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Coverage Control of Mobile Robots with Adaptive Gradient

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Abstract: The aim of this paper is to give a coverage control method for mobile robots whose actuators to drive wheels have saturation in the angular velocities. Since each robot has two independently driven wheels, the ratio of one wheel's angular velocity to the another is required to follow a coverage control law. That is, one or both of the saturated drive inputs deteriorates the coverage performance. To overcome this issue, this paper employs an adaptive gradient in the coverage control law for each robot. The effectiveness of the adaptive gradient is shown by experiments.

Keywords: Coverage control, Mobile robot, Adaptive gradient

1. INTRODUCTION

The coverage problem is to arrange multiple agents including sensors or robots in a given region by using the positions. The distributed algorithms [1] give solutions to the problem. The gradient descent algorithm is a popular one [2]. The distributed adaptive algorithm has been proposed for a type of nonholonomic mobile robots with double integrator dynamics [3]. However, in coverage control methods for mobile robots, there is an issue that control input signals to drive wheels reach some physical limitation, that is, the saturation of the drive inputs. For each of the mobile robots which has two independently driven wheels, one or two of the saturated drive inputs considerably deteriorates the coverage performance.

This paper proposes an adaptive gradient descent algorithm of coverage control for the mobile robots with saturated drive inputs. This paper also gives an adaptive control law to keep the ratio of one wheel's angular velocity to the another. Experiments with the hand made mobile robots [4] show the effectiveness of our coverage control method.

2. MOBILE ROBOT

Consider N mobile robots. The motion equation of i-th mobile robot is

$$\begin{bmatrix} \dot{p}^{\circ}_{x,i}(t) \\ \dot{p}^{\circ}_{y,i}(t) \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i(t) & 0 \\ \sin\theta_i(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}$$

where $(p^{\circ}_{x,i}, p^{\circ}_{y,i})$ is the position that is the middle point between the two wheels, θ_i is the attitude angle, v_i is the translational velocity and ω_i is the angular velocity. Let $(p_{x,i}, p_{y,i})$ be the hand position of the mobile robot. The relationship between $(p^{\circ}_{x,i}, p^{\circ}_{y,i})$ and $(p_{x,i}, p_{y,i})$ is $p_{x,i} = p^{\circ}_{x,i} + h\cos\theta_i$ and $p_{y,i} = p^{\circ}_{y,i} + h\sin\theta_i$ where $h \neq 0$. Then the motion equation with respect to the hand position is

$$\begin{bmatrix} \dot{p}_{x,i}(t) \\ \dot{p}_{y,i}(t) \end{bmatrix} = B_i(\theta_i(t)) \begin{bmatrix} v_i(t) \\ \omega_i(t) \end{bmatrix}$$

where

$$B_i(\theta_i(t)) = \begin{bmatrix} \cos\theta_i(t) & -h\sin\theta_i(t) \\ \sin\theta_i(t) & h\cos\theta_i(t) \end{bmatrix}.$$

Note that $\det B(\theta) = h$. Then we have

$$\dot{p}_i(t) = u_i(t) \tag{1}$$

where $p_i = [p_{x,i} \ p_{y,i}]^\top$ and $u_i = [u_{x,i} \ u_{y,i}]^\top$ is the coverage control input. If we have the control input, then we can generate the translational velocity and the angular velocity such that

$$\begin{bmatrix} v_i(t) \\ \omega_i(t) \end{bmatrix} = B_i(\theta_i(t))^{-1} u_i(t).$$

To drive the wheels, it is necessary to generate the angular velocities of the right and left wheels $(\omega_{R,i}, \omega_{L,i})$, which are the drive inputs, such that

$$\begin{bmatrix} \omega_{R,i}(t) \\ \omega_{L,i}(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{R_i} & \frac{T_i}{2R_i} \\ \frac{1}{R_i} & -\frac{T_i}{2R_i} \end{bmatrix} \begin{bmatrix} v_i(t) \\ \omega_i(t) \end{bmatrix}$$

where T_i is the distance between the two wheels, R_i is the radius of the wheel, and $-\overline{\omega} \leq \{\omega_{R,i}, \omega_{L,i}\} \leq \overline{\omega}$ for a constant $\overline{\omega} > 0$.

3. COVERAGE CONTROL

3.1. Coverage problem

We arrange N mobile robots (1) in a convex region $\mathcal{Q} \subset \mathbb{R}^m$. The evaluation function of coverage control is

$$J(p) := \int_{\mathcal{Q}} \min_{i \in \{1,\ldots,N\}} \|q - p_i\|^2 \phi(q) dq$$

where $q \in \mathcal{Q}$ is any point on $\mathcal{Q}$, $\phi : \mathbb{R}^m \to \mathbb{R}_+$ is the integrable function on $\mathcal{Q}$ and $\|\cdot\|$ denotes the Euclidean norm. The purpose is to minimize $J(p)$.

