VexCL

Vector Expression Template Library for OpenCL

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Meeting C++, 9./10.11.12

- Created for ease of C++ based OpenCL development.
- The source code is publicly available under MIT license.
- This is not a C++ bindings library!

Basic interface

Motivation

Motivation

- Basic interface
- Kernel builder
- Performance
- Implementation details
- Conclusion

¹https://github.com/ddemidov/vexcl

Hello OpenCL: vector sum

Vector sum

Motivation

•00000000

- A, B, and C are large vectors.
- Compute C = A + B.

Overview of OpenCL solution

- Initialize OpenCL context on supported device.
- Allocate memory on the device.
- Transfer input data to device.
- Run your computations on the device.
- Get the results from the device.

1. Query platforms

Motivation

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```
std::vector<cl::Platform> platform;
   cl::Platform::get(&platform);
3
   if ( platform.empty() ) {
      std::cerr << "OpenCL platforms not found." << std::endl;
      return 1;
7
```

Implementation details

Hello OpenCL: vector sum

2. Get first available GPU device

Basic interface

```
cl :: Context context;
   std::vector<cl::Device> device;
   for(auto p = platform.begin(); device.empty() && p != platform.end(); p++) {
10
       std::vector<cl::Device> pldev;
11
       try {
12
           p->getDevices(CL_DEVICE_TYPE_GPU, &pldev);
13
           for(auto d = pldev.begin(); device.empty() && d!= pldev.end(); d++) {
14
                if (!d->getInfo<CL_DEVICE_AVAILABLE>()) continue;
15
               device.push_back(*d);
16
               context = cl :: Context(device);
17
18
        } catch(...) {
19
           device. clear ();
20
21
22
    if (device.empty()) {
23
       std::cerr << "GPUs not found." << std::endl;
24
       return 1;
25
26
```

Motivation

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3. Create kernel source

```
const char source[] =
27
        "kernel void add(\n"
28
                uint n,\n"
29
                global const float *a,\n"
30
             global const float *b,\n"
31
              global float *c\n"
32
        "
                )\n"
33
        "\{ n \}"
34
             uint i = get\_global\_id(0); \n"
35
           if (i < n) \{ n \}
36
           c[i] = a[i] + b[i]; \ n"
37
            }\n"
38
        "}\n";
39
```

4. Compile kernel

Motivation

```
cl::Program program(context, cl::Program::Sources(
40
                1, std::make_pair(source, strlen(source))
41
                ));
42
    try {
43
        program.build(device);
44
    } catch (const cl::Error&) {
45
        std::cerr
46
            << "OpenCL compilation error" << std::endl</p>
47
            << pre>rogram.getBuildInfo<CL_PROGRAM_BUILD_LOG>(device[0])
48
            << std::endl;
49
        return 1:
50
51
    cl::Kernel add_kernel = cl::Kernel(program, "add");
52
```

5. Create command queue

cl::CommandQueue queue(context, device[0]);

Motivation

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Tello OpenCL. Vector sum

6. Prepare input data, transfer it to device

```
const unsigned int N = 1 \ll 20;
54
   std::vector<float> a(N, 1), b(N, 2), c(N);
55
56
   cl :: Buffer A(context, CL_MEM_READ_ONLY | CL_MEM_COPY_HOST_PTR,
57
           a. size() * sizeof(float), a.data());
58
59
   cl :: Buffer B(context, CL_MEM_READ_ONLY | CL_MEM_COPY_HOST_PTR,
60
           b. size() * sizeof(float), b.data());
61
62
   cl::Buffer C(context, CL_MEM_READ_WRITE,
63
           c. size() * sizeof(float));
64
```

7. Set kernel arguments

```
add_kernel.setArg(0, N);
add_kernel.setArg(1, A);
add_kernel.setArg(2, B);
add_kernel.setArg(3, C);
```

Motivation

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8. Launch kernel

queue.enqueueNDRangeKernel(add_kernel, cl::NullRange, N, cl::NullRange);

9. Get result back to host

```
queue.enqueueReadBuffer(C, CL_TRUE, 0, c.size() * sizeof(float), c.data());
std::cout << c[42] << std::endl; // Should get '3' here.
```

Hello VexCL: vector sum

Motivation

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This is much shorter!

std::cout << 3 << std::endl;

Hello VexCL: vector sum

Motivation

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Get all available GPUs

