TUTORIAL 07

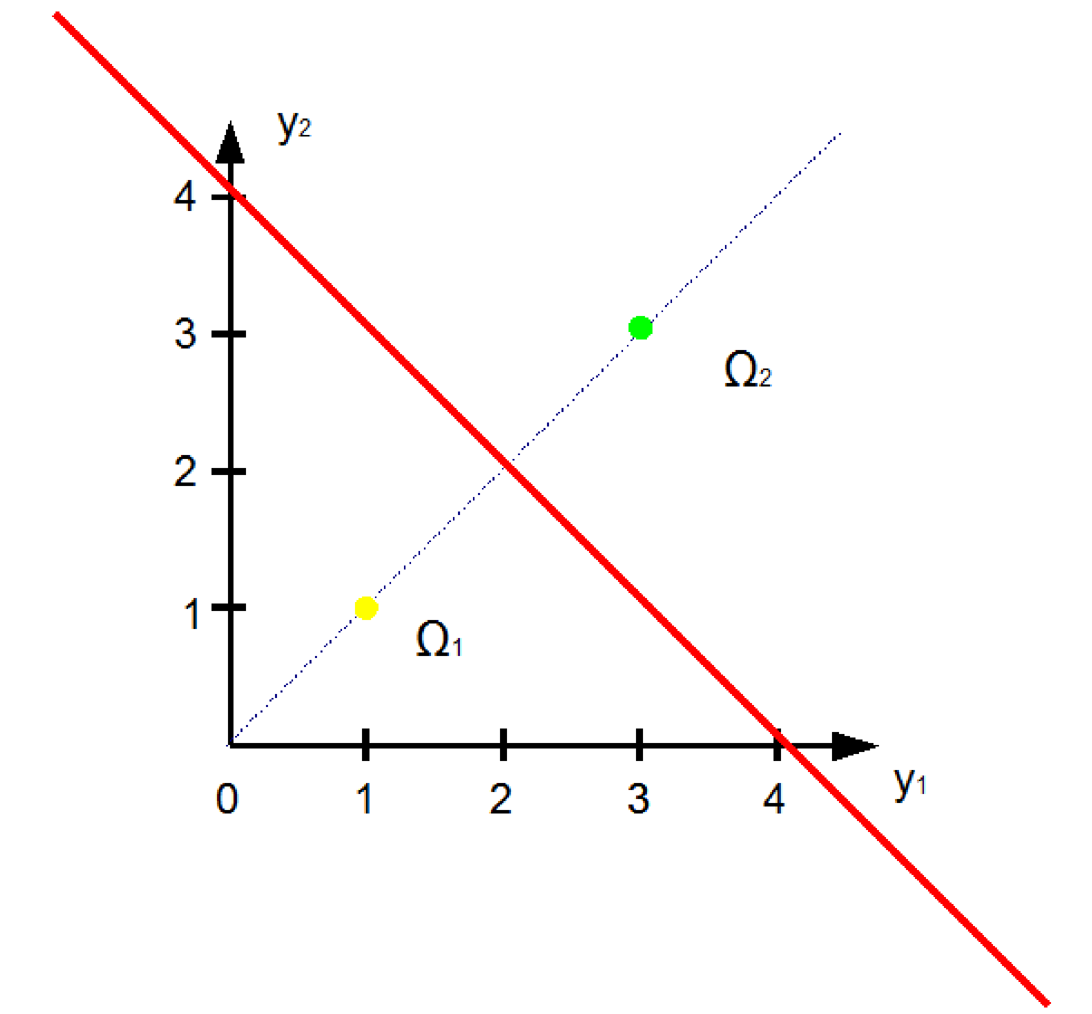
Notiztitel 13.06.2014

Outline:

Pattern classification

- Classification with distance functions:
  - Euclidean distance
  - Mahalanobis distance

