COL333/671: Introduction to AI

Semester I, 2024-25

Local Search Algorithms

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Outline

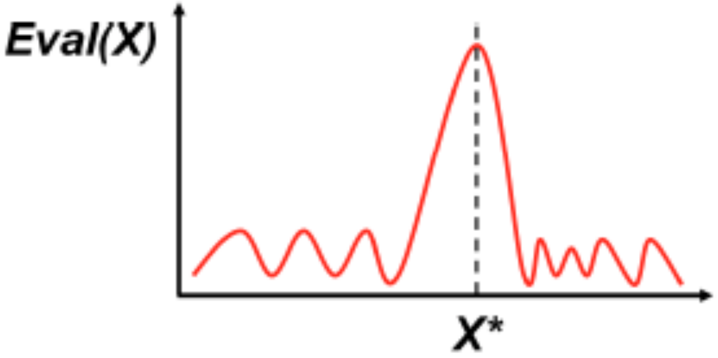
- Last Class
  - Informed Search
- This Class
  - Local Search Algorithms
- Reference Material
  - AIMA Ch. 4.1

Acknowledgement

These slides are intended for teaching purposes only. Some material has been used/adapted from web sources and from slides by Doina Precup, Dorsa Sadigh, Percy Liang, Mausam, Dan Klein, Nicholas Roy and others.

Search Methods for Discrete Optimization

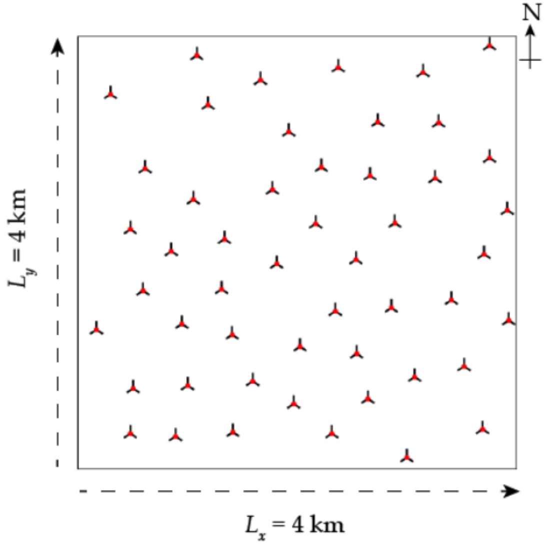
- **Setting**
  - A set of discrete states, *X*.
  - An objective/evaluation function assigns a "goodness" value to a state, *Eval(X)*
  - Problem is to search the state space for the state, *X** that maximizes the objective.



- **Searching for the optimal solution can be challenging. Why?**
  - The number of states is very large.
    - Cannot simply enumerate all states and find the optimal.
  - We can only evaluate the function.
    - Cannot write it down analytically and optimize it directly.

Key Idea
- **Searching for "the optimal" solution is very difficult.**
- **Question is whether we can search for a reasonably good solution.**

Example – Windmill Placements



Problem: Optimizing the locations of windmills in a wind farm

- An area to place windmills.
- Location of windmills affects the others. Reduced efficiency for those in the wake of others.
- Grid the area into bins.
- A large number of configurations of windmills possible.
- Given a configuration we can evaluate the total efficiency of the farm.
- Can neither enumerate all configurations nor optimize the power efficiency function analytically.
- **Goal is to <u>search</u> for the <u>configuration</u> that maximizes the efficiency.**

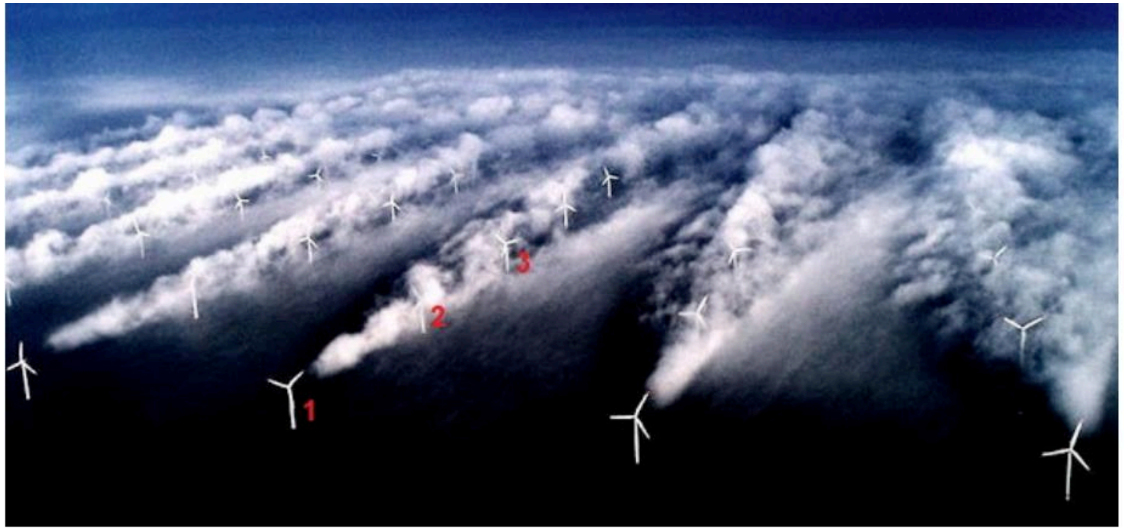
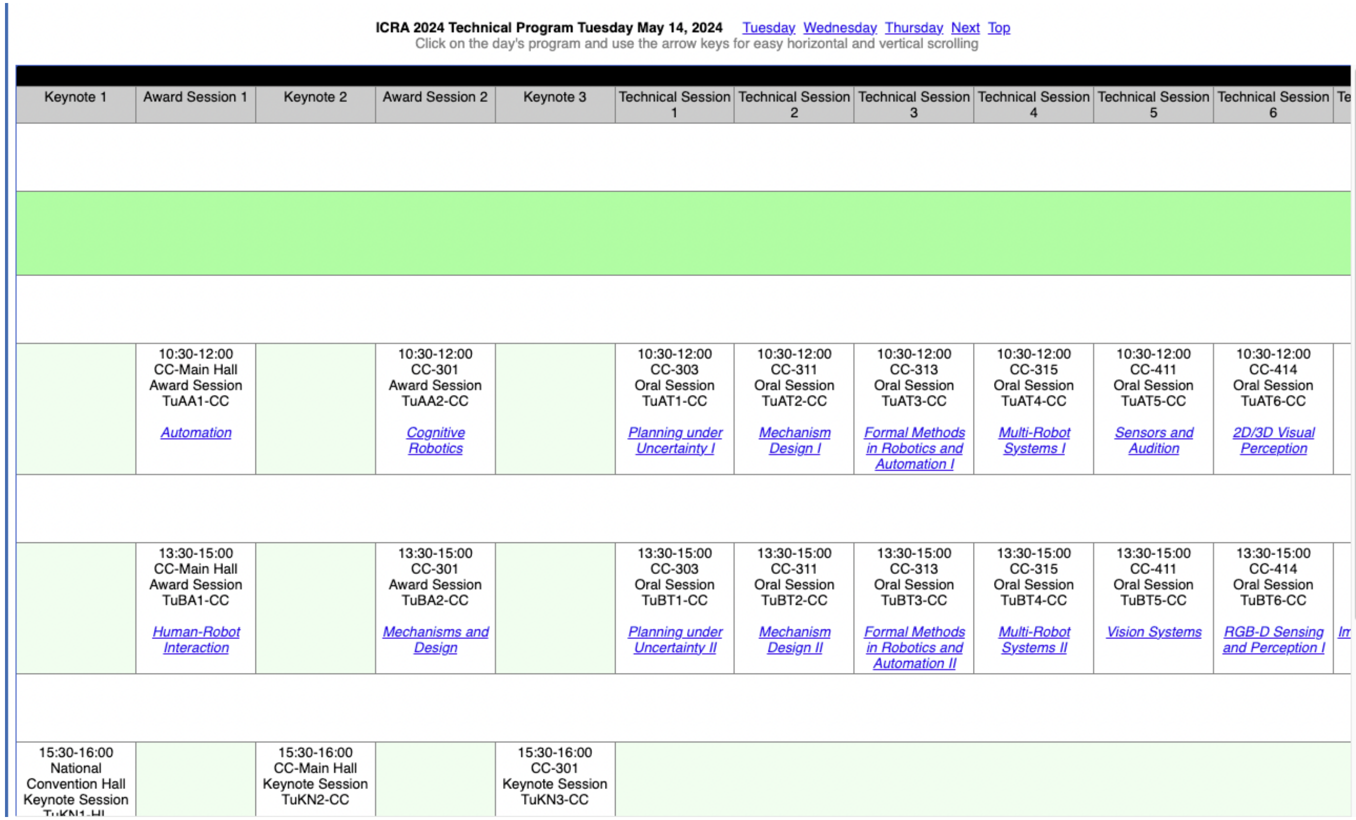


