Learning in Bayes Nets

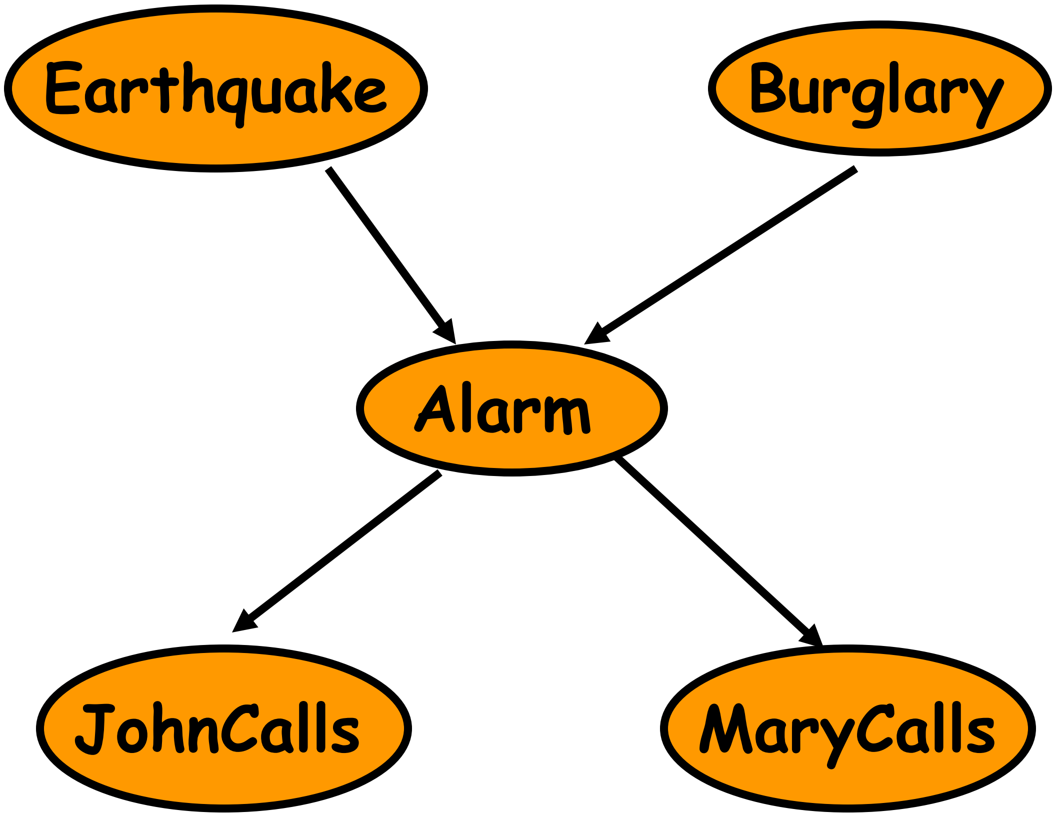
Mausam

(Based on slides by Stuart Russell, Subbarao Kambhampati, Dan Weld)

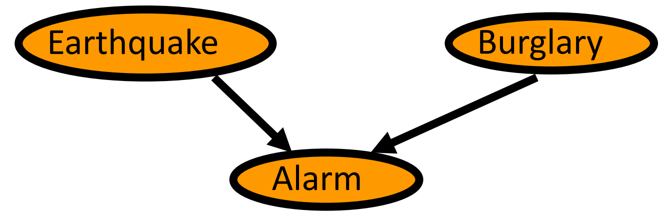
Parameter Estimation

- Learn all the CPTs in a Bayesian Net
- Data → Model → Queries
- Key idea: counting!

