In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
from IPython.core.pylabtools import figsize # import figsize
#figsize(12.5, 4) # 设置 figsize
from scipy.stats import chi2
from scipy.stats import t
from scipy.stats import f
from scipy.stats import norm
```

Linear Fit

$$\boldsymbol{b} = (X^T X)^{-1} X^T \boldsymbol{Y}.$$

$$\mathrm{SS_E} = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} \left(y_i - (b_0 + b_1 x_{1i} + \cdots + b_p x_{pi})\right)^2$$

$$\mathrm{SS_T} = \sum_{i=1}^{n} (Y_i - \overline{Y})^2$$

$$P := \frac{1}{n} \begin{pmatrix} 1 & 1 & \dots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \dots & 1 \end{pmatrix}$$

$$H := X(X^T X)^{-1} X^T,$$

$$\mathrm{SS_T} = \langle \boldsymbol{Y}, (\mathbb{1}_n - P)^T (\mathbb{1}_n - P) \boldsymbol{Y} \rangle = \langle \boldsymbol{Y}, (\mathbb{1}_n - P) \boldsymbol{Y} \rangle$$

$$\begin{aligned} \mathrm{SS_T} &= \langle \boldsymbol{Y}, (\mathbb{1}_n - P) \boldsymbol{Y} \rangle \\ &= \underbrace{\langle \boldsymbol{Y}, (\mathbb{1}_n - H) \boldsymbol{Y} \rangle}_{=\mathrm{SS_E}} + \underbrace{\langle \boldsymbol{Y}, (H - P) \boldsymbol{Y} \rangle}_{=:\mathrm{SS_R}}. \end{aligned}$$

$$R^2 = \frac{\mathrm{SS_R}}{\mathrm{SS_T}}$$

$$S^2 := \frac{\mathrm{SS_E}}{n - p - 1}$$

$$(X^TX)^{-1} = \begin{pmatrix} \xi_{00} & * & \cdots & \cdots & * \\ * & \xi_{11} & \ddots & & * \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & * \\ * & \cdots & \cdots & * & \xi_{pp} \end{pmatrix}$$

$$\text{Var}[B_i] = \xi_{ii}\sigma^2, \qquad\qquad i = 0, \dots, p,$$

Confidence Interval

$$\beta_j = b_j \pm t_{\alpha/2,n-p-1} S\sqrt{\xi_{jj}}, \qquad\qquad j = 0, \dots, p.$$

$$\widehat{\mu}_{Y|\mathbf{x_0}} = \mathbf{x_0}^T \boldsymbol{b} = \mathbf{x_0}^T (X^TX)^{-1} X^T \boldsymbol{Y}$$

$$\mu_{Y|\mathbf{x_0}} = \widehat{\mu}_{Y|\mathbf{x_0}} \pm t_{\alpha/2,n-p-1} S\sqrt{\mathbf{x_0}^T(X^TX)^{-1}\mathbf{x_0}}$$

$$Y \mid \mathbf{x_0} = \widehat{\mu}_{Y|\mathbf{x_0}} \pm t_{\alpha/2,n-p-1} S\sqrt{1 + \mathbf{x_0}^T(X^TX)^{-1}\mathbf{x_0}}.$$

Test for significance of regression

30.6. F-Test for Significance of Regression. Let $x_1, \dots, x_p$ be the predictor variables in a multilinear model (29.1) for Y. Then

$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_p = 0,$$

is rejected at significance level α if the test statistic

$$F_{p,n-p-1} = \frac{\text{SS}_\text{R}/p}{\text{SS}_\text{E}/(n-p-1)} = \frac{\text{SS}_\text{R}/p}{S^2} \tag{30.2}$$

satisfies $F_{p,n-p-1} > f_{\alpha,p,n-p-1}$.

$$\begin{aligned} F_{p,n-p-1} &= \frac{n-p-1}{p}\frac{\text{SS}_\text{R}/\text{SS}_\text{T}}{\text{SS}_\text{E}/\text{SS}_\text{T}} = \frac{n-p-1}{p}\frac{\text{SS}_\text{R}/\text{SS}_\text{T}}{(\text{SS}_\text{T}-\text{SS}_\text{R})/\text{SS}_\text{T}} \\ &= \frac{n-p-1}{p}\frac{R^2}{1-R^2} \end{aligned}$$

In [3]:
```python
plt.rcParams['savefig.dpi'] = 150 #图片像素
plt.rcParams['figure.dpi'] = 150 #分辨率


