COL333/671: Introduction to AI
Semester I, 2022-23

Reinforcement Learning

Rohan Paul

Outline

- Last Class
  - Markov Decision Processes
- This Class
  - Reinforcement Learning
- Reference Material
  - Please follow the notes as the primary reference on this topic. Supplementary reading on topics covered in class from AIMA Ch 21 sections 21.1 – 21.4.

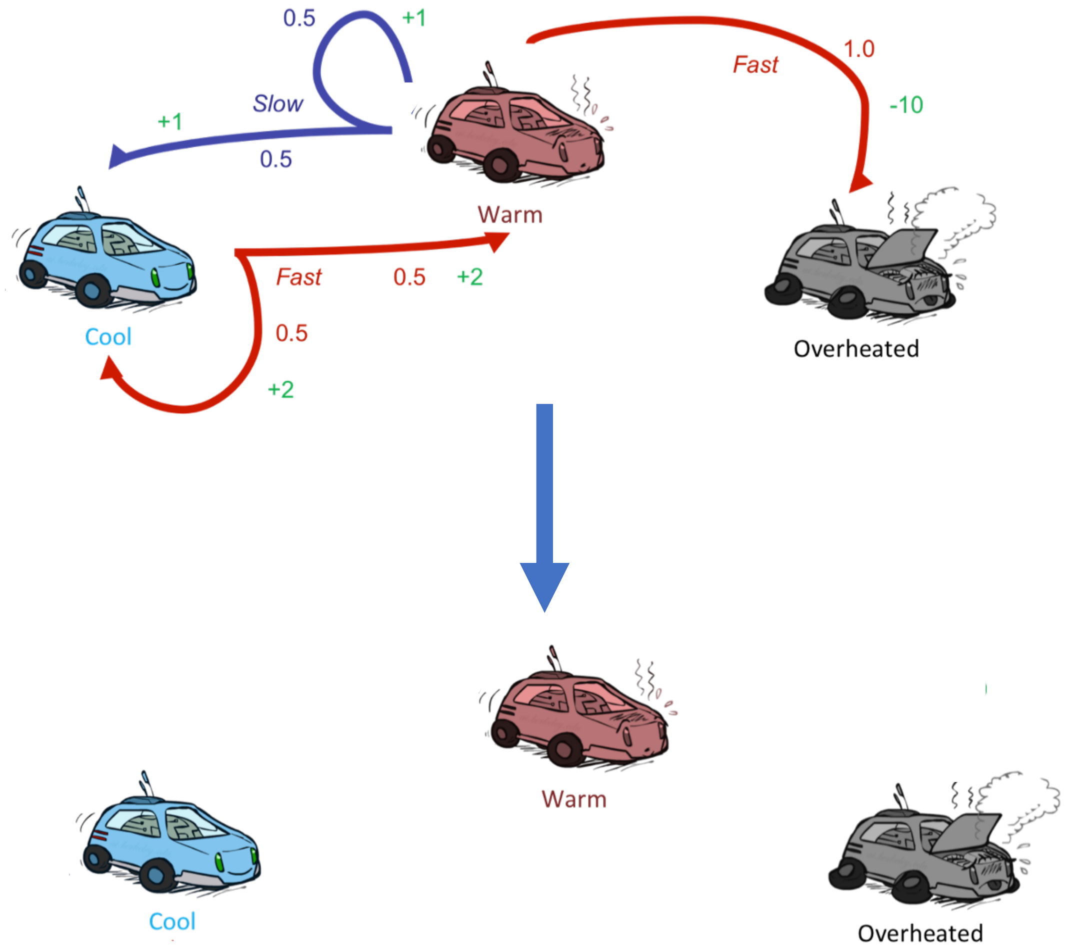
Acknowledgement

These slides are intended for teaching purposes only. Some material has been used/adapted from web sources and from slides by Doina Precup, Dorsa Sadigh, Percy Liang, Mausam, Parag, Emma Brunskill, Alexander Amini, Dan Klein, Anca Dragan, Nicholas Roy and others.

Reinforcement Learning Setup

- Markov decision process (MDP):
  - A set of states $s \in S$
  - A set of actions (per state) A
  - A model T(s,a,s')
  - A reward function R(s,a,s')
- Goal is to determine a policy $\pi(s)$

- Don't know T or R
  - We don't know which states are good or what the actions do
  - Must actually try actions and states to learn their utility.
  - The agent's goal is still to act optimally i.e. determine the optimal policy.



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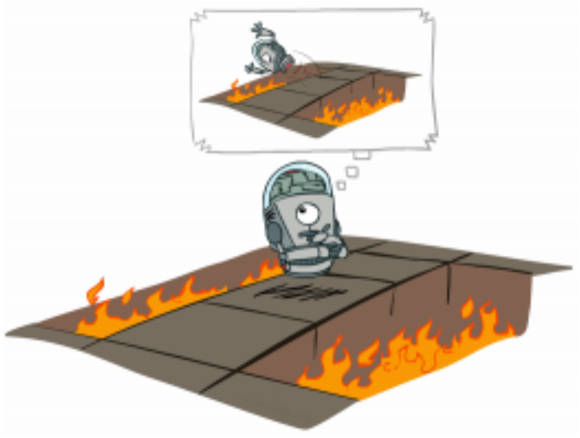
- But we don't know T or R
  - I.e. we don't know which states are good or what the actions do
  - Must actually try actions and states out to learn

In the Car MDP (discussed previously) we now **do not** know the transition model and the rewards. How should the agent act in this setup to maximize expected future rewards.

How should the agent act in this setup so as to maximize expected future rewards. Also, think of the setup as acting in an **"unknown" MDP**

Another View: Offline (MDP) vs Online (RL)

- Offline (MDP)
  - We are given an MDP
  - We use policy or value iteration to learn a policy. This is computed offline.
  - At runtime the agent only executes the computed policy



Offline Solution

The agent solves the MDP and computes a policy. Now it simply acts with it.

- Online (RL)
  - We do not have full knowledge of the MDP
  - We must interact with the world to learn which states are good and which actions eventually lead to good rewards.

Online Learning

The agent interacts with the world and learns that one of the states is bad.

Reinforcement Learning

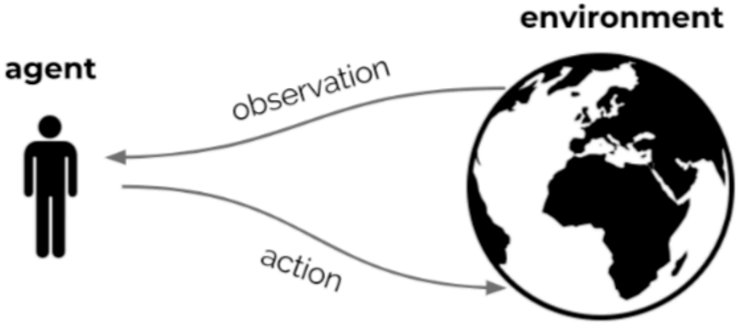
- **Given**
  - Agent, states, actions, immediate rewards and environment.
- **Not given**
  - Transition function (the agent cannot predict which state will it land in once it takes an action)
  - Does not know the reward function. Does not know what reward it will get in a state.
- **Agent’s Task**
  - A policy that maximizes expected rewards (the objective has not changed)

Reinforcement Learning (RL)

- People and animals learn by interacting with our environment
  - It is active rather than passive.
  - Interactions are often sequential — future interactions can depend on earlier ones

