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A local flocking algorithm of multi-agent dynamic systems

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In this paper, the local flocking of multi-agent systems is investigated, which means all agents form some groups of surrounding multiple targets with the partial information exchange. For the purpose of realising local multi-flocking, a control algorithm of local flocking is proposed, which is a biologically inspired approach that assimilates key characteristics of flocking and anti-flocking. In the process of surrounding mobile targets through the control algorithm, all agents can adaptively choose between two work modes to depend on the variation of visual field and the number of pursuing agents with the mobile target. One is a flocking pursuing mode which is that some agents pursue each mobile target, the other is an anti-flocking searching mode that means with the exception of the pursing agents of mobile targets, other agents respectively hunt for optimal the mobile target with a closest principle between the agent and the target. In two work modes, the agents are controlled severally via the different control protocol. By the Lyapunov theorem, the stability of the second-order multi-agent system is proven in detail. Finally, simulation results verify the effectiveness of the proposed algorithm.

Keywords: multi-agent; distributed control; mobile targets; local multi-flocking

1. Introduction

In recent decades, flocking, which is a form of collective behaviour of large numbers of local interacting agents, has attracted scientists' extensive attention in the fields of biophysics, physics, sociology, mathematics, computer science, engineering science and so on (Brambilla, Ferrante, Birattari, & Dorigo, 2013; Chen, Pei, & Zhu, 2008; Cucker & Dong, 2010; Gazi & Fidan, 2013; Jia & Wang, 2014; Jadbabaie, Lin, & Morse, 2003; Luo, Li, & Guan, 2010; Lou & Hong, 2012; Okubo, 1986; Olfati-Saber, 2006; Olfati-Saber & Murray, 2003; Reynolds, 1987; Shi, Wang, Chu & Fu, 2007; Toner, 1998; Vicsek, Cziroók, Ben-Jacob, Cohen, & Shochet, 1995; Virágh et al., 2014; Xi, Shi, & Zhong, 2012). The flocking is always characterised by a group of agents with simple intelligence to reach a consistent speed and a desired distance between agents at some point by means of interaction with their neighbours. In a real-world application, there are many research objectives, which can be regarded as agents such as mobile sensors, mobile robots, unmanned aerial vehicles, etc.

Reynolds (1987) presented a Boid model that can imitate the flocking behaviour of the birds. The model follows three simple rules: (1) better global positional restraint to keep the flock together; (2) repulsion in short distance to avoid collisions with other agents; and (3) velocity matching to align velocity with nearby agents. Vicsek et al. (1995) proposed a basic model of research multi-agent systems that has some key characteristics of complex systems such as dynamical behaviour, local interaction and change of neighbourship, but which was chiefly focused on emergence of flocking (consider only alignment which is also called synchronisation), and yet Toner and Tu (1998) used a continuum mechanics approach to research. For the sake of studying further the alignment of agents, Jadbabaie et al. (2003) proposed a linear Vicsek's model of the direction angel of agent, which can easily handle group synchronisation. Shi et al. (2007) studied a soft control method that intervenes the flocking behaviour of the whole of groups by a virtual shill to achieve synchronisation of flocks. Chen et al. (2008) researched the consensus of swarming behaviour of mass intelligent individuals on account of three initial distributions. Cucker and Dong (2010) proposed a modified model to avoid collisions in flocks and gave the initial condition of agents for the consensus to come true. In Chen et al. (2008), Cucker and Dong (2010), Jadbabaie et al. (2003), Olfati-Saber and Murray (2003), Reynolds (1987), Shi et al. (2007), Toner (1998) and Vicsek et al. (1995), the synchronisation or the consensus, which is an important term in the flocking model, is not the same as flocking. For their studies, the dynamic motion equation is basically first order with multi-agent systems, which can not meet the practical needs.

As a consequence, Olfati-Saber (2006) put forward a theoretical framework for design and analysis of distributed flocking algorithms with multi-agent dynamic systems, as well as performed two-dimensional (2D) and

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three-dimensional flocking. Luo et al. (2010) researched multi-target tracking to permit a group of agents tracking different targets chosen through the distances between agents and targets, but there is a deadlock problem which is how to choose the tracking target when the distance is same between one agent and several targets. Lou and Hong (2012) studied the distributed containment control, which is considered to achieve the flocking for a second-order multi-agent system. Xi et al. (2012) researched the output consensus design problem with high-order linear time-invariant swarm systems.

