1.Suppose we wish to design a minimum-area logic circuit implementing a multi-output boolean function (e.g., the circuit has 4 inputs and 2 outputs).

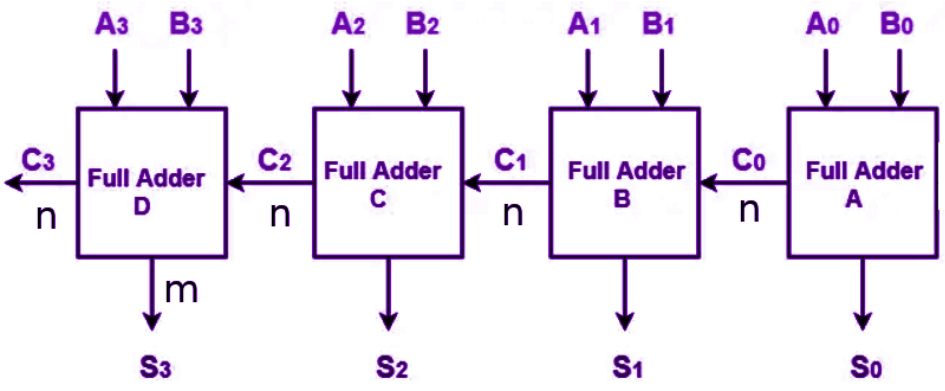
a. Suggest how we can use Karnaugh Maps (K-Maps) to achieve this objective.

Use two K-Maps, one for each output, optimise the resulting expressions independently, and derive two sets of boolean expressions.

b. Point out a disadvantage of minimising the multi-output function using the K-Map approach.

We miss out on optimising across the two boolean functions. For example, there may be common expressions between the two functions, for which logic need not be duplicated.

2. Consider the 4-bit ripple carry adder discussed in class. Suppose the SUM bit in a one-bit full adder stabilises after m nano-seconds, and the CARRY OUT bit stabilises after n nano-seconds. What is the delay of the 4-bit adder? Justify your answer.



Solution 2. **Delay to compute sum (S_3) :**
3n+m (delay to CARRY OUT in first 3 stages, and delay to SUM in last stage). Assuming m>n.

Delay to compute carry (C_3) :
4n (delay to CARRY OUT in all the stages)

Total delay of the 4-bit adder = **max(3n+m, 4n)** or **3n + max(n,m)**

3. In the Priority Encoder we discussed in class, the output was derived from input by just OR-ing the appropriate input variables. Why is this valid, considering that only a subset of table rows were listed in the truth-table? Explain.

If the input variable is 1, then the output is determined, irrespective of the values of the lower priority inputs (the latter are DON'T CARE). So, it is enough to look at a subset of the input rows.

Note we have also considered answers with a truth table and have given them some marks based on the truth table as well. And have defined the Priority encoder.

4. Write a VHDL model with the following behavior: When simulated, the simulation fails to proceed beyond 10 nanoseconds.

design.vhd

```
library IEEE;
use IEEE.std_logic_1164.all;

entity fail is
end fail;

architecture rtl of fail is
   signal a : STD_LOGIC := '0';
   signal b : STD_LOGIC := '0';
   signal c : STD_LOGIC := '0';
Begin
   b <= not a after 10ns;
   c <= not (c and b);
end rtl;
```

testbench.vhd

```
library IEEE;
use IEEE.std_logic_1164.all;

entity testbench is
end testbench;

architecture tb of testbench is
   component fail is
   end component;
begin
   UUT: fail;
   process
   begin
        wait for 20ns;
   end process;
end tb;
```

5. What function of A, B, C, and D does the following CMOS transistor circuit implement? Justify. [Note that crossing wires are connected only when there is a black circle at the intersection]

Should arrive at the minimized expression: A'C'+A'D'+B'C'+B'D'
Your solution can be done in below two ways:-

1) By analyzing the nmos,pmos parts of the circuit. Sufficient justification should be provided as to why you chose a logical operation based on whether the transistors are in series/ parallel in the nmos/ cmos parts.
2) By finding the output Z for all possible values of A,B,C,D. Then create the truth table, draw Kmap and do optimal groupings and arrive at the minimzed expression.

6. The first manned expedition to Mars ("Mangalyaan") found only the ruins of a civilization. From the artifacts and pictures, the explorers deduced that the creatures who produced this civilization were four-legged beings with a tentacle that branched out at the end with a number of grasping "fingers." After much study, the explorers were able to translate Martian mathematics. They found the following equation: 5x2

- 50 x + 125 = 0 with the indicated
solutions x = 5 and x = 8. The value x = 5 seemed legitimate enough, but x = 8 required some explanation. Then the explorers reflected on the way in which Earth's number system developed, and found evidence that the Martian system had a similar history. How many fingers would you say the Martians had? Justify. (From The Bent of Tau Beta Pi, February 1956. Extracted from Wakerly, Digital Design) [Hint: the number of fingers relates to number representation the same way it does for humans.]

The question is: for what base does the quadratic equation ($5x^2$ - 50 x + 125 = 0) have solutions 5 and 8?
Let the answers hold for base n. For this base,
5 refers to number: 5
50 refers to number: 5n + 0
125 refers to number: n^2 + 2n + 5

So, we have the equation (constants 2 and 5 in decimal): $5x^2 – 5nx + n^2 + 2n + 5 = 0$ **[Equation 1]**
According to the question, this equation has solutions x=5 and x=8.

Substituting x = 5 in Equation 1, we obtain the equation (constants in decimals):
125 – 25n + n^2 + 2n + 5 = 0, i.e., n^2 - 23n + 130 = 0 [Solutions n = 10 and n = 13] **[Equation 2]**

Substituting x = 8 in Equation 1, we obtain the equation (constants in decimals):
$320 – 40n + n^2 + 2n + 5 = 0$, i.e., $n^2 - 38n + 325 = 0$ [Solutions n = 13 and n = 25]
[Equation 3]

Base n = 13 satisfies both equations 2 and 3. The aliens have 13 fingers.

Note: We can also reach the answer by writting the equation that has roots 5,8 in decimal system and then comparing that equation with the general equation in 'n' that is written above.