Information Technology Institute

Motion Simulator

Robotics

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Contents

1 Introduction

This project aims to design and manufacture a 3-DOF motion simulator platform. It is divided into three main parts : *modelling, mechanical design and control*.
Motion simulators are used to simulate virtual motion into real life, for example they are used in flight and car simulators to test aircrafts or cars or even to practice flying an airplane before being in an actual one. Another usage of motion simulators which is the topic of this project is game simulation, for example imagine that you play a car-racing game while sitting on this platform you will experience the feeling of being inside the game.

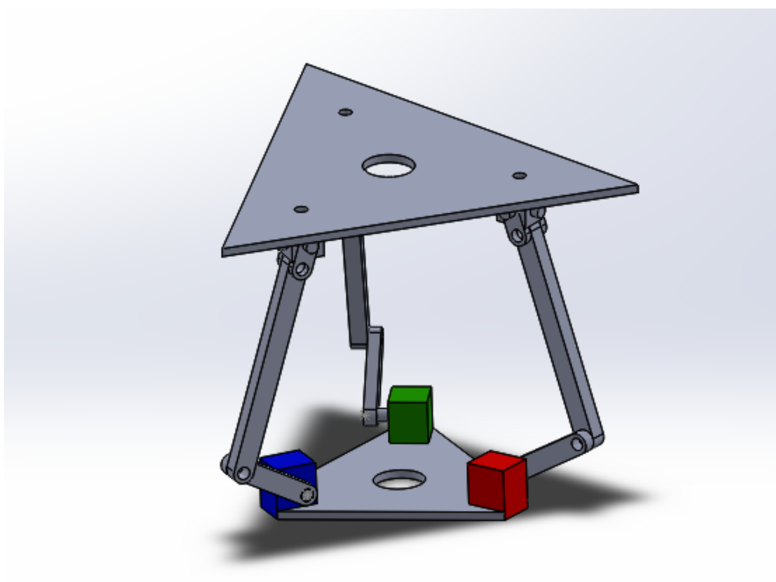
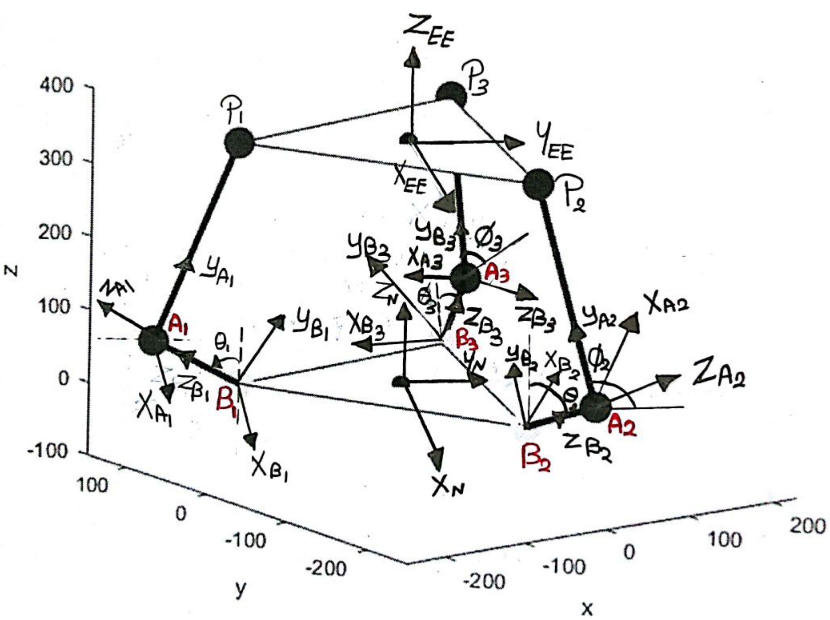


Figure 1: Solidworks model

2 Robot Kinematics

Our motion simulator can be described as a parallel manipulator with 3-DOF, two rotational motions (roll and pitch) and one translational motion in direction of Z-axis. The robot consists of 3 legs connected together through a platform, each leg consists of 2 links connected together through a revolute joint, each leg is connected to a motor through a revolute joint and to the platform through a U-joint.