Figure 5: Turbines experiencing multiple wakes. As an example, turbine 3 is experiencing wake effects from both turbine 1 and 2. Image adopted from [4].

Inspired from this example: https://www.shell.in/energy-and-innovation/ai-hackathon/_jcr_content/par/textimage_1834506119_1619963074.stream/1612943059963/4b0a86b7cc0fe7179148284ffed9ef33524c2816/windfarm-layout-optimisation-challenge.pdf

Example: Conference Scheduling



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	10:30-12:00 CC-Main Hall Award Session TuAA1-CC *Automation*		10:30-12:00 CC-301 Award Session TuAA2-CC *Cognitive Robotics*		10:30-12:00 CC-303 Oral Session TuAT1-CC *Planning under Uncertainty I*	10:30-12:00 CC-311 Oral Session TuAT2-CC *Mechanism Design I*	10:30-12:00 CC-313 Oral Session TuAT3-CC *Formal Methods in Robotics and Automation I*	10:30-12:00 CC-315 Oral Session TuAT4-CC *Multi-Robot Systems I*	10:30-12:00 CC-411 Oral Session TuAT5-CC *Sensors and Audition*	10:30-12:00 CC-414 Oral Session TuAT6-CC *2D/3D Visual Perception*	
	13:30-15:00 CC-Main Hall Award Session TuBA1-CC *Human-Robot Interaction*		13:30-15:00 CC-301 Award Session TuBA2-CC *Mechanisms and Design*		13:30-15:00 CC-303 Oral Session TuBT1-CC *Planning under Uncertainty II*	13:30-15:00 CC-311 Oral Session TuBT2-CC *Mechanism Design II*	13:30-15:00 CC-313 Oral Session TuBT3-CC *Formal Methods in Robotics and Automation II*	13:30-15:00 CC-315 Oral Session TuBT4-CC *Multi-Robot Systems II*	13:30-15:00 CC-411 Oral Session TuBT5-CC *Vision Systems*	13:30-15:00 CC-414 Oral Session TuBT6-CC *RGB-D Sensing and Perception I*	*In*
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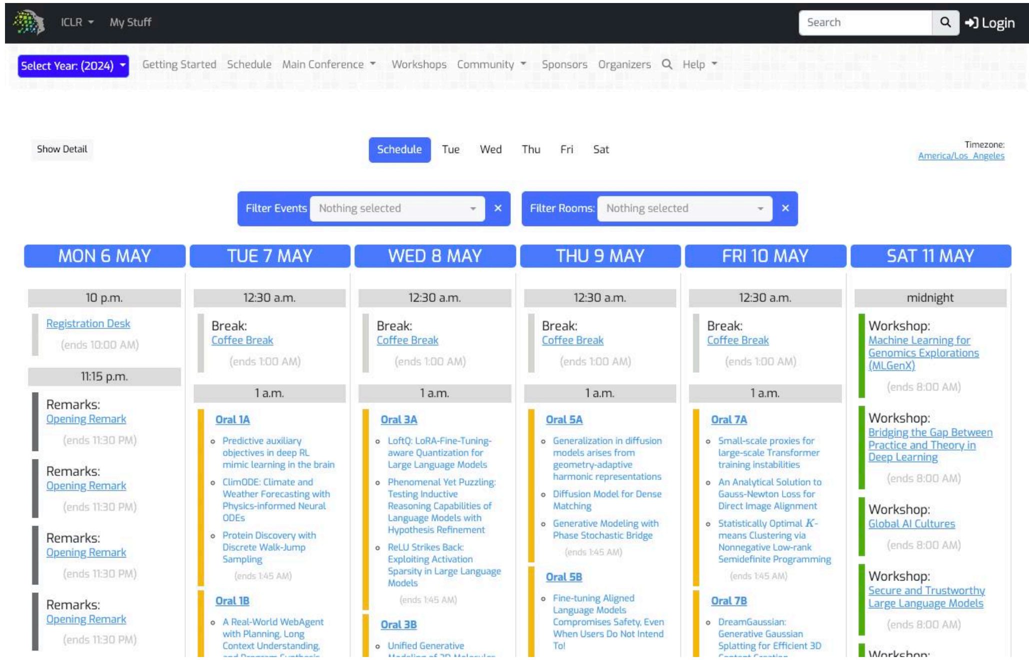
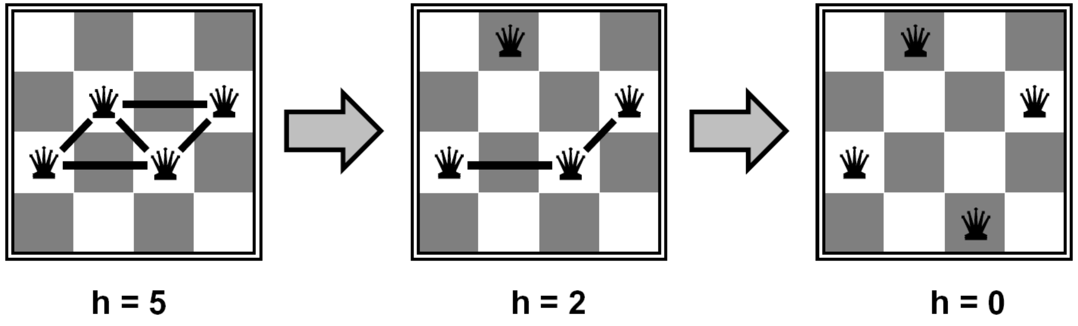


Assign papers that are similar in a session. Avoid conflicts between sessions.