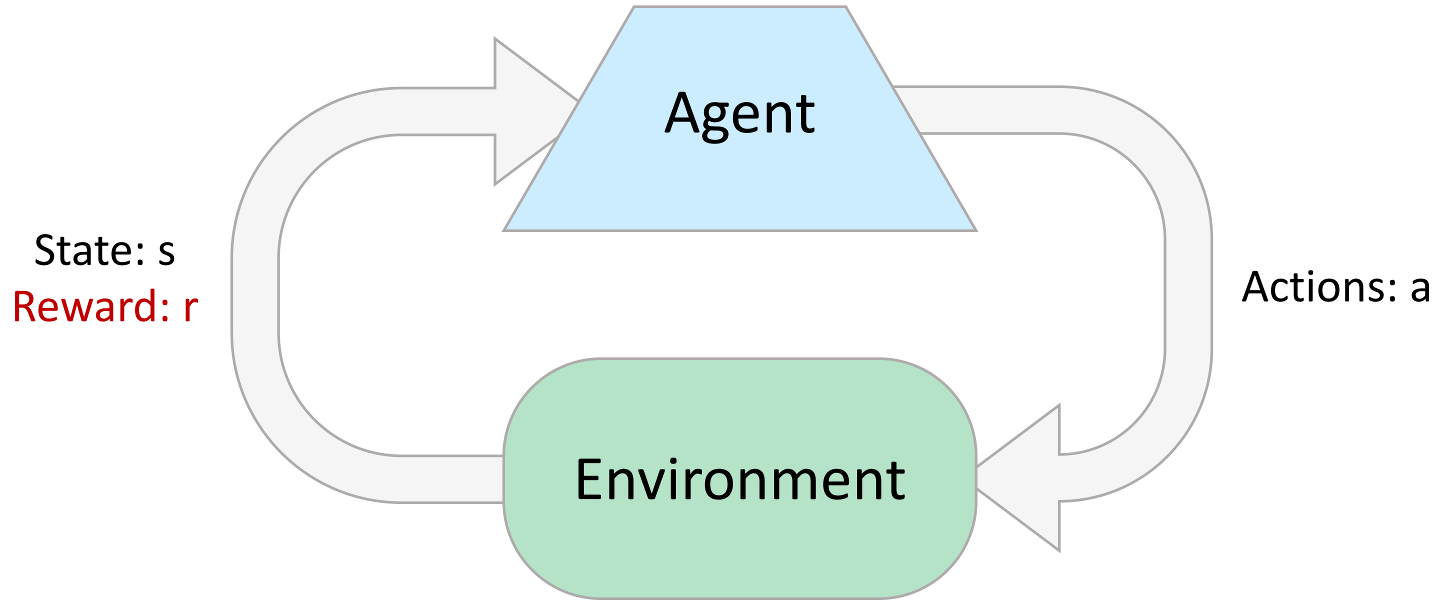
- Reward Hypothesis
  - Any goal can be formalized as the outcome of maximizing a cumulative reward.
  - We can learn without examples of optimal behaviour Instead, we optimise some reward signal

Biological motivation



Goal: optimise sum of rewards, through repeated interaction

Reinforcement Learning: Setup



Key characteristic of reinforcement learning

- **Only evaluative feedback present.**
- **The agent takes and action and is provided feedback (reward).**
- **The agent is not told which action it should take in a state.**

- Receive feedback in the form of rewards
- Agent's utility is defined by the reward function
- Must (learn to) act so as to maximize expected rewards
- All learning is based on observed samples of outcomes!

Classes of Learning Problems

Supervised Learning

Data: (x, y)
x is data, y is label

Goal: Learn function to map
$$x \rightarrow y$$

Apple example:

This thing is an apple.

Unsupervised Learning

Data: x
x is data, no labels!

Goal: Learn underlying structure

Apple example:

This thing is like the other thing.

Reinforcement Learning

Data: state-action pairs

Goal: Maximize future rewards over many time steps

Apple example:

Eat this thing because it will keep you alive.

Figure: Alexander Amini

Examples of RL

- Fly a helicopter → **Reward**: air time, inverse distance, ...
- Manage an investment portfolio → **Reward**: gains, gains minus risk, ...
- Control a power station → **Reward**: efficiency, ...
- Make a robot walk → **Reward**: distance, speed, ...
- Play video or board games → **Reward**: win, maximise score, ...

Examples: Learning to Walk

Initial

[Kohl and Stone, ICRA 2004]

Examples: Learning to Walk

Training

[Kohl and Stone, ICRA 2004]

Examples: Learning to Walk

Finished

[Kohl and Stone, ICRA 2004]

Examples: Game Play

Google Deep Mind

Examples: Healthcare Domain

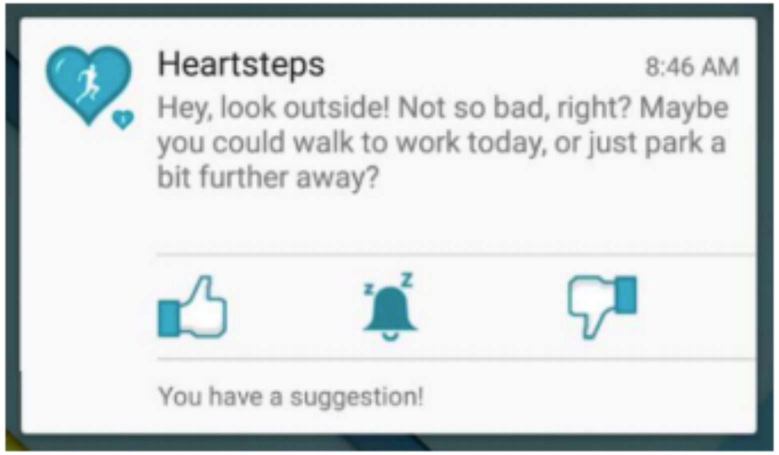
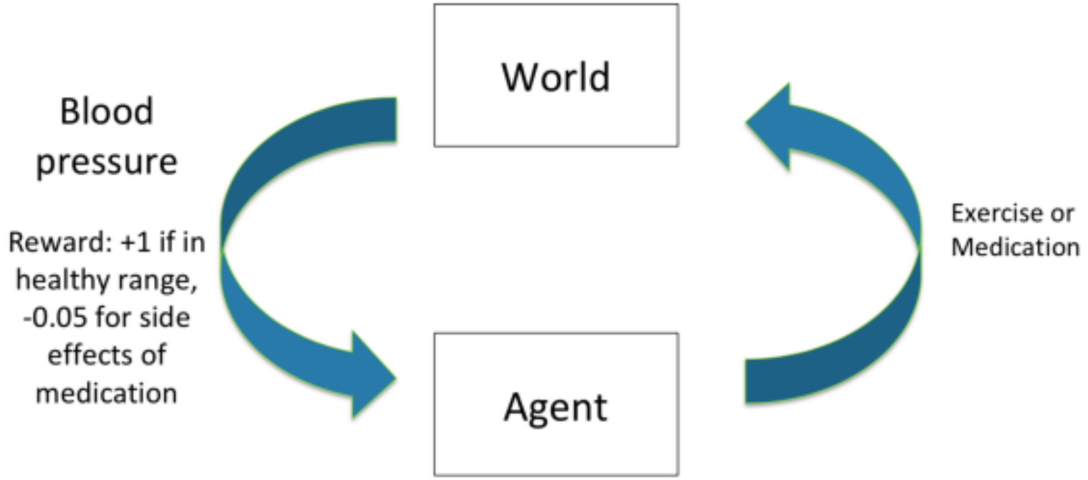