Exercise 17



$y_2 = -y_1 + 4$

b) Derive decision border, mathematically

$$d(\underline{w}_1, \underline{y}) \overset{!}{=} d(\underline{w}_2, \underline{y})$$

$$\sqrt{\sum_{i=1}^{N} (w_{i1} - y_i)^2} = \sqrt{\sum_{i=1}^{N} (w_{i2} - y_i)^2}$$

$$\sum_{i=1}^{N} (w_{i1} - y_i)^2 = \sum_{i=1}^{N} (w_{i2} - y_i)^2$$

$$(w_{11} - y_1)^2 + (w_{21} - y_2)^2 = (w_{12} - y_1)^2 + (w_{22} - y_2)^2$$

...

$$y_1\left(-2w_{11}+2w_{12}\right) = -w_{11}^2 - w_{21}^2 + y_2\left(2w_{21}-2w_{22}\right) + $$
$$+ w_{12}^2 + w_{22}^2$$

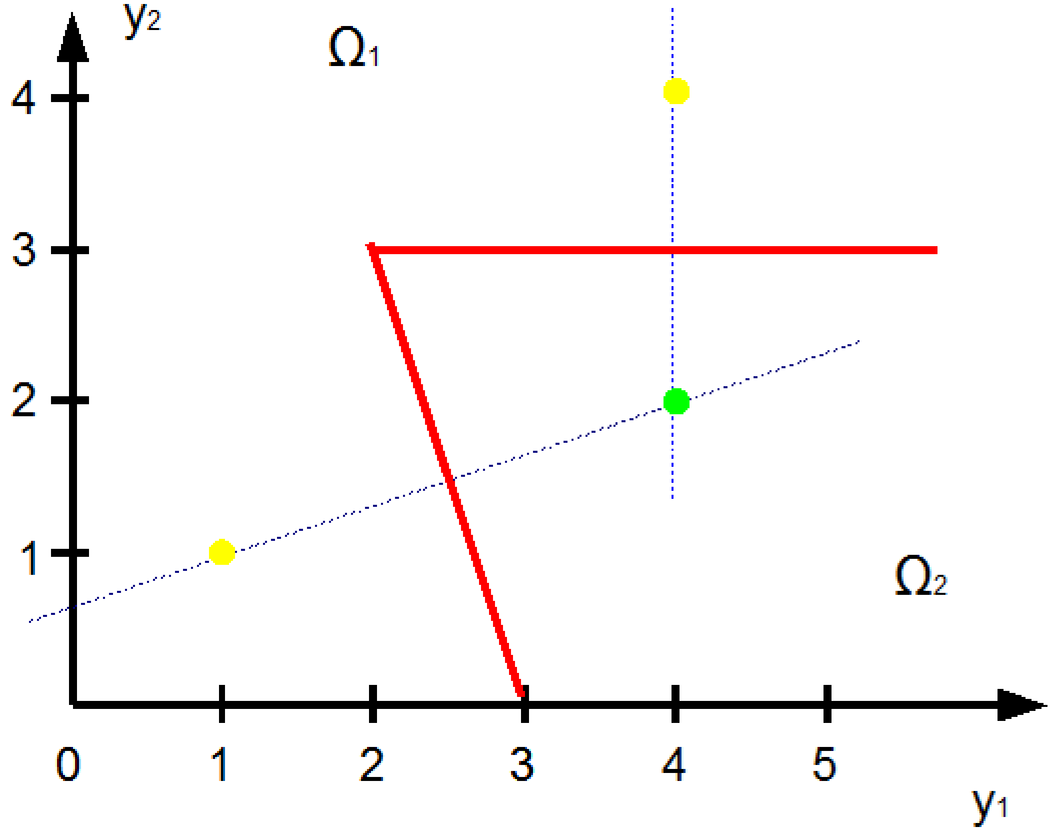
$$y_2 = \frac{w_{12}-w_{11}}{w_{21}-w_{22}}\, y_1 + \frac{w_{11}^2 + w_{21}^2 - w_{12}^2 - w_{22}^2}{2w_{21}-2w_{22}} \quad \leftarrow$$

