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Coverage Control Inspired by Bacterial Chemotaxis

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***Abstract*—This paper considers distributed control of multi-agent systems, inspired by *chemotaxis* of bacteria. The chemotaxis is the biological phenomenon that organisms sense the concentration of a chemical in an environment and move toward or away from the highest concentration point. The problem considered here is a *coverage* problem, *i.e.,* a problem of finding distributed controllers to steer agents so that they are placed uniformly on a given space. As a solution to this problem, we propose distributed controllers based on the mechanism of the chemotaxis. With the proposed controllers, each agent performs forward movement or random rotation depending on the difference between the current achievement degree of the coverage and the previous one. The performance of the proposed controllers is demonstrated by a numerical experiment. This result shows that multi-agent coordination in a distributed manner can be achieved by using the mechanism of the chemotaxis.**

***Keywords*-multi-agent systems; chemotaxis; coverage control;**

I. INTRODUCTION

Chemotaxis is the biological phenomenon that organisms sense the concentration of a chemical in an environment and move toward or away from the highest concentration point as illustrated in Fig. 1. More precisely, the organism moves toward the highest concentration point if the chemical is an attractant, and moves away from there if it is a repellent. There are many organisms exhibiting the chemotaxis, *e.g., Escherichia coli* [1], *Tetrahymena* [2], and *Paramecia* [3].

The chemotaxis of bacteria such as *E. coli* is generated as follows. Consider a bacterium in the concentration field of a chemical as shown in Fig. 1. If the current position is better than the previous one in the sense of the concentration, it goes straight; otherwise, it turns in a random direction. As the result, a biased random walk to a favorable position occurs, and the bacterium reaches there.

From the viewpoint of the control theory, the chemotaxis of bacteria can be interpreted as the process that a controller embedded in a bacterium drives the body according to the above algorithm. The controller is called here the *chemotaxis controller*. The chemotaxis controller is an attractive topic for the following reasons. First, the chemotaxis controller is simple enough to be implemented in biological devices. In fact, as seen above, it only commands either of two types of movements based on the concentration comparison at different locations. This means that the chemotaxis controller

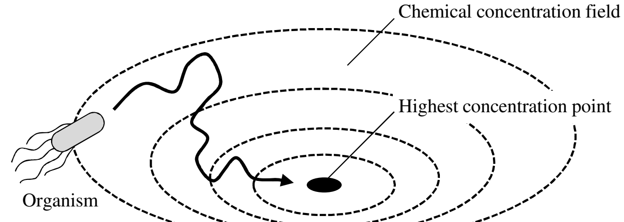


Figure 1. Chemotaxis.

has a great potential for biotechnological applications such as *molecular robotics*. Second, as shown in the fact that *E. coli* survive as a species for a long time, the chemotaxis controller is adaptive and robust against environmental changes. This is useful to make flexible and dependable systems.

Here, we are interested in applying the chemotaxis controller to distributed control of artificial multi-agent systems. In particular, this paper addresses the *coverage control* (see, *e.g.,* [4]). The coverage control is to steer mobile agents so that they are placed uniformly on a given space. This is a key technique for solving modern challenges such as cooperative robotics and mobile sensor networks. Applying the chemotaxis controller to the coverage control is motivated by two reasons. First, by introducing the mechanism of the chemotaxis, we will obtain simple coverage controllers. They are suitable for the above engineering applications where computational resource constraints are imposed for agents. Second, the resulting controllers have not only the engineering applications but also medical ones because they can be implemented in biological devices due to the simplicity. For example, pathogen detection and pinpoint dosing by molecular robots are considered. By directly sending medicines to pathogen, side effects are reduced compared with the diffusion of them in the body.

Thus, the purpose of this paper is to develop distributed coverage controllers inspired by the chemotaxis. To this end, we first formulate a coverage problem as a problem of finding distributed controllers steering agents so that global coverage performance is maximized. As a solution to this problem, we propose distributed controllers based on the chemotaxis controller, where each agent performs forward movement or random rotation depending on the difference between the current local performance of the coverage and the previous one. The performance of the proposed controllers is demonstrated by a numerical experiment. This result shows that multi-agent coordination in a distributed manner can be achieved by the chemotaxis controller.

