

Name: _____

Entry number: _____

There are 3 questions for a total of 10 points.

1. (a) (1 point)
- State true or false
- : The following compound proposition is a tautology.

$$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$$

 (a) True

- (b) (2 1/2 points) Give reason for your answer to part (a).

Solution: We will show that $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ is a tautology using a truth table below.

p	q	r	$(p \vee q)$	$(\neg p \vee r)$	$(p \vee q) \wedge (\neg p \vee r)$	$(q \vee r)$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	T	F	F	F	T
F	T	F	T	T	T	T	T
F	F	T	F	T	F	T	T
F	F	F	F	T	F	F	T

2. (3 points) Let
- $C(p, q, r)$
- denote a compound proposition involving simple propositions
- p, q
- , and
- r
- . Give a compound proposition
- $C(p, q, r)$
- the truth table of which matches the one given below. (
- Note that there may be multiple correct answers for this question*
-)

p	q	r	$C(p, q, r)$
T	T	T	T
T	T	F	F
T	F	T	F
F	T	T	F
T	F	F	T
F	T	F	F
F	F	T	T
F	F	F	T

$$2. C(p, q, r) \equiv (p \wedge q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \vee \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$$

You might have found a more simplified expression for this problem. The purpose of this exercise was to convey that **any** boolean function can be written as a compound propositions involving \neg, \vee, \wedge . There is a standard method for doing this. Consider all table entries that have *T* in the last column. For each such table entry, create a conjunction of variables or negations depending on whether the table entry is T or F. Finally, take a disjunction of all such conjunctions. Try

proving that the compound proposition created using this way will match the given truth table. (Furthermore, note that since \vee can be written using \neg and \wedge . We can write any boolean function using just \neg and \wedge .)

3. (a) (1 point) State true or false: The following two compound propositions are logically equivalent:

- $(p \rightarrow q) \rightarrow (r \rightarrow s)$
- $(p \rightarrow r) \rightarrow (q \rightarrow s)$

(a) False

(b) (2 1/2 points) Give reason for your answer to part (a).

Solution: Consider the following truth values to the propositional variables p, q, r, s :
 - $p = F, q = F, r = T, s = F$.
 For these truth values, $(p \rightarrow q) \rightarrow (r \rightarrow s)$ evaluates to F whereas $(p \rightarrow r) \rightarrow (q \rightarrow s)$ evaluates to T . Hence the compound propositions are not logically equivalent.