The kinematics problem of parallel manipulators is very tedious so we decided to simplify our problem by treating this robot as 3 serial robotic arms connected together through the platform which will be the work space of our robot, these fixation points are fixed in position forming an equilateral triangle and finally we neglected the presence of the U-joint but added the orientation of platform to our calculations. The motor axes of rotation are fixed at 120° rotated from each other creating an equilateral triangle.

Setting 2 coordinate systems for each link and the base frame. In the median of the base triangle the Newtonian frame $X_N Y_N Z_N$ is selected and this is the fixed coordinate system. Z-axis in the fixed frame is vertically upwards, X-axes of the frames at motor points are co-linear with the axes of rotation of the motors and the x-axis of the Newtonian frame is parallel to the axis of the first coordinate system of the first motor(B_1). θ_i's are the 3 active angles of our system created by rotating about (X_i) and it is between the vertical axis of the base frame (Z_N) and (Z_{B_i}) (Z_{B_1} is coincident with the link B_iA_i) . ϕ_i's are the 3 passive angles created by rotating about (X_i) and they are measured between (Y_N) and (Y_{A_i}) (Y_{A_i} is coincident with the link A_iP_i).

3 Kinematics Of 3-DOF Parallel Robot

3.1 Forward Position Kinematics

3.1.1 Vector loop equations:

- ***Link(1)***:

$$|r^{OE} = \frac{-l}{\sqrt{3}}|Y_N + l_1|Z_{B1} + l_2|Y_{A1} \tag{1}$$

- ***Link(2)***:

$$|r^{OE} = \frac{l}{2}|X_N + \frac{l}{2\sqrt{3}}|Y_N + l_1|Z_{B2} + l_2|Y_{A2} \tag{2}$$

- ***Link(3)***:

$$r^{OE} = \frac{-l}{\sqrt{3}}|X_N + \frac{l}{2\sqrt{3}}|Y_N + l_1|Z_{B3} + l_2|Y_{A3} \tag{3}$$

3.1.2 Rotation Matrices

- ***Link(1)***:

$$^{N}R_{B1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{bmatrix}, \quad ^{N}R_{A1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_1 & -\sin\phi_1 \\ 0 & \sin\phi_1 & \cos\phi_1 \end{bmatrix}$$

- ***Link(2)***:

$$
{}^{N}R_{B2_1} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \cos\psi & -\sin\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad {}^{B2_1}R_{B2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2 & -\sin\theta_2 \\ 0 & \sin\theta_2 & \cos\theta_2 \end{bmatrix}
$$

$$
{}^{A2_1}R_{A2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_2 & -\sin\phi_2 \\ 0 & \sin\phi_2 & \cos\phi_2 \end{bmatrix}, \qquad {}^{N}R_{A2_1} = {}^{N}R_{B2_1}
$$

$$
{}^{N}R_{B2} = \begin{bmatrix} \cos\psi & -\cos\theta_2\sin\psi & \sin\theta_2\sin\psi \\ \sin\psi & \cos\theta_2\cos\psi & -\sin\theta_2\cos\psi \\ 1 & 0 & 0 \end{bmatrix}
$$

$$
{}^{N}R_{A2} = \begin{bmatrix} \cos\psi & -\cos\phi_2\sin\psi & \sin\phi_2\sin\psi \\ \sin\psi & \cos\phi_2\cos\psi & -\sin\phi_2\cos\psi \\ 1 & 0 & 0 \end{bmatrix}
$$

