

Entry number: 2017CS10540 Name: POORVA GARG

COL 333/671 Autumn 2019 Minor 2

Welcome to Minor #2. The exam is for 1 hour 5 minutes. Questions numbered 1-20 are two points each. Note that if there are multiple correct choices for a question, write all of them.

Please use only pens while answering questions. Do not use a pencil.

Before starting the exam, close your eyes and take three deep breaths. Your performance in the exam is not an accurate reflection of your understanding of the material. Nevertheless, if you are relaxed, you will likely perform better.

Question Number	Maximum Marks	Marks Obtained
1-20	40	22
21	16	16
22	12	3
23	20	20
24	04	0
25	08	8

69

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1. If I already know a k -sized backdoor (for some constant k) of a SAT problem (w.r.t. a polynomial subsolver A), with n variables and m clauses, then the problem can be solved by A in time

- (A) Polynomial(n, m)
- (B) 2^k .Polynomial(n, m)
- (C) 2^n .Polynomial(n, m)
- (D) None of these

~~B~~

2. Which of these is an instance of data-driven reasoning

- (A) Forward chaining
- (B) Backward chaining
- (C) DPLL

A

3. The heuristic(s) used to pick the best next value to assign for a selected variable in backtracking search for a CSP is (are)

- (A) minimum remaining values
- (B) degree heuristic
- (C) MOM's heuristic
- (D) None of the above

~~C~~

4. Modern DPLL-style SAT solvers first run a procedure to find backdoors, and then set the backdoor variables to simplify the problem and solve it quickly.

- (A) True
- (B) False

~~A~~

5. In a Bayes net which of these is/are allowed

- (A) discrete-valued child of discrete-valued parents
- (B) discrete-valued child of continuous-valued parents
- (C) discrete-valued child of parents some of which are discrete and some continuous
- (D) continuous-valued child of discrete-valued parents

~~D~~

6. Imagine that for my constraint satisfaction problem, I get a slightly modified set of constraints every week. I would like to repair the solution with a minimum number of changes. Which of the following algorithms will be better for solving this sequence of CSPs?

- (A) Backtracking search
- (B) Local search

B

7. Which of the following is propagation of constraints between two unassigned variables?

- (A) Forward Checking
- (B) Arc Consistency

B

8. Arc consistency is equivalent to k -consistency, for $k=$

- (A) 1
- (B) 2
- (C) 3
- (D) None of these

B ✓

9. If an agent's knowledge base has only Horn-clauses and we wish to infer the value of a single literal, then inference using forward chaining is both sound and complete.

- (A) True
- (B) False

A ✓

10. For a Bayes net, which of these is true

- (A) Ontological commitment is a set of facts
- (B) Epistemological commitment is a set of facts
- (C) Ontological commitment is {true, false}
- (D) Epistemological commitment is {true, false}

~~A~~ B C X

11. Bayesian networks are a compact way to represent a

- (A) Joint probability distribution
- (B) Marginal probability distribution
- (C) Conditional probability distribution

A C X

12. Any SAT problem can be converted into a CSP.

- (A) True
- (B) False

A ✓

13. For phase transitions in SAT which of the following are true

- (A) 2-SAT has no phase transitions
- (B) all k -SAT with $k > 2$ demonstrate phase transitions
- (C) $(2+p)$ -SAT does not demonstrate phase transitions for $p < 0.4$

C X

14. Exact inference in Bayesian networks is NP complete.

- (A) True
- (B) False

~~A~~ B ✓

15. The min-conflicts heuristic for a CSP is used to order the nodes in backtracking search.

- (A) True
- (B) False

B ✓

16. Variable elimination algorithm can be seen as an instance of

- (A) divide and conquer
- (B) dynamic programming

B

17. A topological sort will be used as a subroutine for which algorithm(s)?

- (A) WalkSAT
- (B) Tree-CSP Solver
- (C) Variable elimination
- (D) Resolution

B

18. Which of these algorithms can prove that a CSP/SAT problem is unsolvable/unsatisfiable?

- (A) Backtracking search
- (B) DPLL
- (C) GSAT
- (D) DPLL with random restarts after cutoff on number of backtracks

A B

19. Cycle cutsets and backdoors are two names to refer to the same concept.

- (C) True
- (D) False

C

20. WalkSAT is an example of

- (A) Greedy hill climbing with random restarts
- (B) Random walk with random restarts
- (C) Greedy hill climbing and random walk with random restarts
- (D) Random walk

C

21. [16 pts] You are walking in a labyrinth and all of a sudden you find yourself in front of three possible roads: the road on your left is paved with gold, the one in front of you is paved with marble, while the one on your right is made of small stones. Your goal is to choose the best road.

