

A Level Set Method Combined with Gaussian Mixture Model for Image Segmentation

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Abstract. Chan-Vese(CV) model promotes the evolution of level set curve based on the gray distribution inside and outside the curve. It has a better segmentation effect on images with intensity homogeneity and obvious contrast. However, when the gray distribution of image is uneven, the evolution speed of the curve will be significantly slower, and the curve will be guided to the wrong segmentation result. To solve this problem, a method to improve CV model by using of Gaussian mixture model(GMM) is proposed. We use the parameters of the Gaussian submodels to correct the mean value of grayscale inside and outside the curve in the energy function. The target region can be quickly segmented in the images with complex background gray distribution. Experimental results show that the proposed algorithm can significantly reduce the number of iterations and enhance the robustness to noise. The level set curve can quickly evolve into target region in the images with intensity inhomogeneity.

Keywords: Image segmentation · Level set · Chan-Vese model · Gaussian mixture model.

1 Introduction

Level set method is an important research direction in the field of image segmentation in recent years, and has been widely used in various fields, such as medicine [1] and remote sensing, etc. It has an important research value. The Level set method based on contour evolution, such as Snake model and Geometric Active Contour(GAC) model [2], makes curve gradually approach the edge of target by minimizing the energy function of the closed curve [3]. However, such models only use local edge information of image and do not have a good segmentation effect on the images with blurred or discrete edges [4].

For images with weak edges and discontinuous boundaries, Mumford and Shah proposed a new curve evolution model (M-S model), which mainly used the foreground and background gray information of image to realize image segmentation by solving the minimum value of energy function. However, M-S model is over-ideal and computationally complex [5]. On this basis, Chan and Vese simplified the original model and put forward CV model, making the level set curve evolve without depending on the image gradient. They introduced image

grayscale information and successfully applied the model in the remote sensing satellite cloud image segmentation [6], nuclear magnetic resonance image segmentation [7], and CT image segmentation [8], etc. However, CV model requires high uniformity of image gray distribution. In practical applications, if background gray distribution is complex, or the grayscale difference between foreground and background is not obvious, the segmentation effect of CV model will all reduce and even lead to wrong segmentation results. Li [9] introduced penalty terms into the original CV model and tackled the problem of model initialization, reducing the influence of uneven gray distribution during the evolution of level set. However, in each evolution process, the average of foreground and background grayscale is determined by the gray distribution inside and outside the curve. If the two values can be optimized, the evolution speed of level set will be greatly accelerated.

In terms of the optimization and application of CV model, Liu [10] proposed an improved method, which introduced local information to the images with uneven gray distribution after Gaussian filtering, and constructed a CV model that is more suitable for the uneven background. Zhang [11] defined an energy function of the entire image region, and combined it with bias field, level set function and real image signals, proposing a CV model that can segment the images with uneven gray distribution. Li [12] et al. used watershed algorithm to extract useful information of image regions and boundaries for pre-segmentation of image, and then used CV model for further segmentation, verifying the efficiency and accuracy of the algorithm in magnetic resonance images.

The paper introduces the Gaussian mixture model(GMM) to optimize the average of grayscale inside and outside the level set curve. For images with noise and intensity inhomogeneity, a single value is not enough to represent the gray distribution of image's foreground and background. While the distribution of intensity inside and outside the curve can be represented by a GMM. The parameters of model are updated with the evolution of the curve, and a changing foreground and background value can be obtained for every pixel point in the image respectively. It makes the CV model more adaptable to complex background. Through experiments, we find that although the algorithm in this paper increases the complexity of operations, it significantly reduces the number of iterations. And the model can segment images with intensity inhomogeneity faster and more effectively, with better noise suppression effect.

2 Related Works

For images with uneven gray distribution, the traditional CV model only guides curve evolution through the weighted mean of grayscale inside and outside the curve, which is slow in evolution and easy to get wrong segmentation results. A GMM is formed by combining multiple Gaussian functions of multiple probability distributions, it can be used to describe complex changes of grayscale [13]. During the evolution of level set curve, we use GMM's parameters to optimize energy function, and the mean gray value in the function changes according to

different points inside and outside the curve. It makes the curve more adaptive to the changes in complex background.

