Big Quiz 3

Graded

Student

Abhinav Shripad

Total Points

11 / 20 pts

Question 1

Q1 — **10** / 10 pts

+ 0 pts Incorrect

Not regular: Pumping lemma

- ✔ **+ 1 pt** Not regular
- ✔ **+ 2 pts** Correct form of pumping lemma/Myhill Nerode
- ✔ **+ 2 pts** Correct choices for xyz, and i/correct justification for infinitely-many equivalence classes spawned by the M-N relation

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- ✔ **+ 5 pts** Construction for CF: grammar/PDA

Question 2

Q2

1 / 10 pts

- **+ 1 pt** Non-Context Free but Turing decidable

Proof for Non-Context Free

- **+ 3 pts** Applying pumping lemma correctly
- **+ 1.5 pts** choices of xyz, i
- **− 1.25 pts** Missed some cases in applying pumping lemma
- **− 1 pt** chosen xyz do not work in some cases

Turing Machine Description

- **+ 2 pts** Verify equal lengths
- **+ 2.5 pts** Checked for differences
- **− 0.5 pts** Minor issues with verifying equal lengths
- **− 0.5 pts** Minor issues with checking differences

✔ **+ 0 pts** Incorrect

+ 1 pt Point adjustment

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Indian Institute of Technology Delhi

COL352: Introduction to Automata and Theory of Computation **MAJOR QUIZ 3**

DATE: Tuesday the 15th of April 2025 **DURATION:** 45 minutes **MAXIMUM MARKS:** 25

Instructions: Write your name and entry number at the top of each sheet. Use page number 1 and 2 for answering Q1, and 3 and 4 for answering Q2. Answers written on incorrect pages will be marked zero.

Attestation: I agree to abide by the Honour Code of IIT Delhi.

Signature: Abhinav

A1.

$L_1 \Rightarrow$ CFL but not regular.

To show :- CFL, consider a grammar G such that

$(\{S, A, B\}, \{0,1\}, R, S)$ where R has

$$R = \begin{cases} S \longrightarrow AB \mid BA \\ A \longrightarrow 1 \mid 0A0 \mid 0A1 \mid 1A0 \mid 1A1 \\ B \longrightarrow 0 \mid 0B0 \mid 0B1 \mid 1B0 \mid 1B1 \end{cases}$$

Claim :- $L(G) = L_1$

Consider $w = xy \in L_1$, clearly $|x| = |y| > 0$

$\Rightarrow$ $x = (\Sigma)^m 0 (\Sigma)^n$ and $y = (\Sigma)^m 1 (\Sigma)^n$ ... (I)

OR $x = (\Sigma)^m 1 (\Sigma)^n$ and $y = (\Sigma)^m 0 (\Sigma)^n$ ... (II)

(ie x, y differ at $m+1$ position)

WLOG take case 1

$\Rightarrow w = (\Sigma)^m 0 (\Sigma)^n (\Sigma)^m 1 (\Sigma)^n$

$\Rightarrow w = \underbrace{(\Sigma)^m 0 (\Sigma)^m}_{\text{can be derived from B}} \underbrace{(\Sigma)^n 1 (\Sigma)^n}_{\text{can be derived from A}}$

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A1. (contd.)

$\Rightarrow B \xrightarrow{*} x$ and $A \xrightarrow{*} y$

$\Rightarrow S \longrightarrow BA \xrightarrow{*} xy = w \Rightarrow w \in L(G)$

$\Rightarrow$ ~~$L(G) \subseteq L_1$~~ $L_1 \subseteq L(G) \ldots$ (III)

$\|$ly consider $w \in L(G)$, WLOG assume w is derived from $S \longrightarrow AB \Rightarrow w$ is of the form

$$w = \underbrace{(\Sigma)^m 1 (\Sigma)^m}_{A} \underbrace{(\Sigma)^n 0 (\Sigma)^n}_{A}$$

$$= \underbrace{(\Sigma)^m 1 (\Sigma)^n}_{X} \underbrace{(\Sigma)^m 0 (\Sigma)^n}_{Y} = XY \text{ and } X \neq Y \text{ and } |X| = |Y|$$

$\Rightarrow w \in L_1 \Rightarrow L(G) \subseteq L_1 \ldots$ (IV)

From (III) and (IV) $\Rightarrow L(G) = L_1$

To prove:- Non Regular

Use pumping lemma:-

Adversary:- Choose $k \geq 0$ (Assume $k \geq 2$) ($k!$ to be even)

