30. 4. 2014

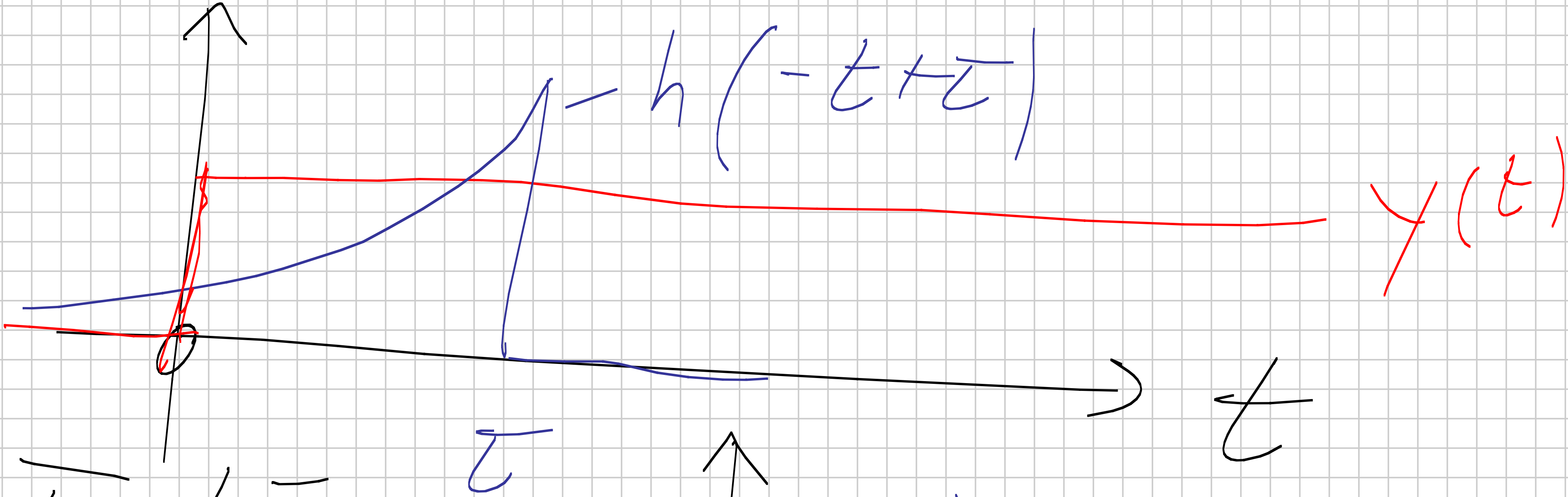
Transformation formula of WT:

$$Y_W(c,\tau) = \frac{1}{\sqrt{c}} \int_{-\infty}^{\infty} y(t)\, \psi\left(\frac{t-\tau}{c}\right) dt$$

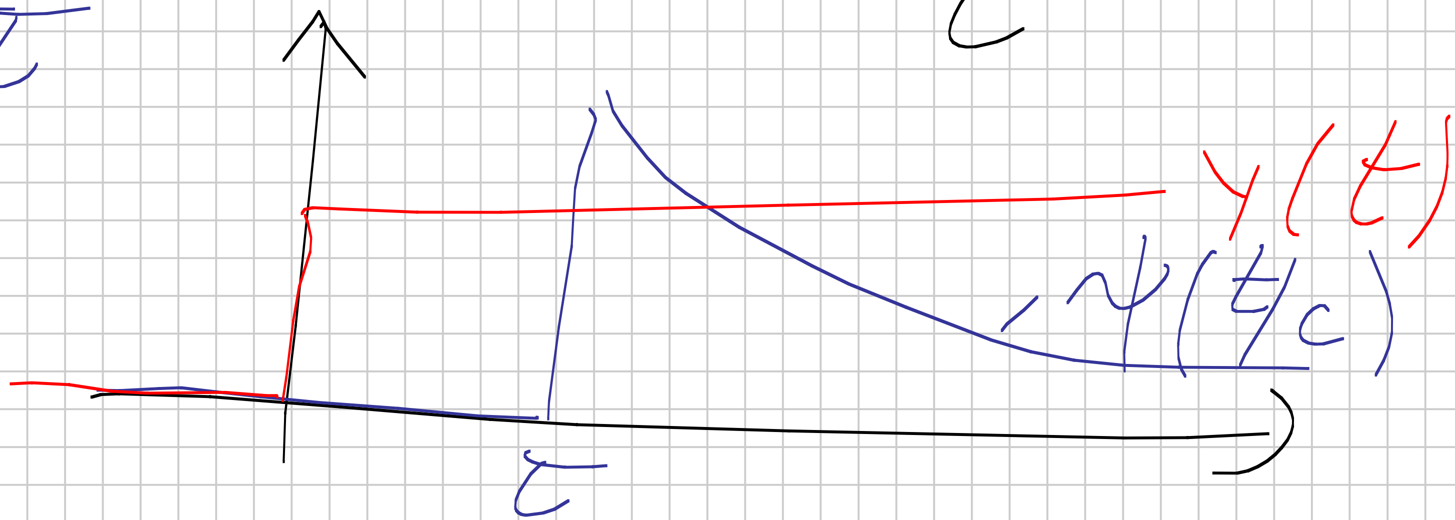
Comparison to convolution:

$$Y = \int_{-\infty}^{\infty} y(\tau) \cdot h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} y(t) \cdot h(\tau - t)\, dt$$



For τ:

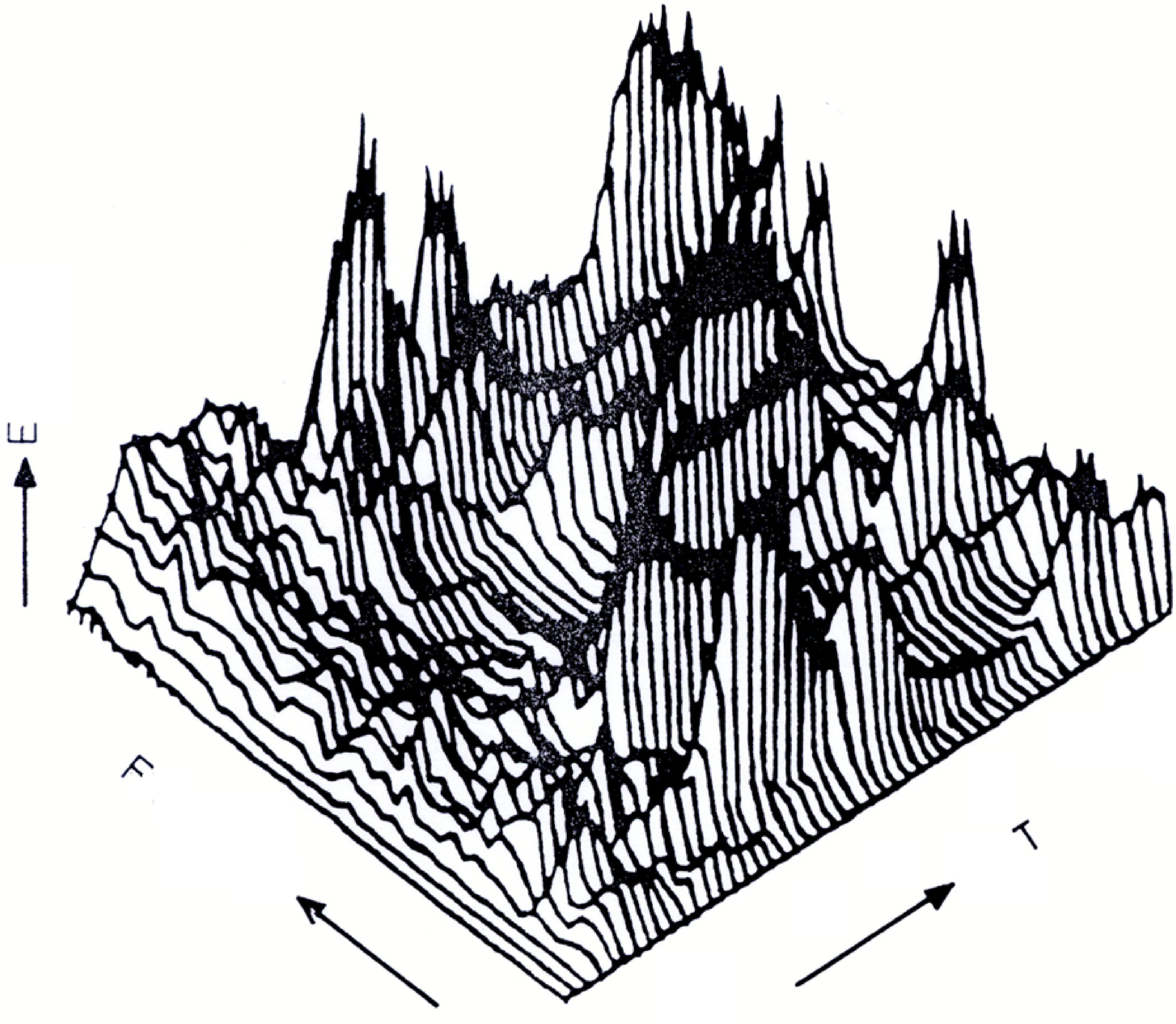


