Name: ______________________

Entry number: ______________________

There are 4 questions for a total of 15 points.

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1. Use ideas developed in the class to calculate the following:
   (a) (1/2 point) Give the value of 5^{547} $(mod\ 15)$.
   (*Note that your answer should be an integer between* 0 *and* 14.)

   (a) __________

   (b) (1/2 point) Give the value of 9^{313} $(mod\ 55)$.
   (*Note that your answer should be an integer between* 0 *and* 54.)

   (b) __________

   (c) (1 point) Find an integer x that simultaneously satisfies the following three linear congruences $x \equiv 2$ $(mod\ 5)$, $x \equiv 2$ $(mod\ 7)$, and $x \equiv 5$ $(mod\ 9)$.
   (*Your answer should be an integer between* 0 *and* 314.)

   (c) __________

2. (3 points) In how many ways can you distribute n indistinguishable apples and one orange to k children such that each child gets at least one fruit? Give reasons.

3. Answer the following questions:

   (a) (1 point) State true or false: Any bipartite graph (L, R, E) with $|L| = |R|$ in which all vertices have degree exactly equal to 5 has a perfect matching.

   (a) ________________

   (b) (3 points) Give reason for your answer to part (a).

4. (6 points) Show that any graph with $2n$ vertices and at least $n^2 + 1$ edges for $n \geq 2$ has a *triangle* (i.e., three vertices v_1, v_2, v_3 such that there is an edge between any pair of vertices among these three).

Extra space