Probability

② $\dfrac{^3C_2 \cdot {^7C_2}}{^{10}C_4} \Rightarrow \dfrac{\dfrac{3\times 2}{2\times 1} \cdot \dfrac{7\times 6}{2\times 1}}{\dfrac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1}} \Rightarrow \dfrac{3\cdot 7\cdot 3}{5\cdot 3\cdot 2\cdot 7} \Rightarrow \dfrac{3}{10}$

⑥ $\dfrac{^3C_0 \cdot {^7C_4}}{^{10}C_4} = \cancel{\dfrac{3\cdot \dfrac{7\times 6\times 5\times 4}{4\times 3\times 2\times 1}}{\dfrac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1}} = \dfrac{3\cdot 7}{5\cdot 3\cdot 2\cdot 7}}$

$= \dfrac{1\cdot \dfrac{7\times 6\times 5\times 4}{4\times 3\times 2\times 1}}{\dfrac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1}} = \dfrac{1\cdot 7\cdot 5}{5\cdot 3\cdot 2\cdot 7} = \dfrac{1}{6}$

① $\dfrac{^3C_1 \cdot {^7C_3}}{^{10}C_4} = \dfrac{3\cdot \dfrac{7\times 6\times 5}{3\times 2\times 1}}{\dfrac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1}} = \dfrac{3\cdot 7\cdot 5}{5\cdot 3\cdot 2\cdot 7} \Rightarrow \dfrac{1}{2}$

③ $\dfrac{^3C_3 \cdot {^7C_1}}{^{10}C_4} = \dfrac{\dfrac{3\times 2\times 1}{3\times 2\times 1}\cdot 7}{\dfrac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1}} = \dfrac{7}{5\cdot 3\cdot 2\cdot 7} \Rightarrow \dfrac{1}{30}$

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[Q.] The amount of time in hours, that computer function before breaking down is continuous random variable with probability density function is given by

$$f(x) = \begin{cases} \theta e^{-\frac{x}{100}} & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}$$

(i) Determine value of θ

(ii) What is probability that computer will function b/w 50 & 150 hours before breaking down?

(iii) What is the probability that it will function less than 100 hours?

Sol^n :: (i) $\Rightarrow \int_0^\infty f(x) = 1$

$\Rightarrow \int_0^\infty \theta e^{-\frac{x}{100}} = 1$

$\Rightarrow \left[\frac{-\theta e^{-\frac{x}{100}}}{\frac{1}{100}}\right]_0^\infty = 1$

$\Rightarrow \left[-100\,\theta e^{-\frac{x}{100}}\right]_0^\infty = 1$

$\Rightarrow [0 - (-100\theta)] = 1$

$\Rightarrow 100\,\theta = 1$

$\boxed{\theta = \frac{1}{100}}$

(ii) $P(50 < x < 150)$

$= \int_{50}^{150} f(x)$

$= \int_{50}^{150} \frac{1}{100} e^{-\frac{x}{100}}$

$= \frac{1}{100}\left[\frac{-e^{-\frac{x}{100}}}{\frac{1}{100}}\right]_{50}^{150}$

$= \left[-e^{-\frac{x}{100}}\right]_{50}^{150}$

$= \left[-e^{-\frac{150}{100}} - \left(-e^{-\frac{50}{100}}\right)\right]$

$= e^{-\frac{1}{2}} - e^{-\frac{3}{2}} \Rightarrow e^{-0.5} - e^{-1.5}$

$= 0.6065 - 0.2231$

$= 0.3834$

(iii) $P(x < 100) = \int_0^{100} f(x)$

$= \int_0^{100} \frac{1}{100} e^{-\frac{x}{100}}$

$= \frac{1}{100}\left[\frac{-e^{-\frac{x}{100}}}{\frac{1}{100}}\right]_0^{100}$

$= \left[-e^{-\frac{x}{100}}\right]_0^{100}$

$= \left[-e^{-\frac{100}{100}} - \left(-e^{-\frac{0}{100}}\right)\right]$

$= 1 - e^{-1}$

$= 1 - 0.3679$

$= 0.6321$

Probability :- Probability is a measure of how likely an event is to occur.

eg:- Today there is a 60% change of rain
, I have fare chance of getting admission

→ If an event is certain to happen, then probability of event is 1 or 100%.

→ If an event will never happen, then probability of the event is 0 or 0%.

→ If an event is just likely to happen or not happen, then probability of event is $\frac{1}{2}$, 0.5 or 50%.

Sample Space :- set of all possible outcome of an experiment is known as Sample Space [redacted]. It is denoted as 'S' i.e S = {H, T}
eg:- If we tossed a coin total no. possible outcomes may be head / tail

Random Expectation :- Expectation which does not result in same outcome. when repeated under same condition. outcomes are uncertain

Event :- Subset of sample space.

1. Simple event :- Event that contains only one outcome.
2. Impossible event (Null event) :- Event that does not contain any outcome
3. Compound event :- Event that contain two or more outcomes
4. Certain / Sure event :- Event that contain all elements of the sample.

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→ Axiomatic probability is just a another way for finding or describing the prob. of event such that:

① The probability of event must is greater than or equal to 0 i.e. $P(E) \geq 0$

② The probability of sample space must be equal to 1 i.e. $P(S) = 1$

Types of sample space

① **Discrete sample space:-** sample space that contains finite or infinite countable no. of outcomes eg:- S = {yes, no}

② **continuous sample space:-** Sample space that contain infinite no. of outcomes which is uncountable. eg:- $S = R^+ = \{$

Probability definition

⇒ (classical definition)

Probability of any event is basically,

$P(A) = \frac{M}{N}$ [where M is favourable outcomes & N is total no. of outcomes]

⇒ $P(S) = 1$, S: sure event

⇒ $P(A \cup B) = P(A) + P(B)$

⇒ $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

Equally likely:- The outcome in a sample space "S" are equally likely if each outcome has same probability of occuring

Mutually exclusive event:- Two event are said to be mutually exclusive if they cannot occur at the same time or simultaneous

Exhaustive event:- A set of event are called exhaustive event if atleast one of them necessarily occurs whenever expectation is perform

complement of an event:- let A be an event, then non occurance A is called its compliment

union of an event:- union of an event are the event of occurance of atleast one event

$A = \{1, 3\}$ $\quad A \cup B = \{1, 3, 5\}$

$B = \{1, 3, 5\}$

$\boxed{A \text{ or } B}$ $\quad \boxed{A + B}$

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Bayes theorem = Bayes theorem describe the probability of occurrence of event related to any condition. Bayes theorem is also known as formula of probability of "cause"

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Intersection of an event :- Intersection of an event are events of simultaneous occurrence of all events.

$A = \{1,3\}$ $\quad A \cap B = \{1,3\}$

$B = \{1,3,5\}$ $\quad \boxed{A \text{ and } B}$ $\quad \boxed{A \cdot B}$

[Q.3c] A continuous random Variable X has the following probability density function

$$f(x) = \begin{cases} \frac{x+1}{2} & -1 < x < +1 \\ 0, & \text{otherwise} \end{cases}$$

$$\int_{-1}^{0.5} \frac{x}{2} + \frac{1}{2} \, dx$$

(i) Determine $P(X < 0.5)$

(ii) Determine $Var(X)$

Sol^n: (i) $P(X<0.5) = \int_{-1}^{0.5} f(x)\,dx = \int_{-1}^{0.5} \frac{x+1}{2}\,dx$

$$= \frac{1}{2}\int_{-1}^{0.5} x\,dx + \frac{1}{2}\int_{-1}^{0.5} dx$$

$$= \frac{1}{2}\left(\frac{x^2}{2}\right)_{-1}^{0.5} + \frac{1}{2}(x)_{-1}^{0.5}$$

$$= \frac{1}{4}\left[(0.5)^2 - (-1)^2\right] + \frac{1}{2}\left[0.5 - (-1)\right]$$

$$= \frac{1}{4}\left[0.25 - 1\right] + \frac{1}{2}(1.5)$$

$$= -0.1875 + 0.75 \Rightarrow 0.5625$$

(ii) $Var(X) = E(x^2) - [E(x)]^2$

$$E(x) = \int x f(x)\,dx = \frac{1}{2}\int_{-1}^{1}(x^2+x)\,dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\left(\frac{1}{3}+\frac{1}{2}\right) - \left(\frac{-1}{3}+\frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{2+3}{6}\right) - \left(\frac{-2+3}{6}\right)\right]$$

$$= \frac{1}{2}\left[\frac{5}{6} - \frac{1}{6}\right]$$

$$= \frac{1}{2}\left(\frac{4^2}{6^3}\right)$$

$\boxed{E(x) = \frac{1}{3}}$

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$$E(x^2) = \int x^2 f(x)\,dx = \frac{1}{2}\int_{-1}^{1} (x^3 + x^2)\,dx$$

$$= \frac{1}{2}\left[\left(\frac{x^4}{4}\right)_{-1}^{1} + \left(\frac{x^3}{3}\right)_{-1}^{1}\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{4} - \frac{1}{4}\right) + \left(\frac{1}{3} + \frac{1}{3}\right)\right]$$

$$= \frac{1}{2}\left(0 + \frac{2}{3}\right) \Rightarrow \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}\right) \Rightarrow \frac{1}{3}$$

$$\boxed{E(x^2) = \frac{1}{3}}$$

$$\text{var}(X) = E(x^2) - [E(x)]^2$$

$$= \frac{1}{3} - \left(\frac{1}{3}\right)^2 \Rightarrow \frac{1}{3} - \frac{1}{9} \Rightarrow \frac{3-1}{9} \Rightarrow \frac{2}{9} \Rightarrow 0.22$$

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Addition theorem

A & B [let A & B be any event]

$P(A)$ $P(B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B)$$ → Mutually exclusive $\boxed{P(A \cap B) = 0}$

Conditional Probability

A & B [let A & B be any two event]

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$ $\boxed{\text{where } P(A) \neq 0}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication theorem [let A & B be any two event]

$P(A \cap B) = P(A) \cdot P(B|A)$ → dependent event

$P(A \cap B) = P(A) \cdot P(B)$ → Independent event

[Q.] A bag contain 3 white, 4 red & 2 green balls. one ball is selected at random from bag. find the probability that the selected ball is

1. white
2. Non-white
3. white or green

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3 – W
4 – R
2 – G

Solⁿ :- here $n = 9$

(1) A: selected ball is white

$P(A) = \frac{m}{n}$

there are 3 white balls $\therefore m = 3$

$P(A) = \frac{\cancel{3}}{\cancel{9}} = \frac{1}{3}$

(2) B: Selected ball is non-white

$P(B) = 1 - P(A)$

$= 1 - \frac{3}{9} \Rightarrow \frac{\cancel{6}}{\cancel{9}} \Rightarrow \frac{2}{3}$

(3) C: selected ball is white or green

$P(C) = \frac{5}{9}$

$P(A \cup C) = P(A) + P(C)$

$= \frac{3}{9} + \frac{2}{9} \Rightarrow \frac{5}{9}$

[Q.] A fair coin is tossed twice. find the probability that the tossed result is

(a) two heads $\boxed{\text{2 heads}}$

(b) Atleast one head.

$S = \{HH, HT, TH, TT\}$

here $n = 4$

(i) $P(A) = \frac{m}{n} = \frac{1}{4}$

(ii) $P(B) = \frac{3}{4}$

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[Q.] The probabilities of two students A & B solving a problem are $\frac{1}{2}$ & $\frac{3}{4}$ respectively. If both of them independently try. What is the probability the problem is solved.

Solⁿ :- A : Student A solve the problem

B : Student B solve the problem

$P(A) = \frac{1}{2}$

$P(B) = \frac{3}{4}$

$$P(\text{Problem is solved}) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{3}{4} - [P(A) \cdot P(B)]$$

[P(A)· P(B) ∴ A & B are independent event]

$$= \frac{1}{2} + \frac{3}{4} - \left(\frac{1}{2} \cdot \frac{3}{4}\right)$$

$$= \frac{7}{8}$$

[Q.] The probability that A is hitted the target is $\frac{1}{3}$. The probability that B is hitted the target is $\frac{1}{4}$. If each of them shoot the target. what is the probability the target is hit by them

Solⁿ :- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{3} + \frac{1}{4} - [P(A) \cdot P(B)]$$

$$= \frac{7}{12} - \frac{1}{12} \Rightarrow \frac{\cancel{6}}{\cancel{12}} \Rightarrow \frac{1}{2}$$

$P(A) = \frac{1}{3}$

$P(B) = \frac{1}{4}$

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(Q) Odds favouring the event of a person hitting a target are :- 3 to 5 . the odds against the event of another person hitting the target are 3 to 2 . if each of them fire at the target,
(1) find the probability that both of them hitted
(2) At least one of them hits it

Sol :

A : The first person hit the target
B : The second person hit the target

$P(B') = 1 - P(B)$
$= 1 - \frac{2}{5}$
$= \frac{3}{5}$

$P(A) = \frac{3}{3+5} = \frac{3}{8}$

$P(A') = \frac{5}{8}$

$P(A') = 1 - P(A)$
$= 1 - \frac{3}{8}$
$= \frac{5}{8}$

$P(B) = \frac{2}{3+2}$
$= \frac{2}{5}$

$P(B') = \frac{3}{5}$

(i) $P(A \cap B) = P(A) \cdot P(B)$
$= \frac{3}{8} \times \frac{2}{5} = \frac{6}{40}$

(ii) $P(\text{at least one}) = 1 - P(\text{none hits the target})$
$= 1 - P(A' \cap B')$
$= 1 - \left(\frac{5}{8} \times \frac{3}{5}\right)$
$= 1 - \frac{15}{40}$
$= \frac{25}{40} \Rightarrow \frac{5}{8}$

1 red

[Q.] A box has one red & 3 white balls. Balls are drawn one after one from the box from. find the probability that the two → (2) balls drawn that would be red. if

(i) Ball drawn 1st is returned to the box before 2nd drawn is made.

(ii) The ball drawn 1st is not returned before the 2nd drawn is made (drawn without replacement)

Soln :-

$P(A) = \frac{1}{4}$ (i) $P(B/A) = \frac{1}{4}$ (ii) $P(B/A) = \frac{0}{3} = 0$

1	3
R	W

(ii) $P(A \cap B) = P(A) \cdot P(B|A)$
$= \frac{1}{4} \cdot \frac{0}{3}$
$= 0$

(i) $P(A \cap B) = P(A) \times P(B/A)$
$= \frac{1}{4} \times \frac{1}{4} \Rightarrow \frac{1}{16}$

[Q.] A card is drawn at random from a pack of card (1) what is the probability that it is a heart

(2) If it is known that card drawn is red what is the probability that it is a heart.