2.1 The CV Model

The energy function of CV model is:

$$F(C, c_1, c_2) = \lambda_1 \int_{inside(C)} |u_0(x, y) - c_1|^2 dx dy + \lambda_2 \int_{outside(C)} |u_0(x, y) - c_2|^2 dx dy + \mu L(C) + \nu A(C), \quad (1)$$

where C refers to the evolution curve of level set, $L(C)$ is the length of the contour line, $A(C)$ means the area of the region within the curve. μ and ν are the coefficients of two terms, and $u_0(x, y)$ is the gray value of a point in the image. c_1 represents the mean gray value of the points inside the curve, and c_2 represents the mean gray value of the points outside the curve. The first two terms of the whole energy function are regularization terms, which are used to regulate evolution curve. The last two are fidelity terms, which are responsible for guiding level set curve to evolve on the target contour [14].

According to level set method, we replace the evolution curve C in energy function with the level set function $\phi(x, y)$. If point (x, y) is on the curve, then $\phi(x, y) = 0$. If the point (x, y) is outside the curve $\phi(x, y) < 0$ while inside the curve $\phi(x, y) > 0$. By solving the Euler-Lagrange equation, the energy function can be minimized, and the corresponding evolution function of CV model is:

$$\frac{\partial \phi}{\partial t} = \delta(\phi) \left\{ \mu \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \nu - \lambda_1 [u_0(x, y) - c_1]^2 + \lambda_2 [u_0(x, y) - c_2]^2 \right\}. \quad (2)$$

Where the calculation method of the contour's inside and outside mean gray value is:

$$c_1 = \frac{\int u_0(x, y) H(\phi) dx dy}{\int H(\phi) dx dy}, \quad (3)$$

$$c_2 = \frac{\int u_0(x, y) [1 - H(\phi)] dx dy}{\int [1 - H(\phi)] dx dy}. \quad (4)$$

Where $H(\phi)$ and $\delta(\phi)$ are the regular forms of Heviside function and Delta function respectively, and the corresponding calculation method is:

$$H(\phi) = \frac{1}{2} \left[1 + \frac{2}{\pi} \arctan \left(\frac{\phi}{\varepsilon} \right) \right], \quad (5)$$

$$\delta(\phi) = H'(\phi) = \frac{\varepsilon}{\pi} * \frac{1}{\phi^2 + \varepsilon^2}, \quad (6)$$

in the formula ε is a constant.

It can be inferred from the evolution function that the internal and external mean gray values change with the evolution of curve. In turn, it affects the

evolution of level set curve, and the final result of evolution depends on the image gray distribution [15]. If the distribution is uneven, both internal and external mean gray values cannot reflect both foreground and background gray distribution well. It is easy to lead the level set curve to the wrong segmentation result.

2.2 Improved CV Model

In respect of the issues above, the paper introduces GMM into the process of curve evolution, optimizing the average grayscale inside and outside the curve in the energy function. The method improves the anti-interference ability of the evolution curve, and guides the level set curve to achieve correct segmentation effect. When describing an image, a gray histogram can generally be used to describe the distribution of each gray level in the image. When the gray distribution of an image is complex, combining multiple Gaussian functions with different weights to form a multiple probability distribution can represent the images gray distribution well [16].

Image Modeling For the images with intensity inhomogeneity, choosing a single gray value as the average grayscale inside and outside the curve cannot represent the gray distribution characteristics of the foreground and background regions. The evolution curve is easy to be led to wrong results. Hence, the GMM is introduced to describe the gray distribution inside and outside the curve, and the parameters of GMM are used to optimize the average grayscale in the CV model's evolution function.

