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RESEARCH ARTICLE

Box Allocation Optimisation in Meal Kit Delivery

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Abstract

This study investigates the Box Allocation Problem (BAP), an underexplored challenge in meal kit delivery operations. BAP seeks to minimise recipe variations in daily allocation decisions while satisfying capacity and eligibility constraints. To address this research gap, we formulate a mathematical model and systematically evaluate both exact and heuristic optimisation methods. The results demonstrate that CBC solver achieves optimal solutions under dynamic conditions, significantly outperforming heuristic approaches such as Tabu Search and Iterative Targeted Pairwise Swap. This research enhances operational efficiency and reduces food waste by enabling quicker, more precise allocation decisions, offering practical optimisation strategies for the rapidly growing meal kit delivery market.

KEYWORDS
Meal kit delivery, Box allocation problem, CBC solver, Heuristics

1 | INTRODUCTION

The meal kit delivery market in the UK has undergone substantial growth in recent years, propelled by escalating consumer demand for convenient, nutritious, and customised meal solutions. With projected revenues reaching $1.4 billion in 2024, this sector has evolved into an intensely competitive landscape that attracts both established enterprises and emerging ventures[24]. Success in this industry hinges critically on sophisticated supply chain management that ensures timely delivery of fresh ingredients. The market's dynamic character introduces multifaceted challenges encompassing inventory optimisation, transportation logistics, and perishable goods management, factors that position supply chain operations as decisive determinants of business viability[13,3]. Furthermore, companies must demonstrate agility in response to demand fluctuations and potential disruptions, necessitating adaptive and responsive supply chain strategies to maintain competitive advantage[23].

A significant operational challenge in this context is the box allocation problem (BAP), which involves strategically assigning recipe boxes (illustrated in Figure 1) to production facilities to minimise interday recipe variations, quantified by the weighted mean average percentage error (WMAPE), while simultaneously satisfying capacity limitations and eligibility requirements[17]. The minimisation of daily factory-level WMAPE yields two crucial operational benefits: primarily, it substantially reduces waste through consistent recipe allocations across facilities, enabling precise ingredient forecasting that mitigates over-procurement and diminishes food spoilage, thereby enhancing environmental sustainability. In addition, it optimises product availability for consumers by maintaining uniform ingredient inventory levels across production sites, thereby minimising stockout risks and facilitating reliable order fulfillment that ultimately enhances customer satisfaction. Despite its pivotal role in operational excellence, BAP remains remarkably underexamined in both academic research and industry-specific literature.

This study addresses the identified research gap by building a comprehensive mathematical formulation of BAP that incorporates the distinctive constraints of factory capacity

FIGURE 1 Recipe box[26]

Abbreviations: B&B, Branch and Bound; BAP, Box Allocation Problem; BPP, Bin Packing Problem; CBC, COIN-OR Branch and Cut; CVRP, Capacitated Vehicle Routing Problem; ITPS, Iterative Targeted Pairwise Swap; TS, Tabu Search; WMAPE, Weighted Mean Absolute Percentage Error.

and recipe eligibility. The research methodically evaluates the performance of exact method, specifically the CBC solver, against heuristic approaches, including the Iterative Targeted Pairwise Swap (ITPS) and Tabu Search (TS), which have demonstrated effectiveness in analogous combinatorial optimisation challenges such as the Bin Packing Problem (BPP) and Vehicle Routing Problem (VRP)[14,15,8]. These heuristic methods, particularly TS, are recognised for their efficacy in exploring expansive solution spaces and transcending local optima[19,16], while B&B guarantees optimality through systematic enumeration and strategic pruning of infeasible solution pathways[18,4]. Through rigorous computational experiments across varied order volumes and operational scenarios, this investigation provides valuable insights into the scalability and adaptability of each approach.

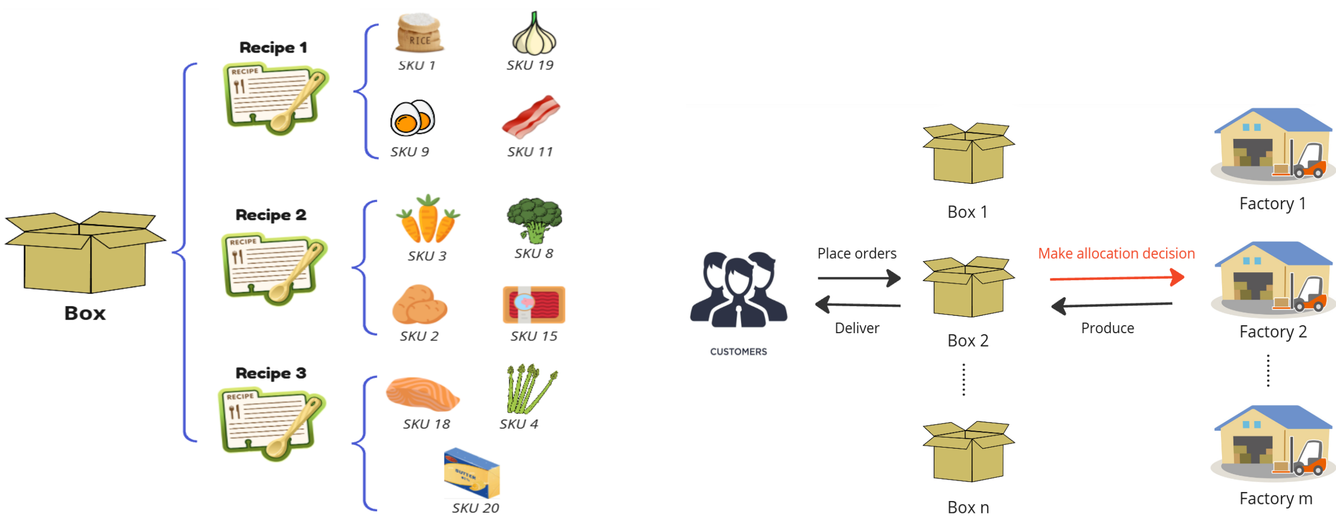
Our findings demonstrate the superior performance of CBC solver in achieving optimal solutions, even under challenging dynamic conditions characterised by fluctuating factory capacities and variable order patterns. By addressing both the theoretical foundations and practical applications of BAP, this research contributes significantly to the supply chain optimisation literature while providing actionable intelligence to meal kit delivery enterprises. The study effectively bridges the knowledge gap, providing scalable solutions that enhance operational efficiency and environmental sustainability in this growing sector.

The subsequent sections of this paper are organised as follows: Section 2 elaborates on BAP's distinctive characteristics and presents its mathematical formulation, contextualizing it through comparison with two related optimisation problems. Section 3 explains the optimisation methods used, including exact and heuristic approaches. Section 4 details the results of numerical experiments, comparing the performance of all methods. Finally, Section 5 examines the implications of our findings, acknowledges research limitations, and proposes promising directions for future investigation.

2 | PROBLEM STATEMENT

2.1 | Problem background

BAP in meal kit delivery involves assigning recipe boxes (Figure 2a) to multiple factories (Figure 2b). The goal is to minimise daily variations in recipe allocations. For instance, if Factory 1 (F1) produced 100 units of Recipe 1 yesterday, the optimal solution would keep allocating around 100 units today.

