

# Stochastic Multi-Hazard Occurrence Model

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## Abstract

The title and so on are just space fillers. Do not reflect a strong preference on my part. Multiple question marks indicate problems or missing content (easy to search in PDF).

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# 1 Introduction – Conceptual

## 1.1 Timeless Networks – From Alex

“What is going to happen when disaster strikes?” is the first order question in risk science. While modelling of disaster scenarios is possible, it poses many challenges; natural hazards are inherently complex, meaning hazard magnitude (i.e. the scale of the event) and the impact (i.e. the damage) are unlikely to follow straightforward relationships (Liu et al., 2016). Firstly, hazard systems include many dependencies and other types of connections between their parts (Dunant, 2021; Liu et al., 2015; Mignan et al., 2014). For example, an earthquake can create thousands of landslides and landslide dams, which themselves, cause disastrous outburst floods, such as has been recently observed in New Zealand during the 2016 Kaikoura earthquake (Dellow et al., 2017a,b; Hughes et al., 2015; Jibson et al., 2018). Current “timeless” risk assessment frameworks lead to inaccurate inferences for dynamic processes, underestimating the potential for extreme events and belittle significant risk. Recently, a potential solution to the shortcoming of complexity in risk systems has been proposed (Dunant et al., 2021a,b), whereby hazards and their interactions can be abstractly represented by networks. However, this is yet to be incorporated into a long term temporal dynamic framework for risk assessment. We aim to incorporate time as a key dimension of the systemic risk assessment and provide novel, more realistic insight into natural disaster resilience.

## 1.2 Time Dependent Networks – Why Not

**From Alex:** Time-dependant networks are widely used in other disciplines (Holme & Saramäki, 2012) including social communication (Pereira et al., 2016), biology (Berdondini et al., 2009), infrastructure networks (Pan & Saramäki, 2011) and ecology (Blonder et al., 2012). Drawing from those disciplines, a holistic solution to the complex problem of “motionable” risk would be to use *dynamic temporal networks* to predict the systemic multi-hazard threat.

**But from Mark:** A temporal network would be unworkable, as they have to be assigned in advance, are limited to discrete time, don’t allow for variable delay, and look like they will actually work slower than discrete simulation.

**Note From David:** *In the Introduction, I am trying to keep ideas on a conceptual level, avoiding technicalities. However, in terms of development of ideas, I think the possibility of Time Dependent Networks should be mentioned at this point and, if possible, a good explanation be given why it will not work. Then we do not need to visit it again later in the document. It is easier to keep the framework in later technical sections simpler and more focussed if we can trim out unnecessary peripheral ideas before we get there.*

*Then in the Overview below, we need to give a conceptual description of how we will deal with time, technical specifics in §2. I understood that this would be done by the addition of a time delay for the “signal” to transmit between certain points or nodes, e.g. how long does it take the water to flow from one reference point in a river (node) to another reference point downstream (another node). This additional time delay might not necessarily be fixed, it might be random about a fixed mean.*

**From David:** I may be completely up the garden path here: Would something like this be feasible: If we were to simulate the model indefinitely, the process would contain multiple “episodes”. Loosely, an episode would be a completed cascade sequence, where it is assumed that, at the end of the episode, no further *triggering* can occur, and all time delayed events (e.g. river flow as above) are completed; the environment is at a stable equilibrium. A new episode

would be initiated by a new volcanic event, earthquake or storm as in Fig 3. This triggering would need to be distinguished, in particular, with damaged infrastructure (e.g. road bridge, factory building) where the damage was caused by a previous event (e.g. earthquake, storm). Essentially, such infrastructure would be more vulnerable to certain events indefinitely until repairs are made; thus their increased vulnerability could span multiple subsequent episodes. It may also be the case that the vulnerability of certain hillside slopes in river valleys may increase/decrease after a landslide (e.g. the slope steepness may change), and this would be a permanent change, or possibly evolving.