Parameter Estimation The paper learns the method of parameters calculation from the Expectation Maximization(EM) algorithm [17], but the results of each iteration are calculated by the level set evolution function. The parameters update depending on the grayscale distribution inside and outside the curve after each evolution. Firstly, the evolution curve of level set is initialized, and we use the K-means algorithm to divide the regions inside and outside the curve into k parts. After segmentation, the gray distribution of each part can be approximated by a Gaussian function [18]. According to k parts of initial segmentation, the internal GMM $G_{inside}(\pi_k, \mu_k, \sigma_k)$ and the external GMM $G_{outside}(\pi_k, \mu_k, \sigma_k)$ of the curve are constructed. Parameters of the model are:

$$\mu_k = \frac{\sum_{i=1}^{N_k} u_i}{N_k}, \quad k = 1, 2, \dots, K, \quad (7)$$

$$\sigma_k^2 = \frac{\sum_{i=1}^{N_k} (u_i - \mu_k)^2}{N_k}, \quad k = 1, 2, \dots, K, \quad (8)$$

$$\pi_k = \frac{N_k}{\sum_{k=1}^K N_k}, \quad k = 1, 2, \dots, K. \quad (9)$$

Where μ_k represents the mean gray value of each submodel, σ_k is the grayscale variance of each submodel, and π_k is the proportion of each submodel in the entire GMM. N_k denotes the number of elements in each submodel, and u_i is the grayscale value of a certain point in the submodel.

Construction of GMM According to the mean and other parameters calculated above, a GMM can be built for the images inside and outside the curve:

$$P(u_i|\theta) = \sum_{k=1}^K \pi_k \frac{1}{\sqrt{2\pi}\sigma_k} \exp \left[-\frac{(u_i - \mu_k)^2}{2\sigma_k^2} \right], \quad (10)$$

where the coefficient $\pi_k \geq 0$ and $\sum_{k=1}^K \pi_k = 1$, $\theta_k = (\mu_k, \sigma_k^2)$ represents the mean and variance of each Gaussian submodel. Fig.1(a) illustrates the level set curve initialized in an image, (b) and (c) demonstrate the probability density functions of each Gaussian submodel inside and outside the curve respectively:

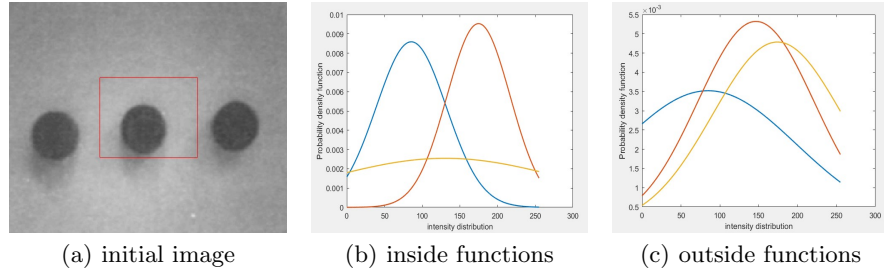


Fig. 1. Probability density function of Gaussian submodel

It should be noted that the number of submodels should be adjusted according to the complexity of the image's gray distribution. Generally, images with a single gray distribution only need three or less submodels, while images with a complex gray distribution need more than five submodels to describe its distribution well.

GMM Optimized CV Model

Model Improvement According to prior knowledge, no matter how complex the background gray distribution is, the gray distribution of target region and background region always maintain significant differences. Under this condition, we can use the GMM of curve's outside region to screen out the most likely submodel of the target region from the internal model:

$$m_{fore} = \arg \max_i \sum_{k=1}^K \pi_{outside,k} (\mu_{inside,i} - \mu_{outside,k})^2. \quad (11)$$

Where $\mu_{outside,k}$ and $\pi_{outside,k}$ denote the mean value and coefficient of the k_{th} Gaussian submodel outside the curve, and $\mu_{inside,i}$ represents the mean value of the i_{th} Gaussian submodel inside the curve. By calculating the weighted difference between the mean value of each Gaussian submodel inside the curve and that of the external GMM, the i_{th} internal region submodel with the largest difference from the background is obtained, and the average grayscale of this submodel $\mu_{m_{fore}}$ is used to represent the gray distribution of the foreground region.

