SMALL HINTS TO HOMEWORK ASSIGNMENTS 2DBI00

This document contains some small hints for the homework. Recommended if you are stuck, before looking at the full answers. You can always ask the instructors for help as well! Answers to the homework will generally appear the evening after the instruction classes.

Class 1: Matrices and vectors

(7) You can write down the (i, j) and (j, i) element of $A + A^T$ and check if they are equal. However, it is more elegant to start with $(A + A^T)^T = \dots$

(11) Start with $(A^TA)^T = \dots$

(12) Take 2 "random" symmetric matrices and see what happens.

Class 2: Vector and matrix norms

(9) Consider vectors with zeros and/or ones as the first step. Try to formulate a result which seems to follow from this, and then try to prove it. This is a challenging exercise, but it requires only Calculus techniques.

Class 6: Inverse matrices

(8) Start with $A \cdot A^{-1} = I$, and take the transpose of both sides.

(10) See also question **(5)** of Class 4.

Class 9: Clustering via the Fiedler vector; Gershgorin's theorem

(15) The first item: do you recall from Chemistry class what substance this is, and what (student) applications this has ...?
The last item has weights, which you can use in the matrix representing your graph. The weights influence the clustering.

Class 10: Linear dependence, orthogonality, bases, and rank

(2) This matrix has only 2 pivots; the rref contains $-\frac{7}{17}$ and $\frac{1}{17}$.

(3) This matrix has rref $= I$ (standard situation).

(6) Take care: a matrix with orthonormal columns is called orthogonal, instead of orthonormal.

(7) First consider the inner product with the second and third column.

(8) Start with a first column of all ones, and then try to create a second that is orthogonal to the first, etc.

(9) What options do we have for the inner product of 2 columns with ± 1's ?

(11) For g,h: compute A^TA and see if it is I (orthonormal basis) or diagonal (orthogonal basis).

Class 11: Singular Value Decomposition (SVD)

(1) To compute $\|A\| = \sigma_1$, or all σ's, or the full SVD by hand: follow the step-by-step plan. Compute $B = A^T A$ and its eigenvalues, and eigenvectors $\mathbf{v}_j$ if necessary. The $\mathbf{u}_j$ follow from $A\mathbf{v}_j = \sigma_j \mathbf{u}_j$.

(2) Important question. Show that $A^T A$ is symmetric, and consider $\mathbf{v}^T A^T A \mathbf{v}$.

(5) What is the transpose $A^T = (U\Sigma V^T)^T$?

(6) If you write out A^2 using $A = U\Sigma V^T$, do some factors cancel out?

(7) Take the (approximate) average column and row and scale (normalize, divide by 2-norm). Also, $\mathbf{u}_2 \perp \mathbf{u}_1$ (note that there are 2 options).

(8) What do you know about left- and right-multiplication?

Class 12: Least squares and fitting

(1) You have to show 2 things:
- $(A^T A)^T = A^T A$, as we have already seen several times
- $\mathbf{x}^T A^T A \mathbf{x} \geq 0$ for all vectors $\mathbf{x}$.

(8) Parts **(a)–(c)**: see Class 10, this is more or less the definition.
From **(c)** to **(d)**: left-multiply by A^T.
From **(d) to (c)**: left-multiply by $\mathbf{v}^T$.
Parts **(d)–(e)**: see Class 10.

Class 13: Rotations, projections, reflections

(4) You will need $\mathbf{v}\mathbf{v}^T\,\mathbf{v}\mathbf{v}^T = \mathbf{v}(\mathbf{v}^T\mathbf{v})\mathbf{v}^T = \|\mathbf{v}\|^2\,\mathbf{v}\mathbf{v}^T$.

(7) The "$\mathbf{v}$" in $I - \mathbf{v}\mathbf{v}^T$ should not be $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ but a vector orthogonal to it. Test your result on both of these vectors.

(8) The "$\mathbf{v}$" in $I - \frac{\mathbf{u}\mathbf{v}^T}{\mathbf{v}^T\mathbf{u}}$ should not be $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ but a vector orthogonal to it.

(10) **(k)** Choose $\mathbf{v} = \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}$ which is of length 1, and then $P = I - \mathbf{v}\mathbf{v}^T$.

(12) Solve $R\mathbf{x} = \mathbf{x}$.
Alternatively, but this may be a bit less practical, choose $\mathbf{v} = \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}$ which is of length 1, and then $R = I - 2\,\mathbf{v}\mathbf{v}^T$.