Effective Coverage Control for Mobile Sensor Networks With Guaranteed Collision Avoidance

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***Abstract*—This paper studies the problem of dynamically covering a given region in the plane using a set of mobile sensor agents. A novel problem formulation is proposed that addresses a number of important multiagent missions. The coverage goal, which is to cover a given search domain using multiple mobile sensors such that each point is surveyed until a certain preset level is achieved, is formulated in a mathematically precise problem statement. A control law is developed that guarantees to meet the coverage goal. This control law is modified to guarantee that a partially connected fleet also attains the coverage goal. Finally, a collision avoidance component is added to the controller to guarantee that the agents do not collide. The new controller is shown to safely achieve coverage. Several numerical examples are provided to illustrate the main results.**

***Index Terms*—Collision avoidance, cooperative systems, coverage control, mobile sensor systems, multisensor systems, sensor networks.**

I. Introduction

RECENT natural disasters such as hurricane Katrina, the earthquake in the Indian subcontinent, and the tsunami in Southeastern Asia have indicated that a speedy and efficient humanitarian and search and rescue responses are crucial in saving human lives. Such humanitarian as well as numerous military operations often involve tasks in adversarial, highly dynamic environments that are hazardous to human operators. Hence, there is a pressing need to develop autonomous multiagent mobile systems that seek to collect and process data under constrained resources such as time restrictions to mission accomplishment, fuel limitations, and dynamic and/or limited communication structures. This paper attempts to address the basic goal of using a mobile sensor network to successfully collect data in an arbitrary environment under more ideal conditions. In particular, we address the problem of surveying a given arbitrarily-shaped mission space without the possibility of agents colliding with each other.

Other problems, however, only require the distribution of a fixed sensor network in the domain to be sampled. In these problems, the two variables of interest are sensor domains (the domain which each sensor is responsible of sampling) and sensor locations. The problem of optimizing sensor locations and sensor domains in fixed sensor networks has been extensively studied in the past and are considered problems in locational optimization [1], [2]. In such problems, the solution is a Voronoi partition [3], where the optimal sensor domain is a Voronoi cell in the partition and the optimal sensor location is a centroid of a Voronoi cell in the partition.

On the other hand, the literature on mobile sensor coverage networks is relatively new due to novel sensor and wireless technologies that only emerged recently. In [4], Cassandras and Li present a survey of the most recent activities in the control and design of both static and dynamic sensor networks. In their design criteria, they consider issues such as maximum coverage, detection of events and minimum communication energy expenditure. Of particular relevance in this paper is Section IV-B in [4], where the authors focus on coverage control missions. By means of illustrative examples, Ganguli *et al.* [5] discuss some challenges in modeling of robotic networks, motion coordination algorithms, sensing and estimation tasks, and complexity of distributed algorithms.

In [6], Li and Cassandras consider a probabilistic network model and a density function to represent the frequency of random events taking place over the mission space. The authors develop an optimization problem that aims at maximizing coverage using sensors with limited ranges, while minimizing communication cost. Starting with initial sensor positions, the authors develop a gradient algorithm to converge to a (local) solution to the optimization problem. The sequence of sensor distributions along the solution is seen as a discrete time trajectory of the mobile sensor network until it converges to the local minimum. A similar goal is the focus of the paper [7], where Zou and Chakrabarty utilize the notion of "virtual forces" to enhance the coverage of a sensor network given an arbitrary initial deployment, and the paper [8], where the goal is to maximize target exposure in surveillance problems. In [9], Cortés *et al.* address the same question, but instead of converging to a local solution of some optimization problem, the trajectory converges to the centroid of a cell in a Voronoi partition of the search domain. The authors propose stable algorithms, in both continuous and discrete time, that converge to the centroids. These algorithms are the dynamic version of the Lloyd algorithm [10], which iteratively achieves the optimal configuration. In [11], the authors use a Voronoi-based polygonal path approach and aim at minimizing exposure of a unmanned aerial vehicle (UAV) fleet to radar. Voronoi-based approaches, however, require exhaustive computational effort to compute the Voronoi cells continuously during a real-time

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implementation of the controllers. In the work presented herein, domain partitioning is not required and, hence, the computational complexity is reduced.

The fundamental difference between our approach and the approaches presented in the previously mentioned literature is in the definition of *mobility*. In the aforementioned research, researchers ask: Given an initial, random deployment of the sensor network, what is its final (optimal or suboptimal) configuration and how to control the network to get there? In other words, they address the redeployment problem to improve network performance, while assuming mobility of the network in achieving redeployment. In this paper, a different question is addressed: Given a sensor network and mission domain—the domain to be sampled—$\mathcal{D}$, develop closed-loop control strategies such that each point in $\mathcal{D}$ is sampled by some agents in the network by an amount of effective coverage equal to C^*? In the discrete setting, the goal may be understood as the collection of at least C^* measurements of a physical quantity at each point in $\mathcal{D}$ using a group of limited range sensors. In the continuous setting, the precise definition of "effective coverage" will be discussed in Section II. The aim is to actively sense the mission domain while the agents are moving in the mission space. This question has been studied in [12] for (optimal and suboptimal) motion planning of multiple spacecraft interferometric imaging systems (MSIIS). While MSIIS involves a slightly modified version of the coverage problem (in that a "sensor" corresponds to measurements made by a spacecraft *pair* and with inter-spacecraft relative positions and velocities as control variables instead of absolute positions and velocities), it is fundamentally a coverage control problem of the type addressed in this paper. In [13], Chakravorty proves that the coverage optimization problem (with fuel expenditure as cost) is computationally intractable by mapping it into a traveling salesman problem and, thus, it is necessary to resort to heuristics. Time optimality was proven to be tractable and solved in [13] for a four spacecraft MSIIS.

Methodologies developed in [12], however, inspire the present approach. In this paper, a convergent feedback control law is proposed that achieves the coverage goal. Note that the proposed methodology is specifically developed for scenarios where there is a uniform probability distribution to event occurrence and, hence, there is no preferred fixed sensor configuration that maximizes detection of an event, and each point in the search domain has to be covered. Examples where this occurs includes the following:

- human- or autonomous vehicle-based search and rescue missions, where each point in the search domain has to be equally surveyed;
- aerial wildfire control in inaccessible and rugged country, where each point in the wildfire region has to be equally "suppressed" using fixed-wing aircraft or helicopters;
- surveillance and aerial mapping;
- underwater sampling and oceanography;
- MSIIS, where each point in the domain (in this case the 2-D u–v frequency domain of an image, see [12] and [13]) has to be equally surveyed by the spacecraft fleet.

The main contribution of this paper is the development of a problem formulation that allows the study of coverage-type problems. We then develop gradient-type kinematic control strategies that ensure coverage of a given domain, including the possibility of a partially connected, bidirectional communication structure. After carefully studying this problem, we adapt the result to guarantee collision avoidance between the agents. We rely on avoidance feedback control strategies originally developed by Leitmann and Skowronski [14], [15]. In their work, the authors consider two agent systems, one of whom desires that no trajectory of the system, emanating from outside a given set, intersects the set, no matter what the admissible actions of the other agent are [14], [15]. For a given class of uncertain dynamical systems, Corless *et al.* [16] derive a class of combined memoryless and adaptive controllers which assure that the system state avoids a prescribed region. They also provide constructive sufficient conditions for the existence of these avoidance controllers. Later these results have been generalized for multiagent (more than two) systems in noncooperative [17] and cooperative settings [18].

This paper is organized as follows. In Section II, the effective coverage goal is formulated and the coverage error is developed. In Section III, a centralized cooperative coverage control strategy is formulated and is proven to drive the coverage error to zero. This is done for a fleet with a fully connected communication network in Section III-A and for a partially connected bidirectional communication structure in Section III-B. Based on the basic control law developed for fully connected networks, in Section IV we add a collision avoidance component that guarantees that no two agents get too close to each other. In Section V, numerical simulations are presented to verify the theory discussed in earlier sections. We conclude this paper with a summary of the main results and a discussion of future research directions in Section VII.

II. Problem Formulation

In this paper, an agent is denoted by $\mathcal{A}$. Let $\mathbb{R}^+ = \{a \in \mathbb{R} : a \geq 0\}$, $\mathsf{Q} = \mathbb{R}^2$ the configuration space of each agent and $\mathcal{D}$ be a compact subset of $\mathbb{R}^2$ which represents a region in $\mathbb{R}^2$ that the network is required to cover. Future research will consider other configuration spaces, in particular the group of planar rigid body motions with $\mathsf{Q} = \mathsf{SE}(2)$. Let the map $\phi : \mathcal{D} \to \mathbb{R}^+$, called a *distribution density function*, represent a measure of information or probability that some event takes place over $\mathcal{D}$. The function ϕ represents all prior information known by the fleet. A large value of ϕ indicates high likelihood of event detection and a smaller value indicates low likelihood. Let N be the number of agents in the fleet and let $\mathbf{q}_i \in \mathsf{Q}$ denote the position of agent $\mathcal{A}_i$, $i \in \mathcal{S} = \{1, 2, 3, \ldots, N\}$. Each agent $\mathcal{A}_i$, $i \in \mathcal{S}$, satisfies the following simple kinematic equations of motion:

$$\dot{\mathbf{q}}_i = \mathbf{u}_i, \quad i \in \mathcal{S} \tag{1}$$

where $\mathbf{u}_i \in \mathbb{R}^2$ is the control velocity of agent $\mathcal{A}_i$.

Define the *instantaneous coverage function* $\bar{A}_i : \mathcal{D} \times \mathsf{Q} \to \mathbb{R}^+$ as a $\mathcal{C}^1$-continuous map that describes how effective an agent $\mathcal{A}_i$ senses a point $\tilde{\mathbf{q}} \in \mathcal{D}$. Without loss of generality, we consider the following sensor model **SM**. Examples of sensors that may be modeled by this mathematical formulation include vision-based (for aerial mapping) and infrared (for temperature distribution measurements) cameras, and radar. We emphasize, however, that *this model is not an assumption for the ensuing theoretical results to be valid*. In the following sections, via straightforward algebraic modifications, similar results can easily be obtained for other sensor models. The important feature of any sensor model allowed by the present paper is the fact that the sensors can have a finite sensor range, which is a practical property of a sensor in real applications. Most of the literature discussed in Section I assumes infinite-range sensors.

Sensor Model SM: In this paper, we consider sensors with the following properties.

SM1: Each agent has a peak *sensing capacity* of M_i exactly at the position $\mathbf{q}_i$ of agent $\mathcal{A}_i$. That is, we have

$$\bar{A}_i(\mathbf{q}_i, \mathbf{q}_i) = M_i > \bar{A}_i(\tilde{\mathbf{q}}, \mathbf{q}_i), \quad \forall \tilde{\mathbf{q}} \neq \mathbf{q}_i.$$

SM2: Each agent's sensor has a circular sensing symmetry about the position $\mathbf{q}_i$, $i \in \mathcal{S}$, in the sense that all points in $\mathcal{D}$ that are on the same circle centered at $\mathbf{q}_i$ are sensed with the same intensity. That is, $\bar{A}_i(\tilde{\mathbf{q}}, \mathbf{q}_i)$ is a constant for all $\tilde{\mathbf{q}} \in \mathcal{D}$ such that $\|\mathbf{q}_i - \tilde{\mathbf{q}}\| = c$ for all constant c, $0 \leq c \leq r_i$, where r_i is the range of the sensor of agent $\mathcal{A}_i$. Hence, we introduce the new function $A_i : \mathbb{R}^+ \to \mathbb{R}^+$ such that $\bar{A}_i(\tilde{\mathbf{q}}, \mathbf{q}_i) = A_i(\|\mathbf{q}_i - \tilde{\mathbf{q}}\|^2)$.

SM3: Each agent has a limited *sensory domain* $\mathcal{W}_i(t)$ with a *sensory range* r_i. The sensory domain of each agent is given by

$$\mathcal{W}_i(t) = \{\tilde{\mathbf{q}} \in \mathcal{D} : \|\mathbf{q}_i(t) - \tilde{\mathbf{q}}\| \leq r_i\}. \tag{2}$$

Mathematically, under assumption **SM2**, this requires that $A_i(\|\mathbf{q}_i(t) - \tilde{\mathbf{q}}\|^2) = 0, \forall \tilde{\mathbf{q}} \in \mathcal{D} \setminus \mathcal{W}_i(t) = \{\bar{\tilde{\mathbf{q}}} : \|\mathbf{q}_i(t) - \bar{\tilde{\mathbf{q}}}\| > r_i\}$Let the union of all coverage regions be denoted by $\mathcal{W}(t) = \cup_{i \in \mathcal{S}} \mathcal{W}_i(t)$.