Each street is protected by a guardian. You talk to the guardians and this is what they tell you:

- The guardian of the gold street: "This road will bring you straight to the center. Moreover, if the stones take you to the center, then also the marble takes you to the center."
- The guardian of the marble street: "Neither the gold nor the stones will take you to the center."
- The guardian of the stone street: "Follow the gold and you'll reach the center, follow the marble and you will not."

- (a) [4.5 pts] We define three variables S, G, M to represent that stone, gold and marble roads will lead to the center. Write down the statement of each guardian in propositional logic.

Gold street : $G \wedge (S \Rightarrow M)$ — ①

Marble street : $\neg G \wedge \neg S$ — ②

Stone street : $G \wedge \neg M$ — ③

4.5

- (b) [11.5 pts] Suppose you know that all the guardians are liars. Now write down everything you know so far in a CNF form. Further, use resolution on the CNF form to deduce which road you should follow. *show your work.*

If all guardians are liars, then their statements are unsatisfiable so we will take negation of all these statements to get valid sentences.

$$\textcircled{1} \neg(G \wedge (S \Rightarrow M))$$

$$\equiv \neg G \vee (\neg(S \Rightarrow M)) \quad \text{De Morgan's rule}$$

$$\equiv \neg G \vee (\neg(\neg S \vee M)) \quad \text{Implication rule}$$

$$\equiv \neg G \vee (S \wedge \neg M) \quad \text{Negation rule}$$

$$\equiv (\neg G \vee S) \wedge (\neg G \vee \neg M) \quad \text{Distributive rule}$$

✓

$$\textcircled{2} \neg(\neg G \wedge \neg S)$$

$$\equiv G \vee S \quad (\text{De Morgan's rule})$$

S

$$\textcircled{3} \neg(G \wedge \neg M)$$

$$\equiv \neg G \vee M \quad (\text{De Morgan's rule})$$

Required CNF: $(\neg G \vee S) \wedge (\neg G \vee \neg M) \wedge (G \vee S) \wedge (\neg G \vee M)$

Let's try for: G

Resolution on $\neg G \vee S$ $\neg G \vee \neg M$ $G \vee S$ $\neg G \vee M$ $\neg G$

Resolution on

$\neg G \vee S$ ————— $\neg G \rightarrow S$
 $\neg G \vee \neg M$ ————— S
 $G \vee S$ ————— $S \vee \neg M$
 $\neg G \vee M$ ————— $S \vee M$
 $\neg G$ ————— S

$S \vee \neg M$ \vee $S \vee M$
 $\equiv S$

No further resolvent would lead to new clauses. Since there is no empty clause in Resolution closure.
 $C \not\equiv \text{false}$

For S
Resolution on

$\neg G \vee S$
 $\neg G \vee \neg M$
 $G \vee S$
 $\neg G \vee M$
 $\neg S$

$S \equiv \text{true}$

$()$ ✓

$$S \equiv \text{true}$$

6.5

For M
Resolution on

$\neg G \vee S$
 $\neg G \vee \neg M$
 $G \vee S$
 $\neg G \vee M$
 $\neg M$

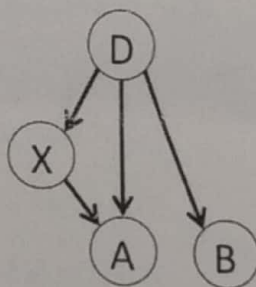
$S \vee S \equiv S$
 $S \vee \neg M$
 S
 $\neg G$
 $S \vee M$

$S \vee S \equiv S$

No further changes would be needed in resolution closure. No empty clause $M \equiv \text{false}$

22. [12 pts] You are given a Bayes net for a disease D and two tests A and B. *show working*

$P(A D, X)$			
$+d$	$+x$	$+a$	0.9
$+d$	$+x$	$-a$	0.1
$+d$	$-x$	$+a$	0.8
$+d$	$-x$	$-a$	0.2
$-d$	$+x$	$+a$	0.6
$-d$	$+x$	$-a$	0.4
$-d$	$-x$	$+a$	0.1
$-d$	$-x$	$-a$	0.9