Figure: Personalized HeartSteps: A Reinforcement Learning Algorithm for Optimizing Physical Activity. Liao, Greenewald, Klasnja, Murphy 2019 arxiv



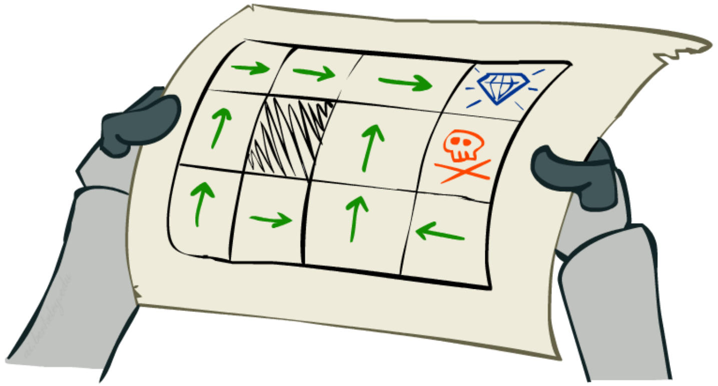
Blood pressure control

Reinforcement Learning: Approaches

- **Different learning agents**
  - **Utility-based agent**
    - Learn a value/utility function on states and use it to select actions that maximize the expected outcome utility.
  - **Q-learning**
    - Learn action-utility function (or Q-function) giving expected utility of taking a given action in a given state.
  - **Reflex agent.**
    - Directly learn a mapping from states to action (policy).
- **Approaches**
  - **Passive Learning**
    - The agent's policy is fixed. the task is to learn the utilities of states (or state-action pairs); this could involve learning a model of the environment.
    - It cannot select the actions during training.
  - **Active Learning**
    - The agent can select actions, it must also learn what actions to take.
    - Issue of exploration: an agent must experience as much as possible of the environment in order to learn how to behave in it.

Passive Reinforcement Learning

- Setup
  - Input: a fixed policy $\pi(s)$
  - Don't know the transitions $T(s,a,s')$
  - Don't know the rewards $R(s,a,s')$
  - The agent executes a set of trials or episodes using its policy $\pi(s)$
  - In each episode or trial it starts in a state and experiences a sequence of state transitions and rewards till it reaches a terminal state.
  - Goal: learn (estimate) the state values $V^{\pi}(s)$



The learner is provided with a policy (cannot change that), using the policy it executes trials or episodes in the world. The goal is to determine the value of states.

Model-Based Reinforcement Learning

- Model-Based Idea
  - Learn an approximate model (R(), T()) based on experiences
  - Compute the value function using the learned model (as if it were correct).

- Step 1: Learn an empirical MDP model
  - Count outcomes s' for each s, a
  - Normalize to give an estimate of $\widehat{T}(s,a,s')$
  - Discover each $\widehat{R}(s,a,s')$ when we experience (s, a, s')

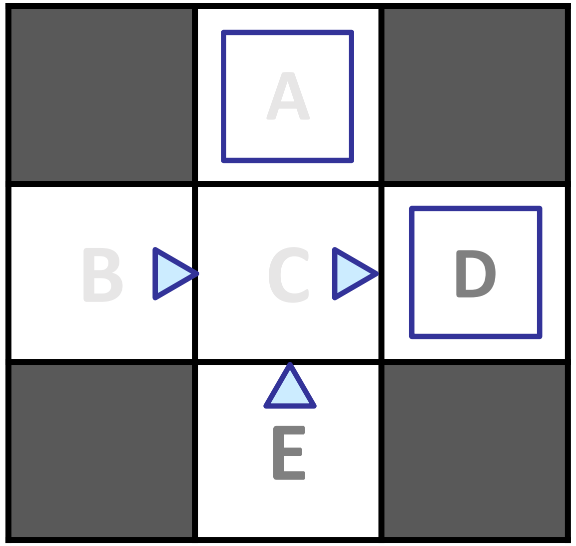
- Step 2: Solve the learned MDP
  - For example, use value iteration, to obtain the final policy.
  - Plug in the estimated T and R in the following equation:

$$\pi^*(s) = \arg\max_a \sum_{s'} \widehat{T}(s,a,s')[\widehat{R}(s,a,s') + \gamma V^*(s')]$$

*Note: going through an intermediate stage of learning the model. In contrast, "**model-free**" approaches do not learn the intermediate model.*

Example: Model-Based Learning

Input Policy π



Assume: γ = 1

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

$$\widehat{T}(s,a,s')$$

T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
…

$$\widehat{R}(s,a,s')$$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
…

Toy example: Model-Based vs Model-Free Estimation

Goal: Compute expected age of COL333/671 students

Known P(A)

$$E[A] = \sum_a P(a) \cdot a \qquad = 0.35 \times 20 + \ldots$$

Without P(A), instead collect samples $[a_1, a_2, \ldots a_N]$

Unknown P(A): "Model Based"

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

Unknown P(A): "Model Free"

$$E[A] \approx \frac{1}{N} \sum_i a_i$$

Eventually, we learn the right model which gives the right estimates.

Bypass the model construction. Averaging works because the samples appear with the right frequencies.

Model-Based vs. Model Free RL

Model-based RL:

- Explore environment and learn model, T=P(**s'**|**s**,**a**) and R(**s**,**a**), (almost) everywhere. Use model to plan a policy (solving the MDP)
- Suitable when the state-space is manageable

Model-free RL:

- Do not learn a model; learn value function or policy directly
- Suitable when the state space is large

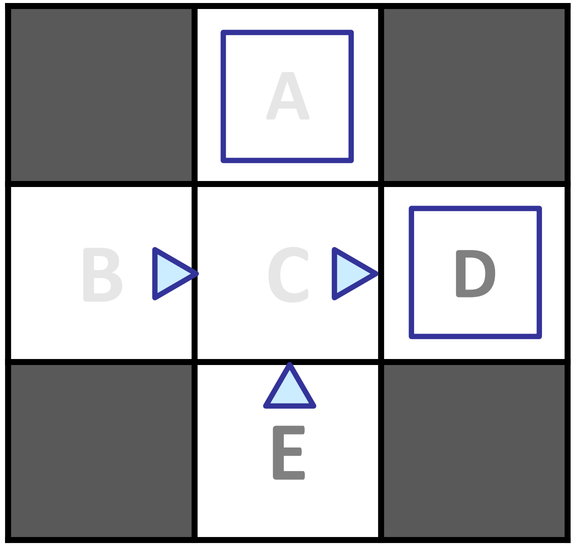
Next, we look at a model free approach to RL

Monte Carlo Methods

- Learning the state-value function for a given policy
  - What is the value of a state?
    - Expected return – expected cumulative future discounted reward
- Key Idea
  - Sample trajectories from the world directly and estimate a value function without a model
  - Simply average the returns observed after visits to that state.
  - As more returns are observed the average should converge to the expected value.
  - Each occurrence of a state in an episode is a called a visit to the state.

