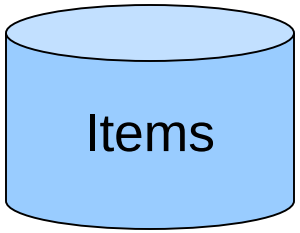
推薦システム
(Recommendation Systems)

Recommendations

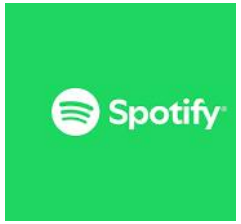
Search

Recommendations



Products, web sites, blogs, news items, …

Examples:

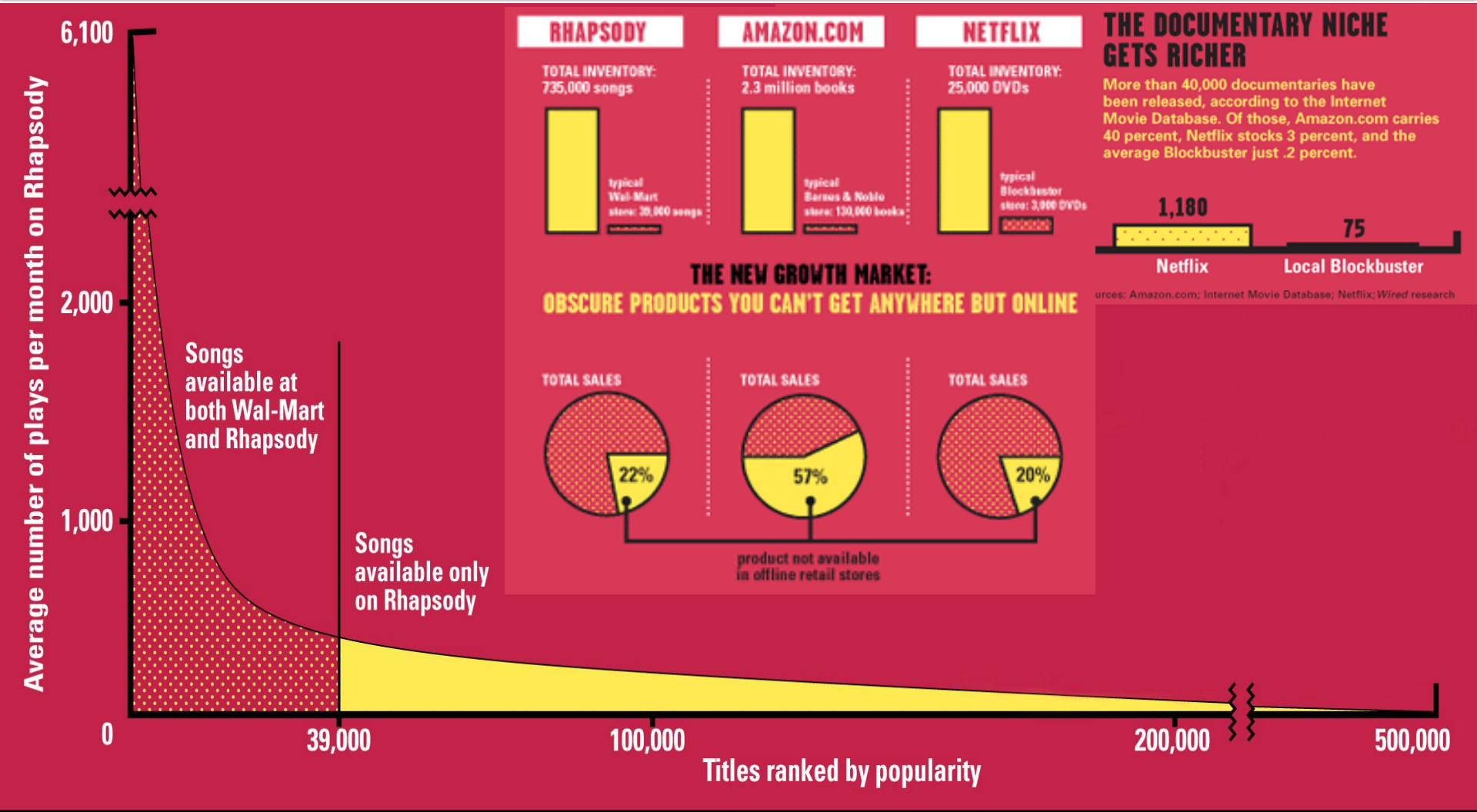


- **Customer X**
  - Buys Metallica CD
  - Buys Megadeth CD
- **Customer Y**
  - Does search on Metallica
  - Recommender system suggests Megadeth from data collected about customer **X, etc.**

From Scarcity to Abundance

- **Shelf space is a scarce commodity for traditional retailers**
  - Also: TV networks, movie theaters,…
- **Web enables near-zero-cost dissemination of information about products**
  - From scarcity to abundance
- **More choice necessitates better filters**
  - Recommendation engines
  - How **Into Thin Air** made **Touching the Void** a bestseller

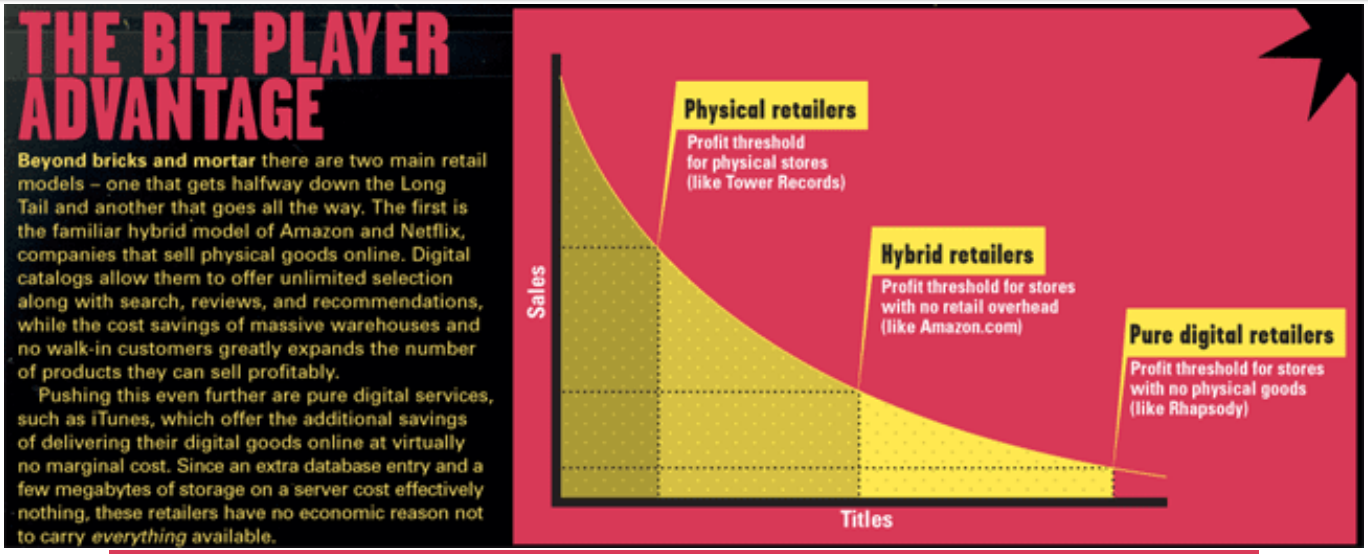
Sidenote: The Long Tail

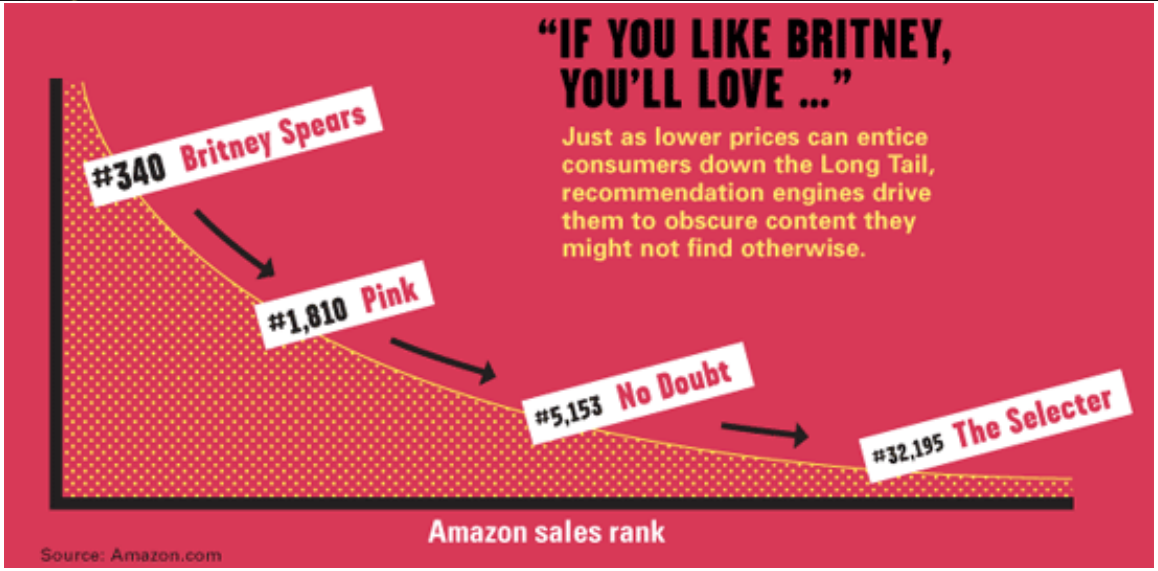


Sources: Erik Brynjolfsson and Jeffrey Hu, MIT, and Michael Smith, Carnegie Mellon; Barnes & Noble; Netflix; RealNetworks

Source: Chris Anderson (2004)

Physical vs. Online





Types of Recommendations

- **Editorial and hand curated (no user input)**
  - List of favorites
  - Lists of “essential” items
- **Simple aggregates**
  - Top 10, Most Popular, Recent Uploads
- **Tailored to individual users (our focus)**
  - Amazon, Netflix, Spotify…

Formal Model

- X = set of **Customers**
- S = set of **Items**

- **Utility function** (効用関数) $u: X \times S \rightarrow R$
  - R = set of ratings
  - R is a totally ordered set
  - e.g., **0-5** stars, real number in **[0,1]**

Utility Matrix (効用行列)

	Avatar	LOTR	Matrix	Pirates
Alice	1		0.2	
Bob		0.5		0.3
Carol	0.2		1	
David				0.4

Key Problems

- **(1) Gathering "known" ratings for matrix**
  - How to collect the data in the utility matrix
- **(2) Extrapolate unknown ratings from the known ones**
  - Mainly interested in high unknown ratings
    - We are not interested in knowing what you don't like but what you like
- **(3) Evaluating extrapolation methods**
  - How to measure success/performance of recommendation methods

(1) Gathering Ratings

- **Explicit**
  - Ask people to rate items
  - Doesn't work well in practice – people can't be bothered
- **Implicit**
  - Learn ratings from user actions
    - E.g., purchase implies high rating
  - What about low ratings?

(2) Extrapolating Utilities

- **Key problem:** Utility matrix ***U*** is **sparse**
  - Most people have not rated most items
  - **Cold start:**
    - New items have no ratings
    - New users have no history
- **Three approaches to recommender systems:**
  - **1)** Content-based (内容に基づいたシステム)
  - **2)** Collaborative filtering (協調フィルタリング)
  - **3)** Latent factor based (潜在因子法)

Content-based Recommendations

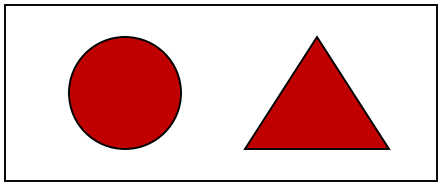
- **Main idea:** Recommend items to customer ***x*** similar to previous items rated highly by ***x***

Example:

- **Movie recommendations**
  - Recommend movies with same actor(s), director, genre, …
- **Websites, blogs, news**
  - Recommend other sites with “similar” content

Plan of Action

likes



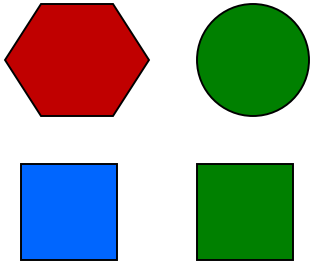
Item profiles

build

Red
Circles
Triangles

User profile

match



recommend

Item Profiles (アイテムプロファイル)

- For each item, create an **item profile**
- **Profile is a set (vector) of features**
  - **Movies:** author, title, actor, director,…
  - **Text:** Set of “important” words in document
- **How to pick important features?**
  - Usual heuristic from text mining is **TF-IDF** (Term frequency * Inverse Doc Frequency)
    - **Term … Feature**
    - **Document … Item**

Sidenote: TF-IDF

F_{ij} = frequency of term (feature) ***i*** in doc (item) ***j***

$$TF_{ij} = \frac{f_{ij}}{\max_k f_{kj}}$$

用語の出現頻度

Sidenote: TF-IDF

f_{ij} = frequency of term (feature) ***i*** in doc (item) ***j***

$$TF_{ij} = \frac{f_{ij}}{\max_k f_{kj}}$$

n_i = number of docs that mention term ***i***
N = total number of docs

$$IDF_i = \log \frac{N}{n_i}$$

How do we compare the weights of different terms?

TF-IDF score: $w_{ij} = TF_{ij} \times IDF_i$

Doc profile = set of words with highest **TF-IDF** scores, together with their scores

User Profiles and Prediction

- **User profile possibilities (ユーザープロファイル):**
  - Weighted average of **rated item profiles**
  - **Variation:** weight by difference from average rating for item
  - …
- **Prediction heuristic:**
  - Given user profile ***x*** and item profile ***i***, estimate

$$u(\boldsymbol{x}, \boldsymbol{i}) = \cos(\boldsymbol{x}, \boldsymbol{i}) = \frac{\boldsymbol{x} \cdot \boldsymbol{i}}{||\boldsymbol{x}|| \cdot ||\boldsymbol{i}||}$$

Example 1: Boolean Utility Matrix

- Items are movies, only feature is "Actor"
  - Item profile: vector with 0 or 1 for each Actor
- Suppose user x has watched 5 movies
  - 2 movies featuring actor A
  - 3 movies featuring actor B
- User profile = mean of item profiles
  - Feature A's weight = 2/5 = 0.4
  - Feature B's weight = 3/5 = 0.6

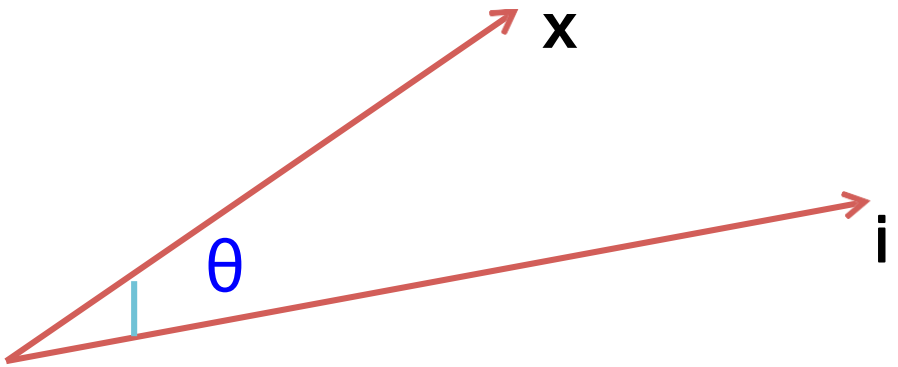
Example 2: Star Ratings

- Same example, 1–5 star ratings
  - Actor A's movies rated 3 and 5
  - Actor B's movies rated 1, 2 and 4

- Useful step: Normalize ratings by subtracting user's mean rating (3)
  - Actor A's normalized ratings = 0, +2
    - Profile weight = (0 + 2)/2 = 1
  - Actor B's normalized ratings = –2, –1, +1
    - Profile weight = –2/3

Making predictions

- User profile **x**, Item profile **i**
- Estimate $u(\boldsymbol{x}, \boldsymbol{i}) = \cos(\boldsymbol{x}, \boldsymbol{i}) = \frac{\boldsymbol{x} \cdot \boldsymbol{i}}{||\boldsymbol{x}|| \cdot ||\boldsymbol{i}||}$



Technically, the cosine distance is actually the angle θ
and the cosine similarity is the angle 180-θ

For convenience, we use cos(θ) as our similarity measure and call it the "cosine similarity" in this context.

Given the user x, we compute the cosine similarity between that user and all the items in the catalog. And then we pick the items with the highest cosine similarity and recommend those to the user.

Pros: Content-based Approach

- **+: No need for data on other users**
  - No sparsity problems
- **+: Able to recommend to users with unique tastes**

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- **+: No need for data on other users**
  - No sparsity problems
- **+: Able to recommend to users with unique tastes**
- **+: Able to recommend new & unpopular items**
  - No first-rater problem
- **+: Able to provide explanations**
  - Can provide explanations of recommended items by listing content-features that caused an item to be recommended

Cons: Content-based Approach

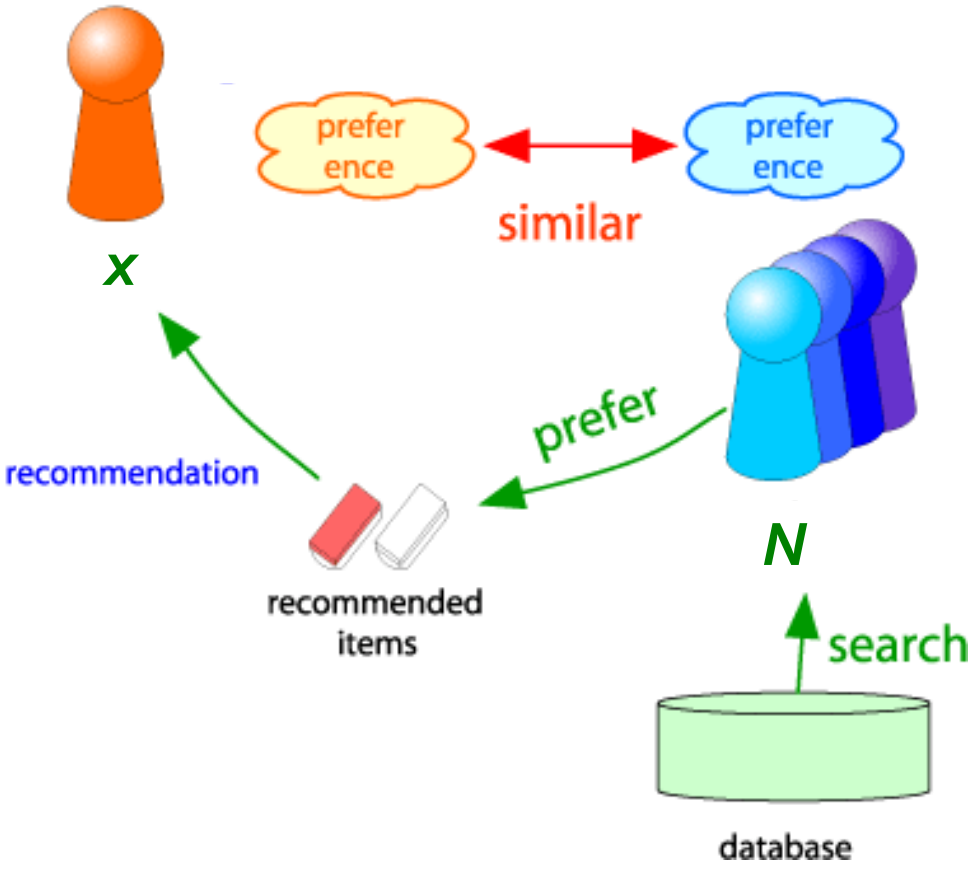
- –: Finding the appropriate features is hard
  - E.g., images, movies, music
- –: Recommendations for new users (cold start)
  - How to build a user profile?

