Major Exam (COL 352)

Give precise arguments. Needlessly long explanations will not fetch any marks. Unless specified otherwise, assume that the alphabet Σ is $\{a, b\}$.

Answer whether the following statements are true or false. You must give a proper justification for your answer. Correct answer without any justification will not fetch any marks. All qustions carry equal marks.

1. For a string w, let w^R denote the reversal of w. If L is a context free language, then the following language L^R is also context free:

$$L^R = \{w^R | w \in L\}.$$

2. Given a language L, define $\mathsf{Mix}(L)$ as the following language:

$$\mathsf{Mix}(L) := \{x_1 x_3 x_5 \dots x_{2n-1} x_{2n} x_{2n-2} \dots x_2 | x_1 x_2 x_3 \dots x_{2n} \in L, n \geq 0\}.$$

   Note that each x_i is either a or b.

   If L is regular, then $\mathsf{Mix}(L)$ is also regular.

3. Refer to the definition of $\mathsf{Mix}(L)$ above. If L is regular, then $\mathsf{Mix}(L)$ is context free.

4. The language $L = \{a^i b^j c^k | i \cdot j = k \bmod 100\}$ is context free.

5. If L_1 and L_2 are recursively enumerable languages, then $L_1 \setminus L_2$ is also recursively enumerable.

6. If L_1 and L_2 are decidable languages, then $L_1 \setminus L_2$ is also decidable.

7. If L_1 is recursively enumerable and L_2 is decidable, then $L_1 L_2$ is recursively enumerable.