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第四章作业思路分享

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目录

公式推导

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The cost:

$$J = \gamma^2 + \beta\gamma T + \frac{1}{3}\beta^2 T^2 + \frac{1}{3}\alpha\gamma T^2 + \frac{1}{4}\alpha\beta T^3 + \frac{1}{20}\alpha^2 T^4$$

α, β, γ Is solved as:

$$\begin{bmatrix} \frac{1}{120}T^5 & \frac{1}{24}T^4 & \frac{1}{6}T^3 \\ \frac{1}{24}T^4 & \frac{1}{6}T^3 & \frac{1}{2}T^2 \\ \frac{1}{6}T^3 & \frac{1}{2}T^2 & T \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} \Delta p \\ \Delta v \\ \Delta a \end{bmatrix} \Rightarrow \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \frac{1}{T^5} \begin{bmatrix} 720 & -360T & 60T^2 \\ -360T & 168T^2 & -24T^3 \\ 60T^2 & -24T^3 & 3T^4 \end{bmatrix} \begin{bmatrix} \Delta p \\ \Delta v \\ \Delta a \end{bmatrix}$$

$$\begin{bmatrix} \Delta p \\ \Delta v \\ \Delta a \end{bmatrix} = \begin{bmatrix} p_f - p_0 - v_0 T - \frac{1}{2}a_0 T^2 \\ v_f - v_0 - a_0 T \\ a_f - a_0 \end{bmatrix}$$

公式推导

For (partially)-free final state problem:

$$given \quad s_i(T), \ i \in I$$

We have boundary condition for other costate:

$$\lambda_j(T) = \frac{\partial h(s^*(T))}{\partial s_j}, \quad for \ j \neq i \qquad (1)$$

Then we solve this problem again.

2) Partially Defined End Translational State: Components of the final state may be left free by $\hat{\sigma}$. These states may correspondingly be specified as free when solving the optimal input trajectory, by noting that the corresponding costates must equal zero at the end time [25]. The closed-form solutions to the six different combinations of partially defined end states are given in Appendix A—in each case, solving the coefficients reduces to evaluating a matrix product.

（2） （3） （4） （5）

[1]A Computationally Efficient Motion Primitivefor Quadrocopter Trajectory Generation

公式推导

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```
syms T pf p0  v0  a0
delta_p = pf - p0 - v0*T - 1/2*a0*T^2;
C=[T^5/120, T^4/24, T^3/6;
   2, 2/T, 0;
   T, 2, 2/T];
inv(C);

v = [delta_p;0;0];
param = C\v;

alpha = param(1)
beta = param(2)
gamma = param(3)
J = gamma^2 + beta*gamma*T + 1/3*beta^2*T^2 + 1/3*alpha*gamma*T^2 + 1/4*alpha*beta*T^3 + 1/20*alpha^2*T^4

S=solve(diff(J,T),T)
```

S =

$$\begin{pmatrix} -\frac{v_0 - \sqrt{v_0^2 - 2\,a_0\,p_0 + 2\,a_0\,\mathrm{pf}}}{a_0} \\ -\frac{2\,v_0 + \sigma_1}{a_0} \\ -\frac{2\,v_0 - \sigma_1}{a_0} \\ -\frac{v_0 + \sqrt{v_0^2 - 2\,a_0\,p_0 + 2\,a_0\,\mathrm{pf}}}{a_0} \end{pmatrix}$$

where

$$\sigma_1 = \sqrt{2}\ \sqrt{2\,v_0^2 - 3\,a_0\,p_0 + 3\,a_0\,\mathrm{pf}}$$

ROS部分

（1）

