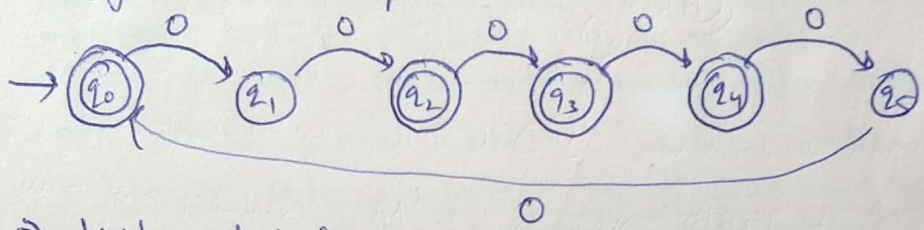
Note (i) Write your answers neatly and precisely in the space provided with with each question including back of the sheet. You won't get a second chance to explain what you have written.
(ii) You can quote any result covered in the lectures without proof but any other claim should be formally justified.

1. Design a DFA for the language $L = \{w \in 0^* \mid |w| \text{ is a multiple of 2 or 3}\}$. Briefly explain the construction. **(5)**

Consider all integers as $6k, 6k+1, 6k+2, 6k+3, 6k+4, 6k+5$ then those integers that are of $6k, 6k+2, 6k+3, 6k+4$ form, they will be accepted.



5

$q_i \Rightarrow |w| \bmod 6 = i$ i.e q_i contains the string whose length was such that $|w| \bmod 6 = i$.

$\delta(q_i, 0) = q_{(i+1) \bmod 6}$

i.e every incoming 0 ~~takes~~ increases length of $|w|$ by 1.

2. Let $L_1, L_2 \subset \Sigma^*$ be infinite languages. **(5+5)**

(a) If $L_1 \cap L_2$ is regular then L_1 and L_2 are regular. Justify or give a counterexample.

0

We know than L_1 is regular and L_2 is regular

so $\bar{L_1}$ and $\bar{L_2}$ are also regular respectively (Regular set is closed under complementation)

$\Rightarrow \bar{L_1} \cup \bar{L_2}$ is also regular (Regular set is closed under union)

$\Rightarrow \overline{\bar{L_1} \cup \bar{L_2}}$ is also regular

and $L_1 \cap L_2 = \overline{\bar{L_1} \cup \bar{L_2}}$. Hence, $L_1 \cap L_2$ is regular.

This is finite!

(b) For $L_1 \subset L_2$, can L_2 be non-regular and L_1 regular ? Provide an example or argue about impossibility.

1

Consider $L_2 = \{0^n 1^n \mid n \geq 1\}$ & $L_1 = \{0^n 1^n \mid 1 \leq n \leq 10^3\}$

Clearly $L_1 \subset L_2$ as L_2 contains all possible strings over 0 & 1.

but we know that L_1 is regular as it is finite. Hence If $L_1 \subset L_2$

It is possible that L_1 is regular & L_2 non regular

3. Consider the language $L = \{0^i \cdot 1^j | i \neq j\}$ for $\Sigma = \{0, 1\}$. Consider the following arguments to show that L is not regular. Point out the fallacy in the proofs (if any) in one sentence.

(a) Since $\{0^i \cdot 1^j | i = j\}$ is not regular (proved in class), it follows from the the closure property of complement of Regular languages that L is not regular. **(2)**

Fallacy is that complement of $L = \{0^i 1^j | i \neq j\}$ is $L' = \Sigma^* - L$ which is not equal to $\{0^i 1^j | i = j\}$. L' is a much wider set.

2

(b) Consider the language $L_< = \{0^i \cdot 1^j | i < j\}$. It can be proved easily by Pumping Lemma that $L_<$ is not regular by choosing a string $0^n \cdot 1^{n+1}$ where n is the constant of the Pumping Lemma and pumping enough 0's so that it exceeds the number of 1's. Similarly, the language $L_> \{0^i \cdot 1^j | i > j\}$ is not regular. Since $L = L_1 \cup L_2$, it follows that L is not regular. **(2)**

Fallacy is that ~~it is not necessary~~ non-regular languages are not closed under union. i.e in $L = L_1 \cup L_2$, L may be regular if L_1 & L_2 are both irregular. eg if L is irregular, $\bar{L}$ is also irregular.

