Flocking of Multi-Agent Non-Holonomic Systems With Proximity Graphs

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***Abstract*—Multi-agent systems are ubiquitous in the real-world and have received an increasing attention by many researchers worldwide. A multi-agent system is composed of many agents interconnected by a communication network. This paper aims to further investigate the flocking and preserving connectedness in multi-agent nonholonomic systems with proximity graphs, in which the positions and the relative distances are not available to the distributed controllers. Several sufficient conditions are derived to resolve the above problem based on the kinematic model and the dynamic model, respectively. These sufficient conditions indicate that, for any given distinct initial positions and connected initial graph, there always exist gains of the linear protocols to preserve the connectedness of the graph and realize flocking. Moreover, under an additional condition on initial heading angles, the similar result is obtained for a nonlinear protocol with the form of Kuramoto model. Finally, numerical simulations are given to validate the above theoretical results.**

***Index Terms*—Complex networks, multi-agent nonholonomic systems, flocking, collision avoidance, proximity graph, preserving connectedness.**

I. Introduction

OVER the last decade, multi-agent systems as a special kind of complex networks have attracted an increasing attention [1]–[4] from many researchers worldwide, which arise from many network systems in the real-world such as flocks [5], [6], a group of vehicles [7], [8], power systems [9] and complex dynamic networks [10]. Flocking problems of multi-agent systems have become a promising emerging research topic in various disciplines including physics [11], [12], theoretical biology [13] and automatic control [14]–[18]. The flocking problem considered in [16] is described as determining a distributed control law such that all agents velocities asymptotically become the same and collisions among them are always avoided. Thus the flocking problem becomes actually the velocity synchronization or consensus with collision avoidance. Many approaches and techniques for flocking have been used to deal with synchronization or consensus problems. As the authors of [1] pointed out, the topic of synchronization of coupled nonlinear oscillators is closely related to the consensus of multi-agent systems. And the synchronization of complex networks has been widely studied (see [10], [19]–[22] and the references therein). Moreover, multi-agent systems have wide applications in various engineering systems, such as formation control [23] and attitude control [24]. It is well known that some natural phenomena show typical flocking behaviors, such as the migration of birds flock and the foraging of ant colonies. In 1987, Reynolds proposed a computer simulation model to characterize a flocking phenomenon, called Boid model [5]. In 1995, Vicsek and his colleagues introduced a simple mathematical model to describe the emergence of self-ordered motion in systems of particles with biologically motivated interaction, called Vicsek model [11]. Over the last few years, there have been numerous results reported on Vicsek model and its various variants [25]–[31].

It should be especially pointed out that theoretical analysis of the original Vicsek model is very difficult since the relationship between neighboring agents is nonlinear and the interconnected graph between agents is dynamically changing [29]. In 2003, Jadbabaie *et al.* [25] investigated the linearized Vicsek model and attained several sufficient conditions for the heading synchronization of the agents. The obtained results show that the heading synchronization can be realized if the considered dynamical neighbor graph or proximity graph can preserve the connectedness or jointly connectedness. Savkin [26] further studied a new heading updating rule and proved that synchronization can be achieved if the union of neighbor graphs is connected infinite times. However, for switching graphs such as proximity graphs, it is often difficult to verify these connectedness conditions, which should be proved rather than assumed. Therefore, an interesting question is: "How can we guarantee that the connectedness assumptions always hold during the whole time interval?". For preserving jointly connectedness over infinitely many time intervals, Tahbaz-Salehi and Jadbabaie [31] used the periodic boundary condition proposed in [11]. They proved that the proximity graph in Vicsek model stays jointly connected infinitely often for almost all initial conditions and the flocking is achieved. The periodic boundary condition implies that all agents move in a finite plane. For Vicsek model with agents moving in an infinite plane, Liu and Guo first proposed some sufficient conditions for flocking and preserving connectedness, which only depend on the system parameters and the initial states [28], [29]. Following this line, some further results were also obtained recently [30], where a random framework of Vicsek model was established without any connectedness assumption.

For continuous-time multi-agent systems, Olfati-Saber and Murray [17] studied the consensus of single-integrator

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multi-agent systems based on the strong connectedness assumption on a directed graph. Ren and Beard further generalized the above results and found that the above multi-agent systems can achieve consensus if there exists a spanning tree in the directed graph [32]. Moreover, Ren and Sorensen [8] applied the consensus algorithm into the formation control of multi-vehicle systems. Thereafter, many kinds of distributed protocols were designed for different multi-agent systems [33]–[39].

Note that preserving connectedness is also an important topic for continuous-time multi-agent systems [16], [40]–[42]. The main idea of the existing results is designing nonlinear protocols using relative distances among agents. But if the information of the relative distances is not available, the flocking problem is still open. It should be noted that there is a relationship between the discrete Vicsek model and the continuous Kuramoto model [31], [43], and the position dynamics of Vicsek model is just the discrete form of a continuous vehicle model.

Stimulated by the work of Liu and Guo [28], [29], this paper aims to investigate flocking and preserving connectedness for a class of continuous multi-agent non-holonomic systems without using relative distances under linear and nonlinear protocols respectively. Based on the initial connectedness assumption, several sufficient conditions are derived. In [28] and [29], the absolute velocity of agents is suggested small enough so that the sufficient conditions are satisfied. In this paper, we will show that no matter how large the absolute velocity is, the flocking can be realized by choosing sufficiently large feedback gain. Furthermore, different from the results in [28] and [29], this paper further guarantees collision avoidance, which plays an important role in proving the heading synchronization. The work of this paper can be regarded as further research of [28] and [29] in the case of continuous-time systems. It is revealed that, for the first time, there always exist the first order and second order linear protocols only using the local heading angles to preserve the connectedness and achieve flocking for any given initial states as long as the initial graph is connected and the initial positions are distinct. Moreover, for the nonlinear protocol with the form of Kuramoto model, similar results are obtained under some suitable limitation on the initial heading angles. Compared with the results of continuous systems in preserving connectedness [16], [40]–[42], in this paper the information transmitted in the proximity graph is only the information of neighbors' heading angles instead of the positions and the relative distances. Finally, numerical simulations are given to validate the effectiveness of the proposed method.

This paper is organized as follows. In Section II, the system description and problem statement are briefly outlined. The flocking problem is investigated for a class of multi-agent non-holonomic systems with linear and nonlinear protocols in Sections III and IV respectively. Conclusions are finally drawn in Section V.

II. System Description and Problem Statement

Consider the multi-agent system composed of n agents moving in the 2-dimensional infinite plane with the same absolute velocity v. Set the Cartesian coordinate system in the phase-plane and denote the position of each agent at time t by $(x_i(t),\ y_i(t))$ $(i = 1, 2, \ldots, n)$. The neighbors of agent i are those which lie in a circle of radius $r > 0$ centered at (x_i, y_i). The radius r is often called sensing radius. Let $z = (x^{\mathrm{T}}, y^{\mathrm{T}})^{\mathrm{T}}$ be the position vector of the whole system, where $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$ and $y = (y_1, y_2, \ldots, y_n)^{\mathrm{T}}$. For any given position vector z, the distance between agents i and j is defined by $d_{ij}(z) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$. The neighbor set of agent i is described by

$$N_i(z) = \{j \neq i \mid\ d_{ij}(z) < r\}. \tag{1}$$

From the above definition, each agent is not a neighbor of itself. This neighbor relationship of the agents can be characterized by an undirected graph $G_z = \{V,\ E_z,\ A_z\}$, where the vertex set $V = \{1, 2, \ldots, n\}$ indexes the agents, the edge set is given by

$$E_z = \{(i, j) \subset V \times V \mid\ d_{ij}(z) < r,\ \ i \neq j\}, \tag{2}$$

and the adjacency matrix $A_z = (a_{ij}(z))$ with n rows and n columns is defined by

$$a_{ii}(z) = 0,\ \ a_{ij}(z) = \begin{cases} 1, & d_{ij}(z) < r, \\ 0, & d_{ij}(z) \geq r, \end{cases} \quad (i \neq j). \tag{3}$$

The above graph is called *proximity graph* [40] or *dynamical graph* [41]. The Laplacian matrix of graph G_z is defined by the $n \times n$ matrix $L_z = (l_{ij}(z))$, where

$$l_{ii}(z) = \sum_{j=1, j\neq i}^{n} a_{ij}(z), \quad l_{ij}(z) = -a_{ij}(z) \quad (i \neq j). \tag{4}$$

For undirected graph G_z, its Laplacian matrix L_z is symmetrical and positive semi-definite. L_z has at least a zero eigenvalue with eigenvector $\mathbf{1}_n$, i.e., $L_z\, \mathbf{1}_n = 0$. The eigenvalues of Laplacian matrix L_z are often called the eigenvalues of graph G_z, which is described by

$$0 = \lambda_1(G_z) \leq \lambda_2(G_z) \leq \cdots \leq \lambda_n(G_z). \tag{5}$$

It is well-known that G_z is connected if and only if $\lambda_2(G_z)$ is positive. $\lambda_2(G_z)$ is usually called *algebraic connectivity* [44].

