Binary Image Analysis
Week 2

Contents

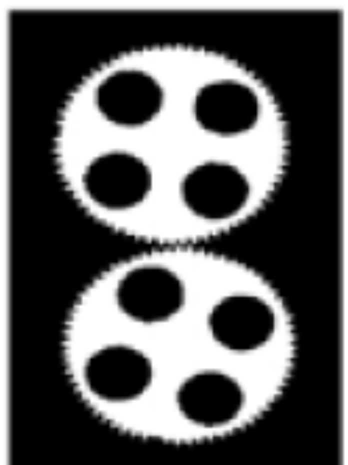
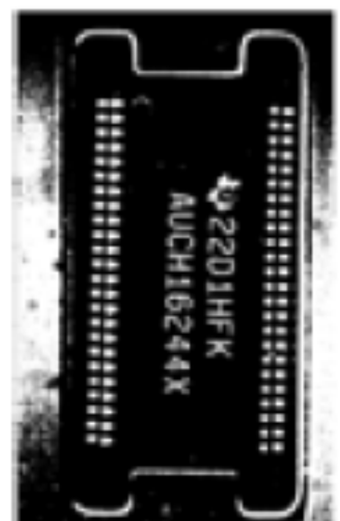
- Image threshold (histogram binarisation):
  how to get a binary image from a colour one ?

- Mathematical Morphology

Why binary ?





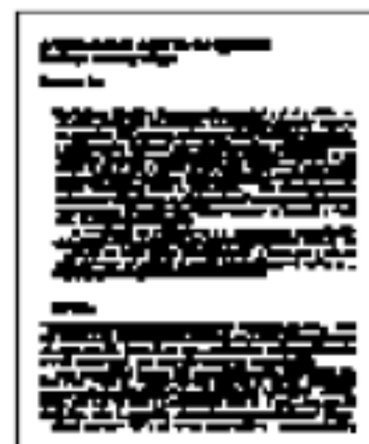


Image threshold (binarization)

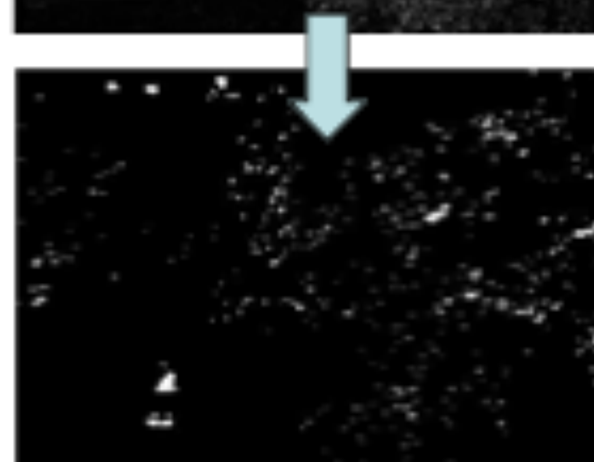
- What is **Thresholding** ?
  - It is a pixel labeling operation.
  - It assigns a binary value to each pixel.
    - **Binary Value 1**: pixels have higher intensity values
    - **Binary Value 0:** pixels have higher intensity values

$\xrightarrow{T=128}$

Thresholding

- Given a grayscale image or an intermediate matrix → threshold to create a binary output.

Example: background subtraction

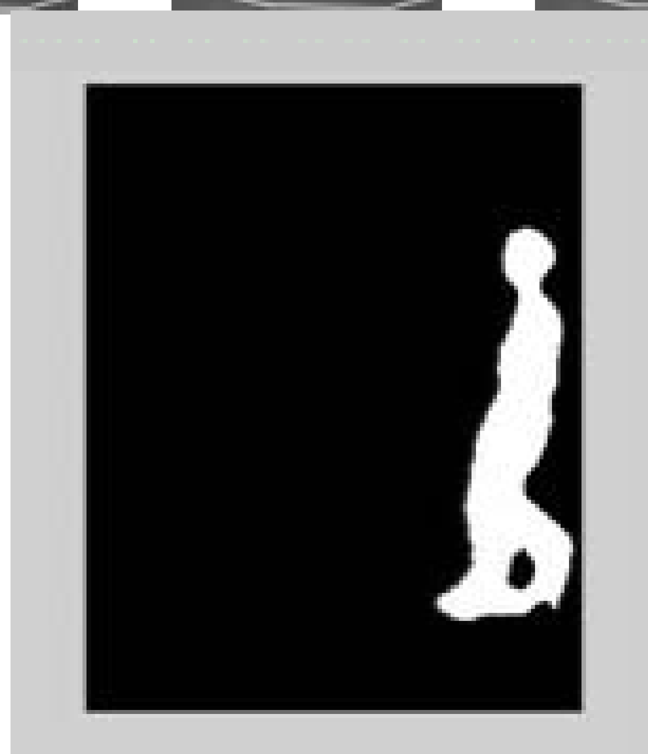


Looking for pixels that differ significantly from the “empty” background.

```
fg_pix = find(diff > t);
```

Example using binary image analysis: Bg subtraction + blob detection

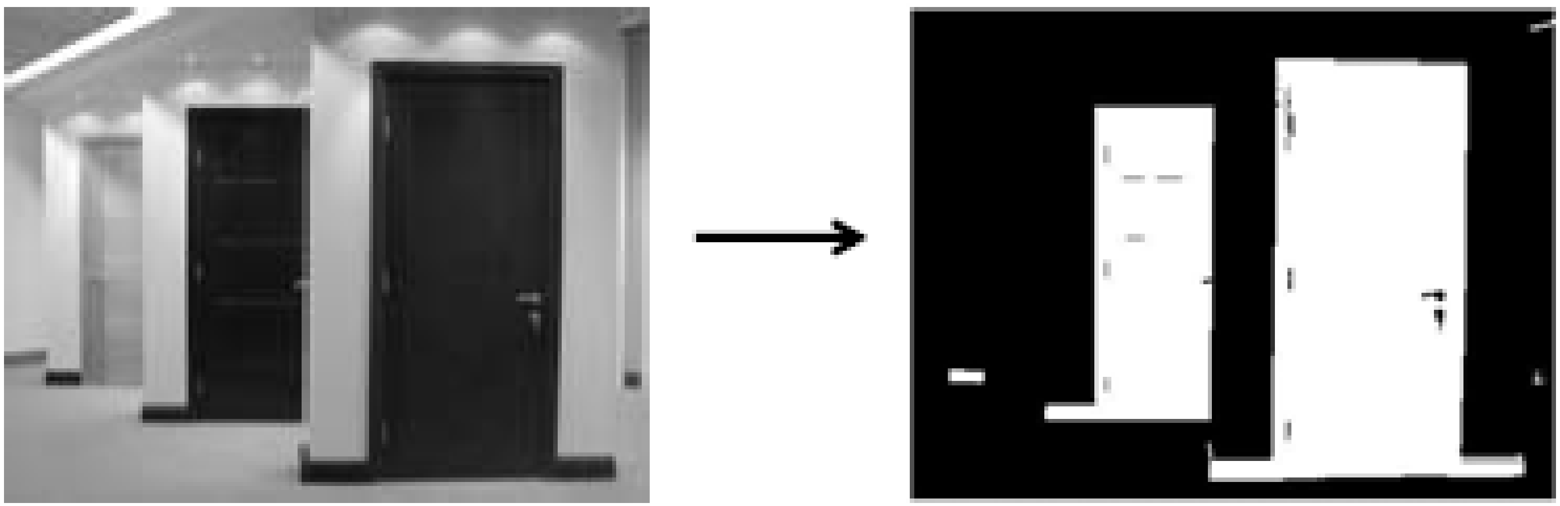
...



Thresholding

- Given a grayscale image or an intermediate matrix → threshold to create a binary output.

Example: intensity-based detection



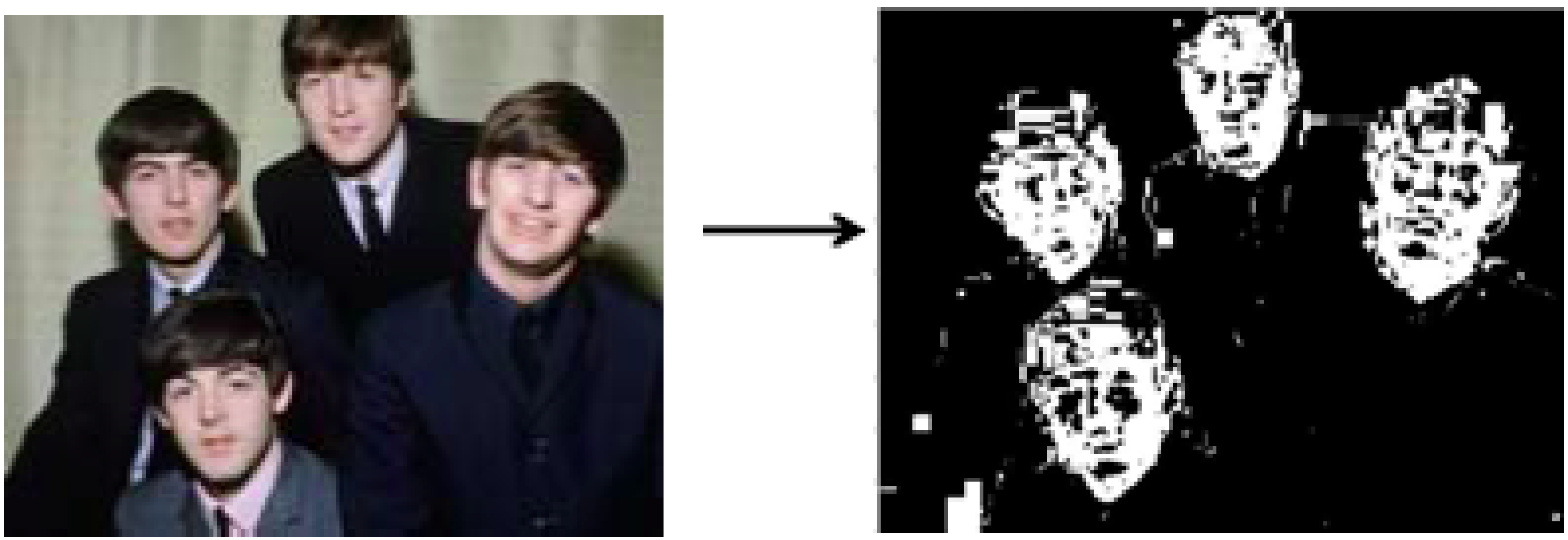
```
fg_pix = find(im < 65);
```

Looking for dark pixels

Thresholding

- Given a grayscale image or an intermediate matrix → threshold to create a binary output.

