Minor Exam

Graded

Student

Abhinav Shripad

Total Points

15 / 20 pts

Question 1

Encoding Question 1

5 / 5 pts

- ✔ **+ 1 pt** Curve encoding: variables clearly defined.
- ✔ **+ 2 pts** Curve encoding: all cases are covered
- ✔ **+ 1 pt** boundary condition is mentioned
- ✔ **+ 1 pt** meeting case is covered
- **+ 0 pts** incorrect

Question 2

Encoding Question 2: Bayesian network

2.5 / 5 pts

- ✔ **+ 1 pt** encoding: variables correctly defined.
- ✔ **+ 1.5 pts** encoding: correctly encoded constraints
- **+ 2.5 pts** correct weight function
- **+ 0 pts** wrong

Question 3

Krom Formula Algorithm

4 / 5 pts

- ✔ **+ 1.5 pts** basic idea is correct
- ✔ **+ 1.5 pts** time complexity is correct
- **+ 1 pt** algorithm is correct
- ✔ **+ 1 pt** used resolution
- **+ 0 pts** wrong

Question 4

UIP points and CDCL

3.5 / 5 pts

- ✔ **+ 1 pt** Sub Question 1 is correct
- **+ 0.5 pts** In subquestion 2: UIP points for decision level 1 and 2 are correct
- ✔ **+ 0.75 pts** In subquestion 2: UIP points for decision level 3 is correct
- ✔ **+ 0.75 pts** In subquestion 2: UIP points for decision level 4 is correct
- ✔ **+ 1 pt** In subquestion 3: correct conflict clauses
- **+ 1 pt** In subquestion 3: correct buckjumping points
- **+ 0.5 pts** for an attempt in subquestion 2 or 3.

COL876: SAT Solvers and Automated Reasoning

Minor Exam
Date: 13/09/2024

Maximum Time: 120 minutes **Maximum Marks: 20**

Please carefully read the instructions below before attempting the exam:

- Write your name and entry number on each sheet.
- You will be provided with rough sheets; however, you are required to write the solutions to the questions in the space provided below. Please ensure your writing is neat. In case of any confusion in the writing, the instructor reserves the right to assume the worst-case scenario and award marks accordingly.
- No clarifications will be given. If you think a question is unclear, write your assumption and then solve the question under your stated assumption.
- There are four questions. Have fun – do not stress out.

Name: Abhinav R. Shripad Entry No. 2022CS11596

Question 1 (5 marks) Consider the $n \times n$ discrete grid. A curve from $(1,1)$ to (n,n) is a set of grid points that includes $(1,1)$ and (n,n) and each point (i,j) on the curve has as the next point either $(i+1,j)$ or $(i,j+1)$ (if these points exist on the grid) but not both. Similarly, a curve from $(1,n)$ to $(n,1)$ is a set of grid points that includes $(1,n)$ and $(n,1)$ and each point (i,j) on the curve has as next point either $(i+1,j)$ or $(i,j-1)$ but not both. Encode the problem into CNF, say F_{CNF}, such that F_{CNF} is satisfiable if and only if a curve from $(1,1)$ to (n,n) and from $(1,n)$ to $(n,1)$ meet. Present such F_{CNF}.

Curve 1 :- from $(1,1)$ to (n,n)
Variable $x_{i,j} \rightarrow$ True iff ~~x~~ $(i,j) \in$ Curve 1

Curve 2 :- from $(n,1)$ to $(1,n)$
Variable $y_{i,j} \rightarrow$ True iff $(i,j) \in$ Curve 2

5 boundary condition
what happen when either i or j is 1.

~~To~~ Third Variable $z_{i,j}$, → True iff
$x_{i,j}$ and $y_{i,j}$ ~~are~~ both true.

