Programming Language Syntax

- LL parsing -

Chapter 2, Section 2.3

Parsing

- Parser
  - in charge of the entire compilation process
    - *Syntax-directed translation*
  - calls the scanner to obtain tokens
  - assembles the tokens into a syntax tree
  - passes the tree to the later phases of the compiler
    - semantic analysis
    - code generation
    - code improvement
  - a parser is a language *recognizer*
  - context-free grammar is a language *generator*

Parsing

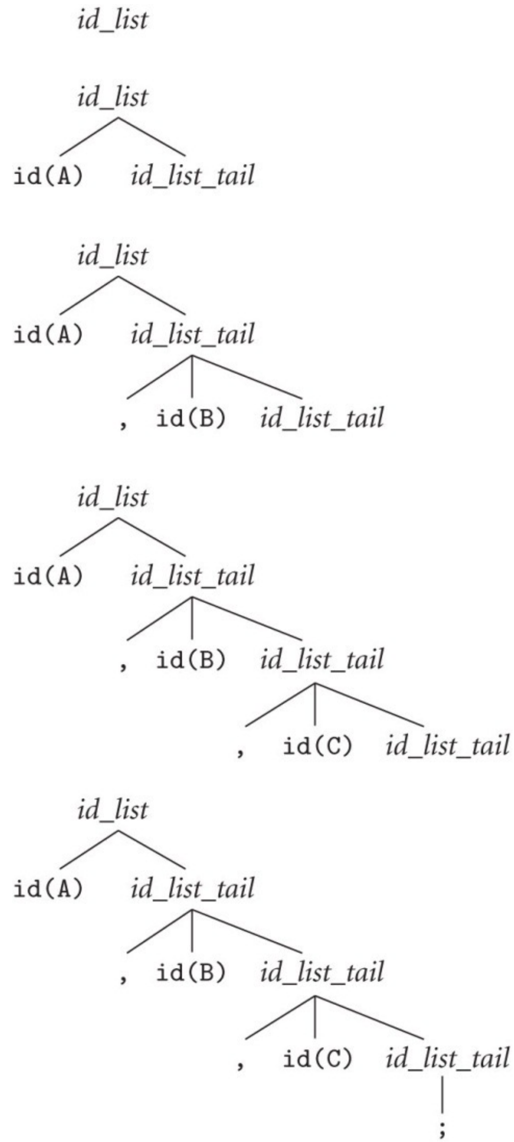
- Context-free language recognition
  - Earley, Cocke-Younger-Kasami alg's
  - $O(n^3)$ time
    - too slow
  - There are classes of grammars with $O(n)$ parsers:
    - **LL**: '**L**eft-to-right, **L**eftmost derivation'.
    - **LR**: '**L**eft-to-right, **R**ightmost derivation′

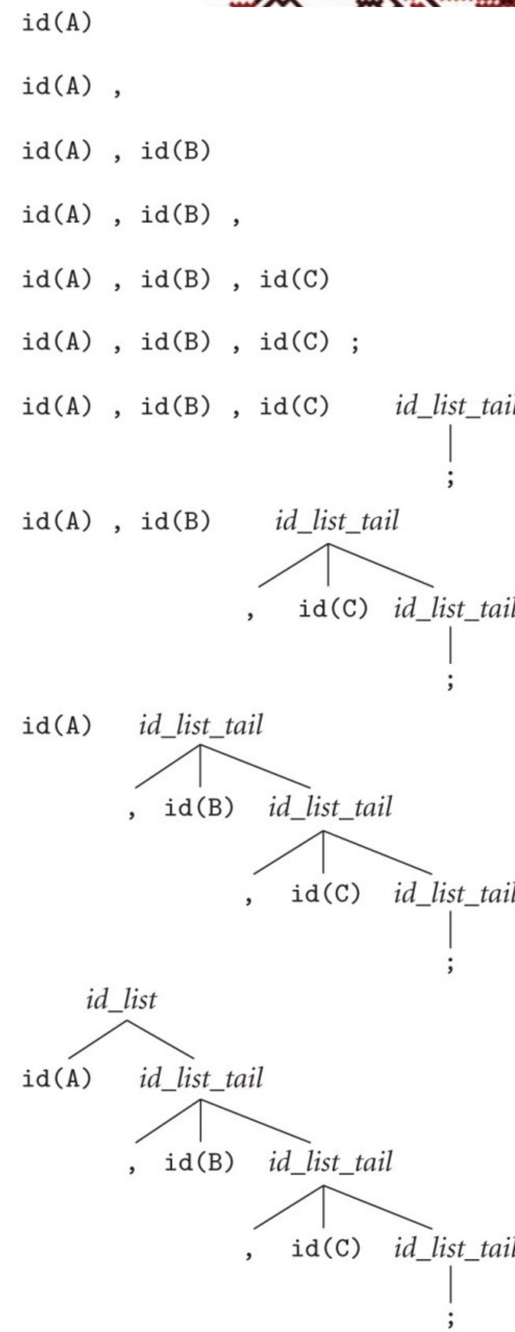
Class	Direction of scanning	Derivation discovered	Parse tree construction	Algorithm used
LL	left-to-right	left-most	top-down	predictive
LR	left-to-right	right-most	bottom-up	shift-reduce

