Space-saving mean value algorithms for quantum circuits

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Abstract

The task of estimating the expectation value of a n-qubit observable $O_1 \otimes \ldots \otimes O_n$ on a low depth quantum circuit is a vital step in many near-term quantum algorithms. We show how this task can be simulated on a reduced quantum circuit in less time than classical algorithms and with fewer qubits than quantum algorithms under certain depth restrictions.

Introduction

- **Quantum circuits**
  - A quantum circuit of depth d, $U = U_d \ldots U_1$, has d sequential layers of single-depth circuits. Each U_i consists of 1 and 2 qubits gates acting on disjoint qubits.
- **2D and 3D circuits**
  - A 2D (3D) quantum circuit can perform 2-qubit gates only between nearest-neighbors on a 2D (3D) lattice.
- **Mean value**
  - Given n-qubit quantum circuit U and single-qubit Hermitian observables $O_1, \ldots, O_n$, the mean value of $O = O_1 \otimes \ldots \otimes O_n$ with respect to $U\left|0^n\right\rangle$ is

$$\mu = \left\langle 0^n\right| U^\dagger O U \left|0^n\right\rangle \tag{1}$$

Additive mean value problem

Given U, $O = O_1 \otimes \ldots \otimes O_n$, and additive error bound δ, output $\tilde{\mu}$ such that $|\mu - \tilde{\mu}| \leq \delta$ w.h.p.

If d is polynomial in n, then this task is BQP-complete. What if d is sub-polynomial?

Question

How efficiently can we additively approximate μ when U is a 2D/3D quantum circuit of low depth?

- We improve on the following in certain depth and circuit geometry contraints
  - **qubit count** against quantum algorithms
  - **estimation runtime/depth** against classical algorithms

Prior work

- Classical algorithms for 2D circuits in $O(\delta^{-2} n 2^{O(d^2)})$ time
- Classical algorithms for 3D circuits of constant depth in $\delta^{-2} 2^{O(n^{1/3})}$ time
- Standard quantum estimation algorithm requires $O(\delta^{-2} d)$ depth

Holographic quantum simulation

- At timestep t, 2D simulator has two subsystems
  - **System** qubits simulate the qubits of the t'th row
  - **Bath** qubits mediate the necessary correlations between system qubits of different time steps
  - Use bath qubits to simulate neighboring rows

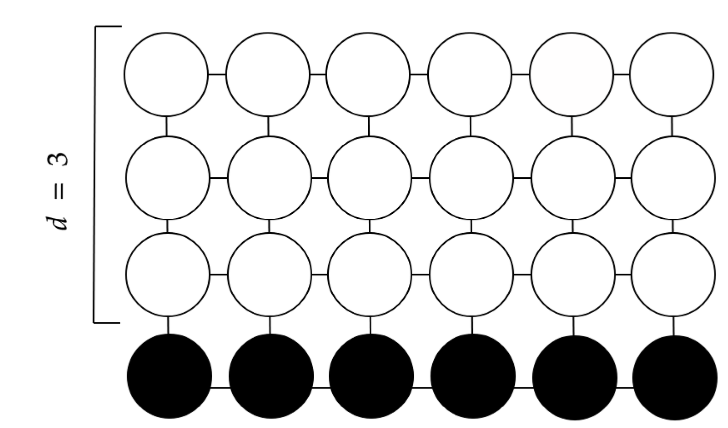

Figure 1: System qubits (black) and bath qubits (white). U has depth $d = 3$, so bath mediates distance d correlations

- Simulator "glides" over rows from bottom to top

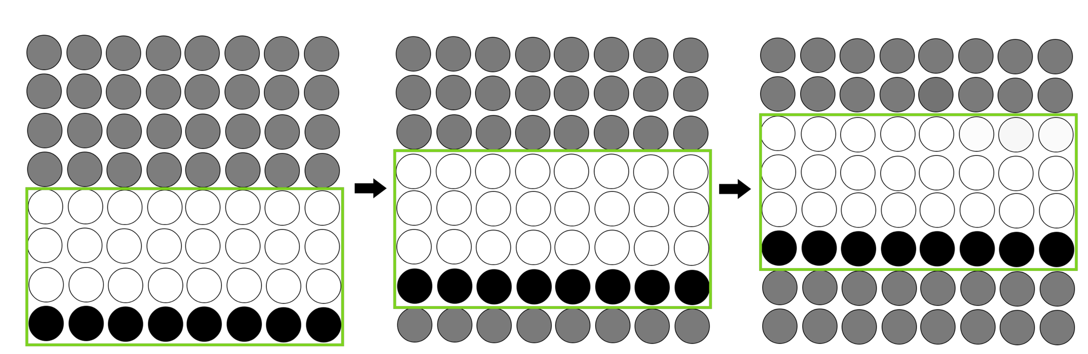

Figure 2: Simulator (green) with system (black) and bath (white) qubits "glides" over 2D grid qubits (gray)

Lightcone finding

- Measurements of row are dependent only on row's **lightcone**: gates within radius d around row
- Lightcone can be computed efficiently before the quantum algorithm

2D mean value approximation

Theorem 1

Given U on $n^{1/2} \times n^{1/2}$ 2D grid, quantum algorithm with $(d+1)\sqrt{n}$ qubits, $O(\delta^{-2} d \sqrt{n})$ depth, and $O(\delta^{-2} n d)$ additional gates that δ-additively approximates μ.