Conditions :-
Base Case :- $(x_{1,1}) \wedge (x_{n,n}) \wedge (y_{1,n}) \wedge (y_{n,1})$ --- (1)

Propogation clauses :-
$\underbrace{(x_{i,j} \longrightarrow x_{i+1,j} \vee x_{i+1,j+1})}_{\text{if } (i,j) \text{ then one of } (i+1,j) \text{ and } (i+1,j+1)} \wedge \underbrace{(\neg x_{i+1,j} \vee \neg x_{i+1,j+1})}_{\text{not both}}$ --- (2)

$\Rightarrow (\neg x_{i,j} \vee x_{i+1,j} \vee x_{i+1,j+1}) \wedge (\neg x_{i+1,j} \vee \neg x_{i+1,j+1})$
for all $i,j \in [1,n]$ --- (2)

Illy for y
$\rightarrow (\neg y_{i,j} \vee y_{i+1,j} \vee y_{i+1,j-1}) \wedge (\neg y_{i+1,j} \vee \neg y_{i+1,j-1})$
for all $(i,j) \in [1,n]$ --- (3)

condition for z
$(x_{i,j} \wedge y_{i,j} \rightarrow z_{i,j}) \Longleftrightarrow (\neg x_{i,j} \vee \neg y_{i,j} \vee z_{i,j})$
$\forall \; i,j \in [1,n]$

atleast one z is true
$\rightarrow \left(\bigvee_{i,j=1,2\dots n} z_{i,j} \right)$ --- (4)

$x_{i,j}$ = false if $i > n$ or $j > n$
Illy $y_{i,j}$

(1) ∧ (2) ∧ (3) ∧ (4)

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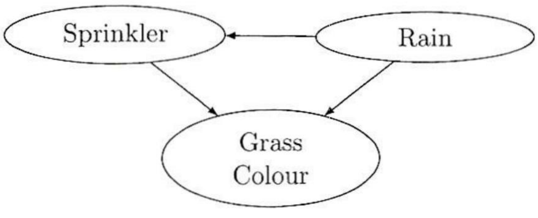
Question 2 (5 marks) We have presented a Bayesian network in Figure 1. The given Bayesian network is different from the Bayesian network taught in the lecture. We are not considering whether the grass is dry or wet; we are interested in the color of the grass, which depends on the state of the sprinkler and the rain.

	Sprinkler	
Rain	T	F
F	0.5	0.5
T	0.1	0.9



	T	F
Rain	0.4	0.6

		Grass Colour		
Sprinkler	Rain	Light	Gray	Dark
F	F	0.7	0.2	0.1
F	T	0.2	0.4	0.4
T	F	0.2	0.3	0.5
T	T	0.1	0.1	0.8

Figure 1: Conditional Probability Table

Your task is to present a CNF encoding F_{CNF} and a weight function W over literals. Similar to the lecture, we would like to have the probability of an event directly proposal to the weight to the formula, that is, $Pr[e] \propto WMC(F \wedge \varphi(e))$ where WMC is the weighted model count and $\varphi(e)$ corresponds to the Boolean formula describing event e.

Variables

I_r, I_s, I_{a1}, I_{a2}, I_{G3} :- Indicator variable for rain, sprinkler, colour of grasses (let them be 1,2,3)

$P(I\_) = 1$

Probability variables

P_R, $P_{\bar{R}}$, P_{RS}, $P_{R\bar{S}}$, $P_{\bar{R}S}$, $P_{\bar{R}\bar{S}}$ and 12 more like P_{RSa1}, $P_{R\bar{S}a2}$ ...... etc

Probablity of these variables are defined as from the table.

F_{CNF} :

$2\frac{1}{2}$

$(I_R \rightarrow P_r) \wedge$

$(\neg I_r \rightarrow P_{\bar{r}}) \wedge$

$(I_r \wedge I_s \rightarrow P_{rs})$

$\wedge (I_r \wedge \neg I_s \rightarrow P_{r\bar{s}}) \wedge (\neg I_r \wedge I_s \rightarrow P_{\bar{r}s})$

$\wedge (\neg I_r \wedge \neg I_s \rightarrow P_{\bar{r}\bar{s}}) \wedge$

$\wedge (I_r \wedge I_s \wedge I_{a1} \rightarrow P_{Rsa1})$

$\wedge (I_r \wedge I_s \wedge I_{a2} \rightarrow P_{rsa2})$

$\wedge (I_r \wedge I_s \wedge I_{a3} \rightarrow P_{rsa3})$

9 more for $R\bar{S}$, $\bar{R}S$, $\bar{R}\bar{S}$

$\wedge (I_{a1} \vee I_{a2} \vee I_{G3})$ # atleastone colour

$\wedge (I_{a1} \vee \neg I_{a2}) \wedge (I_{a1} \vee \neg I_{a3}) \wedge (I_{a2} \vee \neg I_{a1})$