$P(D)$	
$+d$	0.1
$-d$	0.9

$P(X D)$		
$+d$	$+x$	0.7
$+d$	$-x$	0.3
$-d$	$+x$	0.8
$-d$	$-x$	0.2

$P(B D)$		
$+d$	$+b$	0.7
$+d$	$-b$	0.3
$-d$	$+b$	0.5
$-d$	$-b$	0.5

- (a) [3 pts] What is the probability of having disease D and getting a positive result on the test A?
 (b) [4 pts] What is the probability of having disease D given a positive result on the test A?
 (c) [5 pts] What is the probability of having disease D given a positive result on test B, but a negative result on test A?

$$d) P(+a|+d) = \sum_x P(+a|+d, x) = P(+a|+d, +n) + P(+a|+d, -n) = 0.94$$

$$a) P(+d, +a) = \sum_{x, B} P(+d, +a, x, B) = \sum_{x, B} P(+d) P(x|+d) P(B|+d) P(a|x, +d)$$

$$= \sum_d P(+d) \cdot \sum_x P(x|+d) P(+a|x,+d) \cdot \sum_B P(B|+d)$$

$$= (0,1) \sum_x P(x|+d) P(+a|x,+d)$$

$$= (0.1) [P(+x|+d) \cdot P(+a|+x, +d) + P(-x|+d) \cdot P(+a|-x, +d)]$$

$$= (0.1) [(0.7)(0.9) + (0.3)(0.8)] = 0.087 \quad \text{Ans}$$

$$b) P(+d|+a) = \frac{P(+a|+d) P(+d)}{P(+a)}$$

~~≈ 2.04~~

$$= \left(\sum_x P(+a|+d, x) \right) P(+d)$$

$$\frac{\sum_{x, D} P(+a|x, D)}{P(+d)}$$

$$= P(+d) [P(+a|+d, +x) + P(+a|+d, -x)]$$

$$\frac{P(+a|+x, +d) + P(+a|+x, -d) + P(+a|-x, +d) + P(+a|-x, -d)}{P(+x, +d) + P(+x, -d) + P(-x, +d) + P(-x, -d)}$$

$$= \frac{(0.1) [0.9 + 0.8]}{0.9 + 0.8 + 0.6 + 0.1} = 0.0708 \quad \underline{\text{Ans}}$$

$$c) P(+d|+b, -a) = \frac{\sum_x P(+d, +b, -a, x)}{\sum_{x, D} P(+b, -a, x, D)}$$

$$\frac{\sum_{x, D} P(+b, -a, x, D)}{P(+b, -a)}$$

$$= \frac{\sum_x P(+d) P(+b|+d) P(x|+d) P(-a|+d, x)}{\sum_{x, D} P(+d) P(+b|D) P(x|D) P(-a|D, x)}$$

$$= (0.1) (0.7) [(0.7)]$$

23. [20 pts] You are in charge of scheduling for computer science classes that meet Mondays, Wednesdays and Fridays. There are 5 classes that meet on these days and 3 professors who will be teaching these classes. The classes are:

- Class 1 - Intro to Programming (C1): meets from 8:00-9:00am
- Class 2 - Intro to Artificial Intelligence (C2): meets from 8:30-9:30am
- Class 3 - Natural Language Processing (C3): meets from 9:00-10:00am
- Class 4 - Computer Vision (C4): meets from 9:00-10:00am
- Class 5 - Machine Learning (C5): meets from 9:30-10:30am

The professors are:

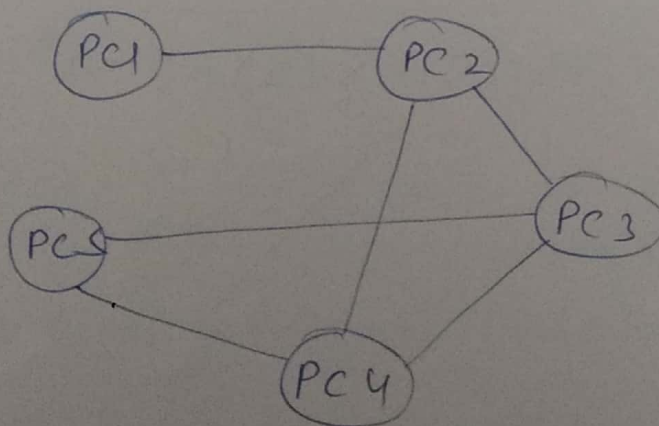
- Professor A, who is available to teach Classes 3 and 4.
- Professor B, who is available to teach Classes 1, 2, 3, 4, and 5.
- Professor C, who is available to teach Classes 2, 3, 4, and 5.

