# Quiz 4

● Graded

**Student**

Abhinav Shripad

**Total Points**

**10 / 10 pts**

**Question 1**

## Q1

**10** / 10 pts

✔ **+ 1 pt** Claiming that CFLs are closed under homomorphism

✔ **+ 2 pts** Defining the correct grammar $G_2$ for $f(L)$, where $f$ is a homomorphism and $L$ is context free

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$f(L) \subseteq L(G_2)$

✔ **+ 0.5 pts** Claiming this

✔ **+ 1.5 pts** Proving the claim

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$L(G_2) \subseteq f(L)$

✔ **+ 0.5 pts** Claiming this

✔ **+ 3.5 pts** Proving the claim

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✔ **+ 1 pt** $f(L) \subseteq L(G_2) \wedge L(G_2) \subseteq f(L) \implies f(L) = L(G_2)$

**+ 0 pts** Totally Incorrect / Not Attempted

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COL352: Introduction to Automata & Theory of Computation QUIZ

Date: 20 March 2025 15 minutes Maximum marks: 15

True. CFL is closed under homomorphism.

Let our CFL has grammar

$G = (R, T, NT, S)$

consider a general homorphism $h$ from $T$ to $\Sigma^*$, ie $h: T \longrightarrow \Sigma^*$.

Construct a grammar $G'$ such that

$G' = (R', \Sigma, NT', S')$, $A' \in NT'$ iff $A \in NT$

where

$A \rightarrow u_1 u_2 \cdots u_k \in R$ then

$A' \rightarrow h(u_1) h(u_2) \cdots h(u_k) \in R'$

llly $A \rightarrow u_1 B u_2 \cdots C u_k \in R$ then

$A' \rightarrow h(u_1) B' h(u_2) \cdots C' \in R'$.

Basically replace terminals by h(terminals) and nonTerminal (say A) by nonTerminal' ($A'$).

Claim:- $w \in L(G) \iff h(w) \in L(G')$.

Proof:- Consider abitrary $w \in L(G)$, it has a derivation as follows in $G$

$S \xrightarrow[G]{*} w$, so every rule used in this derivation has a corresponding rule in $G'$

thus $S' \xrightarrow[G']{*} h(w) \Rightarrow h(w) \in L(G')$.

$\Rightarrow$ $\boxed{h(L(G)) \subseteq L(G')}$ (I)

Consider arbitrary $w' \in L(G')$. it has a derivation in $G'$ as

$S' \xrightarrow[G]{*} w'$, but each rule in $G'$ has nonterminals as $h(u_i)$ or non-Terminal',

thus a corresponding derivation of $G$ with $h(u_i)$ replaced by $u_i$ and nonTerminal' by nonTerminal gives $w \in L(G)$. Clearly $h(w) = w' \in L(G')$

$\Rightarrow$ $\boxed{L(G') \subseteq h(L(G))}$ (II)

From (I) and (II)

Thus $L(G') = h(L(G))$ and $G'$ is a grammar

$\Rightarrow h(L(G))$ is CFL.

$\Rightarrow$ CFL is closed under homomorphism.