COL 352 Intro to Automata and theory of Computation, Tutorial Sheet 4

1. An *oblivious* Turing machine is one such that the movement of the tape head at any step depend only on the length of the input and not the actual input. Show that every TM can be simulated by a 1-tape *oblivious* Turing Machine.

2. Consider the language $L = \{0^i \cdot 1^i, i \geq 0\}$. Can a 1-tape TM recognize this in $O(n)$ time ?

3. If $L \in \mathcal{P}$, then which of the following are in $\mathcal{P}$
(i) $\bar{L}$ (ii) $L \cdot L$

4. Show that any arbitrary boolean function of k variables can be expressed by a CNF formula of atmost 2^k clauses.
Note: When k is constant this is also constant, and therefore the CNF formula in the proof of Cook-Levin thm expressing the transition function of the NDTM is of bounded size as it only involves 4 cells.

5. Can you design an efficient algorithm that satisfies at least 50 % of the clauses in a 3 CNF Boolean formula. How about 66% ?

6. In the proof of Cook-Levin theorem, show that for $x \in L$, and the reduction function f, a certificate y (non-deterministic choices) is mapped to a unique certificate of $f(x)$. Recall that $f(x)$ is a CNF formula and a certificate for CNF formula is a satisfying assignment.

7. Show that if a boolean formula in CNF contains at most one un-negated literal, then the satisfiability problem can be solved in polynomial time.
Note: Such clauses are called Horn clauses

8. The language $TMSAT$ is defined as $\{< x, \alpha, 1^n, 1^t > | \exists u, |u| = n \;\; M_\alpha(x, u)$ is accepted in t steps$\}$.
Show that TMSAT is NP complete and also explain the need for unary representation of n, t.

9. Assuming that 3D matching is NP complete, show that the problem of partition is NP complete.
Hint: Work out the details of the proof outlined in class.

10. In the partition problem, if the sum of the integers are bounded by a polynomial,show that it is possible to design a polynomial time algorithm.

11. Show that the problem of *independent set* is NP complete.

12. Show that the problem of sub-graph isomorphism problem is NP complete. In this problem, the input are two graphs G_1 and G_2, does G_1 contain a copy of G_2 as a subgraph.