Recently, the rules of animal flocking models are blended in the control of autonomous robots (Brambilla et al., 2013). For instance, Gazi and Fidan (2013) developed a direct adaptive fuzzy control for formation control and target tracking in a class of multi-agent systems: formation manoeuvres. Jia and Wang (2014) studied cohesive flocking and formation flocking of multi-robotic fish swimming in the water surface. Virágh et al. (2014) put forward an abstract mathematical model of an autonomous flying robot on the basis of considering several realistic features. Nonetheless, the previous most investigations with respect to flocking have merely considered the problem of single-target tracing or the whole flocking. In practice, it is extremely important for the pursuit and the search of multiple targets such as to save the life in the earthquake and the fire, and to pursue multi-target with manoeuvres and so on. Therefore, it is of great theoretical and practical significance for the research of search and pursuit of multi-target.

In this paper, we present a new concept of local flocking that be made use of studying multi-agent. The local multi-flocking requires that agents have abilities of self-organisation and local communication to select their befitting targets adaptively and then form several local flocks. The key of the problem is working out mainly how to choose and pursue the target effectively for agents to accomplish the total task. The research of local multi-flocking is beneficial to search for mobile targets and to pursue them. Hence, the paper introduces the local flocking algorithm that absorbs the idea of semi-flocking (Semnani & Basir, 2014) with large-scale surveillance systems. Both the flocking (Olfati-Saber, 2006) and the anti-locking (Miao, Khamis, & Kamel, 2010) are reciprocally fused to lead to the basic semi-flocking. The proposed local flocking algorithm gives the variation of visual field of each mobile target to make agents availably choose between the flocking pursuing mode and the anti-flocking searching searching mode to furthermore improve self-organising competences in the pursuit process. In addition, the control algorithm can work out the deadlock problem in Luo et al. (2010).

The outline of the paper is as follows: Section 2 presents some assumption and definitions with prep relevant concepts of graph theory, as well as the double-integrator dynamic models for multiple agents and targets. Section 3 sets forth a local flocking method in detail, and simply introduces the flocking algorithm and the anti-flocking algorithm. Section 4 describes a theorem with the local muti-flocking, and which utilises Lyapunov theorem and LaSalle's invariance principle to certify the stability of the dynamic system with multi-agent. Section 5 discusses simulation results and analysis to evaluate the proposed local flocking algorithm. Finally, the conclusions and recommendations of the paper are given in Section 6.

2. Preliminaries and problem statement

First of all, we introduce some definitions and assumptions as follows:

Definition 1: An undirected graph G is defined as an ordered pair, $G = (V, E)$, where $V = \{1, 2, \dots, n\}$ is a point/vertex set, n denotes the number of vertexes, $E = \{(i, j)\} \subset V \times V$ is an edge set with vertexes of junction, and the vertex pair is unordered, i.e. $(i, j) \in E \Leftrightarrow (j, i) \in E$.

Definition 2: The adjacency matrix $A = [a_{ij}]$ of undirected graph G is defined as $a_{ij} = 1$, if $(i, j) \in E$, $a_{ij} = 0$ otherwise. Here, A is symmetric, i.e. $A^T = A$.

Definition 3: The Laplacian matrix $L \in R^{n\times n}$ is defined as $L = L(G) = [l_{ij}]$, where $l_{ii} = \sum_{p\neq i}^{n} a_{ip}$ and $l_{ij} = -a_{ij}, \forall i \neq j$.

Assumption 1: Consider a multi-agent system with n agents and m targets in a 2D Euclidean space. All agents and targets are regarded as points, meanwhile their dimensions are ignored. Let $r_i, v_i \in R^2$ denote the position and velocity of agent i, as well $r_{tk}, v_{tk} \in R^2$ denote the position and velocity of target $k \in W = \{1, 2, \dots, m\}$ $m < n$, respectively. Let u_i, u_{tk} denote the control input of agent i and target k. Let θ_k denote the variation of visual field of mobile target k

Assumption 2: Suppose that n agents have the same visual range (or interaction range). Let r_c denote the visual radius, then the neighbour set of agent i is defined as

$$N_i = j \in V : \|r_j - r_i\| < r_c.$$

Each agent i can obtain the information of all mobile targets. Whether agent i can pursue the target depends on the number of agents pursuing this target and the variation of visual field of the mobile target as well as the distance between agent i and this target.

The motion of each agent is controlled independently by a control input u in coordination with the motion of other agents. The continuous motion equation (or double-integrator dynamic model) with agent i is given as follows:

$$\begin{cases} \dot{r}_i = v_i \\ \dot{v}_i = u_i \end{cases} \quad i \in V \tag{1}$$

where r_i, v_i, u_i satisfy Assumption 1. In addition, m mobile targets abide by the following equation of motion:

$$\begin{cases} \dot{r}_{tk} = v_{tk} \\ \dot{v}_{tk} = u_{tk} \end{cases} \quad k \in W \tag{2}$$

where r_{ik}, v_{ik}, u_{tk} satisfy Assumption 1.