- ***Link(3)***:

$$
{}^{N}R_{B3_1} = \begin{bmatrix} \cos 2\psi & -\sin 2\psi & 0 \\ \cos 2\psi & -\sin 2\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad {}^{B3_1}R_{B3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_3 & -\sin\theta_3 \\ 0 & \sin\theta_3 & \cos\theta_3 \end{bmatrix}
$$

$$
{}^{A3_1}R_{A3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_3 & -\sin\phi_3 \\ 0 & \sin\phi_3 & \cos\phi_3 \end{bmatrix}, \qquad {}^{N}R_{A3_1} = {}^{N}R_{B3_1}
$$

$$
{}^{N}R_{B3} = \begin{bmatrix} \cos 2\psi & -\cos\theta_3\sin 2\psi & \sin\theta_3\sin 2\psi \\ \sin 2\psi & \cos\theta_3\cos 2\psi & -\sin\theta_3\cos 2\psi \\ 1 & 0 & 0 \end{bmatrix}
$$

$$
{}^{N}R_{A3} = \begin{bmatrix} \cos 2\psi & -\cos\phi_3\sin 2\psi & \sin\phi_3\sin 2\psi \\ \sin 2\psi & \cos\phi_3\cos 2\psi & -\sin\phi_3\cos 2\psi \\ 1 & 0 & 0 \end{bmatrix}
$$

- Link 1:

  Where :

$$Z_{B1} = -\sin\theta_1|Y_N + \cos\theta_1|Z_N \tag{4}$$
$$Y_{A1} = \cos\phi_1|Y_N + \sin\phi_1|Z_N \tag{5}$$

Final Calculations:

- $X_{P1} = 0$
- $Y_{P1} = \frac{-l}{\sqrt{3}} - l_1 \sin\theta_1 + l_2 \cos\phi_1$
- $Z_{P1} = l_1 \cos\theta_1 + l_2 \sin\phi_1$

- Link 2:

  Where :

$$Z_{B2} = \sin\psi \sin\theta_2|X_N - \cos\psi \sin\theta_2|Y_N + \cos\theta_2|Z_N \tag{6}$$
$$Y_{A2} = \sin\psi \sin\phi_2|X_N - \cos\epsilon \sin\phi_2|Y_N + \cos\phi_2|Z_N \tag{7}$$

Final Calculations:

- $X_{P2} = \frac{l}{2} + l_1 \sin\psi \sin\theta_2 - l_2 \cos\phi_2 \sin\psi$
- $Y_{P2} = \frac{l}{2\sqrt{3}} - l_1 \sin\theta_2 \cos\psi + l_2 \cos\phi_2 \cos\psi$
- $Z_{P2} = l_1 \cos\theta_2 + l_2 \sin\phi_2$

- Link 3:

  Where :

$$Z_{B3} = \sin(2\psi)\sin\theta_3|X_N - \cos(2\psi)\sin\theta_3|Y_N + \cos\theta_3|Z_N \tag{8}$$
$$Y_{A3} = -\sin(2\psi)\cos\phi_3|X_N - \cos(2\psi)\cos\phi_3|Y_N + \cos\phi_3|Z_N \tag{9}$$

Final Calculations:

- $X_{P3} = \frac{-l}{\sqrt{3}} + l_1 \sin(2\psi) \sin\theta_3 - l_2 \cos\phi_3 \sin(2\psi)$
- $Y_{P3} = \frac{l}{2\sqrt{3}} - l_1 \sin\theta_3 \cos(2\psi) + l_2 \cos\phi_3 \cos(2\psi)$
- $Z_{P2} = l_1 \cos\theta_3 + l_2 \sin\phi_3$

3.1.3 EE Position Equations

- The EE point is the median of the triangle connecting the 3-edges of the 3-Links.

$$X_{EE} = \frac{X_{p1} + X_{p2} + X_{p3}}{3} \tag{10}$$

$$Y_{EE} = \frac{Y_{p1} + Y_{p2} + Y_{p3}}{3} \tag{11}$$

$$Z_{EE} = \frac{Z_{p1} + Z_{p2} + Z_{p3}}{3} \tag{12}$$

3.1.4 Constrain Equations

The constrain Equation of this robot is that any distance between any two edges is constant and it is equal to the length of the triangle that connects the 3-edges.
So the constrain equations are :

$$C_1 = \sqrt{(X_{p1} - X_{p2})^2 + (Y_{p1} - Y_{p2})^2 + (Z_{p1} - Z_{p2})^2} \tag{13}$$

$$C_2 = \sqrt{(X_{p1} - X_{p3})^2 + (Y_{p1} - Y_{p3})^2 + (Z_{p1} - Z_{p3})^2} \tag{14}$$

$$C_3 = \sqrt{(X_{p3} - X_{p2})^2 + (Y_{p3} - Y_{p2})^2 + (Z_{p3} - Z_{p2})^2} \tag{15}$$