(i) $N = 52$ $P(B) = \frac{13}{52} \Rightarrow \frac{1}{4}$
(ii) $N = 26$ $P(A) = \frac{26}{52} \Rightarrow \frac{1}{2}$

Soln :- A = selected card is a red
B = ~~selected card is a heart~~

(1) $P(A) = \frac{26}{52}$
$P(B) = \frac{13}{52}$

(2) $P(B/A) = \frac{P(A \cap B)}{P(A)}$

$P(A \cap B) = \frac{13}{52}$

$= \frac{\frac{13}{52}}{\frac{26}{52}} = \frac{1}{2}$

R 26 → 13 H, 13 D
B 26 → 13 C, 13 S

$\frac{13}{52} \times \frac{52}{26} = \frac{1}{2}$

4 K
4 Q
4 J
4 A
2 to 10

Bayes Theorem

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$$P(E_i/A) = \frac{P(E_i)\, P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\, P(A|E_i)}$$

[Q.] A manufacturing company produces pipes in 2 plants I and II with daily production of 1500 & 2000 respectively. The fraction defective of the pipes produced by two plants I and II are 0.006 and 0.008 respectively. If a pipe is selected at random from the days production and it is found to be defective. what is the probability that it has come from plant II

Soln :- let E_1 : Pipe produced in plant I

E_2 : pipe produced in plant II

A : defective pipe

$4/7 = 0.571$

$\frac{3}{7} = 0.428$

$P(E_1) = \frac{1500}{3500} \Rightarrow \frac{3}{7} = 0.428$

$P(E_2) = \frac{2000}{3500} \Rightarrow \frac{4}{7} = 0.571$

$P(A|E_1) = 0.006$

$P(A|E_2) = 0.008$

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1)\, P(A|E_1) + P(E_2) \cdot P(A|E_2)}$$

$$= \frac{0.571 \times 0.008}{(0.428 \times 0.006) + (0.571 \times 0.008)} \Rightarrow 0.640$$

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[Q.] A college purchases computer from three supplier A, B & C respectively 60%, 30% & 10% of total requirement for the academic year of the total of their supply 2%, 5% & 8% of the computer are defective. A computer is selected by the college principal at random from lot and is found to be defective. What is the probability that it was supplied by the supplier A?

Soln :-

$P(E_1) = 60\% \Rightarrow 0.6$	$P(A/E_1) = 2\% \Rightarrow 0.02$
$P(E_2) = 30\% \Rightarrow 0.3$	$P(A/E_2) = 5\% \Rightarrow 0.05$
$P(E_3) = 10\% \Rightarrow 0.1$	$P(A/E_3) = 8\% \Rightarrow 0.08$

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)}$$

$$= \frac{(0.6 \times 0.02)}{(0.6 \times 0.02) + (0.3 \times 0.05) + (0.1 \times 0.08)}$$

$$= 0.343$$

[Q] In a bolt factory Machine A, B, C manufacture 25%, 35%, 40% respectively of the total of their output 5%, 4% and 2% are defective. A bolt is drawn and is found to be defective. Find the probability that it was manufacture by machine A, B & C?

Sol^n:- $\boxed{\text{Bolt factory}}$

$P(A) = \frac{25}{100}$, $P(B) = \frac{35}{100}$, $P(C) = \frac{40}{100}$

$P(D/A) = \frac{5}{100}$, $P(D/B) = \frac{4}{100}$, $P(D/C) = \frac{2}{100}$

[∴ where D denote defective bolts]

$P(D) = P(A) \cdot P(D/A) + P(B) \cdot P(D/B) + P(C) \cdot P(D/C)$

$= \frac{25}{100} \cdot \frac{5}{100} + \frac{35}{100} \cdot \frac{4}{100} + \frac{40}{100} \cdot \frac{2}{100} \Rightarrow 0.0345$

$$P(A/D) = \frac{P(A) \cdot P(D/A)}{P(A) \cdot P(D/A) + P(B) \cdot P(D/B) + P(C) \cdot P(D/C)}$$

$$= \frac{\frac{25}{100} \cdot \frac{5}{100}}{0.0345} \Rightarrow \frac{25}{69} \Rightarrow 0.3623$$

$$P(B/D) = \frac{P(B) \cdot P(D/B)}{P(A) \cdot P(D/A) + P(B) \cdot P(D/B) + P(C) \cdot P(D/C)}$$

$$= \frac{\frac{35}{100} \cdot \frac{4}{100}}{0.0345} = \frac{28}{69} \Rightarrow 0.4057$$

$$P(C/D) = \frac{\frac{40}{100} \cdot \frac{2}{100}}{0.0345} = \frac{16}{69} \Rightarrow 0.2318$$

[Q.] A box contains 6 red, 4 white & 5 black balls. A person draws 4 balls from the box at random. Determine the probability that among the balls drawn there is at least one ball of each other.

Sol^n:- $\boxed{\begin{matrix}\text{Red} = 6 \\ \text{white} = 4 \\ \text{Black} = 5\end{matrix}}$

∴ Total no. of balls = 15

case-1:- RRWB

case-2:- WWRB

case-3:- BBRW

then, $\frac{{}^6C_2 \times {}^4C_1 \times {}^5C_1 + {}^6C_1 \times {}^4C_2 \times {}^5C_1 + {}^6C_1 \times {}^4C_1 \times {}^5C_2}{{}^{15}C_4}$

$= \frac{\frac{6 \times 5}{2 \times 1} \cdot 4 \cdot 5 + 6 \cdot \frac{4 \times 3}{2 \times 1} \cdot 5 + 6 \cdot 4 \cdot \frac{5 \times 4}{2 \times 1}}{\frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}}$

$\Rightarrow \frac{\frac{30}{2} \cdot 4 \cdot 5 + 6 \cdot \frac{12}{2} \cdot 5 + 6 \cdot 4 \cdot \frac{20}{2}}{5 \cdot 7 \cdot 13 \cdot 3}$

$\Rightarrow \frac{15 \cdot 4 \cdot 5 + 6 \cdot 6 \cdot 5 + 6 \cdot 4 \cdot 10}{5 \cdot 7 \cdot 13 \cdot 3}$

$\Rightarrow$ [redacted]

$= \frac{720}{1365}$

$= \frac{144}{273} \Rightarrow \frac{48}{91}$

$\Rightarrow 0.5274$

[Q.] A box contain 500 IC chips of which 100 are manufactured by copany A & the rest by company B. It is estimated that 3% of the chips made by company A & 5% made by company B are defective. If a randomly selected chip is found to be defective, find probability that come from company A.

Sol^n:- $P(A) = P$ [IC chips produced by company A] $= \frac{100}{500} \Rightarrow 0.2$

$P(B) = P$ [IC chips produced by company B] $= \frac{400}{500} \Rightarrow 0.8$

E = defective chips

$P(A|E) \leftarrow P(E|A) = 3\% \Rightarrow \frac{3}{100} \Rightarrow 0.03$

$P(B|E) \leftarrow P(E|B) = 5\% \Rightarrow \frac{5}{100} \Rightarrow 0.05$

$P(A/E) =$

$$P(A/E) = \frac{P(A) \cdot P(E|A)}{P(B) \cdot P(E|B) + P(A) \cdot P(E|A)}$$

$$= \frac{0.2 \times 0.03}{0.8 \times 0.05 + (0.2 \times 0.03)} \Rightarrow \frac{0.006}{0.046} \Rightarrow 0.13043$$

[Q] A random variable X has the following probability [redacted] distribution :-

X	-3	6	9
P(x)	1/6	1/2	1/3

(i) Check whether P(x) is probability mass function.
(ii) Determine the expected value of X
(iii) Determine the expected value of (2X+3)

Sol^n:- (i) $\sum P(x) = \frac{1}{6} + \frac{1}{2} + \frac{1}{3} \Rightarrow \frac{1+3+2}{6} \Rightarrow \frac{6}{6} \Rightarrow 1$

$\therefore$ P(x) is probability mass function.

(ii) $E(x) = \sum x \cdot P(x)$
$= \left(-3 \times \frac{1}{6}\right) + \left(6 \times \frac{1}{2}\right) + \left(9 \times \frac{1}{3}\right) \Rightarrow -\frac{3}{6} + \frac{6}{2} + \frac{9}{3} \Rightarrow \frac{-3+18+18}{6} \Rightarrow \frac{33}{6} \Rightarrow 5.5$

(iii) $E(2x+3) = 2\,E(x) + 3$
$= 2(5.5) + 3$
$= 11 + 3 \Rightarrow 14$

Random Variable are of two types :-
(1) Discrete random variable
(2) continuous random variable

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Random Variable :- let S be the sample space associated with the given random experiment. A real valued function defined on S & taking values for R $(-\infty, +\infty)$ is called Random Variable.

(1) The random variables (r.v) are usually denoted by the capital letter x, y, z, etc.

(2) If 'ω' is the outcome of the experiment, then the corresponding real no. is denoted by $x(\omega)$
here $\omega \in S$

This is x of omega

(3) If $x(\omega) = x$, then the random variable can be defined as
$x(\omega) = \{x ; x \in R\}$, $R(-\infty, +\infty)$

Discreate random Variable :- A discrete random Variable is a real valued function defined on a discrete sample space i.e discrete random variable takes atmost a countable number of values.

Then sample space S is discrete sample space $S = \{HH, HT, TH, TT\}$

Random variable eg :- (1) If two balanced coin are tossed at time
(2) An experiment consist of rolling a die

Probability mass fn :- The fn "p" is called probability mass function of random variable x and set $\{x_i, p(x_i)\}$ is called probability distribution of random variable x.
The probability of x_i, $p(x_i)$ satisfy following condition
(1) $p(x_i) \geq 0$
(2) $\sum_{i=1}^{\infty} p(x_i) = 1$

(7)

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[Q.] In tossing two coins let random variable x defined as

X = No. of heads

& x assume value 0, 1, 2   i.e $x = \begin{cases} 0, & \text{if } \{TT\} \\ 1, & \text{if } \{TH, HT\} \\ 2, & \text{if } \{HH\} \end{cases}$

~~then~~ probability of random variable x is :–

$$P(X = x_1) = P(x = 0) = \frac{1}{4}$$
$$P(X = x_2) = P(x = 1) = \frac{2}{4}$$
$$P(X = x_3) = P(x = 2) = \frac{1}{4}$$

we can write using tabular form

$X = x_i$	0	1	2
$P(X = x_i)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

(i) $P(x_i) \geq 0$

(ii) $\sum_{i=1}^{3} P(x_i) = P(x_1) + P(x_2) + P(x_3)$

$= P(0) + P(1) + P(2)$

$= \frac{1}{4} + \frac{2}{4} + \frac{1}{4} = \frac{\cancel{4}}{\cancel{4}} \Rightarrow 1$

$\sum_{i=1}^{3} P(x_i) = 1 \rightarrow$ satisfied

[Q.] In tossing of 3 coins, let random variable be defined as

X = The no. of tails

then X can assume the value 0, 1, 2, 3

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

or $S = \{\omega_1 \times \omega_2 \times \omega_3\} = \{(H,T) \times (H,T) \times (H,T)\}$

$= \{(HH, HT, TH, TT) \times (H,T)\}$

$= \{HHH, HTH, THH, TTH, HHT, HTT, THT, TTT\}$

$P(X = x_1) = P(X = 0) = \frac{1}{8}$

$P(X = x_2) = P(X = 1) = \frac{3}{8}$

$P(X = x_3) = P(X = 2) = \frac{3}{8}$

$P(X = x_4) = P(X = 3) = \frac{1}{8}$

$X = x_i$	0	1	2	3
$P(X = x_i)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

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[Q.] A random variable X has the following probability distribution

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	$10k$	$5k$	k

(i) Find the value of k
(ii) Find $P(X<3)$, $P(2<x<7)$, $P(X>2)$

Soln:- (i) We know that total probability is 1

$\Rightarrow 0+k+2k+2k+3k+10k+5k+k=1$

$\Rightarrow 24k=1$

$\boxed{k = \frac{1}{24}}$

(ii) ∴ The probability distribution of x is given by

x	0	1	2	3	4	5	6	7
$P(x)$	0	$\frac{1}{24}$	$\frac{2}{24}$	$\frac{2}{24}$	$\frac{3}{24}$	$\frac{10}{24}$	$\frac{5}{24}$	$\frac{1}{24}$

(1) $P(X<3) = P(x=0,1,2)$
$= P(x_1=0) + P(x_2=1) + P(x_3=2)$
$= 0 + \frac{1}{24} + \frac{2}{24} \Rightarrow \frac{3}{24}$

(2) $P(2<x<7) = P(x=3,4,5,6)$
$= P(x=3) + P(x=4) + P(x=5) + P(x=6)$
$= \frac{2}{24} + \frac{3}{24} + \frac{10}{24} + \frac{5}{24} \Rightarrow \frac{20}{24}$

(3) $P(x>2) = 1 - P(x\le 2)$
$= 1 - P(x=0,1,2)$
$= 1 - \frac{3}{24} \Rightarrow \frac{21}{24}$

$P(x>2) = P(x=3,4,5,6,$
$= P(x=3) + P(x=4) +$
$P(x=5) + P(x=6$
$P(x=7)$
$= \frac{2}{24} + \frac{3}{24} + \frac{10}{24} + \frac{5}{24} + \frac{1}{2}$
$\Rightarrow \frac{21}{24}$

The probability mass function of a random variable X is zero except at the points $x = 0, 1, 2$ at these points it has the values $p(0) = 3c^3$, $p(1) = 4c - 10c^2$ and

$p(2) = 5c - 1$ for some $c > 1$

(I) Determine the value of c

(II) compute probability $P(x<2)$ and $P(1 < X \leq 2)$

given that, random value X can take value $x = 0, 1, 2$

X	0	1	2
$P(x)$	$3c^3$	$4c-10c^2$	$5c-1$

we know that total probability is 1

$\Rightarrow 3c^3 + 4c - 10c^2 + 5c - 1 = 1$

$3c^3 - 10c^2 + 9c - 2 = 0$

If $c = 1$, we get

$$c-1 \overline{)\, 3c^3 - 10c^2 + 9c - 2 \,}\ 3c^2 - 7c + 2$$
$$3c^3 - 3c^2$$
$$- \quad +$$
$$-7c^2 + 9c$$
$$-7c^2 + 7c$$
$$+ \quad -$$
$$2c - 2$$
$$2c - 2$$
$$- \quad +$$
$$0$$

$\Rightarrow 3(1)^3 - 10(1)^2 + 9(1) - 2 = 0$

$\Rightarrow 3 - 10 + 9 - 2 = 0$

$\Rightarrow 0 = 0$

$\therefore (c-1)$ is factor of the polynomial

thus, above polynomial order of 3 can written as:-

$(c-1)(3c^2 - 7c + 2) = 0$

$(c-1)(3c^2 - 6c - c + 2) = 0$

$(c-1)\{3c(c-2) - 1(c-2)\} = 0$

$(c-1)(3c-1)(c-2) = 0$

$\therefore c = 1, c = \frac{1}{3}, c = 2$

Since, probability X is always (+ve) so, suitable c value is $\boxed{c = \frac{1}{3}}$