Example

4-Queens Problem

- Discrete set of states: 4 queens in 4 columns (4^4 = 256 states)
- Goal is to find a configuration such that there are no attacks.
  - Moving a piece will change the configuration.
- Any configuration can be evaluated using a function
  - h(x) = number of attacks (number of violated binary constraints)
- Search for the configuration that is optimal such that h = 0.



Example

Formally

Variables: x_0, x_1, x_2, x_3 where x_i is the row position of the queen in column i, where $i \in \{0, 1, 2, 3\}$. Assume that there is one queen per column.

Domains: $x_i \in \{0, 1, 2, 3\}\ \forall i$.

Initial state: 4 queens on the board in random row positions.

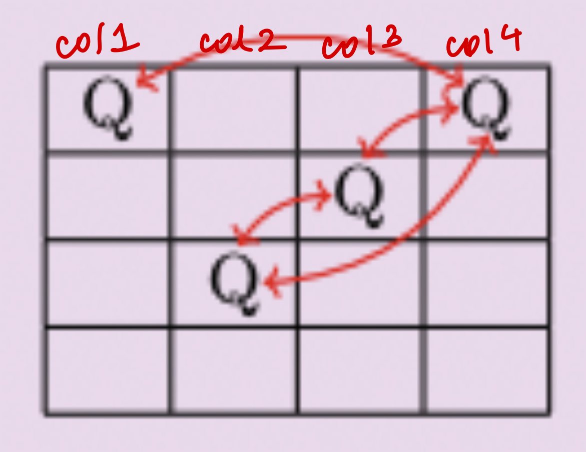
Goal state: 4 queens on the board with no pair of queens attacking each other.

Neighbour relation:

- Version A: move a single queen to a different row in the same column.
- Version B: swap the row positions of two queens.

Cost function: the number of pairs of queens attacking each other, directly or indirectly.

Number of attacks are 4.



Local Search Methods

- **Keep track of a single "current" state**
  - We need a principled way to search/explore the state space hoping to find the state with the optimal evaluation.
  - Do not maintain a search tree as we need the solution not the path that led to the solution.
  - Only maintain a single current state.

What if we get stuck in a loop??

- **Perform local improvements**
  - Look for alternatives in the vicinity of that solution
  - Try to move towards more better solutions.

Hill-climbing Search

Let S be the start node and let G be the goal node.

Let h(c) be a heuristic function giving the value of a node

Let c be the start node

Loop

    Let c’ = the highest valued neighbor of c

    If h(c) ≥ h(c’) then return c

    c = c’

Start at a configuration. Evaluate the neighbors. Move to the highest valued neighbor if its value is higher than the current state. Else stay.

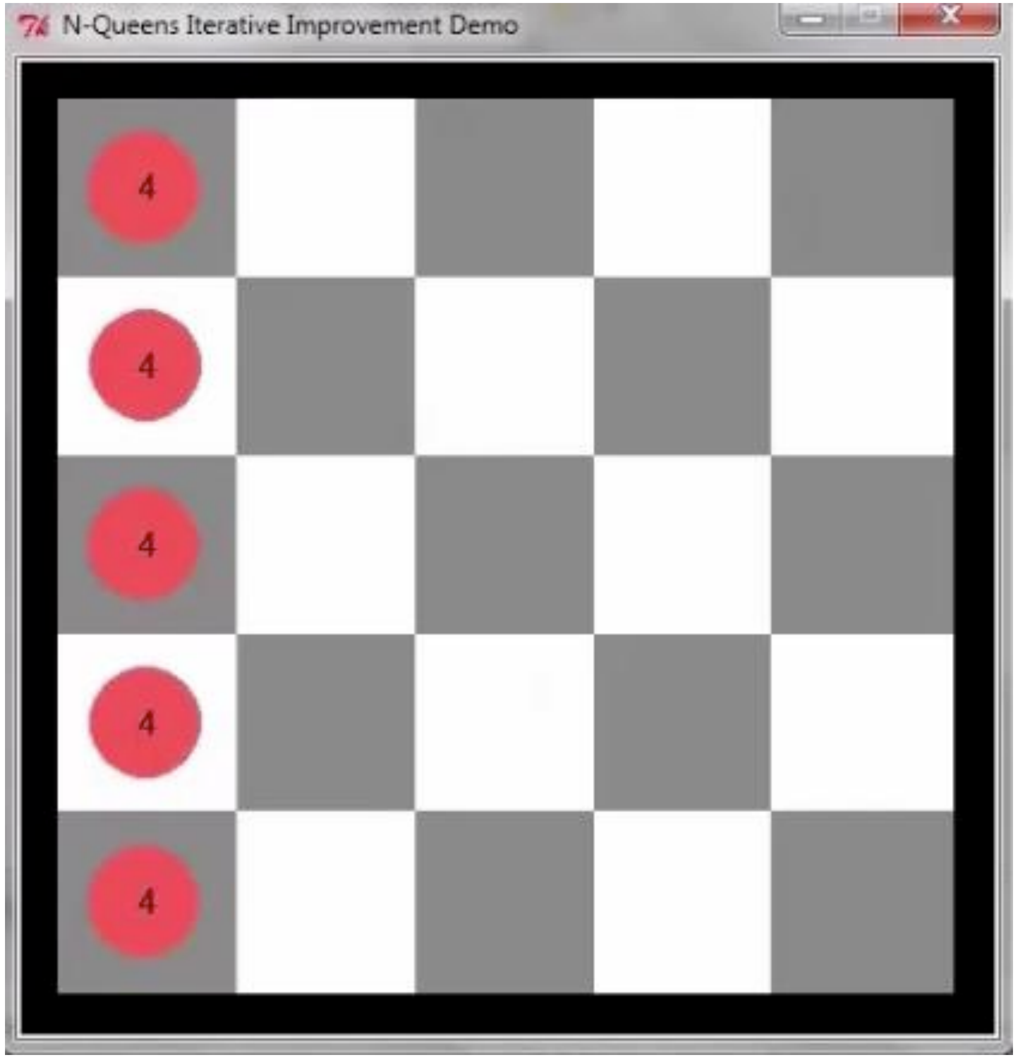
Hill climbing

“Like climbing Everest in thick fog with amnesia”

