COL 351 Lecture 17 2023/02/16

Topic : Assignment Scheduling

Assignment Scheduling

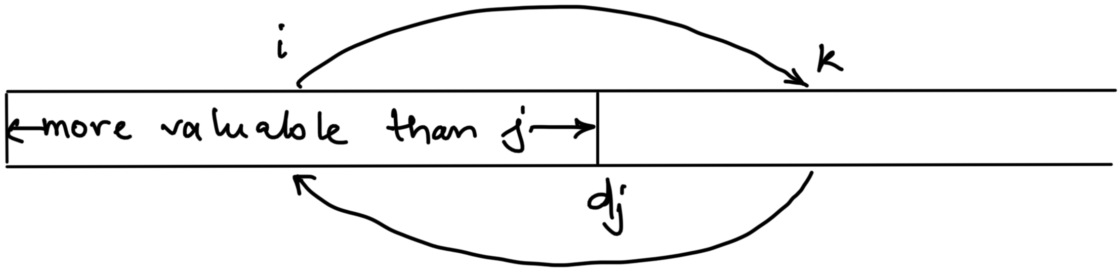
Input: n assignments.
Assignment i has deadline d_i, value p_i
Each assignment takes 1 day.

Output: Subset of assignment to be done + schedule, which maximizes total value.
(You get p_i for each assignment i that you finish before deadine d_i)

Profitable exchanges

① Do assignment j instead of i in the same slot t_i

($p_j > p_i$ and $t_i \le d_j$)



② If $\exists\, i, k$ such that $p_i > p_k$ and $t_i < t_k \le d_i$

then swap positions of i and k.

Candidate algorithm.

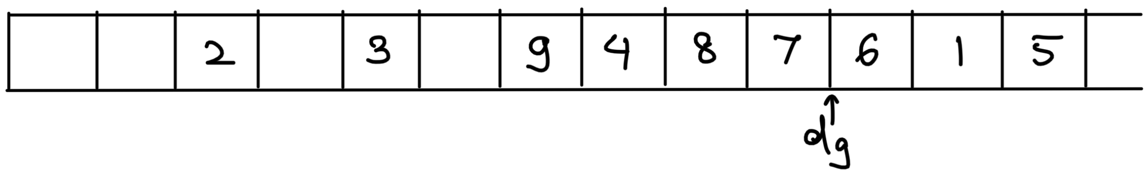
1. Sort assignments in ↓ order of value.
2. For each i in the above sorted order:

    If $\exists$ an available slot for i:

        Schedule i in the latest available slot.

    Else

        Discard i.



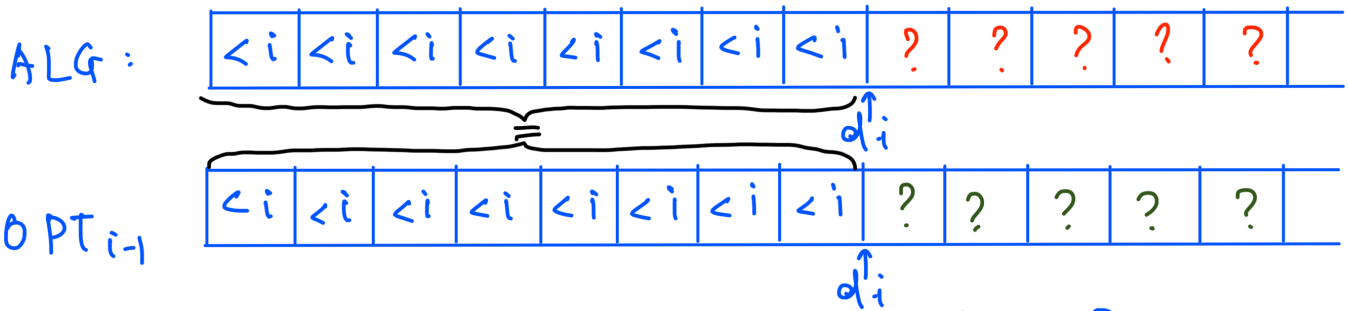
Claim (?) : Suppose $p_1 \ge p_2 \ge \ldots \ge p_n$.

$\forall i$ $\exists$ an opt solution OPT_i which agrees with the ALG's decisions on the first i assignments.

Proof: By induction on i; base case $i=0$ trivial.

