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(#) Numerical Topic: Dimensionality Reduction (PCA) → Principal Component Analysis

[1.] **Problem** = Given data = {2, 3, 4, 5, 6, 7 ; 1, 5, 3, 6, 7, 8}
Compute the principal component using PCA algorithm

(or)

Consider the two dimensional patterns (2,1), (3,5), (4,3), (5,6), (6,7), (7,8).

Compute the principal component using PCA algorithms.

Solution =

Given,

$\boxed{\text{class 1}}$ $X = 2, 3, 4$
$Y = 1, 5, 3$

$\boxed{\text{Class 2}}$ $X = 5, 6, 7$
$Y = 6, 7, 8$

⇒ **Step = 1**

The given features vectors are :-

→ $x_1 = (2,1)$
→ $x_2 = (3,5)$
→ $x_3 = (4,3)$
→ $x_4 = (5,6)$
→ $x_5 = (6,7)$
→ $x_6 = (7,8)$

(ie.) $\begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \end{bmatrix}$

⇒ Step=2

calculate the mean vector (μ)

$$\text{Mean vector}(\mu) = \left(\frac{(2+3+4+5+6+7)}{6}, \frac{(1+5+3+6+7+8)}{6}\right)$$
$$= (4.5, 5)$$

Thus,

$$\text{Mean vector}(\mu) = \begin{bmatrix} 4.5 \\ 5 \end{bmatrix}$$

⇒ Step=3

Subtract mean vector (μ) from the given feature vectors

→ $x_1 - \mu = (2-4.5, 1-5) = (-2.5, -4)$
→ $x_2 - \mu = (3-4.5, 5-5) = (-1.5, 0)$
→ $x_3 - \mu = (4-4.5, 3-5) = (-0.5, -2)$
→ $x_4 - \mu = (5-4.5, 6-5) = (0.5, 1)$
→ $x_5 - \mu = (6-4.5, 7-5) = (1.5, 2)$
→ $x_6 - \mu = (7-4.5, 8-5) = (2.5, 3)$

Feature vectors (x_i) after subtracting mean vector (μ) are :-

$$\begin{bmatrix} -2.5 \\ -4 \end{bmatrix} \begin{bmatrix} -1.5 \\ 0 \end{bmatrix} \begin{bmatrix} -0.5 \\ -2 \end{bmatrix} \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} \begin{bmatrix} 1.5 \\ 2 \end{bmatrix} \begin{bmatrix} 2.5 \\ 3 \end{bmatrix}$$

⇒ Step=4

→ calculate the covariance matrix
→ covariance matrix is given by,

$$\boxed{\text{covariance matrix} = \frac{\Sigma (x_i - \mu)(x_i - \mu)^t}{n}}$$

Now,

$$m_1 = (x_1 - \mu)(x_1 - \mu)^t = \begin{bmatrix} -2.5 \\ -4 \end{bmatrix} \begin{bmatrix} -2.5 & -4 \end{bmatrix} = \begin{bmatrix} 6.25 & 10 \\ 10 & 16 \end{bmatrix}$$

$$m_2 = (x_2 - \mu)(x_2 - \mu)^t = \begin{bmatrix} -1.5 \\ 0 \end{bmatrix} \begin{bmatrix} -1.5 & 0 \end{bmatrix} = \begin{bmatrix} 2.25 & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_3 = (X_3-\mu)(X_3-\mu)^t = \begin{bmatrix} -0.5 \\ -2 \end{bmatrix} \begin{bmatrix} -0.5 & -2 \end{bmatrix} = \begin{bmatrix} 0.25 & 1 \\ 1 & 4 \end{bmatrix}$$

$$m_4 = (X_4-\mu)(X_4-\mu)^t = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} \begin{bmatrix} 0.5 & 1 \end{bmatrix} = \begin{bmatrix} 0.25 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

$$m_5 = (X_5-\mu)(X_5-\mu)^t = \begin{bmatrix} 1.5 \\ 2 \end{bmatrix} \begin{bmatrix} 1.5 & 2 \end{bmatrix} = \begin{bmatrix} 2.25 & 3 \\ 3 & 4 \end{bmatrix}$$

$$m_6 = (X_6-\mu)(X_6-\mu)^t = \begin{bmatrix} 2.5 \\ 3 \end{bmatrix} \begin{bmatrix} 2.5 & 3 \end{bmatrix} = \begin{bmatrix} 6.25 & 7.5 \\ 7.5 & 9 \end{bmatrix}$$

Now,

→ covariance matrix $= \dfrac{(m_1+m_2+m_3+m_4+m_5+m_6)}{6}$

→ On adding the above matrices and Dividing by 6, we get,

$$\text{covariance matrix} = \frac{1}{6} \begin{bmatrix} 17.5 & 22 \\ 22 & 34 \end{bmatrix}$$

$$= \begin{bmatrix} 2.92 & 3.67 \\ 3.67 & 5.67 \end{bmatrix}$$

⇒ **Step=5**

→ Calculate the eigen values and eigen vectors of the covariance matrix

→ λ is an eigen value for a matrix "M". If it is a solution of a characteristic equation $|M-\lambda I| = 0$

So, we have

$$\begin{vmatrix} 2.92 & 3.67 \\ 3.67 & 5.67 \end{vmatrix} - \begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 2.92-\lambda & 3.67 \\ 3.67 & 5.67-\lambda \end{vmatrix} = 0$$

$(2.92-\lambda)(5.67-\lambda) - (3.67 \times 3.67) = 0$

$16.56 - 2.92\lambda - 5.67\lambda + \lambda^2 - 13.47 = 0$

$\lambda^2 - 8.59\lambda + 3.09 = 0$

Solving we get,

$\lambda = 8.22, 0.38$

So, $\lambda_1 = 8.22$ and $\lambda_2 = 0.38$

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then

→ we can use the following equation to find the eigen vector,

$$MX = \lambda X$$

where,

M = Covariance Matrix
X = Eigen vector
λ = Eigen value

$$\begin{bmatrix} 2.92 & 3.67 \\ 3.67 & 5.67 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = 8.22 \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Solving this we get,

$$2.92 X_1 + 3.67 X_2 = 8.22 X_1$$
$$3.67 X_1 + 5.67 X_2 = 8.22 X_2$$

on simplification we get,

$$5.3 X_1 = 3.67 X_2 \quad \text{— (i)}$$
$$3.67 X_1 = 2.55 X_2 \quad \text{— (ii)}$$

Solve (i) & (ii) we get,

$$\text{Eigen vector} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 2.55 \\ 3.67 \end{bmatrix}$$

→ Thus principal component for the given set is,

$$\text{Principal Component} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 2.55 \\ 3.67 \end{bmatrix}$$

→ Lastly we project the data points onto the new subspace as,