When calculating the energy function, by calculating the responsiveness $\gamma_{outside,k}$ of the k_{th} external Gaussian submodel to the point with gray value of u , a dynamic external mean gray value can be obtained for different pixel points. The responsiveness $\gamma_{outside,k}$ and the external average grayscale $\mu_{outside}$ are respectively defined as:

$$\gamma_{outside,k} = \frac{\pi_{outside,k} \varphi(u|\theta_{outside,k})}{\sum_{k=1}^K \pi_{outside,k} \varphi(u|\theta_{outside,k})}, \quad i = 1, 2, \dots, N; \quad k = 1, 2, \dots, K \quad (12)$$

$$\mu_{outside} = \frac{\sum_{k=1}^K \gamma_{outside,k} \mu_{outside,k}}{\sum_{k=1}^K \gamma_{outside,k}}, \quad (13)$$

where $\theta_{outside,k}$ is the k_{th} Gaussian submodel outside the curve, and $\varphi(u|\theta_{outside,k})$ is the probability that the point belongs to the k_{th} submodel of the curve's outside area:

$$\varphi(u|\theta_{outside,k}) = \frac{1}{\sqrt{2\pi}\sigma_{outside,k}} \exp \left[-\frac{(u - \mu_{outside,k})^2}{2\sigma_{outside,k}^2} \right], \quad (14)$$

where, $\sigma_{outside,k}$ represents the variance of the k_{th} Gaussian submodel outside the curve. For the Gaussian submodel with concentrated gray distribution, the variance $\sigma_{outside,k}$ is small, so the responsiveness of the submodel to the points belonging to it may be much greater than other models. In order to ensure that each submodel has an impact on the mean gray value and avoid mistaking foreground pixels for background, the value of responsiveness can be smoothed as:

$$\gamma_{outside,k} = \frac{\gamma_{outside,k} + \frac{a}{\sqrt{\sigma_{outside,k}}}}{1 + \frac{a}{\sqrt{\sigma_{outside,k}}}}. \quad (15)$$

Where, a denotes smooth coefficient, and the smoothing term is inversely proportional to the standard deviation of submodel. When $\sigma_{outside,k}$ is small, the data distribution of submodel is concentrated, and the smoothing term at this time is larger, which reduces the response of the submodels' point to the model. The response generated by the points outside model is increased, so that the calculated external mean value is affected by the Gaussian submodel in which the point is located, and is not excessively close to the grayscale mean of the submodel.

After optimization with GMM, the energy value of each point can be calculated, and a new level set evolution result is obtained. The improved model's

energy function is:

$$\begin{aligned}
F(\phi, c_1, c_2) = & \lambda_1 \int_{inside(\phi)} |u_0(x, y) - \mu_{m_{fore}}|^2 H(\phi) dx dy + \\
& \lambda_2 \int_{outside(\phi)} |u_0(x, y) - \mu_{outside}|^2 [1 - H(\phi)] dx dy + \\
& \mu \int \delta(\phi) |\nabla \phi| dx dy + \nu \int H(\phi) dx dy.
\end{aligned} \tag{16}$$

The level set evolution function corresponding to each point is:

$$\begin{aligned}
\frac{\partial \phi}{\partial t} = & \delta(\phi) \left\{ \mu \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \nu - \lambda_1 [u_0(x, y) - \mu_{m_{fore}}]^2 + \right. \\
& \left. \lambda_2 [u_0(x, y) - \mu_{outside}]^2 \right\},
\end{aligned} \tag{17}$$

Extract the fidelity term from the equation which guides the curve evolution, and analyze the gray value of any pixel point $u_0(x, y)$ in the image: $f(u) = -\lambda_1 [u_0(x, y) - \mu_{m_{fore}}]^2 + \lambda_2 [u_0(x, y) - \mu_{outside}]^2$. Without considering the influence of coefficient, set λ_1, λ_2 as 1. If a certain point belongs to foreground, the grayscale of that point will be significantly different from the background gray value. At this time, u is closer to the optimal submodel's mean gray value m_{fore} within the curve (with the largest difference from the gray value outside the curve). And when m_{fore} completely represents the foreground region's gray distribution, the difference between u and $\mu_{m_{fore}}$ is close to 0. At this time $|u - \mu_{m_{fore}}| < |u - \mu_{outside}|$, the result $f(u)$ is positive and the point evolves into the interior of the curve. If a point belongs to background, its gray value has high responsiveness to the Gaussian submodel which contains it. At this time, the submodel has a greater influence on the calculation result of the external mean gray value, and the value $|u - \mu_{outside}|$ of this point is smaller, which satisfies $|u - \mu_{m_{fore}}| > |u - \mu_{outside}|$. At this time, the result of $f(u)$ is negative and the point evolves to the outside of the curve.