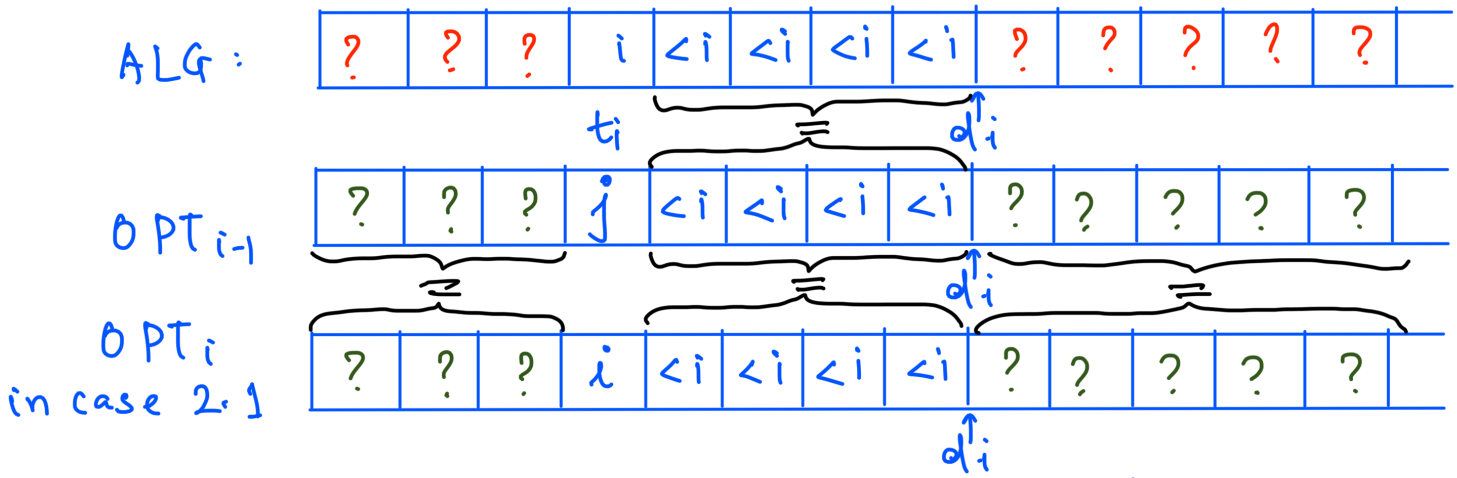
IH: $\exists$ an opt solution OPT_{i-1}, which agrees with the ALG's decisions on the first $i-1$ assignments.

Case 1: ALG decides to skip i.



$\Rightarrow OPT_{i-1}$ can't schedule i. Take $OPT_i = OPT_{i-1}$.

Case 2: ALG schedues i. Let t_i denote the time slot in which ALG schedules i.

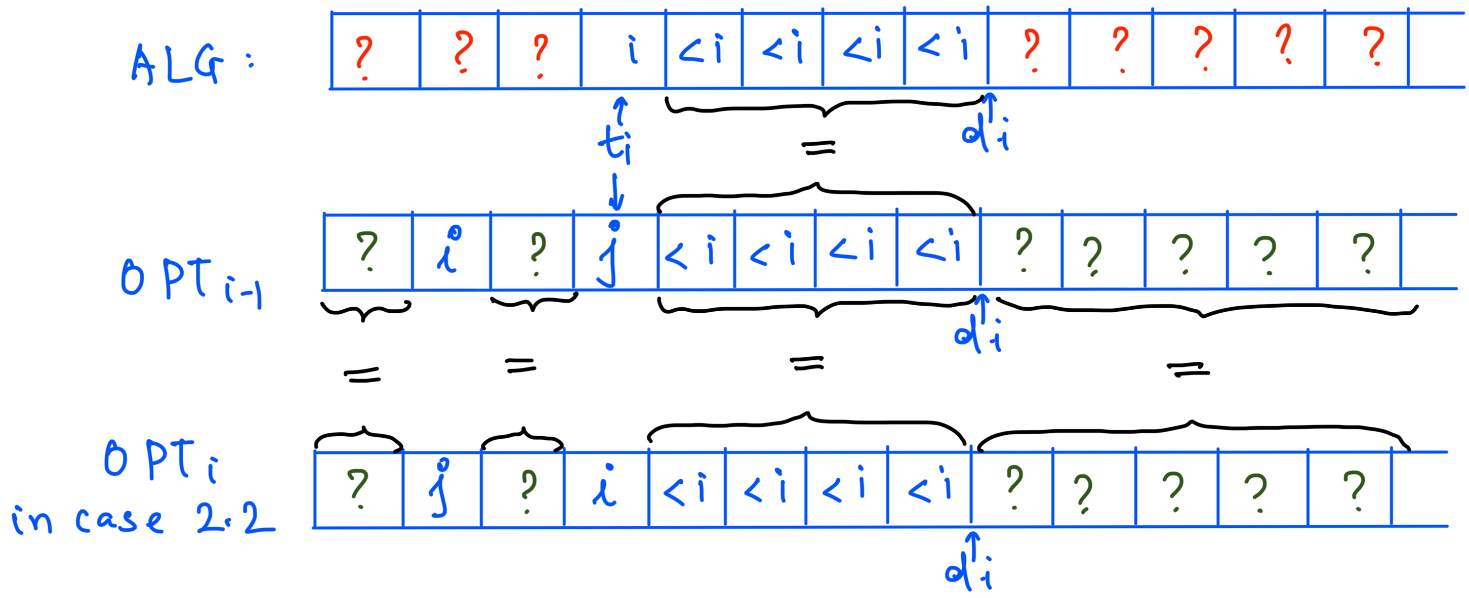


Case 2.0: OPT_{i-1} also schedules i at time $t_i \Rightarrow OPT_i = OPT_{i-1}$

Case 2.1: OPT_{i-1} doesn't schedule i. OPT_{i-1} schedules $j \neq i$ in t_i ($\because j > i$), or keeps t_i empty

OPT_i : same as OPT_{i-1}, except schedule i in slot t_i.

Case 2.2 OPT_{i-1} schedules i in some slot $t \neq t_i$ $\therefore t < t_i$



OPT_i : Same as OPT_{i-1} except contents of slots t and t_i are swapped.

Check

- OPT_i is still feasible.
- OPT_i has same value as OPT_{i-1} $\therefore$ OPT_i is optimal.
- OPT_i agrees with ALG's decisions on the first i assignments.

Exercise : Design an $O(n^2)$ time implementation of the algorithm.