Turjinn

2.1-1 31 41 59 26 41 58

①

②

③

26 31 41 59 41 58

④

26 31 41 41 59 58

26 31 41 41 58 59

2.1-3 Input $A = \{a_1, a_2, \ldots a_n)$ and a value v

output An index i such that $v = A[i]$ or ~~the~~ nil if $v \notin A$

length $= n$

① for $i = 1$ to n

② if $A[i] = v$ then

③ return i

④ end if

⑤ end for

⑥ return nil

2.2-1 $\frac{n^3}{1000} - 100n^2 - 100n + 3 = \Theta(n^3)$

in the search 2.2-3 How many elements? worst case, aver. case.

Yes/No ① average is $\frac{n}{2}$. ② worst is n times

$\Theta(n)$

2-3-1 $A = \{3, 41, 52, 26, 38, 57, 9, 49\}$

3 41   52 26   38 57   9 49

3, 41   26, 52   38, 57   9, 49

3, 26, 41, 52   9, 38, 49, 57

3, 9, 26, 38, 41, 49, 52, 57

2-3-3

$$T(n) = \begin{cases} 2 & \text{if } n = 2 \quad ① \\ 2T(n/2) + n & \text{if } n = 2^k \text{ for } k \ge 1 \quad ② \end{cases}$$

is $T(n) = n \lg n$

① $\cancel{T(n) = n\lg n \text{ if } n = 2}$

$\cancel{T(2) = 2\lg 2 = 2 \cdot 1 = 2}$ so ① right

② $\cancel{T(2^k) = 2^k \lg 2^k}$

$\cancel{\therefore \text{ for } k \ge 1 \Rightarrow T(2^{k+1}) = 2T\left(\frac{2^{k+1}}{2}\right) + 2^{k+1}}$

$\cancel{= 2T(2^k) + 2^{k+1}}$

$\cancel{\therefore T(2^k) = 2^k \lg 2^k \Rightarrow 2(2^k \lg 2^k) + 2^{k+1}}$

$\cancel{= 2}$

x^y

$x^{y+1} / x = x^y$

base case

① 1. $n=2$ so $k=1$ $T(n) = 2 = 2\lg 2$

2. $n=2^2$ so $k=2$ $T(n) = 2T(2^2/2) + 2^2$

$= 2T(2) + 4$

$= 2 \cdot 2 + 4 = 8$

$= 2^2 \lg 2^2$

Special case

② $\because$ if $n = 2^t$ then $2^t \lg 2^t$

$\therefore$ if $n = 2^{t+1}$ then $T(2^{t+1}) = 2T\left(\frac{2^{t+1}}{2}\right) + 2^{t+1}$

$= 2T(2^t) + 2^{t+1}$

$= 2(2^t \lg 2^t) + 2^{t+1}$

$= 2^{t+1} \lg 2^t + 2^{t+1}$

$= 2^{t+1}(\lg 2^t + 1)$

$= 2^{t+1} \lg 2^{t+1}$

3.1-1 prove $Max(f(n), g(n)) = \Theta(f(n) + g(n))$

set $Max(f(n), g(n)) = A$, and positive constants c_1, c_2

$$0 \le \underbrace{c_1(f(n)+g(n)) \le A}_{①} \le \underbrace{c_2(f(n)+g(n))}_{②}$$

for part ① $\because f(n) \ge 0, g(n) \ge 0$, $\therefore f(n) \ge g(n)$ or $g(n) \ge f(n)$

set $c_1 = 0.1$ $0.1(f(n)+g(n)) \le f(n)$, $0.1g(n) \le 0.9f(n)$, is right.

② $c_2 = 1$ $A \le c_2(f(n)+g(n))$ is right.

