COL352 Problem Sheet 3

February 13, 2025

Problem 1. *Let C be a context-free language and R be a regular language. Prove that the language $C \cap R$ is context free.*

Problem 2. *Show that the language $A = \{w \mid w \in \{a, b, c\}^* \text{ and contains equal number of a's, b's, and c's}\}$ is not a CFL.*

Problem 3. *Let $L = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that if A is context-free and B is regular, then L is context-free.*

Problem 4. *Let $\Sigma = \{a, b\}$. Give a CFG generating the language of strings with twice as many a's as b's.*

Problem 5. *Let $L = \{x \# y \mid x, y \in \{0, 1\}^* \text{ and } x \neq y\}$. Show that L is a context-free language.*

Problem 6. *Give unambiguous CFGs for the following languages.*

1. *$\{w \mid \text{in every prefix of } w \text{ the number of } a's \text{ is at least the number of } b's\}$*

2. *$\{w \mid \text{the number of } a's \text{ and } b's \text{ in } w \text{ are equal}\}$*

3. *$\{w \mid \text{the number of } a's \text{ is at least the number of } b's\}$*

Problem 7. *Let $L = \{xy \mid x, y \in \{0, 1\}^* \text{ and } |x| = |y| \text{ but } x \neq y\}$. Show that L is a context-free language.*

Problem 8. *Let $L = \{a^i b^j \mid i \neq j \text{ and } 2i \neq j\}$. Show that L is a context-free language.*

Problem 9. *Consider the grammar $G = (\{S, Z, O\}, \{0, 1\}, R, S)$ described by the rules:*
$R = \{ ZZ \longrightarrow \epsilon, OOO \longrightarrow \epsilon, ZO \longrightarrow OOZ, OZ \longrightarrow ZOO, O \longrightarrow 1, Z \longrightarrow 0 \}$

Given the Grammar $G(s) = (\{S, Z, O\}, \{0, 1\}, R \cup \{S \longrightarrow s\}, S)$ where $s \in \{Z, O\}^*$, provide an efficient algorithm to obtain the length of the shortest word in $\mathcal{L}(G(s))$. **