TUTORIAL 04

Notiztitel 23.05.2014

Outline:

Preprocessing:

- Gabor - Transformation

EXERCISE 11

Gabor elementary functions (2.49)

$$g(\alpha, t) = \Psi_{nm}(t) = e^{\left(-\pi \frac{t^2}{\sigma^2} + j\alpha t\right)}$$

We need the Fourier-Transformation $G(\omega)$ of $g(\alpha, t)$

$$G(\omega) = \int_{-\infty}^{\infty} e^{-\pi \frac{t^2}{\sigma^2}} \cdot e^{j\alpha t} \cdot e^{-j\omega t} \, dt =$$

$$= \int_{-\infty}^{\infty} e^{-\pi \frac{t^2}{\sigma^2}} \cdot e^{j(\alpha - \omega)t} \, dt =$$

$$= \int_{-\infty}^{\infty} e^{-\pi \frac{t^2}{\sigma^2}} \underline{\cos}(\alpha - \omega) t \, dt + \underbrace{j \int_{-\infty}^{\infty} e^{-\pi \frac{t^2}{\sigma^2}} \sin(\alpha - \omega) t \, dt}_{0}$$

$$G(\omega) = \sigma \cdot e^{-\frac{\sigma^2 (\omega - \alpha)^2}{4\pi}}$$

- Effective time extend Δt :

$$(\Delta t)^2 = \frac{\int_{-\infty}^{\infty} (t - \bar{t})^2 \cdot g(t) \cdot g^*(t) \, dt}{\int_{-\infty}^{\infty} g(t) \cdot g^*(t) \, dt} = \frac{A}{B}$$

<u>Integral A</u> : $\bar{t} = 0$ $g(\alpha, t)$ is symmetric to y-axis

$$A = \int_{-\infty}^{\infty} t^2 \cdot \underset{\uparrow}{g(t)} \cdot \underset{\uparrow}{g^*(t)} \, dt =$$

$$= \int_{-\infty}^{\infty} t^2 \cdot e^{\left(-\pi \frac{t^2}{\sigma^2} \cancel{+ j\alpha t}\right)} \cdot e^{\left(-\pi \frac{t^2}{\sigma^2} \cancel{- j\alpha t}\right)} \, dt =$$

$$= \int_{-\infty}^{\infty} t^2 \cdot e^{-2\pi \frac{t^2}{\sigma^2}} \, dt = \frac{\sigma^3}{4 \cdot \sqrt{2} \cdot \pi}$$

<u>Integral B</u>

$$B = \int_{-\infty}^{\infty} g(t) \cdot g^*(t) \, dt =$$

$$= \int_{-\infty}^{\infty} e^{\left(-\pi \frac{t^2}{\sigma^2} \cancel{+ j\alpha t}\right)} \cdot e^{\left(-\pi \frac{t^2}{\sigma^2} \cancel{- j\alpha t}\right)} \, dt$$

$$= \int_{-\infty}^{\infty} e^{-2\pi \frac{t^2}{\sigma^2}} \, dt = \frac{\sigma}{\sqrt{2}}$$

$$(\Delta t)^2 = \frac{A}{B} = \frac{\sigma^3}{4 \cdot \sqrt{2}\, \pi} \cdot \frac{\sqrt{2}}{\sigma} = \frac{\sigma^2}{4 \cdot \pi} \quad \sigma$$

- Effective spectral extend $\Delta \omega$:

$$(\Delta \omega)^2 = \frac{\int_{-\infty}^{\infty} (\omega - \bar{\omega})^2 \cdot G(\omega) \cdot G^*(\omega) \, d\omega}{\int_{-\infty}^{\infty} G(\omega) \cdot G^*(\omega) \, d\omega} = \frac{C}{D}$$