$\therefore$ ① ② $\Rightarrow$ $Max(f(n), g(n)) = \Theta(f(n), g(n))$

3.1-4 Is $2^{n+1} = O(2^n)$

$0 \le 2^{n+1} \le c \cdot 2^n$

$2 \cdot 2^n \le c 2^n$ ~~if~~ $c \ge 2$, $\cancel{2^{n+1} = O(2^n)}$

if $c = 3$ then $2^{n+1} \le 3 2^n$

$\therefore 2^{n+1} = O(2^n)$ is true

$2^{2n} = O(2^n)$

$0 \le 2^{2n} \le c \cdot 2^n$

$\cancel{2n \le c \lg c + n}$

if $2^{2n} = O(2^n)$ is right

$2^{2n} \le c 2^n$

$2n \le \lg c + n$

$n \le \lg c$ can't compare

$\therefore$ is false

3.1-7 $o(g(n)) \cap \omega(g(n))$ is ~~ϕ~~

set to Y

$n_0 > 0$ $n \ge n_0$

① $\left(\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \right)$

$n_0 > 0$ $n \ge n_0$

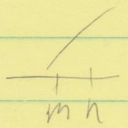
② $\left(\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty \right)$

Let $a \in Y$, then $a = o(g(n)) = \omega(g(n))$

$\because$ ① and ② $0 \ne \infty$

$\therefore o(g(n)) \cap \omega(g(n))$ is empty set

3.2-1 if $f(n)$ and $g(n)$ monotonically $\nearrow$, then $f(n)+g(n)$ and $f(g(n))$ same.



if $f(n)$ and $g(n)$ not negative then $f(n) \cdot f(n) \nearrow$

$\because$ ① $m \le n$ $f(m) \le f(n)$, $g(m) \le g(n)$

$\therefore f(m)+g(m) \le f(n)+g(n)$. so they are monotonically $\nearrow$

$f(g(m)) \le f(g(n))$, $\because f(n)$ is monotonically $\nearrow$, $\therefore f(g(n))$ same.

$f(m) \cdot g(m) \le f(n) \cdot g(m) \le f(n) \cdot g(n)$, $\because f(n) \cdot g(n)$ is monotonically $\nearrow$

3.1-4 is $2^{n+1} = O(2^n)$

$0 \le 2^{n+1} \le c \cdot 2^n$

$2 \cdot 2^n \le c 2^n$ ~~当~~ $c \ge 2$, ~~$2^{n+1} = O(2^n)$~~

if $c = 3$ then $2^{n+1} \le 3 2^n$

$\therefore 2^{n+1} = O(2^n)$ is ture

$2^{2n} = O(2^n)$

$0 \le 2^{2n} \le c \cdot 2^n$

~~$2n \le lg c + n$~~

if $2^{2n} = O(2^n)$ is right

$2^{2n} \le c 2^n$

~~$2n$~~ $n \le$ ~~$lg c + n$~~

$n \le lg_c$ can't compare

$\therefore$ is False

3.1-7 ~~Θ~~ $o(g(n)) \cap w(g(n))$ is ~~$\emptyset$~~

set to Y

$n_0 > 0$ $n \ge n_0$

$n_0 > 0$ $n \ge n_0$

① $\left(\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \right)$

② $\left(\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty \right)$

Let $a \in Y$, then $a = o(g(n)) = w(g(n))$

$\because$ ① and ② $0 \ne \infty$

$\therefore o(g(n)) \cap w(g(n))$ is empty set

3.2-1 if $f(n)$ and $g(n)$ monotonically $\nearrow$, then $f(n) + g(n)$ and $f(g(n))$

same.

if $f(n)$ and $g(n)$ not negative then $f(n) \cdot f(n) \nearrow$

$\because$ ① $m \le n$ $f(m) \le f(n)$, $g(m) \le g(n)$

$\therefore f(m) + g(m) \le f(n) + g(n)$, so they are monotonically $\nearrow$

$f(g(m)) \le f(g(n))$, $\because f(n)$ is monotonically $\nearrow$, $\therefore f(g(n))$ same.

