HIROSHIMA UNIVERSITY

Fundamental Data Science (30104001)

Lecture 3 — Types of data. Descriptive statistics

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Today

- Types of data:
  $\hookrightarrow$ *variable types*.
- Descriptive statistics:
  $\hookrightarrow$ *how to summarize data*.

Types of data

Statistical data

Household data in household survey (Unit: 1000 yen per month)

Household	**Food**	**Education**	**Recreation**	**Size of household**	**Working family***
1	75	40	66	4	1
2	70	80	91	1	0
⋮	⋮	⋮	⋮	⋮	⋮
100	30	51	65	3	1

*Working family: 1 = Worker households; 0 = other households.

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Cells:
Each cell is an observation or datum.

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Variable types — another example

Which variables are qualitative or quantitative?

House Prices and Environmental Conditions data (Takahashi et al., 2000)

ID	Price	Period	Area	Size	JR	Time	Age	Distance
54	3730	0	148	103	0	9	0	13
55	1480	1	110	67	0	3	23	8
64	3590	1	105	100	1	8	8	8

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- **Price**
  Single house price (in $¥10,000$).
- **Period**
  Year (0 = 1998; 1 = 1999).
- **Area**
  Land area (in m^2).
- **Size**
  House are (in m^2).
- **JR**
  Type of nearest station (0 = Not JR; 1 = JR).
- **St. time**
  Time to walk from nearest station/bus stop (in min).
- **Age**
  Time since it was built (in years).
- **Distance**
  Distance from the center of Hiroshima city (in km).

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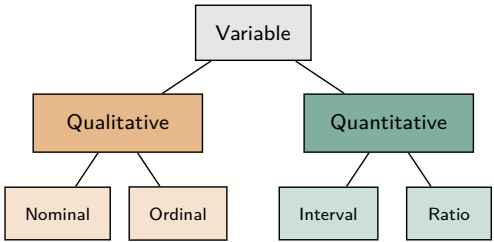
There are four main variable types (Stevens, 1946):

- Nominal scale variables.
- Ordinal scale variables.
- Interval scale variables.
- Ratio scale variables.



Scales of measurement: Nominal scale variables

Also known as categorical scale variables.

There is no particular relationship between the different values (like in size or quality).

Example:

- Eye color
- Gender
- Nationality.

No computations with the values is possible (like sum or mean).

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Scales of measurement: Ordinal scale variables

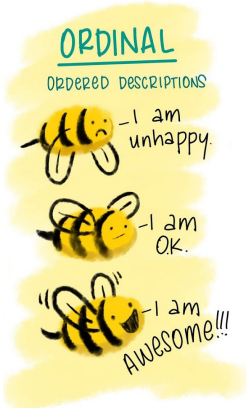
There is a natural, meaningful way to order different values (numbers or labels).

But there is no way to assess the *relative degree of difference* between any two values.

Example:

- Race ranking (1st, 2nd, 3rd, ...).
- Questions with answer options 'completely agree', 'mostly agree', 'mostly disagree', 'completely disagree'. This is known as a Likert item.

No computations with the values is possible (like sum or mean).



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Numerical values such that *differences* between numbers are interpretable.
Addition and subtraction is allowed.

However, interval scale variables lack a "natural" zero value.
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Example: Temperature in degrees celsius.

- We can consider temperature differences, so 15°C is 5°C higher than 10°C.
- However, 0°C **cannot** be interpreted as "no temperature".
  So, one cannot say that 20°C is twice as hot as 10°C.

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Other examples:
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2. Statistical methods are typically quite specific about the type of variables that they allow:
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3. Variables can be ordered based on how easy it is to manipulate them:
   $$\text{Ratio} > \text{Interval} > \text{Ordinal} > \text{Nominal}$$

Descriptive statistics

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As an example, consider the following data:

Housing price data of 158 houses (in ¥10,000) (Takahashi et al., 2000):

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2000	2670	2980	2980	3980	4400	4570	3480	3800	2450	4400	2300	2880	2980	3300	3380	3660	3180	3750	4180
1380	2690	1680	2580	2780	2890	3080	2980	3050	2980	2780	3880	3680	3730	1480	2000	2050	2170	2180	2290
2300	2380	1380	3590	2630	2680	2880	3080	3200	3550	3790	5300	3190	2780	4100	5580	1280	2480	2580	2880
2900	3180	3280	3280	3280	3300	3680	3850	4290	7800	1500	2580	3180	3190	3100	2980	3280	3070	3190	3680
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2300	2380	1380	3590	2630	2680	2880	3080	3200	3550	3790	5300	3190	2780	4100	5580	1280	2480	2580	2880
2900	3180	3280	3280	3280	3300	3680	3850	4290	7800	1500	2580	3180	3190	3100	2980	3280	3070	3190	3680
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We need to find ways to summarize the information in the data.

