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Q1. Let S be the set of students in a class. Each pair of them play a game exactly once (assume that one of the player always wins, there is no possibility of draw in the game). Write the following logical statements using quantifiers. All quantifiers should appear at the beginning of the sentence. You can use the proposition $C(x, y)$ which denotes x defeated y in the game, and $G(x)$ which denotes x is a girl. You can use the usual logical operators ($\wedge, \vee, =, \neq, \rightarrow, \neg$).

(i) No girl lost to a boy.

(1 mark)

$$\forall x \forall y (G(x) \wedge (\neg G(y)) \rightarrow (C(x, y)))$$

$$(\forall x \forall y) \in S \quad (G(x) \wedge (\neg G(y))) \longrightarrow (C(x, y))$$

(ii) There is exactly one girl who defeated all the boys. (2 marks)

$$(\exists x_1 \forall y \forall z) \in S (G(x_1) \wedge (\neg G(y)) \wedge C(x_1, y)) \wedge (G(z) \wedge (\neg G(x_1)) \wedge C(x_1, z)) \rightarrow (y = x_1)$$

(iii) There are two students such that everyone else lost to at least one of these two students. (2 marks)

$$(\exists x_1, x_2 \forall y \neq x_1, x_2) \in S \quad \exists z \in S \quad \neg z_1$$

$$(C(x_1, y) \vee C(x_2, y)) \wedge (C(x_1, z) \vee C(x_2, z))$$

$$(z = x_1) \vee (z = x_2)$$

(iv) No two boys lost to the same girl. (2 marks)

$$(\forall x \forall y \forall z) \in S \quad (G(x) \wedge (\neg G(y)) \wedge (\neg G(z)) \wedge (C(x, y) \wedge C(x, z)))$$

$$\longrightarrow (y = z)$$

(2)

✓

Q2/ (6 marks) You play a game where you start with the number 18. At each step, you perform one of the following 2 operations:

- Multiply the current number by 2.
- Multiply the current number by 3.
- Divide the current number by 2 (if it is even)
- Divide the current number by 3 (if it is divisible by 3)

For example, you can divide 18 by 2 in Step 1 to get 9. In Step 2, you could multiply it by 3 to get 27, and so on. Prove that you will never get the number 96 after 60 steps.

A)

$$18 = 2 \times 3^2$$

$$96 = 2^5 \times 3$$

every n^{th} step can be denoted by $x \begin{pmatrix} x_n & y_n \\ 2 & 3 \end{pmatrix}$ multiply

where (x_n, y_n) can be

- $(1, 0)$
- $(0, 1)$
- $(-1, 0)$
- $(0, -1)$

\therefore after 60 steps we have,

$$96 = 18 \times \begin{pmatrix} x_1 & y_1 \\ 2 & 3 \end{pmatrix} \times \begin{pmatrix} x_2 & y_2 \\ 2 & 3 \end{pmatrix} \dots \begin{pmatrix} x_{60} & y_{60} \\ 2 & 3 \end{pmatrix}$$

$$2^5 \times 3 = 2 \times 3^2 \times \begin{pmatrix} x_1 & y_1 \\ 2 & 3 \end{pmatrix} \times \begin{pmatrix} x_2 & y_2 \\ 2 & 3 \end{pmatrix} \dots \begin{pmatrix} x_{60} & y_{60} \\ 2 & 3 \end{pmatrix}$$

found proof:

2's power should increase by 4
3's power should decrease by 1

there should be atleast 4 $\times \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$ steps
and one $\times \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$ step

remaining steps are 55
to get the final answer as 96 from 18
for every extra $\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$ step we must have one extra $\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$ step
similarly for $\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$ and vice versa

last

Q3. (6 marks) Let n be a positive integer. You are given a set of $n+1$ distinct positive integers, each of which is less than or equal to $2n$. Use mathematical induction to show that there is at least one integer in this set that divides another integer in this set.

A) for $(n=1)$ we have $n+1=2$ $2n=2$
 \therefore possible +ve integer sets are, (with distinct numbers)

is ~~(1,2)~~ ~~(1,2)~~

in each of these cases at least one divides other.

Let us assume this to be true for $1 \leq n \leq n$

now we have to prove for $n+1$ case.

We have $n+2$ distinct +ve integer each $\leq 2(n+1)$.

Now we ~~remove~~

Let us consider two numbers $(2n+1, 2n+2)$

Case (i): - If the maxima of the $n+2$ distinct +ve integers is either of $2n+1$ or $2n+2$ but not both are present, then we remove that number from the set.

$n+2$. Now we have $n+1$ distinct numbers with maximum $\leq 2n$. By above assumption

(~~Case (i)~~) we can find at least one number in this set that divides other.

Case (ii): - If the $(n+2)$ distinct numbers contain both $(2n+1$ and $2n+2)$ we remove both numbers but we are left with n numbers $\leq 2n$.

If we continue reducing the maximum number n by removing numbers