CS 303, Paul Cao,
Topic: String matching algorithms (Horspool, BM) – extra topic

1. Horspool's string searching algorithm

Review of brute force algorithm
pattern: a string of m characters to search for
text: a (long) string of n characters to search in

Brute force algorithm

Step 1: Align pattern at beginning of text
Step 2: Moving from left to right, compare each character of pattern to the corresponding character in text until either all characters are found to match (successful search) or a mismatch is detected
Step 3:While a mismatch is detected and the text is not yet exhausted, realign pattern one position to the right and repeat Step 2

Efficiency: O(mn) in worst case scenario

Idea of better algorithms: Since we know everything about the pattern, maybe we can use the input enhancement idea of preprocessing the pattern.

- Horspool's algorithm simplifies the Boyer-Moore algorithm by using just one table
- Boyer-Moore algorithm preprocesses pattern right to left and store information into two tables
- Knuth-Morris-Pratt (KMP) algorithm preprocesses pattern left to right to get useful information for later searching (not covered)

Horspool's idea: Since we know everything about the short pattern, can we generate a table to tell us how much we can shift under different conditions?

It is a simplified version of Boyer-Moore algorithm:

- preprocesses pattern to generate a **shift table** that determines how much to shift the pattern when a mismatch occurs
- always makes a shift based on the text's character *c* aligned with the last character in the pattern according to the shift table's entry for *c*

Details
Let $p_1..p_m$ be the m characters in the pattern.

...	...	...	...	...	...	c	...	...	...	...	...	...	...
	p_1	p_2	p_3	...	p_{m-1}	p_m							

Look at first (rightmost) character in text that was compared. They may be several scenarios

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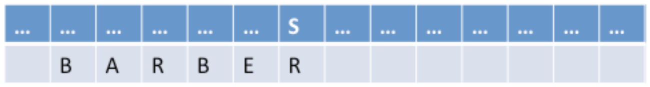
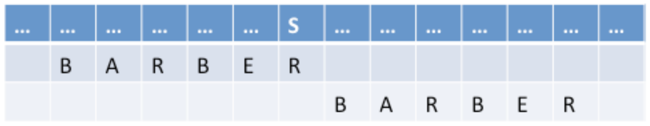
1. The character is not in the pattern (c is not in the pattern)



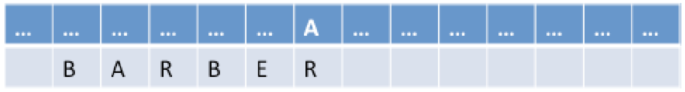
...	...	...	...	...	...	S	...	...	...	...	...	...	...
	B	A	R	B	E	R							

e.g.
Q:/ What should be our action?
A:/ shift the pattern to the right by m positions



...	...	...	...	...	...	S	...	...	...	...	...	...	...
	B	A	R	B	E	R							
							B	A	R	B	E	R	

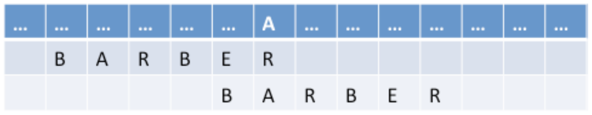
2. The character is in the pattern (but not the rightmost)



...	...	...	...	...	...	A	...	...	...	...	...	...	...
	B	A	R	B	E	R							

e.g.

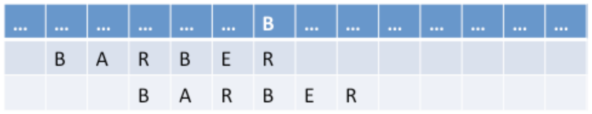
Q:/ What should we do?
A:/ shift the pattern such that the rightmost occurrence of c in the pattern with the c in the text



...	...	...	...	...	...	A	...	...	...	...	...	...	...
	B	A	R	B	E	R							
					B	A	R	B	E	R			

shift: 4

Another example



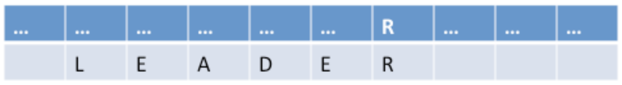
...	...	...	...	...	...	B	...	...	...	...	...	...	...
	B	A	R	B	E	R							
			B	A	R	B	E	R					

shift: 2

Q:/ From the previous two examples, can you decide the number of shifts we should do in scenario 2?

A:/ shift the pattern to the right by the distance of the right most c among the first m-1 characters of the pattern to its last character.

