Minor Exam

Graded

Student

Abhinav Shripad

Total Points

48 / 50 pts

Question 1

Q1

6 / 6 pts

- ✔ **+ 4 pts** Correct construction
- ✔ **+ 2 pts** Justification or proof of equivalence of construction
- **+ 0 pts** Incorrect

Question 2

Q2

5 / 6 pts

- ✔ **+ 3.5 pts** Mentioning that M accepts Σ^*
- **+ 1.5 pts** Claiming that since δ is a total function, runnning any non-empty string s on M will lead to some state $q \in Q$
  (Note that δ being a total function is important, else one cant really define the state reached after reading a string s.)
- ✔ **+ 1 pt** Since any state is an accepting state, hence, any string s will be accepted.
- **+ 0 pts** Totally Incorrect / Not Attempted
- **+ 0.5 pts** Note that δ being a total function is important, else one cant really define the state reached after reading a string s.

1 Should mention this exists since δ is total function

Question 3

Q3 **13** / 14 pts

3.1 **Q3. a)** **3** / 4 pts

+ **4 pts** Correct NFA

✔ + **3 pts** Almost Correct, with very minor mistake

+ **0 pts** Totally Incorrect / Not Attempted

Good Solution

2 Another reason for the contradiction can be that there is no r present in w. That case should be handled too.

3.2 **Q3. b)** **4** / 4 pts

✔ + **4 pts** Correct NFA

+ **3 pts** Almost correct, with very minor mistakes

+ **0 pts** Totally Incorrect / Not Attempted

3.3 **Q3. c)** **6** / 6 pts

✔ + **6 pts** Proved that Regex corresponds to Σ^* and hence, the DFA / NFA consist of just a single state (implying it is minimised)

+ **4.5 pts** Correct equivalent DFA

− **2.5 pts** Not showing any steps for making the equivalent DFA

+ **1.5 pts** Correct Minimised DFA (will consist of just a single state)

+ **0 pts** No marks if NFA in previous part was incorrect

+ **0 pts** Totally Incorrect / Not Attempted

Question 4

Q4 **14** / 14 pts

4.1 **Q4. a)** **8** / 8 pts

✔ + **8 pts** Correct CFG

+ **0 pts** incorrect

4.2 **Q4. b)** **6** / 6 pts

✔ + **1 pt** Correct base case

✔ + **2 pts** Correct Induction hypothesis

✔ + **3 pts** Correct proof of Induction step

+ **0 pts** Incorrect

Question 5

Q5 **10** / 10 pts

5.1 Q5. a) **4** / 4 pts

+ 0 pts Incorrect

✔ **+ 2 pts** Consider a k. Consider a string of the form $a^k b a^{k!+k}$, where $x = \epsilon$, $y = a^k$, and $z = ba^{k!+k}$.

✔ **+ 1 pt** Given $u = a^p, v = a^q, w = a^r$ such that $uvw = a^k$ and $q \neq 0$.

✔ **+ 1 pt** Need i such that xuv^iwz is not a palindrome.

$xuv^iwz = a^{p+qi+r}.b.a^{k!+k}$.

$p + q + r = k$ and $p + qi + r = k! + k$

So $k + (i - 1)q - k! + k$, i.e. $(i - 1)q = k!$. Choose i to be $1 + (k!)/q$ (which must be an integer because $1 \leq q \leq k$, and so q divides $k!$.

5.2 Q5. b) **6** / 6 pts

+ 0 pts Incorrect

✔ **+ 2 pts** PDA has two states and accepts by empty stack. First state: loop for the "first" part of palindrome, push down onto stack

✔ **+ 2 pts** Guess and make a transition on ϵ, 0 and 1 without disturbing stack

✔ **+ 2 pts** Move to second state: loop for the "later" part of the palindrome, pop off the stack as long as input letter matches the top of the stack

Name: Abhinav Rajesh Shripad Entry Number: 2022CS11596

Indian Institute of Technology Delhi

COL352: Introduction to Automata & Theory of Computation **MID-TERM EXAM**

DATE: Tuesday the 25th of February 2025 **DURATION:** 2 hours **MAXIMUM MARKS:** 50

Q1 (6)	Q2 (6)	Q3 (14)	Q4 (14)	Q5 (10)	Total (50)

Instructions: Write your name and entry number on each sheet (rough sheet also). Answer **only** in the given boxes. You can use the last sheet or ask for at most one rough sheet to work stuff out on before writing a clean solution here. If needed, make and state reasonable assumptions.

Attestation: I agree to abide by the Honour Code of IIT Delhi.

