7.3

$\sec\theta = \frac{13}{12}$ $\quad 270° < \theta < 360°$ [4th Quandant]

4th

$L.S = \tan\theta - \operatorname{cosec}\theta$

$= \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta}$

$= \frac{-\frac{5}{13}}{\frac{12}{13}} - \frac{1}{-\frac{5}{13}}$

$= -\frac{5}{12} - \frac{13}{5}$

13×12

$= \frac{-25 - 156}{60}$

$= -\frac{181}{60}$

$\sec\theta = \frac{13}{12}$

$\cos\theta = \frac{12}{13}$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin\theta = \pm\sqrt{1 - \left(\frac{12}{13}\right)^2}$

$= \pm \frac{5}{13}$

$\therefore \sin\theta = -\frac{5}{13}$.

$\sin x + \sin y = 1$ $\quad \cos x + \cos y = 0$

$2\sin\frac{x+y}{2}\cancel{\cos\frac{x-y}{2}} = 1$ — ①

$2\cos\frac{x+y}{2}\cancel{\cos\frac{x-y}{2}} = 0$ — ⑪

$x + y = \pi$

⑪ ÷ ① $\Rightarrow$ $\frac{\cos\frac{x+y}{2}}{\sin\frac{x+y}{2}} = \frac{0}{1}$

$\cot\frac{x+y}{2} = 0$

$\tan\frac{x+y}{2} = \infty$

$= \tan 90°$

$\frac{x+y}{2} = 90°$

$x + y = 180°$

$$\sin 2A = \sin(A+A)$$
$$= \cancel{2}\sin A\cos A + \cos A\sin A$$
$$= 2\sin A\cos A$$

$$\sin A = \sin\left(\frac{A}{2}+\frac{A}{2}\right) = 2\sin\frac{A}{2}\cos\frac{A}{2}$$

$$LS = \frac{\sin\frac{\theta}{2} - \sqrt{1+\sin\theta}}{\cos\frac{\theta}{2} - \sqrt{1+\sin\theta}} = \cot\theta/2$$

$$= \frac{\sin\frac{\theta}{2} - \sqrt{1+\sin 2\cdot\frac{\theta}{2}}}{\cos\theta/2 - \sqrt{1+\sin 2\cdot\theta/2}}$$

$$= \frac{\sin\theta/2 - \sqrt{\sin^2\theta/2 + \cos^2\theta/2 + 2\sin\theta/2\cos\theta/2}}{\cos\theta/2 - \sqrt{\sin^2\theta/2 + \cos^2\theta/2 + 2\sin\theta/2\cos\theta/2}}$$

$$= \frac{\sin\theta/2 - \sqrt{(\sin\theta/2 + \cos\theta/2)^2}}{\cos\theta/2 - \sqrt{(\sin\theta/2 + \cos\theta/2)^2}}$$

$$= \frac{\cancel{\sin\theta/2} - \cancel{\sin\theta/2} - \cos\theta/2}{\cancel{\cos\theta/2} - \sin\theta/2 - \cancel{\cos\theta/2}}$$

$$= \cot\theta/2$$

$$\cos^2\frac{A}{2}\left(1+\tan\frac{A}{2}\right)^2 = 1+\sin A$$

(এটার ও?)

$$\Rightarrow \cos^2\frac{A}{2}\left(1+\frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}\right)^2$$

$$= \cos^2\frac{A}{2}\left(\frac{\sin A/_2+\cos A/_2}{\cos A/_2}\right)^2$$

$$\Rightarrow \cancel{\cos^2\frac{A}{2}} \times \frac{\sin^2 A/_2+\cos^2 A/_2 + 2\sin A/_2 \cos A/_2}{\cancel{\cos^2 A/_2}}$$