As a final note of this section, we would like to stress the novelty of this paper. The idea of applying the chemotaxis controller to the coverage control has been originally proposed in [5]. To our best knowledge, the other related results (except for some variations) have never been obtained. The result [5] does not show that multi-agent coordination can be achieved by the chemotaxis controller. In fact, the controllers proposed in [5] are composed of a chemotaxis-based sub-controller and a flocking sub-controller, and the interaction among agents is performed by the flocking sub-controller. In this sense, our result is distinguished from the existing one. Furthermore, this paper does not just apply the chemotaxis controller to the coverage control. In fact, in such a case, the resulting controllers require the observation of the *global* coverage performance, and thus *distributed* controllers are not obtained. For solving this, we appropriately decompose the global performance index into local ones for each agent by utilizing a special property of the coverage problem, and introduce them to the controllers. This idea has never been seen in the existing studies on applications of the chemotaxis controller, *e.g.,* source seeking [6] and function optimization [7].

Notation: Let $\mathbf{R}$, $\mathbf{R}_{+}$, and $\mathbf{R}_{0+}$ be the real number field, the set of positive real numbers, and the set of nonnegative real numbers, respectively. The zero scalar/vector is denoted by 0. The Euclidian norm of the vector x is represented by $\|x\|$. We denote by $|\mathbf{S}|$ the cardinality of the set $\mathbf{S}$. We use $\mathbf{B}(c,r)$ to represent the closed disk of center c and radius r, *i.e.,* $\mathbf{B}(c,r) := \{x \in \mathbf{R}^2 \,|\, \|x-c\| \le r\}$. Finally, for the vectors $x_1, x_2, \ldots, x_n \in \mathbf{R}^2$ and the set $\mathbf{I} := \{i_1, i_2, \ldots, i_m\} \subseteq \{1,2,\ldots,n\}$, let $[x_i]_{i\in\mathbf{I}} := [x_{i_1}^\top \; x_{i_2}^\top \; \cdots \; x_{i_m}^\top]^\top \in \mathbf{R}^{2m}$. For example, $[x_i]_{i\in\mathbf{I}} = [x_1^\top \; x_3^\top \; x_4^\top]^\top$ for $x_1, x_2, \ldots, x_5$ and $\mathbf{I} = \{1,3,4\}$.

II. Problem Formulation

A. System Description

Consider the multi-agent system in Fig. 2, which is composed of n agents.

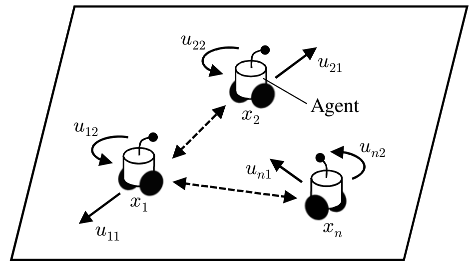


Figure 2. Multi-agent system.

Agent i ($i \in \{1,2,\ldots,n\}$) is described by the discrete-time model of the nonholonomic unicycle:

$$\begin{bmatrix} x_i(t+1) \\ \theta_i(t+1) \end{bmatrix} = \begin{bmatrix} x_i(t) \\ \theta_i(t) \end{bmatrix} + \begin{bmatrix} \cos(\theta_i(t)+u_{i2}(t))u_{i1}(t) \\ \sin(\theta_i(t)+u_{i2}(t))u_{i1}(t) \\ u_{i2}(t) \end{bmatrix} \quad (1)$$

where $x_i(t) \in \mathbf{R}^2$ and $\theta_i(t) \in \mathbf{R}$ are the translational and rotational positions, and $u_{i1}(t) \in \mathbf{R}$ and $u_{i2}(t) \in \mathbf{R}$ are the control inputs corresponding to the translational and rotational velocities. The collection of the control inputs is represented by $u_i(t) \in \mathbf{R}^2$, *i.e.,* $u_i(t) := [u_{i1}(t) \;\; u_{i2}(t)]^\top$. In addition, the collective position of the agents is denoted by $x(t) \in \mathbf{R}^{2n}$, *i.e.,* $x(t) := [x_1^\top(t) \; x_2^\top(t) \; \cdots \; x_n^\top(t)]^\top$.