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One possible way to do this is by plotting the data.

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Visual representations *that allow to quickly grasp the relevant information in the data.*

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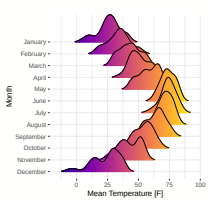

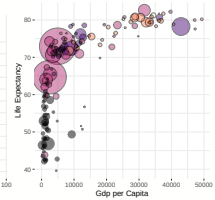

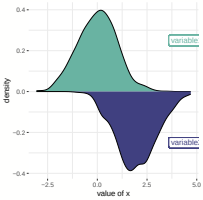

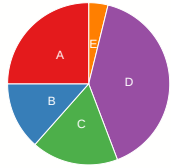

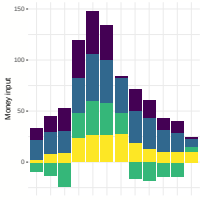

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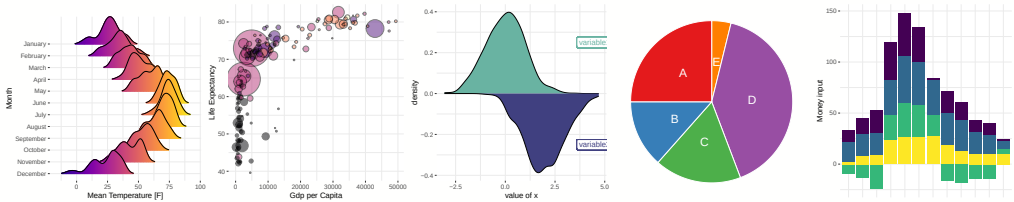

We will only learn about a small few, simple, plots.

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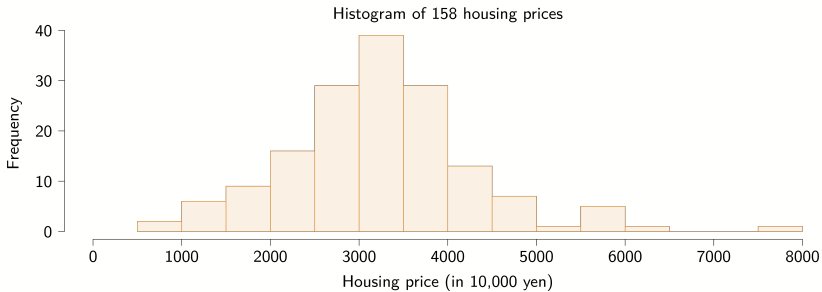

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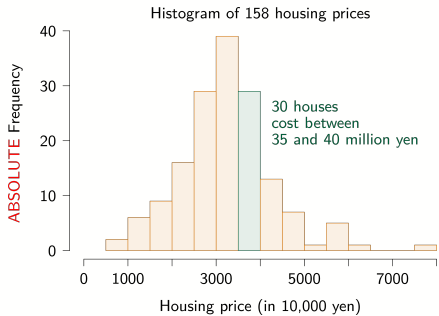

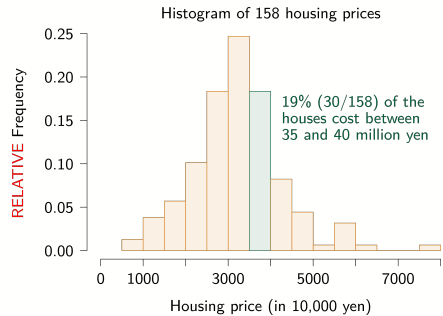

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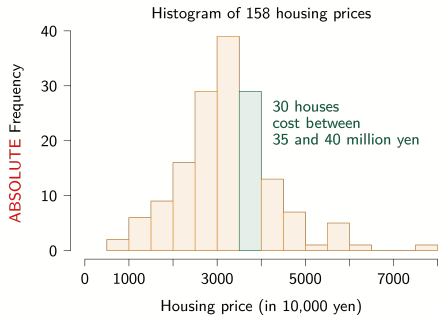