$$1 + \sin 2\cdot A/_2$$

$$= 1+\sin A$$

$$2\sin\frac{\pi}{16} = \sqrt{2-\sqrt{2+\sqrt{2}}}$$

$\sin 2A = \underline{2\sin A\cos A}$

$$R.S = \sqrt{2-\sqrt{2+\sqrt{2}}}$$

$$= \cancel{\sqrt{8-\sqrt{2+\sqrt{2}\times 1}}}$$

$\frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$

$$= \sqrt{2-\sqrt{2\left(1+\frac{1}{\sqrt{2}}\right)}}$$

$1+\cos 2A = 2\cos^2 A$

$$= \sqrt{2-\sqrt{2\left(1+\cos\frac{\pi}{4}\right)}}$$

$1+\cos 2\cdot\frac{\pi}{8} = 2\cos^2\frac{\pi}{8}$

$$= \sqrt{2-\sqrt{2\cdot 2\cos^2\frac{\pi}{8}}}$$

$$= \sqrt{2 - 2\cos\frac{\pi}{8}}$$

$$1 - \cos 2A = 2\sin^2 A$$

$$= \sqrt{2\left(1 - \cos\frac{\pi}{8}\right)}$$

$$1 - \cos\frac{\pi}{\cancel{8}16} = 2\sin^2\frac{\pi}{16}$$

$$= \sqrt{2 \cdot 2\sin^2\frac{\pi}{16}}$$

$$= 2\sin\frac{\pi}{16}$$

$$2\cos\frac{\pi}{16} = \sqrt{2 + \sqrt{2 + \sqrt{2}}}$$

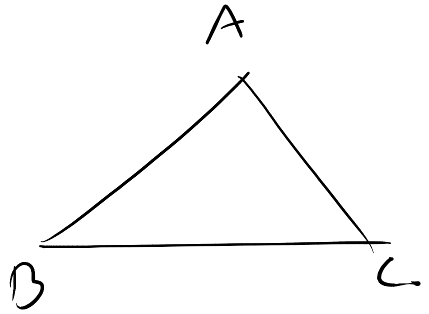
$$\sqrt{2 + \sqrt{2\left(1 + \frac{1}{\sqrt{2}}\right)}}$$ ✓

$$= \sqrt{2 + \sqrt{2\left(1 + \cos\frac{\pi}{4}\right)}}$$

$$= \sqrt{2 + \sqrt{2 \cdot 2\cos^2\frac{\pi}{8}}}$$

$$=$$

7.6



$$A+B+C=\pi$$

$$B+C=\pi-A$$

$$\sin(B+C)=\sin(\pi-A)$$

$$\rightarrow \sin(B+C)=\sin A$$

$\underline{A+B+C=\pi}$ $\quad \cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$.

$$1+\cos 2A=2\cos^2 A$$

$$1-\cos 2A=2\sin^2 A.$$

$$1-2\sin^2 A=\cos 2A$$

$$L.S=\cos A+\cos B+\cos C$$

$$=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}+\cos C$$

$$=2\cos\frac{\pi-C}{2}\cos\frac{A-B}{2}+\cos C$$

$$+\cos\cdot 2\cdot\frac{C}{2}$$

$$=2\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)\cos\frac{A-B}{2}$$

$$=2\sin\frac{C}{2}\cos\frac{A-B}{2}+1-2\sin^2\frac{C}{2}$$

$$=2\sin\frac{C}{2}\left\{\cos\frac{A-B}{2}-\sin\frac{C}{2}\right)+1$$

$$=2\sin\frac{C}{2}\left\{\cos\frac{A-B}{2}-\cos\frac{A+B}{2}\right\}+1$$

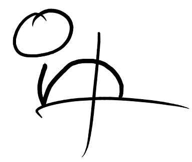
$$=2\sin\frac{C}{2}\left\{2\sin\left(\frac{\frac{A-B}{2}+\frac{A+B}{2}}{2}\right)\sin\left(\frac{\frac{A+B}{2}-\frac{A-B}{2}}{2}\right)\right\}+1$$

$$=2\sin\frac{C}{2}\,2\sin\frac{A}{2}\sin\frac{B}{2}+1$$

$$=4\sin\frac{A}{2}\cdot\sin\frac{B}{2}\sin\frac{C}{2}+1$$

$A+B+C=\pi$

$\tan A+\tan B+\tan C=$
$\tan A\tan B\tan C$



$A+B+C=\pi$

$A+B=\pi-C$

$\tan(A+B)=\tan(\pi-C)$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=\tan(2\cdot\tfrac{\pi}{2}-C)$$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C$$

$\tan A+\tan B=-\tan C+\tan A\tan B\tan C$

$\tan A+\tan B+\tan C=\tan A\tan B\tan C$ ✓

$A+B+C=\frac{\pi}{2}$

$\sin^2A+\sin^2B+\sin^2C+$
$2\sin A\sin B\sin C=1$

$A+B=\frac{\pi}{2}-C$

$\sin(A+B)=\sin(\frac{\pi}{2}-C)$

$\sin(A+B)=\cos C$

$(\sin A\cos B+\cos A\sin B)^2=(\cos C)^2$

$\sin^2A\cos^2B+\cos^2A\sin^2B+2\sin A\cos A\sin B\cos B=\cos^2C$

$+\frac{1}{2}\sin 2A\sin 2B=1-\sin^2C$

$$\sin^2 A(1-\sin^2 B) + (1-\sin^2 A)\sin^2 B + 2\sin A\cos A\sin B\cos B = 1-\sin^2 C$$

$-(\cos A\cos B - \sin A\sin B)$

$-(\cos(A+B))$

$$\sin^2 A - \sin^2 A\sin^2 B + \sin^2 B - \sin^2 A\sin^2 B + 2\sin A\cos A\sin B\cos B = 1-\sin^2 C$$

$$\sin^2 A + \sin^2 B - 2\sin A\sin B(\sin A\sin B - \cos A\cos B) = 1-\sin^2 C$$

$$\sin^2 A + \sin^2 B - 2\sin A\sin B(-\cos(A+B)) = 1-\sin^2 C$$

$$\sin^2 A + \sin^2 B + 2\sin A\sin B\cos\left(\frac{\pi}{2}-C\right) = 1-\sin^2 C$$

$$\sin^2 A + \sin^2 B + \sin^2 C + 2\sin A\sin B\sin C = 1.$$

(Proved)