For the multi-agent system in Fig. 2, we impose the following assumptions:

(A1) Each agent has the information on its own position in the world coordinate frame.

(A2) Each agent can obtain the information on the relative positions of the agents within radius r.

These assumptions are satisfied if each agent has, for instance, a GPS receiver and a stereo camera.

For agent i, we suppose that the local controller

$$L_i : \begin{cases} \xi_i(t+1) = g_1(\xi_i(t), [x_j(t)]_{j\in\mathbf{N}_i(t)}), \\ u_i(t) = g_2(\xi_i(t), [x_j(t)]_{j\in\mathbf{N}_i(t)}) \end{cases} \quad (2)$$

is embedded, where $\xi_i(t) \in \mathbf{R}^m$ is the state, $[x_j(t)]_{j\in\mathbf{N}_i(t)} \in \mathbf{R}^{2|\mathbf{N}_i(t)|}$ is the input, $u_i(t)$ is the output, and $g_1 : \mathbf{R}^m \times \mathbf{R}^{2|\mathbf{N}_i(t)|} \to \mathbf{R}^m$ and $g_2 : \mathbf{R}^m \times \mathbf{R}^{2|\mathbf{N}_i(t)|} \to \mathbf{R}^2$ are functions. The set $\mathbf{N}_i(t) \subseteq \{1,2,\ldots,n\}$ is the index set of the *neighbors*, that is,

$$\mathbf{N}_i(t) := \{j \in \{1,2,\ldots,n\} \,|\, x_j(t) \in \mathbf{B}(x_i(t), r)\}. \quad (3)$$

Note that $i \in \mathbf{N}_i(t)$ for every $t \in \{0,1,\ldots\}$ (agent i is a neighbor of itself). The functions g_1 and g_2 and the initial state $\xi_i(0)$ are assumed to be the same for all the agents. This implies that we deal with the agents in an indiscriminate manner, and as the result, the system becomes scalable. For simplicity of discussion, we further assume

$$\xi_i(0) = 0. \quad (4)$$

B. Coverage Problem

Next, a coverage problem is formulated. Let us introduce the performance index

$$J(x) := \int_{\mathbf{Q}\cap(\cup_{i=1}^n \mathbf{B}(x_i, r/2))} 1\, dq \quad (5)$$

where $\mathbf{Q} \subset \mathbf{R}^2$ is a environment to be covered by n agents. This performance index expresses the area of the union of the disks $\mathbf{B}(x_i, r/2)$ ($i = 1,2,\ldots,n$) in $\mathbf{Q}$. Therefore, x maximizing $J(x)$ is a configuration such that any agent is located at distance r from the others. Then, we consider the following problem.

Problem 1: For the multi-agent system in Fig. 2, suppose that the sensing range $r \in \mathbf{R}_+$ and a convex environment $\mathbf{Q} \subset \mathbf{R}^2$ are given. Find local controllers $L_1, L_2, \ldots, L_n$ (*i.e.,* find functions g_1 and g_2) such that

$$\lim_{t\to\infty} J(x(t)) = \max_{x\in\mathbf{R}^{2n}} J(x) \tag{6}$$

for every initial states $(x_1(0), \theta_1(0)), (x_2(0), \theta_2(0)), \ldots, (x_n(0), \theta_n(0)) \in \mathbf{Q} \times \mathbf{R}$.

Note that there is no trivial solution to Problem 1 though each agent knows its own position in the world coordinate frame as in (A1). In fact, the local controllers L_i $(i = 1, 2, \ldots, n)$ are assumed to be the same as described above, and thus it is impossible to give each desired position to the corresponding local controller in advance. This implies that each agent has to determine the desired position based on the information on the neighbors, which makes the problem challenging.

III. Coverage Controllers Inspired by Bacterial Chemotaxis

In this section, a solution to Problem 1 is given by using the mechanism of the chemotaxis of *E. coli*. We first explain the chemotaxis from the viewpoint of the control theory. Based on this, a solution to Problem 1 is presented.