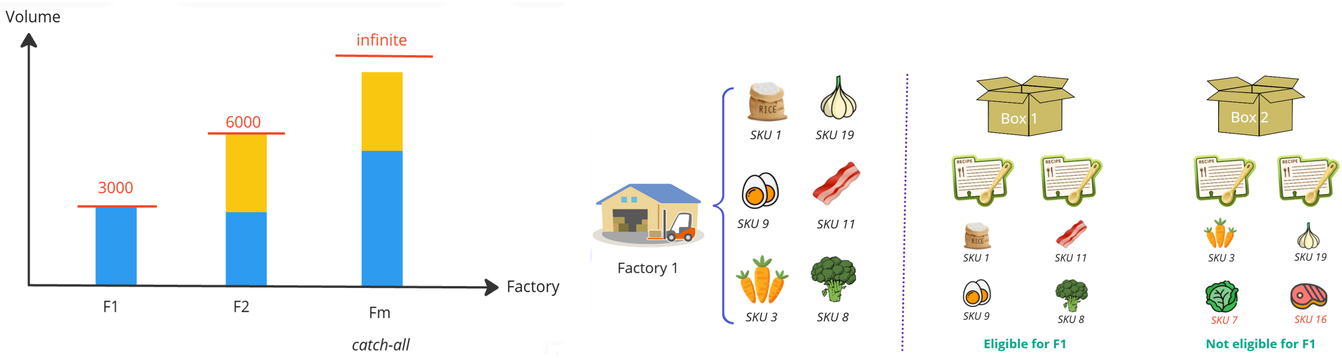


(a) Box structure [17] (b) Allocation process [17]

FIGURE 2 Recipe box allocation

The problem includes two key constraints. First, each factory (except the final simulated factory *m*) has a fixed daily production capacity, limiting the number of boxes it can process. To optimise resource use, solutions must fully meet this capacity (Figure 3a). Second, not all factories can produce every recipe, so allocation must follow the eligibility constraint in Figure 3b, ensuring recipes are assigned only to capable facilities.

A key feature of BAP is its temporal dimension, requiring daily allocation of both actual and simulated orders (Figure 4). This study uses a 15-day planning horizon from lead day 18 (LD18) to lead day 3 (LD3). The process begins with total demand forecasting, followed by continuous monitoring of actual orders. When real orders fall short of projections, simulated orders are generated to represent expected demand. Initially, both real and simulated orders are distributed across production facilities



(a) Capacity constraint [17] (b) Eligibility constraint [17]

FIGURE 3 Main constraints

(*soft allocation*). As the planning horizon progresses, actual orders replace simulated ones. By LD3, the allocation consists only of real orders, transitioning to *hard allocation*, which remains fixed. The company then has three days to produce and deliver the boxes. This approach ensures stable daily allocations while adapting to fluctuating demand, enabling a smooth shift from forecasts to actual order fulfillment.

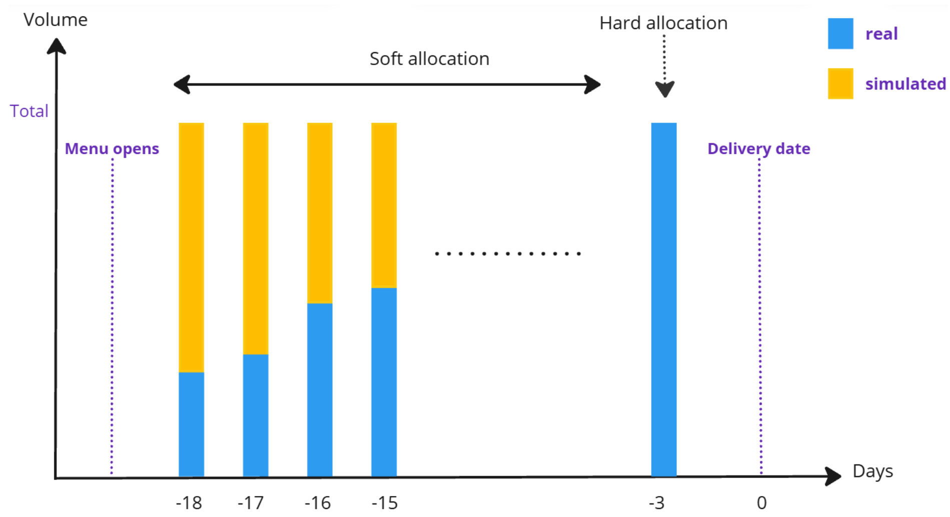


FIGURE 4 Temporal aspect of BAP [17]

2.2 | Mathematical formulation

Weighted Mean Absolute Percentage Error (WMAPE) is a key forecast accuracy metric used in supply chain management and production planning. It measures the sum of absolute differences between actual and forecasted values, divided by the sum of actual values [7], with lower values indicating higher accuracy. This research focuses on minimising WMAPE site in Table 1 to provide granular insights and assess production consistency at the factory level.

TABLE 1 Comparison between WMAPE site and WMAPE global

Aspect	WMAPE site	WMAPE global
Scope	Considers each factory separately	Aggregates data across all factories
Sensitivity	Sensitive to changes in allocation between factories	Not sensitive to changes in allocation between factories
Usefulness	Analyzes stability of production plans at each factory	Provides a broader view of recipe distribution changes

The mathematical formulation of BAP employs the following notation:

- $T = \{-18, -17, \ldots, -3\}$: Set of allocation days, indexed by t.
- $R = \{1, 2, \ldots, n\}$: Set of recipes, indexed by i.
- $F = \{1, 2, \ldots, m\}$: Set of factories, indexed by j.
- $O = \{1, 2, \ldots, k\}$: Set of orders, indexed by o.
- $a_{i,t-1}$: Volume of recipe i at day $t-1$.
- $a_{i,t}$: Volume of recipe i at day t.
- $a_{i,j,t}$: Volume of recipe i allocated to factory j at day t.
- $a_{i,j,t-1}$: Volume of recipe i allocated to factory j at day $t-1$.
- $x_{o,j,t}$: Binary decision variable indicating the allocation of order o to factory j on day t.
- $C_{j,t}$: Capacity of factory j on day t.
- $E_{i,j,t}$: Binary parameter indicating the eligibility of recipe i at factory j on day t.

We formulate BAP as a mixed-integer linear programming problem:

$$\underset{x}{\text{Minimise}}\, f(x) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left|a_{i,j,t} - a_{i,j,t-1}\right|}{\sum_{i=1}^{n} a_{i,j,t}} \tag{2.1}$$

$$\text{subject to} \quad \sum_{o=1}^{k} x_{o,j,t} = C_{j,t}, \qquad \forall j \in F \setminus \{m\}, \quad \forall t \in T, \tag{2.2}$$

$$x_{o,j,t} \leq \min_{i \in o} E_{i,j,t}, \qquad \forall o \in O, \quad \forall j \in F, \quad \forall t \in T, \tag{2.3}$$

$$\sum_{j=1}^{m} x_{o,j,t} = 1, \qquad \forall o \in O, \quad \forall t \in T, \tag{2.4}$$

$$x_{o,j,t} \in \{0, 1\}, \qquad \forall o \in O, \quad \forall j \in F, \quad \forall t \in T. \tag{2.5}$$

The objective function (2.1) minimises WMAPE site by reducing daily fluctuations in recipe allocations across factories while considering the total recipe volume assigned each day. Recipe quantities $a_{i,j,t}$ are indirectly controlled by the binary decision variable $x_{o,j,t}$ through constraints. When $x_{o,j,t}$ determines which factory j receives order o at time t, it simultaneously establishes how recipes must be distributed. The allocation of an order to a specific factory creates a cascade effect that dictates the required recipe quantities, even though $a_{i,j,t}$ does not explicitly contain $x_{o,j,t}$ in its definition.

Constraint (2.2) enforces that each factory operates at its full capacity to avoid resource waste. Every factory (except the special catch-all factory F_m) must receive order quantities that match exactly what that factory can produce. Constraint (2.3) implements the eligibility requirement. An order can only be assigned to a factory if that factory is capable of producing all the recipes needed for that order. This prevents impossible manufacturing assignments. Constraint (2.4) guarantees that each order is assigned to exactly one factory. This prevents order splitting across multiple facilities. Constraint (2.5) specifies that the allocation variables must be binary (0 or 1), meaning an order is either fully assigned to a particular factory or not assigned to it at all. This enforces the discrete nature of order assignments.