# chi square distribution
#percents = [0.995, 0.990, 0.975, 0.950, 0.900, 0.100, 0.050, 0.025, 0.010, 0.005]
#print(np.array([chi2.isf(percents, df=i) for i in range(1, 47)]))
# t distribution
#percents = [0.100, 0.050, 0.025, 0.010, 0.005, 0.001, 0.0005]
#print(np.array([t.isf(percents, df=i) for i in range(1, 46)]))
# F distribution
#alpha = 0.2
#print(np.array([f.isf(alpha, df1, df2) for df1 in range(1, 11) for df2 in range(1, 16)]).reshape(10, -1).T)
# normal distribution
#print(norm.ppf(np.arange(0, 0.99, 0.001).reshape(-1, 10)))
def linearfit(x, y, x_given, alpha,order=None,plot=False):
    (m_x,n_x)=x.shape
    (m_y,n_y)=y.shape
    assert m_x>=n_x and m_y>=n_y and m_x==m_y
    assert x_given.shape == (1,n_x)

    if order == None:
        print('Multilinear Model\n')
        tmp = np.ones((m_x,1),dtype=np.float32)
        X = np.concatenate((tmp,x),axis=1)
        x_given = np.concatenate((np.array([[1.]]),x_given),axis=1)
        Y = y
    else:
        print(f'Polyinomial Model with order {len(order)-1}\n')
        assert n_x == 1

        tmp = np.array([[]])
        X = np.array([[]])

        for i in range(0,len(order)):
            if order[i] == 1:
                if X.size == 0:
                    X = x**i
                    tmp = x_given**i
                else:
                    X = np.concatenate((X,x**i),axis=1)
                    tmp = np.concatenate((tmp,x_given**i),axis=1)

        x_given = tmp
        Y = y
        n_x = X.shape[1]-1

    print(f'X with order {order}:\n {X}\n')
    print(f'x_given:\n {x_given}\n')
    tmp = np.dot(np.linalg.inv(np.dot(X.T,X)),X.T)
    b = np.dot(tmp,Y)
    print(f'b with order {order}:\n {b}\n')
    H = np.dot(X,tmp)
    #print(f'H:\n {H}\n')
    assert np.sum(np.dot(H,X)-X) < 10**-5
    E = Y-np.dot(X,b)
    print(f'E:\n {E}\n')
    sse = np.sum(E**2)
    print(f'sse:\n {sse}\n')
    P=np.ones((m_x,m_x),dtype=np.float32)/m_x
    sst = np.sum((Y-np.dot(P,Y))**2)
    print(f'sst:\n {sst}\n')
    ssr=sst-sse
    print(f'ssr:\n {ssr}\n')
    r_square = ssr/sst
    print(f'r_square:\n {r_square}\n')
    s_square = sse/(m_x-n_x-1)
    print(f's_square:\n {s_square}\n')
    F = (m_x-n_x-1)/n_x*r_square/(1-r_square)
    print(f'F[{n_x},{m_x-n_x-1}]:\n{F}\n')
    print(f'F[{alpha}][{n_x},{m_x-n_x-1}]\n{f.isf(alpha, n_x, m_x-n_x-1)}\n')
    b_var_coeff = np.diagonal(np.linalg.inv(np.dot(X.T,X))).reshape(-1,1)
    print(f'Var[B]/sigma_square\n{b_var_coeff}\n')
    print(f'Var[B] Approximation:\n{b_var_coeff*s_square}\n')
    print(f'Standard Error[B]:\n{(b_var_coeff*s_square)**0.5}\n')
    delta = np.sqrt(s_square)*t.isf(alpha/2,m_x-n_x-1)*b_var_coeff**0.5
    b_lower= b - delta
    print(f'b {1-alpha} lower bound with order {order}:\n {b_lower}\n')
    print(f'b with order {order}:\n {b}\n')
    b_upper= b + delta
    print(f'b {1-alpha} upper bound with order {order}:\n {b_upper}\n')
    mu_Y = np.dot(x_given,b)
    tmp = np.dot(x_given,np.dot(np.linalg.inv(np.dot(X.T,X)),x_given.T))
    delta = np.sqrt(s_square)*t.isf(alpha/2,m_x-n_x-1)*tmp**0.5
    mu_Y_lower = mu_Y - delta
    mu_Y_upper = mu_Y + delta
    print(f'mu_Y|x_given {1-alpha} lower bound:\n {mu_Y_lower}\n')
    print(f'mu_Y|x_given :\n {mu_Y}\n')
```