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(ii)

x	0	1	2
$P(x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{3}$

(i) $p(x<2) = p(x=0,1)$

$= p(x=0) + p(x=1)$

$= \frac{1}{9} + \frac{2}{9} = \frac{3}{9} \Rightarrow \frac{1}{3}$

(ii) $P(1 < x \le 2) = p(x=2) \Rightarrow \frac{2}{3}$

[Q] A random variable X has the following probability distribution

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find k

(ii) Evaluate $p(x<6)$, $p(x\ge 6)$ & $p(0<x<5)$

Solⁿ:-

(i) we know that total probability is 1

$\Rightarrow 0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$

$\Rightarrow 10k^2+9k-1=0$

$\Rightarrow 10k^2+10k-k-1=0$

$\Rightarrow 10k(k+1)-1(k+1)=0$

$\Rightarrow (10k-1)(k+1)=0$

$\therefore k=\frac{1}{10}$ & $k=-1$

Since, $P(x)\ge 0$, the value of k will be $\frac{1}{10}$

$\therefore$ The probability distribution function of random variable X.

x	0	1	2	3	4	5	6	7
$P(x)$	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{17}{100}$

(i) value of $\boxed{k=\frac{1}{10}}$

(ii) $P(x<6) = P(x=0,1,2,3,4,5)$

$= 1 - P(x \geq 6)$

$= 1 - P(x=6,7)$

$= 1 - \left[\frac{2}{100} + \frac{17}{100}\right]$

$= 1 - \frac{19}{100}$

$= \frac{81}{100}$

* $P(x \geq 6) = 1 - P(x<6)$

$= 1 - \frac{81}{100} \Rightarrow \frac{19}{100}$

* $P(0<X<5) = P(x=1,2,3,4)$

$= P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} \Rightarrow \frac{8}{10}$

[Q.] From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. Let the random variable X denote the no. of defective items in the sample. Answer the following when the sample is drawn without replacement

(i) Find probability distribution of X

(ii) Find $P(x \leq 1)$, $P(X<1)$ & $P(0<X<2)$

Sol^n:- (i) let X = No. of defectives

$X = 0, 1, 2, 3$

∴ The probability distribution of X be

X	0	1	2	3
$P(x)$	$\frac{{}^3C_0 \cdot {}^7C_4}{{}^{10}C_4} = \frac{1}{6}$	$\frac{{}^3C_1 \cdot {}^7C_3}{{}^{10}C_4} = \frac{1}{2}$	$\frac{{}^3C_2 \cdot {}^7C_2}{{}^{10}C_4} = \frac{3}{10}$	$\frac{{}^3C_3 \cdot {}^7C_1}{{}^{10}C_4} = \frac{1}{30}$

(ii) $p(x \le 1) = \frac{1}{6} + \frac{1}{2} = \frac{4}{6} = \frac{2}{3}$

* $p(x < 1) = p(x = 0) = \frac{1}{6}$

* $p(0 < x < 2) = p(x = 1) = \frac{1}{2}$

[Q.] A random variable X has following probability distribution function

x	-2	-1	0	1	2	3
$p(x)$	0.1	k	0.2	2k	0.3	k

(i) Find value of k

(ii) Evaluate $p(X < 0)$, $p(x \ge 0)$, $p(-2 < x < 2)$

Soln:- (i) we know that total probability is 1

$\Rightarrow 0.1 + k + 0.2 + 2k + 0.3 + k =$

$\Rightarrow 4k + 0.6 = 1$

$\Rightarrow 4k = 1 - 0.6$

$\Rightarrow 4k = 0.4$

$\Rightarrow k = \frac{0.4}{4.0} \Rightarrow \frac{1}{10}$

$\boxed{k = \frac{1}{10}}$

$\therefore$ The probability distribution function of random variable x is

x	-2	-1	0	1	2	3
$p(x)$	0.1	$\frac{1}{10}$	0.2	$\frac{2}{10}$	0.3	$\frac{1}{10}$

(ii) $p(x < 0) = p(x = -2, -1)$

$= p(x = -2) + p(x = -1)$

$= 0.1 + \frac{1}{10}$

$= \frac{1+1}{10} \Rightarrow \frac{\cancel{2}}{\cancel{10}} \Rightarrow \frac{1}{5}$

* $p(x \geq 0) = p(x = 0, 1, 2, 3)$

$= p(x=0) + p(x=1) + p(x=2) + p(x=3)$

$= 0.2 + \frac{2}{10} + 0.3 + \frac{1}{10}$

$= \frac{2+2+3+1}{10}$

$= \frac{8}{10} \Rightarrow \frac{4}{5}$

* $p(-2 < x < 2) = p(x = -1, 0, 1)$

$= p(x=-1) + p(x=0) + p(x=1)$

$= \frac{1}{10} + 0.2 + \frac{2}{10}$

$= \frac{1+2+2}{10} \Rightarrow \frac{5}{10} \Rightarrow \frac{1}{2}$

[8.] A random variable has the following probability distribution :-

x	1	2	3	4
$p(x)$	$5k$	$4k$	$3k^2$	$7k^2$

(i) Find the value of k

(ii) Evaluate $p(x > 2)$, $p(x < 3)$, $p(1 \leq x < 4)$

Soln:- (i) we know that total probability is 1

$\Rightarrow 5k + 4k + 3k^2 + 7k^2 = 1$

$\Rightarrow 10k^2 + 9k = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0$

$\Rightarrow 10k^2 + 10k - k - 1 = 0$

$\Rightarrow 10k(k+1) - 1(k+1) = 0$

$\Rightarrow (10k-1)(k+1) = 0$

$\therefore k = \frac{1}{10}$ & $k = -1$

Since, $p(x) \geq 0$, the value of k will be $1/10$

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∴ The probability distribution function of random variable x is :-

x	1	2	3	4
$P(x)$	$\frac{5}{10}$	$\frac{4}{10}$	$\frac{3}{100}$	$\frac{7}{100}$

(ii) $P(x>2) = P(x=3,4)$

$= P(x=3) + P(x=4)$

$= \frac{3}{100} + \frac{7}{100} \Rightarrow \frac{10}{100} \Rightarrow \frac{1}{10}$

* $P(x<3) = P(x=1,2)$

$= P(x=1) + P(x=2)$

$= \frac{5}{10} + \frac{4}{10} \Rightarrow \frac{9}{10}$

* $P(1 \le x < 4) = P(x=1,2,3)$

$= P(x=1) + P(x=2) + P(x=3)$

$= \frac{5}{10} + \frac{4}{10} + \frac{3}{100} \Rightarrow \frac{50+40+3}{100} \Rightarrow \frac{93}{100}$

Continuous random variable :- A continuous random variable is a real valued function defined on a continous sample space. It can take all possible values b/w certain limits eg:- If we are interested to know the temperature b/w 9AM to 10AM then random value X = temperature constitute the sample space S & consider any value b/w 9AM–10AM.

Probability density function :- A function f is known as the probability density function, usually denoted by $f(x)$. with following properties (1) $f(x) \geq 0$

(2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

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Q.1 Suppose that x is continuous random variable whose probability density function is given by

$$f(x) = \begin{cases} c(4x - 2x^2) & ; 0 < x < 2 \\ 0 & ; \text{otherwise} \end{cases}$$

(a) Find the value of c

(b) calculate $p(x > 1)$

Soln :- The given p.d.f is

(a) $f(x) = c(4x - 2x^2) \quad , 0 < x < 2$

using property of p.d.f we can write :-

$$\int_0^2 f(x)\,dx = 1$$

$$\int_0^2 c(4x - 2x^2)\,dx = 1$$

$$\Rightarrow c\left[\int_0^2 4x\,dx - \int_0^2 2x^2\,dx\right] = 1$$

$$\Rightarrow c\left[4\left(\frac{x^2}{2}\right)_0^2 - 2\left(\frac{x^3}{3}\right)_0^2\right] = 1$$

$$\Rightarrow c\left[4\left(\frac{4}{2} - 0\right) - 2\left(\frac{8}{3} - 0\right)\right] = 1$$

$$\Rightarrow c\left[8 - \frac{16}{3}\right] = 1$$

$$\Rightarrow c\left[\frac{24 - 16}{3}\right] = 1$$

$$\Rightarrow c\left[\frac{8}{3}\right] = 1$$

$$\boxed{c = \frac{3}{8}}$$

(b) $p(x > 1) =$

$$\Rightarrow \int_1^2 f(x)\,dx$$

$$\Rightarrow \int_1^2 \frac{3}{8}(4x - 2x^2)\,dx$$

$$\Rightarrow \frac{3}{8}\left[4\left(\frac{x^2}{2}\right)_1^2 - 2\left(\frac{x^3}{3}\right)_1^2\right]$$

$$\Rightarrow \frac{3}{8}\left[4\left(2 - \frac{1}{2}\right) - 2\left(\frac{8}{3} - \frac{1}{3}\right)\right]$$

$$\Rightarrow \frac{3}{8}\left[4\left(\frac{3}{2}\right) - 2\left(\frac{7}{3}\right)\right]$$

$$\Rightarrow \frac{3}{8}\left[\frac{12}{2} - \frac{14}{3}\right]$$

$$\Rightarrow \frac{3}{8}\left[\frac{36 - 28}{6}\right]$$

$$\Rightarrow \frac{3}{8} \times \frac{8}{6} \Rightarrow \frac{1}{2}$$

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Q.) A continous random variable X follows the probability law

$$f(x) = Ax^2 \quad , 0 \le x \le 1$$

Determine A & find the probability that

(i) $0.2 \le X \le 0.5$

(ii) $X < 0.3$

(iii) $\frac{1}{4} < X < \frac{1}{2}$

Solⁿ :- By p.d.f

$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_0^1 Ax^2\,dx = 1$

$\Rightarrow A\int_0^1 x^2\,dx$

$\Rightarrow A\left[\frac{x^3}{3}\right]_0^1$

$\Rightarrow A\left[\frac{1}{3} - 0\right]$

$\boxed{A = 3}$

(i) $0.2 \le X \le 0.5$

[redacted] $\int_{0.2}^{0.5} f(x)\,dx$

$\boxed{A = 3}$

$\Rightarrow \int_{0.2}^{0.5} Ax^2\,dx$

$\Rightarrow A\int_{0.2}^{0.5} x^2\,dx$

$\Rightarrow A\left[\frac{x^3}{3}\right]_{0.2}^{0.5}$

$\Rightarrow 3\left[\frac{(0.5)^3}{3} - \frac{(0.2)^3}{3}\right]$

$\Rightarrow \cancel{3}\left[\frac{0.125 - (0.2)^3}{\cancel{3}}\right]$

$\Rightarrow 0.117$

(ii) $X < 0.3$

$\Rightarrow \int_0^{0.3} f(x)\,dx$

$\Rightarrow \int_0^{0.3} Ax^2\,dx$

$\Rightarrow A\left[\frac{x^3}{3}\right]_0^{0.3}$

$\Rightarrow A\left[\frac{(0.3)^3}{3} - (0)\right]$

$\Rightarrow \cancel{3}\left[\frac{(0.3)^3}{\cancel{3}}\right]$

$\Rightarrow 0.027$

(iii) $\frac{1}{4} < X < \frac{1}{2}$

$\Rightarrow \int_{1/4}^{1/2} f(x)\,dx$

$\Rightarrow \int_{1/4}^{1/2} Ax^2$

$\Rightarrow A\int_{1/4}^{1/2} x^2\,dx$

$\Rightarrow A\left[\frac{x^3}{3}\right]_{1/4}^{1/2}$

$\Rightarrow 3\left[\frac{\left(\frac{1}{2}\right)^3}{3} - \frac{\left(\frac{1}{4}\right)^3}{3}\right] \rightarrow \frac{1}{8} - \frac{1}{64}$

$\left(\frac{8-1}{64}\right) \Rightarrow \frac{7}{64}$

$\Rightarrow \cancel{3}\left[\frac{(0.50)^3 - (0.25)^3}{\cancel{3}}\right]$

$\Rightarrow 0.109375$

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[Q.] If X is continuous random variable with probability density function

$$f(x) = \begin{cases} ax & , 0 \le x \le 1 \\ a & , 1 \le x \le 2 \\ -ax+3a & , 2 \le x \le 3 \\ 0 & , \text{otherwise} \end{cases}$$

(i) Find value of a

(ii) compute $p(x \le 1.5)$, $p(1 \le X \le 2.5)$

Sol^n:- WKT, $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\Rightarrow \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax+3a)\,dx = 1$$

$$\Rightarrow a\left(\frac{x^2}{2}\right)_0^1 + a(x)_1^2 + (-a)\left(\frac{x^2}{2}\right)_2^3 + 3a(x)_2^3 = 1$$

$$\Rightarrow a\left(\frac{x^2}{2}\right)_0^1 + a(x)_1^2 - a\left(\frac{9}{2} - \frac{4}{2}\right) + 3a(3-2) = 1$$

$$\Rightarrow a\left(\frac{1}{2} - 0\right) + a(2-1) - a\left(\frac{9}{2} - \frac{4}{2}\right) + 3a(3-2) = 1$$

$$\Rightarrow \frac{a}{2} + a - \frac{5a}{2} + 3a = 1$$

$$\Rightarrow \frac{a+2a-5a+6a}{2} = 1$$

$$\Rightarrow \frac{4a}{2} = 1$$

$$\Rightarrow 4a = 2$$

$$\boxed{a = \frac{1}{2}}$$

$\therefore$ the given p.d.f X is

$$f(x) = \begin{cases} \frac{x}{2} & , 0 \le x \le 1 \\ \frac{1}{2} & , 1 \le x \le 2 \\ -\frac{x}{2} + \frac{3}{2} & , 2 \le x \le 3 \end{cases}$$

(i) $p(X \le 1.5) = \int_{-\infty}^{1.5} f(x)\,dx$

$$= \int_0^1 \frac{x}{2}\,dx + \int_1^{1.5} \frac{1}{2}\,dx$$

$$= \frac{1}{2}\left(\frac{x^2}{2}\right)_0^1 + \frac{1}{2}(x)_1^{1.5}$$

$$= \frac{1}{2}\left(\frac{1}{2} - 0\right) + \frac{1}{2}(1.5-1)$$

$$= \frac{1}{4} + \frac{1}{2}(0.5) \Rightarrow \frac{1}{4} + \frac{\cancel{5}\,1}{\cancel{20}\,4} \Rightarrow \frac{1}{4} + \frac{1}{4} \Rightarrow \frac{\cancel{2}}{\cancel{4}} \Rightarrow \frac{1}{2}$$

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(ii) $p(1 \le x \le 2.5) = \int_1^2 \frac{1}{2}\,dx + \int_2^{2.5} \left(-\frac{x}{2} + \frac{3}{2}\right) dx$

$= \frac{1}{2}(x)_1^2 - \frac{1}{2}\left(\frac{x^2}{2}\right)_2^{2.5} + \frac{3}{2}(x)_2^{2.5}$

$= \frac{1}{2} - \frac{1}{2}\left(\frac{(2.5)^2}{2} - \frac{4}{2}\right) + \frac{3}{2}(2.5-2)$

$= \frac{1}{2} - \frac{1}{2}\left(\frac{(2.5)^2 - 4}{2}\right) + \frac{3}{2}(0.5)$

$= \frac{1}{2} - \frac{2.25}{4} + \frac{3}{4} \Rightarrow \frac{2 - 2.25 + 3}{4} \Rightarrow \frac{2.75}{4} \Rightarrow 0.$

