Assignment IV: Graphical Models

Select the equations or calculate the values that fit in the blanks in the question text. The selected answers ((A)–(D)) or the calculated values must be submitted via the link to the assignment that appears in the "General" channel of "機械学習 2024 KA240201-teams."

Questions 1 and 2

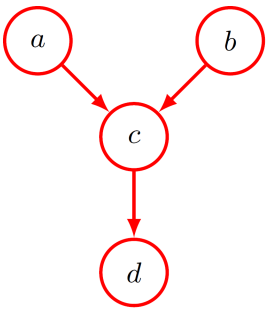

Figure 1

Consider the directed graph shown in Figure 1 in which none of the variables is observed. We first show that $a \perp\!\!\!\perp b \mid \emptyset$. Next, suppose we now observe the variable d. We then show that in general $a \not\perp\!\!\!\perp b \mid d$.

From Figure 1, we see that

$$p(a, b, c, d) = \boxed{(1)}.$$

We can investigate whether a and b are independent by marginalizing the above equation with respect to c and d to give

$$\begin{aligned} p(a, b) &= \sum_c \sum_d p(a, b, c, d) \\ &= \sum_c \sum_d \boxed{(1)} \\ &= p(a)p(b). \end{aligned}$$

Similarly,

$$\begin{aligned} p(a, b|d) &= \frac{\sum_c p(a, b, c, d)}{\sum_a \sum_b \sum_c p(a, b, c, d)} \\ &= \boxed{(2)} \\ &\neq p(a|d)p(b|d). \end{aligned}$$

in general. Note that this result could also be obtained directly from the graph in Figure 1 by using d-separation.

Question 1. Select the equation that fills in the blank (1).

(A) $p(a)p(b)p(c|a,b)p(d|c)$

(B) $p(a|c)p(b|c)p(c|d)p(d)$

(C) $p(a)p(b)p(c|a,b)p(d|a,b,c)$

(D) $p(a|c,d)p(b|c,d)p(c|d)p(d)$

Question 2. Select the equation that fills in the blank (2).

(A) $p(a)p(b)p(d|a,b)/p(d)$

(B) $p(a)p(b)p(d|c)/p(d)$

(C) $p(a|c)p(b|c)p(d|c)/p(d)$

(D) $p(a|c,d)p(b|c,d)p(d|c)/p(d)$

Questions 3 and 4

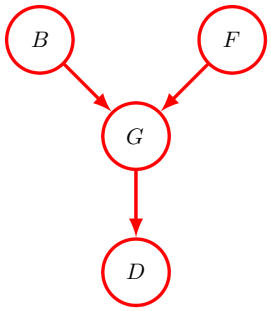

Figure 2

Consider a particular instance of a graph corresponding to a problem with four binary random variables relating to the fuel system on a car with a driver, as shown in Figure 2. The variables are B, which represents the state of a battery that is either charged ($B = 1$) or flat ($B = 0$), F which represents the state of the fuel tank that is either full of fuel ($F = 1$) or empty ($F = 0$), G which is the state of an electric fuel gauge and which indicates that the fuel tank is either full ($G = 1$) or empty ($G = 0$), and D, which is the state of driver's report about the reading on the gauge

and which indicates that the driver reports either full ($D = 1$) or empty ($D = 0$). The battery is either charged or flat, and independently, the fuel tank is either full or empty, with prior probabilities

$$p(B = 1) = 0.9$$
$$p(F = 1) = 0.9.$$

Given the state of the fuel tank and the battery, the fuel gauge reads full with probabilities given by

$$p(G = 1|B = 1, F = 1) = 0.8$$
$$p(G = 1|B = 1, F = 0) = 0.2$$
$$p(G = 1|B = 0, F = 1) = 0.2$$
$$p(G = 1|B = 0, F = 0) = 0.1$$

so this is a rather unreliable fuel gauge. The driver's report is also a bit unreliable, as expressed through the following probabilities

$$p(D = 1|G = 1) = 0.9$$
$$p(D = 0|G = 0) = 0.9.$$

All remaining probabilities are determined by the requirement that probabilities sum to one.

Suppose that the driver tells us that the fuel gauge shows empty, in other words that we observe $D = 0$. The probability that the tank is empty given only this observation, $p(F = 0|D = 0))$, is $\boxed{(3)}$. Similarly, the corresponding probability given also the observation that the battery is flat, $p(F = 0|D = 0, B = 0)$, is $\boxed{(4)}$.

Question 3. Calculate the value that fills in the blank (3). Round the solution to three decimal places.

Question 4. Calculate the value that fills in the blank (4). Round the solution to three decimal places.