- $d \leq n^{1/2}$: simulator uses **fewer qubits** than the standard quantum estimation algorithm
- $d = \Omega(\log n)$: simulator achieves **faster runtime** than classical 2D algorithm
- Initialization
  - At simulation step $t = 1$, initialize system and bath qubits to basis state $|0\rangle$ in a $(d+1) \times n^{1/2}$ grid
- Simulate subcircuit in lightcone
  - Use the lightcone finding procedure to collect necessary gates from U in the lightcone of the system qubits' observables
  - Perform the collected gates and measure system qubits
- Time step transition
  - For subsequent simulation steps $t \in \{2, \ldots, n^{1/2}\}$, reset system qubits to basis state, and swap to the top of the simulator

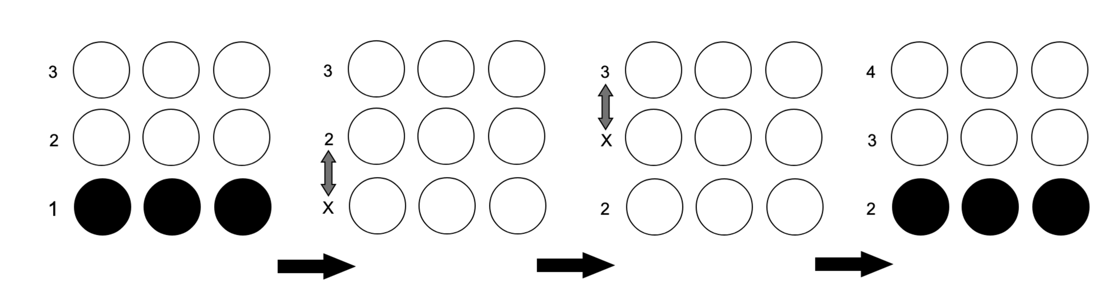

Figure 3: Row 1 is measured, and reset to basis state as "X". Row "X" is sequentially swapped to the top of the simulator until it becomes row 4. In the end, row 2 becomes the system.

- Estimation
  - Multiply measurement outcomes to get value in $\{-1, 1\}$
  - Average outcomes over $\frac{3}{4}\delta^{-2}$ (serial) repetitions to compute $\tilde{\mu}$ with probability exceeding $\frac{2}{3}$

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2D simulation without extra gates

Theorem 2

With U on 2D grid, quantum algorithm with $(d+1)\sqrt{n}$ qubits, $O(\delta^{-2} d \sqrt{n})$ depth, w/o additional gates that δ-additively approximates μ.

- Avoid the additional swap gates during time step transition by folding the simulator in half

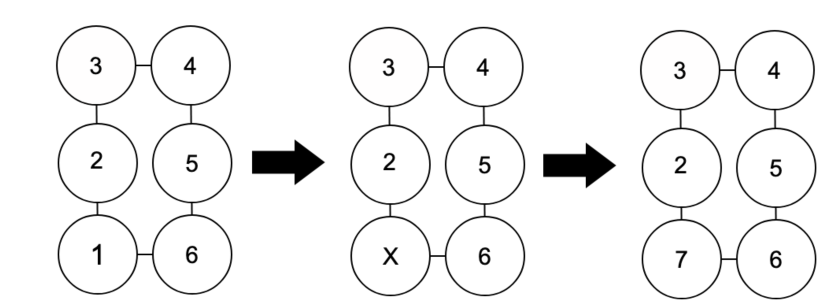

Figure 4: Side view of simulator. Row 6 is folded over as a nearest neighbor to row 1. To transition time steps, the system qubits are reset to "X" and they are replaced by the new row of the next time step.

3D mean value approximation

Theorem 3

With U on 3D grid, quantum algorithm with $(d+1)n^{2/3}$ qubits, $O(\delta^{-2} d n^{1/3})$ depth, that δ-additively approximates μ.

- Simulator is a $(d+1) \times n^{1/3} \times n^{1/3}$ 3D grid
  - **System** qubits are $1 \times n^{1/3} \times n^{1/3}$ region of simulator
  - **Bath** qubits are $d \times n^{1/3} \times n^{1/3}$ region of simulator
- Perform the same simulation task as in 2D
- Achieves lower qubit count when $d \leq n^{1/3}$

References

[1] Isaac H. Kim. Holographic quantum simulation, 2017; arXiv:1702.02093.

[2] Sergey Bravyi, David Gosset and Ramis Movassagh. Classical algorithms for quantum mean values, 2019; arXiv:1909.11485.