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Coverage Control Considering Unknown Moving Obstacles Avoidance

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***Abstract*—This paper presents a new approach for coverage an environment in the presence of unknown moving obstacles. A time varying density function is proposed which represents the importance of each point and causes system to avoid collision with obstacles. A control law is presented to make agents follow the density function and avoid unknown moving obstacles while they provide the optimal coverage. The proposed algorithm is decentralized and less computationally expensive comparing to the previous related approaches.**

***Keywords*—Coverage control; Obstacle avoidance; Time varying density function**

I. INTRODUCTION

Sensor networks have many applications in search and rescue missions, environmental sampling, and data collection [1], [2]. For Mobile sensor networks, a fundamental problem is how to deploy sensors over an environment to sense for special purposes. This mission is named Coverage [3], [4].

There are various attempts and solutions in different coverage problems. In [5] a decentralized control law for multi agent system is proposed to do coverage in an area partitioned into Voronoi diagram and drives the agents toward the centroids of their Voronoi cells. In [6], the authors proposed a control law that safely achieves coverage and avoids agent collisions. In [7], an adaptive and decentralized control law is presented to provide an optimal sensing configuration without a prior knowledge of density function. In [8], an adaptive and decentralized control law is presented to provide an optimal coverage while using potential field method to avoid static obstacles. The proposed control law often fails to find a stable configuration in the presence of moving obstacles. In [9], by using a proper diffeomorphism transformation, the non-convex environment is transformed into a convex region and Voronoi coverage can be applied. We use Voronoi tessellations of the domain in our control law to get optimal coverage. Voronoi tessellations have been used to partition the space where agents are deployed. In this case, each robot should compute the corresponding Voronoi cell. To provide an optimal coverage, system should converge to Centroidal Voronoi Tesselltions (CVT). Many algorithms have been proposed for this case [10]–[13].

There are few approaches for time-varying density function. In [14], authors present an optimal coverage of time varying density function algorithm. In [15], optimal coverage control of density functions in generalized Voronoi tessellations using cartograms is discussed. The density function changes slowly to have a stable algorithm. Each agent should have equal amount of resources in their generalized Voronoi cells to have an optimal coverage. In [16], a Lloyd Newton method was proposed to have an optimal coverage control of time varying density function. The time varying density function is considered as input. The algorithm proposed a centralized control approach which leads to algorithm with more computationally expensive than other comparable algorithms. Lloyd Newton methods often fail to find a stable CVT.

In this paper, we propose a new coverage strategy in the presence of unknown moving obstacles. The proposed control approach leads agents to follow the time varying density function and provide the optimal coverage by moving toward their central of Voronoi tessellations. We choose a proper density function which causes system to avoid collision with obstacles. The proposed algorithm is decentralized and less computationally expensive comparing to the previous related approaches.

The rest of the paper is organized as follows. In Section II, we formulate the problem and then in Section III a proper density function is introduced to avoid obstacles. In Section IV we review the locational optimization problem and the topological concept of Voronoi tessellations. In Section V, a new algorithm is presented which drives the agents to the optimal configuration. Simulation results of the new control law in two scenarios of with and without obstacles are presented in Section VI; also the results are compared with the Lloyd algorithm in the presence of obstacle. Section VII we conclude the paper.

II. PROBLEM STATEMENT

Consider n mobile sensing agents in a convex domain represented by $D \subset \mathbb{R}^N$. $q \in D$ and $q_{obs} \in D$ denote an arbitrary point and obstacle position, respectively. The

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positions of agents are shown by $P=\{p_1, \ \dots \ , p_n\}$. To provide coverage let us consider the following cost function

$$E(P,V,t)=\sum_{i=1}^{n}\int_{V_i} f(\|q-p_i\|)\phi_i(q,q_{obs},t)dq. \quad (1)$$

where t represents time $t \geq 0$, and $\phi_i(q,q_{obs},t)$ is the density function which indicates the importance of different areas over D in time t based on the position of obstacles. Points with greater value of density function are more important to cover. The density function will be defined in the next section. f is a non-decreasing, differentiable function. Let us choose $f(x)=x^2$, so in this case we have

$$E(P,V,t)=\sum_{i=1}^{n}\int_{V_i} \|q-p_i\|^2 \phi_i(q,q_{obs},t)dq. \quad (2)$$

The agents provide optimal coverage of the domain, if configuration of positions of the agents minimizes (2). We develop an algorithm that leads the agents to a configuration that minimizes (2).

