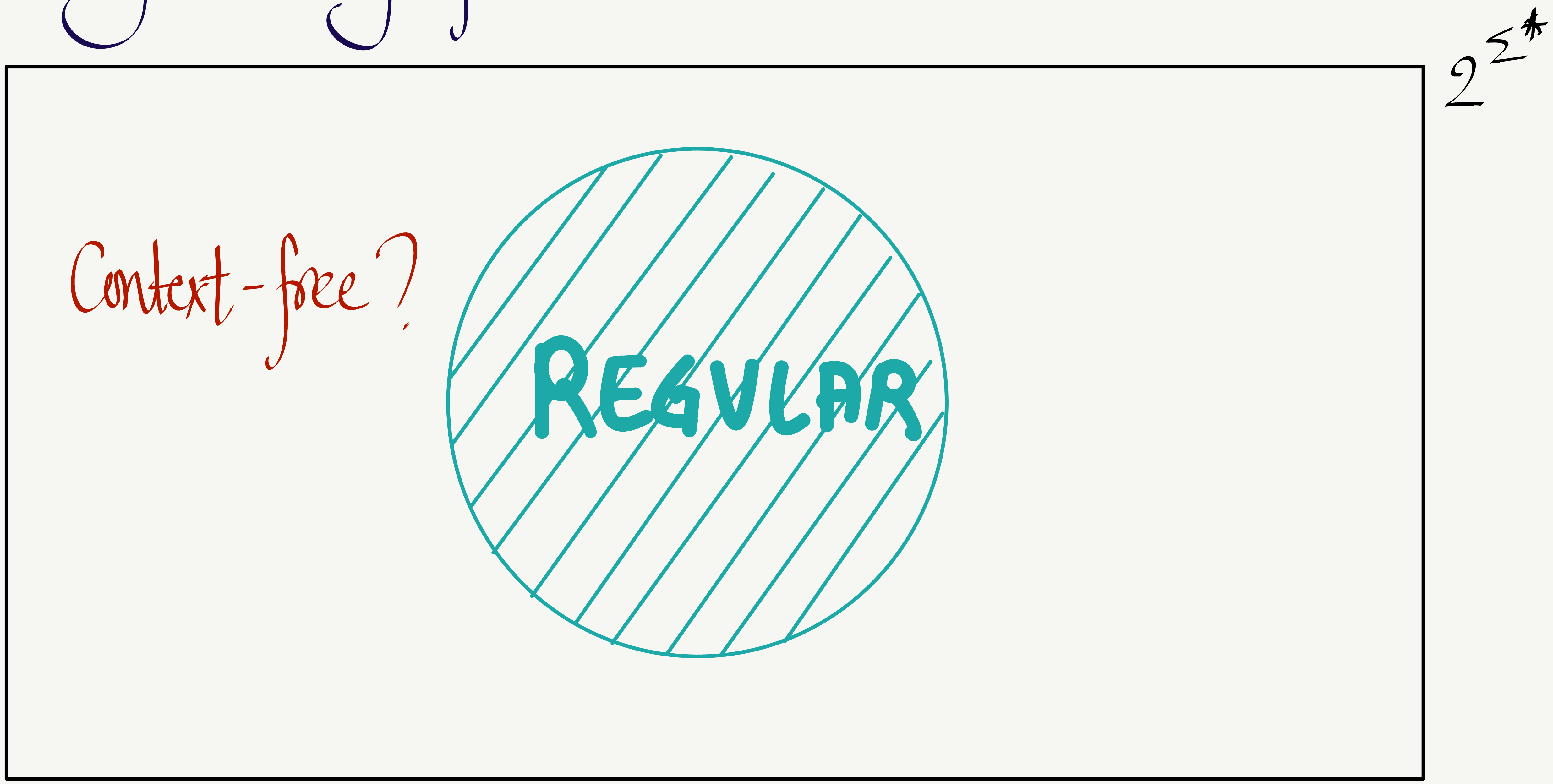
Non-Context Free Languages

Recall: We showed that some non-regular languages are context-free languages. These are generated by context-free grammars, and recognized by pushdown automata.

Today:



Is every non-regular language context-free?

Closure properties:

→ We saw that CFLs are closed under intersection with regular languages

(Cross-product construction, touch the stack only for the CFL)

→ Are CFLs closed under union?

If $\mathcal{L}_1$ and $\mathcal{L}_2$ are CFLs, is $\mathcal{L}_1 \cup \mathcal{L}_2$ also a CFL?

$G_1 = (NT_1, T_1, R_1, S_1)$ $\quad\quad$ $G_2 = (NT_2, T_2, R_2, S_2)$ $\quad$ $T = T_1 \cup T_2$

$G = (NT_1 \cup NT_2 \cup \{S\}, T, R_1 \cup R_2 \cup \{S \to S_1 | S_2\}, S)$ $\quad$ S "fresh"

* Which of the other regular operations is the class of CFLs closed under?

Construct a CFG or a PDA for $\mathcal{L} = \{a^n b^n c^n \mid n \geq 0\}$

$\mathcal{L}' : \{a^n b^{2n} a^n \mid n \geq 0\}$

$\mathcal{L}_{sq} = \{a^{n^2} \mid n \geq 0\}$

$\mathcal{L}'' = \{a^{2n} \mid n \geq 0\}$

We said that DFAs could not "count"

What can PDAs not do?

What CAN they do? Broadly,

- They can count[1] Is $\{a^{n^2} \mid n \geq 0\}$ a CFL?
- They can keep track of pairs[2] Is $\{a^n b^n c^n \mid n \geq 0\}$ a CFL?

1) Not beyond a "linear" count

2) But not if distinct pairs have an "overlap"

How does one show that a language is not context-free?