In this paper, the neighbour graph of all agents is undirected at any time. It is well known that the Laplacian matrix L is invariably symmetric and positive semi-definite for an undirected graph G, which also satisfies the following property:

$$z^T L z = \frac{1}{2} \sum_{i=1}^{n} \sum_{j \in N_i} (z_j - z_i)^2 \tag{3}$$

where $z = \mathrm{col}(z_1, z_2, \ldots, z_n) \in R^{1\times n}$, $z_i \in R(i = 1, 2, \ldots n)$. Particularly, we need the Laplacian matrix of 2D graph that is defined as $\hat{L} = L \otimes I_2$, where $\otimes$ denotes the Kronecker product (Olfati-Saber, 2006).

The objective of this work is to provide a local flocking algorithm having two work modes: the flocking pursuing mode and the anti-flocking searching mode, and the variation of visual field of each mobile target with the number of pursuing agents. Under two different control protocols, the algorithm can realise local muli-flocking to deal with the pursuit and search of multiple targets effectively. Furthermore, we give Theorem 1 with local flocking, which is analysed by Lyapunov theorem and LaSalle's invariance principles. Using simulation experiments, the value of local flocking approach is verified ulteriorly.

3. A local flocking method

In the local multi-flocking behaviour, agents utilise some simple rules and local interactions to acquire a complicated local flocking which is that all agents form some groups of pursuing and searching for multiple targets. These simple rules are able to be represented as an input vector (or a control input u) for each agent. The proposed local flocking method allows agents (or pursuers) to adaptively choose between the flocking pursuing mode and the anti-flocking searching mode. The flocking pursuing mode is to make some agents pursue each mobile target to form small agent groups. The anti-flocking searching mode is to make other agents hunt for optimal the mobile target with the closest principle between the agent and targets. Section 3.3 describes mainly how the control input (or the control protocol) u is determined via the local flocking algorithm, and how the local flocking control algorithm distinguishes from the flocking and anti-flocking rules, as well as the procedure of control algorithm.

3.1 *Flocking algorithm*

Flocking is a form of collective behaviour of autonomous agents, which can achieve a common group objective with any global information. Flocking algorithm is mainly based on cohesion, separation and alignment rules (Reynolds, 1987). Olfati-Saber (Olfati-Saber, 2006) designed two types of flocking algorithms for the different initial distribution with a group of agents. The second flocking algorithm is extensively applied. In this algorithm, each agent has a control input that is composed of three portions

$$u_i = f_i^g + f_i^d + f_i^\gamma \tag{4}$$

where f_i^g is a gradient-based portion, f_i^d is a velocity consensus portion, and f_i^γ is a navigational feedback to lie on a group objective.

Applying the flocking algorithm to surround multiple targets, f_i^γ is defined in accordance with the pursuing targets of agents. When a group of agents only pursue a mobile target at position r_t to move with velocity v_t, f_i^γ is defined in the following equation, also $c_{11} > 0$ and $c_{21} > 0$.

$$f_i^\gamma(r_i, v_i, r_t, v_t) = c_{11}(r_t - r_i) + c_{21}(v_t - v_i) \quad i \in V \tag{5}$$

Here, u_i is as a control input of each agent i. It may be obtained that the mobile target is surrounded ultimately as shown in Figure 1.

Suppose that there are several targets in the workspace. Using the flocking algorithm, all agents work on pursuing the first target. As a result, none remain to pursue other targets to led to the failure of the task. Figure 2 shows this problem as follows. In practical terms, it is necessary to resolve the issue such as searching the life in the earthquake and the fire. Therefore, the Section 3.3 studies chiefly the local flocking of multi-agent to deal with the problem.

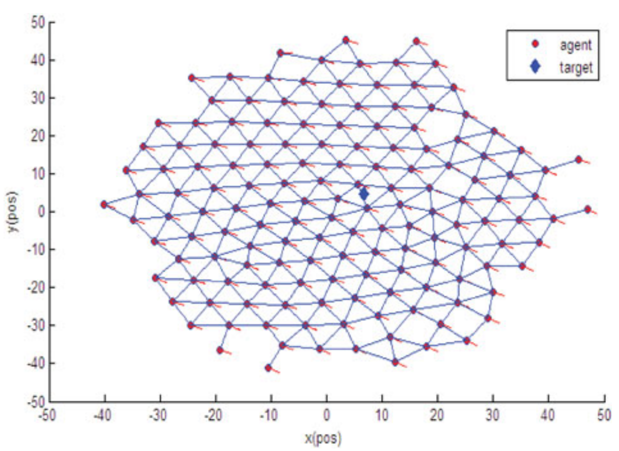

Figure 1. Flocking for $n = 150$ agents.

