Math553

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1 Abstract

2 Introduction

2.1 Konigsberg seven bridge problem

2.2 The problem

I want to travel to Cancún this winter vacation, but it is really an expensive trip, so I hope to solve money. But how can I save money in a trip? What I know is that, I want to go Cancún, and I am living in Pullman now. So my start point is Pullman, WA. And the end point is Cancún. Also, if I need to spend so much money to fly to Cancún, why not travel to some other cities near Cancún? So I also want to go to the Guanajuanto, Mérida and Mexico City, these are also really beautiful or interesting viewpoints. And finally, I still need to go back to Pullman for my next semester's study. It really looks like a weighted undirected graph, so I think maybe I can save money with the knowledge learn in Graph Theory this semester.

3 What is Travelling Salesman Problem

Travelling Salesman Problem, also called TSP. It is trying to solve the problem that there is a salesman that want to travel around the cities and sale his freights. Unfortunately, he need to spend money or time on the way between cities. So, how to use the shortest path to travel around these cities is the main problem. The TSP is like the minimum connector problem but the salesman want to go back home at the end.[**?**]

According to the situation that the cost travel from A to B is not equal to travel from B to A, what we talk about is the asymmetric travelling salesman problem (aTSP) here.

aTSP: Let $V = v_1, \cdots\cdots, v_n$ be a set of cities, $A = (r, s) : r, s$ be the edge set, and d_{rssr} be the cost measure associated with edge $(r, s) \in A$.[**?**]

The aTSP is the problem that we need to find a minimal length closed tour that visits each city once. In my case, cities $v_i \in V$ are given and d_{rs} is the cost I will build with the fee of flight and time spend, then we will have a typical aTSP.

4   How to transform the real-world problem

Now, since I want to solve my save money problem, I need to collect the data at first. I need to know hwo much should I pay for the flight around these cities, or where I want to go don't havev ariport and I need to try some other ways. Then, I need to transform these data to a mathmatical model, which will let me can use mathmatical ways to solve them. So, I started to check the data of fees. However, it's really hard to confirm the fees spend one the way. For example, if I just search the way from Pullman to Cancún, I can get the flight from Pullman to Seattle, and then from Seattle to Cancún. But I know I can drive to Seattle, and the flight from Seattle to Cancún may be cheaper than the first choice. Also, what if I try to fly from Lewiston to the Cancún? It may have some other situation. So I need to mkae some rules for the transform form real-world problem to mathmatical models.
The formula I build at first for the way cost is:

$$C = f + T_1(t) + T_2(s) + e$$

In this formula, C means the total cost of the way. t means the time used on the flight. T_1 is a transformation function that change the time for flight to a score, this transformation function can be really personal like if I just want to let it be a easy problem and don't care about the difference about long time flight and short time flight, then I can set it like a first-order linear function. But if I can only bear short time flight and cannot bear to sit for too long, it can be a quadratic function or even exponential function. And I want to set it like s is the number of transit station in such flight. T_2 is a transformation function that change the number of transit station to a score, it is also really personal. And I want to set it like $50s$ here, since I think the time I spend on one transit station is equal to 50 dollars.

I will all use the air tickets data on December 16, 2019. If people want to use it on the real problem, they just need to calculate the day they will start travel and compute the day they travl between these cities and then check the tickets of flight.

5 How to solve mathematical model

6 How to extend to this model

Since I don't have enough time for build this model and test it, it looks really simple. But I think this can be used for people who want to travel to some place, and want to save time and money at the same time. I will adjustment the parameter of the formula more carefully and think about more factors like the security issue, the time limited for the trip and the limitaion of total spend, and finally expand it to a more complete combinatorial optimization problem. Also, I can write the code for calculate it automatically. And use some ways like Python Spider(a kind of program that can catch the data from the website automatically), to catch the flight information and clean them as the data used in my formula. I hope it can be helpful for people who like traveling.