Integral C

$$G(\omega) = \sigma \cdot e^{-\frac{\sigma^2(\omega-\alpha)^2}{4\pi}}$$

$\bar{\omega} = \alpha$

↖ Gaussian displaced by α

$$C = \int_{-\infty}^{\infty} (\omega-\alpha)^2 \cdot G(\omega) \cdot G^*(\omega)\, d\omega =$$

$$= \int_{-\infty}^{\infty} (\omega-\alpha)^2 \cdot \sigma \cdot e^{-\frac{\sigma^2(\omega-\alpha)^2}{4\pi}} \cdot \sigma \cdot e^{-\frac{\sigma^2(\omega-\alpha)^2}{4\pi}}\, d\omega =$$

$$= \int_{-\infty}^{\infty} (\omega-\alpha)^2 \cdot \sigma^2 \cdot e^{-\frac{\sigma^2(\omega-\alpha)^2}{2\pi}}\, d\omega =$$

Substitute:

$x = \omega - \alpha$

$dx = d\omega$

$$= \int_{-\infty}^{\infty} x^2 \cdot \sigma^2 \cdot e^{-\frac{\sigma^2 x^2}{2\pi}}\, dx = \frac{\pi^2 \cdot \sqrt{2}}{\sigma}$$ //

Integral D

$$D = \int_{-\infty}^{\infty} G(\omega) \cdot G^{*}(\omega)\, d\omega =$$

$$= \int_{-\infty}^{\infty} \sigma^2 \cdot e^{-\frac{\sigma^2 (\omega - \alpha)^2}{2\pi}}\, d\omega \rightarrow \qquad x = \omega - \alpha$$

$$dx = d\omega$$

$$= \int_{-\infty}^{\infty} \sigma^2 \cdot e^{-\frac{\sigma^2 x^2}{2\pi}}\, dx = \sigma \pi \sqrt{2}$$

$$(\Delta\omega)^2 = \frac{C}{D} = \frac{\pi^{\cancel{2}} \cdot \cancel{\sqrt{2}}}{\sigma} \cdot \frac{1}{\sigma \cancel{\pi} \cancel{\sqrt{2}}} = \frac{\pi}{\sigma^2}$$

$$\Delta t \cdot \Delta \omega = \frac{\cancel{6}}{2 \cdot \cancel{\sqrt{11}}} \cdot \frac{\cancel{\sqrt{11}}}{\cancel{6}} = \frac{1}{2}$$

EXERCISE 12

DFT

Consider $e^{j\omega t}$ for the frequency discrimination of the DFT

The basic frequency $\omega = \frac{2\pi}{T}$ ←

Now we split ω into N equal parts :

$$\omega_m = \frac{2\pi}{T} \cdot \frac{m}{N} \qquad (2.19) \qquad \text{DFT}$$

$$= \underbrace{\frac{2\pi}{TN}}_{=\Omega} \cdot m \rightarrow \omega_m = \Omega \cdot m \rightarrow \Omega = \frac{2\pi}{TN}$$

Gabor - Transformation

$$y(k) = \sum_m \sum_n c_{nm} \cdot s(k - m \cdot T_0) \cdot e^{j \Omega n t} \quad \left(\begin{matrix} 2.51 \\ 2.52 \end{matrix} \right)$$