Algorithm Procedure Through above improvement, the CV model combined with GMM can produce a good segmentation effect on the images with complex gray distribution and noise. The specific algorithm steps are as follows:

Algorithm 1. Curve Evolution Based on The Proposed Model

Input: Image I

Output: Level set curve $\phi(x, y)$

Initialize level set curve, divide the region inside and outside the curve into k blocks by K-means algorithm;

Build interior and exterior GMMs and calculate (π_k, μ_k, σ_k) ;

Calculate $\mu_{m_{fore}}$ and $\mu_{outside}$ for the points in the image;

Update the curve according to the calculation result of evolution function $\partial \phi / \partial t$;

Continue iteration until the level set curve no longer converge.

3 Experiments and Analysis

The experiments select images with noise and uneven gray distribution to perform segmentation test. By comparing the segmentation effects of traditional CV model, Local Binary Fitting(LBF) model [19], Chan Vese-Geometric Active Contour(CV-GAC) model [20], Variational Level Set(VLS) model [21], Distance Regularized LS Evolution Method(DRLSE) model [22], Saliency Driven Region-Edge-based Top Down Level Set Evolution(SDREL) model [23] and CV model in this paper, we verify the superiority of our algorithm. The experimental environment is Intel(R)Core(TM)i7-8700 CPU @ 3.20GHz clock /16.0GB memory/MATLAB R2014b. In the experiment, the model parameters are listed as follows: $\lambda_1 = \lambda_2 = 1, \nu = 0.03 \times 255^2, \mu = 1$.

In the experiments, we first discuss final evolution results with different submodel numbers. We mentioned above that according to different initial positions of level set curve and different gray distributions, the number of Gaussian submodels inside and outside should be set differently as well. This operation helps us segment target area faster and more accurately. As experimental results shown in Fig.2, we set different number of Gaussian submodels for images with different gray distributions and get evolution results after iterating 10 times. The gray distribution of the first image is relatively simple. When we set $k_{inside} = 3$ and $k_{outside} = 2$, the level set curve evolves to target area more accurately. While gray distribution of the second image is more complex, thus more submodels are needed to characterize gray distribution for a better segmentation effect. However, an excess of submodels will increase time-consuming of a single iteration, thus extending convergence time. Although results of Fig.2(g) and Fig.2(h) are similar, the latter's time-consuming is 1.23 seconds longer.

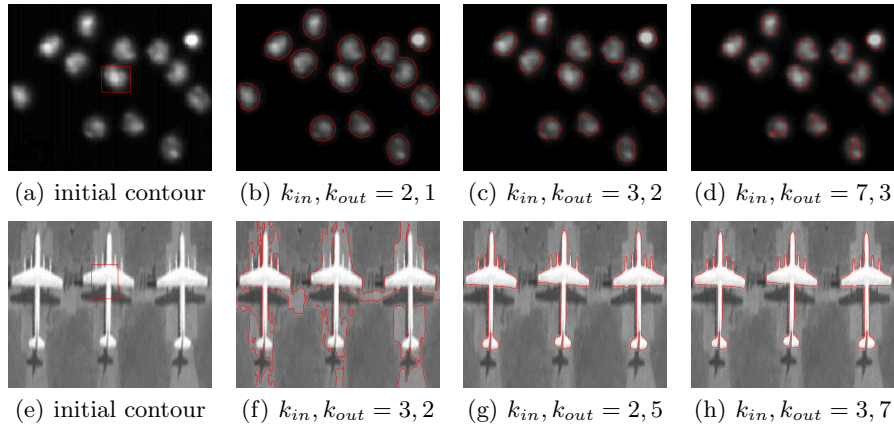


Fig. 2. Results of 10 iterations with different numbers of submodels

In Fig.3, we compare different models' anti-noise performance by images with Gaussian noise (zero mean and standard deviation $\sigma = 0.05$). It can be seen from the results that CV model, CV-GAC model, and LBF model do not have a good segmentation effect on such images. While SDREL model, VLS model, and our model have a better resolving ability for target region and noise, with a better evolution result.