Cons: Content-based Approach

- **–: Finding the appropriate features is hard**
  - E.g., images, movies, music
- **–: Recommendations for new users**
  - **How to build a user profile?**
- **–: Overspecialization**
  - Never recommends items outside user's content profile
  - People might have multiple interests
  - **Unable to exploit quality judgments of other users**

Collaborative Filtering （協調フィルタリング）

- Consider user ***x***
- Find set ***N*** of other users whose ratings are "**similar**" to ***x***'s ratings
- Estimate ***x***'s ratings based on ratings of users in ***N***



Finding "Similar" Users

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

Utility Matrix
効用行列

HP: Harry Potter
TW: Twilight
SW: Star Wars

- Consider users ***x*** and ***y*** with rating vectors r_x and r_y
- We need a similarity metric sim(***x***, ***y***)
- Capture intuition that sim(A,B) > sim(A,C)

Finding "Similar" Users

r_x = [*, _, _, *, ***]
r_y = [*, _, **, **, _]

- Let r_x be the vector of user x's ratings
- **Jaccard similarity measure**
  - **Problem:** Ignores the value of the rating
- **Cosine similarity measure**
  - $\text{sim}(x, y) = \cos(r_x, r_y) = \frac{r_x \cdot r_y}{||r_x|| \cdot ||r_y||}$
  - **Problem:** Treats missing ratings as "negative"
- **Pearson correlation coefficient**
  - S_{xy} = items rated by both users x and y

$$sim(x, y) = \frac{\sum_{s \in S_{xy}} (r_{xs} - \overline{r_x})(r_{ys} - \overline{r_y})}{\sqrt{\sum_{s \in S_{xy}} (r_{xs} - \overline{r_x})^2} \sqrt{\sum_{s \in S_{xy}} (r_{ys} - \overline{r_y})^2}}$$

r_x, r_y as sets:
r_x = {1, 4, 5}
r_y = {1, 3, 4}

r_x, r_y as points:
r_x = {1, 0, 0, 1, 3}
r_y = {1, 0, 2, 2, 0}

$\overline{r_x}$, $\overline{r_y}$ … avg. rating of **x**, **y**

Similarity Metric

Cosine sim:
$$sim(x, y) = \frac{\sum_i r_{xi} \cdot r_{yi}}{\sqrt{\sum_i r_{xi}^2} \cdot \sqrt{\sum_i r_{yi}^2}}$$

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

- **Intuitively we want: sim(*A*, *B*) > sim(*A*, *C*)**
- **Jaccard similarity:** 1/5 < 2/4
- **Cosine similarity:** 0.386 > 0.322 (bigger but not by much)
  - Considers missing ratings as "negative" ratings!
  - **Solution: subtract the (row) mean**

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	2/3			5/3	−7/3		
B	1/3	1/3	−2/3				
C				−5/3	1/3	4/3	
D		0					0

sim A,B vs. A,C:
0.092 > -0.559

Notice cosine sim. is correlation when data is centered at 0

Rating Predictions

From similarity metric to recommendations:

- Let $\boldsymbol{r_x}$ be the vector of user $\boldsymbol{x}$'s ratings
- Let $\boldsymbol{N}$ be the set of $\boldsymbol{k}$ users most similar to $\boldsymbol{x}$ who have rated item $\boldsymbol{i}$
- **Prediction for item *i* of *user x*:**
  - $r_{xi} = \frac{1}{k} \sum_{y \in N} r_{yi}$ **(ignores similarity values b/n users)**
  - $r_{xi} = \frac{\sum_{y \in N} s_{xy} \cdot r_{yi}}{\sum_{y \in N} s_{xy}}$ Shorthand: $s_{xy} = sim(x, y)$
  - Other options?
- **Many other tricks possible…**

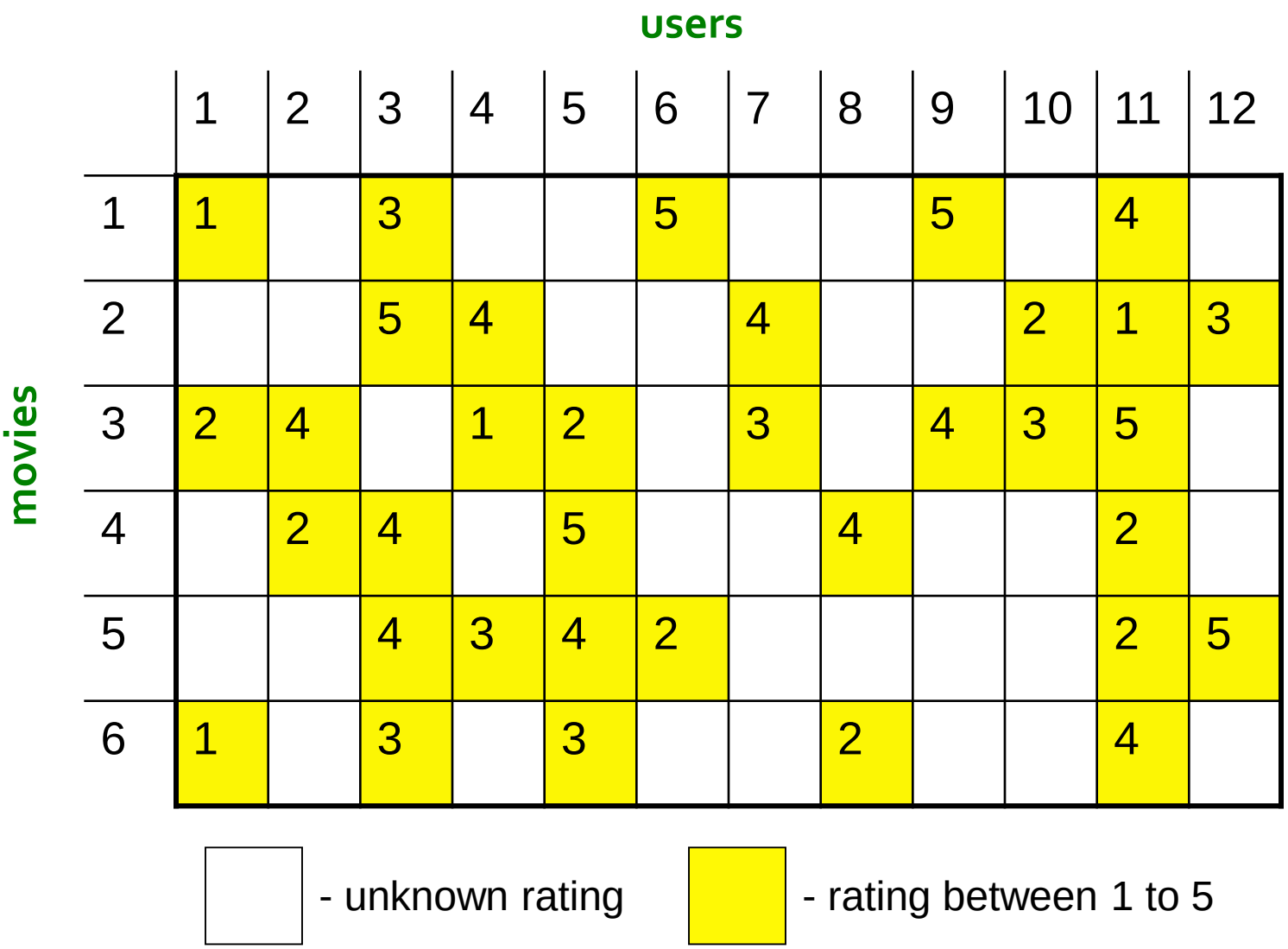
Item-Item Collaborative Filtering

- **So far: User-user collaborative filtering**
- **Another view: Item-item**
  - For item ***i***, find other similar items
  - Estimate rating for item ***i*** based on ratings for similar items
  - Can use same similarity metrics and prediction functions as in user-user model

$$r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

s_{ij}… similarity of items ***i*** and ***j***
r_{xj}…rating of user ***x*** on item ***j***
$N(i;x)$… set items rated by ***x*** similar to ***i***

Item-Item CF (|N|=2)



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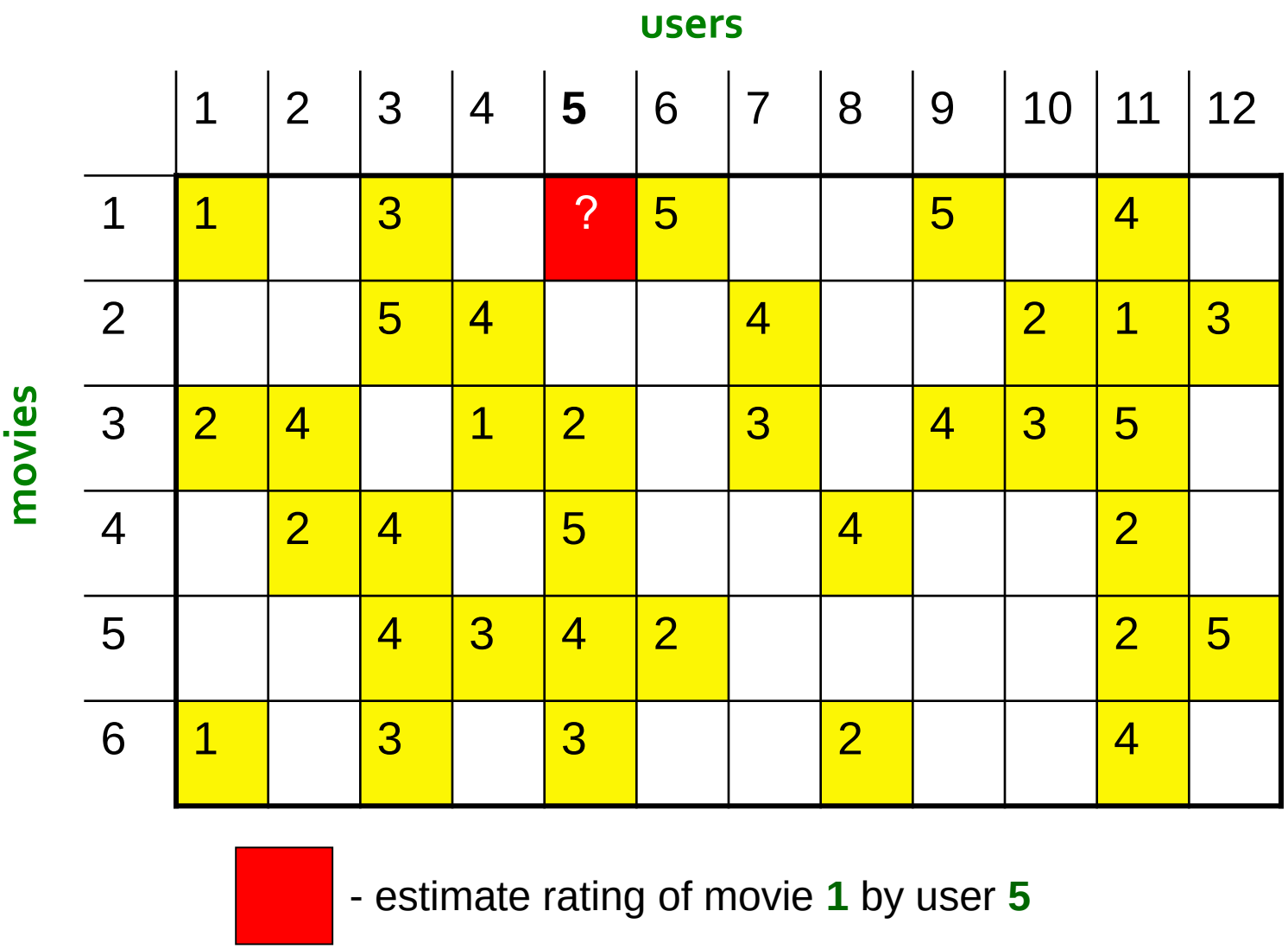

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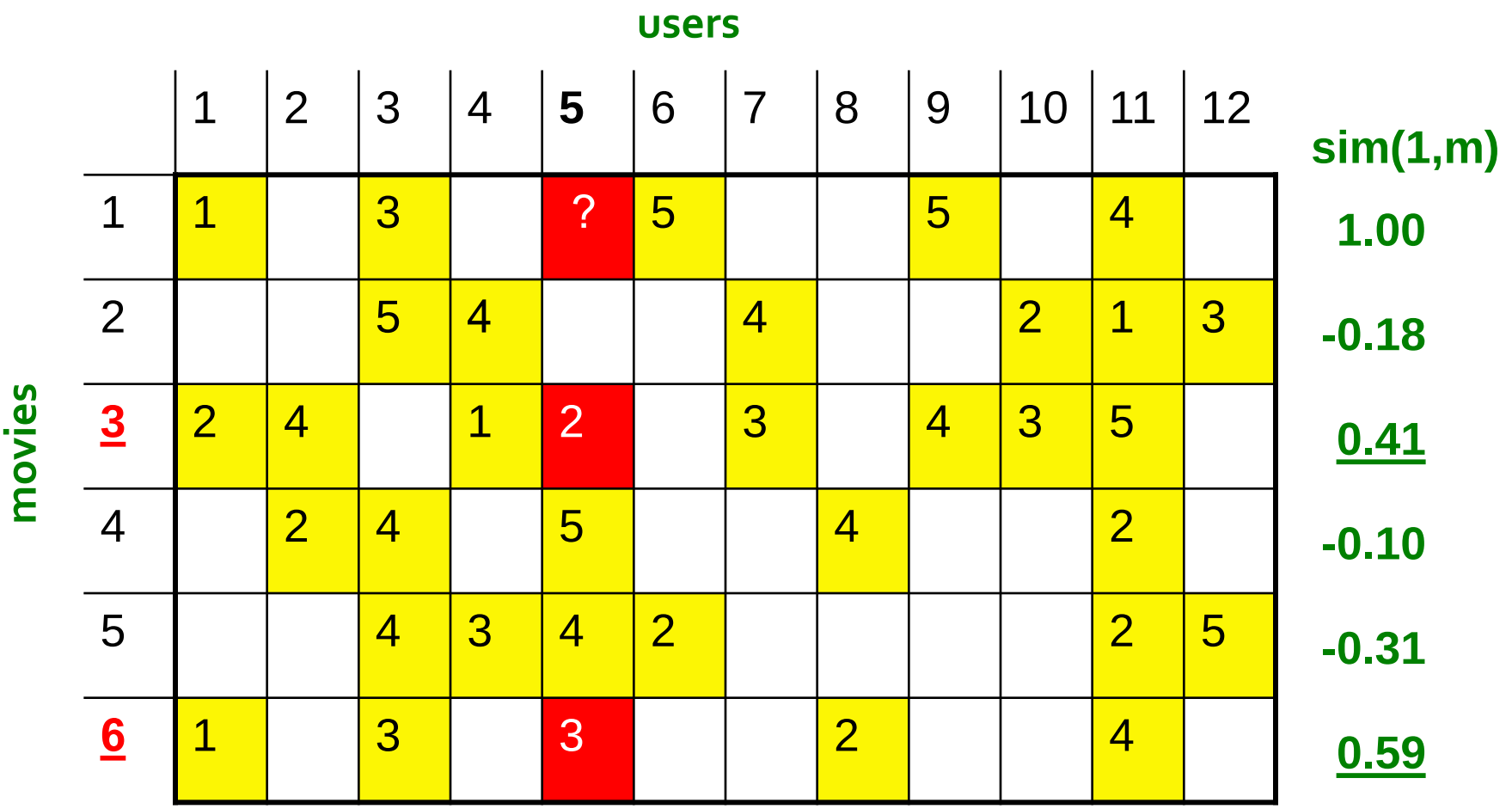

Neighbor selection:
Identify movies similar to movie **1**, **rated by user 5**

Here we use Pearson correlation as similarity:

1) Subtract mean rating m_i from each movie i

$m_1 = (1+3+5+5+4)/5 = 3.6$

row 1: *[-2.6, 0, -0.6, 0, 0, 1.4, 0, 0, 1.4, 0, 0.4, 0]*

2) Compute cosine similarities between rows

Item-Item CF (|N|=2)

users

movies

	1	2	3	4	5	6	7	8	9	10	11	12	sim(1,m)
1	1		3		?	5			5		4		1.00
2			5	4			4			2	1	3	-0.18
3	2	4		1	2		3		4	3	5		0.41
4		2	4		5			4			2		-0.10
5			4	3	4	2					2	5	-0.31
6	1		3		3			2			4		0.59

Compute similarity weights:

$s_{1,3}=0.41$, $s_{1,6}=0.59$

Item-Item CF (|N|=2)

users

movies

	1	2	3	4	5	6	7	8	9	10	11	12	sim(1,m)
1	1		3		2.6	5			5		4		1.00
2			5	4			4			2	1	3	-0.18
3	2	4		1	2		3		4	3	5		0.41
4		2	4		5			4			2		-0.10
5			4	3	4	2					2	5	-0.31
6	1		3		3			2			4		0.59

Predict by taking weighted average:

$r_{1.5}$ = (0.41*2 + 0.59*3) / (0.41+0.59) = 2.6

$$r_{ix} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{jx}}{\sum s_{ij}}$$

Item-Item vs. User-User

	Avatar	LOTR	Matrix	Pirates
Alice	1		0.8	
Bob		0.5		0.3
Carol	0.9		1	0.8
David			1	0.4

- **In practice, it has been observed that item-item often works better than user-user**
- **Why?** Items are simpler, users have multiple tastes

Pros/Cons of Collaborative Filtering

- **+ Works for any kind of item**
  - No feature selection needed

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- **- Cold Start:**
  - Need enough users in the system to find a match
- **- Sparsity:**
  - The user/ratings matrix is sparse
  - Hard to find users that have rated the same items

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- **- Sparsity:**
  - The user/ratings matrix is sparse
  - Hard to find users that have rated the same items
- **- First rater:**
  - Cannot recommend an item that has not been previously rated
  - New items, Esoteric items
- **- Popularity bias:**
  - Cannot recommend items to someone with unique taste
  - Tends to recommend popular items

Hybrid Methods

- **Implement two or more different recommenders and combine predictions**
  - Perhaps using a linear model
- **Add content-based methods to collaborative filtering**
  - Item profiles for new item problem
  - Demographics to deal with new user problem

Global Baseline Estimate

- Estimate Joe's rating for the movie *The Sixth Sense*
  - Problem: Joe has not rated any movie "similar" to *The Sixth Sense*

- Global Baseline approach
  - Mean movie rating (of all movies): **3.7 stars**
  - *The Sixth Sense* is **0.5** stars above avg.
  - Joe rates **0.2** stars below avg.
  - **Baseline estimate: 3.7 + 0.5 – 0.2 = 4 stars**

Combining Global Baseline with CF

- Global Baseline estimate
  - Joe will give *The Sixth Sense* 4 stars
- Local neighborhood (CF/NN)
  - Joe didn't like related (very similar) movie *Signs*
  - Rated it 1 star below his average rating
- Final estimate
  - Joe will rate *The Sixth Sense* 4 –1 = 3 stars