Parsing

- Top-down vs. Bottom-up
- Top-down
  - *predict* based on next token
- Bottom-up
  - *reduce* right-hand side
  - Example:

    `A, B, C;`



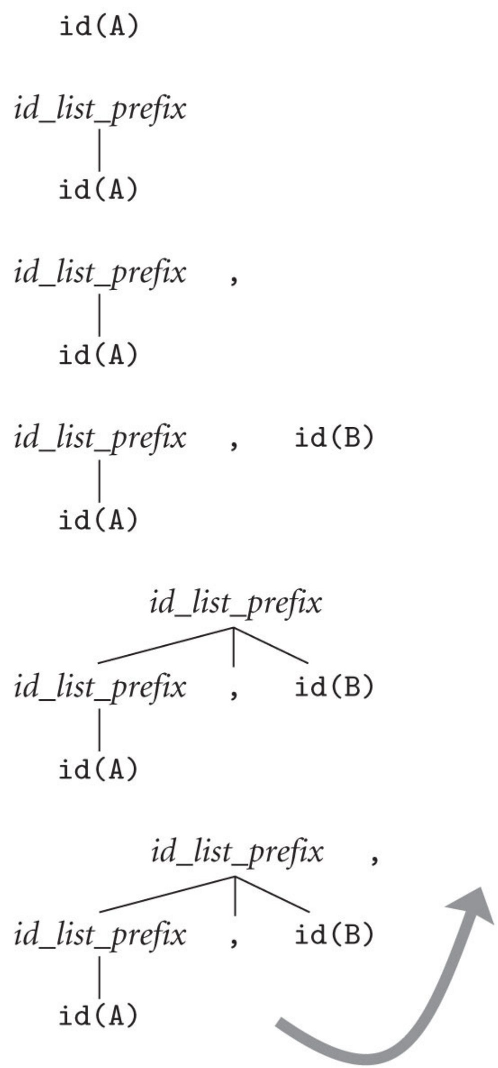


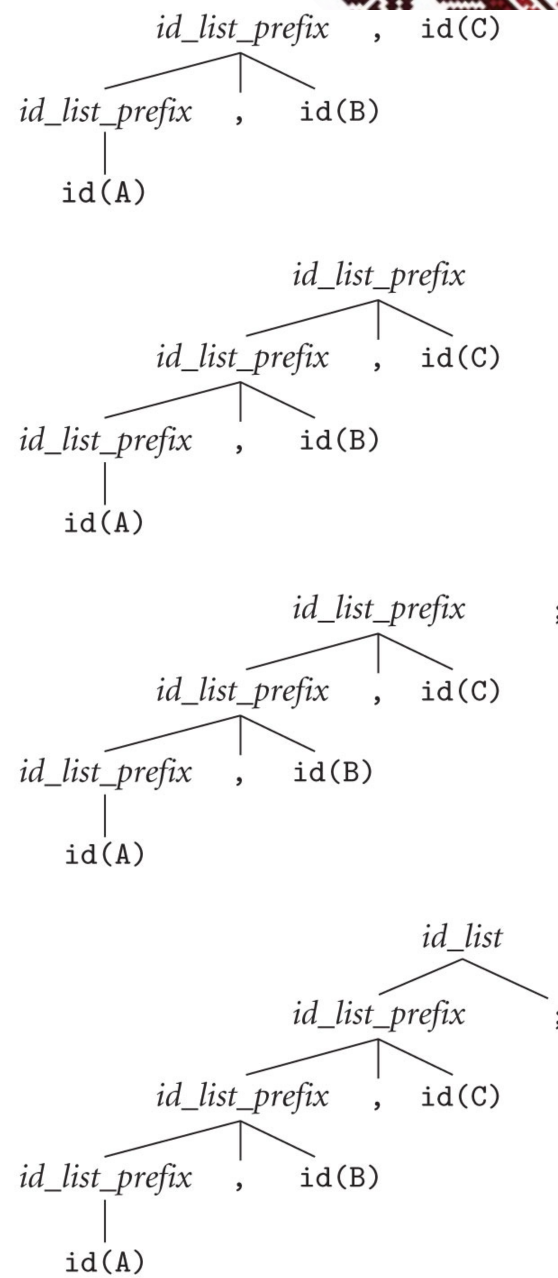
id_list ⟶ `id` *id_list_tail*
id_list_tail ⟶ `,` `id` *id_list_tail*
id_list_tail ⟶ `;`