The changes in vulnerability mentioned above are permanent unless there is repair or the slope steepness is modified artificially. A situation that would not be permanent is where steep slopes are saturated from rainfall, but will naturally dry out over time, and hence their vulnerability to a landslide due to an earthquake, for example, diminishes. Could this be treated in the same way as heightened river flow time between two nodes (mentioned earlier)? Could we simulate a random slope drying time, being a function of the accumulated rainfall since the beginning of the episode; and assume that once it has reached a certain level of dryness, it is not vulnerable to a landslide. If another weather event arrives before the simulated drying time is finished, then the new weather event is part of the current episode.

??? Is the above what is meant by “the right hierarchical scale of the analysis” in the 2nd paragraph of §2.2 ???

Alex, is this feasible in your model/program framework?

### 1.3 Overview of Paper – From Alex

Using existing empirical relationships and process stochastic distributions, we will use a *temporal network model* (at odds with above) to propose a statistical framework for cascading hazard forecasts. Following a temporal simulation of potential cascading hazards interactions, disaster scenarios could be simulated over periods of decades while providing quantifiable uncertainty.

The temporality brings an added complexity to the multi-hazard simulation as nodes are interacting at different times. Using the algorithm, each process cascades and interacts with other nodes, following the initial concept by Dunant et al. (2021b), using a global time and a process time. Global time would be viewed as time moving forward in a global sense and encompasses all processes. A process time is the local time of each of the processes, which may be ahead of the global time by the means of local delays (Fig. 5). When the process time is in line with the global time, the process “occurs”. The proposed stochastic framework is intended to map out the long term consequences of an initial stimuli across the landscape and to study its potential for complex cascading hazard impacts over an extended period of time.

## 2 The Network Model – Technical

### 2.1 Network Definition

**From Mark:** A network is often mathematically called a graph (Bondy & Murty, 1976). As explained by Boccaro (2010, p.325–369), “A graph  $G$  is an ordered pair of disjoint sets  $(V, E)$ , where  $V$  is a set of elements called vertices, *nodes*, or points, and a subset  $E$  of ordered pairs of distinct elements of  $V$ , called directed edges, arcs, or links”. Thus, networks or graphs are objects composed of multiple components (*nodes*) and their interactions via links or ties (also known as *edges*). A network with directional edges is often called a *directed graph* or *digraph*.

These graphs allow one to deal with several issues of complex systems by understanding the causality between the network components (Butts, 2009; Phillips et al., 2015). From Dunant et al. (2021b), “Graphs have been used to study complex systems such as food webs, genome and protein networks, Internet web pages, and human languages (Dorogovtsev & Mendes, 2004)”.

## 2.2 Model Structural Requirements

**From Mark:** The elements comprising the system must be established with the objective of the exercise firmly in mind. The model needs to reflect the known behaviour of the system (Grimm et al., 2005) as well as the end-user aspirations.

Further, the model must be guided by thoroughly quantified output, computing limitations, and a comprehensible way to convey the results. Hence, some underlying processes will not be explicitly included in the model as they would add too much complexity (e.g., internal processes of gravity mass movement or fault network structure underlying the earthquake behaviour (Abe & Suzuki, 2006; Pastén et al., 2016), and not be useful for the final purpose of the method, which is to quantify the risk from multi-hazards in a way that remains comprehensible for the end-user. Our chosen approach to determining the right hierarchical scale of the analysis is to consider only the physical effects that are “observed or felt” (e.g., shaking, gravity mass movement, water depth).

It is stressed that a “conceptualization” is needed whereby the objects of the system are drawn as points (nodes). Our method assumes that an area of interest (AOI) can be considered as a disaster macrosystem composed of *hazard nodes* (or sources, e.g., earthquake, river, slope unit) and *exposed nodes* (e.g., houses, roads). Hence a *node* could represent a fault segment, a contributing source area of landslides, a storm, a river, a road segment, a house, or any other “entity” depending on the specific purpose of the exercise and the natural hazards characteristic of the AOI. The various nodes are considered separate entities with their specific spatial patterns and behaviours. They relate to each other by specific rules based on empirical data, mathematical models, and expert opinion.

