Quiz 4

Graded

Student

Abhinav Shripad

Total Points

15 / 24 pts

Question 1

1 Resolved 10 / 12 pts

+ 2.4 pts Written "I do not know how to approach this problem"

Correct Example Graph

- ✔ + 5 pts For a given fixed α, constructing a graph with a "worse than α" approximation
- ✔ + 2 pts Graph contains only unit capacity edges

+ 5 pts Maximum 5 points will be awarded if no justification is provided with the example graph, even if the example is correct.

Justification

- ✔ + 2 pts Finding a bad Greedy Flow in the graph
- ✔ + 2 pts Finding the max-flow in the graph
- ✔ + 1 pt Showing that the greedy flow has "worse than α" approximation

+ 0 pts Incorrect/Not attempted/Written nothing

– 2 pts Be clear with examples you have to clearly show how max flow and greedy flow is calculated.

1. See not using general graph is creating a issue here now where do I see how your greedy and max flow are coming on which graph actually working -0.5 and -0.5 for greedy and max flow.
2. Again you are just saying s->t path does not exist but never proved it.
3. You could have given clear general graph and then you could have given this graph dereived from general graph although since you wrote explanation for general graph i will draw -1 for not using general graph.
4. from here to where ? v_i+1,i right , but didn't write it it not my evaluators job to guess and if I have to guess I have to draw your marks.
5. see this must have to be your path according to what your explanation says but you chose other path that is why your 2 marks are taken because you are saying something and doing something

Regrade Request — **Submitted on: Nov 10**

Sir I agree with your suggestion that I did not draw a general graph. But why should I ? I gave a exact description of the graph and even the flow in terms of its vertices and edges and capacities. As far as showing no more s-t paths exists after the greedy flow, I wrote why no more path will exists, as it must "cross" the old s-t cut at some point, and thus take an already saturated edge. I again showed this logic on a sample graph with n = 3.

I will also explain the general graph which is what i wrote in the solution. The general graph is a n*n grid with 2 more vertices s and t. s connected to the 1st layer of vertices, and t to the last layer. Each layer is connected to its forward layer and vertices in each layer is connected to the vertex below it in the same layer as well.
The greedy algorithm takes the diagonal path on this graph. This has a flow 1. Trivially the max flow is at-least n.

Then I showed why no more s-t path exists. This is because every s-t path must cross the diagonal atleast once, but since the old s-t path already took the diagonal, no outcoming edge exists from the diagonal.

I have added two more fresh comments, see when you are not creating the graph and just writing the any minute missing may cause huge discrepencies like in comment when you

don't mention v_i,i->v_i+1,j I may chose any edge which may lead to incorrect results . I hope you understood the point.
Thank you

Reviewed on: Nov 11

Question 2

2

Resolved 5 / 12 pts

+ 2.4 pts I don't know the approach

+ 0 pts Incorrect/Not attempted

Correct approach

✔ **+ 1.5 pts** Claiming every edge $e \in \delta + (A)$ is saturated

+ 1.5 pts Claiming every edge $e \in \delta-(A)$ does not carry any flow

✔ **+ 0.5 pts** Claiming similar properties for the cut$(A\prime, B\prime)$

✔ **+ 2 pts** Showing every edge $e \in \delta + (A \cup A\prime)$ is saturated

+ 2 pts Showing every edge $e \in \delta-(A \cup A\prime)$ does not carry any flow

+ 1 pt Therefore, claiming that $(A \cup A\prime, B \cap B\prime)$ is a min-cut

+ 2 pts Showing capacity of the cut $(A \cup A\prime, B \cap B\prime)$ is equal to the value of max-flow

+ 1.5 pts Claiming similar arguments can be used to show that $(A \cap A\prime, B \cup B\prime)$ is a min-cut

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+ 1 pt Point adjustment

Regrade Request **Submitted on: Nov 10**

Sir by f here I mean residual flow left to go on the edge (u,v), i.e. if for flow f, we can send 0 more flow on that edge. Similar notation is used in class notes also. Sir please read the solution considering this notation, you can clearly see that only mistake in my solution is not proving every edge e in delta-(A), ie backedge does not carry any flow, other than that everything is correct i guess. Sir please recheck.

Ahh, okay. But still, you need to prove about the backward flow case too in order to prove it is min cut, and also, some internal flow cases should be considered wrt the new segments, other than the one you took, which contributes to the flow. Updating marks.

Reviewed on: Nov 12

COL351: Analysis and Design of Algorithms
Quiz 4

Name: Abhinav Rajesh Shripad	Oct 29, 2024
Entry number: 2022CS11596	Total points: 24

Please write your answers within the box provided. Answers written outside the boxed region will not be graded.

Problem 1 [12 points]

Consider the following greedy algorithm for the maximum flow problem.

ALGORITHM 1: GREEDY FLOW

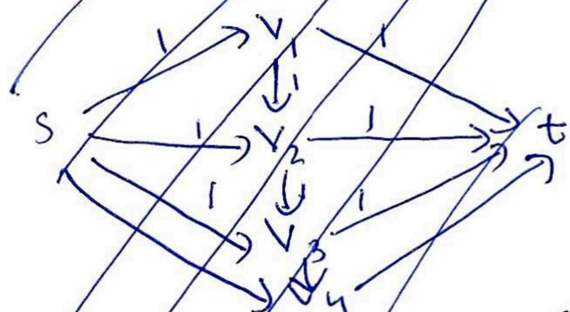
Input: Directed graph $G = (V, E)$, edge capacities $\{u_e\}_{e \in E}$, source $s \in V$, destination $t \in V$.
Output: A flow f.

