Name: ____________________

Entry number: ____________________

There are 2 questions for a total of 10 points.

$n=1 \to m=0$

$1+1 = 2^0 \cdot c$

$T(0) = 1$

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1. (5 points) Consider the following problem from Minor-2. Consider the recursive function:

```
F(n)
  - If (n > 1)
      - F(⌊n/3⌋)
      - F(⌊n/3⌋)
  - Print("Hello World")
```

$n = 3^m$

$T(3^m) = 2^{m+1} - 1$

base

Let $R(n)$ denote the number of times this function prints "Hello World" given the positive integer n as input. Give a closed form expression for $R(n)$ in terms of n. Prove correctness of the expression that you obtain.

Solution: Trying to solve a recurrence relation might be tricky in this case. One trick in such situations is guess-and-check. That is, guess the solution and then verify the solution using mathematical induction. One way to make a good guess is to try writing the solution for a few small values and then try to generalize. Here is the value of $R(n)$ for some small values of n.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$R(n)$	1	3	3	3	3	7	7	7	7	7	7	7	7	7	7	7	7	15	15	15

Note that the change in the value of $R(n)$ happens at $2, 2\cdot 3, 2\cdot 3^2, 2\cdot 3^3, ...$ and the value of $R(n)$ when $2\cdot 3^k \leq n < 2\cdot 3^{k+1}$ seems to be $2^{k+1}-1$. Let us prove this by induction. We first define the propositional function.

$P(k)$: For all n such that $2\cdot 3^k \leq n < 2\cdot 3^{k+1}$, the value of $R(n)$ is $2^{k+2}-1$.

Basis step: $P(0), P(1)$ are true from the above table.

Inductive step: Assume that $P(0), P(1), ..., P(k)$ holds for some arbitrary $k \geq 0$. We will now show that $P(k+1)$ holds.

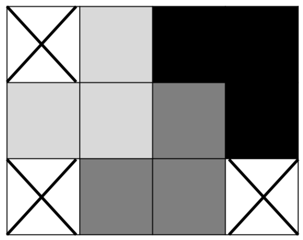
Consider any n such that $2\cdot 3^{k+1} \leq n < 2\cdot 3^{k+2}$. When the function $F(.)$ is called with such a value of n, the function makes two recursive calls with argument $\lfloor n/3 \rfloor$ and prints one "Hello World". Note that since $2\cdot 3^{k+1} \leq n < 2\cdot 3^{k+2}$, we have $2\cdot 3^k \leq \lfloor n/3 \rfloor < 2\cdot 3^{k+1}$. So, using the induction assumption, we get that the total number of "Hello World" that will be printed is $(2^{k+2}-1)\cdot 2 + 1 = 2^{k+3} - 1$.

So, by the principle of Mathematical induction, we conclude that $P(k)$ holds for all $k \geq 0$.

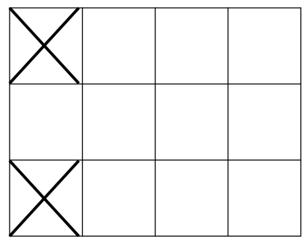
We now just need to express $R(n)$ in terms of n using the above conclusion. This comes out to be $R(n) = 4\cdot 2^{\lfloor \log_3 (n/2) \rfloor} - 1$ for all $n \geq 2$ and $R(1) = 1$.

2. (5 points) For this problem, we first need to define the concept of tiling a grid with triminoes.

   Consider an infinite supply of "L" shaped *triminoes* and an $m \times n$ grid pattern with some grid cells marked with a cross. Such a grid is said to be "tileable" using the triminoes iff it is possible to place one layer of triminoes on the grid such that all grid cells except the ones marked with cross are covered and no part of any trimino is outside the grid. The figure below illustrates this concept.



Grid Tileable (showing possible tiling)



Grid not Tileable

Prove or disprove: For all $n \geq 2$, any $2^n \times 2^n$ grid with precisely one grid cell marked with a cross is tileable using "L" shaped triminoes.