Big quiz 2

Graded

Student

Abhinav Shripad

Total Points

25 / 25 pts

Question 1

Q1

Resolved **10** / 10 pts

- ✔ **+ 3 pts** Defining correct automata for the grammar.
- ✔ **+ 3 pts** Proof that any string accepted by NFA is also accepted by grammar.
- ✔ **+ 4 pts** Proof that any string accepted by grammar is also accepted by NFA.
- **+ 0 pts** Incorrect

Regrade Request **Submitted on: Apr 09**

Sir please tell what is incorrect in the proof ?

I have shown that for any string w, if there exists non terminals A and B such that A --> wB, then B is in delta_cap(w,A), which means that starting from any symbol A if i can get wB using production rules, then in the NFA, i can go from the state A to B by reading w. I also prove the opposite direction for the same.

After proving the above claim, I use the claim to show that if a string w is in the PDA, and thus S->w, by using my claim Epsilon_State is in delta_cap(S,w), but by construction Epsilon_State is the accept state of the NFA, we have w in the language of NFA. This shows that L(PDA) is subset of L(NFA). This also has marks allotted for it in the rubric.

Similarly then I use my claim again to show that if w is in Language of NFA, then Epsilon_State in delta_cap(S,w), thus using my claim S-->wEpsion=w.
Thus again w is in the language of PDA. This again shows that L(NFA) is subset of L(PDA).
This also has marks allotted for it in the rubric.

Sir please look into the proof. I have just explained my proof here. All the logic for the proof is in the Answer Sheet itself.

See first of all the w you are mentioning is a string, it should be an alphabet only, and in your claim in the answer sheet, you are just explaining if there is a B in delta hat, then there will be production in grammar, but it doesn't show that how the grammar is regular. Since the structure of the proof is somewhat similar, you will get some marks.

Reviewed on: Apr 15

Question 2

Q2

15 / 15 pts

✔ **+ 1 pt** L is CF but not regular

✔ **+ 3.5 pts** Grammar

+ 1 pt Partially Correct Grammar

$L \subseteq L(G)$

✔ **+ 3.5 pts** Correct

+ 2.5 pts Minor issue

+ 1 pt Major issue

$L(G) \subseteq L$

✔ **+ 3.5 pts** Correct

+ 2.5 pts Minor issue

+ 1 pt Major issue

Proof for L is not regular

✔ **+ 3.5 pts** Correct

+ 2.5 pts Minor issue

+ 1 pt Major issue

+ 3.5 pts PDA

+ 1 pt Partially Correct PDA

$L(P) \subseteq L$

+ 3.5 pts Correct

+ 2.5 pts Minor issue

+ 1 pt Major issue

$L \subseteq L(P)$

+ 3.5 pts Correct

+ 2.5 pts Minor issue

+ 1 pt Major issue

+ 0 pts Incorrect

1 typo?

Name: Abhinav Rajesh Shripad Entry Number: 2022CS11596

Indian Institute of Technology Delhi
COL352: Introduction to Automata and Theory of Computation **MAJOR QUIZ 2**

DATE: Tuesday the 25th of March 2025 **DURATION:** 45 minutes **MAXIMUM MARKS:** 40

Instructions: Write your name and entry number at the top of each sheet. Use page number 1 and 2 for answering Q1, and 3 and 4 for answering Q2. Answers written on incorrect pages will be marked zero.

Attestation: I agree to abide by the Honour Code of IIT Delhi.

Signature: Abhinav

A1. Consider CFG, $G = (NT, T, R, S)$, construct a

~~NFA M st M = (T, Δ, NT, {S},~~

~~M = (T ∪ {ε~~

NFA M st $M = (\underbrace{T}_{\text{alphabet}}, \Delta, \underbrace{NT \cup \{\varepsilon\}}_{\text{states}}, \underbrace{\{S\}}_{\text{startstate}}, \underbrace{\{\varepsilon\}}_{\text{endstate}})$

where

$$\left.\begin{array}{l} A \to aB \in R \iff (A, a, B) \in \Delta \\ A \to a \in R \iff (A, a, \varepsilon) \in \Delta \end{array}\right\} \text{— (I)}$$

$A \to \varepsilon \in R \iff (A, \varepsilon, \varepsilon) \in \Delta$.

~~Claim:- L(M) = L(G)~~

~~Proof:- Consider a arbitrary w ∈ L(G)~~ where

Claim:- $B \in \hat{\Delta}(w, A) \iff A \xrightarrow{*} wB$, $w \in T^*$ .. (II)

Proof:- If $B \in \hat{\Delta}(w, A)$, then we induct on length of $|w|$. If $|w| = 1$

$\Rightarrow B \in \hat{\Delta}(w, A) = \Delta(w, A) \Rightarrow (A, w, B) \in \Delta$

thus from (I) $\Rightarrow A \to wB \in R$.