```
vex::Context ctx( vex::Filter ::Type(CL_DEVICE_TYPE_GPU) );
   if (!ctx.size()) {
      std::cerr << "GPUs not found." << std::endl;
      return 1;
5
```

Prepare input data, transfer it to device

```
std::vector<float> a(N, 1), b(N, 2), c(N);
vex::vector<float> A(ctx.queue(), a);
```

- vex::vector<**float**> B(ctx.queue(), b);
- vex::vector<float> C(ctx.queue(), N);

Launch kernel, get result back to host

```
C = A + B;
   vex::copy(C, c);
11
    std::cout << c[42] << std::endl;
12
```

- Motivation
- 2 Basic interface
 - Device selection
 - Vector arithmetic
 - Reductions
 - User-defined functions
 - Sparse matrix vector products
 - Stencil convolutions
 - Multivectors & multiexpressions
- Kernel builder
- Performance
- Implementation details
- 6 Conclusion

- Multi-device and multi-platform computations are supported.
- VexCL context is initialized from combination of device filters.
- Device filter is a boolean functor acting on **const** cl::Device&.

Initialize VexCL context on selected devices

vex::Context ctx(vex::Filter :: All);



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- VexCL context is initialized from combination of device filters.
- Device filter is a boolean functor acting on const cl::Device&.

Initialize VexCL context on selected devices

vex::Context ctx(vex::Filter ::Type(CL_DEVICE_TYPE_GPU));



- Multi-device and multi-platform computations are supported.
- VexCL context is initialized from combination of device filters.
- Device filter is a boolean functor acting on **const** cl::Device&.

Initialize VexCL context on selected devices

```
vex::Context ctx(
    vex::Filter ::Type(CL_DEVICE_TYPE_GPU) &&
    vex::Filter ::Platform("AMD")
);
```



- Multi-device and multi-platform computations are supported.
- VexCL context is initialized from combination of device filters.
- Device filter is a boolean functor acting on **const** cl::Device&.

Initialize VexCL context on selected devices

```
vex::Context ctx(
vex::Filter ::Type(CL_DEVICE_TYPE_GPU) &&

[](const cl::Device &d) {
    return d.getInfo<CL_DEVICE_GLOBAL_MEM_SIZE>() >= 4_GB;
});
```



Exclusive device access

Motivation

- vex:: Filter :: Exclusive() wraps normal filters to allow exclusive access to devices.
- Useful for cluster environments.
- An alternative to NVIDIA's exclusive compute mode for other vendors hardware.
- Based on Boost.Interprocess file locks in temp directory.

```
vex::Context ctx( vex:: Filter :: Exclusive (
vex:: Filter :: DoublePrecision &&
vex:: Filter :: Env
));
```

Motivation

- You don't *have to* initialize vex::Context.
- vex::Context is just a convenient container that holds OpenCL contexts and queues.
- vex::Context::queue() returns std::vector<cl::CommandQueue>. This may come from *elsewhere*.

```
std::vector<cl::CommandQueue> my_own_vector_of_opencl_command_queues;
// ...
vex::vector<double> x(my_own_vector_of_opencl_command_queues, n);
```

- Each queue should correspond to a separate device.
- Different VexCL objects may be initialized with different queue lists.
- Operations are submitted to the queues of the vector that is being assigned to.

Hello VexCL example

```
vex::Context ctx( vex::Filter::Name("Tesla") );

vex::vector<float> A(ctx.queue(), N); A = 1;

vex::vector<float> B(ctx.queue(), N); B = 2;

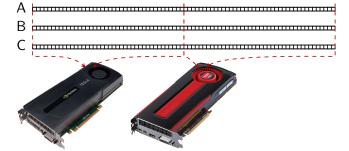
vex::vector<float> C(ctx.queue(), N);

C = A + B;
```

```
С
```

```
Hello VexCL example
```

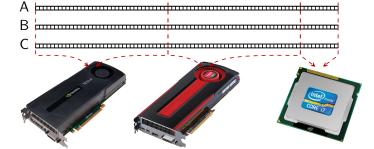
```
vex::Context ctx( vex::Filter::Type(CL_DEVICE_TYPE_GPU) );
   vex::vector < float > A(ctx.queue(), N); A = 1;
   vex:: vector < float > B(ctx.queue(), N); B = 2;
   vex::vector < float > C(ctx.queue(), N);
6
   C = A + B;
```



Vector allocation and arithmetic

Hello VexCL example