Parsing

- Bottom-up
  - better grammar
  - cannot be parsed top-down
  - Example:

    `A, B, C;`





id_list ⟶ *id_list_prefix* ;

id_list_prefix ⟶ *id_list_prefix* , id

⟶ id

Parsing

- LL(k), LR(k)
  - k = no. tokens of look-ahead required to parse
  - almost all real compilers use LL(1), LR(1)
  - LR(0) - *prefix property*:
    - no valid string is a prefix of another valid string

LL Parsing

- LL(1) grammar for calculator language
  - less intuitive: operands not on the same right-hand side
  - parsing is easier (`$$` added to mark the end of the program)

program → *stmt_list* `$$`
stmt_list → *stmt stmt_list* | ε
stmt → `id := ` *expr* | `read id` | `write` *expr*
expr → *term term_tail*
term_tail → *add_op term term_tail* | ε
term → *factor fact_tail*
fact_tail → *mult_op fact fact_tail* | ε
factor → `(` *expr* `)` | `id` | `number`
add_op → `+` | `-`
mult_op → `*` | `/`

- compare with LR grammar:

expr → *term* | *expr add_op term*
term → *factor* | *term mult_op factor*
factor → `id` | `number` | `-` *factor* | (*expr*)
add_op → `+` | `-`
mult_op → `*` | `/`

- Top-down parsers
  - by hand – *recursive descent*
  - table-driven

LL Parsing

- Recursive descent parser
  - one subroutine for each nonterminal
- Example:

```
read A
read B
sum := A + B
write sum
write sum / 2
```

  - Continued on the next slide

```
procedure match(expected)
    if input_token = expected then consume_input_token()
    else parse_error

–– this is the start routine:
procedure program()
    case input_token of
        id, read, write, $$ :
            stmt_list()
            match($$)
        otherwise parse_error

procedure stmt_list()
    case input_token of
        id, read, write : stmt(); stmt_list()
        $$ : skip        –– epsilon production
        otherwise parse_error
```

