TUTORIAL 06
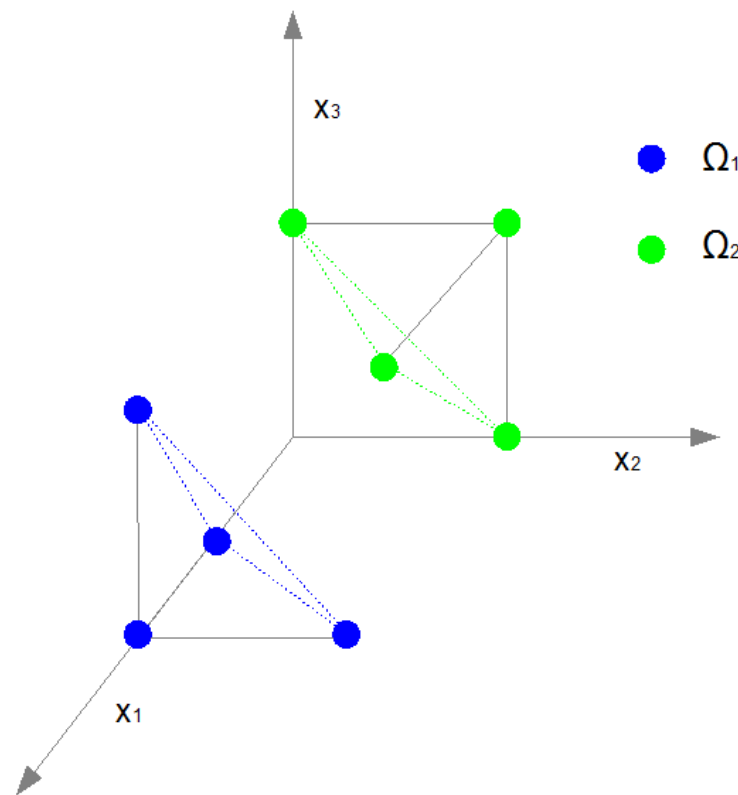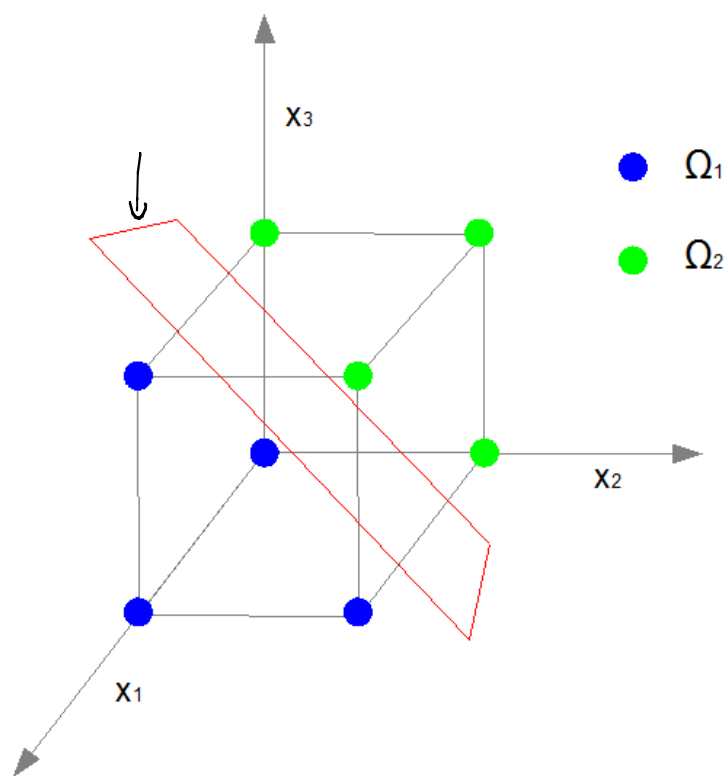
Outline:

- PCA

- Linear Discriminant Analysis (LDA)

EXERCISE 16

a)

b) Statistical features:

- $\underline{m}_x = E\{\underline{x}\} = \frac{1}{k} \sum_{k=1}^{k} \underline{x}(k)$ (Eg 2.64)

$$\underline{m}_\Omega = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$$

- $\underset{\sim}{C}_x = E\left\{ \left(\underline{x} - \underline{m}_x \right) \cdot \left(\underline{x} - \underline{m}_x \right)^T \right\} =$

$= \frac{1}{k} \sum_{k=1}^{k} \left[\underline{x}(k) \cdot x(k)^T \right] - \underline{m}_x \underline{m}_x^T$ (Eq. 2.65)

$$\underset{\sim}{C}_x = \frac{1}{8}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \end{bmatrix} + \right.$$

$$+ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} +$$

$$\left. + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\right) - \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}\begin{bmatrix} 0.5 & 0.5 & 0.5 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\rightarrow$ Eigen - values and Eigen - vectors of $\underset{\sim}{C}_x$

$$\det\left(\underset{\sim}{C}_x - \lambda I\right) \overset{!}{=} 0$$

$$\det \begin{bmatrix} 0.25 - \lambda & 0 & 0 \\ 0 & 0.25 - \lambda & 0 \\ 0 & 0 & 0.25 - \lambda \end{bmatrix}$$