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node
   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
      neighbor ← a highest-valued successor of current
      if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
      current ← neighbor
   end
```

Hill climbing for 4 -queens

- Select a column and move the queen to the square with the fewest conflicts.
- Perform local modifications to the state by changing the position of one piece till the evaluation is minimum.
- Evaluate the possibilities from a state and then jump to that state.



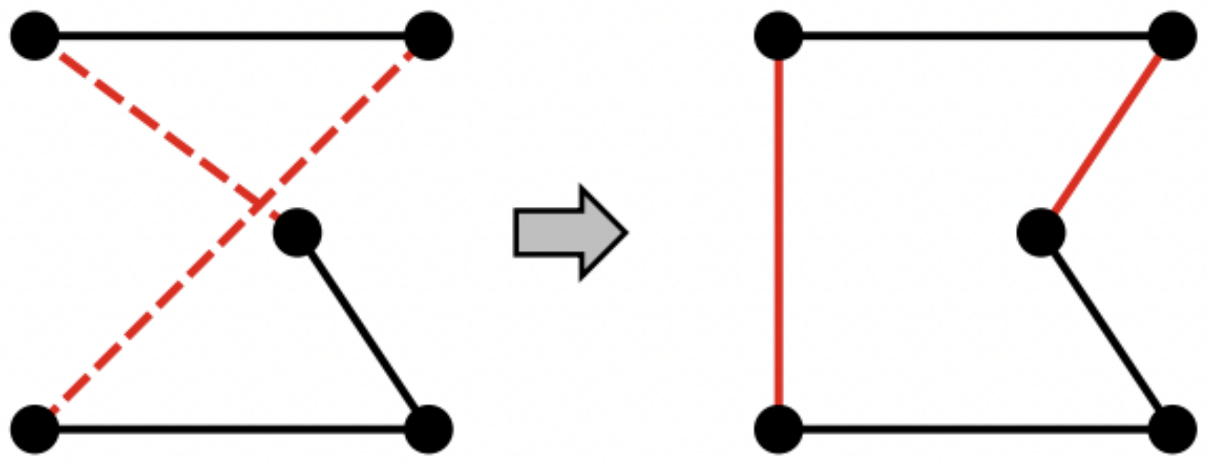
Example

- Local search looks at a state and its <u>local</u> neighborhood.
- <u>Not</u> constructing the entire search tree.
- Consider local modifications to the state. Immediately jump to the next promising neighbor state. Then start again.
- *Highly scalable.*

Example: Idea of local improvements

Locally improving a solution for a Travelling Salesperson Problem.

Start with any complete tour, perform pairwise exchanges

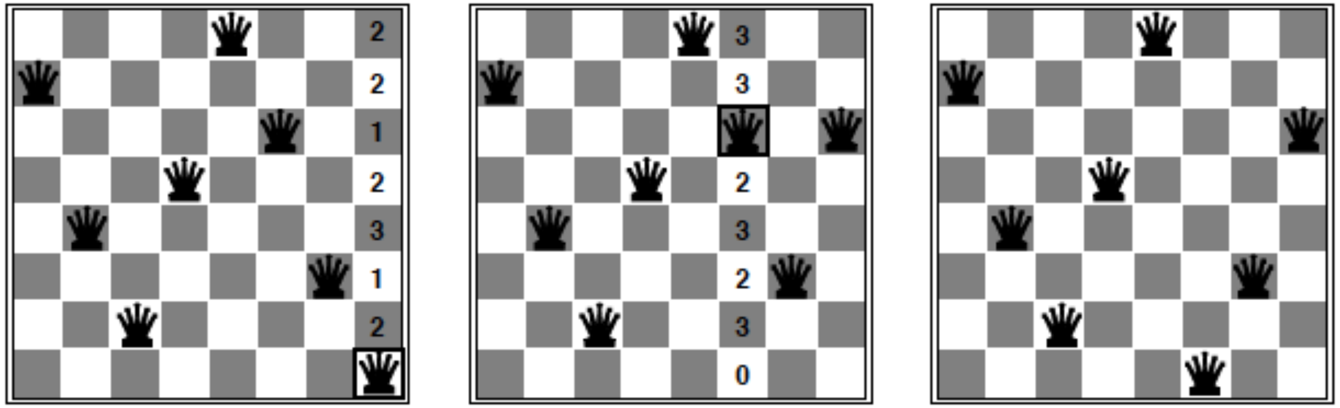


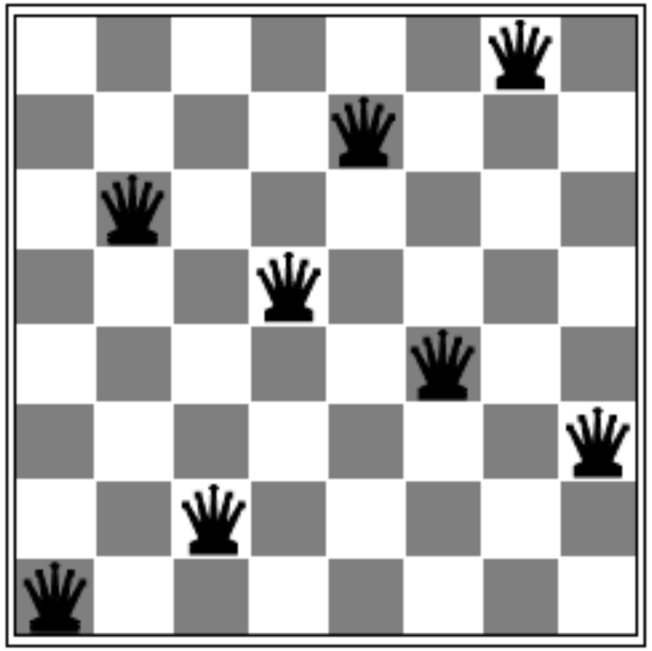
Variants of this approach get within 1% of optimal very quickly with thousands of cities

The idea of making local improvements to a candidate solution is a general and widely applicable technique.

8-Queens Problem

Is this an optimal state?

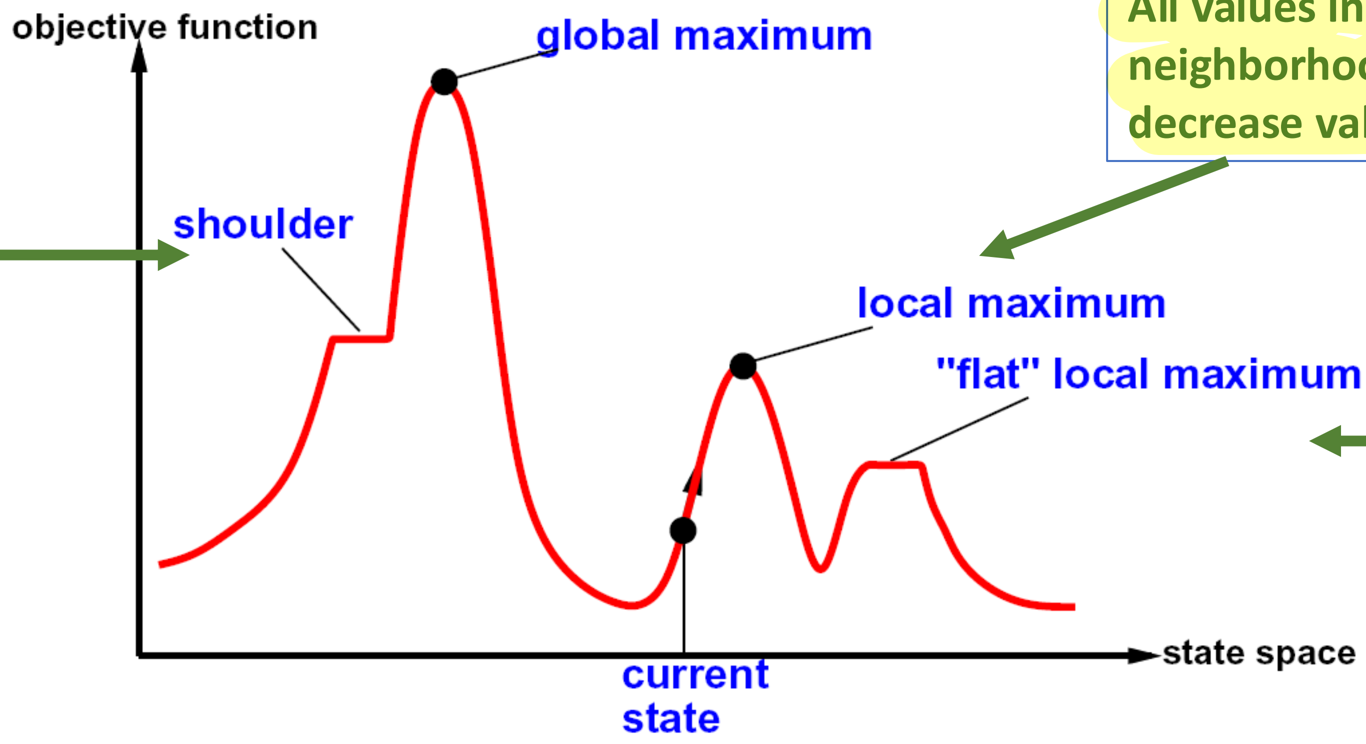




Local minima (h = 1). Every successor has a higher cost.

Issue: search reaches a solution where it cannot be improved - a local minimum.

Core Problem in Local Search



All values in the neighborhood decrease values

Neighborhood values are equivalent, need to search for a longer distance to find a path to a higher value

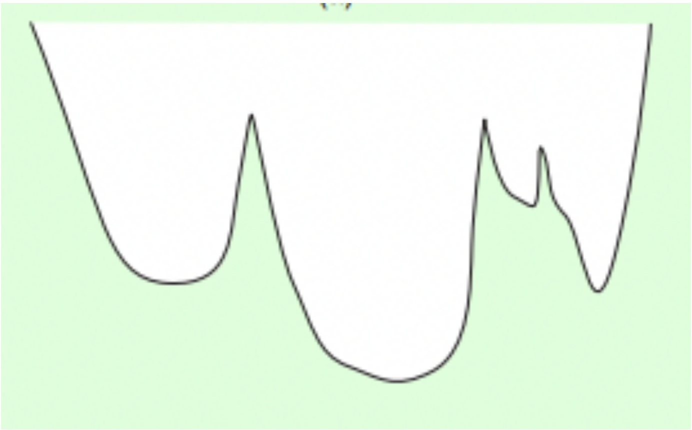
Neighborhood values are equivalent beyond which values only decrease

