COL106 Data Structures and Algorithms

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Applications of Graphs: Search Results Ranking

References

1. Kleinberg, Jon M. "Authoritative sources in a hyperlinked environment." SODA. Vol. 98. 1998.
2. Kleinberg, J. M., Kumar, R., Raghavan, P., Rajagopalan, S., & Tomkins, A. S. (1999). The web as a graph: Measurements, models, and methods. In Computing and Combinatorics: 5th Annual International Conference, COCOON'99 Tokyo, Japan, July 26–28, 1999 Proceedings 5 (pp. 1-17). Springer Berlin Heidelberg.
3. Brin, Sergey, and Lawrence Page. "The anatomy of a large-scale hypertextual web search engine." Computer networks and ISDN systems 30.1-7 (1998): 107-117.
4. The PageRank Citation Ranking: Bringing Order to the Web. by Lawrence Page, Sergey Brin, Rajeev Motwani, and Terry Winograd. Technical Report SIDL-WP-1999-0120, Stanford Digital Library Technologies Project

A Brief History of the Internet

- 1969 – ARPANET project
  - Advanced Research Projects Agency Network (ARPANET)
  - Advanced Research Projects Agency (ARPA) of the United States Department of Defense
- 1981 – expanded into CSNET
  - National Science Foundation (NSF) funded
  - Computer Science Network (CSNET)
- 1985 – NSFNET
  - Internet backbone for government agencies and universities
- 1990 – ARPANET project formally decommissioned
- 1995 – Backbone of the NSFNET decommissioned
  - Growth of commercial Internet
  - Growth of www and web pages

Internet Search

- 1995 – 1998 growth of www
- Proliferation of web sites
- Growth of search engines
- Text based searching
- Find the most relevant pages that match the search query
  - Automated data driven approaches (altavista.com)
  - Manual indexing (yahoo.com)
- Poor search quality

Search Results Ranking Problem

- Billions of web pages
- Millions of queries every day
- Millions of pages matching the query
- Can only show a few results on a page
- Which results to show?
- How to show the most relevant results on top?
- How to rank the search results in decreasing order of relevance?

Modeling Web as a Graph

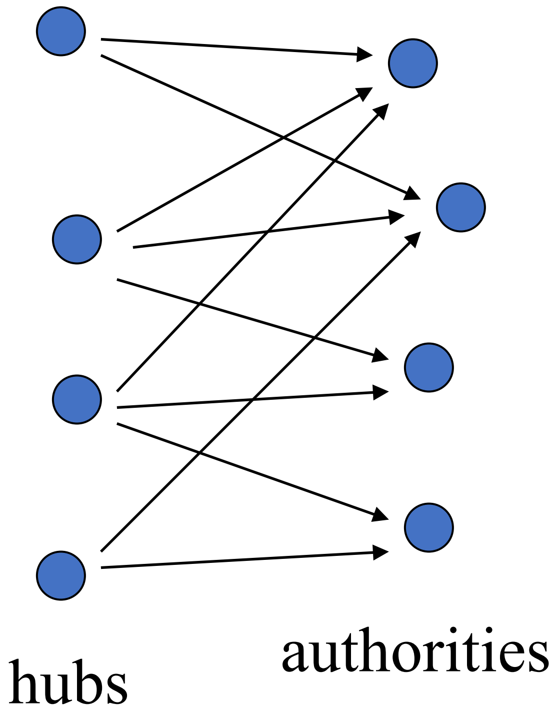
- From a large collection of “relevant” pages, find the most “authorities” pages
- Web as a graph
  - Vertices web-pages
  - Edges are the hyperlinks
- G= (V, E) –Web graph
- V = set of web pages
- (u, v) in E if the page u points to the page v

Authoritative Sources in a Hyperlinked Environment (1998)

- Key ideas:
  - If a page u points to the page v (or $(u, v) \in E$), then u contributes some authority to the page v
  - Especially if u and v belong to different domains
  - E.g, COL106 home page pointing to a Textbook page
- Limitations
  - Navigational links or advertisement links do not confer authority – Remove links within the same domain
  - Authority pages may not point to other authority pages
  - Hub-pages may point to multiple authorities but not to other hubs
  - E.g., a page listing all the car dealers in the vicinity

Bipartite Structure

- Top authorities will be pointed by a lot of hubs
- Top hubs will point to a lot of top authorities
- Can we iterate over this graph to find the authority score and the hub score for all the vertices?



Iterative Algorithm

- Each page p is assigned two non-negative weights, an authority weight x and a hub weight y.
- Update the weights of x and y

Authority Weight: I Operation

$$x^{\langle p \rangle} \leftarrow \sum_{q:(q,p)\in E} y^{\langle q \rangle}$$

Hub Weight : O Operation

$$y^{\langle p \rangle} \leftarrow \sum_{q:(p,q)\in E} x^{\langle q \rangle}$$

These operations add the weights of hubs into the authority weight and add the authority weights into the hub weight, respectively. Alternating these two operations will eventually result in an equilibrium value, or weight, for each page.

Slide adapted from Murtuza Shareef, University of Texas at Austin, USA

Using Matrix Representation

Definition: Adjacency matrix of a graph G = (V. E) is a n x n binary matrix A (n = |V|) such that

$a_{uv} = 1$ if $(u, v) \in E$

Authority Weight: I Operation $$x^{\langle p \rangle} \leftarrow \sum_{q:(q,p)\in E} y^{\langle q \rangle}$$ $$x = A^T y$$

Hub Weight : O Operation $$y^{\langle p \rangle} \leftarrow \sum_{q:(p,q)\in E} x^{\langle q \rangle}$$ $$y = Ax$$

Slide adapted from Murtuza Shareef, University of Texas at Austin, USA

Convergence

- Iteration algorithm converges as k increases. That is, the weights (vectors) converge.

Let G = (V , E) with V = {p1, p2 … pn}

Let A be the adjacency matrix of G.

I and O operations can be written as $x \leftarrow A^T y \quad y \leftarrow Ax$

Let x_i be the authority scores after i iterations.

Let y_i be the hub scores after i iterations.

Operation I $\quad x_i = A^T y_{i-1} \quad x_i = A^T A x_{i-1} \quad x_i = (A^T A)^i x_0$

Operation O $\quad y_i = A x_i \quad y_i = A A^T y_{i-1} \quad y_i = (A A^T)^i y_0$

Slide adapted from Murtuza Shareef, University of Texas at Austin, USA

Eigenvalues and Eigenvectors

- For a square matrix A, v is an eigenvector with an eigen value λ iff

$$Av = \lambda v$$

- For a nxn matrix A, there are generally n eigen vectors and n eigen values
- v is called the principal eigenvector if corresponding λ is the largest

Slide adapted from Murtuza Shareef, University of Texas at Austin, USA

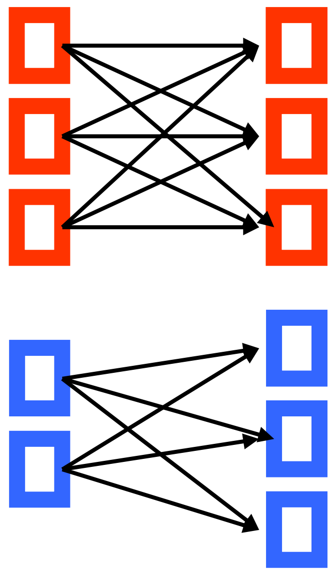
Convergence

- The vectors x_i and y_i converge to x^* and y^* respectively, where x^* and y^* are the principal Eigen vectors of A^TA and AA^T
- Kleinberg says that 20 iterations are sufficient to obtain convergence
- The “principal eigenvector” represents the densest cluster in the focused subgraph
- The non-principal eigenvectors represent less dense areas in the subgraph

