Exercise 37

Prove (with mathematical induction):

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{3\cdot 7}+\ldots\ldots+\frac{1}{(2\cdot n-1)\cdot(2\cdot n+1)}=\frac{n}{(2n+1)} \quad \text{for } n\in N$$

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$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{3\cdot 7}+\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{(2n+1)},\ n\in N$$

1) Base $n=1$, $P(1)$

$$\frac{1}{1\cdot 3}=\frac{1}{2+1};\quad \frac{1}{3}=\frac{1}{3}$$

Inductive Step

2) $n=k$, $P(k)$

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\ldots+\frac{1}{(2k-1)(2k+1)}=\frac{k}{2k+1}$$

3) $n=k+1$, $P(k+1)$

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\ldots+\frac{1}{(2k+1)(2k+3)}=\frac{k+1}{2k+3}$$

So its possible to state that.

$$\frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)}=\frac{k+1}{2k+3};$$

$$\frac{2k^2+3k+1}{(2k+1)(2k+3)}=\frac{(k+1)(2k+1)}{(2k+3)(2k+1)};$$

$$2k^2+3k+1=2k^2+3k+1$$

Conclusion:

Since we have shown that $P(1)$ is true and that $P(k)$ beeing true implies $P(k+1)$ is true, by the principle of math Induction, $P(n)$ is true, $n\in N$.

Exercise 39

Write each proposition without negation:

a. $\neg\forall i\in N\ \exists x\in Z\ [\ x + i > 0\ ]$

b. $\neg\exists x\in Z\ \exists x\in Z\ [\ xy < 0\ ]$

c. $\neg\forall x\in N\ \exists y\in N\ [\ (x = 3) \wedge (y = 5)\ ]$

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a. $\neg\forall i\in N\ \exists x\in Z\ [x+i>0]$

$\Updownarrow$

$\exists i\in N\ \forall x\in Z\ [x+i\leq 0]$

b. Looks like there is mistake in task

$\neg\exists x\in Z\ \exists y\in Z\ [xy<0]$

$\Updownarrow$

$\forall x\in Z\ \forall y\in Z\ [yx\geq 0]$

c. $\neg\forall x\in N\ \exists y\in N\ [(x=3)\wedge(y=5)]$

$\Updownarrow$

$\exists x\in N\ \forall y\in N\ [(x\neq 3)\vee(y\neq 5)$

Exercise 42

We have an array X[1…5] with X[1] = 2, X[2] = 1, X[3] = 3, X[4] = 7, X[5] = 3, and a set V := {1,2,3,4,5}.

Examine whether the next propositions are true or false:

a. The maximum of { X[i] + I | i∈V } = 10

b. The minimum of { X[i] | i∈V \ {4} } equals the maximum of { X[i] | i∈{1,2,3} }

c. The maximum of { X[i] | i∈V ∧ X[i] is odd }

42. $X[1 \ldots 5]$; $X[1]=2$, $X[2]=1$, $X[3]=3$
$X[4]=7$, $X[5]=3$

and $V := \{1,2,3,4,5\}$

a $\max\{X[i]+i \mid i\in V\}=10$

The set of $\{X[i]+i \mid i \in V\} = \{3,6,8,11\}$

So the $\max\{X[i]+i \mid i\in V\} = \max\{3,6,8,11\} = 11$

Consiquently $\max\{X[i]+i \mid i\in V\}=10$ is false

b. $\min\{X[i] \mid i\in V\setminus\{4\}\} = \max\{X[i] \mid i\in\{1,2,3\}\}$

$\{X[i] \mid i\in V\setminus\{4\}\} = \{1,2,3\}$; $\min\{1,2,3\}=1$

$\{X[i] \mid i\in\{1,2,3\}\} = \{1,2,3\}$; $\max\{1,2,3\}=3$

So

$\min\{X[i] \mid i\in V\setminus\{4\}\} = \max\{X[i] \mid i\in\{1,2,3\}\}$ is false

c. $\max\{X[i] \mid i\in V \wedge X[i]\}$ is odd

$\{X[i] \mid i\in V \wedge X[i]\} = \{1,2,3\}$; $\max\{3\}=3$

$\max\{X[i] \mid i\in V \wedge X[i]\}$ is odd is true

Exercise 45

The ‘Pigeonhole principle’ (https://en.wikipedia.org/wiki/Pigeonhole_principle) states that if *n* items are put into *m* containers, with *n* > *m*, then at least one container must contain more than one item.

a) Reformulate the Pigeonhole as a statement about sets and functions.

b) Use the pigeon hole principle to prove the following theorem:
There exists no set of 10 distinct natural numbers between 1 and 100 such that:
no two subsets (of this set of 10 numbers) have the same sum.

b) Find a set of 7 numbers between 1 and 100 such that no two of its subsets have the same sum.

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a) A, B - finite sets, F - function from A to B
So
$\forall(A, B)\ \forall(F: A \to B)(|A| > |B| \Rightarrow \exists(a_1, a_2 \in A): a_1 \neq a_2 \wedge F(a_1) = F(a_2))$

b) exists no sets of 10 distinct natural numbers between 1 and 100 such as no two subsets have the same sum

Prove:
Suppose $\exists S: |S| = 10$. So ~~all~~ exist 2^{10} possible subsets.
Now consider all possible sums of these subsets: (including $\varnothing$ and S)
So the minimal is 0 and max is $100 + 99 + \ldots + 91 = 955$. Consequently we have 956 possible sums.
Hence we have 1024 possible subsets and 956 possible sums.
According Dirichlet principle since set of subsets is larger than set of sums, exist at least two subsets with same sum.

c) Consider S - set, $|S| = 7$ where $S = \{\forall i: 0 \leq i \leq 6 \mid 2^i\}$ so $S = \{2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6\} =$
$= S\{1, 2, 4, 8, 16, 32, 64\}$ This set has a property that no two subsets have the same sum, due to binary representation of the numbers.