Date: 2021/02/24 Duration: 1 hour

Read the instructions carefully.

This quiz is about proving that certain languages are regular. To do this, recall that you have the following tools at your disposal.

1. Existence of a DFA recognizing the language.
2. Existence of an NFA recognizing the language.
3. Closure properties.
4. Existence of a regular expression generating that language. (This is not in the quiz syllabus, so everything can be answered without using regular expressions. However, you are free to use regular expressions in your answer if you want.)

Make a judicious decision of which tool to use (or whether to use a combination of them) to get a clean and short answer (to minimize typing).

1. **[4 marks]** Let $\Sigma = \{0, 1\}$, and let

$$L = \{z \mid \exists x, y \in \Sigma^* \text{ such that } z = xy \text{ and } \#\text{ of 0's in } x = \ \#\text{ of 1's in } y\}.$$

   Prove that L is a regular language.

2. **[6 marks]** Let $L \subseteq \Sigma^*$ be any regular language. Prove that the language

$$L' = \{z \mid \exists x, y \in \Sigma^* \text{ such that } z = xy \text{ and } yx \in L\}$$

   is regular.