```
vex::Context ctx( vex::Filter::DoublePrecision );
   vex::vector < float > A(ctx.queue(), N); A = 1;
   vex:: vector < float > B(ctx.queue(), N); B = 2;
   vex::vector < float > C(ctx.queue(), N);
6
   C = A + B;
```



Implementation details

What may be used in vector expressions?

- All vectors in expression have to be compatible:
 - Have same size
 - Located on same devices
- What may be used:
 - Scalar values
 - Arithmetic, bitwise, logical operators
 - Builtin OpenCL functions
 - User-defined functions

```
std :: vector < float > x(n);
   std::generate(x.begin(), x.end(), rand);
3
   vex::vector < float > X(ctx.queue(), x);
   vex::vector<float> Y(ctx.queue(), n);
   vex::vector<float> Z(ctx.queue(), n);
7
   Y = 42:
   Z = \operatorname{sqrt}(2 * X) + \operatorname{pow}(\cos(Y), 2.0);
```

Reductions

Motivation

- Class vex::Reductor<T, kind> allows to reduce arbitrary vector expression to a single value of type T.
- Supported reduction kinds: SUM, MIN, MAX

Inner product

- vex::Reductor<double, vex::SUM> sum(ctx.queue());
- **double** s = sum(x * y);

Number of elements in x between 0 and 1

- vex::Reductor<size_t, vex::SUM> sum(ctx.queue());
- size_t n = sum((x > 0) && (x < 1));

Maximum distance from origin

- vex::Reductor<double, vex::MAX> max(ctx.queue());
- **double** d = max(sqrt(x * x + y * y));

User-defined functions

Motivation

- Users may define functions to be used in vector expressions:
 - Provide function body
 - Define return type and argument types

Defining a function

- extern const char between_body[] = "return prm1 <= prm2 && prm2 <= prm3;";
- UserFunction

 between_body, bool(double, double, double)> between;

Using a function: number of 2D points in first quadrant

```
size_t points_in_1q( const vex::Reductor<size_t, vex::SUM> &sum,
const vex::vector<double> &x, const vex::vector<double> &y)
{
    return sum( between(0.0, atan2(y, x), M_PI/2) );
}
```

Sparse matrix – vector products

- Class vex::SpMat<T> holds representation of a sparse matrix on compute devices.
- Constructor accepts matrix in common CRS format (row indices, columns and values of nonzero entries).
- SpMV may only be used in additive expressions.

Construct matrix

vex::SpMat<**double>** A(ctx.queue(), n, n, row.data(), col.data(), val.data());

Compute residual value

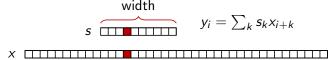
- 2 // vex:: vector < double > u, f, r;
- r = f A * u;

Motivation

double res = $\max(fabs(r));$

Motivation

width



- Simple stencil is based on a 1D array, and may be used for:
 - Signal filters (e.g. averaging)
 - Differential operators with constant coefficients
 - ...

Moving average with 5-points window

```
std::vector<double> sdata(5, 0.2);
vex:: stencil <double> s(ctx.queue(), sdata, 2 /* center */);

y = x * s;
```

User-defined stencil operators

- Define efficient arbitrary stencil operators:
 - Return type
 - Stencil dimensions (width and center)
 - Function body

Example: nonlinear operator

$$y_i = x_i + (x_{i-1} + x_{i+1})^3$$

Implementation

```
extern const char custom_op_body[] =
   "double t = X[-1] + X[1];\n"
   "return X[0] + t * t * t;"

vex::StencilOperator<double, 3 /*width*/, 1 /*center*/, custom_op_body>
   custom_op(ctx.queue());

y = custom_op(x);
```

Motivation

- vex::multivector<T,N> holds N instances of equally sized vex::vector<T>
- Supports all operations that are defined for vex::vector<>.
- Transparently dispatches the operations to the underlying components.
- vex::multivector::operator(uint k) returns k-th component.