An example of such a sensor function is a second-order polynomial function of $s = \|\mathbf{q}_i - \tilde{\mathbf{q}}\|^2$ within the sensor range and zero otherwise. In particular, consider the function

$$A_i(s) = \begin{cases} \frac{M_i}{r_i^4}(s - r_i^2)^2, & \text{if } s \leq r_i \\ 0, & \text{if } s > r_i. \end{cases} \tag{3}$$

All simulations conducted in this paper employ the coverage function given by $A_i(\|\mathbf{q}_i - \tilde{\mathbf{q}}\|^2)$, with A_i as given in (3). One can check that this sensor coverage function satisfies the model properties **SM1–SM3**. An example for the instantaneous coverage function (3) is given in Fig. 1.

Fixing a point $\tilde{\mathbf{q}}$, the *effective coverage* achieved by an agent $\mathcal{A}_i$ surveying $\tilde{\mathbf{q}}$ from the initial time $t_0 = 0$ to time t is *defined* to be

$$\mathcal{T}_i(\tilde{\mathbf{q}}, t) := \int_0^t A_i(\|\mathbf{q}_i(\tau) - \tilde{\mathbf{q}}\|^2) \mathrm{d}\tau$$

and the effective coverage by a subset of agents $\mathcal{A}_\mathcal{K} = \{\mathcal{A}_j \mid j \in \mathcal{K} \subseteq \mathcal{S}\}$ in surveying $\tilde{\mathbf{q}}$ is then given by

$$\mathcal{T}_\mathcal{K}(\tilde{\mathbf{q}}, t) := \int_0^t \sum_{i \in \mathcal{K}} A_i(\|\mathbf{q}_i(\tau) - \tilde{\mathbf{q}}\|^2) \mathrm{d}\tau.$$

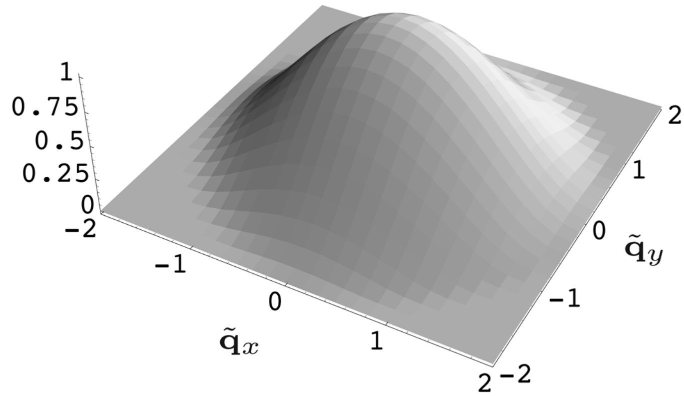


Fig. 1. Instantaneous coverage function A_i with $\mathbf{q}_i = 0$, $M_i = 1$, and $r_i = 2$.

It can easily be checked that $\mathcal{T}_\mathcal{K}(\tilde{\mathbf{q}}, \mathcal{K}, t)$ is a nondecreasing function of time t. In fact, note that

$$\frac{\partial}{\partial t} \mathcal{T}_\mathcal{K}(\tilde{\mathbf{q}}, t) = \sum_{i \in \mathcal{K}} A_i(\|\mathbf{q}_i - \tilde{\mathbf{q}}\|^2) \geq 0.$$

Let C^* be the *desired attained effective coverage* at all points $\tilde{\mathbf{q}} \in \mathcal{D}$. The goal is to attain a *network* coverage of $\mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t) = C^*$, for all $\tilde{\mathbf{q}} \in \mathcal{D}$ at some time t. The quantity C^* guarantees that, when $\mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t) = C^*$, one can judge, with some level of confidence, whether an event happened at $\tilde{\mathbf{q}} \in \mathcal{D}$ or not. Consider the following *error function:*

$$e(t) = \int_\mathcal{D} h(C^* - \mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t)) \phi(\tilde{\mathbf{q}}) \mathrm{d}\tilde{\mathbf{q}} \tag{4}$$

where $h(x)$ is a *penalty function* that is positive definite, twice differentiable, strictly convex on $(0, C^*]$ and that satisfies $h(x) = h'(x) = h''(x) = 0$, for all $x \leq 0$. Positivity and strict convexity in our case mean that $h(x), h'(x), h''(x) > 0$, for all $x \in (0, C^*]$. The penalty function penalizes lack of coverage of points in $\mathcal{D}$. It incurs a penalty whenever $\mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t) < C^*$. Once $\mathcal{T}_\mathcal{S}(\tilde{\mathbf{q}}, t) \geq C^*$ at a point in $\mathcal{D}$, the error at this point is zero no matter how much additional time agents spend surveying that point. The extra time spent there is beneficial, since it results in increasing the probability of event detection, and is hence not penalized. As will be seen, accumulated error will generate an attractive "force" on an agent, while excessive coverage has no effect on the motion. An example for the function $h(x)$ is

$$h(x) = (\max(0, x))^n \tag{5}$$

where $n > 1, n \in \mathbb{R}^+$. The total error is an average over the entire domain $\mathcal{D}$ weighted by the density function $\phi(\tilde{\mathbf{q}})$. When $e(t) = 0$, one says that the *mission is accomplished*.

The error function given in (4) is inspired by a formulation for coverage problems in motion planning of MSIIS. For more on coverage for MSIIS, see [12]. For some special geometries for $\mathcal{D}$, such as rectangular and circular ones, one may attempt to redeploy the agents at particular starting points and systematically sweep the domain. For example, a single agent may spiral from the center of a circular domain until it reaches the boundary, as in [19]. However, for arbitrary geometries, even if convex, such solutions may not be obvious or even exist. Even for circular or rectangular geometries, an application, such as in time-sensitive operations, may not afford a redeployment from an initial arbitrary configuration to an optimal one. In this case, the agents have to immediately respond to a certain application need without redeployment. This is the main reason we pursue the control strategies developed in the rest of this paper.

Remark: In the case when information is inferred dynamically during the surveillance process, the function $\phi : \mathcal{D} \to \mathbb{R}^+$ can be replaced with a time dependent function $\phi_d : \mathcal{D} \times \mathbb{R}^+ \to \mathbb{R}^+$. The map ϕ_d may be viewed as an explicit function of time or, more realistically, as an implicit function of time by having to satisfy some nonautonomous dynamical equation

$$\frac{\partial}{\partial t}\phi_d(\tilde{\mathbf{q}}, t) = f(\tilde{\mathbf{q}}, t).$$

For example, in search and rescue missions, a missing person is often times nonstatic—him/herself trying to find rescuers. The function $f(\tilde{\mathbf{q}}, t)$ will then represent a worst-case scenario of the motion of the missing person, assuming some knowledge of his/her motion such as maximum speed. Another example is in the search of a flight data recorder after an aircraft disaster. With imprecise prior knowledge about the location of the data flight recorder, the search and retrieval operation is initialized with a uniform value for ϕ_d. That is, $\phi_d(\tilde{\mathbf{q}}, 0) = \alpha$ (α being some constant). As the search operation proceeds, indicators, such as the discovery of debris, will modify ϕ_d such that, for example, it has a higher value in the neighborhood of the indicator. The question of dynamic targets will be treated in a future publication.

III. Centralized Control Laws for Dynamic Coverage

In this section, the error function in (4) is used to derive control laws that guarantee, under appropriate assumptions, coverage of the entire domain $\mathcal{D}$ with an effective coverage of C^*. Let $\hat{\mathcal{S}}_i \subseteq \mathcal{S}$ be the set of indices representing all agents in the fleet with which agent $\mathcal{A}_i$ can *mutually* communicate measurement and state histories. Clearly, $i \in \hat{\mathcal{S}}_i$ holds. If $\hat{\mathcal{S}}_i = \mathcal{S}$, the multiagent network is said to be *fully connected*. On the other hand, if $\hat{\mathcal{S}}_i \subset \mathcal{S}$ (strictly), the multiagent network is said to be *partially-connected*. Since bidirectional communication is assumed, then we have

$$j \in \hat{\mathcal{S}}_i \Leftrightarrow i \in \hat{\mathcal{S}}_j. \tag{6}$$

In the next paragraphs, we first consider a control law for the fully connected case. Inspired by this control law, we propose a similar strategy for the partially connected case.

We will make, without any loss of generality, the following natural assumption, whose utility will become obvious later.

- **IC1:** The initial coverage is identically zero

$$\mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, 0) = 0, \ \forall \tilde{\mathbf{q}} \in \mathcal{D}.$$

A. Fully Connected Fleets

Let us first consider the following control law:

$$\bar{\mathbf{u}}_i(t) = -\bar{k}_i \int_{\mathcal{D}} h'(C^* - \mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t)) \times \left.\frac{\partial A_i(s)}{\partial s}\right|_{s=\|\mathbf{q}_i(t)-\tilde{\mathbf{q}}\|^2} (\mathbf{q}_i(t) - \tilde{\mathbf{q}})\phi(\tilde{\mathbf{q}})\mathrm{d}\tilde{\mathbf{q}} \tag{7}$$

where $\cdot$ denotes the inner product and $\bar{k}_i > 0$ are fixed feedback gains. Consider the function $\bar{V} = -e_t(t)$, where $e_t = (de)/(dt)$, and note that $\dot{\bar{V}} = -e_{tt}$, where

$$e_t(t) = -\int_{\mathcal{D}} h'(C^* - \mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t)) \times \left(\sum_{j\in\mathcal{S}} A_j(\|\mathbf{q}_j(t) - \tilde{\mathbf{q}}\|^2)\right) \phi(\tilde{\mathbf{q}})\mathrm{d}\tilde{\mathbf{q}}$$

$$e_{tt} = \int_{\mathcal{D}} h''(C^* - \mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t)) \times \left(\sum_{i\in\mathcal{S}} A_i(\|\mathbf{q}_i(t) - \tilde{\mathbf{q}}\|^2)\right)^2 \phi(\tilde{\mathbf{q}})\mathrm{d}\tilde{\mathbf{q}} + 2\sum_{i\in\mathcal{S}} \bar{k}_i \left[\int_{\mathcal{D}} h'(C^* - \mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t)) \times \left(\left.\frac{\partial A_i}{\partial s}\right|_{s=\|\mathbf{q}_i(t)-\tilde{\mathbf{q}}\|^2} (\mathbf{q}_i(t) - \tilde{\mathbf{q}})\right) \times \phi(\tilde{\mathbf{q}})\mathrm{d}\tilde{\mathbf{q}}\right]^2$$

are the first and second time derivatives of $e(t)$ along the trajectory generated by the control law $\bar{\mathbf{u}}_i$ in (7). Note that in the second equality in the e_{tt} expression, $\bar{\mathbf{u}}_i$ is pulled outside of the integral since $\bar{\mathbf{u}}_i$ is independent of $\tilde{\mathbf{q}}$. We also employed the fact that the integral over the sum is equal to the sum of the integrals. Consider the following condition.

Condition C1: $\mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t) = C^*, \forall \tilde{\mathbf{q}} \in \mathcal{W}_i(t), \forall i \in \mathcal{S}$.

Lemma III.1: If for some $t \geq 0$ Condition **C1** holds, then $e_t(t) = 0$. Conversely, if $e_t(t) = 0$ for some time $t \geq 0$, then Condition **C1** holds.

Proof: By the property **SM3**, Condition **C1** implies that the h' term in the integrand in the expression for e_t is nonzero *possibly* only outside $\mathcal{W} = \cup_{i\in\mathcal{S}}\mathcal{W}_i$ where all coverage functions A_i are zero. Condition **C1**, by construction of the penalty function h, implies that

$$h'(C^* - \mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t)) = 0$$

precisely inside $\mathcal{W}(t)$. Hence, under Condition **C1** $e_t = 0$.

The converse is easily verified by noting that the integrand in the expression for e_t is greater than or equal to zero everywhere in $\mathcal{D}$. Hence the integral over $\mathcal{D}$ has to be greater than or equal to zero. For e_t to be zero, the integrand has to be identically equal to zero everywhere on $\mathcal{D}$, which holds true only if Condition **C1** holds. This completes the proof. ■

From the lemma, $\bar{V} = 0$ if and only if Condition **C1** holds since $\bar{V} = -e_t$. Clearly, e_t is negative semi-definite since the integrand in e_t is positive semi-definite. Hence, $\bar{V} = -e_t \geq 0$ with equality holding if and only if Condition **C1** holds.