For every $e \in E$, initialize $f_e = 0$.
while *there is a path from s to t in G* **do**
  $P \leftarrow$ an arbitrary path from s to t in G
  $\Delta \leftarrow \min_{e \in P} u_e$ `// minimum capacity of any edge in` P
  for *every edge e in P* **do**
    $f_e \leftarrow f_e + \Delta$
    if $u_e = \Delta$ **then**
      remove e from G
    else
      $u_e \leftarrow u_e - \Delta$
return f

Show that GREEDY FLOW fails to compute even a good approximation to the maximum flow even on unit-capacity networks. That is, for any constant $\alpha > 1$, show that there is a flow network G where $u_e = 1$ for every edge $e \in E$ such that the value of the maximum flow is more than α times the value of the flow computed by GREEDY FLOW.

~~let $n = 2\lceil\alpha\rceil$~~
~~construct G with $2n+2$ vertices,~~
~~1 source, 1 sink and $2n$ vertices labelled~~
~~$v_1, v_2 \dots v_n, v_{n+1}, \dots v_{2n}$~~
~~add edge $s \to v_i$ & $v_i \to t$ $\forall i \in [1, 2n]$~~
~~add edge $v_i \to v_{i+1}$ $\forall i \in [1, 2n-1]$~~

Example for $n=2$



~~max~~ max-flow $\geq 2n$, ie a flow exist saturating all $s \to v_i$ and $v_i \to t$

so max-flow atleast n.

If greedy .

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let $n = \lceil \alpha \rceil + 1$

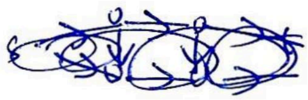
make G with n^2+2 vertices, 1 source 1 sink. let vertices be v_{ij} $i,j \in [1,n]$

add edge $v_{i,j} \to v_{i+1,j}$ $i \in [1,n-1]$, ~~$j \in$~~ and $j \in [1,n]$

and $v_{i,j} \to v_{i,j+1}$ if $i \in [1,n]$ and $j \in [1,n-1]$

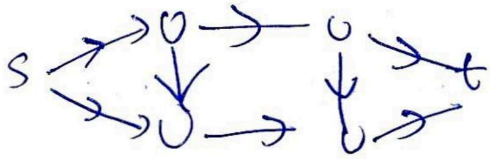
and $s \to v_{1,i}$ and $v_{n,i} \to t$

~~Eg) n = 2~~



all unit capacitated

Eg) $n=2$



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max flow $\geq n$, if we choose a flow via $s \to v_{1,i} \to v_{2,i} \cdots v_{n,i} \to t \;\; \forall i$

flow $= n$ $\to$ max flow $\geq n$ ---- (1)

If greedy chooses path

$s \to v_{1,1} \to v_{2,1} \to v_{2,2} \cdots \to v_{i,i-1} \to v_{i,i} \to$
$\vdots$
$v_{n,n} \to t$

it has flow 1.

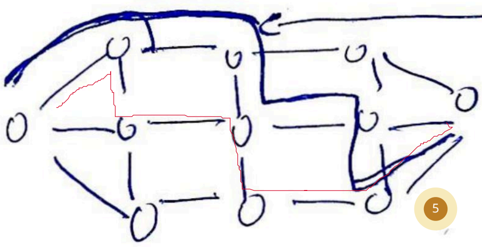
It terminates the algorithm.

$\to$ why $\to$ If any $s \to t$ path exists it must "cross" the previous path.

But all edges coming out of vertices in previous path are saturated. Hence flow by algo is more than α worse than optimum

$\to |f| = 1$ by algo

and $|f|\alpha = \alpha < n \leq$ max-flow - (1) $\quad (n = \lceil 2 \rceil + 1)$



path taken

if any other path it must "cross" this path $\to$ No other path.

Problem 2 [12 points]

Let (A, B) and (A', B') be minimum (s, t)-cuts in some flow network G. Prove that $(A \cap A', B \cup B')$ and $(A \cup A', B \cap B')$ are also minimum (s, t)-cuts in G.

Given :- (A, B), (A', B') min cut for flow f.

$\rightarrow \forall u, v$ s.t. $u \in A$, $v \in B$,

$f_{uv} = 0$ ... (1)

Illy for A', B'. $f_{uv} = 0$ for $u, v \in A', B'$ resp ... (2)

Proof :- ~~For intersection~~ Part (1)

Consider $u \in A \cap A'$ and $v \in B \cup B'$.

Case 1 $v \in B$

Since $u \in A \cap A' \rightarrow u \in A$

$\rightarrow f_{u,v} = 0$ since $(u, v) \in (A, B)$ and then by (1)

Case 2 $v \in B'$

Since $u \in A \cap A' \rightarrow u \in A'$.

$\rightarrow f_{uv} = 0$ since $(u, v) \in (A', B')$ and then by (2)

Proof :- Part (2)

Consider $u \in A \cup A'$ and $v \in B \cap B'$

Case 1 $u \in A$

Since $v \in B \cap B' \rightarrow v \in B$

$\rightarrow f_{uv} = 0$, since $(u, v) \in (A, B)$ and then by (1)

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~~Since~~

Case 2 $u \in A'$

since $v \in B \cap B' \rightarrow v \in B'$

$\rightarrow f_{uv} = 0$, since $(u,v) \in (A', B')$ and then by (2)