Lehrstuhl für STEUERUNGS- UND REGELUNGSTECHNIK	MACHINE LEARNING IN ROBOTICS
Technische Universität München	Exercises 1: Linear Regression
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Exercise 1

Given the dataset shown in table 1 and illustrated in figure 1, we want to predict the output value for $x = 1$.

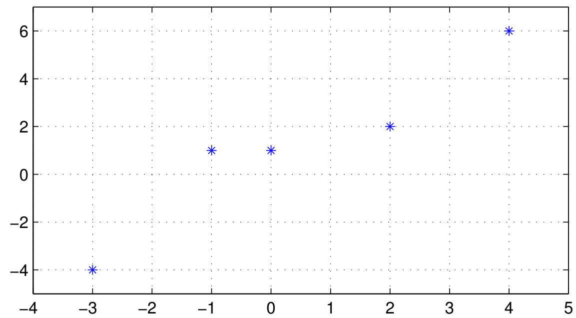

Figure 1: Training dataset

input x	-3	-1	0	2	4
output y	-4	1	1	2	6

Table 1: Data

a) Let's assume $f(x) = w_1 x + w_2 x^2$ is a regression model with unknown parameter vector $\boldsymbol{w} = [w_1 \quad w_2]^T$. Find $\boldsymbol{w}$ which fits the data best in the sense of the Euclidean norm.

b) Predict the output value of the system for $x = 1$.

Exercise 2

Consider the following sum-of-squares error function:

$$E(\boldsymbol{w}) = \frac{1}{2} \sum_{i=1}^{n} \left(f(x^{(i)}, \boldsymbol{w}) - y^{(i)} \right)^2$$

in which the function $f(x^{(i)}, \boldsymbol{w})$ is the polynomial:

$$f(x^{(i)}, \boldsymbol{w}) = w_0 + w_1 \left(x^{(i)}\right) + w_2 \left(x^{(i)}\right)^2 + \ldots + w_m \left(x^{(i)}\right)^m = \sum_{j=0}^{m} w_j \left(x^{(i)}\right)^j \ .$$

Show that the coefficients $\boldsymbol{w} = \{w_k\}$ that minimize this error function are given by the solution to the following set of linear equations

$$\sum_{j=0}^{m} A_{kj} w_j = Y_k$$

where

$$A_{kj} = \sum_{i=1}^{n} (x^{(i)})^{k+j}, \qquad Y_k = \sum_{i=1}^{n} (x^{(i)})^k y^{(i)} \ .$$

Here a suffix i or j denotes the index of a component, whereas $(x)^i$ denotes x to the power of i.

Exercise 3

The kinematic of a differential-drive mobile robot like that in figure 2 is described in the discrete-time by the set of equations

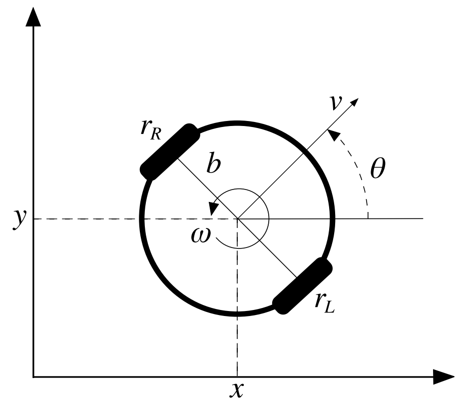

Figure 2: Top-view sketch of a differential-drive mobile robot with relevant variables.

$$\begin{cases} x^{(t+1)} = x^{(t)} + v^{(t)} cos(\theta^{(t)} + \omega^{(t)}\dfrac{\Delta T}{2})\Delta T \\ y^{(t+1)} = y^{(t)} + v^{(t)} sin(\theta^{(t)} + \omega^{(t)}\dfrac{\Delta T}{2})\Delta T \\ \theta^{(t+1)} = \theta^{(t)} + \omega^{(t)}\Delta T \end{cases}$$

where ΔT is the sample time. The relation between the linear v and angular ω velocities of the robot and the velocity of the wheels (ω_R and ω_L) is

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix} = \boldsymbol{W} \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix}$$

Given m motion trajectories $T_r = \left[\left\{ x_1^{(t)}, y_1^{(t)}, \theta_1^{(t)}, \omega_{R,1}^{(t)}, \omega_{L,1}^{(t)} \right\}_{t=0}^{n}, \ldots, \left\{ x_m^{(t)}, y_m^{(t)}, \theta_m^{(t)}, \omega_{R,1}^{(t)}, \omega_{L,1}^{(t)} \right\}_{t=0}^{n} \right]$, estimate the unknown parameters $\boldsymbol{W}$ using least square regression. (Hint: $[w_{11}, w_{12}]$ and $[w_{21}, w_{22}]$ can be separately estimated.)

Exercise 4

Given the dataset shown in table 2, we want to predict the output values for $x_1 = 7, x_2 = 4$. We assume a linear regression model.

input x_1	3	2	1	3
input x_2	5	2	2	3
output y_1	7	4	2	6
output y_2	10	5	9	6

Table 2: Data

a) Let's assume as a regression model:

$$f_1(\boldsymbol{x}) = w_0 x_1 + w_1 x_2$$
$$f_2(\boldsymbol{x}) = w_0^{'} x_1 + w_1^{'} x_2$$

with unknown parameters

$$\boldsymbol{w} = \begin{bmatrix} w_0 & w_0^{'} \\ w_1 & w_1^{'} \end{bmatrix}.$$

Find the best $\boldsymbol{w}$ using the normal equation.

b) Predict the output value of the system for $x_1 = 7, x_2 = 4$.