TUTORIAL 01

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Preprocessing:

Transformations in the frequency domain:

- Laplace Transformation (LT)
- z- Transformation (ZT)
- Fourier Transformation (FT)

Exercise 1

Laplace Transformation

$$Y_L = \int_0^\infty y(t) \cdot e^{-st} \, dt \qquad (Eq. 2.07)$$

$$\frac{\partial}{\partial t}\left[y(t) \cdot e^{-st} \right] = y'(t) \cdot e^{-st} - s \cdot y(t)\, e^{-st}$$

$$\Rightarrow y'(t) \cdot e^{-st} = \frac{\partial}{\partial t}\left[y(t) \cdot e^{-st} \right] + s \cdot y(t) \cdot e^{-st}$$

$$Y'(s) = \int_0^\infty y'(t) \cdot e^{-st} \, dt =$$

$$= \int_0^\infty \frac{\partial}{\partial t}\left[y(t)\cdot e^{-st}\right] + s\,y(t)\cdot e^{-st}\,dt =$$

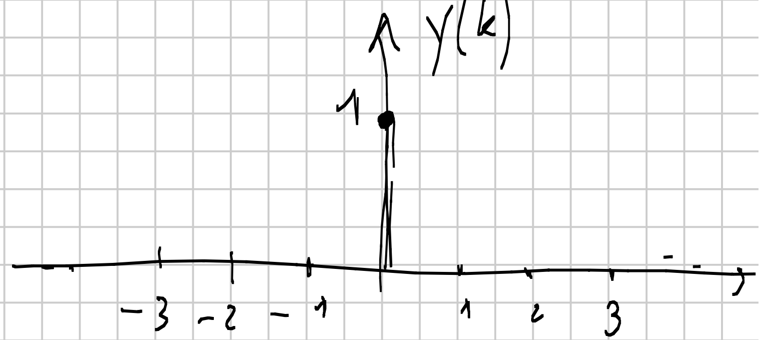
$$= \underline{\int_0^\infty \frac{\partial}{\partial t}\left[y(t)\cdot e^{-st}\right] dt} + s\int_0^\infty y(t)\cdot e^{-st}\,dt =$$

$$= \left[y(t)\cdot e^{-st}\right]_0^\infty + s\int_0^\infty y(t)\cdot e^{-st}\,dt =$$

$$= 0 - y(0) + s\cdot Y_L(s) = \underline{s\cdot Y_L(s) - y(0)}$$

Exercise 2

z-Transformation (E, 2.11)



$$Y(z) = \sum_{k=0}^{\infty} y(k) \cdot z^{-k}$$

a) Impulse: $y(k) = \delta(k) = \begin{cases} 1 & k = 0 \\ 0 & \text{otherwise} \end{cases}$

$$Y(z) = \sum_{k=0}^{\infty} \underline{y(k)} \cdot z^{-k} = \sum_{k=0}^{\infty} \delta(k) \cdot z^{-k} =$$

$$= \delta(0) \cdot z^{0} = 1$$

b) Exponential function : $y(k) = a^k$

$$Y(z) = \sum_{k=0}^{\infty} y(k) \cdot z^{-k} = \sum_{k=0}^{\infty} a^k \cdot z^{-k} =$$

$$= \sum_{k=0}^{\infty} \underline{\left(\frac{a}{z}\right)^k} \qquad q = \frac{a}{z}$$

$$-1 < \frac{a}{z} < 1$$

Geometric series : $\sum_{k=0}^{\infty} q^k = \underbrace{\frac{1}{1-q}}\Big|_{-1 < q < 1}$

$$Y_z(z) = \sum_{k=0}^{\infty} \left(\frac{a}{z}\right)^k = \frac{1}{1-\frac{a}{z}} = \frac{z}{z-a}$$

c) Mix - function : $y(k) = k \cdot a^k$

$$Y(z) = \sum_{k=0}^{\infty} k \cdot a^k \cdot z^{-k} = \underline{\sum_{k=0}^{\infty} k \cdot \left(\frac{a}{z}\right)^k}$$

let

(a)
$$S = \sum_{k=0}^{\infty} k\, q^k = 0 + 1 \cdot q^1 + 2 \cdot q^2 \cdots + m q^m + \cdots$$