In our case:

$$\underline{w}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \underline{w}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$y_2 = \frac{3-1}{1-3}\, y_1 + \frac{1+1-9-9}{2-6} = -y_1 + 4$$

c) KNN decision border



piecewise linear decision border

d) Mahalanobis distance:

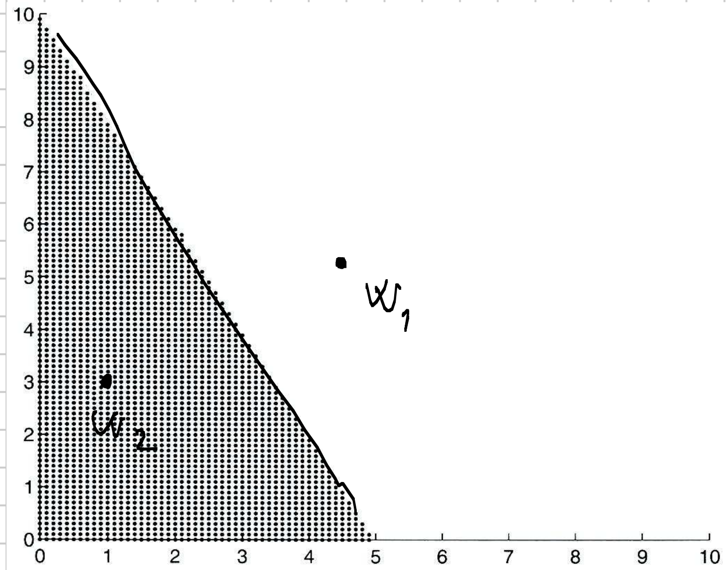
$$d(w, y) = (\underline{w} - \underline{y})^T \cdot C^{-1} (\underline{w} - \underline{y})$$

$$w_1 = \begin{bmatrix} 5 \\ 5 \end{bmatrix} \quad \underset{\sim}{C}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \underline{w}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad \underset{\sim}{C}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

