TUTORIAL 03

Notiztitel 23.05.2014

Outline:

Preprocessing

- time-frequency transformation
    - Wavelet Transformation

EXERCISE 7

Mexican-Hat-Wavelet (Eq 2.29)

$$\psi(t) = \left(1 - \frac{t^2}{\sigma^2}\right) \cdot e^{-\frac{t^2}{2\cdot\sigma^2}}$$

DC component equals zero: $\int_{-\infty}^{\infty} \psi(t)\, dt \stackrel{!}{=} 0$

$$\int_{-\infty}^{\infty} \psi(t)\, dt = \int_{-\infty}^{\infty} \left(1 - \frac{t^2}{\sigma^2}\right) \cdot e^{-\frac{t^2}{2\sigma^2}}\, dt =$$

$$= \int_{-\infty}^{\infty} e^{-\frac{t^2}{2\sigma^2}}\, dt - \frac{1}{\sigma^2} \int_{-\infty}^{\infty} t^2 \cdot e^{-\frac{t^2}{2\sigma^2}}\, dt$$

from Bronstein:

- $$\int_{-\infty}^{\infty} e^{2bx - ax^2}\, dx = \sqrt{\frac{\pi}{a}} \cdot e^{\frac{b^2}{a}} \qquad (a > 0)$$

here $b = 0$ $\quad a = \frac{1}{2\sigma^2}$

Thus: $$\int_{-\infty}^{\infty} e^{-\frac{t^2}{2\sigma^2}}\, dt = \sqrt{\pi\, 2\sigma^2} = \sigma \cdot \sqrt{2\pi}$$

- $$\int_{-\infty}^{\infty} x^2 \cdot e^{-a^2x^2}\, dx = \frac{\sqrt{\pi}}{2 \cdot a^3} \qquad (a > 0)$$

$$a^2 = \frac{1}{2\sigma^2} \rightarrow a = \frac{1}{\sqrt{2}\,\sigma}$$

Thus : $\int_{-\infty}^{\infty} t^2 \cdot e^{-\frac{t^2}{2\sigma^2}} \, dt = \frac{1}{2} \cdot \sqrt{\pi} \cdot \sqrt{2}^3 \sigma^3$

$$\int_{-\infty}^{\infty} \psi(t) \, dt = \sigma \cdot \sqrt{2\pi} - \frac{1}{\sigma^2} \cdot \sqrt{2\pi} \cdot \sigma^3 = \sigma \cdot \sqrt{2\pi} - \sigma\sqrt{2\pi} = 0$$

DC component equals zero

EXERCISE 8

Wavelet Transformation (Eq. 2.28)

$$Y_w(c,\tau) = \frac{1}{\sqrt{c}} \int_{-\infty}^{\infty} y(t) \cdot \psi\left(\frac{t-\tau}{c}\right) dt$$

FOURIER TRANSFORMATION (Eq. 2.14)

$$Y_F(w) = \int_{-\infty}^{\infty} y(t) \cdot e^{-jwt} dt$$

Thus: $$F\left\{\psi\left(\frac{t}{c}\right)\right\} = \Psi(w) = \int_{-\infty}^{\infty} \psi\left(\frac{t}{c}\right) \cdot e^{-jwt} dt$$

Substitution : $x = \frac{t}{c} \Rightarrow \frac{dx}{dt} = \frac{1}{c} \Rightarrow dt = c \cdot dx$

Thus : $\bar{\Psi}(\omega) = \int_{-\infty}^{\infty} \psi(x) \cdot e^{-j\omega c x} \cdot c \cdot dx =$

$\tilde{\omega} = c \cdot \omega$

$$= c \cdot \int_{-\infty}^{\infty} \psi(x) \cdot e^{-j\tilde{\omega} x} \cdot dx =$$

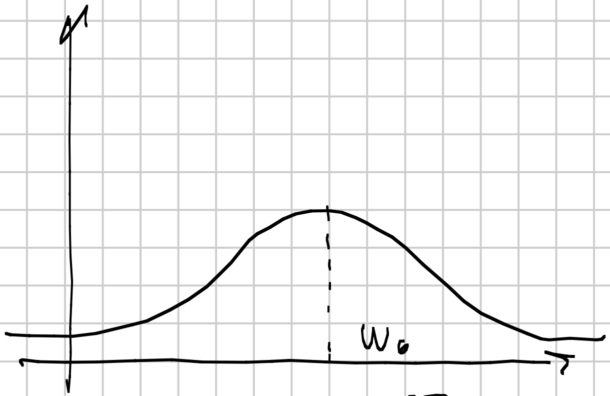
$$= c \; \mathcal{F}\left\{ \psi(x) \right\} = c \cdot \Psi(\tilde{\omega})$$