Suppose that each agent has the nonholonomic constraint for pure rolling and nonslipping with the kinematical model described by

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = v \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix}, \tag{6}$$

$$\dot{\theta}_i = u_i, \tag{7}$$

where v is the constant absolute velocity, u_i is the external control, i.e., the angular velocity of agent i, and $i = 1, 2, \ldots, n$. The dynamics of each agent is just the so-called Dubins' vehicle model (see [45] or [46]) and has been widely investigated in robotics [47], [7], [23].

The flocking problem considered in this paper is defined as follows:

Definition 1: The flocking problem for the above multi-agent system is designing distributed controller u_i with local information such that

$$d_{ij}(z(t)) > 0 \quad \forall t \in [0, +\infty) \tag{8}$$

and

$$\lim_{t \to +\infty} (\theta_i(t) - \theta_j(t)) = 0, \tag{9}$$

for all $1 \leq i, j \leq n$.

In the case of dynamical models, we consider the dynamical model investigated in [48] and [49] as follows:

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = v_i \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix}, \tag{10}$$

$$\dot{\theta}_i = \omega_i, \tag{11}$$

$$\dot{\omega}_i = u_{1i}, \tag{12}$$

$$\dot{v}_i = u_{2i}, \tag{13}$$

where (x_i, y_i) is the position, v_i the absolute velocity, θ_i the heading angle, ω_i the angular velocity, $(u_{1i},\ u_{2i})$ the external control. It is obvious that u_{1i} and u_{2i} are directly proportional to the steering torque and the driving force respectively.

Definition 2: The flocking problem for multi-agent systems described by the dynamical model (10)–(13) is designing distributed controller $u_i = (u_{1i}, u_{2i})$ with local information such that (8), (9) and

$$\lim_{t\to+\infty} (v_i(t) - v_j(t)) = 0 \tag{14}$$

hold for all $1 \le i, j \le n$.

Remark 1: In this paper, we assume the communication network described by the proximity graph $G_{z(t)}$ can only transmit the information of neighbors' heading angles. Thus the local information that can be used by agent i includes the states of itself and its neighbors' heading angles instead of positions and relative distances. How to guarantee the connectedness only using the local heading angles is still an open problem.

Before the main results of this section, we further introduce some basic concepts and results on algebraic graph theory [44].

An *orientation* of an undirected graph X is the assignment of direction to each edge; this means that we declare one end of the edge to be the *head* of the edge and the other to be the *tail*, and view the edge as oriented from its tail to its head. An *oriented graph* is a graph together with a particular orientation. We will sometimes use X^σ to denote the oriented graph determined by the special orientation σ. The *incidence matrix* $D(X^\sigma)$ of an oriented graph X^σ is the $\{0, \pm 1\}$-matrix with rows and columns indexed by the vertices and edges of X, respectively, such that the uf-entry of $D(X^\sigma)$ is equal to 1 if the vertex u is the head of the edge f, -1 if u is the tail of f, and 0 otherwise. If X has n vertices and e edges, then $D(X^\sigma)$ has order $n \times e$. Let σ be an arbitrary orientation of a undirected graph X, and let D be the incidence matrix of X^σ. Then the Laplacian matrix of X is $L = DD^{\mathrm{T}}$ that does not depend on the orientation σ.

III. Synchronization With Collision Avoidance Under Linear Protocols

A. The Case of the Kinematic Model

In this subsection, we consider the linear control protocol proposed by Olfati-Saber and Murray [17]:

$$u_i = k \sum_{j\in N_i(z)} (\theta_j - \theta_i) = k \sum_{j=1}^{n} a_{ij}(z)(\theta_j - \theta_i), \tag{15}$$

where k is a constant feedback gain. Equation (15) means that as long as agent j enters the sensing range of i, the value of θ_j can be transmitted to u_i. Although position vector z appears in the expression of u_i, the values of z and relative distance d_{ij} are not used as feedback information. With the distributed controller (15), the closed-loop system is a nonlinear switching multi-agent system with non-holonomic constraint described by

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = v \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix}, \tag{16}$$

$$\dot{\theta}_i = k \sum_{j=1}^{n} a_{ij}(z)(\theta_j - \theta_i) \quad (i = 1, 2, \ldots, n). \tag{17}$$

For linear subsystem (17), Olfati-Saber and Murray [17] proved that the synchronization can be achieved if the switching graph is connected. However, for the whole nonlinear switching system (16) and (17), no conditions have been found to preserve the connectedness using only the local heading angles. Moreover, collision avoidance has not been considered for the linear protocol (17) yet.

Define $\theta = (\theta_1, \theta_2, \ldots, \theta_n)^{\mathrm{T}}$. Then (17) can be rewritten in the following compact form [17]:

$$\dot{\theta} = -kL_z\theta, \tag{18}$$

where L_z is the Laplacian matrix of graph G_z and determined by the sensing radius r and the position vector z.

This section aims to discover some suitable conditions of system parameters v, r, feedback gain k and initial states $(x_i(0), y_i(0),\ \theta_i(0))$ with $i = 1, 2, \ldots, n$ for preserving connectedness and achieving flocking.

Theorem 1: For closed-loop system (16) and (17), suppose that the initial proximity graph $G_{z(0)}$ is connected and the initial positions of all agents are distinct, namely, the algebraic connectivity $\mu = \lambda_2(G_{z(0)}) > 0$ and $d_{ij}(z(0)) > 0$ for all $i \ne j$. If

$$k > \frac{\sqrt{2}\, v\zeta}{\mu d}, \tag{19}$$

then the proximity graph $G_{z(t)}$ preserves the connectedness on the whole time interval $[0, +\infty)$ and also the flocking problem can be solved with

$$\Delta_t := \max_{i,j} |\theta_i(t) - \theta_j(t)| \le \sqrt{2}\, \zeta\, \mathrm{e}^{-k\mu t}, \tag{20}$$

where

$$\zeta = \sqrt{\sum_{i=1}^{n} \left(\theta_i(0) - \frac{1}{n}\sum_{j=1}^{n}\theta_j(0)\right)^2} \tag{21}$$

and

$$d = \min\left\{ r - \max_{(i,j)\in E_{z(0)}} d_{ij}(z(0)),\ \min_{i,j} d_{ij}(z(0)) \right\} > 0. \tag{22}$$

Remark 2: Theorem 1 reveals that the multi-agent system (16)–(17) can always achieve the flocking by choosing a sufficiently large feedback gain k for any given initial states if all the agents have distinct initial positions and a connected initial proximity graph. In the case that the relative distances between agents are not available, this is the first result reported in the literature on the flocking for the multi-agent nonholonomic systems with initial connectedness.

Remark 3: Theorem 1 only gives a sufficient condition for the flocking and by this theorem the gain k has to be chosen sufficiently large such that (19) holds. Moreover, the gain k is dependent on the initial states of the agents, the connectivity of the initial graph and the sensing radius r. Thus before designing k, the values of the initial states and the algebraic connectivity of the initial graph should be estimated.

Before stating the proof of Theorem 1, several necessary lemmas are given as follows.

Lemma 1: [29]

$$\left\| \begin{bmatrix} \cos\alpha - \cos\beta \\ \sin\alpha - \sin\beta \end{bmatrix} \right\|^2 = 4\left|\sin\frac{\alpha-\beta}{2}\right|^2 \le |\alpha-\beta|^2, \tag{23}$$

where $\|\cdot\|$ denotes Euclidean norm of a vector.

Lemma 2: For any given $t \in [0, +\infty)$, if $d_{ij}(z(t)) \neq 0$, then $\dot{d}_{ij}(z(t)) \le v\Delta_t$.

Proof: From the definition of d_{ij}, one has

$$d_{ij}^2 = [x_i - x_j \quad y_i - y_j] \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix}. \tag{24}$$

Taking the derivative of both sides of (24) with respect to time t, by Lemma 1, one gets

$$\begin{aligned} 2d_{ij}\dot{d}_{ij} &= 2v[x_i - x_j \quad y_i - y_j] \begin{bmatrix} \cos\theta_i - \cos\theta_j \\ \sin\theta_i - \sin\theta_j \end{bmatrix} \\ &\le 2vd_{ij} \left\| \begin{bmatrix} \cos\theta_i - \cos\theta_j \\ \sin\theta_i - \sin\theta_j \end{bmatrix} \right\| \\ &\le 2vd_{ij}|\theta_i - \theta_j|. \end{aligned} \tag{25}$$

Considering $d_{ij}(z(t)) \neq 0$, one has

$$\dot{d}_{ij}(z(t)) \le v|\theta_i(t) - \theta_j(t)| \le v\Delta_t. \tag{26}$$

■

Lemma 3: (Theorem 13.6.2 of [44]) Let X be a graph with n vertices and Y be a graph with n vertices by adding a new edge in X. Then $\lambda_i(X) \le \lambda_i(Y)$ for all i and $\lambda_i(Y) \le \lambda_{i+1}(X)$ for $i < n$.

