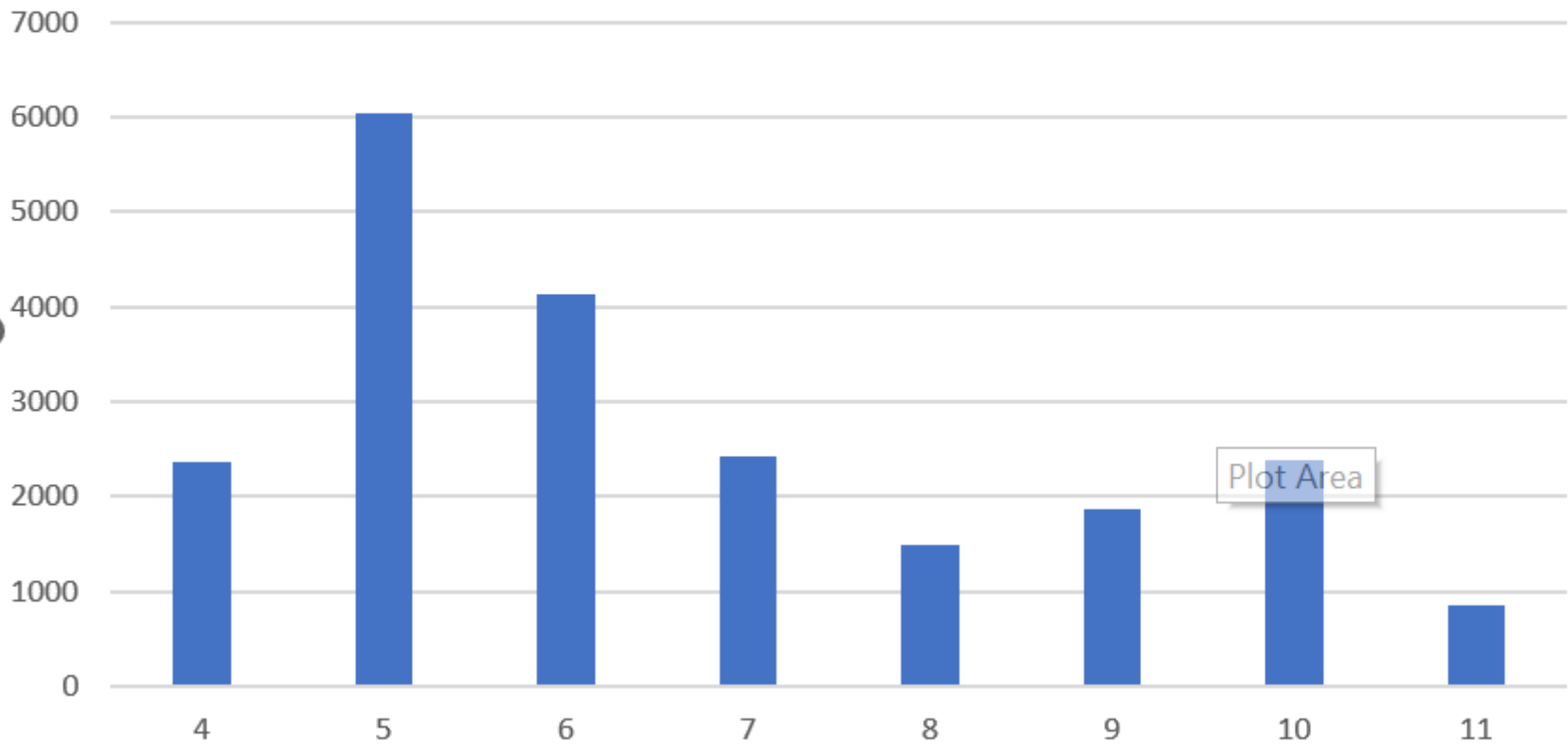


Second Semester GPA vs. Working Memory



Average of Your JEE rank

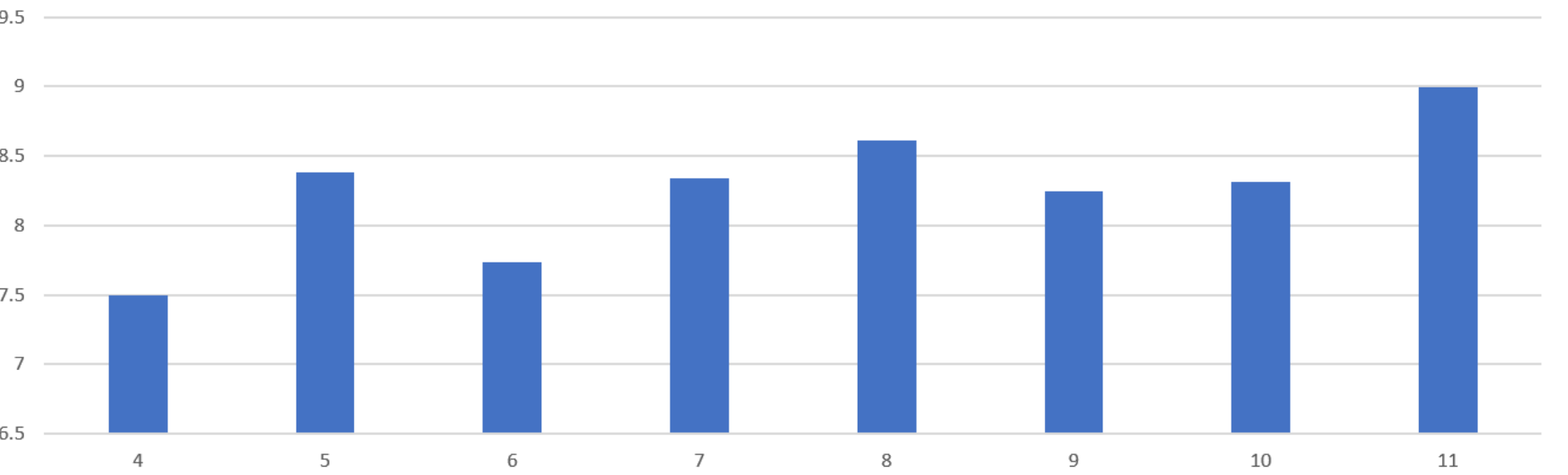
Average JEE Rank for Students with Different Working Memory



Maximum number of digits you could remember. ▼

Average of Your semester GPA in your first semester

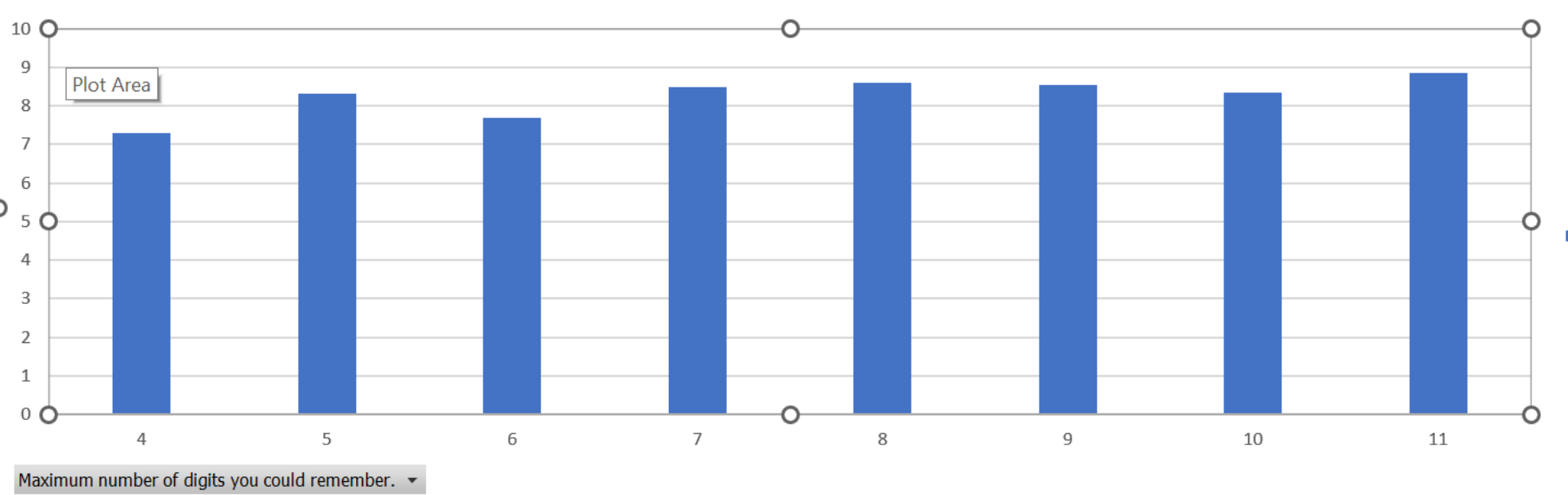
Average First Sem GPA for Students with Different Working Memory



Maximum number of digits you could remember. ▾

Average of Your semester CGPA in your second semester

Average Second Sem GPA for Students with Different Working Memory



Ready for Quiz 1?



- Can only submit response once
- Cannot edit after submitting
- Responses submitted after five minutes of the start of quiz will get a zero mark
- No other queries will be entertained

$f = 20 \text{ mins}$
 $c = u + 20 \text{ mins}$

