PR Example exam

Exam information

11th July 2014

start 11:30

room N1189 N1095

duration 75 minutes

Task 1

a) $$Y_J(\omega) = \int_{-\infty}^{\infty} y(t) \cdot e^{-j\omega t}\, dt$$

$$= \int_0^{t_0} 1 \cdot e^{-j\omega t}\, dt =$$

$$= \left[-\frac{1}{j\omega} e^{-j\omega t} \right]_0^{t_0} = -\frac{1}{j\omega} \cdot e^{-j\omega t_0} + \frac{1}{j\omega} e^0$$

$$= \frac{1}{jw}\left(1 - e^{-jwt_0}\right)$$

b) $$Y_f(w) = \frac{1}{jw}\left(1 - e^{-jwt_0}\right) \qquad w = 2\pi f$$

$$Y_f(f) = \frac{1}{j2\pi f}\left(1 - e^{-j2\pi f t_0}\right)$$

$$\sin(\alpha) = \frac{1}{2j}\left(e^{j\alpha} - e^{-j\alpha}\right)$$

$$Y_f(f) = \frac{1}{\pi f}\sin(\pi f t_0) \cdot e^{-j\pi f t_0}$$

Task 2

a) Euclidean distance : $d(\bar{w}, \bar{y}) = \sqrt{\sum_{i=1}^{n} (w_i - y_i)^2}$

Nearest Neighbour Rule: $\Omega_m = \min_m \{ d(\bar{w}_m, \bar{y}) \}$

$$d(\bar{w}_1, \bar{y}_1) = \sqrt{(1-0)^2 + (1-1)^2 + (1-2)^2} = \sqrt{2} \approx 1.41$$

$$d(\bar{w}_2, \bar{y}_1) = \sqrt{(3-0)^2 + (4-1)^2 + (5-2)^2} = \sqrt{27} \approx 5.20$$

$$\Rightarrow \bar{y}_1 \rightarrow \Omega_1$$

$$d(\bar{w}_1, y_2) = \sqrt{(1-(-1))^2 + (1-6)^2 + (1-8)^2} = \sqrt{78} \approx 8.83$$

$$d(\bar{w}_2, y_2) = \sqrt{(3-(-1))^2 + (4-6)^2 + (5-8)^2} = \sqrt{29} \approx 5.39$$

$$\bar{y}_2 \rightarrow \Omega_2$$

b)

$$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} y_1^2 \\ y_2^2 \\ y_1 \\ y_2 \\ 1 \end{bmatrix}$$

$$\bar{e} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \underset{\sim}{W} \cdot \begin{bmatrix} 4 & 25 & 2 & 5 & 1 \end{bmatrix}^T$$

$$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 2 \cdot 4 + 4 \cdot 25 + 2 \cdot 2 + 1 \cdot 5 + 1 \\ 4 + 25 + 2 + 5 + 2 \end{bmatrix} = \begin{bmatrix} 118 \\ 38 \end{bmatrix}$$

Classification rule : $\Omega_i = \max_i \{d_i\}$

$$\bar{e} \to \Omega_1$$

c) $\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} y_1^2 & y_2^2 & y_1 & y_2 & 1 \end{bmatrix}^T$

Class border:

$$2y_1^2 + 4y_2^2 + 2y_1 + y_2 + 1 = \underline{y_1^2} + \underline{y_2^2} + \underline{y_1} + \underline{y_2} + 2$$

$$2y_1^2 - y_1^2 + 4y_2^2 - y_2^2 + 2y_1 - y_1 + \underline{y_2} - \underline{y_2} + 1 = 2$$

$$y_1^2 + 3y_2^2 + y_1 \quad + \underset{\left(\frac{4}{4}\right)}{1} \xrightarrow{\ -\frac{3}{4}\ } = \underset{\left(\frac{8}{4}\right)}{2} \Longrightarrow \qquad = \frac{5}{4}$$

$$y_1^2 + 3y_2^2 + y_1 + \underline{\frac{1}{4}} = \frac{5}{4}$$

$$\left(y_1 + \underline{\frac{1}{2}}\right)^2 + 3y_2^2 = \frac{5}{4} \quad \leftarrow$$

Topological form of our decision border is an ELLIPSE , $C: \left(-\frac{1}{2}, 0\right)$