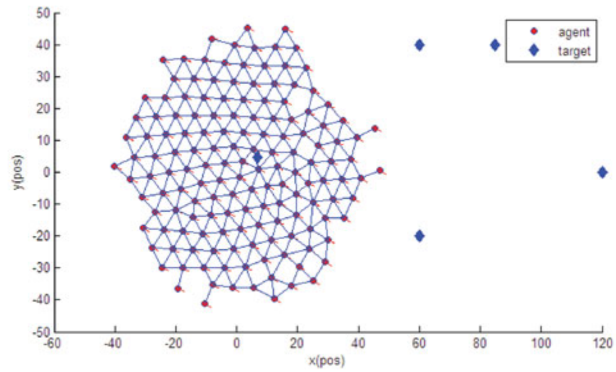

Figure 2. Flocking algorithm applies in pursuing multi-target.

3.2 Anti-flocking algorithm

In nature, in order to survive preferably, some social animals select group activities, which is the above-mentioned flocking behaviour. However, there are some animals that, for survival, are solitary, such as tigers, fighting fishes and the like. In Giraldeau (2008), the behaviour of solitary animals is beneficial to all species members to forage on a large scale with neighbour information, which is called anti-flocking behaviour.

Anti-flocking algorithm (Miao et al., 2010) abides by the following three rules as shown in Figure 3: (1) separation: stay away from the nearest obstacle to keep safe distance; (2) de-centring: attempt to move apart from its neighbours; and (3) selfishness: move to a direction so as to maximise own gains (i.e. pursuers obtain more evaders in the process of pursuit).

3.3 A local flocking algorithm

In the flocking algorithm introduced in Section 3.1, each agent applies a control protocol $u_i = f_i^g + f_i^d + f_i^\gamma$. In the local flocking algorithm, the implemented control protocol is similar to the former with the exception of the third portion f_i^γ, which is modified in accordance with the different behaviour of agents, i.e. the flocking pursuing behaviour, the anti-flocking searching behaviour.

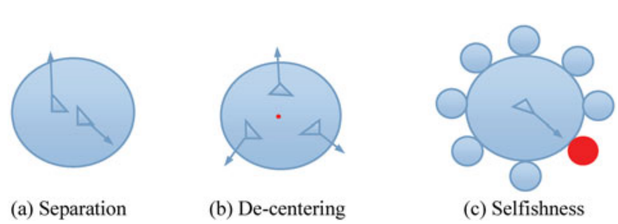

Figure 3. Three rules of anti-flocking. (a) Separation, (b) decentring and (c) selfishness.

The control protocol of each agent is the following control function:

$$u_i = \sum_{j\in N_i} \varphi_\alpha(\|r_j - r_i\|_\sigma)e_{ij} + \sum_{j\in N_i} a_{ij}(v_j - v_i) + f_i^\gamma(r_i, v_i, r_{t1}\ldots r_{tm}, v_{t1}\ldots v_{tm}) \quad i \in V \qquad (6)$$

where the N_i satisfies Assumption 1, $\varphi_\alpha(x)$ is an action force function as follows:

$$\varphi_\alpha(x) = \rho h(x/r_\alpha)\varphi(x - d_\alpha) \qquad (7)$$

where $r_\alpha = \|r\|_\sigma$ and $d_\alpha = \|r_j - r_i\|_\sigma \forall j \in N_i$ are constant parameters. In Equation (7), $\|x\|_\sigma = \frac{1}{\varepsilon}(\sqrt{1+\varepsilon\|x\|^2} - 1)$ $\varepsilon > 0$ is the $\sigma-$norm of a vector. $\rho h(x)$ is the following bump function:

$$\rho h(x) = \begin{cases} 1 & x \in [0, h) \\ \frac{1}{2}\left[1 + \cos\left(\pi\frac{(x-h)}{(1-h)}\right)\right] & x \in [h, 1] \\ 0 & \text{otherwise} \end{cases}, \quad h \in (0, 1).$$

$\varphi(x)$ is the following sigmoidal function:

$$\varphi(x) = \frac{1}{2}[(a+b)\sigma_1(x+c) + (a-b)],$$

where $\sigma_1(x) = x/\sqrt{1+x^2}$, $0 < a \le b$, $c = |a-b|\sqrt{4ab}$ to assure $\varphi(0) = 0$. In Equation (7). e_{ij} is defined as $e_{ij} = \sigma_\varepsilon(r_j - r_i) = \frac{(r_j - r_i)}{\sqrt{1+\varepsilon\|r_j - r_i\|^2}}$, where $\varepsilon \in (0, 1)$.