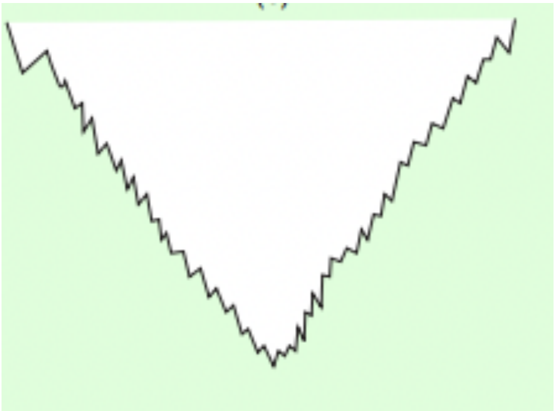
- Hill climbing prone to local maxima. Neighbors may not be of higher value. Search will stop at a sub-optimal solution
- Locally optimal actions may not lead to the globally optimal solution

Escaping local minima: Adding randomness

- **Random Re-starts**
  - A series of searches from randomly generated initial states.
- **Random Walk**
  - Pick ”any” candidate move (whether improves the solution or not).

Q: Which method to use for the following cost surfaces? Random re-starts or random walk?

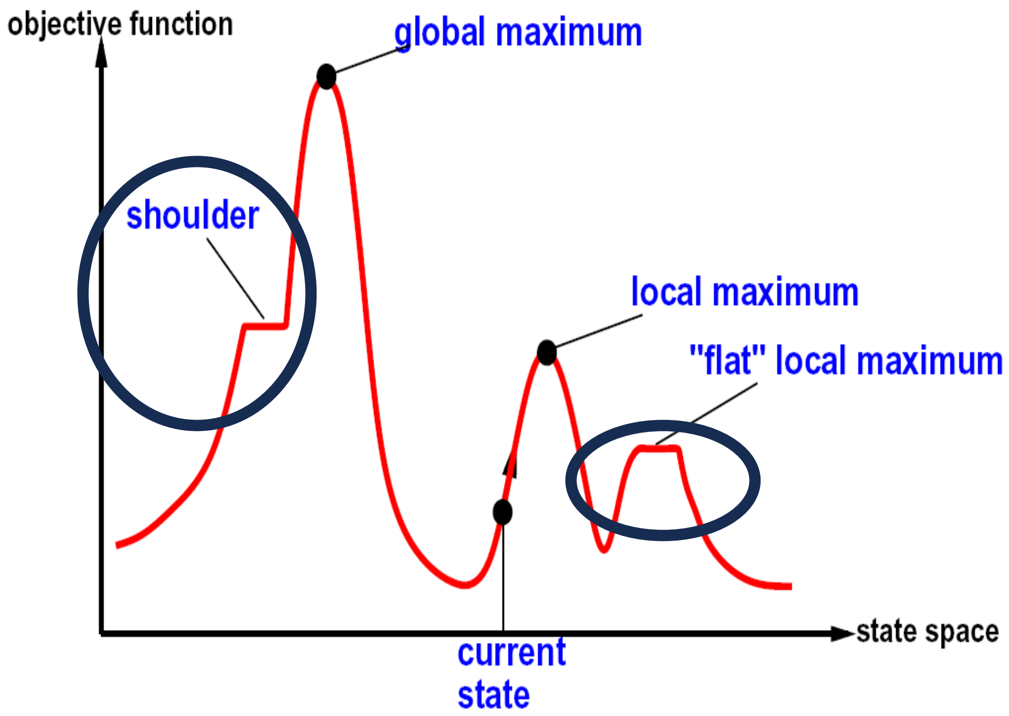




Escaping local minima: Adding randomness

- **Escaping flat local minima (shoulders)**
  - When local search reaches a flat area, that is, when all the neighbours have the same cost as the current state, it terminates right away
  - Keep moving - strategy
    - Make sideways moves for a few steps.

- ***Stochastic* Hill Climbing**
  - Instead of picking the *best move, pick any* move that produces an *improvement*.



Looking for Solution from Multiple Points

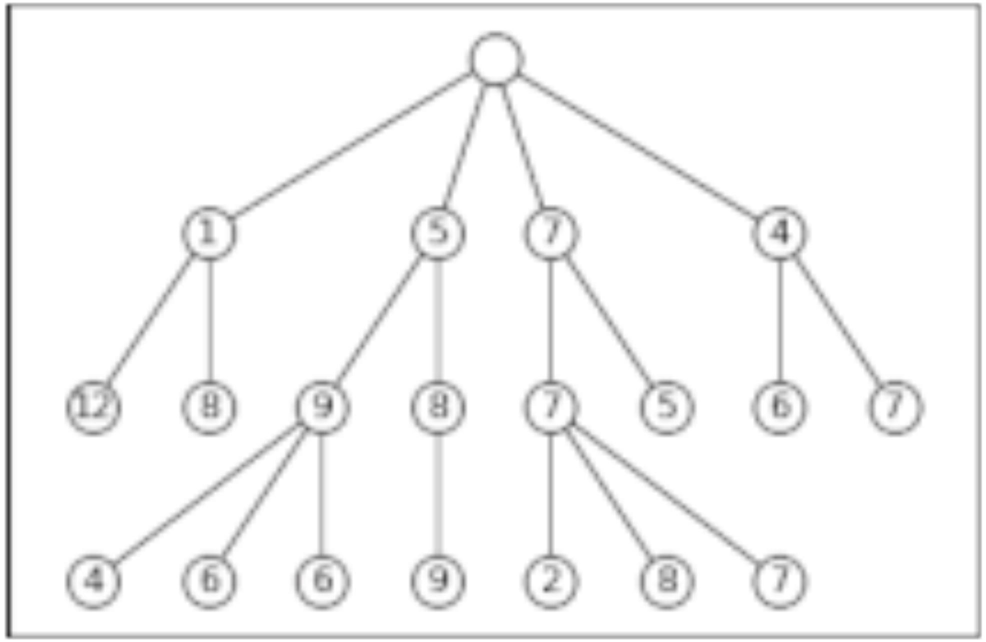
- **Local Beam Search**
- Algorithm
  - Track k states (rather than 1).
  - Begin with k randomly sampled states.
  - Loop
    - Generate successors of each of the k-states
    - If anyone has the goal, the algorithm halts
    - Otherwise, select only the k-best successors from the list and repeat.

one state can give more than 1 good successor.

- Note:
  - Each run is not independent, information is passed between parallel search threads.
  - Promising states are propagated. Less promising states are not propagated.
  - Problem: states become concentrated in a small region of space.

Beam Search is a General Search Technique

- Beam search is a general idea (see right figure).