A. Chemotaxis Control System

1) System Description: The chemotaxis control system of *E. coli* is shown in Fig. 3. This is composed of the body P, the controller K, and the chemical concentration field F.

The physical dynamics of the body P is described by (1) where $x_i(t)$, $\theta_i(t)$, $u_{i1}(t)$, and $u_{i2}(t)$ correspond to the translational and rotational positions and control inputs (which are the translational and rotational velocities) of *E. coli*. The positions and the collective control input are represented by $x_c(t) \in \mathbf{R}^2$, $\theta_c(t) \in \mathbf{R}$, and $u_c(t) \in \mathbf{R}^2$.

As a model considered in [8], the controller K is given by

$$K : \begin{cases} \xi_c(t+1) = y(t), \\ u_c(t) = \begin{cases} \begin{bmatrix} v \\ 0 \end{bmatrix} & \text{if } \xi_c(t) \le y(t), \\ \begin{bmatrix} v \\ \frac{2}{3}\pi \end{bmatrix} & \text{if } \xi_c(t) > y(t), w(t) = 1, \\ \begin{bmatrix} v \\ -\frac{2}{3}\pi \end{bmatrix} & \text{if } \xi_c(t) > y(t), w(t) = -1 \end{cases} \end{cases} \tag{7}$$

where $\xi_c(t) \in \mathbf{R}$ is the state, $y(t) \in \mathbf{R}_{0+}$ is the input, $u_c(t)$ is the output, $v \in \mathbf{R}_+$ is a constant number, and $w(t) \in \{-1, 1\}$ is a random number following the Bernoulli distribution with outcomes 1 and -1 and equal probabilities. The initial state is assumed to be zero, *i.e.,* $\xi_c(0) = 0$.

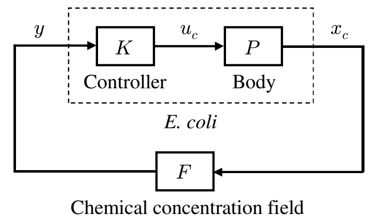


Figure 3. Chemotaxis control system of *E. coli*.

The chemical concentration field F is of the form

$$F : y(t) = f(\|x_c(t)\|) \tag{8}$$

where $x_c(t)$ is the input, $y(t)$ is the output, which corresponds to the concentration at the position $x_c(t)$, and $f : \mathbf{R}_{0+} \to \mathbf{R}_{0+}$ is a monotonically decreasing function. In this field, the concentration is highest at the origin and becomes lower as the distance from the origin increases.

In this system, the controller K works as follows. Since $\xi_c(t) = y(t-1)$ from (7), the concentration at the previous position is saved to $\xi_c(t)$. Thus, $\xi_c(t) \le y(t)$ means that the concentration at the current position is higher than or equal to that at the previous position. In this case, the controller K commands the straight movement as shown in Fig. 4 (a) because the resulting position might be a higher concentration point from the monotonically decreasing property of f. On the other hand, if $\xi_c(t) > y(t)$, then K commands the combination of the rotational and straight movements as shown in Fig. 4 (b) and (c). The reason is that the body might move away from high concentration points if it moves in the current direction. The rotation direction is randomly determined by $w(t)$.

2) Illustrative Example: We give an example to demonstrate the chemotaxis control system for $v := 0.3$ and

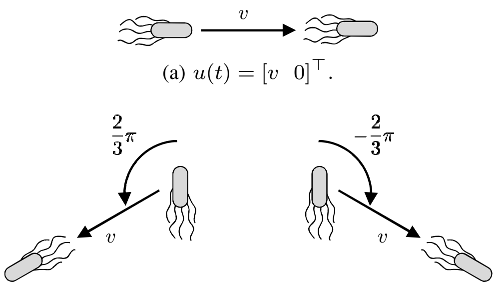
$$f(\|x_c\|) := \frac{1.25 \times 10^{-12}}{\pi} e^{-(2.5\times 10^{-5})\|x_c\|^2}. \tag{9}$$

Fig. 5 illustrates the trajectory of the body from the initial state $(x_c(0), \theta_c(0)) := ([-5 \;\; -6]^\top, (2/3)\pi)$, where x_{c1} and x_{c2} are the first and second elements of x_c, the ellipsoids express the positions, and the circles and the arcs represent the contour lines of $f(\|x_c\|)$. We see that the body moves to an area around the highest concentration point.