(a) [6 pts] Formulate this problem as a CSP. Define the variables, domains and constraints.

Let ~~PC1, PC2, PC3~~ be the var
 let PC_x denote the professor who is going to teach class C_x
 Defining Domain of each variable to be A, B, C
 $Variables = \{PC_1, PC_2, PC_3, PC_4, PC_5\}$
 Domains of each variable $\{A, B, C\}$
 Constraints $PC_1 \neq A, PC_1 \neq C, PC_2 \neq A, PC_3 \neq A, PC_5 \neq A, PC_1 \neq PC_2, PC_2 \neq PC_3, PC_2 \neq PC_4, PC_3 \neq PC_4, PC_3 \neq PC_5, PC_4 \neq PC_5$

6

(b) [4 pts] Draw the constraint graph of the problem.



4

(c) [5 pts] First enforce all unary constraints, if any, in your CSP. Then run the arc consistency algorithm. Show the domains of each variable after completing these operations.

After unary constraints

PC1	PC2	PC3	PC4	PC5
$\{B\}$	$\{B, C\}$	$\{A, B, C\}$	$\{A, B, C\}$	$\{B, C\}$

Arc consistency

PC1 \neq PC2	$\{B\}$	$\{C\}$	$\{A, B, C\}$	ABC	BC
PC2 \neq PC3	B	C	AB	ABC	BC
PC2 \neq PC4	$\{B\}$	$\{C\}$	$\{AB\}$	$\{ABC\}$	$\{BC\}$
PC3 \neq PC4	-	-	-	-	-

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(d) [3 pts] Is your CSP tree-structured? If not, mention the size of the smallest cycle cutset. Which variables are part of the smallest cycle cutset?

CSP is not tree structured. It has a loop PC2, PC3, PC4.

Smallest cycle cutset has size 1: PC3.

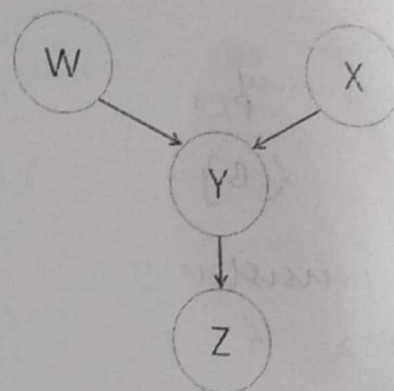
3

(e) [2 pts] Give any one solution to the CSP.

PC1 = B
 PC2 = C
 PC3 = A
 PC4 = B
 PC5 = C

2

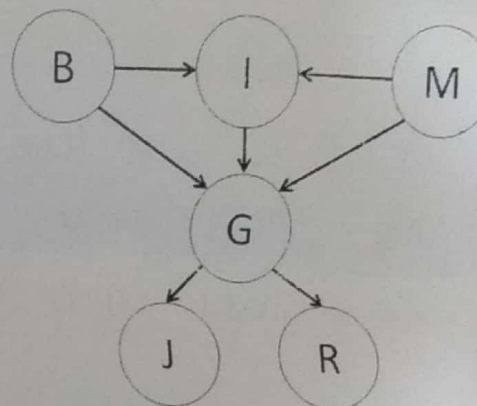
24. [4 points] For the Bayes Net shown in the figure, the random variables W, X, Y and Z take 3, 4, 5 and 2 values respectively. What is the least number of parameters needed to represent the joint distribution of W, X, Y and Z.



To represent $W = 1$ $P(W)$
 $X = 1$ $P(X)$
 $Y = 4$ $P(Y|W, X)$
 $Z = 2$ $P(Z|Y)$

Total parameters required = 8.

25. [8 points] In the Bayes net



(a) Is B independent of I? No. ✓

(b) Is B independent of M? Yes. ✓

(c) Is J conditionally independent of R given G? Yes. ✓

(d) Is J conditionally independent of I given B and M? No. ✓

(e) Is B conditionally independent of M given I? No. ✓

(f) Is B conditionally independent of M given J? No. ✓

(g) Is J conditionally independent of I given G? Yes. ✓

(h) Is J conditionally independent of R given B, I, and M? No. ✓

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