Slide adapted from Murtuza Shareef, University of Texas at Austin, USA

Eigenvectors and Tightly Knit Communities

- The algorithm favors dense community of hubs and authorities



Eigenvectors and Tightly Knit Communities

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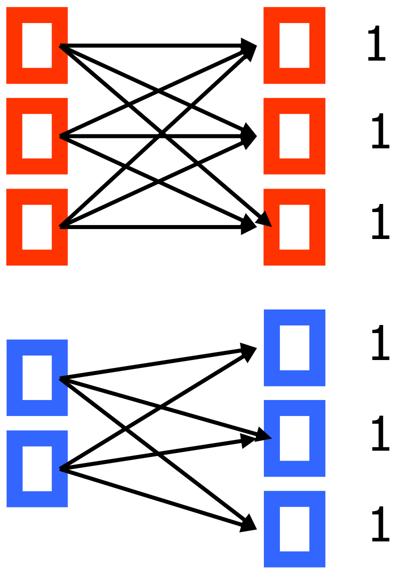

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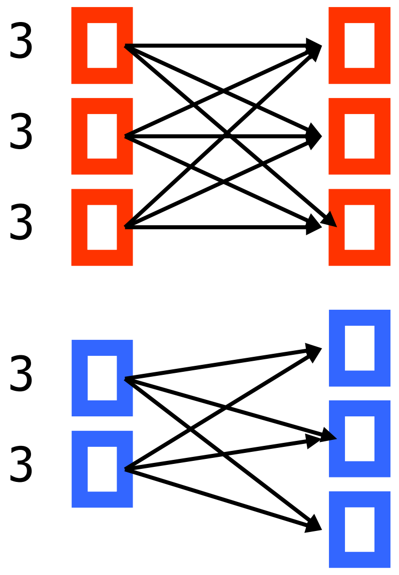

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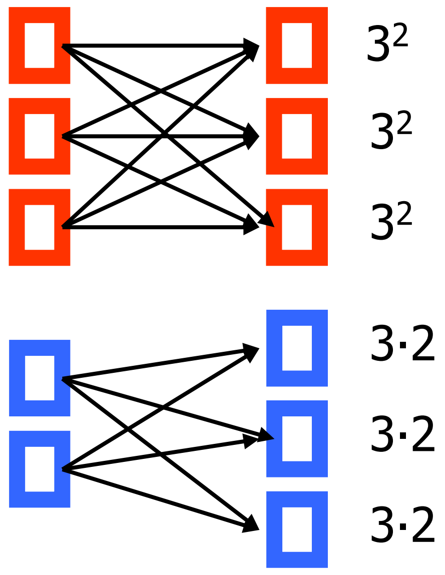

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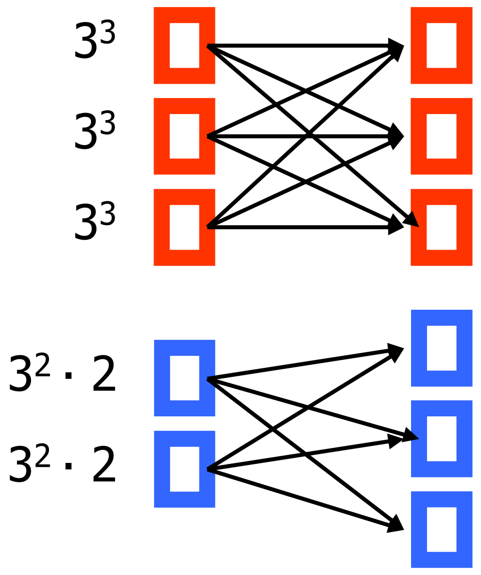
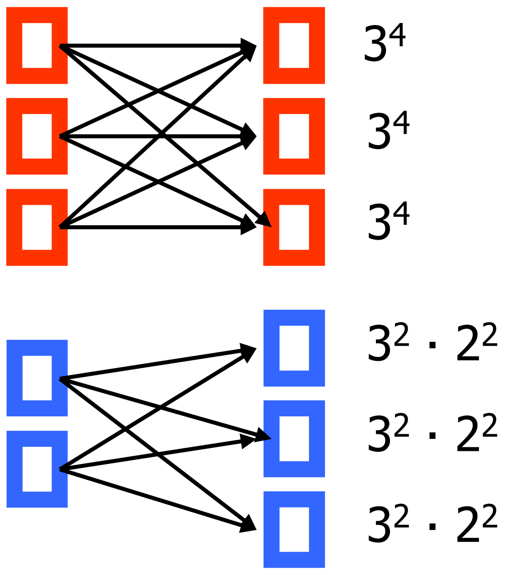


Eigenvectors and Tightly Knit Communities

- The algorithm favors dense community of hubs and authorities



Eigenvectors and Tightly Knit Communities

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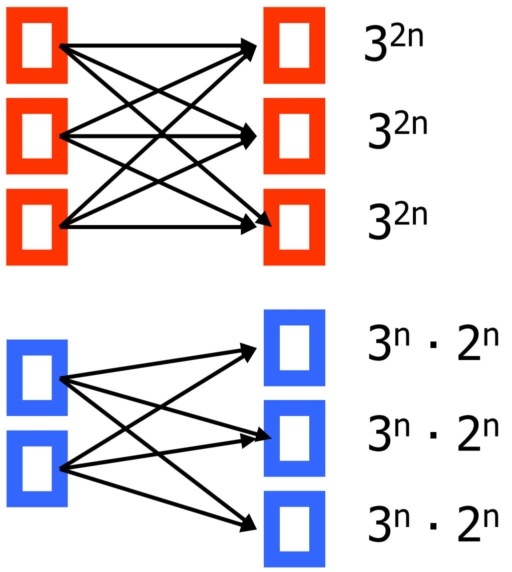
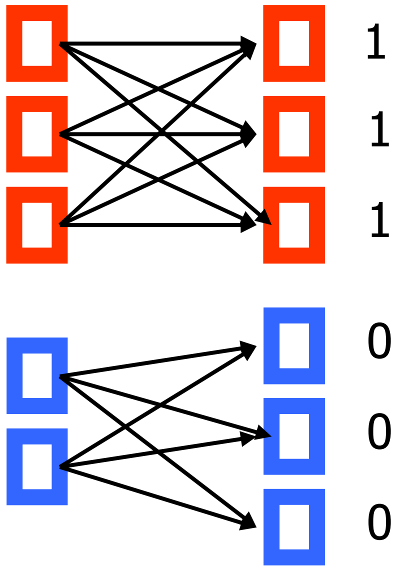


