Theory exrecises

Exercise 37

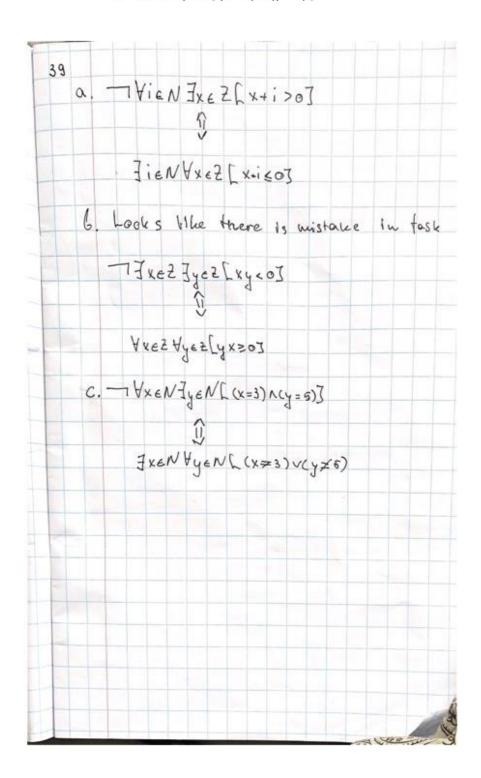
Prove (with mathematical induction):

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{3 \cdot 7} + \dots + \frac{1}{(2 \cdot n - 1) \cdot (2 \cdot n + 1)} = \frac{n}{(2n+1)} \quad \text{for } n \in \mathbb{N}$$

Exercise 39

Write each proposition without negation:

- a. $\neg \forall i \in N \exists x \in Z [x+i>0]$
- b. $\neg \exists x \in Z \exists x \in Z [xy < 0]$
- c. $\neg \forall x \in N \exists y \in N [(x = 3) \land (y = 5)]$



Exercise 42

We have an array X[1...5] with X[1] = 2, X[2] = 1, X[3] = 3, X[4] = 7, X[5] = 3, and a set $V := \{1,2,3,4,5\}$.

Examine whether the next propositions are true or false:

- a. The maximum of $\{X[i] + I \mid i \in V\} = 10$
- b. The minimum of { X[i] | i \in V \ {4} } equals the maximum of { X[i] | i \in {1,2,3} }
- c. The maximum of { $X[i] \mid i \in V \land X[i] \text{ is odd }$ }

4	d.	X [1 6]; X [1] = 2, X [2] = 1, X [3] = 3
		and V:= {1,2,3,4,5}
		nax X[i] + i i \ V] = 10
		The set of [x[i]+i iev]=[3,6,8,11] So the max [xci]+i iev]=max [3,6,8,4]=11 Consiquently max [xci]+i lev]=10 is fels
	6.	min [V[i] iev \ [4] = max [X[i] ie [1, 2, 3] }
		\x \(\) \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\
		{xLi] ie {1,2,3}} = {1,2,3}; max {1,2,3}=3
		So min [reiz 1 e y 1, 2, 3 } is false
	c.	max [XCI] IEVA XCIZ] Is odd
		[x [i] i e V x X [i] } = { max [3] = 3
		max [x[i] leva x[i] is odd Is true

Exercise 45

The 'Pigeonhole principle' (https://en.wikipedia.org/wiki/Pigeonhole principle) states that if n items are put into m containers, with n > m, then at least one container must contain more than one item.

- a) Reformulate the Pigeonhole as a statement about sets and functions.
- b) Use the pigeon hole principle to prove the following theorem: There exists no set of 10 distinct natural numbers between 1 and 100 such that: no two subsets (of this set of 10 numbers) have the same sum.
- b) Find a set of 7 numbers between 1 and 100 such that no two of its subsets have the same sum.