Extended-Kalman-Filter-CHCV

March 12, 2018

```
In [ ]: import numpy as np
        %matplotlib inline
        import matplotlib.dates as mdates
        import matplotlib.pyplot as plt
        from scipy.stats import norm
        from sympy import Symbol, symbols, Matrix, sin, cos
        from sympy import init_printing
        init_printing(use_latex=True)
```

1 Extended Kalman Filter Implementation for Constant Heading and Velocity (CHCV) Vehicle Model in Python

Wikipedia writes: In the extended Kalman filter, the state transition and observation models need not be linear functions of the state but may instead be differentiable functions.

$\boldsymbol{x}_k = g(\boldsymbol{x}_{k-1}, \boldsymbol{u}_{k-1}) + \boldsymbol{w}_{k-1}$

$\boldsymbol{z}_k = h(\boldsymbol{x}_k) + \boldsymbol{v}_k$

Where w_k and v_k are the process and observation noises which are both assumed to be zero mean Multivariate Gaussian noises with covariance matrix Q and R respectively.

The function g can be used to compute the predicted state from the previous estimate and similarly the function h can be used to compute the predicted measurement from the predicted state. However, g and h cannot be applied to the covariance directly. Instead a matrix of partial derivatives (the Jacobian matrix) is computed.

At each time step, the Jacobian is evaluated with current predicted states. These matrices can be used in the Kalman filter equations. This process essentially linearizes the non-linear function around the current estimate.

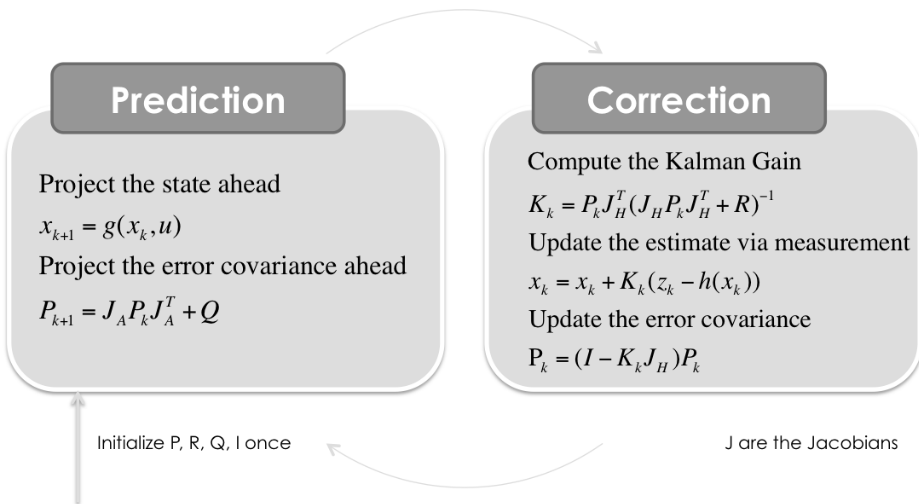
1.1 State Vector - Constant Heading and Velocity Model (CHCV)

Situation covered: You have a position sensor, which measures the vehicle position (x & y) and you try to estimate the heading and the velocity.

Constant Heading, Constant Velocity Model

$$x_k = \begin{bmatrix} x^* \\ y^* \\ \psi \\ v \end{bmatrix} = \begin{matrix} \text{Position X} \\ \text{Position Y} \\ \text{Heading} \\ \text{Velocity} \end{matrix}$$

*=actually measured values in this implementation example!