Next, consider the time derivative of $\bar{V}$, which is given by $\dot{\bar{V}} = -e_{tt}$. Under the control law (7) and the previous expression for e_{tt}, note, by similar arguments as those given before, that $\dot{\bar{V}} \leq 0$ with equality holding if and only if Condition **C1** holds. The proof of this statement also relies on the fact that h is a strictly convex function on $(0, C^*]$ and that $h''(0) = 0$, which is true by construction of h. This implies that the function $\bar{V}$ is a Lyapunov-type function that guarantees that the system always

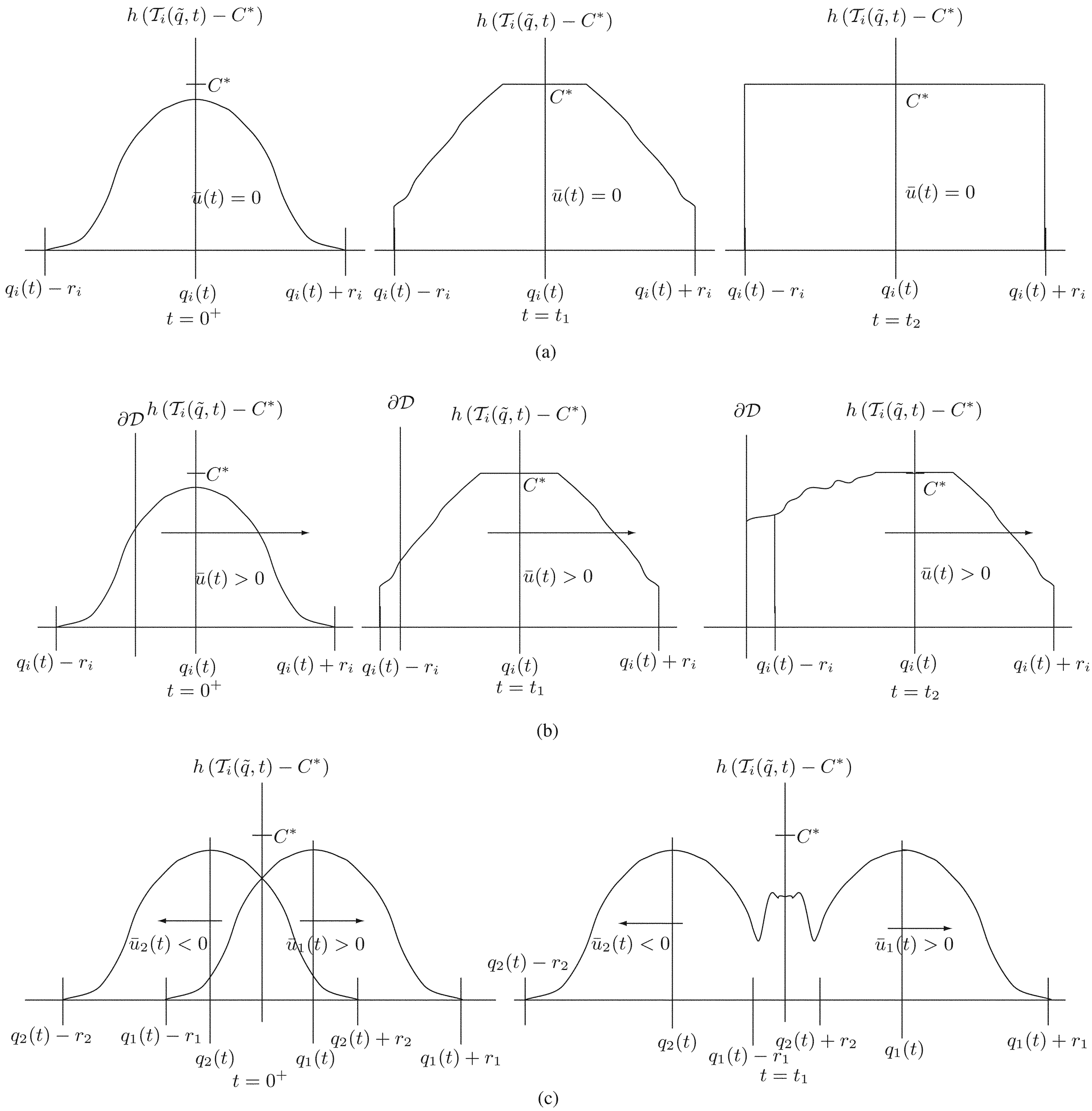


Fig. 2. 1-D illustration of the zero control strategy and the response of the system to the control law $\bar{\mathbf{u}}_i$ to Condition **IC2**. (a) Behavior of the zero control strategy. (b) Initial agent configuration that satisfies Condition **IC2** through interaction with the boundary. (c) Initial agent configuration that satisfies Condition **IC2** through interaction with other agents. Distribution for two neighboring agents is shown.

converges to the state described in Condition **C1**. This proves the following result.

Lemma III.2: Under the control law (7), a fully connected multiagent sensor network will converge to the state described in Condition **C1**.

By construction of the function h and similar arguments as before, under the control law given in (7), $e_t < 0$ away from the state described by Condition **C1**. We then have the following lemma.

Lemma III.3: The control law $\bar{\mathbf{u}}_i$ in (7) guarantees that $e_t < 0$ away from the state described by Condition **C1**. At the state **C1**, the control law $\bar{\mathbf{u}}_i$ in (7) is identically zero.

This Lemma tells us that the control law $\bar{\mathbf{u}}_i$ in (7) causes the error to decrease as long as the Condition **C1** does not hold.

Note that in the worst case performance with unactuated agents, the agents' positions are fixed. Say the control is set to zero for all $t \geq t_1$ for some $t_1 \geq 0$. The error inside all agents' sensory domains, $\mathcal{W}(t)$, keeps decreasing for all $t \geq t_1$ until we have

$$\mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t) \geq C^*, \forall \tilde{\mathbf{q}} \in \mathcal{W}_i(t) = \mathcal{W}_i(t_1).$$

At that point, the term $\sum_{i\in\mathcal{S}} A_i(\|\mathbf{q}_i(t)-\tilde{\mathbf{q}}\|^2)$ is not zero for all $\tilde{\mathbf{q}} \in \mathcal{D}$ exactly where $\mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t) \geq C^*$ and is zero exactly where $\mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t) < C^*$, for all $\tilde{\mathbf{q}} \in \mathcal{D}$. Hence $e_t = 0$ while $e \neq 0$. From this, note that the zero control, which sets the second term in the equation for e_{tt} equal to zero, is guaranteed to converge to the state described in Condition **C1**. The behavior of the zero control law is illustrated in Fig. 2(a). *This is not necessarily true for the control law* $\bar{\mathbf{u}}_i$ in (7). An example where the control law (7) drives the error to zero without satisfying the Condition **C1** is given in Section V.

Under a dynamic nonzero control, such as that described in (7), the agents are in constant motion with $e_t < 0$ (i.e., error is always decreasing) as long as the Condition **C1** is not satisfied. In fact, this control law applied to agent $\mathcal{A}_i$ utilizes the gradient of the error distribution inside $\mathcal{W}_i(t)$ to move in directions with maximum error. Hence, it locally seeks to maximize coverage. The gradient nature of (7) is what motivates it in this work. This choice of controller is obviously better than the zero control strategy, though it does not guarantee effective coverage of at least C^* where an agent has been. This is of no concern, since this lack of full effective coverage implies that $e \neq 0$, which will induce some agent, as discussed later in our control strategy, to return and recover these partially covered regions. Optimal controllers will be the focus of future research.

We now study the behavior of the control law (7). First consider the following class of initial agent locations.

IC2: All agents are initialized such that the following hold:
1) the interior of each agent's sensory domain intersects with the boundary of $\mathcal{D}$, $\partial\mathcal{D}$; i.e., $\mathcal{W}(0) \cap \partial\mathcal{D} \neq \emptyset$;
2) *or* the interior of the sensory domains of *at least two agents* interfere (not allowing exact overlapping)

$$\|\mathbf{q}_i(0) - \mathbf{q}_j(0)\| < r_i + r_j$$

for some $i, j \in \mathcal{S}$.

This class of initial conditions includes natural cases where a fleet begins a mission outside the domain $\mathcal{D}$ or when the fleet deploys from a flocking configuration. Note that if **IC2** is violated, then the system with control law (7) is guaranteed to converge to the state of Condition **C1**. To see why this is true, note that the gradient of the coverage function $(\partial A_i)/(\partial s)$ is an odd function about the agent location $\mathbf{q}_i$ (that is, $(\partial A_i)/(\partial s)|_{s=s_1} = -(\partial A_i)(\partial s)|_{s=-s_1}$). If $\mathcal{W}$ were completely inside $\mathcal{D}$ (i.e., $\mathcal{W} \cap \partial\mathcal{D} = \emptyset$) or no two agents intersect (i.e., $\mathcal{W}_i \cap \mathcal{W}_j = \emptyset$, for all $i \neq j \in \mathcal{S}$) then, under assumption **IC1**, the integral in (7) is zero since the integrand is an odd function that is being integrated over a symmetric domain $\mathcal{W}_i$ (Property **SM2**). Since the integral of an odd function over a symmetric domain is zero, then the control (7) is zero. Later in the paper, a remedy is proposed using switching to perturb the system away from Condition **C1** whenever the controller is zero.

On the other hand, satisfying the class of initial Conditions **IC2**, the control law (7) is guaranteed to be nonzero. Fig. 2(b) is for the initial condition where an agent starts with its sensory domain intersecting the boundary of $\mathcal{D}$. Fig. 2(c) is for the initial condition where two agents start with an overlapping sensory domains (but not at the same location). In either these cases, the integration over $\mathcal{D}$ is performed over nonsymmetric subsets of each $\mathcal{W}_i$. Moreover, the integrand in (7) is generally not odd and, therefore, $\bar{\mathbf{u}}_i \neq 0$ as shown in Fig. 2(b) and (c). Each agent, then, is set to motion and will retain breaking the symmetry of the error distribution for all future time until, at a later time (not shown in the figure), it satisfies the Condition **C1** again. Hence, by Lemma III.3, the control law guarantees that $e_t < 0$.

Lemma III.2 is important if, in addition to Condition **C1**, the error $e(t) = 0$. At that point the goal of achieving a minimum effective coverage of C^* is attained everywhere in $\mathcal{D}$. However, the previous result does not guarantee full coverage of $\mathcal{D}$ using the control law (7) alone. Hence, we propose the following control strategy.

Control Strategy: Under the control law (7), all agents in the system are in continuous motion (see previous remarks) as long as the state described in Condition **C1** is avoided. Whenever Condition **C1** holds with nonzero error $e(t) \neq 0$, the system has to be perturbed by switching to some other control law $\bar{\bar{\mathbf{u}}}_i$ that ensures violating Condition **C1**. Once away from condition **C1**, the controller is switched back to the nominal control $\bar{\mathbf{u}}_i$ in (7). Only when both **C1** and $e(t) = 0$ are satisfied is when there is no need to switch to $\bar{\bar{\mathbf{u}}}_i$. Thus, the goal is to propose a simple linear feedback controller that guarantees driving the system away from Condition **C1**. We will often refer to any such control law by *symmetry-breaking* controller since it attempts to move the agents to positions where the oddness of the coverage functions inside all $\mathcal{W}_i$ is violated and the control law $\bar{\mathbf{u}}_i$ is no longer zero.

We now consider a simple symmetry-breaking control law. In the remainder of the paper, we may also refer to this control law as a perturbation since it aims at perturbing the system away from Condition **C1**. The controller presented here is analogous to that found in [9], but is designed based on the performance error function $e(t)$ in (4). Define the time varying set

$$\mathcal{D}_e(t) = \{\tilde{\mathbf{q}} \in \mathcal{D} : T_{\mathcal{S}}(\tilde{\mathbf{q}}, t) < C^*\}. \tag{8}$$

Let $\overline{\mathcal{D}}_e(t)$ be the closure of $\mathcal{D}_e(t)$. For each agent $\mathcal{A}_i$, let $\widetilde{\mathcal{D}}_e^i(t)$ denote the set of points in $\overline{\mathcal{D}}_e(t)$ that minimize the distance between $\mathbf{q}_i(t)$ and $\bar{\mathcal{D}}_e(t)$. That is, we have

$$\widetilde{\mathcal{D}}_e^i(t) = \{\bar{\tilde{\mathbf{q}}} \in \overline{\mathcal{D}}_e(t) : \bar{\tilde{\mathbf{q}}} = \mathrm{argmin}_{\tilde{\mathbf{q}} \in \bar{\mathcal{D}}_e(t)} \|\mathbf{q}_i(t) - \tilde{\mathbf{q}}\|\}.$$

Note that this is just one particular choice and other choices for the set $\widetilde{\mathcal{D}}_e^i(t)$ are possible. For example, another trivial choice is to set $\widetilde{\mathcal{D}}_e^i(t) = \overline{\mathcal{D}}_e(t)$. The previous choice is efficient since the perturbation maneuver seeks the minimum-distance for redeployment.

Let t_s be the time at which Condition **C1** holds and $e(t_s) > 0$, while $e_t \neq 0$, for all $t < t_s$. That is, t_s is the time of entry into the state described in Condition **C1** with nonzero error. At t_s, for each agent $\mathcal{A}_i$, consider a point $\tilde{\mathbf{q}}_i^*(t_s) \in \widetilde{\mathcal{D}}_e^i(t_s)$. Note that the set $\widetilde{\mathcal{D}}_e^i(t_s)$ may include more than a single point. An example of this is when the condition on initial positions **IC2** is violated. In that case, one may assign a fixed rule for picking up a point $\tilde{\mathbf{q}}_i^*(t_s)$ in $\widetilde{\mathcal{D}}_e^i(t_s)$. This choice is immaterial in the present discussion and we will assume there is a single such point. Call the rule that picks a unique $\tilde{\mathbf{q}}_i^*$ for each agent $\mathcal{A}_i$ Rule **R1**.

