CSL202: Discrete Mathematical Structures (Semester-I-2018-19) Quiz-10

Name: _____

Entry number: _____

There are 2 questions for a total of 10 points.

1. Let S be the subset of the set of ordered pairs of integers defined recursively by

   Basis step: $(0, 0) \in S$

   Inductive step: If $(a, b) \in S$, then $(a + 2, b + 3) \in S$ and $(a + 3, b + 2) \in S$.

   Answer the following questions:

   (a) (1 point) List the elements of S produced by the first five applications of the recursive definition.

   (b) (3 points) Use structural induction to show that 5 divides $a + b$ when $(a, b) \in S$.

2. (6 points) Prove or disprove: There exists an integer $k > 1$ such that 7^k ends with 0001 (in its decimal representation).
(*Hint that you may or may not use: Consider powers of* 7 *modulo* 10000 *and apply the Pigeonhole principle.*)

$7^0, 7^1, 7^2 \ldots\ldots 7^{10000}$