```python
    print(f'mu_Y|x_given {1-alpha} upper bound:\n {mu_Y_upper}\n')

    tmp = np.dot(x_given,np.dot(np.linalg.inv(np.dot(X.T,X)),x_given.T))+1
    delta = np.sqrt(s_square)*t.isf(alpha/2,m_x-n_x-1)*tmp**0.5
    Y = mu_Y
    Y_lower = Y - delta
    Y_upper = Y + delta
    print(f'Y|x_given {1-alpha} lower bound:\n {Y_lower}\n')
    print(f'Y|x_given :\n {Y}\n')
    print(f'Y|x_given {1-alpha} upper bound:\n {Y_upper}\n')

    if plot == True:
        if order!=None:
            x_min = min(x)
            x_max = max(x)
            x_interval  = np.arange(x_min,x_max,(x_max-x_min)/1000)
            y_interval  = np.zeros(x_interval.shape,dtype=np.float32)
            j=0

            for i in range(0,len(order)):
                if order[i] == 1:
                    y_interval += b[j][0]*x_interval**i
                    j += 1

            plt.plot(x_interval,y_interval,c='r')
            plt.scatter(x.T[0],y.T[0],c='b')
        else:
            print('Since the multilinear plot can hardly show anything clear, so no multilinear plot\n')

    #plt.scatter(x.T[0],E.T[0])
    #plt.scatter(x.T[0],y.T[0])
    #plt.show()
```

In [3]:
```python
x = [ [1.35, 1.90, 1.70, 1.80, 1.30, 2.05, 1.60, 1.80, 1.85, 1.40]
    , [90, 30, 80, 40, 35, 45, 50, 60, 65, 30]
    ]


y = [
      [17.9, 16.5, 16.4, 16.8, 18.8, 15.5, 17.5, 16.4, 15.9, 18.3]
    ]

x_given = [ [1.5]
          , [70]
          ]


x = [[5, 7.5, 10, 12.5, 15, 17.5, 20]]
y = [[1, 2.2, 4.9, 5.3, 8.2, 10.7, 13.2]]
x_given = [[0]]

x = np.array(x).T
x_given = np.array(x_given).T
y = np.array(y).T
print(f'x:\n {x}\n')
print(f'y:\n {y}\n')
print(f'x_given:\n {x_given}\n')
```

```
x:
 [[ 5. ]
 [ 7.5]
 [10. ]
 [12.5]
 [15. ]
 [17.5]
 [20. ]]

y:
 [[ 1. ]
 [ 2.2]
 [ 4.9]
 [ 5.3]
 [ 8.2]
 [10.7]
 [13.2]]

x_given:
 [[0]]
```

In [ ]:
```
x = [ [1.35, 1.90, 1.70, 1.80, 1.30, 2.05, 1.60, 1.80, 1.85, 1.40]
    , [90, 30, 80, 40, 35, 45, 50, 60, 65, 30]
    ]


y = [
      [17.9, 16.5, 16.4, 16.8, 18.8, 15.5, 17.5, 16.4, 15.9, 18.3]
    ]

x = np.array(x).T
x_given = np.array(x_given).T
y = np.array(y).T
print(f'x:\n {x}\n')
print(f'y:\n {y}\n')
```

In [4]:
```
linearfit(x=x, y=y, x_given=x_given, alpha=0.05,order=[1,1,1],plot=True)
```

```
Polyinomial Model with order 2

X with order [1, 1, 1]:
 [[  1.     5.    25.  ]
 [  1.     7.5   56.25]
 [  1.    10.   100.  ]
 [  1.    12.5  156.25]
 [  1.    15.   225.  ]
 [  1.    17.5  306.25]
 [  1.    20.   400.  ]]

x_given:
 [[1 0 0]]