after n iterations

Eigenvectors and Tightly Knit Communities

- The algorithm favors dense community of hubs and authorities



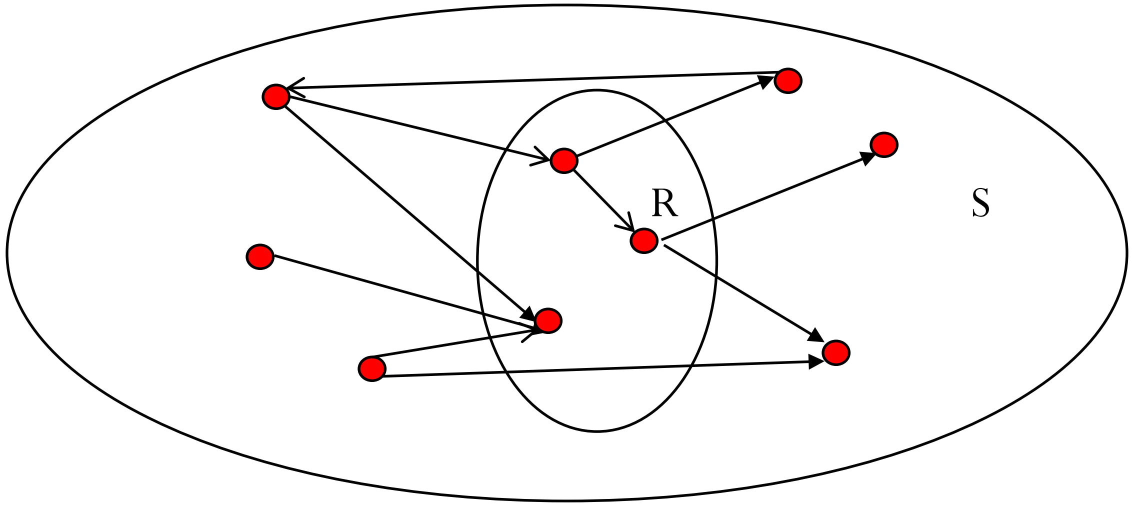
after normalization with the max element as n $\rightarrow \infty$

Hyperlink-Induced Topic Search (HITS) Algorithm

- First determines a set of relevant pages for the query called the *base* set *S*
- Analyze the link structure of the web subgraph defined by *S* to find authority and hub pages in this set
- Displays top authorities in the results

Slide adapted from Murtuza Shareef, University of Texas at Austin, USA

- For a specific query *Q*, let the set of documents returned by a standard search engine be called the *root* set *R*.
- Initialize *S* to *R*.
- Add to *S* all pages pointed to by any page in *R*.
- Add to *S* all pages that point to any page in *R*.



Slide adapted from Murtuza Shareef, University of Texas at Austin, USA

Subgraph reduction

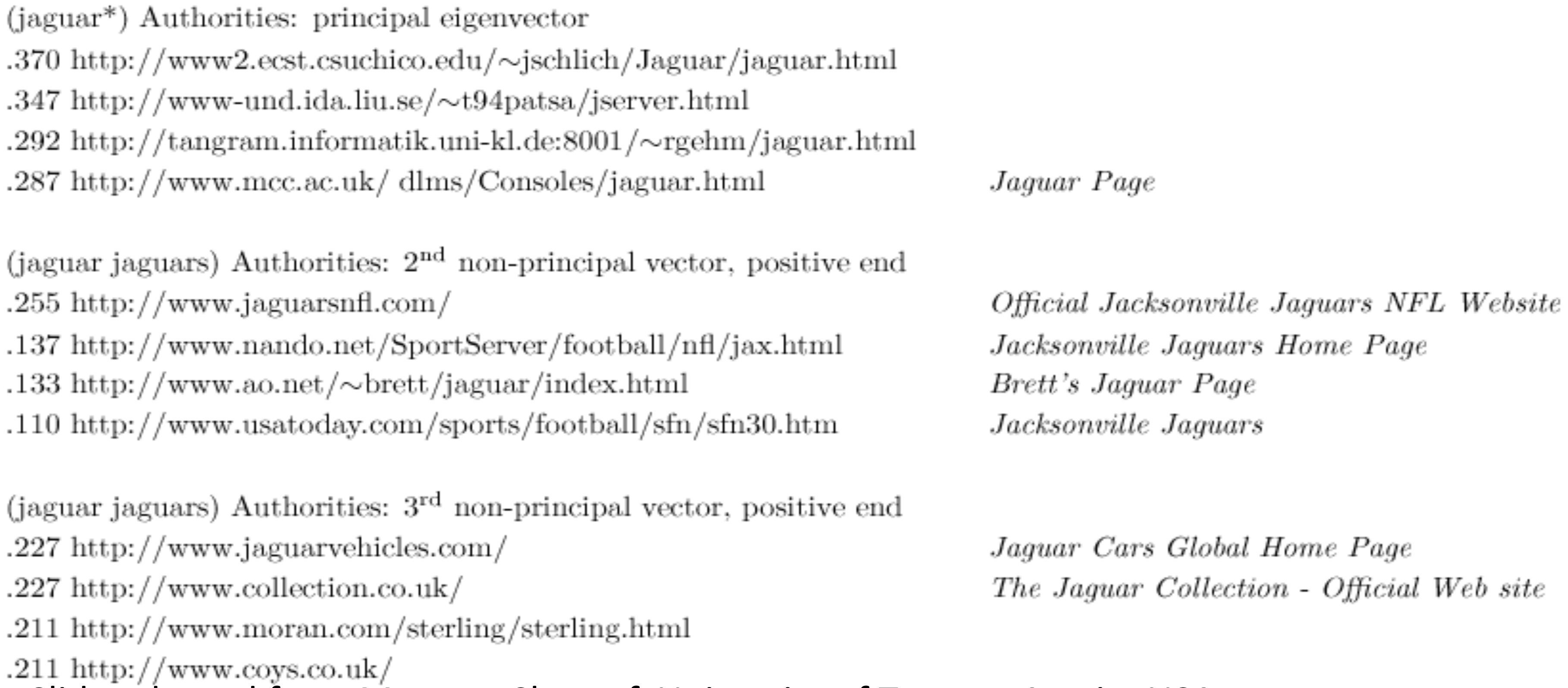
- Remove links that serve purely a navigational function
    - For example links within the same domain name
    - E.g., IIT Delhi to IIT Delhi website link
- Remove links that are mentioned in more than m pages (m=4-8).

Slide adapted from Murtuza Shareef, University of Texas at Austin, USA

Multiple sets of Hubs and Authorities

<u>Experimental result 1</u>

- For the query “jaguar”, the strongest collections of authoritative sources concerned the Atari Jaguar product, the NFL football team from Jacksonville, and the automobile.



(jaguar*) Authorities: principal eigenvector

.370	http://www2.ecst.csuchico.edu/~jschlich/Jaguar/jaguar.html	
.347	http://www-und.ida.liu.se/~t94patsa/jserver.html	
.292	http://tangram.informatik.uni-kl.de:8001/~rgehm/jaguar.html	
.287	http://www.mcc.ac.uk/ dlms/Consoles/jaguar.html	*Jaguar Page*

(jaguar jaguars) Authorities: 2nd non-principal vector, positive end

.255	http://www.jaguarsnfl.com/	*Official Jacksonville Jaguars NFL Website*
.137	http://www.nando.net/SportServer/football/nfl/jax.html	*Jacksonville Jaguars Home Page*
.133	http://www.ao.net/~brett/jaguar/index.html	*Brett's Jaguar Page*
.110	http://www.usatoday.com/sports/football/sfn/sfn30.htm	*Jacksonville Jaguars*

