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$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{3\cdot 7}+\dots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{(2n+1)}, \quad n\in N$$

1) Base $n=1$, $P(1)$

$$\frac{1}{1\cdot 3}=\frac{1}{2+1}; \quad \frac{1}{3}=\frac{1}{3}$$

Inductive Step

2) $n=k$, $P(k)$

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\dots+\frac{1}{(2k-1)(2k+1)}=\frac{k}{2k+1}$$

3) $n=k+1$, $P(k+1)$

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\dots+\frac{1}{(2k+1)(2k+3)}=\frac{k+1}{2k+3}$$

So its possible to state that.

$$\frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)}=\frac{k+1}{2k+3};$$

$$\frac{2k^2+3k+1}{(2k+1)(2k+3)}=\frac{(k+1)(2k+1)}{(2k+3)(2k+1)},$$

$$2k^2+3k+1=2k^2+3k+1$$

Conclusion:

Since we have shown that $P(1)$ is true and that $P(k)$ beeing true implies $P(k+1)$ is true, by the principle of math Induction, $P(n)$ is true, $n\in N$.