

ଉଦାହରଣ:

ସ୍ୱାଭାବିକ → ଡିଗ୍ରୀ ଲେଖ

$$\vec{\nabla} = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$$

ϕ

Gradient of ϕ $\text{Grad } \phi = \vec{\nabla} \phi$

$$\phi = 3x^2yz$$

$$\begin{aligned} \vec{\nabla} \phi &= \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) 3x^2yz \\ &= \hat{i} \frac{\partial}{\partial x} 3x^2yz + \hat{j} \frac{\partial}{\partial y} 3x^2yz + \hat{k} \frac{\partial}{\partial z} 3x^2yz \\ &= \hat{i} 3yz \frac{\partial}{\partial x} x^2 + \hat{j} 3x^2 \frac{\partial}{\partial y} y + \hat{k} 3x^2y \frac{\partial}{\partial z} z \\ &= \hat{i} 3yz \cdot 2x + \hat{j} 3x^2 \cdot 1 + \hat{k} 3x^2y \cdot 1 \\ &= 6xyz \hat{i} + 3x^2 \hat{j} + 3x^2y \hat{k} \\ &= 6(1) \times 2(-1) \hat{i} + 3(1)^2(-1) \hat{j} + 3(1)^2 \cdot 2 \hat{k} \\ &= -12 \hat{i} + 3 \hat{j} + 6 \hat{k} \\ &= \sqrt{(-12)^2 + 3^2 + 6^2} \end{aligned}$$

$\phi = 3x^2yz$

$(1, 2, -1)$ ଡିଗ୍ରୀ
Grad କର
 $x=1$
 $y=2$

ଉଚ୍ଚ ଡିଗ୍ରୀ:-

$$f'(u) = \frac{\partial f}{\partial u} = \frac{1}{2u} \quad f'(u) = 0$$

କେଉଁ ଡିଗ୍ରୀ ଲେଖ କର
→ ଓ ଲେଖ ଯାହା ଗୁଣିତରୁ ୧ରୁ ୧

Question $\phi = xyz^3$ $(1, 1, -1)$ ଓ Grad କର
 $-\hat{i} + \hat{j} + 3\hat{k}$

Divergence:

ଓଡ଼ିଆ → ଇଂରାଜୀ

$$\vec{\nabla} \cdot \vec{\varphi} \quad (2i+5j) \quad (6) \quad 2-5$$

$$\text{div } \varphi = \vec{\nabla} \cdot \vec{\varphi}$$

$$\vec{\varphi} = \underline{3xy\hat{i} + 6z\hat{k} + 3z^3\hat{j}}$$

div (1,1,1)

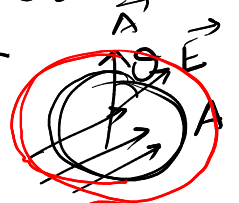
$$\text{div } \varphi = \vec{\nabla} \cdot \vec{\varphi}$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) \cdot (3xy\hat{i} + 3z^3\hat{j} + 6z\hat{k})$$

$$= \frac{\partial}{\partial x} 3xy + \frac{\partial}{\partial y} 3z^3 + \frac{\partial}{\partial z} 6z$$

$$= 3y + 0 + 6$$

$$(1,1,1) \rightarrow 3 \times 1 + 6 = 9$$



$$\text{div } \vec{A} = \vec{\nabla} \cdot \vec{A}$$

କିଛି ଗୁଣ:

(i) $\text{div} > 0$ ଯଦି

ତେବେ, କେଉଁ ସମ୍ପ୍ରାପ୍ତ ହେଉଛି

ସମସ୍ତ ସ୍ଥାନରେ, କ୍ଷେତ୍ର ଗୁଣ

(ii) $\text{div} < 0$ ଯଦି,

ତେବେ, କେଉଁ ସମ୍ପ୍ରାପ୍ତ ହେଉଛି

ସମସ୍ତ ସ୍ଥାନରେ, କ୍ଷେତ୍ର ଗୁଣ

(iii) $\text{div} = 0$

ତେବେ, କେଉଁ କ୍ଷେତ୍ର ଗୁଣ

ସମ୍ପ୍ରାପ୍ତ ହେଉଛି

କ୍ଷେତ୍ର ଗୁଣ

Question:

$$\vec{\varphi} = \underline{xy\hat{i} - x^3z^3\hat{j} + x^5\hat{k}}$$

(1,-1,1) ବିନ୍ଦୁରେ କ୍ଷେତ୍ର ଗୁଣ କେତେ ହେବ?

Curl: $\vec{A} \rightarrow \vec{A}'$ (division \rightarrow $\frac{\text{ସଂକୀର୍ଣ୍ଣ}}{\text{ସଂକୀର୍ଣ୍ଣ}}$? କାହା ଦୃଷ୍ଟିରୁ ନ?

$$\vec{\nabla} \times \vec{\varphi}$$

$$\vec{\varphi} = 3xy\hat{i} + y\hat{j} + \hat{k}$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\text{curl } \varphi = \vec{\nabla} \times \vec{\varphi}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3xy & y & 1 \end{vmatrix}$$

$$= \hat{i} \left\{ \frac{\partial}{\partial y} 1 - \frac{\partial}{\partial z} y \right\} - \hat{j} \left\{ \frac{\partial}{\partial x} 1 - \frac{\partial}{\partial z} 3xy \right\} + \hat{k} \left\{ \frac{\partial}{\partial x} y - \frac{\partial}{\partial y} 3xy \right\}$$

$$= \hat{i} \{0\} - \hat{j} \{0\} + \hat{k} \{0 - 3x\}$$

$$(1, 1, 1)$$

$$= -3x\hat{k}$$

$$= -3x\hat{i} \times \hat{k}$$

$$= -3\hat{k}$$

$$|\text{curl}| = \sqrt{(-3)^2} = 3 \neq 0$$

ଅସଂସ୍ଥିତ

ଉଦାହରଣ:

- ① $\text{curl} = 0$ ଯଦି କେବଳ ସଂକୀର୍ଣ୍ଣ
- ② curl ଅସଂସ୍ଥିତ ଯଦି କେବଳ ସଂକୀର୍ଣ୍ଣ

(1, 2, 3) $\rightarrow \vec{\varphi} = 3x\hat{i} - 3y\hat{j} - 5z\hat{k}$

କେବଳ $\vec{\varphi}$ ଦି. ସଂକୀର୍ଣ୍ଣ ନୁହେଁ

$$-8\hat{i} - 8\hat{j} - 6\hat{k}$$