$$\Omega \leq \frac{2\pi}{\tau_0} \quad (\text{Eq. } 2.48)$$

with $\underline{\tau_0 = NT}$

$$\Rightarrow \Omega \leq \frac{2\pi}{NT}$$

length of the frame is sampling points · sampling frequency

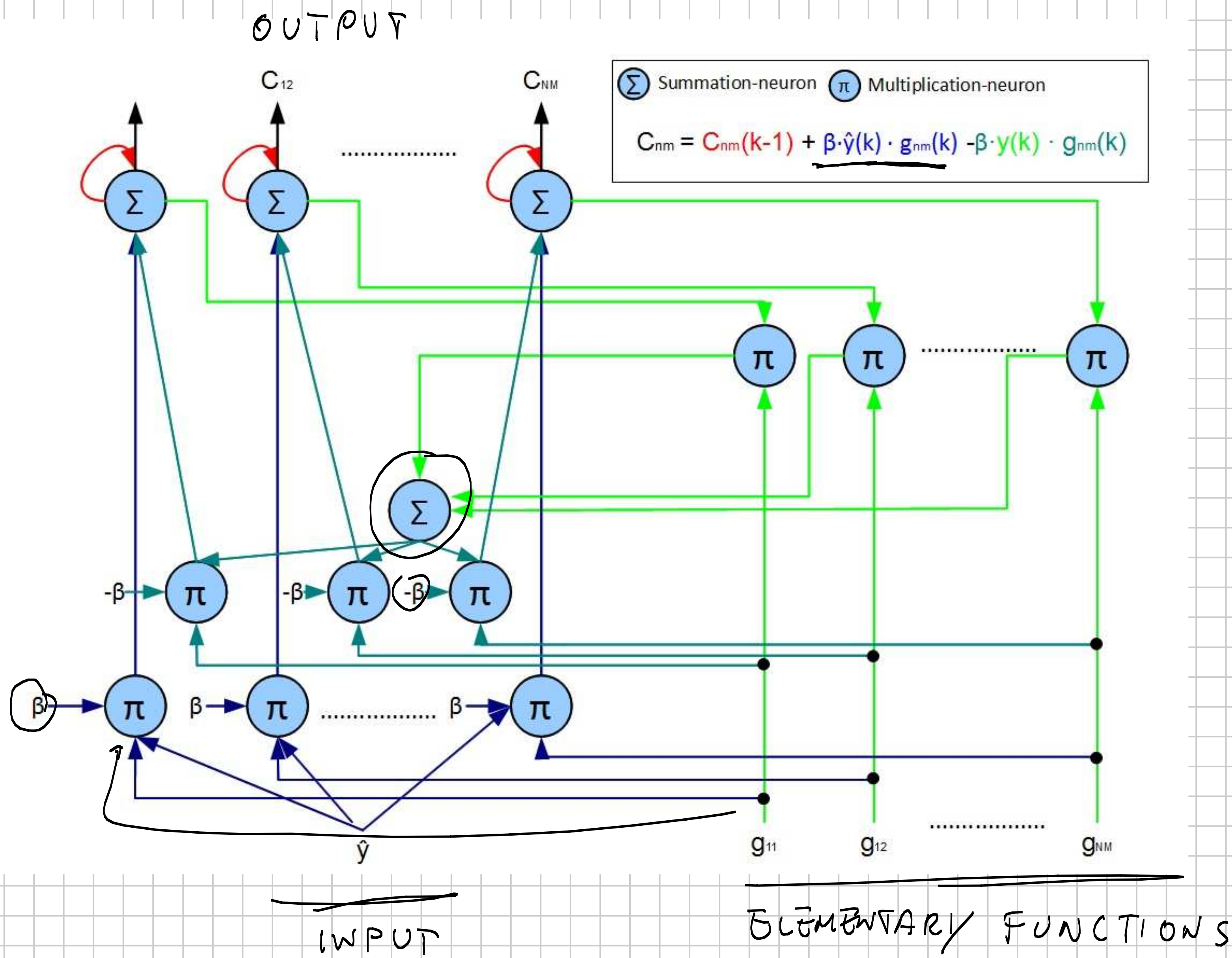
DFT an GT have the same frequency discrimination if the window size N is equal

Exercise 13

Learning rule for Gabor - coefficients (Eq. 2.63)

$$c_{nm}(k) = c_{nm}(k-1) + \beta\Big[\hat{y}(k) - \underbrace{\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} c_{nm}\cdot g_{nm}(k)}_{y(k)}\Big]\cdot g_{nm}(k)$$

$$= c_{nm}(k-1) + \beta\,\hat{y}(k)\cdot g_{nm}(k) - \beta\,y(k)\cdot g_{nm}(k)$$


OUTPUT
C12
CNM
Summation-neuron
Multiplication-neuron
Cnm = Cnm(k-1) + β·ŷ(k) · gnm(k) -β·y(k) · gnm(k)
-β
β
ŷ
g11
g12
gNM
INPUT
ELEMENTARY FUNCTIONS