(jaguar jaguars) Authorities: 3rd non-principal vector, positive end

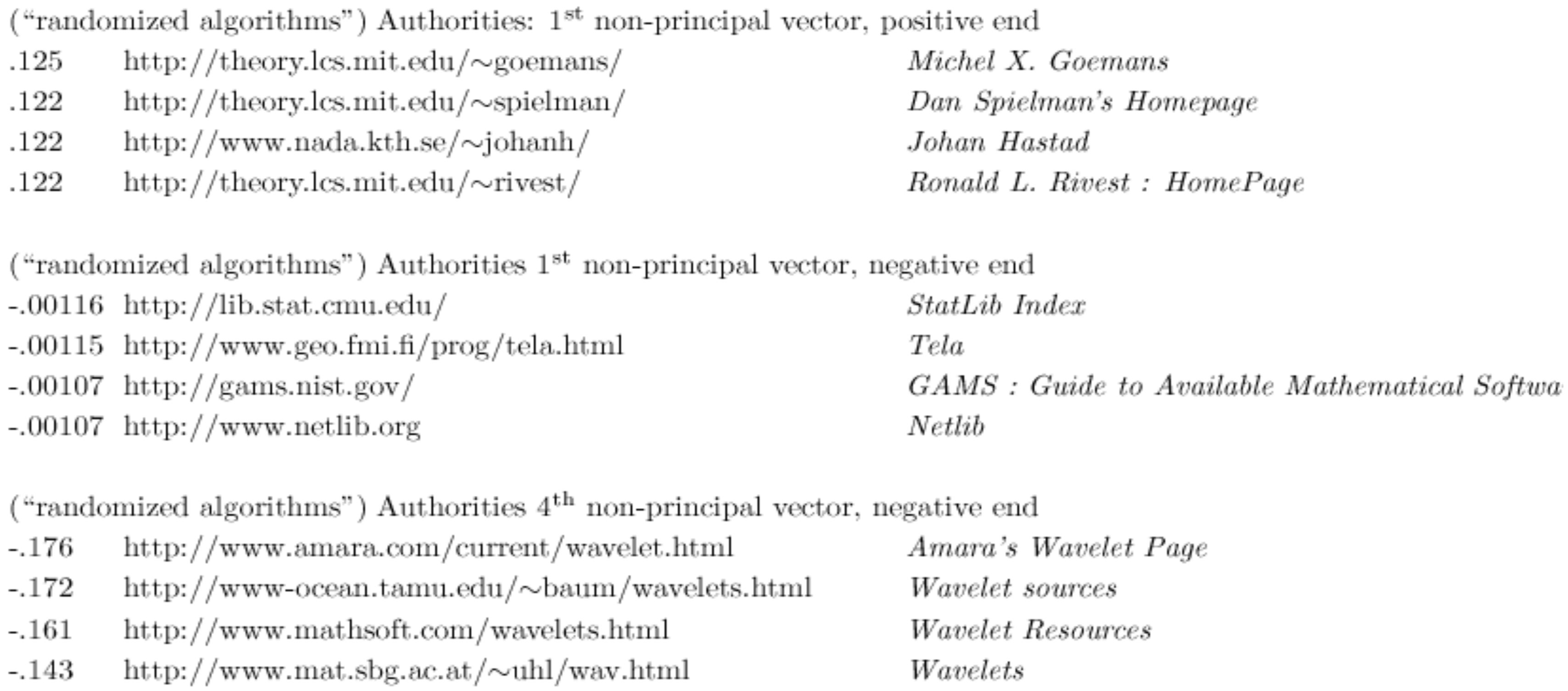
.227	http://www.jaguarvehicles.com/	*Jaguar Cars Global Home Page*
.227	http://www.collection.co.uk/	*The Jaguar Collection - Official Web site*
.211	http://www.moran.com/sterling/sterling.html	
.211	http://www.coys.co.uk/	

Slide adapted from Murtuza Shareef, University of Texas at Austin, USA

Multiple sets of Hubs and Authorities

Experimental result 2

- For the query “randomized algorithms”, none of the strongest collections of hubs and authorities are precisely on the query topic. They include home pages of theoretical computer scientists, compendia of mathematical software and pages on wavelets.



(“randomized algorithms”) Authorities: 1st non-principal vector, positive end

.125	http://theory.lcs.mit.edu/~goemans/	*Michel X. Goemans*
.122	http://theory.lcs.mit.edu/~spielman/	*Dan Spielman's Homepage*
.122	http://www.nada.kth.se/~johanh/	*Johan Hastad*
.122	http://theory.lcs.mit.edu/~rivest/	*Ronald L. Rivest : HomePage*

(“randomized algorithms”) Authorities 1st non-principal vector, negative end

-.00116	http://lib.stat.cmu.edu/	*StatLib Index*
-.00115	http://www.geo.fmi.fi/prog/tela.html	*Tela*
-.00107	http://gams.nist.gov/	*GAMS : Guide to Available Mathematical Softwa*
-.00107	http://www.netlib.org	*Netlib*

(“randomized algorithms”) Authorities 4th non-principal vector, negative end

-.176	http://www.amara.com/current/wavelet.html	*Amara's Wavelet Page*
-.172	http://www-ocean.tamu.edu/~baum/wavelets.html	*Wavelet sources*
-.161	http://www.mathsoft.com/wavelets.html	*Wavelet Resources*
-.143	http://www.mat.sbg.ac.at/~uhl/wav.html	*Wavelets*

Slide adapted from Murtuza Shareef, University of Texas at Austin, USA

The Page, Brin’s et al. Page Rank Algorithm

PageRank: History

- PageRank was developed by Larry Page (hence the name *Page*-Rank) and Sergey Brin

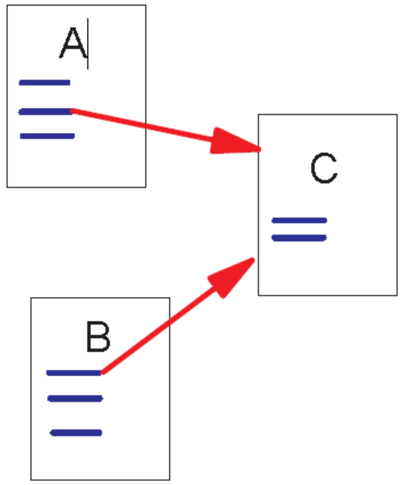
- It is first as part of a research project about a new kind of search engine. That project started in 1995 and led to a functional prototype in 1998.

- Shortly after, Page and Brin founded Google.

Slides adapted from various Internet sources

Link Structure of the Web

- 150 million web pages → 1.7 billion links



Backlinks and Forward links:

➢A and B are C's backlinks

➢C is A and B's forward link

Intuitively, a webpage is important if it has a lot of backlinks.

