# EE310 – Chapter 3

Diodes

Lecture Slides

Instructor:
Prof. Chu Ryang Wie

# Chap. 3    Diodes
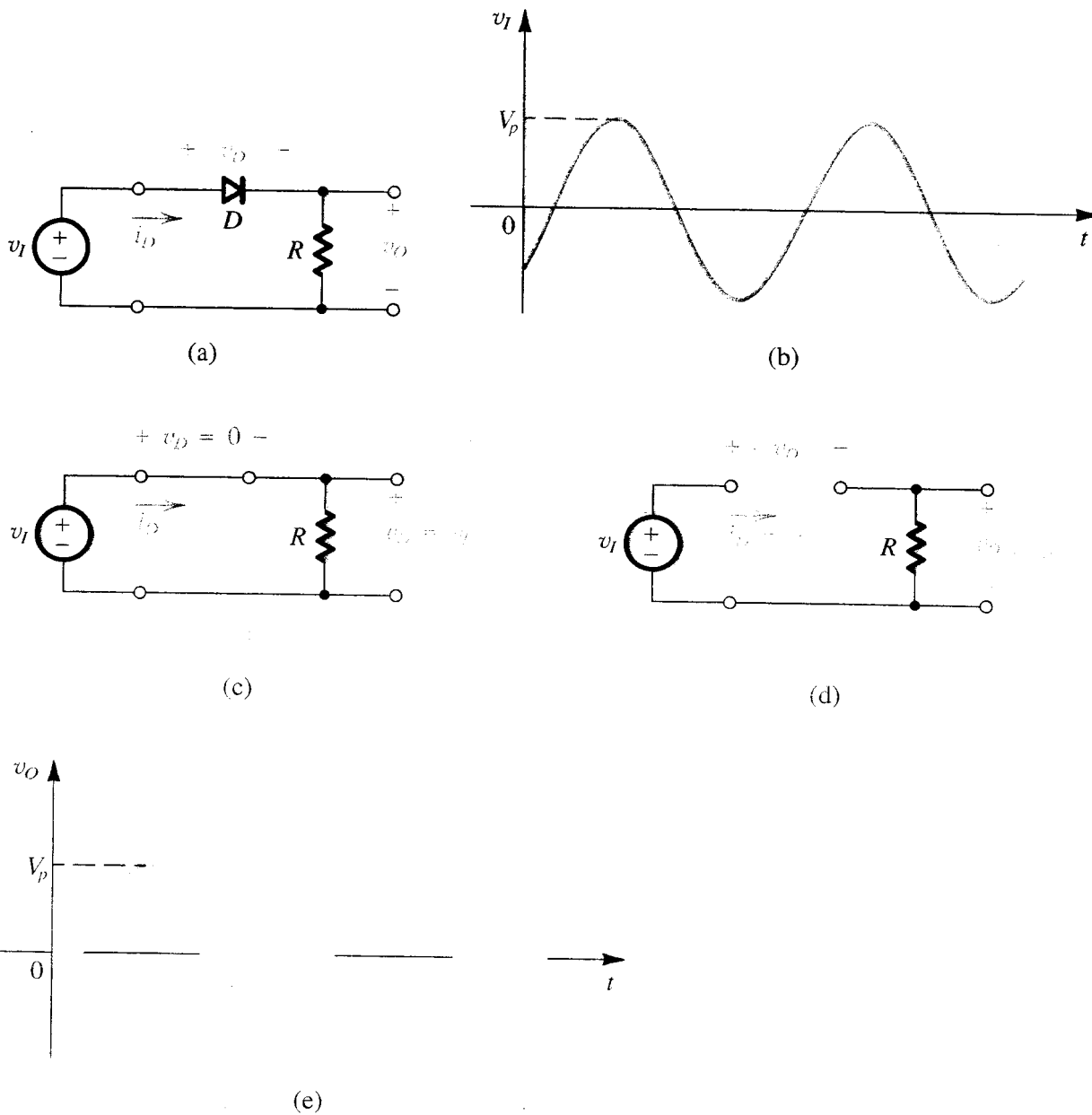
## 3.1  Ideal Diode

### I-V Characteristic

Anode $\xrightarrow{i}$ Cathode

+ $v$ −

(a)

$i$

- - - Reverse bias $\longrightarrow$ | $\longleftarrow$ Forward bias - - -

(Open)       (Short)

0       $v$

(b)

+ $v$ −

(c)

$\xrightarrow{i}$

(d)
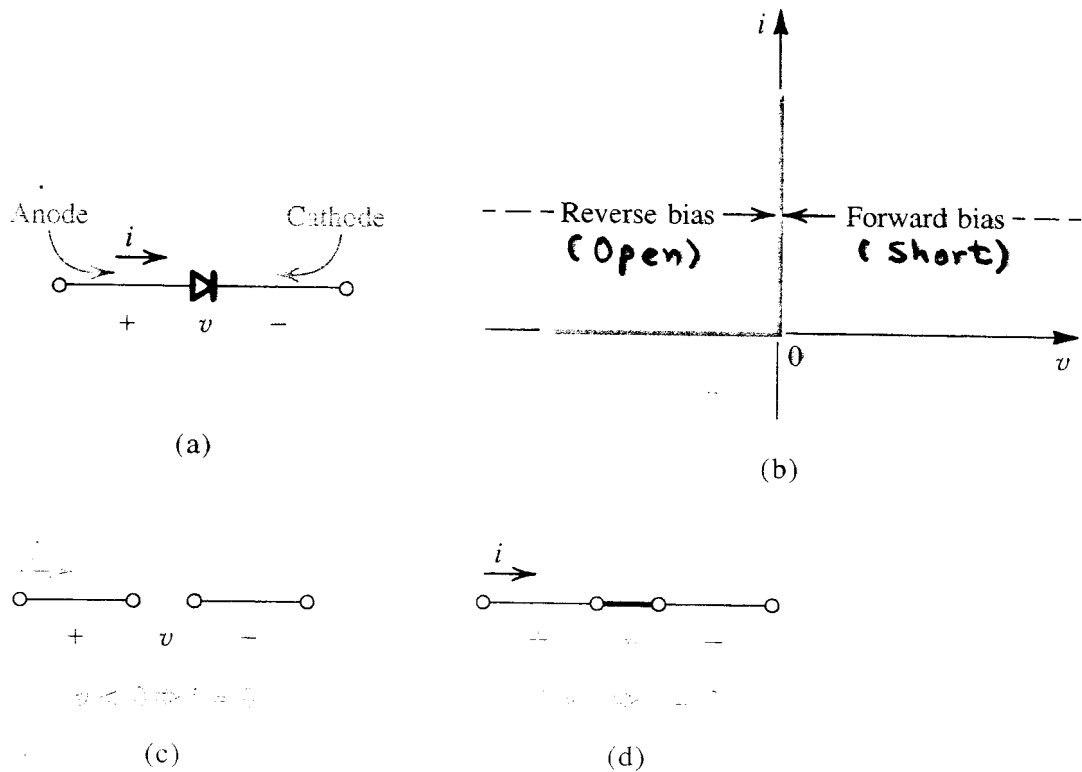
The ideal diode: **(a)** diode circuit symbol; **(b)** $i–v$ characteristic; **(c)** equivalent circuit in the reverse direction; **(d)** equivalent circuit in the forward direction.

# Application — Rectifier



(a)

(b)

(c)

(d)



(e)

**(a)** Rectifier circuit. **(b)** Input waveform. **(c)** Equivalent circuit when $v_I \geq 0$. **(d)** Equivalent circuit when $v_I \leq 0$. **(e)** Output waveform.

3.3 In the circuit of Fig. 3.3(a), let $v_I$ have a peak value of 10 V and $R = 1$ kΩ. Find the peak value of $i_D$ and the dc component of $v_O$.

Ans. 10 mA; 3.18 V

Sol)

$$\hat{V}_I = 10V \qquad R = 1K\Omega \qquad \hat{i}_D = (\quad) \qquad \overline{V_0} = (\quad)$$

$$\hat{i}_D = \frac{\hat{V}_I}{R} = \frac{10V}{1K\Omega} = 10mA$$

$$\overline{V_0} = DC \ comp \ of \ V_0 = \frac{\int_0^T V_0 \, dt}{T} = \frac{\int_0^\pi V_p \sin\theta \, d\theta}{2\pi} = \frac{2V_p}{2\pi} = \frac{10V}{\pi}$$
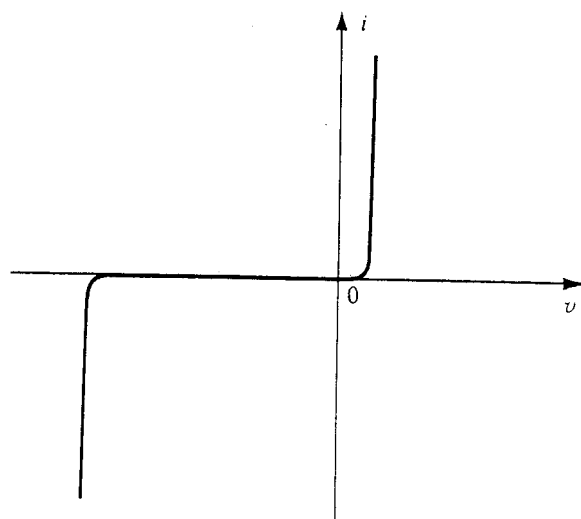
# 3.2 Diode Terminal Characteristic



**FIGURE 3.7** The $i$–$v$ characteristic of a silicon junction diode.
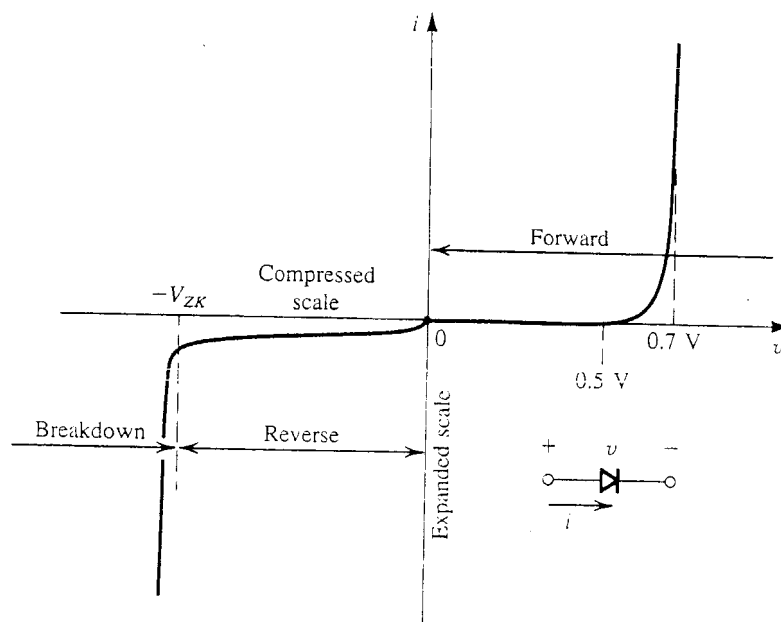


**FIGURE 3.8** The diode $i$–$v$ relationship with some scales expanded and others compressed in order to reveal details.

### 3.2.1 The Forward-Bias Region

$$i = I_s(e^{v/mV_T} - 1)$$

where, $V_T = \dfrac{kT}{q} = 25\,mV$ at $T = 300K$ (R.T.)

$m = $ ideality factor $= 1 - 2$

(IC) (Discrete)

For $i \gg I_s$, then $i \approx I_s e^{v/mV_T}$

$$\iff v = mV_T \ln \dfrac{i}{I_s} = 2.3\,mV_T \log \dfrac{i}{I_s}$$

ex) $i_1 @ V_1$, $i_2 @ V_2$ then, $V_2 - V_1 = 2.3\,mV_T \log \dfrac{i_2}{i_1}$

For example, $i_2 = 10 \times i_1$, then $V_2 - V_1 = 2.3\,mV_T$

$\Rightarrow$ Diode voltage increases $2.3\,mV_T$ volts per each _decade_ increase in current.

Note $2.3\,mV_T = 57.5\,mV \sim 115\,mV$ at RT

$(m = 1)$ $(m = 2)$

ex) $i \approx 0$ for $v \le 0.5V$

$\therefore v = 0.5V$ is called cut-in voltage

$i = $ fully conducting for $v \approx 0.6 \sim 0.8\,V$ !

$\therefore v \approx 0.7V$ is turn-on voltage

**3.6** Consider a silicon diode with $n = 1.5$. Find the change in voltage if the current changes from 0.1 mA to 10 mA.

Ans. 172.5 mV

$$n = 1.5 \qquad i_1 = 0.1 \, mA \longrightarrow i_2 = 10 \, mA$$

$$V_2 - V_1 = (\qquad)$$

Sol)
$$V_2 - V_1 = 2.3 \, n V_T \log \frac{i_2}{i_1} = 2.3 \, n V_T \log \frac{10 \, mA}{0.1 \, mA}$$

$$= 2.3 \times 1.5 \times 25 \, mV \times \log 100 = 172.5 \, mV$$

**3.8** Using the fact that a silicon diode has $I_S = 10^{-14}$ A at 25°C and that $I_S$ increases by 15% per °C rise in temperature, find the value of $I_S$ at 125°C.

