$$\tan\theta = \frac{x\sin\varphi}{1-x\cos\varphi}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{x\sin\varphi}{1-x\cos\varphi}$$

$$\sin\theta - x\sin\theta\cos\varphi = \cos\theta\, x\sin\varphi$$

$$\sin\theta = x\sin\theta\cos\varphi + x\cos\theta\sin\varphi$$

$$\sin\theta = x\{\sin(\theta+\varphi)\} \quad \text{--- (I)}$$

$$\tan\varphi = \frac{y\sin\theta}{1-y\cos\theta}$$

$$\sin\varphi = y\{\sin\theta\cos\varphi + \cos\theta\sin\varphi\}$$

$$\sin\varphi = y\{\sin(\theta+\varphi)\} \quad \text{--- (II)}$$

$$\text{(I)} \div \text{(II)}$$

$$\frac{\sin\theta}{\sin\varphi} = \frac{x}{y} \text{ (proved)}$$

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$$\tan\theta = \frac{a\sin x + b\sin y}{a\cos x + b\cos y}$$

$$a\sin(\theta - x) - b\sin(\theta - y) = 0$$

$$\frac{\sin\theta}{\cos\theta} = \frac{a\sin x + b\sin y}{a\cos x + b\cos y}$$

$$a\sin\theta\cos x + b\sin\theta\cos y = a\sin x\cos\theta + b\sin y\cos\theta$$

$$\rightarrow a\{\sin\theta\cos x - \sin x\cos\theta\} = b\{\sin y\cos\theta - \sin\theta\cos y\}$$

$$a\sin(\theta - x) = b\sin(\theta - y)$$

$$a\sin(\theta - x) - b\sin(\theta - y) = 0$$

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$$A+B = \frac{\pi}{4} \qquad (1+\tan A)(1+\tan B) = 2$$

$$A = \frac{\pi}{4} - B$$

$$\tan A = \tan\left(\frac{\pi}{4} - B\right)$$

$$\tan A = \frac{\tan\frac{\pi}{4} - \tan B}{1 + \tan\frac{\pi}{4}\tan B}$$

$$\tan A = \frac{1-\tan B}{1+\tan B}$$

$$\tan A + \tan A\tan B = 1 - \tan B$$

$\tan A + \tan A \tan B = 1 - \tan B$

$\tan A = 1 - \tan B - \tan A \tan B$

$\tan A = 1 - \tan B(1 + \tan A)$

$\rightarrow 1 + \tan A = 2 - \tan B(1 + \tan A)$

$1(1 + \tan A) + \tan B(1 + \tan A) = 2$

$(1 + \tan A)(1 + \tan B) = 2$

(proved)

$\sin(A - B - C)$

$= \sin\{A - (B + C)\}$

$= \sin A \cos(B + C) - \cos A \sin(B + C)$

$= \sin A\{\cos B \cos C - \sin B \sin C\} - \cos A\{\sin B \cos C + \cos B \sin C\}$

$= \sin A \cos B \cos C - \sin A \sin B \sin C - \cos A \sin B \cos C - \cos A \cos B \sin C$

7.3

$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$

$\tan C + \tan D =$

$\cos 40 + \cos 80 + \cos 160$

$$\cos 27 + \sin 27 = \sqrt{2}\cos 18$$

$$L.S = \cos 27 + \sin 27 = \cos(90-63) + \sin 27$$

$$= \sin 63 + \sin 27$$

$$= 2\sin\frac{63+27}{2}\cos\frac{63-27}{2}$$

$$= 2\sin 45 \cos 18^\circ$$

$$= \sqrt{2}\cos 18^\circ$$

$2+3+4+5$

$2+5+3+4$

$$\frac{\sin\theta + \sin 5\theta + \sin 9\theta + \sin 13\theta}{\cos\theta + \cos 5\theta + \cos 9\theta + \cos 13\theta} = \tan 7\theta$$