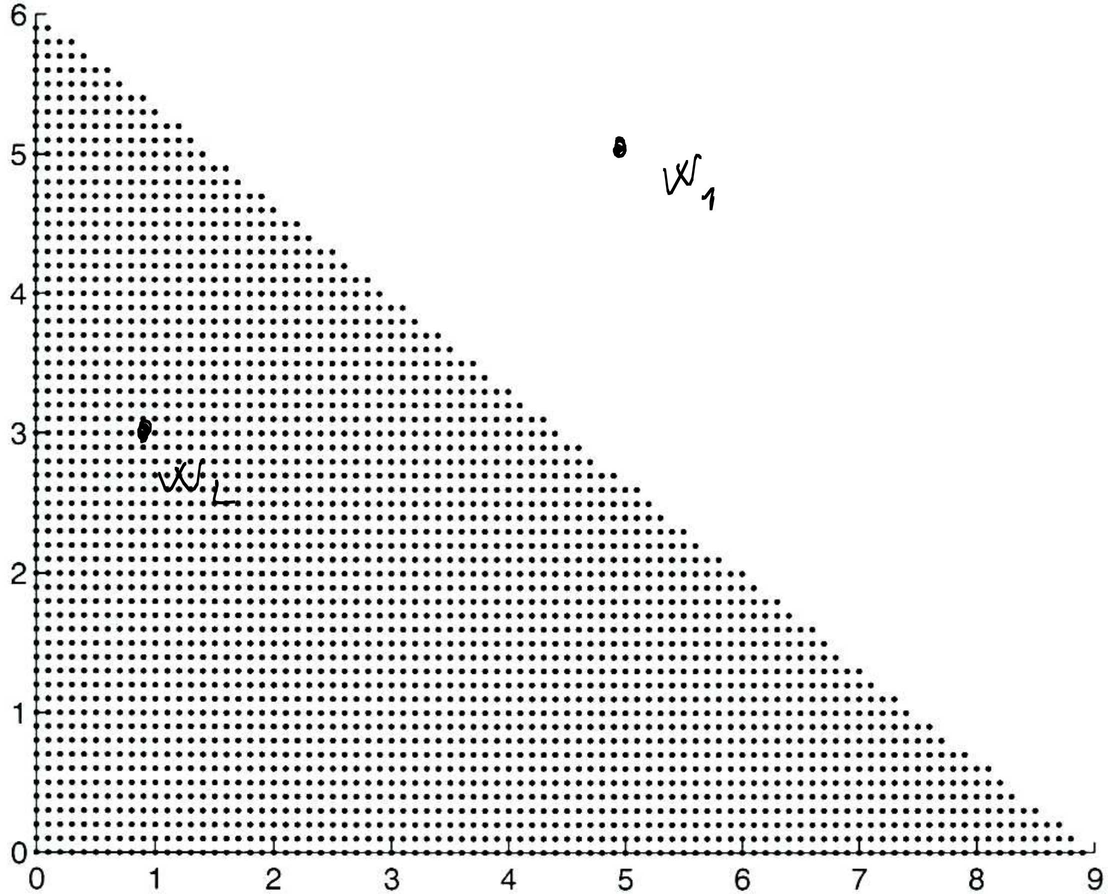
equal C

C identity

=> euclidean distance



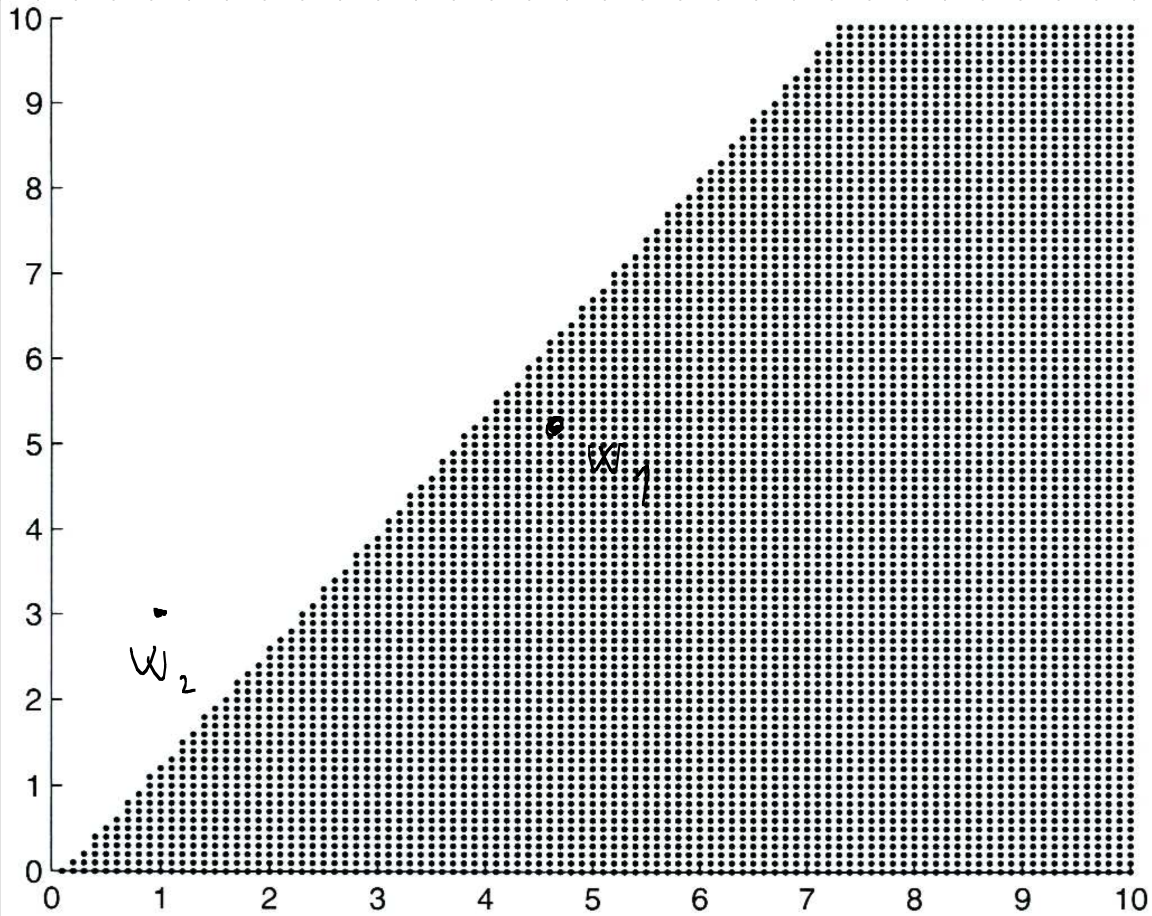
$$C_1 = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \qquad C_2 = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$