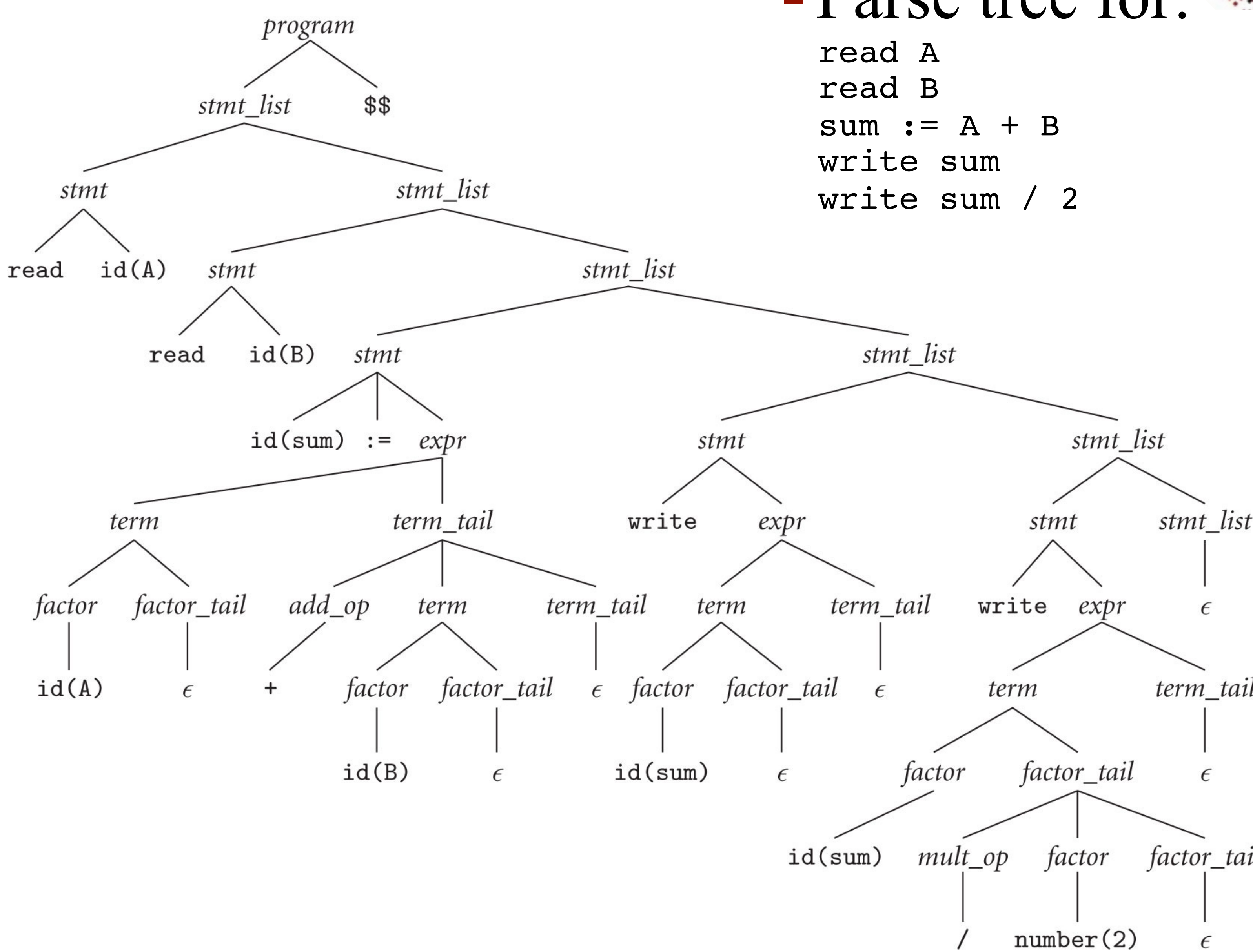
LL Parsing

```
procedure stmt()
    case input_token of
        id : match(id); match(:=); expr()
        read : match(read); match(id)
        write : match(write); expr()
        otherwise parse_error

procedure expr()
    case input_token of
        id, number, ( : term(); term_tail()
        otherwise parse_error

procedure term_tail()
    case input_token of
        +, - : add_op(); term(); term_tail()
        ), id, read, write, $$ :
            skip        -- epsilon production
        otherwise parse_error

procedure term()
    case input_token of
        id, number, ( : factor(); factor_tail()
        otherwise parse_error

procedure factor_tail()
    case input_token of
        *, / : mult_op(); factor(); factor_tail()
        +, -, ), id, read, write, $$ :
            skip        -- epsilon production
        otherwise parse_error

procedure factor()
    case input_token of
        id : match(id)
        number : match(number)
        ( : match((); expr(); match())
        otherwise parse_error

procedure add_op()
    case input_token of
        + : match(+)
        - : match(-)
        otherwise parse_error

procedure mult_op()
    case input_token of
        * : match(*)
        / : match(/)
        otherwise parse_error
```

LLL Parsing

- Parse tree for:

```
read A
read B
sum := A + B
write sum
write sum / 2
```

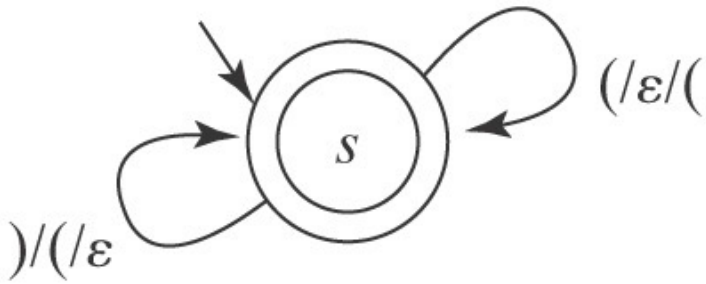


LL Parsing

- Table-driven LL parsing:
  - repeatedly look up action in 2D table based on:
    - current leftmost non-terminal and
    - current input token
  - actions:
    - (1) match a terminal
    - (2) predict a production
    - (3) announce a syntax error

LL Parsing

- Table-driven LL parsing:
  - Push-down automaton (PDA)
    - Finite automaton with a stack
    - Example: balanced parentheses: input / pop / push



  - Parsing stack: containing the expected symbols
    - initially contains the starting symbol
    - predicting a production: push the right-hand side in reverse order