Lemma 4: Consider the switching differential equation (18) and denote $\alpha = (1/n)\mathbf{1}^T\theta(0)$ and $\delta(t) = \theta(t) - \alpha\mathbf{1}$. Then one has

$$\|\delta(t)\| \le \|\delta(0)\|, \quad \Delta_t \le \sqrt{2}\|\delta(0)\|, \quad \forall t \in [0, +\infty), \tag{27}$$

where Δ_t is denoted in (20). Moreover, if the initial graph $G_{z(0)}$ is connected and $E_{z(0)} \subset E_{z(t)}$ for all $t \in [0, \eta]$, then one gets

$$\|\delta(t)\| \le \mathrm{e}^{-k\mu t}\|\delta(0)\|, \quad \Delta_t \le \sqrt{2}\mathrm{e}^{-k\mu t}\|\delta(0)\|, \forall t \in [0, \eta], \tag{28}$$

where η is a constant and μ is the algebraic connectivity of $G_{z(0)}$.

Proof: In section VIII of [17], it was proved that $\dot{\delta} = -kL_z\delta$. Using the method in the proof of Theorem 8 of [17], we let

$$V(\delta) = \frac{\delta^T\delta}{2} = \frac{1}{2}|\delta|^2.$$

Thus

$$\dot{V}(\delta(t)) = -k\delta^T(t)L_{z(t)}\delta(t) \le 0, \tag{29}$$

which implies $V(\delta(t)) \le V(\delta(0))$, that is, $\|\delta(t)\| \le \|\delta(0)\|$. It is easy to see

$$\begin{aligned} |\theta_i(t) - \theta_j(t)| &\le |\theta_i(t) - \alpha| + |\theta_j(t) - \alpha| \\ &\le \sqrt{2}\sqrt{(\theta_i(t) - \alpha)^2 + (\theta_j(t) - \alpha)^2} \\ &\le \sqrt{2}\|\delta(t)\|. \end{aligned} \tag{30}$$

Hence it follows that the second inequality of (27) holds. If the initial graph $G_{z(0)}$ is connected and $E_{z(0)} \subset E_{z(t)}$ for all $t \in [0, \eta]$, then by Lemma 3, one gets

$$\lambda_2(G_{z(t)}) \ge \lambda_2(G_{z(0)}) = \mu.$$

For any $t \in [0, \eta]$, one has

$$\dot{V}(\delta(t)) \le -k\lambda_2(G_{z(t)})\|\delta(t)\|^2 \le -2k\mu V(\delta(t)). \tag{31}$$

According to (31), by simple calculation, one obtains

$$V(\delta(t)) \le \mathrm{e}^{-2k\mu t}V(\delta(0)),$$

which implies the first inequality of (28). By (30), it follows the second inequality of (28). ■

Lemma 5: Suppose that the initial positions of all agents are different from each other, namely, $d_{ij}(z(0)) > 0$ for any $i \neq j$, and

$$k > \frac{\sqrt{2}\, v\|\delta(0)\|}{\mu \min\limits_{i,j} d_{ij}(z(0))}. \tag{32}$$

Then multi-agent system (16) and (17) can avoid collision on the time interval $[0, (k\mu)^{-1}]$, namely, $d_{ij}(z(t)) > 0$ for any $t \in [0, (k\mu)^{-1}]$.

Proof: For (16) and (17), suppose that there exist $\tilde{i}$, $\tilde{j}$, and $\tilde{t} \in [0, (k\mu)^{-1}]$ satisfying $d_{\tilde{i}\tilde{j}}(z(\tilde{t})) = 0$. Then, by (16), one has

$$\begin{aligned} 0 &= \begin{bmatrix} x_{\tilde{i}}(\tilde{t}) - x_{\tilde{j}}(\tilde{t}) \\ y_{\tilde{i}}(\tilde{t}) - y_{\tilde{j}}(\tilde{t}) \end{bmatrix} \\ &= \begin{bmatrix} x_{\tilde{i}}(0) - x_{\tilde{j}}(0) \\ y_{\tilde{i}}(0) - y_{\tilde{j}}(0) \end{bmatrix} + v\int_0^{\tilde{t}} \begin{bmatrix} \cos\theta_{\tilde{i}}(t) - \cos\theta_{\tilde{j}}(t) \\ \sin\theta_{\tilde{i}}(t) - \sin\theta_{\tilde{j}}(t) \end{bmatrix} \mathrm{d}t, \end{aligned} \tag{33}$$

which implies

$$\begin{bmatrix} x_{\tilde{i}}(0) - x_{\tilde{j}}(0) \\ y_{\tilde{i}}(0) - y_{\tilde{j}}(0) \end{bmatrix} = -v\int_0^{\tilde{t}} \begin{bmatrix} \cos\theta_{\tilde{i}}(t) - \cos\theta_{\tilde{j}}(t) \\ \sin\theta_{\tilde{i}}(t) - \sin\theta_{\tilde{j}}(t) \end{bmatrix} \mathrm{d}t. \tag{34}$$

Combining (34), (23) and (27), one gets

$$\begin{aligned} d_{\tilde{i}\tilde{j}}(0) &\le v\int_0^{(k\mu)^{-1}} |\theta_{\tilde{i}}(t) - \theta_{\tilde{j}}(t)|\mathrm{d}t \\ &\le v\int_0^{(k\mu)^{-1}} \Delta_t \mathrm{d}t \le \sqrt{2}\, v(k\mu)^{-1}\|\delta(0)\|, \end{aligned} \tag{35}$$

which contradicts the known assumption condition (32). ■

Proof of Theorem 1: By (19), there exists a sufficiently small number $\varepsilon > 0$ satisfying

$$k > \frac{\sqrt{2}\, v\zeta}{\mu(d - \varepsilon\sqrt{2}\,\zeta v)} > \frac{\sqrt{2}\, v\zeta}{\mu d}, \tag{36}$$

where $\zeta = \|\delta(0)\|$ from (21). It is easy to verify that the infinite time interval $[0,\ +\infty)$ can be expressed as

$$[0, +\infty) = \bigcup_{m=0}^{+\infty} [0, (k\mu)^{-1} + m\varepsilon]. \tag{37}$$

Thus, one only needs to prove that the results of Theorem 1 hold on the time intervals $[0, (k\mu)^{-1} + m\varepsilon]$ for $m = 0, 1, 2, \ldots$. In the following, the mathematical induction is used to prove Theorem 1.

For the case of $m = 0$, by Lemma 5, one has $d_{ij}(z(t)) > 0$ on the time interval $[0, (k\mu)^{-1}]$. From Lemma 2, one gets $\dot{d}_{ij}(z(t)) \le v\Delta_t$. Therefore, for any $t \in [0, (k\mu)^{-1}]$ and any edge $(i, j) \in E_{z(0)}$, according to (19), (22) and (27), one can easily obtain

$$\begin{aligned} d_{ij}(z(t)) &= d_{ij}(z(0)) + \int_0^t \dot{d}_{ij}(z(s))\mathrm{d}s \\ &\le d_{ij}(z(0)) + \int_0^t v\Delta_s \mathrm{d}s \\ &\le d_{ij}(z(0)) + v\sqrt{2}\,(k\mu)^{-1}\|\delta(0)\| \\ &< d_{ij}(z(0)) + d < r. \end{aligned} \tag{38}$$

That is, $(i, j) \in E_{z(t)}$ for any $t \in [0, (k\mu)^{-1}]$. Thus, $E_{z(0)} \subset E_{z(t)}$, which implies $G_{z(t)}$ is also connected. By Lemma 4, (20) holds on the time interval $[0, (k\mu)^{-1}]$.