WMAPE global is computed using the following equation:

$$WMAPE_{\text{global}} = \frac{\sum_{i=1}^{n}\left|a_{i,t} - a_{i,t-1}\right|}{\sum_{i=1}^{n} a_{i,t}}, \tag{2.6}$$

which establishes a theoretical lower bound for evaluating allocation solution optimality. When decomposed at the site level, this formula transforms into:

$$WMAPE_{\text{global}} = \frac{\sum_{i=1}^{n}\left|\sum_{j=1}^{m} a_{i,j,t} - \sum_{j=1}^{m} a_{i,j,t-1}\right|}{\sum_{i=1}^{n} a_{i,t}}, \tag{2.7}$$

By applying the triangle inequality, we derive the fundamental relationship:

$$WMAPE_{\text{global}} \leq \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left|a_{i,j,t} - a_{i,j,t-1}\right|}{\sum_{i=1}^{n} a_{i,t}}(WMAPE_{\text{site}}) \tag{2.8}$$

Table 2 presents exemplary order data for two consecutive days, while Table 3 quantifies the corresponding WMAPE site and global. This result reveals that despite using the same order data, WMAPE global (0.5) is lower than WMAPE site (1.14), confirming the theoretical relationship established in equation (2.8).

TABLE 2 Exemplary orders of LD15 and LD14

LD15		
Order ID	**Recipe IDs**	**Assigned Factory**
1	1, 10	F1
2	2, 3, 5	F1
3	4, 6	F2
4	2, 3, 7	F2
5	3, 5, 9	F3

LD14		
Order ID	**Recipe IDs**	**Assigned Factory**
1	2, 5	F1
2	2, 6, 7	F3
3	3, 5, 9	F1
4	4, 6, 8	F2
5	5, 9, 10	F2

TABLE 3 WMAPE site and WMAPE global

Recipe	Factory	LD15 ($a_{i,j,t-1}$)	LD14 ($a_{i,j,t}$)	Absolute difference
1	F1	1	0	1
2	F1	1	1	0
2	F2	1	0	1
2	F3	0	1	1
3	F1	1	1	0
3	F2	1	0	1
3	F3	1	0	1
4	F2	1	1	0
5	F1	1	2	1
5	F2	0	1	1
6	F2	1	1	0
6	F3	0	1	1
7	F2	1	0	1
7	F3	0	1	1
8	F2	0	1	1
9	F1	0	1	1
9	F2	0	1	1
9	F3	1	0	1
10	F1	1	0	1
10	F2	0	1	1
SUM			**14**	**16**
WMAPE site = 16/14 = 1.14				

Recipe	LD15 ($a_{i,t-1}$)	LD14 ($a_{i,t}$)	Absolute difference
1	1	0	1
2	2	2	0
3	3	1	2
4	1	1	0
5	2	3	1
6	1	2	1
7	1	1	0
8	0	1	1
9	1	2	1
10	1	1	0
SUM		**14**	**7**
WMAPE global = 7/14 = 0.5			

The company's ultimate goal is to minimise the area under the curve shown in Figure 5, which represents the WMAPE site when comparing the final day's allocation with all previous days. However, directly optimising this metric is impractical due to forecast uncertainties before LD3. These uncertainties arise from uncontrollable factors, including the variability of simulated orders, customer modifications to existing real orders, and the continuous arrival of new actual orders throughout the planning period. To tackle this challenge, the company adopts a more practical approach by minimising allocation variations between consecutive days, as defined in Equation 2.1. This strategy reframes BAP as a *proxy optimisation problem*, where solving a simpler, related problem serves as a substitute for optimising the more complex objective[27]. By reducing day-to-day allocation discrepancies, this approach effectively lowers cumulative error over the entire planning period, making it a feasible way to achieve the company's broader strategic goal.

BAP shares fundamental similarities with two well-known combinatorial optimisation problems: BPP and CVRP. These problems rely on a network structure, where nodes represent key entities (factories, storage bins, or vehicles) and edges define decision variables (order allocations, item assignments, or routing sequences). Allocations are subject to capacity constraints to ensure efficient resource utilisation, as illustrated in Figure 6. Despite these shared elements, each problem has a distinct mathematical formulation. BPP minimises the number of bins while keeping total item weight or volume within capacity limits. CVRP focuses on reducing total travel costs or distances while ensuring vehicle capacity is not exceeded. BAP introduces the unique objective of minimising temporal variations in recipe allocations across factories.

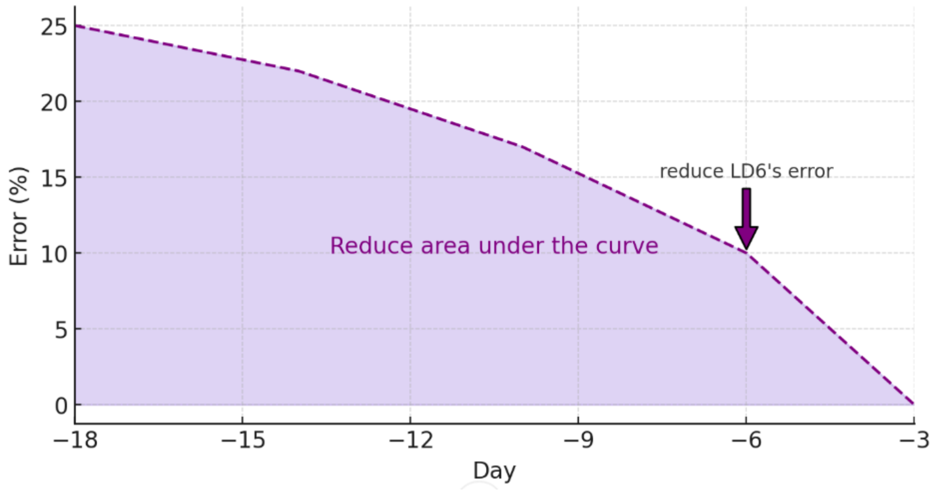


FIGURE 5 Objective of proxy optimisation

Their similarities highlight opportunities for cross-domain optimisation techniques. The B&B method from BPP has proven effective across various allocation problems [18]. Likewise, heuristic approaches such as 2-Opt and Tabu Search from CVRP have influenced allocation strategies in multiple domains [15,10]. However, adapting these methods for BAP requires significant modifications to address its unique focus on allocation consistency, specialised constraints (such as recipe eligibility), and the need for stable day-to-day allocations. These distinctions position BAP beyond traditional BPP and CVRP models, necessitating problem-specific mathematical formulations while strategically integrating established optimisation techniques.

