QIP Technical Addendum

December 24, 2019

We'll omit proofs of our claims in this technical addendum and only present results. In this addendum, we'll show conjectured upper and lower bounds for T-state conversions where we'll consider T-states the "resource." Similar improvements can be seen for other important states such as $|C^{n-1}Z\rangle := C^{n-1}Z|+^{\otimes n}\rangle$.

Definition 0.1. (Normalized Pauli Spectrum) For a n-qubit pure state $|\psi\rangle$,

$$\text{NormSpec}|\psi\rangle = \left\{ \frac{|\langle\psi|P|\psi\rangle|^2}{\langle\psi|\psi\rangle^2 \cdot 2^n}, \quad \forall\, P \in \{I, X, Y, Z\}^{\otimes n} \right\}$$

Note that this is a valid probability distribution of size 4^n.

Definition 0.2. (Pauli Entropy Monotone) Define the Pauli Entoropy Monontontone as:

$$\omega_\alpha(|\psi\rangle) = H_\alpha(\text{NormSpec}(|\psi\rangle)) - n$$

where H_α is the Rényi entropy of order α.

We begin by considering catalytic resource conversions, i.e. those that are allowed to bring in a resource state to aid in state-to-state conversion but must return it at the end of the computation.

Recall that $|T\rangle := T|+\rangle$ and

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

For the following conversion, we are lower bounding the quantity r asymptotically in n. We present the values of r realizing our bounds, previously known bounds, and the best known algorithm.

$$rn|T\rangle \Rightarrow n|\psi\rangle$$

$\|\psi\rangle$	ω_α	Prev. Lower Bound	Best known algo.
$\|CS\rangle$	3	2.9681*	3
$\|CCS\rangle$	4.843	4.53328*	7
$\|C^3S\rangle$	6.689	4	11
$\|CCZ\rangle$	3.724	3.63356*	4
$\|C^3Z\rangle$	5	5.12122*	6
$\|C^4Z\rangle$	6.765	5	12
$\|W_3\rangle$	3.723	3.63356	4
$\|W_4\rangle$	5	4.99907	5

$$n|\psi\rangle \Rightarrow r'n|T\rangle$$

$\|\psi\rangle$	ω_α (r')	Prev. Upper Bound (r')	Best known algo. (r')	α
$\|CS\rangle$	2	2	1	∞
$\|CCS\rangle$	2.67	3	0.5	3.43
$\|C^3S\rangle$	1.546	3.82743	0.25	2.7
$\|CCZ\rangle$	3	3	2	∞
$\|C^3Z\rangle$	3.118	4	1	2.94
$\|C^4Z\rangle$	1.619	3.8233*	0.5	2.74
$\|W_3\rangle$	3	3	3	∞
$\|W_4\rangle$	4	4	4	∞