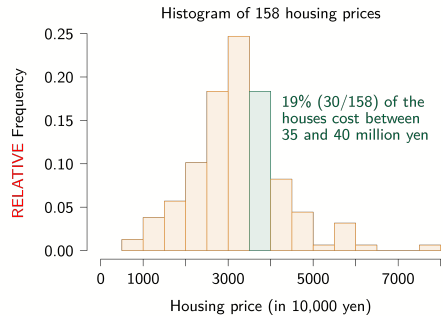

bin height $=$ number of observations in each bin

bin height $= \frac{\text{bin count}}{\text{sample size}}$

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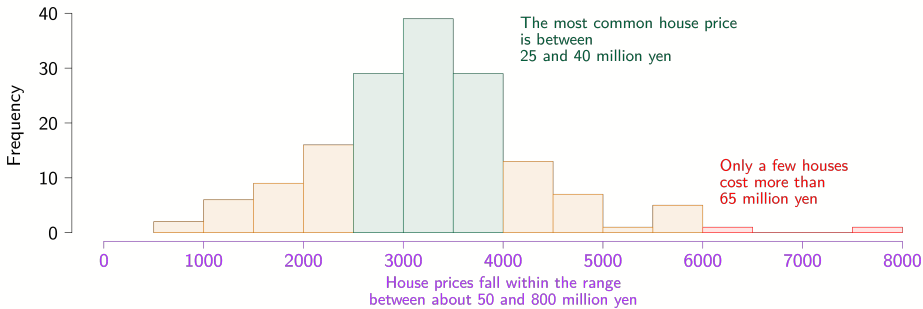

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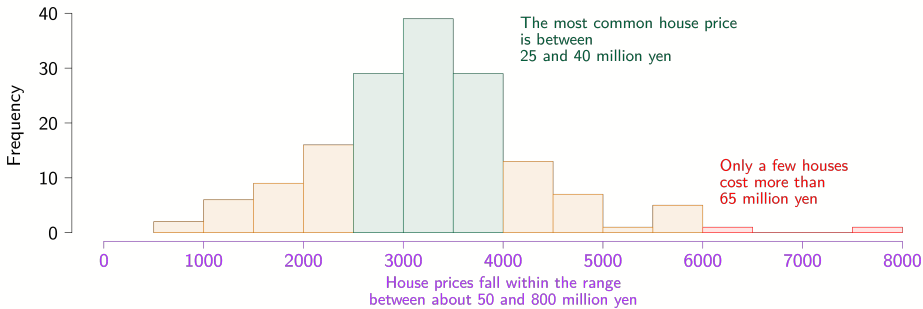

We can tell all this even without looking at the numbers in the data!

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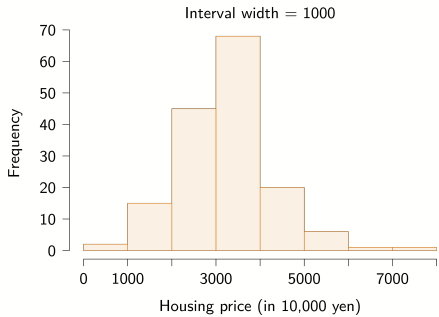

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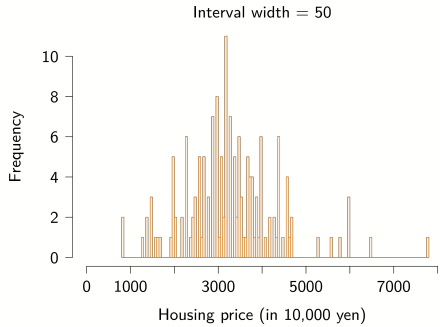

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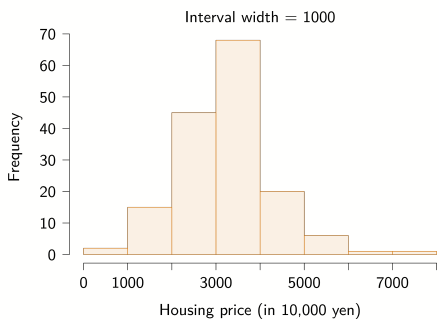

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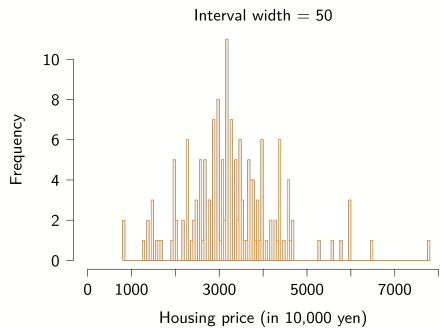

A sensible intermediate number of bins (not too few, not too many) is the best.