Consider the control law

$$\bar{\bar{\mathbf{u}}}_i(t) = -\bar{\bar{k}}_i(\mathbf{q}_i(t) - \tilde{\mathbf{q}}_i^*(t_s)). \tag{9}$$

We claim that $\tilde{\mathbf{q}}_i^*(t_s)$ is fixed for some time interval $t_s < t < \hat{t}_s$. This claim will be justified shortly. Under the regime when $e_t = 0$ and $e(t) > 0$, this control law is a simple linear feedback controller and will drive each agent in the fleet towards its associated $\tilde{\mathbf{q}}_i^*(t_s)$. Note that it is possible that $\tilde{\mathbf{q}}_i^*(t_s) = \tilde{\mathbf{q}}_j^*(t_s)$

for some pair $i \neq j \in \mathcal{S}$. By simple linear systems theory, the feedback control law (9) will result in having $\mathbf{q}_i(\hat{t}_s)$, for some $i \in \mathcal{S}$, be inside a ball of radius $\varepsilon < r_i$ at some time $\hat{t}_s > t_s$. Hence, the point $\mathbf{q}_i^*$ is guaranteed to lie *strictly* inside the sensory range of agent $\mathcal{A}_i$.

We now justify that the point $\tilde{\mathbf{q}}_i^*$ by construction does not move as a function of time. Let $\hat{t}_s$ be the first instance of any one agent's sensory domain overlapping with $\tilde{\mathbf{q}}_i^*$ (i.e., $\tilde{\mathbf{q}}_i^* \in \mathcal{W}_i$ for some $i \in \mathcal{S}$). Once an agent $\mathcal{A}_i$ reaches a set $\{\tilde{\mathbf{q}} \in \mathcal{D} : \|\tilde{\mathbf{q}} - \tilde{\mathbf{q}}_i^*(t_s)\| \leq r_i\}$, Condition **C1** is violated due to the oddness of the integrand of e_t at $t = \hat{t}_s$ within $\mathcal{W}_i$. At that point, we have regained $e_t(t) \neq 0$ for some time $\hat{t}_s + \epsilon > \hat{t}_s$ and the control is switched back to $\bar{\mathbf{u}}_i$ in (7). However, up to time $\hat{t}_s$, by definition, we are guaranteed that symmetry is unbroken and the uncovered nearest points set $\tilde{\mathcal{D}}_e^i(t)$ is unaltered for time $t_s < t < t_s^*$. This proves that $\tilde{\mathbf{q}}_i^*(t_s)$ is fixed during the phase of operation of $\bar{\bar{\mathbf{u}}}_i$.

Remark: Note that Condition **C1** with $e \neq 0$ may occur for one or more agents but not *all* agents. In this case, the function $\bar{V}$ will still be strictly decreasing. However, one realizes that an agent that satisfies an agent-wise version of **C1** at some time will be virtually unproductive in covering $\mathcal{D}$, which is an inefficiency. In this case, we apply the proposed remedy (switching to another symmetry breaking control law) to any such agents to improve the efficiency of the fleet, while the rest of fleet is employing the controller $\bar{\mathbf{u}}_i(t)$. This is indeed what we employ in the simulations in Section III-B. •

The previous discussion proves the following result.

Theorem III.1 (Fully Connected): Under sensor model Properties **SM1–SM3** and **IC1**, the control law

$$\mathbf{u}_i^*(t) = \begin{cases} \bar{\mathbf{u}}_i, & \text{if } \mathbf{C1} \text{ does not hold} \\ \bar{\bar{\mathbf{u}}}_i, & \text{if } \mathbf{C1} \text{ holds} \end{cases} \tag{10}$$

drives the error $e(t) \to 0$ as $t \to \infty$.

Remark: Note that if Condition **C1** holds, then the control law $\bar{\bar{\mathbf{u}}}_i$ is in effect. Once it converges to $\tilde{\mathbf{q}}_i^*(t_s)$, if in addition $e = 0$, then the goal is met and the control converges to $\bar{\bar{\mathbf{u}}} = 0$. If $e \neq 0$, the controller switches back to $\bar{\mathbf{u}}_i$. Switching will recur until $e = 0$. •

Comment on the Switching Control Strategy: As is the case with switched systems, one has to ask if infinite switching between the nominal and symmetry-breaking (or the perturbation) control law is possible. In this paper, infinite switching is not possible. Since after every single switch (from $\bar{\bar{\mathbf{u}}}_i$ to $\bar{\mathbf{u}}_i$) an area of measure larger than zero is covered and since the domain $\mathcal{D}$ is compact, there will be no infinite switching. Clearly, if $\mathcal{D}$ is open or unbounded, there is no guarantee that after each switch the nonzero measure area covered by the formation will eventually cover the entire domain. This is the main reason for requiring that $\mathcal{D}$ be compact.

B. Partially Connected Networks

Now assume that agent $\mathcal{A}_i$ receives information from agents $\mathcal{A}_j$ only, $j \in \hat{\mathcal{S}}_i \subset \mathcal{S} \setminus \{i\}$. Inspired by the error function (4), consider the *estimated error function of agent* $\mathcal{A}_i$

$$\hat{e}_i(t) = \int_{\mathcal{D}} h(C^* - \mathcal{T}_i(\tilde{\mathbf{q}}, t) - \mathcal{T}_{\hat{\mathcal{S}}_i}(\tilde{\mathbf{q}}, t))\phi(\tilde{\mathbf{q}})\mathrm{d}\tilde{\mathbf{q}}.$$

Note that the error associated with agent $\mathcal{A}_i$ depends only on its own state history and that of all other agents from which it receives information. The following two remarks are crucial for the next theorem.

Remark: Note that

$$\hat{e}_i(t) \geq e(t)$$

equality holding if and only if $\hat{\mathcal{S}}_i = \mathcal{S} \setminus \{i\}$. •

Remark: In the case of bidirectional communication (i.e., if $j \in \hat{\mathcal{S}}_i$ then $i \in \hat{\mathcal{S}}_j$) then $\hat{e}_i = \hat{e}_j$. •

Theorem III.2 (Partially Connected with Bidirectional Communication): The control law

$$\mathbf{u}_l^*(t) = \begin{cases} \bar{\mathbf{u}}_l, & \text{if } \mathbf{C1} \text{ does not hold} \, \forall j \in \hat{\mathcal{S}}_l \\ \bar{\bar{\mathbf{u}}}_l, & \text{if } \mathbf{C1} \text{ holds} \, \forall j \in \hat{\mathcal{S}}_l \end{cases} \tag{11}$$

where

$$\bar{\mathbf{u}}_l(t) = \bar{k}_l \int_{\mathcal{D}} h'(C^* - \mathcal{T}_{\hat{\mathcal{S}}_l}(\tilde{\mathbf{q}}, t)) \times \left.\frac{\partial A_l(s)}{\partial s}\right|_{s=\|\mathbf{q}_l(t)-\tilde{\mathbf{q}}\|^2} (\mathbf{q}_l(t) - \tilde{\mathbf{q}})\phi(\tilde{\mathbf{q}})\mathrm{d}\tilde{\mathbf{q}} \tag{12}$$

and

$$\bar{\bar{\mathbf{u}}}_l(t) = -\bar{\bar{k}}_l(\mathbf{q}_l(t) - \tilde{\mathbf{q}}_l^*) \tag{13}$$

drives the error $\hat{e}_l(t) \to 0$ and, therefore, $e_l(t) \to 0$ as $t \to \infty$.

Proof: Define the function $\hat{V}_i = -\hat{e}_t$ and note that $\dot{\hat{V}} = -\hat{e}_{tt}$ where we get expressions similar to those obtained in the proof of the previous theorem. The rest of the proof follows exactly that of the fully-connected case to prove that $\hat{e}_l \to 0$ as $t \to \infty$. Since $\hat{e}_l \geq e_l$, the result follows immediately. See the remark in the following for an intuitive explanation. ■

Remark: Here, we have effectively split the network into $\bar{N}$ subsystems, each seeking to cover the entire domain $\mathcal{D}$ with effective coverage C^*. Hence, when each subsystem converges with $\hat{e}_i = 0$, we have a total effective coverage of $\bar{N}C^*$ at each $\tilde{\mathbf{q}} \in \mathcal{D}$. This, however, is not efficient both fuel- and time-wise. A remedy is to reset desired coverage level from C^* to $C^*/\bar{N}$ such that the total desired coverage achieved by the entire network is C^*. This helps reduce the time taken to accomplish the mission using lower levels of control effort. This, however, assumes knowledge of how many subsystems $\bar{N}$ exist in the entire fleet. •

IV. Effective Coverage With Collision Avoidance

We now return to the fully connected fleet and consider a control strategy that ensures that no two agents are too close to collide. Following the methodology proposed in [18], we consider the following functions:

$$P_{ij}(\mathbf{q}_i, \mathbf{q}_j) = \left(\min\left\{0, \frac{\|\mathbf{q}_i - \mathbf{q}_j\|^2 - R^2}{\|\mathbf{q}_i - \mathbf{q}_j\|^2 - r^2}\right\}\right)^2 \tag{14}$$

where $R > r > 0$, R is the radius of a disc centered at the agent's location in which other agents can be detected by the agent, and r is the smallest safe distance between agents. The derivative of these *mutual avoidance functions* is given by [18]

$$\frac{\partial P_{ij}}{\partial \mathbf{q}_i} = \begin{cases} 0, & q_{ij} \geq R \\ \frac{4(R^2-r^2)\left(q_{ij}^2-R^2\right)(\mathbf{q}_i-\mathbf{q}_j)}{\left(q_{ij}^2-r^2\right)^3}, & R > q_{ij} > r \\ \text{not defined}, & q_{ij} = r \\ 0, & q_{ij} < r \end{cases} \tag{15}$$

where $q_{ij} = \|\mathbf{q}_i - \mathbf{q}_j\|$. This will be used as the control "force" acting on each agent to guarantee collision avoidance. Note that it is only when $r < \|\mathbf{q}_i - \mathbf{q}_j\| < R$ that the mutual repulsive force between the agents is nonzero. Moreover, observe that

$$P_{ij} = P_{ji}, \frac{\partial P_{ij}}{\partial \mathbf{q}_i} = -\frac{\partial P_{ij}}{\partial \mathbf{q}_j}. \tag{16}$$

A. *Collision-Free Nominal Controller* $\bar{\mathbf{u}}_i$

Let the nominal control law be given by

$$\bar{\mathbf{u}}_i(t) = \bar{\mathbf{u}}_i^{\mathrm{cov}}(t) + \bar{\mathbf{u}}_i^{\mathrm{col}}(t) \tag{17}$$

where

$$\bar{\mathbf{u}}_i^{\mathrm{cov}}(t) = -\bar{k}_i^{\mathrm{cov}} \int_{\mathcal{D}} h'(C^* - \mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t)) \times \left.\frac{\partial A_i(s)}{\partial s}\right|_{s=\|\mathbf{q}_i(t)-\tilde{\mathbf{q}}\|^2} \cdot (\mathbf{q}_i(t) - \tilde{\mathbf{q}})\phi(\tilde{\mathbf{q}})\mathrm{d}\tilde{\mathbf{q}} \tag{18}$$

is the same nominal coverage control law as before, and

$$\bar{\mathbf{u}}_i^{\mathrm{col}}(t) = -\bar{k}_i^{\mathrm{col}} \sum_{j=1,\neq i}^{N} \frac{\partial P_{ij}}{\partial \mathbf{q}_i}. \tag{19}$$

In the previous two equations, $\bar{k}_i^{\mathrm{cov}} > 0$ and $\bar{k}_i^{\mathrm{col}} > 0$ are feedback gains.

In this section, we derive conditions that guarantee both effectively covering the domain $\mathcal{D}$ and a collision-free maneuver. We proceed as before by developing a Lyapunov-like function that guarantees convergence to the zero coverage error state. We consider each controller separately first and then combine the result in a theorem at the end of this section.

