C240424_7

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2025-01-05

Exercise 7-1

```r
LI <- c(8, 8, 10, 10, rep(c(12, 14, 16), each = 3),
        18, 20, 20, 20, 22, 22, 24, 26, 28, 32, 34, rep(38, 3))
y <- c(rep(0, 13), 1, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 1, 1, 0)
cancer.fit <- glm(y ~ LI, family=binomial)
summary(cancer.fit)
```

```
##
## Call:
## glm(formula = y ~ LI, family = binomial)
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -3.77714    1.37862  -2.740  0.00615 **
## LI           0.14486    0.05934   2.441  0.01464 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 34.372  on 26  degrees of freedom
## Residual deviance: 26.073  on 25  degrees of freedom
## AIC: 30.073
##
## Number of Fisher Scoring iterations: 4
```

```r
confint(cancer.fit)
```

```
## Waiting for profiling to be done...
```

```
##                  2.5 %     97.5 %
## (Intercept) -6.9951909 -1.4098443
## LI           0.0425232  0.2846668
```

a. $\because P(Y=1) = \pi = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}, \alpha \approx -3.78, \beta \approx 0.145$
$\therefore \hat{P}(Y=1) = \frac{e^{-3.78+0.145*26}}{1+e^{-3.78+0.145*26}} \approx 0.4975 \approx 0.5$

```r
LI_test <- 26
intercept <- coef(cancer.fit)[1]
beta <- coef(cancer.fit)[2]
P_Y1 <- exp(intercept + beta * LI_test) / (1 + exp(intercept + beta * LI_test))
P_Y1
```

```
## (Intercept)
##   0.4973257
```

b. $\because \frac{\pi}{1-\pi} = e^{\alpha+\beta x}$
$\therefore e^{\beta} = e^{0.145} = 0.1156 \approx 0.116$
The odds of a success are multiplied by $e^{\beta}(\approx 0.116)$ when LI is increased by 1

c. The log-odds of remission are added by 0.145 per 1 unit increase of LI.

d. $\because$ Rate of Change $= \beta\hat{\pi}(x)[1-\hat{\pi}(x)]$
$\hat{\pi}(18) = \frac{e^{-3.78+0.145*18}}{1+e^{-3.78+0.145*18}} = 0.2369268$
$\therefore 0.145 * 0.247(1-0.247) \approx 0.026$

```r
LI_test2 <- 18
P2_Y1 <- exp(intercept + beta * LI_test2) / (1 + exp(intercept + beta * LI_test2))
P2_Y1
```

```
## (Intercept)
##   0.2369268
```

```r
dP <- beta * P2_Y1 * (1 - P2_Y1)
round(dP, 3)
```

```
##    LI
## 0.026
```

Exercise 7-2

a. Wald test for Hypothesis: $\beta = 0$:
$z_W = \frac{\hat{\beta}_{width}}{SE(\hat{\beta}_{width})} = 0.145/0.06 = 2.4167$
Since p value $< .05$, assuming default significance level of $\alpha = .05$, we reject the null hypothesis.

b. Wald 95% CI for odds ratio $\sim e^{\beta}$:

```r
exp(confint.default(cancer.fit))
```

```
##                   2.5 %    97.5 %
## (Intercept) 0.001535048 0.3412682
## LI          1.028968667 1.2984476
```

Which means that for every 1% increase in LI, the odds of remission increase by a factor between about approximately 1.03 and 1.3.

c. Likelihood-ratio test for Hypothesis: $\beta = 0$:

```r
drop1(cancer.fit, test = "LRT")
```

```
## Single term deletions
##
## Model:
## y ~ LI
##        Df Deviance    AIC    LRT Pr(>Chi)
## <none>      26.073 30.073
## LI      1   34.372 36.372 8.2988 0.003967 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$\chi^2(1) = 8.2988$, at significance level $\alpha = .05$, we reject the null hypothesis as p-value $\approx .004 < .05$.

d. Likelihood-ratio CI for e^{β}:

```r
exp(confint(cancer.fit))
```

```
## Waiting for profiling to be done...

##                    2.5 %    97.5 %
## (Intercept) 0.0009162778 0.2441813
## LI          1.0434402672 1.3293190
```

Which means that for every 1% increase in LI, the odds of remission increase by a factor between about approximately 1.04 and 1.33.

Exercise 7-3

To be solved in/after lecture 9.