Toy Example: Monte Carlo Method

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

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C, east, D, -1
D, exit, x, +10

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Episode 4

E, north, C, -1
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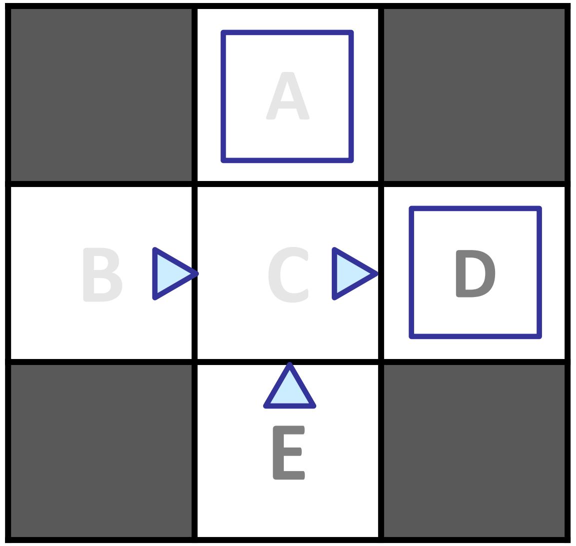
Value/utility of state c

$V^{\pi}(C) = ((9 + 9 + 9 + (-11))/4)$
$= 4$

Toy Example: Monte Carlo Method

Input Policy π



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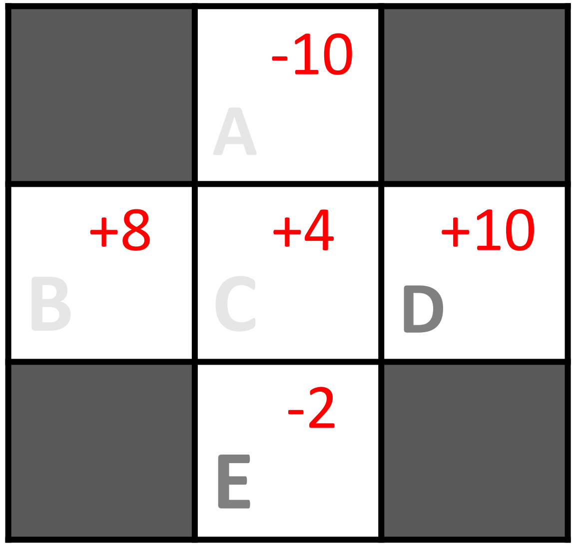
Output Values



First-Visit Monte Carlo (FVMC) Policy Evaluation

Initialize the value function arbitrarily.

Loop over the episode from the end till the start.

Account for the discounting of the reward

Except if (unless) the state has been visited from time 0 till (t-1) append and average out the results.
I.e., only update the value estimate if this is the first visit to the state.

First-visit MC prediction, for estimating $V \approx v_\pi$

Input: a policy π to be evaluated
Initialize:
  $V(s) \in \mathbb{R}$, arbitrarily, for all $s \in \mathcal{S}$
  $Returns(s) \leftarrow$ an empty list, for all $s \in \mathcal{S}$

Loop forever (for each episode):
  Generate an episode following π: $S_0, A_0, R_1, S_1, A_1, R_2, \ldots, S_{T-1}, A_{T-1}, R_T$
  $G \leftarrow 0$
  Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:
    $G \leftarrow \gamma G + R_{t+1}$
    Unless S_t appears in $S_0, S_1, \ldots, S_{t-1}$:
      Append G to $Returns(S_t)$
      $V(S_t) \leftarrow \text{average}(Returns(S_t))$

Pseudo-code from Sutton and Barto (Reinforcement Learning) Ch 5 (Sec. 5.1)

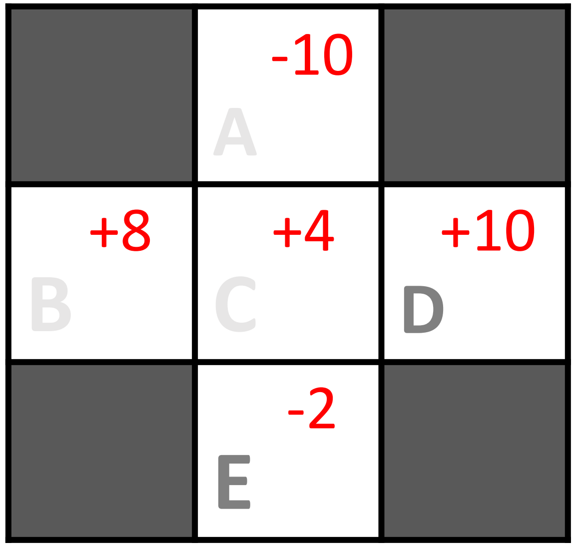
First-Visit Monte Carlo (FVMC)

- First-Visit Monte Carlo (FVMC)
  - Averages the returns following the first visit to a state s in the episode.
- Every-visit Monte Carlo (EVMC)
  - Averages returns following all the visits to s.
- Convergence
  - FVMC – error falls as 1/N(s). Needs lots of data
  - EVMC - error falls quadratically, slightly better data efficiency.

Monte Carlo Methods: Pros and Cons

- Pros
  - Does not estimate a model. It does not require any knowledge of T(), R().
  - With large number of runs it computes the correct average values, using just sample transitions
- Cons
  - Each state must be learned separately, loses the state connection information.
    - Estimate of one state is not taking advantage of the estimates of the other states.
    - Note: Bellman equations tell us that value function for states has a recursive relationship.
  - Could only be used in an episodic setting.

How can the Bellman equation be incorporated in this learning?



- **Problem: we have lost the connection between states.**
- If B and E both go to C under this policy, how can their values be different?

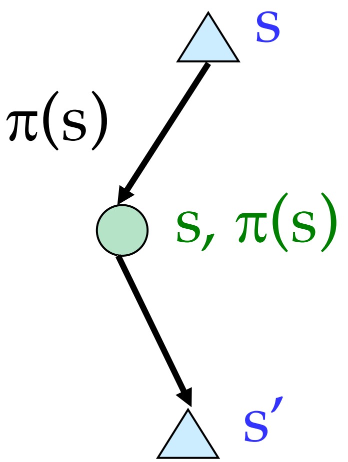
Temporal Difference (TD) Learning

- Model-Free combination of
  - Monte Carlo (learning from sample trajectories/experience) and
  - Dynamic programming (via Bellman Equations)
- Incorporate Bootstrapping
  - Update value function estimates of a state based on others
    - Adjust the value function estimate using the Bellman Equation relationship between the value function of successor states.
    - More data-efficient than a Monte Carlo method
- Setting
  - Can be used in an *episodic* or *infinite-horizon non-episodic* settings
    - Immediately updates the estimate of V(s) after each (s, a, s’, r) tuple.

“If one had to identify one idea that is central and novel to reinforcement learning, it would undoubtedly be temporal-difference (TD) learning”, Sutton and Barto 2017

Temporal Difference (TD) Learning

- Temporal difference learning of values
  - Policy still fixed
  - Move values toward the value of the successor that is encountered
  - Keep a running average
  - Don't need to store all the experience to build models. It is a model-free approach.