Now : $\bar{\Psi}^{-1}(\omega) = c \cdot \Psi(\tilde{\omega}) = c \cdot \Psi(c \cdot \omega)$

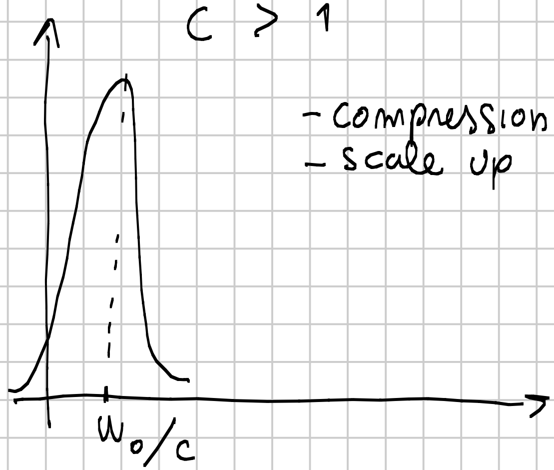
For the spectrum of ψ:

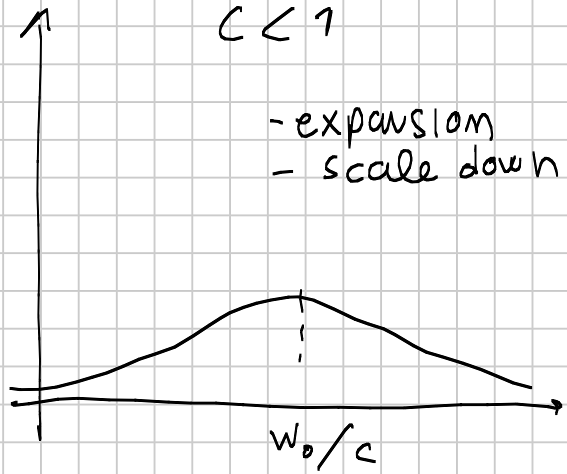
1. high c ($c = f_0/f$ small frequencies) the spectrum is compressed in ω-direction and at the same time is scaled by factor c

2. small c (high frequencies) the spectrum is stretched in ω-direction but at the same time is scaled down



ORIGINAL SPECTRUM





EXERCISE 9

Mexican - Hat - Wavelet (2.29)

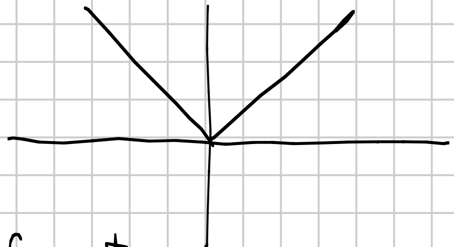
$$\psi(t) = \left(1 - \frac{t^2}{\sigma^2}\right) \cdot e^{-\frac{t^2}{2\sigma^2}} \leftarrow$$

Fourier Transformation (2.14)

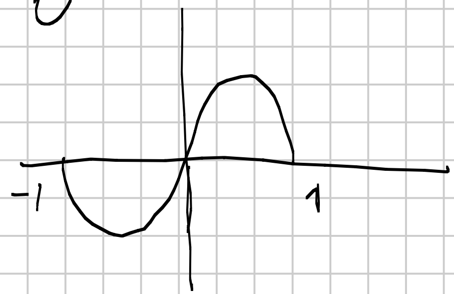
$$Y_F(\omega) = \int_{-\infty}^{\infty} y(t) \cdot e^{-j\omega t} \, dt$$

$$\mathcal{F}\{\psi(t)\} = \Psi(\omega) = \int_{-\infty}^{\infty} \left(1 - \frac{t^2}{\sigma^2}\right) \cdot e^{-\frac{t^2}{2\sigma^2}} \cdot e^{-j\omega t} \cdot dt =$$

$$= \int_{-\infty}^{\infty} \left(1 - \frac{t^2}{\sigma^2}\right) \cdot e^{-\frac{t^2}{2\sigma^2}} \cos \omega t \; - \underbrace{j \int_{-\infty}^{\infty} \left(1 - \frac{t^2}{\sigma^2}\right) \cdot e^{-\frac{t^2}{2\sigma^2}} \sin \omega t \, dt}_{0} =$$