equal covariance matrices

linear decision border

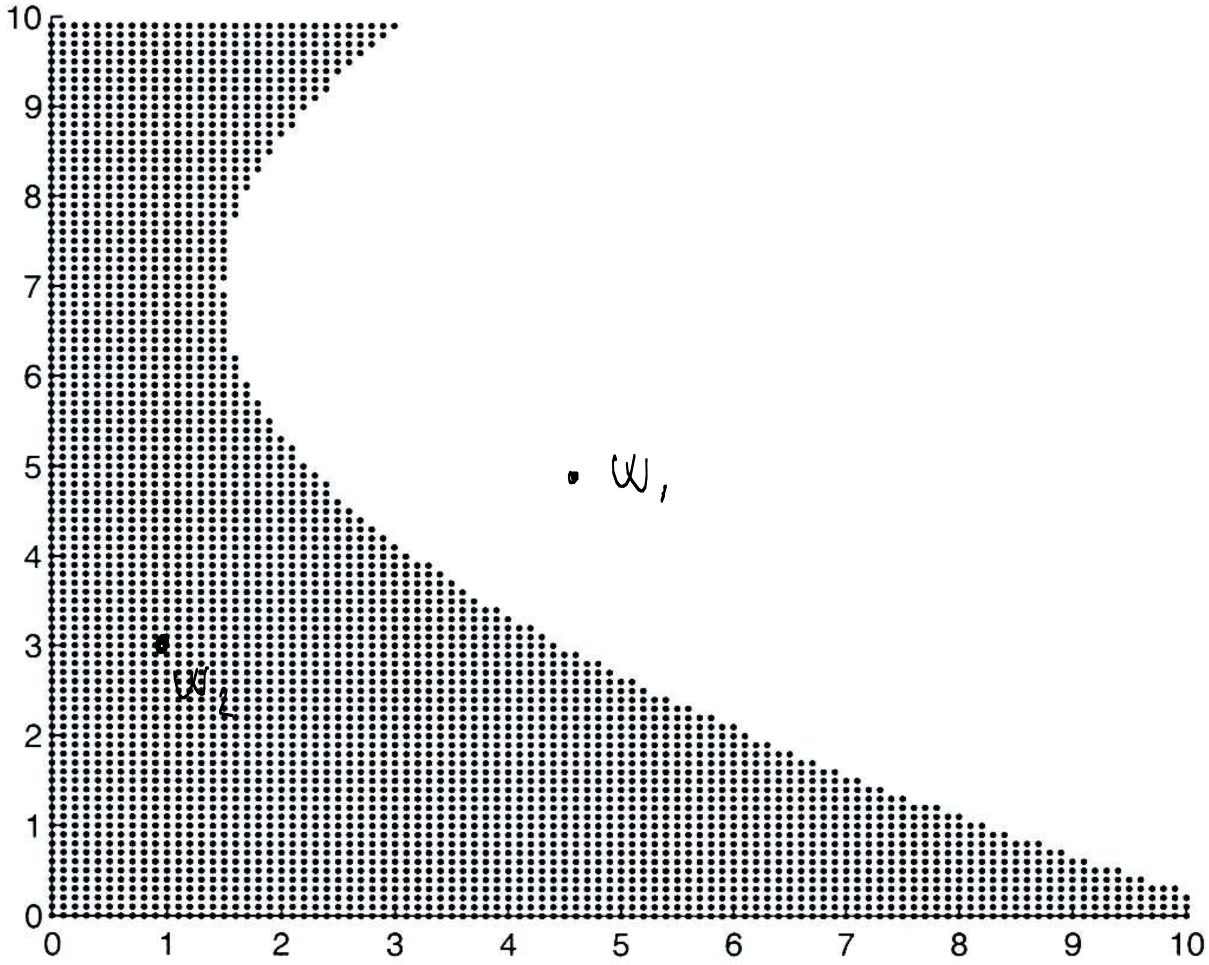
$$C_1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad C_2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$