[Q] A continuous random variable x has a probability density function
$f(x) = 3x^2 \quad 0 \le x \le 1$
Find a & b such that
(i) $p(x \le a) = p(x > a)$ &
(ii) $p(x > b) = 0.05$

Sol^n: (i) Since $p(x \le a) = p(x > a)$ each probability is equal to $\frac{1}{2}$
(Because total probability is 1)

$\therefore p(x \le a) = \frac{1}{2}$ & $p(x > a) = \frac{1}{2}$

$\Rightarrow \int_{-\infty}^{a} f(x)\,dx = \frac{1}{2}$

$\Rightarrow \int^{a} f(x)\,dx = \frac{1}{2}$

$\Rightarrow \int_0^{a} 3x^2\,dx = \frac{1}{2}$

$\Rightarrow 3\left[\frac{x^3}{3}\right]_0^a = \frac{1}{2}$

$\Rightarrow a^3 = \frac{1}{2}$

$\boxed{a = \left(\frac{1}{2}\right)^{\frac{1}{3}}}$

(ii) $p(x > b) = 0.05$

$\Rightarrow \int_b f(x)\,dx = 0.05$

$\Rightarrow \int_b^1 3x^2\,dx = 0.05$

$\Rightarrow 3\left[\frac{x^3}{3}\right]_b^1 = 0.05$

$\Rightarrow 1 - b^3 = 0.05$

$\boxed{b = (0.95)^{\frac{1}{3}}}$

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[Q.1] The diameter of an electric cable say x is assumed to be continuous random variable with p.d.f

$$f(x) = 6x(1-x), \quad 0 \le x \le 1$$

(i) check that $f(x)$ is p.d.f

(ii) determine the no. 'b' such that $P(X<b) = P(X>b)$

Sol:- Now, for $f(x)$ is a p.d.f

(a)

$$\int_0^1 f(x)\,dx = 1$$

$$\Rightarrow \int_0^1 6x(1-x)\,dx$$

$$\Rightarrow 6\int_0^1 (x-x^2)\,dx$$

$$\Rightarrow 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$\Rightarrow 6\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$\Rightarrow \cancel{6}\left[\frac{3-2}{\cancel{6}}\right] \Rightarrow 1$$

So, $\int_0^1 f(x)\,dx = 1$, then $f(x)$ is p.d.f

(b) $P(X<b) = P(X>b)$

$$\Rightarrow \int_0^b f(x)\,dx = \int_b^1 f(x)\,dx$$

$$\Rightarrow \int_0^b 6x(1-x)\,dx = \int_b^1 6x(1-x)\,dx$$

$$\Rightarrow 6\int_0^b (x-x^2)\,dx = 6\int_b^1 (x-x^2)\,dx$$

$$\Rightarrow 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^b = 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_b^1$$

$$\Rightarrow \cancel{6}\left[\frac{3x^2 - 2x^3}{\cancel{6}}\right]_0^b = \cancel{6}\left[\frac{3x^2 - 2x^3}{\cancel{6}}\right]_b^1$$

$$\Rightarrow \left[3x^2 - 2x^3\right]_0^b = \left[3x^2 - 2x^3\right]_b^1$$

$$\Rightarrow \left[3b^2 - 2b^3 - (0)\right] = (1) - (3b^2 - 2b^3)$$

$$\Rightarrow 3b^2 - 2b^3 = 1 - 3b^2 + 2b^3$$

$$\Rightarrow 6b^2 - 4b^3 = 1$$

$$\Rightarrow 4b^3 - 6b^2 + 1 = 0$$

$$\Rightarrow b = 0.5,\ 1.366,\ -0.366$$

But $0 \le x \le 1$, then $\boxed{b = 0.5 \text{ i.e } \frac{1}{2}}$

$6b^2 - 4b^3 = 1$

$-(6b^2 - 4b^3) - 1 = 0$

$4b^3 - 6b^2 + 1 = 0$

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[Q.] If a random variable X has the following probability density function.

$$f(x) = \begin{cases} k \cdot e^{-3x}, & \text{for } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

(i) find value of k, (ii) Distribution function of X

(iii) determine $p(0.5 < x \le 1)$

Soln :- (i) we know that $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\Rightarrow \int_0^{\infty} k \cdot e^{-3x}\,dx = 1$$

$$\Rightarrow k \int_0^{\infty} e^{-3x}\,dx = 1$$

$$\Rightarrow k \left[\frac{e^{-3x}}{-3}\right]_0^{\infty} = 1$$

$$\Rightarrow k \left[\frac{e^{-\infty}}{-3} - \frac{e^0}{-3}\right] = 1$$

$$\Rightarrow \frac{k}{-3}\left[\frac{1}{e^{\infty}} - 1\right] = 1$$

$$\Rightarrow \frac{k}{-3}[0-1] = 1$$

$$\Rightarrow \frac{k}{3} = 1$$

$$\boxed{k=3}$$

∴ given p.d.f is

$$f(x) = \begin{cases} 3e^{-3x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

(ii) Distribution function of x :-

for $x > 0$

$$F(x) = \int_0^x f(t)\,dt$$

$$= \int_0^x 3e^{-3t}\,dt$$

$$= 3\left[\frac{e^{-3t}}{-3}\right]_0^x$$

$$= -\left[e^{-3x} - e^0\right]$$

$$= 1 - e^{-3x}$$

∴ distribution function of x is

$$F(x) = \begin{cases} 1 - e^{-3x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

(iii) $p(0.5 \le x \le 1)$

$$\Rightarrow \int_{0.5}^{1} 3e^{-3x}\,dx \Rightarrow 3\left[\frac{e^{-3x}}{-3}\right]_{0.5}^{1}$$

$$\Rightarrow -\left[e^{-3} - e^{-3(0.5)}\right]$$

$$\Rightarrow -\left[0.0498 - 0.2231\right]$$

$$\Rightarrow 0.173$$

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Distribution function :- let x be random variable on sample space 'S' then function $F(x) = P(X \le x)$ is called distribution function or cummulative distribution function of random variable x.

Discrete distribution function :- let x be discrete random variable with countable no. of values x_1, x_2, x_n with corresponding probability $P(x_1), P(x_2), \dots P(x_n)$. then discrete distribution function is defined as,

$$\boxed{F(x) = \Sigma\, p(x_i)} \quad (x_i \le x)$$

continous distribution function :- let x be continous random variable with probability density function $f(x)$, then function

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$$ is called continous distribution function of random variable x.

[Q.] The distribution function of random variable X is given by

$$f(x) = \begin{cases} 0, & \text{for } x < -1 \\ \frac{x+1}{2}, & \text{for } -1 \le x < 1 \\ 1, & \text{for } x \ge 1 \end{cases}$$

Find $P\left(-\frac{1}{2} < x < \frac{1}{2}\right)$ & $P(2 < x < 3)$

Sol:-

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx \Rightarrow P\left(-\frac{1}{2} < x < \frac{1}{2}\right)$$

$$\Rightarrow \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} x\,dx + \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} dx$$

$$\Rightarrow \frac{1}{2}\left(\frac{x^2}{2}\right)_{-\frac{1}{2}}^{\frac{1}{2}} + \frac{1}{2}(x)_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$\Rightarrow \frac{1}{4}\left[\left(\frac{1}{2}\right)^2 - \left(\frac{-1}{2}\right)^2\right] + \frac{1}{2}\left[\left(\frac{1}{2} - \frac{1}{2}\right)\right]$$

$$\Rightarrow \frac{1}{4}\left[\left(\frac{1}{4} - \frac{1}{4}\right)\right] + \frac{1}{2}[(0)]$$

$$\Rightarrow 0$$

$$\Rightarrow P(2 < x < 3)$$

$$\int_{2}^{3} f(x)\,dx$$

$$\Rightarrow \frac{1}{2}\int_{2}^{3} x\,dx + \frac{1}{2}\int_{2}^{3} dx$$

$$\Rightarrow \frac{1}{2}\left(\frac{x^2}{2}\right)_2^3 + \frac{1}{2}(x)_2^3$$

$$\Rightarrow \frac{1}{4}(x^2)_2^3 + \frac{1}{2}(x)_2^3$$

$$\Rightarrow \frac{1}{4}\left[(3)^2 - (2)^2\right] + \frac{1}{2}(3-2)$$

$$\Rightarrow \frac{1}{4}[9-4] + \frac{1}{2}(1)$$

$$\Rightarrow \frac{1}{4}(5) + \frac{1}{2}$$

$$\Rightarrow \frac{5}{4} + \frac{1}{2} \Rightarrow \frac{5+2}{4} \Rightarrow \frac{7}{4}$$

[Q.] A random Variable X has the following probability function

$X = x$	-2	-1	0	1	2	3
$P(X=x)$	0.1	k	0.2	$2k$	0.3	k

(i) find value of k
(ii) construct cumulative distribution function.

Soln:- (i) we know that total probability is 1

$\Rightarrow 0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$\Rightarrow 0.6 + 4k = 1$

$\Rightarrow 4k = 1 - 0.6$

$\Rightarrow 4k = 0.4$

$\Rightarrow k = \frac{0.4}{4}$ $\therefore \boxed{k = \frac{1}{10}}$

(ii) The probability distribution of random Variable x is :-

$X = x$	-2	-1	0	1	2	3
$P(X=x)$	0.1	$\frac{1}{10}$	0.2	$\frac{2}{10}$	0.3	$\frac{1}{10}$

[Q.] The length (in hours) X of certain type of light bulb may be Supposed to be continous random variable with probability density function

$$f(x) = \begin{cases} \frac{a}{x^3}, & 1500 < x < 2500 \\ 0, & \text{otherwise} \end{cases}$$

(i) Determine the constant a
(ii) Distribution function of X
(iii) Compute the probability of the event $1700 \le X \le 1900$

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Sol^n:- By p.d.f

(i) $\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_{0}^{\infty} \frac{a}{x^3}\,dx = 1$

$\Rightarrow a\int_{0}^{\infty} \frac{1}{x^3}\,dx = 1$

$\Rightarrow a\int_{0}^{\infty} x^{-3}\,dx = 1$

$\Rightarrow a\left[-3x^{-4}\right]_0^{\infty} = 1$

$\Rightarrow a\left[\frac{-3}{x^4}\right]_0^{\infty} = 1$

$\Rightarrow -3a\left[\frac{1}{x^4}\right]_0^{\infty} = 1$

$\Rightarrow -3a\left[\frac{1}{\infty} - \frac{1}{0}\right] = 1$

$\Rightarrow -3a\left[0 - \frac{1}{0}\right] = 1$

$\Rightarrow 3a = 1$

$\boxed{a = \frac{1}{3}}$

(ii) distribution function of x :-

for $1500 < x < 2500$ $\quad f(x) = \int_{\infty}^{x} f(t)\,dt$

$f(x) = \int_{1500}^{2500} \frac{a}{x^3}$

$= \frac{1}{3}\int_{1500}^{2500} \frac{1}{t^3}$

$= \frac{1}{3}\int_{1500}^{2500} t^{-3}$

$= \frac{1}{3}\left[-3t^{-4}\right]_{1500}^{2500}$

$= \frac{1}{3}\left[\frac{-3}{t^4}\right]_{1500}^{2500}$

$= \frac{-\cancel{3}}{\cancel{3}}\left[\frac{1}{t^4}\right]_{1500}^{2500}$

$= -1\left[\frac{1}{(2500)^4} - \frac{1}{(1500)^4}\right]$

$= \frac{1}{(1500)^4} - \frac{1}{(2500)^4}$

$= 1.975\times10^{-13} - 2.56\times10^{-14}$

$= 1.7193\times10^{-13}$

(iii) $1700 \le x \le 1900$

$\int_{1700}^{1900} \frac{\frac{1}{3}}{x^3}\,dx \Rightarrow \frac{1}{3}\int_{1700}^{1900} \frac{1}{x^3}$

$\Rightarrow \frac{1}{3}\left[\frac{-3}{x^4}\right]_{1700}^{1900}$

$= \frac{-3}{3}\left[\frac{1}{x^4}\right]_{1700}^{1900}$

$= -1\left[\frac{1}{(1900)^4} - \frac{1}{(1700)^4}\right]$

$\Rightarrow \frac{1}{(1700)^4} - \frac{1}{(1900)^4}$

$\Rightarrow 1.1973\times10^{-13} - 7.6733\times10^{-14}$

$\Rightarrow 4.299\times10^{-14}$

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[Q] A random Variable X has the following probability distribution function.

x	-2	-1	0	1	2	3
$P(x)$	0.1	K	0.2	$2K$	0.3	K

(i) Find the value of K

(ii) Evaluate $P(x<0)$, $P(x \geq 0)$, $P(-2 < x < 2)$

Solⁿ:- (i) $\sum_{i=1}^{\infty} P(x_i) = 1$

$\Rightarrow 0.1 + K + 0.2 + 2K + 0.3 + K = 1$

$\Rightarrow 0.6 + 4K = 1$

$\Rightarrow 4K = 1 - 0.6$

$\Rightarrow 4K = 0.4$

$\boxed{K = 0.1}$

x	-2	-1	0	1	2	3
$P(x)$	0.1	0.1	0.2	0.2	0.3	0

(ii) $P(x<0) = \frac{0.2 + 0.1}{3} \Rightarrow 0.1$

$P(x \geq 0) = \frac{0.2 + 0.1 + 0.3}{3} \Rightarrow 0.2$

$P(-2 < x < 2) = \frac{0.1 + 0.2 + 0.2}{3} \Rightarrow 0.1$

[Q] A random Variable has the following probability distribution

x	1	2	3	4
$P(x)$	$5K$	$4K$	$3K^2$	$7K^2$

(i) Find value of K

(ii) $P(x<2)$, $P(x<3)$, $P(1 \leq x < 4)$

Solⁿ:- (i) $5K + 4K + 3K^2 + 7K^2 = 1$

$10K^2 + 9K = 1$

$10K^2 + 9K - 1 = 0$

$10K^2 + 10K - K - 1$

$10K(K+1) - 1(K+1) = 0$

$K = -1, \frac{1}{10}$

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(ii)

x	1	2	3	4
$P(x)$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{3}{100}$	$\frac{7}{100}$

$$P(x>2) = \frac{\frac{3+7}{100}}{2} = \frac{1}{20}$$

$$\frac{\frac{10}{100}}{2} = \frac{1}{20}$$

$$P(x<3) = \frac{\frac{1}{2}+\frac{2}{5}}{2} = \frac{9}{20}$$

$$\frac{\frac{5+4}{10}}{2} = \frac{\frac{9}{10}}{2} \Rightarrow \frac{9}{20}$$

$$P(1 \le x < 4) = \frac{\frac{1}{2}+\frac{2}{5}+\frac{3}{100}}{3} = \frac{31}{100}$$