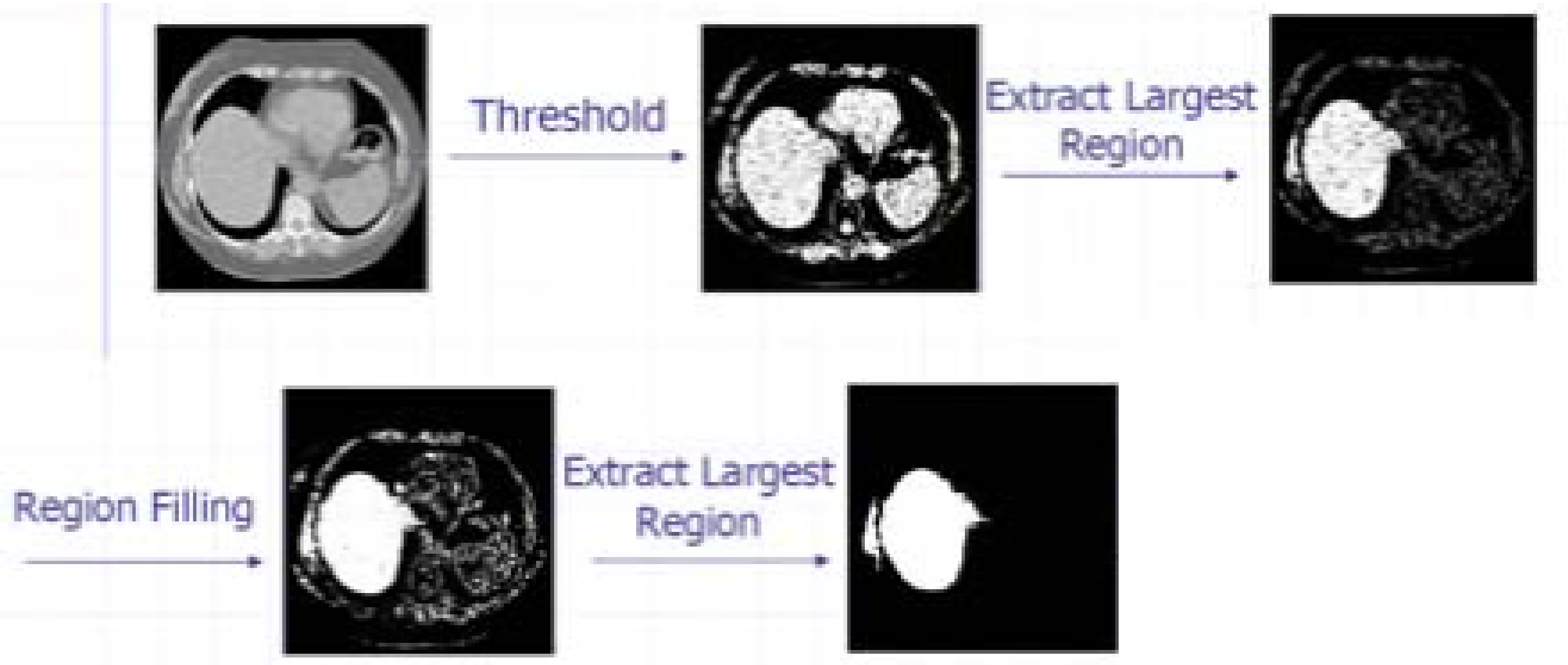
Example: color-based detection



```
fg_pix = find(hue > t1 & hue < t2);
```

Looking for pixels within a certain hue range.

Example using binary image analysis: segmentation of a liver



- Mathematical Formulation

$$B(r,c) = \begin{cases} 1 & if \quad I(r,c) \geq T \\ 0 & if \quad I(r,c) < T \end{cases}$$

  - (r,c) : row and column
  - $I(.,.)$: gray-level intensity image
  - T : intensity threshold
  - $B(.,.)$: binary intensity image

How to select an appropriate threshold ?

r \ c	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0									5	8	9					
1											9	8	9			
2										9	8	8	7	9	8	
3												8	9	6	9	9
4						5	5	5								
5						3	2	8	6							
6						3	8	8	6							
7					6	8	7	7								
8																
9																
10																
11																
12							6	1	1	1	4					
13							2	3	4	3	4					
14							2	2	3	3	2					
15							4	4	4	4						

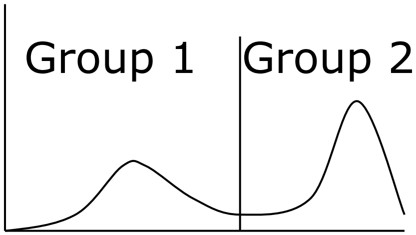
$I(.,.)$

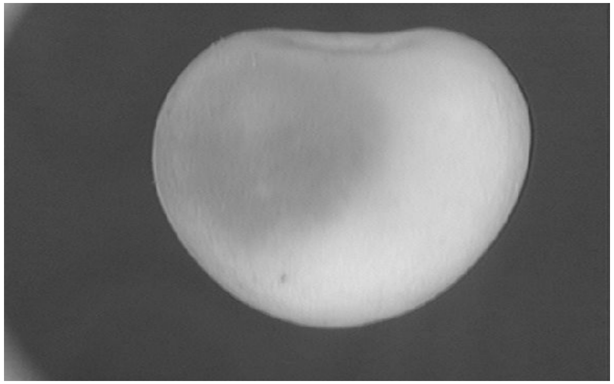
$$T = 5$$

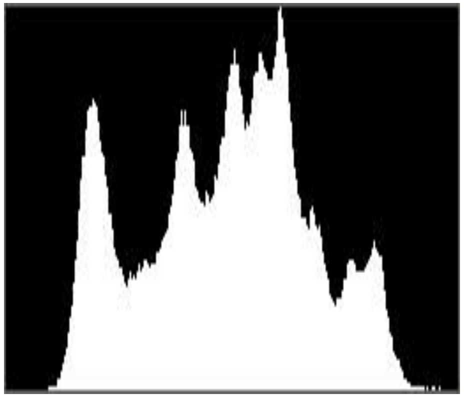
r \ c	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0									1	1	1					
1											1	1	1			
2										1	1	1	1	1	1	
3												1	1	1	1	1
4						1	1	1								
5								1	1							
6							1	1	1							
7					1	1	1	1								
8																
9																
10																
11																
12							1									
13																
14																
15																

$B(.,.)$

- Approach: Histogram analysis







Where to pick an optimal threshold?

$h(m)$

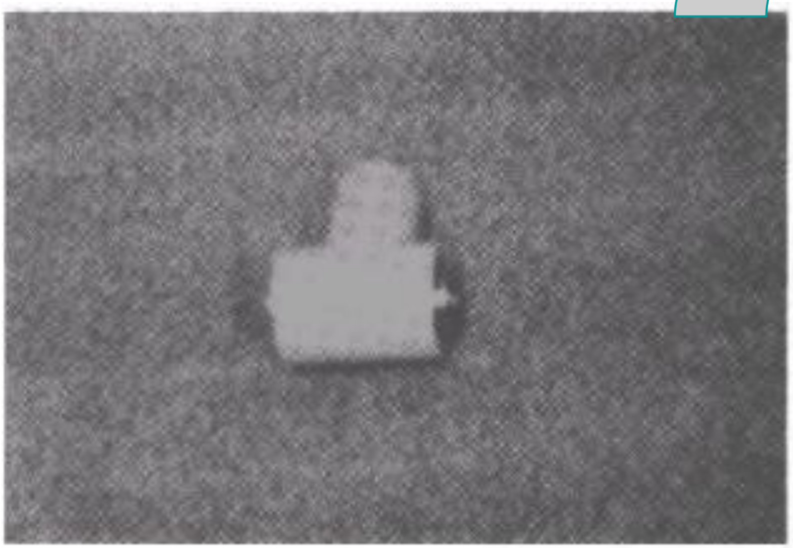


Figure 2.6 BNC T-connector against a dark background.