Consider the function $\bar{V} = -e_t - \varpi P$, where

$$P = \sum_{i=1}^{N} \sum_{j=1,\neq i}^{N} P_{ij}$$

and $\varpi > 0$ is a weighting parameter. Computing the time derivative of $\bar{V}$ along the flow of the system (1) under the control law (17), we obtain

$$\dot{\bar{V}} = -e_{tt} - \varpi \dot{P} \tag{20}$$

where

$$\begin{aligned} e_{tt} &= \bar{V}^1 + 2\sum_{j=1}^{N} \frac{\bar{\mathbf{u}}_j^{\mathrm{cov}}}{\bar{k}_j^{\mathrm{cov}}} \cdot \bar{\mathbf{u}}_j \\ &= \bar{V}^1 + 2\sum_{j=1}^{N} \frac{1}{\bar{k}_j^{\mathrm{cov}}} \left[\left\|\bar{\mathbf{u}}_j^{\mathrm{cov}}\right\|^2 + \bar{\mathbf{u}}_j^{\mathrm{cov}} \cdot \bar{\mathbf{u}}_j^{\mathrm{col}}\right] \end{aligned}$$

and

$$\begin{aligned} \dot{P} &= \sum_{i=1}^{N} \sum_{j=1,\neq i}^{N} \frac{\partial P_{ij}}{\partial \mathbf{q}_i} \bar{\mathbf{u}}_i + \sum_{i=1}^{N} \sum_{j=1,\neq i}^{N} \frac{\partial P_{ij}}{\partial \mathbf{q}_j} \bar{\mathbf{u}}_j \\ &= \sum_{i=1}^{N} \sum_{j=1,\neq i}^{N} \frac{\partial P_{ij}}{\partial \mathbf{q}_i} \bar{\mathbf{u}}_i + \sum_{i=1}^{N} \sum_{j=1,\neq i}^{N} \frac{\partial P_{ji}}{\partial \mathbf{q}_j} \bar{\mathbf{u}}_j \\ &= \sum_{i=1}^{N} \sum_{j=1,\neq i}^{N} \frac{\partial P_{ij}}{\partial \mathbf{q}_i} \bar{\mathbf{u}}_i + \sum_{i=1}^{N} \sum_{j=1,\neq i}^{N} \frac{\partial P_{ij}}{\partial \mathbf{q}_i} \bar{\mathbf{u}}_i \\ &= -2\sum_{i=1}^{N} \frac{1}{\bar{k}_j^{\mathrm{col}}} \bar{\mathbf{u}}_i^{\mathrm{col}} \cdot \left(\bar{\mathbf{u}}_i^{\mathrm{col}} + \bar{\mathbf{u}}_i^{\mathrm{cov}}\right). \end{aligned}$$

In the previous equations $\bar{V}^1$ is given by

$$\bar{V}^1 = \int_{\mathcal{D}} h''(C^* - \mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t)) \times \left(\sum_{j \in \mathcal{S}} A_j(\|\mathbf{q}_j(t) - \tilde{\mathbf{q}}\|^2)\right)^2 \phi(\tilde{\mathbf{q}})\mathrm{d}\tilde{\mathbf{q}} \geq 0. \tag{21}$$

After a lengthy but straightforward computation one can show that

$$\dot{\bar{V}} = -\bar{V}^1 - 2\sum_{i=1}^{N} \begin{bmatrix} \tilde{\bar{\mathbf{u}}}_i^{\mathrm{cov}} & \tilde{\bar{\mathbf{u}}}_i^{\mathrm{col}} \end{bmatrix} \bar{\mathbf{Q}}_i \begin{bmatrix} \tilde{\bar{\mathbf{u}}}_i^{\mathrm{cov}} \\ \tilde{\bar{\mathbf{u}}}_i^{\mathrm{col}} \end{bmatrix}$$

where

$$\tilde{\bar{\mathbf{u}}}_i^{\mathrm{col}} = \frac{\bar{\mathbf{u}}_j^{\mathrm{col}}}{\sqrt{\bar{k}_i^{\mathrm{col}}}}, \quad \tilde{\bar{\mathbf{u}}}_i^{\mathrm{cov}} = \frac{\bar{\mathbf{u}}_i^{\mathrm{cov}}}{\sqrt{\bar{k}_i^{\mathrm{cov}}}}$$

and $\bar{\mathbf{Q}}_i$ is a constant matrix given by

$$\bar{\mathbf{Q}}_i = \begin{bmatrix} \mathbf{I}_{2\times 2} & \frac{\bar{\alpha}_i}{2}\mathbf{I}_{2\times 2} \\ \frac{\bar{\alpha}_i}{2}\mathbf{I}_{2\times 2} & \varpi \mathbf{I}_{2\times 2} \end{bmatrix}$$

with

$$\bar{\alpha}_i = \frac{\bar{k}_i^{\mathrm{cov}} + \varpi \bar{k}_i^{\mathrm{col}}}{\sqrt{\bar{k}_i^{\mathrm{cov}} \bar{k}_i^{\mathrm{col}}}} > 0.$$

Note that $\bar{\mathbf{Q}}_i$ has two eigenvalues at

$$\frac{1}{2}\left(1 + \varpi + \sqrt{1 - 2\varpi + \varpi^2 + \bar{\alpha}_i^2}\right)$$

and two eigenvalues at

$$\frac{1}{2}\left(1+\bar{w}-\sqrt{1-2\bar{w}+\bar{w}^2+\bar{\alpha}_i^2}\right).$$

To guarantee that $\bar{\mathbf{Q}}_i$ is positive semi-definite then we have to set $\bar{\alpha}_i^2 - 4\bar{w} \le 0$. However, by definition of $\bar{\alpha}_i$, this condition is satisfied if and only if $\bar{\beta}_i^2 = (\bar{k}_i^{\rm cov})/(\bar{k}_i^{\rm col}) = \bar{w}$, which sets $\bar{\alpha}_i = 2\sqrt{\bar{w}}$. Setting $\bar{\alpha}_i = 2\sqrt{\bar{w}}$, we obtain

$$\dot{\bar{V}} = -\bar{V}^1 - 2\sum_{i=1}^{N}\left\|\tilde{\bar{\mathbf{u}}}_i^{\rm cov} + \sqrt{\bar{w}}\tilde{\bar{\mathbf{u}}}_i^{\rm col}\right\|^2. \tag{23}$$

Before we proceed, we first introduce a modified version of Condition **C1**.

Condition C1′: $\mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}},t) = C^*, \forall\tilde{\mathbf{q}} \in \mathcal{W}_i(t), \forall i \in \mathcal{S}$ *and* all agents having relative distances $\|\mathbf{q}_i(t) - \mathbf{q}_j(t)\| > R$, for all $i,j \in \mathcal{S}$.

Lemma IV.1: If $\bar{\beta}_i^2 = (\bar{k}_i^{\rm cov})/(\bar{k}_j^{\rm col}) = \bar{w}$ for all $i \in \mathcal{S}$ and with initial configuration such that $\|\mathbf{q}_i(0) - \mathbf{q}_j(0)\| > r$, then under the control law (17) a fully connected multiagent sensor network will *safely* converge to the state described in **C1′**.

Proof: Note that the function $\bar{V}$ is equal to zero if and only if Condition **C1** is satisfied (as in earlier sections of this paper) *and* $P = 0$. $P = 0$ if and only if all agents are outside each others' detection ranges determined by the detection distance R. Checking the expression for $\dot{\bar{V}}$ we note that the term $\bar{V}^1 \ge 0$ with equality holding if and only if **C1** is satisfied. From the condition $\bar{k}_j^{\rm cov} = \bar{w}\bar{k}_j^{\rm col}$ and, hence, (23), we note that $\dot{\bar{V}} \le 0$ with equality holding if and only if $\bar{V}^1$, $\tilde{\bar{\mathbf{u}}}_i^{\rm cov}$ and $\tilde{\bar{\mathbf{u}}}_i^{\rm col}$ are zero. The quantities $\bar{V}^1$ and $\tilde{\bar{\mathbf{u}}}_i^{\rm cov}$ are zero if and only if **C1** is satisfied, and $\tilde{\bar{\mathbf{u}}}_i^{\rm col}$ is zero if and only if $P = 0$. (Note that $\tilde{\bar{\mathbf{u}}}_i^{\rm cov}$ and $\tilde{\bar{\mathbf{u}}}_i^{\rm col}$ cannot be in the null space of $\bar{\mathbf{Q}}_i$ and $\bar{V}^1$ zero unless $\tilde{\bar{\mathbf{u}}}_i^{\rm cov} = \tilde{\bar{\mathbf{u}}}_i^{\rm col} = 0$ because if $\tilde{\bar{\mathbf{u}}}_i^{\rm cov}$ is non-zero, then so must $\bar{V}^1$.) This proves convergence to the condition **C1′**.

With initial conditions $\|\mathbf{q}_i(0) - \mathbf{q}_j(0)\| > r$, the system is safely initialized. By construction of the avoidance functions P_{ij}, if the safety region (the circle of radius r centered at all agents' centers of masses) is to be violated, then $P_{ij} \to \infty$ as $\|\mathbf{q}_i - \mathbf{q}_j\| \to r^+$. This implies that the function $\bar{V} \to \infty$. However, this is a contradiction since the function $\bar{V}$ is finite and is strictly decreasing. ■

Remark: If Condition **C1** alone is satisfied and some relative distances are such that $r < \|\mathbf{q}_i(t) - \mathbf{q}_j(t)\| < R$, then due to the repulsive nature of P_{ij} the agents repel each other until all agent pairs satisfy $\|\mathbf{q}_i(t) - \mathbf{q}_j(t)\| > R$, at which point $P = 0$. If **C1** is not satisfied but $P = 0$, the agents move according to the nominal controller $\bar{\mathbf{u}}_i^{\rm cov}$ only as in previous sections. •

Remark: Note here that we are not restricting the motion of the agents to remain inside $\mathcal{D}$. In the case when, say, R is larger than the domain size (defined, say, as the maximum Euclidean distance between any two points in $\mathcal{D}$) the repulsive force will push some agents outside the boundary of $\mathcal{D}$, which we allow in this paper. At this point, along with the satisfaction of Condition **C1**, the control will be switched to the perturbation control law $\bar{\bar{\mathbf{u}}}_i$ discussed in Section IV-B. •

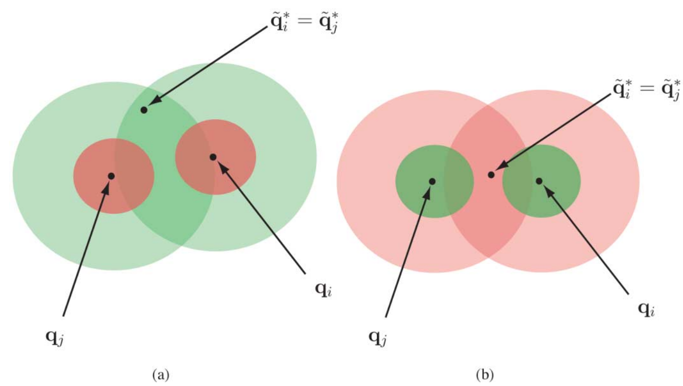


Fig. 3. Red represents the safety zone (of radius r) around each agent and green represents the sensory domain of each agent. (a) Two agents seeking the same $\tilde{\mathbf{q}}^*$ with $r < c_i$. (b) Two agents seeking the same $\tilde{\mathbf{q}}^*$ with $r > c_i$.

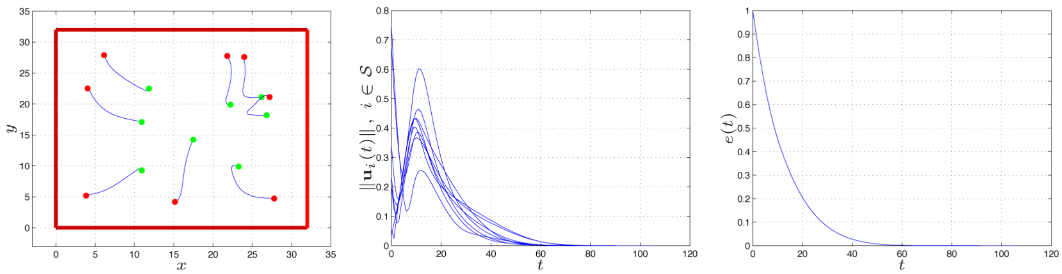
B. Collision-Free Perturbation Controller $\bar{\bar{\mathbf{u}}}_i$

Recall that the condition **C1′** does not necessarily imply that $e = 0$. If **C1′** is satisfied but $e \neq 0$, then we need to apply a switching controller to violate Condition **C1′**. The control strategy proposed in Section III-A for the perturbation control law $\bar{\bar{\mathbf{u}}}_i$ aims at moving at least one agent that perturbs the system from Condition **C1**. In this section, we propose a similar control strategy where each agent is commanded to move in a linear fashion to some point $\tilde{\mathbf{q}}_i^*$. In Section III-A, one may allow $\tilde{\mathbf{q}}_i^* = \tilde{\mathbf{q}}_j^*$ for some $i \neq j$. In that case, the two agents may converge to the same point, which will result in a collision.

For collision avoidance, we present one control strategy that relies on the same approach as that discussed in Section III-A and originally proposed in [18]. However, we cannot achieve having two agents $\mathcal{A}_i$ and $\mathcal{A}_j$ that have the same destination $\tilde{\mathbf{q}}_i^* = \tilde{\mathbf{q}}_j^*$ to both converge to the same point because that inherently implies violation of the unsafe set around both agents. Mathematically, the control strategy proposed earlier will entail that $\lim_{t\to\infty}\|\mathbf{q}_i(t) - \mathbf{q}_j(t)\| = 0$, which violates the collision avoidance condition that $\|\mathbf{q}_i - \mathbf{q}_j\| > r$.