Herein, the *nodes* are considered *connected* if the geographical effect of one node overlaps that of another node. For example, if an earthquake “effect” (i.e., shaking) can impact a house, the earthquake node and the house node are considered connected, hence an *edge* exists between the two (extract from Dunant et al. (2021b)).

The initial (simplified) design for the node necessary for the temporal multi-hazard risk assessment of the Rangitaiki-Tarawera catchment is presented in the following paragraphs, starting with the design of the nodes and followed by their interactions in the form of edges.

## 2.3 Nodes

1. The nodes are based on Davies et al. (2020). An example of potential hazards leading to flooding is in Fig 1.
2. Proposed node locations in a virtual catchment are shown in Fig 2.

The nodes taken into account are based on Davies et al. (2020), and summarized in Fig 1.

The initial node design proposed is summarized in Figure 2. The author used a virtual catchment to organise the node as, either, spatially explicit where the location of the node is spatially tied to the phenomenon it conceptualizes (e.g. a slope unit or catchment area), or nominal where the node is not tied spatially and could be placed at a random location. The nominal node is often associated with a regional effect that has a geographical footprint large

enough that no discrete location is required for multi-hazard modelling use (e.g. even if an earthquake has a specific source location, the effect is regional hence the node doesn't need specific coordinates). Effectively, the earthquake node will include location through its links to other nodes (distance, orientation, site-specific effects?). Volcano-tectonic earthquakes will be a separate node.

The current attribution of the node is in its early design and a few questions are still to be answered. Indeed, the attribute given to the node during the modelling (e.g. dimensions, empirical relationship, threshold values, time step etc.) must remain consistent with the geomorphological features that the node is attempting to represent. A few example questions are:

1. Is the rainfall pattern consistent throughout the area of study?
  - To be decided. Effectively, we will probably have a single 'storm' node that covers the time element, with different spatial effects. For the graph model, the latter can be represented as a two-parameter distribution. We may need a correlation structure, which could be introduced by generating a 'storm intensity' and then using some sort of random variation, generating catchment rainfalls sequentially, based on neighboring catchments.
2. Can the flow and sediment load be accurately predicted from one river node to the next if the node represents 1km of river segments?
  - This is a (graph) model calibration task. In the upper reaches, the catchment details might decide the river segments.
3. Are 1km river segments the right dimension to accurately represent the sediment runoff volumes from the slope unit(s)?
  - See above
4. What is the minimum size of the slope unit/catchment to accurately investigate runoff volumes (aka SU to river edge)?
  - Suggest looking at DEM as a first step

It is also important to acknowledge that the fewer nodes are required, the most efficient the multi-hazard modelling will be as less computation might be required. Accuracy may not suffer, as beyond a certain point, the precision will be spurious. Greater efficiency means better grasp of aleatory uncertainty.

## 2.4 Connections Between Nodes – Edges

1. We consider a more general system of nodes and connections as in Fig 3 to that for flooding in Fig 1.
2. We need to distinguish between spatial connections (edges), see Fig 4; and temporal connections, see Fig 5.

The connections between nodes are called *edges* and they can be roughly translated as the impact or effect of one node on another node. This includes

1. if there is any effect,
2. the time delay involved, and
3. the magnitude of any effect.

A theoretical framework for the connections relevant to the project is presented in Figure 3.

An initial attempt to apply the connectivity presented in Figs 1 & 3 has been applied to the nodes presented in Fig 2 as a simplified model catchment. Fig 4 is an example of how the nodes could be connected from a spatial perspective.

Initial thoughts on the requirements needed to develop the multi-hazard algorithm for the specific example of the case study are presented Table 3. The requirements could be in the form of GIS footprints/layers, empirical relationships, stochastic distributions and threshold values that would be used to assess conditional statement to control the flow of “quantities” (e.g. discharge, sediment volumes, landslide volumes etc.).