Ans. $1.17 \times 10^{-8}$ A

$$I_S = 10^{-14} A \, @ \, 25°C \qquad \frac{\Delta I_S}{\Delta T} = \frac{0.15 \, I_S}{°C}$$

$$I_S = (\qquad) @ 125°C$$

Sol)
$$I_S (125°C) = (1 + 0.15) \, I_S (124°C)$$

$$= (1 + 0.15) \times (1 + 0.15) \, I_S (123°C)$$

$$= (1 + 0.15)^{100} \, I_S (25°C)$$

$$= 1.15^{100} \times 10^{-14} A = 1.17 \times 10^{-8} A$$

# $I_S$ doubles every 5°C
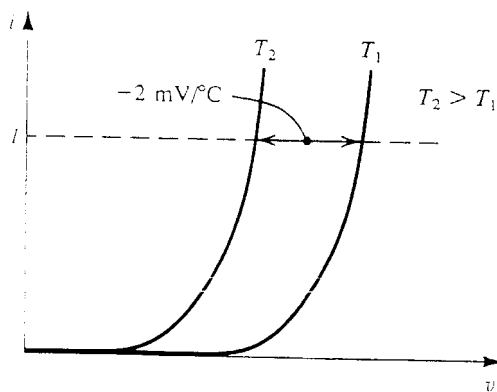


FIGURE 3.9 Illustrating the temperature dependence of the diode forward characteristic. At a constant current, the voltage drop decreases by approximately 2 mV for every 1°C increase in temperature.

### 3.2.2 The Reverse-Bias Region $\qquad$ $V < 0$

$$i = I_s\left(e^{V/nV_T} - 1\right) \simeq -I_s \equiv \text{reverse saturation current}$$

For real devices, often $i = i_R \gg I_s$

ex) a small-signal diode with $I_s = 10^{-14} - 10^{-15} A$

Can show $i_R = 10^{-9} A$ ! due to <u>Leakage</u>

$$i_R \longrightarrow \mathbf{2x \ every \ 10°C}$$

<u>Note</u> $I_s \rightarrow 2x$ every $5°C$

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**3.9** The diode in the circuit of Fig. E3.9 is a large high-current device whose reverse leakage is reasonably independent of voltage. If $V = 1$ V at 20°C, find the value of $V$ at 40°C and at 0°C.
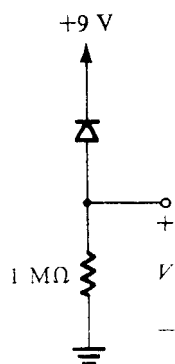


**FIGURE E3.9**

Ans. 4 V; 0.25 V

$i_R =$ indept. of $V$

$V = 1\,V$ @ $20°C$

$V = (\quad)$ @ $40°C$, $0°C$

Sol)

$$V = i_R \times 1 M\Omega$$

$20°C$: $\quad 1V = i_R \times 1M$

$\therefore i_R = 1\mu A$

$40°C$: $\quad 20°C \rightarrow 30°C \rightarrow 40°C$

$i_R = \quad 2x \qquad 2x$

$= 4 \times 1\mu A \qquad \therefore V = 4\mu A \times 1M\Omega$

$= 4V$

$0°C$: $\quad 20°C \rightarrow 10°C \rightarrow 0°C$

$i_R = \quad \frac{1}{2}x \qquad \frac{1}{2}x \quad = \frac{1}{4} \times 1\mu A$

$V = \frac{1}{4}\mu A \times 1M\Omega = 0.25V$

Fig. 8 $V_{ZK} =$ "Knee" voltage (Breakdown Voltage)

Breakdown is NOT destructive unless
the power dissipated <u>exceeds</u> the safe level.

# 3.3 Forward Characteristics – Models!

## 1) Exponential Model
— Most accurate
— Most complicated to use.

Fig. 10 $\quad I_D = I_s\, e^{V_D/mV_T} \quad \cdots$ (diode model)

$$I_D = \frac{V_{DD} - V_D}{R} \quad \cdots \text{(circuit)}$$

(First) Graphical Solution = Fig. 11

(Second) Iterative Solution

① Let $V_D = 0.7V$ (diode)

② Calc $I_D = \dfrac{V_{DD} - 0.7}{R}$ (circuit)

③ Refine $V_D = 2.3\, m V_T \log \dfrac{I_D}{I_s}$ (diode)
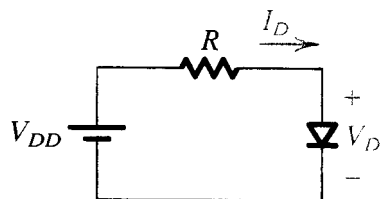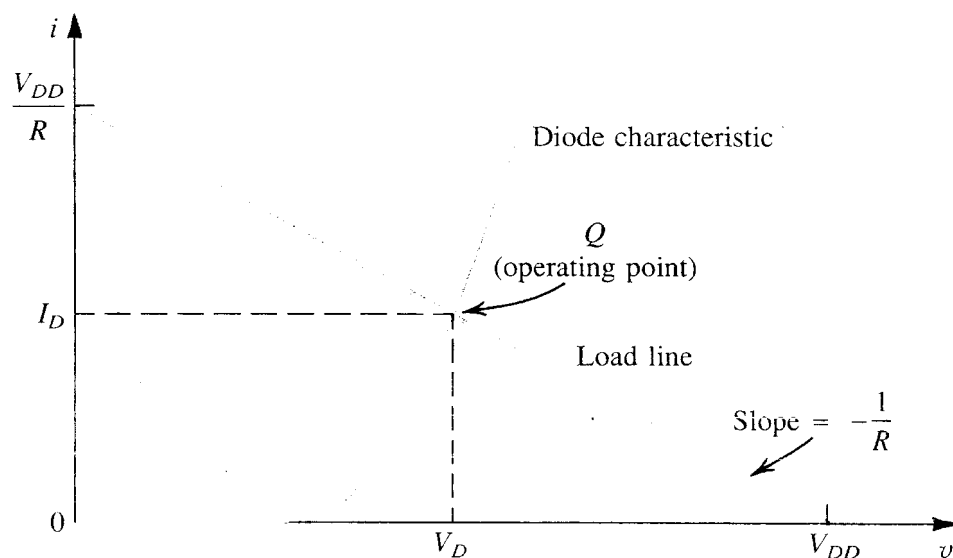
# Exponential Model



FIGURE 3.10  A simple circuit used to illustrate the analysis of circuits in which the diode is forward conducting.



Graphical analysis of the circuit in Fig. 3.10 using the exponential diode model.

3.10 For the circuit in Fig. 3.10, find $I_D$ and $V_D$ for the case $V_{DD} = 5$ V and $R = 10$ k$\Omega$. Assume that the diode has a voltage of 0.7 V at 1-mA current and that the voltage changes by 0.1 V/decade of current change. Use (a) iteration, (b) the piecewise-linear model with $V_{D0} = 0.65$ V and $r_D = 20$ $\Omega$, (c) the constant-voltage-drop model with $V_D = 0.7$ V.

Sol)  Circuit:   $V_{DD} = 5V$    $R = 10K$

Diode:   $\dfrac{\Delta V_D}{\Delta I_D} = \dfrac{0.1V}{decade}$   $\therefore 2.3\, m V_T = 0.1 V$

$0.7V @ 1mA \rightarrow 1mA = I_s e^{0.7V/mV_T}$

$= I_s e^{0.7 \times 2.3/0.1}$

$\therefore I_s = 1mA\, e^{-16.1}$

Iteration:

Let $V_D = 0.7V \rightarrow I_D = \dfrac{5-0.7}{10k} = 0.43\,mA$ (circuit)

$\rightarrow 0.43mA = 1mA\, e^{-16.1}\, e^{23 V_D}$  $\therefore V_D = 0.663$ (diode)

$\rightarrow I_D = \dfrac{5-0.663}{10k} = 0.434\,mA$  (circuit)

D3.12 Design the circuit in Fig. E3.12 to provide an output voltage of 2.4 V. Assume that the diodes available have 0.7-V drop at 1 mA and that $\Delta V = 0.1$ V/decade change in current.

+10 V

R

$V_o$

FIGURE E3.12

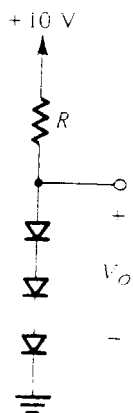Ans. $R = 760$ $\Omega$

Sol) Diode: $0.1V/decade$, $0.7V$ @ 1mA

Design for $V_o = 2.4V \rightarrow R = \dfrac{10-2.4}{I_D}$

$I_D = (\quad)$ @ $V_D = \dfrac{2.4}{3} = 0.8V$?

Diode has $I_s = 1mA\, e^{-16.1}$, $2.3 V_T = 0.1 V$

$I_D = I_s e^{V_D/mV_T} = 1mA\, e^{-16.1}\, e^{2.3 \times 0.8/0.1}$

$\therefore I_D = 9.974\,mA$ @ $V_D = 0.8V$

$\therefore R = \dfrac{10-2.4}{9.974} = 0.762\,K\Omega$

## 2) Piecewise Linear Model
( — 2nd most accurate
  — 2nd most involved to use! )  Fig.12

$$i_D = \begin{cases} \dfrac{V_D - V_{DO}}{r_D} & \text{for } V_D \geq V_{DO} \\ \\ 0 & \text{for } V_D \leq V_{DO} \end{cases}$$

## 3) Const. Voltage Drop Model
( — 3rd most accurate
  — 3rd most involved! )  Fig.15

$$V_D = 0.7 \text{ V}$$

## 4) Ideal Diode Model  — most simple!

$$i_D = \begin{array}{l} \text{short } (V_D > 0) \\ \text{Open } (V_D < 0) \end{array}$$

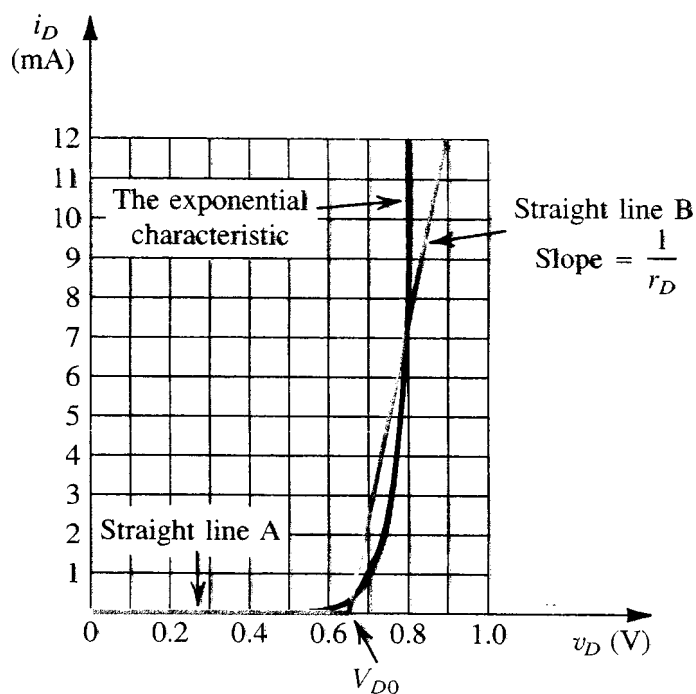## 5) Small — signal Model

For small signals, Diode is modeled as

$$i_d = \frac{V_d}{r_d} \qquad \text{where } r_d \equiv \frac{m V_T}{I_D}$$
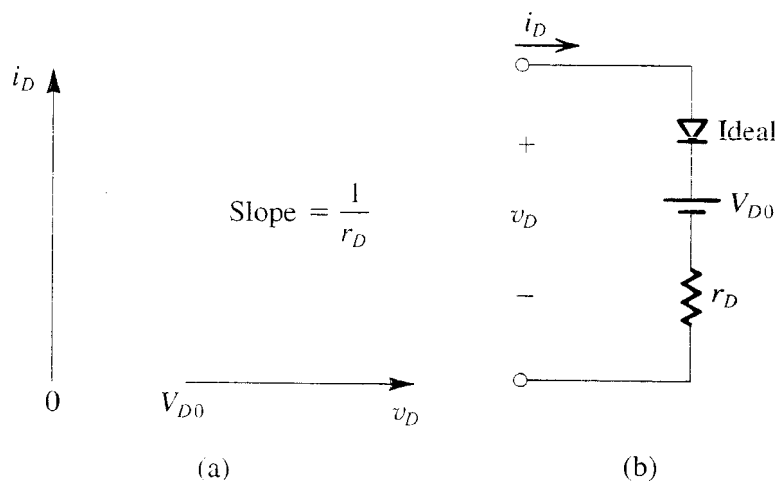
Fig.17

$$V_D + v_d$$

$$I_D + i_d$$

# Piecewise Linear Model



The exponential characteristic

Straight line B

Slope = $\dfrac{1}{r_D}$

Straight line A

$V_{DO}$

Approximating the diode forward characteristic with two straight lines: the piecewise-linear model.