b with order [1, 1, 1]:
 [[-1.03571429]
 [ 0.31285714]
 [ 0.02      ]]
```

Model Comparison

30.14. Partial F-Test for Model Sufficiency. Let $x_1, \dots, x_p$ be possible predictor variables for Y and (30.5) and (30.6) the full and reduced models, respectively. Then

$$H_0: \text{the reduced model is sufficient}$$

is rejected at significance level α if the test statistic

$$F_{p-m,n-p-1} = \frac{n-p-1}{p-m} \frac{\text{SS}_{\text{E;reduced}} - \text{SS}_{\text{E;full}}}{\text{SS}_{\text{E;full}}} \tag{30.7}$$

satisfies $F_{p-m,n-p-1} > f_{\alpha,p-m,n-p-1}$.

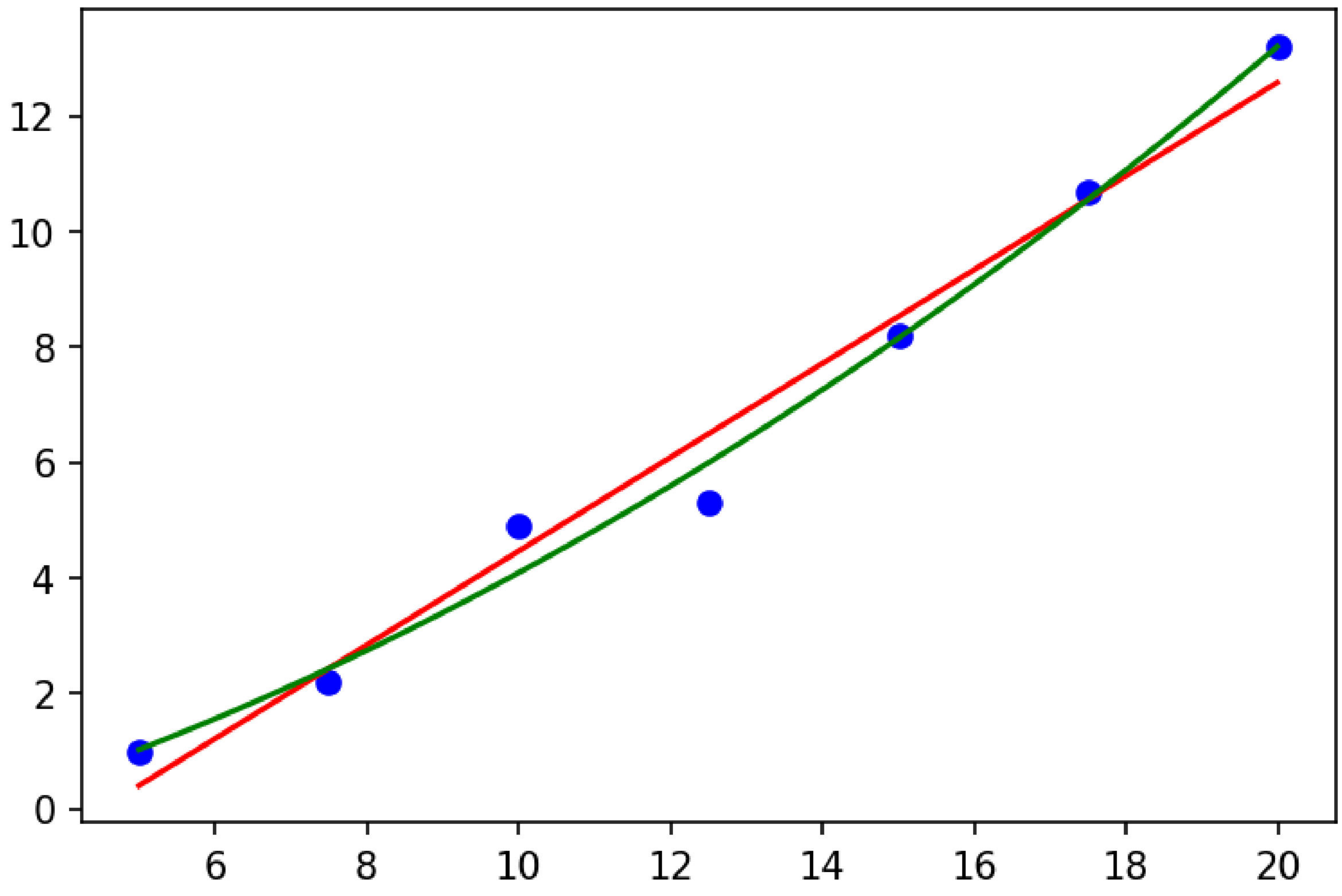
In [5]:
```
#figsize(12.5, 4) # 设置 figsize
plt.rcParams['savefig.dpi'] = 150 #图片像素
plt.rcParams['figure.dpi'] = 150 #分辨率

def modelcomparison(x,y,order1,order2,alpha):
    (m_x,n_x)=x.shape
    (m_y,n_y)=y.shape
    assert m_x>=n_x and m_y>=n_y and m_x==m_y and n_x==1
    assert len(order1)==len(order2)
    print(f'Polyinomial Models\n')