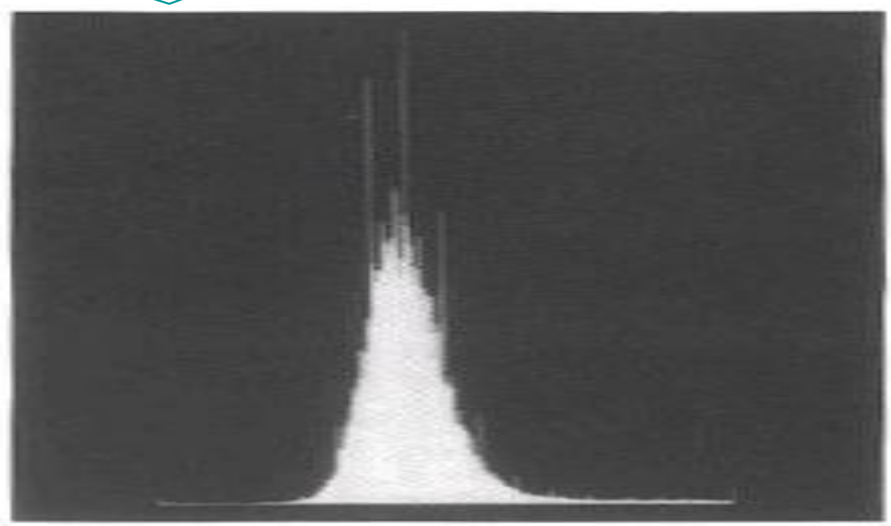
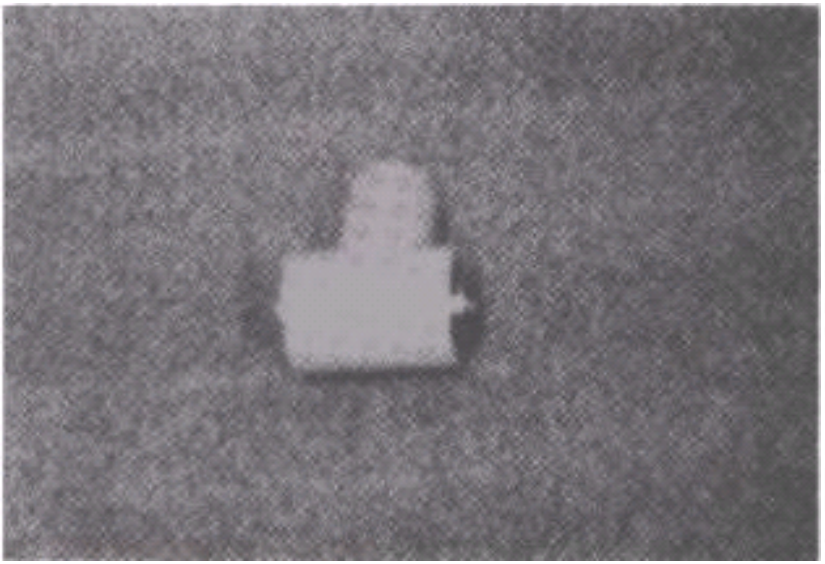


Figure 2.7 Histogram of the BNC T-connector image of Fig. 2.6.

Histogram



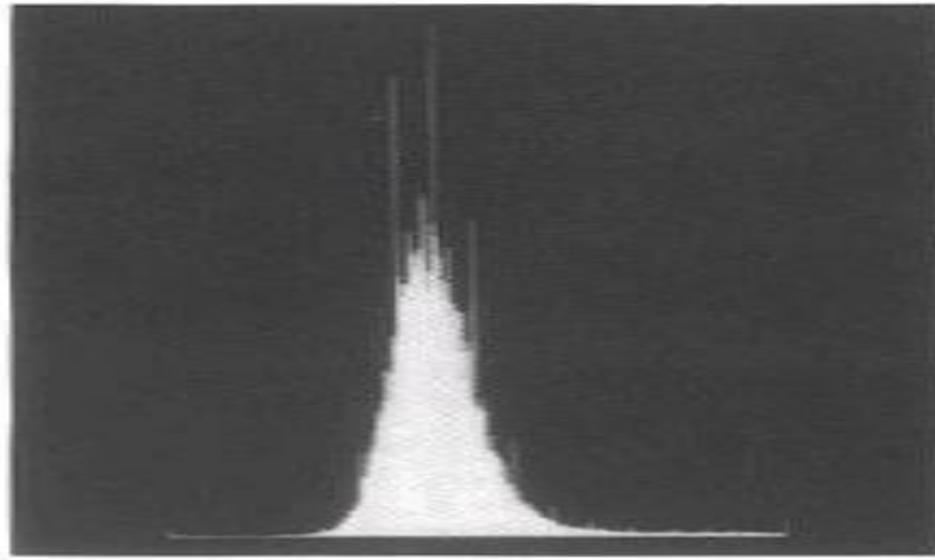
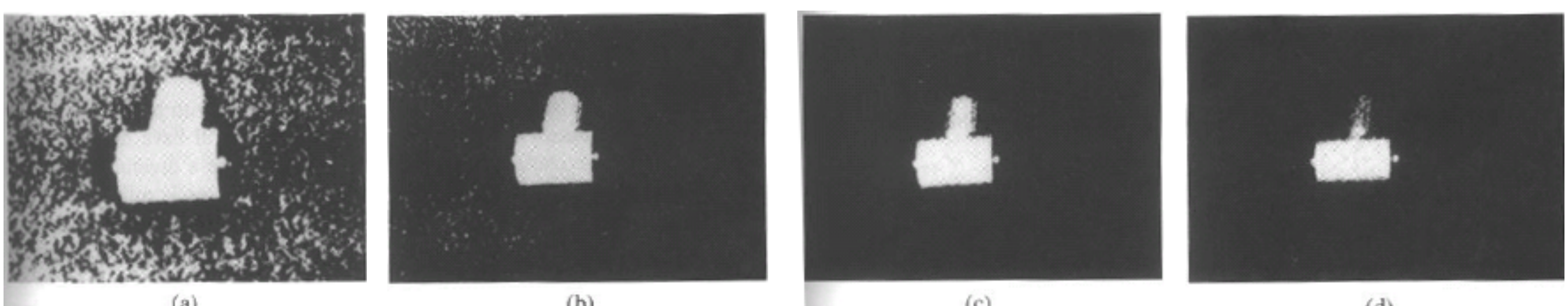


Figure 2.7 Histogram of the BNC T-connector image of Fig. 2.6.



(a) (b) (c) (d)

T=110 **T=130** **T=150** **T=170**

Basic Global Thresholding

Simple Idea:

1. Select an initial estimate for T.
2. Segment image using T into two groups G1 and G2.
3. Compute the average gray level value as μ_1 and μ_2
4. Compute a new threshold value:

$$T = \frac{1}{2}(\mu_1 + \mu_2)$$

5. Repeat steps 2 through 4 until the difference in T in successive iteration is smaller than a predefined parameter T_0

How to find an optimal threshold ?

- There is no generic solution.
- The optimal solution is often problem-dependent;

- A heuristic binarisation algorithm based on K-L divergence:

  – fitting a 2-mode Gaussian mixture model to the histogram: minimizing K-L divergence.

Region Property Computation

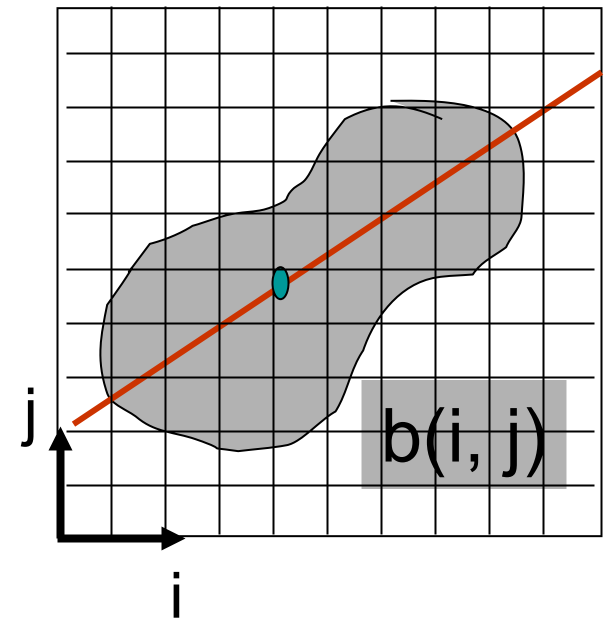
Region property computation

- For each connected region, we can compute many of its properties, e.g.
  - Areas
  - Perimeter
  - Centre of gravity
  - circularity,
  - Major axis
  - Minor axis
  - mean and standard deviation of radial distance
  - bounding box
  - extremal axis length from bounding box
  - **second order moments** (row, column, mixed)
  - …

Region properties

- Assume:

    b(i, j) is discrete
    only one object



- Areas (0-th Moment) $A = \sum\sum b_{ij}$

- Center of Mass (First Moment)

$$\bar{x} = \frac{1}{A}\sum\sum i\, b_{ij} \qquad \bar{y} = \frac{1}{A}\sum\sum j\, b_{ij}$$

- Second Moments:

$$a' = \sum\sum i^2\, b_{ij} \qquad b' = 2\sum\sum ij\, b_{ij} \qquad c' = \sum\sum j^2\, b_{ij}$$