Burglars and Earthquakes



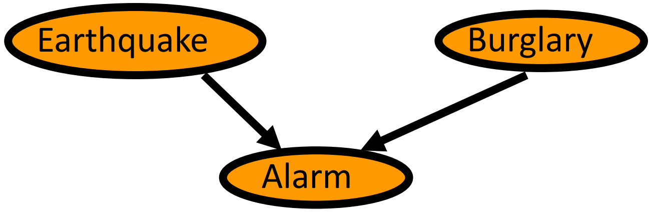
Counting



E	B	A	#
0	0	0	1000
0	0	1	10
0	1	0	20
0	1	1	100
1	0	0	200
1	0	1	50
1	1	0	0
1	1	1	5

	Pr(A|E,B)
e,b	
$e,\bar{b}$	
$\bar{e},b$	
$\bar{e},\bar{b}$	

Counting



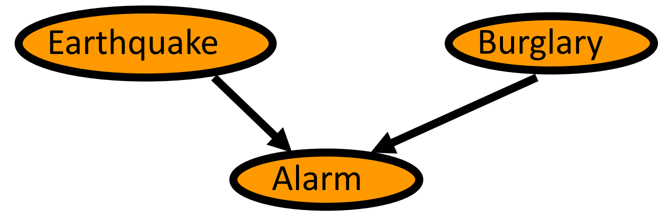
E	B	A	#
0	0	0	1000
0	0	1	10
0	1	0	20
0	1	1	100
1	0	0	200
1	0	1	50
1	1	0	0
1	1	1	5

	Pr(A|E,B)
e,b	
$e,\overline{b}$	
$\overline{e},b$	
$\overline{e},\overline{b}$	

$P(a \mid \overline{e}, \overline{b}) = ?$

$= 10/1010$

Counting

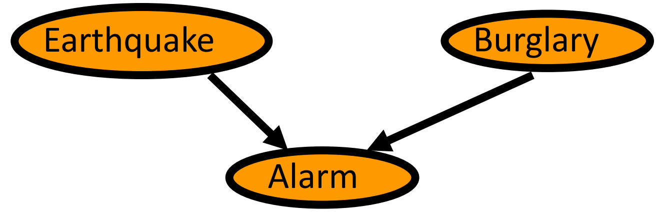


E	B	A	#
0	0	0	1000
0	0	1	10
0	1	0	20
0	1	1	100
1	0	0	200
1	0	1	50
1	1	0	0
1	1	1	5

	Pr(A|E,B)
e,b	
$e,\bar{b}$	
$\bar{e},b$	
$\bar{e},\bar{b}$	~0.01

$P(a\,|\,\bar{e}, b) = ?$

$= 100/120$



Counting

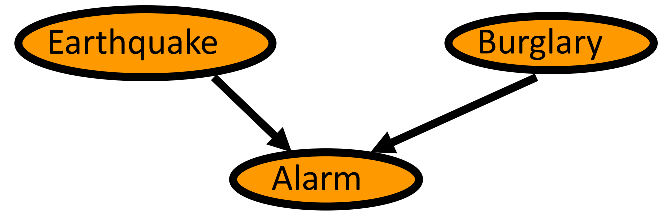
E	B	A	#
0	0	0	1000
0	0	1	10
0	1	0	20
0	1	1	100
1	0	0	200
1	0	1	50
1	1	0	0
1	1	1	5

	Pr(A|E,B)
e,b	
$e,\bar{b}$	
$\bar{e},b$	0.83
$\bar{e},\bar{b}$	~0.01

$P(a \mid e, \bar{b}) = ?$

$= 50/250$

Counting



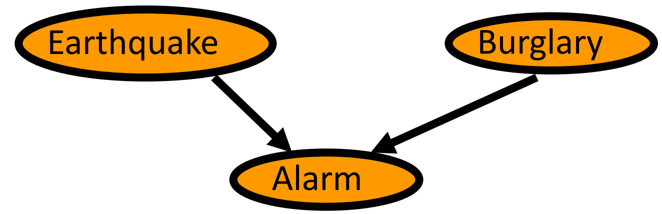
E	B	A	#
0	0	0	1000
0	0	1	10
0	1	0	20
0	1	1	100
1	0	0	200
1	0	1	50
1	1	0	0
1	1	1	5