$\cdot$



$=$

because we have a combination of a function symmmetrical to y-axis and the sine function

$$\int_{-\infty}^{\infty} = 0$$

$$= \int_{-\infty}^{\infty} e^{-\frac{t^2}{2\sigma^2}} \cdot \cos \omega t \, dt = \frac{1}{\sigma^2} \int_{-\infty}^{\infty} t^2 \cdot e^{-\frac{t^2}{2\sigma^2}} \cos \omega t \, dt$$

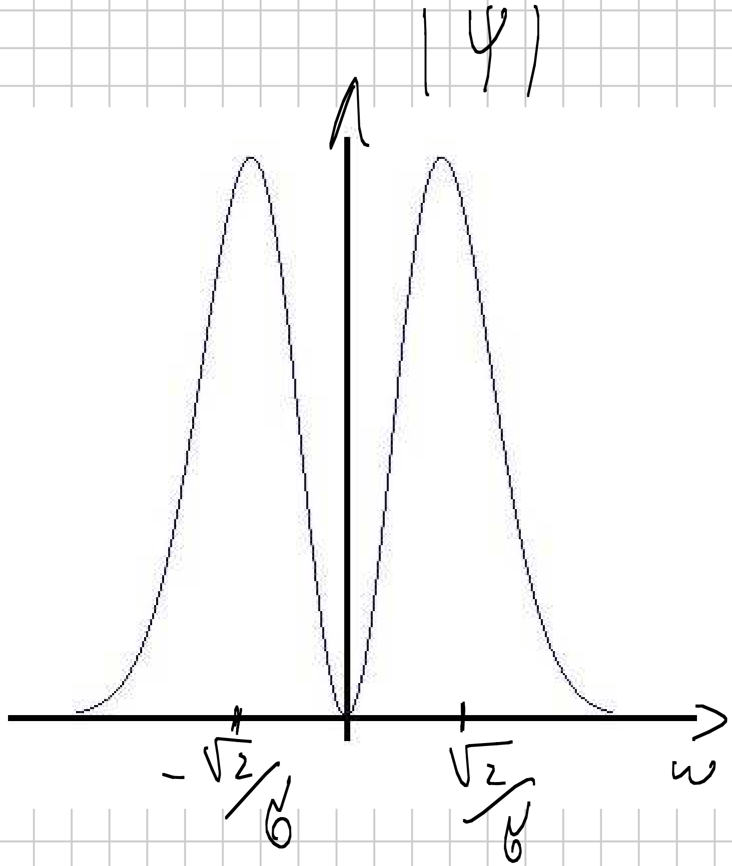
$$\cdot \int_{-\infty}^{\infty} e^{-a^2 t^2} \cdot \cos bt \, dt = \frac{\sqrt{\pi}}{a} \cdot e^{-\frac{b^2}{4a^2}} \quad (\mathrm{I})$$

$$\cdot \int_{-\infty}^{\infty} t^2 \cdot e^{-a^2 t^2} \cos bt \, dt = \frac{\sqrt{\pi}}{2a^3} \left(1 - \frac{b^2}{2a^3}\right) \cdot e^{-\frac{b^2}{4a^2}} \quad (\mathrm{II})$$

$$b = \omega \qquad a^2 = \frac{1}{2\sigma^2} \rightarrow a = \frac{1}{\sqrt{2}\,\sigma} \Rightarrow a^3 = \frac{1}{2 \cdot \sqrt{2} \cdot \sigma^3}$$