$$\lambda_1 = 0.25$$
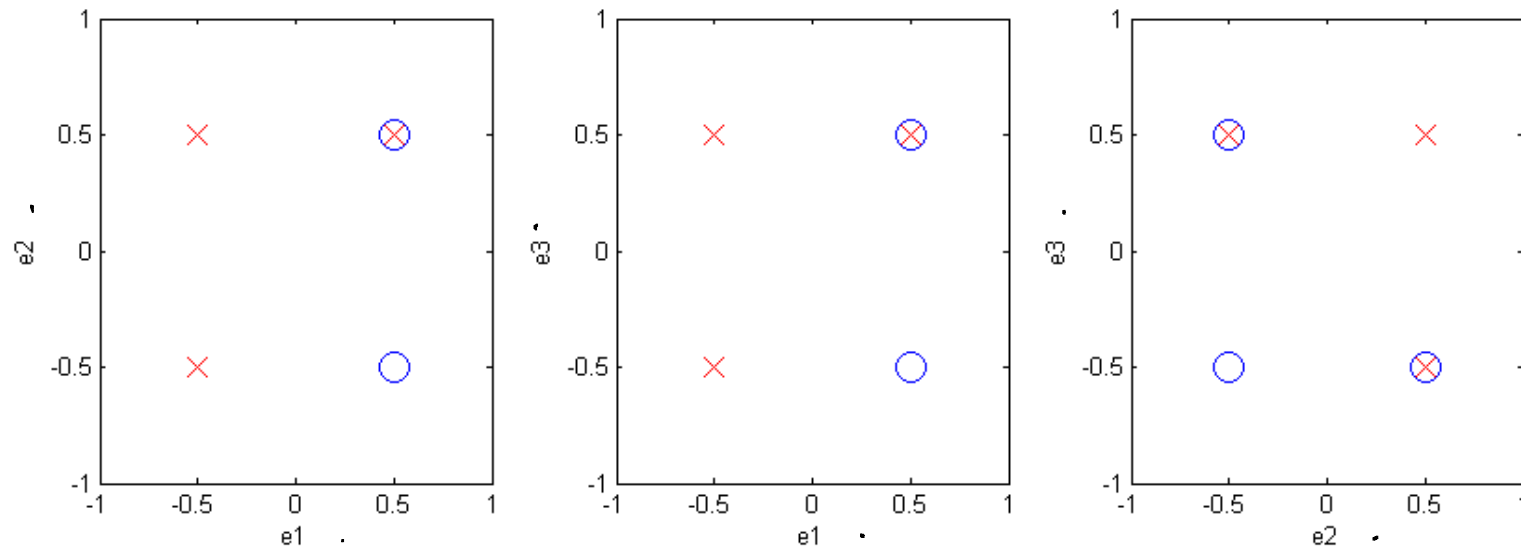$$\lambda_2 = 0.25$$
$$\lambda_3 = 0.25$$

$$\underline{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\underline{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\underline{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- Transform according to Eq. 2.71

$$\underline{y}(k) = \underset{\sim}{\underline{U}} \left(\underline{x}(k) - \underline{m}_x \right)$$

c) The patterns cannot be separated
in the reduced feature space.

$\times \ \Omega_1$

$\bigcirc \ \Omega_2$

d) LDA processing — statistical features.

- a priory - probability : (2.78) $\quad p_M = \dfrac{K_M}{K}$

$$p_{\Omega_1} = \dfrac{4}{8} = \dfrac{1}{2} \qquad p_{\Omega_2} = \dfrac{1}{2}$$

- class - centre $\quad (2.79) \qquad \underline{m}_M = \dfrac{1}{K_M} \sum\limits_{k=1}^{\overline{K}_M} \underline{x}(k)$

$$\underline{m}_{\Omega_1} = \dfrac{1}{4}\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \qquad\qquad \underline{m}_{\Omega_2} = \dfrac{1}{4}\begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$$

- Class - covariance matrix $\quad (2.80)$

$$\underline{C}_M = \dfrac{1}{K_M} \sum\limits_{k=1}^{K_M} \underline{x}(k) \cdot \underline{x}(k)^T - \underline{m}_M \, \underline{m}_M^T$$

$$\underset{\sim}{C}_{\Omega_1} = \frac{1}{16} \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix} \qquad \underset{\sim}{C}_{\Omega_2} = \frac{1}{16} \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

- Overall centre for all patterns : (2.81)

$$\underline{m} = \sum_{n=1}^{N} P_m \cdot \underline{m}_n \quad \Rightarrow \quad \underline{m}_\Omega = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

— Instruction for LDA - procedure (pp. 37 - 38)

Step 1

- Intra class scattering matrix (2.82)

$$\underset{\sim}{C}_a = \sum_{n=1}^{N} P_m \cdot \underset{\sim}{C}_m \qquad \underset{\sim}{C}_a = \frac{1}{16} \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

- Inter clan scattering matrix (2.83)

$$\underset{\sim}{C}_b = \sum_{M=1}^{N} p_M \left(\underline{m}_M - \underline{m} \right) \left(\underline{m}_M - \underline{m} \right)^T$$

$$\underset{\sim}{C}_b = \frac{1}{16} \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

Step 2

Eigen-values and Eigen-vectors of $\underset{\sim}{C}_a$

$$\underset{\sim}{\Lambda}_a = \frac{1}{16} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \qquad \underset{\sim}{U}_a = \begin{bmatrix} -0.57 & -0.15 & 0.80 \\ 0.58 & -0.77 & 0.22 \\ 0.57 & 0.62 & 0.54 \end{bmatrix}$$

$$\underset{\sim}{\Phi}_a = \underset{\sim}{\Lambda}_a^{-\frac{1}{2}} \underset{\sim}{U}_a \qquad\qquad (2.92)$$

$$\overline{\underline{\Phi}}_a = \begin{bmatrix} -2.31 & -0.62 & 3.21 \\ 1.16 & -1.54 & 0.54 \\ 1.16 & 1.23 & 1.07 \end{bmatrix}$$