- Instead of considering all solutions at a level, consider only the top-k.
- Note: usually our memory is finite in size, there is an upper bound on the number of states that can be kept.
- In general, it is an approximate search method.

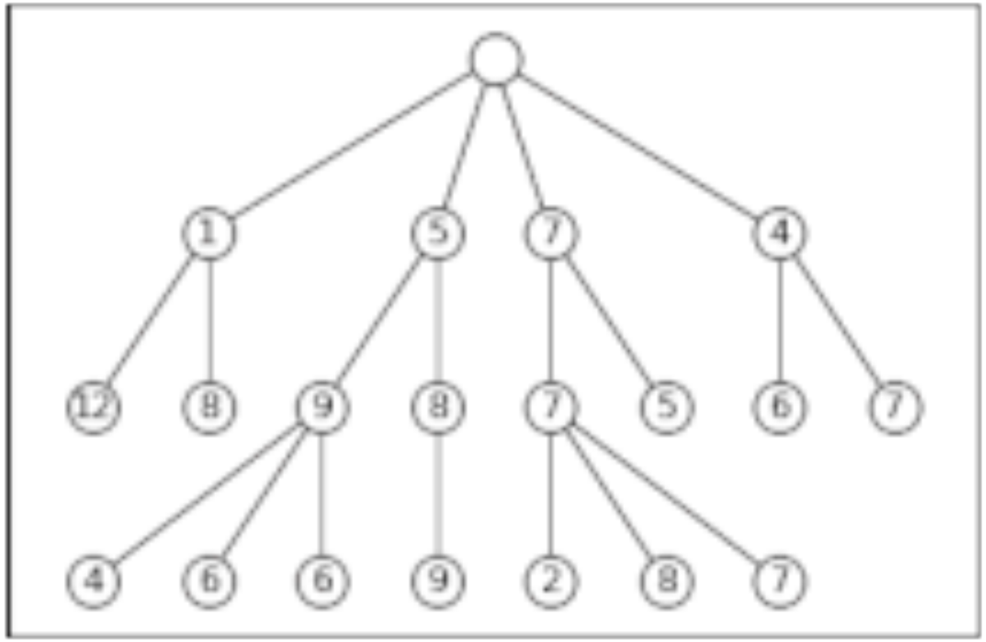


Beam search is a general idea. Here, shown in the context of a tree search. Beam size is 3. For local search we don't construct the full tree.

Source: wikipedia

"Stochastic" Beam Search

- **Local beam search**
  - Problem: states become concentrated in a small region of space
  - Search degenerates to hill climbing
- ***Stochastic* beam search**
  - Instead of taking the best k states
  - Sample k states from a distribution
  - Probability of selecting a state *increases* as the *value* of the state.



Instead of top k, sample k given a probability.

Source: wikipedia

Simulated Annealing

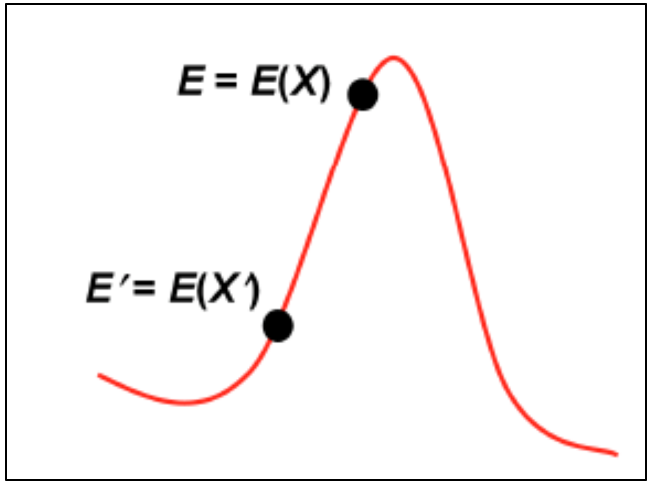
- In case of an improving move – move there.
- But allow some apparently **bad moves** - to escape local maxima.
- **Decrease** the size and the frequency of bad moves over time.

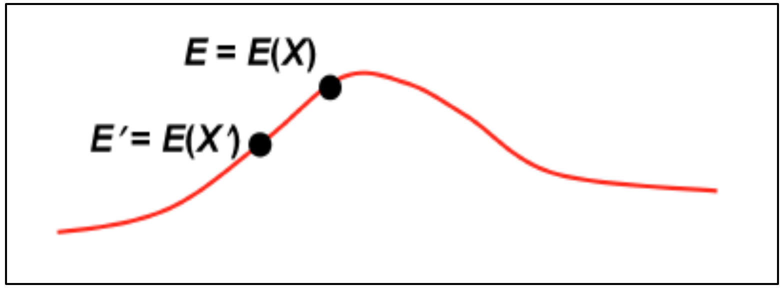
- Algorithm sketch
  1. Start at initial configuration X of value E (high is good)
  2. Repeat:
     (a) Let X_i be a *random neighbor* of X and E_i be its value
     (b) If $E < E_i$ then let $X \leftarrow X_i$ and $E \leftarrow E_i$
     (c) Else, *with some probability* p, still accept the move: $X \leftarrow X_i$ and $E \leftarrow E_i$
- Best solution ever found is always remembered

A form of Monte-Carlo Search. Move around the environment to explore it instead of systematically sweeping. Powerful technique for large domains.

Simulated Annealing: How to decide *p*?

- Considering a move from state of value E to a lower valued state of E'. *That is considering a sub-optimal move (E is higher than E').*