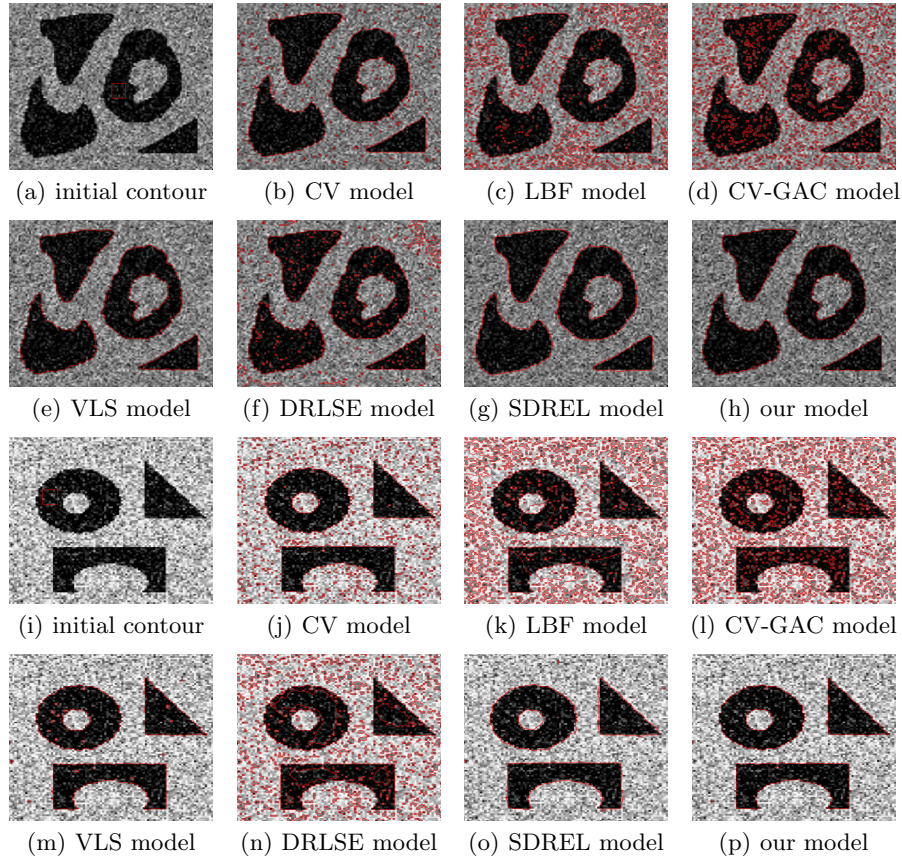


Fig. 3. Segmentation results of nosiy images

As shown in Fig.4, the gray distribution of images is uneven. The level set curves of some models tend to misjudge noise as small targets in the process of evolution, or mistake some foreground regions as background, thus form a segmentation gap. After several iterations, the evolution of level set curve stopped gradually. It can be seen that our model has superiority in segmenting images with uneven gray distribution.