Next, for the case of any $m > 0$, suppose that the graph $G_{z(t)}$ is connected, collision does not occur and (20) holds for any $t \in [0, (k\mu)^{-1} + m\varepsilon]$. In the following, we consider the case of $m+1$, that is, the above results also hold for any $[0, (k\mu)^{-1} + (m + 1)\varepsilon]$. We claim that collision does not occur on the time interval $[0, (k\mu)^{-1} + (m + 1)\varepsilon]$ either. Indeed, suppose there exist two agents $\tilde{i}, \tilde{j}$, and some time $\tilde{t} \in [(k\mu)^{-1} + m\varepsilon, (k\mu)^{-1} + (m + 1)\varepsilon]$ satisfying $d_{\tilde{i}\tilde{j}}(z(\tilde{t})) = 0$. Then, similar to (35), one has

$$\begin{aligned} &d_{\tilde{i}\tilde{j}}(z(0)) \\ &\le v \int_0^{(k\mu)^{-1}+(m+1)\varepsilon} |\theta_{\tilde{i}}(t) - \theta_{\tilde{j}}(t)|\mathrm{d}t \\ &\le v \int_0^{(k\mu)^{-1}+(m+1)\varepsilon} \Delta_t \mathrm{d}t \\ &= v \int_0^{(k\mu)^{-1}+m\varepsilon} \Delta_t \mathrm{d}t + v \int_{(k\mu)^{-1}+m\varepsilon}^{(k\mu)^{-1}+(m+1)\varepsilon} \Delta_t \mathrm{d}t \\ &\le v\sqrt{2}\,\|\delta(0)\| \int_0^{(k\mu)^{-1}+m\varepsilon} \mathrm{e}^{-k\mu t}\mathrm{d}t + v\sqrt{2}\,\|\delta(0)\|\varepsilon \\ &\le v\sqrt{2}\,\|\delta(0)\|((k\mu)^{-1} + \varepsilon), \end{aligned} \tag{39}$$

which contradicts (36).

For any $t \in [(k\mu)^{-1} + m\varepsilon, (k\mu)^{-1} + (m+1)\varepsilon]$ and any edge $(i, j) \in E_{z(0)}$, it is easy to deduce

$$\begin{aligned} &d_{ij}(z(t)) \\ &= d_{ij}(z(0)) + \int_0^t \dot{d}_{ij}(z(s))\mathrm{d}s \\ &\le d_{ij}(z(0)) + \int_0^t v\Delta_s \mathrm{d}s \\ &\le d_{ij}(z(0)) + \int_0^{(k\mu)^{-1}+m\varepsilon} v\Delta_s \mathrm{d}s \\ &\quad + \int_{(k\mu)^{-1}+m\varepsilon}^{(k\mu)^{-1}+(m+1)\varepsilon} v\Delta_s \mathrm{d}s \\ &\le d_{ij}(z(0)) + v\sqrt{2}\|\delta(0)\| \\ &\quad \times \int_0^{(k\mu)^{-1}+m\varepsilon} \mathrm{e}^{-k\mu t}\mathrm{d}t + v\sqrt{2}\|\delta(0)\|\varepsilon \\ &\le d_{ij}(z(0)) + v\sqrt{2}\|\delta(0)\|((k\mu)^{-1} + \varepsilon) \\ &\le d_{ij}(z(0)) + d < r. \end{aligned} \tag{40}$$

That is, $(i, j) \in E_{z(t)}$ for any $t \in [(k\mu)^{-1} + m\varepsilon, (k\mu)^{-1} + (m + 1)\varepsilon]$. Thus one gets $E_{z(0)} \subset E_{z(t)}$. By Lemma 4, $G_{z(t)}$ is connected and (20) holds for any $t \in [0, (k\mu)^{-1} + (m + 1)\varepsilon]$.

Therefore, according to the mathematical induction, Theorem 1 is proved. ■

Remark 4: The proof idea using the mathematical induction comes from [28] and [29] on the discrete Vicsek model. But in our result, the considered model is a continuous multi-agent non-holonomic system. Some key techniques shown in Lemma 2, 4, 5 and (37) are adopted so that the mathematical induction can be used for the continuous systems. Moreover, the collision avoidance is considered under the linear protocol for the first time and it plays an important role in the proof of the heading synchronization for the multi-agent non-holonomic system.

Remark 5: Theorem 1 can be generalized to a nonlinear form in N-dimensional space:

$$\dot{X}_i = f(\phi_i), \tag{41}$$

$$\dot{\phi}_i = k \sum_{j=1}^{n} a_{ij}(X)(\phi_j - \phi_i)\ (i = 1, 2, \ldots, n), \tag{42}$$

where f is a smooth function, X_i the N-dimensional position vector and ϕ_i the M-dimensional vector expected to be synchronized for each agent i, $X = (X_1^{\mathrm{T}}, X_2^{\mathrm{T}}, \ldots, X_n^{\mathrm{T}})^{\mathrm{T}}$. Indeed, the linear protocol (42) implies $\phi_i(t)$ is uniformly bounded. Thus for any given initial values, $\phi_i(t)$ lies inside a compact set S for all $t \in [0, +\infty)$. By the smoothness of f and Lagrange Mean Value Theorem, there is a constant K such that

$$\|f(\phi_i) - f(\phi_j)\| \le K\|\phi_i - \phi_j\| \tag{43}$$

in S. So, we can obtain a similar result as Lemma 2 replacing v and Δ_t by K and $\max_{i,j}\|\phi_i(t) - \phi_j(t)\|$ respectively. With the similar procedure, we can generalize Theorem 1 to the broader nonlinear system (41) (42) that includes more kinds of multi-agent non-holonomic systems. For examples, the 2-input multi-agent system and the 3-dimensional multi-agent system with the dynamics

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = v_i \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix}, \tag{44}$$

$$\dot{v}_i = k \sum_{j \in N_i} (v_j - v_i) \tag{45}$$

$$\dot{\theta}_i = k \sum_{j \in N_i} (\theta_j - \theta_i) \tag{46}$$

and

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \end{bmatrix} = v \begin{bmatrix} \sin\psi_i \cos\theta_i \\ \sin\psi_i \sin\theta_i \\ \cos\psi_i \end{bmatrix}, \tag{47}$$

$$\dot{\theta}_i = k \sum_{j \in N_i} (\theta_j - \theta_i), \tag{48}$$

$$\dot{\psi}_i = k \sum_{j \in N_i} (\psi_j - \psi_i) \tag{49}$$

respectively.

Remark 6: Let $f(\phi_i)$ in (41) be ϕ_i. Then (41) and (42) are reduced to the double-integrator multi-agent system discussed in [16]. Our protocol (42) is just the first term of protocol (3) in [16] except a gain k. For maintaining connectedness and avoiding collision, the second term of (3) in [16] is designed with a artificial potential function with respect to the relative distances. The main design idea is from the classical Lyapunov function. Different from [16], in this paper, the positions and the relative distances are not available. Our results show that one can add a sufficient large gain k in the first term to preserve the connectedness and realize flocking, where k is dependant on the initial states. The flocking in [16] is global, but it is not in this paper since the gain k is dependant on the estimate of the initial states. Moreover, from the proof of Theorem 1, one can see $E_{z(0)} \subset E_{z(t)}$, which means the edge set can only grow. Thus the evolution of the proximity graph is the same as that of [16].

Numerical Example 1: We consider a 8-agent system with the kinematical model (16) and (17). Let the sensing radius be $r = 4$ and the absolute velocity $v = 0.8$. We randomly choose the initial heading angles in $[0, 2\pi]$ and the initial positions in the square $[0,\ 10] \times [0,\ 10]$ as

$$\theta(0) = [3.0927,\ 2.7810,\ 4.3544,\ 1.4031,\ 1.7899,\ 4.0765,\ 1.9622,\ 5.2434],$$

$$\begin{bmatrix} x^{\mathrm{T}}(0) \\ y^{\mathrm{T}}(0) \end{bmatrix} = \begin{bmatrix} 1.4024 & 1.3984 & 3.9984 & 5.3385 & 1.9653 & 2.1315 & 0.4158 & 4.7561 \\ 3.7419 & 5.5623 & 5.8020 & 6.9329 & 4.4725 & 5.9405 & 0.7064 & 7.8531 \end{bmatrix}.$$

It is easy to check that the initial proximity graph is a connected graph with algebraic connectivity $\mu = 0.8853$. Fig. 1 shows the trajectories of agents in 10 seconds with different values of k. In Fig. 1 the dashed lines denote the edges of the initial graph, which is connected obviously. By Theorem 1, we get that the flocking problem is solved as $k > 8.9657$ and the convergence rate of the flocking can be described by

$$\max_{i,j} |\theta_i(t) - \theta_j(t)| \le \sqrt{2}\,\zeta\, \mathrm{e}^{-0.8853kt}.$$

The simulations show that the flocking is not achieved as $k = 0.04$ and $k = 0.08$. And the critical value of k realizing flocking is about 0.14, which shows that the condition of Theorem 1 is only sufficient. How to theoretically obtain the critical value of k is an important and difficult problem. We will investigate it in our future work. From Fig. 2, we can see all the headings of the agents tend to the same constant. Fig. 3 is the time-response curve of the minimum distance among all the agents, which shows that no collision occur.