    X1 = np.array([[]])
    X2 = np.array([[]])

    for i in range(0,len(order1)):
        if order1[i] == 1:
            if X1.size == 0:
                X1 = x**i
            else:
                X1 = np.concatenate((X1,x**i),axis=1)
        if order2[i] == 1:
            if X2.size == 0:
                X2 = x**i
            else:
                X2 = np.concatenate((X2,x**i),axis=1)


    Y = y

    print(f'X1:\n {X1}\n')
    print(f'X2:\n {X2}\n')
    tmp = np.dot(np.linalg.inv(np.dot(X1.T,X1)),X1.T)
    b1 = np.dot(tmp,Y)
    print(f'b1 with order {order1}:\n {b1}\n')
    H1 = np.dot(X1,tmp)
    tmp = np.dot(np.linalg.inv(np.dot(X2.T,X2)),X2.T)
    b2 = np.dot(tmp,Y)
    print(f'b2 with order {order2}:\n {b2}\n')
    H2 = np.dot(X2,tmp)
    assert np.sum(np.dot(H1,X1)-X1) < 10**-5
    assert np.sum(np.dot(H2,X2)-X2) < 10**-5
    E1 = Y-np.dot(X1,b1)
    E2 = Y-np.dot(X2,b2)
    #print(f'E:\n {E1}\n')
    sse1 = np.sum(E1**2)
    sse2 = np.sum(E2**2)
    print(f'sse1:\n {sse1}\n')
    print(f'sse2:\n {sse2}\n')
    m = sum(order1)-1
    p = sum(order2)-1
    F = (m_x-p-1)/(p-m)*(sse1-sse2)/sse2
    print(f'F[{p-m},{m_x-p-1}]:\n{F}\n')
    print(f'F[{alpha}][{p-m},{m_x-p-1}]\n{f.isf(alpha, p-m, m_x-p-1)}\n')


    x_min = min(x)
    x_max = max(x)
    y_min = min(y)
    y_max = max(y)
    x_interval  = np.arange(x_min,x_max,(x_max-x_min)/1000)
    y1_interval  = np.zeros(x_interval.shape,dtype=np.float32)
    y2_interval  = np.zeros(x_interval.shape,dtype=np.float32)

    j1=0
    j2=0
    for i in range(0,len(order1)):
        if order1[i] == 1:
            y1_interval += b1[j1][0]*x_interval**i
            j1 += 1
        if order2[i] == 1:
            y2_interval += b2[j2][0]*x_interval**i
            j2 += 1
    plt.plot(x_interval,y1_interval,c='r')
    plt.plot(x_interval,y2_interval,c='g')
    plt.scatter(x.T[0],y.T[0],c='b')
```

In [6]:
```python
x = [[5, 7.5, 10, 12.5, 15, 17.5, 20]]
y = [[1, 2.2, 4.9, 5.3, 8.2, 10.7, 13.2]]
x_given = [[0]]
#x = [[5, 5, 10, 10, 15, 15, 20, 20, 25, 25]]
#y = [[14.0, 12.5, 7.0 ,5.0, 2.1, 1.8, 6.2, 4.9, 13.2, 14.6]]
#x_given = [[0]]

x = np.array(x).T
y = np.array(y).T
print(f'x:\n {x}\n')
print(f'y:\n {y}\n')
```

```
x:
 [[ 5. ]
 [ 7.5]
 [10. ]
 [12.5]
 [15. ]
 [17.5]
 [20. ]]

y:
 [[ 1. ]
 [ 2.2]
 [ 4.9]
 [ 5.3]
 [ 8.2]
 [10.7]
 [13.2]]
```

```
In [7]: modelcomparison(x=x,y=y,order1=[1,1,0],order2=[1,1,1],alpha=0.05)
```

```
Polyinomial Models

X1:
 [[ 1.   5. ]
 [ 1.   7.5]
 [ 1.  10. ]
 [ 1.  12.5]
 [ 1.  15. ]
 [ 1.  17.5]
 [ 1.  20. ]]