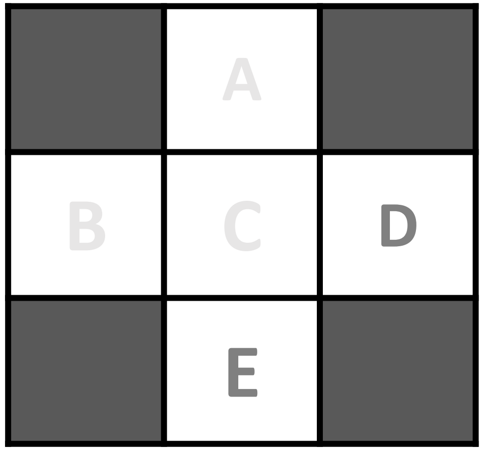
Sample of V(s): $sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$ — What is observed

Update to V(s): $V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + (\alpha)sample$ — Modify the old value

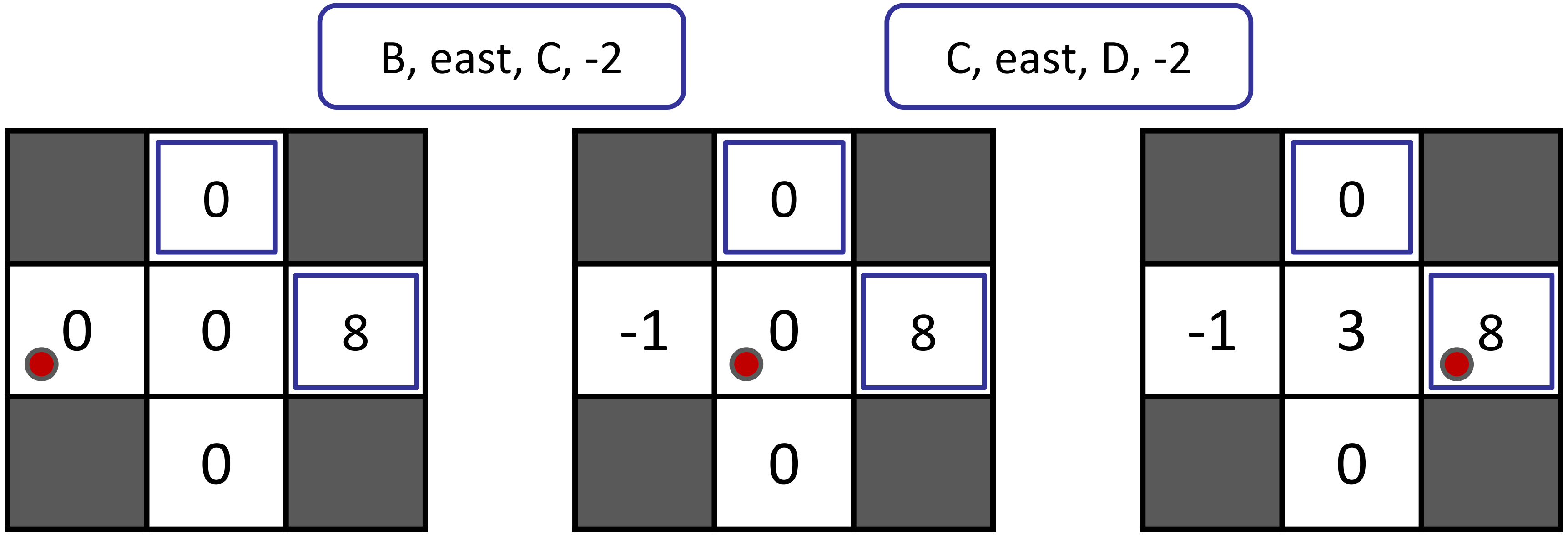
Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$ — Sign and magnitude of the difference.

Temporal Difference Learning: Example

States



Observed Transitions



Assume: $\gamma = 1$, $\alpha = 1/2$

$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^{\pi}(s')\right]$$

Temporal Difference

- The updates are based on the difference in value functions at each time step, the TD error,

$$\delta_t = r_t + \gamma V^\pi(s_{t+1}) - V^\pi(s_t)$$

  hence the name *temporal difference learning*.
- α is the learning rate.
- TD can be generalised to n-step returns:

$$R_t^{(n)} = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \cdots + \gamma^n V_t(s_{t+n})$$

TD Learning (intuitively)
- Nudge our prior estimate of the value function for a state using the given experience.
- Shift the estimate based on the error in what we are experiencing and what estimate we had before
- Weighted by the learning rate.

$$V^\pi(s_t) = V^\pi(s_t) + \alpha\,(\underbrace{[r_t + \gamma V^\pi(s_{t+1})]}_{\text{TD target}} - V^\pi(s_t))$$

TD Value Learning: Problems

- TD Value Learning
  - Model-free approach to perform policy evaluation
  - Incorporates Bellman updates with running sample averages
- Output of TD Value Learning
  - TD Value learning outputs the value function
- *How to turn the learned values into a new policy?*
  - Can use the following relationships:

$$\pi(s) = \arg\max_a Q(s,a)$$

$$Q(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V(s') \right]$$

- Problem: we don't have T and R
- Next: Can we learn the Q-values and not values? Then select actions for the new policy using the relationships above.

Estimating the value function may be useful on its own.

Example 6.1: Driving Home Each day as you drive home from work, you try to predict how long it will take to get home. When you leave your office, you note the time, the day of week, the weather, and anything else that might be relevant. Say on this Friday you are leaving at exactly 6 o'clock, and you estimate that it will take 30 minutes to get home. As you reach your car it is 6:05, and you notice it is starting to rain. Traffic is often slower in the rain, so you reestimate that it will take 35 minutes from then, or a total of 40 minutes. Fifteen minutes later you have completed the highway portion of your journey in good time. As you exit onto a secondary road you cut your estimate of total travel time to 35 minutes. Unfortunately, at this point you get stuck behind a slow truck, and the road is too narrow to pass. You end up having to follow the truck until you turn onto the side street where you live at 6:40. Three minutes later you are home. The sequence of states, times, and predictions is thus as follows:

State	*Elapsed Time (minutes)*	*Predicted Time to Go*	*Predicted Total Time*
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
exiting highway	20	15	35
2ndary road, behind truck	30	10	40
entering home street	40	3	43
arrive home	43	0	43

The rewards in this example are the elapsed times on each leg of the journey.[1] We are not discounting ($\gamma = 1$), and thus the return for each state is the actual time to go from that state. The value of each state is the *expected* time to go. The second column of numbers gives the current estimated value for each state encountered.

[1]If this were a control problem with the objective of minimizing travel time, then we would of course make the rewards the *negative* of the elapsed time. But because we are concerned here only with prediction (policy evaluation), we can keep things simple by using positive numbers.

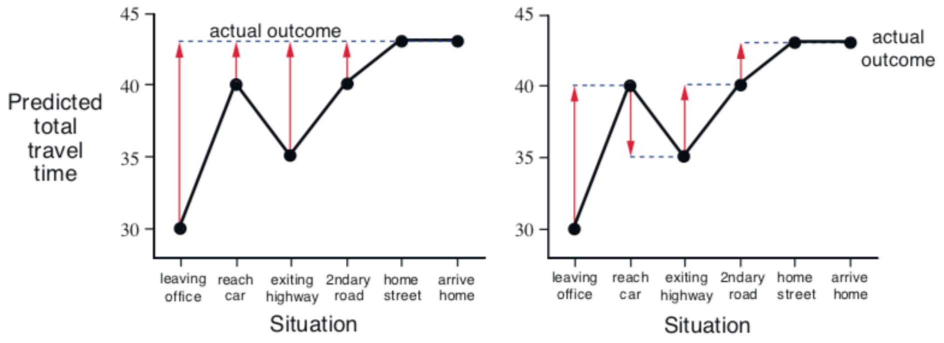


Figure 6.1: Changes recommended in the driving home example by Monte Carlo methods (left) and TD methods (right).