(b)
$$S \cdot q = 0 + 1 \cdot q^2 + 2 \cdot q^3 \cdots$$

$$\underline{S - S \cdot q = q^1 + q^2 + \cdots q^n = \left(\sum_{k=0}^{\infty} q^k\right) - 1}$$

$$S - S \cdot q = S(1-q) = \frac{q}{1-q} \Rightarrow S = \underline{\underline{\frac{q}{(1-q)^2}}} \qquad -1 < q < 1$$

$$Y_z(z) = \sum_{k=0}^{\infty} k \cdot \left(\frac{a}{z}\right)^k = \frac{a/z}{\left(1-\frac{a}{z}\right)^2} = \frac{za}{(z-a)^2}$$

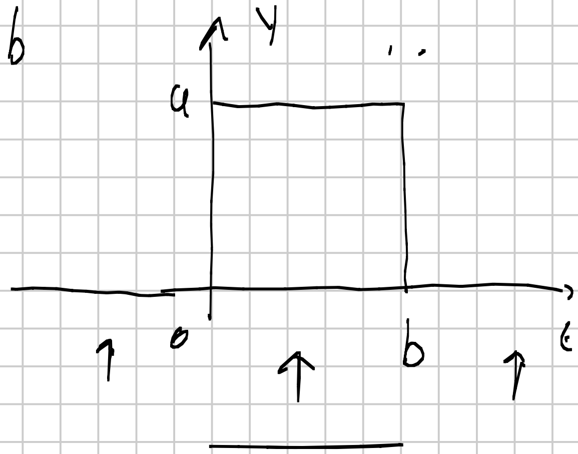
Exercise 3

Fourier Transformation (Eq. 2.14)

$$Y_F(\omega) = \int_{-\infty}^{\infty} y(t) \cdot e^{-j\omega t} \, dt \qquad \omega = 2\pi f$$

$$Y_F(f) = \int_{-\infty}^{\infty} y(t) \cdot e^{-j2\pi f t} \, dt$$

· Function $y(t) = \begin{cases} a & 0 \le t \le b \\ 0 & \text{otherwise} \end{cases}$



$$Y(f) = \int_{-\infty}^{\infty} \underline{y(t)} \cdot e^{-j2\pi f t}\, dt =$$

$$= \underbrace{\int_{-\infty}^{0} 0 \cdot e^{-j2\pi f t}\, dt}_{0} + \underbrace{\int_{0}^{b} a \cdot e^{-j2\pi f t}\, dt}_{} + \underbrace{\int_{b}^{\infty} 0 \cdot e^{-j2\pi f t}\, dt}_{0} =$$

$$= \underbrace{\int_{0}^{b} a \cdot e^{-2\pi f t}\, dt}_{} = a \left[\frac{1}{-j2\pi f} \cdot e^{-j2\pi f t} \right]_{t=0}^{b} =$$

$$= \frac{-a}{j2\pi f} \cdot e^{-j2\pi f b} - \frac{-a}{j2\pi f} \cdot e^{-j2\pi f 0} =$$

$$= \frac{a}{j2\pi f} \left(1 - e^{-j2\pi f b} \right) = \underline{\frac{a}{j2\pi f} \left(e^{j\pi f b} - e^{-j\pi f b} \right) e^{-j\pi f b}}$$

with $\sin \alpha = \frac{1}{2j} \left(e^{j\alpha} - e^{-j\alpha} \right)$

$$Y_r(f) = \underbrace{\frac{a}{\pi f} \cdot \sin(\pi f b)}_{\text{Magnitude}} \cdot \underbrace{e^{-j\pi f b}}_{\text{phase}}$$

Absolute value: $\left| Y_F(f) \right| = \left| \frac{a}{\pi f} \sin(\pi f b) \cdot \cancel{e^{-j \pi f b}} \right|$

$$= \frac{a}{\pi f} \cdot \left| \sin(\pi f b) \right| = a\, b \cdot \frac{\left| \sin(\pi f b) \right|}{\pi f b} =$$

$$= a \cdot b \cdot \left| si(\pi f b) \right|$$

Sinc - function