$$y(t) \longrightarrow \underbrace{\boxed{h(t) = \psi\left(t/c\right)}}_{\text{Filter}} \longrightarrow Y(c,\tau)$$

Interpretation of WT-result:

Result of STFT:

Spectogram


E
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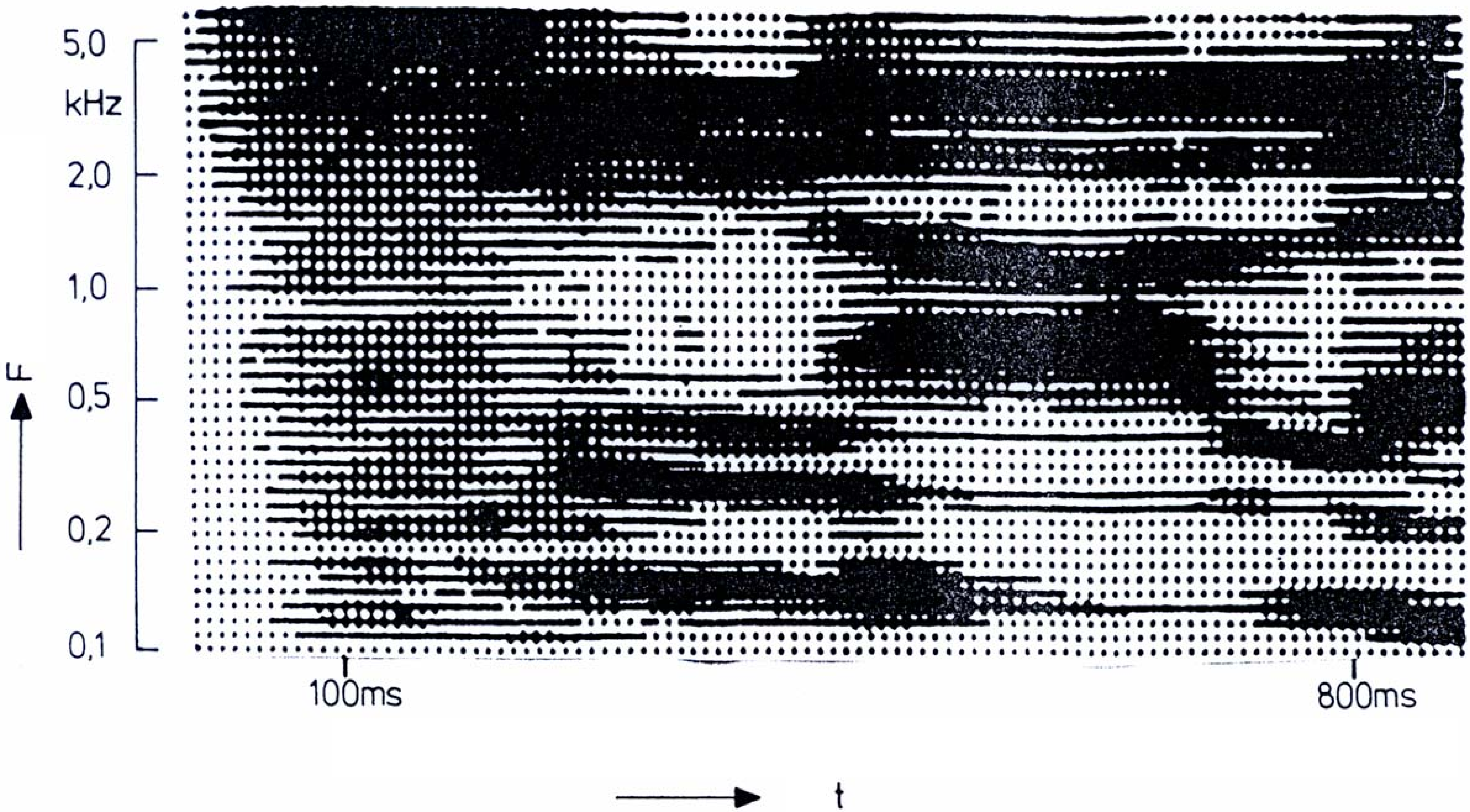
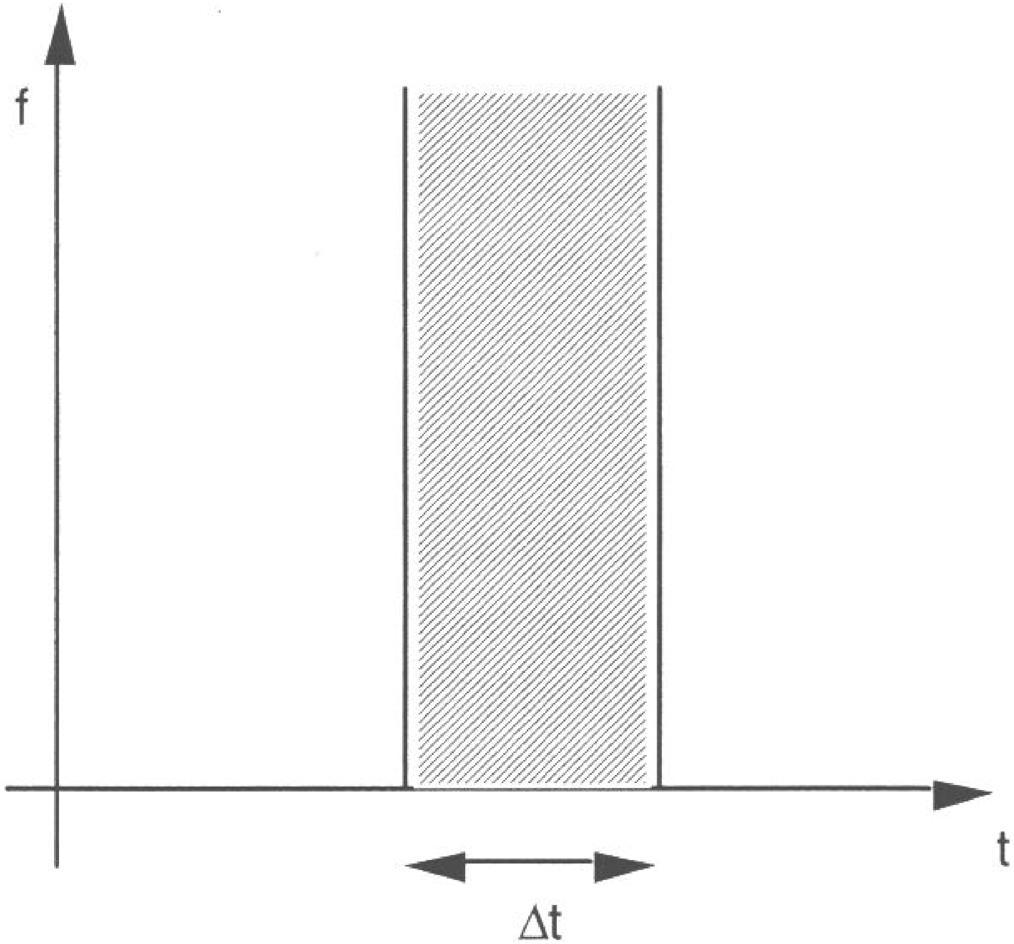