Me:- $S = xyz = 0^{k+k!} 1 1 0^{k+k!}$ clearly this in L_1

Me:- choose $x = 0^{k+k!} 11$, $y = 0^k$, $z = \epsilon$

Adversary:- $uvw = 0^k$, $|v| = \lambda$ (say) > 0

Me:- $i = 1 + k!/\lambda \in \mathbb{N}$, then xuv^iwz

$$= 0^{k+k!} 11 0^{k+k!} \notin L_1$$

$\Rightarrow$ $\boxed{L_1 \text{ is not regular}}$

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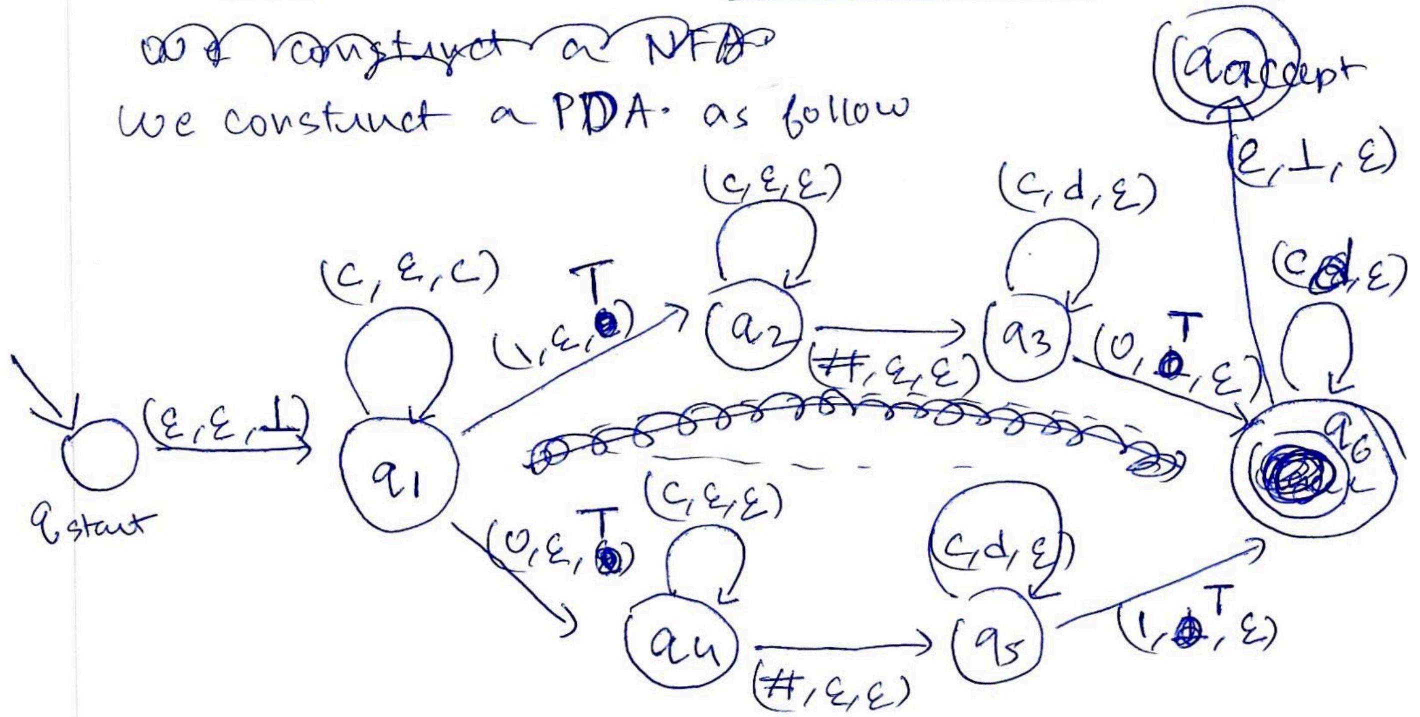
A2. ~~Regular~~ CFL but not regular.

~~we construct a NFA~~

We construct a PDA as follow



A string can take 2 paths on this PDA, the q_{start}, q_1, q_2, q_3, q_6, q_{accept} or the below one. both are symmetric WLOG if it takes upper path ⇒ it pushes any character n times to stack then reads 1 (push T) to go to q_2 then stays at 1 till reading #, then pops n character, then if it reads 0 then go to ~~accept and~~ then it pops of T (signifying same place at 1 at x) then in q_6 it reads character and go to q_{accept} when stack empties, Thus making sure $|x| = |y|$ but $x[n] \neq y[n] \Rightarrow x \neq y$.

⇒ L_2 is CFL.

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A2. (contd.)

To show :- L_2 is not regular

Advesary :- $k \geq 0$

Me :- $0^k \# 1^k$ and

$x = 0^k \#$, $y = 1^k$ $z = \epsilon$ $(|y| \geq k)$

Advesary :- $uvw = 1^k$,

Me :- choose $i = 2$, then

$0^k \# uv^2 z$ but $|ux^2 z| > k$

$\Rightarrow |ux^2 z| \neq |0^k|$ thus $\notin L_2$

$\Rightarrow L_2$ is not regular.