Within the framework of previous sections, one may consider any one of several control strategies that achieve the purpose of perturbing the system away from **C1** while at the same time guaranteeing collision avoidance. For example, if $r < c_i$, then one can apply the same control strategy as in Section III-A, but now we only require that at least one of the two agents, say $\mathcal{A}_i$, of interest be within a distance of c_i or less from the point $\tilde{\mathbf{q}}_i^* = \tilde{\mathbf{q}}_j^*$ since this guarantees violation of **C1** without violating the agents' safety sets. The reason one needs to assume that $r < c_i$ is that the distances to the target point from either agent could be less than c_i or c_j (and hence covering $\tilde{\mathbf{q}}_i^* = \tilde{\mathbf{q}}_j^*$ and violating **C1**) while the inter-agent distance is larger than r. This is shown graphically in Fig. 3(a).

Another simpler control strategy that does not require that $r < c_i$ is as follows. We again assume that the Rule **R1** introduced in Section III-A assigns a unique point $\tilde{\mathbf{q}}_i^*$ for each agent $\mathcal{A}_i$. This rule may result in $\|\tilde{\mathbf{q}}_i^* - \tilde{\mathbf{q}}_j^*\| < r$, which in turn may result in conflicting goals. This is because two agents may seek the same point $\tilde{\mathbf{q}}^*$. However, this may be impossible to achieve along with avoidance of each others' safety zones. In particular, this will happen if $r > c_i$ [see Fig. 3(b)] since there



(a) (b) (c)

Fig. 4. Fleet motion in the plane (start at green dot and end at red dot), control effort and error function for a "dense" network with control switching. (a) Fleet motion in the plane. (b) Control effort $\|\mathbf{u}_i\|$, $i \in \mathcal{S}$. (c) Error $e(t)$.

is no guarantee that the two agents will not converge to a state of equilibrium where control effort due to collision avoidance and control effort that seeks $\tilde{\mathbf{q}}^*$ balance each other out perfectly without actually covering $\tilde{\mathbf{q}}^*$. Whenever Condition **C1′** is met, a centralized computer assigns to each agent $\mathcal{A}_i$ a maximum of one unique point $\tilde{\mathbf{q}}_i^*$ according to the Algorithm **ALG-1**. The algorithm seeks to ensure that no two points $\tilde{\mathbf{q}}_i^*$ and $\tilde{\mathbf{q}}_j^*$ are equal if **R1** results in such equalities. The Rule **R1** can be the minimum distance rule as that used in Section III-A. Recall that t_s is the switch time as before.

ALG-1.

1) **Compute** $\tilde{\mathbf{q}}_i^*$ for each agent $\mathcal{A}_i$, $i \in \mathcal{S}$, using Rule **R1**.

2) **Reindex** all agents such that

$$\|\tilde{\mathbf{q}}_1^* - \mathbf{q}_1(t_s)\| \leq \|\tilde{\mathbf{q}}_2^* - \mathbf{q}_2(t_s)\| \leq \cdots \leq \|\tilde{\mathbf{q}}_N^* - \mathbf{q}_N(t_s)\|.$$

3) **Let** $\tilde{\mathcal{S}} = \{1\}$.

4) **For** $i = 2, \ldots, N$, **if** $\|\tilde{\mathbf{q}}_i^* - \tilde{\mathbf{q}}_j^*\| > R$, for all $j \in \{1, \ldots, i-1\}$, **then** $\tilde{\mathcal{S}} \mapsto \tilde{\mathcal{S}} \cup \{i\}$.

Algorithm Description: Since Condition **C1′** does not guarantee $e = 0$, Rule **R1** guarantees that for each $\mathcal{A}_i$ there exists a point $\tilde{\mathbf{q}}_i^*$ at which $\mathcal{T}(\tilde{\mathbf{q}}, t_s) < C^*$. These points are computed for all agents in Step 1). Step 2) aims at prioritizing the agents according to how close each one is to its computed destination from Step 1). The closer an agent is to its destination, the higher priority it receives. Other criteria for priority assignment may be considered. Step 3) defines a new set $\tilde{\mathcal{S}}$ that will be iteratively constructed in Step 4). Step 4) aims at ensuring that no two agents possess the same destination point $\tilde{\mathbf{q}}^*$. This ensures that, for example, the situation shown in Fig. 3(b) does not occur. The set $\tilde{\mathcal{S}}$ corresponds to the agents which will be commanded to move to new points that aim at violating **C1′**. Hence, the input to the algorithm is the set of positions of all agents at t_s and its output is the set of agents to be moved to violate **C1′** along with their destination points. Note again that the set $\tilde{\mathcal{S}}$ is guaranteed to be nonempty since it is assumed that the algorithm is invoked only when **C1′** is satisfied *and* $e \neq 0$. It is guaranteed to at least return $\tilde{\mathcal{S}} = \{1\}$ whose destination $\tilde{\mathbf{q}}_1^*$ is uniquely computed by **R1**.

We now let the perturbation control law $\bar{\mathbf{u}}_i$ be given by

$$\begin{aligned} \bar{\mathbf{u}}_i(t) &= \bar{\mathbf{u}}_i^{\mathrm{pert}}(t) + \bar{\mathbf{u}}_i^{\mathrm{col}}(t), \forall i \in \tilde{\mathcal{S}} \\ \bar{\mathbf{u}}_i(t) &= \bar{\mathbf{u}}_i^{\mathrm{col}}(t), \forall i \in \mathcal{S} \setminus \tilde{\mathcal{S}} \end{aligned} \tag{24}$$

where $\bar{\bar{k}}_i^{\mathrm{pert}} > 0$ and $\bar{\bar{k}}_i^{\mathrm{col}} > 0$ are feedback gains and

$$\bar{\mathbf{u}}_i^{\mathrm{pert}}(t) = -\bar{\bar{k}}_i^{\mathrm{pert}}(\mathbf{q}_i - \tilde{\mathbf{q}}_i^*), \quad \forall i \in \tilde{\mathcal{S}} \tag{25}$$

is the same perturbation control law as before but only applied to agents whose indexes belong to $\tilde{\mathcal{S}}$, and

$$\bar{\mathbf{u}}_i^{\mathrm{col}}(t) = -\bar{\bar{k}}_i^{\mathrm{col}} \sum_{j=1,\neq i}^{N} \frac{\partial P_{ij}}{\partial \mathbf{q}_i}, \quad i \in \mathcal{S}. \tag{26}$$

We now have all the necessary ingredients to prove the following lemma.

Lemma IV.2: If $\bar{\bar{\beta}}_i^2 = (\bar{\bar{k}}_i^{\mathrm{col}})/(\bar{\bar{k}}_i^{\mathrm{pert}}) = \bar{\bar{w}}$, for some $\bar{\bar{w}} > 0$ and for all $i \in \mathcal{S}$, and starting at any instance t_s where **C1′** is satisfied, then under the control law (24) a fully connected multiagent sensor network will *safely* converge to $\mathbf{q}_i = \tilde{\mathbf{q}}_i^*$, $i \in \tilde{\mathcal{S}}$, and $P = 0$.

Proof: Consider the function

$$\bar{\bar{V}} = \sum_{i \in \tilde{\mathcal{S}}} \|\mathbf{q}_i - \tilde{\mathbf{q}}_i^*\|^2 + \bar{\bar{w}} P$$

and note that this function is zero precisely when $P = 0$ and when $\mathbf{q}_i = \tilde{\mathbf{q}}_i^*$, $i \in \tilde{\mathcal{S}}$. A straightforward computation reveals that

$$\dot{\bar{\bar{V}}} = -2 \sum_{i \in \mathcal{S} \setminus \tilde{\mathcal{S}}} \left\| \tilde{\bar{\bar{\mathbf{u}}}}_i^{\mathrm{col}} \right\|^2 - 2 \sum_{i \in \tilde{\mathcal{S}}} \begin{bmatrix} \tilde{\bar{\bar{\mathbf{u}}}}_i^{\mathrm{pert}} & \tilde{\bar{\bar{\mathbf{u}}}}_i^{\mathrm{col}} \end{bmatrix} \bar{\bar{\mathbf{Q}}}_i \begin{bmatrix} \tilde{\bar{\bar{\mathbf{u}}}}_i^{\mathrm{pert}} \\ \tilde{\bar{\bar{\mathbf{u}}}}_i^{\mathrm{col}} \end{bmatrix}$$

where

$$\begin{aligned} \tilde{\bar{\bar{\mathbf{u}}}}_i^{\mathrm{col}} &= \frac{\bar{\mathbf{u}}_i^{\mathrm{col}}}{\sqrt{\bar{\bar{k}}_i^{\mathrm{col}}}} \\ \tilde{\bar{\bar{\mathbf{u}}}}_i^{\mathrm{pert}} &= \frac{\bar{\mathbf{u}}_i^{\mathrm{pert}}}{\sqrt{\bar{\bar{k}}_i^{\mathrm{pert}}}} \\ \bar{\bar{\mathbf{Q}}}_i &= \begin{bmatrix} \mathbf{I}_{2\times 2} & \frac{\bar{\bar{\alpha}}_i}{2}\mathbf{I}_{2\times 2} \\ \frac{\bar{\bar{\alpha}}_i}{2}\mathbf{I}_{2\times 2} & \bar{\bar{w}}\mathbf{I}_{2\times 2} \end{bmatrix} \end{aligned}$$

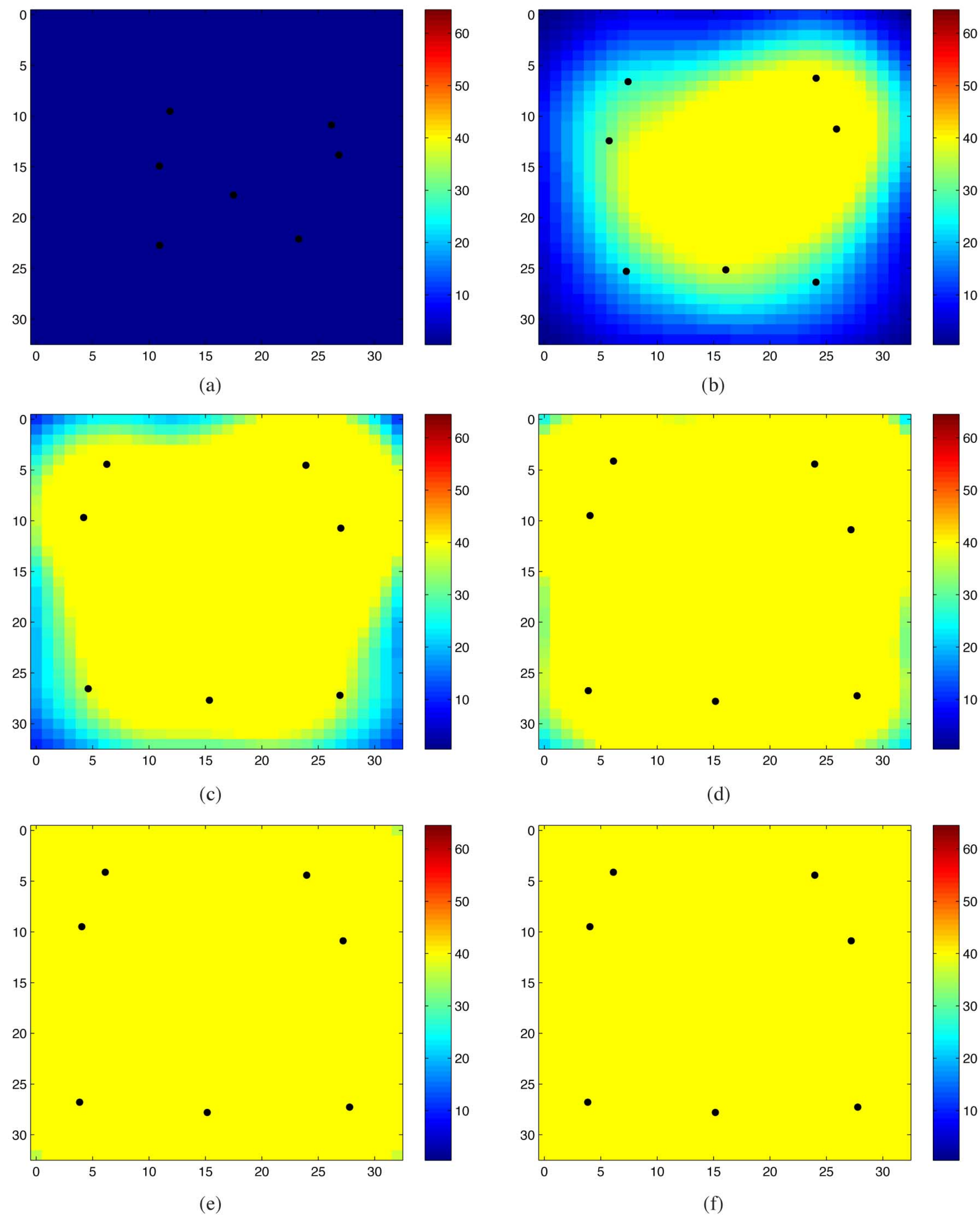


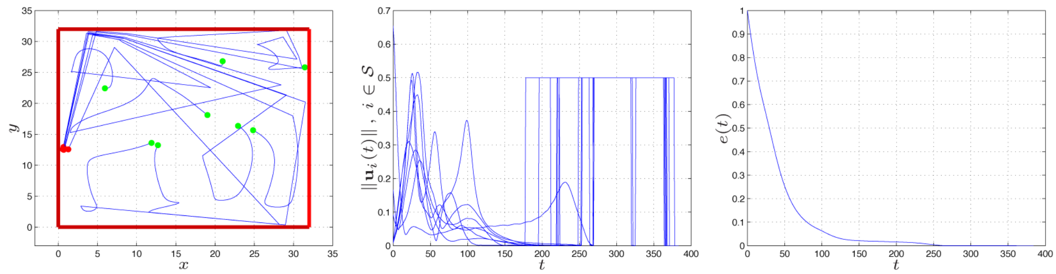
Fig. 5. Effective coverage (dark blue for low and yellow for full coverage) and fleet configuration at $t = 0, 21, 43, 64, 86, 107$ for a "dense" network with no control switching. (a) Coverage at $t = 0$. (b) Coverage at $t = 21$. (c) Coverage at $t = 43$. (d) Coverage at $t = 64$. (e) Coverage at $t = 86$. (f) Coverage at $t = 107$.

