COL351: Analysis and Design of Algorithms

Tutorial Sheet - 6

September 17, 2022

Hashing

Question 1 Let $U = [1, M]$ be a universe, and $S \subsetneq U$ be a fixed set of size n. Present a construction of a random hash function $H : U \to [0, n-1]$ such that

(i) For each $i \in [0, n-1]$, the expected size of $T[i]$ is $O(1)$.

(ii) For each $x \in U$, the time to check membership of x in S is $\Omega(n)$.

Question 2 Let $U = [1, M]$ be a universe, and let $S = \{s_1, \ldots, s_n\}$ be a subset of U of size n such that each s_i is a uniformly random element of U independent of other s_j's. Let H be a hash function such that $H(x) = x \bmod n$.

(i) Show that the expected size of $\left(\max_{i=0}^{n-1} T[i]\right)$ is $O(\log n)$.

(ii) Argue that the expected value of maximum time taken to verify the membership of elements of U in S is $O(\log n)$.

(iii) If we redefine $H(x)$ as $x(\bmod n^2)$, then prove that with probability at least $1/2$, there will be no collisions under H.
Hint: Use Markov's inequality.

Quiz 2

Question 1 Let $X, Y, Z \in \{0,1\}^n$ be three n-length strings. Describe an $O(n^2)$ time algorithm to compute largest k such that there exists a k-length string that is substring of X, Y, and Z.

Question 2 Let $G = (V, E)$ be a weighted digraph with no cycle of negative weight, and let $S \subseteq V$ be a set of size k. A path P is said to be an S-path if the internal vertices of P lie in S.

Describe an $O(kn^2)$ time algorithm to compute a binary matrix B such that $B[i, j] = 1$ if and only if there exists an S-path of negative weight from vertex i to vertex j in G.

Quiz 1

Question 1 Compute the optimal prefix free encoding for a character set consisting of 5 letters $\{a, b, c, d, e\}$ with frequencies as: $a : 1,\ b : 1,\ c : 2,\ d : 3,\ e : 5$.

Question 2 Prove or disprove the following statement: "Any n vertex graph with $n - 1$ bridges has a unique MST".

Question 3 Let $G = (V, E)$ be a DAG and let $s \in V$ be a vertex in G. Design an optimal algorithm to verify that there is a unique path between all pairs in $\{s\} \times V$, and analyse its time complexity.