	Pr(A|E,B)
e,b	
$e,\bar{b}$	0.2
$\bar{e},b$	0.83
$\bar{e},\bar{b}$	~0.01

$P(a|e, b) = ?$

$= 5/5$

Counting



E	B	A	#
0	0	0	1000
0	0	1	10
0	1	0	20
0	1	1	100
1	0	0	200
1	0	1	50
1	1	0	0
1	1	1	5

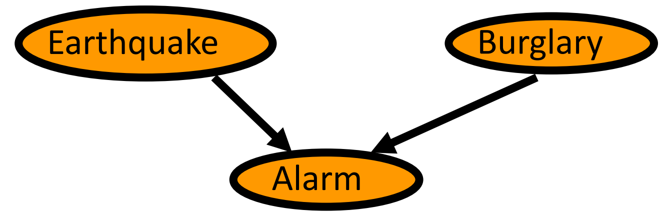
	Pr(A|E,B)
e,b	1
$e,\bar{b}$	0.2
$\bar{e},b$	0.83
$\bar{e},\bar{b}$	~0.01

Bad idea to have prob as 0 or 1
- stumps Gibbs sampling
- low prob states become impossible

Solution: Smoothing

- Why?
  - To deal with events observed zero times.
  - “event”: a particular ngram
- How?
  - To shave a little bit of probability mass from the higher counts, and pile it instead on the zero counts
- Laplace Smoothing/Add-one smoothing
  - assume each event was observed at least once.
  - add 1 to all frequency counts
- Add m instead of 1 (m could be > or < 1)

Counting w/ Smoothing



E	B	A	#
0	0	0	1000+1
0	0	1	10+1
0	1	0	20+1
0	1	1	100+1
1	0	0	200+1
1	0	1	50+1
1	1	0	0+1
1	1	1	5+1

	Pr(A|E,B)
e,b	0.86
$e,\bar{b}$	~0.2
$\bar{e},b$	~0.83
$\bar{e},\bar{b}$	~0.01

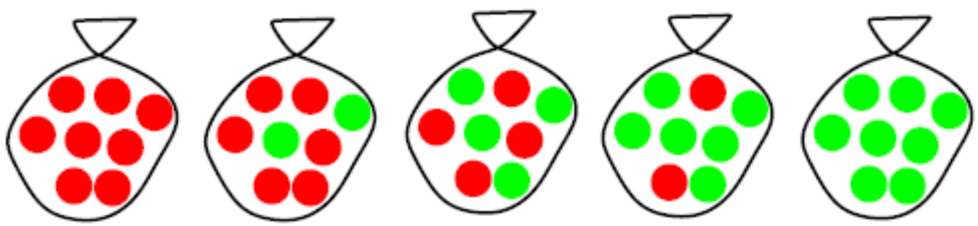
ML vs. MAP Learning

- ML: maximum likelihood (what we just did)
  - find parameters that maximize the prob of seeing the data D
  - $\text{argmax}_\theta \, P(D|\,\theta)$
  - easy to compute (for example, just counting)
  - assumes uniform prior
- Prior: your belief before seeing any data
  - Uniform prior: all parameters equally likely
- MAP: maximum a posteriori estimate
  - maximize prob of parameters after seeing data D
  - $\text{argmax}_\theta \, P(\theta|D) = \text{argmax}_\theta \, P(D|\theta)P(\theta)$
  - allows user to input additional domain knowledge
  - better parameters when data is sparse…
  - reduces to ML when infinite data

Example

Suppose there are five kinds of bags of candies:

10% are h_1: 100% cherry candies
20% are h_2: 75% cherry candies + 25% lime candies
40% are h_3: 50% cherry candies + 50% lime candies
20% are h_4: 25% cherry candies + 75% lime candies
10% are h_5: 100% lime candies



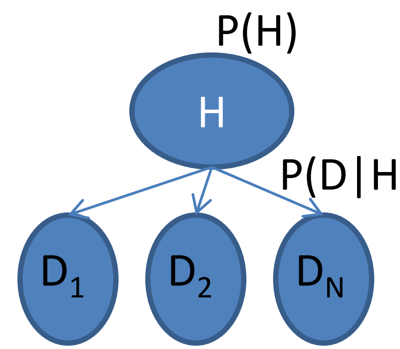
Then we observe candies drawn from some bag: 

What kind of bag is it? What flavour will the next candy be?

