COL 352 Introduction to Automata and Theory of Computation

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March 20, 2023

Lecture 23:Turing Machines: Variants, CT Thesis (Part 2)

k-tape Turing machines

Theorem

Every k-tape Turing machine has an equivalent 1-tape Turing machine.

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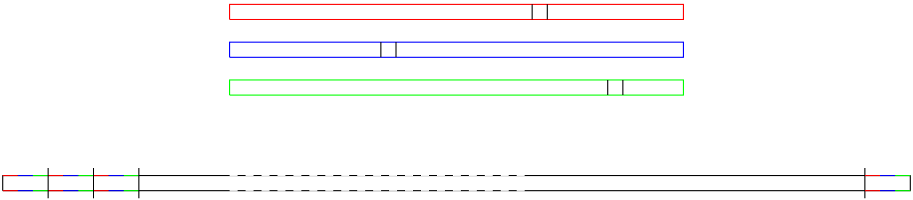

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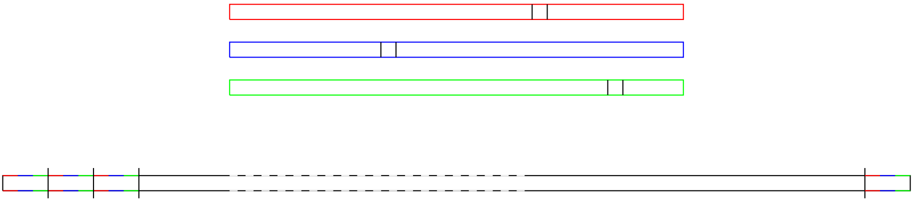

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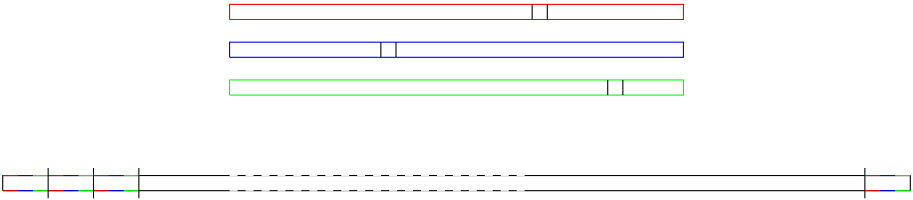

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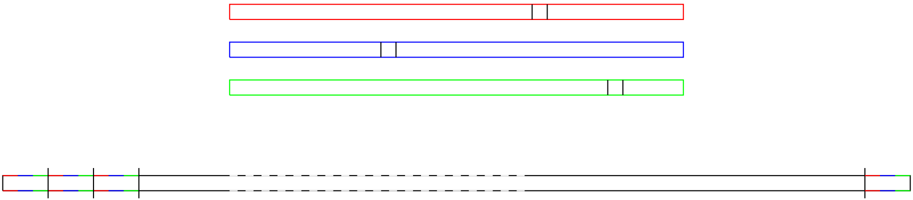

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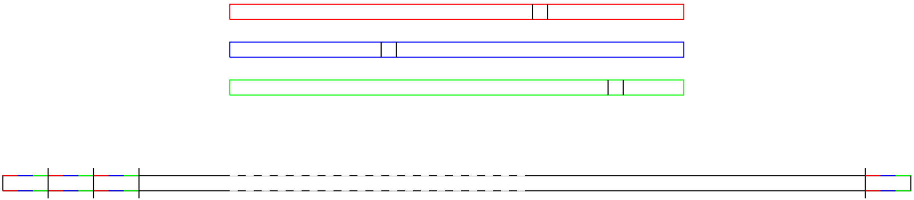

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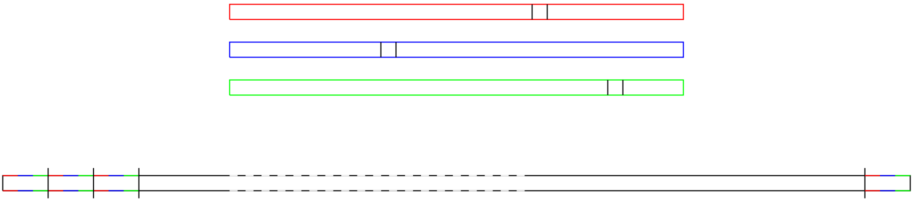

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$\overline{\Gamma}$ symbols used to denote tape head positions.

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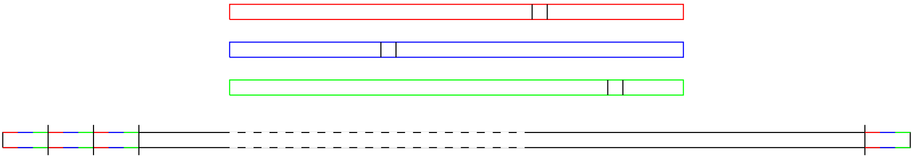

To simulate 1 step of M, M' works follows:

reads the tape left to right once, remembering the marked symbols in its states,

uses δ to determine the next state,

sweeps the input left to right again to update marked symbols.