From Value Iteration to Q-Value Iteration

- **Value Iteration**
  - Start with $V_0(s) = 0$
  - Given V_k, calculate the V_{k+1} for all states as:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

- **Q-Value Iteration**
  - Start with $Q_0(s,a) = 0$
  - Given Q_k, calculate the depth Q_{k+1} q-values for all q-states:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \underbrace{\left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]}_{\text{Sample}}$$

Q-Learning

- Sample-based Q-value iteration

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]$$

- Estimate Q(s,a) values as:
  - Receive a sample (s,a,s',r)
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate $sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$
  - Incorporate the new estimate into a running average:

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[sample\right]$$

Q-Learning: Procedure

- **Forall s, a**
  - **Initialize Q(s, a) = 0**
- **Repeat Forever**

  Where are you? s

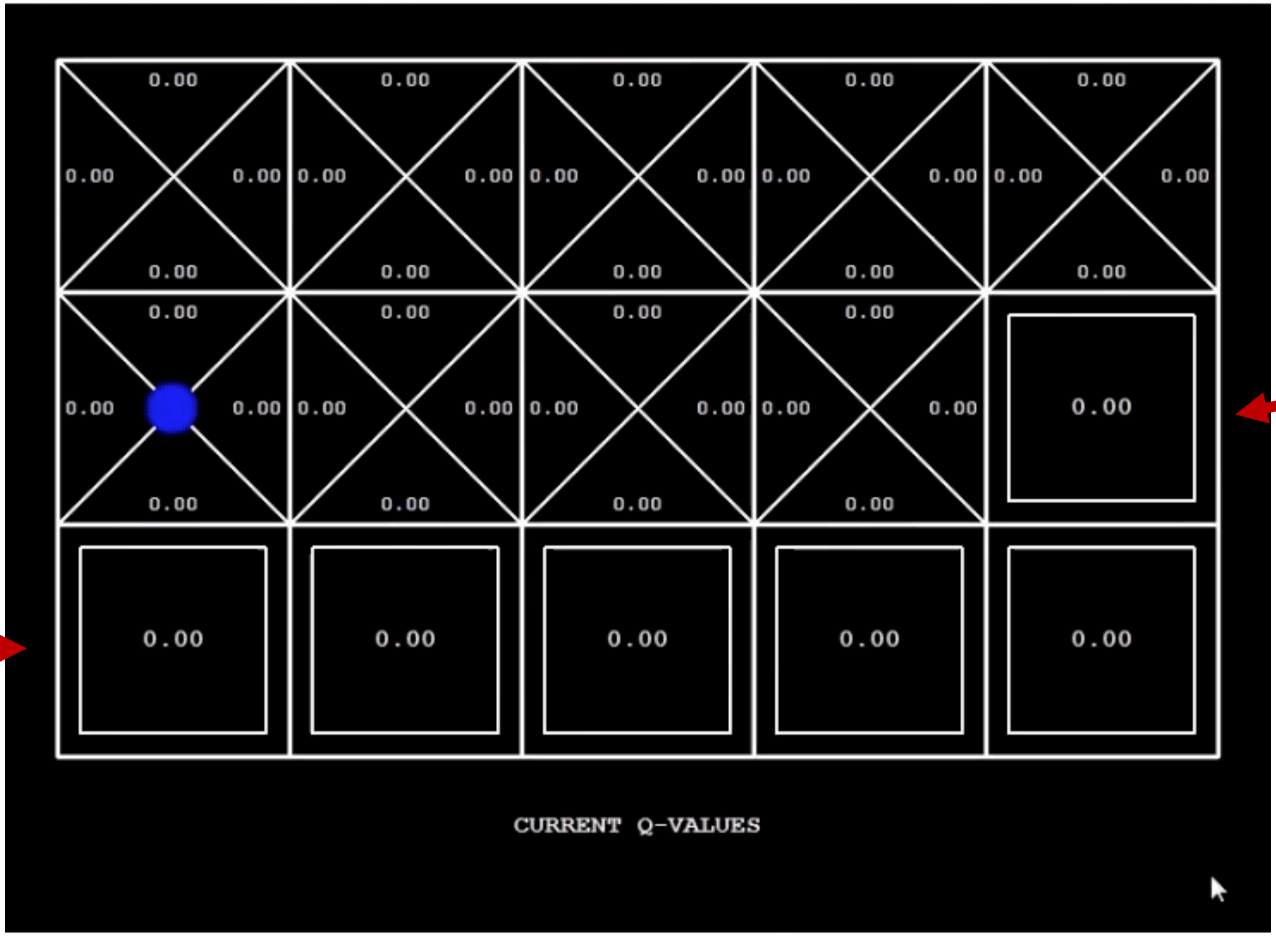
  Choose some action a

  Execute it in real world: *(s, a, r, s')*

  Do update:

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[r + \gamma \max_{a'} Q(s',a')\right]$$

Q-Learning



Exit with +10 reward

Cliff at the bottom, negative reward here.

The agent does not know the rewards a-priori. Learns the effect of the east action over time. Only the actions we do are updated. Occasionally falls in the cliff and gets the negative reward. Note that the max of the Q values is propagated (green values) to other states as it is approximating the optimal Q value.

Q-Learning: Properties

- Off-policy learning
  - Q-learning converges to optimal policy -- even if the agent is acting *sub-optimally.*

- Some technical conditions:
  - Exploration is enough
  - In the limit does not matter how the actions are selected.

SARSA

- State-Action-Reward-State-Action (SARSA)
  - Update using (s, a, r, s', a')
- SARSA Update equation $Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[r + \gamma\, Q(s',a')\right]$
- Note
  - SARSA waits until an action is taken and backs up the Q-value for that action. Learns the Q-function from actual transitions.
  - On-policy algorithm
  - More realistic if the policy is partly controlled by other agents. Learns from more realistic values.

Active Reinforcement Learning

- **Q-learning so far**
  - Q-learning allows action selection because we are learning the Q(s,a) function.
  - This means there is a policy that can be derived from the Q function learned.
  - Should the agent follow this policy exactly or should it explore at times?
- **Active RL**
  - The agent can select actions
  - Actions play two roles
    - A means to collect reward (exploitation)
    - **Help in acquiring a model of the environment (exploration)**
- **Exploration vs. Exploitation trade-off**
    - Act according to the current optimal (based on Q-Values)
    - Pick a different action to explore.
    - Example: a new tea stall opens in IIT (should you try the new one or stick to the old one? Goal is to maximize tea utility over time

Exploration Strategy

- **How to force exploration?**
- **An ε-greedy approach**
  - Every time step: either pick a random action or act on the current policy
  - With (small) probability ε, pick a random action
  - With (large) probability **1-ε**, act based on the current policy (based on the current Q –values in the table that the agent is updating)
- **What is the problem with ε-greedy?**
  - It takes a long time to explore.
  - Exploration is *not directed* towards states of which we have less information.

Exploration Functions

- How to **direct exploration** towards state-action pairs that are less explored?

- **Exploration function**
  - Trades off *exploitation* (preference for high values of u) vs. *exploration* (preference for actions that have not been tried out as yet).
  - f(u,n): Increasing in u and decreasing in n.

$$f(u, n) = u + k/n$$

- **What is the exploration function trying to achieve?**
  - The exploration function provides an optimistic estimate of the best possible value attainable in any state.
  - Makes the agent think that there is high reward propagated from states that are explored less.