Learning Inference

Full Bayesian learning

View learning as Bayesian updating of a probability distribution over the hypothesis space



i.i.d

H is the hypothesis variable, values $h_1, h_2, \ldots$, prior $\mathbf{P}(H)$

jth observation d_j gives the outcome of random variable D_j
training data $\mathbf{d} = d_1, \ldots, d_N$

Given the data so far, each hypothesis has a posterior probability:

$$P(h_i|\mathbf{d}) = \alpha P(\mathbf{d}|h_i)P(h_i)$$

where $P(\mathbf{d}|h_i)$ is called the likelihood

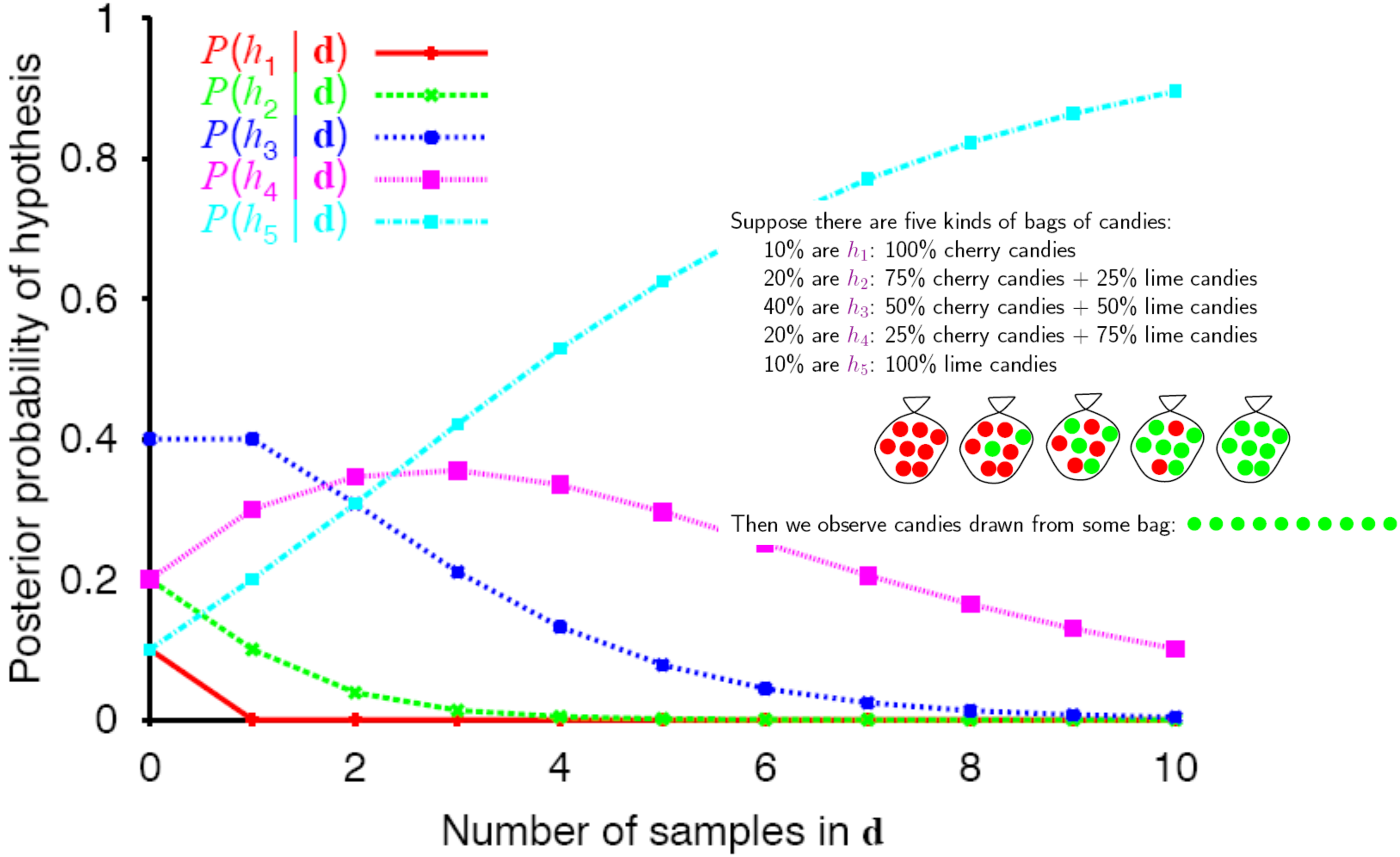
Predictions use a likelihood-weighted average over the hypotheses:

$$\mathbf{P}(X|\mathbf{d}) = \Sigma_i\, \mathbf{P}(X|\mathbf{d}, h_i)P(h_i|\mathbf{d}) = \Sigma_i\, \mathbf{P}(X|h_i)P(h_i|\mathbf{d})$$

No need to pick one best-guess hypothesis!

$$P(X|d) = \Sigma\, P(X|h_i)\, P(h_i|d)$$

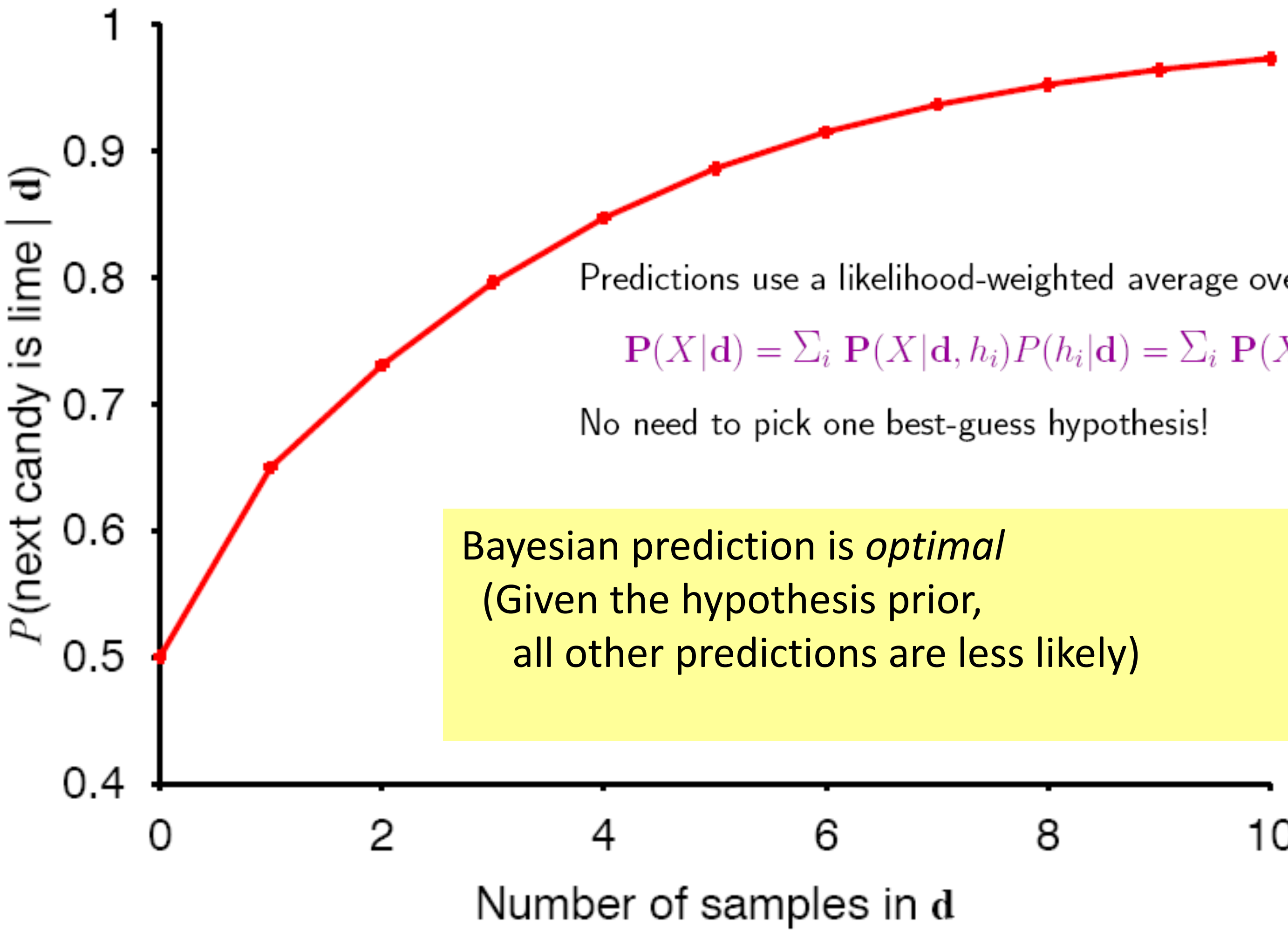
Posterior probability of hypotheses



True hypothesis eventually dominates…

probability of indefinitely producing uncharacteristic data →0

Prediction probability