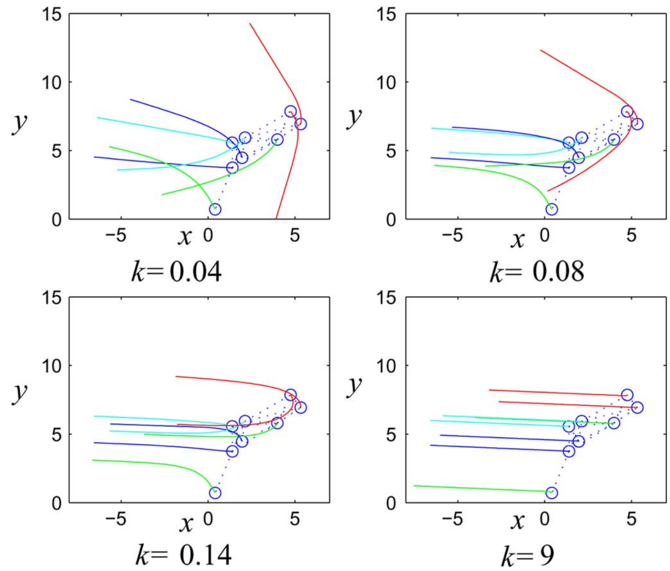

Fig. 1. The initial positions and the trajectories with different values of gain k.

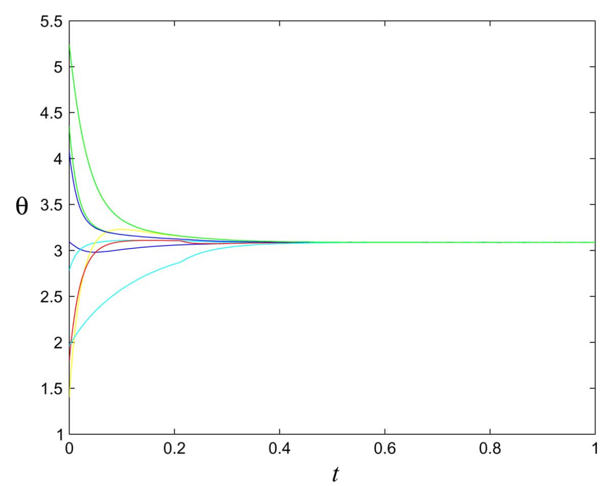

Fig. 2. The time-response curves of the headings.

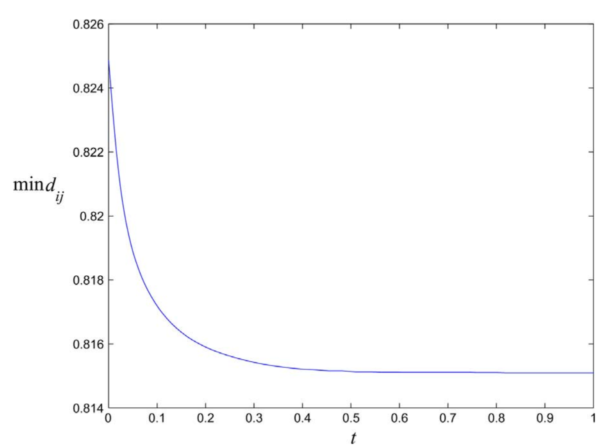

Fig. 3. The time-response curves of the minimum distance among agents.

B. The Case of the Dynamic Model

Consider the multi-agent nonholonomic system (10)–(13). We design the distributed linear protocol as

$$u_{1i} = -2k\omega_i + \frac{k^2}{n}\sum_{j\in N_{ij}(z)}(\theta_j-\theta_i), \tag{50}$$

$$u_{2i} = k\sum_{j\in N_{ij}(z)}(v_j-v_i) \quad (i=1,2,\ldots,n), \tag{51}$$

where $k>0$. Then the closed-loop system is

$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\end{bmatrix} = v_i\begin{bmatrix}\cos\theta_i\\ \sin\theta_i\end{bmatrix}, \tag{52}$$

$$\dot{\theta}_i = \omega_i, \tag{53}$$

$$\dot{\omega}_i = -2k\omega_i + \frac{k^2}{n}\sum_{j=1}^{n}a_{ij}(z)(\theta_j-\theta_i), \tag{54}$$

$$\dot{v}_i = k\sum_{j=1}^{n}a_{ij}(z)(v_j-v_i) \quad (i=1,2,\ldots,n), \tag{55}$$

where $z=(x_1,y_1,x_2,y_2,\ldots,x_n,y_n)$. Equations (53)–(55) can be rewritten as the compact forms

$$\begin{bmatrix}\dot{\theta}\\ \dot{\omega}\end{bmatrix} = \begin{bmatrix}0 & I_n\\ -\frac{k^2}{n}L_z & -2kI_n\end{bmatrix}\begin{bmatrix}\theta\\ \omega\end{bmatrix} \tag{56}$$

$$\dot{v} = -kL_z v, \tag{57}$$

where $\theta=[\theta_1,\theta_2,\ldots,\theta_n]^{\mathrm{T}}$, $\omega=[\omega_1,\omega_2,\ldots,\omega_n]^{\mathrm{T}}$ and $v=[v_1,v_2,\ldots,v_n]^{\mathrm{T}}$.

For the case of dynamical model, we replace Lemma 2 with the following result:

Lemma 6: Let $\Delta_t^{\theta}=\max_{i,j}\{|\theta_i-\theta_j|\}$, $\Delta_t^{v}=\max_{i,j}\{|v_i-v_j|\}$, $\bar{v}(t)=\max_{1\le i\le n}v_i(t)$ and $\Psi_t=\Delta_t^{v}+\bar{v}(0)\Delta_t^{\theta}$. For any given $t\in[0,+\infty)$, if $d_{ij}(z(t))\neq 0$, then $\dot{d}_{ij}(z(t))\le\Psi_t$.

Proof: It is easy to see $\Delta_t^{v}=\bar{v}(t)-\underline{v}(t)$, where $\underline{v}(t)=\min_{1\le i\le n}v_i(t)$. Denote $S_t=\{l\ |v_l(t)=\bar{v}(t),\ 1\le l\le n\}$. Since $\bar{v}(t)$ is a nonsmooth max-function, we calculate its Dini derivative by Lemma 2.9 in [50] or Lemma 2.2 in [51] as follows:

$$\begin{aligned}D^{+}\bar{v}(t) &= \max_{i\in S_t}\{\dot{v}_i(t)\}\\ &= \max_{i\in S_t}\left\{k\sum_{j=1}^{n}a_{ij}(z)(v_j(t)-\bar{v}(t))\right\}\\ &\le 0.\end{aligned} \tag{58}$$

Thus $\bar{v}(t)$ is nonincreasing, which implies $\bar{v}(t)\le\bar{v}(0)$. Taking the derivative of each side of (24) with respect to time t yields

$$\begin{aligned}2d_{ij}\dot{d}_{ij} &= 2[x_i-x_j \quad y_i-y_j]\begin{bmatrix}v_i\cos\theta_i-v_j\cos\theta_j\\ v_i\sin\theta_i-v_j\sin\theta_j\end{bmatrix}\\ &\le 2d_{ij}\left\|\begin{bmatrix}(v_i-v_j)\cos\theta_i+v_j(\cos\theta_i-\cos\theta_j)\\ (v_i-v_j)\sin\theta_i+v_j(\sin\theta_i-\sin\theta_j)\end{bmatrix}\right\|\\ &\le 2d_{ij}(|v_i-v_j|+v_j(t)|\theta_i-\theta_j|)\\ &\le 2d_{ij}(\Delta_t^{v}+\bar{v}(0)\Delta_t^{\theta})\\ &= 2d_{ij}\Psi_t.\end{aligned} \tag{59}$$

Considering $d_{ij}(z(t))\neq 0$, one has $\dot{d}_{ij}(z(t))\le\Psi_t$. ■

For (56), we take the transformation adopted in [53] and [52] as follows:

$$\begin{bmatrix}\theta\\ \tilde{\omega}\end{bmatrix} = \begin{bmatrix}I_n & 0\\ I_n & \frac{1}{k}I_n\end{bmatrix}\begin{bmatrix}\theta\\ \omega\end{bmatrix}. \tag{60}$$

Then (56) is transformed into

$$\begin{bmatrix}\dot{\theta}\\ \dot{\tilde{\omega}}\end{bmatrix} = -k\tilde{L}_z\begin{bmatrix}\theta\\ \tilde{\omega}\end{bmatrix}, \tag{61}$$

where

$$\tilde{L}_z = \begin{bmatrix}I_n & -I_n\\ -I_n+\frac{1}{n}L_z & I_n\end{bmatrix} \tag{62}$$

is a Laplacian matrix of a directed weighted graph $\tilde{G}_z$ with $2n$ vertexes. It is easy to see $\mathbf{1}_{2n}^{T}\tilde{L}_z=0$ due to $\mathbf{1}_n^{T}L_z=0$. Thus $\tilde{G}_z$ is a balanced directed graph.