Extended Kalman Filter Step

```
In [ ]: numstates=4 # States
```

1.1.1 Developing the math behind dynamic model

All symbolic calculations are made with Sympy. Thanks!

```
In [ ]: vs, psis, dts, xs, ys, lats, lons = symbols('v \psi T x y lat lon')

        gs = Matrix([[xs+vs*dts*cos(psis)],
                     [ys+vs*dts*sin(psis)],
                     [psis],
                     [vs]])
        state = Matrix([xs,ys,psis,vs])
```

1.2 Dynamic Function g

This formulas calculate how the state is evolving from one to the next time step

```
In [ ]: gs
```

1.2.1 Calculate the Jacobian of the Dynamic function g with respect to the state vector x

```
In [ ]: state
```

```
In [ ]: gs.jacobian(state)
```

It has to be computed on every filter step because it consists of state variables!

1.3 Initial Uncertainty P_0

Initialized with 0 means you are pretty sure where the vehicle starts

```
In [ ]: P = np.eye(numstates)*1000.0
        print(P, P.shape)
```

1.4 Process Noise Covariance Matrix Q

"The state uncertainty model models the disturbances which excite the linear system. Conceptually, it estimates how bad things can get when the system is run open loop for a given period of time." - Kelly, A. (1994). A 3D state space formulation of a navigation Kalman filter for autonomous vehicles, (May). Retrieved from http://oai.dtic.mil/oai/oai?verb=getRecord&metadataPrefix=html&identifier=ADA282853

Because this depends on the dt, it has to be calculated in every filterstep.

1.5 Real Measurements

```
In [ ]: #path = './../RaspberryPi-CarPC/TinkerDataLogger/DataLogs/2014/'
        datafile = '2014-03-26-000-Data.csv'

        date, \
        time, \
        millis, \
        ax, \
        ay, \
        az, \
        rollrate, \
        pitchrate, \
        yawrate, \
        roll, \
        pitch, \
        yaw, \
        speed, \
        course, \
        latitude, \
        longitude, \
        altitude, \
        pdop, \
        hdop, \
        vdop, \
        epe, \
        fix, \
        satellites_view, \
        satellites_used, \
        temp = np.loadtxt(datafile, delimiter=',', unpack=True,skiprows=1)

        print('Read \'%s\' successfully.' % datafile)

        # A course of 0ř means the Car is traveling north bound
        # and 90ř means it is traveling east bound.
        # In the Calculation following, East is Zero and North is 90ř
        # We need an offset.
        course =(-course+90.0)
```

1.5.1 Calculate dt for Measurements

```
In [ ]: dt = np.hstack([0.02, np.diff(millis)])/1000.0 # in s
```

1.6 Measurement Function h

Matrix J_H is the Jacobian of the Measurement function h with respect to the state. Function h can be used to compute the predicted measurement from the predicted state.

If a GPS measurement is available, the following function maps the state to the measurement.

```
In [ ]: hs = Matrix([[xs],
                     [ys]])
        hs
```

```
In [ ]: JHs=hs.jacobian(state)
        JHs
```

If no GPS measurement is available, simply set the corresponding values in J_h to zero.

1.7 Measurement Noise Covariance R

"In practical use, the uncertainty estimates take on the significance of relative weights of state estimates and measurements. So it is not so much important that uncertainty is absolutely correct as it is that it be relatively consistent across all models" - Kelly, A. (1994). A 3D state space formulation of a navigation Kalman filter for autonomous vehicles, (May). Retrieved from http://oai.dtic.mil/oai/oai?verb=getRecord&metadataPrefix=html&identifier=ADA282853

```
In [ ]: varGPS = 6.0 # Standard Deviation of GPS Measurement
        R = np.diag([varGPS**2.0, varGPS**2.0])

        print(R, R.shape)
```

1.8 Identity Matrix

```
In [ ]: I = np.eye(numstates)
        print(I, I.shape)
```

1.9 Approx. Lat/Lon to Meters to check Location

```
In [ ]: RadiusEarth = 6378388.0 # m
        arc= 2.0*np.pi*(RadiusEarth+altitude)/360.0 # m/ř

        dx = arc * np.cos(latitude*np.pi/180.0) * np.hstack((0.0, np.diff(longitude))) # in m
        dy = arc * np.hstack((0.0, np.diff(latitude))) # in m

        mx = np.cumsum(dx)
        my = np.cumsum(dy)

        ds = np.sqrt(dx**2+dy**2)

        GPS=(ds!=0.0).astype('bool') # GPS Trigger for Kalman Filter
```