[Q.] A continuous random variable x follows the probability law

$$f(x) = Ax^2, \quad 0 \le x \le 1$$

Determine A and find probability that

(i) $0.2 \le x \le 0.5$

(ii) $x < 0.3$

(iii) $\frac{1}{4} < x < \frac{1}{2}$

Soln:-

$$f(x) = \int_0^1 Ax^2\,dx = 1$$

$$= A\int_0^1 x^2\,dx = 1$$

$$= A\left[\frac{x^3}{3}\right]_0^1 = 1$$

$$= A \times \frac{1}{3} = 1$$

$$\boxed{A = 3}$$

(ii)
$$f(x) = \int_0^{0.3} 3x^2\,dx$$

$$= 3\left[\frac{x^3}{3}\right]_0^{0.3}$$

$$= \frac{3}{3}\left[(0.3)^3 - (0)^3\right]$$

$$= 0.027$$

(iii)
$$f(x) = \int_{\frac{1}{4}}^{\frac{1}{2}} 3x^2\,dx$$

$$= 3\left[\frac{x^3}{3}\right]_{\frac{1}{4}}^{\frac{1}{2}} \Rightarrow \frac{3}{3}\left[\left(\frac{1}{2}\right)^3 - \left(\frac{1}{4}\right)^3\right]$$

$$\Rightarrow \frac{1}{8} - \frac{1}{64} \Rightarrow \frac{8-1}{64} \Rightarrow \frac{7}{64}$$

(i)
$$f(x) = \int_{0.2}^{0.5} 3x^2\,dx \Rightarrow 3\int_{0.2}^{0.5} x^2\,dx \Rightarrow 3\left[\frac{x^3}{3}\right]_{0.2}^{0.5} \Rightarrow 3\left[\frac{(0.5)^3}{3} - \frac{(0.2)^3}{3}\right]$$

$$\Rightarrow \frac{3}{3}\left[(0.5)^3 - (0.2)^3\right]$$

$$\Rightarrow 1\,(0.117) \Rightarrow 0.117$$

Mathematical Expection of Discrete Random Variable

→ Mathematical expectation of discrete random variable is obtained by multiplying each probable value of random variable by its corresponding probability & then adding these products.

$$E(x) = x_1 P(x_1) + x_2 p(x_2) + \dots + x_n P(x_n)$$

$$= \sum_{i=1}^{n} x_i P(x_i)$$

Mathematical Expectation of function h(x) of X :-

→ let x be discrete random variable with respective probability $P(x)$, then the mathematical expectation of any function h defined on X is given by $E[h(x)] = \sum h(x) P(x)$

Result

[1.] For a random variable X, the mean is given by

$$E(x) = \sum x P(x)$$

[2.] For the random variable the variance

$$Var(x) = E[x - E(x)]^2$$

$$= E(x)^2 + [E(x)]^2 - 2[E(x)]^2$$

$$Var(x) = E(x)^2 - [E(x)]^2$$

[3.] The square root of variance is the standard deviation, which is given by $SD(x) = \sqrt{Var(x)}$

[4.] If X is random variable and a, b are constant then we have the following :-

(a) $E(a) = a$	(d) $Var(a) = 0$
(b) $E(ax) = a E(x)$	(e) $Var(ax) = a^2 Var(X)$
(c) $E(aX+b) = a E(x) + b$	(f) $Var(aX+b) = a^2 Var(X)$

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Note :-

→ We know that

$Var(X) > 0$

$E(X^2) - [E(x)]^2 > 0$

$E(X^2) > [E(X)]^2$

[Q.] For the following probability distribution, find the following :-

x	0	1	2	3
P(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

1. $E(X)$
2. $Var(X)$
3. $S.D(X)$
4. $E(2x+3)$
5. $Var(2x+3)$
6. $SD(2x+3)$

Sol:-

(1) $E(X) = \left(0 \times \frac{1}{8}\right) + \left(1 \times \frac{3}{8}\right) + \left(2 \times \frac{3}{8}\right) + \left(3 \times \frac{1}{8}\right)$

$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} \Rightarrow \frac{12}{8} \Rightarrow \frac{3}{2}$

(2) $E(X^2) = \Sigma x^2 P(x)$

$= \left(0^2 \times \frac{1}{8}\right) + \left(1^2 \times \frac{3}{8}\right) + \left(2^2 \times \frac{3}{8}\right) + \left(3^2 \times \frac{1}{8}\right)$

$= 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8} \Rightarrow \frac{24}{8} \Rightarrow 3$

(2) $Var(X) = E(X^2) - [E(X)]^2$

$= 3 - \left(\frac{3}{2}\right)^2$

$= 3 - \frac{9}{4} \Rightarrow \frac{12-9}{4} \Rightarrow \frac{3}{4}$

(3) $SD(X) = \sqrt{\frac{3}{4}}$ $\because$ $\boxed{SD = \sqrt{Var(X)}}$

(4) $E(2x+3) = 2 \times \frac{3}{2} + 3 \Rightarrow 6$ $\because$ $\boxed{E(ax+b) = a\,E(x) + b}$

(5) $Var(2x+3) = \cancel{4} \times \frac{3}{\cancel{4}} \Rightarrow 3$

(6) $SD(2x+3) = \sqrt{3}$ $\because$ $\boxed{SD(2x+3) = \sqrt{Var(2x+3)}}$

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[Q] For the following probability distribution. find the following :-

x	1	5
$P(x)$	$2/3$	$1/3$

① $E(x)$
② $E(2x+4)$
③ $E(3x^2-5)$
④ $Var(x)$
⑤ $Var(3x)$
⑥ $Var(2x+4)$
⑦ $Var\left(\frac{-x}{2}\right)$
⑧ $Var(-x+2)$
⑨ $SD(x)$
⑩ $SD(-5x+2)$

Sol :- ① $E(x) = \left(1\times\frac{2}{3}\right) + \left(5\times\frac{1}{3}\right)$
$= \frac{2}{3}+\frac{5}{3} \Rightarrow \frac{7}{3}$

② $E(2x+4) = 2\times\frac{7}{3}+4$
$= \frac{14}{3}+4 \Rightarrow \frac{12+14}{3} \Rightarrow \frac{26}{3}$

③ $E(3x^2-5)$

$E(x^2) = \left(1^2\times\frac{2}{3}\right) + \left(5^2\times\frac{1}{3}\right)$
$= \frac{2}{3}+\frac{25}{3} \Rightarrow \frac{27}{3} \Rightarrow 9$

$E(3x^2-5) = 3\times9-5$
$= 27-5 \Rightarrow 22$

④ $Var(x) = 9-\left(\frac{7}{3}\right)^2$
$= 9-\frac{49}{9} \Rightarrow \frac{81-49}{9} \Rightarrow \frac{32}{9}$

⑤ $Var(3x) = 3^2\times\frac{32}{9}$
$= \cancel{9}\times\frac{32}{\cancel{9}} \Rightarrow 32$

⑥ $Var(2x+4) = 4\times\frac{32}{9}$
$= \frac{128}{9}$

⑦ $Var\left(\frac{-x}{2}\right) = \frac{1}{\cancel{4}}\times\frac{\cancel{32}\,8}{9}$
$= \frac{8}{9}$

⑧ $Var(-x+2) = (-1)^2\times\frac{32}{9} \Rightarrow \frac{32}{9}$

⑨ $SD(x) = \sqrt{\frac{32}{9}} \Rightarrow \frac{4\sqrt{2}}{3}$

⑩ $SD(-5x+2) = \sqrt{(-5)^2\times\frac{32}{9}}$
$= \sqrt{25\times\frac{32}{9}}$
$= \sqrt{\frac{800}{9}} \Rightarrow \frac{20\sqrt{2}}{3}$

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Q.1 Suppose that X is a continuous random Variable whose p.d.f is given by

$$f(x) = \begin{cases} (4x - 2x^2) & ; 0 \le x < 2 \\ 0 & ; \text{otherwise} \end{cases}$$

Find $E(X)$

Sol^n :-

$$E(X) = \int_0^2 x f(x)\, dx$$

$$= \int_0^2 x(4x - 2x^2)\, dx$$

$$= \int_0^2 4x^2 - 2x^3\, dx$$

$$= 4\left[\frac{x^3}{3}\right]_0^2 - \cancel{2}\left[\frac{x^4}{\cancel{4}_2}\right]_0^2$$

$$= 4\left[\frac{8}{3}\right] - \left[\frac{16}{2}\right]$$

$$= \frac{32}{3} - 8 \Rightarrow \frac{8}{3}$$

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[Q.] A random Variable X has the following probability function:

$X = x$	-2	-1	0	1	2	3
$P(X=x)$	0.1	K	0.2	$2K$	0.3	K

(1) Find the value of K

(2) Construct the commulative distribution function $F(x)$ & draw its graph.

Soln :- $\Sigma p(x) = 1$

(i) $0.1 + K + 0.2 + 2K + 0.3 + K = 1$

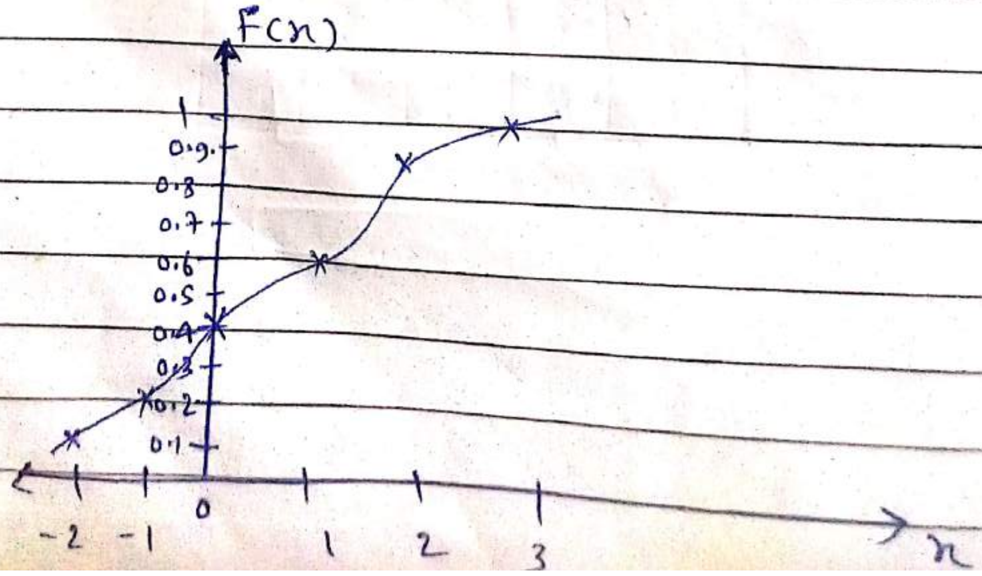
$4K = 1 - 0.6$

$4K = 0.4$

$K = \frac{0.4}{4} \frac{1}{10} \Rightarrow 0.1$

(ii)

x	$p(x)$	$F(x)$	$P(X \le x)$
-2	0.1	0.1	$P[X \le -2]$
-1	0.1	0.2	$P[X \le -1]$
0	0.2	0.4	$P[X \le 0]$
1	0.2	0.6	$P[X \le 1]$
2	0.3	0.9	$P[X \le 2]$
3	0.1	1	$P[X \le 3]$



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Joint Probability Function (or Bivariate Probability function) :- let X & Y be two discrete random variables which defined on sample space Ω of random experiment

. let $P(x,y)$ be function such that

$$P(x,y) = P[X=x, Y=y]$$

then $P(x,y)$ is called bivariate probability function of X & Y. if it satisfy the following three conditions :-

1. $0 \le P(x,y) \le 1$
2. $\sum\sum_{x,y \in \Omega} P(x,y) = 1$
3. $P[(X,Y) \in A] = \sum\sum_{x,y \in A} P(x,y)$

Marginal Probability f^n :- let $P(x)$ be marginal probability function of X and $P(y)$ be marginal probability function of Y obtained from joint probability function $P(x,y)$

$$P(x) = \sum_y P[X=x, Y=y] = \sum_y P(x,y)$$
$$P(y) = \sum_x P[X=x, Y=y] = \sum_x P(x,y)$$

Independent Random Variables :- Two random variable X & Y are said to be independent if & only if $P(x,y) = P(x).P(y)$ for all value X & Y.

Addition theorem for two discrete random Variable :- let X & Y be two discrete random variable with expectation $E(X)$ & $E(Y)$. then the expectation of sum of these random variable is given by

$$E(X+Y) = E(X) + E(Y)$$

Multiplication theorem for two independent random variables :- let X & Y be two independent random variables with expectation $E(X)$ & $E(Y)$. then the expectation of product of these random variables is given by

$$E(XY) = E(X) \cdot E(Y)$$

then, $E(X_1, X_2 \ldots\ldots X_n) = E(X_1).E(X_2)\ldots.E(X_n)$

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Covariance :- let X & Y be two random variables. then covariance b/w X & Y is given by,

$$\operatorname{cov}(X,Y) = E\left[(X-E(X))(Y-E(Y))\right] = E(XY) - E(X)E(Y)$$

Coefficient of correlation :- The correlation coefficient b/w two random variables X & Y is defined as

$$\gamma = \frac{\operatorname{cov}(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}} = \frac{\operatorname{cov}(X,Y)}{SD(X)\ SD(Y)}$$

→ **Result :-** For two independent random variable X & Y

① $\operatorname{cov}(X,Y) = 0$

② correlation coefficient = 0

[Q.] Suppose X & Y are any 2 discrete random variables having following probability distribution.