Signature: Abhinav

Q1. (6 marks) Define a 1NFA, which is an NFA with one initial state. A 1NFA is given by $M = (Q, \Sigma, \Delta, q_0, F)$, where Q is the set of states, $\Delta \subseteq (Q \times (\Sigma \cup \{\varepsilon\}) \times Q)$, $q_0 \in Q$ is the initial state, and F is the set of final states. A string $w \in \Sigma^*$ is accepted by M iff there is some path (according to the transitions in Δ) on w from q_0 to some $f \in F$ in M. Show that every NFA has an equivalent 1NFA.

Consider a generalized NFA $= (Q, \Sigma, \Delta, Q_0, F)$
, create a equivalent 1NFA $= (Q \cup q_0, \Sigma, \Delta', q_0 F)$
where $\Delta' = \Delta \cup \{ (q_0, \varepsilon, q) \mid q \in Q_0 \}$.

Claim:- Language accepted by NFA is same as that accepted by 1NFA.

Consider arbitary $w \in \Sigma^*$, then

$$\Delta'(\{q_0\}, w) = \Delta'(\{q_0\}, \varepsilon w)$$
$$= \Delta'(\Delta(q_0, \varepsilon), w)$$
$$= \Delta'(Q_0, w) = \Delta(Q_0, w)$$

so states w end up in 1NFA is same as the states it end up in the NFA. Since set of accepting state is same, we have

$$w \in L(NFA) \iff \Delta(Q_0, w) \cap F \neq \phi$$
$$\iff \Delta'(q_0, w) \cap F \neq \phi$$
$$\iff w \in L(1NFA)$$

$\Rightarrow L(NFA) = L(1NFA)$

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Q2. (6 marks) A DFA is given by $M = (Q, \Sigma, \delta, q_0, F)$, where Q is the set of states, δ is a total function from $Q \times \Sigma$ to Q, $q_0 \in Q$ is the initial state, and F is the set of final states. Consider a DFA M where $F = Q$. What language does M accept? Prove your answer formally.

$L(M) = \Sigma^*$

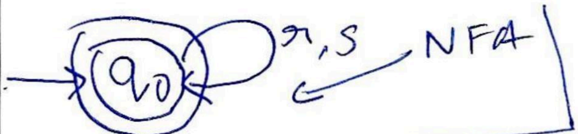
Since M is a DFA, $\forall w \in \Sigma^*$ has a valid run on it, thus

$\hat{\delta}(q_0, w) \in Q = F \Rightarrow \hat{\delta}(q_0, w) \in F \; \forall w$

Since w was arbitrary $L(M) = \Sigma^*$

Q3. (14 marks) Consider the language L over $\Sigma = \{r, s\}$ described by $(r+s)^* rs(r+s)^* + s^*r^*$.

(a) (4 marks) Convert the regular expression into an NFA which accepts L.



Claim:- $L = \Sigma^*$

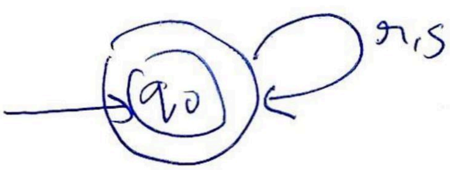
let $L_1 = (r+s)^* rs (r+s)^*$

$L_2 = s^* r^*$

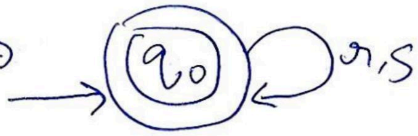
Claim:- $\overline{L_1} = L_2$, consider $w \in \overline{L_1}$. In w consider the first occurence of "r", Now consider first "s" after this "r", This means it has a "s" followed by "r".

Continued on soln to Q3 (c)

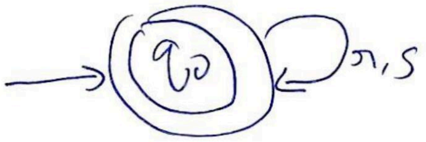
(b) (4 marks) Draw an NFA recognizing L which has a single initial state and does not have any ε-transitions.



(c) (6 marks) Use the subset construction and obtain a DFA equivalent to the NFA in Q3.b. Minimize this DFA. Show all steps clearly.

Since NFA $\Rightarrow$ 

It doesnt change in subset constuction

DFA = 

It is clearly the minimum DFA.

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q3(a) Continuation

$\Rightarrow$ w has 'rs' as substring $\Rightarrow w \in L_1$ as $L_1 = \Sigma^* rs \Sigma^*$ contradiction.

$\Rightarrow$ No 's' in w after the first 'r'.

$\Rightarrow$ All 's' in w is before the first 'r'.

$\Rightarrow w = s^n r^m \Rightarrow \overline{L_1} = s^* r^* = L_2$

Thus clearly $\overline{L_1} = L_2 \Rightarrow L_1 + L_2 = \Sigma^*$

Thus the NFA clearly accepts Σ^*.