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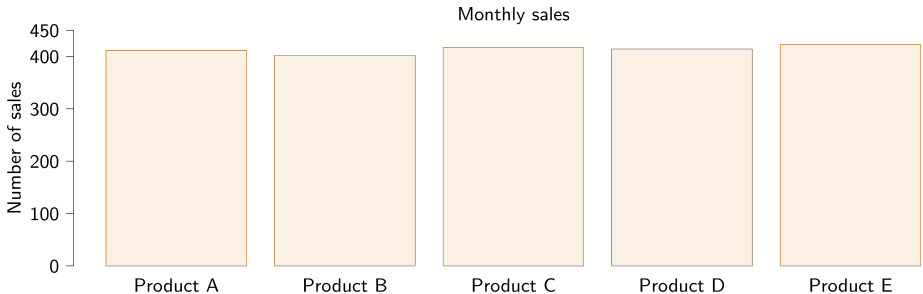

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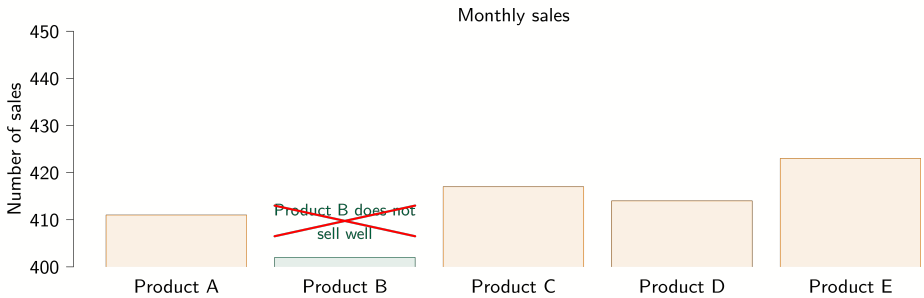

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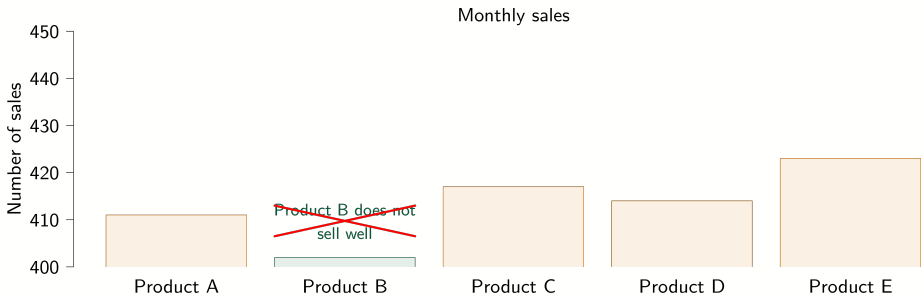
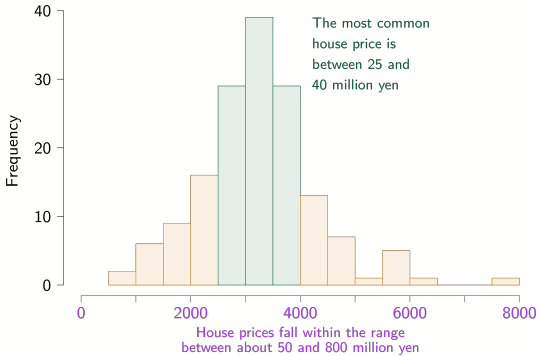


Changing the appearance of the y-axis changed the heights of the bars.
This should **not** be done! Be aware that some advertisement companies do use such tricks...

Descriptive statistics — Using numbers

Plotting your data is great!



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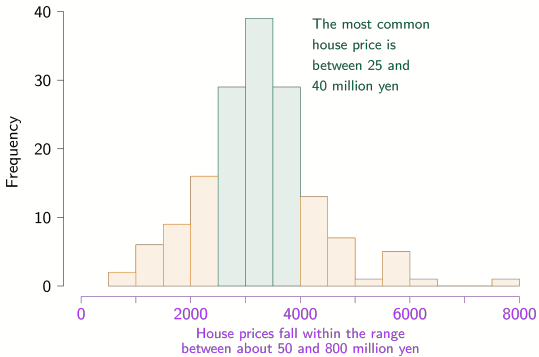

But plots provide only a broad overview.

To draw more precise information, we need numbers.

Descriptive statistics — Using numbers

Descriptive statistic:

A value that represents a particular characteristic of the data distribution.
Easy to communicate and objective.