CF: Common Practice

Before:

$$r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

- Define **similarity s_{ij}** of items ***i*** and ***j***
- Select ***k*** nearest neighbors ***N(i; x)***
  - Items most similar to ***i***, that were rated by ***x***
- Estimate rating r_{xi} as the weighted average:

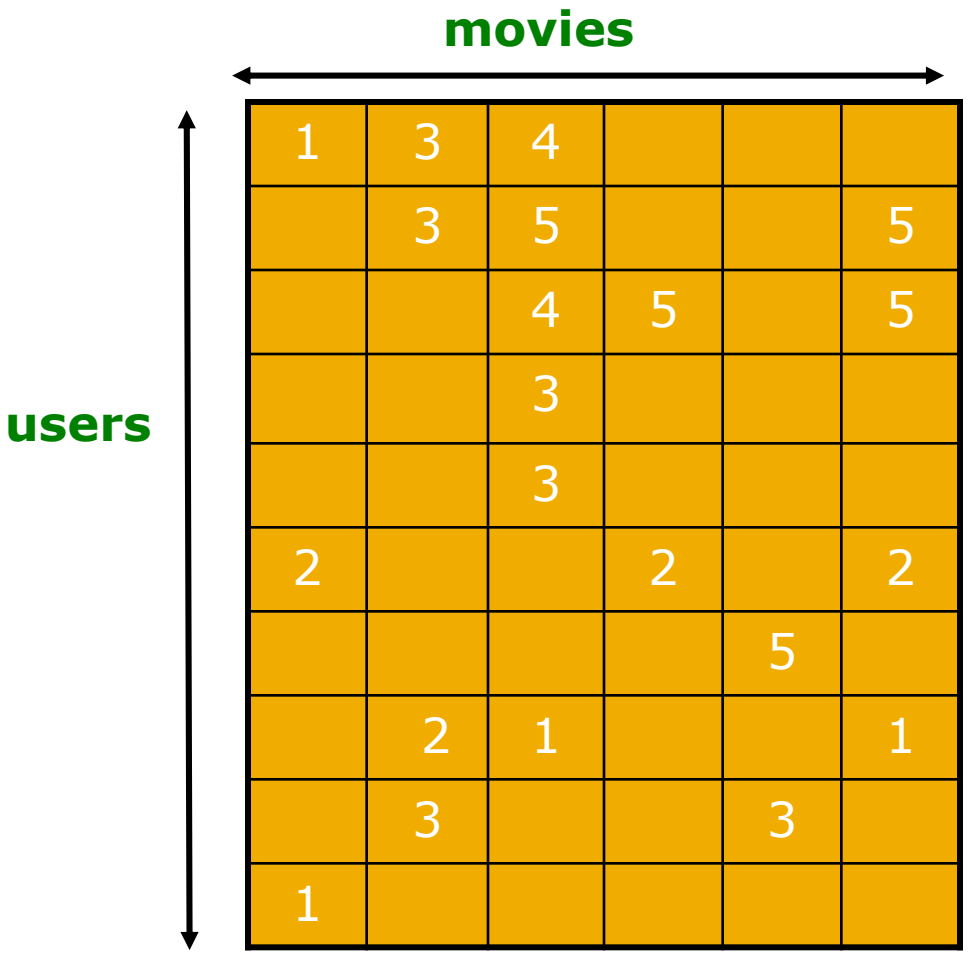
$$r_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} s_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} s_{ij}}$$

baseline estimate for r_{xi}

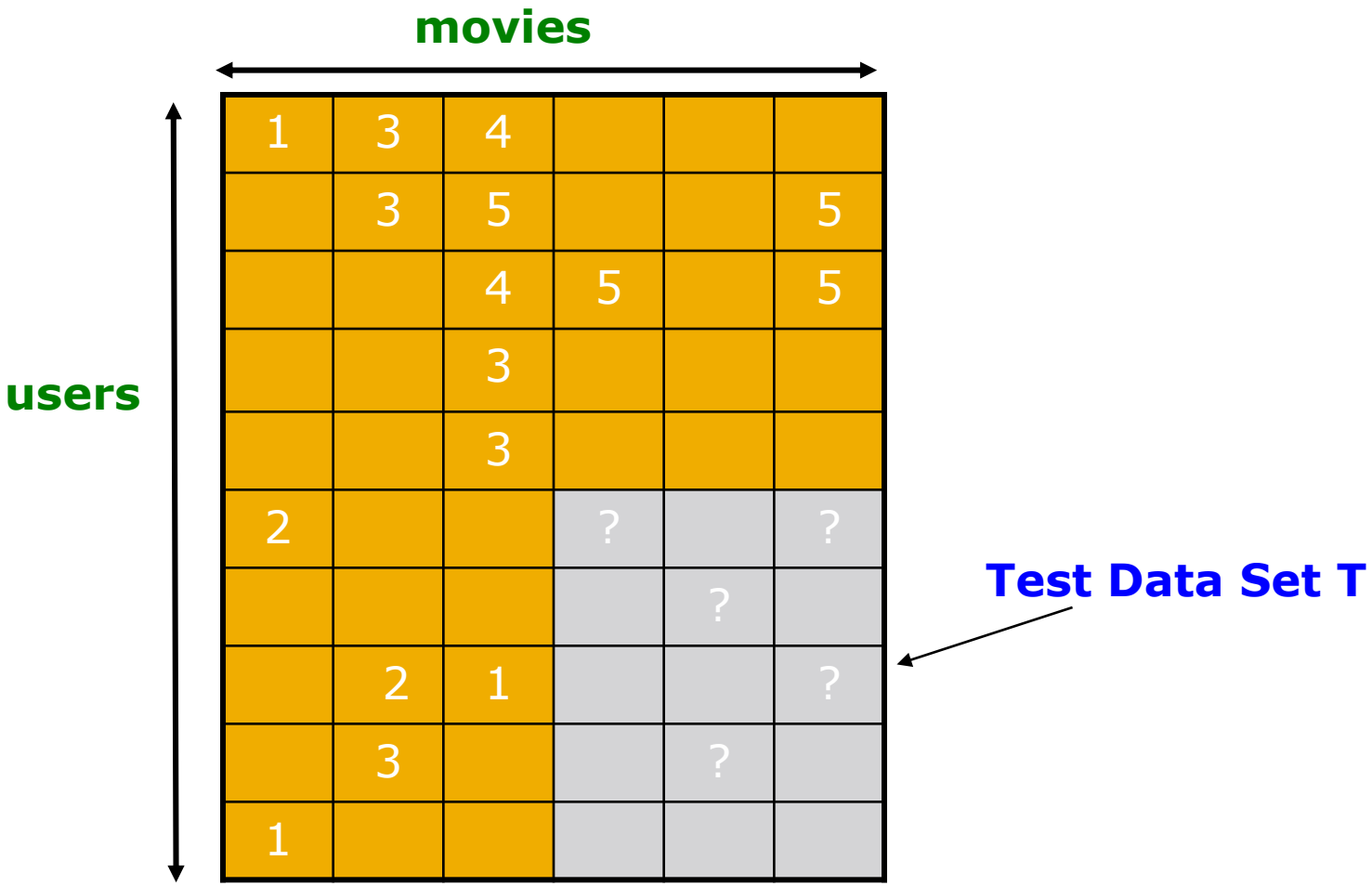
$$b_{xi} = \mu + b_x + b_i$$

- μ = overall mean movie rating
- b_x = rating deviation of user ***x***
  = (*avg. rating of user* ***x***) **– μ**
- b_i = rating deviation of movie ***i***

Evaluation



Evaluation



Evaluating Predictions

- Compare predictions against withheld known ratings (test set *T*)

- Root–mean–square error (RMSE)

$$\sqrt{\frac{\sum_{(x,i)\in T}(r_{xi}-r^*_{xi})^2}{N}}$$

where $N = |T|$

r_{xi} is the predicted rating

r_{xi}^* is the actual rating

Problems with Error Measures

- **Narrow focus on accuracy sometimes misses the point**
  - Prediction Diversity
  - Prediction Context
  - Order of predictions

Problems with Error Measures

- **Narrow focus on accuracy sometimes misses the point**
  - Prediction Diversity
  - Prediction Context
  - Order of predictions
- **In practice, we care only to predict high ratings:**
  - RMSE might penalize a method that does well for high ratings and badly for others
  - Alternative: precision at top k

    Percentage of predictions in the user's top k withheld ratings

Collaborative Filtering: Complexity

- Expensive step is finding ***k*** most similar customers: **O(|X|)**
- **Too expensive to do at runtime**
  - Could pre-compute
- Naïve pre-computation takes time **O(k ·|X|)**
    - X … set of customers
- **We already know how to do this!**
  - Near-neighbor search in high dimensions (**LSH**)
  - Clustering
  - Dimensionality reduction

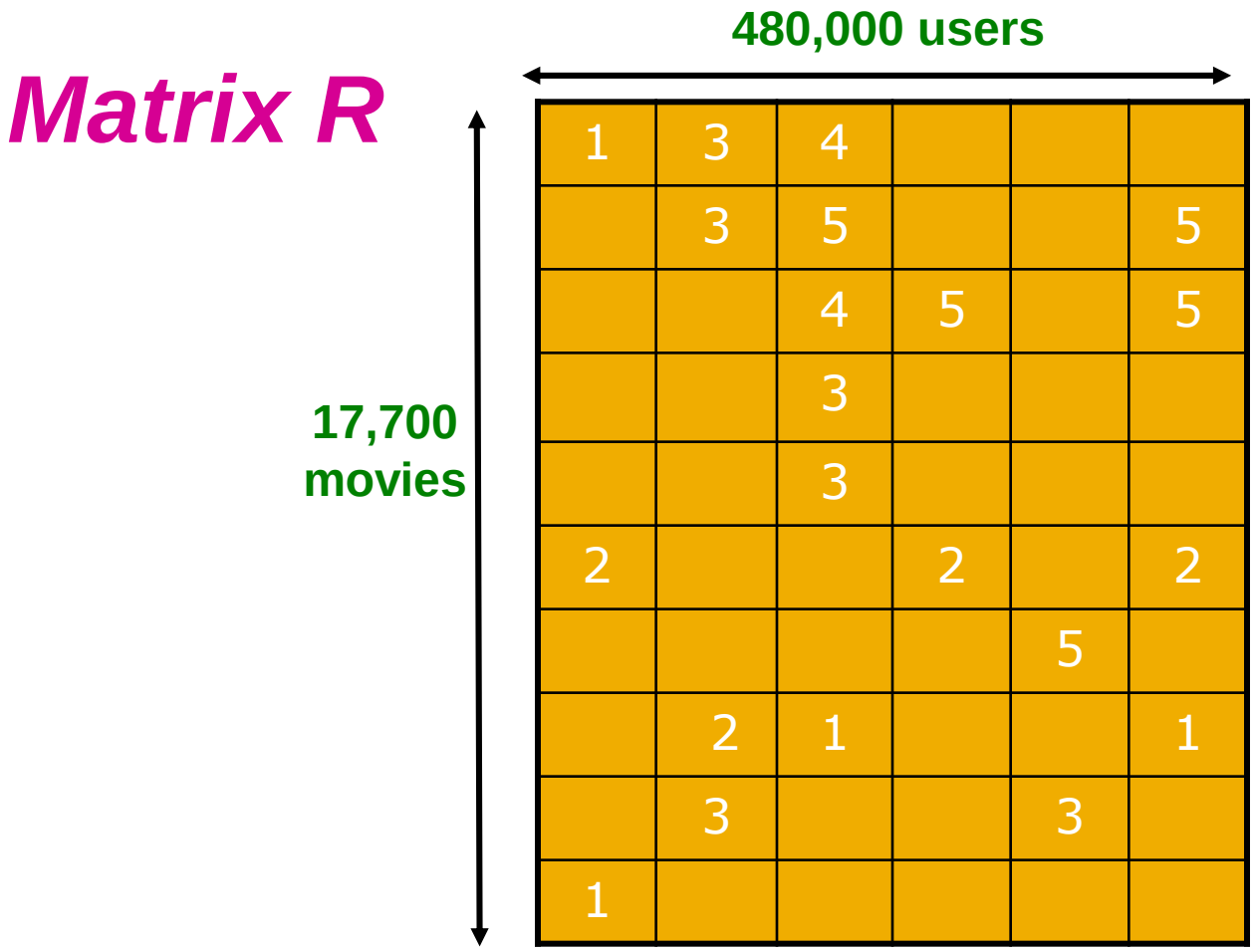
Tip: Add Data

- **Leverage all the data**
  - Don't try to reduce data size in an effort to make fancy algorithms work
  - Simple methods on large data do best
- **Add more data**
  - e.g., add IMDB data on genres (for movies)
- **More data beats better algorithms**

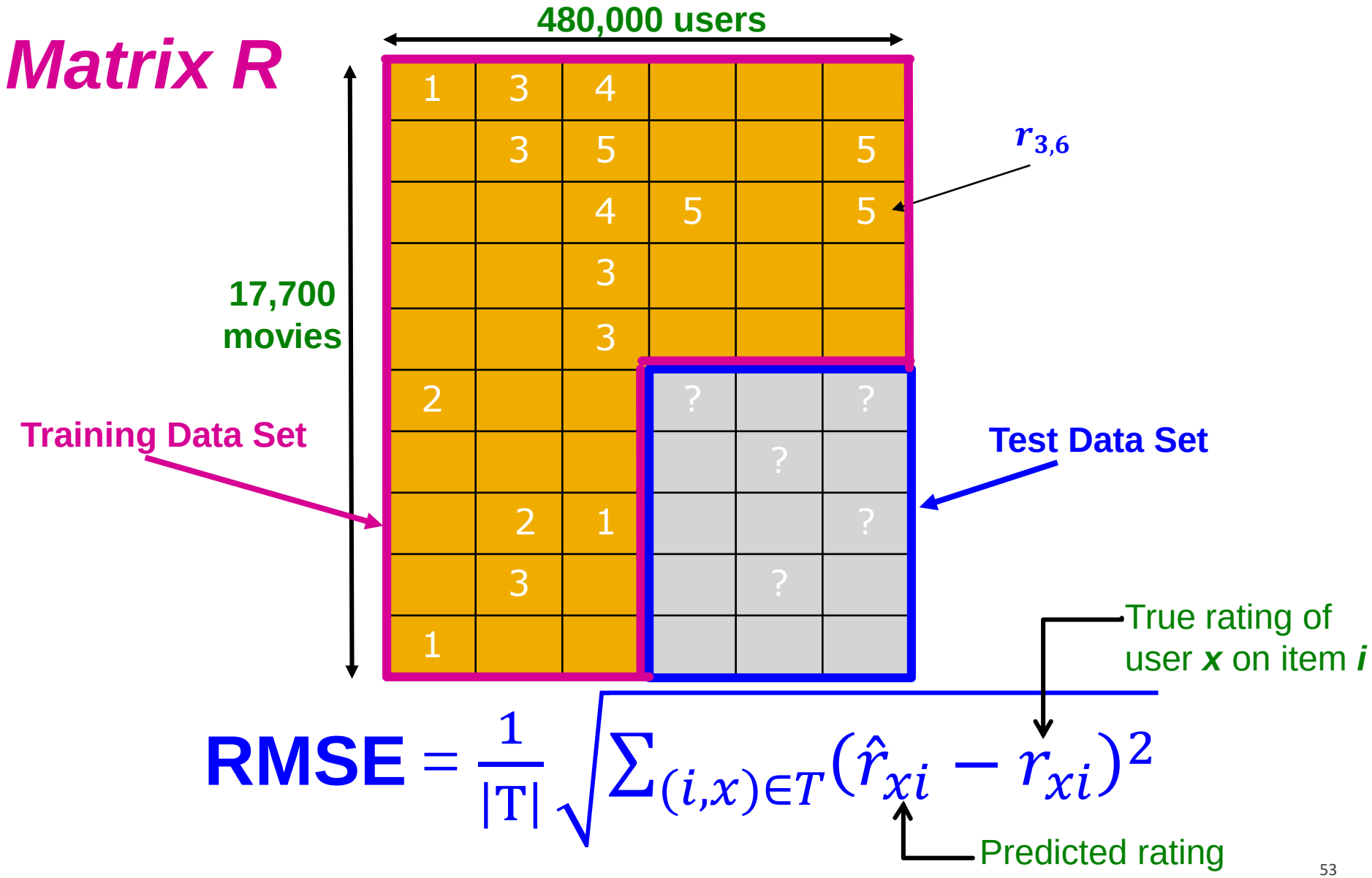
The Netflix Prize

- **Training data**
  - 100 million ratings, 480,000 users, 17,770 movies
  - 6 years of data: 2000-2005
- **Test data**
  - Last few ratings of each user (2.8 million)
  - **Evaluation criterion:** Root Mean Square Error (RMSE) $= \frac{1}{|\mathrm{T}|}\sqrt{\sum_{(i,x)\in \mathrm{T}}(\hat{r}_{xi} - r_{xi})^2}$
  - **Netflix's system RMSE: 0.9514**
- **Competition**
  - 2,700+ teams
  - **$1 million** prize for 10% improvement on Netflix

The Netflix Utility Matrix *R*

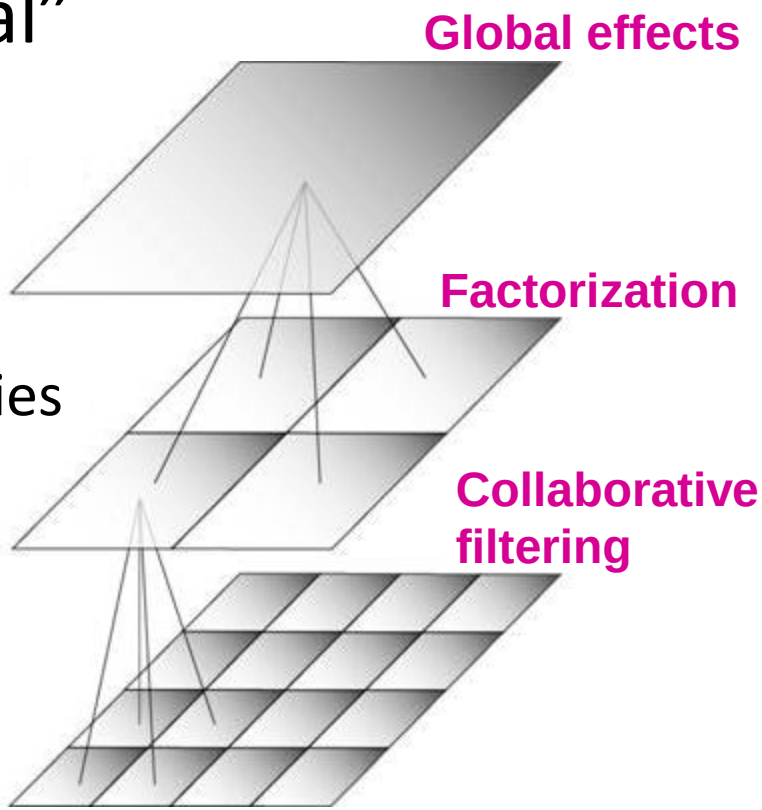


Utility Matrix *R*: Evaluation



BellKor Recommender System

- **The winner of the Netflix Challenge!**
- **Multi-scale modeling of the data:**
  Combine top level, "regional" modeling of the data, with a refined, local view:
  - **Global:**
    - Overall deviations of users/movies
  - **Factorization:**
    - Addressing "regional" effects
  - **Collaborative filtering:**
    - Extract local patterns



Modeling Local & Global Effects

- **Global:**
  - Mean movie rating: **3.7 stars**
  - *The Sixth Sense* is **0.5** stars above avg.
  - Joe rates **0.2** stars below avg.