Lemma 7: If graph G_z is connected, then the mirror graph $\hat{G}_z$ of $\tilde{G}_z$ is connected, namely,

$$\lambda_2\left(\frac{1}{2}\left(\tilde{L}_z+\tilde{L}_z^{T}\right)\right) > 0. \tag{63}$$

Proof: It is easy to check that

$$\begin{aligned}\mathrm{rank}\tilde{L}_z &= \mathrm{rank}\begin{bmatrix}I_n & -I_n\\ -I_n+\frac{1}{n}L_z & I_n\end{bmatrix}\\ &= \mathrm{rank}\begin{bmatrix}I_n & -I_n\\ \frac{1}{n}L_z & 0\end{bmatrix}\\ &= n+\mathrm{rank}L_z,\end{aligned} \tag{64}$$

which implies $\mathrm{rank}\tilde{L}_z=2n-1$ if and only if $\mathrm{rank}L_z=n-1$. Thus the balanced directed graph $\tilde{G}_z$ is strongly connected due to the connectedness of G_z. Hence the mirror graph $\hat{G}$ of $\tilde{G}_z$ is connected by the definition of mirror graph (Definition 2 of [17]). By Theorem 7 of [17], $(1/2)(\tilde{L}_z+\tilde{L}_z^{T})$ is a valid Laplacian matrix of the mirror graph $\hat{G}_z$. Therefore, by the connectedness of $\hat{G}_z$, we get (63). ■

Remark 7: Different from the protocols in [53] and [52], the external input u_{i1} shown in (50) adds the gain k^2/n, which results the free parameter k in (61). With this technique, the method used in the above section can be extended to the case of the dynamical model. In [52], the authors used a graph method to reveal the relationship between the connectedness of G_z and that of $\hat{G}_z$. But from (64), we can see this relationship can be easily obtained using the algebraic method.

Lemma 8: Consider the switching differential equation (56) with the feedback gain $k>1$. Denote $\alpha^{\theta}=(1/n)\mathbf{1}_n^{T}\theta(0)$ and $\delta^{\theta}(t)=\theta(t)-\alpha^{\theta}\mathbf{1}_n$. Then

$$\Delta_t^{\theta}\le\sqrt{2}\|\delta^{\theta}(0)\|+\frac{1}{\sqrt{2n}}|\mathbf{1}_n^{T}\omega(0)|+\|\omega(0)\| =: A. \tag{65}$$

Moreover, if the initial graph $G_{z(0)}$ is connected and $E_{z(0)}\subset E_{z(t)}$ for all $t\in[0,\eta]$, then one gets

$$\Delta_t^{\theta}\le A\mathrm{e}^{-k\hat{\mu}t}, \forall t\in[0,\eta], \tag{66}$$

where η is a constant,

$$\hat{\mu}=\lambda_2\left(\frac{1}{2}\left(\tilde{L}_{z(0)}+\tilde{L}_{z(0)}^{T}\right)\right) > 0 \tag{67}$$

and $\tilde{L}_z$ is shown in (62).

Proof: Set

$$\tilde{\alpha}(t)=\frac{1}{2n}\left(\mathbf{1}_n^T\theta(t)+\mathbf{1}_n^T\tilde{\omega}(t)\right),\quad \tilde{\delta}(t)=\begin{bmatrix}\theta\\ \tilde{\omega}\end{bmatrix}-\tilde{\alpha}(t)\mathbf{1}_{2n}. \tag{68}$$

Considering $\mathbf{1}_{2n}^T\tilde{L}_z=0$ and (56), we have

$$\dot{\tilde{\delta}}(t)=-k\tilde{L}_{z(t)}\tilde{\delta}(t). \tag{69}$$

Let $V(\tilde{\delta})=(1/2)\tilde{\delta}^T\tilde{\delta}$. Then

$$\begin{aligned}\dot{V}(\tilde{\delta}(t))&=-k\tilde{\delta}^T(t)\tilde{L}_{z(t)}\tilde{\delta}(t)\\ &=-k\tilde{\delta}^T(t)\frac{\tilde{L}_{z(t)}+\tilde{L}_{z(t)}^T}{2}\tilde{\delta}(t).\end{aligned} \tag{70}$$

One can see $(\tilde{L}_{z(t)}+\tilde{L}_{z(t)}^T)/2$ is positive semi-definite since it is just the Laplacian matrix of the mirror graph $\hat{G}_z$. Thus we have $\dot{V}(\tilde{\delta}(t))\le 0$ due to (70). With the similar procedure of the proof of Lemma 4, we obtain

$$\Delta_t^\theta\le\sqrt{2}\|\tilde{\delta}(0)\|. \tag{71}$$

It is easily obtained that

$$\begin{aligned}\tilde{\delta}(0)&=\begin{bmatrix}\theta(0)-\frac{1}{2n}\left(2\mathbf{1}_n^T\theta(0)+\frac{1}{k}\mathbf{1}_n^T\omega(0)\right)\mathbf{1}_n\\ \theta(0)+\frac{1}{k}\omega(0)-\frac{1}{2n}\left(2\mathbf{1}_n^T\theta(0)+\frac{1}{k}\mathbf{1}_n^T\omega(0)\right)\mathbf{1}_n\end{bmatrix}\\ &=\begin{bmatrix}\delta^\theta(0)-\frac{1}{2nk}\mathbf{1}_n^T\omega(0)\mathbf{1}_n\\ \delta^\theta(0)+\frac{1}{k}\omega(0)-\frac{1}{2nk}\mathbf{1}_n^T\omega(0)\mathbf{1}_n\end{bmatrix},\end{aligned} \tag{72}$$

which implies $\|\tilde{\delta}(0)\|\le A$ due to $k>1$. Thus (65) holds. In the rest of the proof, we assume the initial graph $G_{z(0)}$ is connected and $E_{z(0)}\subset E_{z(t)}$ $(\forall t\in[0,\eta])$. Denote the edge sets of $\tilde{G}_z$ and $\hat{G}_z$ by $\tilde{E}_z$ and $\hat{E}_z$ respectively. By (62), one obtain $\tilde{E}_z(t)\subset\tilde{E}_z(0)$ $(\forall t\in[0,\eta])$ due to $E_{z(0)}\subset E_{z(t)}(\forall t\in[0,\eta])$. Considering the relationship between a directed graph and its mirror graph, one has $\hat{E}_z(t)\subset\hat{E}_z(0)$ $(\forall t\in[0,\eta])$. Therefore,

$$\begin{aligned}\dot{V}(\tilde{\delta}(t))&\le -k\lambda_2\left(\frac{\tilde{L}_{z(t)}+\tilde{L}_{z(t)}^T}{2}\right)\|\tilde{\delta}(t)\|^2\\ &\le -k\hat{\mu}V(\tilde{\delta}(t)),\end{aligned} \tag{73}$$

which yields (66). ■

Remark 8: If all the initial angular velocities of the agents are zero, i.e., $w(0)=0$, then $A=\sqrt{2}\|\delta^\theta(0)\|$, which is consistent to the results of Lemma 4.

Lemma 9: Denote $\alpha^v=(1/n)\mathbf{1}^Tv(0)$, $\delta^v(t)=v(t)-\alpha^v\mathbf{1}$ and let $k>1$. Then

$$\Psi_t:=\Delta_t^v+\bar{v}(0)\Delta_t^\theta\le B,\quad \forall t\in[0,\,+\infty), \tag{74}$$

where $B=\sqrt{2}\|\delta^v(0)\|+\bar{v}(0)A$. Moreover, if the initial graph $G_{z(0)}$ is connected and $E_{z(0)}\subset E_{z(t)}$ for all $t\in[0,\eta]$, then one gets

$$\Psi_t\le B\mathrm{e}^{-k\bar{\mu}t},\quad \forall t\in[0,\eta], \tag{75}$$

where η is a constant and $\bar{\mu}=\min\{\mu,\hat{\mu}\}$.

Proof: Applying Lemma 4 to (57) and considering Lemma 8, one can prove the lemma. ■

Theorem 2: For the closed-loop system (56) and (57), suppose that the initial proximity graph $G_{z(0)}$ is connected and the initial positions of all agents are distinct. If

$$k>\max\left\{\frac{B}{\bar{\mu}d},\,1\right\}, \tag{76}$$

then the proximity graph $G_{z(t)}$ preserves the connectedness on the whole time interval $[0,+\infty)$ and also the flocking can be achieved with $\Psi_t\le B\mathrm{e}^{-k\bar{\mu}t}$ $\forall t\in[0,+\infty]$, where B and $\bar{\mu}$ are shown in Lemma 9, d in (22) and Ψ_t in (74).

