CS-302 Theory of Computation
Assignment 2

Name: ____

1. **(2 points)** Suppose Σ is an alphabet. It is obviously possible for two strings x and y over Σ to satisfy the condition $xy = yx$, since this condition is always satisfied if $y = \lambda$. Is it possible under the additional restriction that x and y are both nonempty? Either prove that this cannot occur, or describe precisely the circumstances under which it can.

2. **(2 points)** Let L be a language. It is clear from the definition that $L^+ \subseteq L^*$. Under what circumstances are they equal?

3. **(3 points)** For a finite set S, denote by $|S|$ the number of elements of S. Is it always true that for finite languages L_1 and L_2, $|L_1 L_2| = |L_1| \times |L_2|$? (For example, if L_1 has 3 elements and L_2 has 4, does the concatenation $L_1 L_2$ always have 12 elements?) Either prove it or find a counterexample.

4. **(6 points)** In each of the following cases, give an example of languages L_1 and L_2 that satisfy both the condition $L_1 L_2 = L_2 L_1$ and the given additional condition.

   a) Neither language is a subset of the other and neither language is $\{\lambda\}$.

   b) L_1 is a proper nonempty subset of L_2 and $L_1 \neq \{\lambda\}$.

5. **(3 points)** Let L_1 and L_2 be two languages over some alphabet Σ. Consider the two languages:

$$L_1{}^* \cap L_2{}^* \qquad (L_1 \cap L_2)^*$$

State the relationship between the two languages. (Are they always equal? If not, is one always a subset of the other?) Give reasons for your answers, including counterexamples if appropriate.

6. **(4 points)** In each part below, find an example of languages L_1 and L_2 over $\{0,1\}$ that satisfy the given condition(s).

   a) $(L_1 \cup L_2)^* \neq L_1{}^* \cup L_2{}^*$.

   b) L_1 is not a subset of L_2 and L_2 is not a subset of L_1 and $(L_1 \cup L_2)^* \neq L_1{}^* \cup L_2{}^*$.