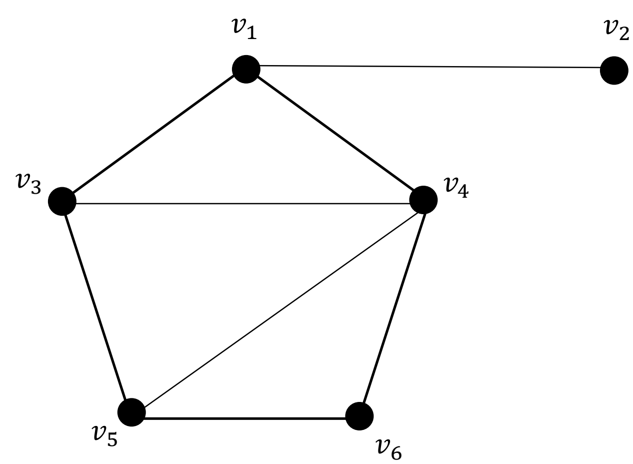
Graph Theory Fall 2019

Assignment 4

Due at 5:00 pm on Wednesday, October 16

1. Suppose $G$ is a forest of 7 trees, each with 5 vertices. Find the dimensions of $\mathcal{C}(G)$ and $\mathcal{B}(G)$.

2. Let $G$ be the graph depicted below. You should decide on an ordering of the edges early in the process of answering the parts to this question.

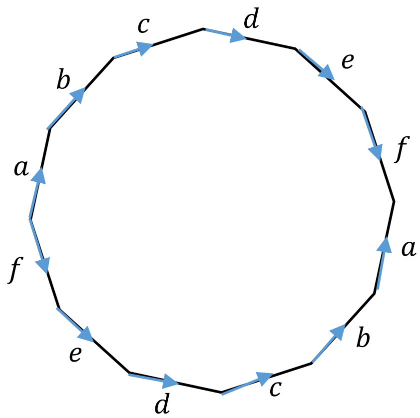



A) Find the dimension of and a basis for $\mathcal{B}(G)$, the cut space (or bond space).
B) Find the dimension of and a basis for $\mathcal{C}(G)$, the cycle space
C) Determine if there is a nonzero vector in the cut space that is also in the cycle space. If so, provide an example of such a vector.

3. Draw $K_7$ on a torus (or the schematic representation where opposite sides of a rectangle are identified.)

4. Use an edge counting argument to show $K_8$ cannot be drawn on a torus. Recall that $K_8$ has 8 vertices, $\binom{8}{2} = 28$ edges, and for a torus, $n - m + f = 0$. The faces would all have to have at least three edges.

5. Consider the following schematic diagram for a surface (it has 12 sides and opposite sides are identified and oriented in parallel directions):




   A. What is its Euler characteristic $\chi$? If there is a vertex at every corner, how many distinct vertices are there, after accounting for forced identifications? There are six edges and one face, so this should be enough information to compute $n - m + f = 2 - 2\chi$. Remember that $\chi$ counts the "holes" in the $h$-holed torus.

   B. What happens if we re-orient one copy of side $a$ leaving all other sides in their original orientation? What effect does this have on the number of distinct vertices, if there is a vertex at every corner? Notice that there would still be six edges and one face. Would $n - m + f$ change?

6. Determine, with justification, whether the Petersen graph (drawn below) is a planar graph. (Observation: Pete has no triangles or 4-cycles.)