1 Problem Formulation

We consider two different vehicle dynamics. First, the double integrator with parallel to the axis input accelerations u_x and u_y described by

$$x = \begin{pmatrix} p_x \\ v_x \\ p_y \\ v_y \end{pmatrix}, \dot{x} = \frac{dx}{dt} = \begin{pmatrix} x_2 \\ u_x \\ x_4 \\ u_y \end{pmatrix}, \quad \|(v_x, v_y)\| \leq v_{max}, \|(u_x, u_y)\| \leq u_{max},$$

where p_x, p_y and v_x, v_y are the x and y positions and velocities respectively. We assume both the maximum velocity and acceleration as bounded.

The second type of vehicle follows the Dubins car's dynamic

$$x = \begin{pmatrix} p_x \\ p_y \\ \theta \\ v \end{pmatrix}, \dot{x} = \frac{dx}{dt} = \begin{pmatrix} x_4 \cos(x_3) \\ x_4 \sin(x_3) \\ u_\theta \\ u_v \end{pmatrix}, \quad v_{min} \leq \|(v_x, v_y)\| \leq v_{max}, |u_\theta| \leq u_{\theta max}, |u_v| \leq u_{smax}$$

here θ is the heading angle, v is the car speed and p_x and p_y are the x and y positions. In this case we set bounds for the maximum car acceleration u_v, and for the maximum turn rate u_θ. Additionally, we impose vehicle speed maximum and minimum, the last one particularly relevant for flying vehicles like airplanes. (Should we change the Dubins variable name v to s, as v is consider for us as velocity vector not speed?)

2 Methodology - Coverage Controller

Define $p_{ij} := p_i - p_j$, and denote by $P_{\partial\Omega}(p_i)$ the closest point of $\partial\Omega$ to p_i (i.e., the projection of p_i on $\partial\Omega$). Also, define $h_i := p_i - P_{\partial\Omega}(p_i)$, and denote by $[[h_i]]$ the signed distance of p_i from $\partial\Omega$.

The proposed control force is given as

$$u_i = \begin{pmatrix} u_{ix} \\ u_{iy} \end{pmatrix} = \underbrace{-\sum_{j\neq i}^{N} f_I(\|p_{ij}\|) \frac{p_{ij}}{\|p_{ij}\|}}_{\text{Inter Vehicle}} - \underbrace{f_h([[h_i]]) \frac{h_i}{[[h_i]]}}_{\text{Domain Vehicle}} - \underbrace{\frac{1}{N} \sum_{j\neq i}^{N} f_v(\|p_{ij}\|) v_{ij}}_{\text{Velocity Alignment}} + \underbrace{f_s(\|v_i\|) \frac{v_i}{\|v_i\|}}_{\text{Speed Alignment}} \tag{1}$$

I change the old f_v for f_s (because of speed word)

As done in [1] we choose $f_v(\|p_{ij}\|) = C_{al} e^{-\frac{\|p_{ij}\|}{l_{al}}}$, where C_{al} and l_{al} are constants associated to the velocity alignment strength and range respectively.

Note for Professors:
As we are using the speed alignment term to mimic the domain's speed. We could change the speed alignment term to get a more "homogeneous" model that generalizes the current one:

$$u_i = \underbrace{-\sum_{j\neq i}^{N} f_I(\|p_{ij}\|) \frac{p_{ij}}{\|p_{ij}\|}}_{\text{Inter Vehicle Position}} - \underbrace{f_h([[h_i]]) \frac{h_i}{[[h_i]]}}_{\text{Domain Vehicle Position}} - \underbrace{\frac{1}{N} \sum_{j\neq i}^{N} f_{vI}(\|p_{ij}\|) v_{ij}}_{\text{Inter Vehicle Velocity Alignment}} - \underbrace{f_{vh}([[h_i]]) w_i}_{\text{Domain Vehicle Velocity Alignment}}$$

where $w_i = v_i - v_{domain}$.