Where

$$\boxed{C_1 = C_2 = C_3 = d} \tag{16}$$

Where **d** is the length of the triangle.
By solving the 3-previous equations using **Newton Raphson** , the passive angles $\phi's$ can be obtained.
Applying $\theta's$ and $\phi's$ in the Vector Loop equations , the positions of the 3-edges and EE can be obtained.

3.2 Solid-Work Model Verification

Comparsion		Solid-Work Result			Matlab Results		
		ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_3
θ_1	30°	82°	76.63°	82.8°	80.63°	77.47°	83.746°
θ_2	40°						
θ_3	20°						
θ_1	0°	88.86°	88.86°	88.69°	90.85°	90.85°	90.85°
θ_2	0°						
θ_3	0°						
θ_1	90°	68.244°	68.244°	68.244°	69.98°	69.98°	69.98°
θ_2	90°						
θ_3	90°						
θ_1	70°	69.45°	84.32°	74.63°	69.76°	85.73°	75.78°
θ_2	10°						
θ_3	50°						

Table 1: Model Verification

3.3 Inverse Position Kinematics

3.3.1 Orientation Equations

- Due to the platform orientation around x-axis and y-axis (Pitch and Roll) , the position of 3-edges of the triangle can be obtained as a function in the EE position and the platform orientation (α and β).
- α is the angle due to the orientation around x-axis of the fixed frame.
- β is the angle due to the orientation around y-axis of the fixed frame.

Link(1):

- $X_{P1} = 0$
- $Y_{P1} = Y_{EE} - \frac{d\cos(\alpha)}{\sqrt{3}}$
- $Z_{P1} = Z_{EE} + \frac{d}{2}\frac{\sin(\beta)+\sin(\alpha)}{\sqrt{3}}$

Link(2):

- $X_{P2} = X_{EE} + \frac{d}{2}\cos(\beta)$
- $Y_{P2} = \frac{-Y_{p2}}{2} - \frac{3Y_{EE}}{2}$
- $Z_{P2} = Z_{EE} + \frac{d}{2}\frac{-\sin(\beta)+\sin(\alpha)}{\sqrt{3}}$

Link(3):

- $X_{P3} = X_{EE} - \frac{d}{2}\cos(\beta)$
- $Y_{P3} = Y_{p2}$
- $Z_{P3} = Z_{EE} + d\frac{\sin(\alpha)}{\sqrt{3}}$

3.3.2 Passive angle equations

$$\boxed{Z_{pi} = f(\theta, \phi)} \tag{17}$$

The previous equation shows that $\phi's$ function in $\theta's$ and Z_{pi}.
So the equations will be:

- Link(1):

$$\phi_1 = sin^{-1}(\frac{Z_{p1} - l_1 cos(\theta_1)}{l_2}) \tag{18}$$

- Link(2):

$$\phi_2 = sin^{-1}(\frac{Z_{p2} - l_1 cos(\theta_2)}{l_2}) \tag{19}$$

- Link(3):

$$\phi_3 = sin^{-1}(\frac{Z_{p3} - l_1 cos(\theta_3)}{l_2}) \tag{20}$$

3.3.3 Position Equations

$$\frac{-l}{\sqrt{3}} - l_1 \sin\theta_1 + l_2 \cos\phi_1 - Y_{P1} = 0 \tag{21}$$

$$\frac{l}{2\sqrt{3}} - l_1 \sin\theta_2 \cos\psi + l_2 \cos\phi_2 \cos\psi - Y_{P2} = 0 \tag{22}$$

$$\frac{l}{2\sqrt{3}} - l_1 \sin\theta_3 \cos(2\psi) + l_2 \cos\phi_3 \cos(2\psi) - Y_{P3} = 0 \tag{23}$$

by substituting equations 18,19,20 in equations 21,22,23 ,we can get the $\theta's$ and $\phi's$.

