Date: 2021/03/15 Duration: 1 hour

Read the instructions carefully.

1. **[5 points]** Prove that the language

$$\{x \mid x \text{ is the binary representation of } n!, \text{ without leading 0's, for some } n \in \mathbb{N}\}$$

is not regular.

2. **[5 points]** Let $L_k \subseteq \{0,1\}^*$ be the language defined as

$$L_k = \{xy \mid x,\, y \in \{0,1\}^k, \text{ and the bitwise-AND of } x \text{ and } y \text{ is } 0^k\}.$$

Observe that L_k is finite, and hence, regular. Prove that for all k, any DFA that recognizes L_k has at least 2^k states.

(As a challenge, prove that even an NFA that recognizes L_k must have at least 2^k states. Warning: this is a moderately hard problem and is NOT a part of this quiz. Discuss this on your favorite forum, but only after the mid-term exams.)