In addition, a_{ij} is the element of the adjacency matrix A as Definition 2 in the consensus portion of Equation (6).

The agent pursues the target to rest with the following two key factors: (1) the distance between the agent and the target; (2) the number of pursuing agents with the target, denoted by n_t, which is influenced by the variation of visual field of the target. Here, we define $u_i^\gamma = f_i^\gamma(r_i, v_i, r_{t1}\ldots r_{tm}, v_{t1}\ldots v_{tm})$, and give the procedure of local flocking algorithm as follows:

(1) To initialise parameters: setting n, m, r_c among Assumptions 1, 2, and the initial position and velocity of agents and targets;

(2) If $\|r_{tk} - r_i\| \le \theta_k$, agent i enters into the flocking pursuing mode. Simultaneously, to count

$$u_{i,tk}^\gamma = \frac{c_{12}(r_{tk} - r_i) + c_{22}(v_{tk} - v_i)}{n_{tk}},$$

where θ_k is called the variation of visual field of mobile target k, which is an exponential function that declines with $n_{tk} > 0$, $k \in (1, 2, \ldots, m)$, defined as $\theta_k = \lambda e^{-\frac{n_{tk}}{\lambda}}$, $\lambda \in R$, n_{tk} denotes the number of the pursuing agents of target k, $c_{12} > 0, c_{22} > 0$;

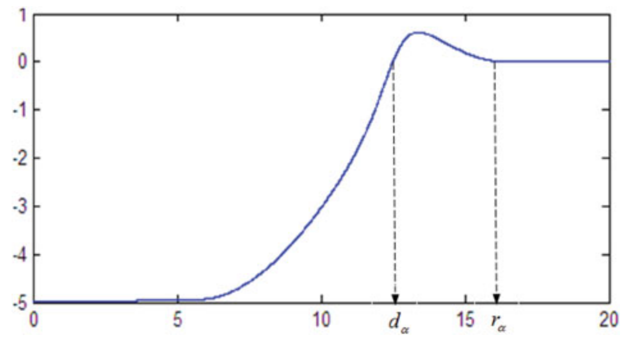

Figure 4. The action force function.

(3) To ensure the navigational feedback $u_i^\gamma = \sum_{k=1}^{m} u_{i,tk}^\gamma$. At this point, under flocking pursuing mode, in Equation (6) the control protocol u_i is changed as the following form:

$$u_i = \sum_{j\in N_i} \varphi_\alpha(\|r_j - r_i\|_\sigma)e_{ij} + \sum_{j\in N_i} a_{ij}(r)(v_j - v_i) + \sum_{k=1}^{m} u_{i,tk}^\gamma \tag{8}$$

(4) If $\|r_{tk} - r_i\| > \theta_k$ or $u_i^\gamma == 0$, agent i enters into the anti-flocking searching mode. At this time, agent i searches for a new target of the minimal distance with the closest principle between agent i and targets; to update the following u_i^γ: $u_i^r = c_{13}(r_{i,t\min} - r_i) + c_{23}(v_{i,t\min} - v_i)$, where $r_{i,t\min}$, $v_{i,t\min}$ denote severally the position and velocity of the closest target of agent i; $c_{13} > 0$, $c_{23} > 0$. Hence, under anti-flocking searching mode, u_i is the following equation:

$$u_i = \sum_{j\in N_i} \varphi_\alpha\left(\|r_j - r_i\|_\sigma\right) e_{ij} + \sum_{j\in N_i} a_{ij}(r)(v_j - v_i) + c_{13}(r_{i,t\min} - r_i) + c_{23}(v_{i,t\min} - v_i) \tag{9}$$

Note: In the control protocol Equation (6), $f_i^g = -\nabla_{r_i} V(r) = \sum_{j\in N_i} \varphi_\alpha(\|r_j - r_i\|_\sigma)e_{ij}$ is a gradient-based portion (Olfati-Saber, 2006). The action function $\varphi_\alpha(x)$ defined by the front Equation (7) is described as Figure 4.

The group potential function $\psi(x) = \int_{d_\alpha}^{x} \varphi_\alpha(s)ds$ is shown in Figure 5, where d_α is the σ−norm of a global minimum $x = d$. Figure 5 shows that $\psi(x)$ is a smooth group potential function with a global minimum at $x = d$.

