COL 352 Introduction to Automata and Theory of Computation

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Lecture 29: Rice's Theorem

More undecidable languages

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Undecidability is the not an exception - it is the rule!
Rice's theorem: A systematic way of proving undecidability of languages.

Property P

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Let P be a non-trivial property of Turing recognizable languages. Let $\mathcal{L}_P = \{M \mid L(M) \in P\}$. Then $\mathcal{L}_P$ is undecidable.

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We say that a property P is trivial if either $\mathcal{L}_P = \varnothing$ or $\mathcal{L}_P$ is the set of all the Turing recognizable languages.

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   Therefore, P is not trivial.

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Therefore, P is in fact all Turing recognizable languages.

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