L.S

$$= \frac{\sin\theta + \sin 13\theta + \sin 5\theta + \sin 9\theta}{\cos\theta + \cos 13\theta + \cos 5\theta + \cos 9\theta}$$

$$= \frac{2\sin\frac{\theta+13\theta}{2}\cos\frac{\theta-13\theta}{2} + 2\sin\frac{5\theta+9\theta}{2}\cos\frac{5\theta-9\theta}{2}}{2\cos\frac{\theta+13\theta}{2}\cos\frac{\theta-13\theta}{2} + 2\cos\frac{5\theta+9\theta}{2}\cos\frac{5\theta-9\theta}{2}}$$

$$= \frac{2\sin 7\theta\cos 6\theta + 2\sin 7\theta\cos 2\theta}{2\cos 7\theta\cos 6\theta + 2\cos 7\theta\cos 2\theta}$$

$$= \frac{\cancel{2}\sin 7\theta(\cos 6\theta + \cos 2\theta)}{\cancel{2}\cos 7\theta(\cos 6\theta + \cos 2\theta)}$$

$$= \tan 7\theta$$

(Proved)

প্রমাণ কর:)

$$\tan\frac{45+\theta}{2}\tan\frac{45-\theta}{2} = \frac{\sqrt{2}\cos\theta-1}{\sqrt{2}\cos\theta+1}$$

$$L.S = \tan\frac{45+\theta}{2}\tan\frac{45-\theta}{2}$$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$$= \frac{2\sin\frac{45+\theta}{2}\sin\frac{45-\theta}{2}}{2\cos\frac{45+\theta}{2}\cos\frac{45-\theta}{2}}$$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\cos(A+B)+\cos(A-B) = 2\cos A\cos B$

$$= \frac{\cos\left(\frac{45+\theta}{2}-\frac{45-\theta}{2}\right) - \cos\left(\frac{45+\theta}{2}+\frac{45-\theta}{2}\right)}{\cos\left(\frac{45+\theta}{2}+\frac{45-\theta}{2}\right)+\cos\left(\frac{45+\theta}{2}-\frac{45-\theta}{2}\right)}$$

$\cos(A-B) - \cos(A+B) = 2\sin A\sin B$

$$= \frac{\cos\left(\frac{90}{2}\right) - \cos\left(\frac{2\theta}{2}\right)}{\cos\frac{90}{2}+\cos\frac{2\theta}{2}}$$

$2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$$= \frac{\frac{1}{\sqrt{2}} - \cos\theta}{\frac{1}{\sqrt{2}}+\cos\theta} = \frac{\frac{1-\sqrt{2}\cos\theta}{\sqrt{2}}}{\frac{1+\sqrt{2}\cos\theta}{\sqrt{2}}}$$

$$= \frac{1-\sqrt{2}\cos\theta}{1+\sqrt{2}\cos\theta}$$

$\sin x = m\sin y$ $\tan\frac{1}{2}(x-y) = \frac{m-1}{m+1}\tan\frac{1}{2}(x+y)$

$$\rightarrow \frac{\sin x}{\sin y} = \frac{m}{1}$$

যোজন বিয়োজন

$$\rightarrow \frac{\sin x - \sin y}{} = \frac{m-1}{}$$

$\sin x + \sin y \qquad m+1$

$$\Rightarrow \frac{2 \sin \frac{x+y}{2} \cos \frac{x+y}{2}}{2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}} = \frac{m-1}{m+1}$$

$$\Rightarrow \tan \frac{x-y}{2} \quad \cot \frac{x+y}{2} = \frac{m-1}{m+1}$$

$$\Rightarrow \tan \frac{x-y}{2} = \frac{m-1}{m+1} \tan \frac{x+y}{2}.$$

$\alpha + \beta = \theta \qquad \cos\alpha = k \cos\beta$

$$\tan \frac{1}{2}(\alpha - \beta) = \frac{1-k}{1+k} \cot \frac{1}{2}\theta$$

$A \neq B$

প্রমাণ কর যে,

$A + B = \frac{\pi}{2}$

$\sin A + \cos A = \sin B + \cos B$

$\sin A - \sin B = \cos B - \cos A$

$(\theta - \varphi)$ সূক্ষ্মকোণ,

$\sin\theta + \sin\varphi = \sqrt{3}(\cos\varphi - \cos\theta)$

প্রমাণ কর, $\sin 3\theta + \sin 3\varphi = 0$