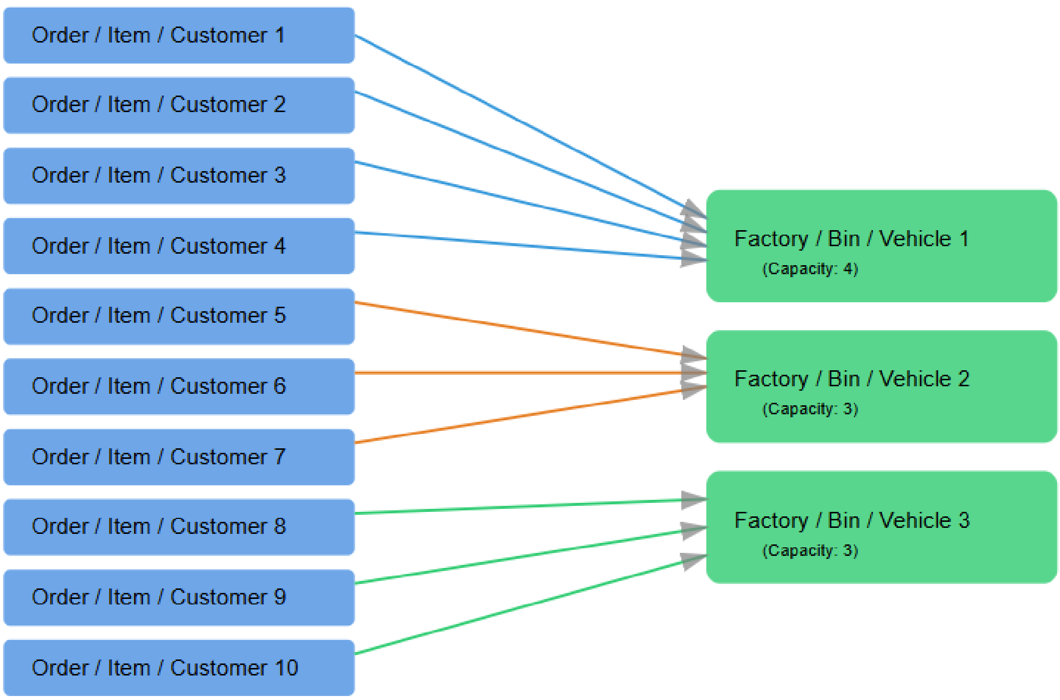


FIGURE 6 Solution of BAP and its similarities to BPP and CVRP

3 | PROBLEM SOLVING

The company seeks an efficient approach to minimise recipe discrepancies between two allocation solutions. Figure 7 illustrates different swap types used in heuristics, where blue boxes represent real orders and orange boxes indicate simulated orders. To simplify the process, this study adopts the 1:1 swap method. The solving approach consists of the following steps:

- Exact method: The CBC solver is directly applied to current orders to determine the optimal allocation that minimises recipe differences compared to the previous day's allocation.
- Heuristics: The process begins with a greedy algorithm to construct an initial feasible solution. Orders are first allocated to F1, prioritising those with fewer eligible factories (F1-F3 first, then F1-F2-F3) until capacity is reached. Remaining orders with eligible factories F1-F2-F3 and F2-F3 are then allocated to F2 until its capacity is full. Any unallocated orders are assigned to F3, the catch-all factory with no constraints. ITPS and TS are then applied to refine the initial allocation.
- Evaluation: The performance and scalability of each method are assessed by comparing the final WMAPE site against WMAPE global and analyzing optimisation times for different order volumes. The best method is then used in temporal tests to evaluate its effectiveness over a 15-day planning period and its adaptability to dynamic changes.

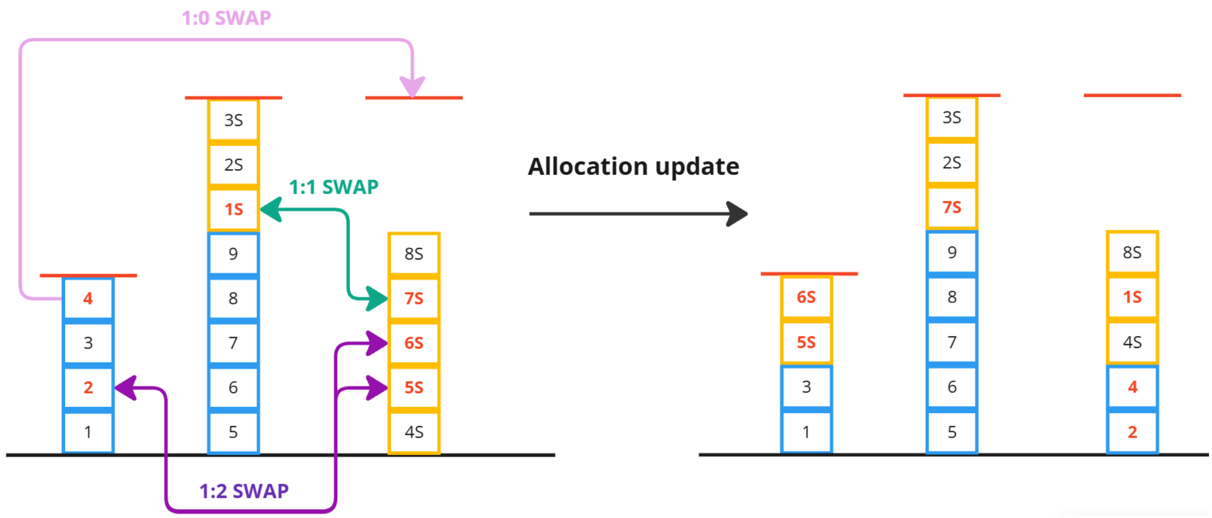


FIGURE 7 Types of box swaps

This article first explores the application of the exact method to address BAP. The CBC solver, developed as part of the Computational Infrastructure for Operations Research (COIN-OR) project, provides a robust open-source solution for MILP problems[6]. Its seamless integration with the COIN-OR linear programming solver and cut generation library makes it particularly effective for handling complex optimisation challenges. In this case, CBC solver can solve BAP in just a few steps. For instance, consider a hypothetical order quantity of 100,000. The process unfolds as follows:

1. The model is loaded and initialised with 210,302 rows, 300,600 columns, and 1,359,124 elements. Preprocessing is applied, fixing 110,000 variables and reducing the problem size. The continuous relaxation is solved, yielding an objective value of 4,646.
2. The Feasibility Pump heuristic (Table 4) is employed to quickly find an initial integer-feasible solution. Then, Coefficient Diving (Table 4) is applied, successfully identifying the optimal solution. For most order quantities below 100,000, the Feasibility Pump alone is sufficient to find the optimal solution immediately.
3. A limited B&B process is used to resolve any remaining general integer variables that were not fixed by heuristics. This step focuses on a reduced problem size, branching only on the unfixed general integer variables.
4. A mini B&B is conducted to refine the solution further. Although this step does not lead to an improved solution, it plays a crucial role in validating the quality of the heuristic-based solution.
5. The process concludes without extensive branching, as the optimal solution is proven at the root node following the limited branching and mini B&B steps.

TABLE 4 Primal heuristics in CBC

Category	Feasibility Pump	Coefficient Diving
Overview	Introduced by Fischetti et al.[5], this heuristic rapidly finds feasible solutions by alternating between LP-relaxation and integer solutions.	Based on works by Berthold[2], and Paulus and Krause[20], this heuristic prioritises variables with minimal constraint violations.
Steps	1. Start with the LP relaxation solution. 2. Round to the nearest integer-feasible point. 3. Find the closest LP solution to this integer point. 4. Repeat until a feasible solution is found or a stopping criterion is met.	1. Select variables based on *locks* (potential constraint violations). 2. Prioritise variables with minimal locks and smallest fractionality. 3. Iteratively bound selected variables to guide the solution toward feasibility.
Key advantage	The *pumping* action between LP-feasible and integer-feasible solutions enables fast convergence to high-quality feasible solutions.	This targeted approach efficiently finds optimal or near-optimal solutions early in the process.

Regarding two heuristics, Iterative Targeted Pairwise Swap (ITPS) is inspired by 2-Opt method commonly used for VRP. While 2-Opt swaps adjacent elements in a route, ITPS swaps two orders between factories to minimise WMAPE site. ITPS has three key features: it only accepts improvements, it uses recipe differences to select candidates for swaps, and it ensures the solution remains feasible by swapping only two orders at a time.