Exploratory Q-Learning

- **In Q-learning**
  - Explicitly encode the value of exploration in the Q-function
- **Exploratory Q-Learning**

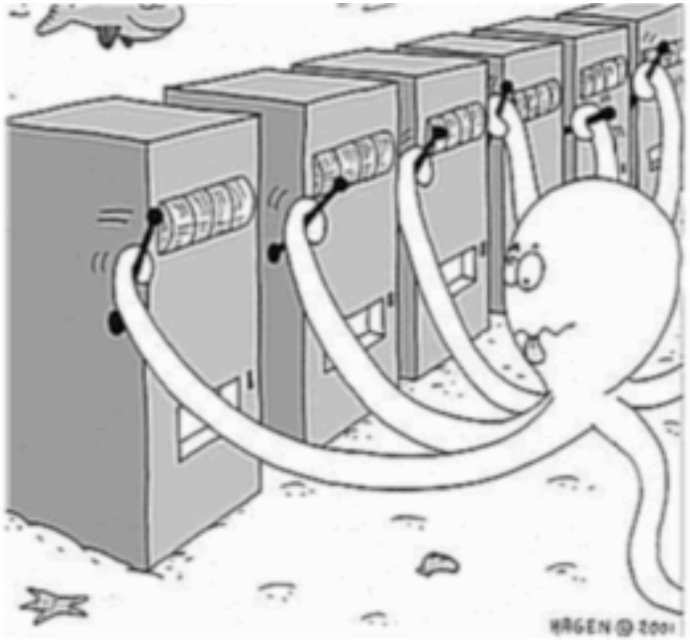
$$Q(s,a) \leftarrow_{\alpha} R(s,a,s') + \gamma \max_{a'} \underbrace{f(Q(s',a'),N(s',a'))}_{\textit{Modified update}}$$

- **Effects**
  - The lower the N(s', a') is the higher is the exploration bonus.
  - The exploration bonus makes those states favorable which lead to unknown states (propagation).
  - Will have a cascading effect when an exploration action is there.

Multi-arm Bandits

Multi-armed bandits are equivalent to a one state MDP. Goal is to learn a policy that picks actions such that in the expected rewards are maximized.

- Multi-armed bandit is a tuple of $(\mathcal{A}, \mathcal{R})$
- $\mathcal{A}$: known set of m actions (arms)
- $\mathcal{R}^a(r) = \mathbb{P}[r \mid a]$ is an unknown probability distribution over rewards
- At each step t the agent selects an action $a_t \in \mathcal{A}$
- The environment generates a reward $r_t \sim \mathcal{R}^{a_t}$
- Goal: Maximize cumulative reward $\sum_{\tau=1}^{t} r_\tau$



Source: Emma Brunskill (CS232 Course) Lecture 10

Toy Example: *Treatment planning*

- Consider deciding how to best treat patients with broken toes
- Imagine have 3 possible options:
  - Do Surgery
  - Perform buddy taping the broken toe with another toe,
  - Do Nothing
- Outcome measure / reward is binary variable: whether the toe has healed (reward +1) or not healed (reward 0) after 6 weeks, as assessed by x-ray

- We can model this problem as a multi-arm bandit problem with 3 arms
- Imagine true (unknown) Bernoulli reward parameters for each arm (actions) are
  - surgery: $Q(a^1) = \theta_1 = .95$
  - buddy taping: $Q(a^2) = \theta_2 = .9$
  - doing nothing: $Q(a^3) = \theta_3 = .1$

Source: Emma Brunskill (CS232 Course) Lecture 10

Toy Example: *Treatment Planning*

- We consider algorithms that estimate $\hat{Q}_t(a) \approx Q(a) = \mathbb{E}[R(a)]$
- Estimate the value of each action by Monte-Carlo evaluation

$$\hat{Q}_t(a) = \frac{1}{N_t(a)} \sum_{t=1}^{T} r_t \mathbb{1}(a_t = a)$$

Avg. the rewards for each action.

- The **greedy** algorithm selects the action with highest value

$$a_t^* = \arg\max_{a \in \mathcal{A}} \hat{Q}_t(a)$$

- The ϵ-**greedy** algorithm proceeds as follows:
  - With probability $1 - \epsilon$ select $a_t = \arg\max_{a \in \mathcal{A}} \hat{Q}_t(a)$
  - With probability ϵ select a random action

Exploration vs. Exploitation trade off! If the reward variance is high then the epsilon-greedy approach works better than simply greedy.

Source: Emma Brunskill (CS232 Course)

Upper Confidence Bound

- Greedy actions are those that look best at the present, but some of the other actions may be better.
- Epsilon-greedy forces the non-greedy actions to be tried.
  - Indiscriminately, with no preference for those actions that are nearly greedy or particularly uncertain.
- It would be better to select among the non-greedy actions according to their potential for being optimal.
  - Take into account how close their estimates are to being maximal and the uncertainties in their estimates.

Upper Confidence Bound

- Upper Confidence Bound (UCB) for action selection
  - Square-root term is a measure of uncertainty or variance in the estimate of the action's value.
    - When a is selected then $N_t(a)$ is incremented. The uncertainty reduces as denominator increases.
    - Each time when a is not selected then t increases but $N_t(a)$ does not. The uncertainty estimate increases (numerator).
  - Natural logarithm
    - Increases get smaller over time but are unbounded; all actions will eventually be selected.
    - Actions with lower value estimates will be selected with lower frequency.

$$a_t = \arg\max_{a \in \mathcal{A}} [\hat{Q}(a) + \sqrt{\frac{2 \log t}{N_t(a)}}]$$

Upper Confidence Bound

- True (unknown) parameters for each arm (action) are
  - surgery: $Q(a^1) = \theta_1 = .95$
  - buddy taping: $Q(a^2) = \theta_2 = .9$
  - doing nothing: $Q(a^3) = \theta_3 = .1$
- UCB1 (Auer, Cesa-Bianchi, Fischer 2002)
  1. Sample each arm once
     - Take action a^1 (r ~Bernoulli(0.95)), get $+1$, $\hat{Q}(a^1) = 1$
     - Take action a^2 (r ~Bernoulli(0.90)), get $+1$, $\hat{Q}(a^2) = 1$
     - Take action a^3 (r ~Bernoulli(0.1)), get 0, $\hat{Q}(a^3) = 0$
  2. Set $t = 3$, Compute upper confidence bound on each action

$$UCB(a) = \hat{Q}(a) + \sqrt{\frac{2 \log t}{N_t(a)}}$$

  3. $t = 3$, Select action $a_t = \arg\max_a UCB(a)$,
  4. Observe reward 1
  5. Compute upper confidence bound on each action

Source: Emma Brunskill (CS232 Course)

Example

Example of K-arm bandits in practice. Figure is for a 10-arm bandit test bed that learns the rewards for the 10 options. The average reward collected goes up as the learning progresses.