```
pos = pos + vel * delta_time + acc_input * delta_time * delta_time / 2.0;
vel = vel + acc_input * delta_time; // // v = v0 + a*t
```

$$A=\begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\quad B=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

（2）

$$e^{At}=I+\frac{At}{1!}+\frac{(At)^2}{2!}+\frac{(At)^3}{3!}+\cdots+\frac{(At)^k}{k!}+\cdots$$

（3）

（4）

ROS部分

$$J = T + \left(\frac{1}{3}\alpha_1{}^2T^3 + \alpha_1\beta_1T^2 + \beta_1{}^2T\right) + \left(\frac{1}{3}\alpha_2{}^2T^3 + \alpha_2\beta_2T^2 + \beta_2{}^2T\right)$$
$$+ \left(\frac{1}{3}\alpha_3{}^2T^3 + \alpha_3\beta_3T^2 + \beta_3{}^2T\right)$$

$$\begin{pmatrix}\Delta p_x\\ \Delta p_y\\ \Delta p_z\\ \Delta v_x\\ \Delta v_y\\ \Delta v_z\end{pmatrix} = \begin{pmatrix} p_{xf} - v_{x0}T - p_{x0}\\ p_{yf} - v_{y0}T - p_{y0}\\ p_{zf} - v_{x0}T - p_{z0}\\ v_{xf} - v_{x0}\\ v_{yf} - v_{y0}\\ v_{zf} - v_{z0}\end{pmatrix}$$

$$\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ \beta_1\\ \beta_2\\ \beta_3\end{pmatrix} = \begin{pmatrix} -\frac{12}{T^3} & 0 & 0 & \frac{6}{T^2} & 0 & 0\\ 0 & -\frac{12}{T^3} & 0 & 0 & \frac{6}{T^2} & 0\\ 0 & 0 & -\frac{12}{T^3} & 0 & 0 & \frac{6}{T^2}\\ \frac{6}{T^2} & 0 & 0 & -\frac{2}{T} & 0 & 0\\ 0 & \frac{6}{T^2} & 0 & 0 & -\frac{2}{T} & 0\\ 0 & 0 & \frac{6}{T^2} & 0 & 0 & -\frac{2}{T}\end{pmatrix} \begin{pmatrix}\Delta p_x\\ \Delta p_y\\ \Delta p_z\\ \Delta v_x\\ \Delta v_y\\ \Delta v_z\end{pmatrix}$$

J =

$$T + T\,\sigma_6^2 + T\,\sigma_5^2 + T\,\sigma_4^2 + \frac{T^3\,\sigma_3^2}{3} + \frac{T^3\,\sigma_2^2}{3} + \frac{T^3\,\sigma_1^2}{3} - T^2\,\sigma_6\,\sigma_3 - T^2\,\sigma_5\,\sigma_2 - T^2\,\sigma_4\,\sigma_1$$

where

$$\sigma_1 = \frac{12\,(\mathrm{Pz}_0 - \mathrm{Pzf} + T\,\mathrm{Vz}_0)}{T^3} - \frac{6\,(\mathrm{Vz}_0 - \mathrm{Vzf})}{T^2}$$

$$\sigma_2 = \frac{12\,(\mathrm{Py}_0 - \mathrm{Pyf} + T\,\mathrm{Vy}_0)}{T^3} - \frac{6\,(\mathrm{Vy}_0 - \mathrm{Vyf})}{T^2}$$

$$\sigma_3 = \frac{12\,(\mathrm{Px}_0 - \mathrm{Pxf} + T\,\mathrm{Vx}_0)}{T^3} - \frac{6\,(\mathrm{Vx}_0 - \mathrm{Vxf})}{T^2}$$

$$\sigma_4 = \frac{6\,(\mathrm{Pz}_0 - \mathrm{Pzf} + T\,\mathrm{Vz}_0)}{T^2} - \frac{2\,(\mathrm{Vz}_0 - \mathrm{Vzf})}{T}$$

$$\sigma_5 = \frac{6\,(\mathrm{Py}_0 - \mathrm{Pyf} + T\,\mathrm{Vy}_0)}{T^2} - \frac{2\,(\mathrm{Vy}_0 - \mathrm{Vyf})}{T}$$

$$\sigma_6 = \frac{6\,(\mathrm{Px}_0 - \mathrm{Pxf} + T\,\mathrm{Vx}_0)}{T^2} - \frac{2\,(\mathrm{Vx}_0 - \mathrm{Vxf})}{T}$$

ROS部分

多项式的伴随矩阵

计算与多项式 $(x-1)(x-2)(x+3)=x^3-7x+6$ 对应的伴随矩阵。

```
u = [1 0 -7 6];
A = compan(u)
```

```
A = 3x3

     0     7    -6
     1     0     0
     0     1     0
```

A 的特征值是多项式的根。

```
eig(A)
```

```
ans = 3x1

   -3.0000
    2.0000
    1.0000
```

ROS部分

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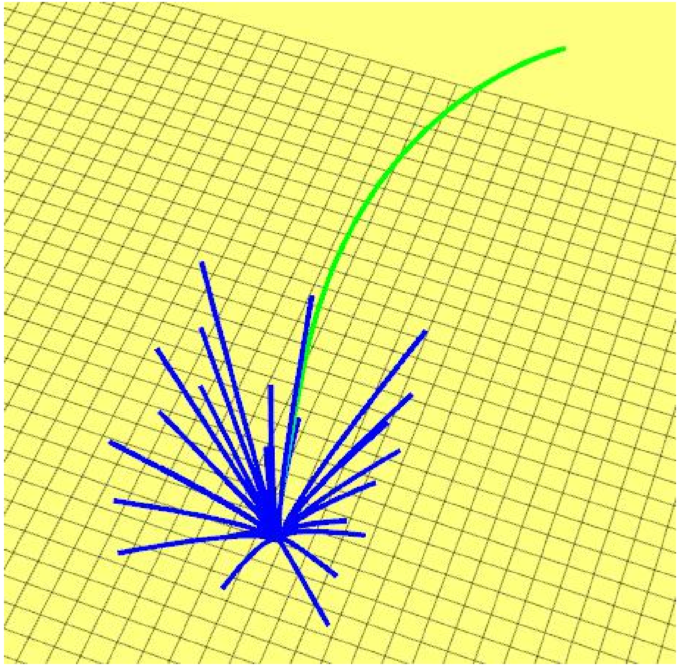
$$\begin{pmatrix} \frac{1}{6}T^3 & 0 & 0 & \frac{1}{2}T^2 & 0 & 0 \\ 0 & \frac{1}{6}T^3 & 0 & 0 & \frac{1}{2}T^2 & 0 \\ 0 & 0 & \frac{1}{6}T^3 & 0 & 0 & \frac{1}{2}T^2 \\ \frac{1}{2}T^2 & 0 & 0 & T & 0 & 0 \\ 0 & \frac{1}{2}T^2 & 0 & 0 & T & 0 \\ 0 & 0 & \frac{1}{2}T^2 & 0 & 0 & T \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} \Delta p_x \\ \Delta p_y \\ \Delta p_z \\ \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{pmatrix} \quad (1)$$

（3）

（2）

（4）

ROS部分



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感谢各位聆听!

Thanks for Listening