Note: a’,b’,c’ are defined w.r.t origin

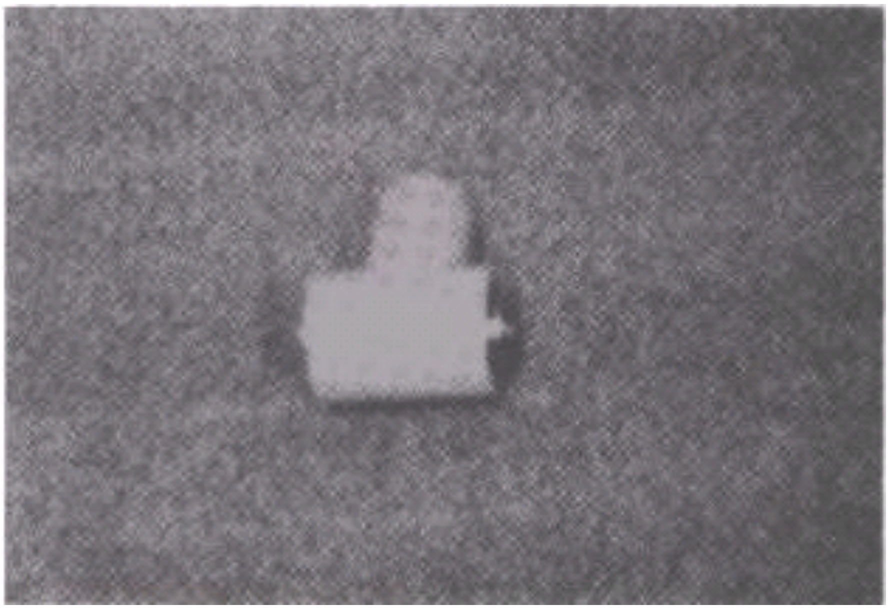
Mathematical Morphology

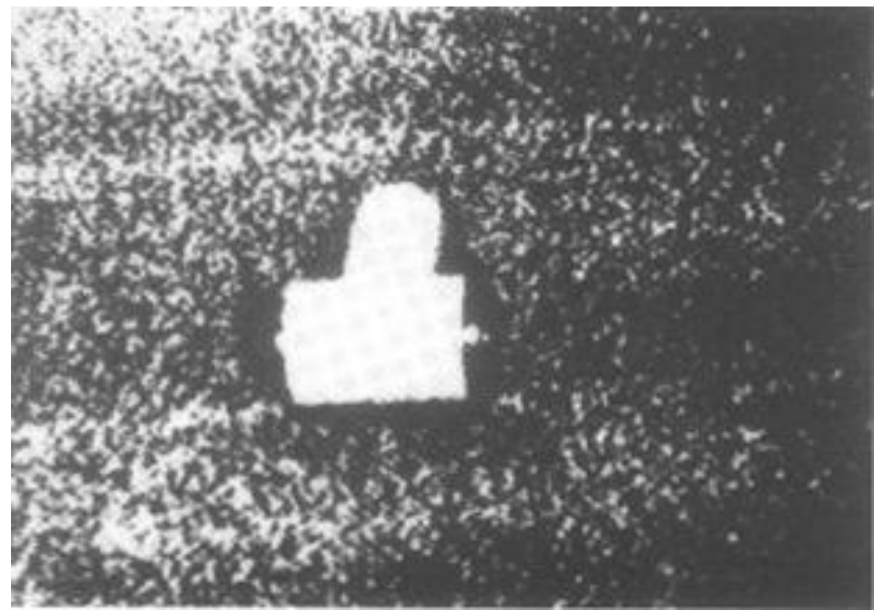
- **Morphology**: originally, a branch of biology science that studies the form and structure of animals and plants

- Mathematical Morphology in image processing is used to extract image components for representation and description of region shape, such as boundaries, skeletons, and the convex hull.

Motivation: remove small noise regions

- Example result by a binarisation algorithm





Mathematical Morphology

In binary image processing, mathematical morphology consists of two basic operations,

dilation , erosion

and several composite operations

closing , opening, . . .

LE 9.1
three basic
cal operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

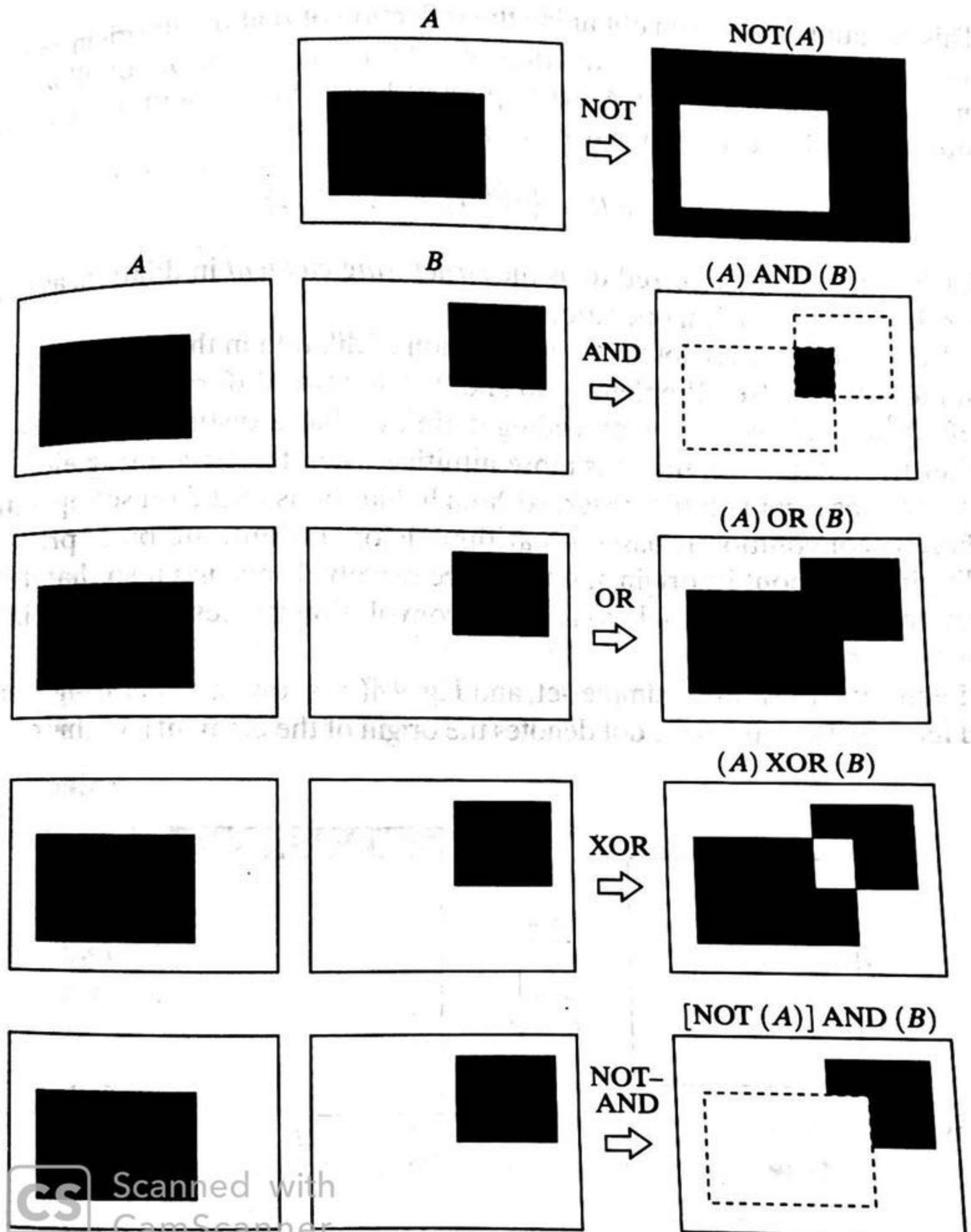


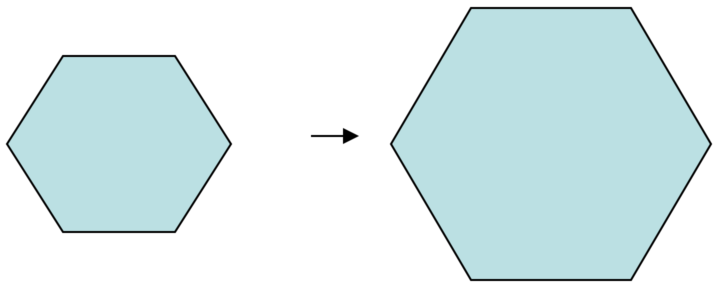
FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Dilation

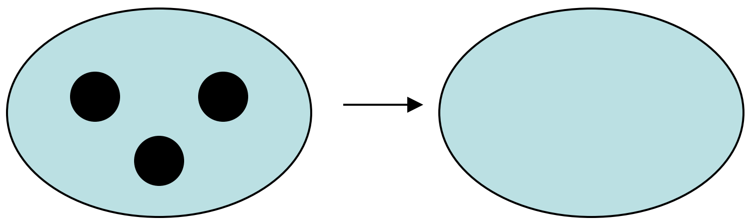
Dilation expands the connected sets of 1s of a binary image.

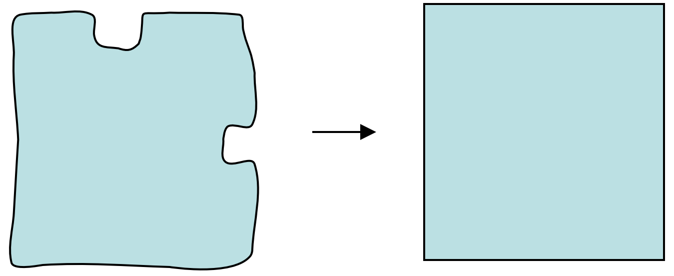
It can be used for

1. growing features



2. filling holes and gaps

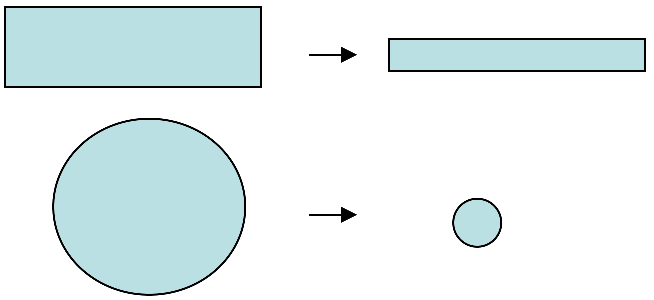




Erosion

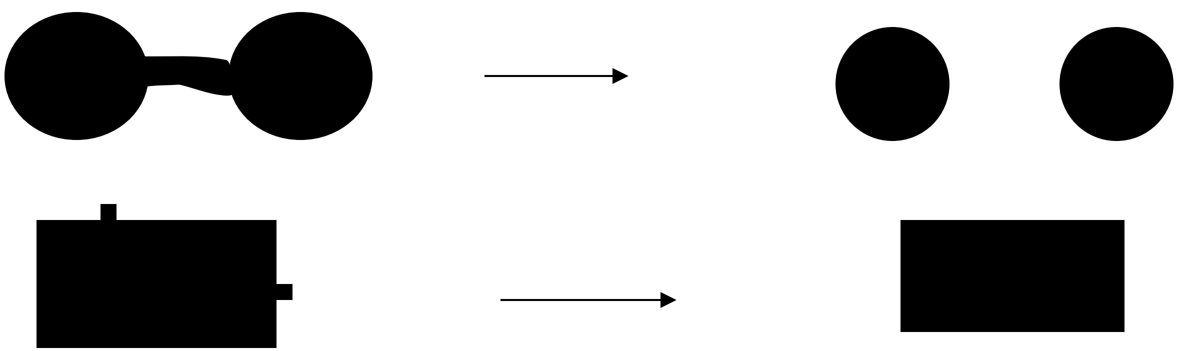
Erosion shrinks the connected sets of 1s of a binary image.

