A3 sol

(win 2022

## $Q_1$

(a) s(n) and p() always use the global a:

$s(0)$ — $a_g = 0$

$f()$ : $s(1)$ : $a_g = 1$

$p()$ : print 1

$p()$: print 1

$g()$ : $s(2)$ : $a_g = 2$

$p()$ : print 2

$p()$ : print 2

output: 1, 1, 2, 2

(b) most recent a : $a_g$

$s(0)$ : $a_g = 0$

$f()$ : $s(1)$ : $a_g = 1$

$p()$ : print 1

$p()$ : print 1

$g()$ : most recent a : $a_e$

$s(2)$ : $a_e = 2$

$p()$ : print 2

most recent a : $a_g$

$p()$ : print 1

output : 1, 1, 2, 1

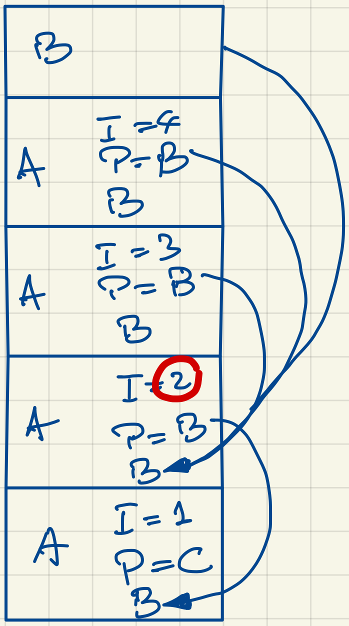
Q2 We add the print statements below:

```python
def A(I, P):
    def B():
        print(I)
    print("A call:  I = " + str(I) + "  B = " + str(B) + "  P = " + str(P))
    if I > 3:
        P()
    elif I > 2:
        A(4, P)
    elif I > 1:
        A(3, B)
    else:
        A(2, B)
def C():
    print(0)
print("C = ", end="")
print(C)
A(1, C)
```

The output is:

```
C = <function C at 0x7fd978159048>
A call:  I = 1  B = <function A.<locals>.B at 0x7fd978159268>  P = <function C at 0x7fd978159048>
A call:  I = 2  B = <function A.<locals>.B at 0x7fd9781596a8>  P = <function A.<locals>.B at 0x7fd978159268>
A call:  I = 3  B = <function A.<locals>.B at 0x7fd978159730>  P = <function A.<locals>.B at 0x7fd9781596a8>
A call:  I = 4  B = <function A.<locals>.B at 0x7fd9781597b8>  P = <function A.<locals>.B at 0x7fd9781596a8>
2
```

The stack:




Python uses deep binding, which is why the deeper I = 2 is printed.

Q3 (a) - call by value

$$1+3 \equiv +\ 1\ 3 \equiv$$

$$(\lambda mnab.\ ma\,(n\,a\,b))(\lambda fc.\ fc)(\lambda fc.\ f(f(f\,c))) \Rightarrow_\beta$$

$$(\lambda nab.\ (\lambda fc.fc)\,a\,(n\,a\,b))(\lambda fc.\ f(f(f\,c))) \Rightarrow_\beta$$

$$(\lambda nab.\ (\lambda c.\ ac)(n\,a\,b))(\lambda fc.\ f(f(fc))) \Rightarrow_\beta$$

$$(\lambda nab.\ a(n\,a\,b))(\lambda fc.\ f(f(fc))) \Rightarrow_\beta$$

$$\lambda ab.\ a((\lambda fc.\ f(f(fc)))\,a\,b) \Rightarrow_\beta$$

$$\lambda ab.\ a((\lambda c.\ a(a(ac)))\,b) \Rightarrow_\beta$$

$$\lambda ab.\ a\,(a(a(a\,b))) \equiv 4$$

- call by name

$$1+3 \equiv +\ 1\ 3 \equiv$$

$$(\lambda mnab.\ ma\,(n\,a\,b))(\lambda fc.\ fc)(\lambda fc.\ f(f(f\,c))) \Rightarrow_\beta$$