### 3 Examples

1. Result of a simulation using plausible parameter values for probability distributions described in the Appendix.
2. The above may show little triggering between hazards?
3. Can we give another example with a few fiddled event rates where we get triggering? We may be able to show that the fiddled rates are quite implausible, hence the likelihood of such a sequence happening is quite small. I suppose it depends on the relationship between the aleatory and epistemic uncertainty.
4. Are there other combinations of parameters where the methodology breaks, giving bizarre results?

# A Details of Probability Distributions

This appendix gives details of event occurrence time distributions, and other observed effects. These distributions are used to simulate event times and related effects (e.g. volcanic ash deposits, earthquake ground shaking, etc).

Note that parameter meanings in any given subsection below do not necessarily carry over to another subsection. The subject areas being considered are too wide to utilise a unique variable name for each specific aspect. This also enables notation within a given area, e.g. volcanos, to be more consistent with the literature on volcanos.

## A.1 Volcano Tephra

Mark: please check refs to eq nos below, broken in Word doc

We model the volcano explosion process with a stochastic renewal process (Bebbington & Lai, 1996) by assuming a Weibull distribution with distribution function

$$F(r) = 1 - \exp [-(\alpha r)^\beta] , \quad (1)$$

where  $\alpha$  and  $\beta$  need to be estimated. Then, if the last explosion occurred at time  $s$ , the time of the following explosion is at  $s + R$ , where

$$R = \frac{1}{\alpha}(\log_e X)^{1/\beta} , \quad (2)$$

and  $X$  is a uniform  $[0, 1]$  random variable.

The amount (thickness) of ash that arrives in a catchment at location  $x$  (using the centroid???, is location  $x$  a node or a point within an area represented by a node ...) can be modelled using Kawabata et al. (2013), where

$$\bar{T}(d, \theta) = \frac{\gamma}{d^{-\eta}} \exp \{-vSd[1 - \cos(\theta - \phi)]\} \quad (3)$$

is the mean tephra thickness at a location a distance  $d$  from the volcano at an angle of  $\theta$  off the dispersal axis  $\phi$ . The actual tephra deposit can be considered a lognormal random variable with mean given by Eq 3 and a coefficient of variation (standard deviation divided by mean) of approximately 0.5 (Kawabata et al., 2013). So the actual thickness can be simulated as

$$T = \exp(0.22Z + \bar{T}) ,$$

where  $Z$  is a normal (0,1) random variable. Note the following:

1. The parameter  $\gamma$  is the expected thickness at 1km along the dispersal axis, which can be obtained from the volume and the expected wind via numerical integration (or just summation) of (1.3)???
2. The combination (product??)  $vS$  is a combination of diffusivity  $S$  (grainsize) and wind speed  $v$ ??? (shouldn't wind speed be velocity??, direction must be important). Typical values are approximately  $S = 1.5$  for a  $v = 50\text{km/hr}$  ??? wind (Kawabata et al., 2013).

Mark: is my interpretation correct for  $v$  &  $S$ ??, or is  $S$  a subscript??

3. The decay parameter  $\eta$  can be estimated by

$$\eta = 2.5 - 0.05H$$

(González-Mellado & De la Cruz-Reyna, 2010), where  $H$  is the column height in km.



Season		$\mu_u$	$\mu_v$	$\sigma_u$	$\sigma_v$	$\rho$
Spring	Sep–Nov	21.1	0.69	10.8	10.0	-0.11
Summer	Dec–Feb	20.8	0.55	13.6	14.1	-0.28
Autumn	Mar–May	23.7	0.88	13.7	12.5	-0.22
Winter	Jun–Aug	27.6	0.26	12.1	10.3	-0.17
Overall Average		23.3	0.60	12.9	11.9	-0.21

Table 1: Seasonal volcano parameter values used in simulations. Given the low variability, we could just use the overall average.