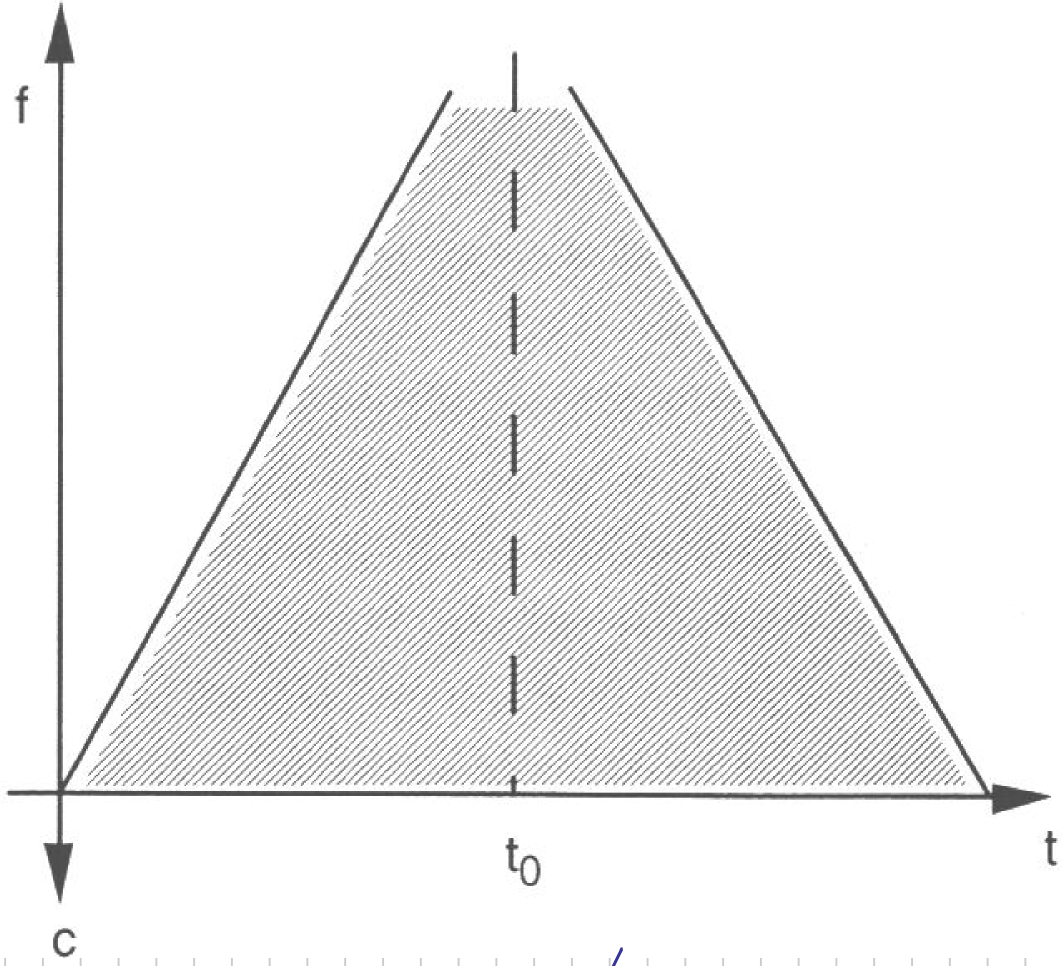
Result for WT = Scalogram

1st example: result for dirac-impulse $\delta(t-\tau)$

Spectrogram:

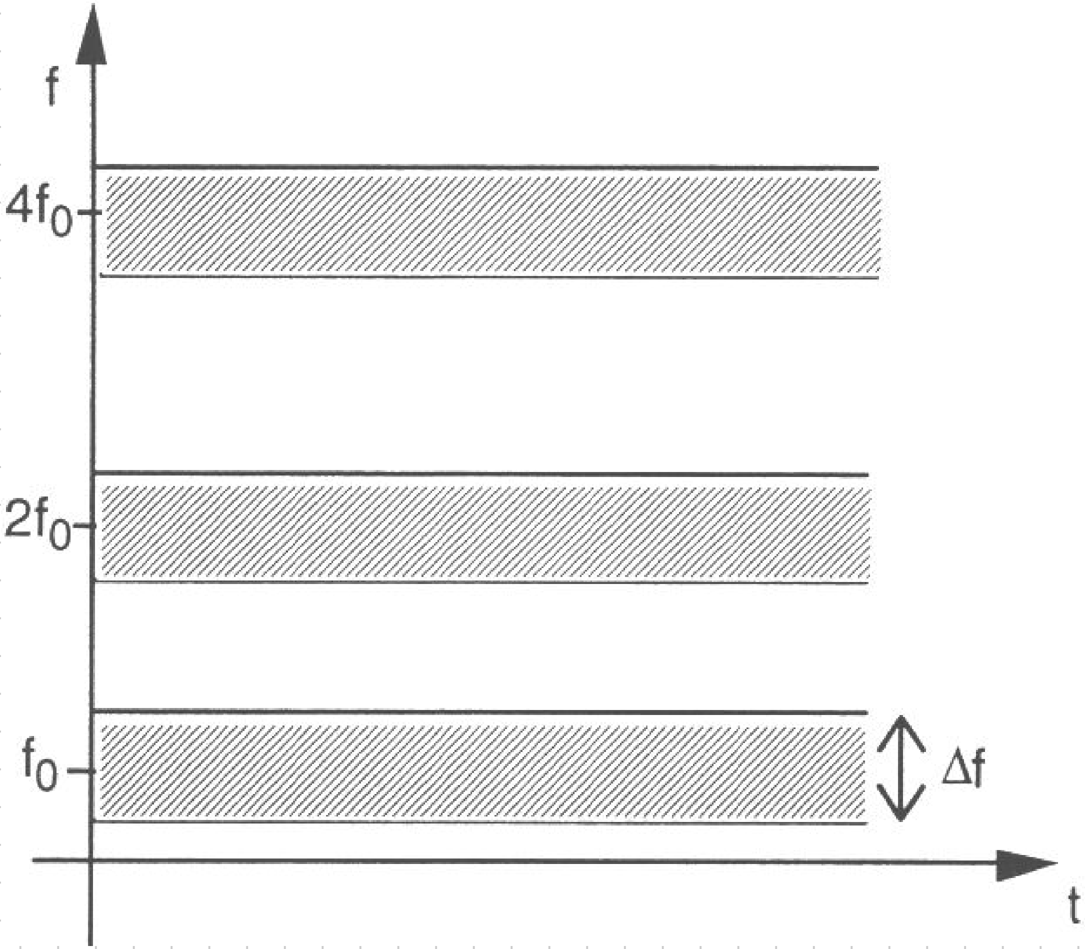


Spectrogram

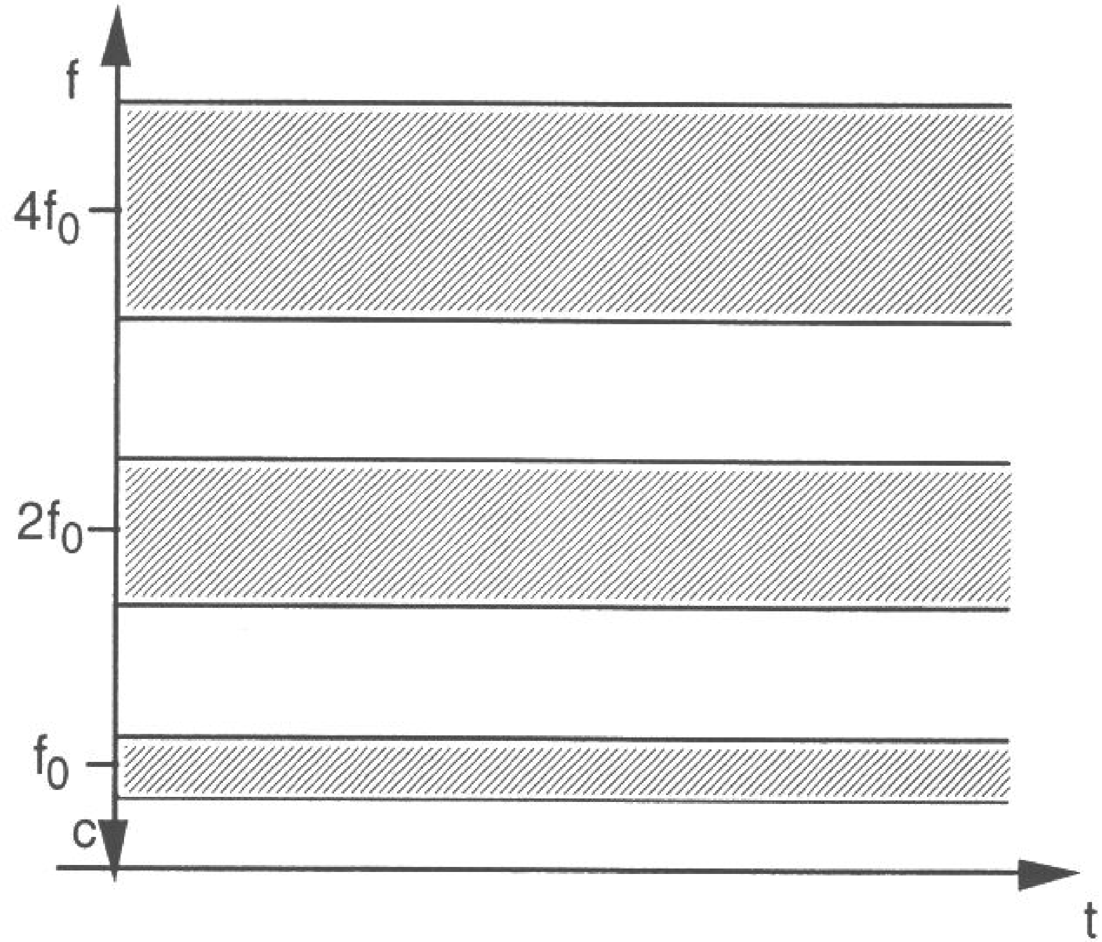


Scalogram

2nd example: signal of superposed sine-waves of frequencies: $f_0, 2f_0, 4f_0$



Spectogram



Scalogram

Implementation of the WT:

1. option: discretization of time and scaling:

$$a_n = a_0^n \qquad \tau_m = m \cdot T \cdot a_0^n$$

$$t = k \cdot T$$

leads to following discrete wavelets:

$$\psi(k, n, m) = \frac{1}{\sqrt{a_0^n}} \psi\left(\frac{kT - mTa_0^n}{a_0^n}\right)$$

$$= \frac{1}{\sqrt{c_0^n}} \psi\left(\frac{kT}{c_0^n} - mT\right)$$

and new discrete WT:

$$Y_{DW}(n,m) = \frac{1}{\sqrt{c_0^n}} \sum_{k=0}^{K-1} y(k)\,\psi\left(\frac{kT}{c_0^n} - mT\right)$$

2nd option via FFT:

(1) Fourier - transformation of signal $y(k)$, using FFT

(2) Selection of a discrete scaling factor c_n

(3) Scale the basis wavelet with c_n and subsequent take FFT of this signal

(4) multiplication results from

steps (1) and (3)

(5) inverse transformation results in $y_w(c_n, \tau)$ for the specific scaling factor c_n

(6) back to step (2) until all discrete scaling factors are processed.

2.2.2.2 Gabor Transformation (GT)

- time-frequency transformation

$$y(t) \Rightarrow Y(t, \Omega)$$

- popular, because similarity to biological information processing

Derivation of formula for GT can be done starting with STFT:

$$Y(\omega,\tau) = \int_{-\infty}^{\infty} y(t)\cdot s(t-\tau)\, e^{-j\omega t}\, dt$$