X \ Y	0	1	2	P(x)
0	8/72	?	10/72	1/3
1	12/72	9/72	?	1/2
2	?	3/72	?	?
P(y)	1/3	?	?	1

(i) complete the table
(ii) Expectation of X & Y
(iii) Covariance b/w X & Y
(iv) check whether X & Y are independent

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Solⁿ :- (i)

X \ Y	0	1	2	P(x)
0	8/72	6/72	10/72	24/72
1	12/72	9/72	15/72	36/72
2	4/72	3/72	5/72	12/72
P(y)	24/72	18/72	30/72	1 (72/72)

(ii) Marginal distribution of X

x	0	1	2
P(x)	24/72	36/72	12/72

Marginal distribution of Y

y	0	1	2
P(y)	24/72	18/72	30/72

(iii) $E(X) = \Sigma x P(x)$

$= 0 \times \frac{24}{72} + 1 \times \frac{36}{72} + 2 \times \frac{12}{72}$

$= \frac{60}{72}$

$E(Y) = \Sigma y P(y)$

$= 0 \times \frac{24}{72} + 1 \times \frac{18}{72} + 2 \times \frac{30}{72}$

$= \frac{78}{72}$

$\text{cov}(X,Y) = E(XY) - E(X)E(Y)$

$= \left(0 \times 0 \times \frac{8}{72} + 0 \times 1 \times \frac{6}{72} + 0 \times 2 \times \frac{10}{72} + 1 \times 0 \times \frac{12}{72} + 1 \times 1 \times \frac{9}{72} + 1 \times 2 \times \frac{15}{72} + 2 \times 0 \times \frac{4}{72} + 2 \times 1 \times \frac{3}{72} + 2 \times 2 \times \frac{5}{72}\right) - \left(\frac{60}{72} \times \frac{78}{72}\right)$

$= \left(\frac{9}{72} + \frac{30}{72} + \frac{6}{72} + \frac{20}{72}\right) - \left(\frac{60 \times 78}{72 \times 72}\right)$

$= \frac{65}{72} - \frac{65}{72}$

$= 0$

(iv) X & Y are independent as $\text{cov}(X,Y) = 0$

[Q.] The joint probability distribution of 2 discrete random variables X & Y is partially given in following table.

b	a = 0	a = 1	a = 2	$P(Y=b)$
-1	—	—	—	$1/2$
1	—	$1/2$	—	$1/2$
$P(X=a)$	$1/6$	$2/3$	$1/6$	1

(i) complete the table
(ii) are X & Y are independent or dependent

Soln:-

b	a = 0	a = 1	a = 2	$P(Y=b)$
-1	$1/6$	$1/6$	$\frac{1}{6}$	$1/2$
1	0	$1/2$	0	$1/2$
$P(X=a)$	$1/6$	$2/3$	$1/6$	1

$E(X) = \Sigma x\, P(x)$
$= 0\times\frac{1}{6} + 1\times\frac{2}{3} + 2\times\frac{1}{6}$
$= \frac{2}{3}+\frac{1}{3} \Rightarrow \frac{3}{3} \Rightarrow 1$

x	0	1	2
P(x)	$1/6$	$2/3$	$1/6$

$E(Y) = \Sigma y\, P(y)$
$= -1\times\frac{1}{2} + 1\times\frac{1}{2}$
$= -\frac{1}{2}+\frac{1}{2} \Rightarrow 0$

y	-1	1
P(y)	$1/2$	$1/2$

Independent

$Cov(X,Y) = E(XY) - [E(X)\cdot E(Y)]$
$= \Big(-1\times0\times\frac{1}{6} + -1\times1\times\frac{1}{6} + -1\times2\times\frac{1}{6} + 1\times0\times0 + 1\times1\times\frac{1}{2} + 1\times2\times0\Big) - (1\times0)$
$= \left(-\frac{1}{6}-\frac{1}{3}+\frac{1}{2}\right) - (1\times0)$
$= -\frac{1}{6}-\frac{1}{3}+\frac{1}{2} \Rightarrow 0$

$\frac{-1-2}{6}$
$= -\frac{3}{6}+\frac{1}{2}$
$= \frac{-3+3}{6} \Rightarrow 0$

PAGE :

[Q.] check if X and Y are independent, compute γ & find $E(X+Y)$

Y \ X	-1	0	1	Total
-1	0	0.1	0.1	0.2
0	0.2	0.2	0.2	0.6
1	0	0.1	0.1	0.2
Total	0.2	0.4	0.4	1

Sol^n:- $E(X) = -1\times0.2 + 0\times0.4 + 1\times0.4$

$= -0.2+0.4 \Rightarrow 0.2$

x	-1	0	1
P(x)	0.2	0.4	0.4

$E(Y) = -1\times0.2 + 0\times0.6 + 1\times0.2$

$= -0.2+0.2 \Rightarrow 0$

y	-1	0	1
P(y)	0.2	0.6	0.2

→ $cov(X,Y) = E(XY) - E(X)\,E(Y)$

$= (-1\times1\times0 + -1\times0\times0.1 + -1\times1\times0.1 + 0\times-1\times0.2 +$
$0\times0\times0.2 + 0\times1\times0.2 + 1\times-1\times0 + 1\times0\times0.1 +$
$1\times1\times0.1) - (0.2\times0)$

$= -0.1 + 0.1 \Rightarrow 0$

∴ X & Y are independent

→ $\gamma = \dfrac{cov(X,Y)}{SD(X)\;SD(Y)}$

$= 0$

→ $E(X+Y) = E(X) + E(Y)$

$= 0.2 + 0$

$= 0.2$

[Q.7] For the following bivariate probability distribution of X & Y

- find probability of :-

(i) $P(X \le 1, Y = 2)$

(ii) $P(X \le 1)$

(iii) $P(Y = 3)$

(iv) $P(Y \le 3)$

(v) $P(X < 3, Y \le 4)$

X \ Y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

Soln :- (i) $P(X \le 1, Y = 2) = \frac{1}{16}$, $P(X = 0, Y = 2) + P(X = 1, Y = 2) \Rightarrow 0 + \frac{1}{16} = \frac{1}{16}$

(ii) $P(X \le 1) = 0 + 0 + \frac{1}{32} + \frac{2}{32} + \frac{2}{32} + \frac{3}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} =$

$$\frac{1+2+2+3+2+2+4+4+4+4}{32} \Rightarrow \frac{28}{32} \Rightarrow \frac{7}{8}$$

(iii) $P(Y = 3) = \frac{1}{32} + \frac{1}{8} + \frac{1}{64} = \frac{2+8+1}{64} \Rightarrow \frac{4}{64}$

(iv) $P(Y \le 3) = 0 + 0 + \frac{1}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{32} + \frac{1}{32} + \frac{1}{64}$

$= \frac{23}{64}$

(v) $P(X < 3, Y \le 4) = \frac{1}{32} + \frac{2}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{8} + \frac{1}{32} + \frac{1}{32} + \frac{2}{64}$

$= \frac{3}{32} + \frac{2}{16} + \frac{2}{8} + \frac{2}{32} + \frac{2}{64}$

$= \frac{9}{16}$

Q.7 Given the following bivariate probability distribution obtain

(i) Marginal distribution of X & Y

(ii) conditional distribution of X given Y=2

(iii) conditional distribution of Y given X=0

Y \ X	-1	0	1
0	1/15	2/15	1/15
1	3/15	2/15	1/15
2	2/15	1/15	2/15

Ans:-

(i)

X	-1	0	1
P(X)	6/15	5/15	4/15

[redacted]

$\frac{1}{15} + \frac{3}{15} + \frac{2}{15} \Rightarrow \frac{6}{15}$

$\frac{2}{15} + \frac{2}{15} + \frac{1}{15} \Rightarrow \frac{5}{15}$

$\frac{1}{15} + \frac{1}{15} + \frac{2}{15} \Rightarrow \frac{4}{15}$

Y	0	1	2
P(Y)	4/15	6/15	5/15

[redacted]

$\frac{1}{15} + \frac{2}{15} + \frac{1}{15} \Rightarrow \frac{4}{15}$

$\frac{3}{15} + \frac{2}{15} + \frac{1}{15} \Rightarrow \frac{6}{15}$

$\frac{2}{15} + \frac{1}{15} + \frac{2}{15} \Rightarrow \frac{5}{15}$

(ii)

$$\left.\begin{aligned} P(X/Y) &= \frac{P(X \cap Y)}{P(Y)} \\ P(X/Y=2) &= \frac{P(X \cap Y=2)}{P(Y=2)} \end{aligned}\right\} \text{Formulae}$$

$$P(X=-1/Y=2) = \frac{P(X=-1 \cap Y=2)}{P(Y=2)} = \frac{\frac{2}{15}}{\frac{5}{15}} = \frac{2}{\cancel{15}} \times \frac{\cancel{15}}{5} \Rightarrow \frac{2}{5}$$

$$P(X=0/Y=2) = \frac{P(X=0 \cap Y=2)}{P(Y=2)} = \frac{\frac{1}{15}}{\frac{5}{15}} = \frac{1}{\cancel{15}} \times \frac{\cancel{15}}{5} \Rightarrow \frac{1}{5}$$

$$P(X=1/Y=2) = \frac{P(X=1 \cap Y=2)}{P(Y=2)} = \frac{\frac{2}{15}}{\frac{5}{15}} = \frac{2}{\cancel{15}} \times \frac{\cancel{15}}{5} \Rightarrow \frac{2}{5}$$

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(iii) $P(Y=0 / X=0) = \frac{P(Y=0 \cap X=0)}{P(X=0)} = \frac{\frac{2}{15}}{\frac{5}{15}} = \frac{2}{\cancel{15}} \times \frac{\cancel{15}}{5} \Rightarrow \frac{2}{5}$

$P(Y=1 / x=0) = \frac{P(Y=1 \cap X=0)}{P(X=0)} = \frac{\frac{2}{15}}{\frac{5}{15}} = \frac{2}{\cancel{15}} \times \frac{\cancel{15}}{5} \Rightarrow \frac{2}{5}$

$P(Y=2 / x=0) = \frac{P(Y=2 \cap X=0)}{P(X=0)} = \frac{\frac{1}{15}}{\frac{5}{15}} = \frac{1}{\cancel{15}} \times \frac{\cancel{15}}{5} \Rightarrow \frac{1}{5}$

[Q.7] Find (i) Marginal distribution of X & Y
(ii) condition distribution of Y given X=2

Y \ X	1	2	3
1	1/12	1/6	0
2	0	1/9	1/5
3	1/18	1/4	2/15

$\frac{1}{12} + \frac{1}{18} = \frac{3+2}{36} \Rightarrow \frac{5}{36}$

$\frac{1}{6} + \frac{1}{9} + \frac{1}{4} = \frac{6+4+9}{36} \Rightarrow \frac{19}{36}$

$0 + \frac{1}{5} + \frac{2}{15} = \frac{3+2}{15} \Rightarrow \frac{\cancel{5}}{\cancel{15}} \Rightarrow$

Soln :- (i)

X	1	2	3
P(X)	5/36	19/36	1/3

$\frac{1}{12} + \frac{1}{6} + 0 = \frac{1+2}{12} \Rightarrow \frac{\cancel{3}}{\cancel{12}} \Rightarrow \frac{1}{4}$

Y	1	2	3
P(Y)	1/4	14/45	79/180

$0 + \frac{1}{9} + \frac{1}{5} = \frac{5+9}{45} \Rightarrow \frac{14}{45}$

$\frac{1}{18} + \frac{1}{4} + \frac{2}{15} = \frac{10+45+24}{180} \Rightarrow \frac{79}{180}$

(ii) $P(Y=1 / x=2) = \frac{P(Y=1 \cap x=2)}{P(x=2)} = \frac{\frac{1}{6}}{\frac{19}{36}} = \frac{1}{\cancel{6}} \times \frac{\cancel{36}^{6}}{19} \Rightarrow \frac{6}{19}$

$P(Y=2 / x=2) = \frac{P(Y=2 \cap x=2)}{P(x=2)} = \frac{\frac{1}{9}}{\frac{19}{36}} = \frac{1}{\cancel{9}} \times \frac{\cancel{36}^{4}}{19} \Rightarrow \frac{4}{19}$

$P(Y=3 / x=2) = \frac{P(Y=3 \cap x=2)}{P(x=2)} = \frac{\frac{1}{4}}{\frac{19}{36}} = \frac{1}{\cancel{4}} \times \frac{\cancel{36}^{9}}{19} \Rightarrow \frac{9}{19}$

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Stochastic Process

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[1.] A stochastic process $X = \{x(t), t \in T\}$ is a collection of random Variables

[2.] we often interpret t as time & call $x(t)$ the state of process at time t

[3.] If index set T is countable set, we call X a discrete time stochastic process.

[4.] If T is continuous, we call it a continuous time stochastic process.

[5.] The set T is called indexing set or parameter space, where $t \in T$ may denote time, length, distance or any other quantity

[6.] S is a set of all possible values of random variable where, $X(t) : t \in T$ is called state space.

[7.] The set T & S could be arbitrary but in practice they are discrete or continuous.

[8.] The time index index may be either discrete such as, $n = 0, 1, 2, \dots$ or continuous such as $t \in R = [0, \infty]$

[9.] The possible values of these random Variables are called states of process. And set of all states is called the state space.

classification of Stochastic Process :-

[1.] Discrete time and discrete State space

[2.] Discrete time and continuous state space

[3.] continuous time and discrete state space

[4.] continuous time and continuous State space

Absorbing state :- once the process enters the state forever the process will be there itself then it is called absorbing state.

Markov chains

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[1.] let $X = \{X_n = 0, 1, 2, \ldots\}$ be stochastic process with finite or countable state space.

[2.] if $X_n = i$, then the process is called to be in state "i" at time "n"

[3.] Suppose that whenever the process is in state "i" there is fixed probability P_{ij} that it will next time be in state j

[4.] whenever the Indexing set T is countable we call it as chain

[Q.] computation of K^{th} order transition probability of markov chain

let $S = \{1, 2, 3, 4\}$ and

$$P = \begin{array}{c|cccc} i \backslash j & 1 & 2 & 3 & 4 \\ \hline 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 2 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 3 & 0 & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 4 & 0 & 0 & 0 & 1 \end{array}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B / B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = \frac{P(A \cap B / B)}{P(B)}$$

In tpm each row in a probability distribution

let $S = (1, 0, 0, 0)$ be distribution of X_0.

Soln :- (1) $P[X_1 = 3] = P[X_1 = 3, X_0 = 1] + P[X_1 = 3, X_0 = 2] + P[X_1 = 3, X_0 = 3] + P[X_1 = 3, X_0 = 4]$

$= P[X_0 = 1]\, P[X_1 = 3 \mid X_0 = 1] + P[X_0 = 2]\, P[X_1 = 3 \mid X_0 = 2] + P[X_0 = 3]\, P[X_1 = 3 \mid X_0 = 3] + P[X_0 = 4]\, P[X_1 = 3 \mid X_0 = 4]$

$= (1 \times 0) + (0 \times \frac{1}{4}) + (0 \times \frac{1}{4}) + (0 \times 0)$

$= 0 + 0 + 0 + 0$

$= 0$

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(2) $P_{13}^{(2)} = P$[Being in state 1, going to state 3 after 2 transition]

$P_{13}^{(2)} = P[X_{n+2} = 3 \mid X_n = 1]$

$P_{13}^{(2)} = P[X_{n+2} = 3, X_{n+1} = 1 \mid X_n = 1] + P[X_{n+2} = 3, X_{n+1} = 2 \mid X_n = 1] + P[X_{n+2} = 3, X_{n+1} = 3 \mid X_n = 1] + P[X_{n+2} = 3, X_{n+1} = 4 \mid X_n = 1]$

$= \sum_{K=1}^{4} P[X_{n+2} = 3, X_{n+1} = K \mid X_n = 1]$

$= \sum_{K=1}^{4} P[X_{n+2} = 3, X_{n+1} = K \mid X_n = 1] \; P[X_{n+1} = K \mid X_n = 1]$

$\therefore P_{13}^{(2)} = \sum_{K=1}^{4} P[X_{n+2} = 3 \mid X_{n+1} = K] \; P[X_{n+1} = K \mid X_n = 1]$

$= P_{11} P_{13} + P_{12} P_{23} + P_{13} P_{33} + P_{14} P_{43}$

$= \left(\frac{1}{2} \times 0\right) + \left(\frac{1}{2} \times \frac{1}{4}\right) + \left(0 \times \frac{1}{4}\right) + (0 \times 0)$

$= 0 + \frac{1}{8} + 0 \times 0$

$= \frac{1}{8}$

$\boxed{\text{Chapman - Kolmogorov eq}^n}$:-

[Q:] Let X_n, $n \geq 0$ be Markov chain with a state space $\{1, 2, 3\}$ and t.p.m

$$\boxed{P_{ij}^{(m+n)} = \sum_{K \in S} P_{iK}^{(m)} P_{Kj}^{(n)}}$$

$$P = \begin{bmatrix} 0.3 & 0.7 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \end{bmatrix}$$. Find $P[X_4 = 3 \mid X_2 = 1]$