```
vex::multivector<double, 2> X(ctx.queue(), N), Y(ctx.queue(), N);
  vex::Reductor<double, vex::SUM> sum(ctx.queue());
  vex::SpMat<double> A( ctx.queue(), ... );
  std :: array < double, 2 > v;
5
  // ...
 X = \sin(v * Y + 1);   // X(k) = \sin(v[k] * Y(k) + 1);
 v = sum(between(0, X, Y)); //v[k] = sum(between(0, X(k), Y(k)));
                                //X(k) = A * Y(k);
  X = A * Y;
```

Sometimes an operation cannot be expressed with simple multivector arithmetics.

Example: rotate 2D vector by an angle

$$y_0 = x_0 \cos \alpha - x_1 \sin \alpha$$
,

$$y_1 = x_0 \sin \alpha + x_1 \cos \alpha$$
.

- Multiexpression is a tuple of normal vector expressions
- Its assignment to a multivector is functionally equivalent to componentwise assignment, but results in a single kernel launch.

Multiexpressions

Motivation

Multiexpressions may be used with multivectors:

and with tied vectors:

• Any expression that is assignable to a vector is valid in a multiexpression.

Copies between host and device memory

- vex::vector<double> X;
- std::vector<double>x;
- double c_array[100];

Simple copies

- vex::copy(X, x); // From device to host.
- vex::copy(x, X); // From host to device.

STL-like range copies

- vex::copy(X.begin(), X.end(), x.begin());
- vex::copy(X.begin(), X.begin() + 100, x.begin());
- $vex::copy(c_array, c_array + 100, X.begin());$

Inspect or set single element (slow)

- vex::copy(X, x);
- assert(x[42] == X[42]);
- X[0] = 0;

- Motivation
- 2 Basic interface
- 3 Kernel builder
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*Restrictions applied

Motivating example

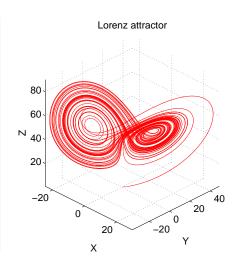
- Let's solve an ODE!
- Let's do it with Boost.odeint!
- Lorenz attractor system:

$$\dot{x} = -\sigma(x - y),$$

$$\dot{y} = Rx - y - xz,$$

$$\dot{z} = -bz + xy.$$

 We want to solve large number of Lorenz systems, each for a different value of R.



1. System functor

```
typedef vex::vector<double>
                                       vector_type;
1
   typedef vex::multivector<double, 3> state_type;
3
   struct lorenz_system {
       const vector_type &R;
       lorenz_system(const vector_type &R): R(R) {}
       void operator()(const state_type &x, state_type &dxdt, double t) {
           dxdt = std:: tie(
9
                       sigma * (x(1) - x(0)),
10
                       R * x(0) - x(1) - x(0) * x(2),
11
                       x(0) * x(1) - b * x(2)
12
                    );
13
14
15
```

2. Integration

```
state_type X(ctx.queue(), n);
    vector_type R( ctx.queue(), r );
3
    // ... initialize X and R here ...
5
    odeint::runge_kutta4<
6
            state_type, double, state_type, double,
7
            odeint::vector_space_algebra, odeint::default_operations
            > stepper;
9
10
    odeint::integrate_const(stepper, lorenz_system(R), X, 0.0, t_max, dt);
11
```

That was easy!

2. Integration

```
state_type X( ctx.queue(), n );
vector_type R( ctx.queue(), r );

// ... initialize X and R here ...

odeint::runge_kutta4<
state_type, double, state_type, double,
odeint::vector_space_algebra, odeint::default_operations
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That was easy! And fast!

2. Integration

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That was easy! And fast! But,

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2. Integration
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            > stepper:
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    odeint:: integrate_const (stepper, lorenz_system(R), X, 0.0, t_max, dt);
11
```

- That was easy! And fast! But,
 - Runge-Kutta method uses 4 temporary state variables (here stored on GPU).
 - Single Runge-Kutta step results in several kernel launches.

What if we did this manually?

- Create single monolithic kernel that does one step of Runge-Kutta method.
- Launch the kernel in a loop.
- This is ≈ 10 times faster!