1.10 Initial State

```
In [ ]: x = np.matrix([[mx[0], my[0], 0.5*np.pi, 0.0]]).T
        print(x, x.shape)
```

1.10.1 Put everything together as a measurement vector

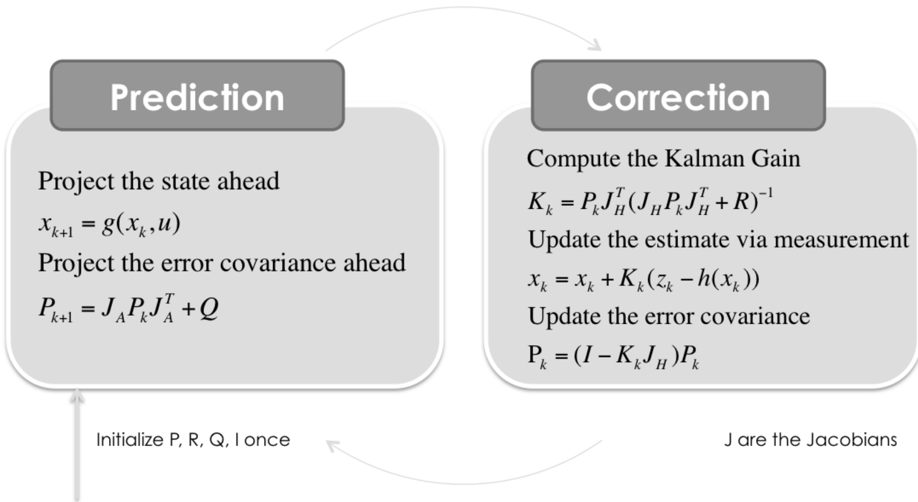
```
In [ ]: measurements = np.vstack((mx, my))
        # Lenth of the measurement
        m = measurements.shape[1]
        print(measurements.shape)
```

```
In [ ]: # Preallocation for Plotting
        x0, x1, x2, x3 = [], [], [], []
        Zx, Zy = [], []
        Px, Py, Pdx, Pdy = [], [], [], []
        Kx, Ky, Kdx, Kdy = [], [], [], []

        def savestates(x, Z, P, K):
            x0.append(float(x[0]))
            x1.append(float(x[1]))
            x2.append(float(x[2]))
            x3.append(float(x[3]))
            Zx.append(float(Z[0]))
            Zy.append(float(Z[1]))
            Px.append(float(P[0,0]))
            Py.append(float(P[1,1]))
            Pdx.append(float(P[2,2]))
            Pdy.append(float(P[3,3]))
            Kx.append(float(K[0,0]))
            Ky.append(float(K[1,0]))
            Kdx.append(float(K[2,0]))
            Kdy.append(float(K[3,0]))
```

2 Extended Kalman Filter

$$x_k = \begin{bmatrix} x \\ y \\ \psi \\ v \end{bmatrix} = \begin{bmatrix} \text{Position X} \\ \text{Position Y} \\ \text{Heading} \\ \text{Velocity} \end{bmatrix} = \underbrace{\begin{matrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{matrix}}_{\text{Python Nomenclature}}$$

```
In [ ]: for filterstep in range(m):

            # Time Update (Prediction)
            # ========================
            # Project the state ahead
            # see "Dynamic Matrix"
```