Predictions use a likelihood-weighted average over the hypotheses:

$$\mathbf{P}(X|\mathbf{d}) = \Sigma_i\ \mathbf{P}(X|\mathbf{d}, h_i)P(h_i|\mathbf{d}) = \Sigma_i\ \mathbf{P}(X|h_i)P(h_i|\mathbf{d})$$

No need to pick one best-guess hypothesis!

Bayesian prediction is *optimal*
(Given the hypothesis prior,
all other predictions are less likely)

ML vs. MAP Learning

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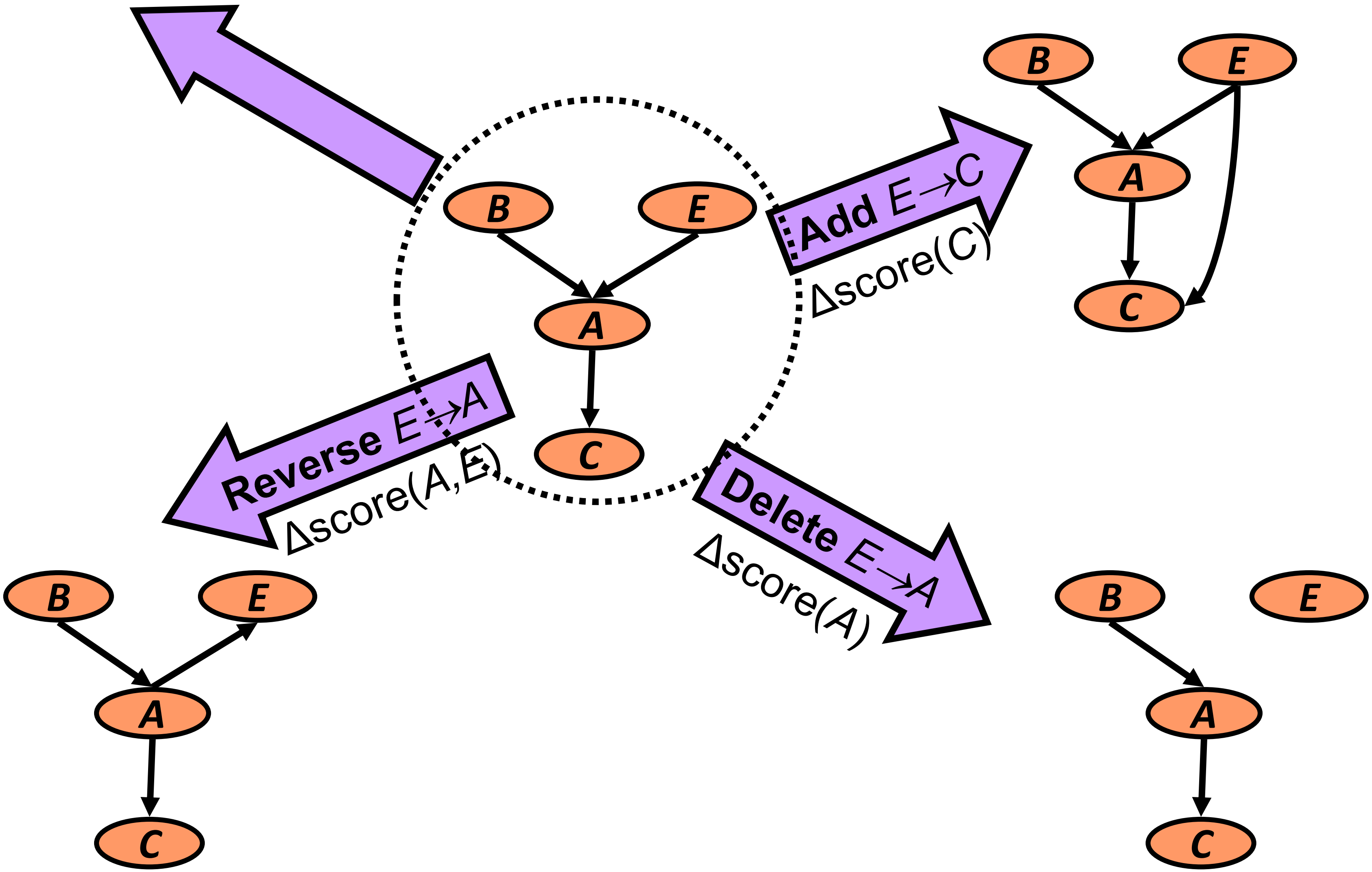
Learning the Structure