LL Parsing

- Table-driven LL parsing:

```
terminal = 1 . . number_of_terminals
non_terminal = number_of_terminals + 1 . . number_of_symbols
symbol = 1 . . number_of_symbols
production = 1 . . number_of_productions

parse_tab : array [non_terminal, terminal] of record
    action : (predict, error)
    prod : production
prod_tab : array [production] of list of symbol
–– these two tables are created by a parser generator tool

parse_stack : stack of symbol

parse_stack.push(start_symbol)
loop
    expected_sym : symbol := parse_stack.pop()
    if expected_sym ∈ terminal
        match(expected_sym)                          –– as in Figure 2.17
        if expected_sym = $$ then return             –– success!
    else
        if parse_tab[expected_sym, input_token].action = error
            parse_error
        else
            prediction : production := parse_tab[expected_sym, input_token].prod
            foreach sym : symbol in reverse prod_tab[prediction]
                parse_stack.push(sym)
```

LL Parsing

- LL(1): `parse_tab` for parsing for calculator language
- productions: 1..19
- '–' means error
- `prod_tab` (not shown) gives RHS

1	*program* → *stmt_list* `$$`
2,3	*stmt_list* → *stmt* *stmt_list* | ε
4,5,6	*stmt* → `id` `:=` *expr* | `read` `id` | `write` *expr*
7	*expr* → *term* *term_tail*
8,9	*term_tail* → *add_op* *term* *term_tail* | ε
10	*term* → *factor* *fact_tail*
11,12	*fact_tail* → *mult_op* *fact* *fact_tail* | ε
13,14,15	*factor* → `(` *expr* `)` | `id` | `number`
16,17	*add_op* → `+` | `-`
18,19	*mult_op* → `*` | `/`

Top-of-stack nonterminal	Current input token: id	number	read	write	:=	(	)	+	-	*	/	$$
program	1	–	1	1	–	–	–	–	–	–	–	1
stmt_list	2	–	2	2	–	–	–	–	–	–	–	3
stmt	4	–	5	6	–	–	–	–	–	–	–	–
expr	7	7	–	–	–	7	–	–	–	–	–	–
term_tail	9	–	9	9	–	–	9	8	8	–	–	9
term	10	10	–	–	–	10	–	–	–	–	–	–
factor_tail	12	–	12	12	–	–	12	12	12	11	11	12
factor	14	15	–	–	–	13	–	–	–	–	–	–
add_op	–	–	–	–	–	–	–	16	17	–	–	–
mult_op	–	–	–	–	–	–	–	–	–	18	19	–

LL Parsing

- Example:

```
read A
read B
sum := A + B
write sum
write sum / 2
```

Parse stack	Input stream	Comment
program	read A read B ...	initial stack contents
stmt_list $$	read A read B ...	predict *program* ⟶ *stmt_list* $$
stmt stmt_list $$	read A read B ...	predict *stmt_list* ⟶ *stmt stmt_list*
read id *stmt_list* $$	read A read B ...	predict *stmt* ⟶ read id
id *stmt_list* $$	A read B ...	match read
stmt_list $$	read B sum := ...	match id
stmt stmt_list $$	read B sum := ...	predict *stmt_list* ⟶ *stmt stmt_list*
read id *stmt_list* $$	read B sum := ...	predict *stmt* ⟶ read id
id *stmt_list* $$	B sum := ...	match read
stmt_list $$	sum := A + B ...	match id
stmt stmt_list $$	sum := A + B ...	predict *stmt_list* ⟶ *stmt stmt_list*
id := *expr stmt_list* $$	sum := A + B ...	predict *stmt* ⟶ id := *expr*
:= *expr stmt_list* $$	:= A + B ...	match id
expr stmt_list $$	A + B ...	match :=
term term_tail stmt_list $$	A + B ...	predict *expr* ⟶ *term term_tail*
factor factor_tail term_tail stmt_list $$	A + B ...	predict *term* ⟶ *factor factor_tail*
id *factor_tail term_tail stmt_list* $$	A + B ...	predict *factor* ⟶ id
factor_tail term_tail stmt_list $$	+ B write sum ...	match id
term_tail stmt_list $$	+ B write sum ...	predict *factor_tail* ⟶ ε
add_op term term_tail stmt_list $$	+ B write sum ...	predict *term_tail* ⟶ *add_op term term_tail*
+ *term term_tail stmt_list* $$	+ B write sum ...	predict *add_op* ⟶ +
term term_tail stmt_list $$	B write sum ...	match +
factor factor_tail term_tail stmt_list $$	B write sum ...	predict *term* ⟶ *factor factor_tail*
id *factor_tail term_tail stmt_list* $$	B write sum ...	predict *factor* ⟶ id
factor_tail term_tail stmt_list $$	write sum ...	match id