B. Proposed Controllers

Now, a solution to Problem 1 is given.

As seen above, the chemotaxis controller K drives the body to the highest concentration point. Namely, the body reaches the maximum point of the function f by K. This suggests us to replace $y(t)$ in (7) with $J(x(t))$ and use the resulting controller as the local controllers L_i $(i = 1, 2, \ldots, n)$ to achieve (6). However, since $J(x)$ depends on the positions



(a) $u(t) = [v \;\; 0]^\top$.

(b) $u(t) = [v \;\; \frac{2}{3}\pi]^\top$.

(c) $u(t) = [v \;\; -\frac{2}{3}\pi]^\top$.

Figure 4. Movements commanded by controller K.

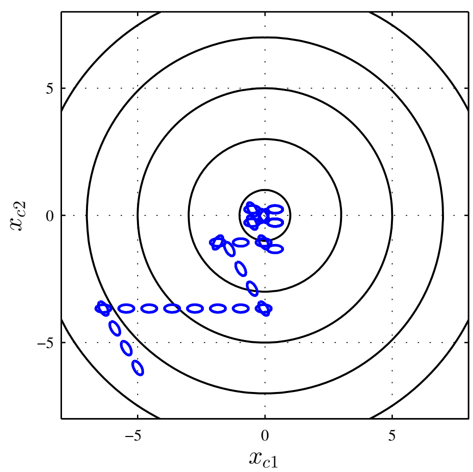


Figure 5. Example of trajectory of body in chemotaxis control system.

of all the agents, the resulting controller is not distributed. Hence, we decompose $J(x)$ into local performance indices which can be calculated from the positions of the neighbors.

According to this idea, we decompose $J(x)$ into

$$J_i([x_j]_{j\in\mathbf{N}_i}) := \int_{\mathbf{Q}\cap(\cup_{j\in\mathbf{N}_i}\mathbf{B}(x_j,r/2))} 1\,dq - \int_{\mathbf{Q}\cap(\cup_{j\in\mathbf{N}_i\setminus\{i\}}\mathbf{B}(x_j,r/2))} 1\,dq \quad (i=1,2,\ldots,n). \tag{10}$$

The local performance index $J_i([x_j]_{j\in\mathbf{N}_i})$ expresses the area of the region which is only covered by agent i.

Then, the following result is obtained.

Theorem 1: Consider the global performance index $J(x)$ in (5) and the local performance index $J_i([x_j]_{j\in\mathbf{N}_i})$ in (10). Then

$$J(x') - J(x) = J_i([x_j]'_{j\in\mathbf{N}_i}) - J_i([x_j]_{j\in\mathbf{N}_i}) \tag{11}$$

for every $i \in \{1,2,\ldots,n\}$, where $x' \in \mathbf{R}^{2n}$ and $[x_j]'_{j\in\mathbf{N}_i} \in \mathbf{R}^{2|\mathbf{N}_i|}$ are x and $[x_j]_{j\in\mathbf{N}_i}$ in which x_i changes to x_i'.

Proof: By the definition of $J_i([x_j]_{j\in\mathbf{N}_i})$, we have

$$J_i([x_j]_{j\in\mathbf{N}_i}) = \int_{\mathbf{Q}\cap(\cup_{j=i}^{n}\mathbf{B}(x_j,r/2))} 1\,dq - \int_{\mathbf{Q}\cap(\cup_{j\neq i}\mathbf{B}(x_j,r/2))} 1\,dq. \tag{12}$$

Therefore, it follows from (5) that

$$J(x) = J_i([x_j]_{j\in\mathbf{N}_i}) + \int_{\mathbf{Q}\cap(\cup_{j\neq i}\mathbf{B}(x_j,r/2))} 1\,dq. \tag{13}$$

This yields (11), which completes the proof. ■

Theorem 1 shows that $J(x)$ and $J_i([x_j]_{j\in\mathbf{N}_i})$ correspond to the potential function and the utility function in the potential games [9]. That is, when agent i unilaterally moves, the change in $J_i([x_j]_{j\in\mathbf{N}_i})$ is equal to that in $J(x)$. This implies that the coverage will be completed by maximizing each $J_i([x_j]_{j\in\mathbf{N}_i})$.