- Problem: learn the structure of Bayes nets
- Search thru the space…
  - of possible network structures!
  - Heuristic search/local search
- For each structure, learn parameters
- Pick the one that fits observed data best
  - Caveat – won't we end up fully connected????

When scoring, add a penalty

$\propto$ model complexity

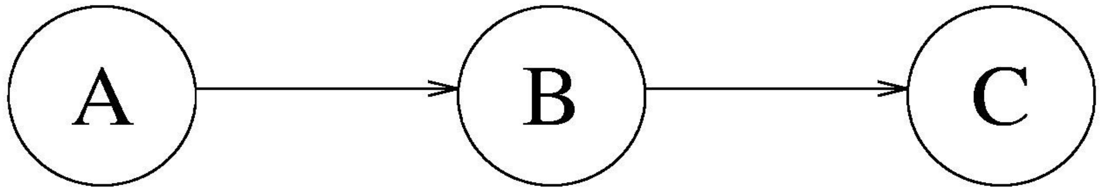
Local Search



How to learn when some data missing?

- Expectation Maximization (EM)

Example



Examples:	0	1	1
	1	0	0
	1	1	1
	1	?	0

Initialization: $P(B|A) =$ $P(C|B) =$

$P(A) =$ $P(B|\neg A) =$ $P(C|\neg B) =$

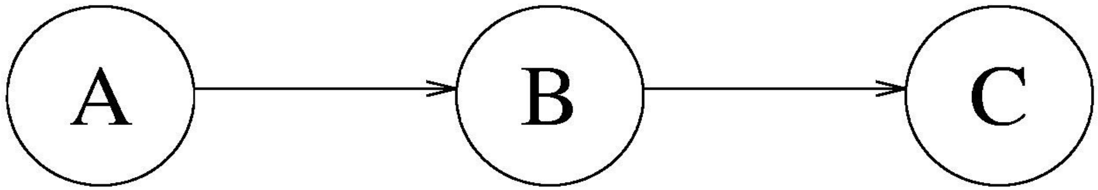
Chicken & Egg Problem

- If we knew the missing value
  - It would be easy to learn CPT

- If we knew the CPT
  - Then it'd be easy to infer the (probability of) missing value

- But we do not know either!