3.2. Gradient descent method

To minimize $J(p)$, we adopt the control law by using the gradient descent method such that

$$u(t) = -\alpha(t) G(p(t)) \frac{\partial J(p(t))}{\partial p} \tag{2}$$

† Yuki Nakai is the presenter of this paper.

where $\alpha \in \mathbb{R}^{N\times N}$ is an adaptive step size matrix, $G(p) \in \mathbb{R}^{N\times N}$ is a positive definite matrix for any $p = [p_1^\top \cdots p_N^\top]^\top \in \mathbb{R}^{Nm}$ and $\frac{\partial J(p)}{\partial p}$ is the gradient of $J(p)$. We give the adaptive step size matrix with the diagonal structure $\alpha = \mathrm{diag}\{\alpha_1, \ldots, \alpha_N\}$ with

$$\alpha_i(t) = \frac{\overline{\omega}}{\max\{\overline{\omega}, |\omega_{R,i}(t)|, |\omega_{L,i}(t)|\}}\ (i = 1, \ldots, N).$$

This adaptive matrix works effectively if the absolute value of a drive input reach $\overline{\omega}$. Then the ratio of $\omega_{R,i}$ to $\omega_{L,i}$ is kept properly to realize coverage control.

3.3. Voronoi partitions

To arrange the robots, we divide $\mathcal{Q}$ into the Voronoi partition $\mathcal{V}(p) = \{\mathcal{V}_1(p), \ldots, \mathcal{V}_N(p)\}$ generated by the hand points p where $\mathcal{V}_i(p)$ is the Voronoi region with respect to p_i defined by

$$\{q \in \mathcal{Q} : \|q - p_i\| \leq \|q - p_j\|, \forall j \in \{1, \ldots, N\}, i \neq j\}.$$

3.4. Centroidal Voronoi partition

The partial derivative of $J(p)$ with respect to p_i is

$$\frac{\partial J(p(t))}{\partial p_i} = 2M(\mathcal{V}_i(p(t)))\Big(p_i - C\big(\mathcal{V}_i(p(t))\big)\Big) \quad (3)$$

where

$$M(\mathcal{V}_i(p)) = \int_{\mathcal{V}_i(p)} \phi(q)dq,$$
$$C(\mathcal{V}_i(p)) = \frac{1}{M(\mathcal{V}_i(p))}\int_{\mathcal{V}_i(p)} q\phi(q)dq$$

are the mass and the center of gravity position, respectively, when considering the Voronoi region $\mathcal{V}_i(p)$ as a rigid body and $\phi(q)$ as its density function. We use the coverage control input (2) with (3) and

$$G(p(t)) = \frac{K}{2}(\mathrm{diag}\{M(\mathcal{V}_1(p(t))), \ldots, M(\mathcal{V}_N(p(t)))\})^{-1}$$

where K is the design parameter.

4. EXPERIMENT

The number of mobile robots is $(N =)$ 4, $\mathcal{Q}$ is a 2 $[\mathrm{m}] \times 2$ $[\mathrm{m}]$ square region. The limit of the angular velocity of the wheel is $\overline{\omega} = 4\pi$ $[\mathrm{rad/s}]$, and the design parameter is $K = 2$. The distance between the position of a mobile robot and the hand position is $h = 0.1$ $[\mathrm{m}]$. Then a trajectory of the mobile robots with the Voronoi partitions is shown in Fig.1 where $\times$ and $\circ$ represent the start and goal points, respectively. We can see that the goal points cover the square region. The time histories of the angular velocities of the wheels are shown in Fig.2. We can see that the angular velocities are within the range $[-\overline{\omega}, \overline{\omega}]$ and converge to 0.

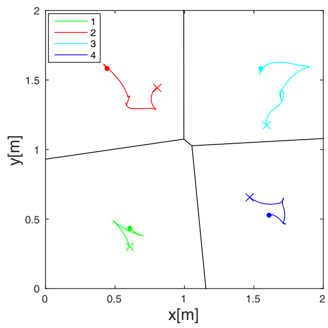

Fig. 1 Trajectories of vehicle.

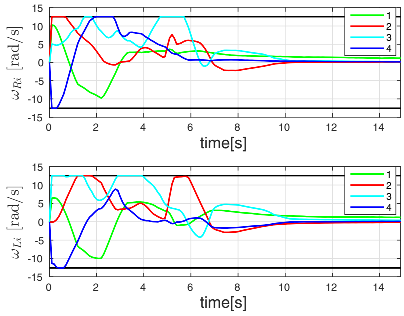

Fig. 2 Time histories of angular velocities of right and left wheels.

5. CONCLUSION

This paper discussed coverage control of mobile robots with the gradient decent method including the adaptive step size matrix to prevent the influence on the saturation of the drive inputs. From the experimental results, we confirmed that the hand points covered the square region by keeping the ratio of one wheel's angular velocity to the another while the angular velocities of the wheels were within the specified range.

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