X2:
 [[  1.     5.    25.  ]
 [  1.     7.5   56.25]
 [  1.    10.   100.  ]
 [  1.    12.5  156.25]
 [  1.    15.   225.  ]
 [  1.    17.5  306.25]
 [  1.    20.   400.  ]]

b1 with order [1, 1, 0]:
 [[-3.66071429]
 [ 0.81285714]]

b2 with order [1, 1, 1]:
 [[-1.03571429]
 [ 0.31285714]
 [ 0.02      ]]

sse1:
 2.531071428571428

sse2:
 1.2185714285714286

F[1,4]:
4.308323563892143

F[0.05][1,4]
7.708647422176786
```



Evidence of lack of fit

$$\mathrm{SS_E} = \sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(Y_{ij} - (b_0 + b_1 x_i)\right)^2$$

$$\mathrm{SS_{E;pe}} := \sum_{i=1}^{k}\sum_{j=1}^{n_i}(Y_{ij} - \overline{Y}_i)^2$$

$$\mathrm{SS_{E;lf}} := \mathrm{SS_E} - \mathrm{SS_{E;pe}}\,.$$

Number 2 used in the following context is due to the fact that the linear model only use two estimators. Howeve, there can also be more

28.3. Test for Lack of Fit. Let $x_1, \ldots, x_k$ be regressors and $Y_{i1}, Y_{i2}, \ldots, Y_{in_i}$, $i = 1, \ldots, k$, the measured responses at each of the regressors. Let $\mathrm{SS_{E;pe}}$ and $\mathrm{SS_{E;lf}}$ be the pure error and lack-of-fit sums of squares for a linear regression model. Then

$$H_0: \text{the linear regression model is appropriate}$$

is rejected at significance level α if the test statistic

$$F_{k-2,n-k} = \frac{\mathrm{SS_{E;lf}}/(k-2)}{\mathrm{SS_{E;pe}}/(n-k)}$$

satisfies $F_{k-2,n-k} > f_{\alpha,k-2,n-k}$.

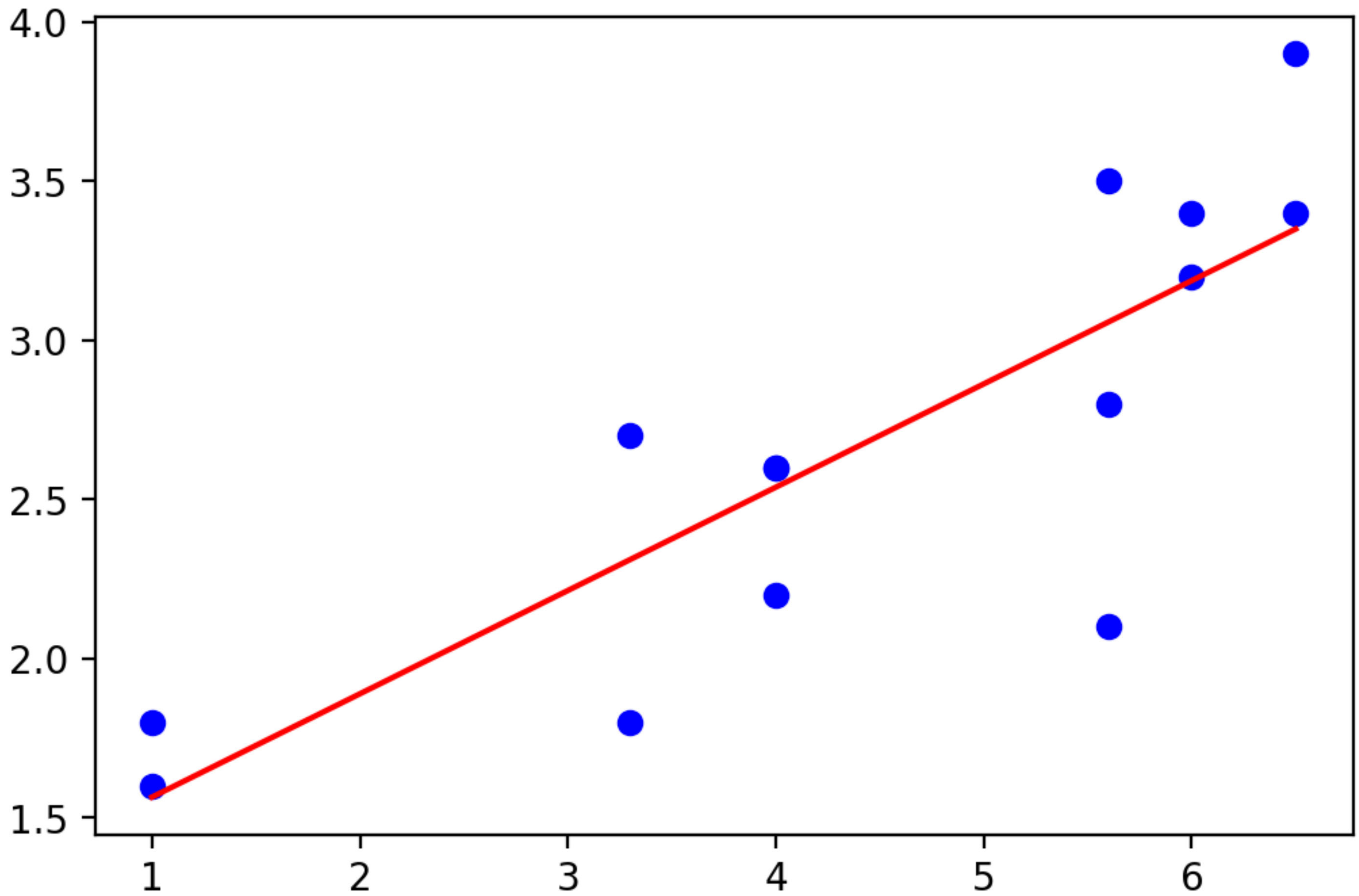
In [8]:
```python
plt.rcParams['savefig.dpi'] = 200 #图片像素
plt.rcParams['figure.dpi'] = 200 #分辨率

