# Tutorial-6

Graded

**Student**

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**Total Points**

**3 / 3 pts**

**Question 1**

**(no title)** **3** / 3 pts

**+ 0.6 pts** Written ”I do not know how to approach this problem”

- ✔ **+ 1 pt** Mentioning Claim 1
- ✔ **+ 0.5 pts** Proof idea for Claim 1
- ✔ **+ 1.5 pts** High-level proof for optimality of greedy

COL351: Analysis and Design of Algorithms
Tutorial 6

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|---|---|
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| Entry number: 2022CS11596 | Group: 3 |

Let us assume greedy doesn't always give optimal solution.

Let $L$ be smallest number such that greedy sol$^n$ $\neq$ optimal sol$^n$.

Let $L = a_0 a_1 a_2 \cdots a_n$ be the greedy solution, ie $a_i$ is the number of coin of denomination $2^i$.

and let $a_0' a_1' \cdots a_n'$ be the optimal solution.

Observe that for $i \neq 1000$, $a_i' < 2$, because if not we can decrease $a_i'$ by 2 and increase $a'_{i+1}$ by 1 and improve the optimal soln.

Let $i$ be the first index where $a_i \neq a_i'$

consider the number in binary representations

$M = (a_0 a_1 \cdots a_{i-1})_2$, ~~obviously M sol~~

and $N = (a_i \cdots a_n)_2$, obviously $M < L$ and $N < L$

observe that $(a_0' a_1' \cdots a_{i-1}')$ and $(a_i' a_{i+2}' \cdots a_n')$ are also ONE of the allocation of coins for $M$ and $N$.

(because $a_j = a_j'$ for $j = 0, \cdots i-1$)

since $(a_i')$ is optimal for $L$

$\to \sum_{j=0}^{n} a_j' \leq \sum_{i=0}^{n} a_j$

this implies that either

$$\left(\sum_{j=0}^{i-1} a_j' < \sum_{j=0}^{i-1} a_j\right) \text{ OR } \left(\sum_{j=i}^{n} a_j' < \sum_{j=i}^{n} a_j\right)$$

Thus showing greedy of either M or N is not optimal.

Since $M < L$ and $N < L$

$\to$ contradicting the minimality of $L$.

$\to$ Greedy works

R.W.

$\underbrace{a_1 \; a_2 \; a_3 \cdots a_i \; a_n}$

$a_1 \; a_2 \; a_3 \cdots a_i' \cdot a_n$

$2^n \leq L$