Slope = $\dfrac{1}{r_D}$

$i_D$

Ideal

$v_D$

$V_{D0}$

$r_D$

$V_{D0}$

$v_D$

(a)

(b)

Piecewise-linear model of the diode forward characteristic and its equivalent circuit representation.
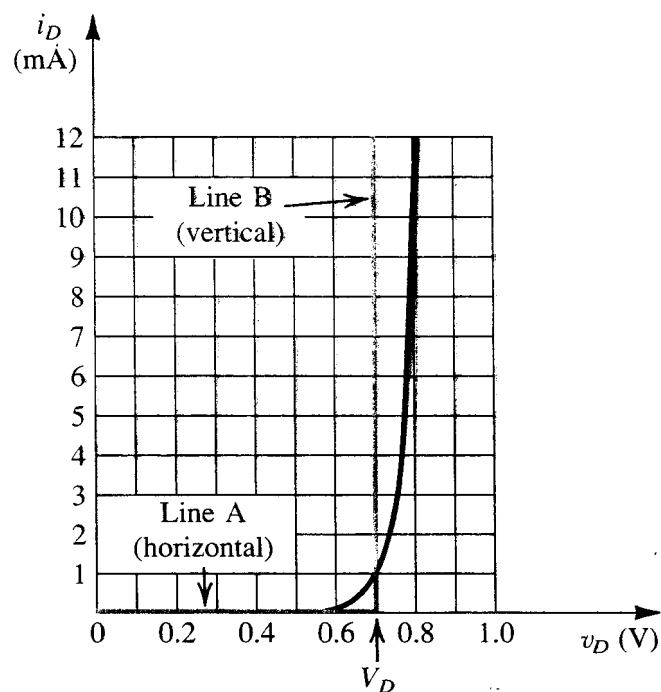
# Constant Voltage Drop Model



Development of the constant-voltage-drop model of the diode forward characteristics. A vertical straight line (B) is used to approximate the fast-rising exponential. Observe that this simple model predicts $V_D$ to within $\pm 0.1$ V over the current range of 0.1 mA to 10 mA.
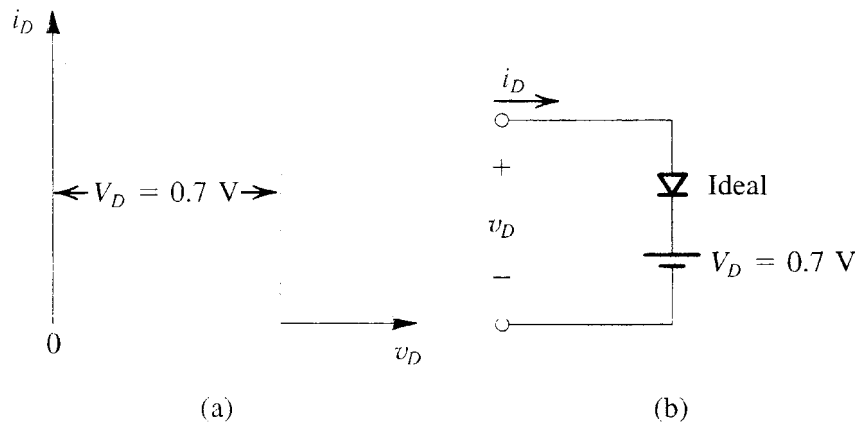


(a)

(b)
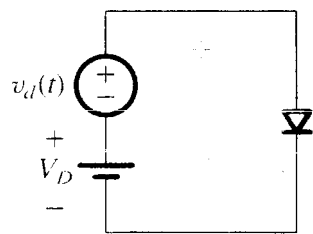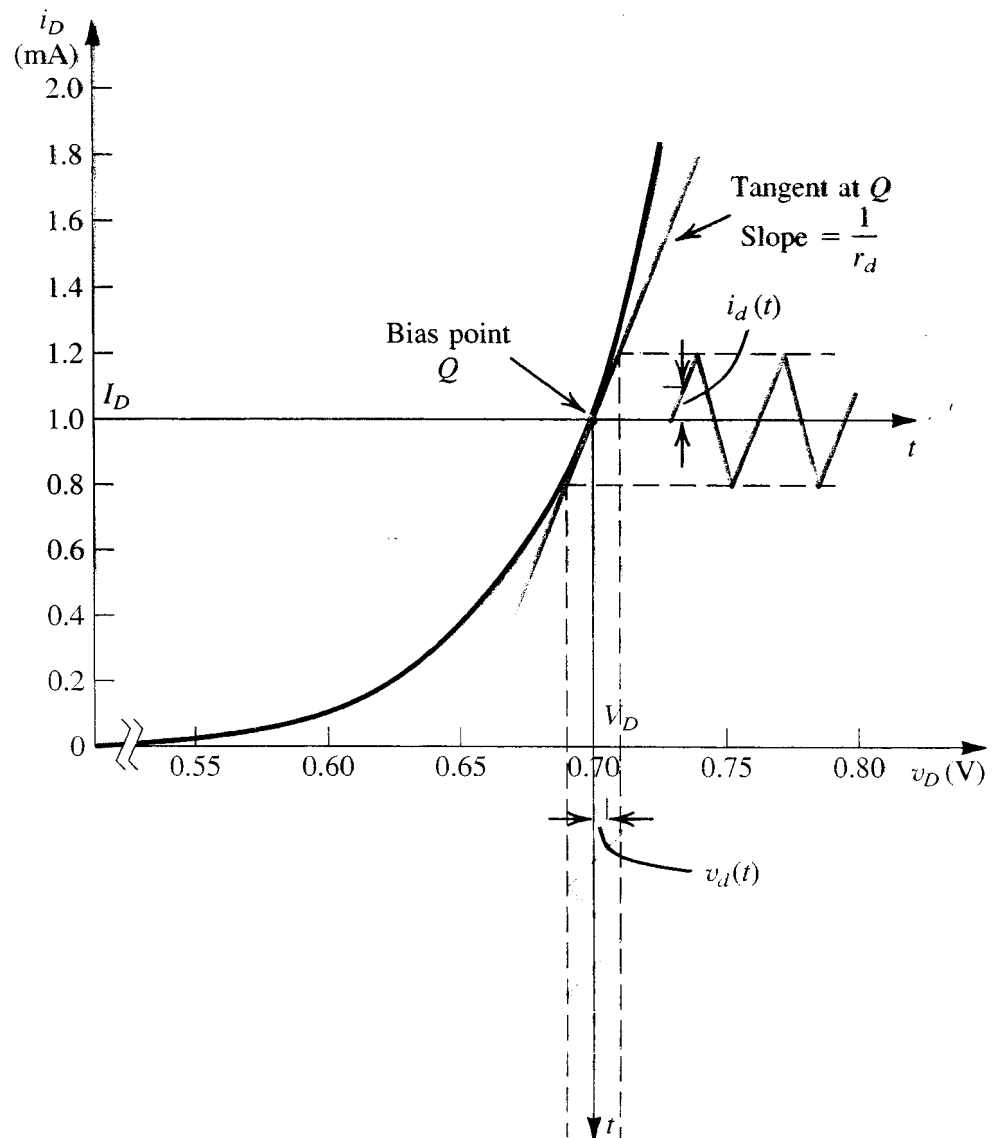
The constant-voltage-drop model of the diode forward characteristics and its equivalent-circuit representation.

# Small-signal Model



Tangent at $Q$
Slope $= \dfrac{1}{r_d}$

Bias point
$Q$

$i_d(t)$

$v_d(t)$

(a)

(b)

Development of the diode small-signal model. Note that the numerical values shown are for a diode with $n = 2$.

**3.10** For the circuit in Fig. 3.10, find $I_D$ and $V_D$ for the case $V_{DD} = 5$ V and $R = 10$ kΩ. Assume that the diode has a voltage of 0.7 V at 1-mA current and that the voltage changes by 0.1 V/decade of current change. Use (a) iteration, (b) the piecewise-linear model with $V_{D0} = 0.65$ V and $r_D = 20$ Ω, (c) the constant-voltage-drop model with $V_D = 0.7$ V.

Ans. (a) 0.434 mA, 0.663 V; (b) 0.434 mA, 0.659 V; (c) 0.43 mA, 0.7 V

**Sol)**

(b) Piecewise Linear:     $V_{D0} = 0.65V$     $r_D = 20\,\Omega$

Diode:     $i_D = \dfrac{V_D - 0.65}{20\Omega}$ ⎫

Circuit:     $i_D = \dfrac{5 - V_D}{10k\Omega}$ ⎬ $\xrightarrow{\text{Solve}}$ $i_D = 0.434\,mA$

$V_D = 0.659\,V$

(c) Constant Voltage Drop:     $V_D = 0.7V$

Circuit:     $i_D = \dfrac{5 - 0.7V}{10k\Omega} = 0.43\,mA$

**3.15** Consider a diode with $n = 2$ biased at 1 mA. Find the change in current as a result of changing the voltage by (a) −20 mV, (b) −10 mV, (c) −5 mV, (d) +5 mV, (e) +10 mV, and (f) +20 mV. In each case, do the calculations (i) using the small-signal model and (ii) using the exponential model.

Ans. (a) −0.40, −0.33 mA; (b) −0.20, −0.18 mA; (c) −0.10, −0.10 mA; (d) +0.10, +0.11 mA; (e) +0.20, +0.22 mA; (f) +0.40, +0.49 mA

**Sol)**     Diode: $n = 2$, $I_D = 1\,mA$ $\rightarrow$ $r_d = \dfrac{nV_T}{I_D} = \dfrac{2 \times 25mV}{1mA} = 50\,\Omega$

$V_{D2} - V_{D1} =$ given     $I_{D2} - I_{D1} = ?$

(a) $V_{D2} - V_{D1} = -20mV$

Small-signal: $i_d = \dfrac{-20mV}{50\Omega} = -0.4\,mA$

Exponential: $\dfrac{I_{D2}}{I_{D1}} = \dfrac{I_S e^{V_{D2}/nV_T}}{I_S e^{V_{D1}/nV_T}} = e^{(V_{D2} - V_{D1})/nV_T}$

$\rightarrow I_{D2} = I_{D1} e^{(V_{D2} - V_{D1})/nV_T} = 1mA\, e^{-20mV/2 \times 25mV} = 0.67\,mA$

∴ $I_{D2} - I_{D1} = 0.67 - 1 = -0.33\,mA$

(c) $V_{D2} - V_{D1} = -5mV$     Small-signal $i_d = \dfrac{-5mV}{50\Omega} = -0.1\,mA$

Exponential $I_{D2} = 1mA\, e^{-5mV/2 \times 25mV} = 0.9\,mA$ ∴ $\Delta I_D = -0.1\,mA$

# Summary of Diode Models

**TABLE 3.1   Modeling the Diode Forward Characteristic**

| Model | Graph | Equations | Circuit | Comments |
|---|---|---|---|---|
| Exponential |  | $i_D = I_S e^{v_D / n V_T}$ <br> $v_D = 2.3 n V_T \log\left(\dfrac{i_D}{I_S}\right)$ <br> $V_{D2} - V_{D1} = 2.3 n V_T \log\left(\dfrac{I_{D2}}{I_{D1}}\right)$ <br> $2.3 n V_T = 60$ mV for $n = 1$ <br> $2.3 n V_T = 120$ mV for $n = 2$ |  | $I_S = 10^{-12}$ A to $10^{-15}$ A, depending on junction area <br> $V_T \cong 25$ mV <br> $n = 1$ to $2$ <br> Physically based and remarkably accurate model <br> Useful when accurate analysis is needed |
| Piecewise-linear (battery-plus-resistance) |  | For $v_D \le V_{D0}$: <br> $\quad i_D = 0$ <br> For $v_D \ge V_{D0}$: <br> $\quad i_D = \dfrac{1}{r_D}(v_D - V_{D0})$ |  | Choice of $V_{D0}$ and $r_D$ is determined by the current range over which the model is required. <br> For the amount of work involved, not as useful as the constant-voltage-drop model. <br> Used only infrequently. |
| Constant-voltage-drop (or the "0.7-V model") |  | For $i_D > 0$: <br> $\quad v_D = 0.7$ V |  | Easy to use and very popular for the quick, hand analysis that is essential in circuit design. |
| Ideal-diode |  | For $i_D > 0$: <br> $\quad v_D \approx 0$ |  | Good for determining which diodes are conducting and which are cutoff in a multiple-diode circuit. <br> Good for obtaining very approximate values for diode currents, especially when the circuit voltages are much greater than $V_D$. |
| Small-signal |  | For small signals superimposed on $V_D$ and $I_D$: <br> $\quad i_d = v_d / r_d$ <br> $\quad r_d = n V_T / I_D$ <br> (For $n = 1$, $v_d$ is limited to 5 mV; for $n = 2$, 10 mV) |  | Useful for finding the signal component of the diode voltage (e.g., in the voltage-regulator application). <br> Serves as the basis for small-signal modeling of transistors (Chapters 4 and 5). |

# 3.4 Zener Diode

## - Reverse Breakdown Diode

i) Onset of Breakdown  (Fig. 21)

$V_{ZK}$ = Knee Voltage , $I_{ZK}$ = Knee current

ii) Manufacturer Datasheet gives

$V_Z$ at $I_{ZT}$

ex) $V_Z = 6.8$ V at $I_{ZT} = 10$ mA

ex) $V_Z$ = a few V ~ a few 100 V

ex) A 0.5-W, 6.8-V zener can conduct $I_Z < 70$ mA

iii) ac

$\Delta V = r_Z \Delta I$    around the Q-point

→  $V_Z = V_{Z0} + r_Z I_Z$   (dynamic model)(Fig. 22)

iv) Zener diode is used in Shunt Regulator

T-dependence:   $\dfrac{dV_Z}{dT} = (\quad) \dfrac{mV}{^\circ C}$

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**3.17** A zener diode whose nominal voltage is 10 V at 10 mA has an incremental resistance of 50 Ω. What voltage do you expect if the diode current is halved? Doubled? What is the value of $V_{Z0}$ in the zener model?