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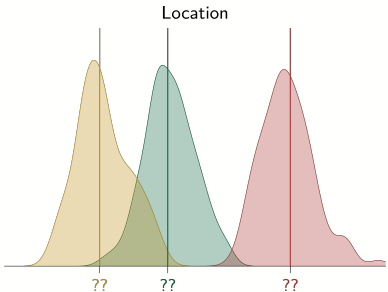

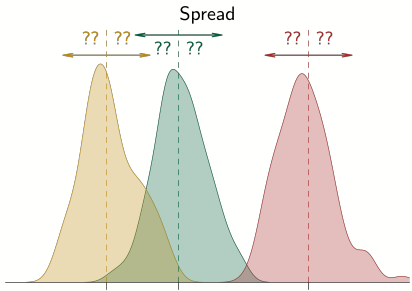

Descriptive statistics — Using numbers

There are *many* measures of location and spread available.
We will only discuss a few.

Descriptive statistics — Using numbers

There are *many* measures of location and spread available.
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Location: (today)

- Mean
- Median
- Mode

Spread: (next lecture)

- Variance
- Standard deviation
- Range
- Quartiles
- Interquartile range

Measure of location: Mean

The sample mean of n data points $x_1, \ldots, x_n$ from variable x is

$$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}.$$

Individual	Variable x
1	x_1
2	x_2
$\vdots$	$\vdots$
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In words,

Sample mean = add all the data up, and divide by the sample size.

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In words,

Sample mean = add all the data up, and divide by the sample size.

Example:

Individual	Weight (kg)
1	60
2	52
3	44
4	74
5	60

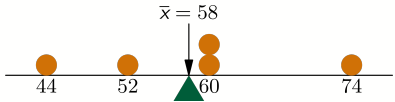
- $n = 5$
- Variable $x =$ weight in Kg
- $x_1 = 60$, $x_2 = 52$, ..., $x_5 = 60$

$$\overline{x} = \frac{60+52+44+74+60}{5} = 58$$

Measure of location: Mean

The mean is…

- …actually the center of gravity for the data:



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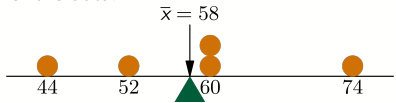

- …a suitable representative value for the location of a quantitative variables.
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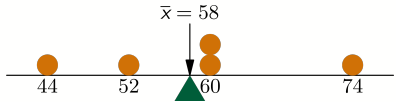

- …a suitable representative value for the location of a quantitative variables.
  For example: height, weight, annual income, year, temperature (°C).

- …**not** a suitable representative value for the location of a qualitative variable.
  For example, what would be the mean of "blood type" (A, B, AB, O)?

  Also for numeric qualitative variables, the mean makes no sense.
  For example, what would be the mean of "blood type" (1 = A, 2 = B, 3 = AB, 4 = O)?
  Here, the numbers are just *labels*. They have **no** numerical meaning.

Measure of location: Median

The median of a set of observations is the middle value.

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To find the median, do this:

- Sort the values from smallest to largest.
- If there is an **odd number** of values, the median is the middle value.
- If there is an **even number** of values, the median is the mean of the two most middle values.

Measure of location: Median

Example — odd number of values.

Q: What is the median of the values $\{5, 1, 5, 0, 3\}$?

Measure of location: Median

Example — odd number of values.

Q: What is the median of the values $\{5, 1, 5, 0, 3\}$?
A: First, sort the values in increasing order:

$$0, 1, \mathbf{3}, 5, 5.$$

The middle value is 3, thus the median is 3.

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Measure of location: Median

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The two most middle values are 1 and 4 and their mean is $\frac{1+4}{2} = 2.5$.
Thus the median is 2.5.

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Thus the median is 2.5.

Example:

Individual	Weight (kg)
1	60
2	52
3	44
4	74
5	60

Sort the values in increasing order:

$$44, 52, \mathbf{60}, 60, 74.$$

The middle value is 60, thus the median is 60.

Measure of location: Median

The median is a suitable representative value for the location of a quantitative variable.

And, interestingly,

> *The median is also suitable for the location of an ordinal variable.*

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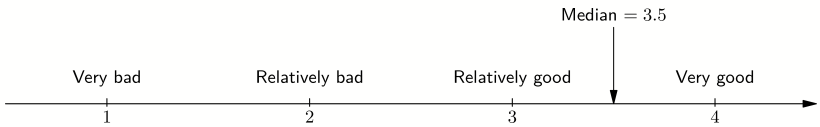
Example:
A question with four answer options (1 = very bad; 2 = relatively bad; 3 = relatively good; 4 = very good) was given to a group of people. The median score in the sample is 3.5.