4 Forward Velocity Kinematics

In order to solve the velocity kinematics problem, the velocity of the end effector must be found using the angular speed of the motors as the input. After differentiating the constraint equations using the following equations :

$$\frac{\partial C_1}{\partial t} = \frac{\partial C_1}{\partial \theta_i}\frac{\partial \theta_i}{\partial t} + \frac{\partial C_1}{\partial \phi_i}\frac{\partial \phi_i}{\partial t} \tag{24}$$

$$\frac{\partial C_2}{\partial t} = \frac{\partial C_2}{\partial \theta_i}\frac{\partial \theta_i}{\partial t} + \frac{\partial C_2}{\partial \phi_i}\frac{\partial \phi_i}{\partial t} \tag{25}$$

$$\frac{\partial C_3}{\partial t} = \frac{\partial C_3}{\partial \theta_i}\frac{\partial \theta_i}{\partial t} + \frac{\partial C_3}{\partial \phi_i}\frac{\partial \phi_i}{\partial t} \tag{26}$$

Now these equations become 3 linear equations in $\dot{\phi}'s$ (passive angles' angular velocities) because in these equations the end effector position, motor angles and passive angles are givens to these equations , by solving them together we obtain $\dot{\phi}'s$. Now it's necessary to differentiate the end effector position equations for each link in order to obtain the velocity of the end effector of the whole platform.
As,

$$\dot{X_{EE}} = \frac{\dot{X_{p_1}} + \dot{X_{p_2}} + \dot{X_{p_3}}}{3} \tag{27}$$

$$\dot{Y_{EE}} = \frac{\dot{Y_{p_1}} + \dot{Y_{p_2}} + \dot{Y_{p_3}}}{3} \tag{28}$$

$$\dot{Z_{EE}} = \frac{\dot{Z_{p_1}} + \dot{Z_{p_2}} + \dot{Z_{p_3}}}{3} \tag{29}$$

5 Inverse Velocity Kinematics

The inverse velocity kinematic problem can be solved by getting the values of angular velocity ($\dot{\theta}_i$).
By differentiating the EE position equations and the constrain equations w.r.t θ and ϕ.

5.1 Constrain equations differentiation

$$\frac{\partial C_1}{\partial t} = \frac{\partial C_1}{\partial \theta_i}\frac{\partial \theta_i}{\partial t} + \frac{\partial C_1}{\partial \phi_i}\frac{\partial \phi_i}{\partial t} \tag{30}$$

$$\frac{\partial C_2}{\partial t} = \frac{\partial C_2}{\partial \theta_i}\frac{\partial \theta_i}{\partial t} + \frac{\partial C_2}{\partial \phi_i}\frac{\partial \phi_i}{\partial t} \tag{31}$$

$$\frac{\partial C_3}{\partial t} = \frac{\partial C_3}{\partial \theta_i}\frac{\partial \theta_i}{\partial t} + \frac{\partial C_3}{\partial \phi_i}\frac{\partial \phi_i}{\partial t} \tag{32}$$

5.2 EE position equations differentiation

$$\frac{\partial X_{EE}}{\partial t} = \frac{1}{3}(\frac{\partial (X_{p1} + X_{p2} + X_{p3})}{\partial \theta_i}\frac{\partial \theta_i}{\partial t} + (\frac{\partial (X_{p1} + X_{p2} + X_{p3})}{\partial \phi_i}\frac{\partial \phi_i}{\partial t}) \tag{33}$$

$$\frac{\partial Y_{EE}}{\partial t} = \frac{1}{3}(\frac{\partial (Y_{p1} + Y_{p2} + Y_{p3})}{\partial \theta_i}\frac{\partial \theta_i}{\partial t} + (\frac{\partial (Y_{p1} + Y_{p2} + Y_{p3})}{\partial \phi_i}\frac{\partial \phi_i}{\partial t}) \tag{34}$$

$$\frac{\partial Z_{EE}}{\partial t} = \frac{1}{3}(\frac{\partial (Z_{p1} + Z_{p2} + Z_{p3})}{\partial \theta_i}\frac{\partial \theta_i}{\partial t} + (\frac{\partial (Z_{p1} + Z_{p2} + Z_{p3})}{\partial \phi_i}\frac{\partial \phi_i}{\partial t}) \tag{35}$$