Q4. (14 marks)

(a) (8 marks) Write a context-free grammar for the language $L \subseteq \{a, b\}$ defined as follows: $w \in L$ iff w contains an equal number of 'a's and 'b's, but does not contain the substring 'ab'. **Hint:** Construct a grammar with more than one non-terminal symbol.

Consider $G = (T = \{a, b\}, NT = \{S\}, R, S)$

where $R = \{ S \rightarrow \varepsilon \mid bSa \}$

(b) (6 marks) Prove that this CFG generates L. Clearly state your induction hypothesis.

consider a $w \in L$. Consider the first occurence of "a" in w. Now consider the first occurence of "b" after this first "a". This would make a 'ab' substring.

$\Rightarrow$ No "b" after first "a" $\Rightarrow$ All "b" before first "a" $\Rightarrow w = b^m a^n$, since $\#b(w) = \#a(w)$

$\Rightarrow m = n \Rightarrow L = \{ b^n a^n \mid n \geq 0 \}$.

consider $w \in L(G)$ which is generated using n^{th} step derivation. Since there is only 1 rule in G, it is

$$S \xrightarrow{1} bSa \xrightarrow{2} bbSaa \rightarrow \cdots \xrightarrow{n} b^n S a^n \xrightarrow{n+1} b^n \varepsilon a^n$$

$\Rightarrow w = b^n a^n \Rightarrow L \subseteq L(G)$

Illy consider arbitary $w \in L(G)$, thus $w = b^n a^n$, clearly $w \in L$ as $\#b(w) = \#a(w)$ and no 'ab' in w. $\Rightarrow L(G) \subseteq L$. Thus

$L(G) = L$.

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Q5. (10 marks) Consider $L = \{w \mid w \text{ is a palindrome}\} \subseteq \{a, b\}^*$.

(a) (4 marks) Use the pumping lemma to show that $\overline{L} = \{w \mid w \text{ is not a palindrome}\} \subseteq \{a, b\}^*$ is not regular.

Advesary :- Choose $k \geq 0$

Me :- let $x = \varepsilon$, $y = a^k$ $z = ba^{k+k!}$, clearly $xyz \in \overline{L}$

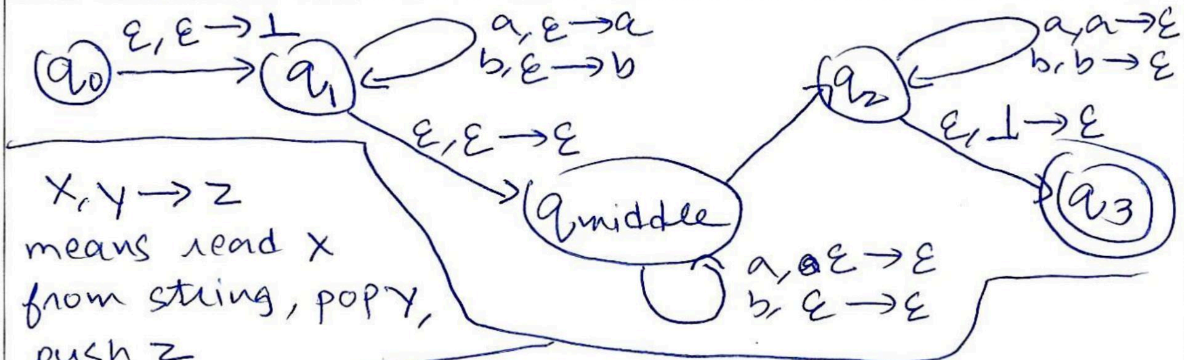
Advesary :- Choose $u = a^p, v = a^q, w = a^r$ where $p, q, r \geq 0$ and $p+q+r = k$, $q > 0$

Me :- ~~Choose~~ Clearly $q \leq k$, choose $n = 1 + k!/q \in \mathbb{N}$

then $xuv^n wz = a^{k+k!}\, b\, a^{k+k!} \notin \overline{L}$

I won the game. $\overline{L}$ is not regular.

(b) (6 marks) Construct a PDA for L, and informally but rigorously argue that it accepts L.



Clearly $L = \{ww^R \mid w \in \Sigma^*\} \cup \{waw^R \mid w \in \Sigma^*\} \cup \{wbw^R \mid w \in \Sigma^*\}$

because all even length palindrome are of the form ww^R and all odd length are of the form waw^R or wbw^R. ww^R is accepted by M because $(q_{middle}, S) \subseteq \Delta(q_0, w, \varepsilon)$ where $S = w^R$, //ly $(q_3, \varepsilon) \subseteq \Delta(q_{middle}, w^R, w^R)$ because we pop exactly what we read thus $(q_3, \varepsilon) \in \Delta(q_0, ww^R, \varepsilon)$.

//ly we can argue for waw^R and wbw^R as the middle a, b is read and 'ignored' by q_{middle}.

This page will not be graded; do rough work here.