PageRank: A Simplified Version

$$R(u) = c \sum_{v \in B_u} \frac{R(v)}{N_v}$$

- u: a web page
- B_u: the set of u's backlinks
- N_v: the number of forward links of page v
- c: the normalization factor to make $||R||_{L1} = 1$ ($||R||_{L1} = |R_1 + \ldots + R_n|$)

Using Matrix Representation

Definition: Adjacency matrix of a graph G = (V. E) is a n x n binary matrix A (n = |V|) such that

$a_{uv} = 1$ if $(u, v) \in E$

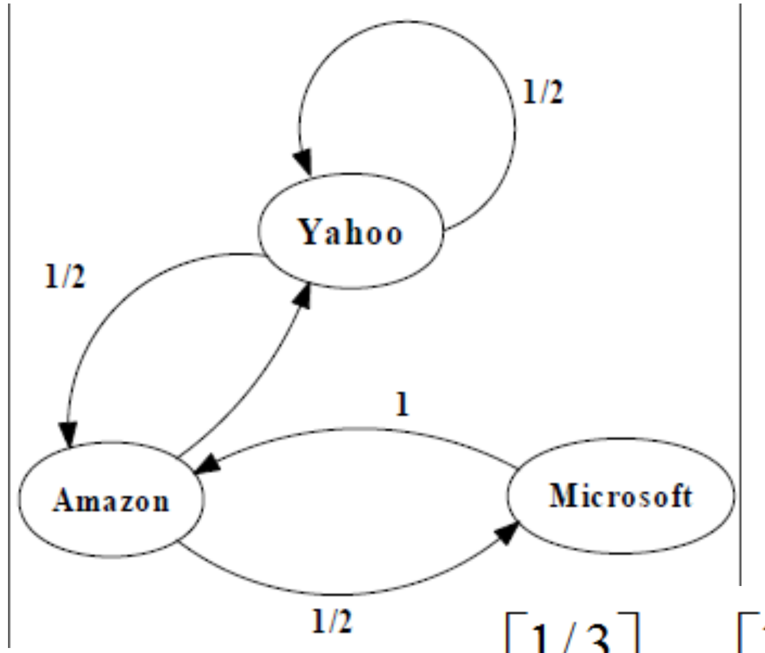
Let w = A 1^n

Elements of w represent the row sum of A

Divide the rows of A by corresponding element in w to get the matrix B

Solve for: $r = cB^T r$

An example of Simplified PageRank



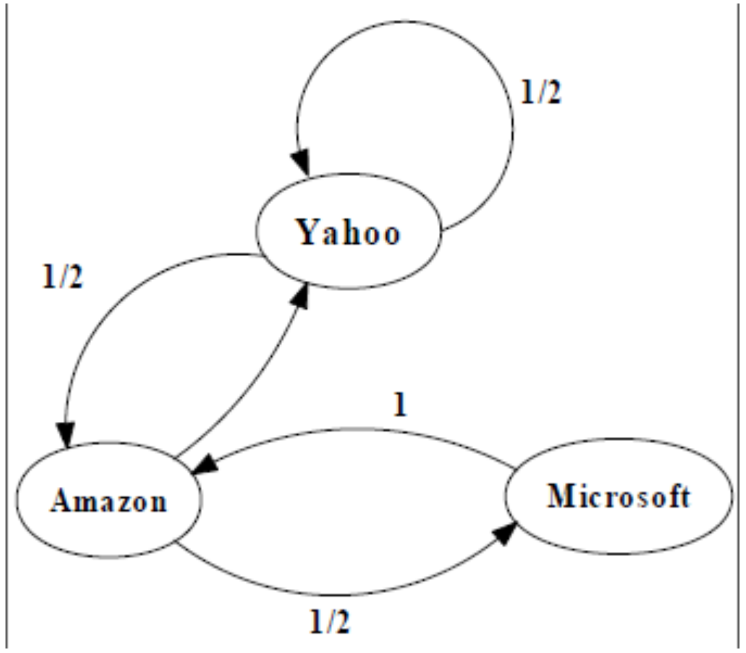
$$B^T = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1/3 \\ 1/2 \\ 1/6 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

PageRank Calculation: first iteration

An example of Simplified PageRank



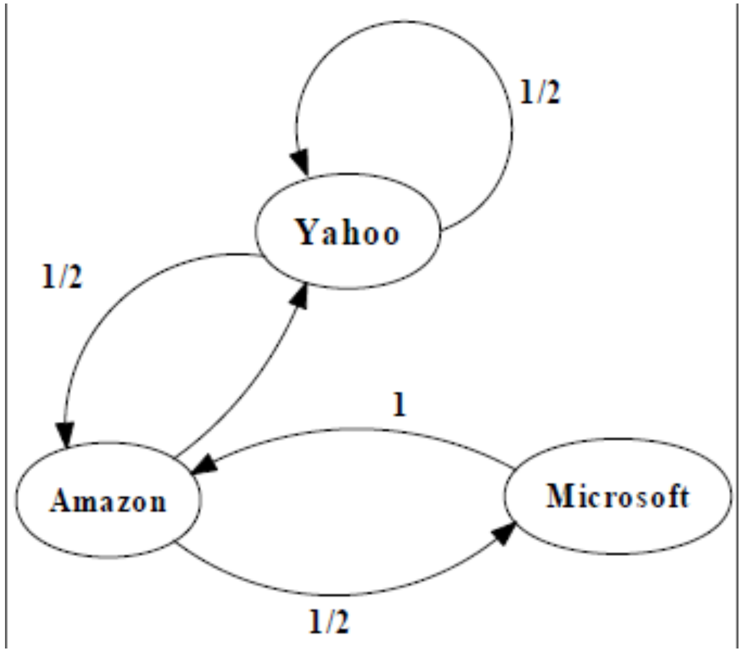
$$B^T = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 5/12 \\ 1/3 \\ 1/4 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/2 \\ 1/6 \end{bmatrix}$$

PageRank Calculation: second iteration

An example of Simplified PageRank



$$B^T = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix}$$

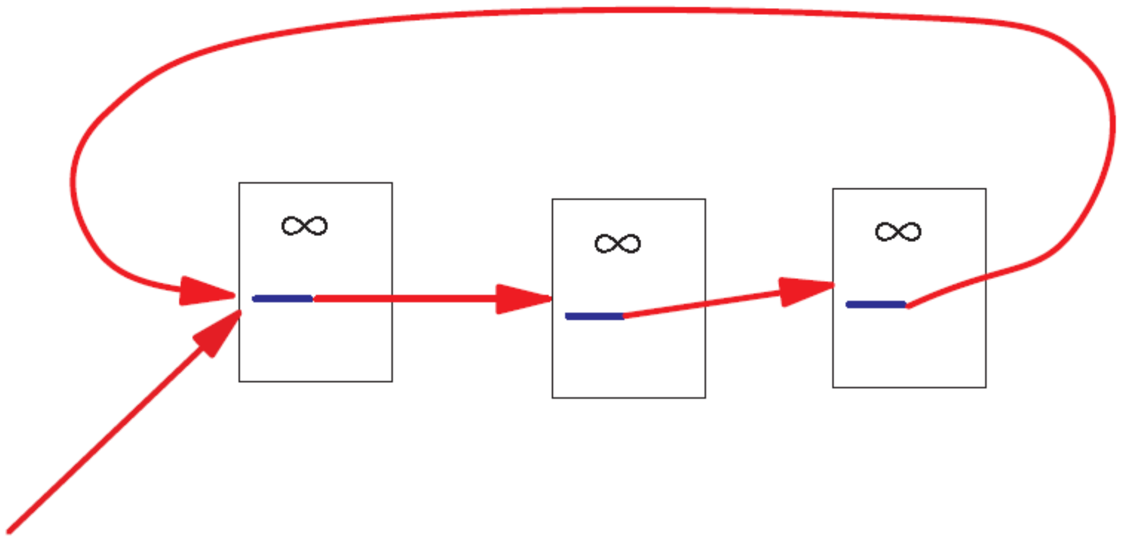
$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 3/8 \\ 11/24 \\ 1/6 \end{bmatrix} \begin{bmatrix} 5/12 \\ 17/48 \\ 11/48 \end{bmatrix} \ldots \begin{bmatrix} 2/5 \\ 2/5 \\ 1/5 \end{bmatrix}$$