```
double3 lorenz_system(double r, double sigma, double b, double3 s) {
        return (double3)(
            sigma * (s.v - s.x).
            r * s.x - s.y - s.x * s.z,
            s.x * s.v - b * s.z
    kernel void lorenz_ensemble(
        ulong n, double sigma, double b,
10
       const global double *R,
11
        global double *X,
12
        global double *Y,
13
        global double *Z
14
15
16
       double r;
17
       double3 s, dsdt, k1, k2, k3, k4;
18
19
        for(size_t gid = get_global_id(0); gid < n; gid += get_global_size(0))
20
            r = R[gid];
21
            s = (double3)(X[gid], Y[gid], Z[gid]);
22
23
            k1 = dt * lorenz_system(r, sigma, b, s);
24
            k2 = dt * lorenz\_system(r, sigma, b, s + 0.5 * k1);
25
            k3 = dt * lorenz_system(r, sigma, b, s + 0.5 * k2);
26
            k4 = dt * lorenz\_system(r, sigma, b, s + k3);
28
            s += (k1 + 2 * k2 + 2 * k3 + k4) / 6;
29
30
            X[gid] = s.x; Y[gid] = s.y; Z[gid] = s.z;
31
32
33
```

What if we did this manually?

Motivation

- Create single monolithic kernel that does one step of Runge-Kutta method.
- Launch the kernel in a loop.
- This is ≈ 10 times faster! But.

```
double3 lorenz_system(double r, double sigma, double b, double3 s) {
        return (double3)(
            sigma * (s.v - s.x).
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    kernel void lorenz_ensemble(
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11
        global double *X,
12
        global double *Y,
13
        global double *Z
14
15
16
       double r;
17
       double3 s, dsdt, k1, k2, k3, k4;
18
19
        for(size_t gid = get_global_id(0); gid < n; gid += get_global_size(0))
20
            r = R[gid]:
21
            s = (double3)(X[gid], Y[gid], Z[gid]);
22
23
            k1 = dt * lorenz_system(r, sigma, b, s);
24
            k2 = dt * lorenz\_system(r, sigma, b, s + 0.5 * k1);
25
            k3 = dt * lorenz_system(r, sigma, b, s + 0.5 * k2);
26
            k4 = dt * lorenz\_system(r, sigma, b, s + k3);
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31
32
33
```

Motivation

- Create single monolithic kernel that does one step of Runge-Kutta method.
- Launch the kernel in a loop.
- This is ≈ 10 times faster! But.
- We lost the generality odeint offers!

```
double3 lorenz_system(double r, double sigma, double b, double3 s) {
        return (double3)(
            sigma * (s.v - s.x).
            r * s.x - s.y - s.x * s.z,
            s.x * s.v - b * s.z
    kernel void lorenz_ensemble(
        ulong n, double sigma, double b,
10
       const global double *R.
11
        global double *X,
12
        global double *Y,
13
        global double *Z
14
15
16
       double r;
17
       double3 s, dsdt, k1, k2, k3, k4;
18
19
        for(size_t gid = get_global_id(0); gid < n; gid += get_global_size(0))
20
            r = R[gid]:
21
            s = (double3)(X[gid], Y[gid], Z[gid]);
22
23
            k1 = dt * lorenz_system(r, sigma, b, s);
24
            k2 = dt * lorenz\_system(r, sigma, b, s + 0.5 * k1);
25
            k3 = dt * lorenz_system(r, sigma, b, s + 0.5 * k2);
26
            k4 = dt * lorenz\_system(r, sigma, b, s + k3);
28
            s += (k1 + 2 * k2 + 2 * k3 + k4) / 6
            X[gid] = s.x; Y[gid] = s.y; Z[gid] = s.z;
31
32
33
```

- Output the sequence of arithmetic expressions of an algorithm.
- Onstruct OpenCL kernel from the captured sequence.
- ???
- Use the kernel!

Motivation

1. Declare functor operating on vex::generator::symbolic<> values