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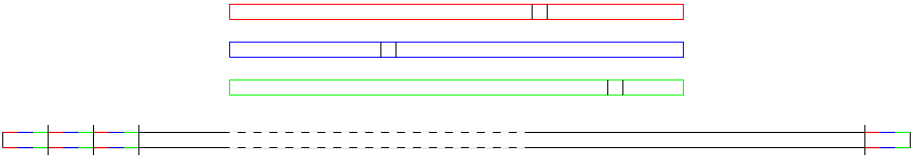

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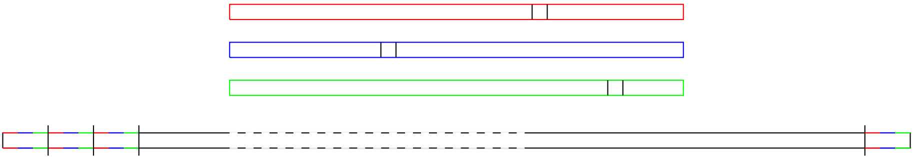

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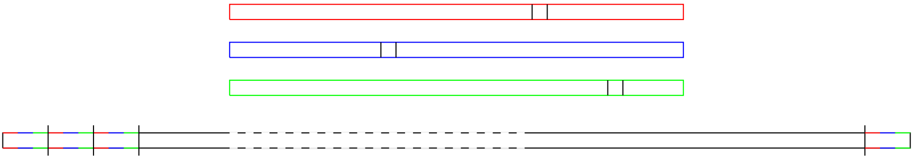

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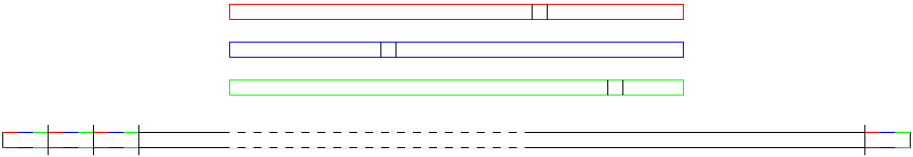

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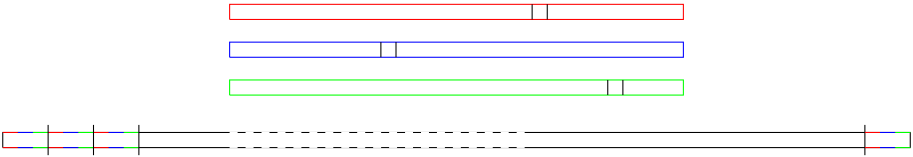

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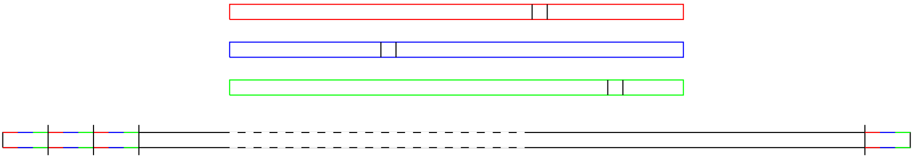

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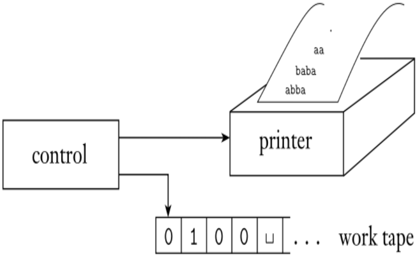

- Turing machine with an attached printer.
- **Exercise:** Formally define it!

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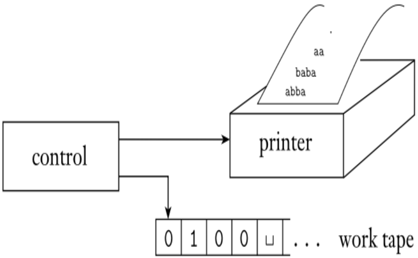

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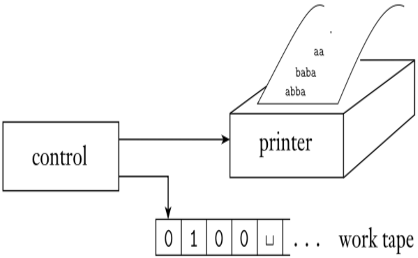

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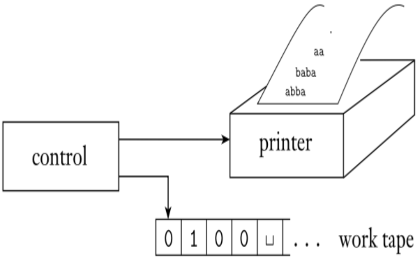

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- If the enumerator doesn't halt, it may print an infinite list of strings.
- The language enumerated by E is the collection of all the strings that it eventually prints out.
- E may generate the strings of the language in any order, possibly with repetitions.

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2. If any computations accepts, print out the corresponding s_j.

$\square$

Remark: Turing Recognizable = Recursively Enumerable languages.