Tabu Search (TS) is a metaheuristic based on adaptive memory techniques that uses a memory structure called the *tabu list* to guide the search process more effectively. The term *tabu*, which originates from Polynesian cultures, refers to actions that are temporarily forbidden. In TS, tabu moves are those that are blocked to prevent cycling and encourage the exploration of new areas in the solution space. A key concept in TS is *tenure*, which is the duration a move remains in the tabu list. The tenure controls how long a move stays forbidden, preventing short-term cycling and allowing previously explored moves to be revisited after enough exploration. TS balances intensification (choosing the best move) and diversification (through random swaps), ensuring a comprehensive search of the solution space.

4 | NUMERICAL EXPERIMENTS

4.1 | Experimental design

Simulated data replicating the characteristics of real data is generated to evaluate the effectiveness of three proposed methods. The resulting order list follows realistic rules: 30% of the total orders are eligible for F1, 60% for F2, and all orders are eligible for F3, which has no constraints. When the proportion of real orders increases over time, all real orders from the previous day are carried over to the current day with their IDs unchanged, facilitating allocation tracking. Simulated orders are generated anew each day to make up for any shortfall in the total quantity, resulting in dynamic IDs. Each order consists of 1 to 4 recipes chosen from a total of 100 available recipes, with eligibility based on their grouping. Recipes in Group 1 (1–29) are eligible exclusively for F1, Group 2 (30–49) qualifies for both F1 and F2, Group 3 (50–89) is eligible only for F2, and Group 4 (90–100) is eligible solely for F3. An order qualifies for a factory only if all its recipes meet the factory's eligibility. However, for *F3-only orders*, any recipe can be included as long as the order contains at least one recipe from Group 4. Table 5 presents exemplary order data along with solutions that satisfy the capacity constraints: 25% of the total quantity for F1 and 50% for F2. Importantly, the allocated factories for four existing real orders on LD12 remain unchanged, ensuring a minimum WMAPE site.

To evaluate the performance of the proposed methods, three key indicators are used. First, the optimality gap measures the difference between the MAPE site and global, with WMAPE global serving as the lower bound. The CBC solver is expected to match WMAPE site to WMAPE global, as the exact method can find the optimal solution. Second, the improvement percentage calculates the WMAPE site improvement over the initial solution for heuristic methods, enabling comparison of their performance, as they may not always achieve the optimal result. Finally, computational time is assessed by the duration each method takes to find the best solution, with a maximum limit of 10 minutes (600 seconds). All tests are implemented in Python on a PC with an Intel(R) Core(TM) i5-10210U CPU @ 1.60GHz and 8 GB of RAM, and the code is available on GitHub.

TABLE 5 Exemplary orders and solutions

LD12 (46% real orders)			
Order ID	**Recipe IDs**	**Is real**	**Eligible factories**
1	30	True	F1, F2, F3
2	8, 5, 24	True	F1, F3
3	22	True	F1, F3
4	87	True	F2, F3
5	52, 51, 55, 63	False	F2, F3
6	82, 88	False	F2, F3
7	85	False	F2, F3
8	84, 76	False	F2, F3
9	93, 1, 36, 76	False	F3
10	28, 95, 20	False	F3

LD11 (52% real orders)			
Order ID	**Recipe IDs**	**Is real**	**Eligible factories**
1	30	True	F1, F2, F3
2	8, 5, 24	True	F1, F3
3	22	True	F1, F3
4	87	True	F2, F3
5	74	True	F2, F3
6	85	False	F2, F3
7	89, 73, 86	False	F2, F3
8	54, 52	False	F2, F3
9	100, 99	False	F3
10	91, 13	False	F3

Solution for LD12	
Factory	**Allocated orders**
F1	2, 3
F2	1, 4, 5, 6, 8
F3	7, 9, 10

Solution for LD11	
Factory	**Allocated orders**
F1	2, 3
F2	1, 4, 5, 7, 8
F3	6, 9, 10

4.2 Results

4.2.1 Benchmark test

The following results are based on 10,000 orders over two consecutive days: LD12 (46% real orders) and LD11 (52% real orders). B&B uses the allocation decision from LD12 to directly determine the best allocation for LD11, minimising the WMAPE site between two days (without swaps, as in heuristics). For heuristics, the number of iterations significantly affects their ability to find the optimal solution within a reasonable time. Therefore, an iteration test was conducted on 30 different sets of 10,000 orders, identifying 1,500 iterations for ITPS and 500 iterations for TS as the optimal numbers. With these iterations, the allocations for LD11 before and after applying ITPS and TS are presented in Table 6, which shows that B&B is the best method, providing the optimal solution in the shortest time. Between two heuristics, TS slightly outperforms ITPS in both solution quality and speed.

B&B	
LD12	
F1	2500 orders, 2500 real
F2	5000 orders, 1890 real
F3	2500 orders, 210 real
LD11	
F1	2500 orders, 2500 real
F2	5000 orders, 2593 real
F3	2500 orders, 107 real
WMAPE site	0.054
WMAPE global	0.054
optimisation time	2.89 seconds

	ITPS	**TS**
LD12		
F1	2500 orders, 2500 real	
F2	5000 orders, 1890 real	
F3	2500 orders, 210 real	
LD11 (Before)		
F1	2500 orders, 2500 real	
F2	5000 orders, 2422 real	
F3	2500 orders, 278 real	
LD11 (After)		
F1	2500 orders, 2500 real	2500 orders, 2500 real
F2	5000 orders, 2436 real	5000 orders, 2429 real
F3	2500 orders, 264 real	2500 orders, 271 real
WMAPE site		
Before	0.074	0.074
After	0.054	0.054
Improvement	26.21%	26.97%
WMAPE global		
	0.054	
optimisation time	19.45 seconds	15.35 seconds

TABLE 6 Benchmark test results

4.2.2 | Scalability test

In this test, each method is applied to five different order quantities, ranging from 10,000 to 100,000, over two days: LD12 and LD11. Figure 8 compares the optimisation times of the three algorithms, while Figures 9, 10, and 11 illustrate the gap between WMAPE site and WMAPE global for each method. The results confirm that B&B is the best method, consistently delivering optimal solutions across all quantities in the shortest time. A notable observation is that, as the total order quantity increases, both WMAPEs decrease. This can be attributed to the principles of aggregate forecasting, which suggest that forecasts are more accurate for groups of items than for individual items, as variability within a group tends to cancel out, leading to more stable and reliable predictions[22]. Furthermore, as the order quantity increases, individual fluctuations have a smaller impact on the overall error, whereas, in smaller samples, changes in each order have a proportionally larger effect.