It can be used for



1. shrinking features

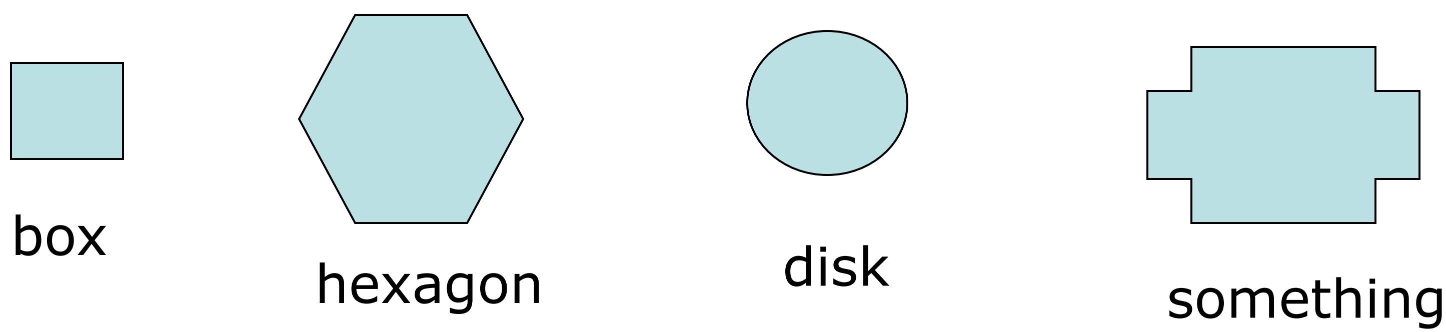
2. Removing bridges, branches and small protrusions



Structuring Elements

A structuring element is a shape mask used in the basic morphological operations.

They can be any shape and size that is digitally representable, and each has an origin.



box(length,width)

disk(diameter)

Opening and Closing

- Closing is the composite operation of dilation followed by erosion (with the same structuring element)
- Opening is the composite operation of erosion followed by dilation (with the same structuring element)

- Detailed explanations

  - Dilation
  - Erosion
  - Opening
  - Closing

Matlab

- `N = hist(Y,M)`
- `L = bwlabel (BW,N);`
- `STATS = regionprops(L,PROPERTIES) ;`
  - `'Area'`
  - `'Centroid'`
  - `'BoundingBox'`
  - `'Orientation‘, …`
- `IM2 = imerode(IM,SE);`
- `IM2 = imdilate(IM,SE);`
- `IM2 = imclose(IM, SE);`
- `IM2 = imopen(IM, SE);`

DILATION

dilate(B,S) takes binary image B, places the origin of structuring element S over each 1-pixel, and ORs the structuring element S into the output image at the corresponding position.

Dilation

- **Dilation** is the set of all points in the image, where the origin of the structuring element “hits” the foreground.
- Consider each foreground pixel in the input image
  - If the structuring element origin hits the foreground image, “OR” the structuring element to the image.
- **Input:**
  - **Binary Image**
  - **Structuring Element, containing only 1s!!**

Example for Dilation

Input image

0	0	0	0	1	1	1	0	1	1

Structuring Element

1	1*	1

Output Image

	0								

Example for Dilation

Input image

0	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0							

Example for Dilation

Input image

0	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	1						

Example for Dilation

Input image

0	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	1	1					

Example for Dilation

Input image

0	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	1	1	1				

Example for Dilation

Input image	0	0	0	0	1	1	1	0	1	1
Structuring Element						1	1	1		
Output Image		0	0	1	1	1	1			

Example for Dilation

Input image

0	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	1	1	1	1	1		

Example for Dilation

Input image

0	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	1	1	1	1	1	1	

Example for Dilation

Input image	0	0	0	0	1	1	1	0	1	1
Structuring Element								1	1	1
Output Image		0	0	1	1	1	1	1	1	1