Convergence after some iterations

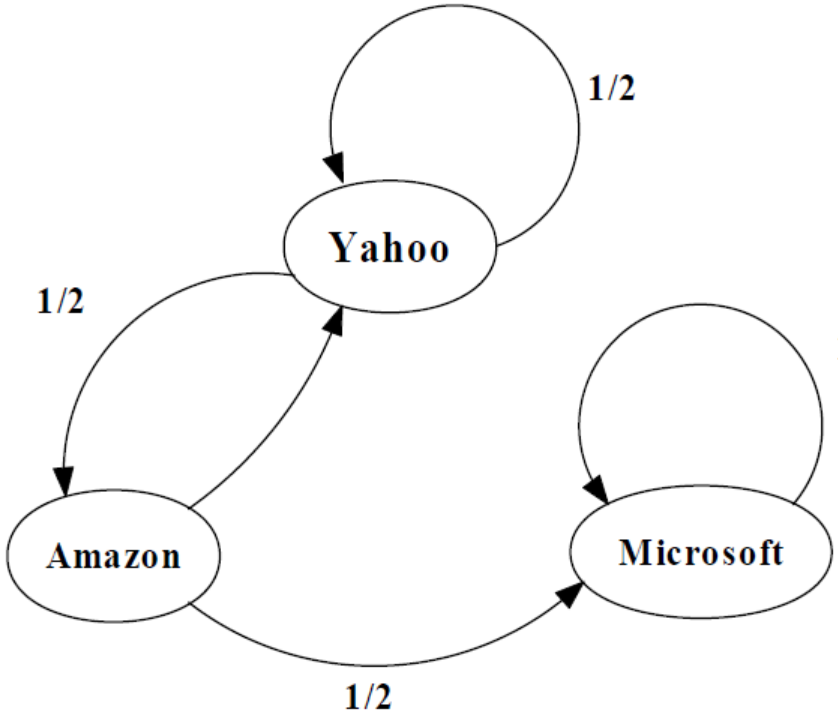
A Problem with Simplified PageRank

A loop:



During each iteration, the loop accumulates rank but never distributes rank to other pages!

An example of the Problem

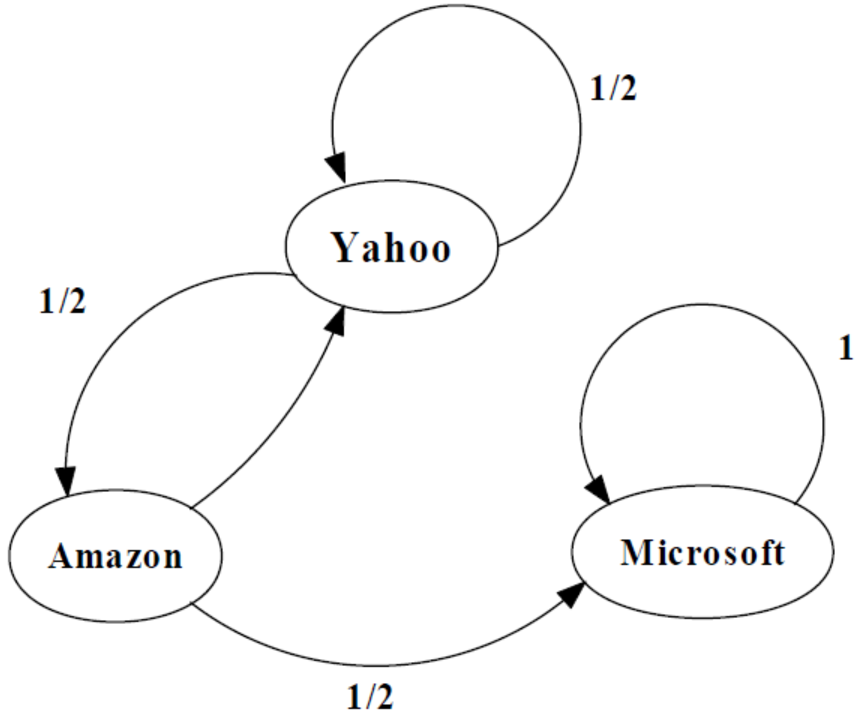


$$B^T = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1/3 \\ 1/6 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

An example of the Problem

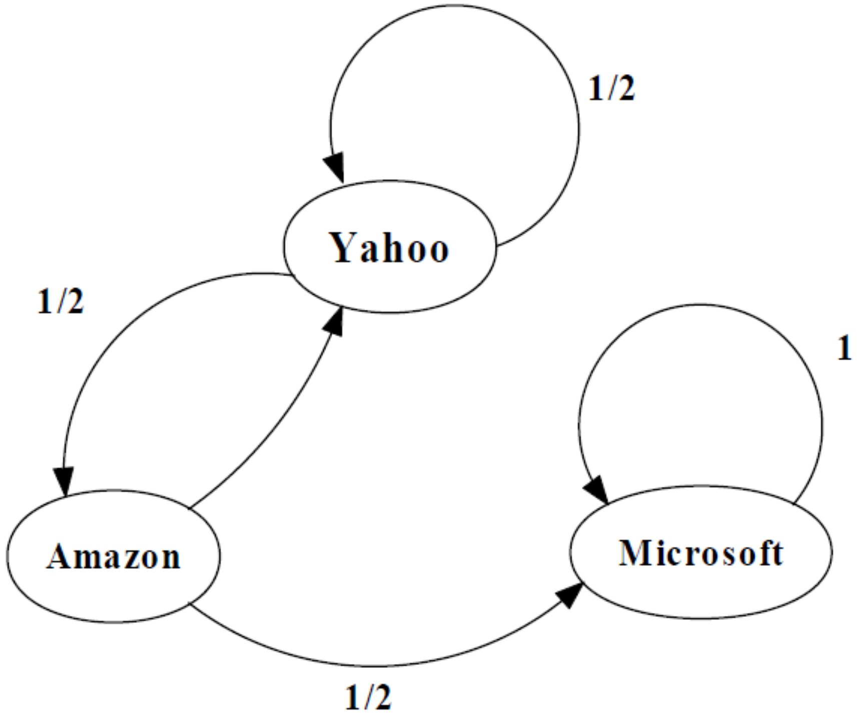


$$B^T = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1/4 \\ 1/6 \\ 7/12 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/6 \\ 1/2 \end{bmatrix}$$

An example of the Problem



$$B^T = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 5/24 \\ 1/8 \\ 2/3 \end{bmatrix} \begin{bmatrix} 1/6 \\ 5/48 \\ 35/48 \end{bmatrix} \ldots\ldots \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Random Walks in Graphs