Assume true for $|w| = 1, 2 \cdots, n-1$ ~~for~~ ~~a~~ for $|w| = n$

A1. (contd.)

let $w = vc$, $|v| = n-1$, $c \in T$

$\Rightarrow B \in \hat{\Delta}(w, A) = \hat{\Delta}(vc, A)$
$= \hat{\Delta}(c, \hat{\Delta}(v, A))$
$= \Delta(c, D)$ where $D \in \hat{\Delta}(v, A)$ for some D.

$\Rightarrow D \rightarrow cB$ and since $|v| = n-1$, by induction hypothesis, $A \xrightarrow{*} vD$ thus $A \xrightarrow{*} vc = w$

Thus we have shown 1 direction of (II).

Consider $w \in T^*$ s.t. $A \xrightarrow{*} wB$, we again induct on $|w|$, Base case $|w| = 1$ Thus $A \rightarrow wB$ but from (I), $B \in \Delta(w, A)$ clearly true by construction of Δ.

Assume true for $|w| = 1, \dots n-1$ for $|w| = n$ let $|w| = vc$, $|v| = n-1$, $c \in T$.

$\Rightarrow \exists D \in NT$ s.t. $A \xrightarrow{*} v\underline{D} \rightarrow v\underline{c}B$ ... (III)

since $|v| = n-1 \Rightarrow D \in \hat{\Delta}(v, A)$ and clearly from (III) $D \rightarrow cB \Rightarrow B \in \Delta(c, D)$

thus clearly $B \in \hat{\Delta}(c, \hat{\Delta}(v, A)) = \hat{\Delta}(vc, A)$

Hence Claim Proved.

Now consider $w \in L(G) \Leftrightarrow S \xrightarrow{*} w = w\varepsilon$
$\Leftrightarrow \{\varepsilon\} \in \hat{\Delta}(w, S)$ from Claim
$\Leftrightarrow w \in L(M)$

$\Rightarrow \boxed{L(G) = L(M)}$ since M is NFA $\Rightarrow L(M)$ is regular

$\Rightarrow \boxed{L(G) \text{ is regular}}$ Hence Proved.

A2. $L = \{a^i b^j \mid 2i \le 2j \le 3i\}$

$\boxed{\text{Answer :- CFL, not Regular}}$

For CFL consider the grammar $G = (\{S, S_2\}, \{a, b\}, R, \{S\})$

where Rules R are $S \to aaSbbb \mid S_2 \mid \varepsilon$

and $S_2 \to aS_2bb \mid \varepsilon$

Claim:- $L(G) = L$.

consider $w \in L(G)$, which is derived using the rule $S \to aSbbb$ x times followed by $S_2 \to aS_2bb$ y times (If $x = 0$, $w = \varepsilon$, but trivially $w \in L$)

thus we have $S \xrightarrow{x} a^{2x} S b^{3x} \to a^{2x} S_2 b^{3x}$ --- (I)

$\downarrow y$

$= a^{2x} a^y b^{2y} b^{3x} \leftarrow a^{2x} a^y S_2 b^{2y} b^{3x}$

thus $a^i b^j = a^{2x+y} b^{2y+3x} \Rightarrow i = 2x+y \quad j = 2y+3x$

Clearly $2(2x+y) \le 2(2y+3x) \le 3(2x+y)$

$\Rightarrow 2i \le 2j \le 3i \Rightarrow L(G) \subseteq L$ --- (II)

Now consider $w \in L \Rightarrow w = a^i b^j$ s.t. $2i \le 2j \le 3i$

let $x = j - i \ge 0$ (by $i \le j$) and $x \in \mathbb{N} \cup \{0\}$

and $y = 3i - 2j \ge 0$, thus clearly

w can be generated by G exaclty as (I)

for choosen x and y. $\Rightarrow L \subseteq L(G)$. (III)

From (II) and (III) we get $\boxed{L = L(G)}$

$\Rightarrow$ L is CFL.

A2. (contd.)

To ~~dis~~prove ~~to~~ L is non-regular, we use pumping lemma.

~~Consider~~

Advesary :- Choose k

Me :- Choose $S = a^{2k} b^{3k}$ with

$x = a^k$ $y = a^k$ $z = b^{3k}$

Advesary :- Choose $uvw = a^k = y$

Me :- let $|v| = \lambda$

then

$xuv^iwz = a^{2k+(i-1)\lambda} b^{3k} = a^i b^j$

choose $i = 100 + \lceil \frac{k}{\lambda} \rceil$

then we have $i > j$ thus

$xuv^iwz \notin L$

$\Rightarrow$ L is not regular.