Zener : 10 V at 10 mA ,  $r_Z = 50 \Omega$

$V_Z = (\quad)$ at 20 mA ;  $(\quad)$ at 5 mA

Sol)  $V_Z = V_{Z0} + I_Z r_Z$

$10V = V_{Z0} + 50\Omega \times 10$ mA

∴ $V_{Z0} = 9.5$ V

For $I_Z = 20$ mA

$V_Z = 9.5 + 20$ mA $\times 50\Omega = 10.5$ V

For $I_Z = 5$ mA

$V_Z = 9.5 + 5$ mA $\times 50\Omega = 9.75$ V

**3.19** A shunt regulator utilizes a zener diode whose voltage is 5.1 V at a current of 50 mA and whose incremental resistance is 7 Ω. The diode is fed from a supply of 15-V nominal voltage through a 200-Ω resistor. What is the output voltage at no load? Find the line regulation and the load regulation.
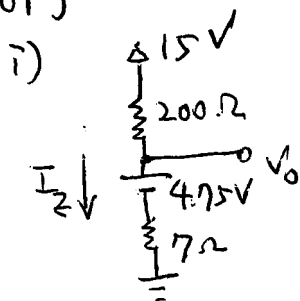
# Shunt Regulator

Zener: 5.1V at 50mA
$r_z = 7\,\Omega$
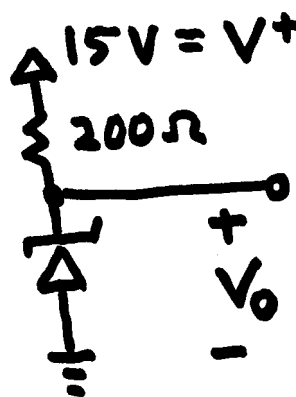
i) $V_0 = ($     $)$ with no load.

ii) Load Regulation $\dfrac{dV_0}{dI_L} = ($     $) \dfrac{mV}{mA}$

iii) Line Regulation $\dfrac{\Delta V_0}{\Delta V^+} = ($     $) \dfrac{mV}{V}$

## Sol)

i)

$V_z = 5.1V = V_{z0} + 50\,mA \times 7\,\Omega$   $\therefore V_{z0} = 4.75V$
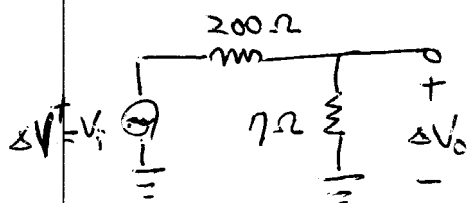
$I_z = \dfrac{15 - 4.75}{200 + 7} = 49.5\,mA$

$V_0 = 15 - 200\,\Omega \times 49.5\,mA = 5.1\,V$

ii) Load Regulation $= \dfrac{\Delta V_0}{\Delta I_L}$
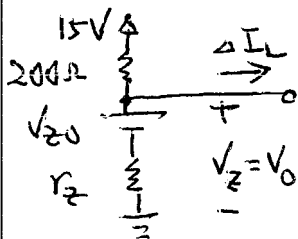
$\dfrac{\Delta V_0}{\Delta I_L} = -\dfrac{\Delta V_0}{\Delta I_z} = -\dfrac{1mA \times 7\Omega}{1mA} = -7\,\dfrac{mV}{mA}$

iii) Line Regulation $= \dfrac{\Delta V_0}{\Delta V^+}$ — change in output voltage
— fluctuation in Line voltage $= V_i$

$\dfrac{\Delta V_0}{\Delta V^+} = \dfrac{7}{200 + 7} = 0.0338 = 33.8\,\dfrac{mV}{V}$
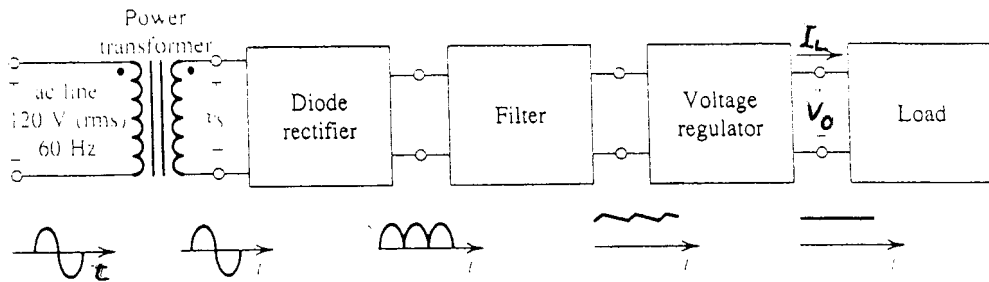
# 3.5 Rectifier Circuit



FIGURE 3.24  Block diagram of a dc power supply.

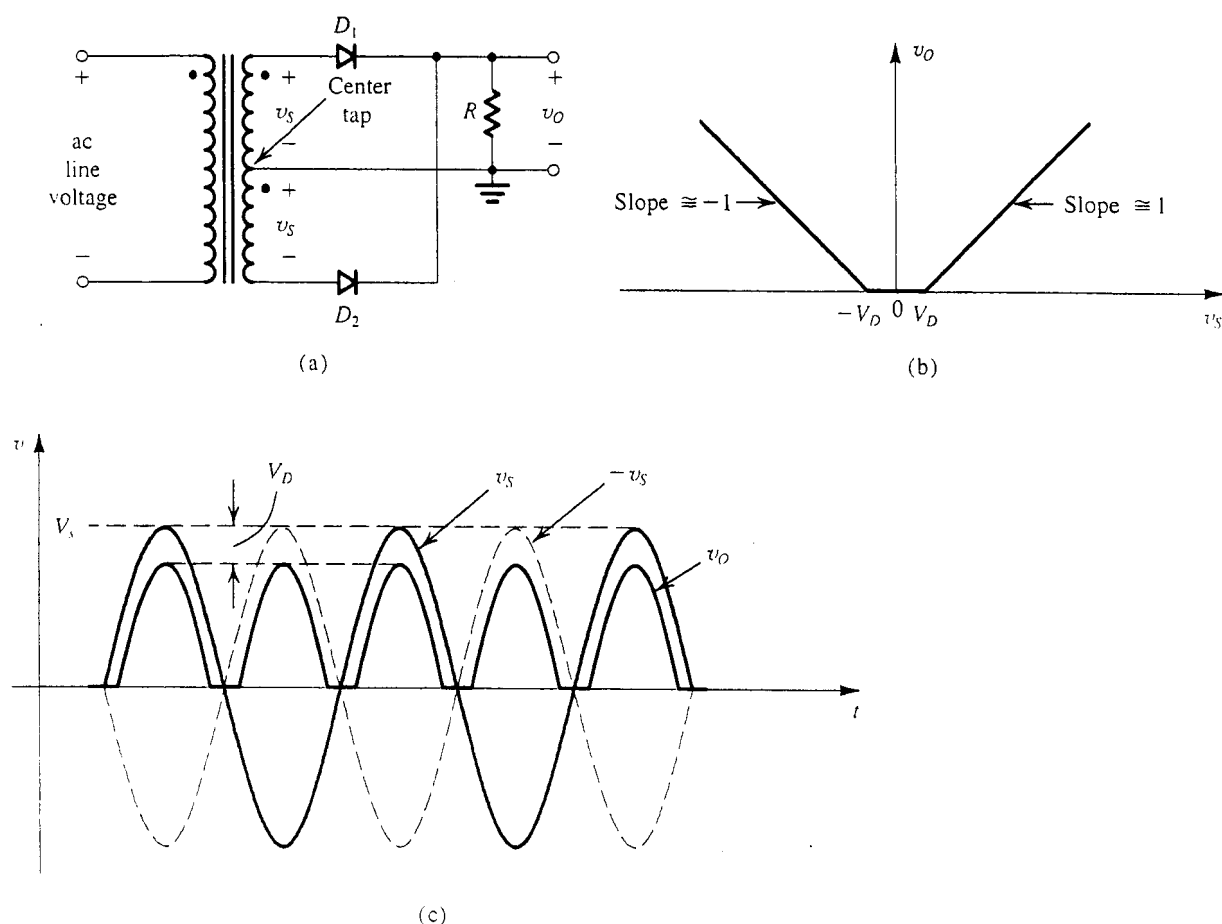# ( DC Power Supply )

# (1) Half-wave Rectifier  (Fig.25)

$$V_0 \cong V_S - V_{DO}$$

$$PIV = \text{peak Inverse Voltage}$$
$$= V_S$$

# (2) Full-Wave Rectifier



(a)

(b)

(c)

**FIGURE 3.26** Full-wave rectifier utilizing a transformer with a center-tapped secondary winding: (a) circuit; (b) transfer characteristic assuming a constant-voltage-drop model for the diodes; (c) input and output waveforms.

## (Fig. 26)

$$V_o = V_S - V_D$$

$$PIV = V_S - (-V_S + V_D) = 2V_S - V_D$$

$$\underset{\substack{(Anode) \\ D1}}{\qquad} \underset{\substack{(Cathode) \\ D1}}{\qquad}$$
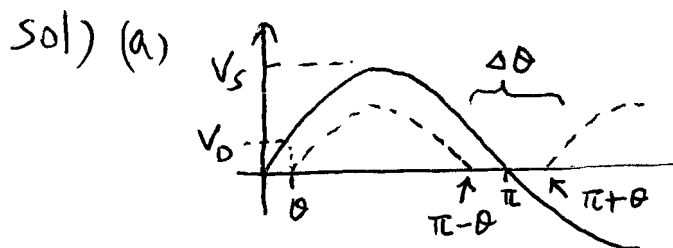
**3.21** For the full-wave rectifier circuit in Fig. 3.26(a), neglecting the effect of $r_D$, show the following: (a) The output is zero for an angle of $2 \sin^{-1} (V_D/V_s)$ centered around the zero-crossing points of the sine-wave input. (b) The average value (dc component) of $v_O$ is $V_O \approx (2/\pi)V_s - V_D$. (c) The peak current through each diode is $(V_s - V_D)/R$. Find the fraction (percentage) of each cycle during which $v_O > 0$, the value of $V_O$, the peak diode current, and the value of PIV, all for the case in which $v_S$ is a 12-V (rms) sinusoid, $V_D \approx 0.7$ V, and $R = 100\ \Omega$.

Ans. 97.4%; 10.1 V; 163 mA; 33.2 V

$$V_s = 12 - V_{(rms)}, \quad V_D = 0.7V, \quad \text{neglect } r_D, \quad R = 100\,\Omega$$

(a) $V_0 = 0$ for $\Delta\theta = 2\sin^{-1}(\frac{V_D}{V_s})$  (b) $V_{0,avg} = \frac{2}{\pi}V_s - V_D$

(c) Find $I_{D,max} = \frac{V_s - V_D}{R}$, angle for $V_0 > 0$, $V_{0,avg}$, PIV

Sol) (a)



$$V_s \sin\theta = V_D \quad \therefore \theta = \sin^{-1}(\frac{V_D}{V_s})$$

$$\Delta\theta = 2\theta = 2\sin^{-1}(\frac{V_D}{V_s})$$

(b) $$V_{0,avg} = \frac{1}{\pi}\int_{\theta}^{\pi-\theta}(V_s\sin\phi - V_D)d\phi = \frac{1}{\pi}[-V_s\cos\phi - V_D\phi]_{\theta}^{\pi-\theta}$$

$$= \frac{2}{\pi}V_s\cos\theta - \frac{(\pi-2\theta)}{\pi}V_D \approx \frac{2}{\pi}V_s - V_D$$

(c)
i) $I_{D,max} = \frac{V_s - V_D}{R} = \frac{12\sqrt{2} - 0.7}{100} = 163\text{ mA}$

ii) $V_0 > 0$ for angle $= 2(\pi - 2\theta) = 2(\pi - 2\sin^{-1}(\frac{V_D}{V_s}))$

angle fraction $= \frac{\text{angle}}{2\pi} = 1 - \frac{2}{\pi}\sin^{-1}(\frac{0.7}{12\sqrt{2}}) = 97.4\%$

iii) $V_{0,avg} = \frac{2}{\pi}V_s - V_D = \frac{2}{\pi}12\sqrt{2} - 0.7 = 10.1\text{ V}$

iv) $PIV = (V_s - V_D) + V_s = 2 \times 12\sqrt{2} - 0.7 = 33.2\text{ V}$

# (3) The Bridge Rectifier

$$\text{Fig.27} \quad V_0 = V_S - V_D(\text{of } D_1) - V_D(\text{of } D_2) = V_S - 2V_D$$
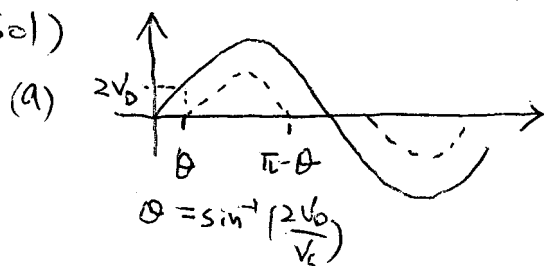
$$PIV = V_S - V_D$$

**3.22** For the bridge rectifier circuit of Fig. 3.27(a), use the constant-voltage-drop diode model to show that (a) the average (or dc component) of the output voltage is $V_O \simeq (2/\pi)V_s - 2V_D$ and (b) the peak diode current is $(V_s - 2V_D)/R$. Find numerical values for the quantities in (a) and (b) and the PIV for the case in which $v_S$ is a 12-V (rms) sinusoid, $V_D \simeq 0.7$ V, and $R = 100 \, \Omega$.