  ⇒ Baseline estimation:

  ***Joe* will rate *The Sixth Sense* 4 stars**
- **Local neighborhood (CF/NN):**
  - *Joe* didn't like related movie *Signs*
  - **⇒ Final estimate:**

    ***Joe* will rate *The Sixth Sense* 3.8 stars**

Recap: Collaborative Filtering (CF)

- Derive unknown ratings from those of "**similar**" movies (item-item variant)
- Define **similarity measure s_{ij}** of items ***i*** and ***j***
- Select ***k***-nearest neighbors, compute the rating
  - ***N(i; x):*** items most similar to ***i*** that were rated by ***x***

$$\hat{r}_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

s_{ij}… similarity of items ***i*** and ***j***
r_{xj}…rating of user ***x*** on item ***j***
N(i;x)… set of items similar to item ***i*** that were rated by ***x***

Modeling Local & Global Effects

- **In practice we get better estimates if we model deviations:**

$$\hat{r}_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} s_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} s_{ij}}$$

baseline estimate for r_{xi}

$$\boldsymbol{b_{xi} = \mu + b_x + b_i}$$

$\boldsymbol{\mu}$ = overall mean rating
$\boldsymbol{b_x}$ = rating deviation of user $\boldsymbol{x}$
= (*avg. rating of* user $\boldsymbol{x}$) $\boldsymbol{-\mu}$
$\boldsymbol{b_i}$ = (*avg. rating of* movie $\boldsymbol{i}$) $\boldsymbol{-\mu}$

Problems/Issues:

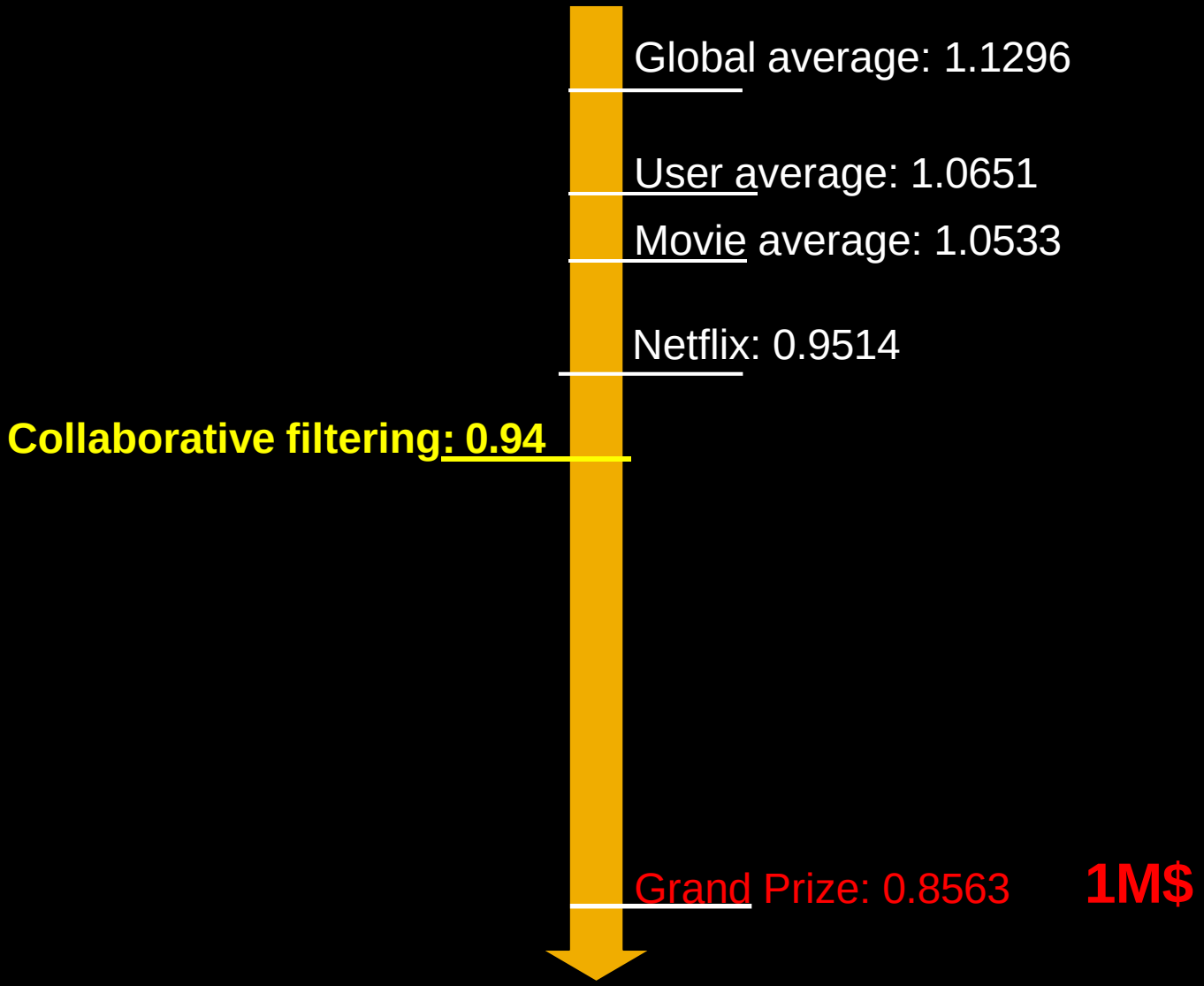
1) Similarity measures are "arbitrary"

2) Pairwise similarities neglect interdependencies among users

3) Taking a weighted average can be restricting

Solution: **Instead of s_{ij} use w_{ij} that we estimate directly from data**

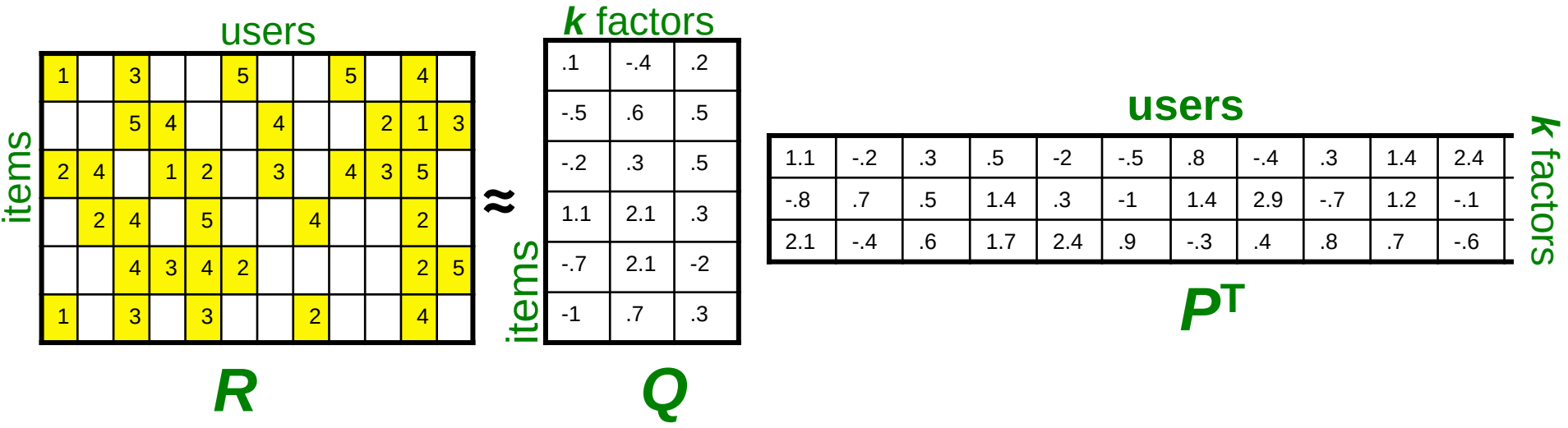
RMSE of Various Methods



Latent Factor Models (潜在因子法)

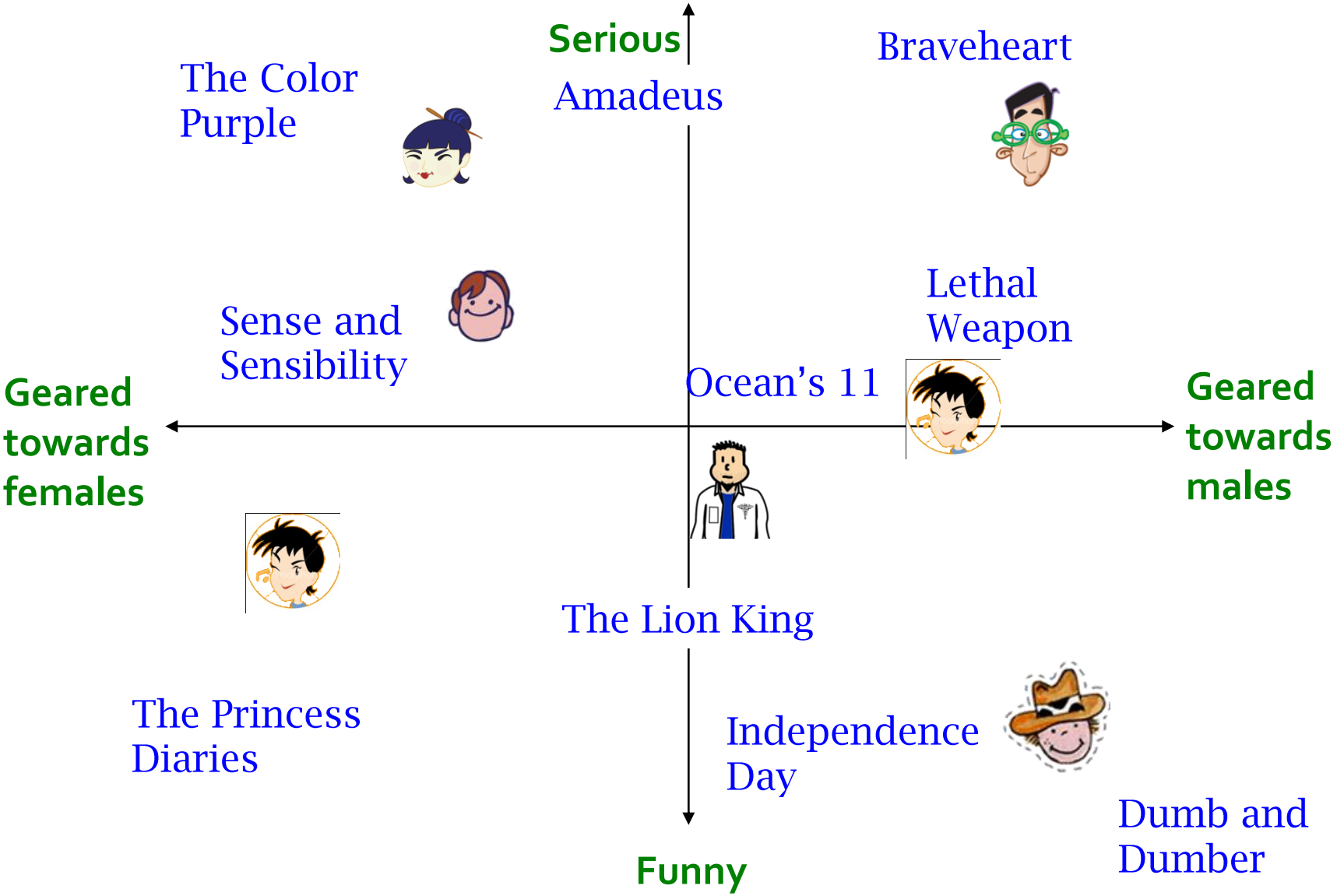
SVD: $A = U \Sigma V^T$

- "SVD" on Netflix data: $\mathbf{R} \approx \boldsymbol{Q} \cdot \boldsymbol{P}^T$



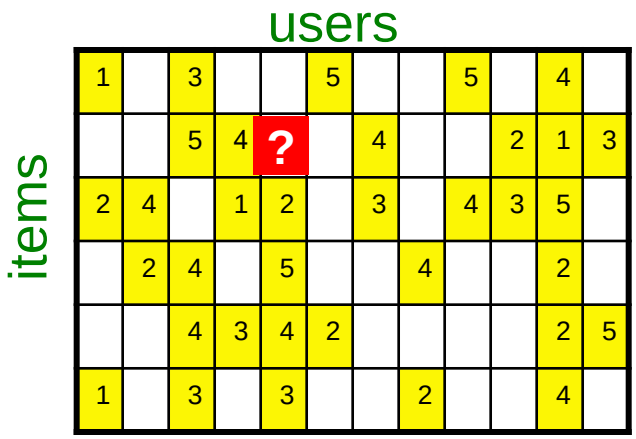
- For now let's assume we can approximate the rating matrix $\boldsymbol{R}$ as a product of "thin" $\boldsymbol{Q} \cdot \boldsymbol{P}^T$
  - $\boldsymbol{R}$ has missing entries but let's ignore that for now!
    - Basically, we will want the reconstruction error to be small on known ratings and we don't care about the values on the missing ones

Latent Factor Models (e.g., SVD)



Ratings as Products of Factors

- **How to estimate the missing rating of user *x* for item *i*?**



≈

$$\hat{r}_{xi} = q_i \cdot p_x = \sum_f q_{if} \cdot p_{xf}$$

q_i = row i of Q
p_x = column x of P^T

items / factors			
	.1	-.4	.2
	-.5	.6	.5
	-.2	.3	.5
	1.1	2.1	.3
	-.7	2.1	-2
	-1	.7	.3

Q

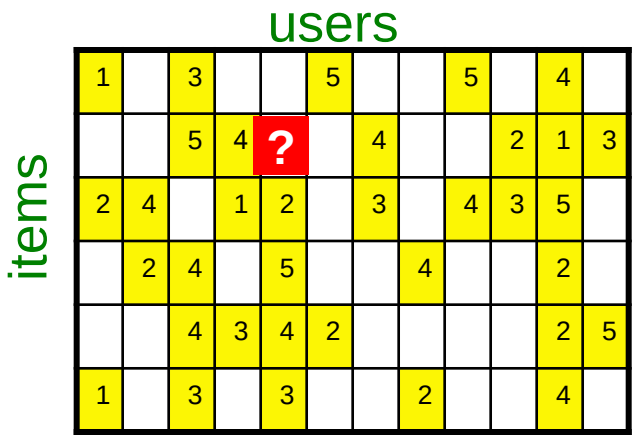
•

factors / users												
	1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
	-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
	2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

P^T

Ratings as Products of Factors

- **How to estimate the missing rating of user *x* for item *i*?**



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.1	-.4	.2
-.5	**.6**	**.5**
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

items, factors

Q

•

users

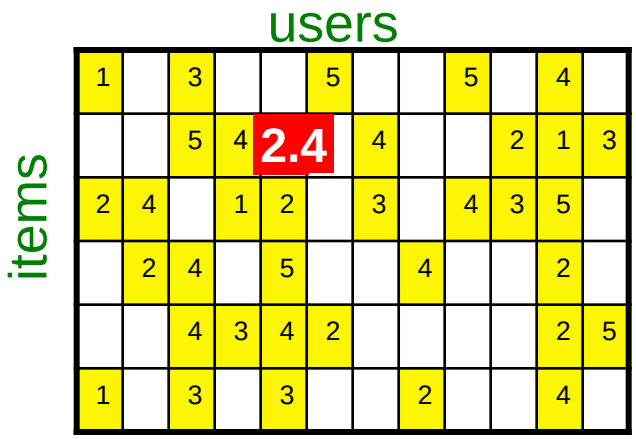
1.1	-.2	.3	.5	**-2**	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	**.3**	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	**2.4**	.9	-.3	.4	.8	.7	-.6	.1

factors

P^T

Ratings as Products of Factors

- **How to estimate the missing rating of user *x* for item *i*?**



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q_i = row i of Q
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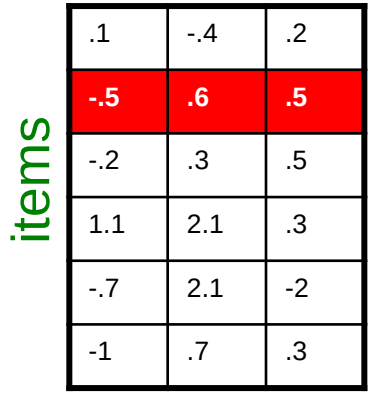