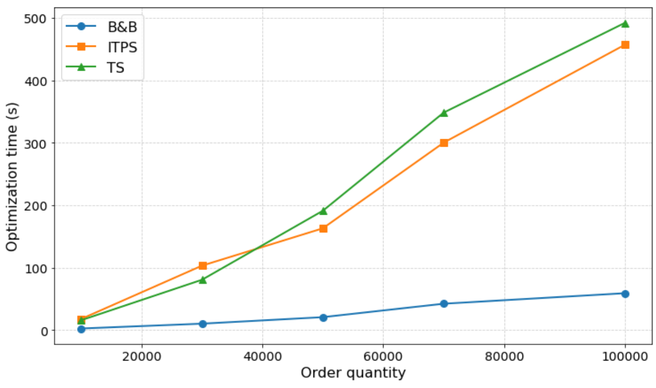


FIGURE 8 Comparison of optimisation time

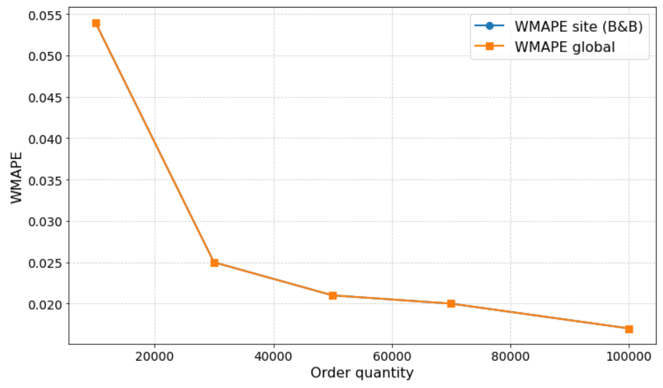


FIGURE 9 WMAPEs of B&B

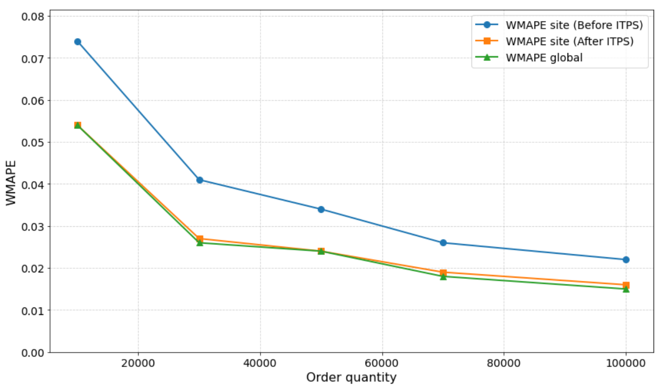


FIGURE 10 WMAPEs of ITPS

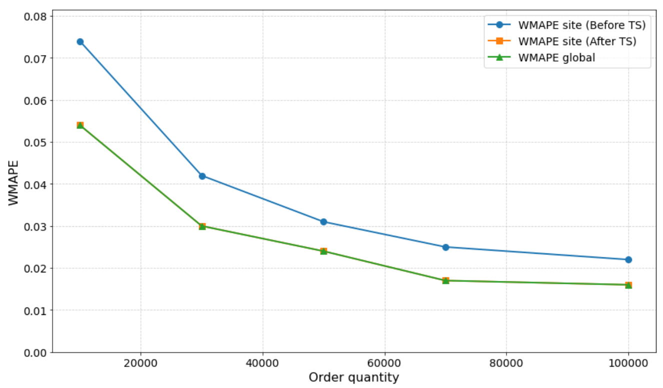


FIGURE 11 WMAPEs of TS

4.2.3 | Temporal fixed test

In this test, the best method will be used to address the temporal aspect of BAP. Specifically, B&B will continuously make allocation decisions for a 15-day planning period under ideal conditions, where there are no changes in factory capacities or real orders, and the total order quantity for each day is 10,000. Figures 12 show the increasing proportions of real orders over the days. As LD3 approaches, the order composition between consecutive days becomes more similar, since each day includes real orders from the previous day. This also explains the decreasing trend in the error graphs.

Figure 13 shows the WMAPE site when B&B continuously optimises allocation between two consecutive days. For instance, LD16 is optimised based on the B&B allocation of LD17, LD15 is optimised based on LD16's allocation, and so on. It is evident that, except for the initial days when the proportion of real orders is still low, B&B consistently provides optimal solutions from LD14 to LD3. The WMAPE site (Greedy) line illustrates that, without B&B, the recipe differences between two consecutive days are significantly higher.

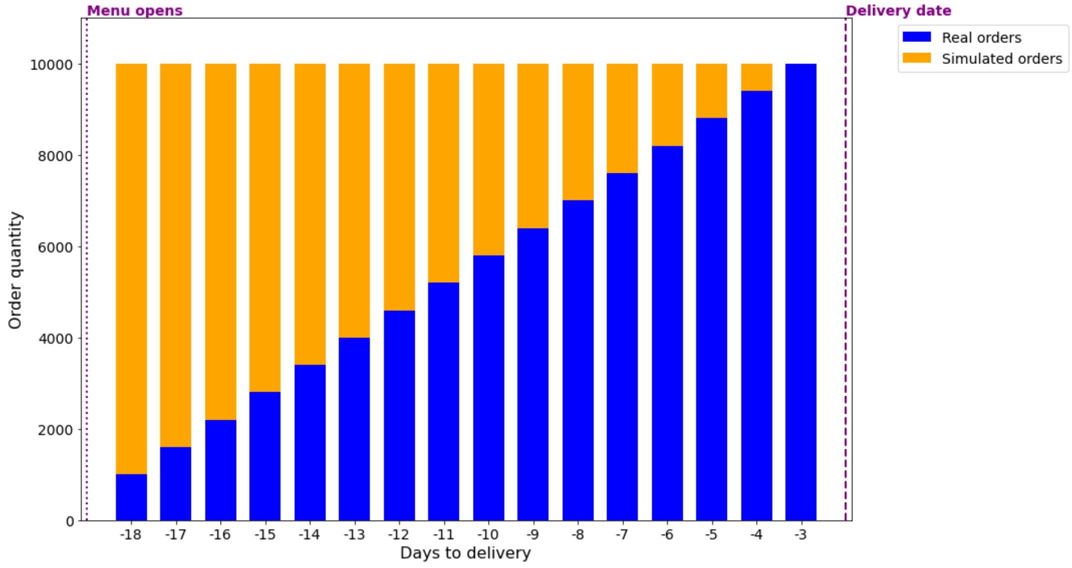


FIGURE 12 Composition of total orders through days

After B&B completes the order allocation for all days, each day's decision is compared with the final day's allocation, as shown in Figure 14. This comparison assesses whether the day-by-day optimisation provides a smooth transition from soft allocations to hard allocations. The gradual decrease in WMAPE site indicates that B&B has effectively contributed to stable production planning, minimising abrupt changes in the allocated recipe quantity. The ultimate goal is to reduce the area under the WMAPE site curve, ideally aligning it with the area under the WMAPE global curve. Although there is a big gap between the WMAPE global and WMAPE site (B&B) lines in the early days, this gap narrows over time. As a result, the area under the WMAPE site (B&B) curve is smaller than that under the WMAPE site (Greedy) curve, showing B&B's effectiveness in achieving proxy optimisation.

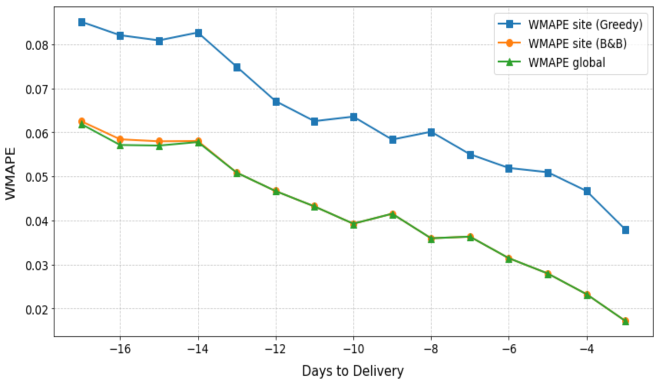


FIGURE 13 WMAPE site and WMAPE global through days

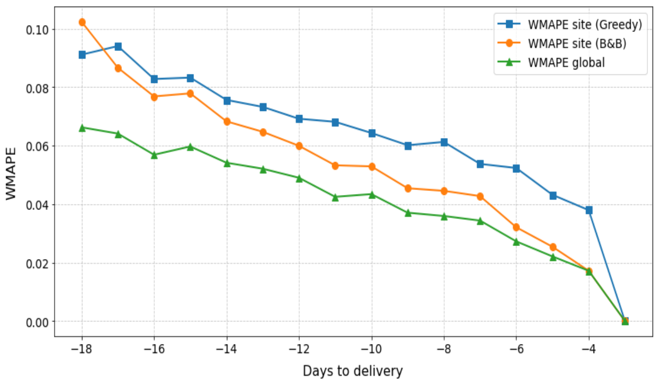


FIGURE 14 Each day's allocation versus LD3's allocation

4.2.4 | Temporal variation test

In the previous test, factory capacities and real orders are assumed to remain constant throughout the entire planning period. However, in reality, the following scenarios may occur:

1. **Capacity change**: When F1's capacity drops from 3,000 to 1,000 orders on LD10, there is a sudden increase in WMAPE site, as shown in Figure 15. This spike happens because orders have to be quickly redistributed among factories. B&B handles this well, quickly adjusting the allocations and achieving the optimal WMAPE site by LD9. In contrast, the greedy method shows a large gap between WMAPE site and WMAPE global after the capacity change, showing its lack of

flexibility. The comparison of each day's allocations with the final day's allocation in Figure 16 shows that WMAPE site of B&B gradually moves toward the optimal value, especially from LD7 onward. The sharp drop in error between LD10 and LD11 signals a clear shift in how the allocations are handled, with decisions before LD10 differing greatly from LD3. This test demonstrates B&B's ability to effectively respond to a major change.