def lackoffit(x, y, alpha,order=None,plot=False):
    (m_x,n_x)=x.shape
    (m_y,n_y)=y.shape
    assert m_x>=n_x and m_y>=n_y and m_x==m_y

    if order == None:
        print('No data for order, error\n')
        quit()

    else:
        print(f'Polyinomial Model with order {len(order)-1}\n')
        assert n_x == 1

        X = np.array([[]])

        for i in range(0,len(order)):
            if order[i] == 1:
                if X.size == 0:
                    X = x**i
                else:
                    X = np.concatenate((X,x**i),axis=1)

        Y = y
        n_x = X.shape[1]-1

    data = {}
    keys = list(x.T[0])
    values = list(y.T[0])
    for i in range(0,len(x)):
        if keys[i] not in data:
           data[keys[i]]=[values[i]]
        else:
           data[keys[i]].append(values[i])

    sse_pe = 0
    for key in data:
        value = np.array(data[key]).reshape(-1,1)
        sse_pe += np.sum((value-value.mean())**2)

    k = len(data)
    p = sum(order)


    print(f'X with order {order}:\n {X}\n')
    tmp = np.dot(np.linalg.inv(np.dot(X.T,X)),X.T)
    b = np.dot(tmp,Y)
    print(f'b with order {order}:\n {b}\n')
    H = np.dot(X,tmp)
    #print(f'H:\n {H}\n')
    assert np.sum(np.dot(H,X)-X) < 10**-5
    E = Y-np.dot(X,b)
    #print(f'E:\n {E}\n')
    sse = np.sum(E**2)
    print(f'sse:\n {sse}\n')
    print(f'sse_pe:\n {sse_pe}\n')
    sse_if = sse - sse_pe
    print(f'sse_if:\n {sse_if}\n')
    F = (sse_if/(k-p))/(sse_pe/(m_x-k))
    print(f'F[{k-p},{m_x-k}]:\n{F}\n')
    print(f'F[{alpha}][{k-p},{m_x-k}]\n{f.isf(alpha, k-p,m_x-k)}\n')


    if plot == True:
        if order!=None:
            x_min = min(x)
            x_max = max(x)
            x_interval  = np.arange(x_min,x_max,(x_max-x_min)/1000)
            y_interval  = np.zeros(x_interval.shape,dtype=np.float32)
            j=0

            for i in range(0,len(order)):
                if order[i] == 1:
                    y_interval += b[j][0]*x_interval**i
                    j += 1

            plt.plot(x_interval,y_interval,c='r')
            plt.scatter(x.T[0],y.T[0],c='b')
        else:
            print('Since the multilinear plot can hardly show anything clear, so no multilinear plot\n')