- If (E – E') is large:
  - Likely to be close to a promising maximum.
  - Less inclined to to go downhill.
- If (E – E') is small:
  - The closest maximum may be shallow
  - More inclined to go downhill is not as bad.





Simulated Annealing: Selecting Moves

- If the new value E_i is ***better*** than the old value E, move to X_i

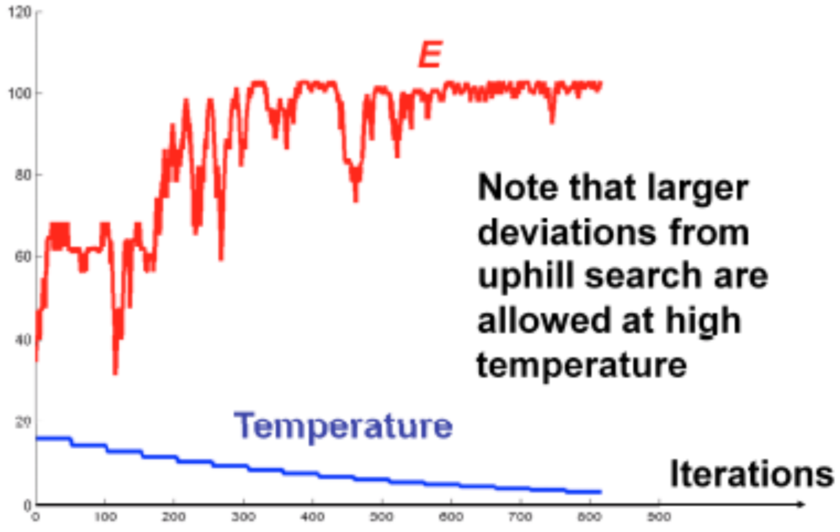
- If the new value is ***worse*** (E_i < E) then move to the neighboring solution as per *Boltzmann* distribution.

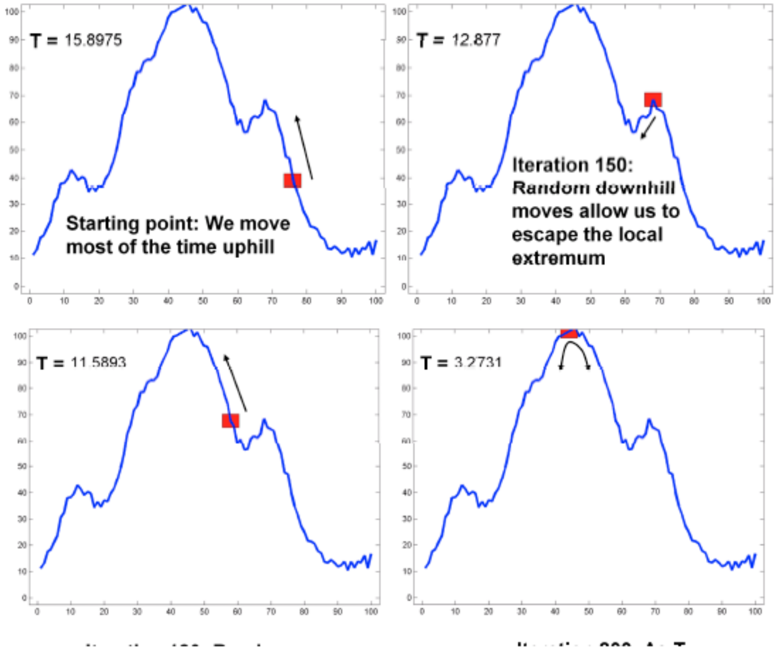
$$\exp\left(-\frac{E - E_i}{T}\right)$$

- Temperature (T>0)
  - **T is high**, exp is ~0, acceptance probability is ~1, high probability of acceptance of a worse solution.
  - **T is low**, the probability of moving to a worse solution is ~ 0, low probability of acceptance of a worse solution.
  - Schedule T to reduce over time.

Simulated Annealing

- **T is high**
  - The algorithm is in an *exploratory* phase
  - Even bad moves have a high chance of being picked
- **T is low**
  - The algorithm is in an *exploitation* phase
  - The “bad” moves have very low probability
- **If T is decreased slowly enough**
  - Simulated annealing is guaranteed to reach the best solution in the limit.



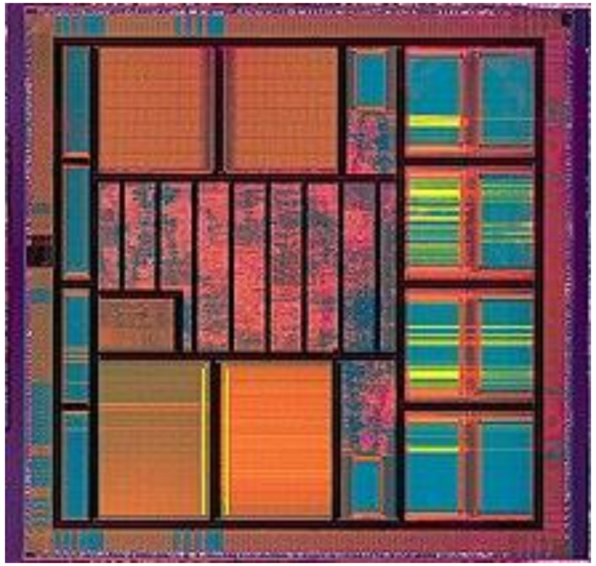


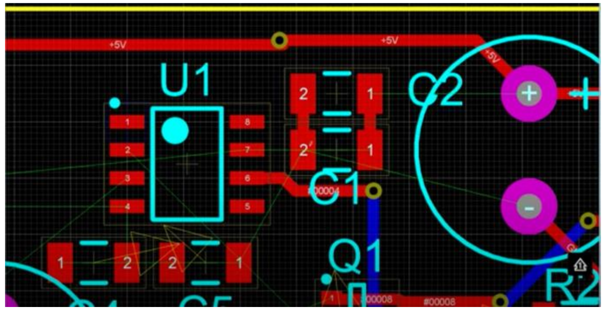
Able to escape local maxima.

Adding (some) memory: Tabu Search

- Local search loses track of the global cost landscape.
  - May frequently come back to the same state
- Introduce "memory" to prevent re-visits.
  - Maintain a finite-sized "tabu" list which remembers recently visited states so that one does not go towards them.
  - If a state proposed in the neighbourhood is in the tabu list do not go.

Motivating example: PCB layout with lower wire overlaps.





Search with Memory