$\wedge (I_{a2} \vee \neg I_{a3}) \wedge (I_{G3} \vee \neg I_{a1}) \wedge (I_{a3} \vee \neg I_{a2})$

→ Exactly one colour

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Question 3 (5 marks) A clause is called *Krom* if it contains **at most two** literals. A Krom formula is a formula formed by the conjunction of Krom clauses. Using the resolution, develop a polynomial-time algorithm for determining the satisfiability of Krom formulas. The algorithm should accept a Krom formula K (a set of clauses) as input and return **SAT** if K is satisfiable; otherwise, it should return **UNSAT**. Provide the algorithm along with its time complexity in terms of the number of variables of K.

let

$$F_1 = \bigwedge_{i=1}^{n} (x \vee y_i) \bigwedge_{j=1}^{m} (\neg x \vee z_j)$$

be a formula

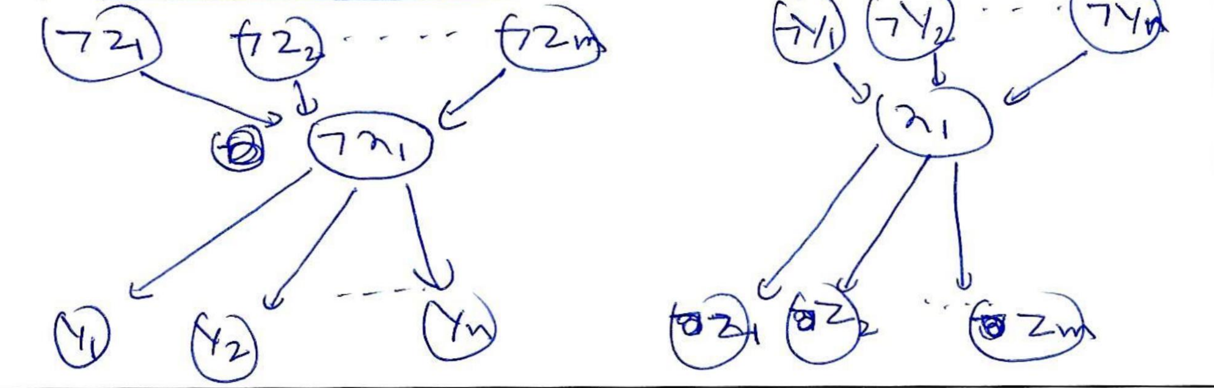
let F_2 be another CNF such that

$$F_2 = \bigwedge_{i=1}^{n-1} (\neg y_{i+1} \vee y_i) \bigwedge_{j=1}^{m-1} (\neg z_j \vee z_{j+1}) \wedge (\neg z_m \vee y_n) \wedge (\neg z_1 \vee \neg y_1)$$

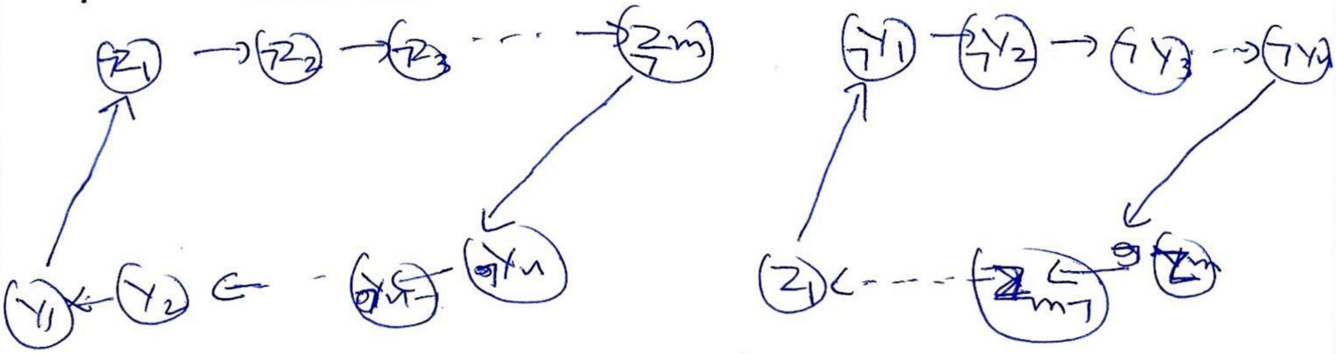
~~Observe that either both F_1 and F_2 are sat or are both unsat.~~

Observe that both F_1 and F_2 are sat or both unsat.