Example



Examples:

0	1	1
1	0	0
1	1	1
1	?	0

Initialization: $P(B|A) = 0$ $\quad P(C|B) = 0$

$P(A) = 0.75$ $\quad P(B|\neg A) = 0$ $\quad P(C|\neg B) = 0$

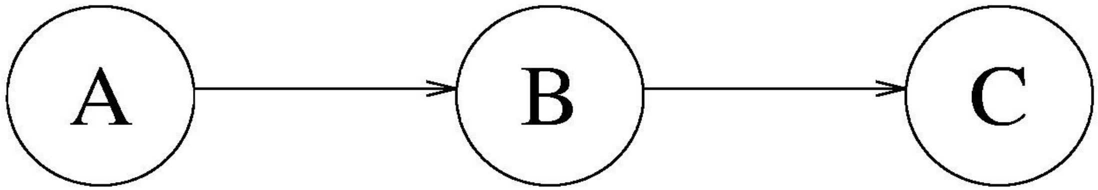
E-step: $P(? = 1) = P(B|A, \neg C) = \frac{P(A,B,\neg C)}{P(A,\neg C)} = \ldots = 0$

M-step: $P(B|A) =$ $\quad P(C|B) =$

$P(A) =$ $\quad P(B|\neg A) =$ $\quad P(C|\neg B) =$

E-step: $P(? = 1) =$

Example



Examples:

A	B	C
0	1	1
1	0	0
1	1	1
1	? 0	0

Initialization: $P(B|A) = 0$ $\quad P(C|B) = 0$

$P(A) = 0.75$ $\quad P(B|\neg A) = 0$ $\quad P(C|\neg B) = 0$

E-step: $P(? = 1) = P(B|A, \neg C) = \frac{P(A,B,\neg C)}{P(A,\neg C)} = \ldots = 0$

M-step: $P(B|A) = 0.33$ $\quad P(C|B) = 1$

$P(A) = 0.75$ $\quad P(B|\neg A) = 1$ $\quad P(C|\neg B) = 0$

E-step: $P(? = 1) =$

Expectation Maximization

- **Guess** probabilities for nodes with **missing values** (e.g., based on other observations)
- **Compute the probability distribution** over the missing values, given our guess
- **Update the probabilities** based on the guessed values
- **Repeat** until convergence

- Guaranteed to converge to local optimum

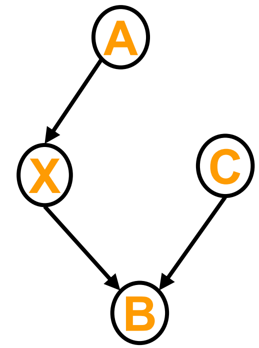
Learning Summary

- **Known structure, fully observable**: only need to do parameter estimation
- **Unknown structure, fully observable:** do heuristic/local search through structure space, then parameter estimation
- **Known structure, missing values:** use expectation maximization (EM) to estimate parameters

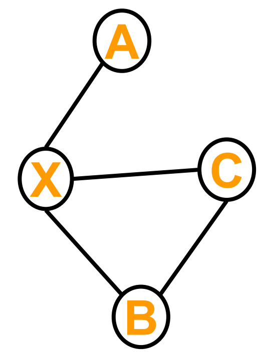
- **Known structure, hidden variables:** apply adaptive probabilistic network (APN) techniques
- **Unknown structure, hidden variables:** too hard to solve!

Other Graphical Models

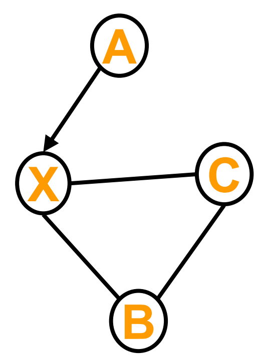
- Directed
  - Bayesian Networks



- Undirected
  - Markov Network (Markov Random Field)
  - BN → MN (moralization: marry all co-parents)



- Mixed
  - Chain Graph



Other Graphical Models