Question 1

let x_i be $\{0, 1\}$

then

$$F_{CNF} = \bigwedge_{1 \le i < j < k \le 5} \left((\neg x_i \vee \neg x_j \vee \neg x_k) \right)$$

Why it is true?

(3)

→ $\neg x_i \neg x_j \neg x_k \rightarrow x_i + x_j + x_k \le 2$
ie not all 1

→ So for $1\ x_1 + x_2 + x_3 + x_4 + x_5 \le 2$

if $\exists\ i, j, k$ such that $x_i + x_j + x_k \ge 3$

then that individual clause is false.

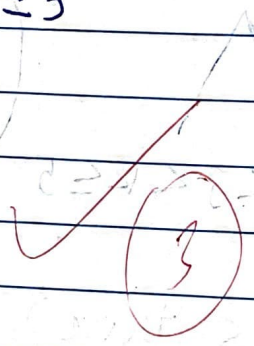
Question 2

$$\rightarrow \sum_{i=1}^{5} x_i \geq 2$$

$$\longleftrightarrow \boxed{\sum_{i=1}^{5} (7x_i) \leq 3}$$

Now using same idea as Question 1

$$F_{CNF} = \bigwedge_{1 \leq i < j < k < l \leq 5} \left((x_i \vee x_j \vee x_k \vee x_l) \right)$$



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Problem 3

$n_{hcnbgpl}$ → If house h has colour c, lives a guy with nationality n, drinks beverage b, cigar g, pet p, location l, then 1.

ij_{kl} → If type i & j (house/colour) of variety k and l respectively is true then 1.

eg nc → danish guy in pink house.
(↑ dane, pink (nationality/colour))

So we know that

(1) hn_{kn} ⟷ atleast($n_{kcnbgpl}$ ---- , 1)
all possible variations over c, B, g, P, l

∧ atmost($n_{kcnbgpl}$, ---- , 1)

Similarly for all pairs among hcNBgPl

Constraint ②

each guy/cigar/beverage owns exactly one → cigar, beverage etc

→ atleast (ij_{kl}, ... , 1)

∧ atmost (ij_{kl}, ... , 1)

for all i, j. (for fixed i, j k, l vary over all possible values i, j can take).

No owners have same pet/smoke.

Constraint ③

Information given in the problem.

$\rightarrow (nc_{Brit, red}) \wedge (np_{swede, dog})$

$\wedge (nb_{Dane, tea}) \wedge$

$\wedge \left(\bigvee_{i=1}^{4} (cl_{green, i} \wedge cl_{white, i+1}) \right)$ ← green house on left of white house

$\wedge (cb_{green, cofee})$ ∧

$\wedge (gp_{pallmall, birds})$

$\wedge (cg_{yellow,dunhill}) \wedge (lb_{3,milk})$

$\wedge (nl_{norwegian,1})$

$\wedge \left(\bigvee_{i=1}^{4} (lp_{i,cats} \wedge lg_{i+1,blends}) \right)$ ← cat next to blends guy

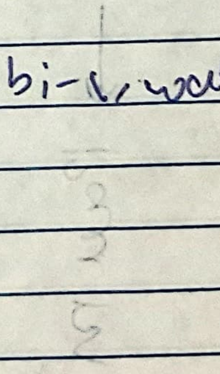
$\wedge \left(\bigvee_{i=1}^{4} (lp_{i,horse} \wedge lg_{i+1,dunhill}) \right)$ ← house next to dunhill

$\wedge (gb_{bluemaster,beer}) \wedge (ng_{german,prince})$

$\wedge \left(\bigvee_{i=1}^{4} (lc_{i,blue} \wedge ln_{i+1,norwegian}) \right)$ ← blue ^ norwegian next to bl.

$\wedge \left(\bigvee_{i=1}^{4} (lg_{i,blends} \wedge lb_{i+1,water}) \right.$

8 $\vee \left(\bigvee_{i=2}^{5} (lg_{i,blends} \wedge lb_{i-1,water}) \right)$ } → blends has neighbour water.



Question 3 Continued

Fish owner → German

Solution is:-

House 1 → Norwegian, Yellow, Dunhill, Water Cats

House 2 → Dane, Blue, Blends, tea, Horse

House 3 → Brit, Red, Pallmall, Milk, Birds

House 4 → German, Green, Prince, Coffee, Fish

House 5 → Swede, White, Bluemaster, Beer, Dogs.