    #plt.scatter(x.T[0],E.T[0])
    #plt.scatter(x.T[0],y.T[0])
    #plt.show()
```

In [9]:
```python
x = [
      [1.0, 1.0, 3.3, 3.3, 4.0, 4.0, 4.0, 5.6, 5.6, 5.6, 6.0, 6.0, 6.5, 6.5]
    ]


y = [
      [1.6, 1.8, 1.8, 2.7, 2.6, 2.6, 2.2, 3.5, 2.8, 2.1, 3.4, 3.2, 3.4, 3.9]
    ]

x = np.array(x).T
y = np.array(y).T
print(f'x:\n {x}\n')
print(f'y:\n {y}\n')
```

```
x:
 [[1. ]
 [1. ]
 [3.3]
 [3.3]
 [4. ]
 [4. ]
 [4. ]
 [5.6]
 [5.6]
 [5.6]
 [6. ]
 [6. ]
 [6.5]
 [6.5]]

y:
 [[1.6]
 [1.8]
 [1.8]
 [2.7]
 [2.6]
 [2.6]
 [2.2]
 [3.5]
 [2.8]
 [2.1]
 [3.4]
 [3.2]
 [3.4]
 [3.9]]
```

In [10]:
```
lackoffit(x=x, y=y, alpha=0.05,order=[1,1],plot=True)
```

```
Polyinomial Model with order 1

X with order [1, 1]:
 [[1.  1. ]
 [1.  1. ]
 [1.  3.3]
 [1.  3.3]
 [1.  4. ]
 [1.  4. ]
 [1.  4. ]
 [1.  5.6]
 [1.  5.6]
 [1.  5.6]
 [1.  6. ]
 [1.  6. ]
 [1.  6.5]
 [1.  6.5]]

b with order [1, 1]:
 [[1.23963312]
 [0.32444129]]

sse:
 2.120946906084485

sse_pe:
 1.6566666666666667

sse_if:
 0.4642802394178185

F[4,8]:
0.5604992829993785

F[0.05][4,8]
3.837853354555897
```



Multilinear plot 3D

In [11]:

```python
#figsize(12.5, 4) # 设置 figsize
plt.rcParams['savefig.dpi'] = 150 #图片像素
plt.rcParams['figure.dpi'] = 150 #分辨率

from mpl_toolkits import mplot3d
import matplotlib.pyplot as plt
import numpy as np


delta =0.1
def plot3D(delta=0.01,n1=20000,n2=20000,X=np.arange(1,2.1, 0.01),Y=np.arange(30,90, 0.1)):

    X, Y = np.meshgrid(X, Y, sparse=False)
    fig=plt.figure(num=0,figsize=(10, 10))
    ax = plt.axes(projection='3d')
    #ax=fig.gca(projection="3d")
    #ax.view_init(elev=elev, azim=azim)
    [b0,b1,b2]= [24.7489, -4.1593, -0.014895]

    Z = b0 + b1*X + b2*Y
    #ax.scatter(x.T[0],x.T[1],y.T[0], c='r', marker='o')
    #ax.plot_surface(X, Y, Z, cmap='Greys')
    ax.plot_wireframe(X, Y, Z, rstride=100, cstride=10)
    X = x.T[0]
    Y = x.T[1]
    Z = y.T[0]
    index = (b0 + b1*X + b2*Y<=Z)

    ax.scatter(X[index],Y[index],Z[index], c='r', marker='o')
    ax.scatter(X[index!=1],Y[index!=1],Z[index!=1], c='g', marker='o')
```

In [12]:

```python
x = [ [1.35, 1.90, 1.70, 1.80, 1.30, 2.05, 1.60, 1.80, 1.85, 1.40]
    , [90, 30, 80, 40, 35, 45, 50, 60, 65, 30]
    ]


y = [
      [17.9, 16.5, 16.4, 16.8, 18.8, 15.5, 17.5, 16.4, 15.9, 18.3]
    ]

x = np.array(x).T
y = np.array(y).T
print(f'x:\n {x}\n')
print(f'y:\n {y}\n')
```

```
x:
 [[ 1.35 90.  ]
 [ 1.9  30.  ]
 [ 1.7  80.  ]
 [ 1.8  40.  ]
 [ 1.3  35.  ]
 [ 2.05 45.  ]
 [ 1.6  50.  ]
 [ 1.8  60.  ]
 [ 1.85 65.  ]
 [ 1.4  30.  ]]

y:
 [[17.9]
 [16.5]
 [16.4]
 [16.8]
 [18.8]
 [15.5]
 [17.5]
 [16.4]
 [15.9]
 [18.3]]
```

In [13]:
```
plot3D()
```