Ans. 9.4 V; 156 mA; 16.3 V

Show (a) $V_{0,avg} = \dfrac{2}{\pi}V_S - 2V_D$ , (b) $I_{D,max} = \dfrac{V_S - 2V_D}{R}$

Calc (c) $V_{0,avg}$, $I_{D,max}$, PIV for $V_S = 12V_{(rms)}$, $V_D = 0.7$, $R = 100$

Sol)

(a)



$\theta = \sin^{-1}\left(\dfrac{2V_D}{V_S}\right)$

$\therefore V_S \sin\theta = 2V_D$

$$V_{0,avg} = \frac{\displaystyle\int_{\theta}^{\pi-\theta}(V_S\sin\phi - 2V_D)\,d\phi}{\pi}$$

$$= \frac{2V_S\cos\theta - 2V_D(\pi-2\theta)}{\pi}$$

$$\simeq \frac{1}{\pi}(2V_S - 2V_D)$$

(b) $I_{D,max} = \dfrac{V_{0,peak}}{R} = \dfrac{V_S - 2V_D}{R}$

(c) $V_{0,avg} \simeq \dfrac{2}{\pi}(12\sqrt{2} - 0.7) = 9.4V$

$I_{D,max} = \dfrac{12\sqrt{2} - 2\times 0.7}{100\,\Omega} = 156mA$

$PIV = V_S - V_D = 12\sqrt{2} - 0.7 = 16.3V$

**D3.79** It is required to design a full-wave rectifier circuit using the circuit of Fig. 3.26 to provide an average output voltage of:

(a) 10 V
(b) 100 V

In each case find the required turns ratio of the transformer. Assume that a conducting diode has a voltage drop of 0.7 V. The ac line voltage is 120 V rms.

**D3.80** Repeat Problem 3.79 for the bridge rectifier circuit of Fig. 3.27.

Diode $V_D = 0.7 V$

ac Line  120 V(rms)

Build Bridge Rectifier
for (a) $V_{0,avg} = 10V$

(b) $V_{0,avg} = 100V$

⟹ **Transformer Turn Ratio?**

Sol) $V_{0,avg} = \frac{2}{\pi} V_s - 2 V_D$

(a)

$10V = \frac{2}{\pi} V_s - 2 \times 0.7 \qquad \therefore V_s = 17.914 \, V$

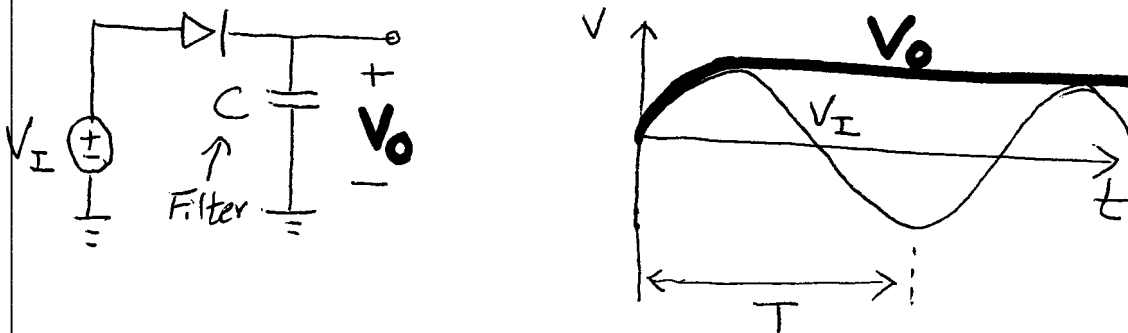⟹ Turn Ratio $= \dfrac{120 \sqrt{2}}{17.91} = 9.477$ to 1

(b)

$100V = \frac{2}{\pi} V_s - 2 \times 0.7 \qquad \therefore V_s = 159.3 \, V$

⟹ Turn Ratio $= \dfrac{120 \sqrt{2}}{159.3} = 1.065$ to 1

# (4) Filtered Rectifier



Real Circuit — Fig.29

(Leakage from $C$ via $R$)

**Fig.29:** Once $V_0 = V_p$ is reached $\Rightarrow$ $D = $ open as $V_I < V_p$

$\Rightarrow$



initial condition $V_0(0) = V_p$

$V_0(t) = V_p e^{-t/CR}$

$\Rightarrow$ For $T \ll CR$, $\quad V_0(t) \simeq V_p (1 - \frac{t}{CR})$

# Properties of Filtered Rectifier (Fig.29)

i) Ripple Voltage $\equiv V_r$

ii) DC Output Voltage $\equiv V_{0,avg} = \frac{V_p + (V_p - V_r)}{2} = V_p - \frac{1}{2}V_r$

iii) Conduction Angle $= \Delta\theta \equiv \omega \Delta t$

iv) Average Diode Current $= i_{D,avg}$ during $\Delta t$

v) Peak $\quad\quad\quad '' \quad\quad\quad\quad = i_{D,max}$

## Fig. 29

i) Ripple Voltage $= V_r$

$$V_0(T) \equiv V_p - V_r \qquad V_0(T) = V_p\left(1 - \frac{T}{CR}\right)$$

$$\therefore \boxed{V_r = V_p \frac{T}{CR}} \qquad I_L \equiv \frac{V_p}{R} = \text{leakage current}$$

$$f = \frac{1}{T}$$

iii) Conduction Angle $= \Delta\theta = w\Delta t$

$$V_p \cos\Delta\theta = V_p - V_r \qquad \cos\Delta\theta \simeq 1 - \frac{1}{2}\Delta\theta^2$$

$$\therefore \Delta\theta = w\Delta t = \sqrt{\frac{2V_r}{V_p}}$$

iv) $i_{D,avg}$ during conduction : $\qquad i_{D,avg} = i_{C,avg} + i_L$

$$i_{C,avg} = \frac{Q}{\Delta t} = \frac{CV_r}{\Delta t} = \pi I_L \sqrt{\frac{2V_p}{V_r}}$$

$$I_L = \frac{V_p}{R}$$

$$\therefore i_{D,avg} = i_{C,avg} + I_L = I_L\left(1 + \pi\sqrt{\frac{2V_p}{V_r}}\right)$$

v) Peak diode current, $i_{D,max}$

$$i_D = i_C + i_L = C\frac{dV_i}{dt} + i_L$$

$$i_{D,max} = C\frac{dV_i}{dt}\bigg|_{t=-\Delta t} + i_L \qquad \text{and} \quad V_i(t) = V_p \cos(wt)$$

$$= C w V_p \sin(w\Delta t) + i_L \simeq C w V_p (w\Delta t) + I_L$$

$$i_{D,max} = I_L\left(1 + 2\pi\sqrt{\frac{2V_p}{V_r}}\right)$$

**D3.24** Consider a bridge-rectifier circuit with a filter capacitor $C$ placed across the load resistor $R$ for the case in which the transformer secondary delivers a sinusoid of 12 V (rms) having a 60-Hz frequency and assuming $V_D = 0.8$ V and a load resistance $R = 100\ \Omega$. Find the value of $C$ that results in a ripple voltage no larger than 1 V peak-to-peak. What is the dc voltage at the output? Find the load current. Find the diodes' conduction angle. What is the average diode current? What is the peak reverse voltage across each diode? Specify the diode in terms of its peak current and its PIV.



$12\,V_{rms}$
$60 Hz$
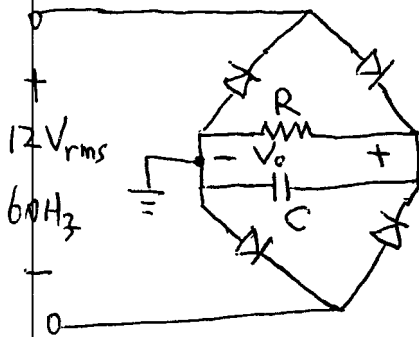
$R = 100\,\Omega \qquad V_D = 0.8\ V$

i) $C = (\quad)$ for $V_r \le 1\ V_{p-p}$

ii) $V_{0,avg} = (\quad)$     iii) $I_L = (\quad)$

iv) $\Delta\theta = W\Delta t = (\quad)$

v) $i_{D,avg} = (\quad)$     vi) $PIV = (\quad)$

vii) $i_{D,max} = (\quad)$

**Sol)** $V_o(t) = V_p\, e^{-t/RC}$,   $V_p = V_s - 2V_D$,   $V_s = 12\sqrt{2}$

i) $V_o(\frac{T}{2}) = V_p - V_r$,   $V_o(\frac{T}{2}) \simeq V_p(1 - \frac{T}{2RC})$   $\therefore V_r = V_p \frac{T}{2RC} \le 1V$

$$C \ge \frac{V_p}{f \cdot 2R} = \frac{12\sqrt{2} - 2\times 0.8}{60\,Hz \times 2 \times 100\,\Omega} = 1.281\times 10^{-3} F = 1281\,\mu F$$

ii) DC $V_{0,avg} = V_p - \frac{1}{2}V_r = (12\sqrt{2} - 1.6) - \frac{1}{2}\cdot 1V = 14.87\ V$

iii) $I_L = \frac{V_p}{R} = \frac{12\sqrt{2} - 2\times 0.8}{100\,\Omega} = 0.15\,A$

iv) $\Delta\theta = \sqrt{\frac{2V_r}{V_p}} = \sqrt{\frac{2\times 1V}{12\sqrt{2} - 1.6}} = 0.36\ rad$   or   $20.7°$

v) $i_{D,avg} = I_L(1 + \pi\sqrt{\frac{2V_p}{V_r}}) = 0.15(1 + \pi\sqrt{\frac{2(12\sqrt{2}-1.6)}{1}}) = 2.0\,A$

vi) $i_{D,max} = I_L(1 + 2\pi\sqrt{\frac{2V_p}{V_r}}) = 0.15(1 + 2\pi\sqrt{\frac{2(12\sqrt{2}-1.6)}{1}}) = 2.76\,A$

vii) $PIV = V_s - V_D = 12\sqrt{2} - 0.8 = 16.2$

Diode Spec: $\begin{cases} \text{Current Rating} = 3.5 - 4.0\,A \\ PIV = 20\ V \end{cases}$

# 3.7 Physical Operation of Diodes
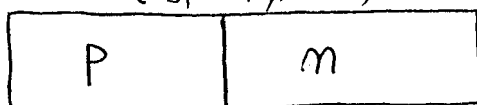
3.22

(1) Basic Semiconductor Concepts

i) PN Junction

(Si crystal)

| P | n |
|---|---|

Large amount of ( Positive | negative ) charge carriers
holes          electrons
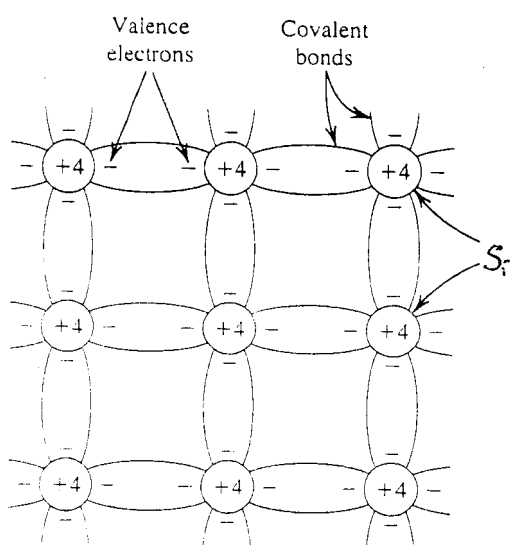
ii) Intrinsic Si = chemically pure Si crystal
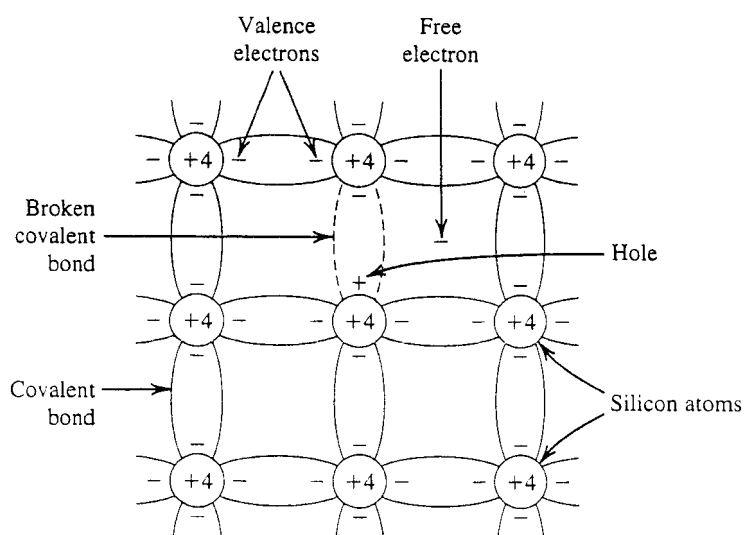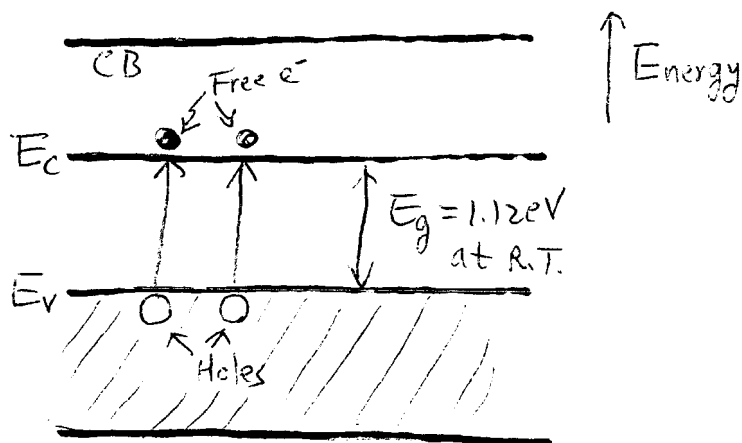
Real Space



FIGURE 3.40    T = 0 K


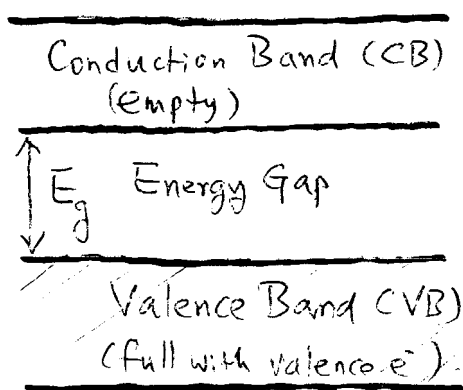
FIGURE 3.41    T > 0 K, Broken Bonds

Energy Diagram



Conduction Band (CB)
(Empty)

$E_g$ Energy Gap

Valence Band (VB)
(full with valence $e^-$)

CB

Free $e^-$

$E_c$

$E_g = 1.12 eV$ at R.T.