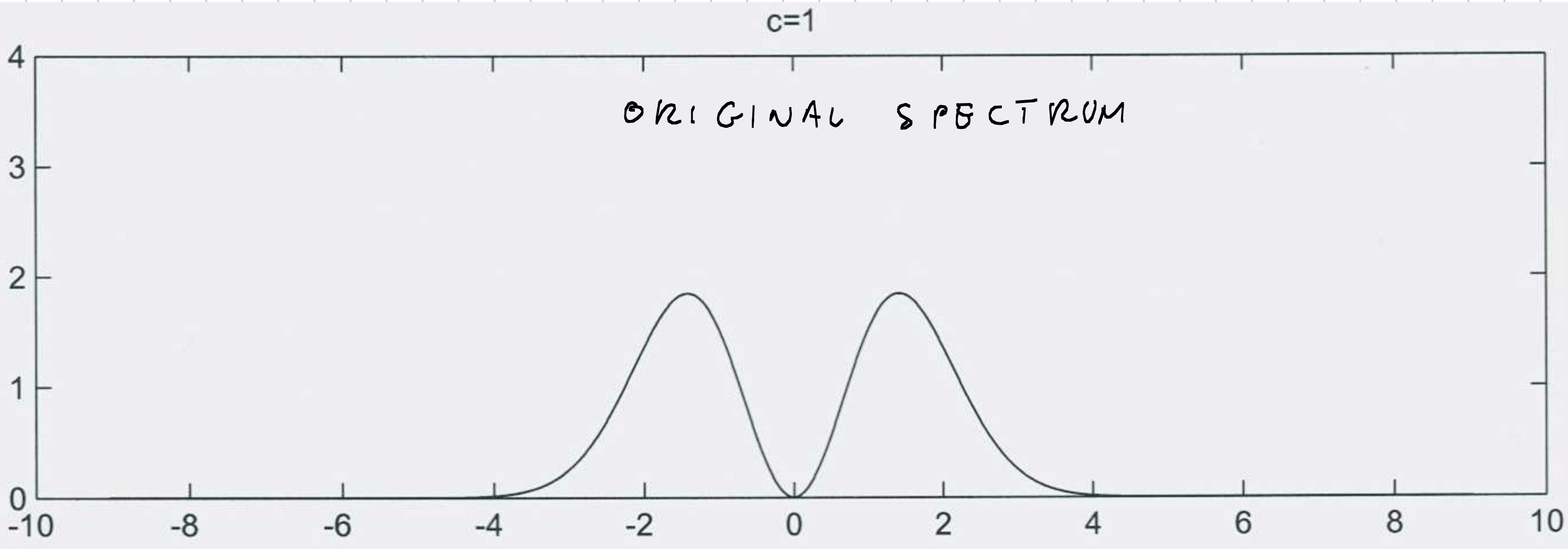
$$\Psi(\omega) = \sqrt{2\pi} \cdot \omega^2 \sigma^3 \cdot e^{-\frac{\omega^2 \sigma^2}{2}}$$

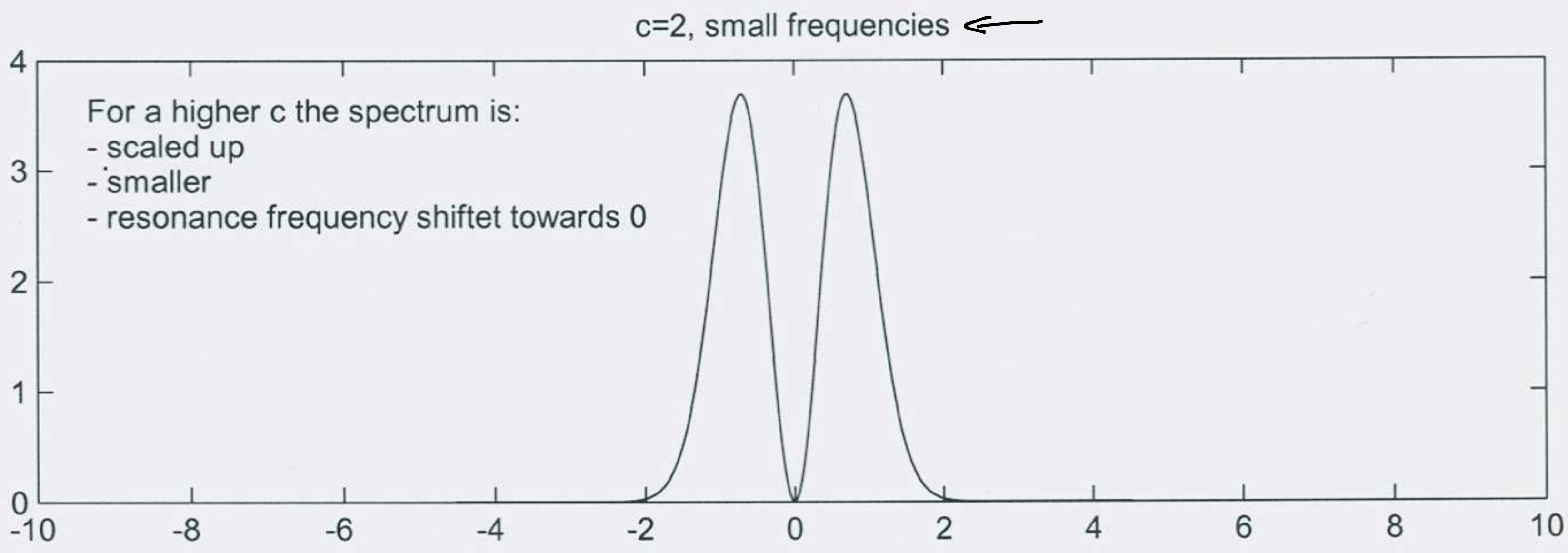
is curve of the form $\omega^2 \cdot e^{-\omega^2}$