Q

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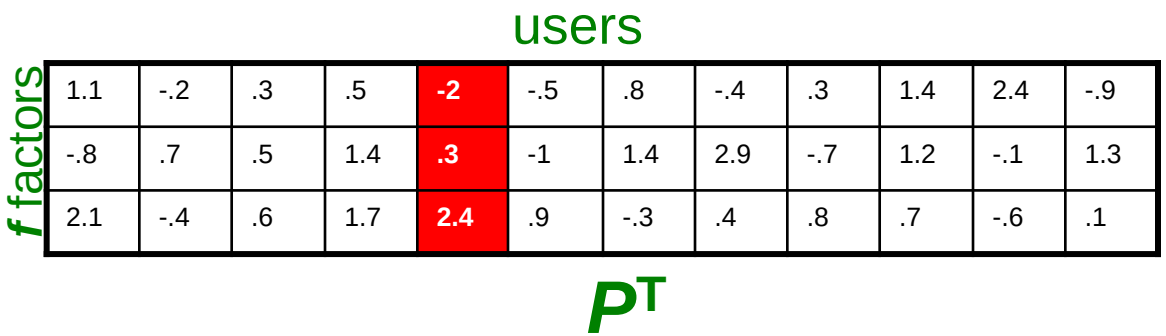
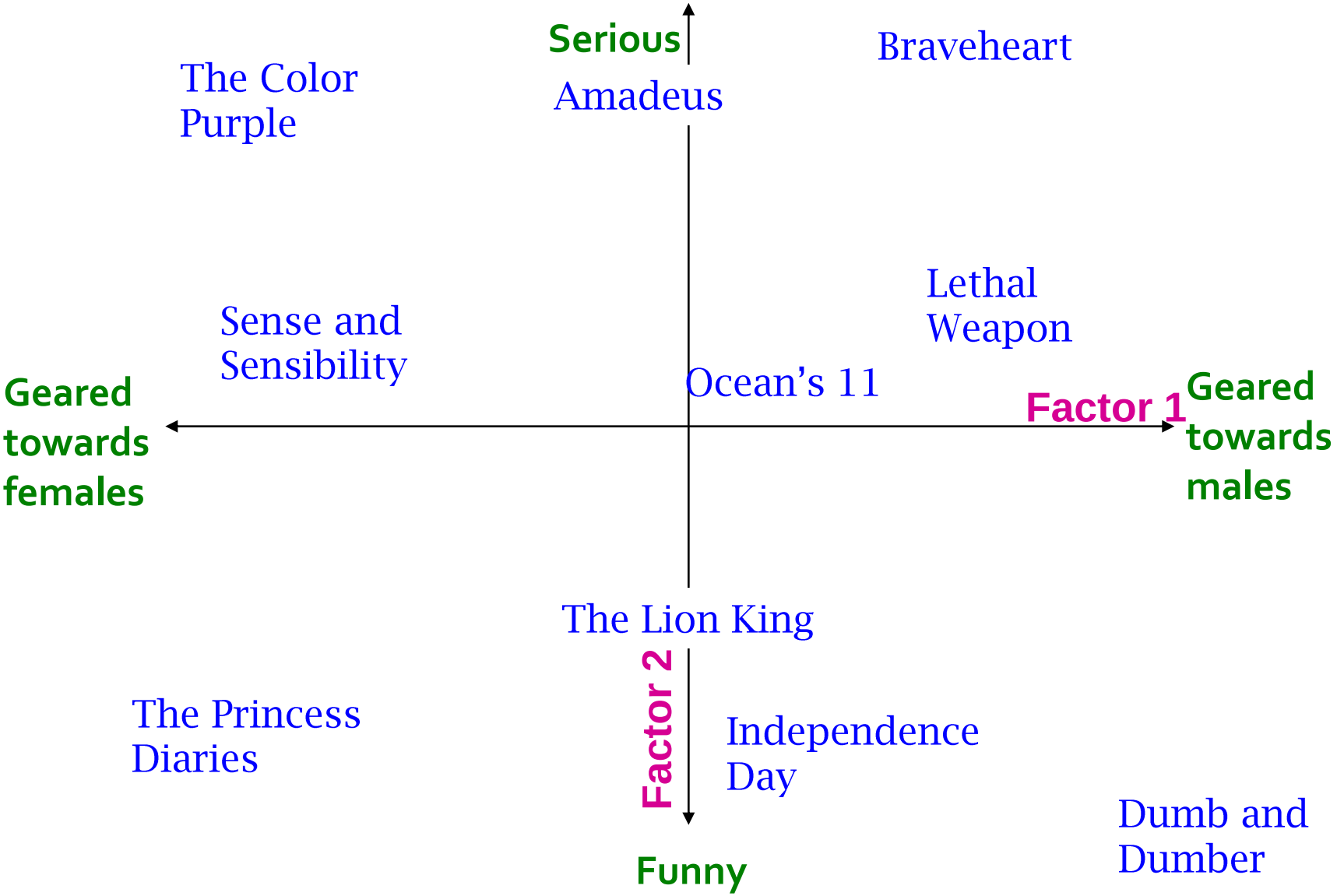
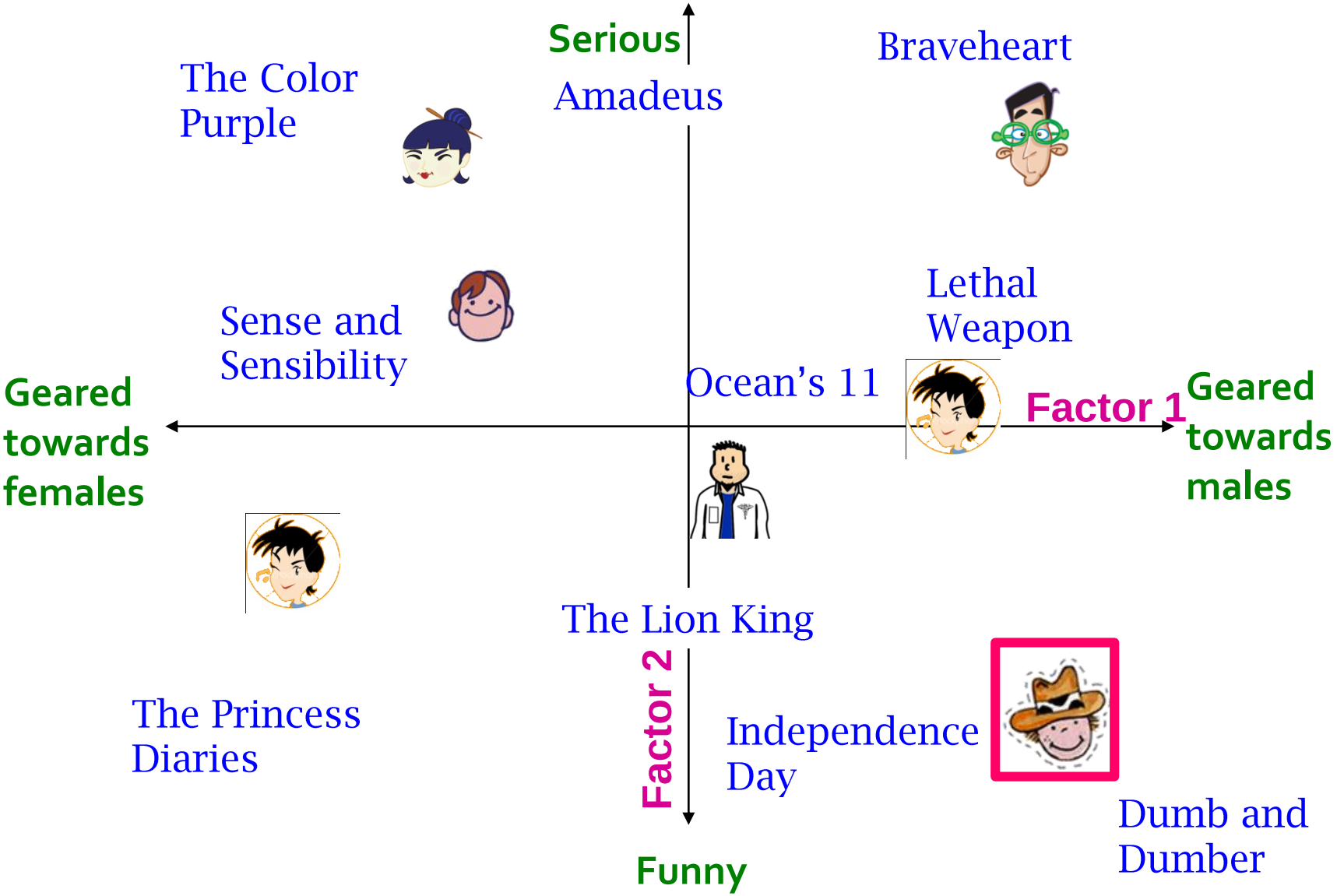


P^T

Latent Factor Models

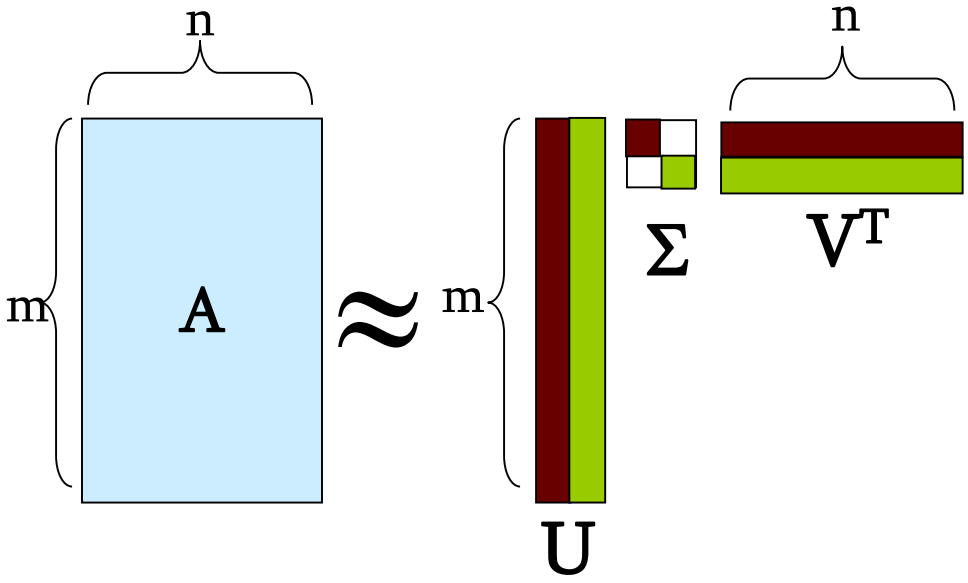


Latent Factor Models



Links to SVD (from your LA course)

- **Remember SVD:**
  - **A**: Input data matrix
  - **U**: Left singular vecs
  - **V**: Right singular vecs
  - **Σ**: Singular values



- **So in our case:**

  "SVD" on Netflix data: $R \approx Q \cdot P^T$

  $A = R,\ Q = U,\ P^T = \Sigma V^T$

$$\hat{r}_{xi} = q_i \cdot p_x$$

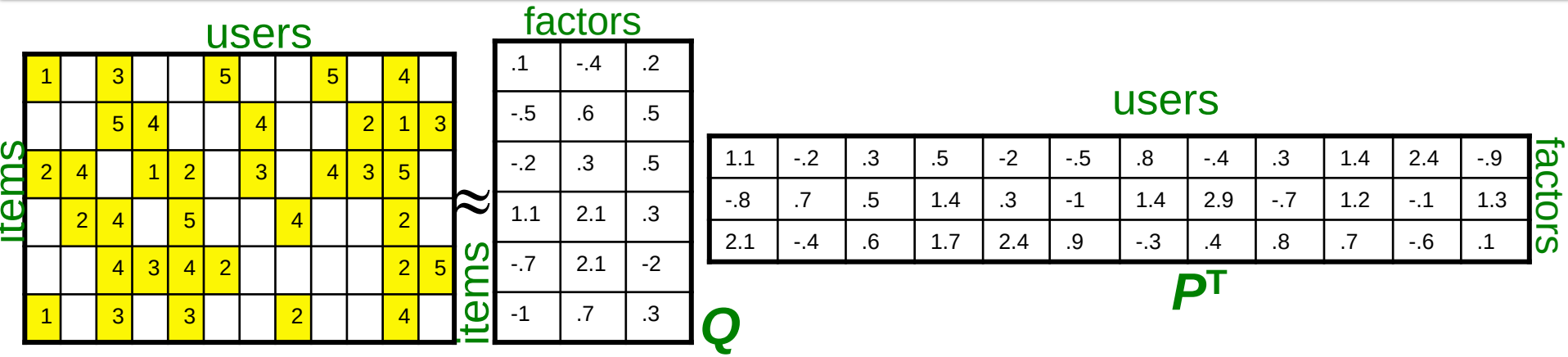
SVD: More good stuff

- **We already know that SVD gives minimum reconstruction error (Sum of Squared Errors):**

$$\min_{U,\mathrm{V},\Sigma} \sum_{ij \in A} \left(A_{ij} - [U\Sigma V^{\mathrm{T}}]_{ij}\right)^2$$

- **Note two things:**
  - **SSE** and **RMSE** are monotonically related:
    - $RMSE = \frac{1}{c}\sqrt{SSE}$ **Great news: SVD is minimizing RMSE**
  - **Complication:** The sum in SVD error term is over all entries (no-rating is interpreted as zero-rating). But our ***R*** has missing entries!

Latent Factor Models



1		3			5			5		4	
		5	4			4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

.1	-.4	.2
-.5	.6	.5
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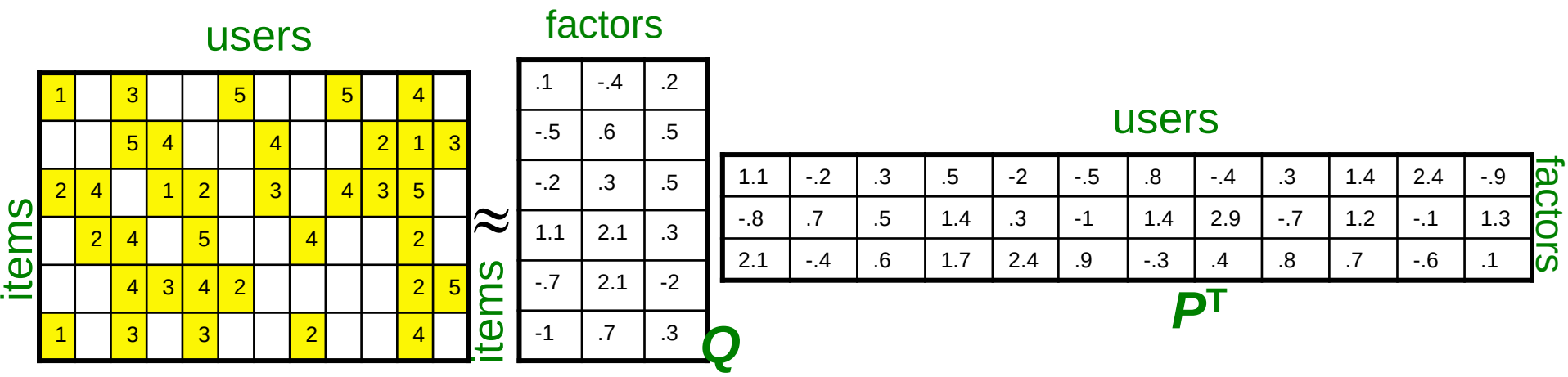
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

- **SVD isn't defined when entries are missing!**
- **Use specialized methods to find *P, Q***
  - $\min_{P,Q} \sum_{(i,x)\in R} \left(r_{xi} - q_i \cdot p_x\right)^2$ $\qquad \hat{r}_{xi} = q_i \cdot p_x$
  - **Note:**
    - We don't require cols of ***P, Q*** to be orthogonal/unit length
    - ***P, Q*** map users/movies to a latent space
    - The most popular model among Netflix contestants

Latent Factor Models

- **Our goal is to find P and Q such tat:**

$$\min_{P,Q} \sum_{(i,x)\in R} \left(r_{xi} - q_i \cdot p_x\right)^2$$



1		3			5			5		4	
		5	4			4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

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1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
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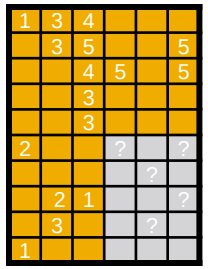
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

Back to Our Problem

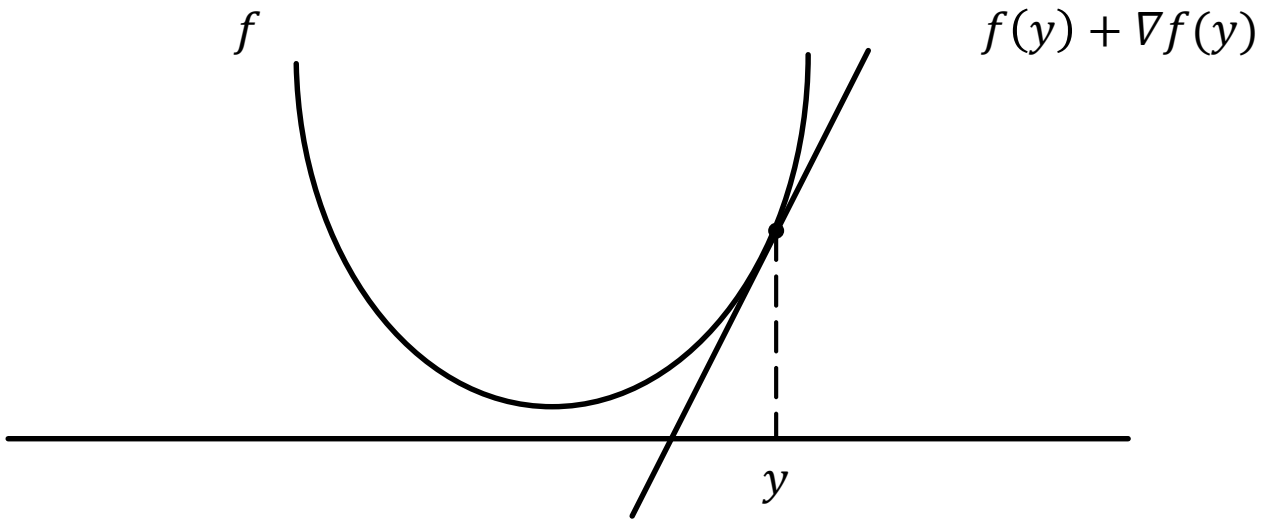
- **Want to minimize SSE for unseen test data**
- **Idea: Minimize SSE on training data**
  - Want large ***k*** (# of factors) to capture all the signals
  - But, **SSE** on **test data** begins to rise for ***k*** > 2

- This is a classical example of **overfitting:**
  - With too much freedom (too many free parameters) the model starts fitting noise
    - That is it fits too well the training data and thus **not generalizing** well to unseen test data

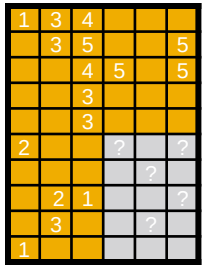


Detour: Minimizing a function

- **A simple way to minimize a function $f(x)$:**
  - Compute the derivative ∇f
  - **Start at some point y and evaluate $\nabla f(y)$**
  - **Make a step in the reverse direction of the gradient: $y = y - \nabla f(y)$**
  - **Repeat until converged**



Dealing with Missing Entries



- **To solve overfitting we introduce regularization:**
  - Allow rich model where there are sufficient data
  - Shrink aggressively where data are scarce

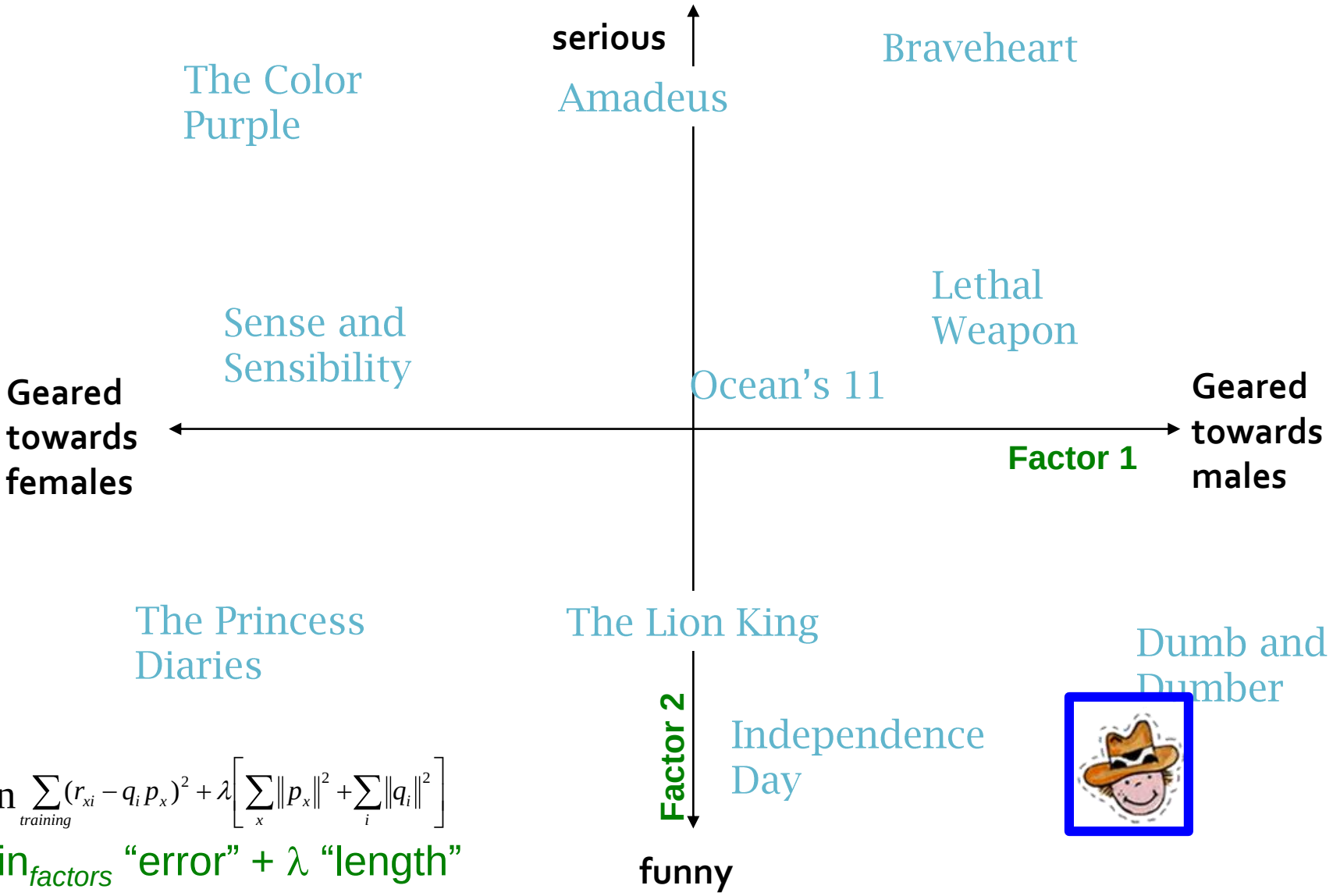
$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \left[\lambda_1 \sum_x \|p_x\|^2 + \lambda_2 \sum_i \|q_i\|^2 \right]$$