Extended Kalman Filter Step

```
x[0] = x[0] + dt[filterstep]*x[3]*np.cos(x[2])
x[1] = x[1] + dt[filterstep]*x[3]*np.sin(x[2])
x[2] = (x[2]+ np.pi) % (2.0*np.pi) - np.pi
x[3] = x[3]

# Calculate the Jacobian of the Dynamic Matrix A
# see "Calculate the Jacobian of the Dynamic Matrix with respect to the state vector
a13 = -dt[filterstep]*x[3]*np.sin(x[2])
a14 = dt[filterstep]*np.cos(x[2])
a23 = dt[filterstep]*x[3]*np.cos(x[2])
a24 = dt[filterstep]*np.sin(x[2])
JA = np.matrix([[1.0, 0.0, a13, a14],
                [0.0, 1.0, a23, a24],
                [0.0, 0.0, 1.0, 0.0],
                [0.0, 0.0, 0.0, 1.0]])


# Calculate the Process Noise Covariance Matrix
sGPS     = 0.5*8.8*dt[filterstep]**2  # assume 8.8m/s2 as maximum acceleration
sCourse  = 2.0*dt[filterstep] # assume 0.5rad/s as maximum turn rate
sVelocity= 35.0*dt[filterstep] # assume 8.8m/s2 as maximum acceleration

Q = np.diag([sGPS**2, sGPS**2, sCourse**2, sVelocity**2])

# Project the error covariance ahead
P = JA*P*JA.T + Q

# Measurement Update (Correction)
# ===============================
# Measurement Function
```

```
    hx = np.matrix([[float(x[0])],
                    [float(x[1])]])

    if GPS[filterstep]: # with 10Hz, every 5th step
        JH = np.matrix([[1.0, 0.0, 0.0, 0.0],
                        [0.0, 1.0, 0.0, 0.0]])
    else: # every other step
        JH = np.matrix([[0.0, 0.0, 0.0, 0.0],
                        [0.0, 0.0, 0.0, 0.0]])

    S = JH*P*JH.T + R
    K = (P*JH.T) * np.linalg.inv(S)

    # Update the estimate via
    Z = measurements[:,filterstep].reshape(JH.shape[0],1)
    y = Z - (hx)                         # Innovation or Residual
    x = x + (K*y)

    # Update the error covariance
    P = (I - (K*JH))*P

    # Save states for Plotting
    savestates(x, Z, P, K)
```

2.1 Lets take a look at the filter performance

```
In [ ]: def plotP():
            fig = plt.figure(figsize=(16,9))
            plt.semilogy(range(m),Px, label='$x$')
            plt.step(range(m),Py, label='$y$')
            plt.step(range(m),Pdx, label='$\psi$')
            plt.step(range(m),Pdy, label='$v$')

            plt.xlabel('Filter Step')
            plt.ylabel('')
            plt.title('Uncertainty (Elements from Matrix $P$)')
            plt.legend(loc='best',prop={'size':22})
```

2.1.1 Uncertainties in P

```
In [ ]: plotP()
```

```
In [ ]: fig = plt.figure(figsize=(6, 6))
        im = plt.imshow(P, interpolation="none", cmap=plt.get_cmap('binary'))
        plt.title('Covariance Matrix $P$ (after %i Filter Steps)' % (m))
        ylocs, ylabels = plt.yticks()
        # set the locations of the yticks
        plt.yticks(np.arange(6))
```

```
# set the locations and labels of the yticks
plt.yticks(np.arange(5),('$x$', '$y$', '$\psi$', '$v$', '$\dot \psi$'), fontsize=22)

xlocs, xlabels = plt.xticks()
# set the locations of the yticks
plt.xticks(np.arange(6))
# set the locations and labels of the yticks
plt.xticks(np.arange(5),('$x$', '$y$', '$\psi$', '$v$', '$\dot \psi$'), fontsize=22)

plt.xlim([-0.5,4.5])
plt.ylim([4.5, -0.5])

from mpl_toolkits.axes_grid1 import make_axes_locatable
divider = make_axes_locatable(plt.gca())
cax = divider.append_axes("right", "5%", pad="3%")
plt.colorbar(im, cax=cax)


plt.tight_layout()
```

