# COL351: Analysis and Design of Algorithms

Tutorial Sheet - 11

November 11, 2022

**Question 1** You are given a graph $G = (V, E)$ and an integer $k$. A set $I \subseteq V$ is defined to be *Strongly Independent* if for any two nodes $v, u \in I$, the edge $(v, u)$ does not belong to $E$, and there is also no path of 2 edges from $u$ to $v$, i.e., there is no node $w$ such that both $(u, w) \in E$ and $(w, v) \in E$. The Strongly Independent Set problem is to decide whether $G$ has a strongly independent set of size $k$. Prove that the Strongly Independent Set problem is NP-complete.

**Question 2** *Group Interval Scheduling (GIS) Problem* is defined as follows. You have a processor that is available to run jobs under the constraint that it can work on only one job at any single point of time. Jobs in this model, however, are more complicated - each job requires a set of intervals of time during which it needs to use the processor. For example, a single job could require the processor from 10 AM to 11 AM, and again from 2 PM to 3 PM. If you accept this job, it ties up with the processor during those two hours, but could still accept jobs that need any other time periods (including the hours from 11 to 2).

You are given a set of $n$ jobs ($J$), each specified by a set of time intervals, and the question is: For a given number $k$, is it possible to accept at least $k$ of the jobs so that no two of the accepted jobs have any overlap in time? Show that Group Interval Scheduling (GIS) is NP-complete.

**Question 3** Consider the Group Interval Scheduling (GIS) problem defined in the above question. For any given job $j \in J$, let $\Delta_j$ be the number of jobs in $J \setminus \{j\}$ that overlaps with $j$. Present a $(\Delta + 1)$-approximation algorithm for GIS problem, where $\Delta = \max_{j \in J} \Delta_j$.

*Hint:* Recursively pick a job that overlaps with minimum number of jobs.

**Question 4** Show that SAT $\leq_P$ 3-SAT.

**Question 5** Design an $O(2^k \; poly(n))$ time algorithm for $k$-vertex-cover problem: *Given a graph $G$ on $n$ vertices check if there exists a vertex cover of size at most $k$ in $G$.*