LL Parsing

- Example:

```
read A
read B
sum := A + B
write sum
write sum / 2
```

Parse stack	Input stream	Comment
term_tail stmt_list $$	`write sum write` ...	predict *factor_tail* ⟶ ϵ
stmt_list $$	`write sum write` ...	predict *term_tail* ⟶ ϵ
stmt stmt_list $$	`write sum write` ...	predict *stmt_list* ⟶ *stmt stmt_list*
`write` *expr stmt_list* $$	`write sum write` ...	predict *stmt* ⟶ `write` *expr*
expr stmt_list $$	`sum write sum / 2`	match `write`
term term_tail stmt_list $$	`sum write sum / 2`	predict *expr* ⟶ *term term_tail*
factor factor_tail term_tail stmt_list $$	`sum write sum / 2`	predict *term* ⟶ *factor factor_tail*
`id` *factor_tail term_tail stmt_list* $$	`sum write sum / 2`	predict *factor* ⟶ `id`
factor_tail term_tail stmt_list $$	`write sum / 2`	match `id`
term_tail stmt_list $$	`write sum / 2`	predict *factor_tail* ⟶ ϵ
stmt_list $$	`write sum / 2`	predict *term_tail* ⟶ ϵ
stmt stmt_list $$	`write sum / 2`	predict *stmt_list* ⟶ *stmt stmt_list*
`write` *expr stmt_list* $$	`write sum / 2`	predict *stmt* ⟶ `write` *expr*
expr stmt_list $$	`sum / 2`	match `write`
term term_tail stmt_list $$	`sum / 2`	predict *expr* ⟶ *term term_tail*
factor factor_tail term_tail stmt_list $$	`sum / 2`	predict *term* ⟶ *factor factor_tail*
`id` *factor_tail term_tail stmt_list* $$	`sum / 2`	predict *factor* ⟶ `id`
factor_tail term_tail stmt_list $$	`/ 2`	match `id`
mult_op factor factor_tail term_tail stmt_list $$	`/ 2`	predict *factor_tail* ⟶ *mult_op factor factor_tail*
`/` *factor factor_tail term_tail stmt_list* $$	`/ 2`	predict *mult_op* ⟶ `/`
factor factor_tail term_tail stmt_list $$	`2`	match `/`
`number` *factor_tail term_tail stmt_list* $$	`2`	predict *factor* ⟶ `number`
factor_tail term_tail stmt_list $$		match `number`
term_tail stmt_list $$		predict *factor_tail* ⟶ ϵ
stmt_list $$		predict *term_tail* ⟶ ϵ
$$		predict *stmt_list* ⟶ ϵ

LL Parsing

- How to build the table:
  - FIRST(α) – tokens that can start an α
  - FOLLOW(A) – tokens that can come after an A

$$\texttt{EPS}(\alpha) \equiv \text{if } \alpha \Longrightarrow^{*} \varepsilon \text{ then } \texttt{true} \text{ else } \texttt{false}$$
$$\texttt{FIRST}(\alpha) \equiv \{c \mid \alpha \Longrightarrow^{*} c\beta\}$$
$$\texttt{FOLLOW}(A) \equiv \{c \mid S \Longrightarrow^{+} \alpha A c\beta\}$$
$$\texttt{PREDICT}(A \rightarrow \alpha) \equiv \texttt{FIRST}(\alpha) \cup \text{if } \texttt{EPS}(\alpha) \text{ then } \texttt{FOLLOW}(A) \text{ else } \emptyset$$

  - If a token belongs to the predict set of more than one production with the same left-hand side, then the grammar is not LL(1)
  - Compute: pass over the grammar until nothing changes
  - Algorithm and examples on the next slides

LL Parsing

- Constructing `EPS, FIRST, FOLLOW, PREDICT`

$program \longrightarrow stmt\_list$ `$$`	`$$` $\in$ FOLLOW($stmt\_list$)
$stmt\_list \longrightarrow stmt\ \ stmt\_list$	
$stmt\_list \longrightarrow \epsilon$	EPS($stmt\_list$) = true
$stmt \longrightarrow$ `id :=` $expr$	`id` $\in$ FIRST($stmt$)
$stmt \longrightarrow$ `read id`	`read` $\in$ FIRST($stmt$)
$stmt \longrightarrow$ `write` $expr$	`write` $\in$ FIRST($stmt$)
$expr \longrightarrow term\ \ term\_tail$	
$term\_tail \longrightarrow add\_op\ \ term\ \ term\_tail$	
$term\_tail \longrightarrow \epsilon$	EPS($term\_tail$) = true
$term \longrightarrow factor\ \ factor\_tail$	
$factor\_tail \longrightarrow mult\_op\ \ factor\ \ factor\_tail$	
$factor\_tail \longrightarrow \epsilon$	EPS($factor\_tail$) = true
$factor \longrightarrow$ `(` $expr$ `)`	`(` $\in$ FIRST($factor$) and `)` $\in$ FOLLOW($expr$)
$factor \longrightarrow$ `id`	`id` $\in$ FIRST($factor$)
$factor \longrightarrow$ `number`	`number` $\in$ FIRST($factor$)
$add\_op \longrightarrow$ `+`	`+` $\in$ FIRST(add_op)
$add\_op \longrightarrow$ `-`	`-` $\in$ FIRST(add_op)
$mult\_op \longrightarrow$ `*`	`*` $\in$ FIRST($mult\_op$)
$mult\_op \longrightarrow$ `/`	`/` $\in$ FIRST($mult\_op$)