3. The rightmost characters do match



...	...	...	...	...	...	R	...	...	...
	L	E	A	D	E	R			

e.g.
Q:/ Action?
A:/ Keep comparing to the left.

Q:/ However, if we see a mismatch on the left, what should we do?

scenario 3.1: there are no c's among the first m-1 characters ⇒ shift m positions to the right
e.g.

...	...	...	...	M	E	R	...	...	...	...	...	...	...
	L	E	A	D	E	R							
							L	E	A	D	E	R	

scenario 3.2: there are other c's in the first m-1 characters. ⇒ shift the pattern to the right by the distance of the right most c among the first m-1 characters of the pattern to its last character.
e.g.

...	...	...	...	M	E	R	...	...	...	...	...	...	...
	B	A	R	B	E	R							
				B	A	R	B	E	R				

Overall, the shift table can be computed using the following formula. Note the c is the character in the text, not in the pattern.

t(c)= { distance from *c*'s rightmost occurrence in pattern among its first *m*-1 characters to its right end
pattern's length *m*, otherwise

We scan the pattern before search begins and store the result in a table called **shift table**
e.g. pattern is BAOBAB
Shift table can be built as

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	_
1	2	6	6	6	6	6	6	6	6	6	6	6	6	3	6	6	6	6	6	6	6	6	6	6	6	6

B	E	S	S	_	K	N	E	W	_	A	B	O	U	T	_	B	A	O	B	A	B	S
B	A	O	B	A	B																	
						B	A	O	B	A	B											
								B	A	O	B	A	B									
														B	A	O	B	A	B			
																B	A	O	B	A	B	

SUCCESSFUL SEARCH!

Efficiency: Worst case scenario: still O(mn)

- text: all n zeros: 000000000000000
- pattern: 1 followed by m-1 zeros

Shift table:

0	1
1	m-1

But Horspool's performance is faster on random text compared with brute force.

BM algorithm improves from Horspool's algorithm

- Horspool: only compare the characters of pattern and text one by one ⇨ one character pattern where basically every character is considered to be independent from each other.
- Boyer-Moore: can we find more patterns? ⇨ k-character patterns
- Use two tables
    - Bad-symbol table
    - Good-suffix table

Similar to Horspool, BM tries to compare the pattern from the right to the left but uses a more complicated way to decide how much to shift.

1. Bad symbol table

Purpose of this table: decide how much to shift based upon the **character of the text** that caused the mismatch. (not the character in the text aligned with the last char of the pattern like in Horspool)

Scenario 1:

If the last character is a mismatch ⇨ same as Horsloop (using the shift table)

e.g.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	_
4	2	6	6	1	6	6	6	6	6	6	6	6	6	6	6	6	3	6	6	6	6	6	6	6	6	6

…	…	…	…	…	…	S	…	…	…	…	…	…	…
	B	A	R	B	E	R							
							B	A	R	B	E	R	

…	…	…	…	…	…	A	…	…	…	…	…	…	…
	B	A	R	B	E	R							
					B	A	R	B	E	R			

Scenario 2:

If there are some k (0<=k<m) of the pattern's characters are matched successfully before a mismatch is found.

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$$\begin{array}{cccccccccc} s_1 & \ldots & & c & s_{i-k} & \ldots & s_i & \ldots & s_n & \textbf{text} \\ & & & \neq & = & & = & & & \\ & p_1 & \ldots & p_{m-k-1} & p_{m-k} & \ldots & p_m & & & \textbf{pattern} \end{array}$$

We can compare the effect of Horspool and BM

Example1

BM: since S is not in the pattern, we can shift 4 positions directly

Shift=4=t(S)-2=t(S)-k

…	…	…	…	S	E	R	…	…	…	…	…	…	…
	B	A	R	B	E	R							
					B	A	R	B	E	R			

Example 2

BM: since A is in the pattern, we will shift the next rightmost A to align with the current mismatched A. ▭

2 positions Shift=2=t(A)-2=t(A)-k

…	…	…	…	A	E	R	…	…	…	…	…	…	…
	B	A	R	B	E	R							
			B	A	R	B	E	R					

Example 3

BM: since there is no other E to the left in the pattern, we can shift one.
Shift=1 (because t(E)-2≤0)

Q:/ Can you summarize the way BM shifts with a bad symbol?
A:/ Shift to match the "bad symbol". ▭bad-symbol table

- bad-symbol table indicates how much to shift based on text's character causing a mismatch.
- bad-symbol shift $d_1 = \max\{t_1(c) - k, 1\}$ ▭ same idea as horspool.