LEHRSTUHL FÜR MENSCH-MASCHINE-KOMMUNIKATION

**Pattern Recognition**

Tutorial No. 5
06.06.2014

### *Exercise 14*

Given is a transformation of the form $\underline{y} = \Phi \cdot \underline{x}$, with the matrix $\Phi$ which looks like:

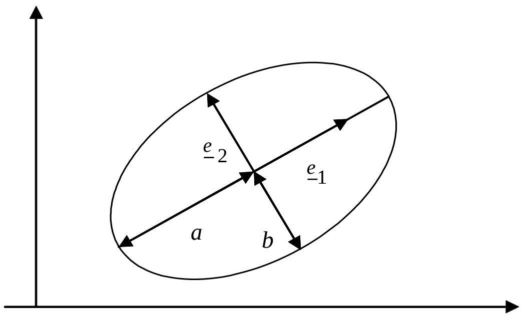
$$\Phi = \left[\underline{e}_1, \underline{e}_2, \ldots, \underline{e}_N\right]^T$$

the vectors $\underline{e}_i$ are normalised eigenvectors.

a) Show, that this transform is a rotation of the coordinate system, resulting in a new coordinate system, build by the vectors $\underline{e}_i$
b) Is the coordinate system expanded or compressed by this transform?
c) Compute the transformation matrix $\Phi$ for the 2-dimensional case, if the coordinate system has to be rotated by an angle of $\varphi$.

### *Exercise 15*

Given is an ellipse, as shown in the following figure:




a) Give reasons, why the eigenvectors of the covariance matrix for an infinite amount of points inside the ellipse correspond to the shown vectors $\underline{e}_1$ and $\underline{e}_2$. Use the fact, that a transformation based on a principle components analysis would change the ellipsis into a horizontal position, centred in the origin of the coordinate system.

b) Give reasons for the statement in a) by analytically computing the covariance matrix for the points inside of the transformed ellipse.