Example for Dilation

Input image

0	0	0	0	1	1	1	0	1	1

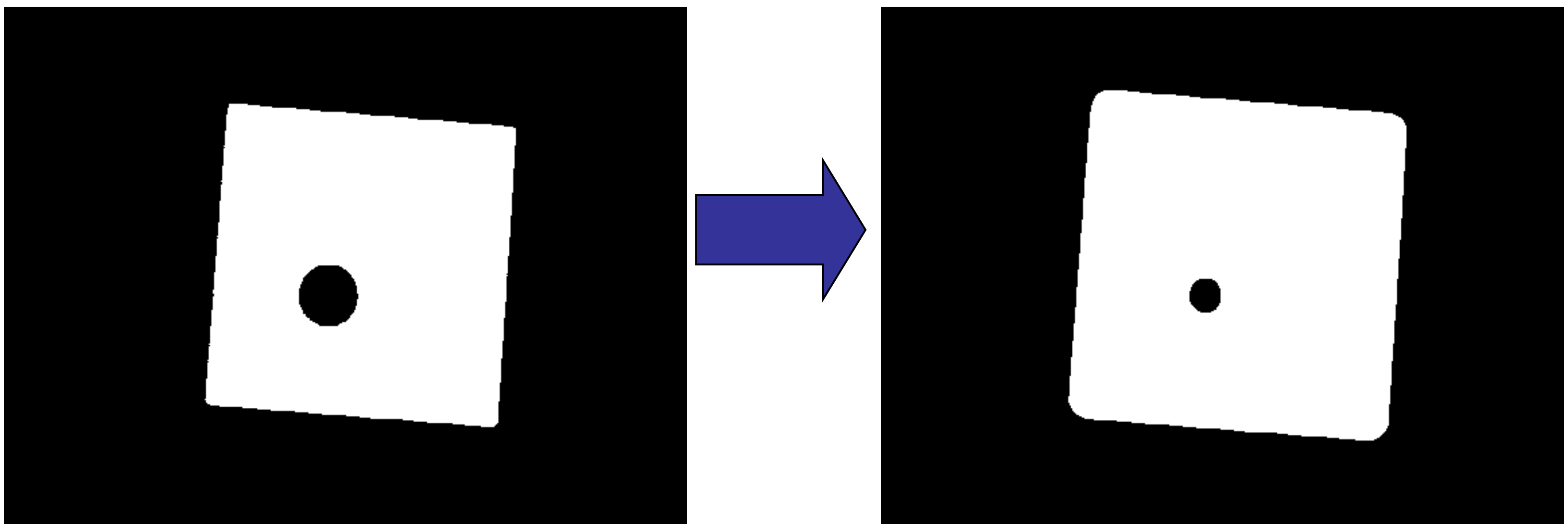
Structuring Element

1	1	1

Output Image

0	0	0	1	1	1	1	1	1	1

Another Dilation Example

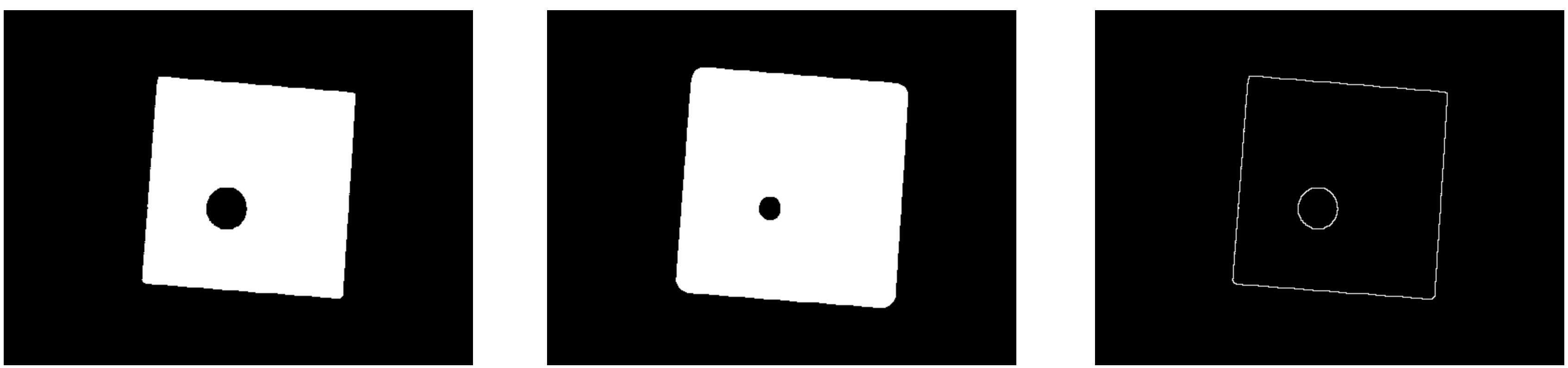


- Image get lighter, more uniform intensity

Edge detection example

- Edge Detection

1. Dilate input image
2. Subtract input image from dilated image
3. Edges remain!

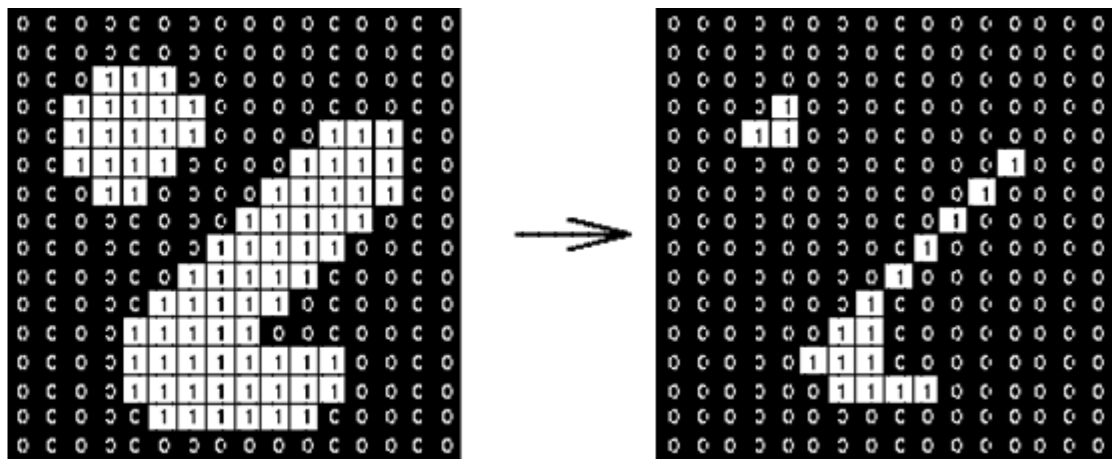


EROSION

erode(B,S) takes a binary image B, places the origin of structuring element S over every pixel position, and ORs a binary 1 into that position of the output image, if and only if every position of S (with a 1) covers a 1 in B.

Erosion

- **Erosion** is a basic morphological operation



- Applied **Structuring Element**:

1	1	1
1	1	1
1	1	1

Set of coordinate points =

{ (-1, -1), (0, -1), (1, -1),

(-1, 0), (0, 0), (1, 0),

(-1, 1), (0, 1), (1, 1) }

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0								

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0							

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0						

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0	0					

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0	0	1				

Example for Erosion

Input image	1	0	0	0	1	1	1	0	1	1
Structuring Element						1	1	1		
Output Image		0	0	0	0	1	0			

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0	0	1	0	0		

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

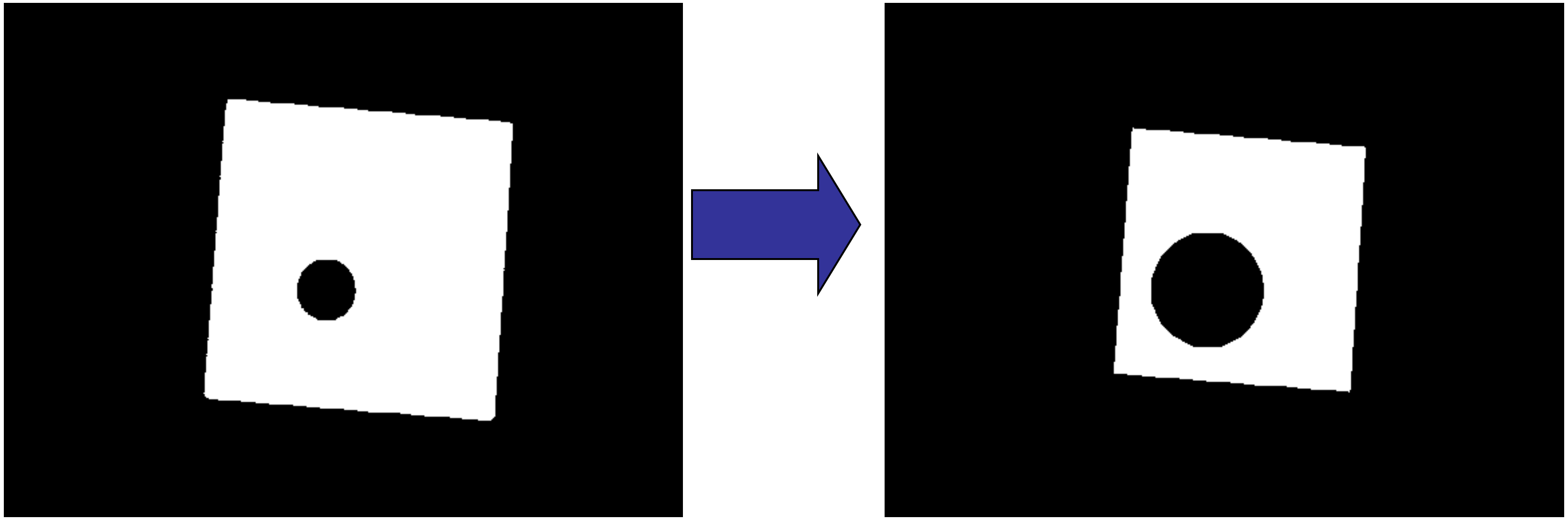
Structuring Element

1	1	1

Output Image

	0	0	0	0	1	0	0	0	

An example of erosion

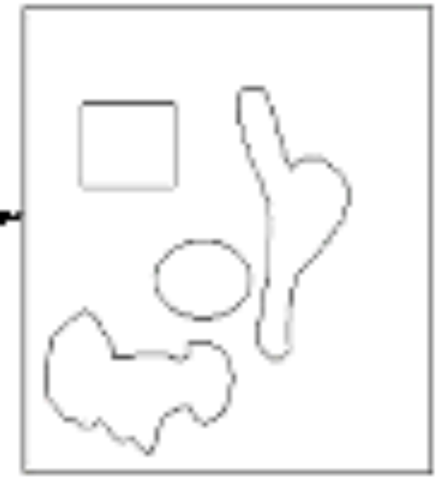


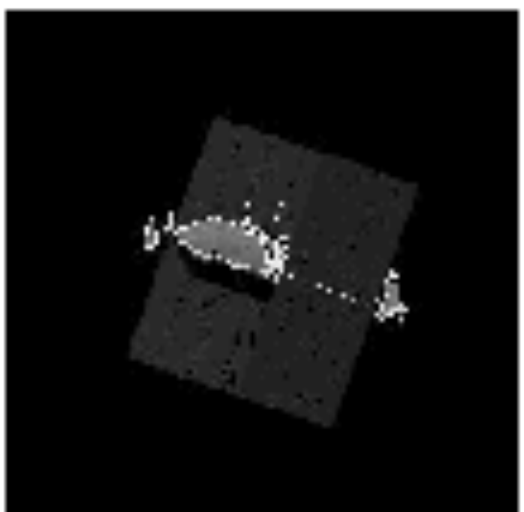
Binary image contour extraction

image

```
Erosed=erode(image);
Contour=
  double(image)-double(Erosed);
Contour=~Contour;
```

Contour





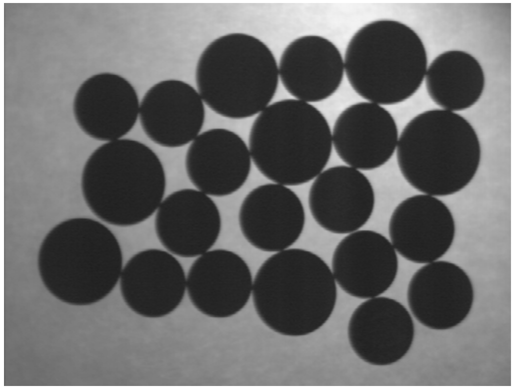
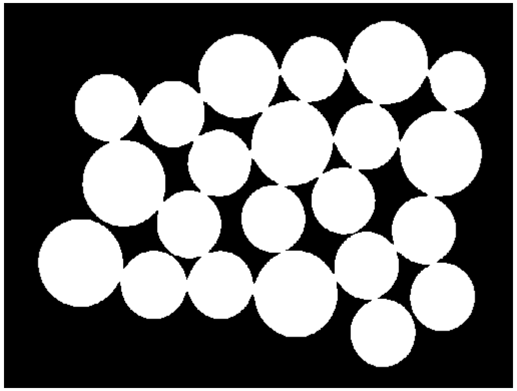
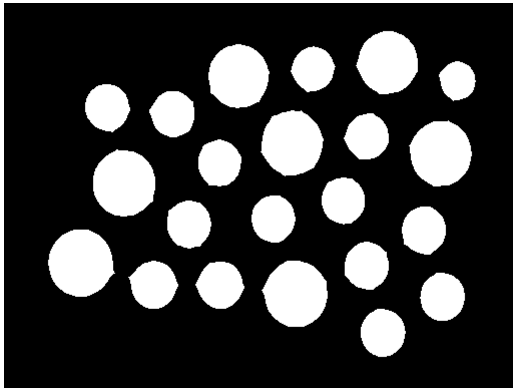
Satellite image with contour.

Erosion can be used to find contour

Dilation can be also used for it.

Counting Coins

- Counting coins is difficult because they touch each other!
- Solution: Binarization and Erosion separates them!

OPENING

Opening & Closing

- Important operations
- Derived from the two basic operations
  - Dilation
  - Erosion
- Usually applied to binary images.
- Opening and closing are dual operations

Opening & Closing

- Opening is the *dual* of closing:
-
  - *i.e.* opening the foreground pixels with a particular structuring element

  - is equivalent to closing the background pixels with the same element.

