Quiz 2 - 060225

Graded

Student

Abhinav Shripad

Total Points

15 / 15 pts

Question 1

Q1

15 / 15 pts

Approach 1: $(p+q)^* \subseteq (p^*+q^*)^*$ and $(p^*+q^*)^* \subseteq (p+q)^*$

- ✔ **+ 2 pts** Claiming $R_1 \subseteq R_2 \implies R_1^* \subseteq R_2^*$
- ✔ **+ 4 pts** Proving $(p+q)^* \subseteq (p^*+q^*)^*$
- ✔ **+ 4 pts** Proving $p^*+q^* \subseteq (p+q)^*$
- ✔ **+ 5 pts** Showing $p^*+q^* \subseteq ((p+q)^*)^* = (p+q)^*$ as $R^{**} = R^*$

Approach 2: Using Induction on length of string

+ 1 pt Mentioning Strong Induction on length of String

+ 1.5 pts $(p+q)^* \subseteq (p^*+q^*)^*$: Base Case

+ 6 pts $(p+q)^* \subseteq (p^*+q^*)^*$: Inductive Step

+ 1.5 pts $(p^*+q^*)^* \subseteq (p+q)^*$: Base Case

+ 4 pts $(p^*+q^*)^* \subseteq (p+q)^*$: Inductive Step

+ 1 pt Mentioning $(p+q)^* \subseteq (p^*+q^*)^* \wedge$
$(p^*+q^*)^* \subseteq (p+q)^* \implies (p+q)^* = (p^*+q^*)^*$

+ 0 pts Totally Incorrect / Unattempted

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COL352: Introduction to Automata & Theory of Computation

QUIZ 2

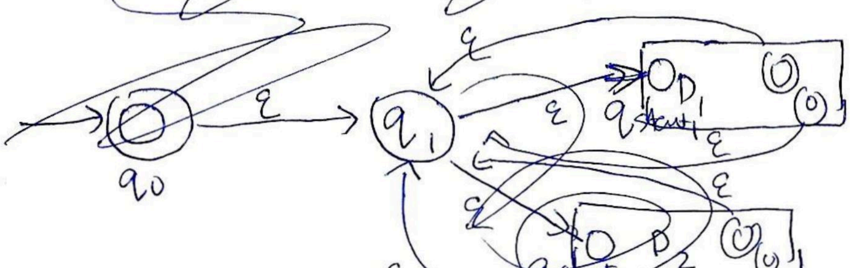
Date: 06/02/2025 15 minutes Maximum marks: 15

~~let $s \in (p+q)^*$, then $s = a_1 a_2 \cdots a_n$~~

~~where $a_i \in p$~~

~~let D_1 be DFA for p, D_2 for q,~~

~~As covered in class, NFA for $(p+q)^*$ is given as~~



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let $s \in (p+q)^*$, then $s = a_1 a_2 \cdots a_n, n \geq 0$

with $a_i \in (p+q) \Rightarrow a_i \in p$ OR q

$\Rightarrow a_i \in p^*$ OR $q^* \Rightarrow a_i \in p^* + q^*$ $(p \in p^*, q \in q^*)$

$\Rightarrow s = a_1 a_2 \cdots a_n$ with $a_i \in p^* + q^*$

$\Rightarrow s \in (p^* + q^*)^*$, since s was arbitrary

$\Rightarrow (p+q)^* \subseteq (p^* + q^*)^* \cdots$ (I)

Consider $s \in (p^* + q^*)^*$

$\Rightarrow s = b_1 b_2 \cdots b_n, n \geq 0$ s.t. $b_i \in p^* + q^*$

if $b_i \in p^* \Rightarrow b_i = c_{i,1}, c_{i,2} \cdots c_{i,z_i}$ with

$c_{i,j} \in p$, ||ly if $b_i \in q^*$, then $c_{i,j} \in q$

$\Rightarrow b_i \in (p+q)^*$

$\Rightarrow s = b_1 b_2 \cdots b_n$, when $b_i \in (p+q)^*$ $\boxed{\text{PTO}}$

$\Rightarrow s \in ((p+q)^*)^* = (p+q)^*$

Since s was arbitrary $\Rightarrow (p^*+q^*)^* \subseteq (p+q)^* \cdots$ (II)

From (1) and (2) $\Rightarrow (p^*+q^*)^* = (p+q)^*$

R.W $(p+q)^* = (p^*+q^*)^*$

let