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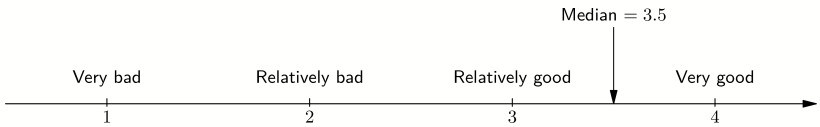


We can then say that the middle of the data is in between *Relatively good* and *Very good*.

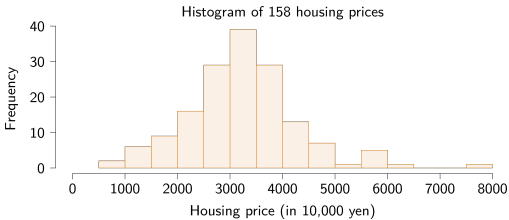
Exercises (1)

Individual	Weight (kg)
1	60
2	52
3	44
4	74
5	60

$\overline{x} = 58$ kg
median $= 60$ kg

1. Calculate the mean and median of six people when a sixth observation, 61 (kg), is added to the table above.
2. Calculate the mean and median of six people when the sixth observation was accidentally added as 1000 (kg) to the table above.
3. Consider the difference between the mean and the median from the results of questions 1 and 2.

Measure of location: Mean vs median



For the *housing price* data we have that:

- Mean $= 3291.12$.
- Median $= 3195$.

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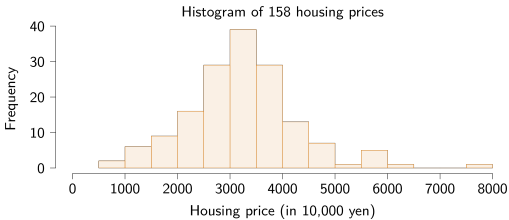

In this case, the mean and the median are *almost* the same.
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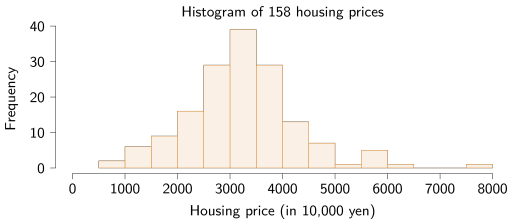

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But, they are indeed different.

Which value should be preferred?
What are the differences between the mean and the median?

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Answer

- $\overline{x} = \frac{60+52+44+74+60+61}{6} = 58.5.$

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- $\overline{x} = \frac{60+52+44+74+60+61}{6} = 58.5$.
- Ordered data: $44, 52, 60, 60, 61, 74$.
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Answer

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3. Consider the difference between the mean and the median from the results of questions 1 and 2.

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Answer

		One more data value	
	Original data for five people	Question 1 (61 kg)	Question 2 (1000 kg)
Mean	58	58.5	215
Median	60	60	60

The mean is much more sensitive to **extreme values** than the median.

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	Original data for five people	Question 1 (61 kg)	Question 2 (1000 kg)
Mean	58	58.5	215
Median	60	60	60

The mean is much more sensitive to **extreme values** than the median.

Extreme values are also known as **outliers**.
In case you want to avoid the effect of outliers, it's better to use the median instead of the mean!!

Notes about outliers

Example:
Consider the following exam scores from five students:

$$40, 40, 40, 40, 100.$$

Choosing between the mean or the median also depends on what *question* one is trying to answer.

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Choosing between the mean or the median also depends on what *question* one is trying to answer.

Question	Answer
"What is the middle of the ordered scores?"	Median = 40
"What is the average score?"	Mean = $\frac{40+40+40+40+100}{5} = 52$

Notes about outliers

Example:
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Choosing between the mean or the median also depends on what *question* one is trying to answer.

Question	Answer
"What is the middle of the ordered scores?"	Median = 40
"What is the average score?"	Mean = $\frac{40+40+40+40+100}{5} = 52$

Even with outliers, choosing between the mean or the median may depend based on what kind of "middle" score we want to know.

Mean vs median — symmetric distributions

For symmetric distributions, the mean and the median are very close to each other.