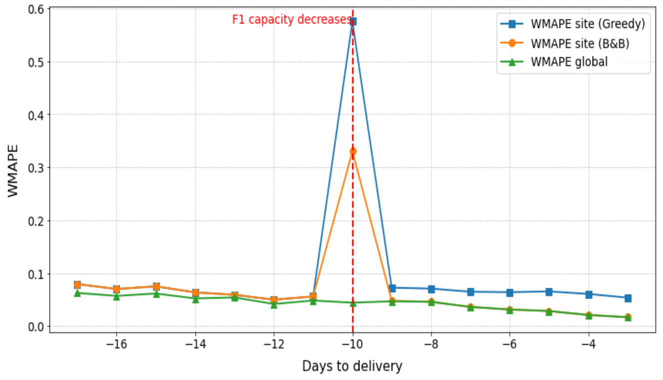


FIGURE 15 WMAPE site and WMAPE global through days

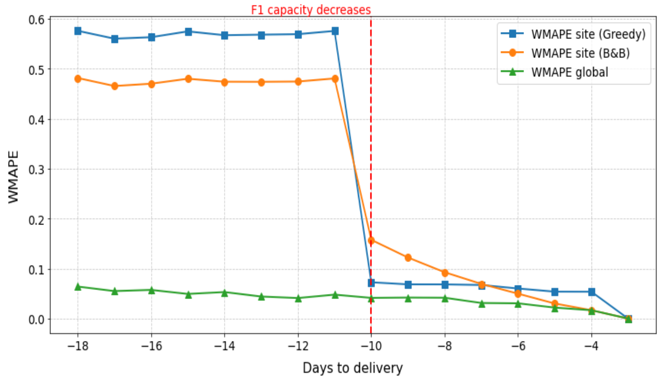


FIGURE 16 Each day's allocation versus LD3's allocation

2. **Order changes**: This study assumes that each day, customers delete 5% of real orders and adjust recipes in 30% of them. Figure 17 shows how these changes cause fluctuations in WMAPEs. Without modifications, both WMAPE site and WMAPE global follow a clear downward trend. However, when changes occur, their values fluctuate significantly depending on the extent of adjustments. Despite this variability, the graph confirms that B&B consistently achieves the optimal WMAPE site in all cases.

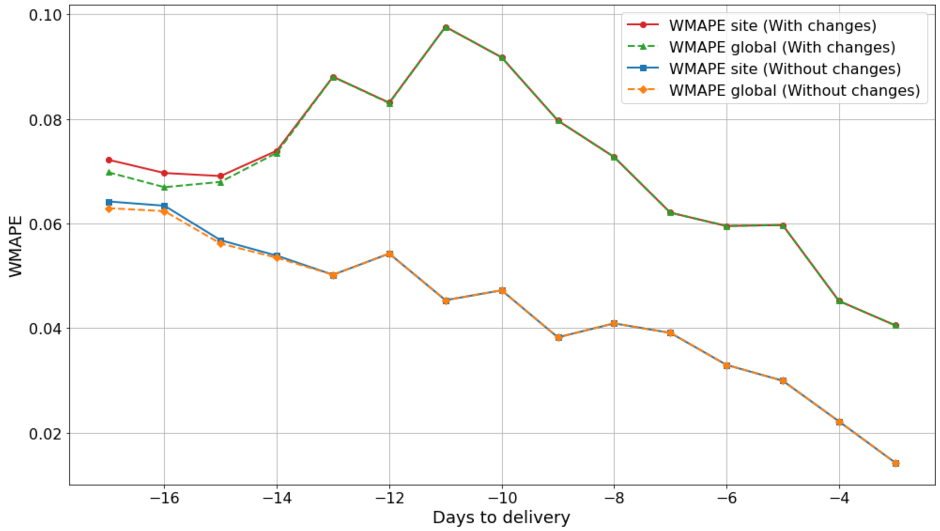


FIGURE 17 WMAPE site and WMAPE global with and without order changes

In this scenario, a simple allocation strategy can also be used since IDs of real orders remain unchanged across days. For example, if order O1 is assigned to F1 and O2 to F2 on LD14, the same assignments can carry over to LD15. This ID-based approach ensures consistency and minimises allocation changes for recurring orders. However, challenges arise when recipes change within an order, potentially making it not eligible for its original factory. Deleted orders add further complications, as past real order IDs may no longer exist. Figures 18 and 19 highlight these issues by comparing the ID-based method with B&B. The results show that B&B consistently delivers optimal solutions, aligning WMAPE site with WMAPE global, regardless of order changes. In contrast, the ID-based method can match WMAPE global only on some final days when the proportion of real orders is high. When order changes occur, the gap between WMAPE site

(ID-based) and WMAPE global widens, demonstrating the limitations of simple allocation methods and the advantages of more advanced techniques like B&B.

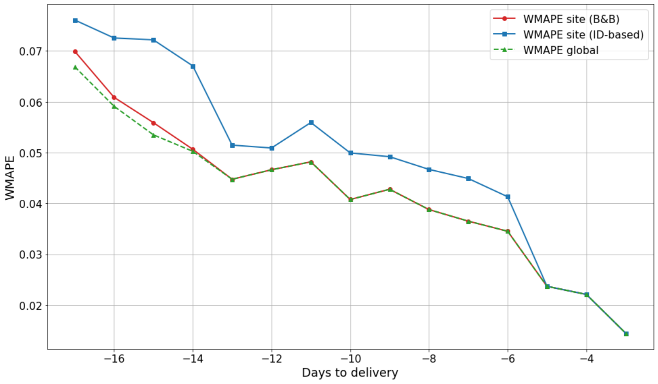


FIGURE 18 B&B and ID-based allocation without order changes

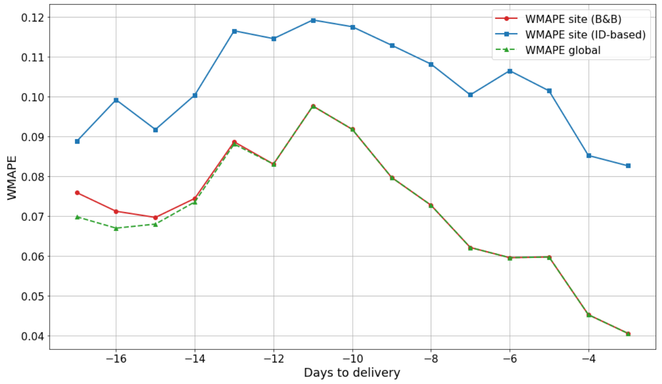


FIGURE 19 B&B and ID-based allocation with order changes

5 | CONCLUSIONS

This study investigates BAP in meal kit delivery, leveraging insights from BPP and CVRP. A mathematical model is formulated and evaluated using both exact and heuristic methods to optimise allocation decisions. Results show that B&B with primal heuristics in CBC solver consistently delivers optimal solutions, significantly outperforming heuristic approaches such as ITPS and TS. Among heuristics, TS achieved near-optimal results comparable to B&B. Scalability tests confirm that all methods efficiently process up to 100,000 orders within 10 minutes, demonstrating their viability for large-scale operations. Additionally, larger order volumes further reduced errors, suggesting that consolidating orders into a single allocation cycle enhances accuracy.