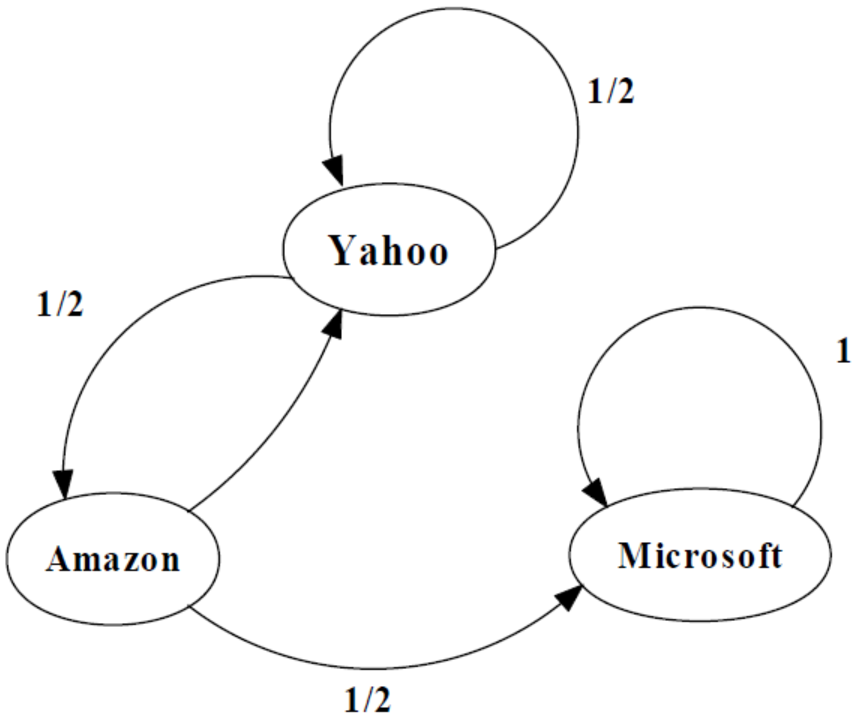
- The Random Surfer Model
  - The simplified model: the standing probability distribution of a random walk on the graph of the web. simply keeps clicking successive links at random
- The Modified Model
  - The modified model: the “random surfer” simply keeps clicking successive links at random, but periodically “gets bored” and jumps to a random page based on the distribution of E

Modified Version of PageRank

$$R'(u) = c_1 \sum_{v \in B_u} \frac{R'(v)}{N_v} + c_2 E(u)$$

E(u): a distribution of ranks of web pages that "users" jump to when they "gets bored" after successive links at random.

An example of Modified PageRank



$$B^T = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

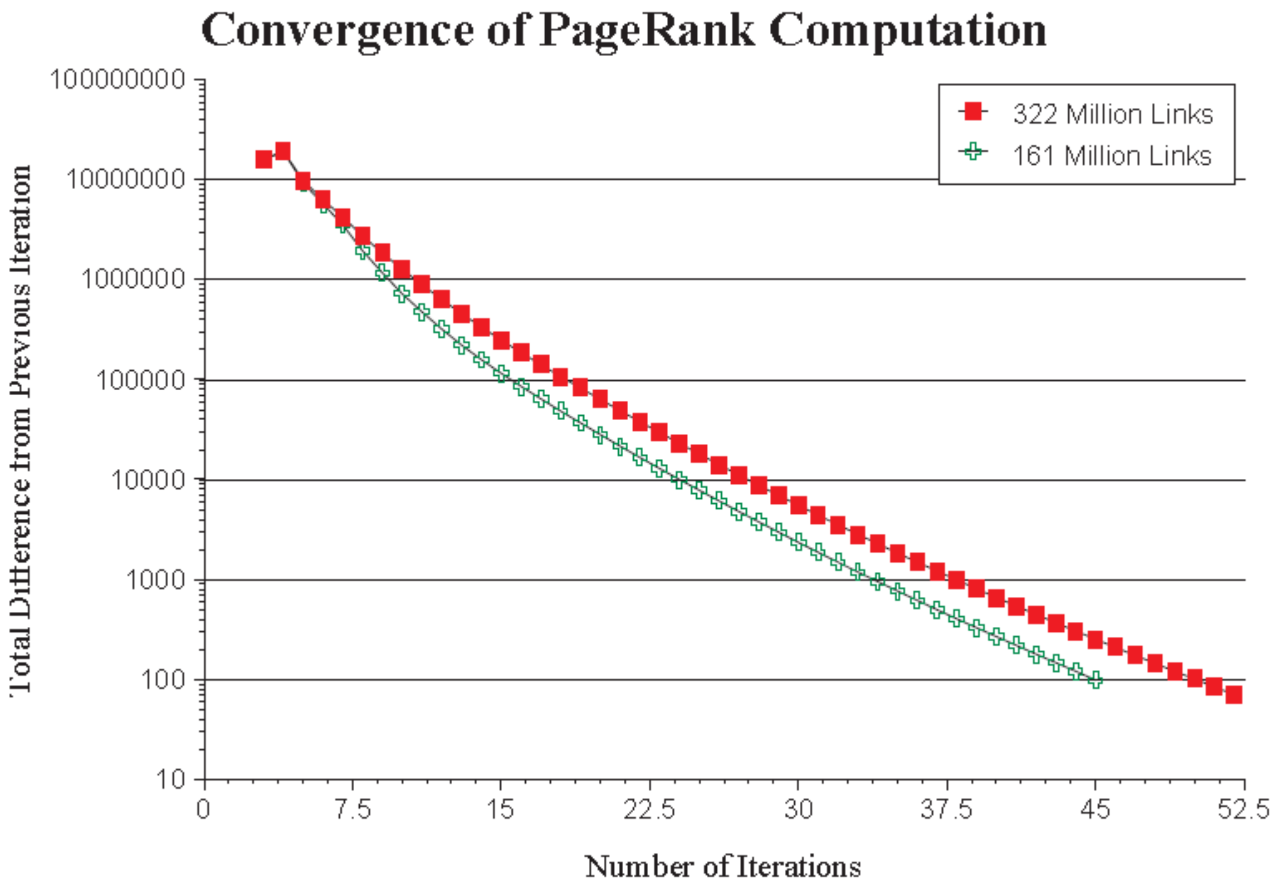
$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$C_1 = 0.8 \qquad C_2 = 0.2$

$$\begin{bmatrix} 0.333 \\ 0.333 \\ 0.333 \end{bmatrix} \begin{bmatrix} 0.333 \\ 0.200 \\ 0.467 \end{bmatrix} \begin{bmatrix} 0.280 \\ 0.200 \\ 0.520 \end{bmatrix} \begin{bmatrix} 0.259 \\ 0.179 \\ 0.563 \end{bmatrix} \cdots \begin{bmatrix} 7/33 \\ 5/33 \\ 21/33 \end{bmatrix}$$

Convergence Property

- PR (322 Million Links): 52 iterations
- PR (161 Million Links): 45 iterations
- Scaling factor is roughly linear in *logn*