LLL Parsing

- Algorithm for constructing `EPS, FIRST, FOLLOW, PREDICT`

(Continued on the next slide)

```
–– EPS values and FIRST sets for all symbols:
    for all terminals c, EPS(c) := false; FIRST(c) := {c}
    for all nonterminals X, EPS(X) := if X ⟶ ϵ then true else false; FIRST(X) := ∅
    repeat
        ⟨outer⟩ for all productions X ⟶ Y1 Y2 ... Yk,
            ⟨inner⟩ for i in 1 . . k
                add FIRST(Yi) to FIRST(X)
                if not EPS(Yi) (yet) then continue outer loop
            EPS(X) := true
    until no further progress

–– Subroutines for strings, similar to inner loop above:

    function string_EPS(X1 X2 ... Xn)
        for i in 1 . . n
            if not EPS(Xi) then return false
        return true
```

LL Parsing

- Algorithm for constructing `EPS, FIRST, FOLLOW, PREDICT`

```
function string_FIRST(X1  X2  ...  Xn)
    return_value := ∅
    for i in 1 .. n
        add FIRST(Xi) to return_value
        if not EPS(Xi) then return

-- FOLLOW sets for all symbols:
    for all symbols X, FOLLOW(X) := ∅
    repeat
        for all productions A ⟶ α B β,
            add string_FIRST(β) to FOLLOW(B)
        for all productions A ⟶ α B
                or A ⟶ α B β, where string_EPS(β) = true,
            add FOLLOW(A) to FOLLOW(B)
    until no further progress

-- PREDICT sets for all productions:
    for all productions A ⟶ α
        PREDICT(A ⟶ α) := string_FIRST(α) ∪ (if string_EPS(α) then FOLLOW(A) else ∅ )
```

LL Parsing

EPS(A) is true iff
A ∈ {*stmt_list*, *term_tail*, *factor_tail*}

- Example: the sets `EPS, FIRST, FOLLOW, PREDICT`

FIRST

- *program* {id, read, write, $$}
- *stmt_list* {id, read, write}
- *stmt* {id, read, write}
- *expr* {(, id, number}
- *term_tail* {+, -}
- *term* {(, id, number}
- *factor_tail* {*, /}
- *factor* {(, id, number}
- *add_op* {+, -}
- *mult_op* {*, /}

FOLLOW

- *program* ∅
- *stmt_list* {$$}
- *stmt* {id, read, write, $$}
- *expr* {), id, read, write, $$}
- *term_tail* {), id, read, write, $$}
- *term* {+, -,), id, read, write, $$}
- *factor_tail* {+, -,), id, read, write, $$}
- *factor* {+, -, *, /,), id, read, write, $$}
- *add_op* {(, id, number}
- *mult_op* {(, id, number}

PREDICT

1. *program* ⟶ *stmt_list* $$ {id, read, write, $$}
2. *stmt_list* ⟶ *stmt* *stmt_list* {id, read, write}
3. *stmt_list* ⟶ ϵ {$$}
4. *stmt* ⟶ id := *expr* {id}
5. *stmt* ⟶ read id {read}
6. *stmt* ⟶ write *expr* {write}
7. *expr* ⟶ *term* *term_tail* {(, id, number}
8. *term_tail* ⟶ *add_op* *term* *term_tail* {+, -}
9. *term_tail* ⟶ ϵ {), id, read, write, $$}
10. *term* ⟶ *factor* *factor_tail* {(, id, number}
11. *factor_tail* ⟶ *mult_op* *factor* *factor_tail* {*, /}
12. *factor_tail* ⟶ ϵ {+, -,), id, read, write, $$}
13. *factor* ⟶ (*expr*) {(}
14. *factor* ⟶ id {id}
15. *factor* ⟶ number {number}
16. *add_op* ⟶ + {+}
17. *add_op* ⟶ - {-}
18. *mult_op* ⟶ * {*}
19. *mult_op* ⟶ / {/}