So we need wind speed & direction (i.e. velocity), column height and eruptive volume. The column height (km) is consistent with a Weibull distribution (Eq 1) with  $\alpha = 0.93$  and  $\beta = 0.19$ , so  $H$  can be generated from Eq 2. The bulk volume (cubic km) of ash (tephra) can be generated from the relationship (Mastin et al., 2009)

$$V = 2.6 \exp(0.065H - 1.69). \quad (4)$$

If we want to add an error term, multiply  $V$  in Eq 4 by a lognormal random variable, simulated as  $M = \exp(2Z)$ , where  $Z \sim N(0, 1)$ .

Wind speed and direction: get bivariate distribution for  $(u, v) = (\text{toeast}, \text{tonorth})$  components of wind velocity at circa 12km altitude. The model is bivariate normal, i.e.

$$(u, v)' = N(\mu, \Sigma),$$

where  $\mu = (\mu_u, \mu_v)'$  is the mean, and

$$\Sigma = \begin{pmatrix} \sigma_{uu} & \sigma_{uv} \\ \sigma_{uv} & \sigma_{vv} \end{pmatrix}$$

is the covariance matrix. We can reparametrize the latter as

$$\Sigma = \begin{pmatrix} \sigma_u^2 & \rho\sigma_u\sigma_v \\ \rho\sigma_u\sigma_v & \sigma_v^2 \end{pmatrix},$$

where  $u \sim N(\mu_u, \sigma_u^2)$ ,  $v \sim N(\mu_v, \sigma_v^2)$  and  $\rho$  is the correlation between  $u$  and  $v$ . It is then easy to simulate a draw  $(U, V)$  from the bivariate distribution via the equations

$$U = \mu_u + Z_1\sigma_u$$

and

$$V = \mu_v + \rho\frac{\sigma_v}{\sigma_u}(U - \mu_u) + Z_2\sqrt{(1 - \rho^2)\sigma_v^2},$$

where  $Z_i \sim N(0, 1)$  are standard normal random variables. Seasonal variability is modelled through parameters values (in m/s) as in Table 1.

Then

$$\phi = \arctan(-U/V),$$

(clockwise from East), and

$$vS = \frac{5.1}{50}\sqrt{U^2 + V^2}.$$

looks like  $vS$  should be  $v_s$ ??? If so,  $v_s$  might look better

## A.2 Debris Avalanches

**Stuart???** Assume that dome growth may or may not occur between significant explosions. Time between explosions already modelled. If dome growth, distribution for rate of growth. Random dome collapse as function of volume, or deterministically at next explosion. Given volume of debris, where does it run to (probability distribution – does it hit nearby stream or infrastructure nodes.)

Season		$p$	$\mu$	$\sigma$
Spring	(Sep–Nov)	0.43	1.62	1.01
Summer	(Dec–Feb)	0.40	1.56	1.06
Autumn	(Mar–May)	0.35	1.73	1.09
Winter	(Jun–Aug)	0.39	1.80	1.07

Table 2: Parameters for the precipitation component:  $p$  is the probability of rain on any given day, and  $\mu$  &  $\sigma$  are the parameter values of the utilised lognormal distribution.

### A.3 Precipitation/Storms

Precipitation feeds streams, sediment mobilization, landslides and surface flooding

Storms (i.e. wind etc.) are direct hazards. Can be ignored.

Probability distribution for 24 hour(???) precipitation at location. Then do random distribution for reference cell, probably seasonal.

Each day, the probability of rain is  $p$  (probability  $1 - p$  of no rain). This varies by season and is tabulated in Table 2. On days when there is rainfall, the amount (in mm), is modelled by a lognormal distribution with parameters  $\mu$  &  $\sigma$ , also in Table 2.

The amount of rainfall per day is then simulated as

$$R = I(U < p) \exp(\mu + \sigma Z)$$

where  $U$  is a uniform(0,1) random variable, and  $Z$  is a normal(0,1) random variable.

Comment: There is little to no spatial variation in rainfall within a 24 hour period, so we will ignore it and use a spatially constant rainfall rate.

## A.4 Sediment Mobilization

State of catchment node = thickness of tephra.

