Assignment I: Probability Distribution

Select the equations that fit in the blanks in the question text. The selected answers ((A)–(D)) must be submitted via the link to the assignment that appears in the "General" channel of "機械学習 2024 KA240201-teams."

Questions 1 and 2

We verify that the Bernoulli distribution $\mathrm{Bern}(x|\mu) = \mu^x(1-\mu)^{1-x}$ satisfies the following properties

$$\begin{aligned} \mathbb{E}[x] &= \mu \\ \mathrm{var}[x] &= \mu(1-\mu). \end{aligned}$$

From the definition of the Bernoulli distribution we have

$$\begin{aligned} \mathbb{E}[x] &= \sum_{x\in\{0,1\}} xp(x|\mu) = 0 \cdot \boxed{\ (1)\ } + 1 \cdot \boxed{\ (2)\ } = \mu \\ \mathrm{var}[x] &= \sum_{x\in\{0,1\}} (x-\mu)^2 p(x|\mu) = \mu^2 \boxed{\ (1)\ } + (1-\mu)^2 \boxed{\ (2)\ } \\ &= \mu^2(1-\mu) + (1-\mu)^2\mu = \mu(1-\mu). \end{aligned}$$

Question 1. Select the equation that fills in the blank (1).

(A) $p(x|\mu = 0)$

(B) $p(x = 0|\mu)$

(C) $p(x|\mu = 1)$

(D) $p(x = 1|\mu)$

Question 2. Select the equation that fills in the blank (2).

(A) $p(x|\mu = 0)$

(B) $p(x = 0|\mu)$

(C) $p(x|\mu = 1)$

(D) $p(x = 1|\mu)$

Questions 3 and 4

We prove that the following identity

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{M} & -\mathbf{M}\mathbf{B}\mathbf{D}^{-1} \\ -\mathbf{D}^{-1}\mathbf{C}\mathbf{M} & \mathbf{D}^{-1} + \mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{B}\mathbf{D}^{-1} \end{pmatrix} \tag{a}$$

by premultiplying both sides by the matrix

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \tag{b}$$

and making use of the definition $\mathbf{M} = (\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1}$.

Premultiplying the left hand side of (a) by the matrix (b) trivially gives the identity matrix. On the right hand side consider the four blocks of the resulting partitioned matrix:

upper left

$$\mathbf{A}\mathbf{M} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}\mathbf{M} = (\boxed{\ (3)\ })(\boxed{\ (3)\ })^{-1} = \mathbf{I}$$

upper right

$$\begin{aligned} & - \mathbf{A}\mathbf{M}\mathbf{B}\mathbf{D}^{-1} + \mathbf{B}\mathbf{D}^{-1} + \mathbf{B}\mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{B}\mathbf{D}^{-1} \\ &= -(\boxed{\ (3)\ })(\boxed{\ (3)\ })^{-1}\boxed{\ (4)\ } + \boxed{\ (4)\ } = \mathbf{0} \end{aligned}$$

lower left

$$\mathbf{C}\mathbf{M} - \mathbf{D}\mathbf{D}^{-1}\mathbf{C}\mathbf{M} = \mathbf{C}\mathbf{M} - \mathbf{C}\mathbf{M} = \mathbf{0}$$

lower right

$$-\mathbf{C}\mathbf{M}\mathbf{B}\mathbf{D}^{-1} + \mathbf{D}\mathbf{D}^{-1} + \mathbf{D}\mathbf{D}^{-1}\mathbf{C}\mathbf{M}\mathbf{B}\mathbf{D}^{-1} = \mathbf{D}\mathbf{D}^{-1} = \mathbf{I}$$

Thus the right hand side also equals the identity matrix.

Question 3. Select the equation that fills in the blank (3).

(A) $\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}$

(B) $\mathbf{B}\mathbf{D}^{-1}\mathbf{C}$

(C) $\mathbf{M} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}$

(D) $\mathbf{A}$

Question 4. Select the equation that fills in the blank (4).

(A) $\mathbf{M}\mathbf{B}$

(B) $\mathbf{AM}$

(C) $\mathbf{BD}^{-1}$

(D) $\mathbf{D}^{-1}\mathbf{C}$