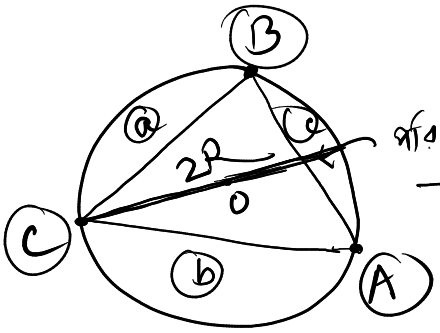
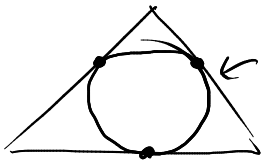
ত্রিকোণমিতি



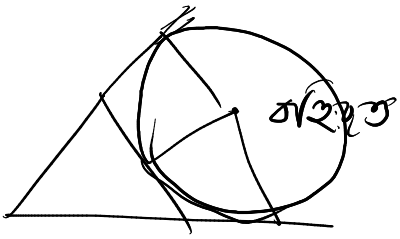
পরিবৃত্ত (যে বৃত্ত কোনো ত্রিভুজের বাহু তিনটি স্পর্শ করে)



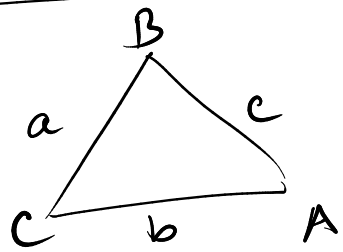
যে বৃত্ত ত্রিভুজের বাহুত্রয় স্পর্শ করে।

সাইন সূত্র:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

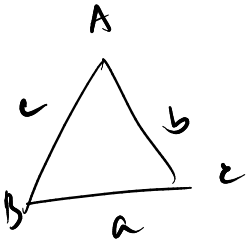


কসাইন সূত্র:



$$\cos A = \frac{b^2+c^2-a^2}{2ab} \qquad \cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$



(i) $a = b\cos C + c\cos B$

(ii) $b = c\cos A + a\cos C$

(iii) $c = b\cos A + a\cos B$

$\sin\frac{A}{2}$

$1+\cos 2A = 2\cos^2 A$

$1+\cos A = 2\cos^2\frac{A}{2}$

$1-\cos 2A = 2\sin^2 A$

$1-\cos A = 2\sin^2\frac{A}{2}$

$\cos\frac{A}{2}$

$$1-\frac{b^2+c^2-a^2}{2bc} = 2\sin^2\frac{A}{2}$$

$$\frac{2bc-(b^2+c^2-a^2)}{2bc} = 2\sin^2\frac{A}{2}$$

$$\frac{2bc-b^2-c^2+a^2}{2bc} = 2\sin^2\frac{A}{2}$$

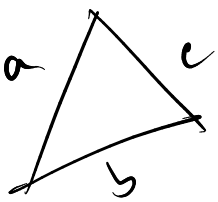
$$\frac{-(b-c)^2+a^2}{2bc} = 2\sin^2\frac{A}{2}$$

$$\sqrt{\frac{a^2-(b-c)^2}{4bc}} = \sin\frac{A}{2}$$

$$\sqrt{\frac{(a+b-c)(a-b+c)}{4bc}} = \sin\frac{A}{2}$$

$$\sin\frac{A}{2} = \sqrt{\frac{(a+b+c-2c)(a+b+c-2b)}{4bc}}$$

$$= \sqrt{\frac{(2s-2c)(2s-2b)}{4bc}}$$



$s = \frac{a+b+c}{2}$

$a+b+c = 2s$

$$\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$$

$$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$= \sqrt{\frac{2(s-c)\,2(s-b)}{4bc}}$$

$$\sin\frac{A}{2} = \sqrt{\frac{(s-c)(s-b)}{bc}}$$

$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

$$\tan\frac{A}{2} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}$$

$$= \frac{\sqrt{\frac{(s-b)(s-c)}{bc}}}{\sqrt{\frac{s(s-a)}{bc}}}$$

$$= \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$= \sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2(s-a)^2}}$$

$$= \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s(s-a)}$$

$$\tan\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\tan\frac{C}{2} = \sqrt{\frac{(s+a)(s-b)}{s(s-c)}}$$

$$\sqrt{s(s-a)(s-b)(s-c)} = \Delta$$ (พื้นที่)

$$\tan\frac{A}{2} = \frac{\Delta}{s(s-a)}$$

$$\tan\frac{B}{2} = \frac{\Delta}{s(s-b)}$$

$$\tan\frac{C}{2} = \frac{\Delta}{s(s-c)}$$

Important:

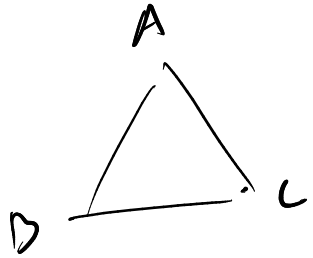
ABC ত্রিভুজে $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

$b = \sin B \, 2R$

$\frac{b-c}{b+c} = \tan\frac{B-C}{2} \tan\frac{A}{2}$.

L.S $= \frac{b-c}{b+c} = \frac{2R\sin B - 2R\sin C}{2R\sin B + 2R\sin C}$

$= \frac{\sin B - \sin C}{\sin B + \sin C}$



$A + B + C = \pi$

$B + C = \pi - A$

$= \cot\frac{B+C}{2} \tan\frac{B-C}{2}$

$= \cot\left(\frac{\pi - A}{2}\right) \tan\frac{B-C}{2}$

$= \cot\left(\frac{\pi}{2} - \frac{A}{2}\right) \tan\frac{B-C}{2}$

$= \tan\frac{A}{2} \tan\frac{B-C}{2}$

$b = 2R\sin B$

//

ABC ত্রিভুজে $a(\cos C - \cos B) = 2(b-c)\cos^2\frac{A}{2}$

L.S $= a(\cos C - \cos B)$

$= a\cos C - a\cos B$

$= (a\cos C + c\cos A) - c\cos A - (a\cos B + b\cos A) + b\cos A$

$= b - c - c\cos A + b\cos A$

$= (b-c) + \cos A(b-c)$

$\sin A$

$a\cos C + b\cos A$

A কোণের মান নির্ণয় কর।

$(a+b+c)(b+c-a)=3bc$ ✓

$\Rightarrow \{(b+c)+a\}\{(b+c)-a\}=3bc$

$\Rightarrow (b+c)^2-a^2=3bc$

$\Rightarrow b^2+2bc+c^2-a^2=3bc$

$\Rightarrow b^2+c^2-a^2=bc$

$\Rightarrow \frac{b^2+c^2-a^2}{2bc}=\frac{bc}{2bc}$

$\Rightarrow \cos A=\frac{1}{2}$

$\cos A=\cos 60^\circ$

$A=60^\circ$

$=(b-c)\{1+\cos A\}$

$=(b-c)\,2\cos^2\frac{A}{2}$

$=2(b-c)\cos^2\frac{A}{2}$

(প্রমাণিত)

H.W

$p=\sin 2\alpha$, $q=\sin 2\beta$, $r=\cos 2\alpha$, $s=\cos 2\beta$, $t=\sin 2\gamma$

ক. $\triangle ABC$ এর তিনটি বাহুর দৈর্ঘ্য যথাক্রমে a, b, c হলে, $a^2+b^2-c^2=\sqrt{2}ab$.