III. Obstacle Avoidance

The objective is to provide a decentralized coverage while avoid moving obstacle. Number of obstacles, their location, and time of their presence are not known a prior. In this paper, we assume that each agent is able to estimate the position and the velocity of the obstacles which are in the sensing range of the agent.

We indicate R_i as the sensing range of the agent i. The total density function for each agent, ϕ_i, is chosen as follows

$$\phi_i(q,q_{obs},t)=\sum_{j=1}^{l}\delta_j\phi_j(q,q_{obs},t)+\beta \quad (3)$$

where ϕ_j is

$$\phi_j(q,q_{obs},t)=\frac{1}{\exp(-(q-q_{obs}(t))^2)} \quad (4)$$

δ_j is a constant parameter indicating the importance of the density function, ϕ_j, which is based on jth obstacle location. l denotes the number of obstacles which is sensed by agent i, in its sensing range R_i. Using β, constant parameter, provides a uniform coverage over the environment to increase the probability of detecting new obstacles. It should be noted that (4) reaches a minimum value at the location of the obstacle, in contrast the points without obstacles have high importance and will be covered by agents.

IV. Review Of Centroidal Voronoi Tessellations

Voronoi tessellations minimizes the objective function (2) by choosing V_i as follows [5]:

$$V_i=\{x\in D \mid \|x-p_i\|\leq\|x-p_j\|, i\neq j\} \quad (5)$$

Let us define mass, moment, and central of mass of Voronoi regions, respectively by these equations:

$$M_{V_i}=\int_{V_i}\phi_i(q,q_{obs},t)dq \quad (6)$$

$$L_{V_i}=\int_{V_i}q\phi_i(q,q_{obs},t)dq \quad (7)$$

$$C_{V_i}=\frac{L_{V_i}}{M_{V_i}} \quad (8)$$

It should be noted that, (6), (7), (8) show that, despite of the previous approaches, C_{V_i} is a time varying function.

The partial derivative of cost function can be written as:

$$\frac{\partial E}{\partial p_i}=\int_{V_i}\frac{\partial}{\partial p_i}f(\|q-p_i\|)\phi_i(q,q_{obs},t)dq.$$

by choosing $f(x)=x^2$, one can get

$$\frac{\partial E}{\partial p_i}=\int_{V_i}-2(q-p_i)\phi_i(q,q_{obs},t)dq.$$

using (6), (7), (8) one can obtain

$$\frac{\partial E}{\partial p_i}=2M_{V_i}(p_i-C_{V_i}) \quad (9)$$

to minimize the cost function, each agent should be driven to center of its Voronoi region.

V. Algorithm For Coverage In The Presence Of Unknown Moving Obstacles

In [14], for agents with the first order model

$$\dot{p}_i=u_i$$

control signal

$$u_i=-k(p_i-C_{V_i})$$

is proposed to provide optimal configuration when the density function is time invariant, where k is a positive gain.

By employing the introduced control law, one can get:

$$\frac{d}{dt}E(P)=\sum_{i=1}^{n}\frac{\partial}{\partial p_i}E(P)\dot{p}_i$$
$$=\sum_{i=1}^{n}2M_{V_i}(p_i-C_{V_i})(-k(p_i-C_{V_i}))$$
$$=-2k\sum_{i=1}^{n}M_{V_i}\left\|p_i-C_{V_i}\right\|^2$$

By using LaSalle's invariance principle, it can be shown that the system converges to its CVT, asymptotically.

