TUTORIAL 08

Notiztitel 13.06.2014

Outline:

Pattern Classification:

- Classification with decision functions

Exercise 19

a) Classification (Eq 3.15) $\rightarrow \Omega_m = \max_m \{ d(w_m, y) \}$

Δ_1 : $d_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \Delta_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow \Omega_1$ ✓

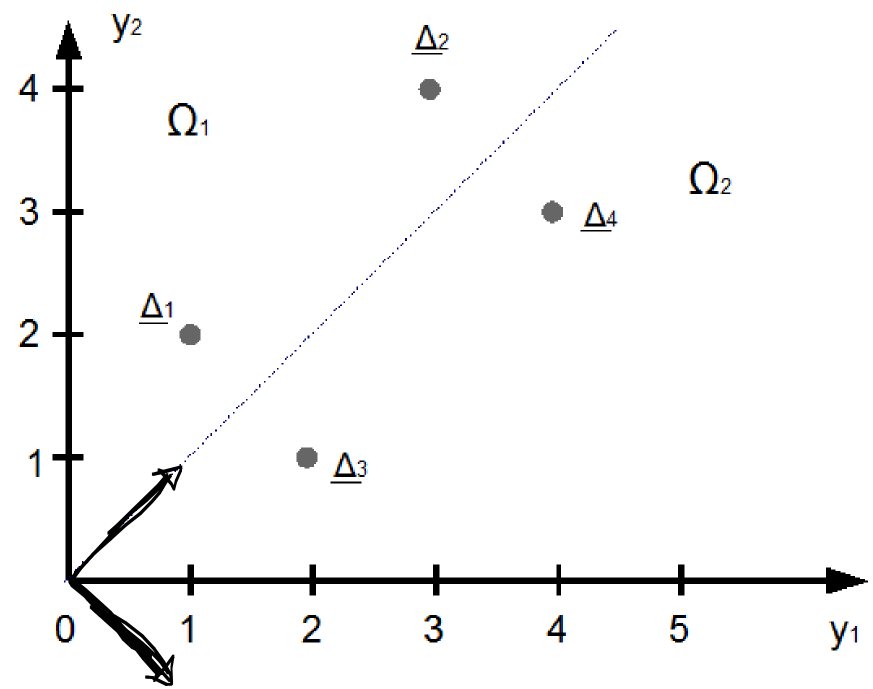
Δ_2 : $d_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \Delta_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \Rightarrow \Omega_1$ ✓

Δ_3 : $d_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \Delta_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow \Omega_2$ ✓

Δ_4 : $d_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \Delta_4 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \Rightarrow \Omega_2$ ✓

Classes are separated correctly

b) Decision border $d_1 = d_2$ with $\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \Rightarrow y_2 = y_1$



c) PCA $\Rightarrow$ $$\underline{\Delta}' = \underline{U} \cdot \left[\underline{\Delta} - \underline{m}_x \right]$$

average value $$\underline{m}_x = \frac{1}{K} \sum_{k=1}^{K} \underline{\Delta}_k \qquad (\text{Eq } 2.64)$$

$$\Rightarrow \underline{m}_x = \frac{1}{4} \left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix} \right) = \begin{bmatrix} 2.5 \\ 2.5 \end{bmatrix}$$

Covariance matrix C_x

$$C_x = \frac{1}{K} \cdot \sum_{k=1}^{K} \left[\underline{\Delta}_k \cdot \underline{\Delta}_k^T \right] - \underline{m}_x \underline{m}_x^T =$$

$$= \frac{1}{4} \left[\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} \begin{bmatrix} 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 3 \end{bmatrix} \right] -$$

$$- \begin{bmatrix} 2.5 \\ 2.5 \end{bmatrix} \begin{bmatrix} 2.5 & 2.5 \end{bmatrix} =$$

$$= \frac{1}{4}\left[\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 9 & 12 \\ 12 & 16 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 16 & 12 \\ 12 & 9 \end{bmatrix}\right] - \begin{bmatrix} 6.25 & 6.15 \\ 6.25 & 6.15 \end{bmatrix}$$

$$= \begin{bmatrix} 1.25 & 0.75 \\ 0.75 & 1.25 \end{bmatrix}$$

Eigenvalues of C_x

$$\det(\underset{\sim}{C} - \lambda \underset{\sim}{I}) = 0 \Rightarrow \begin{vmatrix} 1.25 - \lambda & 0.75 \\ 0.75 & 1.25 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (1.25 - \lambda)^2 - 0.5625 = 0$$

$$\Rightarrow 1.5625 - 2.5\lambda + \lambda^2 - 0.5625 = 0$$

$$\lambda^2 - 2.5\lambda + 1 = 0$$

$$\lambda_{1/2} = 1.25 \pm \sqrt{1.5625 - 1} = 1.25 \pm 0.75$$

$$\underline{\lambda_1 = 2.0} \qquad \lambda_2 = 0.5$$

Consider $\lambda_2 \to$ eigenvector:

$$\begin{pmatrix} 1.25 - 0.5 & 0.75 \\ 0.75 & 1.25 - 0.5 \end{pmatrix} \cdot \underline{e} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$