Implication graph of F_1



Implication graph of F_2



both implication graphs have same SCCs (except n node).

No. of clause added = $O(n+m)$.

Idea is instead of add m^2 edges to graph as it resolution, add n

Algorithm:-

~~F := remove pure literal & tautology (F)~~ # $O(m)$

if F contain l and $\neg l$ as ~~pure literal~~ single clause

return UNSAT # $O(m)$

if F empty:

return SAT # $O(1)$

F := remove_pureliteral_~~and~~_and_tautology(F) # $O(m)$

for l and $\neg l$ in F:

replace any 2 clause (one with l, other with $\neg l$) as per above algo)

F = ------ ; return SAT_SOLVER(F)

every instance of removal of variable ⇒ increase in num. of clause linear. In n ~~clauses~~ variable, m ~~variable~~ clause

$O(\underbrace{2m}\cdot\underbrace{2n} + m\cdot 2m\cdot 2n)$ Polynomial

2m: max number of occurence of a literal

2n: increase in length

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Question 4 (5 marks) The implication graph generated in a CDCL solver is shown in Figure 2. Given Figure 2, answer the following:

1. Assign decision level to every node (write neatly within the node).
2. Write unique implication points (UIPs) for **each** decision level.
3. Provide the conflict clause learned using the first UIP strategy and the last UIP strategy, along with their corresponding backjumping points.

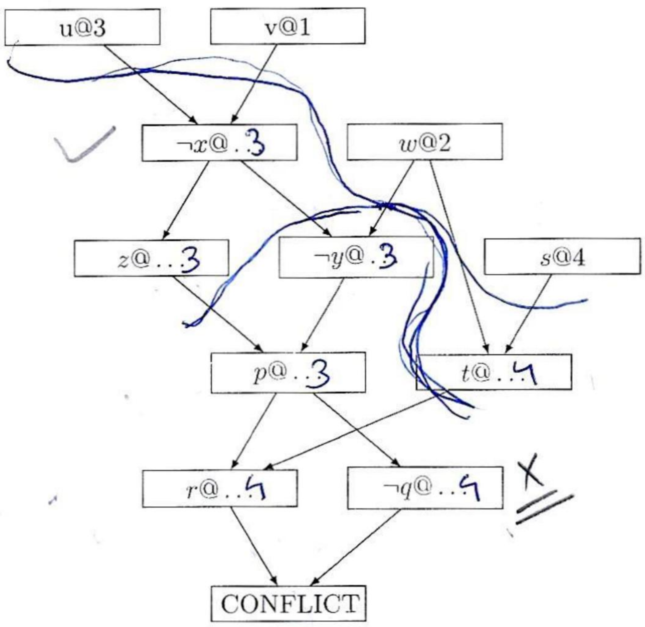

Figure 2: CDCL implication graph for some formula.

UIP's

decision level 1 :- v@1 , ¬x@3 , p@3

decision level 2 :- w@2 , r@4

∴ decision level 3 :- $u@3, \neg x@3, p@3$

decision level 4 :- $s@4, t@4, r@4$

1½

(3) D level 1

1st UIP :- NOT in conflict side :- $u@3, v@1, w@2,$
$s@4, t@4,$ ~~conflict~~

learned clause :- $(\neg v \vee \neg w \vee \neg u \vee \neg t)$

last UIP :- in conflict side :- $r@4, \neg q@4,$ conflict

learned clause :- $\neg p \vee \neg t$

level 2

1st UIP :- NOT in conflict side :- $u@3, v@1, \neg x@3\ z@3$
$w@2, s@4, t@4$

learned clause :- $(x \vee \neg z \vee \neg w \vee \neg t)$

last UIP :- in conflict side :- conflict

learned clause :- $(\neg r \vee q)$

level 3

1st UIP :- not in conflict side :- $u@3, v@1, w@2, s@4, t@4$

learned clause :- $(\neg v \vee \neg w \vee \neg u \vee \neg t)$

last UIP :- in conflict side :- , $r@4, \neg q@4$

learned clause :- $(\neg p \vee \neg t)$

level 4 Last

1st UIP :- in conflict side :- $t@4, r@4$

learned clause :- $(\neg s \vee \neg w \neg p)$

1

last UIP :- in conflict side :- CONFLICT

learned clause :- $(\neg r \vee q)$