"error" — "length"

λ_1, λ_2 … user set regularization parameters

Note: We do not care about the "raw" value of the objective function, but we care in P,Q that achieve the minimum of the objective

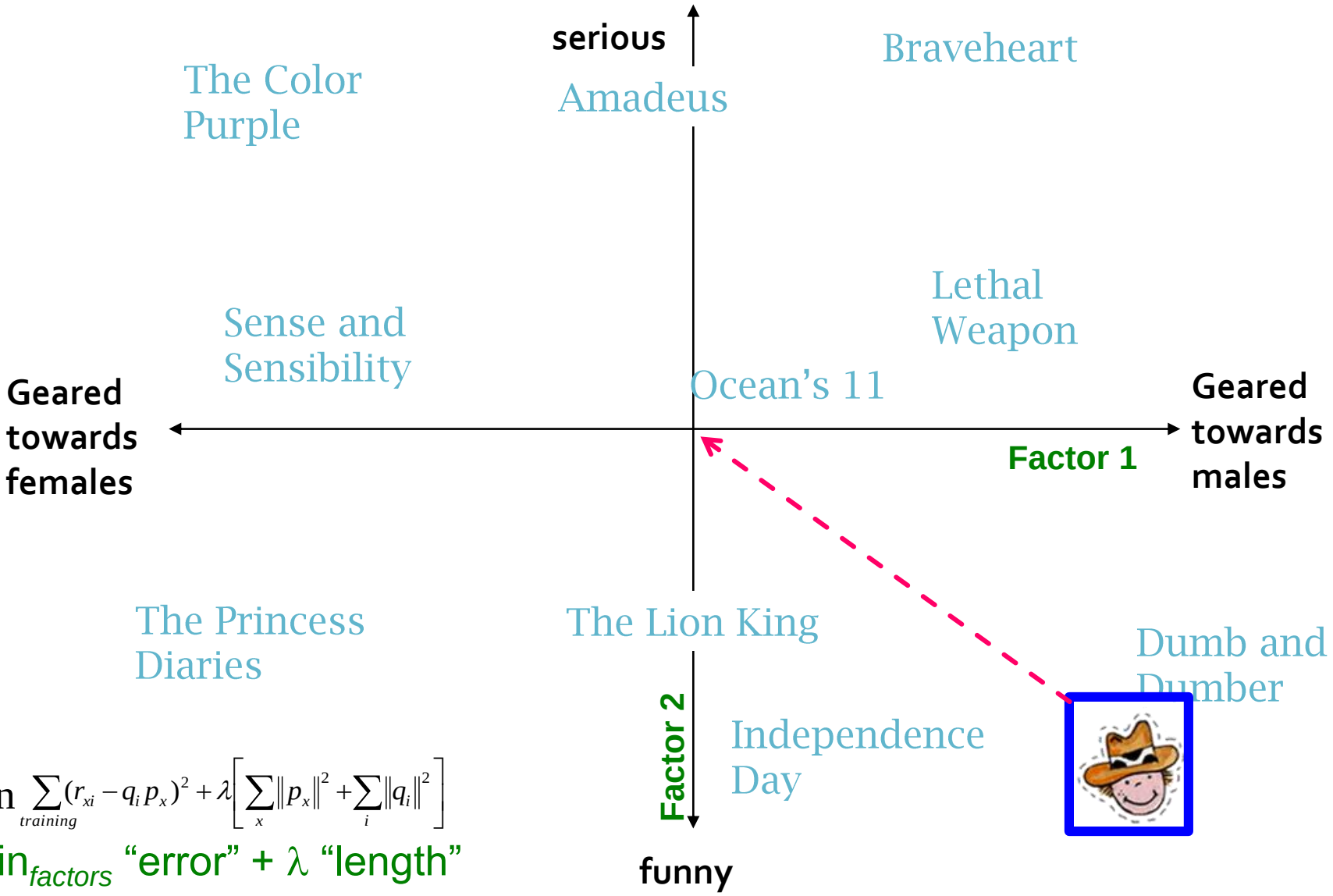
The Effect of Regularization



$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \lambda \left[\sum_x \|p_x\|^2 + \sum_i \|q_i\|^2 \right]$$

$\min_{factors}$ "error" + λ "length"

The Effect of Regularization



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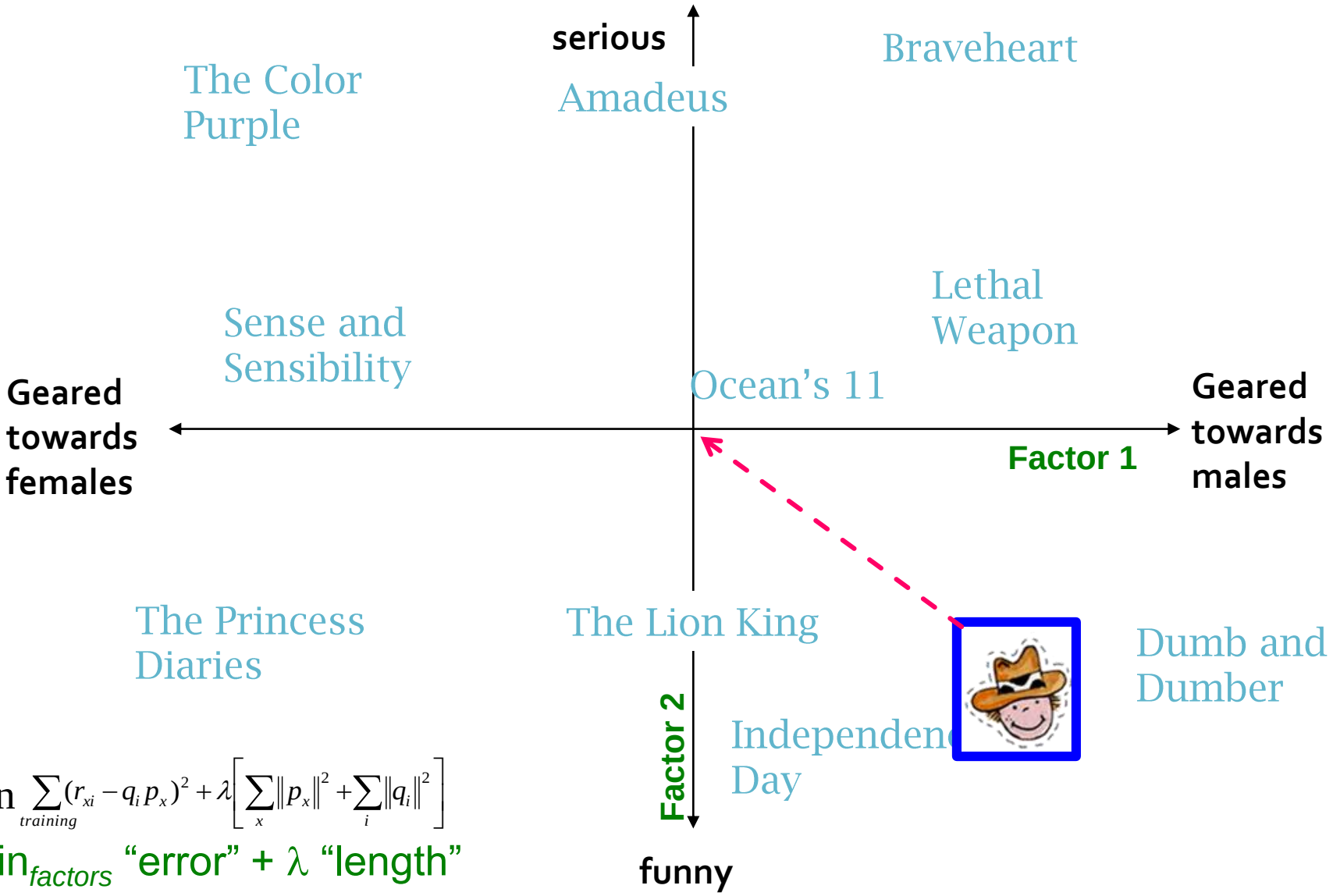
The Effect of Regularization



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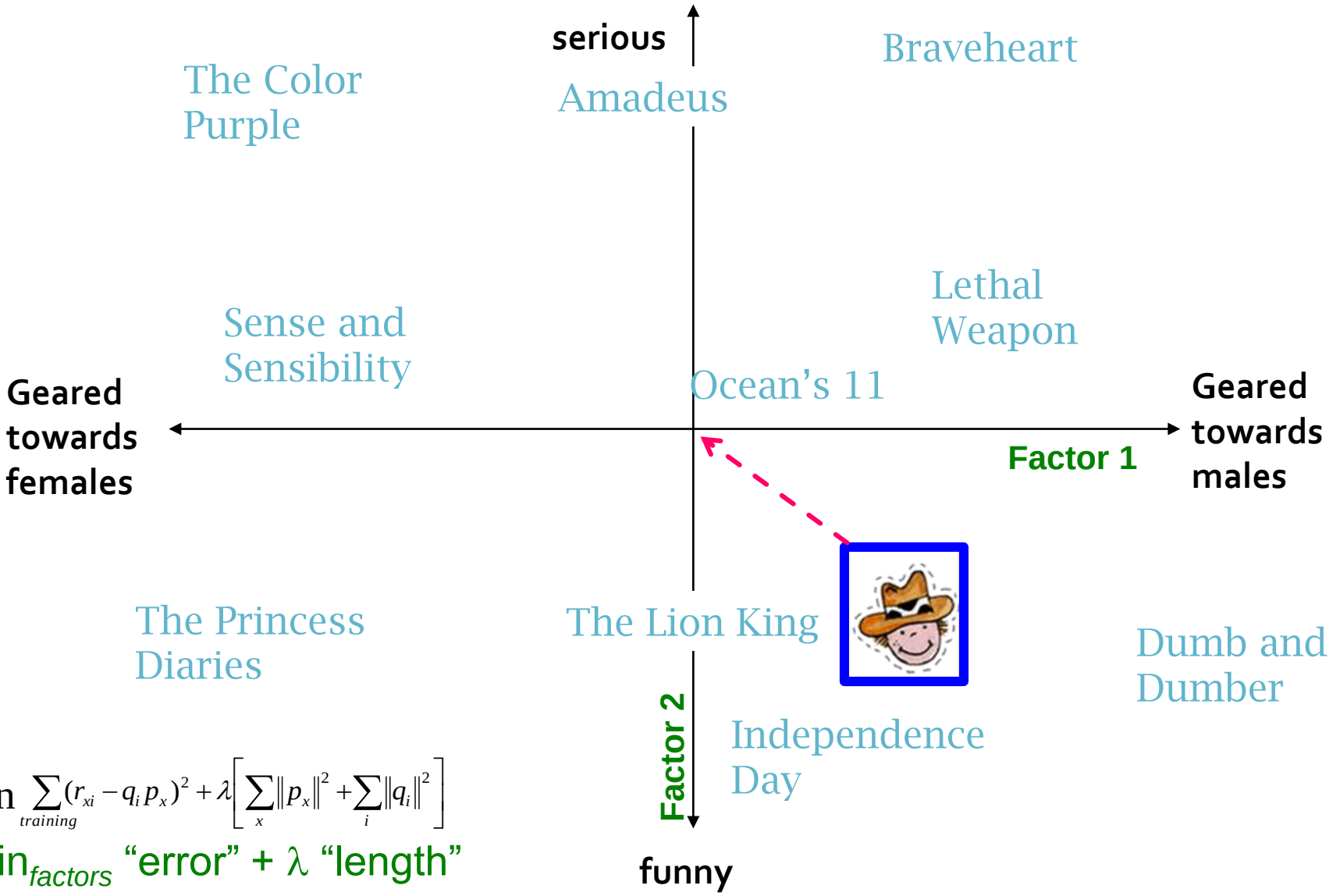
The Effect of Regularization



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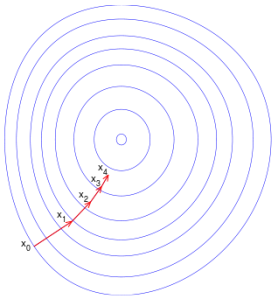
$\min_{factors}$ "error" + λ "length"

Stochastic Gradient Descent



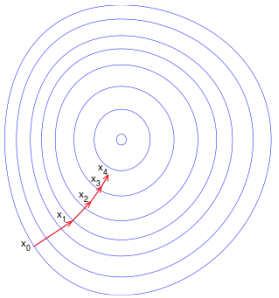
- **Want to find matrices *P* and *Q*:**

$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \left[\lambda_1 \sum_x \|p_x\|^2 + \lambda_2 \sum_i \|q_i\|^2 \right]$$

- **Gradient descent:**
  - Initialize ***P*** and ***Q*** (using SVD, pretend missing ratings are 0)
  - Do gradient descent:
    - $P \leftarrow P - \eta \cdot \nabla P$
    - $Q \leftarrow Q - \eta \cdot \nabla Q$
    - where ∇Q is gradient/derivative of matrix ***Q***:
      $\nabla Q = [\nabla q_{if}]$ and $\nabla q_{if} = \sum_{x,i} -2(r_{xi} - q_i p_x) p_{xf} + 2\lambda_2 q_{if}$
      - Here q_{if} is entry ***f*** of row q_i of matrix ***Q***
  - **Observation: Computing gradients is slow!**

How to compute gradient of a matrix?
Compute gradient of every element independently!

Stochastic Gradient Descent



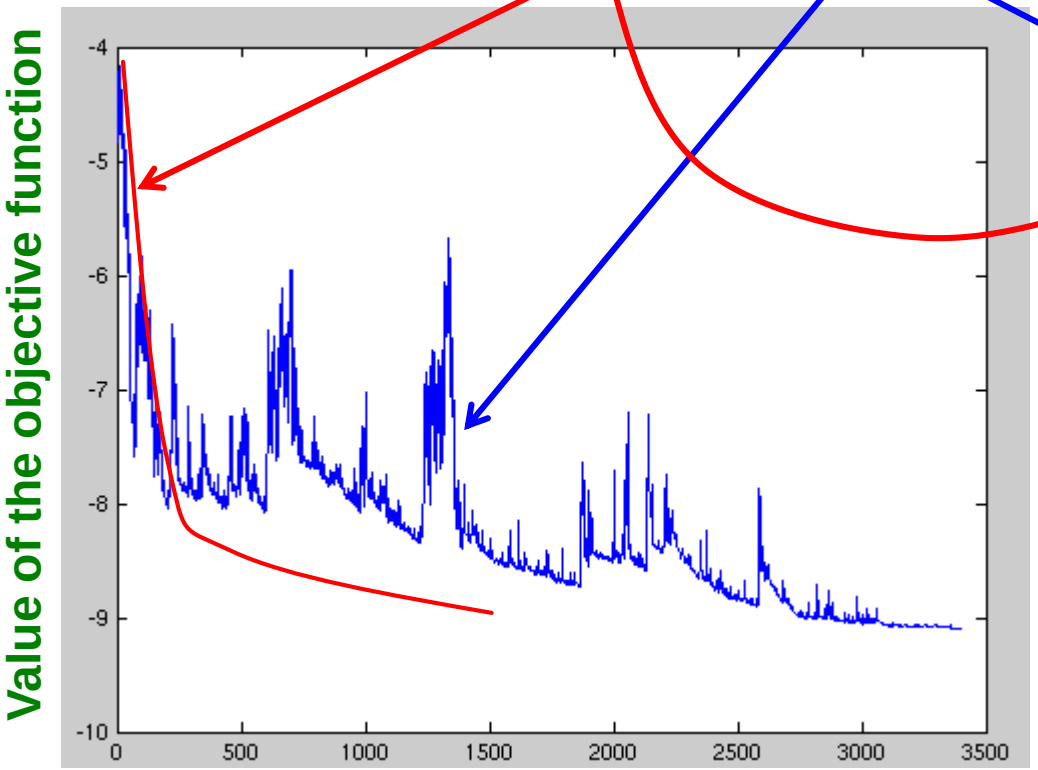
- **Gradient Descent (GD) vs. Stochastic GD**
  - **Observation:** $\nabla Q = [\nabla q_{if}]$ where

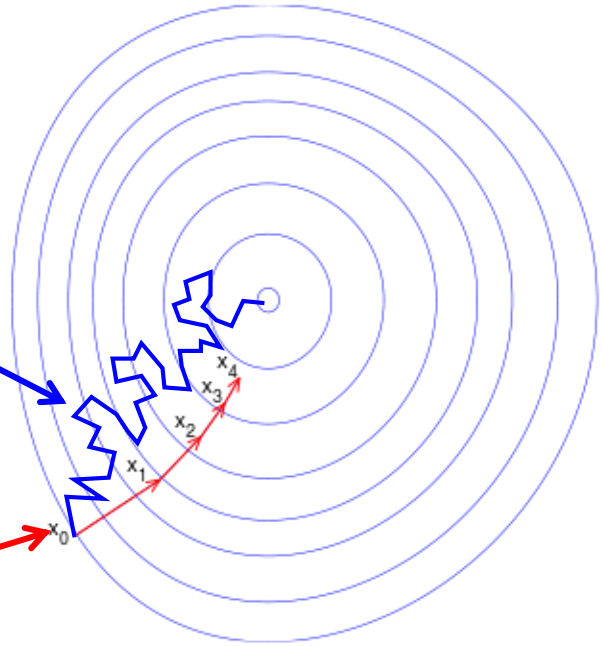
$$\nabla q_{if} = \sum_{x,i} -2(r_{xi} - q_{if}p_{xf})p_{xf} + 2\lambda q_{if} = \sum_{x,i} \nabla \boldsymbol{Q}(\boldsymbol{r_{xi}})$$

    - Here $\boldsymbol{q_{if}}$ is entry $\boldsymbol{f}$ of row $\boldsymbol{q_i}$ of matrix $\boldsymbol{Q}$
  - $\boldsymbol{Q} = \boldsymbol{Q} - \eta\nabla\boldsymbol{Q} = \boldsymbol{Q} - \eta\left[\sum_{x,i} \nabla \boldsymbol{Q}(\boldsymbol{r_{xi}})\right]$
  - **Idea:** Instead of evaluating gradient over all ratings evaluate it for each individual rating and make a step
- **GD:** $\boldsymbol{Q} \leftarrow \boldsymbol{Q} - \eta\left[\sum_{\boldsymbol{r_{xi}}} \nabla \boldsymbol{Q}(\boldsymbol{r_{xi}})\right]$
- **SGD:** $\boldsymbol{Q} \leftarrow \boldsymbol{Q} - \mu\nabla\boldsymbol{Q}(\boldsymbol{r_{xi}})$
  - **Faster convergence!**
    - Need more steps but each step is computed much faster

SGD vs. GD

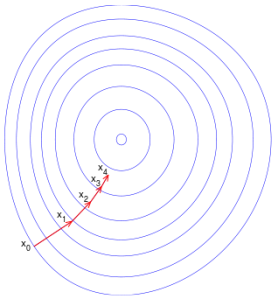
- **Convergence of GD vs. SGD**





GD improves the value of the objective function at every step.
SGD improves the value but in a "noisy" way.
GD takes fewer steps to converge but each step takes much longer to compute.
In practice, **SGD** is much faster!