In the case of time varying density function, the given control law, cannot guarantee the convergence of the system into the optimal configuration, since

$$\frac{d}{dt}E(P,t)=\sum_{i=1}^{n}\frac{\partial}{\partial p_i}E(P,t)\dot{p}_i+\frac{\partial}{\partial t}E(P,t) \tag{10}$$
$$=-2k\sum_{i=1}^{n}M_{V_i}\left\|p_i-C_{V_i}\right\|^2$$
$$+\sum_{i=1}^{n}\int_{V_i}\left\|q-p_i\right\|^2\frac{\partial\phi_i}{\partial t}(q,t)dq\le 0$$

Inequality (10) is not always hold. A new control law is proposed to cope with this problem. Since the CVT function is nonlinear, a nonlinear optimization method, i.e., a combination of time-varying Newton's method and Broyden method is employed to compute CVT.

A. Time Varying Newton's Method

In [17], authors used classical Newton's method to compute local minimum of a differentiable function $f(P)$ as follows

$$\dot{P}=-\alpha f(P)\left(\frac{\partial f(P)}{\partial P}\right)^{-1}$$

where α is a constant, indicating step size, and $P=[p_1,\ldots,p_n]$

In this paper, we consider the problem of finding a time varying local minimum of differentiable function $f(P,t)$. To compute the time varying local minimum of function, one should use the proposed time varying Newton method as follows

$$\frac{df(P,t)}{dt}=\frac{\partial f(\mathrm{P},t)}{\partial P}\dot{P}+\frac{\partial f(P,t)}{\partial t}=-\alpha f(P,t) \tag{11}$$

$$\dot{P}=\left(\frac{\partial f(P,t)}{\partial P}\right)^{-1}\left(-\alpha f(P,t)-\frac{\partial f(P,t)}{\partial t}\right) \tag{12}$$

In the next subsection, coverage control law for the case of first order model is proposed.

B. The Proposed Coverage Control

Let us consider a first order model for each agent.

$$\dot{P}=u$$

where $u=[u_1,\ldots,u_n]$, under the control signal,

$$u=\left(\frac{\partial f(P,t)}{\partial P}\right)^{-1}\left(-\alpha f(\mathrm{P},t)-\frac{\partial f(P,t)}{\partial t}\right) \tag{13}$$

where $f(P,t)=(P-C_V(t))$, $C_V=[C_{V_1},\ldots,C_{V_n}]$ and the agents will be driven to the centers of their Voronoi tessellations.

It is worth noting that despite the introduced approaches in the literature, the proposed control given in (13) is a decentralized control strategy. To compute the center of Voronoi tessellation of each agent, C_{V_i}, each agent only needs local information due to the properties of the density function of each agent, ϕ_i.

Proof: The stability of the closed loop system, induced by (13) can be analyzed with the Lyapunov function

$$V=\frac{1}{2}f^2(P,t)$$

Taking the time derivative of the Lyapunov function, one can get

$$\frac{d}{dt}V=\frac{d}{dt}\left(\frac{1}{2}f^2(P,t)\right) \tag{14}$$

substituting (11) into (14), one has

$$\frac{d}{dt}V=f(P,t)\left(-\alpha f(P,t)\right)=-\alpha f^2(P,t)\le 0 \tag{15}$$