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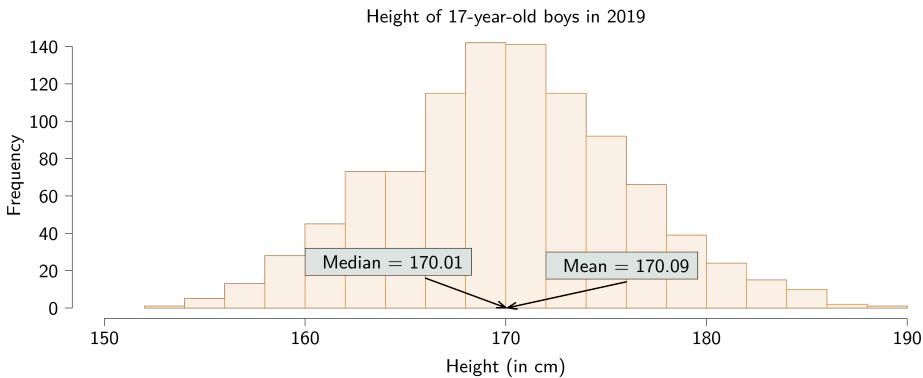

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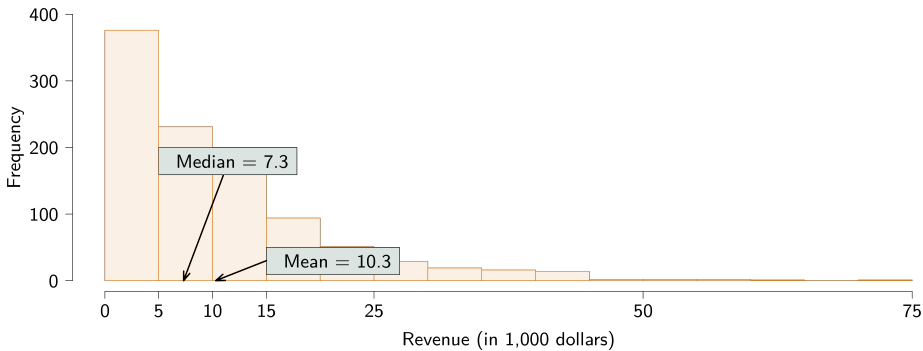

Measure of location: Mean vs median — Which one?

Ordinal variable:
Use the median (the mean is not defined in this case).

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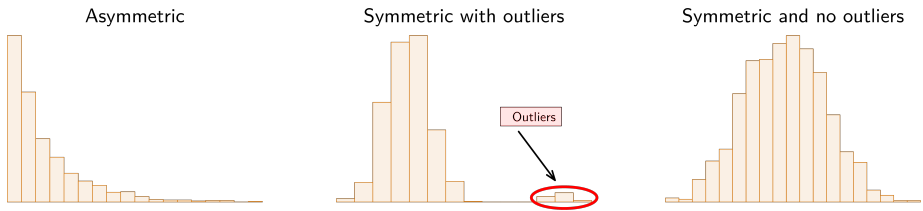
The best advice is to plot the scores and check for symmetry and/or outliers.



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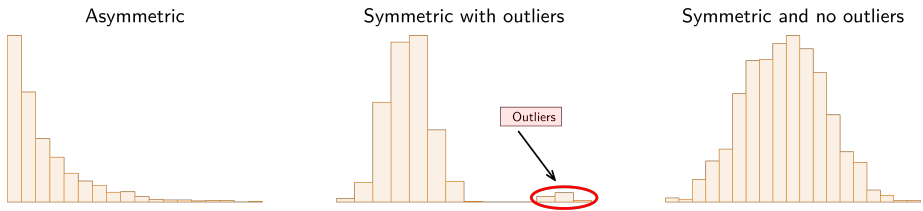

- Distribution asymmetric *or* there are outliers: Use the median (*but you can still also report the mean*).
- Distribution roughly symmetric and with no outliers: Use the mean (*but you can still also report the median*). In this case the mean and the median will be very similar.

Otherwise, the decision to choose between the mean on the median may depend on the question at hand.

Measure of location: Mean vs median — Neither

For some distributions **neither** the mean **nor** the median are suitable representatives of the location of a distribution…

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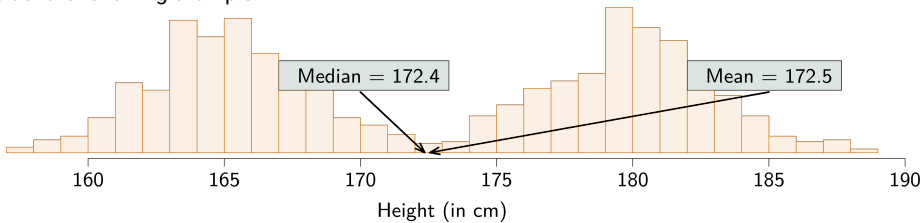