linear decision border

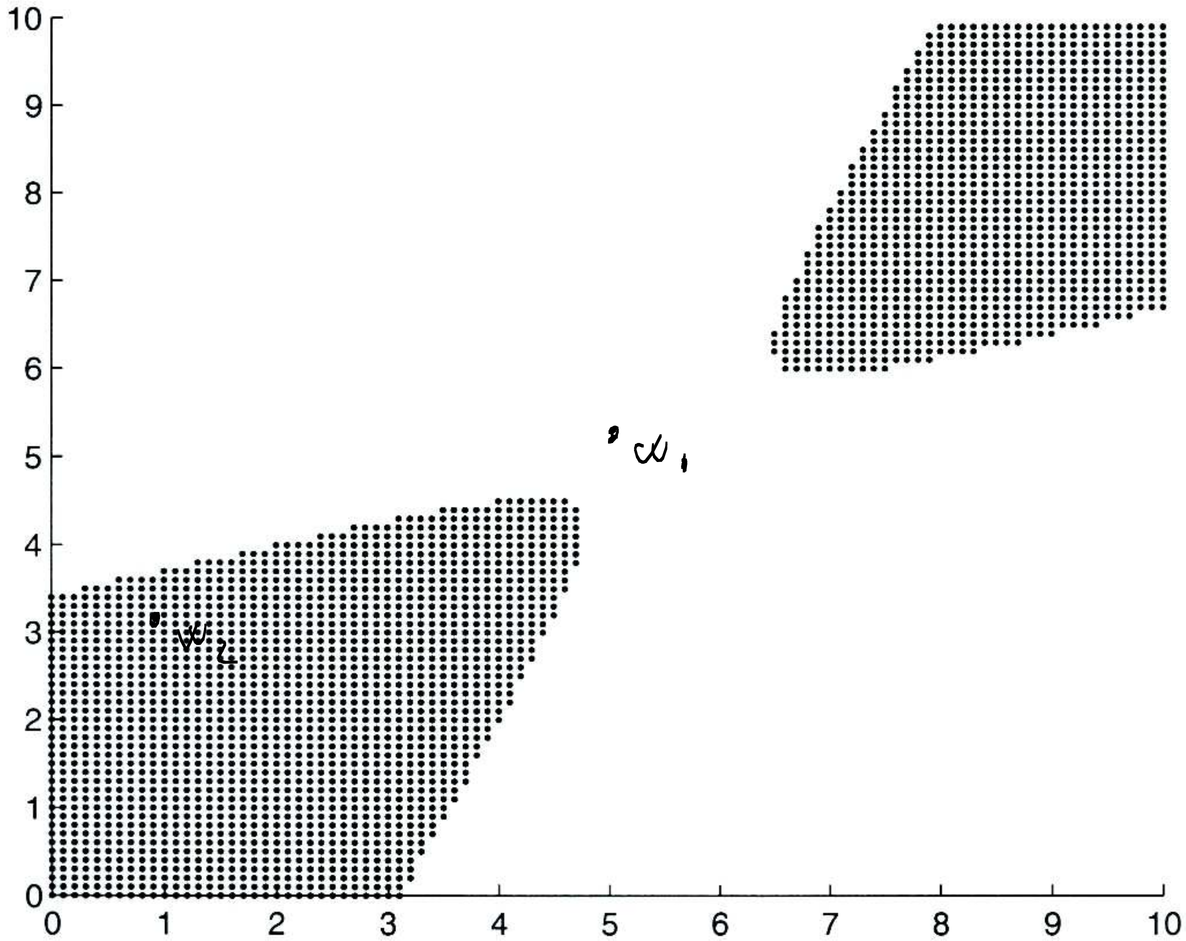
equal cov. matrix

$$C_1 = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \qquad C_2 = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$



different covariance matrices

$$C_1 = \begin{bmatrix} 1 & 0.5 \\ 2 & 0.7 \end{bmatrix} \qquad C_2 = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$$



different covariance matrices

Exercise 18

2 - class problem

$w_{\Omega_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $w_{\Omega_2} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ Reference

