COL 352 Introduction to Automata and Theory of Computation
Minor 1, Sem II 2015-16, Max 40, Time 1 hr

Name ______________________ **Entry No.** ______________________**Group**

Note (i) Write your answers neatly and precisely in the space provided with with each question including back of the sheet. You won't get a second chance to explain what you have written.
(ii) You can quote any result covered in the lectures without proof but any other claim should be formally justified.

1. Let L be a language over $\{0,1\}$ that consists of strings having even number of zeroes *or* odd number of ones. For example, $1010, 1101 \in L$ and $101 \notin L$. $(5 \times 2\ )$
(i) Design a DFA for L. (No correctness proof required)

(ii) Hence or otherwise write a regular expression for this language. (No correctness proof required)

Even no. of 0's $r_1 = 1^* \cdot (01^*0)^* 1^*$
Odd no. of 1's $r_2 = 0^* \cdot 10^*(10^*1)^* 0^*$
Ans is $r_1 + r_2$.

Comment Although proof was not required, some basis must have been given for the r.e. Simply writing a long r.e. without saying what it represents was not acceptable. I also found several answers trying to capture all paths which was not done in class (it comes from writing and solving state equations). Such answers were also penalized.

2. Are the following languages regular ? Justify. (5×2)

   (a) $L = \bigcap_{i=1}^{i=\infty} L_i$ where L_i is regular.

   Not regular. We have already seen in class that $L = \bigcup_{i=1}^{i=\infty} L_i$ is not regular. Since regular languages are closed under complementation, $L = \bigcup_{i=1}^{i=\infty} L_i = \overline{\bigcap \overline{L_i}}$.
   If $\bigcap_{i=1}^{i=\infty} L_i$ is regular, then $\bigcap_{i=1}^{i=\infty} \overline{L_i}$ is regular and consequently its negation is regular which will contradict that $L = \bigcup_{i=1}^{i=\infty} L_i$ may not be regular.
   Comment By rewriting $L = \bigcap_{i=1}^{i=\infty} L_i$ as $\bar{L} = \bigcup_{i=1}^{i=\infty} \bar{L}_i$ and then invoking the previous result on infinite union is not the same unless done with some extra care. You have to first show that $L = \bigcup_{i=1}^{i=\infty} L_i$ is not regular, implies that $L = \bigcup_{i=1}^{i=\infty} \bar{L}_i$ is not regular and so on.

   We are using the mechanism of *reduction* to show that the current statement (say A) would imply that a known statment (B) is true. Since B is not true so A cannot be true.

   The statement B proved in class had the form $L = \bigcup_{i=1}^{i=\infty} L_i$ so any other form (including negation) must be explicitly dealt with.

   (b) $L = \{a_1 a_2 \ldots a_k | a_i \in \Sigma\}$ where $a_1 \cdot a_3 \cdot a_5 \ldots a_{k-1} \in L_1$ $a_2 \cdot a_4 \ldots a_k \in L_2$ and L_1, L_2 are regular.

L is regular. We can design a DFA similar to the product construction, with an extra bit in the state space to alternate between M_1 and M_2 (corresponding to L_1 and L_2). So $Q = Q_1 \times Q_2 \times \{1, 2\}$ and $\delta((q_1, q_2, 1), a) = (\delta_1(q_1, a), q_2, 2)$ and similarly for $(q_1, q_2, 2)$. For acceptance, it must be $F_1 \times F_2 \times 1$ (must end on the final state of M_2.

Comment If you only used some intuitive constructions with block diagrams and non-det transitions without giving the formal state-space and transition function, it was not acceptable.

3. Let $L_{pal} = \{w \in (0+1)^* | w \text{ is a palindrome }\}$. A palindrome is a string which equals its reversal. For example, 01010 is a palindrome but 01011 is not a palindrome Use pumping lemma to show that L_{pal} is not regular. **(8)**

Let L_{pal} be regular and n be the constant of the pumping lemma for L_{pal}. Choose the following string $z = 0^n \cdot 1^n \cdot 1^n \cdot 0^n$. Then $u \cdot v \cdot w = z$ where $|v| = 0^k$ for some $1 \leq k \leq n$. and $uv^iw \in L_{pal}$ for all i. This is not possible since the string will have unequal number of 0's in the beginning and end which cannot form a palindrome.

4. Let $L = \{0^k | k \text{ is prime or even }\}$. Is L regular ? Justify **(12)**

The given language is not regular. Let us assume that 2 is not prime to keep the argument simpler. Let $L_1 = \{0^k | k \text{ is even }\}$ and $L_2 = \{0^k | k \text{ is prime }\}$ So $L = L_1 \cup L_2$.

Let us assume that L is regular. Then $L - L_1 = L \cap \overline{L_1} = L_2$. We know that L_2 is not regular (primes done in class). So from closure properties we arrive at a contradiction about L_2. Therefore L cannot be regular.

By incuding 2 in L_2, we need an extension of the result on primes, namely, $L_3 = \{0^k | k \text{ is prime greater than 2 }\}$ is also not regular. This also follows from closure property since $\{2\}$ is regular, $L_3 \cup \{2\}$ will be regular - contradiction.

Alternate approach using PL only - no closure properties are used

Choose a string 0^n where $n > 2$ is an odd prime. Then $u \cdot v \cdot w = 0^n$ where $|u| = x \quad |v| = y \geq 1 \quad w = n - x - y$. Then for any $i \geq 0$, 0^{n_i} is in L where $n_i = x + iy + n - x - y = iy - n - y = y(i-1) + n$.

Choose $i = 2n + 1$. So $n_i = (n)(2y + 1)$. Then n_i is not prime and it is also not even since it is a product of two odd numbers. This leads to a contradiction.