Big Quiz 1

Graded

Student

Abhinav Shripad

Total Points

25 / 25 pts

Question 1

Q1

10 / 10 pts

Regex

- ✔ **+ 4 pts** $L' = a^n.\{a^p\}^*$
- ✔ **+ 1.5 pts** Proving $L' \subseteq L$
- ✔ **+ 2.5 pts** Proving $L \subseteq L'$
- ✔ **+ 2 pts** Case when $p < 0$

DFA

- **+ 4 pts** DFA Construction
- **+ 1.5 pts** Proving $L_{DFA} \subseteq L$
- **+ 2.5 pts** Proving $L \subseteq L_{DFA}$
- **+ 2 pts** Case when $p < 0$

- **+ 0 pts** Incorrect

Question 2

Q2

15 / 15 pts

- ✔ **+ 10 pts** Correct unambiguous context-free grammer
- ✔ **+ 2 pts** Stating every formula has a unique derivation
- ✔ **+ 1 pt** Correct base case for induction
- ✔ **+ 1 pt** Correct Induction hypothesis
- ✔ **+ 1 pt** Correct Induction step
- **+ 2 pts** Correct ambiguous grammer
- **+ 0 pts** Incorrect context-free grammer

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Indian Institute of Technology Delhi

COL352: Introduction to Automata and Theory of Computation MAJOR QUIZ 1

DATE: Thursday the 13th of February 2025 DURATION: 45 minutes MAXIMUM MARKS: 40

Instructions: Write your name and entry number at the top of each sheet. Use page number 1 and 2 for answering Q1, and 3 and 4 for answering Q2. Answers written on incorrect pages will be marked zero.

Attestation: I agree to abide by the Honour Code of IIT Delhi.

Signature: Abhinav

A1.

We prove L is regular by giving it a regular expression. (Assuming $p \geq 0$, $n \geq 0$)

Claim:- $a^n (a^p)^* = R$, then $L(R) = L$

Proof:- Consider arbitrary string $s \in L(R)$

$\Rightarrow s = a^n (a^p)^m$ for some $m \geq 0$

$\Rightarrow |s| = |a^n \cdot (a^p)^m| = |a^n| + |a^{pm}|$

$= n + pm$ ~~$\Rightarrow n+pi$~~

$\in \{n+pi \mid i \geq 0\}$

$\Rightarrow s \in L$, since s was arbitrary, we get

~~$L(R) \subseteq$~~ $L(R) \subset L \cdots$ (I)

Consider arbitrary $s \in L$, $\Rightarrow |s| = n+pi$ for some $i \geq 0$, since $\Sigma = \{a\}$

$\Rightarrow s = a^{n+pi} = a^n (a^p)^i$

$\Rightarrow s \in a^n (a^p)^*$, since s was arbitrary

we get $L \subset L(R) \cdots$ (II)

by (I) and (II) $L = L(R) \Rightarrow$ L has a regex for it

$\Rightarrow$ L is regular language

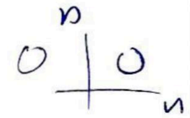
A1. (contd.)

Since we assume $n \geq 0$ and $p \geq 0$

if $n \leq 0$ but $p \geq 0$, then the language is same as the one where n is replaced by $n + ip$ where i is smallest number such that $n+ip \geq 0$, thus again regular.

if $p \leq 0$, then the set is clearly finite. and every finite language is regular.

$\Rightarrow$ L is always regular.



R.W.

$\wedge, (,), \vee, \neg,$

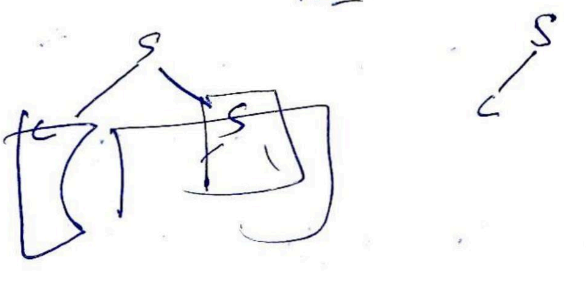
V = variable

R.W

$S = (C) \mid (C) \wedge S$

$C = A \mid A \vee C$

$A = \neg V \mid V$



Name: Abhinav R. Shripad   Entry Number: 2022CS11596

A2. let V be the set of variables.

Define a grammar G as = (T, NT, S, R)

as $T = \{\wedge, \vee, (,), \neg, v \mid \text{where } v \in V\}$

$NT = \{C, A, S\}$

$S = S$ (Start symbol)

and Rules = R are

$$S \longrightarrow (\underline{C}) \mid (\underline{C}) \wedge \underline{S} \quad \ldots \text{(I)}$$

$$C \longrightarrow \underline{A} \mid \underline{A} \vee \underline{C} \quad \ldots \text{(II)}$$

$$A \longrightarrow v \mid \neg v \quad \forall v \in V. \ldots \text{(III)}$$

(C generates individual clause, A generates a single literal which is variable OR ~~not~~ $\neg$ variable, paranthuses are there appropriately)

~~Claim:- G is unambigious~~

Claim:- Set of strings generated by starting at A is unambigious . . . (IV)

Proof:- A only generates terminals, ~~and all the strings~~ ~~generate~~ thus it is unmabigious.

Claim:- Set of strings generated by starting at C is unambigious . . (V)

Let s be string generated by starting at C having 2 derivations ~~and~~ ~~s~~ ~~has~~ d_1 and d_2 and d_1 has the least depth possible in the derivation.

A2. (contd.)

If d_1 and d_2 both used 1st rule is $C \to A$, then we would have ambigious string starting from A. Contradiction.

If d_1 uses $C \to A$ and d_2 uses $C \to A \vee C$ then s derived using $C \to A$ can be either

$s := C \to A \to v \mid \neg v \Rightarrow |s| \leq 2$ — (VI)

but s derived using $A \vee C$ has length

$$|s| = |A \vee C| = |A| + |\vee| + |C| \geq 1 + 1 + |C| = 2 + |C| > 3$$

Contradiction to (VI), Illy if d_1 uses $C \to A \vee C$ and d_2 is $C \to A$.

d_2 is $C \to A$.

Thus if d_1 and d_2 both uses $C \to A \vee C$.

let $d_1 := C \to A \vee C_1$, $d_2 := C \to A \vee C_2$

then the derivation from C_1 and C_2 is also ambigious, and has smaller structure than d_1. Contradiction. $\Rightarrow$ Strings generated by starting at C is unambigious. *

$\boxed{\text{Claim:- Strings generated by starting at S is unambigious}}$

Let s be string having 2 derivation. Say d_1, d_2. d_1 has smallest structure/depth. If $d_1 \to (C)$ and $d_2 \to (C)$ both, then contradiction as then ~~would be~~ strings generated by starting at C would be unambigious.

If $d_1 \to (C)$ and $d_2 \to (C) \wedge S$, then again as above we get contradiction from length of string.

Illy for $d_1 \to (C) \wedge S$ and $d_2 \to (C)$.

If $d_1 \to (C) \wedge S$ and $d_2 \to (C) \wedge S$, then second S is ambigious too. Contradiction to d_1 being smallest ambigious structure.