Proof: The theorem can be easily proved following the procedure of the proof of Theorem 1 with $v\sqrt{2}\zeta$ and Δ_t replaced by B and Ψ_t respectively. ■

Remark 9: A contribution of this subsection lies in introducing gain k in the distributed protocol (50), which can be used to assign flocking rate just as the single-integrator case if the flocking is realized. Another contribution is proving the flocking for sufficiently large k, which means a gain realizing flocking does exist. But the value of k obtained by Theorem 2 may be conservative. In the following numerical example, we only show the effect of k on the flocking and the flocking rate.

Numerical Example 2: We consider a multi-agent systems composed of 10 unicycles with different masses m_i and moments of inertia J_i:

$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\end{bmatrix}=v_i\begin{bmatrix}\cos\theta_i\\ \sin\theta_i\end{bmatrix}, \tag{77}$$

$$\dot{\theta}_i=\omega_i, \tag{78}$$

$$J_i\dot{\omega}_i=T_i, \tag{79}$$

$$m_i\dot{v}_i=F_i, \tag{80}$$

where T_i and F_i are the steering torque and the driving force respectively, $i=1,2,\ldots,10$. By (50) and (51), the external steering torque and the external driving force of unicycle i are designed as

$$T_i=-2J_ik\omega_i+J_i\frac{k^2}{10}\sum_{j\in N_{ij}(z)}(\theta_j-\theta_i), \tag{81}$$

$$F_i=m_ik\sum_{j\in N_{ij}(z)}(v_j-v_i), \tag{82}$$

where k is a sufficiently large positive number. Let the initial positions be (10,3), (5,2), (1,4), (2,2), $(8,-3)$, $(-3,-4)$, (8,3), (2,5), $(-5,3)$ and $(-8,9)$. Let the sensing radius be $r=8$. Then the initial graph is connected. Fig. 4 shows the position trajectories with different values of k in 10 seconds. As $k\ge 8$, the flocking is realized. The time-response curves of the heading angles and the absolute velocities are shown in Figs. 5 and 6 respectively. One can see that the convergent speed as $k=14$ is greater than that as $k=8$.

In the simulations, the initial headings, angular velocities and absolute velocities are

$$\theta(0)=\left[0,\,\frac{\pi}{6},\,\frac{\pi}{3},\,\frac{\pi}{2},\,\frac{2\pi}{3},\,\frac{5\pi}{6},\,\pi,\,\frac{7\pi}{6},\,\frac{4\pi}{3},\,\frac{3\pi}{2}\right]^{\mathrm{T}},$$

$$\omega(0)=[1,\,-1,\,2,\,3,\,1,\,4,\,2,\,4,\,1,\,-2]^{\mathrm{T}},$$

and

$$v(0)=[1,\,5,\,2,\,4,\,3,\,2,\,1,\,0,\,2,\,4]^{\mathrm{T}}$$

respectively.

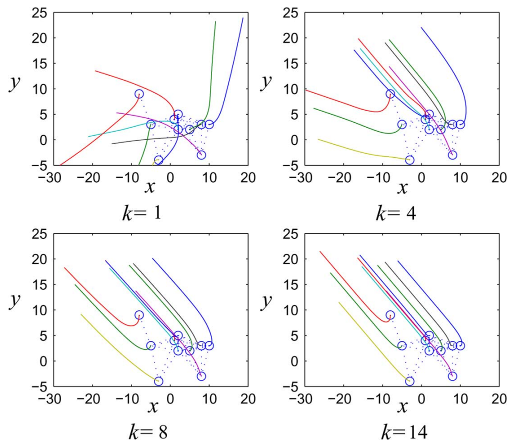

Fig. 4. The Initial positions and the trajectories with different values of k.

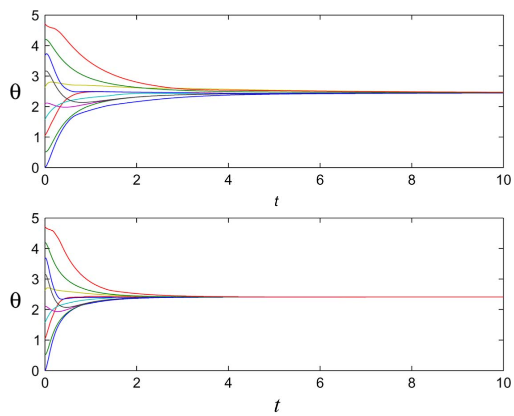

Fig. 5. The time-response curves of headings with different values of k.

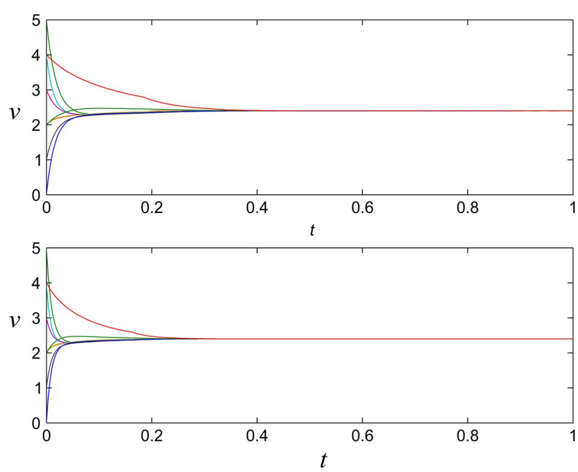

Fig. 6. The time-response curves of absolute velocities with different values of k.

IV. Flocking Under Nonlinear Protocols

In this section, we consider the multi-agent non-holonomic system (6) and (7) under the nonlinear protocol with the form of Kuramoto model:

$$u_i = k \sum_{j \in N_i(z)} \sin(\theta_j - \theta_i) \quad (i = 1, 2, \ldots, n). \tag{83}$$

The closed-loop system is described by

$$\begin{bmatrix} \dot{x}_i(t) \\ \dot{y}_i(t) \end{bmatrix} = v \begin{bmatrix} \cos\theta_i(t) \\ \sin\theta_i(t) \end{bmatrix}, \tag{84}$$

$$\dot{\theta}_i(t) = k \sum_{j=1}^{n} a_{ij}(z) \sin(\theta_j(t) - \theta_i(t)), \tag{85}$$

which is just the continuous model of Vicsek model (see [43] and [31]).

Remark 10: If the sensing radius r is sufficiently large such that the graph is a complete graph, then system (85) becomes the original Kuramoto model [54] with all natural frequencies being zero. If the graph has a fixed topology, then system (85) becomes the generalized Kuramoto model discussed by Jadbabaie *et al.* [36]. In fact, system (85) can be regarded as a switching Kuramoto model with a switching law determined by the proximity graph.

It should be pointed out that only Lemma 4 has a close relationship with the angle updating (17) in the proof of Theorem 1. Thus the estimation of synchronization rate plays a key role in the proof of Theorem 1. Fortunately, the synchronization rate can be easily obtained for Kuramoto model by using the method in [36]. In the following, similar to Theorem 1 in [36], Lemma 10 estimates the synchronization rate of the switching Kuramoto model (85).

Lemma 10: Consider the nonlinear switching differential equation (85). Let $\alpha = (1/n)\mathbf{1}_n^{\mathrm{T}}\theta(0)$, $\delta(t) = \theta(t) - \alpha\mathbf{1}_n$ and μ be the algebraic connectivity of the initial graph $G_{z(0)}$. Suppose $|\theta_i(0) - \theta_j(0)| < \pi$ for any $i, j = 1, 2, \ldots, n$, then one has

$$\|\delta(t)\| \leq \|\delta(0)\|, \quad \Delta_t \leq \sqrt{2}\|\delta(0)\|, \quad \forall t \in [0, +\infty). \tag{86}$$

Moreover, if the initial graph $G_{z(0)}$ is connected and $E_{z(0)} \subset E_{z(t)}$ for any $t \in [0, \eta]$, then one gets

$$\begin{aligned} &\|\delta(t)\| \leq \mathrm{e}^{-k\tilde{\mu}t}\|\delta(0)\|, \\ &\Delta_t \leq \sqrt{2}\mathrm{e}^{-k\tilde{\mu}t}\|\delta(0)\|, \forall t \in [0, \eta], \end{aligned} \tag{87}$$

where η is a constant and $\tilde{\mu} = \mu(\sin\Delta_0)/\Delta_0 > 0$.

