COL351 Holi2023: Tutorial Problem Set 10

1. Recall the `CircuitSAT` problem discussed in class. The input to the problem is a boolean circuit over variables $x_1, \ldots, x_n$, where each input of each gate is a variable, a constant (`true` or `false`), or the output of an "earlier" gate. One gate is designated as the output gate, and we are required to determine whether there exists a boolean assignment to the variables that makes the output of the circuit `true` (a.k.a. satisfying assignment). The `CircuitSATSearch` problem has the same input as `CircuitSAT`, but we are required to output a satisfying assignment if such an assignment exists, and otherwise output "NO". Design a Cook-reduction from `CircuitSATSearch` to `CircuitSAT`.

2. A *dominating set* of a graph is a subset D of vertices such that every vertex that is not in D is adjacent to some vertex in D. The input to the `DominatingSet` problem is a graph G and an integer k, and the output is whether G has a dominating set of size at most k. Prove that `DominatingSet` is in `NP`, and that `VertexCover` is Karp-reducible to `DominatingSet`.

3. The input to the `DominatingSetSearch` problem is a graph G and an integer k, and the output is a dominating set of G having at most k vertices, if such a set exists, and "NO" otherwise. Design a Cook-reduction from `DominatingSetSearch` to `DominatingSet`.

4. Let U be a finite set and, let $\mathcal{S} \subseteq 2^U$ be a collection of subsets of U. We say that a set $H \subseteq U$ is a *hitting set* of $\mathcal{S}$ if for all $A \in \mathcal{S}$, we have $H \cap A \neq \emptyset$, or in other words, H intersects every set in $\mathcal{S}$. In the `HittingSet` problem, we are given U, $\mathcal{S} \subseteq 2^U$, and an integer k as input. We are required to determine whether there is a hitting set of $\mathcal{S}$ having size at most k. Prove that `HittingSet` is in `NP`, and that `VertexCover` is Karp-reducible to `HittingSet`.

5. In the `HittingSetOpt` problem, we are given a finite set U and a collection $\mathcal{S} \subseteq 2^U$ of subsets of U, and we are required to output a minimimum cardinality hitting set of $\mathcal{S}$. Design a Cook-reduction from `HittingSetOpt` to `HittingSet`.

6. Design a Karp-reduction from `HittingSet` to `CircuitSAT`.

7. Recall that in the `GraphIso` problem, we are given two undirected graphs and we wish to determine whether they are isomorphic. Design a Karp-reduction from `GraphIso` to `CircuitSAT`.