CSL COL Intro to Automata and Theory of Computation, Tutorial Sheet 1

1. Design DFA for the following languages over $\{0, 1\}$
   (a) The set of all strings such that every block of of five consecutive symbols have at least two 0's.
   (b) The set of all strings beginning with a 1 which interpreted as an integer is congruent to zero modulo 5.
   (c) The set of strings with an equal number of 0's and 1's such that each prefix has atmost one more 0 than 1's and at most one more 1 than 0's.
   (d) The set of strings not containing the substring 110.

2. Design NFA for the following languages
   (a) The set of strings over $\{0, 1\}$ such that *some* pair of 0's is seprated by a string of length $4i$ $i \geq 0$.
   (b) The set of strings over $\{a, b\}$ that have the same value when multiplied from left to right as from left to right. The rules of multiplication are $a \times a = b, b \times b = a, a \times b = b, b \times a = b$. Note that $((a \times b) \times b) = a$ and $(a \times (b \times b)) = b$, i.e. they are not same, i.e. it is not associative.
   (c) The set of strings of the form $\{x \cdot w \cdot x^R | x, w$ are strings over 0,1 of non-zero length$\}$

3. Prove of disprove the following about regular expressions r, s, t where $r = s$ implies $L(r) = L(s)$
   (a) $r(s + t) = rs + rt$
   (b) $(r^*)^* = r^*$
   (c) $(r^* s^*)^* = (r + s)^*$
   (d) $(r + s)^* = r^* + s^*$

4. Which of the following are regular sets - Prove them
   (a) $\{0^{2^n} | n \geq 1\}$
   (b) $0^m 0^n 0^{m+n} | m, n \geq 1\}$
   (c) $\{0^n | n$ is prime $\}$
   (d) The set of all strings with equal number of 0s and 1's.
   (e) Set of all palindromes over 0,1.
   (f) $\{x x^R w | x, w \in (0 + 1)^+\}$

5. Let L be a regular set. Which of the following are regular
   (a) $\{a_1 a_3 \ldots a_{2n-1} | a_1 a_2 a_3 \ldots a_{2n}$ is in $L\}$
   (b) $MAX(L) = \{x$ is in $L|$ no extension of x is in $L\}$
   (c) $L^R = \{x | x^R$ is in $L\}$
   (d) $\frac{1}{2}(L) = \{x|$ for some y such that $|x| = |y|, xy \in L\}$.

6. A set of integers is *linear* if it is of the form $\{c + p \cdot i | i \geq 0\}$. A set is *semilinear* if it is a finite union of linear sets. Let $R \subset 0^*$ be regular. Prove that R is semilinear.

7. What is the relationship between class of regular sets and the least class of languages closed under union, intersection and complement containing all finite sets ?

8. What will be the statement of the converse of Pumping Lemma for regular language ?
   What does it imply for the language $\{0^i \cdot 1^j \cdot 2^k\}$ where $i = 0$ or $j = k$?