4. Stability analysis of local flocking

In the above section, we have put forward the control algorithm to realise that several mobile targets are pursued and surrounded via local multi-flocking. In this section, we

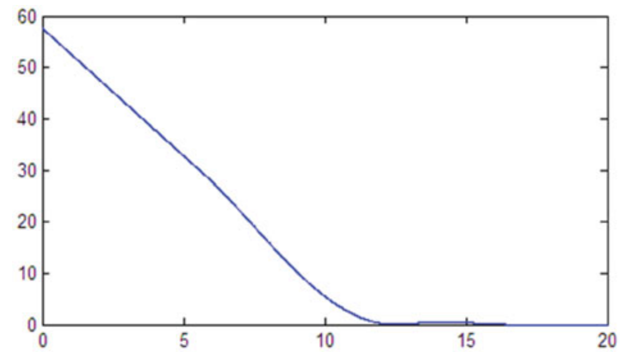

Figure 5. The group potential function.

give theorem with the local flocking, which is proven by Lyapunov theorem and LaSalle's invariance principle.

Theorem 1: Suppose that the initial velocities are mismatched, and the initial positions of a swarm agent obey the Gaussian distribution. Consider the dynamical system Equation (1) with the control protocols, the following statement holds:

(i) No collision will happen with each agent;
(ii) Agents compose local flocks pursuing and surrounding their mobile targets;
(iii) Agents with the same mobile target match velocity;
(iv) Agents with the same target compose a flock having a local minimum energy.

Proof: Consider the flocking pursuing mode, assume that a mobile target is pursued via w agents. Let $\hat{r}_i = r_i - r_{tk}$ and $\hat{v} = v_i - v_{tk}$, then

$$\begin{cases} \dot{\hat{r}}_i = \hat{v}_i \\ \dot{\hat{v}}_i = u_i - u_{tk} \end{cases} \quad i = 1, 2, \ldots w \tag{10}$$

where $\hat{r} = \mathrm{col}(\hat{r}_1, \hat{r}_2 \ldots, \hat{r}_w)$, $\hat{v} = \mathrm{col}(\hat{v}_1, \hat{v}_2 \ldots, \hat{v}_w)$. Thus, the control protocol Equation (8) for agent i will be changed into

$$u_i = \sum_{j\in N_i} \varphi_\alpha(\|\hat{r}_{ij}\|_\sigma)e_{ij} + \sum_{j\in N_i} a_{ij}(\hat{v}_j - \hat{v}_i) - \frac{1}{n_{tk}}(c_{12}\hat{r}_i + c_{22}\hat{v}_i). \tag{11}$$

The collective potential function $V_i(r)$ can be rewritten as

$$\hat{V}_i(\hat{r}_{ij}) = V_i(r) = \sum_{j\notin N_i(t), j\neq i} \psi(r_\alpha) + \sum_{j\in N_i(t)} \psi(\|\hat{r}_{ij}\|_\sigma). \tag{12}$$

Consider the following energy function as the Lyapunov function

$$Q_k(\hat{r},\hat{v}) = Q_k(r,v) = \frac{1}{2}\sum_{i=1}^{w}(U_i(\hat{r}) + \hat{v}_i^T\hat{v}_i) \tag{13}$$

where

$$U_i(\hat{r}) = \sum_{j=1,j\neq i}^{w}\psi(\|\hat{r}_{ij}\|_\sigma) + \frac{c_{12}}{n_{tk}}\hat{r}_i^T\hat{r}_i = \hat{V}_i(\hat{r}_{ij}) + \frac{c_{12}}{n_{tk}}\hat{r}_i^T\hat{r}_i \tag{14}$$

According to the symmetry of the pairwise potential $\psi(x)$ and the adjacent matrix $A(t)$ in Definition 2, one has

$$\frac{1}{2}\sum_{i=1}^{w}\dot{U}_i(\hat{r}) = \sum_{i=1}^{w}\left(\hat{v}_i^T\nabla_{\hat{r}_i}\hat{V}_i(\hat{r}_{ij}) + \frac{c_{12}}{n_{tk}}\hat{v}_i^T\hat{r}\right). \tag{15}$$

Therefore, on the basis of Equations (3) and (15), we have

$$\begin{aligned}\dot{Q}_k &= \frac{1}{2}\sum_{i=1}^{w}\dot{U}_i(\hat{r}) + \sum_{i=1}^{w}\hat{v}_i^T\dot{\hat{v}} \\ &= -\hat{v}^T\left[\left(L + \frac{c_{22}}{n_{tk}}I_w\right)\otimes I_2\right]\hat{v} \le 0.\end{aligned} \tag{16}$$

Here, $\dot{Q}_k \le 0$ implies that $Q_k(t) := Q_k(\hat{r},\hat{v})$ is a decreasing function of time t in consequence of $Q_k(t) \le Q_{k0}$ for $t \ge 0$, where $Q_{k0} = Q_k(0)$. From Equations (13) and (14), we have $\frac{c_{12}}{n_{tk}}\hat{r}_i^T\hat{r}_i \le 2Q_{k0}$ for agent i, which ensures local flocking. In the flocking pursuing mode, the analytic process is similar. Therefore, part (ii) of Theorem 1 is proven.