$$(\lambda nab.\ (\lambda fc.fc)\,a\,(n\,a\,b))(\lambda fc.\ f(f(f\,c))) \Rightarrow_\beta$$

$$\lambda ab.\ (\lambda fc.\ fc)\,a\,((\lambda fc.\ f(f(fc)))\,a\,b) \Rightarrow_\beta$$

$$\lambda ab.\ (\lambda c.\ ac)((\lambda fc.\ f(f(fc)))\,a\,b) \Rightarrow_\beta$$

$$\lambda ab.\ a((\lambda fc.\ f(f(fc)))\,a\,b) \Rightarrow_\beta$$

$$\lambda ab.\ a\,((\lambda c.\ a(a(ac)))\,b) \Rightarrow_\beta$$

$$\lambda ab.\ a\,(a(a(a\,b))) \equiv 4$$

(b) - call by value

$0 * 2 \equiv * \; 0 \; 2 \equiv$

$(\lambda mna.\, m(na))(\lambda fc.c)(\lambda fc.\, f(fc)) \Rightarrow_\beta$

$(\lambda na.\, (\lambda fc.c)(na))(\lambda fc.\, f(fc)) \Rightarrow_\beta$

$(\lambda na.\, (\lambda c.c))(\lambda fc.\, f(fc)) \Rightarrow_\beta$

$\lambda a.\, \lambda c.c \equiv \lambda ac.c \equiv 0$

- call by name

$0 * 2 \equiv * \; 0 \; 2 \equiv$

$(\lambda mna.\, m(na))(\lambda fc.c)(\lambda fc.\, f(fc)) \Rightarrow_\beta$

$(\lambda na.\, (\lambda fc.c)(na))(\lambda fc.\, f(fc)) \Rightarrow_\beta$

$\lambda a.\, (\lambda fc.c)((\lambda fc.\, f(fc))a) \Rightarrow_\beta$

$\lambda a.\, \lambda c.c \equiv \lambda ac.c \equiv 0$

Q4

- call by value

$$XOR\ T\ T \equiv$$
$$(\lambda xy.x\,(y\,F\,T)\,y)\,T\,T \Rightarrow_\beta$$
$$(\lambda y.\,T\,(y\,F\,T)\,y)\,T \equiv$$
$$(\lambda y.(\lambda xy.x)\,(y\,F\,T)\,y)\,T \Rightarrow_\alpha$$
$$(\lambda y.(\lambda xz.x)\,(y\,F\,T)\,y)\,T \Rightarrow_\beta$$
$$(\lambda y.(\lambda z.\,y\,F\,T)\,y)\,T \Rightarrow_\beta$$
$$(\lambda y.\,y\,F\,T)\,T \Rightarrow_\beta$$
$$T\,F\,T \equiv$$
$$(\lambda xy.x)\,(\lambda xy.y)\,T \Rightarrow_\beta$$
$$(\lambda y.(\lambda xy.y))\,T \Rightarrow_\beta$$
$$\lambda xy.y \equiv F$$

- call by name

$$XOR\ T\ T \equiv$$
$$(\lambda xy.x\,(y\,F\,T)\,y)\,T\,T \Rightarrow_\beta$$
$$(\lambda y.\,T\,(y\,F\,T)\,y)\,T \Rightarrow_\beta$$
$$T\,(T\,F\,T)\,T \equiv$$
$$(\lambda xy.\,x)\,(T\,F\,T)\,T \Rightarrow_\beta$$
$$(\lambda y.\,T\,F\,T)\,T \Rightarrow_\beta$$
$$T\,F\,T \equiv$$
$$(\lambda xy.x)\,(\lambda xy.y)\,T \Rightarrow_\beta$$
$$(\lambda y.(\lambda xy.y))\,T \Rightarrow_\beta$$
$$\lambda xy.y \equiv F$$