$E_v$

Holes

↑ Energy

# iii) Diffusion and Drift

## Diffusion

Free particles (electrons, holes) show a <u>net flow</u> if its concentration distribution is <u>not</u> uniform.

$$\text{ex) if } \frac{dn}{dx} \neq 0, \text{ then electrons diffuse.}$$

Diffusion current density $[A/cm^2]$

$$J_{n-diff} = q D_n \frac{dn}{dx}$$

$q = 1.6 E-19$ Coul

$D_n =$ diffusion const.

$$J_{p-diff} = -q D_p \frac{dp}{dx}$$

## Drift

If E-field is present, then charged particles (e,h) flow.

$$J_{n-drift} = q n \mu_n E$$

$\mu_n =$ electron mobility $\left[\frac{cm^2}{V \cdot s}\right]$

$V_{n,drift} = \mu_n E =$ drift speed of electron

$$J_{p-drift} = q p \mu_p E$$

Total Drift Current:
$$J_{drift} = J_{n-drift} + J_{p-drift}$$
$$= q(n\mu_n + p\mu_p)E \equiv \frac{E}{\rho}$$

$$\rho \equiv \frac{1}{q(n\mu_n + p\mu_p)} = \text{Resistivity } [\Omega-cm]$$

$$\frac{D_n}{\mu_n} = V_T = \frac{D_p}{\mu_p} \qquad (\text{Einstein Relation})$$
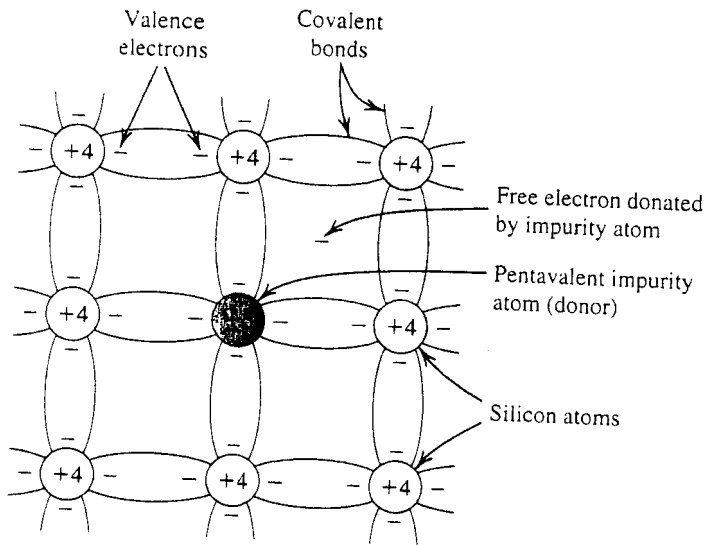
| n-type Si | p-type Si |



FIGURE 3.43    **m-type Si**



FIGURE 3.44    **p-type Si**

Labels for left diagram: Valence electrons, Covalent bonds, Free electron donated by impurity atom, Pentavalent impurity atom (donor), Silicon atoms

Labels for right diagram: Valence electrons, Covalent bonds, Silicon atom, Trivalent impurity atom (acceptor), Hole

Left energy band diagram:

CB

$n = N_D$

free electrons

$E_c$ ——————

$E_D = $ Donor Level

(Donors)

Conc. $= N_D$

$E_v$ ————————

$E_g$

VB

Right energy band diagram:

CB

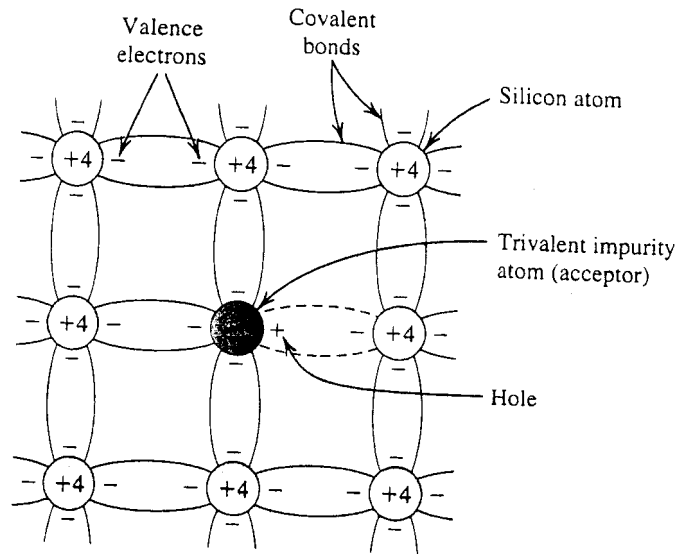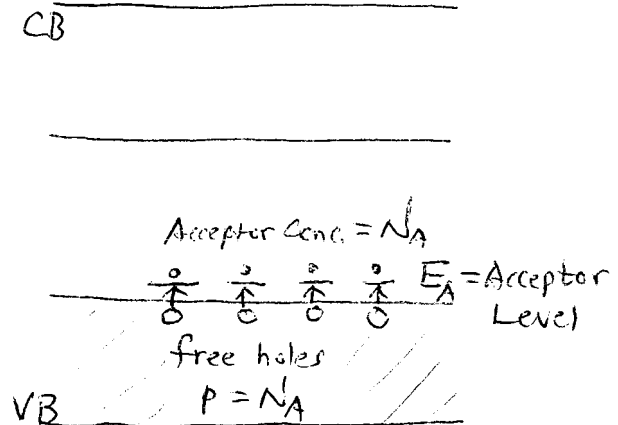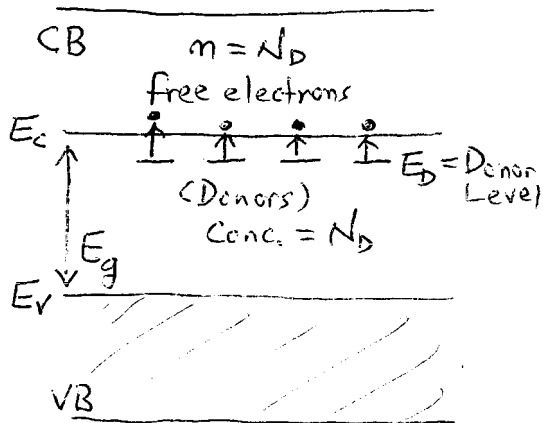Acceptor Conc. $= N_A$

$E_A = $ Acceptor Level

free holes

$p = N_A'$

VB

$$np = n_i^2$$

ex) $n_{no} = N_D$,  $p_{no} = \dfrac{n_i^2}{N_D}$

n-type / equil.

ex) $p_{po} = N_A$  $n_{po} = \dfrac{n_i^2}{N_A}$

3.29 Calculate the intrinsic carrier density $n_i$ at 250 K, 300 K, and 350 K.

Sol) $n_i = \sqrt{BT^3 e^{-E_g/kT}}$

$B = 5.4 \times 10^{31}$

$E_g = 1.12 eV$ for Si

$kT = 20.8, 25, 29.2 meV$
(250k) (300) (350)

T=250: $n_i = \sqrt{5.4E31 \times 250^3 \times e^{-1.12/0.0208}} = 1.5 \times 10^8 \, cm^{-3}$

300: $n_i = \sqrt{'' \times 300^3 \times e^{-1.12/0.025}} = 1.5 \times 10^{10}$

350: $n_i = \sqrt{'' \times 350^3 \times e^{-1.12/0.0292}} = 4.18 \times 10^{11} \, cm^{-3}$

3.30 Consider an n-type silicon in which the dopant concentration $N_D$ is $10^{17}/cm^3$. Find the electron and hole concentrations at 250 K, 300 K, and 350 K. You may use the results of Exercise 3.29.

Sol) $N_D = 10^{17} \, cm^{-3}$

T=250K: $n = N_D = 10^{17} cm^{-3}$, $p = \dfrac{n_i^2}{N_D} = \dfrac{(1.5E8)^2}{1E17} = 0.225 \, cm^{-3}$

T=300K: $n = N_D = 10^{17} cm^{-3}$, $p = \dfrac{(1.5E10)^2}{1E17} = 2,250 \, cm^{-3}$

T=350K: $n = 10^{17} cm^{-3}$, $p = \dfrac{(4.18E11)^2}{1E17} = 1.75 \times 10^6 \, cm^{-3}$

3.31 Find the resistivity of (a) intrinsic silicon and (b) p-type silicon with $N_A = 10^{16}/cm^3$. Use $n_i = 1.5 \times 10^{10}/cm^3$, and assume that for intrinsic silicon $\mu_n = 1350 \, cm^2/V \cdot s$ and $\mu_p = 480 \, cm^2/V \cdot s$ and for the doped silicon $\mu_n = 1110 \, cm^2/V \cdot s$ and $\mu_p = 400 \, cm^2/V \cdot s$. (Note that doping results in reduced carrier mobilities.)

Sol)

(a) Intrinsic: $\dfrac{1}{\rho} = q(n_i \mu_n + n_i \mu_p) = 1.6E-19 \times 1.5E10 \times (1350 + 480)$

$= 4.392 \times 10^{-6} \, \dfrac{1}{\Omega-cm}$

$\therefore \rho = 2.28 \times 10^5 \, \Omega-cm$

(b) p-type: $N_A = 10^{16} \, cm^{-3} = p$    $n = \dfrac{n_i^2}{p} = \dfrac{(1.5E10)^2}{1E16} = 2.25E4 \, cm^{-3}$

$\dfrac{1}{\rho} = q(n\mu_n + p\mu_p) = 1.6E-19 \times (2.25E4 \times 1100 + 1E16 \times 400)$

$= 0.64 \, \dfrac{1}{\Omega-cm}$

$\therefore \rho = \dfrac{1}{0.64} \, \Omega-cm = 1.56 \, \Omega-cm$

# 3.7.2 PN Junction under Open-Circuit Condition

(1) Diffusion Current $I_D$ : $\frac{dp}{dx} \neq 0$, $\frac{dn}{dx} \neq 0$ across the junction

(2) Depletion Region or Space Charge Region

Around the junction, ionized impurity charges become uncovered, or exposed, because the mobile charges ($e^-$ or $h^+$) leave that region.

The space charge (of impurities) in the depletion region Leads to, Electric Field, which leads to, Potential Barrier.

(3) Drift Current $I_S$: The E-field in the depletion region causes certain carriers to cross the junction.

(      ) from $p$-region to $n$-region

(      ) $n \longrightarrow p$

Equilibrium: $I_D = I_S$

(4) Junction Built-in Potential : $V_o = V_T \ln \frac{N_A N_D}{n_i^2} = 0.6 - 0.8V$
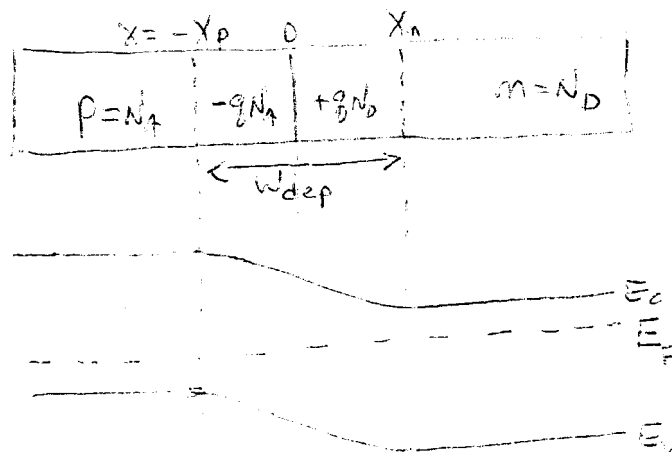
(5) Width of Depletion Region

$$q N_A X_p = q N_D X_n \quad \cdots (1)$$

$$W_{dep} = X_n + X_p = \sqrt{\frac{2\varepsilon_{Si}}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_0} \quad \cdots (2)$$

$$= 0.1 - 1.0 \, \mu m$$

From (1) and (2),

$$X_n = \frac{N_A}{N_A+N_D} W_{dep}, \qquad X_p = \frac{N_D}{N_A+N_D} W_{dep}$$

3.32 For a *pn* junction with $N_A = 10^{17}/cm^3$ and $N_D = 10^{16}/cm^3$, find, at $T = 300$ K, the built-in voltage, the width of the depletion region, and the distance it extends in the $p$ side and in the $n$ side of the junction. Use $n_i = 1.5 \times 10^{10}/cm^3$.