$Y_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $Y_2 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ patterns classify

a) $d(w_{\Omega_1}, Y_1) = \sqrt{\sum_{i=1}^{N} (w_i - Y_i)^2} = \sqrt{(1-1)^2 + (1-2)^2} = \sqrt{0+1} = 1$

$d(w_{\Omega_2}, Y_1) = \sqrt{(3-1)^2 + (3-2)^2} = \sqrt{4+1} \simeq 2.24$

$d(w_{\Omega_1}, Y_1) < d(w_{\Omega_2}, Y_1)$

Nearest Neighbour (NN)

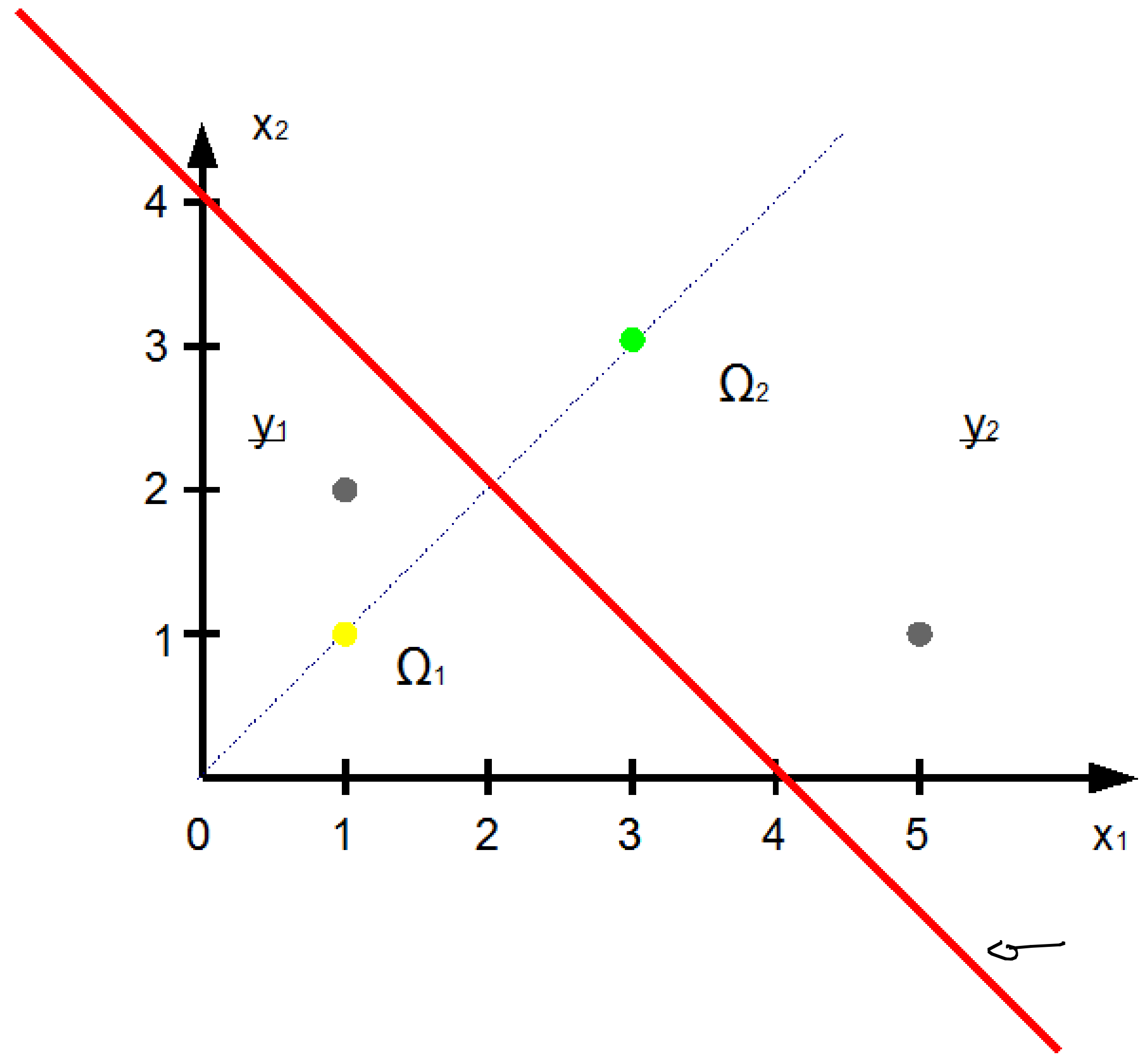
$$\Omega^*_m = \min_m \{ d(w_m, Y_1) \} \qquad (Eq.\ 3.9)$$

