LEHRSTUHL FÜR MENSCH-MASCHINE-KOMMUNIKATION

Pattern Recognition

Tutorial No. 2
16.05.2014

Exercise 4

Show the orthogonality of the following sets of base-functions:

a) The set of the discrete Fourier-transform:

$$\sum_{k=0}^{N-1} \exp\left(j\frac{2\pi n}{N}k\right) \cdot \exp\left(-j\frac{2\pi m}{N}k\right) = \begin{cases} N & \text{for } n = m \\ 0 & \text{otherwise} \end{cases}$$

b) The set of the discrete cosine-transform:

$$\sum_{k=0}^{N-1} \cos\frac{2\pi n}{N}k \cdot \cos\frac{2\pi m}{N}k = \begin{cases} N/2 & \text{for } n = m \\ 0 & \text{otherwise} \end{cases}$$

Exercise 5

Derive the formulas for the DFT, using the orthogonality from Exercise 4) and the DFT-base-functions, for which yields:

$$y(k) = \frac{1}{N}\sum_{m=0}^{N-1} Y(m) \cdot \exp\left(j\frac{2\pi k}{N}m\right)$$

Exercise 6

Determine the N-point-DFT of the following functions:

a) $y(k) = \begin{cases} a & \text{for } 0 \le k \le N-1,\ N \text{ arbitrary} \\ 0 & \text{else} \end{cases}$

b) $y = \cos\frac{2\pi}{N} \cdot k$

c)