2.2 State Vector

```
In [ ]: def plotx():
            fig = plt.figure(figsize=(16,16))

            plt.subplot(311)
            plt.step(range(len(measurements[0])),x0-mx[0], label='$x$')
            plt.step(range(len(measurements[0])),x1-my[0], label='$y$')

            plt.title('Extended Kalman Filter State Estimates (State Vector $x$)')
            plt.legend(loc='best',prop={'size':22})
            plt.ylabel('Position (relative to start) [m]')

            plt.subplot(312)
            plt.step(range(len(measurements[0])),x2, label='$\psi$')
            plt.step(range(len(measurements[0])),
                     (course/180.0*np.pi+np.pi)%(2.0*np.pi) - np.pi,
                     label='$\psi$ (from GPS as reference)')
            plt.ylabel('Course')
            plt.legend(loc='best',prop={'size':16})

            plt.subplot(313)
            plt.step(range(len(measurements[0])),x3, label='$v$')
            plt.step(range(len(measurements[0])),speed/3.6,
                     label='$v$ (from GPS as reference)')
            plt.ylabel('Velocity')
            #plt.ylim([0, 30])
            plt.legend(loc='best',prop={'size':16})
```

```
plt.xlabel('Filter Step')

plt.savefig('Extended-Kalman-Filter-CHCV-State-Estimates.png',
            dpi=72, transparent=True, bbox_inches='tight')
```

```
In [ ]: plotx()
```

2.3 Position x/y

```
In [ ]: #%pylab --no-import-all
```

```
In [ ]: def plotxy():

            fig = plt.figure(figsize=(16,9))

            # EKF State
            plt.quiver(x0,x1,np.cos(x2), np.sin(x2), color='#94C600',
                       units='xy', width=0.05, scale=0.5)
            plt.plot(x0,x1, label='EKF Position', c='k', lw=5)

            # Measurements
            plt.scatter(mx[::5],my[::5], s=50, label='GPS Measurements', marker='+')
            #cbar=plt.colorbar(ticks=np.arange(20))
            #cbar.ax.set_ylabel(u'EPE', rotation=270)
            #cbar.ax.set_xlabel(u'm')

            # Start/Goal
            plt.scatter(x0[0],x1[0], s=60, label='Start', c='g')
            plt.scatter(x0[-1],x1[-1], s=60, label='Goal', c='r')

            plt.xlabel('X [m]')
            plt.ylabel('Y [m]')
            plt.title('Position')
            plt.legend(loc='best')
            plt.axis('equal')
            #plt.tight_layout()

            plt.savefig('Extended-Kalman-Filter-CHCV-Position.png',
                        dpi=72, transparent=True, bbox_inches='tight')
```

```
In [ ]: plotxy()
```

2.3.1 Detailed View

```
In [ ]: def plotxydetails():
            fig = plt.figure(figsize=(12,9))

            plt.subplot(221)
            # EKF State
```

```
#plt.quiver(x0,x1,np.cos(x2), np.sin(x2), color='#94C600', units='xy', width=0.05, s
plt.plot(x0,x1, label='EKF Position', c='g', lw=5)

# Measurements
plt.scatter(mx[::5],my[::5], s=50, label='GPS Measurements', alpha=0.5, marker='+')
#cbar=plt.colorbar(ticks=np.arange(20))
#cbar.ax.set_ylabel(u'EPE', rotation=270)
#cbar.ax.set_xlabel(u'm')

plt.xlabel('X [m]')
plt.xlim(70, 130)
plt.ylabel('Y [m]')
plt.ylim(140, 200)
plt.title('Position')
plt.legend(loc='best')


plt.subplot(222)

# EKF State
#plt.quiver(x0,x1,np.cos(x2), np.sin(x2), color='#94C600', units='xy', width=0.05, s
plt.plot(x0,x1, label='EKF Position', c='g', lw=5)

# Measurements
plt.scatter(mx[::5],my[::5], s=50, label='GPS Measurements', alpha=0.5, marker='+')
#cbar=plt.colorbar(ticks=np.arange(20))
#cbar.ax.set_ylabel(u'EPE', rotation=270)
#cbar.ax.set_xlabel(u'm')

plt.xlabel('X [m]')
plt.xlim(160, 260)
plt.ylabel('Y [m]')
plt.ylim(110, 160)
plt.title('Position')
plt.legend(loc='best')
```

```
In [ ]: plotxydetails()
```

