Lec2_report

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2024-12-5

Exercise 2-1

1.

Response variables are (a) Attitude toward gun control, (b) Heart disease, (c) Vote for President. The rest of the variables are explanatory variables.

2.

Scales of measurements: (a) nominal, (b) ordinal, (c) ordinal, (d) nominal, (e) nominal, (f) ordinal.

Exercise 2-2

(a) The student's number of correct answers on the exam follows a Binomial distribution with n = 100 and $\pi = 0.25$.

(b) Set Y as the student's number of correct answers on the exam. From (a), $Y \sim binomial(n = 100, \pi = 0.25)$.

That is, $E(Y) = n\pi = 25$, $\sigma(Y) = \sqrt{Var(Y)} = \sqrt{n\pi(1-\pi)} = 4.33$.

When the sample size is large enough, binomial distribution is approximately normal, i.e., $Y \simeq N(\mu = 25, \sigma = 4.33)$ (see slide 17).

Here, $P(y \geq 50) = 1 - P(y < 50) \simeq 1 - \Phi(\frac{y-25}{4.33} < \frac{50-25}{4.33}) = 1 - \Phi(5.77) \simeq 0$. Therefore, the result of getting at least 50 correct responses would be surprising since it is almost impossible.

Exercise 2-3

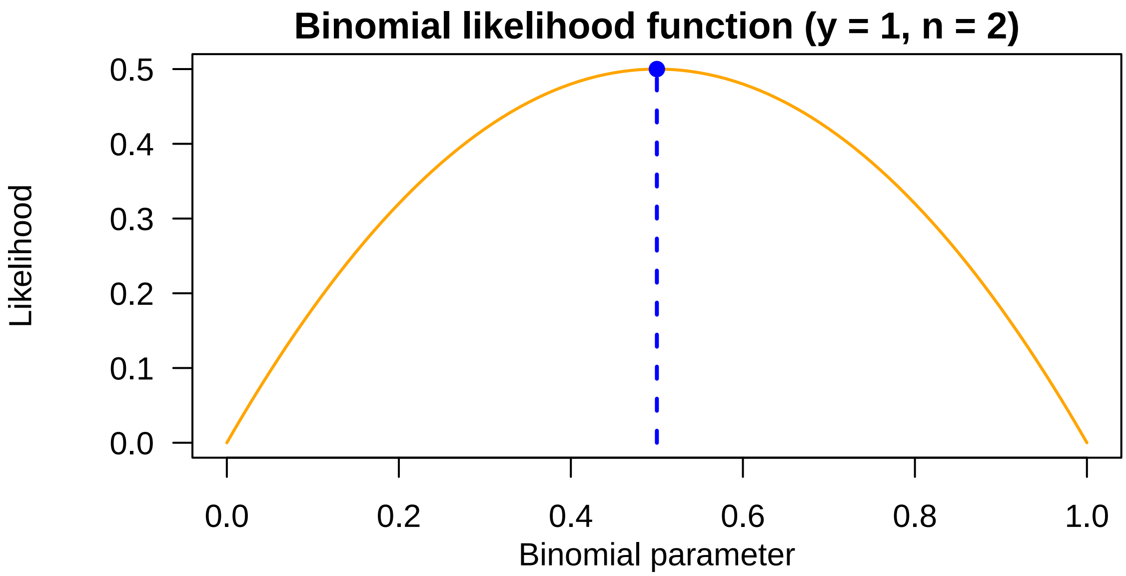
(a)$Y \sim binomial(n = 2, \pi = 0.50)$. Therefore, the probabilities for all possible Y can be summarized as below table:

Y	Prob.
0	0.25
1	0.50
2	0.25

Here, the mean is $n\pi = 2 \times 0.50 = 1$, and the standard deviation is $\sqrt{n \times \pi \times (1-\pi)} = \sqrt{2 \times 0.5 \times 0.5} = 0.71$.

(b) The likelihood function of Y is $l(\pi) = \binom{n}{y}\pi^y(1-\pi)^{n-y} = \binom{2}{1}\pi^1(1-\pi)^{2-1} = 2\pi(1-\pi)$.

When you plot this:



From the plot, the function $l(\pi) = 2\pi(1-\pi)$ takes its maximum value at $\pi = 0.50$.

Exercise 2-4

a. $\hat{\pi} = 486/1374 = 0.354$.
Wald 99%CI:$0.354 \pm z_{0.001/2} \times \sqrt{\hat{\pi}(1-\hat{\pi})/n} = 0.354 \pm 2.58\sqrt{(0.354 \times 0.646)/1374} = (0.321, 0.387)$.

b. H_0: $\pi = 0.5$, $H_1 : \pi \neq 0.5$.
$z_W = \frac{\hat{\pi}-0.5}{\sqrt{\hat{\pi}(1-\pi)/n}} = -0.146/0.012901 = -11.34$.
$P-$value $= 2 \times P(Z < -11.34) < 0.001$.
Therefore, at $\alpha = 5$, we reject H_0.

Exercise 2-5

a. $\sigma(\hat{\pi}) = \sigma(\frac{y}{n}) = \sigma(\frac{1}{n}y) = \frac{1}{n}\sigma(y) = \frac{1}{n}\sqrt{n\pi(1-\pi)} = \sqrt{\pi(1-\pi)/n}$.
(see slide page 40).

b. When $\pi \approx 0$ or $\pi \approx 1$, then
$\pi(1-\pi) \approx 0 \times 1 = 0$ OR $\pi(1-\pi) \approx 1 \times 0 = 0$.
Therefore, in both cases,
$\sigma(\hat{\pi}) = \sqrt{\frac{\pi(1-\pi)}{n}} \approx \sqrt{\frac{0}{n}} = 0$.
That is, $\hat{\pi}$ has little variability.