This implies that the time derivative of the Lyapunov function is non-increasing, i.e., $\dot{V}\le 0$. Taking in to account this assertion and that the Lyapunov function is lower-bounded, V approaches a limit. Furthermore, the time derivative is uniformly continuous and so by Barbalat's lemma $\lim_{t\to\infty}\dot{V}=0$. Thus $\lim_{t\to\infty}\alpha f^2(P,t)=0$, and we have

$$P\to C_V$$
■

Remark. In practice, calculation of Jacobian matrix, $J=\left(\frac{\partial f(P,t)}{\partial P}\right)$, can be expensive in space dimension higher than one and the Jacobian matrix in (13) may be singular. Thus, one can use Broyden's method to approximate Jacobian matrix [18]. In order to approximate the Jacobian matrix, one should discretize (13). Using Euler forward difference, the discrete model can be obtained as follows:

$$P^{k+1} = P^k - (J^k)^{-1}(P^k, t^k)\left(hf(\mathrm{P}^k, t^k) + \tau \frac{\partial f}{\partial t}(\mathrm{P}^k, t^k) \right)$$

where τ is step size and $h = \tau\alpha$. Let J^k be the Jacobian approximation at iteration k, and let $s^k = P^{k+1} - P^k$, then the updated Jacobian matrix approximation, J^{k+1}, can be written as follows

$$J^{k+1} = J^k + \frac{(y^k - J^k s^k)(s^k)^T}{\left\| s^k \right\|_2^2}$$

where $y^k = f(P^{k+1}, t^{k+1}) - f(P^k, t^k)$. One can use Sherman-Morrison Formula to update directly the inverse of the Jacobian matrix [17].

$$(J^{k+1})^{-1} = (J^k)^{-1} + \frac{\left(s^k - (J^k)^{-1} y^k\right)\left((s^k)^T (J^k)^{-1}\right)}{(s^k)^T (J^k)^{-1} s^k}$$

VI. SIMULATION RESULTS

In this section, we present the simulation results of the new control law for seven agents in two scenario of with and without of unknown moving obstacle. We also compare our control law operation with Lloyd algorithm in the presence of unknown moving obstacle.Let us choose the environment a square of $D = [0,20] \times [0,20]$.The initial location of our agents are randomly chosen over the environment. The unknown obstacle enters the environment and passes through it. Fig. 1 shows the initial position of seven agents. In time 8.2s and 14.2s, Fig. 2 and Fig. 3 show the location of agents, black squares, and their Voronoi cells with moving obstacle, respectively. We study the operation of algorithm in absence of obstacle. Fig. 4 and Fig. 5 show the locations and the Voronoi cells of agents in the absence of obstacle in time 8.2s and 14.2s respectively. Comparing the simulations with the case of unknown moving obstacle presence, one can easily find the differences in Voronoi cells and location of agents. It can be concluded that the changes in density function make our control law to have different operation in various cases. We also show the advantage of our algorithm respect to Lloyd method in the presence of unknown moving obstacle. Fig. 6 and Fig. 7 show the simulation results of our algorithm and Lloyd method. As it can be seen, in Lloyd method, agents are not able to avoid the obstacle and there is a collision between one of the agents and the obstacle in time 4.1s, but in our method the agent avoid obstacle and changes its path. Fig. 8 shows the operation of our algorithm in presence of two obstacles.

VII. CONCLUSION

In this paper, a coverage control was proposed to lead agents to cover the environment in the presence of unknown moving obstacles. We choose a proper density function which causes system to avoid collision with obstacles. The density function must be continuously differentiable. The proposed algorithm is decentralized and less computationally expensive comparing with previous related approaches. The simulation results show the effective performance of our method in avoiding unknown moving obstacle.

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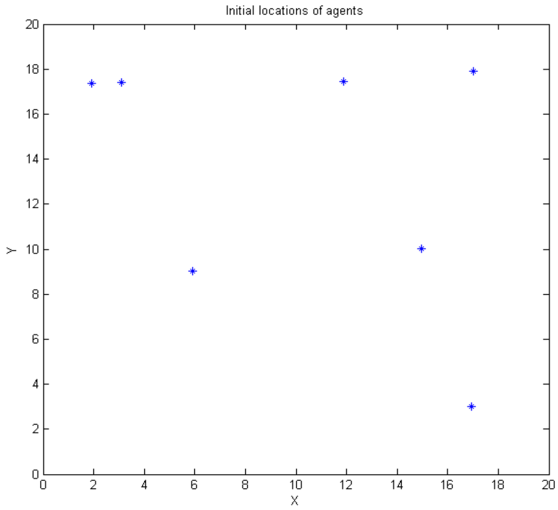