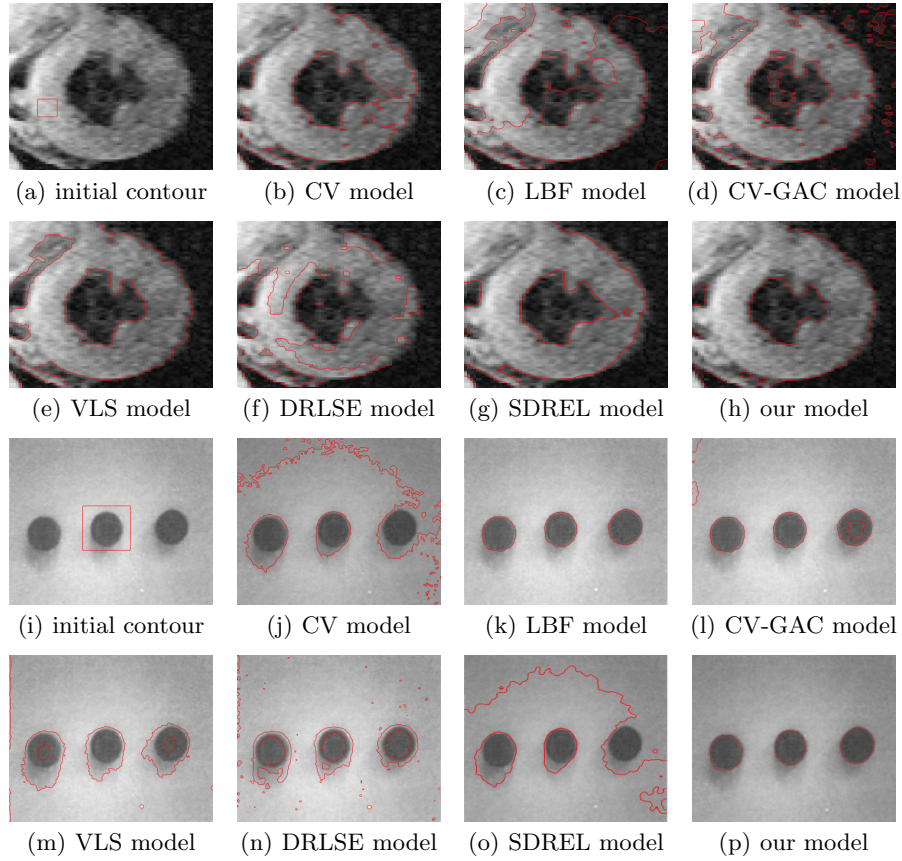


Fig. 4. Segmentation results of images with intensity inhomogeneity

For some images with complex gray distribution, although the curves can evolve to target region, the number of iterations and the time consumed of different models have a great difference. It can be seen from the Fig.5 that although the time our model consuming in a single iteration increases slightly, the total number of iterations is smaller and the total time required for convergence is less than the other models.

4 conclusion

In this paper, a level set method combined with GMM for image segmentation is proposed. The GMM is used to describe gray distribution characteristics inside and outside the curve. The improved model optimizes the mean gray value inside and outside the curve in the energy function, which makes the overall segmentation effect of CV model greatly improved.

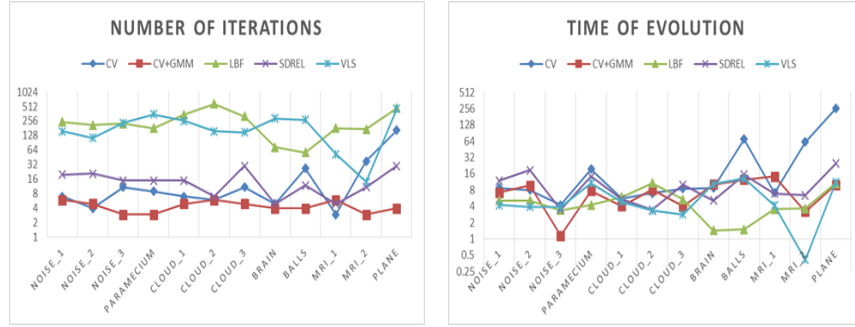


Fig. 5. Iteration number and evolution time of different models

Experimental results show that the improved CV model can not only segment target regions from the images with intensity inhomogeneity, but also reduce iteration number and evolution time, improving the anti-noise capability of the model. By analysis, we prove that the proposed model has superiority over general CV model. However, it depends too much on the curve's initial position, and requires images with contrasting foreground and background. To overcome these shortcomings is the next research direction.

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References

1. Devraj, M., Amitava, C., Madhubanti, M.: Robust medical image segmentation using partial swarm optimization aided level set based global fitting energy active contour approach. *Engineering Applications of Artificial Intelligence* **35**, 199–214 (2014)
2. Ren, J.J., He, M.Y.: Level Set Method Of Image Segmentation Based On Improved CV Model Of 3-D Histogram. *Infrared and Millimeter Waves* **27**(1), 72–76 (2008)
3. Chen B., Zhang M., Chen W., Pan B., Li L.C., Wei X. (2018) A Novel Adaptive Segmentation Method Based on Legendre Polynomials Approximation. In: Lai JH. et al. (eds) *Pattern Recognition and Computer Vision. PRCV 2018. Lecture Notes in Computer Science*, vol 11256. Springer, Cham
4. Fu, J.M., Xu, M.Y.: An Improved C-V Level Set Image Segmentation Model Based on Rapid Narrow-Band Method. *Metallurgical and Mining Industry* **7**, 339–344 (2015)
5. Yang, X., Gao, X.B., Tao, D.C.: An Efficient MRF Embedded Level Set Method for Image Segmentation. *IEEE Transactions on Image Processing* **24**(1), 9–21 (2015)
6. Song, Y., Wu, Y.Q., Bi, S.B.: Satellite remote sensing cloud image segmentation using edge corrected CV model. *Acta Optica Sinica* **34**(9), (2014)