3 Conclusion

As you can see, complicated analytic calculation of the Jacobian Matrices, but it works pretty well.

3.1 Write Google Earth KML

3.1.1 Convert back from Meters to Lat/Lon (WGS84)

```
In [ ]: latekf = latitude[0] + np.divide(x1,arc)
        lonekf = longitude[0]+ np.divide(x0,np.multiply(arc,np.cos(latitude*np.pi/180.0)))
```

3.1.2 Create Data for KML Path

Coordinates and timestamps to be used to locate the car model in time and space The value can be expressed as yyyy-mm-ddThh:mm:sszzzzzz, where T is the separator between the date and the time, and the time zone is either Z (for UTC) or zzzzzz, which represents śhh:mm in relation to UTC.

```
In [ ]: import datetime
        car={}
        car['when']=[]
        car['coord']=[]
        car['gps']=[]
        for i in range(len(millis)):
            d=datetime.datetime.fromtimestamp(millis[i]/1000.0)
            car["when"].append(d.strftime("%Y-%m-%dT%H:%M:%SZ"))
            car["coord"].append((lonekf[i], latekf[i], 0))
            car["gps"].append((longitude[i], latitude[i], 0))
```

```
In [ ]: from simplekml import Kml, Model, AltitudeMode, Orientation, Scale, Style, Color
```

```
In [ ]: # The model path and scale variables
        car_dae = r'https://raw.githubusercontent.com/balzer82/Kalman/master/car-model.dae'
        car_scale = 1.0

        # Create the KML document
        kml = Kml(name=d.strftime("%Y-%m-%d %H:%M"), open=1)

        # Create the model
        model_car = Model(altitudemode=AltitudeMode.clamptoground,
                                    orientation=Orientation(heading=75.0),
                                    scale=Scale(x=car_scale, y=car_scale, z=car_scale))

        # Create the track
        trk = kml.newgxtrack(name="EKF", altitudemode=AltitudeMode.clamptoground,
                             description="State Estimation from Extended Kalman Filter with CHCV

        # Attach the model to the track
        trk.model = model_car
        trk.model.link.href = car_dae

        # Add all the information to the track
        trk.newwhen(car["when"])
        trk.newgxcoord(car["coord"])

        # Style of the Track
        trk.iconstyle.icon.href = ""
        trk.labelstyle.scale = 1
        trk.linestyle.width = 4
        trk.linestyle.color = '7fff0000'
```

```
# Add GPS measurement marker
fol = kml.newfolder(name="GPS Measurements")
sharedstyle = Style()
sharedstyle.iconstyle.icon.href = 'http://maps.google.com/mapfiles/kml/shapes/placemark_

for m in range(len(latitude)):
    if GPS[m]:
        pnt = fol.newpoint(coords = [(longitude[m],latitude[m])])
        pnt.style = sharedstyle

# Saving
#kml.save("Extended-Kalman-Filter-CTRV.kml")
kml.savekmz("Extended-Kalman-Filter-CHCV.kmz")
```

```
In [ ]: print('Exported KMZ File for Google Earth')
```

To use this notebook as a presentation type:

`jupyter-nbconvert --to slides Extended-Kalman-Filter-CHCV.ipynb --reveal-prefix=reveal.js --post serve`

Questions? [@Balzer82](https://twitter.com/balzer82)