Soln:- $P[X_4 = 3 \mid X_2 = 1] = P_{13}^{(2)} = P_{13}^{(1+1)} = \sum_{K \in S} P_{1K}^{(1)} P_{K3}^{(1)}$

$= P_{11} P_{13} + P_{12} P_{23} + P_{13} P_{33}$

$= (0.3 \times 0) + (0.7 \times 0.8) + (0 \times 1)$

$= 0 + 0.56 + 0 \Rightarrow 0.56$

[Q.) let X_n, $n \geq 0$ be markov chain with state space $\{1,2\}$ and t.p.m

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

Find

(i) $P[X_4 = 1 \mid X_2 = 0]$

(ii) $P[X_5 = 2 \mid X_2 = 1]$

(iii) $P[X_1 = 2]$ given the initial probability are $\frac{1}{3}, \frac{1}{3}$ & $\frac{1}{3}$

Soln:- (i) $P[X_4 = 1 \mid X_2 = 0] = P_{01}^{(2)} = P_{01}^{(1+1)}$

$$= \sum_{k \in S} P_{0k}^{(1)} P_{k1}^{(1)}$$

$$= P_{01} P_{11} + P_{02} P_{21}$$

$$= \left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{2} \times 0\right)$$

$$= \frac{1}{6}$$

(ii) $P[X_5 = 2 \mid X_2 = 1] = P_{12}^{(3)}$

$$= P_{12}^{(2+1)}$$

$$= P_{12}^{(2)} \cdot P^{(1)}$$

$$= \sum_{k \in S} P_{1k}^{(2)} P_{k2}^{(1)}$$

$$= P_{10}^{(2)} P_{02}^{(1)} + P_{11}^{(2)} P_{12}^{(1)} + P_{12}^{(2)} P_{22}^{(1)} \quad \text{--- (1)}$$

consider, $P_{10}^{(2)} = \sum_{k \in S} P_{1k}^{(1)} P_{k0}^{(1)}$

$$= P_{10}^{(1)} P_{00}^{(1)} + P_{11}^{(1)} P_{10}^{(1)} + P_{12}^{(1)} P_{20}^{(1)}$$

$$= \left(\frac{1}{3} \times 0\right) + \left(\frac{1}{3} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times 0\right)$$

$$= \frac{1}{9} \quad \text{--- (2)}$$

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Consider, $P_{11}^{(2)} = \sum_{k\in S} P_{1k}^{(1)} P_{k1}^{(1)}$

$= P_{10}^{(1)} P_{01}^{(1)} + P_{11}^{(1)} P_{11}^{(1)} + P_{12}^{(1)} P_{21}^{(1)}$

$= \left(\frac{1}{3}\times\frac{1}{2}\right) + \left(\frac{1}{3}\times\frac{1}{3}\right) + \left(\frac{1}{3}\times 0\right)$

$= \frac{1}{6} + \frac{1}{9} \Rightarrow \frac{3+2}{18} \Rightarrow \frac{5}{18}$ — (3)

consider $P_{12}^{(2)} = \sum_{k\in S} P_{1k}^{(1)} P_{k2}^{(1)}$

$= P_{10}^{(1)} P_{02}^{(1)} + P_{11}^{(1)} P_{12}^{(1)} + P_{12}^{(1)} P_{22}^{(1)}$

$= \left(\frac{1}{3}\times\frac{1}{2}\right) + \left(\frac{1}{3}\times\frac{1}{3}\right) + \left(\frac{1}{3}\times 1\right)$

$= \frac{1}{6} + \frac{1}{9} + \frac{1}{3}$

$\Rightarrow \frac{3+2+6}{18} \Rightarrow \frac{11}{18}$ — (4)

put (2), (3), (4) in (1)

$\Rightarrow \frac{1}{9}\times\frac{1}{2} + \frac{5}{18}\times\frac{1}{3} + \frac{11}{18}\times 1$

$\Rightarrow \frac{1}{18} + \frac{5}{54} + \frac{11}{18} \Rightarrow \frac{41}{54}$

(iii) $P[X_1 = 2] = \sum_{k\in S} P[X_1 = 2, X_0 = k]$

$= P[X_1 = 2, X_0 = 0] + P[X_1 = 2, X_0 = 1] +$
$P[X_1 = 2, X_0 = 2]$

$= P[X_0 = 0]\, P[X_1 = 2 \mid X_0 = 0] + P[X_0 = 1]\, P[X_1 = 2 \mid X_0 = 1]$
$+ P[X_0 = 2]\, P[X_1 = 2 \mid X_0 = 2]$

$= P[X_0 = 0]\, P_{02} + P[X_0 = 1]\, P_{12} + P[X_0 = 2]\, P_{22}$

$= \left(\frac{1}{3}\times\frac{1}{2}\right) + \left(\frac{1}{3}\times\frac{1}{3}\right) + \left(\frac{1}{3}\times 1\right)$

$= \frac{1}{6} + \frac{1}{9} + \frac{1}{3}$

$= \frac{11}{18}$

[Q.] let X_n, $n \geq 0$ be a markov chain with state space $\{1,2,3,4\}$ with t.p.m

δ is uniform $= \frac{1}{4}$

$$P = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

(i) $P[X_1 = 1]$

(ii) $P[X_2 = 1]$

(iii) $P[X_5 = 3, X_3 = 2 \mid X_1 = 3]$

(i) $\sum_{k \in S} P[X_1 = 1, X_0 = k]$

$= \sum_{k \in S} P[X_1 = 1 \mid X_0 = k] \; P[X_0 = k]$

$= P[X_1 = 1 \mid X_0 = 1] \; P[X_0 = 1] + P[X_1 = 1, X_0 = 2] \; P[X_0 = 2] +$
$P[X_1 = 1 \mid X_0 = 3] \; P[X_0 = 3] + P[X_1 = 1, X_0 = 4] \; P[X_0 = 4]$

$= \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} + 0 \times \frac{1}{4} + 0 \times \frac{1}{4}$

$= \frac{1}{16} + \frac{1}{8} \Rightarrow \frac{1+2}{16}$

$\Rightarrow \frac{3}{16}$

$$P^2 = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} + \frac{1}{8} & \frac{1}{16} + \frac{1}{8} & \frac{1}{16} + \frac{1}{16} + \frac{1}{8} & \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} + \frac{1}{8} & \frac{1}{8} + \frac{1}{8} & \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} + \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} + \frac{1}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{16} & \frac{3}{16} & \frac{1}{4} & \frac{3}{8} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{4} & \frac{3}{8} \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{3}{8} \end{bmatrix}$$

(ii) $P[X_2 = 1] = \sum_{k \in S} P[X_2 = 1, X_0 = k]$

$= \sum_{k \in S} P[X_2 = 1 \mid X_0 = k] \; P[X_0 = k]$

$= \sum_{k \in S} P^{(2)}_{k1} \quad \boxed{P(X_0 = k)}$

$= P^{(2)}_{11} \; P(X_0 = 1) + P^{(2)}_{21} \; P(X_0 = 2) + P^{(2)}_{31} \; P(X_0 = 3) + P^{(2)}_{41} \; P(X_0 = 4)$

$= \frac{1}{4}\left(\frac{3}{16} + \frac{1}{8} + 0 + \frac{1}{4}\right)$

$= \frac{1}{4} \times \left(\frac{3+2+4}{16}\right)$

$= \frac{1}{4} \times \frac{9}{16}$

$= \frac{9}{64}$

(iii) $P[X_5 = 3, X_3 = 2 \mid X_1 = 3]$

$P[X_5 = 3, X_3 = 2 \mid X_1 = 3]$ $\qquad S = \{1, 2, 3, 4\}$

$= \dfrac{P(X_5 = 3, X_3 = 2, X_1 = 3)}{P(X_1 = 3)}$

$= \dfrac{P(X_5 = 3 \mid X_3 = 2, X_1 = 3) \; P(X_3 = 2, X_1 = 3)}{P(X_1 = 3)}$

$= P(X_5 = 3 \mid X_3 = 2) \; P(X_3 = 2 \mid X_1 = 3)$

$= P^{(2)}_{23} \; P^{(2)}_{32}$

$= \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{16}$

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Counting Process :- stochastic process is called counting process if $N(t)$ represents no. of events occurred upto t

• counting process is called poisson process if it follows poisson distribution with parameter λ and t

$$\therefore \boxed{[N(t) \sim P(\lambda t)]}$$

[Q1] The following data gives the time taken (in min) by 11 students to develop an algorithm for specific problem.

~~21~~ ~~26~~ ~~29~~ ~~20~~ ~~16~~ ~~17~~ ~~16~~ ~~8~~ ~~20~~ ~~14~~ ~~18~~

(i) Determine the quartiles and inter quartile range of the data
(ii) construct the box and whisker plot for the data
(iii) mention the outlier present if any.

Solⁿ :- Arrange data in ascending order :-

8 14 16 16 17 18 20 $\overset{20}{\wedge}$ 21 26 29

$$Q_1 = \left[\frac{1 \times (11+1)}{4}\right]^{th} \text{ value}$$

$$= 3^{rd} \text{ value} \Rightarrow 16$$

$$Q_2 = \left[\frac{2 \times (11+1)}{4}\right]^{th} \text{ value}$$

$$= \left[\frac{24}{4}\right]^{th} \text{ value}$$

$$= 6^{th} \text{ value} \Rightarrow 18$$

$$Q_3 = \left[\frac{3 \times (11+1)}{4}\right]^{th} \text{ value}$$

$$= 9^{th} \text{ value}$$

$$= 21$$

$$IQR = Q_3 - Q_1$$

$$= 21 - 16$$

$$= 5$$

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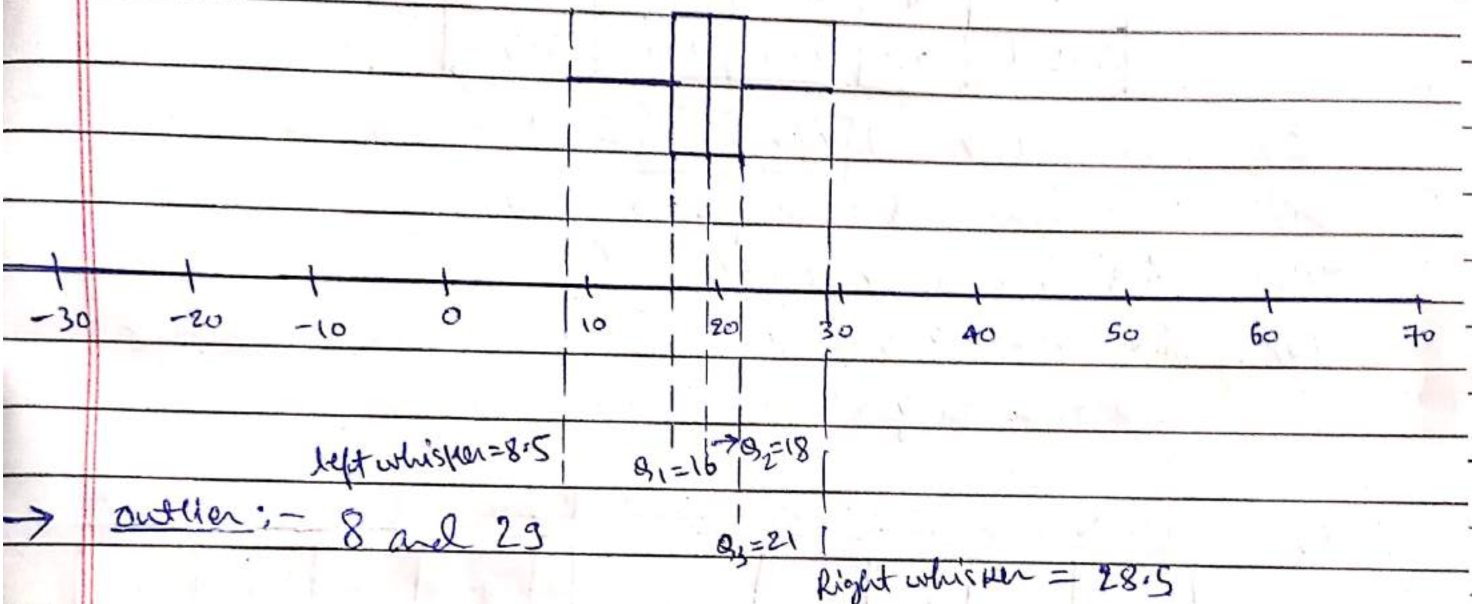
left whisker/ lower inner fence = $Q_1 - 1.5 \times IQR$

$= 16 - 1.5 \times 5 \Rightarrow 8.5$

Right whisker/ upper inner fence = $Q_3 + 1.5 \times IQR$

$= 21 + 1.5 \times 5 \Rightarrow 28.5$

→ Box & whisker plot



→ Outlier :- 8 and 29

3.6] correlation coefficient :- The correlation coefficient is statical concept which helps in establishing a relationship b/w predicted & actual value obtained in statical experiment. The calculated value of correlation coefficient explains the exactness b/w predicted & actual values. correlation coefficient is given by,

$$\gamma = \frac{cov(x,y)}{sd(x) \cdot sd(y)}$$

condition for independent of two random variable X & Y.

→ If Both X and Y are independent variables that means there is no correlation b/w them. thus, $\gamma = 0$

[3.c] let X and Y be any two random variables each taking three values $-1, 0$ and 1 and having the following joint probability distribution :-

X \ Y	-1	0	1	Total
-1	0	0.1	0.1	0.2
0	0.2	0.2	0.2	0.6
1	0.	0.1	0.1	0.2
Total	0.2	0.4	0.4	1.0

(i) Show that X and Y have different expectations.
(ii) Prove that X & Y are uncorrelated
(iii) Find Var(X) & Var(Y)
(iv) given that $Y=0$, what is the conditional probability distribution of X?

