2202 COL 352 Quiz1

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TOTAL POINTS

10 / 10

QUESTION 1

True or False 6 pts

1.1 Containment Regular language **3 / 3**

   + 0 pts Incorrect/No Explanation

   + 1 pts Partially correct with some right ideas

   ✓ **+ 3 pts** *Correct*

1.2 **3 / 3**

   ✓ **- 0 pts** *Correct*

   - 3 pts Incorrect

QUESTION 2

2 Divisibility by 4 **4 / 4**

   ✓ **+ 4 pts** *Completely Correct*

   + 0 pts Completely Incorrect/Unattempted

   + 3 pts Correct construction, but mistakes in proof of correctness

   + 2 pts Correct construction, but no proof of correctness

   + 1 pts There is an attempt, but construction is wrong

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(COL 352) Introduction to Automata and Theory of Computation   Jan 25, 2023

Quiz 1

Duration: 40 minutes   **(10 points)**

Beware: Be clear in your writing. If you use a statement proved in class or in the problem set, then write down the entire statement before using it. You will not get a new sheet, so make sure you are certain when you write something (maybe use a dark pencil). Make a judicious decision of which tool(s) to use to get a clean and short answer that fits in the space. If you cheat, you will surely get an F in this course.

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1. $(2 \times 3 = 6$ **points**$)$ For the questions that follow answer whether they are True/False with a brief justification. Each question carries 4 points. Simply writing True or False will not get you any points.

   (a) If L is a regular language and $L' \subseteq L$, then L' is also a regular language.

   <u>FALSE.</u>

   Consider the language Σ^* (set of all strings over the characters Σ_i)
   By closure properties we know that Σ^* is regular since Σ is regular
   (finite characters). For simplicity take $\Sigma_i = \{0,1\}$. Now, note that
   any language over $\Sigma_i \subseteq \Sigma^*$.
   However, we have already proved an impossibility result in class
   that $\exists$ languages over Σ which are <u>not regular</u>. Take any such
   language L'. Now $L' \subseteq \Sigma^*$ but L is <u>NOT</u> regular - Hence disproved.
   (For example: $L = 0^* 1^*$, $L' = \{0^n 1^n \mid n \geq 0\}$, L' is not regular but
                                                        L is, and $L' \subseteq L$)

   (b) Any finite language is a regular language.

   <u>Sol.</u> TRUE. Any finite language is a regular language.
   We can enumerate all strings of the language as
   $S_1, S_2, \ldots S_n$ (say $|L| = n$). For each $s_i \in L$,
   say $S_i = x_1^i x_2^i \ldots x_m^i$ (say)
   $\therefore$ In the ~~DFA~~ NFA we can add states as:

   $\therefore$ Now $\exists$ a run which <u>accepts the string</u> S_i.
   We do this for all string S_i (which are finite).

   This NFA/DFA
   accepts all
   $x \in L$

   $\therefore$ Finite language
   is
   regular

2. **(4 points)** Construct a DFA that works over the alphabet $\{0, 1, 2\}$ which recognizes base-3 representations of numbers divisible by 4. Give a brief proof of correctness of your construction.

Sol. Let the DFA $\equiv (Q, \Sigma, \delta, q_0, F)$, where :

> Let $\#w$ represent base-3 value of string.

$$Q = \{Q_0 \text{ ??? }\} \quad Q = \{0, 1, 2, \cancel{\text{???}} 3\}$$

$$\Sigma = \{0, 1, 2\}$$

$$\delta(x, a) = \cancel{(\text{??? ??? ???})} (3\#x + \#a) \bmod 4$$

$$q_0 = 0$$

$$F = \{0\}$$

Proof of correctness: To prove the correctness, we claim that

$$\cancel{\text{??????}} \hat{\delta}(0, w) = \#w \bmod 4$$

Proof. By induction on $|w|$.

Base case $|w| = 0 \implies w = \epsilon$

$$\therefore \hat{\delta}(0, \epsilon) = \delta(0, \epsilon) = 0 = \#0 \bmod 4$$
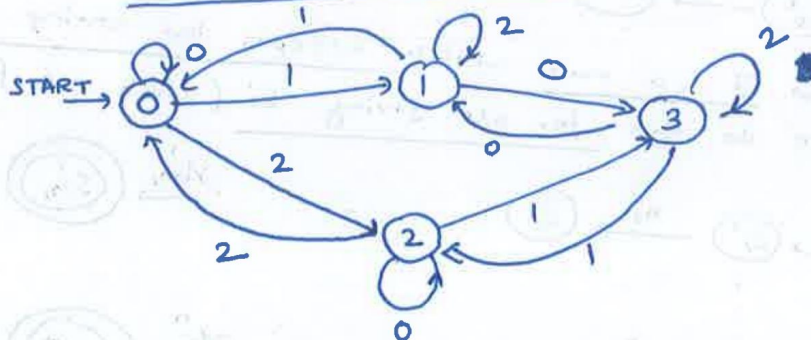
I.H. Assume true for all $|w| < k$.

I.S. Consider any $|\bar{w}| = k$ string w. Then, let $\bar{w} = w \cdot a$

$$
\begin{aligned}
\hat{\delta}(0, wa) &= \delta\left(\hat{\delta}(0, w), a\right) && (\text{by defn}) \\
&= \delta(\#w \bmod 4, a) && (\text{by I.H}) \\
&= (3\#w \bmod 4 + \#a) \bmod 4 && (\text{by defn of } \delta) \\
&= ((3\#w + \#a) \bmod 4) \bmod 4 && (\text{by modular arithmetic}) \\
&= \underline{\#\bar{w} \bmod 4} && \left(\text{since } \#\bar{w} = 3\#w + \#a \atop (\text{base 3 notation})\right)
\end{aligned}
$$

Hence, since $\hat{\delta}(0, w) = \#w \bmod 4 \;\forall w$, and 0 is the only accepting state, DFA accepts w iff $\#w \bmod 4 = 0$, which is as required. ∎

Diagram:

The DFA for the above problem.