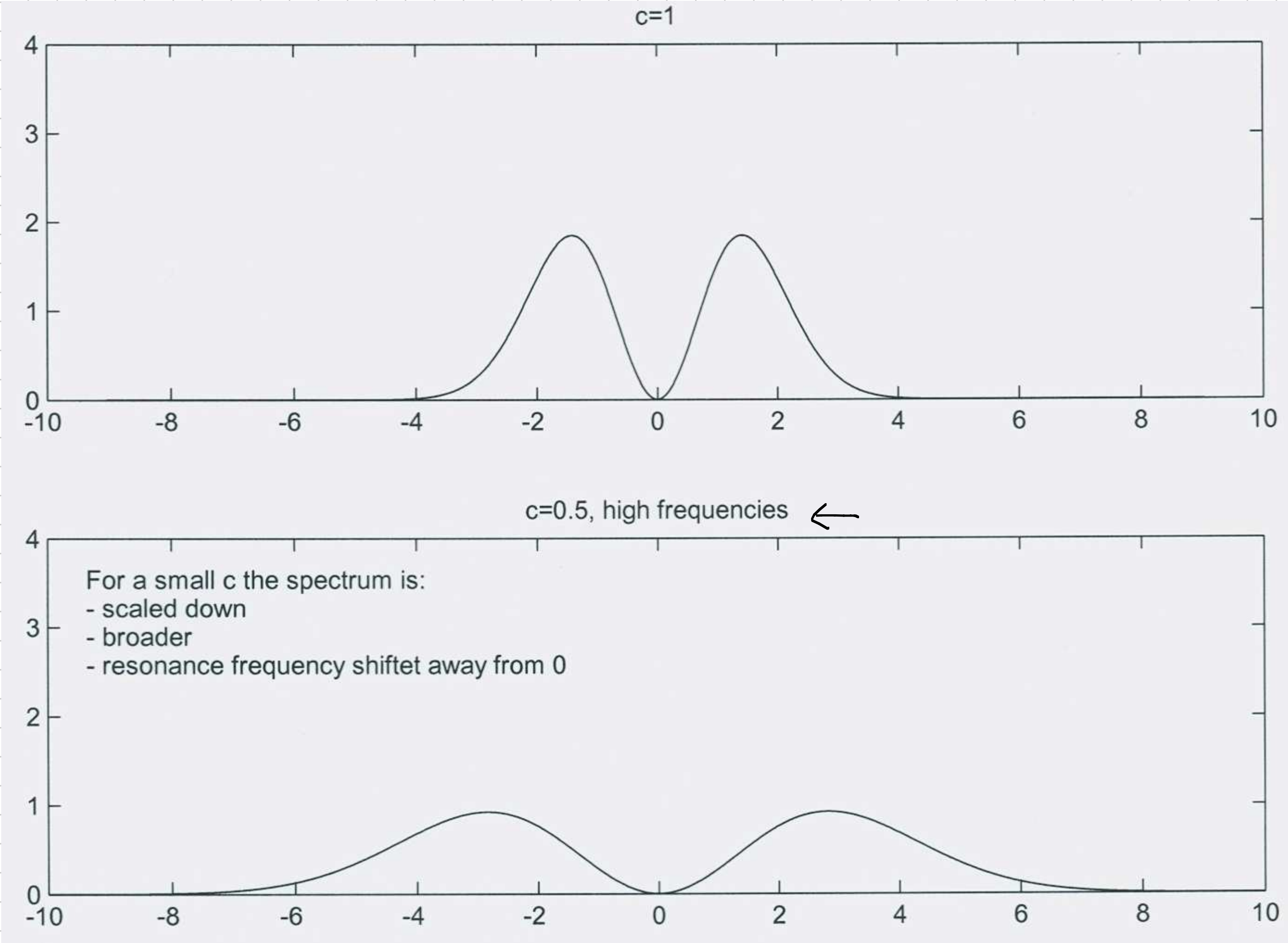


$\Rightarrow$ characteristic of a bandpass with middle frequency

$$\omega = \frac{\sqrt{2}}{\sigma}$$


c=1
ORIGINAL SPECTRUM
4
3
2
1
0
-10
-8
-6
-4
-2
0
2
4
6
8
10


c=2, small frequencies
For a higher c the spectrum is:
- scaled up
- smaller
- resonance frequency shiftet towards 0
4
3
2
1
0
-10
-8
-6
-4
-2
0
2
4
6
8
10


c=1
4
3
2
1
0
-10
-8
-6
-4
-2
0
2
4
6
8
10
c=0.5, high frequencies
For a small c the spectrum is:
- scaled down
- broader
- resonance frequency shiftet away from 0
4
3
2
1
0
-10
-8
-6
-4
-2
0
2
4
6
8
10

EXERCISE 10

Mexican - Hat - Wavelet (2.29)

$$\psi(t) = \left(1 - \frac{t^2}{\sigma^2}\right) \cdot e^{-\frac{t^2}{2\sigma^2}}$$

Wavelet Transformation (2.28)

$$Y_w(c, \tau) = \frac{1}{\sqrt{c}} \int_{-\infty}^{\infty} y(t) \cdot \psi\left(\frac{t-\tau}{c}\right) dt$$

$$y(t) = \delta(t - \tau_0)$$

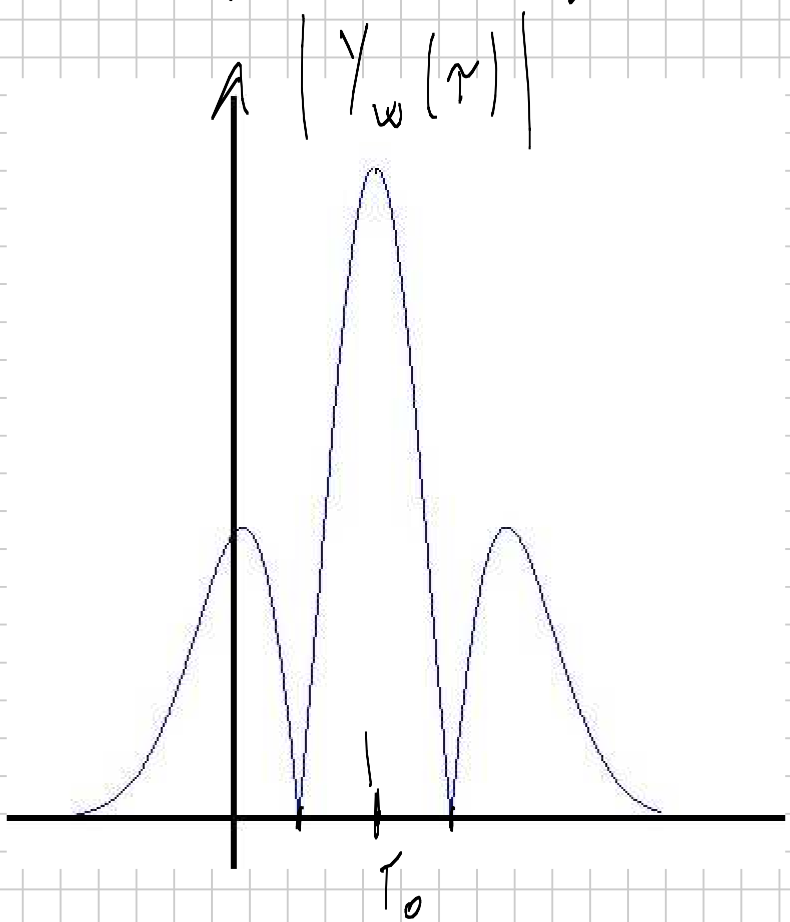
Thus:

$$Y_w(c, \tau) = \frac{1}{\sqrt{c}} \int_{-\infty}^{\infty} \delta(t - \tau_0) \cdot \psi\left(\frac{t - \tau}{c}\right) dt =$$

$$= \frac{1}{\sqrt{c}} \psi\left(\frac{\tilde{\tau}_0 - \tilde{\tau}}{c}\right)$$

$$Y_w(c, \tilde{\tau}) = \frac{1}{\sqrt{c}} \left(1 - \frac{(\tilde{\tau}_0 - \tilde{\tau})^2}{c^2 \cdot \sigma^2}\right) \cdot e^{-\frac{(\tilde{\tau}_0 - \tilde{\tau})^2}{2c^2 \cdot \sigma^2}}$$

Wavelet itself is a Mexican-Hat function of τ shifted by τ_0

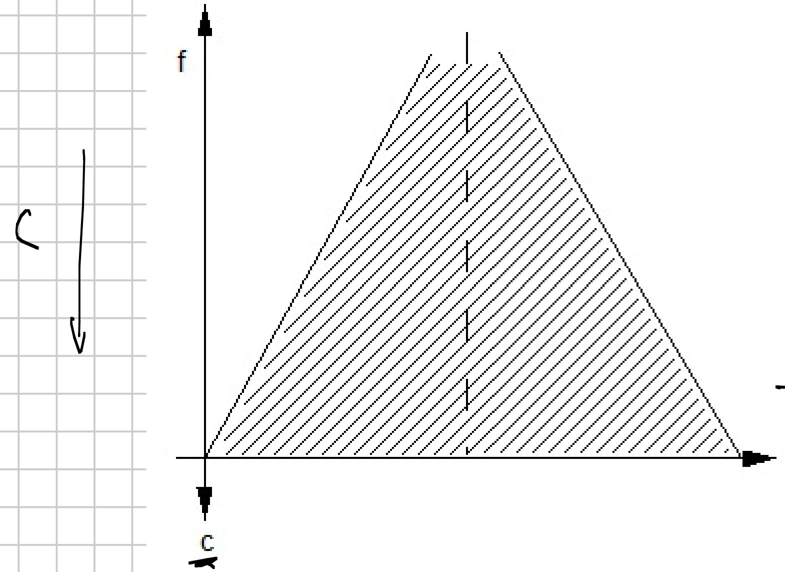


→ Maximum at $\tau = \tau_0$

→ zero points at $\tau = \tau_0 \pm \sigma c$

→ fixed c

SCALOGRAM



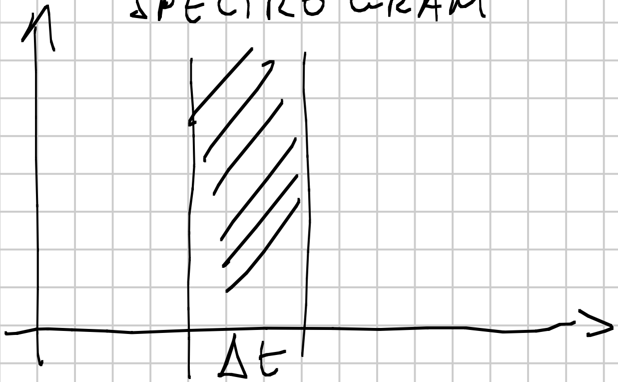
→ good time resolution

bad freq. resolution

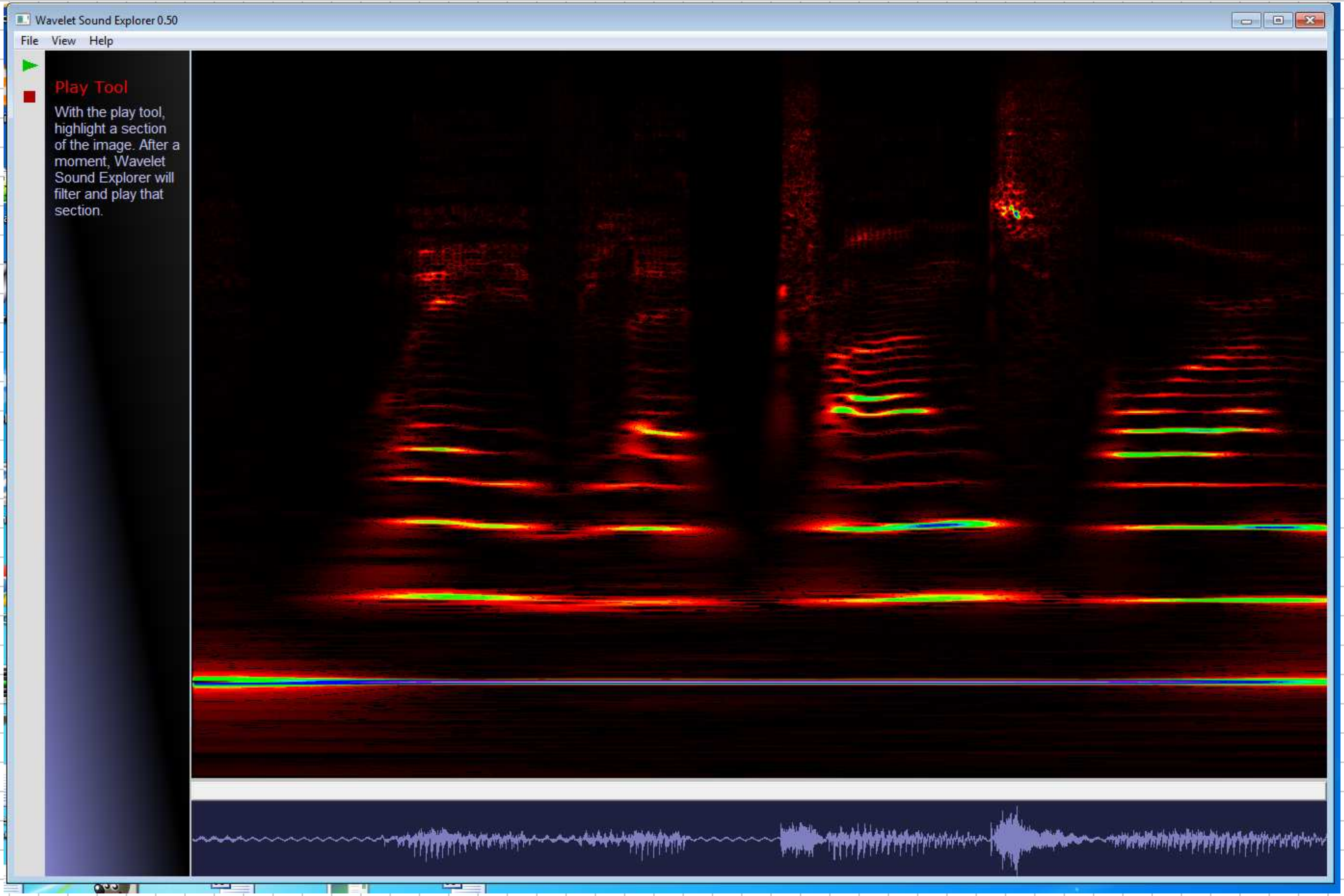
→ good freq. resolution

– bad time resolution

SPECTROGRAM




Wavelet Toolbox Main Menu
File
Window
Help
One-Dimensional
Wavelet 1-D
Wavelet Packet 1-D
Continuous Wavelet 1-D
Complex Continuous Wavelet 1-D
Continuous Wavelet 1-D (Using FFT)
Two-Dimensional
Wavelet 2-D
Wavelet Packet 2-D
Three-Dimensional
Wavelet 3-D
Multiple 1-D
Multisignal Analysis 1-D
Multivariate Denoising
Multiscale Princ. Comp. Analysis
Wavelet Design
New Wavelet for CWT
Specialized Tools 1-D
SWT Denoising 1-D
Density Estimation 1-D
Regression Estimation 1-D
Wavelet Coefficients Selection 1-D
Fractional Brownian Generation 1-D
Specialized Tools 2-D
True Compression 2-D
SWT Denoising 2-D
Wavelet Coefficients Selection 2-D
Image Fusion
Display
Wavelet Display
Wavelet Packet Display
Extension
Signal Extension
Image Extension
Close



Wavelet Sound Explorer 0.50
File View Help
Play Tool
With the play tool, highlight a section of the image. After a moment, Wavelet Sound Explorer will filter and play that section.