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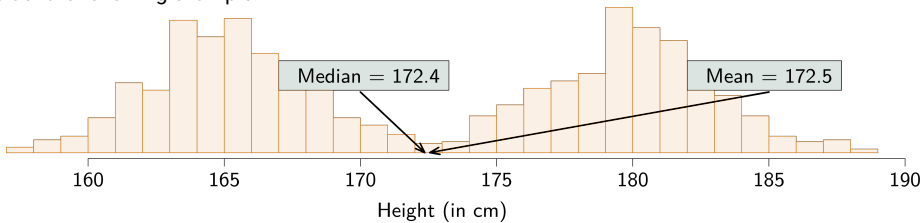
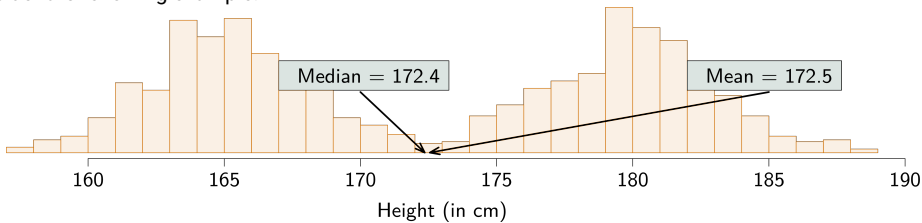


This may be an example of a mixture of two distributions with different characteristics.
Example: Height data with women (left side) and men (right side).

Measure of location: Mean vs median — Neither

For some distributions **neither** the mean **nor** the median are suitable representatives of the location of a distribution…

Consider the following example:



This may be an example of a mixture of two distributions with different characteristics.
Example: Height data with women (left side) and men (right side).

In such cases it would be better to study each distribution separately:

Neither the mean nor the median offer a good representation of the location of this distribution.

Measure of location: Mode

Mode:

The mode is the most frequently observed value.

Measure of location: Mode

Mode:

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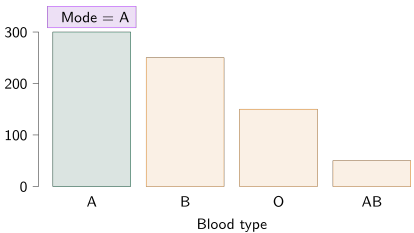
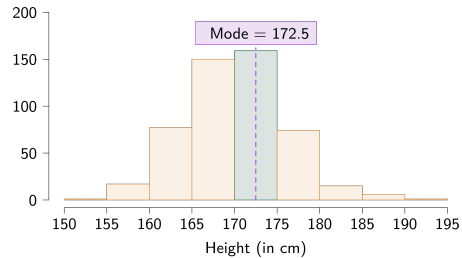
The mode can be found as follows:

For qualitative variables:
Find the category with the largest count.



For quantitative variables:
Look at the histogram and…
…find the midpoint of the highest bar in the histogram, or
…find the mean value of the data in the highest bar.



Measure of location: Mode

Note that:

- The mode is a representative value of the location of the distribution, where 'representative' = most frequent.
- The mode can be found for either qualitative or quantitative variables.

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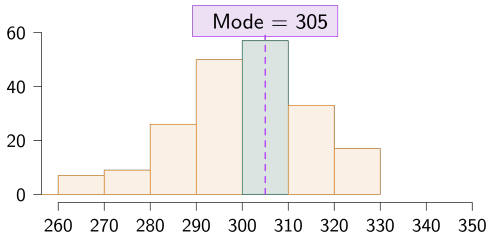
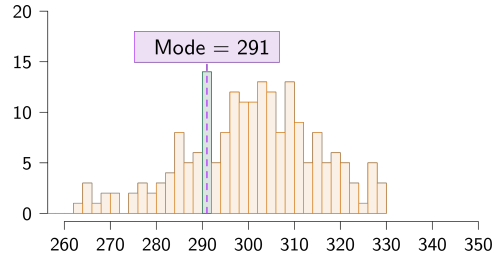
- The mode is a representative value of the location of the distribution, where 'representative' = most frequent.
- The mode can be found for either qualitative or quantitative variables.

However and for quantitative variables, how to set the interval width of the histogram can influence the mode:





Summary

To describe the characteristics of our data we can:

- Use numerical summaries (e.g., mean, median, mode).
- Create plots.

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To describe the characteristics of our data we can:

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This is not an either/or decision:

You can, and should, use both!