Proof: Let $\mathcal{S} = \{\theta \| \theta_i - \theta_j| < \pi, \ 1 \leq i < j \leq n\}$ and denote $S_t^{\theta} = \{l \ | \theta_l(t) = \bar{\theta}(t), \ 1 \leq l \leq n\}$. For any $\theta(t) \in \mathcal{S}$, it is easy to check

$$\begin{aligned} D^+\bar{\theta}(t) &= \max_{i \in S\theta_t}\{\dot{\theta}_i(t)\} \\ &= \max_{i \in S_t^{\theta}} \left\{ k \sum_{j=1}^{n} a_{ij}(z)\sin(\theta_j(t) - \bar{\theta}(t)) \right\} \\ &\leq 0. \end{aligned} \tag{88}$$

Thus with the initial condition, $\bar{\theta}(t)$ is nonincreasing. Similarly, $\underline{\theta}(t)$ is nondecreasing. Thus Δ_t is nonincreasing, which implies $\Delta_t \leq \Delta_0$ for all $t > 0$. From [36], (85) can be rewritten as a matrix form

$$\dot{\theta} = -kB_z \sin(B_z^{\mathrm{T}}\theta), \tag{89}$$

where B_z is the incidence matrix of an oriented graph of G_z satisfying $L_z = B_z B_z^{\mathrm{T}}$. Let $V(\delta) = (\delta^T \delta)/2 = (1/2)|\delta|^2$. Then, similar to the proof of Theorem 1 of [36], one obtains

$$\dot{V}(\delta) = -k\delta^T B_z \sin(B_z^{\mathrm{T}}\delta) = -k\delta^T B_z W_z(\theta) B_z^{\mathrm{T}}\delta \leq 0, \quad (90)$$

where

$$W_z(\theta) = \mathrm{diag}\left\{\frac{\sin(\theta_i - \theta_j)}{\theta_i - \theta_j};\ (i,j) \in E_z\right\} \geq 0.$$

Thus, $V(\delta(t)) \leq V(\delta(0))$, which implies (86). If the initial graph $G_{z(0)}$ is connected and $E_{z(0)} \subset E_{z(t)}$ for any $t \in [0, \eta]$, then $\lambda_2(G_{z(t)}) \geq \lambda_2(G_{z(0)}) = \mu$ from Lemma 3. Considering that $(\sin x)/x$ is a decreasing function on $[0, \pi]$ and $\Delta_t \leq \Delta_0$, one gets

$$W_z(\theta) \geq \frac{\sin \Delta_t}{\Delta_t} I_{e_z} \geq \frac{\sin \Delta_0}{\Delta_0} I_{e_z}.$$

From (90), one has

$$\begin{aligned}\dot{V}(\delta) &\leq -k\frac{\sin \Delta_0}{\Delta_0}\delta^T B_z B_z^{\mathrm{T}}\delta \\ &= -k\frac{\sin \Delta_0}{\Delta_0}\delta^T L_z \delta \leq -2k\tilde{\mu}V(\delta), \quad (91)\end{aligned}$$

which implies that (87) holds. ■

Similarly, based on Lemma 10 and following the same proof procedure of Theorem 1, one can easily obtain the following theorem.

Theorem 3: For multi-agent system (84) and (85), suppose that the initial graph $G_{z(0)}$ is connected, the initial positions of all agents are distinct and the initial headings satisfy $|\theta_i(0) - \theta_j(0)| < \pi$ for any $i, j = 1, 2, \ldots, n$. If

$$k > \frac{\sqrt{2}\, v\zeta\Delta_0}{\mu d \sin \Delta_0},$$

then graph $G_{z(t)}$ preserves the connectedness on $[0, +\infty)$ and the flocking can be achieved with an exponential rate.

Remark 11: In Theorem 3, multi-agent system (84) and (85) can achieve the flocking under the limitation on the initial heading angles. However, generally speaking, the flocking problem will become very difficult without this limitation because there may exist some stable non-synchronization equilibrium points even for fixed network structure [55]. Therefore, the dynamical analysis of multi-agent systems with Kuramoto model is still an open problem for general case. However, if there exist integers l_i $(i = 1, 2, \ldots, n)$ such that $\tilde{\theta}_i = \theta_i + 2l_i\pi$ $(i = 1, 2, \ldots, n)$ satisfy the initial heading limitation of Theorem 3, then the synchronization with collision avoidance can also be achieved. In this case, $\theta_i - \theta_j$ converges to $2(l_i - l_j)\pi$ instead of 0, which means all the agents tend to the same direction.

Numerical Example 3: For the multi-agent system with Kuramoto model, a 5-agent system is considered with the initial positions $(\cos(i\pi/6), \sin(i\pi/6))$ for $i = 0, \pm 1, \pm 2$, the initial absolute velocity 0.1 and the initial headings $\pi/3,\ \pi/6, 0,\ -\pi/6,\ -\pi/3$. Simulations show that the 5-agent system with Kuramoto model can achieve flocking as $k \geq 0.4$. In Fig. 7, the trajectories of positions in 30 seconds are shown with different values of k. The sufficient condition for k obtained by Theorem 3 is $k > 3.5102$. Fig. 8 shows the time-response curves of the headings. If the initial angles are given by $\pi/3,\ \pi/6,\ 0,\ 11\pi/6$ and $5\pi/3$, then the limitation condition of Theorem 3 can not be satisfied. However, by Remark 11, the flocking can also be realized. The trajectories are the same as shown in the last figure of Fig. 7. In this case, one gets $\theta_i(t) \to 0$ for $i = 1, 2, 3$ and $\theta_j(t) \to 2\pi$ for $j = 4, 5$, which are shown in Fig. 9 with $k = 3.6$.

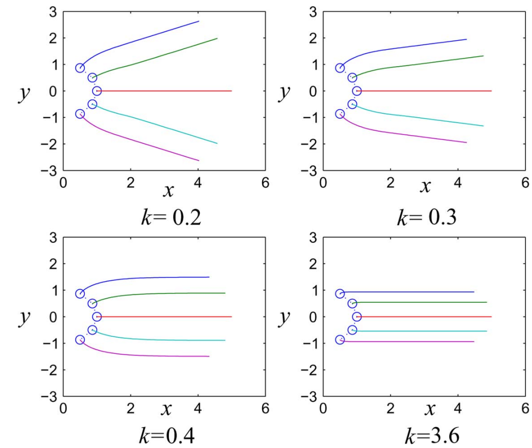

Fig. 7. The Initial positions and the trajectories with different values of k.

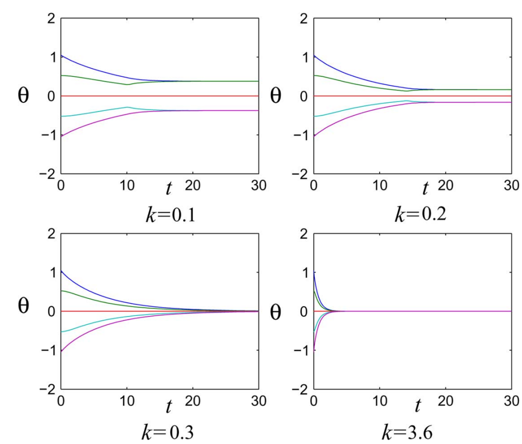

Fig. 8. The time-response curves of the headings with different values of k.

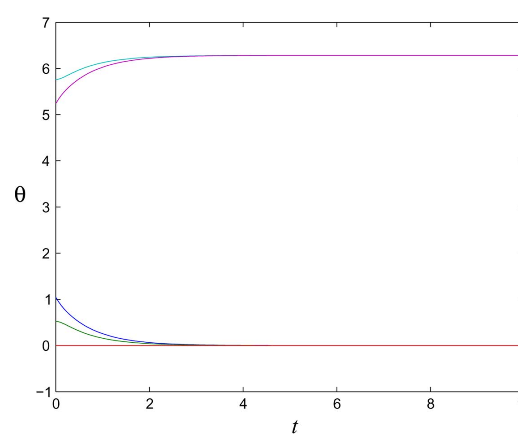
Fig. 9. The time-response curves of the headings with $k = 3.6$ and the initial headings: $\pi/3,\ \pi/6,\ 0,\ 11\pi/6,\ 5\pi/3$.

V. Conclusion

This paper has investigated the flocking problem based on a kinematic model and a dynamic model for the multi-agent nonholonomic systems. It has been revealed that there always exist linear protocols only using local heading angles to preserve the connectedness and realize the flocking for the considered multi-agent systems as long as the initial graph is connected and the initial positions are distinct. Moreover, for the nonlinear protocol described by the switching Kuramoto model, similar result has been obtained under a limitation on the initial angles. Finally, numerical examples have been given to verify the obtained theoretical results. In the future work, we will investigate the critical value of the gain realizing flocking.

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