Ans. 728 mV; 0.32 μm; 0.03 μm and 0.29 μm

| $P = N_A = 10^{17}$ | $n = N_D = 10^{16}$ |
|---|---|

$T = 300K \quad n_i = 1.5 \times 10^{10} \, cm^{-3}$

$\varepsilon_{Si} = 11.07 \varepsilon_0 = 1.04 \times 10^{-12} \, F/cm$

$V_0 = (\quad), \quad W_{dep} = (\quad), \quad X_n = (\quad), \quad X_p = (\quad)$

Sol)

$$V_0 = V_T \ln \frac{N_A N_D}{n_i^2} = 0.025 \ln \frac{10^{17} \times 10^{16}}{2.25 \times 10^{20}} = 0.726 V$$

$$W_{dep} = \sqrt{\frac{2\varepsilon_{Si}}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_0} = \sqrt{\frac{2 \times 1.04 E{-}12}{1.6 E{-}19}(10^{17}+10^{16})0.728}$$
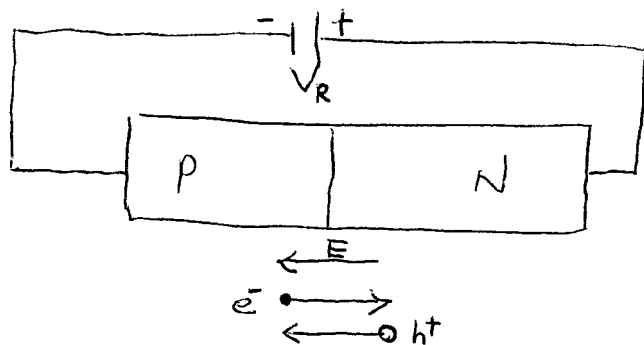
$$= 3.2 \times 10^{-5} cm = 0.32 \mu m$$

$$X_n = \frac{N_A}{N_A+N_D} W_{dep} = \frac{10^{17}}{10^{17}+10^{16}} \cdot 0.32 = 0.29 \mu m$$

$$X_p = \frac{N_D}{N_A+N_D} W_{dep} = W_{dep} - X_n = 0.32 - 0.29 = 0.03 \mu m$$

# 3.7.3 PN Junction under Reverse Bias



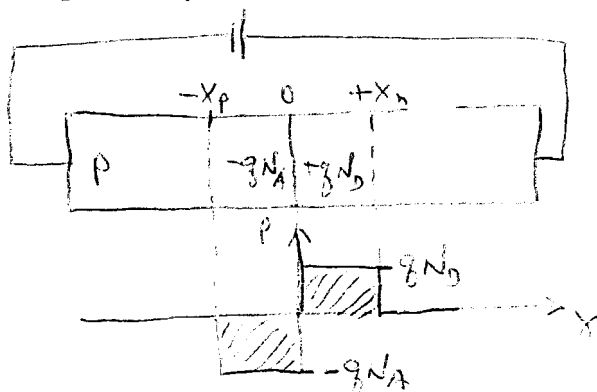Dribt current across junction $= I_S =$ indept. of $V$
$\qquad = const.$

$V_R$ increases the potential barrier from $V_o$ to $V_o + V_R$

$\therefore \quad I_S = const.$

$\quad I_D \downarrow$

$\quad I_{Rev} = I_S - I_D \approx I_S$

## Depletion Capacitance



In general,

$$C_J = \frac{C_{J0}}{\left(1 + \frac{V_R}{V_o}\right)^m}$$

$m =$ Junction grading coef.

$m = \frac{1}{2}$ for abrupt junction

$m = \frac{1}{3}$ for linear "

Space charge $= Q_J = q\, N_D X_n A = q\, N_D \frac{N_A}{N_A + N_D} W_{dep} A$

where $W_{dep} = \sqrt{\frac{2\varepsilon_{si}}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_o + V_R)}$

Junction Cap $= C_J = \frac{dQ_J}{dV_R} = q \frac{N_A N_D}{N_A \cdot V_o} \frac{dW_{dep} A}{dV_R}$

$$\boxed{C_J = A\frac{\varepsilon_{si}}{W_{dep}} = \frac{C_{J0}}{\left(1 + \frac{V_R}{V_o}\right)^{1/2}}}$$

where $= A\sqrt{\frac{2\varepsilon_{si}}{q}\left(\frac{N_A N_D}{N_A + N_D}\right)\frac{1}{V_o}}$

**3.33** For a *pn* junction with $N_A = 10^{17}/cm^3$ and $N_D = 10^{16}/cm^3$, operating at $T = 300$ K, find (a) the value of $C_{j0}$ per unit junction area ($\mu m^2$ is a convenient unit here) and (b) the capacitance $C_j$ at a reverse-bias voltage of 2 V, assuming a junction area of 2500 $\mu m^2$. Use $n_i = 1.5 \times 10^{10}/cm^3$, $m = \frac{1}{2}$, and the value of $V_0$ found in Exercise 3.32 ($V_0 = 0.728$ V).

**Ans.** (a) 0.32 fF/$\mu m^2$; (b) 0.41 pF

abrupt grading, $m = \frac{1}{2}$

| $p = N_A$ $= 10^{17}$ | $n = N_D$ $= 10^{16}$ |
|---|---|

$\leftarrow A = 2500\,\mu m^2, \quad V_0 = 0.728\,V$

(a) $\dfrac{C_{j0}}{A} = ( \quad )$ , (b) $C_j = ( \quad ) @ V_R = 2V$

**Sol)**

(a) $\dfrac{C_{j0}}{A} = \dfrac{\epsilon_{si}}{W_{dep}(V_R=0)}$

$W_{dep}(V_R=0) = \sqrt{\dfrac{2\epsilon_{si}}{q}\left(\dfrac{1}{N_A}+\dfrac{1}{N_D}\right)V_0}$

$= \sqrt{\dfrac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}}(10^{-17}+10^{-16})\,0.728}$

$\dfrac{C_{j0}}{A} = \dfrac{\epsilon_{si}}{W_{dep}} = \dfrac{1.04 \times 10^{-12}\,F/cm}{0.32 \times 10^{-4}\,cm}$

$= 0.32\,\mu m$

$= 3.25 \times 10^{-8}\,F/cm^2 = 3.25 \times 10^{-16}\,F/\mu m^2$

(b) $C_j = A\dfrac{C_{j0}/A}{\sqrt{1+\dfrac{V_R}{V_0}}} = 2500\,\mu m^2 \cdot \dfrac{3.25 \times 10^{-16}\,F/\mu m^2}{\sqrt{1+\dfrac{2}{0.728}}} = 4.1 \times 10^{-13}\,F$
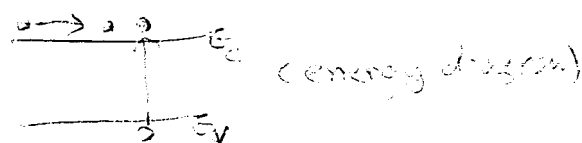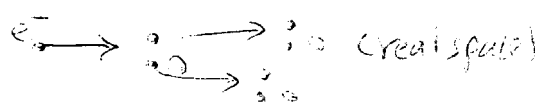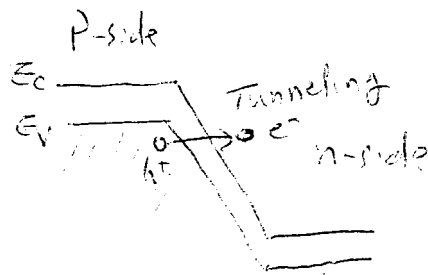
# 3.7.4 Breakdown Region

Under large reverse bias,
pN junction breaks down via

(i) Zener effect
usually $V_z < 5V$

or
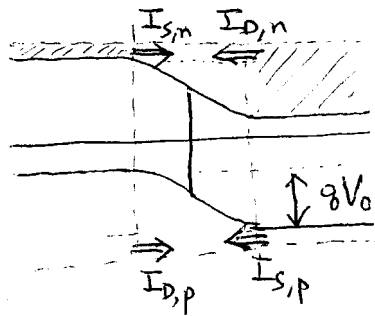
(ii) Avalanche effect
usually $V_z > 7V$
High carrier Kinetic Energy
$(\frac{1}{2}mv^2 > E_g)$

# 3.7.5 Forward Biased PN Junction

(1) PN Junction under Zero Bias : Energy Band Diagram

$$I_S = I_D$$

$$\therefore I = I_D - I_S = 0$$

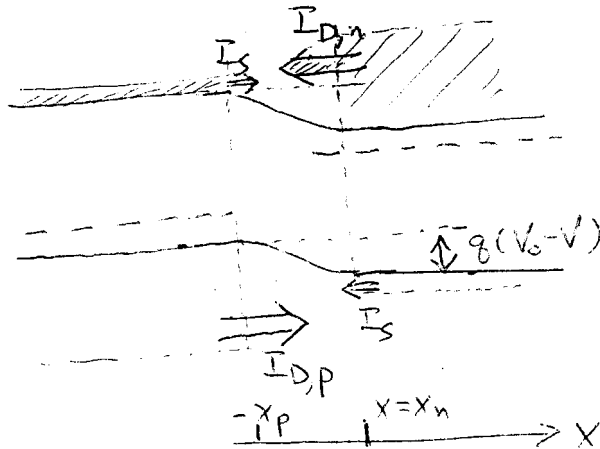(2) Forward bias $V$ lowers the Pontential Barrier from $V_0$ to $V_0 - V$

Fig. 50

(3) I-V Characterstics

$$P_n(x_n) = P_{no} e^{V/V_T}$$

$$P_n(x) - P_{no} = [P_n(x_n) - P_{no}] e^{-(x-x_n)/L_p}$$

$$L_p = \sqrt{\tau_p D_p} = \text{Diffusion Length}$$

$$J_p = -q D_p \frac{dP_n(x)}{dx}\bigg|_{x=x_n} = q \frac{D_p}{L_p} P_{no} (e^{V/V_T} - 1)$$

$$= \text{Hole currents injected from p-side to n-side}$$

By the same way,

$$J_n = \text{Electron current injected from n-side to p-side}$$

$$= q D_n \frac{dn_p(x)}{dx}\Big|_{x=-x_p} = q \frac{D_n}{L_n} n_{po}(e^{V/V_T}-1)$$

Therefore,

$$I = A(J_n + J_p) = Aq\left(\frac{D_n}{L_n}n_{po} + \frac{D_p}{L_p}p_{no}\right)(e^{V/V_T}-1)$$

$$\equiv I_S(e^{V/V_T}-1)$$

where, $\quad I_s = Aqn_i^2\left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D}\right)$

## (4) Diffusion Capacitance

Minority carrier (hole) charge injected and stored in N-side:

$$Q_p = Aq\int_0^\infty [P_n(x) - P_{no}]dx = Aq[P_n(x_n) - P_{no}]L_p$$

$$= Aq\, P_{no}(e^{V/V_T}-1)L_p = \frac{L_p^2}{D_p}I_p = \tau_p I_p$$

The same way, $\quad Q_n = \tau_n I_n = $ minority electron charge
stored in p-side

$$Q = \tau_p I_p + \tau_n I_n \equiv \tau_T(I_p + I_n) = \tau_T I$$

$$C_d = \frac{dQ}{dV} = \frac{Q}{V_T} = \frac{\tau_T I}{V_T} \qquad \text{because} \quad \frac{dI_p}{dV} = \frac{I_p}{V_T} \quad \text{etc}$$

**3.120.** In a forward-biased *pn* junction show that the ratio of the current component due to hole injection across the junction to the component due to electron injection is given by

$$\frac{I_p}{I_n} = \frac{D_p}{D_n}\frac{L_n}{L_p}\frac{N_A}{N_D}$$

Evaluate this ratio for the case $N_A = 10^{18}/cm^3$, $N_D = 10^{16}/cm^3$, $L_p = 5\ \mu m$, $L_n = 10\ \mu m$, $D_p = 10\ cm^2/s$, $D_n = 20\ cm^2/s$, and hence find $I_p$ and $I_n$ for the case in which the diode is conducting a forward current $I = 1$ mA.

