Polynomial Operations

Introduction

You learned in a college algebra or calculus course that a polynomial, $p(x)$, in one variable, x, is an expression of the form:

$$p(x) = a_0 + a_1 x + \cdots + a_n x^n$$

where a_i are real (or complex) numbers and n is a non-negative integer. If $p(x) = a_0$, then $p(x)$ is called a *constant* polynomial. If $p(x)$ is a non-zero constant polynomial, the degree of $p(x)$ is defined to be 0. Even though, in mathematics, the degree of the zero polynomial is undefined, for the purpose of this program, we consider the degree of such polynomials to be zero. If $p(x)$ is not constant and $a_n \neq 0$, then n is called the *degree* of $p(x)$, that is, the degree of a non-constant polynomial is defined to be the exponent of the highest power of x.

The basic operations performed on polynomials are add, subtract, multiply, divide, and evaluate a polynomial at a given point. For example, suppose that

$$p(x) = 1 + 2x + 3x^2$$

and

$$q(x) = 4 + x$$

The degree of $p(x)$ is 2 and the degree of $q(x)$ is 1. Moreover,

$p(2) = 1 + 2 \cdot 2 + 3 \cdot 2^2 = 17$

$p(x) + q(x) = 5 + 3x + 3x^2$

$p(x) - q(x) = -3 + x + 3x^2$

$p(x) \times q(x) = 4 + 9x + 14x^2 + 3x^2$

Next, we define the basic operations performed on polynomials, namely addition, subtraction, and multiplication. Suppose that $p(x)$ and $q(x)$ are the following polynomials:

$$p(x) = a_0 + a_1 x + \cdots + a_{n-1} x^{n-1} + a_n x^n$$

$$q(x) = b_0 + b_1 x + \cdots + b_{m-1} x^{m-1} + b_m x^m$$

Let $t = \max(m, n)$. Then

$$p(x) + q(x) = c_0 + c_1 x + \cdots + c_{t-1} x^{t-1} + c_t x^t$$

where, for $i = 0, 1, 2, \cdots, t$,

$$c_i = \begin{cases} a_i + b_i & \text{if } i \leq \min(n, m) \\ a_i & \text{if } i > m \\ b_i & \text{if } i > n \end{cases}$$

The difference, $p(x) + q(x)$, of $p(x)$ and $q(x)$ can be defined similarly. It follows that the degree of the polynomials is less than or equal to $\max(m, n)$.

The product $p(x) \times q(x)$, of $p(x)$ and $q(x)$ is defined as follows:

$$p(x) \times q(x) = d_0 + d_1 x + \cdots + d_{n+m} x^{n+m}$$

The coefficient d_k, for $k = 0, 1, 2, \cdots, n + m$, is given by the formula $d_k = a_0 \cdot b_k + a_1 \cdot b_{k-1} + \cdots + a_k \cdot b_0$ where is either a_i or b_i does not exist, it is assumed to be zero. For example, $d_0 = a_0 b_0$, $d_0 = a_0 b_1 + a_1 b_0$ and so on $d_{n+m} = a_n b_m$.

Problem Statement

The purpose of this programming assignment is to design and implement the class `Polynomial` to perform the various polynomial operations in a program. To be specific, this program implements the following operations on polynomials:

1. Evaluate a polynomial at a given value.
2. Add two polynomials.
3. Subtract two polynomials.
4. Multiply two polynomials.

Design Remarks

- It is assumed that the coefficients of polynomials are integers.
- To store a polynomial, your program should use a linked list. You may use the `LinkedList` class developed in class, the STL `list` type, or the `OrderedLinkedList` developed in class (recommended).
- A traditional list representation of a polynomial has one node for each term that has a nonzero coefficient. Terms with zero coefficients are not represented (traditionally). If you find it easier to represent all terms (including terms with zero coefficients), I have no objection. But my hope is that you would challenge yourself.
- If you follow the traditional list representation of a polynomial, then the data field of each node includes a coefficient and exponent field. Hence, you may want to consider a record **Term** that contains the coefficient and power of x for each term that has a nonzero coefficient.
- The terms of a polynomial can be stored in an increasing or in a decreasing order of exponent.
- Provide a driver for the `Polynomial` class that is menu driven. The menu displays all options and allows the user to perform the correspondent operation.

Program Input

The program gets its input from a data file with data organized as follows:

- Each line in the input file represents a polynomial.
- Each line contains an even number of integers.
- A nonzero term in a polynomial is represented using a pair of integers. The components of the pair are separated by a single blank.
- The first component in a pair represents the coefficient and the second component represents the power.

Create your own text file containing a minimum of two polynomials. Here are the first two lines in a sample input file and they represent the example polynomials $\boldsymbol{p}(\boldsymbol{x})$ and $\boldsymbol{q}(\boldsymbol{x})$of the Introduction section:

```
1 0 2 1 3 2
4 0 1 1
```

Program Output

The output of the program depends on the user's choice. Your program should:

- Display a menu allowing the user to select one of the operations listed above. Of course, you should provide an additional option to allow the user to quit your program.
- According to user's selection, your program calls the correspondent function to perform the operation. Write each term of a polynomial in the form: `coef x^exp`.
- Your program exits only when the user chooses to do so.

Submission Instructions

- Zip **ALL** files in the project in one zip file. The zip file must include all necessary files to run and test your program including but not limited to the input data file, the proper header and cpp files for the linked list class of your choice as well as your program.
- Rename the zip file using the following naming convention: **LastName-FristName.zip**
- Submit the zip file in the drop box dedicated for this assignment.