```
typedef vex::generator::symbolic < double > sym_vector;
typedef std::array < sym_vector, 3 > sym_state;

struct lorenz_system {
    const sym_vector &R;
    lorenz_system(const sym_vector &R) : R(R) {}

    void operator()(const sym_state &x, sym_state &dxdt, double t) const {
        dxdt[0] = sigma * (x[1] - x[0]);
        dxdt[1] = R * x[0] - x[1] - x[0] * x[2];
        dxdt[2] = x[0] * x[1] - b * x[2];
}

typedef vex::generator::symbolic < double > sym_vector;
typedef std::array < sym_vector;
typedef std::array <
```

2. Record one step of Runge-Kutta method

std::ostringstream lorenz_body;

Motivation

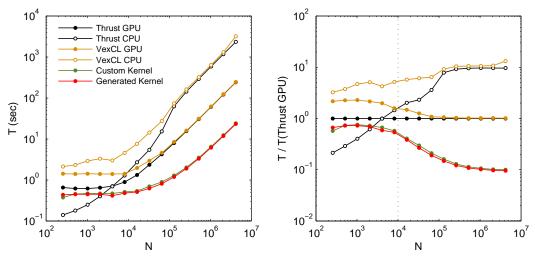
```
vex::generator::set_recorder(lorenz_body);
3
    sym_state sym_S = \{\{\}\}
        sym_vector::VectorParameter,
        sym_vector::VectorParameter,
        sym_vector::VectorParameter }};
    sym_vector sym_R(sym_vector::VectorParameter, sym_vector::Const);
9
    odeint::runge_kutta4<
10
            sym_state, double, sym_state, double,
11
            odeint::range_algebra, odeint::default_operations
12
            > stepper;
13
14
    lorenz_system sys(sym_R);
15
    stepper.do_step(std::ref(sys), sym_S, 0, dt);
16
```

3. Generate and use OpenCL kernel

The restrictions

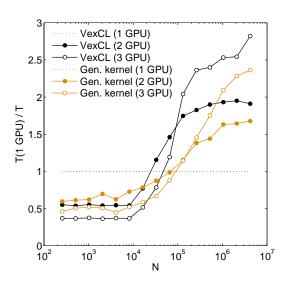
- Algorithms have to be embarassingly parallel.
- Only linear flow is allowed (no conditionals or data-dependent loops).
- Some precision may be lost when converting constants to strings.
- Probably some other corner cases. . .

The performance results



GPU: NVIDIA Tesla C2070 CPU: Intel Core i7 930

- Larger problems may be solved on the same system.
- Large problems may be solved faster.

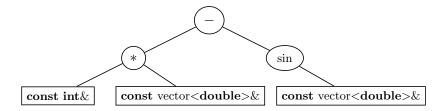


- Motivation
- 2 Basic interface
- 3 Kernel builder
- Performance
- Implementation details
- 6 Conclusion

- VexCL is an expression template library
- Each expression in the code results in an expression tree evaluated at time of assignment.
 - No temporaries are created
 - Single kernel is generated and executed

Example expression

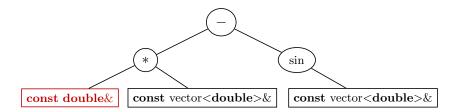
 $x = 2 * y - \sin(z);$



- VexCL is an expression template library
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Example expression

 $x = 2.0 * y - \sin(z);$



Kernel generation

The expression

```
x = 2 * y - \sin(z);
```

Define VEXCL_SHOW_KERNELS to see the generated code.

...results in this kernel:

Performance tip

- No way to tell if two terminals refer to the same data!
- Example: finding number of points in 1st quadrant

Naive

- **return** sum($0.0 \le \tan 2(y, x) \&\& \tan 2(y, x) \le M_PI/2$);
 - x and y are read twice
 - atan2 is computed twice

Using custom function

return sum(between(0.0, atan2(y, x), $M_PI/2$));

Custom kernels

It is possible to use custom kernels with VexCL vectors

```
vex::vector<float> x(ctx.queue(), n);
1
2
    for(uint d = 0; d < ctx.size(); d++) {
        cl::Program program = build_sources(ctx.context(d),
            "kernel void dummy(ulong size, global float *x) {\n"
               x[\text{get\_global\_id }(0)] = 4.2; n"
            "}\n");
        cl::Kernel dummy(program, "dummy");
9
10
        dummy.setArg(0, static_cast < cl_ulong > (x.part_size(d)));
11
        dummy.setArg(1, x(d));
12
13
        ctx.queue(d).enqueueNDRangeKernel(dummy, cl::NullRange, x.part_size(d), cl::NullRange);
14
15
```

Conclusion and Questions

Motivation

- VexCL allows to write compact and readable code without sacrificing too much performance.
- Multiple compute devices are employed transparently.
- Supported compilers (don't forget to enable C++11 features):
 - GCC v4.6
 - Clang v3.1
 - MS Visual C++ 2010 (partially)

https://github.com/ddemidov/vexcl



Conjugate gradients method