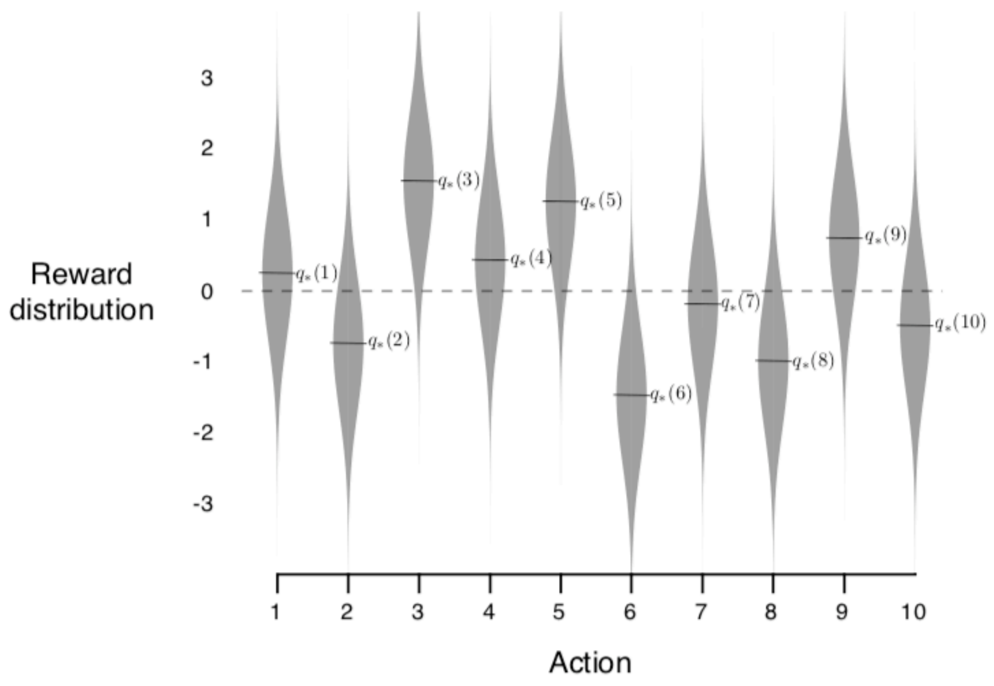
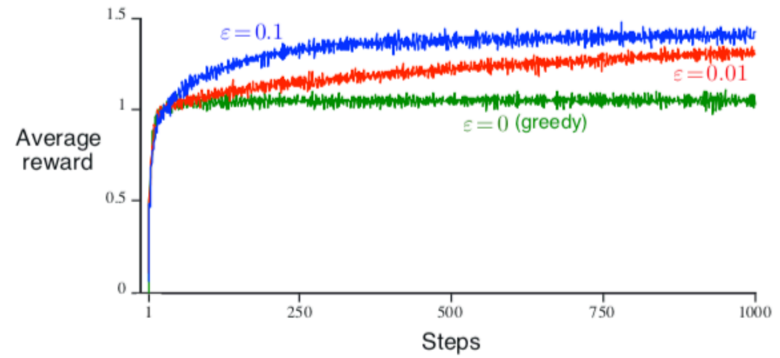


Figure 2.1: An example bandit problem from the 10-armed testbed. The true value $q_*(a)$ of each of the ten actions was selected according to a normal distribution with mean zero and unit variance, and then the actual rewards were selected according to a mean $q_*(a)$, unit-variance normal distribution, as suggested by these gray distributions.



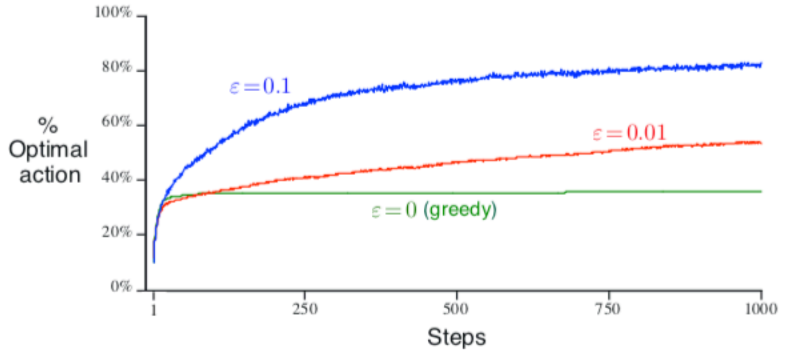


Figure 2.2: Average performance of ε-greedy action-value methods on the 10-armed testbed. These data are averages over 2000 runs with different bandit problems. All methods used sample averages as their action-value estimates.

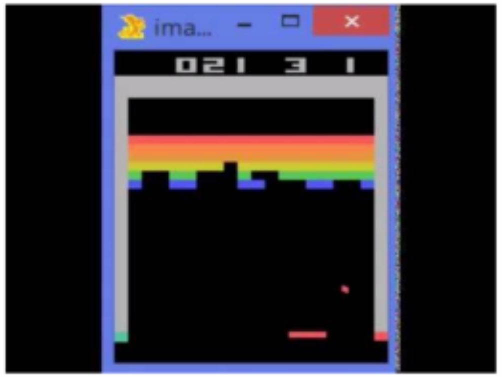
Figure from S&B Book Chapter 2

Problem of Generalization

- Problem when the state space is very large
  - Visiting all states is not possible at training time.
  - Memory issue: cannot fit the q-table in memory.
  - Need a succinct representation of the state.

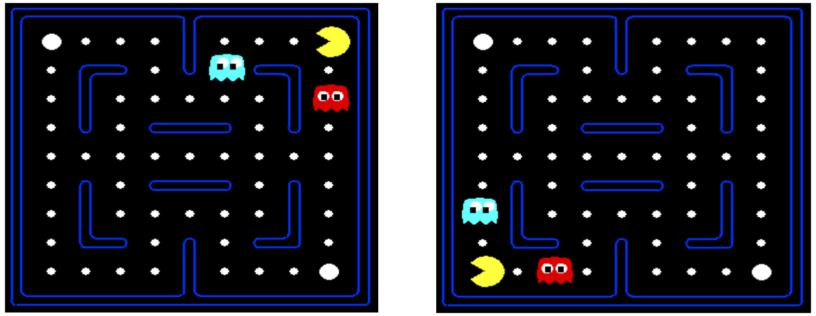
- Need for Generalization
  - Learn from *small number* of training states encountered in training
  - Generalize that experience to *new*, *similar situations* that not encountered before

- Feature-based Representations
  - Features or properties are functions from states to real numbers (often 0/1) that capture important properties of the state.
  - Example features that can be computed from the state
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts



State space is raw pixels, the state space size is large, cannot experience all states.

$$\left(256^{100\times 200}\right)^3$$



The states are similar. But for Q-learning it will be a different entry. It does not know that in essence these states are the same.

Linear Value Functions

- Using a feature representation, a q function (or value function) can be expressed for any state using a set of weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Now the goal of Q-learning is to estimate these weights from experience. Once the weights are learned the resulting Q-values will hopefully be close to the true Q values.

- Main consequence:
  - A new state that shares features with a previously seen state will have the same Q-values.
- Disadvantage
  - If there are less features, actually different states (good and bad states) may start looking the same.

Approximate Q-learning

- Q-learning with linear Q-functions: $Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$

  transition $= (s, a, r, s')$

  $Q(s,a) \leftarrow Q(s,a) + \alpha\,[\text{difference}]$ **Exact Q function updates**

  $w_m \leftarrow w_m + \alpha \left[r + \gamma \max_{a} Q(s',a') - Q(s,a) \right] f_m(s,a)$ **Approximate Q function updates**

  "target" "prediction"

- Interpretation:
  - Adjust weights of active features.
  - If the difference is positive (what we get is higher than previous) and the feature value is 1 then the weight increases and the q value increases. No change if the feature value is 0.

Conclusions

- Learning from reinforcement applies to problems where there is no knowledge of which states have good rewards and how the actions lead to new states,
- Various approaches for performing RL: estimating a model, estimating the value function or directly optimizing the policy (not covered yet). Variation with whether we learn from the whole episode or from each transition.
- Data is essentially experience of the agent. There is evaluative feedback not prescriptive feedback. RL is different from supervised learning.
- Fundamental tradeoff between exploration and exploitation. Generalization is a challenge in large state spaces.