Soln: Marginal distribution of x

x	-1	0	1
$P(x)$	0.2	0.6	0.2

Marginal distribution of Y.

y	-1	0	1
$P(y)$	0.2	0.4	0.4

(i) $E(x) = \Sigma x P(x)$

$= -1\times 0.2 + 0\times 0.6 + 1\times 0.2$

$= -0.2 + 0 + 0.2$

$= 0$

$E(y) = \Sigma y\, P(y)$

$= -1\times 0.2 + 0\times 0.4 + 1\times 0.4$

$= -0.2 + 0 + 0.4$

$= 0.2$

(ii) $\text{cov}(X,Y) = E(XY) - E(X)E(Y)$

$$= \Big(-1\times-1\times0 + -1\times0\times0.1 + -1\times1\times0.1 + 0\times-1\times0.2 + 0\times0\times0.2 + 0\times1\times0.2 + 1\times-1\times0 + 1\times0\times0.1 + 1\times1\times0.1\Big) - \Big(0\times0.2\Big)$$

$$= (0 + 0 + (-0.1) + 0 + 0 + 0 + 0 + 0 + 0.1) - (0)$$

$$= (0 + 0 - \cancel{0.1} + 0 + 0 + 0 + 0 + 0 + \cancel{0.1}) - (0)$$

$$\Rightarrow 0$$

$\boxed{\text{cov}(X,Y) = 0}$ Proved that X & Y are uncorrelated

(iii)

$E(X') = \Sigma x^2 P(x)$

$$= [(-1)^2 \times 0.2] + [(0)^2 \times 0.6] + [(1)^2 \times 0.2]$$

$$\Rightarrow 0.2 + 0 + 0.2 \Rightarrow 0.4$$

→ $\text{Var}(x) = E(X') - [E(X)]^2$

$$= 0.4 - (0)^2 \Rightarrow 0.4$$

$E(Y') = \Sigma y^2 P(y)$

$$= [(-1)^2 \times 0.2] + [(0)^2 \times 0.4] + [(1)^2 \times 0.4]$$

$$= 0.2 + 0 + 0.4 \Rightarrow 0.6$$

→ $\text{Var}(Y) = E(Y') - [E(Y)]^2$

$$= 0.6 - (0.2)^2$$

$$= 0.6 - 0.04 \Rightarrow 0.56$$

(iv) $P(X = -1, 0, 1 / Y = 0)$

$P(X=-1 / Y=0) = \dfrac{P(X=-1 \cap Y=0)}{P(Y=0)} = \dfrac{0.1}{0.4} \dfrac{\times 10}{\times 10} \Rightarrow \dfrac{1}{4}$

$P(X=0 / Y=0) = \dfrac{P(X=0 \cap Y=0)}{P(Y=0)} = \dfrac{0.2}{0.4} \dfrac{\times 10}{\times 10} \Rightarrow \dfrac{2}{4}$

$P(X=1 / Y=0) = \dfrac{P(X=1 \cap Y=0)}{P(Y=0)} = \dfrac{0.1}{0.4} \dfrac{\times 10}{\times 10} \Rightarrow \dfrac{1}{4}$

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[2.b] A problem in statistics is given to two students Sunil & Ram. The chances of solving the problem by these students are $\frac{1}{4}$ & $\frac{3}{4}$ respectively. What is the probability that the problem will be solved if both try independently?

Solⁿ:- let
$P(A) = P$ [Solving the problem by student A] $= \frac{1}{4}$
$P(B) = P$ [Solving the problem by student B] $= \frac{3}{4}$

P [The problem solved, if both them try independently]

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$$
$$= \frac{1}{4} + \frac{3}{4} - \frac{1}{4} \cdot \frac{3}{4}$$
$$= \frac{4}{4} - \frac{3}{16} \Rightarrow \frac{16-3}{16} \Rightarrow \frac{13}{16}$$
$$P(A \cup B) = 0.8125$$

[4.a] Define Bernoulli variate with eg. Explain P.m.f of binomial distribution, list some of situations where binomial distribution is applicable.

Solⁿ:- Bernoulli Variate :- A random Variable taking only two value
(i) 1 & 0 with probability P and (1-P) is known as Bernoulli Variate.
eg:- Tossing a coin & getting head or tail is a bernoulli trial. If X is getting head, then X takes two value. 1 = getting head &
0 = getting tail

then,
$$P(X=1) = \frac{1}{2} \quad \& \quad P(X=0) = 1 - \frac{1}{2} \Rightarrow \frac{1}{2}$$

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Binomial Distribution :- A discrete random variable X has a binomial distribution with parameter n & p, where, n = 1, 2, ... $0 \le p \le 1$ if its p.m.f is given by,

$$\boxed{P(X=x) = \binom{n}{x} p^{x} q^{n-x} ; \quad q = 1-p, \; x = 0, 1, 2, \ldots n}$$

Binomial distribution is applicable in following situation :-

Tossing a coin
Throwing a dice
collection of opinion

The mean life of electric bulbs manufactured by a firm is 1200 hrs with a standard deviation of 200 hrs. with the assumption of the life of bulbs follows normal distribution, answer the following :-

In a box of 10,000 bulbs, how many bulbs are expected to have life of 1050 hrs or more?
Find the percentage of bulbs which are expected to fuse before 1500 hrs of service ?
calculate the probability that the mean life of the bulbs lies b/w 800 hrs to 1500 hrs ?
How many bulbs are expected to have life b/w 1050 hrs and 1150 hrs ?

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Sol^n:- let X = life of electric bulbs

$\mu = 1200$, $\sigma = 200$

(i) $P(X \geq 1050) = P\left[\frac{X-\mu}{\sigma} \geq \frac{1050-1200}{200}\right]$

$= P[Z > -0.75]$

$= 1 - P[Z \leq -0.75]$

$= 1 - 0.2266$

$= 0.7734$

→ No. of bulbs have life of more than 1050 hrs is

$= 10,000 \times 0.7734$

$= 7734$

(ii) $P(X \leq 1500) = P\left(\frac{X-\mu}{\sigma} < \frac{1500-1200}{200}\right)$

$= P(Z < 1.5)$

$= 0.9332$

→ % of bulb expected to fuse before 1500 hrs is 93.32 %

(iii) $P(800 < X < 1050) = P\left(\frac{800-1200}{200} < Z < \frac{1050-1200}{200}\right)$

$= P(-2 < Z < -0.75)$

$= P[Z = -0.75] - P[Z = -2]$

$= 0.2266 - 0.0228$

$= 0.2038$

(iv) $P(1050 < X < 1150) = P\left[\frac{1050-1200}{200} < Z < \frac{1150-1200}{200}\right]$ [redacted]

$=$ [redacted] $P(-0.75 < Z < -0.25)$

$= P[Z = -0.25] - P[Z = -0.75]$

$= 0.4012 - 0.2266 \Rightarrow 0.1747$

∴ No. of bulb expected to life b/w 1050 hrs & 1150 hrs is $= 10,000 \times 0.1747$

$= 1747$

[5.6] Two random variable X & Y has the following bivariate Probability distribution.

X \ Y	1	2	3	4	5	6
0	0	0	$1/32$	$2/32$	$2/32$	$3/32$
1	$1/16$	$1/16$	$1/8$	$1/8$	$1/8$	$1/8$
2	$1/32$	$1/32$	$1/64$	$1/64$	0	$2/64$

determine

① Marginal density of X & Y

② $P(X \le 1, Y \le 4)$

③ $P(X = 1 / Y = 4)$ and $P(X = 2 / Y = 6)$

Solⁿ :- ①

Marginal density of X

X	0	1	2	Total
P(x)	$8/32$	$10/16$	$8/64$	$\frac{16+40+8}{64} = \frac{64}{64} \Rightarrow 1$

Marginal density of Y

Y	1	2	3	4	5	6	Total
P(Y)	$3/32$	$3/32$	$11/64$	$13/64$	$6/32$	$16/64$	$\frac{64}{64} = 1$ $\frac{6+6+11+13+12+16}{64} = \frac{64}{64} \Rightarrow 1$

(ii) $P(X \le 1, Y \le 4) = P[X = 0, 1 ; Y = 1, 2, 3, 4]$

$P(X = 0, Y = 1, 2, 3, 4) = 0 + 0 + \frac{1}{32} + \frac{2}{32} = 3/32$

$P(X = 1, Y = 1, 2, 3, 4) = 1/16 + \frac{1}{16} + \frac{1}{8} + \frac{1}{8} \Rightarrow \frac{6}{16}$

$\therefore P(X \le 1, Y \le 4) = 3/32 + 6/16 \Rightarrow 15/32 \Rightarrow 0.47$

determine the median of following data representing IQ score of CS students
82 95 86 104 125 137 113 145 131
First arrange data in ascending order :- 82 86 95 104 113 125 131 137 145
$n = 9$ since n is odd. then, median $= \left[\frac{(n+1)}{2}\right]$ th position $\Rightarrow \frac{10}{2} \Rightarrow 5$th position $\Rightarrow 113$

$$P(X=1 / y=4) = \frac{P(X=1 \cap Y=P)}{P(Y=4)} = \frac{1/8}{13/64} = \frac{1}{\cancel{8}} \times \frac{\cancel{64}^{8}}{13} \Rightarrow \frac{8}{13} \Rightarrow 0.62$$

$$P(X=2 / Y=6) = \frac{P(X=2 \cap Y=6)}{P(Y=6)} = \frac{2/64}{16/64} = \frac{2}{\cancel{64}} \times \frac{\cancel{64}}{\cancel{16}\,8} \Rightarrow \frac{2}{16} \Rightarrow 0.125$$

define unbiased estimator. show that for a normal distribution, the sample mean, $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, is an unbiased estimator of μ.

- Unbiased estimator :- An estimator T_n is said to be an unbiased estimator of the parameter θ. If $E(T_n) = \theta$.

eg :- The sample mean $\bar{x}$ is an unbiased estimator of μ.

Proof :- If $X \sim ND(\mu, \sigma^2)$

$$E(x_i) = \mu$$
$$V(x_i) = \sigma^2$$

$$E(\bar{x}) = E\left(\frac{\Sigma x_i}{n}\right) = \frac{1}{n} E(x_1 + x_2 + \cdots + x_n)$$
$$= \frac{1}{n}\{E(x_1) + E(x_2) + \cdots + E(x_n)\}$$
$$= \frac{1}{n}\{\mu + \mu \ldots + n \text{ times}\}$$
$$= \frac{1}{\cancel{n}} \cancel{n} \cdot \mu$$
$$= \mu$$

$\boxed{E(\bar{x}) = \mu}$ $\rightarrow$ The sample mean $\bar{x}$ is an unbiased estimator of mean normal distribution μ.

Uniform Distribution [Discrete case] :- Suppose $S = \{\omega_1, \omega_2, \dots, \omega_n\}$ is a Sample space with n equally likely outcomes. Then probability of

$P(\omega_k)$, $k = 1, 2, 3, \dots, n$

$$\boxed{P(\omega_k) = \frac{1}{n}}$$

eg :- ① Tossing a coin and the probability of getting head & tail are uniform

② Throwing an unbiased dice and the probability of getting no. 1, 2, 3, 4, 5, 6 is $\frac{1}{6}$

Mean :- $E(X) = \Sigma x\, P(x)$

$$\boxed{E(X) = \frac{n+1}{2}}$$

$$= \sum_{x=1}^{n} x \left(\frac{1}{n}\right)$$

$$= \frac{1}{n}(1+2+3+\dots+n)$$

$$= \frac{1}{\cancel{n}} \frac{\cancel{n}(n+1)}{2} \Rightarrow \frac{n+1}{2}$$

Variance :- $\text{Var}(X) = E(X^2) - [E(X)]^2$

$=$

$$E(X^2) = \Sigma x^2 P(x)$$

$$= \sum_{x=1}^{n} x^2 \left(\frac{1}{n}\right)$$

$$= \frac{1}{n}(1^2 + 2^2 + 3^2 + \dots + n^2)$$

$$= \frac{1}{\cancel{n}} \frac{\cancel{n}(n+1)(2n+1)}{6}$$

$$= \frac{(n+1)(2n+1)}{6}$$

$$2 \,\underline{|\, 6, 2}$$
$$\quad\; 3, 2$$

$$\text{Var}(X) = \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 \qquad \Rightarrow 12$$

$$= \frac{2(n+1)(2n+1) - 3(n+1)^2}{12} \qquad \Rightarrow \frac{n^2-1}{12}$$

$$= \frac{2(2n^2 + n + 2n + 1) - 3(n^2 + 2n + 1)}{12}$$

$$= \frac{4n^2 + 2n + 4n + 2 - 3n^2 + 6n + 3}{12}$$

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uniform distribution [continuous case] :-

A random variable X is said to be uniformly distributed over interval $[\alpha, \beta]$ if its p.d.f is given by,

$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & , \text{if } \alpha \le x \le \beta \\ 0 & , 0 \end{cases}$$

The uniform distribution arises in practice when we have certain random variable is equally likely to be near any value in the interval $[\alpha, \beta]$

eg:- If X is [redacted] uniformly distributed over the interval $[0, 10]$, compute the probability that

ⓐ $2 < x < 9$

ⓑ $1 < x < 4$

ⓒ $x < 5$

ⓓ $x > 6$

Ans:-

ⓐ $P(2 \le x \le 9) = \dfrac{9-2}{10-0} = \dfrac{7}{10}$

ⓑ $P(1 \le x < 4) = \dfrac{4-1}{10-0} = \dfrac{3}{10}$

ⓒ $P\{x < 5\} = \dfrac{5-0}{10-0} \Rightarrow \dfrac{5}{10}$

ⓓ $P\{x > 6\} = \dfrac{10-6}{10-0} \Rightarrow \dfrac{4}{10}$

Mean & Variance of Uniform distribution with interval $[a,b]$:-

① **Mean :-**

$$E(X) = \int_a^b x\, f(x)\, dx$$

$$= \int_a^b x \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{b-a}\left[\frac{b^2}{2} - \frac{a^2}{2}\right]$$

$$= \frac{1}{b-a}\left[\frac{b^2-a^2}{2}\right]$$

$$= \frac{1}{2}\frac{(b^2-a^2)}{(b-a)}$$

$$= \frac{1}{2}\frac{\cancel{(b-a)}(b+a)}{\cancel{(b-a)}} \Rightarrow \frac{b+a}{2}$$

$$\boxed{E(X) = \frac{b+a}{2}}$$

② **Variance :-**

$$Var(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \int_a^b x^2 f(x)\, dx$$

$$= \int_a^b x^2 \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$= \frac{1}{b-a}\left[\frac{b^3}{3} - \frac{a^3}{3}\right]$$

$$= \frac{1}{b-a}\left[\frac{b^3-a^3}{3}\right]$$

$$= \frac{1}{3}\frac{(b^3-a^3)}{(b-a)}$$

$$= \frac{1}{3}\frac{\cancel{(b-a)}(b^2+ba+a^2)}{\cancel{(b-a)}} \Rightarrow \frac{b^2+ba+a^2}{3}$$

$$\boxed{E(X^2) = \frac{b^2+ab+a^2}{3}}$$

$$Var(X) = \frac{b^2+ba+a^2}{3} - \left(\frac{b+a}{2}\right)^2$$

$$= \frac{b^2+ba+a^2}{3} - \frac{(b+a)^2}{4}$$

$$= \frac{4(b^2+ba+a^2) - 3(b+a)^2}{12}$$

$$= \frac{4b^2+4ba+4a^2-3(a^2+b^2+2ab)}{12}$$

$$= \frac{4b^2+4ba+4a^2-3a^2-3b^2-6ab}{12}$$

$$= \frac{b^2-2ba+a^2}{12}$$

$$= \frac{1}{12}(b-a)^2$$

$$= \frac{(b-a)^2}{12}$$

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Moment generating function (M.g.f) :-

The m.g.f of uniform distribution is defined with interval $[a,b]$ is

$$M_x(t) = E\left[e^{tx}\right]$$

$$= \int_a^b e^{tx} \cdot \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a} \int_a^b e^{tx}\, dx$$

$$= \frac{1}{b-a} \left[\frac{e^{tx}}{t}\right]_a^b$$

$$= \frac{1}{b-a} \left[\frac{e^{tb}}{t} - \frac{e^{ta}}{t}\right]$$

$$= \frac{1}{b-a} \left[\frac{e^{tb} - e^{ta}}{t}\right]$$

$$= \frac{e^{tb} - e^{ta}}{t(b-a)}, \quad b > a$$