

COL 351 Quiz 1B

PrathamAgrawal

TOTAL POINTS

9 / 10

QUESTION 1

1 Question 1 9 / 10

✓ + 3 pts *Correct Greedy Algorithm Idea*

✓ + 1 pts *Proper Algorithm Description*

+ 4 pts *Proof Mostly correct, but some Mistake/Informalness in Proof*

✓ + 5 pts *Proof Mostly correct, Slight Mistake/Informalness in Proof*

+ 6 pts *Correct Proof*

- 1 pts *Only proved for first element. Did not extend it using induction*

+ 0 pts *Incorrect / Not Attempted*

1 First need to show that there exists an optimal solution in which s_1 is picked

Quiz I (COL 351)

Name **Pratham Agrawal**

Entry No. **2021CS10891**

Give precise arguments. Needlessly long explanations will not fetch any marks.

You have n friends, call them $1, \dots, n$. Person i has *unfriendliness* value s_i . You are given a target T . You need to divide these n friends into teams of two each (assume n is even, and so there will be $n/2$ teams). A team consisting of i and j is said to be good if $s_i + s_j \leq T$. Give a greedy algorithm to divide the n friends into teams of two each such that the number of good teams is maximized.

Greedy Algorithm:

sort in increasing order of unfriendliness value's s_i .
it becomes s_1, s_2, \dots, s_n in sorted order.

starting from s_1 , we find the friend with maximum s_j s.t.
 $s_1 + s_j \leq T$. Then, we continue from the left. number of
such pairs found will be the maximum answer possible.

Proof of Correctness:

Example $\rightarrow 1 \ 2 \ 4 \ 2 \ 8 \ 7 \ 6 \ 5 \quad T=9$

sort $\rightarrow 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \quad T=9$

1. ~~sort~~ order doesn't matter
as we are just choosing 2 friends. \therefore answer = 4.
 \therefore sorting doesn't affect the problem.

\rightarrow 2. let s_1, s_2, \dots, s_n be the sorted order.

if s_1 is matched with s_j (according to my greedy algo G)

$s_1 + s_j \leq T$ and s_j is the max such value.

i.e. $s_1 + s_{j+1} > T, \dots, s_1 + s_{j+2} > T, \dots, s_1 + s_n > T$

\rightarrow in the optimal solⁿ O , say s_1 is matched to s_k where $k \neq j$.

if k clearly $k < j$. as if $k > j \rightarrow s_1 + s_k > T$.

if $k < j \rightarrow s_1 + s_k \leq T$. Now we reduce our sample spaceⁿ
to s_2, \dots, s_{k-1} . However if we chose s_j instead of
 s_k , our sample spaceⁿ would have been s_2, \dots, s_{j-1} .

$\therefore s_1 \subseteq s_2$ (subset notation). To prove, our greedy G

is as much optimal as O , we need to show, number of
pairs in $s_2 \geq$ number of pairs in s_1 .

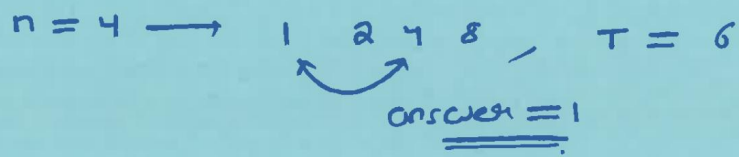
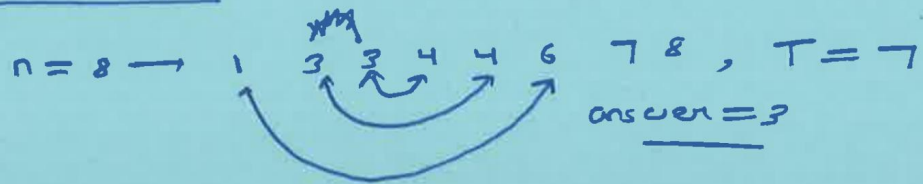
since $s_1 \subseteq s_2 \rightarrow$ all possible pairs in s_1 are also possible
in s_2 , because s_2 contains all values of s_1 . \therefore no. of

pairs in $s_2 \geq$ number of pairs in s_1 . $\therefore G$ is as optimal

as O . \therefore our greedy algo. G works.

* if $k = j$,
we can use
induction for
new sample space
 $\{s_2, \dots, s_{j-1}\}$

more examples:



1 3 4 5 8 2
1 2 3 4 5 8 $T = 7$