COL351: Analysis and Design of Algorithms

Tutorial Sheet - 10

October 28, 2022

Question 1 The basic rule for blood donation are: A patient of blood group A can receive only blood of group A or O. A patient of blood group B can receive only blood of group B or O. A patient of blood group O can receive only blood of group O. A patient of blood group AB can receive blood of any group.

Let s_O, s_A, s_B, s_{AB} denote the supply in whole units of the different blood types in a hospital for the coming week. Assume that the hospital knows the projected demand for each blood type d_O, d_A, d_B, and d_{AB} for the coming week. Give a max-flow based algorithm to check if the supply would meet the projected demand.

Question 2 Let $G = (V, E)$ be a directed graph, and (s, t) be a vertex pair. Two paths from s to t are said to be *internally-vertex-disjoint* if they do not share any vertex except end-points s and t. Present an $O(mn)$ algorithm to compute the maximum number of vertex disjoint paths from s to t.
Hint: An (s, t)-max-flow on edges of unit capacity can be computed in $O(mn)$ time.

Question 3 Let $X = (x_{ij})$ be a square matrix of size n storing positive real numbers. It is given that the sum of elements of each column as well as each row is a positive integer. Prove that elements of X can be replaced by integers without changing any column sum or row sum.

Question 4 You have a collection of n software applications, $1, \ldots, n$, running on an old system; and now you would like to port some of these to a new system. If you move application i to the new system, you expect a net (monetary) benefit of $b_i \geqslant 0$. The different software applications interact with one another; if applications i and j have extensive interaction, then the you will incur an expense if you move one of i or j to the new system but not both – let's denote this expense by $x_{ij} \geqslant 0$. So if the situation were really this simple, you would just port all n applications, achieving a total benefit of $\sum_{i=1}^{n} b_i$. Unfortunately, there's a problem. Due to small but fundamental incompatibilities between the two systems, there's no way to port application 1 to the new system; it will have to remain on the old system. Nevertheless, it might still pay off to port some of the other applications.

Your task is the following: which of the remaining applications, if any, should be moved? Design an algorithm to find a set $S \subseteq \{2, \ldots, n\}$ for which the sum of the benefits minus the expenses of moving the applications in S to the new system is maximized.
Hint: The value of (s, t)-max-flow in a graph is same as the capacity of (s, t)-min-cut.

Question 5 There are n clients $(c_1, \ldots, c_n)$ who want to be connected to one of the k mobile towers $(m_1, \ldots, m_k)$ in a town. You are given the (x,y) coordinates of each client and each tower, a distance parameter d, and a load parameter L. Design a polynomial time algorithm to decide if every client can be connected simultaneously to some mobile tower subject to the following constraints.

1. Each client is connected with exactly one of the mobile towers, and a client can only be connected to tower that is within distance d.
2. No more than L clients can be connected to any single mobile tower.