Opening

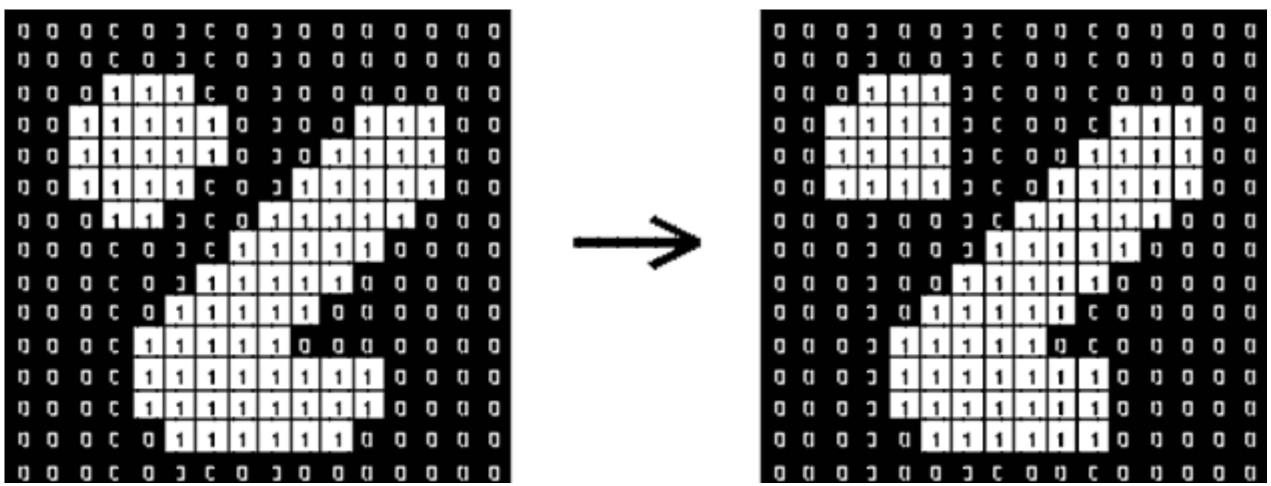
- Similar to Erosion
  - Spot and noise removal
  - Less destructive
- First Erosion, followed by dilation
- the *same structuring element for both operations.*
- Input:
  - Binary Image
  - Structuring Element, containing only 1s!

Opening

- Opening is **idempotent:** Repeated application has no further effects!

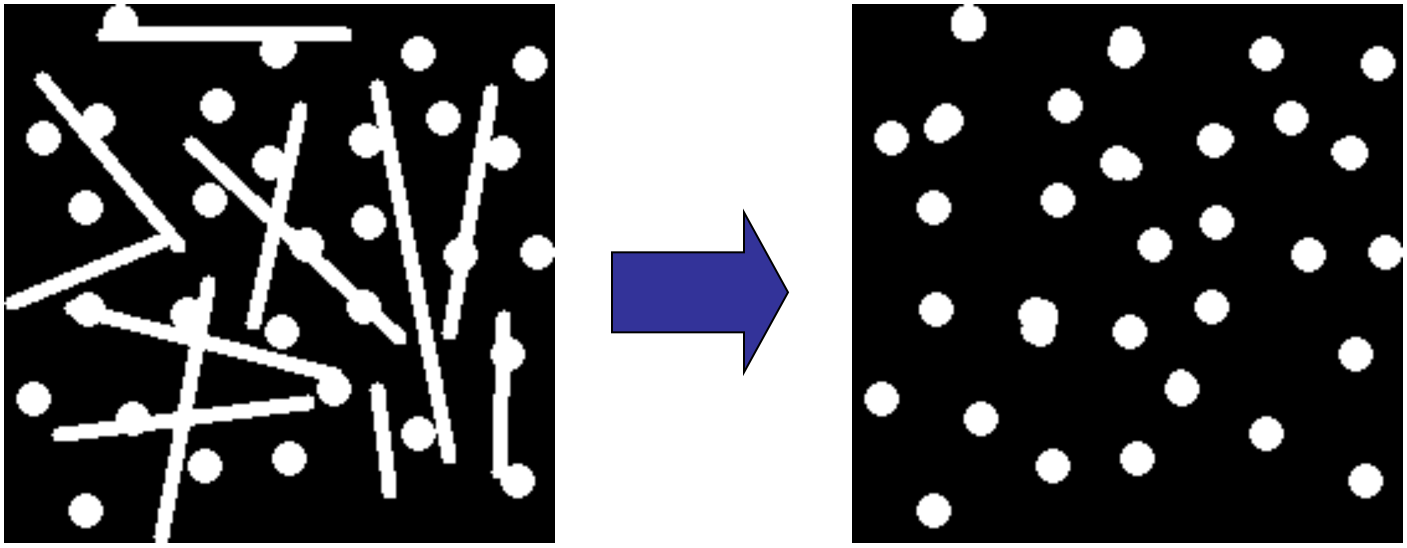
Opening

- Structuring element: 3x3 square



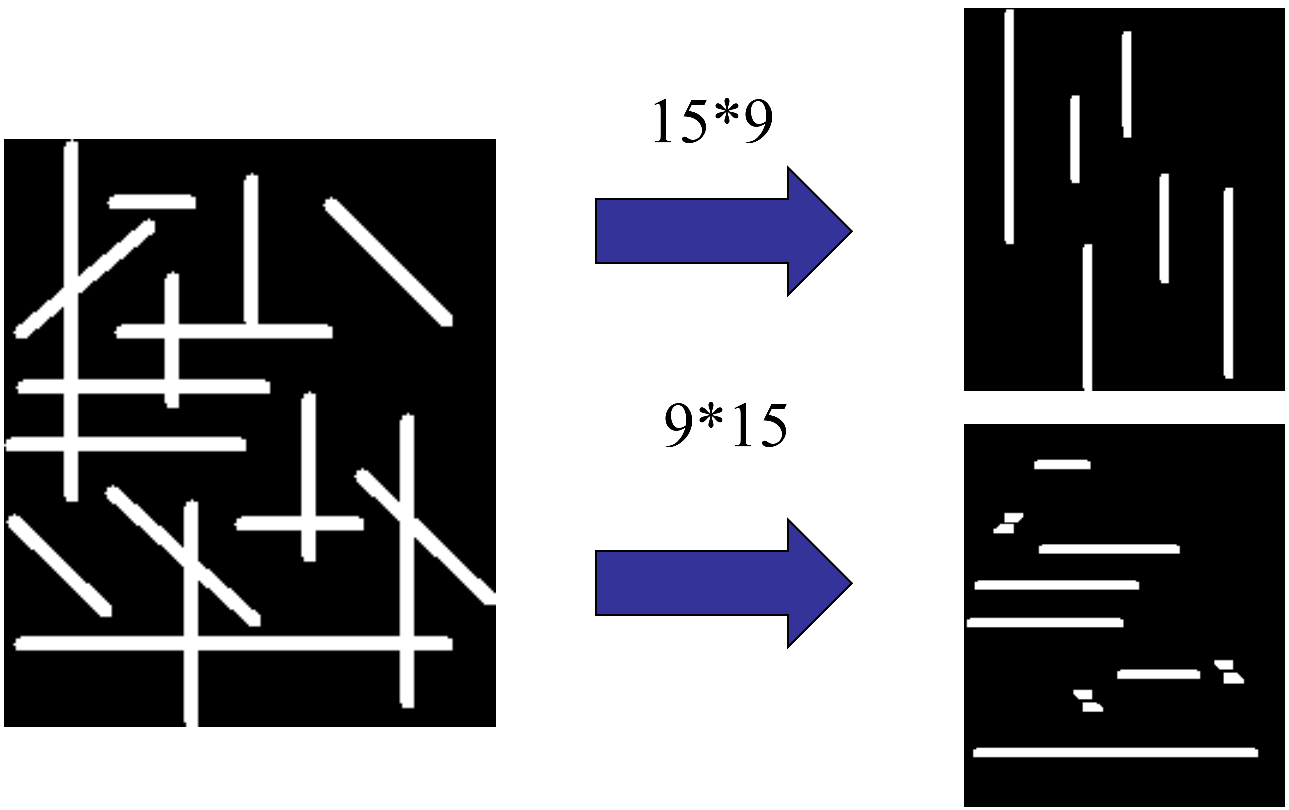
Opening Example

- Opening with a 11 pixel diameter disc



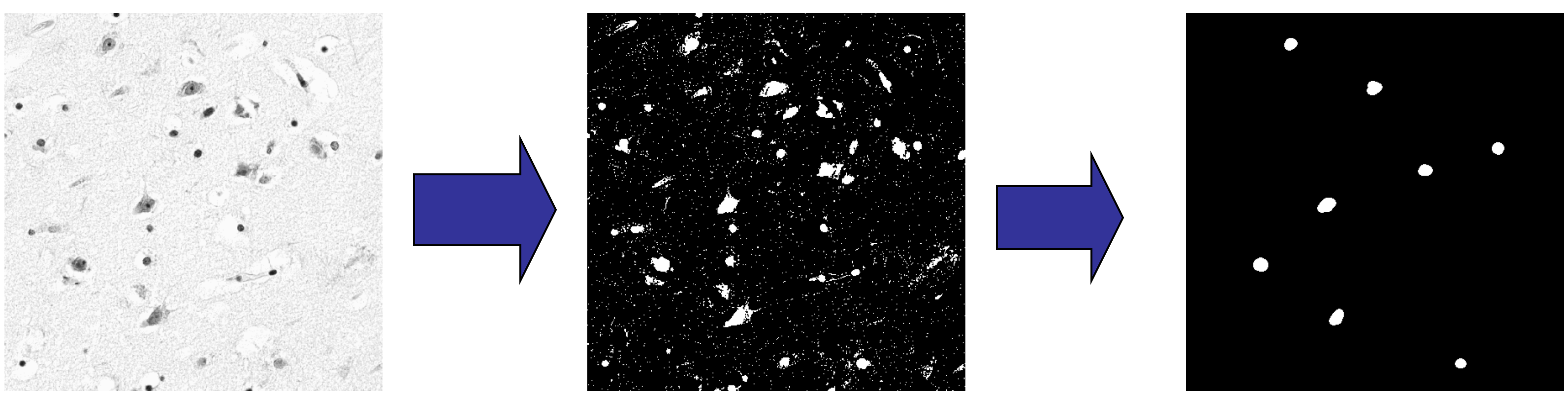
Opening Example

- 3x9 and 9x3 Structuring Element



Use Opening for Separating Blobs

- Use large structuring element that fits into the big blobs
- Structuring Element: 11 pixel disc