By using $J_i([x_j]_{j\in\mathbf{N}_i})$, our solution to Problem 1 is given as follows:

$$g_1(\xi_i(t), [x_j(t)]_{j\in\mathbf{N}_i(t)}) := J_i([x_j(t)]_{j\in\mathbf{N}_i(t)}), \tag{14}$$

$$g_2(\xi_i(t), [x_j(t)]_{j\in\mathbf{N}_i(t)}) := \begin{cases} \begin{bmatrix} v \\ 0 \end{bmatrix} & \text{if } \xi_i(t) \le J_i([x_j(t)]_{j\in\mathbf{N}_i(t)}), \\ \begin{bmatrix} v \\ \frac{2}{3}\pi \end{bmatrix} & \text{if } \xi_i(t) > J_i([x_j(t)]_{j\in\mathbf{N}_i(t)}),\; w_i(t) = 1, \\ \begin{bmatrix} v \\ -\frac{2}{3}\pi \end{bmatrix} & \text{if } \xi_i(t) > J_i([x_j(t)]_{j\in\mathbf{N}_i(t)}),\; w_i(t) = -1 \end{cases} \tag{15}$$

where $\xi_i(t)$ is assumed to be scalar, *i.e.*, $m := 1$, and $w_i(t) \in \{-1, 1\}$ is a random number following the Bernoulli distribution with outcomes 1 and -1 and equal probabilities.

The proposed controller L_i plays a similar role to that of the chemotaxis controller K. Since $\xi_i(t) = J_i([x_j(t-1)]_{j\in\mathbf{N}_i(t-1)})$ from (2) and (14), the value of the local performance index at the previous time step is saved to $\xi_i(t)$. So, $\xi_i(t) \le J_i([x_j(t)]_{j\in\mathbf{N}_i(t)})$ implies that the current local performance is higher or equal to the previous one. In this case, the proposed controller L_i moves agent i in the current direction. Meanwhile, if $\xi_i(t) > J_i([x_j(t)]_{j\in\mathbf{N}_i(t)})$, then L_i rotates agent i $(2/3)\pi$ radian and moves it forwardly. The direction of the rotation is randomly determined by $w_i(t)$.

IV. Numerical Experiment

Consider the multi-agent system in Fig. 2 with $n := 9$. The environment and the sensing range are given by $\mathbf{Q} := [0, 5]^2$ and $r := 1.75$. We use the local controllers L_i $(i = 1, 2, \ldots, 9)$ given by (2), (10), (14), and (15) with $v := 0.05$.

Fig. 6 illustrates the time series of the agent positions, where the small circles and the line segments represent the translational and rotational positions and the large circles