and

$$\bar{\bar{\alpha}}_i = \frac{\bar{\bar{k}}_i^{\mathrm{col}} + \bar{\bar{w}}\bar{\bar{k}}_i^{\mathrm{pert}}}{\sqrt{\bar{\bar{k}}_i^{\mathrm{col}}\bar{\bar{k}}_i^{\mathrm{pert}}}}.$$

As before, one can easily check that $\bar{\bar{\alpha}}_i^2 - 4\bar{\bar{w}} \leq 0$. However, by definition of $\bar{\bar{\alpha}}_i$, this condition is satisfied if and only if $\bar{\bar{\beta}}_i^2 = (\bar{\bar{k}}_i^{\mathrm{cov}})/(\bar{\bar{k}}_i^{\mathrm{cov}}) = \bar{\bar{w}}$, which sets $\bar{\bar{\alpha}}_i = 2\sqrt{\bar{\bar{w}}}$. Setting $\bar{\bar{\alpha}}_i = 2\sqrt{\bar{\bar{w}}}$, we guarantee that $\dot{\bar{\bar{V}}} \leq 0$ with equality holding if and only if $P = 0$ and $\mathbf{q}_i = \tilde{\mathbf{q}}_i^*$, $i \in \tilde{\mathcal{S}}$.

At t_s, the system satisfies the condition $\mathbf{C1'}$ and, hence, $\|\mathbf{q}_i(t_s) - \mathbf{q}_j(t_s)\| > r$. Therefore, the system is safely initialized at switching. By construction of the avoidance functions P_{ij}, if the safety region (the circle of radius r centered at all agents' centers of masses) is to be violated, then $P_{ij} \to \infty$ as $\|\mathbf{q}_i - \mathbf{q}_j\| \to r^+$. This implies that the function $\bar{\bar{V}} \to \infty$.



(a) (b) (c)

Fig. 6. Fleet motion in the plane (start at green dot and end at red dot), control effort and error function for an "intermediate-range" network with control switching. (a) Fleet motion in the plane. (b) Control effort $\|\mathbf{u}_i\|, i \in \mathcal{S}$. (c) Error $e(t)$.

However, this is a contradiction since the function $\bar{\bar{V}}$ is finite and is strictly decreasing. ∎

Lemmas IV.1 and IV.2 give us the following result. Similar to the case without collision avoidance, infinite switching can not occur due to compactness of the domain $\mathcal{D}$ and the way we designed the switching control.

Theorem IV.1 (Fully Connected, With Collision Avoidance): Under sensor model properties **SM1–SM3** and **IC1**, the control law

$$\mathbf{u}_i^*(t) = \begin{cases} \bar{\mathbf{u}}_i, & \text{if } \mathbf{C1}' \text{ does not hold} \\ \bar{\bar{\mathbf{u}}}_i, & \text{if } \mathbf{C1}' \text{ holds} \end{cases} \tag{27}$$

drives the error $e(t) \to 0$ as $t \to \infty$ without agents violating the safety zones of each other, where $\bar{\mathbf{u}}_i$ and $\bar{\bar{\mathbf{u}}}_i$ are given by (17) and (24).

Remark: While in this paper there are no constraints imposed on the control effort, the magnitude of the control $\bar{\mathbf{u}}_i$ or $\bar{\bar{\mathbf{u}}}_i$ is always finite. First, the coverage component of the control law $\bar{\mathbf{u}}_i^{\text{cov}}$ is finite since the function h', the derivative of A_i, and the relative vector between $\mathbf{q}_i$ and $\tilde{\mathbf{q}}$ (since the domain is assumed compact) in the integrand in (18) are all finite. For the perturbation control law $\bar{\bar{\mathbf{u}}}_i$, for finite feedback control gains, the relative vector between $\mathbf{q}_i$ and $\tilde{\mathbf{q}}^*$ is finite since the domain is assumed compact. Hence, the proposed perturbation control law is finite. Finally, the collision avoidance control component $\bar{\mathbf{u}}_i^{\text{col}}$ or $\bar{\bar{\mathbf{u}}}_i^{\text{col}}$ is always finite. This is easily seen by examining (15). Hence, the overall control law is guaranteed to be finite. •

V. Simulation Results

In this section, we provide simulation results for the fully connected network case, where each agent satisfies the dynamics (1). Assume that $\mathcal{D}$ is a square region of side length $d = 32$ units length. There are eight agents $(N = 8)$ with a randomly selected initial deployment as shown in Fig. 5(a). Let the desired effective coverage C^* be 40 (corresponds to the yellow scale in all coverage figures in this section). Here we use the control law in (7) with control gains $\bar{k}_i = 0.00001, i = 1, \ldots, 7$. In this example we do not employ switching control since the coverage regions of each agent are large enough for Condition **C1**. This example shows that switching may possibly not be required for applications where the domain is "densely" and evenly populated with mobile sensors. Sensor density is defined as the number of sensors divided by the area of the domain. The domain is evenly populated if the sensors' initial locations are evenly distributed over the domain. Finally, we assume that we do not have any *a priori* known information as to the possibility of finding the missing person and, hence, we set $\phi(\tilde{\mathbf{q}}) = q, \forall \tilde{\mathbf{q}} \in \mathcal{D}$. For the sensor model, we have set $M_i = 1$, $r_i = 14$, for all $i = 1, \ldots, 8$.

We used a simple trapezoidal method to compute integration over $\mathcal{D}$ and a simple first order Euler scheme to integrate with respect to time. The results are shown in Figs. 4 and 5. The total error $e(t)$ is shown in Fig. 4(c). Note that the error converges to zero. Fig. 5 shows the time evolution of the 2-D function $h(C^* - \mathcal{T}_{\mathcal{S}}(\tilde{\mathbf{q}}, t))$. We note that coverage converges to C^* as desired. Hence, we see that we achieve satisfactory coverage using the control law (7) without switching control. Note that we have normalized error by dividing by $(C^*)^n d^2$ so that the initial error (which is precisely the volume under the graph of the function C^*, for all $\tilde{\mathbf{q}} \in \mathcal{D}$), where d is the side length of $\mathcal{D}$ and $n = 2$ is defined in (5). The 120-s simulation took 29 s on a 2-GHz Intel CPU with 2 GB of RAM.

The second example is one with sensor ranges of $r_i = 8$ with the same parameters as the last example. In this example, we employ the switching controller (with $\bar{\bar{k}}_i = 0.5$) from Theorem III.1. An agent is set to switch to the linear feedback control law whenever Condition **C1** applies to it alone and not to the entire fleet. As mentioned before, this is a more efficient strategy than having the entire fleet satisfy the condition for switching to occur. The results are shown in Figs. 6 and 7. We note here that switching is necessary to achieve coverage. Also note that as the coverage error decreases, agents tend to flock since the agents' desired target points $\tilde{\mathbf{q}}_i^*$ tend to agree. As will be seen in Section VI, the algorithm that incorporates a collision avoidance mechanism generally does not result in flocking. The 400-s simulation took 1 min and 25 s on a 2-GHz Intel CPU with 2 GB of RAM.

Finally, we show simulation results for the control strategy with guaranteed collision avoidance. We consider a square domain of side length $d = 64$. In this simulation, we seek to cover the domain with four agents, initially randomly deployed. The desired coverage is again $C^* = 40$. The sensor ranges are all $r_i = 12$, minimum desired distance between any agent pair for safety is $r = 6$ and the agent detection range is $R = 18$. The controller gains are $\bar{k}_i^{\text{cov}} = 1 \times 10^{-5}$, $\bar{\bar{k}}_i^{\text{pert}} = 0.5$ for all agents, and $\bar{\bar{k}}_i^{\text{col}} = 1 \times 10^{-3}$. Hence, we pick $\bar{w} = 0.001$ and $\bar{\bar{w}} = 0.002$ to guarantee convergence. Clearly, as can be seen from Figs. 8(c) and 9 the domain is completely covered with zero coverage error. Moreover, collision avoidance is achieved since the relative distance between any pair of agents never drops below

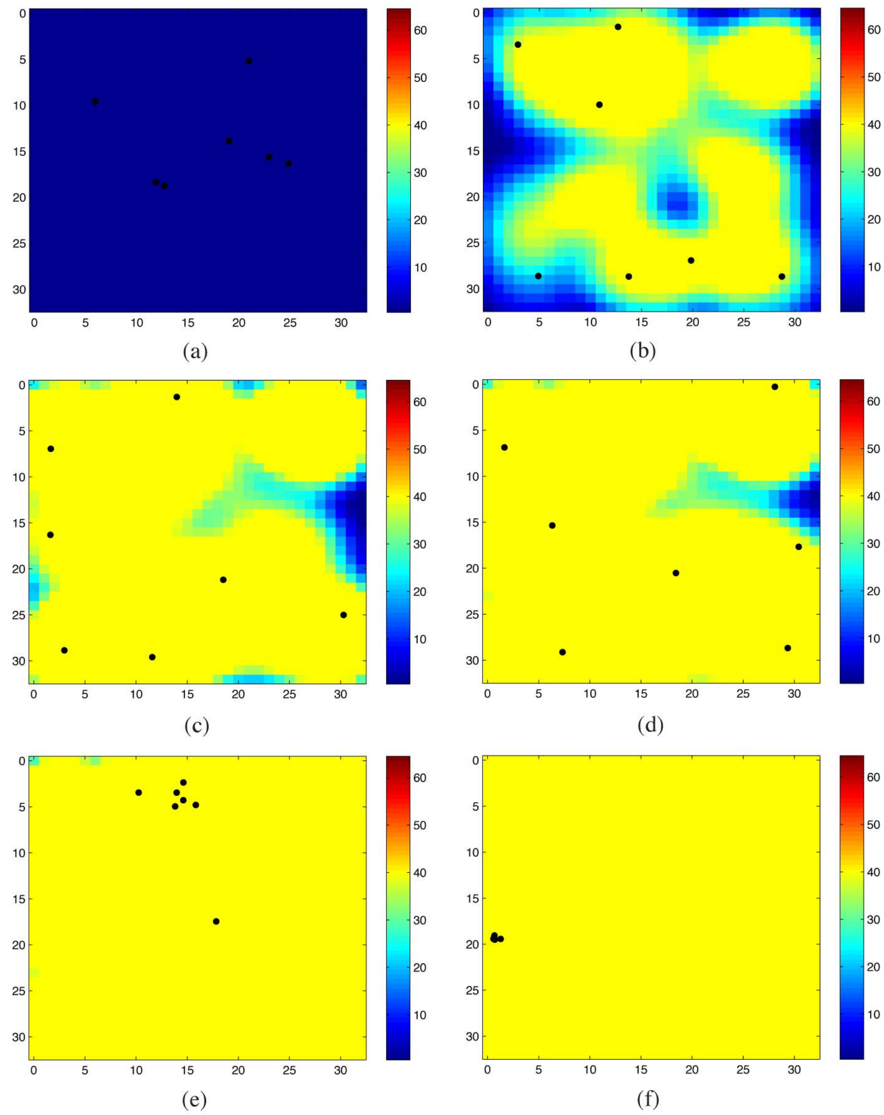


Fig. 7. Effective coverage (dark blue for low and yellow for full coverage) and fleet configuration at $t = 0, 77, 154, 231, 308, 3858$ for an "intermediate-range" network with control switching. (a) Coverage at $t = 0$. (b) Coverage at $t = 77$. (c) Coverage at $t = 154$. (d) Coverage at $t = 231$. (e) Coverage at $t = 308$. (f) Coverage at $t = 385$.