PageRank v.s. HITS

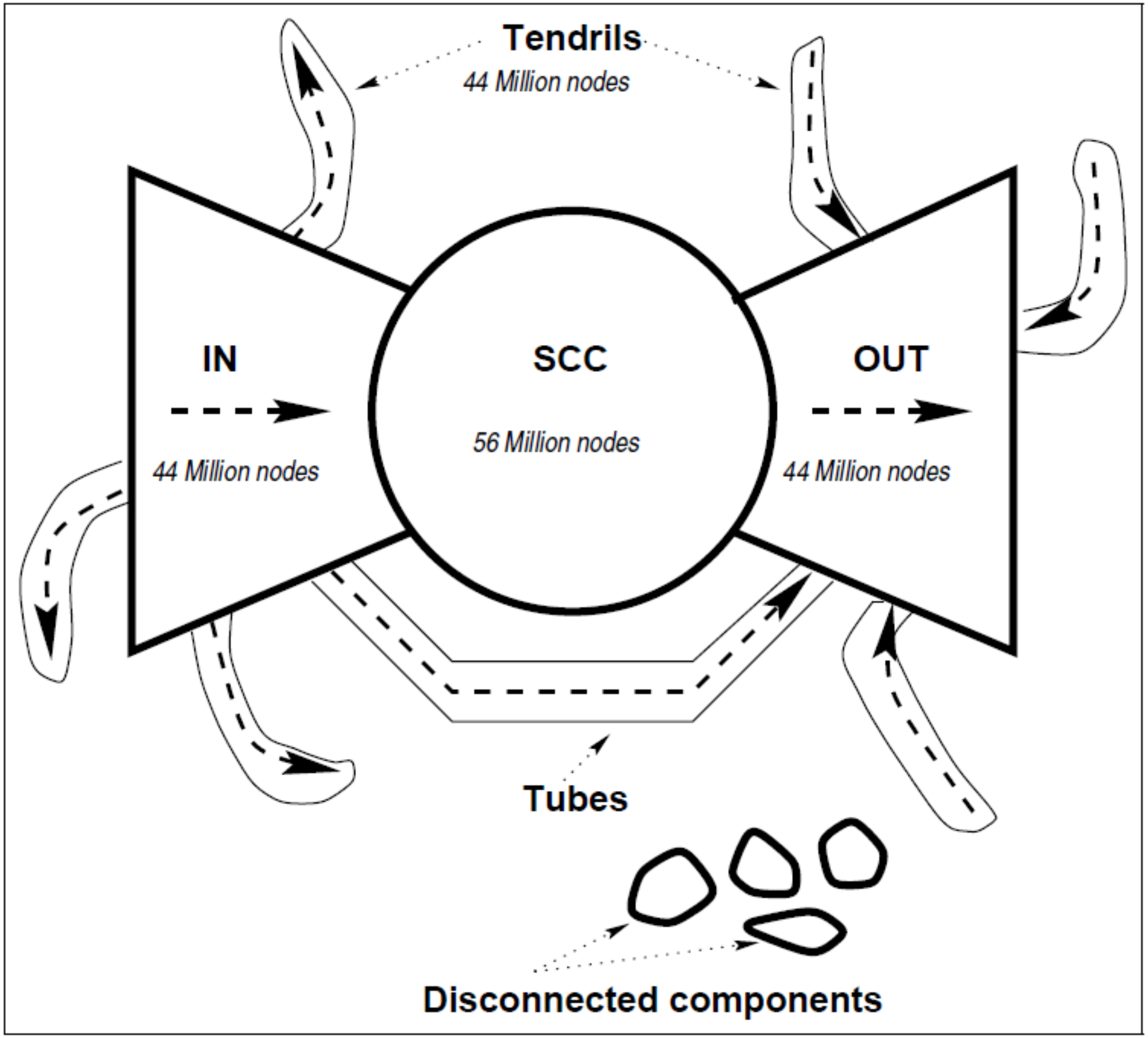
- PageRank
  - Computed for all web pages stored prior to the query
  - Computes authorities only
  - Fast to compute

- HITS
  - Performed on the subset generated by each query.
  - Computes authorities and hubs
  - Easy to compute, real-time execution is hard.

Which one is more suitable for large scale data set??

Web as a Graph

How does the Internet www look like?


Tendrils
44 Million nodes
IN
44 Million nodes
SCC
56 Million nodes
OUT
44 Million nodes
Tubes
Disconnected components

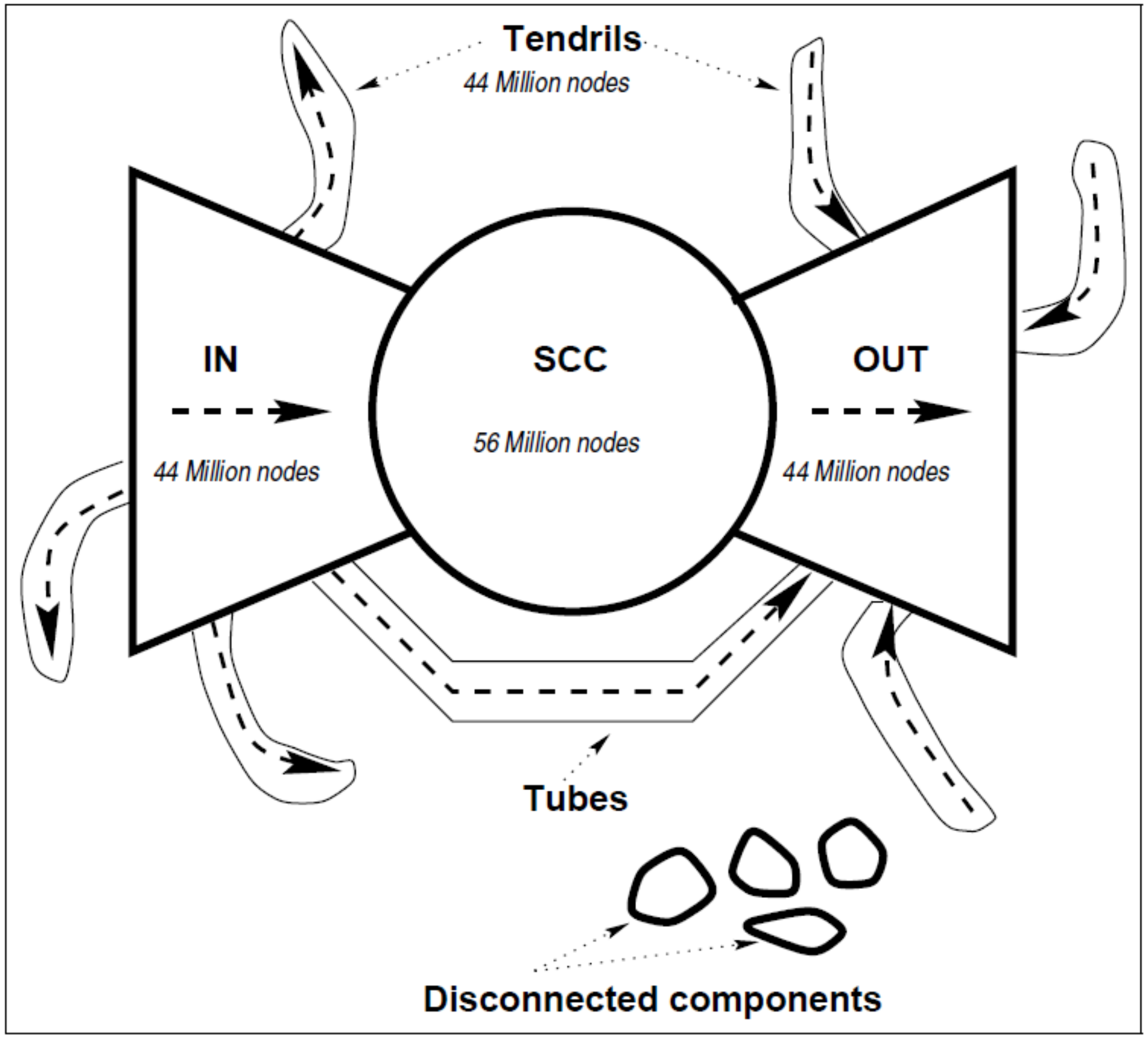


Figure 4: The web as a bowtie. SCC is a giant strongly connected component. IN consists of pages with paths to SCC, but no path from SCC. OUT consists of pages with paths from SCC, but no path to SCC. TENDRILS consists of pages that cannot surf to SCC, and which cannot be reached by surfing from SCC.

Thank You