now: discrete version

$$\tau_m = m \cdot \tau_0$$

$$t = k \cdot T \qquad \omega_n = \frac{2\pi n}{T \cdot N} \ (\mathrm{DFT})$$

discrete STFT:

$$Y(n,m) = \sum_{k=-\infty}^{\infty} y(k) \cdot s(k - mT_0)\, e^{-j\frac{2\pi n}{N}k}$$

Reconstruction of original discrete time signal: Inverse Transformation

$$Y(k,m) = \frac{1}{N} \sum_{n=-\infty}^{\infty} y(n,m)\, e^{\frac{2\pi k}{N} \cdot n}$$

entire signal reconstruction:

$$y(k) = \sum_{m=-\infty}^{\infty} S^{-1}(k - mT_0)\, y(k,m)$$

$$= \sum_{m=-\infty}^{\infty} S'(k - m\tau_0) \frac{1}{N} \sum_{n=-\infty}^{\infty} Y(n,m) e^{j\frac{2\pi k}{N} n}$$

$$= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} Y(n,m) \cdot \underbrace{S(k - m\tau_0) e^{j\frac{2\pi k}{N} \cdot n}}_{g_{nm}(k)}$$

in general:
$$y(k) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} c_{nm} \cdot g_{nm}(k)$$

with
$$g_{nm}(k) = \delta(k - mT_0) \cdot e^{-j\frac{2\pi n}{N}k}$$

$\Rightarrow$ go back to time-continuous space by setting $t = kT \rightarrow k = t/T$

$$g_{nm}(t) = s(t - mT_0)\, e^{j\frac{2\pi n}{NT}\cdot t}$$

$$= s(t - mT_0)\, e^{j\Omega n t}$$

with: $\Omega = \dfrac{2\pi}{N\cdot T}$

$\Rightarrow$ $\overline{\text{Final}}$ version of GT:

$$y(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} G_{nm} \cdot g_{nm}(t)$$

Gabor coefficients

Gabor basis functions

with

$$g_{nm}(t) = s(t - mT_0)\, e^{j\Omega n t}$$

interpretation of Ω:

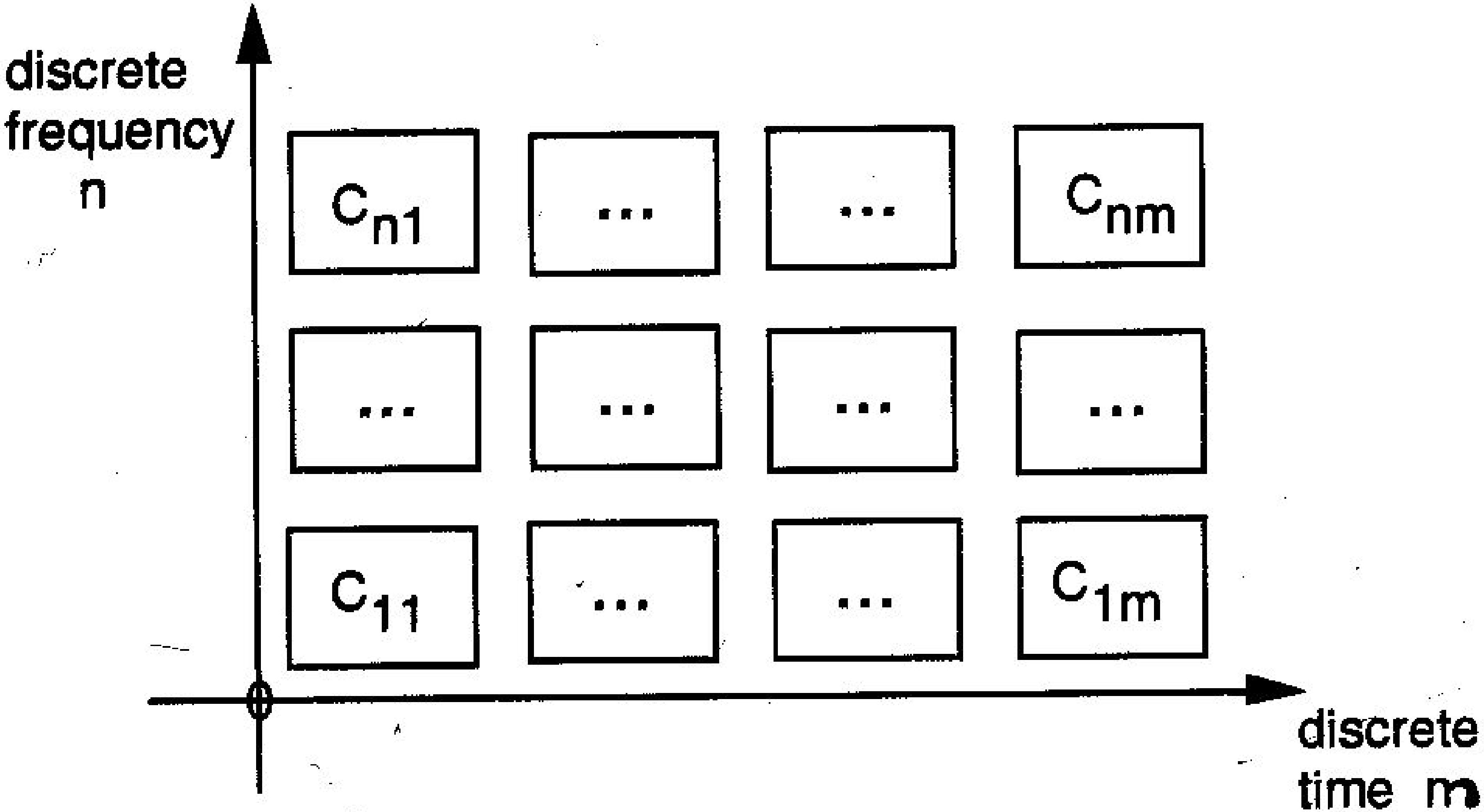
Re-consider DFT: $\omega_n = \frac{2\pi}{T} \cdot \frac{n}{N}$

now $\Omega \cdot n = \omega_n = \frac{2\pi}{N \cdot T} n = \frac{2\pi}{T} \cdot \frac{n}{N}$

Interpretation of Gabor Coefficients:

(1) as weighting factors of the Gabor basis functions

(2) as energy coefficients, similar to the result of a STFT


discrete
frequency
n
c_{n1}
...
...
c_{nm}
...
...
...
...
c_{11}
...
...
c_{1m}
discrete
time m