

$$(\lambda f g h. f g (h h)) (\lambda x y. x) h (\lambda x. x x)$$

$$(\lambda \underline{f}. \lambda g. \lambda h. f g (h h)) (\lambda x. \lambda y. \underline{x}) h (\lambda x. x x)$$

$$\xRightarrow[\beta]{N, A} (\lambda g. \lambda h. (\lambda x. \lambda y. \underline{x}) \underline{g} (h h)) h (\lambda x. x x)$$

$$\Rightarrow_{\alpha} (\lambda \underline{g}. \lambda h. (\lambda x. \lambda y. x) g (h h)) h (\lambda x. x x)$$

$$\xRightarrow[\beta]{N} (\lambda \underline{h}. (\lambda x. \lambda y. x) h (h h)) (\lambda x. x x)$$

$$\xRightarrow[\beta]{N} (\lambda x. \lambda y. x) h ((\lambda x. x x) (\lambda x. x x))$$

$$\xRightarrow[\beta]{N} (\lambda \underline{y}. h) ((\lambda x. x x) (\lambda x. x x))$$

$$\xRightarrow[\beta]{N} h$$

N - normal
A - applicative

$$\xRightarrow[\beta]{A} (\lambda g. \lambda h. (\lambda y. \underline{g}) (h h)) h (\lambda x. x x)$$

$$\xRightarrow[\beta]{A} (\lambda \underline{y}. \lambda h. g) h (\lambda x. x x)$$

$$\Rightarrow_{\alpha} (\lambda \underline{g}. \lambda h. g) h (\lambda x. x x)$$

$$\xRightarrow[\beta]{A} (\lambda \underline{h}. h) (\lambda x. x x)$$

$$\xRightarrow[\beta]{A} h$$

$$\underbrace{(\lambda x. (\lambda y. y) z x)}_{\text{green}} \underbrace{(\lambda w. (\lambda x. w) h)}_{\text{green}}$$

$$\xRightarrow{\beta} (\lambda y. y) z (\lambda w. (\lambda x. w) h)$$

$$\xRightarrow{\beta} z (\lambda w. (\lambda x. w) h)$$

$$\xRightarrow{\beta} z (\lambda w. w)$$

$$\xRightarrow{\beta} (\lambda x. z x) (\lambda w. (\lambda x. w) h)$$

$$\xRightarrow{\beta} (\lambda x. z x) (\lambda w. w)$$

$$\xRightarrow{\beta} z (\lambda w. w)$$