Fig. 1. The initial position of seven agents

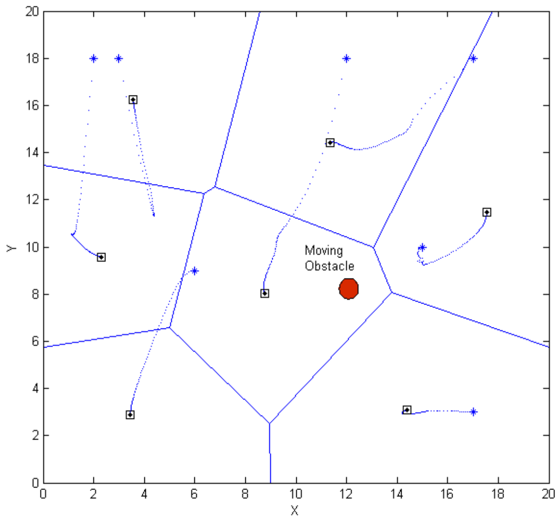

Fig. 2. The Locations and Voronoi cells of agents in presence of moving obstacle at 8.2s

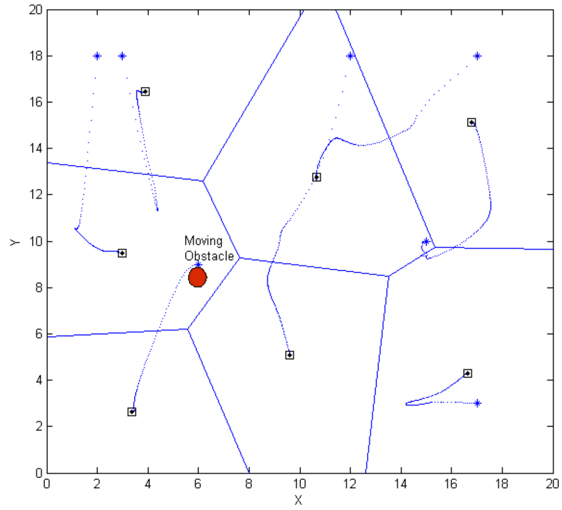

Fig. 3. The Locations and Voronoi cells of agents in presence of moving obstacles at 14.2s

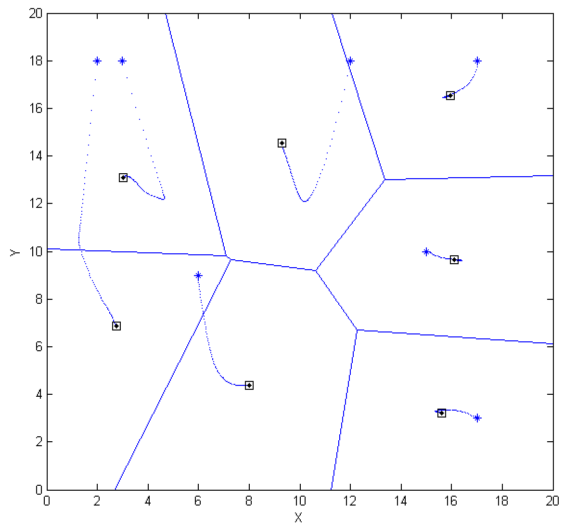

Fig. 4. The Locations and Voronoi cells of agents in absence of moving obstacle at 8.2s