Stochastic Gradient Descent

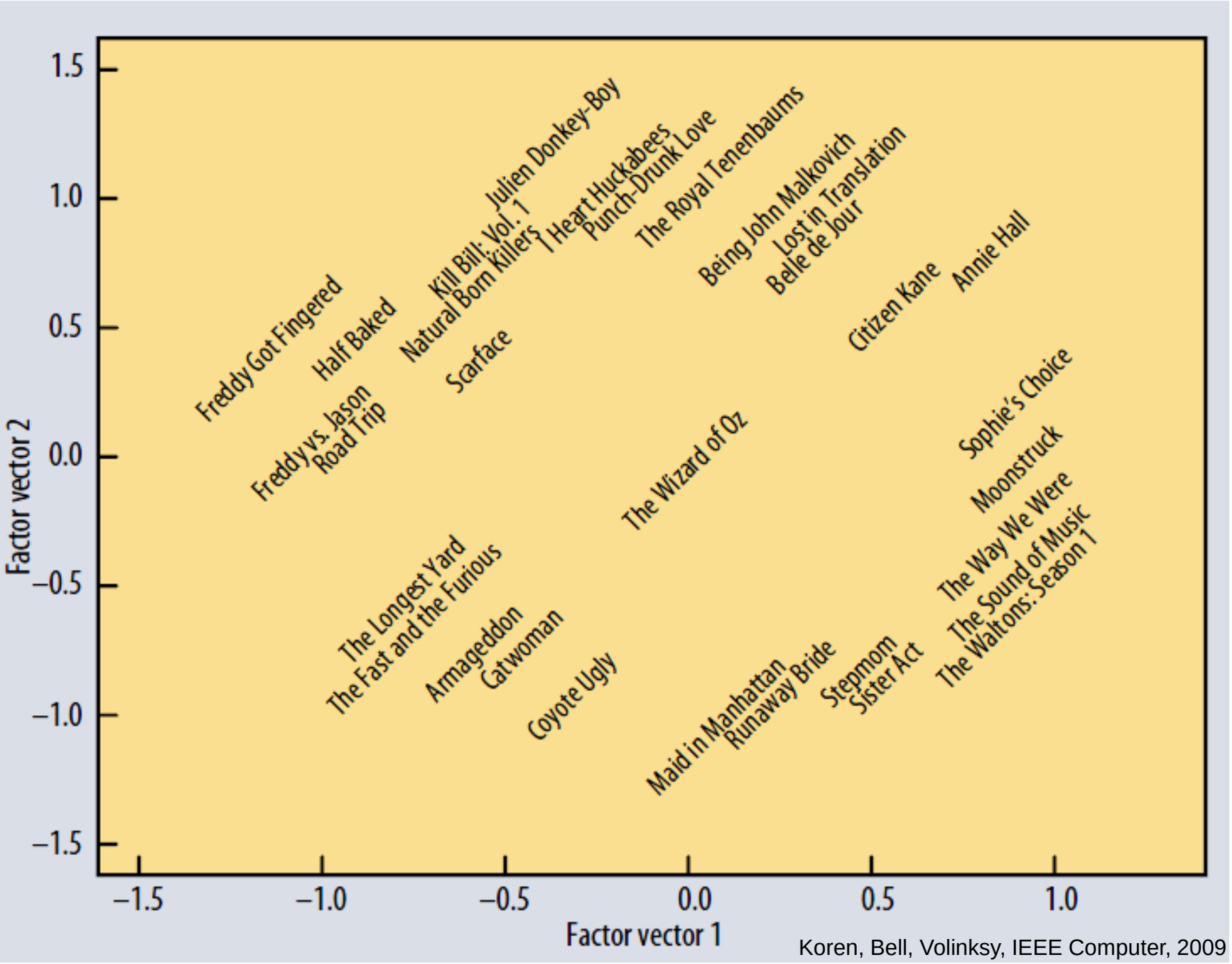


- **Stochastic gradient descent:**
  - Initialize ***P*** and ***Q*** (using SVD, pretend missing ratings are 0)
  - Then iterate over the ratings (multiple times if necessary) and update factors:

    For each r_{xi}:

    - $\varepsilon_{xi} = 2(r_{xi} - q_i \cdot p_x)$ (derivative of the "error")
    - $q_i \leftarrow q_i + \mu_1 (\varepsilon_{xi}\, p_x - \lambda_2\, q_i)$ (update equation)
    - $p_x \leftarrow p_x + \mu_2 (\varepsilon_{xi}\, q_i - \lambda_1\, p_x)$ (update equation)

    μ … learning rate
- **2 for loops:**
  - For until convergence:
    - For each $\mathbf{r_{xi}}$
      - Compute gradient, do a "step"



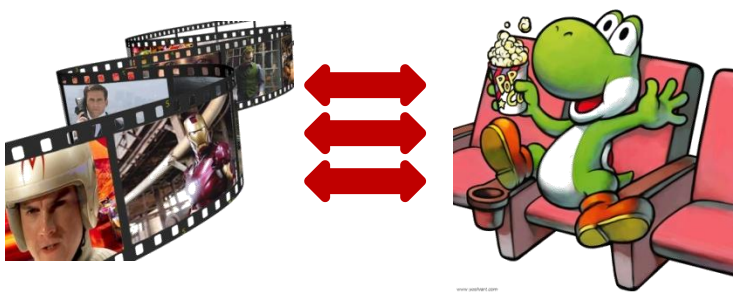
Koren, Bell, Volinksy, IEEE Computer, 2009

Modeling Biases and Interactions

user bias

movie bias

user-movie interaction



Baseline predictor

- Separates users and movies
- Benefits from insights into user's behavior
- Among the main practical contributions of the competition

User-Movie interaction

- Characterizes the matching between users and movies
- Attracts most research in the field
- Benefits from algorithmic and mathematical innovations

- μ = overall mean rating
- b_x = bias of user x
- b_i = bias of movie i

Putting It All Together

$$r_{xi} = \mu + b_x + b_i + q_i \cdot p_x$$

- μ: Overall mean rating
- b_x: Bias for user *x*
- b_i: Bias for movie *i*
- $q_i \cdot p_x$: User-Movie interaction

- **Example:**
  - Mean rating: $\mu = 3.7$
  - You are a critical reviewer: your ratings are 1 star lower than the mean: $b_x = -1$
  - Star Wars gets a mean rating of *0.5* higher than average movie: $b_i = +0.5$
  - Predicted rating for you on Star Wars:
    $= 3.7 - 1 + 0.5 = 3.2$

Fitting the New Model

- **Solve:**

$$\min_{Q,P} \sum_{(x,i)\in R} \left(r_{xi} - (\mu + b_x + b_i + q_i\, p_x)\right)^2$$

goodness of fit

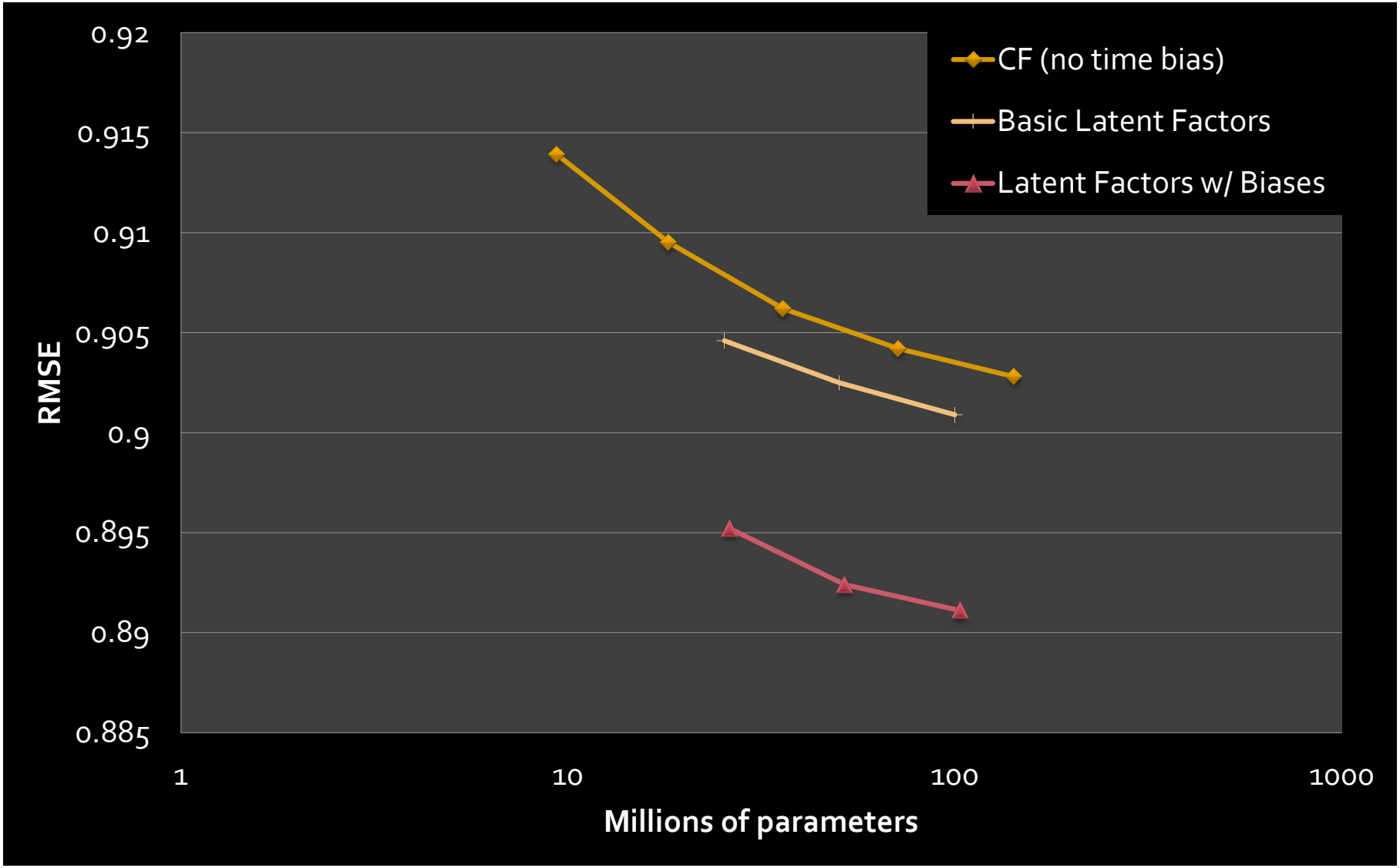
$$+ \left(\lambda_1 \sum_i \|q_i\|^2 + \lambda_2 \sum_x \|p_x\|^2 + \lambda_3 \sum_x \|b_x\|^2 + \lambda_4 \sum_i \|b_i\|^2 \right)$$

regularization

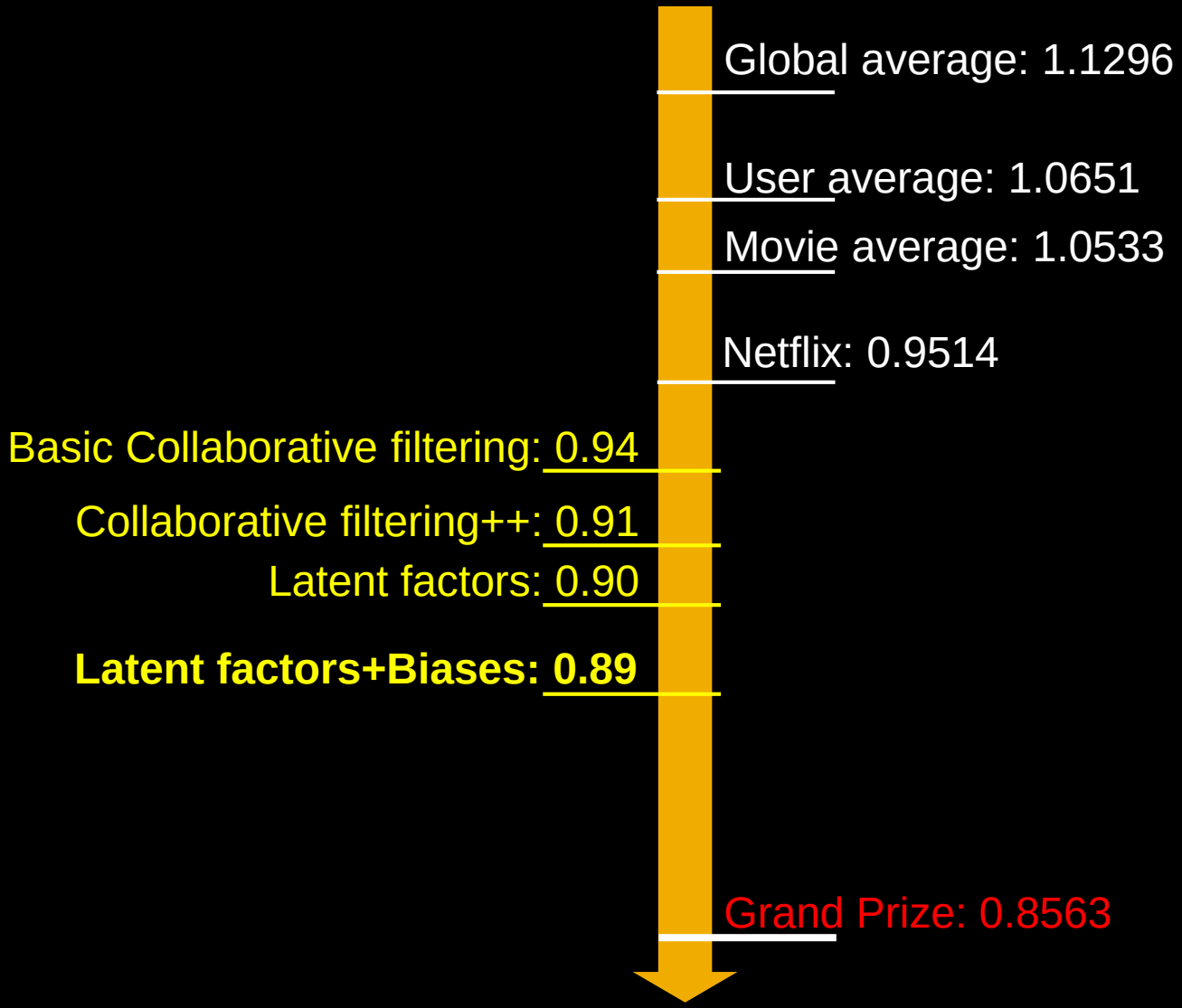
λ is selected via grid-search on a validation set

- **Stochastic gradient descent to find parameters**
  - **Note:** Both biases b_x, b_i as well as interactions q_i, p_x are treated as parameters (we estimate them)

Performance of Various Methods

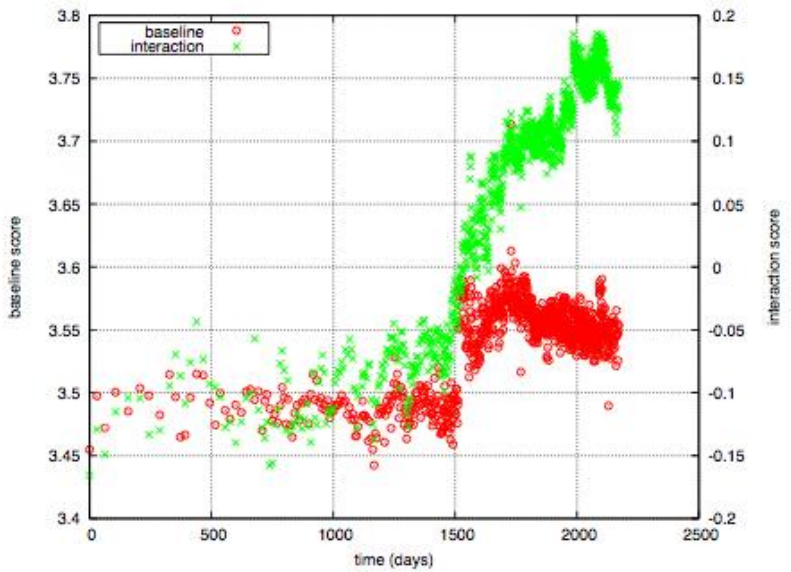


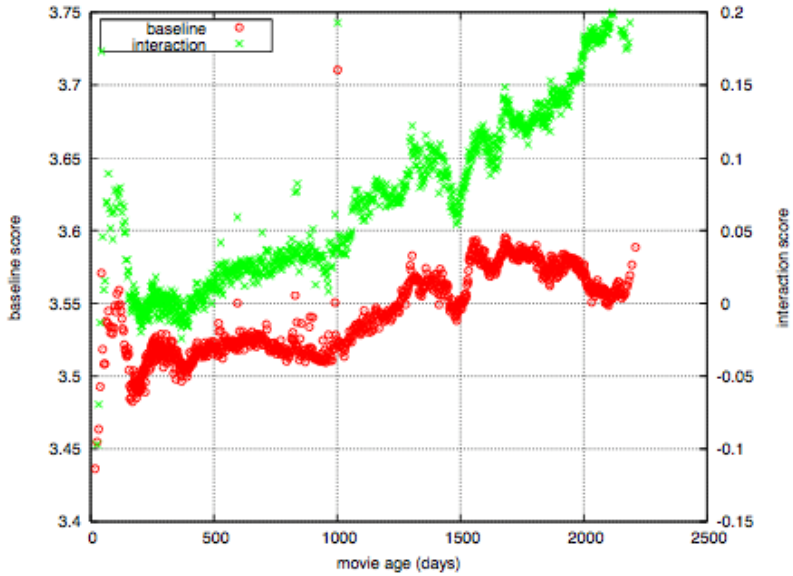
Performance of Various Methods



Temporal Biases Of Users

- **Sudden rise in the average movie rating** (early 2004)
  - Improvements in Netflix
  - GUI improvements
  - Meaning of rating changed
- **Movie age**
  - Users prefer new movies without any reasons
  - Older movies are just inherently better than newer ones





Y. Koren, Collaborative filtering with temporal dynamics, KDD '09

Temporal Biases & Factors

- **Original model:**

  $r_{xi} = \mu + b_x + b_i + q_i \cdot p_x$

- **Add time dependence to biases:**

  $r_{xi} = \mu + b_x(t) + b_i(t) + q_i \cdot p_x$

  - Make parameters b_x and b_i to depend on time
  - **(1)** Parameterize time-dependence by linear trends
    (2) Each bin corresponds to 10 consecutive weeks