7. Li, C.M., Huang, R.: A Level Set Method for Image Segmentation in the Presence of Intensity Inhomogeneities with Application to MRI. *IEEE Transactions on Image Processing* **20**(7), 2007–2016 (2011)
8. Juneja, P., Kashyap, R.: Optimal Approach For CT Image Segmentation Using Improved Energy Based Method. *International Journal of Control Theory and Applications* **9**, 599–608 (2016)
9. Li, C.M., Xu, C.Y., Gui, C.F.: Distance regularized level set evolution and its application to image segmentation. *IEEE Trans on Image Processing* **19**(12), 3243–3254 (2010)
10. Liu, S.G., Peng, Y.L.: A local region-based Chan-Vese model for image segmentation. *Pattern Recognition* **45**: 2769–2779 (2012)
11. Zhang K.H., Zhang, L.: A Level Set Approach to Image Segmentation With Intensity Inhomogeneity. *IEEE Transactions on Cybernetics* **46**(2) : 546–557 (2016)
12. Li, N., Liu, M.M., Li, Y.F.: Image Segmentation Algorithm Using Watershed Transform and Level Set Method. In: *IEEE International Conference on Acoustics, Speech and Signal Processing*, pp. 613–616. ICASSP '07, Honolulu (2007)
13. Wang, H., Gao, J., Yu, L., Hu, Y., Wang, Z.: Combined improved Frequency-Tuned with GMM algorithm for moving target detection. In: *2017 IEEE International Conference on Mechatronics and Automation (ICMA)*, pp. 1848–1852. IEEE, Takamatsu (2017)
14. He, J.F., Guan, G.H., Yi, S.L.: Image Segmentation of CV Model Based on Curve Area Constraint. *International Conference on Mechatronics and Intelligent Robotics*, pp. 502–509. (2018)
15. Yang, M.Y., Ding, H., Zhao, B.: Chan-Vese model image segmentation with neighborhood information. *Journal of Computer-Aided Design & Computer Graphics* **23**(3): 413–418 (2011)
16. Ma, J.Y., Jiang, J.J., Liu, C.Y., Li, Y.S.: Feature guided Gaussian mixture model with semi-supervised EM and local geometric constraint for retinal image registration. *Information Sciences* **417**: 128–142 (2017)
17. Hayit, G., Amit, R., Jacob, G.: Constrained Gaussian Mixture Model Framework for Automatic Segmentation of MR Brain Images. *IEEE Transactions on Medical Imaging* **25**(9): 1233–1245 (2006)
18. Rother, C., Kolmogorov, V., Blake, A.: Grabcut: interactive foreground extraction using iterated graph cuts. *Acm Transactions on Graphics* **23**(3): 309–314 (2004)
19. Li, C., Kao, C., Gore, J.C., Ding, Z.: Implicit Active Contours Driven by Local Binary Fitting Energy. In: *2007 IEEE Conference on Computer Vision and Pattern Recognition*, pp. 1–7. IEEE, Minneapolis (2007)
20. Lin, Y., Tong, L., Level Set Image Segmentation of CV-GAC Model. In: *2018 13th International Conference on Computer Science & Education (ICCSE)*, pp. 1–5. IEEE, Colombo (2018)
21. Wang, L.L., Hu, L., Wang, X.Y.: A bias correction variational level set image segmentation model combining structure extraction. In: *2017 2nd International Conference on Image, Vision and Computing (ICIVC)*, pp. 327–331. IEEE, Chengdu (2017)
22. Wang, X.C., Shan, J.X., Niu, Y.M., Tan, L.W., Zhang, S.X.: Enhanced distance regularization for re-initialization free level set evolution with application to image segmentation. *Neurocomputing* **141**: 223–235(2014)
23. Zhi, X.H., Shen, H.B.: Saliency driven region-edge-based top down level set evolution reveals the asynchronous focus in image segmentation. *Pattern Recognition* **80**: 241–255(2018)