The CBC solver proved highly effective in minimising day-to-day recipe variations, ensuring a smooth transition from soft to hard allocations. Variation tests indicate that B&B effectively reallocates orders when factory capacities shift, ensuring full capacity utilisation and preventing production imbalances. It also maintained optimal performance despite order deletions and customer-driven recipe changes, demonstrating resilience in dynamic conditions. Comparisons between B&B and the ID-based method highlight the limitations of rule-based approaches, underscoring the need for advanced techniques in environments with frequent operational changes.

However, this study has some limitations. The current approach optimises allocation based only on the previous day's data, lacking a long-term planning strategy. Future research should explore multi-day optimisation frameworks to enhance accuracy and efficiency. In addition, metaheuristics such as Genetic Algorithms and Large Neighborhood Search, which involve more complex swaps, remain unexplored and could further improve solution quality and computational efficiency.

By addressing an underexamined problem in meal kit delivery, this research contributes new insights to the supply chain optimisation literature while solving a practical business problem. The proposed methods enhance operational efficiency and reduce food waste, supporting sustainability efforts in the industry. Future work should refine these approaches to improve scalability, adaptability, and long-term strategic planning in meal kit delivery operations.

References

1. Battiti, R. and Tecchiolli, G. (1994) 'The reactive tabu search', *ORSA Journal on Computing*, 6(2), pp. 126-140. Available at: https://doi.org/10.1287/ijoc.6.2.126.

2. Berthold, T. (2006) *Primal heuristics for mixed integer programs*. Diploma thesis. Technische Universität Berlin. Available at: https://www.researchgate.net/publication/258846101_Primal_Heuristics_for_Mixed_Integer_Programs.

3. Clear Spider (2024) *How Meal Kit Delivery Services Manage Their Supply Chains*. Available at: https://clearspider.net/blog/meal-kit-delivery-supply-chains/.

4. Clausen, J. (1999) 'Branch and bound algorithms-principles and examples', Department of Computer Science, University of Copenhagen, pp. 1-30. Available at: https://imada.sdu.dk/~jbj/DM85/TSPtext.pdf.

5. Fischetti, M., Glover, F. and Lodi, A. (2005) ‘The feasibility pump’, *Mathematical Programming*, 104(1), pp. 91-104. Available at: https://doi.org/10.1007/s10107-004-0570-3.

6. Forrest, J. and Lougee-Heimer, R. (2005) ‘CBC user guide’, in *Emerging theory, methods, and applications*. INFORMS, pp. 257-277. Available at: https://doi.org/10.1287/educ.1053.0020.

7. GeeksforGeeks (2021) *How to Calculate Weighted MAPE in Excel?* Available at: https://www.geeksforgeeks.org/how-to-calculate-weighted-mape-in-excel/.

8. Gendreau, M., Hertz, A. and Laporte, G. (1994) ‘A Tabu Search Heuristic for the Vehicle Routing Problem’, *Management Science*, 40(10), pp. 1276-1290. Available at: https://doi.org/10.1287/mnsc.40.10.1276.

9. Gendreau, M. (2003) ‘An introduction to tabu search’, in Glover, F. and Kochenberger, G.A. (eds.) Handbook of metaheuristics. Boston: Springer, pp. 37-54. Available at: https://doi.org/10.1007/0-306-48056-5_2.

10. Glover, F. (1986) ‘Future paths for integer programming and links to artificial intelligence’, *Computers & Operations Research*, 13(5), pp. 533-549. Available at: https://doi.org/10.1016/0305-0548(86)90048-1.

11. Glover, F. (1989) ‘Tabu search—part I’, ORSA Journal on Computing, 1(3), pp. 190-206. Available at: https://courses.cs.umass.edu/cics521-cg/docs/tabu-glover-1.pdf.

12. Glover, F., and Laguna, M. (1997) Tabu search. Boston: Springer Science & Business Media. Available at: http://dx.doi.org/10.1007/978-1-4615-6089-0.

13. Langham Logistics (2020) *The Complex Logistics Behind Meal-Kit Services*. Available at: https://www.elangham.com/2020/06/23/whats-for-dinner-the-complex-logistics-behind-meal-kit-services/.

14. Laporte, G. and Nobert, Y. (1987) ‘Exact algorithms for the vehicle routing problem’, *Annals of Discrete Mathematics*, 132, pp. 147-184. Available at: https://doi.org/10.1016/S0304-0208(08)73235-3.

15. Lin, S. (1965) ‘Computer solutions of the traveling salesman problem’, *Bell System Technical Journal*, 44(10), pp. 2245-2269. Available at: https://doi.org/10.1002/j.1538-7305.1965.tb04146.x.

16. Lodi, A., Martello, S. and Vigo, D. (1999) ‘Heuristic and metaheuristic approaches for a class of two-dimensional bin packing problems’, *INFORMS Journal on Computing*, 11(4), pp. 345-357. Available at: https://doi.org/10.1287/ijoc.11.4.345.

17. Loic, G. (2024) Internal material.

18. Martello, S. and Toth, P. (1990) *Knapsack Problems: Algorithms and Computer Implementations*. Wiley. Available at: http://old.math.nsc.ru/LBRT/k5/knapsack_problems.pdf.

19. Osman, I.H. (1993) ‘Metastrategy simulated annealing and tabu search algorithms for the vehicle routing problem’, *Annals of Operations Research*, 41(4), pp. 421-451. Available at: https://doi.org/10.1007/BF02023004.

20. Paulus, M.B. and Krause, A. (2023) ‘Learning To Dive In Branch And Bound’, *arXiv preprint arXiv:2301.09943*. Available at: https://doi.org/10.49550/arXiv.2301.09943.

21. Radcliffe-Brown, A.R. (1939) *Taboo*. Cambridge: Cambridge University Press. Available at: https://www.amazon.co.uk/Taboo-Frazer-Lecture-R-Radcliffe-Brown/dp/1107695791.

22. Reid, R. Dan and Sanders, Nada R. (2012) *Operations Management: An Integrated Approach, 5th Edition*. Available at: https://www.oreilly.com/library/view/operations-management-an/9781118122679/ch8-sec004.html.

23. RMS Omega (2024) *4 Challenges For Meal Kit Delivery Operations*. Available at: https://rmsomega.com/4-challenges-for-meal-kit-delivery-operations/.

24. Statista (2024) *Meal Kits in the UK - Overview*. Available at: https://www.statista.com/topics/8895/meal-kits-in-the-uk/.

25. Toth, P. and Vigo, D. (2002) *The Vehicle Routing Problem*, SIAM Monographs on Discrete Mathematics and Applications. Available at: https://api.semanticscholar.org/CorpusID:209742615.

26. Woop (2024) *Home*. Available at: https://woop.co.nz/.

27. Zangl, G., Graf, T. and Al-Kinani, A. (2006) ‘Proxy Modeling in Production optimisation’, *Paper presented at the SPE Europec/EAGE Annual Conference and Exhibition*, Vienna, Austria, 12-15 June. Available at: https://doi.org/10.2118/100131-MS.

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