$f(m) \cdot g(m) \le f(n) \cdot g(m) \le f(n) \cdot g(n)$, $\because f(n) \cdot g(n)$ is monotonically $\nearrow$

P 3-2 $k \geq 1$ $\epsilon > 0$ $c > 1$

A	⇒	O	o	Ω	ω	Θ	of B
ⓐ $\lg^k n$		Yes	No	No	No	No	$n^{\epsilon > 0}$
ⓑ n^k		Yes	No	No	No	No	c^n
ⓒ $\sqrt{n}$	All	No	∵ $\sin n$	No	No	No	$n^{\sin n}$
ⓓ 2^n		Yes	Yes	No	No	No	$2^{\frac{n}{2}}$
ⓔ $n^{\lg c}$		Yes	No	Yes	No	Yes	$c^{\lg n}$ A=B
ⓕ $\lg(n!)$		Yes	No	Yes	No	Yes	$\lg(n^n)$ A=B

$\leq$ O $\lg^k n\, O(n)$ $n^k\, O(n^k)$, $2^n\, O(c)$,

o and ω in the end, because not sure how to do, not on note.

$\geq$ Ω in ⓔ ⓕ AΘB, so other was opposite to O

$=$ Θ only ⓔ ⓕ

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o $2n^2 = O(n^2)$, but ① $2n \neq O(n^2)$, $2n = o(n^2)$, but $2n^2 \neq o(n^2)$

$f(n) < c\,g(n)$

$n \geq n_0$ need have condition ①, so ⓑ is No

can't same, so ⓔ ⓕ is No

ⓐ is $\lg^k n < n^{\epsilon c}$ set $k=1$ $\epsilon=1$ $an < n^1$, set $k=100$ $\epsilon=1$ $2n \not< n^1$. No

ⓓ O Yes, Θ No ⇒ o is Yes

ω $g(n) \in o(f(n))$, ~~No~~ Ω need Yes.

P 3-3 $g_1 = \Omega(g_2)$ $f(n) = \Theta(g(n))$

(a)

$2^{\sqrt{2\lg n}}, 2^n, 2^{2n+1}$

1 $2^{\lg n}$, $(\sqrt{2})^{\lg n}$, $(\frac{3}{2})^n$, 2^{2^n}, $n \cdot 2^n$, 1, $2^{\lg n}$, e^n, $4^{\lg n}$

2 $\lg(\lg^* n)$, $\lg^2 n$, $\lg(n!)$, $\ln \ln n$, $\lg^* n$, $\ln n$, $(\lg n)^{\lg n}$

$\sqrt{\lg n}$, $\lg^*(\lg n)$, $n \lg n$

2-1 n

3 n^2, n^3, $n^{1/\lg n}$, $n^{\lg \lg n}$

4 $n!$, $(\lg n)!$, $(n+1)!$

clean (1) 1, $(\sqrt{2})^{\lg n}$, $(\frac{3}{2})^n$, | $2^{\lg n}$, 2^{2^n}, $n \cdot 2^n$, $2^{\lg n}$, e^n, $4^{\lg n}$

$2^{\sqrt{2\lg n}}$, 2^n, $2^{2^{n+1}}$

(1) $1, (\sqrt{2})^{\lg n}, (\frac{3}{2})^n, 2^{\sqrt{2\lg n}}, 2^{\lg n}, 2^{\lg n}, 4^{\lg n}, 2^n, 2^{2n}, 2^{2^{n+1}}, n \cdot 2^n, e^n$

clean (2) $\ln n$, $\ln \ln n$, $\sqrt{\lg n}$, $\lg^*(\lg n)$, $\lg^*(\lg^* n)$. | $\lg^2 n$, $\lg(n!)$, $(\lg n)^{\lg n}$ $n \lg n$

$\lg^* n$

(2) $\ln n, \ln \ln n, \sqrt{\lg n}, \lg^* n, \lg^*(\lg n), \lg^*(\lg n), \lg^*(\lg^* n), n \lg n, \lg(n!), (\lg n)^{\lg n}, \lg^2 n$

(2-1) n

(3) $n^{\lg \lg n}$, $n^{\frac{1}{\lg n}}$, n^2, n^3

(4) $(\lg n)!$, $n!$, $(n+1)!$

$g_1 = \Omega g_2 \cdots$, in the same list. $f(n) = \Theta g(n)$

(b) $f(n) = \left((\sqrt{2})^{\lg n}\right)^{\sin n}$