```
Solve linear equations system Au = f
   void cg::solve(const vex::SpMat<double> &A, const vex::vector<double> &f, vex::vector<double> &u) {
       // Member fields:
2
       // vex::vector<double> r, p, q;
       // Reductor<double,MAX> max; Reductor<double,SUM> sum;
5
       double rho1 = 0, rho2 = 1;
6
       r = f - A * u:
7
8
       for(int iter = 0; max(fabs(r)) > 1e-8 \&\& iter < n; iter++) {
9
           rho1 = sum(r * r);
10
11
           if (iter == 0) {
12
               p = r;
           } else {
14
               double beta = rho1 / rho2;
15
               p = r + beta * p;
17
18
           q = A * p;
19
20
           double alpha = rho1 / sum(p * q);
21
22
           u += alpha * p;
23
           r -= alpha * q;
24
25
           rho2 = rho1;
26
27
28
```

The generated kernel (is ugly)

```
1 kernel void lorenz(
2 ulong n,
3 global double+ p_var0.
4 global double+ p_varl
s global double+ p.var2.
   global const double* p_var3

    size_t idx = get_global_id(0);

10 if (idx < n) {
11 double var0 = p_var0[idx];
12 double varl = p_varl[idx];
13 double var2 = p_var2[idx];
4 double var3 = p_var3[idx];
15 double var4:
16 double var5:
17 double var6:
18 double var7:
10 double var8
20 double var9:
21 double var10:
22 double varl1:
   double var12;
24 double var13:
25 double var14:
   double var15:
27 double var16:
20 double var17:
29 double var18:
var4 = (1.0000000000000e+01 * (var1 - var0));
var5 = (((var3 * var0) - var1) - (var0 * var2));
yar6 = ((var0 * var1) - (2.666666666666e+00 * var2));
var7 = ((1.00000000000000e+00 * var0) + (5.000000000000e-03 * var4));
var8 = ((1.0000000000000e+00 * var1) + (5.000000000000e-03 * var5));
var9 = ((1.00000000000000e+00 * var2) + (5.000000000000e-03 * var6));
var10 = (1.0000000000000e+01 * (var8 - var7));
yr var11 = (((var3 * var7) - var8) - (var7 * var9));
var12 = ((var7 + var8) - (2.6666666666666+00 + var9));
var7 = (((1.00000000000000000+00 * var0) + (0.0000000000000+00 * var4)) + (5.0000000000000-03 * var10));
40 \text{ var8} = (((1.000000000000000+00 * \text{var1}) + (0.0000000000000+00 * \text{var5})) + (5.00000000000000-03 * \text{var11}));
var9 = (((1.0000000000000000+00 * var2) + (0.0000000000000+00 * var6)) + (5.00000000000000-03 * var12));
42 var13 = (1.0000000000000e+01 * (var8 - var7));
41 var14 = (((var3 * var7) - var8) - (var7 * var9));
44 var15 = ((var7 * var8) - (2.666666666666e+00 * var9));
 \text{var8} = ((((1.000000000000000000+00 * \text{var1}) + (0.00000000000000+00 * \text{var5})) + (0.0000000000000+00 * \text{var11})) + (1.000000000000000-02 * \text{var14})) 
40 var16 = (1.0000000000000e+01 * (var8 - var7));
49 var17 = (((var3 * var7) - var8) - (var7 * var9));
so var18 = ((var7 * var8) - (2.666666666666e+00 * var9));
var0 = (((((1.000000000000+0) * var0) + (1.6666666666-03 * var4)) + (3.333333333-03 * var10)) + (3.333333333-03 * var13)) + (1.66666666666-03 * var16))
var1 = (((((1.0000000000000+0) * var1) + (1.66666666666-03 * var5)) + (3.333333333-03 * var11)) + (3.3333333333-03 * var14)) + (1.666666666666-03 * var17))
var2 = (((((1.00000000000000+0*var2) + (1.6666666666-03*var6)) + (3.333333333-03*var12)) + (3.3333333333-03*var15)) + (1.666666666666-03*var18));
54 p_var0[idx] = var0;
ss p_varl[idx] = varl;
   p_{\text{avar2}}[idx] = var2;
57
```