45.

a) A, B - finite sets, F - function from A to B

So

$\forall(A, B)\ \forall(F: A \to B)(|A| > |B| \Rightarrow \exists(a_1, a_2 \in A): a_1 \neq a_2 \wedge F(a_1) = F(a_2))$

b) exists no sets of 10 distinct natural numbers between 1 and 100 such as no two subsets have the same sum

Prove:

Suppose $\exists S: |S| = 10$. So ~~all~~ exist 2^{10} possible subsets.

Now consider all possible sums of these subsets: (including $\varnothing$ and S)

So the minimal is 0 and max is $100 + 99 + \ldots + 91 = 955$. Consiquently we have 956 possible sums.

Hence we have 1024 possible subsets and 956 possible sums.

According Dirichlet principle since set of subsets is larger then set of sums, exist at least two subsets with same sum.

c) Consider S - set, $|S| = 7$ where $S = \{\forall i: 0 \le i \le 6 \mid 2^i\}$ so $S = \{2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6\} =$

$= \{1, 2, 4, 8, 16, 32, 64\}$ This set has a property that no two subsets have the same sum, due to binary representation of the numbers.