Major Exam (COL 352)

Give precise arguments. Needlessly long explanations will not fetch any marks. Answer each question on a fresh page.

1. (**10 marks**) For a language L, define $\tilde{L}$ to be the following language:

$$\tilde{L} = \{s_1 s_2 : s_2 s_1 \in L\},$$

where s_1, s_2 are strings in Σ^*. In other words, a string is in $\tilde{L}$ if and only if we can break it into parts s_1, s_2 such that $s_2 s_1$ is in the language L. Show that if L is regular then $\tilde{L}$ is also regular. You can assume $\Sigma = \{0, 1\}$.

2. (**10 marks**) Suppose you are given a program P which behaves as follows: there is a regular language L (which you don't know the details of) and when you give a string x as input to P, P outputs either 'yes' or 'no'. It outputs 'yes' exactly when the string x is in the language L. Note that you can access P only through such means (in other words, think of P as a "black box" and you can only call it with some input x). However, you know one more piece of information about L: you know of a number n such that a DFA for L exists with n states (again, you don't know the DFA, just the number n). Describe a procedure which will figure out whether the language L is infinite. How many calls to P will this procedure make ? You can assume that the alphabet Σ is $\{0, 1\}$.

3. Prove that the following languages are context free:

   (a) (**5 marks**) $L = \{0^n 1^m : 10n < m < 11n\}$.

   (b) (**5 marks**) $L = \{0^n 1^m : m \neq 10n \text{ and } m \neq 11n\}$.

4. (**10 marks**) Let Σ be the alphabet $\{x, y, z, w\}$. Let L be the set of all strings in Σ^* such that the number of occurences of letter y is exactly three times the number of occurences of letter x, and the number of occurences of w is exactly four times the number of occurences of the letter z. Show that L is not context free.

5. (**10 marks**) Let L be the language consisting of strings of the form $x \sqcup x$, where $\sqcup$ denotes the blank symbol and x is a binary string.

   Describe a **single tape** turing machine which decides this language. The Turing machine should use the same alphabet $\{0, 1\}$ (along with the start symbol and the blank symbol). You can use the Turing Machines W_a, L, R as described in lectures. Although your description of the Turing Machines should be in terms of these Turing Machines (do not write a table of transitions), you should also explain in English.

6. (**10 marks**) Consider the following language: $L = \{\langle M_1 \rangle, \langle M_2 \rangle : M_1, M_2 \text{ are Turing machines on alphabet } \{0, 1\} \text{ and there is no binary string } x \text{ such that } M_1 \text{ and } M_2 \text{ both halt on } x\}$. Show that L is undecidable. Is L recursively enumerable ? Give reasons.