Where $\dot{X}_{EE}$, $\dot{Y}_{EE}$ and $\dot{Z}_{EE}$ are the input to the inverse velocity kinematic problem . So, by solving this 6-equations using fsolve function , $\dot{\theta}_i$ and $\dot{\phi}_i$ can be obtained.

6 Simulation

6.1 Trajectory Planning

In this section it is supposed to simulate the motion of a ball on the platform using the robot kinematics. In order to simulate the ball motion, the differential equations describing the ball motion is added to the rest of the system's equation.

$$a_1 \frac{d^2x}{dt^2} + \frac{dx}{dt}\left(a_2 + a_3\left|\frac{dx}{dt}\right|\right) + x\left(\frac{d\beta}{dt}\right)^2 = -g\sin\beta \tag{36}$$

$$a_1 \frac{d^2y}{dt^2} + \frac{dy}{dt}\left(a_2 + a_3\left|\frac{dy}{dt}\right|\right) + y\left(\frac{d\alpha}{dt}\right)^2 = -g\sin\alpha \tag{37}$$

As,

$$a_1 = 1 + \frac{J_b}{m_b(r_b)^2} \qquad a_2 = \frac{K_b}{m_b r_b} \qquad a_3 = \frac{K_{cx}}{m_b}$$

Where,

- m_b is the ball mass.
- r_b is the ball radius.
- J_b is the moment of inertia of the ball.
- K_b is the coefficient of friction of the ball.

By changing the orientation of the platform (by changing values of $\alpha \& \beta$) with respect to time in order to set a rectangular and a circular trajectories for the ball then solving the previous equations for these inputs to specify the position of the ball on the plate.

6.1.1 Circular Trajectory

System Input Signal:

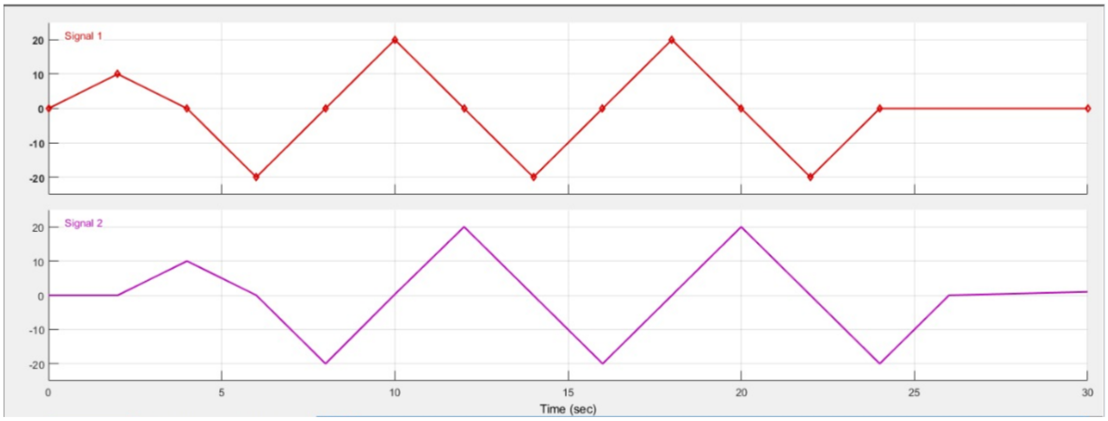


Figure 2: Orientation angles input

Path:

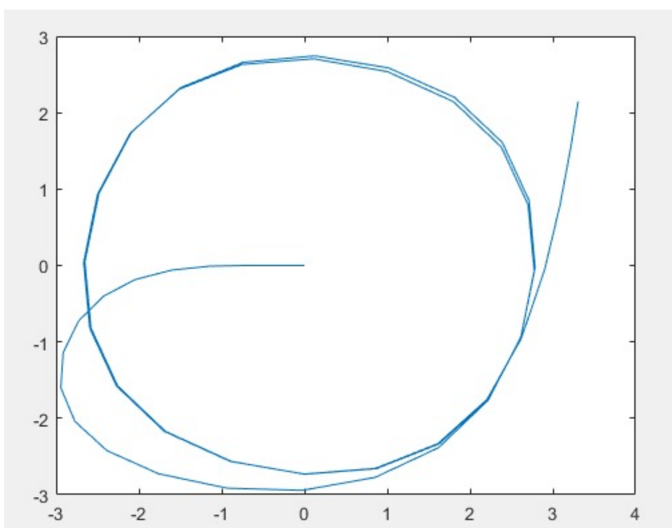


Figure 3: Circular trajectory

6.1.2 Rectangular Trajectory

System Input Signal:

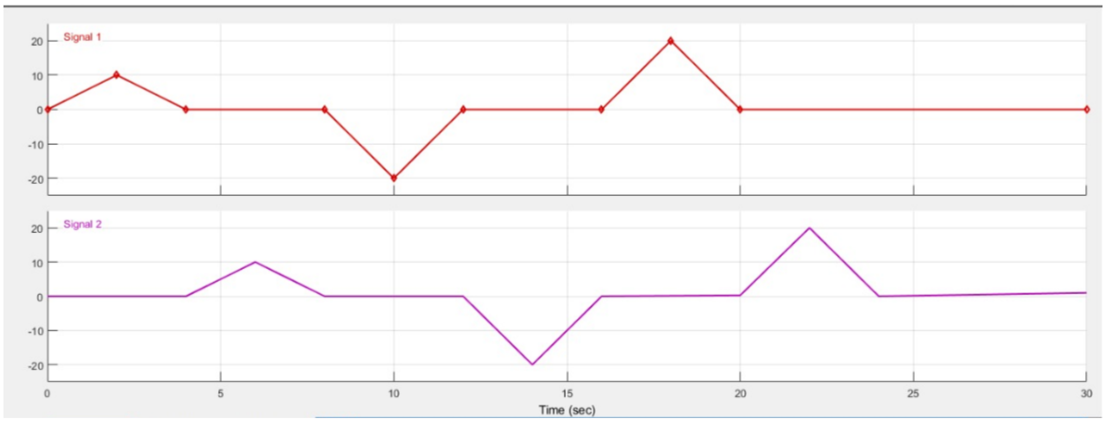


Figure 4: Orientation angles input

Path:

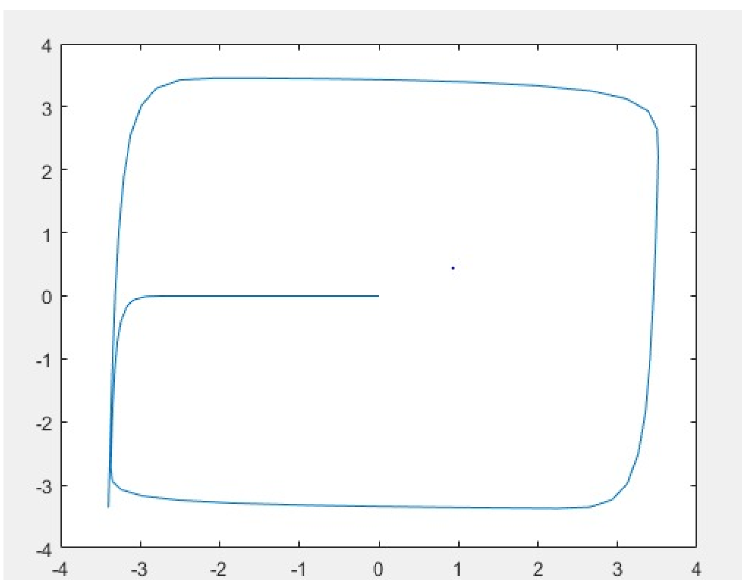


Figure 5: Rectangle trajectory

7 References

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