Since $Q_k(t) > 0$ and $\dot{Q}_k \le 0$, $\Omega = \{(\hat{r}^T,\hat{v}^T)^T \in R^{2\times 2w} : Q_k \le Q_{k0}\}$ is a invariant set. According to LaSalle's invariance principle, the state of all agents start from Ω converge to the largest set $S = \{(\hat{r}^T,\hat{v}^T)^T \in R^{2\times 2w} : \dot{Q}_k = 0\}$. From Equation (16), we acquire

$$\begin{aligned}\dot{Q}_k &= -\hat{v}^T\left[\left(L + \frac{c_{22}}{n_{tk}}I_w\right)\otimes I_2\right] \\ \hat{v} &= -\hat{v}^T(L\otimes I_2)\hat{v} - \frac{c_{22}}{n_{tk}}\hat{v}^T\hat{v}.\end{aligned}$$

Therefore, $\dot{Q}_k = 0$ is equivalent to $\hat{v} = \mathrm{col}(\hat{v}_1,\hat{v}_2\ldots,\hat{v}_w) \equiv \mathrm{col}(0,0\ldots,0)$, which happens only when $v_1 \equiv v_2 \equiv \cdots \equiv v_w \equiv v_{tk}$.

In the meantime, existing a local minimum value $\hat{r}$, then a local flocking has the minimum energy. Parts (iii) and (iv) are proven.

Using the proof by contradiction, we prove part (i). Suppose that there are at least two agents colliding at time t. Then

$$\begin{aligned}Q(\hat{r},\hat{v}) &= \sum_{k=1}^{m}Q_k(\hat{r},\hat{v}) = \frac{1}{2}\sum_{k=1}^{m}\sum_{i=1}^{w}U_i(\hat{r}) + \frac{1}{2}\sum_{i=1}^{n}\hat{v}_i^T\hat{v}_i \\ &= \frac{1}{2}\sum_{k=1}^{m}\sum_{i=1}^{w}\left(\hat{V}_i(\hat{r}_{ij}) + \frac{c_{12}}{n_{tk}}\hat{r}_i^T\hat{r}_i\right) + \frac{1}{2}\sum_{i=1}^{n}\hat{v}_i^T\hat{v}_i \\ &\ge \frac{1}{2}\sum_{k=1}^{m}\sum_{i=1}^{w}\hat{V}_i(\hat{r}_{ij}) \ge Q_0\end{aligned}$$

which contradict the condition $Q(\hat{r},\hat{v}) \le Q_0 = \sum_{k=1}^{m}Q_{k0}$, where Q_0 denotes the initial energy of the total multi-agent system. Finally, no two agents collide with each other. □

Through the proof and analysis of Theorem 1, under the control protocols, the dynamical system of multi-agent is stability, which can come true local multi-flocking pursuing and surrounding several mobile targets by means of the local flocking algorithm.

5. Simulation of local multi-flocking

In this section, the effectiveness of the proposed algorithm of this paper is verified by the simulation. Assume that the condition of the successful pursuit is to completely surround the mobile targets. When all mobile targets are pursued successfully, it means that all agents have finished the assigned mission.

The initial velocity coordinates are chosen at random on the interval $[0, 1]$. Other parameters are given as follows:

$$d = 8, r_c = 10,\ \lambda = 20;\quad r_{t1} = [-50, 20],$$
$$r_{t2} = [0, -20], r_{t3} = [40, 40];\quad v_{t1} = v_{t2} = v_{t3} = [1, 1].$$

The initial positions of 150 agents submit to the standard normal distribution as shown in Figure 6, in which the initial group communication topology is not connected. In

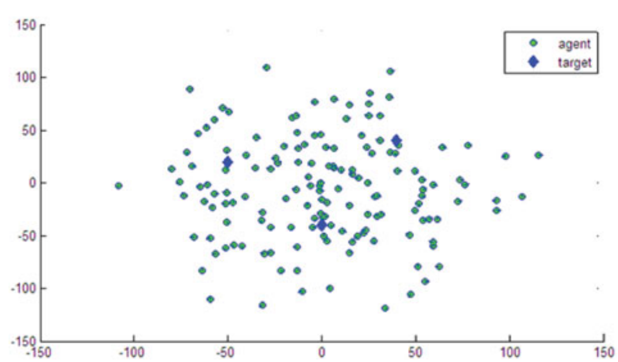

Figure 6. Initial distribution.