$\angle C=?$

খ. যদি, $p+q=c$, $r+s=d$ হয়, তবে দেখাও যে, $\cos(2\alpha+2\beta)=\frac{d^2-c^2}{d^2+c^2}$

গ. $\alpha+\beta+\gamma=\pi$ হলে প্রমাণ কর, $p^2+q^2+t^2=2-2\cos 2\alpha\cos 2\beta\cos 2\gamma$

$f(\theta) = \cos\frac{\theta}{2} - \sin\frac{\theta}{2}$

$f(\pi/4) = \sqrt{2}\sin\pi/8$

$f(\pi/4) = \cos\frac{\pi}{8} - \sin\frac{\pi}{8}$ $\quad \sin(-\sin\theta)$

$\cos(\frac{\pi}{2} \pm \theta)$

$\frac{\pi}{2} + \theta = \frac{\pi}{8}$

$\theta = \frac{\pi}{4} - \frac{\pi}{2}$

$= \frac{\pi - 4\pi}{8}$

$= \frac{3\pi}{8}$

$= \cos(\frac{\pi}{2} - \frac{3\pi}{8}) - \sin\frac{\pi}{8}$

$= \sin(+\frac{3\pi}{8}) - \sin\frac{\pi}{8}$

$= 2\cos\frac{3\pi/8 + \pi/8}{2}\sin\frac{3\pi/8 - \pi/8}{2}$

$= 2\cos\frac{\pi}{4}\sin\frac{\pi}{8}$

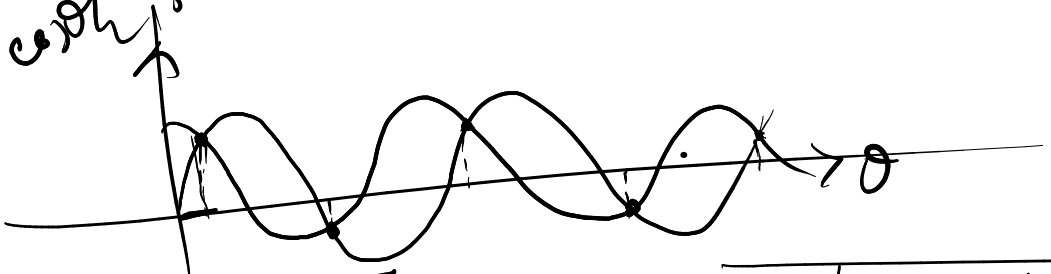
$= 2 \cdot \frac{1}{\sqrt{2}}\sin\frac{\pi}{8}$

$= \sqrt{2}\sin\pi/8$

$f(\theta) = 0$

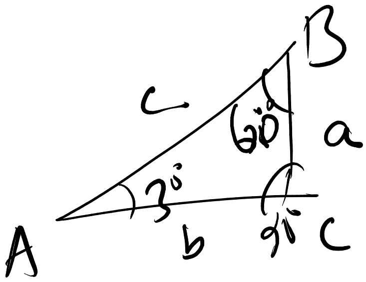
$\cos\theta/2 - \sin\theta/2 = 0$

$\cos\theta/2 = \sin\theta/2$



θ	0	45	60	90
sinθ/2				

θ	
cosθ/2	



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 30} = \frac{b}{\sin 60} = \frac{c}{\sin 90}$$

$$\frac{a}{1/2} = \frac{b}{\sqrt{3}/2} = \frac{c}{1}$$

$$2a = \frac{2b}{\sqrt{3}} = c$$

$a = \frac{b}{\sqrt{3}}$

$c = \frac{2b}{\sqrt{3}}$

$$a : b : c = \frac{b}{\sqrt{3}} : b : \frac{2b}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} : 1 : \frac{2}{\sqrt{3}}$$

$$= 1 : \sqrt{3} : 2$$

[[illegible]]

$a = \sqrt{b^2 + bc + c^2}$

$a^2 = b^2 + bc + c^2$

~~$a^2 + b^2$~~

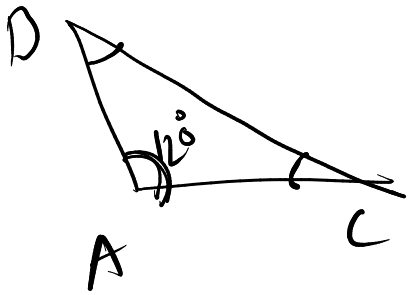
$b^2 + c^2 - a^2 = -bc$

$\frac{b^2 + c^2 - a^2}{2bc} = -\frac{bc}{2bc}$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos A = -\frac{1}{2}$

$A = 120^\circ$



$B + C = 60^\circ$