2.1 Choosing the Adequate Cucker Smale Parameters

We assume as premise that the major velocity alignment effects should be for those vehicles within an r_d radius neighborhood. It seems wide enough to guarantee flocking behavior without causing group inertia that may slow down the domain coverage aim. In order to so, we impose a tenth decay on the alignment strength every r_d, i.e. $l_{al} = -\frac{r_d}{\ln(0.1)}$.

3 Thresholding Coverage Control Force

As we assume constrained input forces, we to need modify the proposed coverage control force when necessary. For the double integrator model the given coverage control force $u = (u_x, u_y)$ is projected onto the set of admissible forces using the mapping,

$$\hat{u} = \begin{cases} u & \text{if } \|u\| \leq u_{max}, \\ u_{max}\frac{u}{\|u\|} & \text{otherwise.} \end{cases}$$

On the other hand, in order to get the appropriate Dubins car control force we use the relation

$$\frac{d}{dt}\begin{pmatrix} v_x \\ v_y \end{pmatrix} = R(v,\theta)\begin{pmatrix} u_\theta \\ u_v \end{pmatrix};\ R(v,\theta) := \begin{pmatrix} v\sin(\theta) & \cos(\theta) \\ v\cos(\theta) & -\sin(\theta) \end{pmatrix}, \tag{2}$$

which allow us to represent the set of admissible forces from the xy perspective as the region

$$S = \left\{ R(v,\theta)\begin{pmatrix} u_\theta \\ u_v \end{pmatrix} : \begin{pmatrix} u_\theta \\ u_v \end{pmatrix} \in [-u_{\theta max}, u_{\theta max}] \times [-u_{vmax}, u_{vmax}] \right\}$$

we set $\begin{pmatrix} \hat{u}_x \\ \hat{u}_y \end{pmatrix} = sup\left\{ t \in \mathbb{R} : t\begin{pmatrix} u_x \\ u_y \end{pmatrix} \in S \right\}\begin{pmatrix} u_x \\ u_y \end{pmatrix}$, see Fig. 1. Finally, we get the associated Dubins car input force by inverting (2) as $\begin{pmatrix} \hat{u}_\theta \\ \hat{u}_v \end{pmatrix} = R^{-1}(v,\theta)\begin{pmatrix} \hat{u}_x \\ \hat{u}_y \end{pmatrix}$.

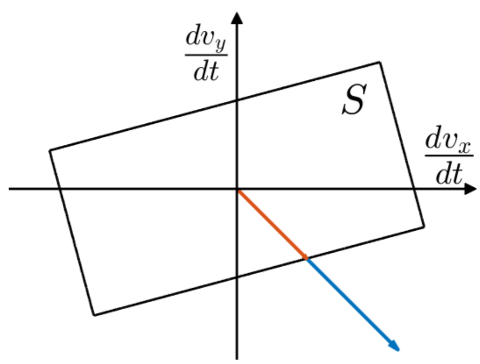

Figure 1: Thresholding Dubins car force in rectangular coordinates.

4 Two Comments on the Asymptotic Behavior

It is clear that thresholding the force the theoretical guarantees may not necessary hold anymore, however, when close to the desired operation point the coverage input forces are small enough to not be thresholded and it is feasible for the controller apply the required coverage force, implying the theoretical results are locally valid.

When vehicles are getting to the desired behavior the contribution of the velocity alignment term seems to be negligible respect to the others (We have to check this fact empirically, I am not sure this is the case). Therefore, the stability results without considering it are likely to hold. (Prof Razvan: You mentioned we have stability results for the Cucker-Smale type model then we may not need this comment, I am starting to read some work on it. I am including this because it was Prof Mo's suggestion but not clear for me now).

References

[1] RC Fetecau and A Guo. A mathematical model for flight guidance in honeybee swarms. *Bulletin of mathematical biology*, 74(11):2600–2621, 2012.