LEHRSTUHL FÜR MENSCH-MASCHINE-KOMMUNIKATION

Pattern Recognition

Tutorial No. 8
13.06.2014

Exercise 19

In a two-dimensional, two-class problem the pattern vectors are:

$$\Omega_1 = \{\underline{\Delta}_1, \underline{\Delta}_2\} = \left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\4\end{bmatrix}\right\}$$

$$\Omega_2 = \{\underline{\Delta}_3, \underline{\Delta}_4\} = \left\{\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}4\\3\end{bmatrix}\right\}$$

a) Can the two classes be separated with a decision function $\underline{d} = W\,\underline{\Delta} = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\underline{\Delta}$?
b) Derive and plot the decision border for the decision function from the task above.

Now a PCA is applied to the problem to reduce the feature space to one dimension.

c) Derive the required transformation matrix U and transform the patterns to the reduced, one-dimensional feature space $\underline{\Delta}'$.
d) Plot the patterns in the new feature space.
e) Can the patterns be separated with a decision function $\underline{d} = W\,\underline{\Delta}' = \begin{bmatrix}-1\\0\end{bmatrix}\underline{\Delta}'$?

Unfortunately the system was badly designed and therefore has a malfunction: The information of the second feature dimension can not be used. Thus your new patterns only contain the first feature dimension:

$$\Omega_1 = \{\underline{\Delta}_1, \underline{\Delta}_2\} = \{[1], [3]\}$$

$$\Omega_2 = \{\underline{\Delta}_3, \underline{\Delta}_4\} = \{[2], [4]\}$$

f) Can the patterns be separated with the decision function $\underline{d} = W\,\underline{\Delta} = \begin{bmatrix}-1\\0\end{bmatrix}\underline{\Delta}$?
g) Derive the decision border for this decision function.
h) Now the pattern vectors are extended, such that $\underline{\Delta}^* = \begin{bmatrix}\underline{\Delta}\\1\end{bmatrix}$. Can the patterns be separated with the extended decision function $\underline{d} = W\,\underline{\Delta}^* = \begin{bmatrix}0 & 3{,}5\\1 & 0\end{bmatrix}\underline{\Delta}^*$?
i) Derive the decision border for this decision function.
j) Now the pattern vectors are extended, such that $\underline{\Delta}^* = [y_1^3 \quad y_1^2 \quad y_1 \quad 1]$. Can the patterns be separated with the extended polynomial decision function
$\underline{d} = W\,\underline{\Delta}^* = \begin{bmatrix}-1{,}33 & 0 & 0 & 15\\0 & -10 & 22{,}67 & 0\end{bmatrix}\underline{\Delta}^*$?
k) Derive the decision border for this decision function.