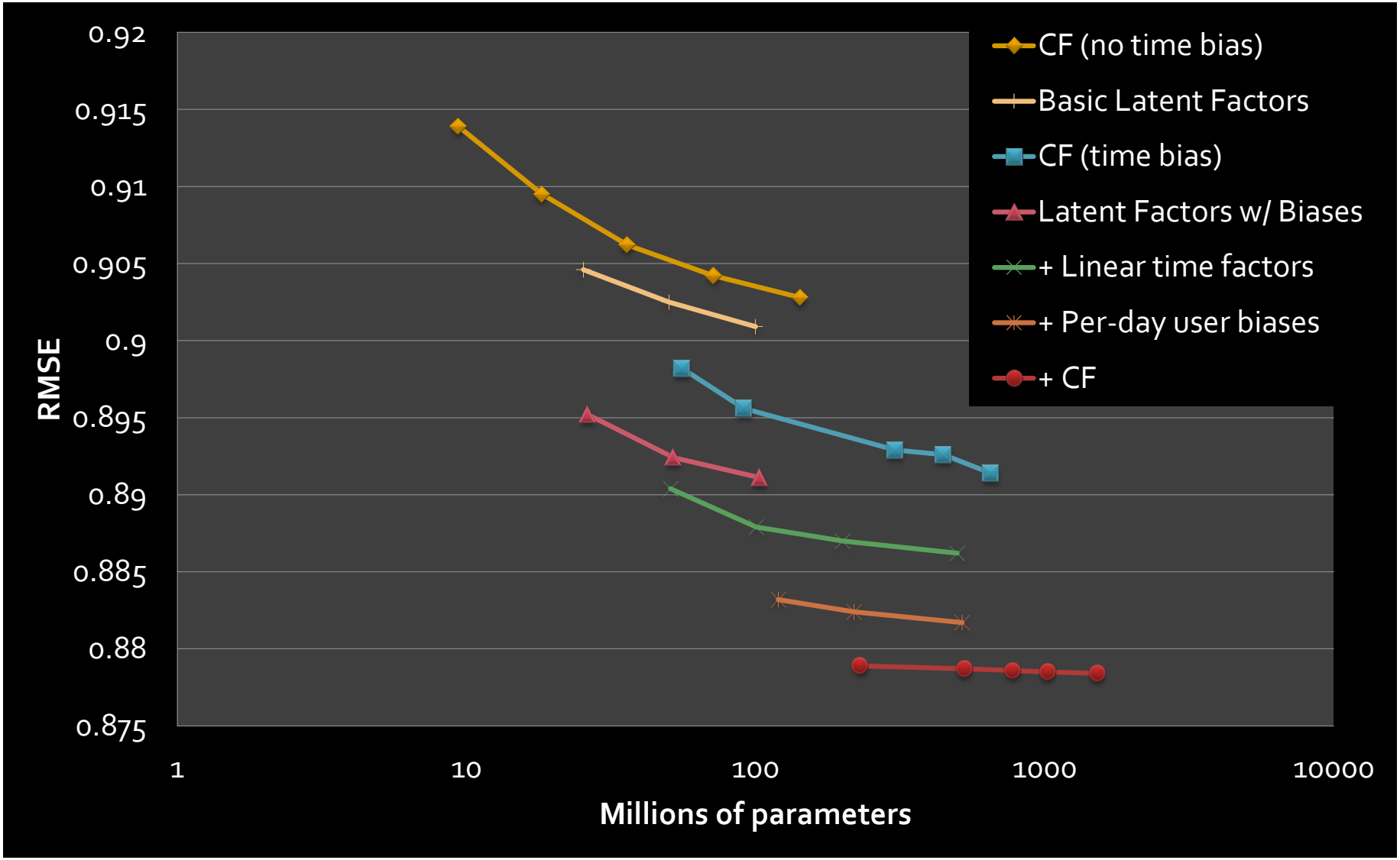
    $$b_i(t) = b_i + b_{i,\mathrm{Bin}(t)}$$

- **Add temporal dependence to factors**

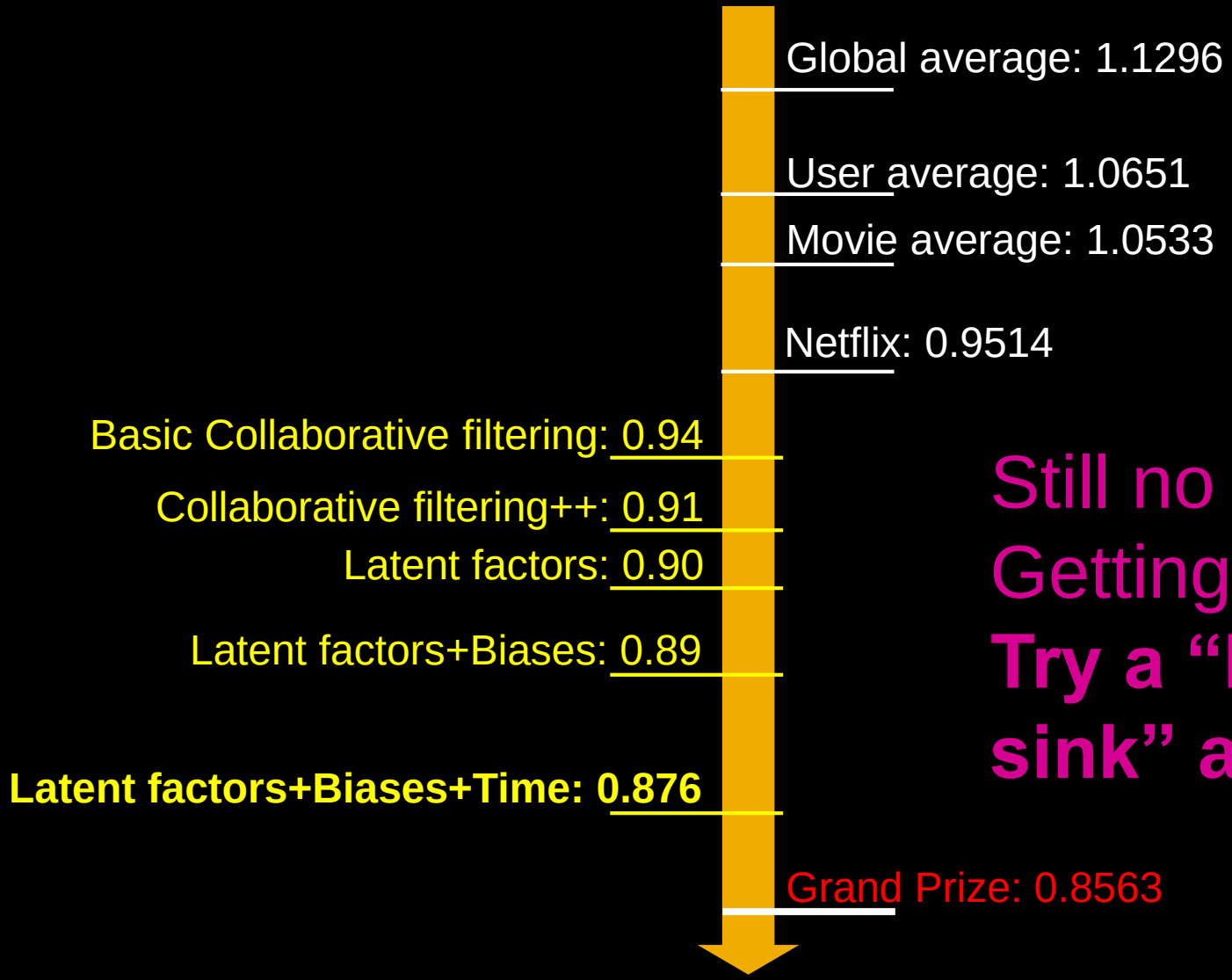
  - $p_x(t)$... user preference vector on day t

Y. Koren, Collaborative filtering with temporal dynamics, KDD '09

Adding Temporal Effects



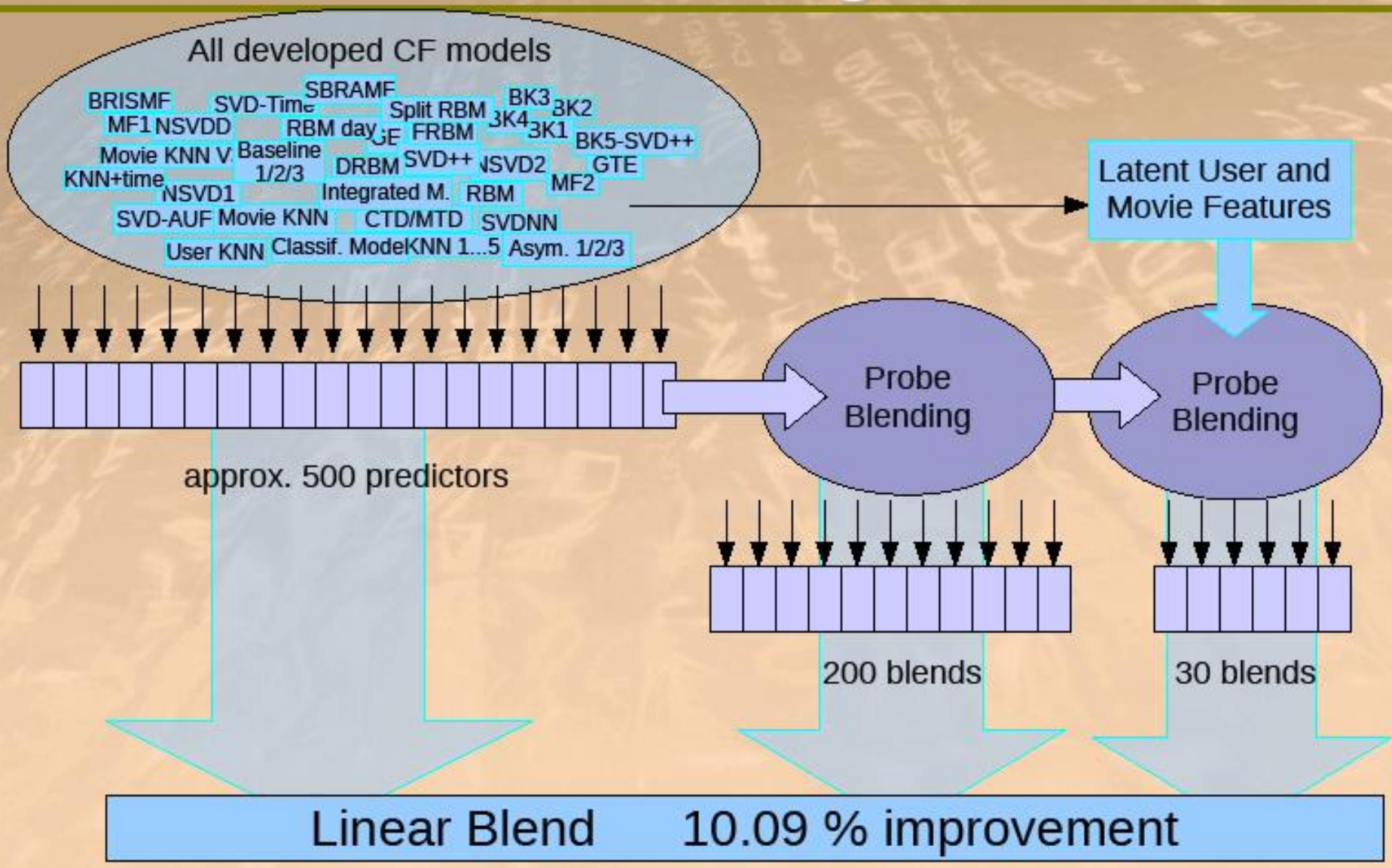
Performance of Various Methods



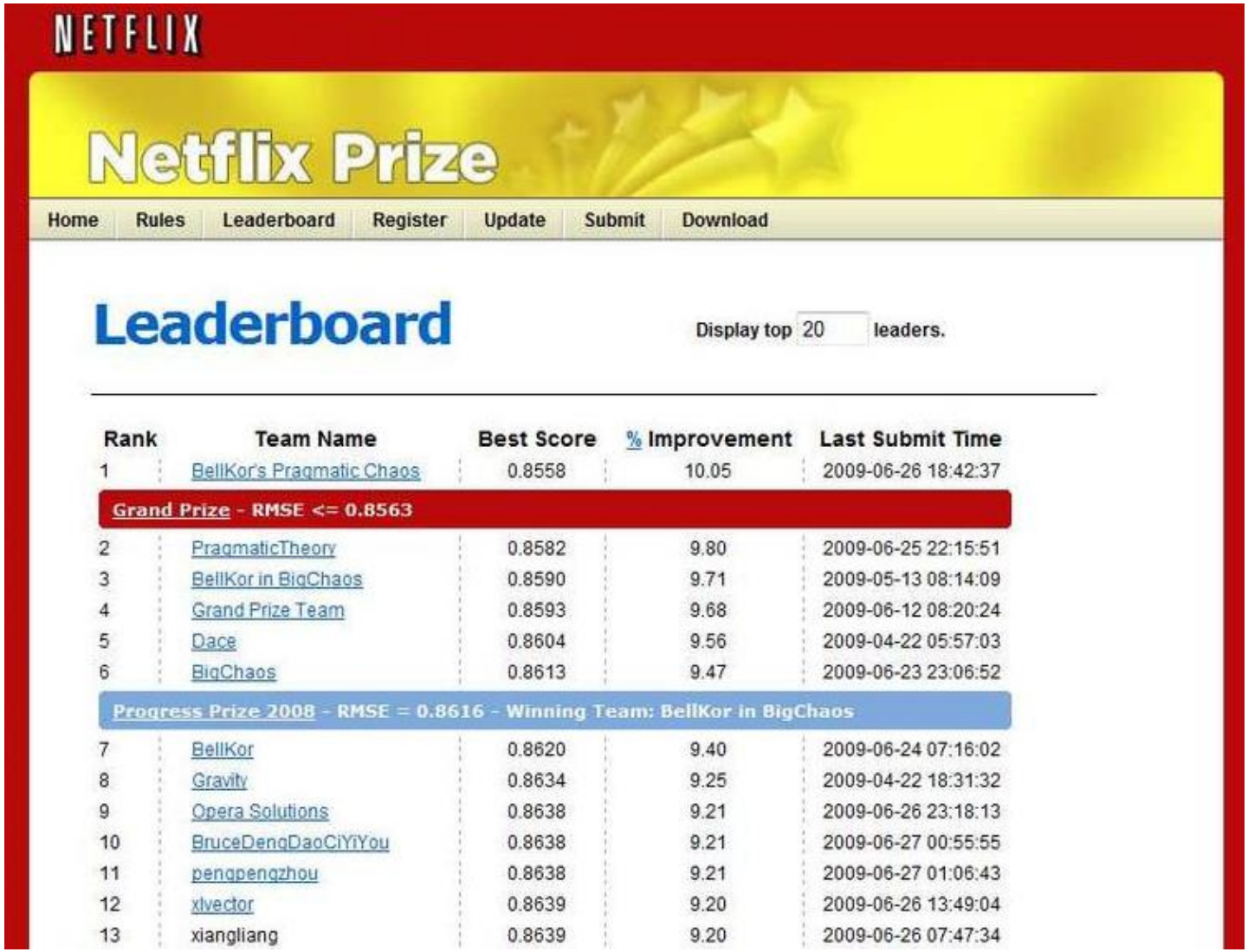
Still no prize!
Getting desperate.
Try a "kitchen sink" approach!

The big picture

Solution of BellKor's Pragmatic Chaos



Standing on June 26th 2009



NETFLIX

Netflix Prize

Home | Rules | Leaderboard | Register | Update | Submit | Download

Leaderboard

Display top 20 leaders.

Rank	Team Name	Best Score	% Improvement	Last Submit Time
1	BellKor's Pragmatic Chaos	0.8558	10.05	2009-06-26 18:42:37
Grand Prize - RMSE <= 0.8563				
2	PragmaticTheory	0.8582	9.80	2009-06-25 22:15:51
3	BellKor in BigChaos	0.8590	9.71	2009-05-13 08:14:09
4	Grand Prize Team	0.8593	9.68	2009-06-12 08:20:24
5	Dace	0.8604	9.56	2009-04-22 05:57:03
6	BigChaos	0.8613	9.47	2009-06-23 23:06:52
Progress Prize 2008 - RMSE = 0.8616 - Winning Team: BellKor in BigChaos				
7	BellKor	0.8620	9.40	2009-06-24 07:16:02
8	Gravity	0.8634	9.25	2009-04-22 18:31:32
9	Opera Solutions	0.8638	9.21	2009-06-26 23:18:13
10	BruceDengDaoCiYiYou	0.8638	9.21	2009-06-27 00:55:55
11	pengpengzhou	0.8638	9.21	2009-06-27 01:06:43
12	xlvector	0.8639	9.20	2009-06-26 13:49:04
13	xiangliang	0.8639	9.20	2009-06-26 07:47:34

June 26th submission triggers 30-day "last call"

The Last 30 Days

- **Ensemble team formed**
  - Group of other teams on leaderboard forms a new team
  - Relies on combining their models
  - Quickly also get a qualifying score over 10%
- **BellKor**
  - Continue to get small improvements in their scores
  - Realize that they are in direct competition with Ensemble
- **Strategy**
  - Both teams carefully monitoring the leaderboard
  - Only sure way to check for improvement is to submit a set of predictions
    - This alerts the other team of your latest score

24 Hours from the Deadline

- **Submissions limited to 1 a day**
  - Only 1 final submission could be made in the last 24h
- **24 hours before deadline…**
  - **BellKor** team member in Austria notices (by chance) that **Ensemble** posts a score that is slightly better than BellKor's
- **Frantic last 24 hours for both teams**
  - Much computer time on final optimization
  - Carefully calibrated to end about an hour before deadline
- **Final submissions**
  - **BellKor** submits a little early (on purpose), 40 mins before deadline
  - **Ensemble** submits their final entry 20 mins later
  - **….and everyone waits….**

NETFLIX

Netflix Prize

COMPLETED

Home | Rules | Leaderboard | Update | Download

Leaderboard

Showing Test Score. Click here to show quiz score

Display top 20 leaders.

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	BellKor's Pragmatic Chaos	0.8567	10.06	2009-07-26 18:18:28
2	The Ensemble	0.8567	10.06	2009-07-26 18:38:22
3	Grand Prize Team	[illegible]	[illegible]	[illegible]
4	Opera Solutions and Vandelay United	0.8588	9.84	2009-07-10 01:12:31
5	Vandelay Industries !	0.8591	9.81	2009-07-10 00:32:20
6	PragmaticTheory	0.8594	9.77	2009-06-24 12:06:56
7	BellKor in BigChaos	0.8601	9.70	2009-05-13 08:14:09
8	Dace	0.8612	9.59	2009-07-24 17:18:43
9	Feeds2	0.8622	9.48	2009-07-12 13:11:51
10	BigChaos	0.8623	9.47	2009-04-07 12:33:59
11	Opera Solutions	0.8623	9.47	2009-07-24 00:34:07
12	BellKor	0.8624	9.46	2009-07-26 17:19:11
Progress Prize 2008 - RMSE = 0.8627 - Winning Team: BellKor in BigChaos				
13	xiangliang	0.8642	9.27	2009-07-15 14:53:22
14	Gravity	0.8643	9.26	2009-04-22 18:31:32
15	Ces	0.8651	9.18	2009-06-21 19:24:53
16	Invisible Ideas	0.8653	9.15	2009-07-15 15:53:04
17	Just a guy in a garage	0.8662	9.06	2009-05-24 10:02:54
18	J Dennis Su	0.8666	9.02	2009-03-07 17:16:17
19	Craig Carmichael	0.8666	9.02	2009-07-25 16:00:54
20	acmehill	0.8668	9.00	2009-03-21 16:20:50
Progress Prize 2007 - RMSE = 0.8723 - Winning Team: KorBell				

Million $ Awarded Sept 21st 2009