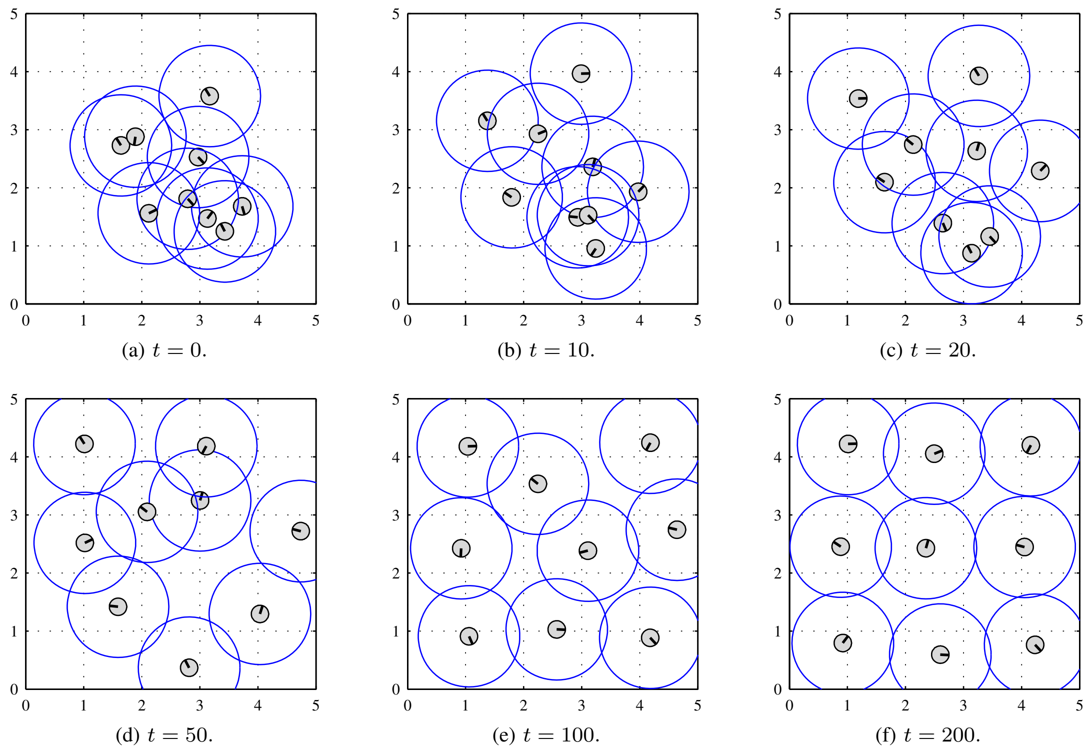


Figure 6. Time series of agent positions.

express $\mathbf{B}(x_i, r/2)$ $(i = 1, 2, \ldots, 9)$. Fig. 7, on the other hand, illustrates the time evolution of the global performance index $J(x(t))$. These show that the coverage is achieved by the proposed controllers.

In addition, Fig. 8 shows the time evolution of the local performance indices $J_i([x_j(t)]_{j \in \mathbf{N}_i(t)})$ $(i = 1, 2, \ldots, 9)$. It turns out that each agent moves so as to increase the local performance index for itself.

V. Conclusion

This paper has developed coverage controllers inspired by the bacterial chemotaxis. By decomposing a global performance index into local ones for each agent, we have obtained distributed controllers for the coverage. The performance of the proposed controllers has been evaluated by a numerical experiment. We hope that this result will be a foundation for multi-agent coordination by the chemotaxis controller.

Although a coverage problem has been addressed in this paper, other motion coordination tasks, *e.g.,* rendezvous [10], should also be considered in the future. In such a case, the method proposed in this paper will be useful.

Acknowledgments

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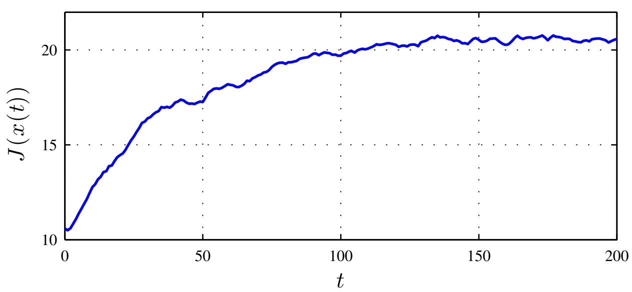

Figure 7. Time evolution of global performance index.

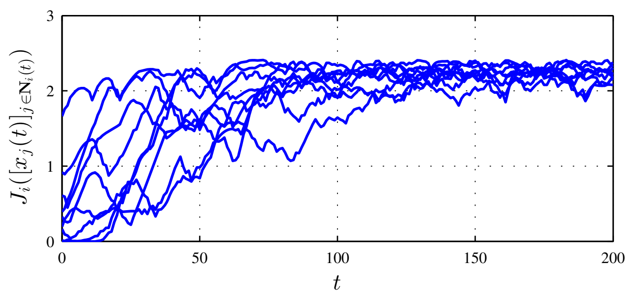

Figure 8. Time evolution of local performance indices.

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