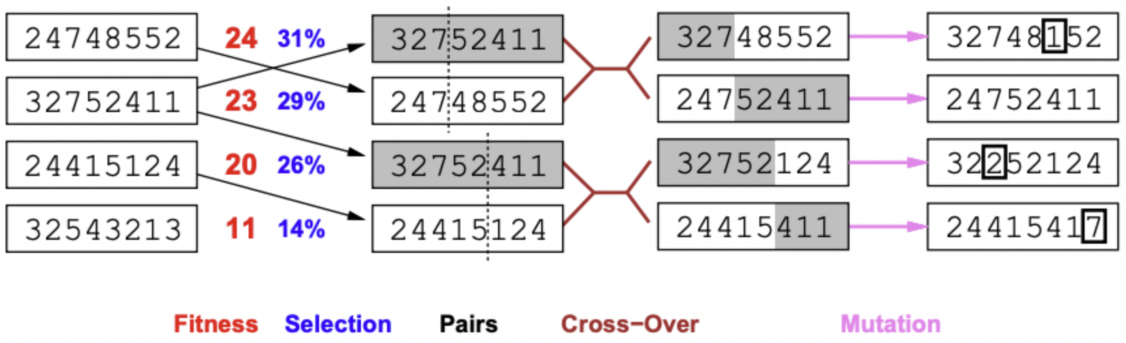
- Tabu Search
  - Maintain a tabu list of the k last assignments.
  - Don’t allow an assignment that is already on the tabu list.
  - If k = 1, we don’t allow an assignment of to the same value to the variable chosen.
  - Maintain a finite-sized tabu list (a form of local memory) which remembers recently visited states so that one does not go towards them.
  - Note: Tabu search allows for sub-optimal moves.
- Types of memory rules
  - Short-term: immediate states visited in the past.
  - Longer-term: guide the search towards certain regions of the search – where all have we explored in the past.
- Generalise to – searching locally by growing a tree for a short horizon and then picking a move (combining local and tree search).

Genetic Algorithms

- Idea
  - Variant of stochastic beam search: progression is by modifying a state.
  - Combine two states to generate the successor.
  - A mechanism to propose next moves in a different way.

- Ingredients
  - Coding of a solution into a string of symbols or bit-string
  - A fitness function to judge the worth of a state (or configuration)
  - A population of states (or configurations)

= stochastic local beam search + generate successors from **pairs** of states



Many variations:

how selection will be applied, what type of cross-over operators will be used, etc.

Selection of individuals according to a fitness function and pairing

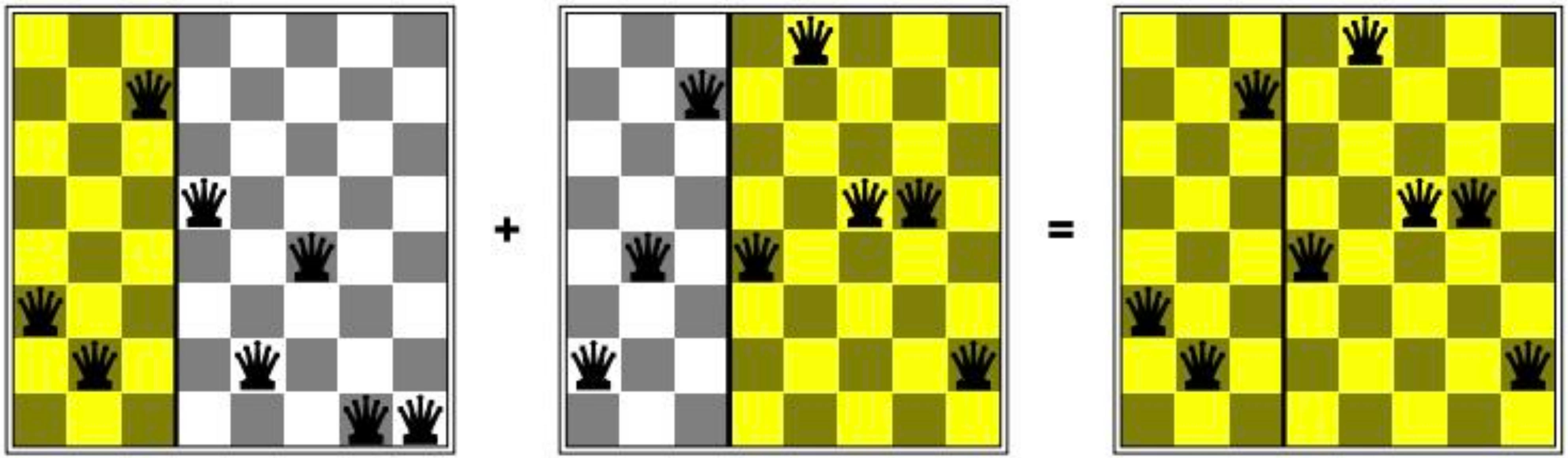
Calculation of the breaking points and recombination

According to a given probability elements in the string are modified.

Genetic Algorithms

https://rednuht.org/genetic_cars_2/

View as a way to propose moves – in an evolutionary way.



Advantage: ability to combine large blocks that evolved independently, impact the granularity of search.