$$Y_1 \Rightarrow \Omega_1$$

$$Y_2: \quad d(w_{\Omega_1}, Y_2) = \sqrt{(1-5)^2 + (1-1)^2} = \sqrt{4^2} = 4$$

$$d(w_{\Omega_2}, Y_2) = \sqrt{(3-5)^2 + (3-1)^2} = \sqrt{8} \simeq 2.83$$

$$d(w_{\Omega_2}, Y_2) < d(w_{\Omega_1}, Y_2) \Rightarrow \Omega_2$$


x_2
4
3
2
1
0
1
2
3
4
5
x_1
$\underline{y}_1$
$\underline{y}_2$
Ω_1
Ω_2

b) Mahalanobis distance

$$d(\underline{w}, \underline{y}) = (\underline{w} - \underline{y})^T \cdot \underset{\sim}{C}^{-1} (\underline{w} - \underline{y}) \leftarrow (\text{Eq. } 3.6)$$

Y_1 : $$d(w_{\Omega_1}, Y_1) = \begin{bmatrix} 1-1 \\ 1-2 \end{bmatrix}^T \begin{bmatrix} -2.3 & 1.7 \\ 6.7 & -3.3 \end{bmatrix} \begin{bmatrix} 1-1 \\ 1-2 \end{bmatrix} =$$

$$= \begin{bmatrix} -6.7 & 3.3 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \underline{-3.3}$$

$$d(w_{\Omega_2}, Y_1) = \begin{bmatrix} 3-1 \\ 3-2 \end{bmatrix}^T \begin{bmatrix} -0.4 & 0.6 \\ 0.6 & -0.4 \end{bmatrix} \begin{bmatrix} 3-1 \\ 3-2 \end{bmatrix} =$$

$$= \begin{bmatrix} -0.2 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \underline{0.4}$$

$\underline{NN}$: $d(w_{\Omega_1}, y_1) < d(w_{\Omega_2}, y_1) \Rightarrow y_1 \Rightarrow \Omega_1$

y_2 : $d(w_{\Omega_1}, y_2) = -37.3$ $\qquad d(w_{\Omega_2}, y_2) = -8$

$y_2 \Rightarrow \Omega_1$