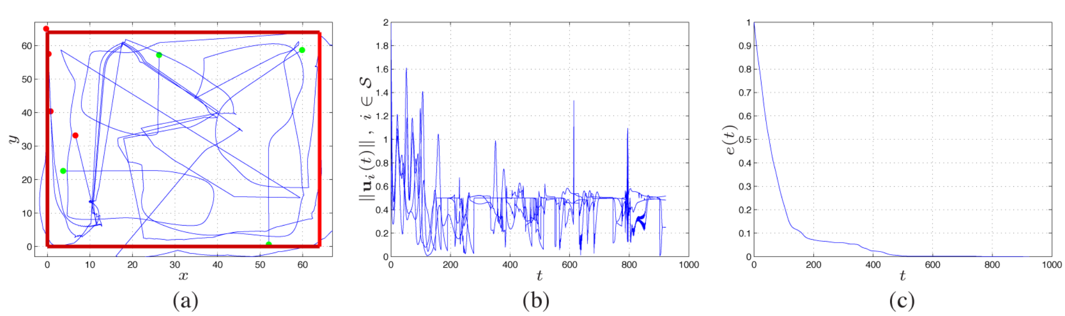


Fig. 8. Fleet motion in the plane (start at green dot and end at red dot), control effort and error function for a network with guaranteed collision avoidance. (a) Fleet motion in the plane. (b) Control effort $\|\mathbf{u}_i\|, i \in \mathcal{S}$. (c) Error $e(t)$.

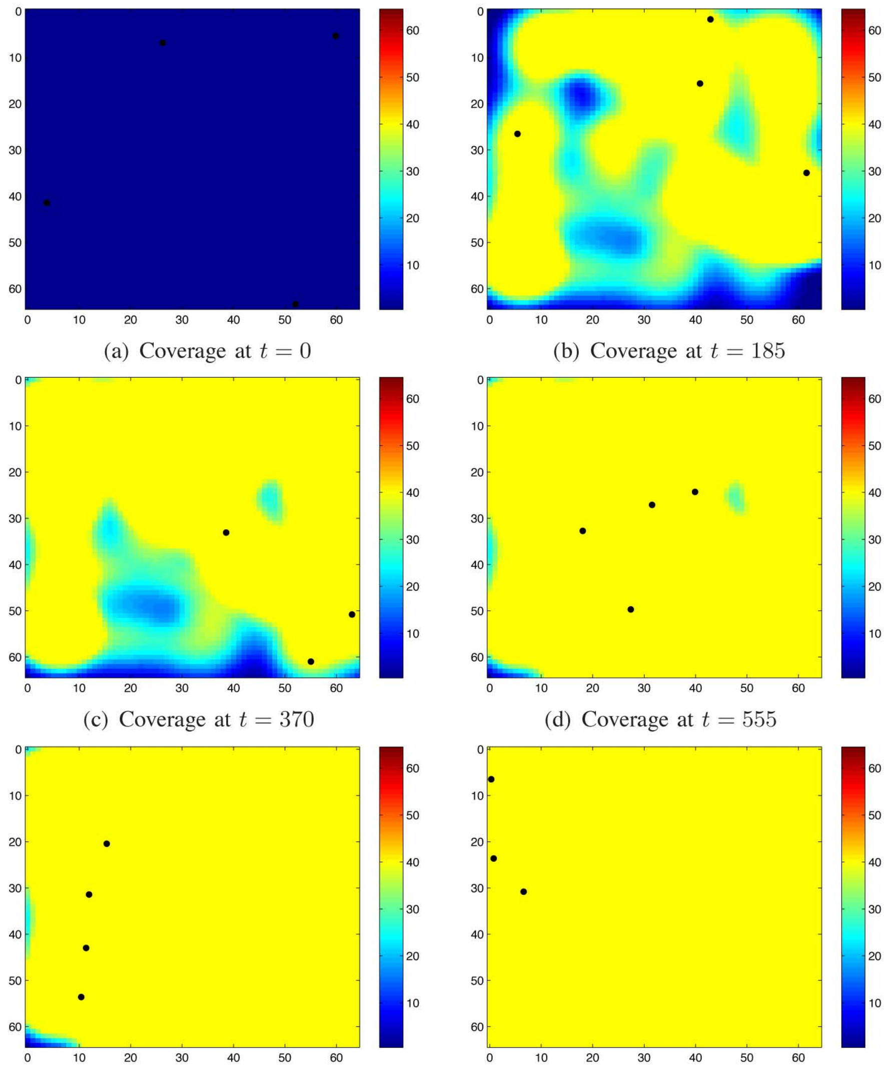
the minimum desired of six length units. This is illustrated in Fig. 10. The 1000-s simulation took 4 min and 47 s on a 2-GHz Intel CPU with 2 GB of RAM.

VI. Further Discussion and Future Extensions

In this section, we highlight some important aspects of the proposed control algorithms and its extensions.

A. *Computational Complexity*

The control algorithms developed in this paper require the evaluation of a double integral at each time step. In practice, the domain $\mathcal{D}$ is discretized into a set of n cells and, hence, the numerical integration over the domain is reduced to a n^2 summation assuming a simple trapezoidal rule. In the integrand of the control law, there is a single summation over all vehicles. Hence,



(a) Coverage at $t = 0$ (b) Coverage at $t = 185$

(c) Coverage at $t = 370$ (d) Coverage at $t = 555$

(e) Coverage at $t = 740$ (f) Coverage at $t = 925$

Fig. 9. Effective coverage (dark blue for low and yellow for full coverage) and fleet configuration at $t = 0, 185, 370, 555, 740, 925$ for a network with guaranteed collision avoidance. (a) Coverage at $t = 0$. (b) Coverage at $t = 185$. (c) Coverage at $t = 370$. (d) Coverage at $t = 555$. (e) Coverage at $t = 740$. (f) Coverage at $t = 925$.

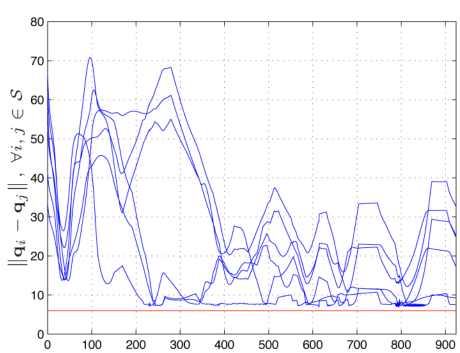


Fig. 10. Relative distances between all agent pairs in the fleet. The red line represents the minimum distance allowed between any agent pair.

our approach requires computations at the order $\mathcal{O}(n^2 + N)$. On the other hand, any other Voronoi-based approach to coverage (recall that the Voronoi-based method of [9] is only for redeployment problems and not full domain *dynamic* coverage) requires $\mathcal{O}(N \log N)$, where N is the number of Voronoi partitions which is also equal to the number of vehicles in this paper [20]. Hence, the total computational burden is $\mathcal{O}(n^2 + N \log N)$ since we need to compute the Voronoi partition at each point in time *and* perform a 2-D integration over the domain partitioned into N subdomains. This clearly indicates that the computational burden required by a Voronoi-based approach is higher than the method proposed herein. We emphasize, however, that previous research that rely on the use of Voronoi partitioning for sensor redeployment problems, to the best of our knowledge, does not report computational burden of Voronoi-based algorithms. Moreover, our assessment, as given before, is that simply redeploying (which is a simpler task than full surveillance of a domain) using a Voronoi partitioning approach is computationally more costly than the proposed control laws.

Other dynamic coverage methods such as simultaneous localization and mapping (SLAM) literature are Kalman-filter-based methods for dynamic surveillance of a domain $\mathcal{D}$. See for example [21], [22] and references therein. To the best of our knowledge, there is no reporting in the literature on these methods regarding the computational burden involved in these problems. By inspecting the control laws derived in the literature, such methods still involve double integrals of 2-D variables, which again result in $\mathcal{O}(n^2)$ operation. While our approach results in scalar evaluations in the integrand of the control laws developed in this paper, the control laws involved in, say, [21], [22] involve matrix operations, which result in additional computational operations that are avoided in the method presented in this paper.

To conclude, the method presented in this paper is computationally more efficient than the methods developed for other effective coverage control problems that involve domain surveillance. Current and future work will focus on improving the computational efficiency of **ALG-1**.

B. General Vehicle Dynamics

In this paper, we have considered a simple kinematic model for the vehicle dynamics. However, vehicle dynamics are usually nonlinear and second order of the form

$$\begin{aligned} \dot{\mathbf{q}}_i &= \mathbf{u}_i \\ \dot{\mathbf{u}}_i &= \mathbf{a}_i + \mathbf{f}_i(\mathbf{q}_i, \mathbf{u}_i) \end{aligned} \tag{28}$$

where $\mathbf{a}_i$ is the control acceleration. The authors have already utilized (in a paper currently under review) the control laws developed in this paper for $\dot{\mathbf{v}}_i$ to obtain a dynamic control law for $\mathbf{a}_i$ for the previous model by dynamic inversion. Dynamic inversion assumes exact knowledge of the system dynamics represented by the drift term $\mathbf{f}_i$. However, nonlinear robust control methods can be used to robustly ensure coverage under model uncertainties. Robust control is beyond the scope of this paper and robust control techniques are currently under investigation to take into account unmodeled and uncertain dynamics. Finally, another issue that is under consideration is the case where the dynamics are subject to some nonholonomic constraints [23] as is the case with wheeled type robots, aircraft, and underwater vehicles. Starting with the control laws developed in this paper, we are currently considering adaptations of the coverage control law developed in this paper such that the resulting motion satisfies any nonholonomic constraints imposed on the dynamics.

C. Communication Structures and Network Complexity

In this paper, we have considered two information flow structures. The first is where the communication network is all-to-all. Each vehicle receives and sends information to all vehicles in the network. Such a scenario is relevant to "small world" networks where communication channels are of a sufficient quality and strength such that no or minute information loss occurs. The second scenario is where not all vehicles can communicate, but if a vehicle pair communicate, they do so in a bidirectional communication mode. This scenario may be applicable in the less likely situation where the network fleet is split up into subgroups, with each group operating at a unique communication frequency that is different from other subgroups.

To the best of the authors' knowledge the question of a dynamically changing communication structure for *dynamic coverage* control problems has never been addressed. In the stochastic *estimation* setting with *static* vehicle sensor networks for the *consensus* problem, some results and conditions have been established for a dynamically varying communication structure network (see [24], [25], and references therein). This is, however, a different problem from the dynamic coverage control problem addressed in this paper. Addressing the effective coverage control problem from a stochastic estimation point of view, in [26], the author proposes a Kalman-filter-based approach for distributed process estimation that follows similar paths to [21], [24], and [25] for the dynamic coverage control problem. This approach may allow for some important results for dynamic coverage control using dynamic information structures similar to that of [24] and [25]. This is currently a subject of current research being addressed by the authors.

D. Other Open Problems

Some other open problems include the following.

- As mentioned in an earlier remark, some applications involve the search for or identification of dynamic objects (as in underwater surveillance). In this case, the function $\psi : \mathcal{D} \to \mathbb{R}^+$ in (4) can be replaced with a time dependent function $\psi_d : \mathcal{D} \times \mathbb{R}^+ \to \mathbb{R}^+$, which may be viewed as an explicit or implicit function of time. The question of dynamically changing state of awareness is one of the open problems that should be addressed in future research.
- Global coverage seems to require centralized implementations. For a fleet to be able to ensure global coverage, a central processing station needs to exist to keep track of the domain geometry and regions with lack of coverage. The interplay between global coverage and scheme decentralization is not well understood. The authors will also attempt in future work to better understand decentralized schemes and how decentralization affects global coverage.
- Optimal coverage in the sense of time to mission completion, fuel expenditure, or number of agents, has been studied in [12] and [13] for multiple satellite imaging systems. This topic is hard to address and is far from being concluded. The authors will seek to address aspects of optimal coverage, possibly with pursuit-evasion games, in this research project. Pursuit-evasion arises in some military situations where the fleet is pursuing unfriendly underwater vehicles, or is performing search and retrieval or surveillance while evading unfriendly units.

VII. Conclusion

In this paper, we formulated a coverage control problem that addresses a wide variety of multiagent system applications. The goal is to achieve coverage of each point in the search domain by an effective coverage of C^*. We explored the fully connected network, where all agents have access to the state histories of all other agents in the group. This result was modified for a class of bidirectional partially connected networks. We proposed a control law that ensures that the coverage error converges to zero for both communication structures. A collision avoidance component was appended to the control law and the closed loop system was shown to achieve full coverage of the mission space safely. The control law for the fully connected fleet was validated numerically for various model parameters, including collision avoidance. Finally, we commented on some practical implementation issues, and discussed open and future research directions.

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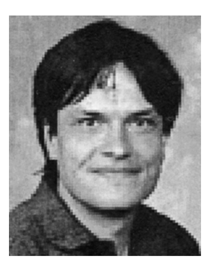

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