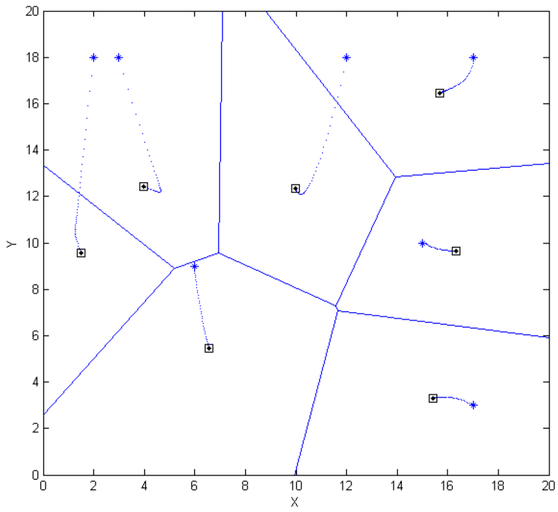

Fig. 5. The Locations and Voronoi cells of agents in absence of moving obstacle at 14.2s

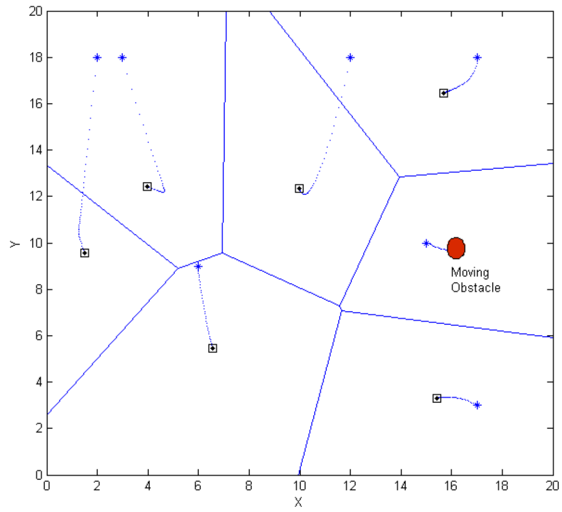

Fig. 7. The Locations and Voronoi cells of agents in Lloyd method and inability to avoid obstacle.

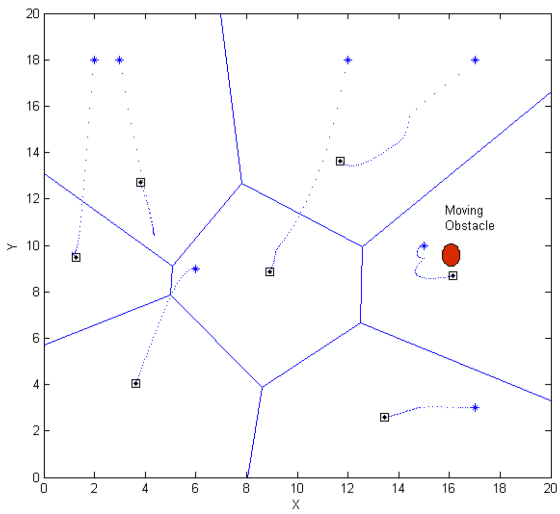

Fig. 6. The Locations and Voronoi cells of agents in our method.

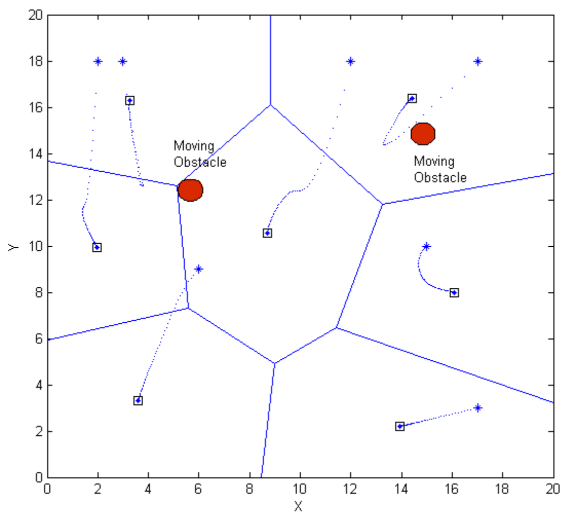

Fig. 8. Voronoi cells of agents in the presence of two moving obstacles