$I = 1\ mA = $ Forward Current

| $P = N_A = 10^{18}$ | $n = N_D = 10^{16}$ |
|---|---|

$L_n = 10$ , $\qquad L_p = 5\ um$

$D_n = 20$ , $\qquad D_p = 10\ cm^2/s$

$I_n = (\qquad) \qquad I_p = (\qquad)$

Sol)

$$\frac{I_p}{I_n} = \frac{A J_p}{A J_n} = \frac{q\frac{D_p}{L_p}\frac{n_i^2}{N_D}(e^{V/V_T}-1)}{q\frac{D_n}{L_n}\frac{n_i^2}{N_A}(e^{V/V_T}-1)} = \frac{\frac{D_p}{L_p N_D}}{\frac{D_n}{L_n N_A}} = \frac{D_p}{D_n}\frac{L_n}{L_p}\frac{N_A}{N_D}$$

$$\frac{I_p}{I_n} = \frac{D_p}{D_n}\frac{L_n}{L_p}\frac{N_A}{N_D} = \frac{10}{20}\cdot\frac{10}{5}\cdot\frac{10^{18}}{10^{16}} = 100$$

Roughly speaking, $\quad I_p : I_n \approx N_A : N_D$

$\therefore I_p = 100\ I_n$

$I = I_p + I_n = 101\ I_n = 1\ mA$

$\therefore I_n = 0.01\ mA$

$\qquad I_p = 0.99\ mA$

---

**3.108)** Holes are being steadily injected into a region of *n*-type silicon (connected to other devices, the details of which are not important for this question). In the steady state, the excess-hole concentration profile shown in Fig. P3.108 is established in the *n*-type silicon region. Here "excess" means over and above the concentration $p_{n0}$. If $N_D = 10^{16}/cm^3$, $n_i = 1.5 \times 10^{10}/cm^3$, and $W = 5\ \mu m$, find the density of the current that will flow in the *x* direction.
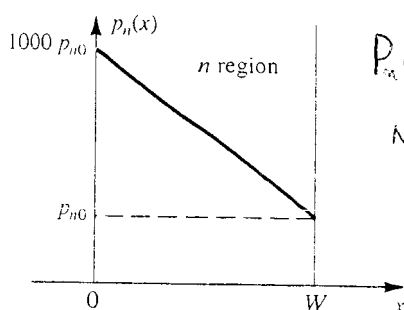
**FIGURE P3.108**

$P_n(x) - P_{no} = [P_n(o) - P_{no}](1 - \frac{x}{w})$
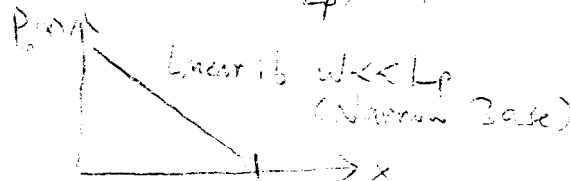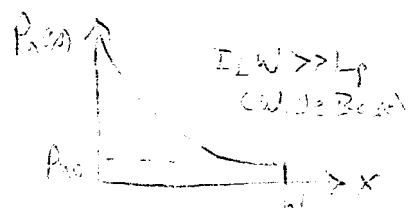
$N_D = 10^{16}\ cm^3$, $W = 5\ um$

$D_p = 12\ cm^2/s$

$J_p = (\qquad)$

Sol) $J_p = -q D_n \left.\frac{dP_n(x)}{dx}\right|_{x=0}$

$$= \frac{q D_n}{W}[P_n(o) - P_{no}] = \frac{q D_n}{W} \cdot 999 \cdot \frac{n_i^2}{N_D} = \frac{1.6\times10^{-19}\times12}{5\times10^{-4}}\times999\times\frac{2.25\times10^{20}}{10^{16}}$$

$$= 8.63\times10^{-9}\ A/cm^2$$

(Note) $P_n(x) - P_{no} = [P_n(o) - P_{no}]e^{-x/L_p}$

$$\approx [P_n(o) - P_{no}](1 - \frac{x}{L_p})\quad if\quad x \ll L_p$$

3.34 A diode has $N_A = 10^{17}/cm^3$, $N_D = 10^{16}/cm^3$, $n_i = 1.5 \times 10^{10}/cm^3$, $L_p = 5~\mu m$, $L_n = 10~\mu m$, $A = 2500~\mu m^2$, $D_p$ (in the $n$ region) = 10 cm$^2$/V·s, and $D_n$ (in the $p$ region) = 18 cm$^2$/V·s. The diode is forward biased and conducting a current $I = 0.1$ mA. Calculate: (a) $I_S$; (b) the forward-bias voltage $V$; (c) the component of the current $I$ due to hole injection and that due to electron injection across the junction; (d) $\tau_p$ and $\tau_n$; (e) the excess hole charge in the $n$ region $Q_p$, and the excess electron charge in the $p$ region $Q_n$, and hence the total minority stored charge $Q$, as well as the transit time $\tau_T$; and (f) the diffusion capacitance.

Ans. (a) $2 \times 10^{-15}$ A; (b) 0.616 V; (c) 91.7 $\mu A$, 8.3 $\mu A$; (d) 25 ns, 55.6 ns; (e) 2.29 pC, 0.46 pC, 2.75 pC, 27.5 ns; (f) 110 pF

Forward Current $I = 0.1~mA$

$P = N_A = 10^{17}$ | $n = N_D = 10^{16}$ ← $A = 2500~\mu m^2$

$n_i = 1.5 \times 10^{10}~cm^{-3}$

$L_n = 10~\mu m$    $L_p = 5~\mu m$

$D_n = 18$ ,    $D_p = 10~cm^2/s$

(a) $I_s = ($   $)$

(b) $V = ($   $)$

(c) $I_n = ($   $)$, $I_p = ($   $)$

(d) $\tau_n = ($   $)$, $\tau_p = ($   $)$

(e) $Q_p = ($   $)$   $Q_n = ($   $)$   $Q = Q_n + Q_p$

$\tau_t = ($   $)$        $= ($   $)$

(f) $C_d = \dfrac{Q}{V_T} = ($   $)$

Sol) (a) $I_s = A\, q\, n_i^2 \left( \dfrac{D_p}{L_p N_D} + \dfrac{D_n}{L_n N_A} \right) = (2500 \times 10^{-8})\, 1.6 \times 10^{-19} \times 2.25 \times 10^{20} \left( \dfrac{10}{(5 \times 10^{-4}) \times 10^{16}} + \dfrac{18}{(10 \times 10^{-4}) \times 10^{17}} \right)$

$= 2 \times 10^{-15} A$

(b) $I = I_s e^{V/V_T}$    $V = V_T \ln \dfrac{I}{I_s} = 0.025 \ln \dfrac{0.1 \times 10^{-3}}{2 \times 10^{-15}} = 0.616 V$

(c) $\dfrac{I_p}{I_n} = \dfrac{D_p}{D_n} \dfrac{L_n}{L_p} \dfrac{N_A}{N_D} = \dfrac{10}{18} \dfrac{10}{5} \dfrac{10^{17}}{10^{16}} = 11.11$ ,    $I = I_p + I_n = 12.11\, I_n = 0.1~mA$

$\therefore I_n = \dfrac{0.1}{12.11}~mA = 8.3~mA$

$I_p = 11.11\, I_n = 91.7~mA$

(d) $\tau_n = \dfrac{L_n^2}{D_n} = \dfrac{(10 \times 10^{-4}~cm)^2}{18~cm^2/s} = 55.6~nS$

$\tau_p = \dfrac{L_p^2}{D_p} = \dfrac{(5 \times 10^{-4})^2}{10} = 25~nS$

(e) $Q_p = I_p \tau_p = (91.7 \times 10^{-6})(25 \times 10^{-9}) = 2.29 \times 10^{-12}~C = 2.29~pC$

$Q_n = I_n \tau_n = (8.3 \times 10^{-6})(55.6 \times 10^{-9}) = 0.46~pC$
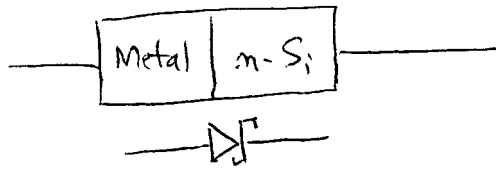
$Q = Q_n + Q_p = 2.29 + 0.46 = 2.75~pC$

$\tau_t = \dfrac{Q}{I} = \dfrac{2.75 \times 10^{-12}}{0.1 \times 10^{-3}} = 27.5~nS$

(f) $C_d = \dfrac{\tau_t\, I}{V_T} = \dfrac{Q}{V_T} = \dfrac{2.75 \times 10^{-12}}{0.025} = 110~pF$

# 3.8 Special Diode Types

1. Schottky Barrier Diode (SBD or SD)



(i) $\begin{cases} Si & SD & V_0 = 0.3 - 0.5 \; V \\ Si & PN & V_0 = 0.6 - 0.8 \; V \end{cases}$
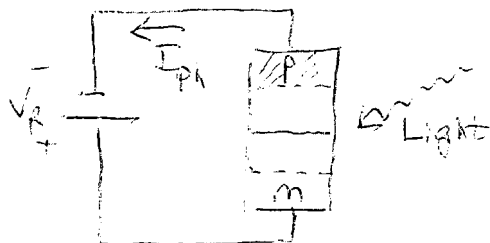
GaAs SD $V_0 = 0.7 \; V$

(ii) SD = majority carrier device (fast)

PN = minority carrier injection (slow)

2. Varactor Diode = Reverse-biased PN diode used as Capacitor

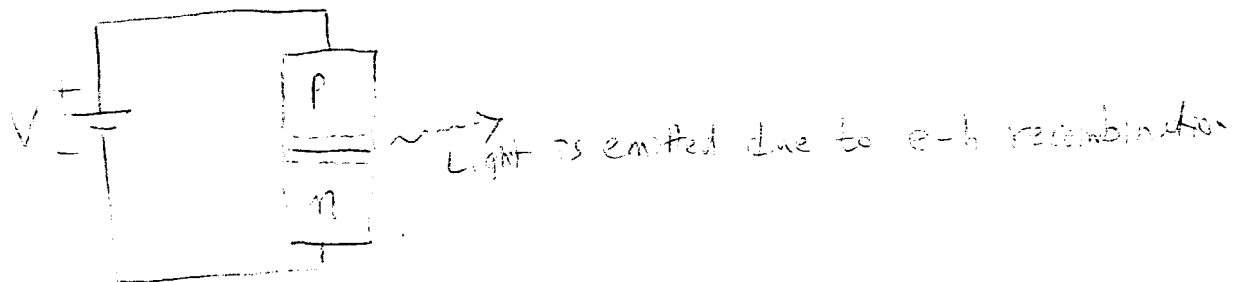$$C_j = \frac{C_{j0}}{\left(1 + \frac{V_R}{V_0}\right)^m}$$

- $C_j$ is variable by $V_R$
- useful in Radio tuner
- $m = 3$ or $4$ :

3. Photodiode = reverse-biased (GaAs) PN diode or PIN diode



$I_{ph}$ = photocurrent is proportional to Light intensity

4. LED = Forward-biased PN diode



Light is emitted due to e-h recombination

# 3.9 Spice Diode Model



$$i_D = I_S \left( e^{v_D / nV_T} - 1 \right)$$

$$C_D = C_d + C_j = \frac{\tau_T}{V_T} I_S \, e^{v_D / nV_T} + C_{j0} \Big/ \left( 1 - \frac{v_D}{V_0} \right)^m$$
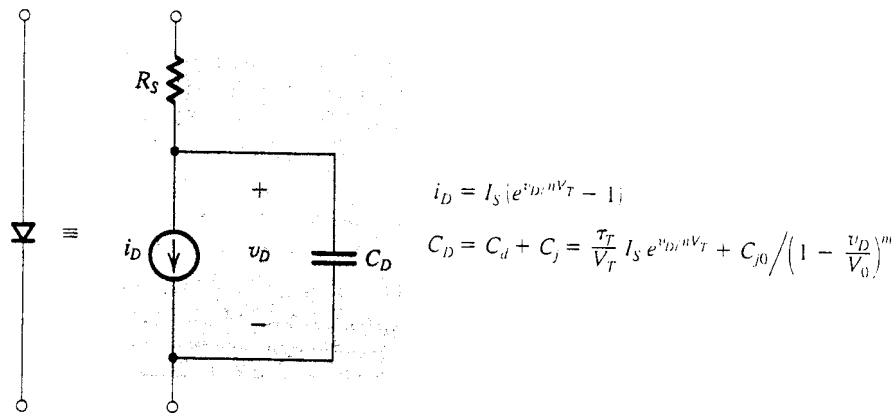
**FIGURE 3.51**  The SPICE diode model.

**TABLE 3.3**  Parameters of the SPICE Diode Model (Partial Listing)

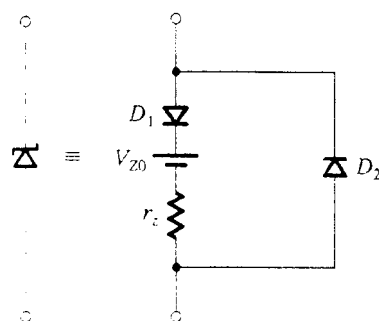| SPICE Parameter | Book Symbol | Description | Units |
|---|---|---|---|
| IS | $I_S$ | Saturation current | A |
| N | $n$ | Emission coefficient | |
| RS | $R_S$ | Ohmic resistance | $\Omega$ |
| VJ | $V_0$ | Built-in potential | V |
| CJ0 | $C_{j0}$ | Zero-bias depletion (junction) capacitance | F |
| M | $m$ | Grading coefficient | |
| TT | $\tau_T$ | Transit time | s |
| BV | $V_{ZK}$ | Breakdown voltage | V |
| IBV | $I_{ZK}$ | Reverse current at $V_{ZK}$ | A |



**FIGURE 3.52**  Equivalent-circuit model used to simulate the zener diode in SPICE. Diode $D_1$ is ideal and can be approximated in SPICE by using a very small value for $n$ (say $n = 0.01$).