# Pattern Recognition

Tutorial No. 4
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## *Exercise 11*

One of the major discoveries of Gabor was the fact, that the product of the time and frequency extend $\Delta t \cdot \Delta\omega$ is minimized by the family of the Gabor-elementary-functions. These functions are expressed by a complex exponential function using gaussians:

$$g(\alpha, t) = \exp\left(\frac{\pi t^2}{\sigma^2}\right) \cdot \exp\left(j\alpha t\right) = \exp\left(-\frac{\pi t^2}{\sigma^2} + j\alpha t\right)$$

The effective extend $\Delta x$ of a function f(x) is the square-root of the variance of the power-distribution:

$$(\Delta x)^2 = \frac{\int_{-\infty}^{\infty} (x - \bar{x})^2 \cdot f(x) f^*(x)\, dx}{\int_{-\infty}^{\infty} f(x) f^*(x)\, dx}$$

where $\mathbf{f}^*$ denotes the conjunctive complex value of **f** and $\bar{x}$ denotes the mean-value of **x**. Show, that for Gabor-functions the inequality

$$\Delta t \cdot \Delta \omega \geq \frac{1}{2}$$

always equals 1/2.

## *Exercise 12*

Compare the sampling frequency $\Delta\omega$ of the Gabor-transform, that yields $\Omega \leq \frac{2\pi}{\tau_0}$ where $\tau_0$ denotes the length of the actual frame, with the discrete fragmentations of the frequencies of the DFT. For which case are both fragmentations identical?

## *Exercise 13*

Construct a multilayer neural network for the training procedure of the Gabor-coefficients using a perceptron. The input layer consists of the Gabor-elementary functions and the signal $\hat{y}$. The output layer computes all of the Gabor-coefficients.