LL Parsing

- Problems trying to make a grammar LL(1)
  - *left recursion*: $A \Longrightarrow^{+} A\alpha$
    - example – cannot be parsed top-down

$$\begin{aligned} &id\_list \rightarrow id\_list\_prefix\ \texttt{;} \\ &id\_list\_prefix \rightarrow id\_list\_prefix\ \texttt{,}\ \texttt{id} \\ &id\_list\_prefix \rightarrow \texttt{id} \end{aligned}$$

    - solved by *left-recursion elimination*

$$\begin{aligned} &id\_list \rightarrow \texttt{id}\ id\_list\_tail \\ &id\_list\_tail \rightarrow \texttt{,}\ \texttt{id}\ id\_list\_tail \\ &id\_list\_tail \rightarrow \texttt{;} \end{aligned}$$

  - General left-recursion elimination:

$$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \dots \mid A\alpha_n \mid \beta_1 \mid \beta_2 \mid \dots \mid \beta_m$$

replaced by:

$$\begin{aligned} &A \rightarrow \beta_1 B \mid \beta_2 B \mid \dots \mid \beta_m B \\ &B \rightarrow \alpha_1 B \mid \alpha_2 B \mid \dots \mid \alpha_n B \mid \varepsilon \end{aligned}$$

LL Parsing

- Problems trying to make a grammar LL(1)
  - *common prefixes*
    - example

$$\mathit{stmt} \rightarrow \texttt{id} \ \texttt{:=} \ \mathit{expr}$$
$$\mathit{stmt} \rightarrow \texttt{id} \ \texttt{(} \ \mathit{argument\_list} \ \texttt{)}$$

    - solved by *left-factoring*

$$\mathit{stmt} \rightarrow \texttt{id} \ \mathit{stmt\_list\_tail}$$
$$\mathit{stmt\_list\_tail} \rightarrow \texttt{:=} \ \mathit{expr}$$
$$\mathit{stmt\_list\_tail} \rightarrow \texttt{(} \ \mathit{argument\_list} \ \texttt{)}$$

- Note: Eliminating left recursion and common prefixes does NOT make a grammar LL; there are infinitely many non-LL languages, and the automatic transformations work on them just fine

LL Parsing

- Problems trying to make a grammar LL(1)
  - the *dangling else* problem
  - prevents grammars from being LL(k) for any k
  - Example: ambiguous (Pascal)

$$\mathit{stmt} \rightarrow \texttt{if}\ \ \mathit{cond}\ \ \mathit{then\_clause}\ \ \mathit{else\_clause}\ \mid\ \mathit{other\_stmt}$$

$$\mathit{then\_clause} \rightarrow \texttt{then}\ \ \mathit{stmt}$$

$$\mathit{else\_clause} \rightarrow \texttt{else}\ \ \mathit{stmt}\ \mid\ \varepsilon$$

$$\texttt{if}\ C_1\ \texttt{then}\ \texttt{if}\ C_2\ \texttt{then}\ S_1\ \texttt{else}\ S_2$$

LL Parsing

- Dangling else problem
  - Solution: unambiguous grammar
  - can be parsed bottom-up but not top-down
    - there is no top-down grammar

$$stmt \rightarrow balanced\_stmt \mid unbalanced\_stmt$$

$$balanced\_stmt \rightarrow \texttt{if}\ cond\ \texttt{then}\ balanced\_stmt\ \texttt{else}\ balanced\_stmt \mid other\_stmt$$

$$unbalanced\_stmt \rightarrow \texttt{if}\ cond\ \texttt{then}\ stmt \mid \texttt{if}\ cond\ \texttt{then}\ balanced\_stmt\ \texttt{else}\ unbalanced\_stmt$$

LL Parsing

- Dangling else problem
  - Another solution - *end-markers*

$$stmt \rightarrow \texttt{IF}\ cond\ \ then\_clause\ \ else\_clause\ \texttt{END}\ |\ other\_stmt$$
$$then\_clause \rightarrow\ \texttt{THEN}\ \ stmt\_list$$
$$else\_clause \rightarrow\ \texttt{ELSE}\ \ stmt\_list\ |\ \varepsilon$$

- Modula-2, for example, one says:

```
if A = B then
     if C = D then E := F end
else
     G := H
end
```

- Ada: `end if`
- other languages: `fi`

LL Parsing

- Problem with end markers: they tend to bunch up

```
if A = B then …
else if A = C then …
else if A = D then …
else if A = E then …
else ...
end end end end
```

  - To avoid this: `elsif`

```
if A = B then …
elsif A = C then …
elsif A = D then …
elsif A = E then …
else ...
end
```