GMC Sample Problems

"Dedicated Students from Gunn High School"

February 2022

1 Sample Problem Solutions

Problem (Sweet Bell). Let $a \oplus b = ab + 1$. Compute $3 \oplus (4 \oplus 5)$.

Problem Credit: Arul Mathur
Solution: $3 \oplus (4 \oplus 5) = 3 \oplus (4 \cdot 5 + 1) = 3 \oplus 21 = 3 \cdot 21 + 1 = \boxed{64}$

Problem (Jalapeño). A box holding 32 identical chocolates weighs 680 grams. Once we remove 9 chocolates, the box now weighs half a kilogram. How much does the box weigh on its own?

Problem Credit: Alan Lee
Solution: Let the weight of a chocolate be c grams and let the weight of the box be b grams. We know that $b+32c = 680$ and $b+9c = 500$, and solving, we get that $b = \boxed{40}$.

Problem (Jalapeño). Richard drops a ball from a height of 12 feet and it bounces. Each bounce is $\frac{2}{3}$ the height of the bounce before. Find the total vertical distance, in feet, the ball is going to travel.

Problem Credit: Andrew Peng
Solution: Initially, the ball drops 12 feet. Every successive bounce is $\frac{2}{3}$ the height of the one before it, so the next bounce is 8 feet high for a total vertical distance of 16 feet. This becomes an infinite geometric series with common ratio $\frac{2}{3}$, summing to 48. Including the initial drop of 12 feet, the total vertical distance travelled is $\boxed{60}$ feet.

Problem (Cayenne). An integer n has 302481 digits in base-9261. What is the difference between the largest and smallest number of digits that n could have in base 21?

Problem Credit: Ayush Aggarwal
Solution: Note that $9261 = 21^3$. Each digit besides the leftmost digit of n represents 3 digits in base 21. The leftmost digit is nonzero and can represent $1, 2,$ or 3 digits. Thus, the difference between the largest and smallest number of digits is $3 - 1 = \boxed{2}$.

Problem (Habanero). Let S be the sum of all 25-digit numbers containing 1 1, 2 2s, 3 3s, 4 4s, 5 5s, 4 6s, 3 7s, 2 8s, and 1 9. Find the largest k such that 3^k divides S.

Problem Credit: Roger Fan

Solution: Consider each digit individually. Given a random such 25-digit number, the expected value of any digit will be 5. If A is the expected value, or the average, of the 25-digit numbers, then $A = 555...555$, where there are 25 5s.

There are $N = \binom{25}{1,2,3,4,5,4,3,2,1} = \frac{25!}{1!2!3!4!5!4!3!2!1!}$ total such numbers, and since the average of the numbers is A, $S = A \cdot N$.

$A = 55...55$ is not divisible by 3 since the sum of its digits is 125. (You can also represent it as the sum of a geometric series and lift the exponent.) Also, careful analysis of $N = \binom{25}{1,2,3,4,5,4,3,2,1}$ shows that it has 10 factors of 3 in the numerator and 5 in the denominator, so N has 5 factors of 3.

Our answer is thus $\boxed{5}$.

Problem (Wild Card). The value of the expression $1 * 9 * 881 * 41$ can be expressed in the form $a^b - c^b$ such that a, b, and c are all positive integers and b is maximized. Find $a + b + c$.

Remark. *Please try to do this problem on your own first. We've been trying to convince Ayush that it's not a doable problem for the past month.*

Problem Credit: Ayush Aggarwal

Solution: Any ordinary kindergartener would immediately notice that $881 = 4^4 + 5^4$. Also, it is disgustingly obvious that $41 = 4^2 + 5^2$ and that $9 = 4 + 5$. We then have that $1*9*881*41 = (5-4)(5+4)(5^2+4^2)(5^4+4^4) = 5^8 - 4^8$. To be sure we have the largest b, we simply bash out every case for every b until b becomes sufficiently large, a casual pasttime that would take an infant give or take 3 minutes. Thus our answer is easily $\boxed{17}$.