LEHRSTUHL FÜR MENSCH-MASCHINE-KOMMUNIKATION

Pattern Recognition

Tutorial No. 7
13.6.2014

Exercise 17

For a two class problem the two reference vectors are $w_{\Omega 1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $w_{\Omega 2} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$.

a) Graphically derive the decision border for a Nearest Neighbor (NN) classifier with the Euclidean distance.
b) Mathematically derive the decision border for a NN classifier with the Euclidean distance.
c) Now a KNN classifier is used, where the first class is represented with the two reference vectors $w_{\Omega 1}^1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $w_{\Omega 1}^2 = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$, and the second class is represented with one reference vector $w_{\Omega 2} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Derive the decision border graphically.
d) Graphically analyze the effect of different covariance matrices for a NN classifier with the Mahalanobis distance. Try out the same covariance matrix for both classes, different covariance matrices for the two classes, the identity matrix and diagonal matrices.

Exercise 18

For a two class problem the two reference vectors are $w_{\Omega 1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $w_{\Omega 2} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$, the corresponding covariance matrices (and their inverses are) $C_1 = \begin{bmatrix} 1 & 0.5 \\ 2 & 0.7 \end{bmatrix}$, $C_2 = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$, $C_1^{-1} = \begin{bmatrix} -2.3 & 1.7 \\ 6.7 & -3.3 \end{bmatrix}$, and $C_2^{-1} = \begin{bmatrix} -0.4 & 0.6 \\ 0.6 & -0.4 \end{bmatrix}$.
The two unknown vectors $y_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$and $y_2 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ shall be classified.

a) Classify the two vectors with a NN classifier and the Euclidean distance.
b) Classify the patterns with a NN classifier and the Mahalanobis distance.