Use Opening to extract corners

Original

Opening

Corners

1. What kind of structuring element was used in the opening?
2. How did we get the corners?

CLOSING

Closing

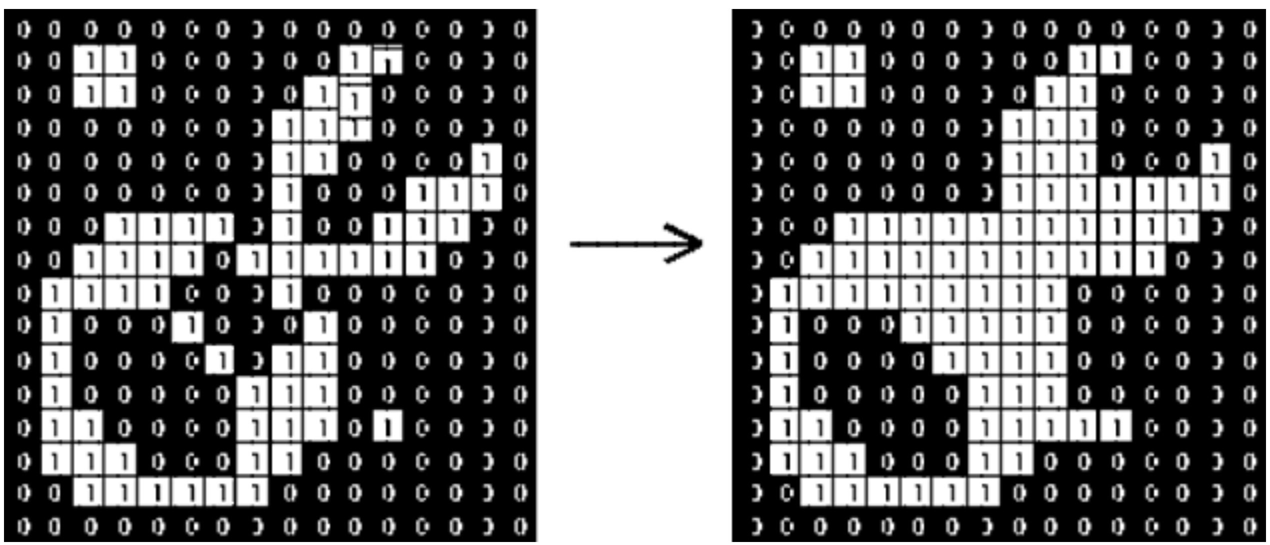
- Similar to Dilation
  - Removal of holes
  - Tends to enlarge regions, shrink background
- Closing is defined as a Dilatation, followed by an Erosion *using the same structuring element for both operations.*
- Dilation next erosion!
- Input:
  - Binary Image
  - Structuring Element, containing only 1s!

Closing

- Closing is **idempotent:** Repeated application of ‘closing’ has no further effects!

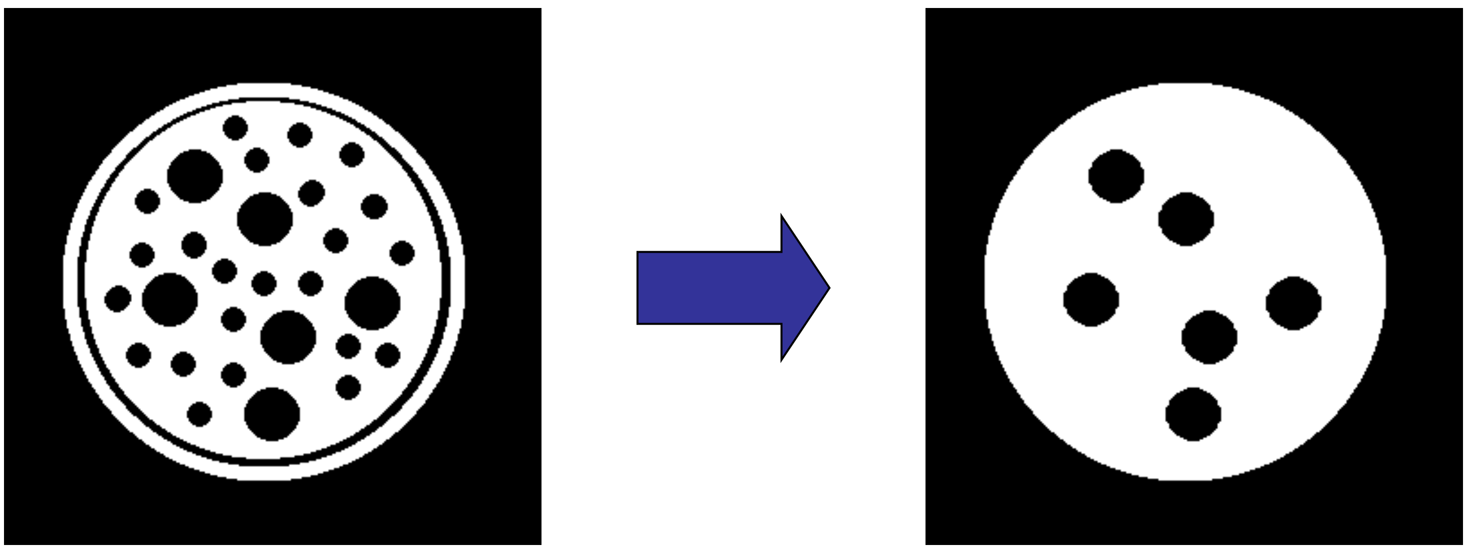
Closing

- Structuring element: 3x3 square



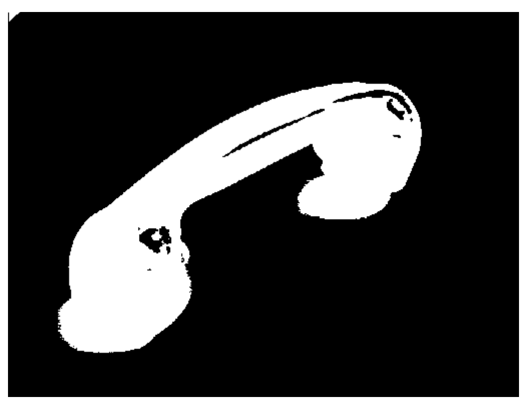
Closing Example

- Closing operation with a 22 pixel disc
- Closes small holes in the foreground



Closing Example

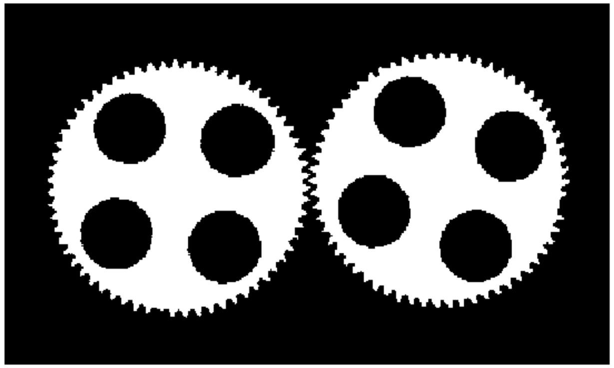
1. Threshold
2. Closing with disc of size 20



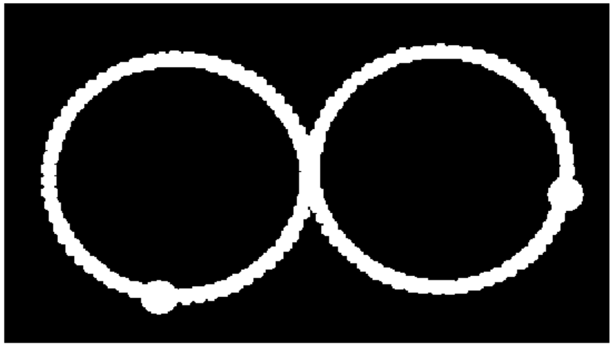
Threshold

closing

Sample Application: Gear-tooth Inspection



original binary image



detected defects

Sample Application: PCB board defect detection

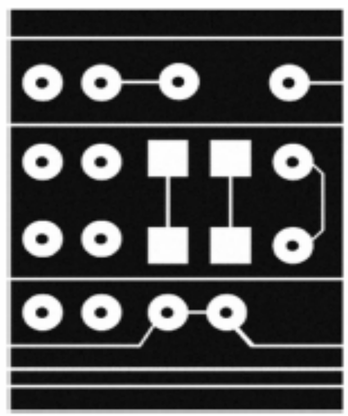


Fig. 1 Template Greyscale PCB Image

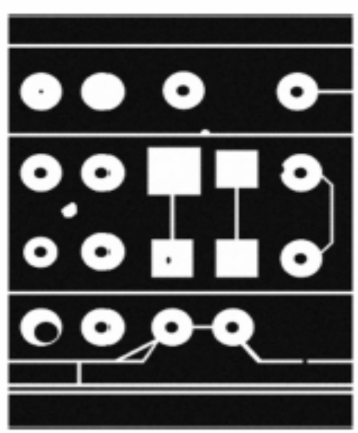


Fig. 2 Test Greyscale PCB Image