# Guidelines for writing a mathematical proof

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The material on this page is adapted from Sec 2.7 of "E. Lehman, F. T. Leighton, and A. R. Meyer. Mathematics for Computer Science, Lecture notes from Fall 2010, MIT Open Courseware."

Below we discuss Lehman et. al.'s suggestions for what a good written proof looks like. For the purposes of this class all the bulletpoints **in bold** (#1, #3, #8, and #9) will be considered mandatory, i.e., omitting them will lead to penalty in exam situations. The bulletpoints in *italics* are recommended and can help improve clarity.

1. **State your plan**: Is this is a proof by contradiction, by induction? What do you intend to do here? Write it clearly at the beginning.

2. *Linear flow*: New thoughts or arguments or concepts should follow each other in the correct sequence. It's good to ask: (i) Have I already explained what is needed to understand this? (ii) Is what I am writing necessary for this proof at this stage? That will help you decide whether this is the right place to write something.

3. **A proof is an essay not a calculation**. A long sequence of expressions with no words does not constitute a proof. Explain your steps. Since students trained in India are particularly susceptible to this problem, we will be particularly harsh in grading if we see this.

4. *Avoid excessive symbols*. Use words when possible. Switch to symbols when the use of words is leading to confusion.

5. *Think about what new notation to introduce*. When new notation or symbols are needed, introduce them but think about what you're doing. Is this variable already used for something else? Should it be upper case or lower? Do I really really need it? Also, *tell* the reader what the new notation means.

6. *Revise and simplify*. Re-read the proof and see if it can be presented more simply. This may not be possible in exam situations due to time pressure but if you do it often enough in tutorials or other practice sessions it will become a habit.

7. *Structure long proofs*. If the proof is long break it down into smaller lemmas and prove them one at a time. Finally tie it together by arguing how the lemmas together prove the bigger result.

8. **Nothing is "obvious"**. Words like "obviously" and "clearly" have no place in a mathematical proof or a mathematical discussion. They show laziness or arrogance or, worst of all, that you are unsure and trying to bully the reader into believing what you are saying. Any appearance of one of these or similar terms will be treated harshly in exams.

9. **Finish**. Once all the arguments are made write a line or two to end the proof and actually finish it, e.g., "since the derivative is positive for all positive x and the function is 0 at x = 7, therefore we can say that the function is positive for x > 7".

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