# COL876 Quiz

Quiz 1.
Tuesday, 20 August

**Instructions:**

1. You have 90 minutes to attempt this quiz.
2. There are three questions.
3. No clarifications will be given. If you think a question is unclear, write your assumption and then solve the question under your stated assumption.
4. Have fun  do not stress out.

## Question 1 (3 Marks)

Let $F$ be $x_1 + x_2 + x_3 + x_4 + x_5 <= 2$, where each $x_i \in \{0, 1\}$. Can you transform $F$ into a equisatisfiable formula in conjunction normal form, $F_{CNF}$? If yes, write $F_{CNF}$.

## Question 2 (3 Marks)

Let $F$ be $x_1 + x_2 + x_3 + x_4 + x_5 >= 2$, where each $x_i \in \{0, 1\}$. Can you transform $F$ into a equisatisfiable formula in conjunction normal form, $F_{CNF}$? If yes, write $F_{CNF}$.

## Question 3 (9 Marks)

Consider the following puzzle:
There are five houses in five different colors. In each house lives a man with a different nationality. The five owners drink a certain type of beverage, smoke a certain brand of cigar, and keep a certain pet. No owners have the same pet, smoke the same brand of cigar or drink the same beverage, that is, there are 5 pets, 5 brand of cigar, and 5 types of beverage.
**The question is: "Who owns the fish?"**

Additionally, we have the following hints:

- The Brit lives in the red house.
- The Swede keeps dogs as pets.
- The Dane drinks tea.
- The green house is on the left of the white house.

- The green house's owner drinks coffee.
- The person who smokes Pall Mall rears birds.
- The owner of the yellow house smokes Dunhill.
- The man living in the center house drinks milk.
- The Norwegian lives in the first house.
- The man who smokes Blends lives next to the one who keeps cats.
- The man who keeps the horse lives next to the man who smokes Dunhill.
- The owner who smokes Bluemasters drinks beer.
- The German smokes Prince.
- The Norwegian lives next to the blue house.
- The man who smokes Blends has a neighbor who drinks water.

In order to solve the puzzle, you have to make some assumptions: (i) The owner is the resident of each house, (ii) One of the residents owns the fish, (iii) The term neighbor in the last hints refers only to a directly adjacent neighbor, (iv) The houses are on the same side of the street, (v) They are next to each other, and are ordered from left to right as you face them rather than standing in front of a house facing the street (i.e. facing the same direction as the house).

You need to solve this puzzle by first expressing it as a CNF formula $F_{CNF}$ , such that the satisfying assignment of the formula corresponds to possible solutions of the puzzle. Specifically, from the satisfying assignment of the corresponding CNF formula, we should be able to identify the owner of the fish.

You need to write $F_{CNF}$. If in your encoding you need to use an "at least $K$" or "at most $K$" constraint, where $K$ is a non-negative integer, you can simply write $atleast(x_1, x_2, \ldots, x_n, K)$ or $atmost(x_1, x_2, \ldots, x_n, K)$ respectively, where each $x_i$ is your propositional variable.