~~to we can get this as if L is irregular $\bar{L}$ is also irregular~~

$\Rightarrow L' = L \cup \bar{L} = \Sigma^*$ is also irregular which is clearly not the case.

2

(c) *Using Pumping Lemma* Consider a string $z = 0^n \cdot 1^{n+k}$ where n is the constant of the Pumping Lemma and k is an integer $1 \leq k \leq n$. In the partiion $z = u \cdot v \cdot w$, note that uv consists only of 0's, so choose $v = 0^k$. Then $uv^2w = 0^{n+k} \cdot 1^{n+k} \notin L$ and therefore a contradiction. **(2)**

We cannot choose v by our own choice. ~~As~~ The statement is $\exists$ a partition $u \cdot v \cdot w$. ~~So~~ So, we need to prove for every possible p & not ~~for~~ only $v = 0^k$.

2

(d) In case, you find all the proofs are incorrect, then either show that L is regular or give a correct proof that L is not regular. (Otherwise you just mention one of the previous proof that is correct). **(6)**

Consider $L' = \{0^* 1^*\}$ & $L = \{0^i \cdot 1^j | i = j\}$

Now $L_2 = L' - L \Rightarrow L_2 = \{0^i \cdot 1^j | i \neq j\}$ (By construction)

Let L_2 be regular, then

$L' - L_2 = L$ should be regular

as $L' - L_2 = L' \cap \bar{L}_2$ [Both L' & $\bar{L}_2$ is regular then $L' \cap \bar{L}_2$ is regular]

But we know that L is not regular as by pumping lemma for a string $0^n 1^n$ with pumping constant n, I can pump more ~~zer~~ 0's which are not accepted.

Hence, By contradiction L_2 is not regular.

6

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4. Let L be a regular language over $\{0,1\}$ and consider the set of strings $S = \{y|y\cdot(01^*01+010^*) \in L\}$.
(i) What can you say about S - is it always regular ? Justify or give a counterexample. (10)
(ii) What can you say about $S' = \{y|y\cdot 0^i\cdot 1^i \in L, i \geq 1\}$ (3)

1

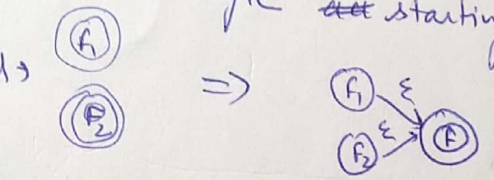
(i) First we know that $(01^*01 + 010^*)$ is a regular expression
So, we will have a Machine M that can represent $(01^*01 + 010^*)$. ~~[illegible]~~

Confusing arguments

Not proper construction

With the use of ε-transitions we can always have this machine with a single ~~[illegible]~~ starting state and single final state.

M?



. Similarly for start.
A common state ε-translate to all starting states.

existence by contradiction?

So, If $y\cdot(01^*01 + 010^*) \in L$ and L is regular, we must have a machine M' that can accept y. If it were not so, then, the only way to recognize $y\cdot(01^*01+010^*)$ was through the ~~[illegible]~~ single start state. Before that there must exist some machine to take y from starting state ~~[illegible]~~ of $\boxed{M'.M}$ to Starting state of M.

what construct is this?

So, S is always regular.

→ Alternatively, consider $S = y\cdot L'$ where L' is regular.
Then $S'' = y\cdot\overline{L'}$ should also be regular [complement of regular language is also regular]
In that case $S \cup S'' = (y\cdot L') \cup (y\cdot\overline{L'}) = y\cdot(L' \cup \overline{L'})$
$= y\cdot\Sigma^*$ is also regular
So, basically y concatenated to any string is regular.
Σ^* also contains ε (epsilon = Empty string)
$\Rightarrow y\cdot\varepsilon$ is regular $\Rightarrow y$ is regular.

(ii) In this case we can't say that S' is regular as we can take

0

$y = \{0^k | k = \text{prime}\}$. This will give ~~[illegible]~~ $L = \{0^i 1^j | i \geq 3, j \geq$
Clearly y is non-regular and L is regular. Our previous argument can't hold because we can't construct a Machine for