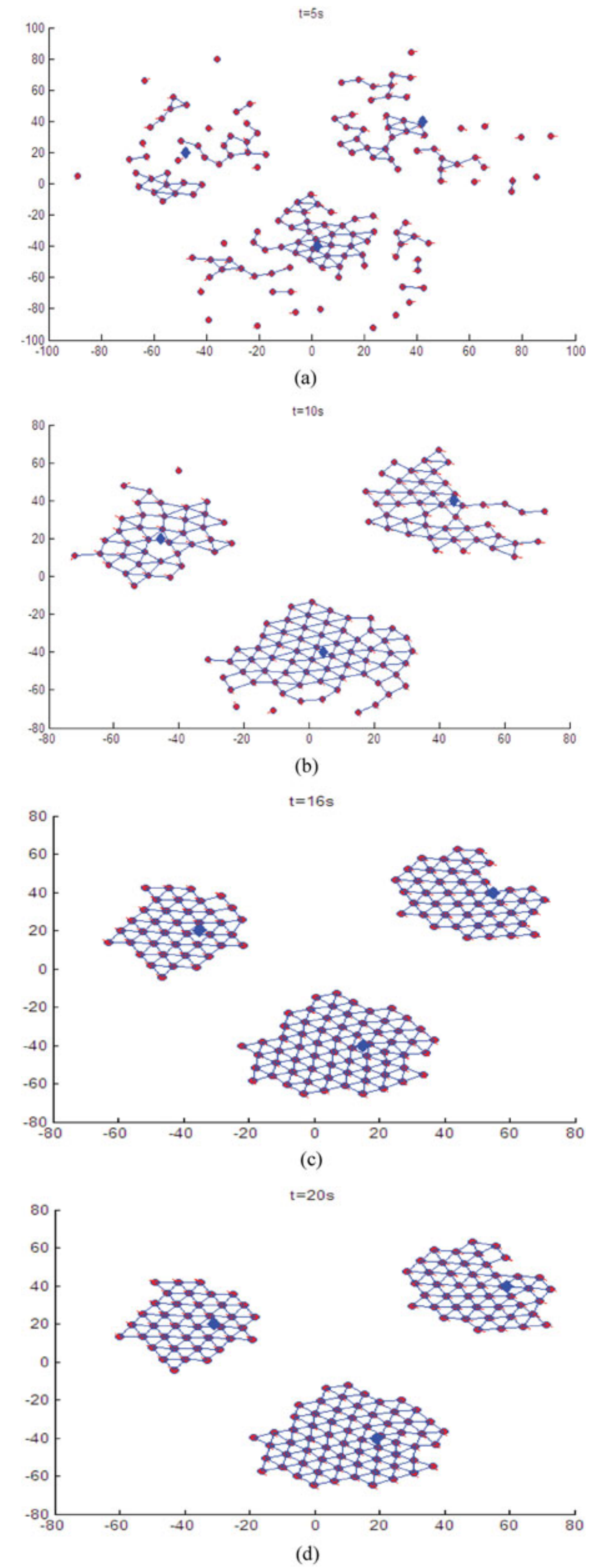

Figure 7. The process of local muli-flocking for $n = 150$ pursuing targets.

the local flocking algorithm, when $t = 5$ s, some agents have been connected and pursued the targets as shown in Figure 7(a). When $t = 10$ s, the local multi-flocking has been formed clearly as shown in Figure 7(b), and two mobile targets have been surrounded by two local flocks. When $t = 16$ s, all agents with the same mobile targets have generated a stable topology, and which have surrounded their respective target as shown in Figure 7(c). From Figure 7(c), it is clearly shown that local multi-flocking has been completely formed and three targets also have been respectively surrounded via 150 agents.

Simulation results indicate that the proposed control algorithm is able to make a swarm of agents from local multi-flocking to pursue and to surround mobile targets, which can be expanded in scale.

6. Conclusion

This paper came up with the local flocking algorithm which has the flocking pursuing mode and the anti-flocking searching mode for the problem of local multi-flocking. It can achieve the search and pursuit of multiple mobile targets. In these two types of switching work mode, different control protocols are utilised for the agents. In addition, the stability of multi-agent system was analysed via the proposed Theorem 1 and the relevant proof. Simulation results verified the availability of the proposed approach.

As a future work, we will consider the existing case of complicated gambling between agents and targets in the local flocking algorithm, and improving the efficiency of fulfilling a task through applying means that predict the next position of mobile targets. And then, it will be able to guide local flocks pursue targets faster.

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