Assuming a thickness of sediment  $T$ , and an amount of rainfall  $R$ , we need a function for how much sediment is mobilized into the stream segment (along with the rainfall) and how much remains. There are three equations needed: one for how much rainfall runs off (the rest is absorbed), one for the saturation level of the ground, and one for how much sediment (read tephra) is carried with the runoff.

### Tarawera Rainfall Runoff

Runoff into rivers is calculated using a non-linear reservoir hydrological model (NLRRM). Each subcatchment (attached shapefiles for Rangitaiki and Tarawera) are represented as a reservoir with non-linear discharge. The discharge ( $q$ ) is a function of the catchment storage depth ( $S$ ) and coefficients  $K$  and  $p$ , where

$$q = \left( \frac{S}{K} \right)^{1/p}.$$

The storage depth is calculated using the F1-RSA method. It consists of a primary runoff rate (F1) and a saturated runoff rate (FSA). Essentially, some rainfall infiltrates into the ground until saturated, which reduces the amount of runoff. Once the enough rainfall has accumulated to saturate the soil, the runoff rate increases. The effective rainfall,  $r$ , is given as a proportion to rainfall intensity ( $I$ , mm/h):

$$r = \frac{I}{I + R_{sa}},$$

where  $S_{sat}$  is the total storage of water and  $R_{sa}$  is the storage amount for saturation.

The total storage,  $S$ , is then the sum of total storage and effective rain, minus the runoff

$$S = S_{sat} + r - q;$$

are mm the units for  $S_{sat}$ ???

In the algorithm,  $r$  is calculated first, then  $q$  (using  $S$  values from previous timestep), then  $S$ . The timestep is the duration between rainfall data points, see Fig 6.

The total discharge from each catchment is then the discharge ( $q$ ) multiplied by catchment area, divided by the time increment plus any baseflow.

In the case study model, this discharge is then provided to the morphodynamic model as an inflow to the river. We take this approach to model the effect of sedimentation.

However, in NLRRM, the water is routed downstream using a kinematic routing method to provide/forecast streamflow heights and flowrates. This is a very simple method, but provides good quantitative results to downstream flow gauge records. The NLRRM approximation uses a simplified parabolic shape for the channel cross section.

So we have an effective rainfall equation

$$R_{eff} = \begin{cases} c_1 R & S_{sat} \leq R_{sa} , \\ c_1 R + (c_2 - c_1)(S_{sat} - R_{sa}) & S_{sat} > R_{sa} , \end{cases}$$

where  $R = R(t)$  is the rainfall in time step  $t$ ,  $R_{sa}$  is a constant (the level of saturation at which rainfall runs off at a higher rate), and  $S_{sat} = S_{sat}(t)$  is the current saturation level of the ground.

In each catchment, water runs off at a (lower) rate  $c_1$  until the ground is saturated, then runs off at a higher rate  $c_2$ . So the model needs to keep track of the saturation level in each catchment,

$$S_{sat}(t + 1) = S_{sat} + R_{eff} - q$$

where the discharge to the stream  $Q$  is a non-linear function of the effective rainfall and coefficients  $K$ ,  $p$ :

$$Q = \left( \frac{R_{\text{eff}}}{K} \right)^{1/p}$$

For our purposes (see attached data),  $p = 1$ , and so  $K$  can be absorbed in  $c_1$  and  $c_2$ .

### Modified Universal Soil Loss Equation

The Modified Universal Soil Loss Equation (MUSLE) is used to estimate sediment yield per catchment. The equation is:

$$Y = 11.8(Q_T Q_{\text{max}})^{0.56} \times K \times \text{LS} \times C \times P,$$

where  $Y$  is the soil loss within a period (often expressed in tons),  $Q_T$  is the total discharge ( $\text{m}^3$ ) throughout the period,  $Q_{\text{max}}$  is the maximum discharge ( $\text{m}^3/\text{s}$  rate??) during the studied period,  $K$  is the soil erodability  $\text{LS}$  is a slope length and steepness factor (dimensionless),  $C$  is a land usage factor (dimensionless),  $P$  is an erosion control factor (dimensionless, often assumed as 1).

The parameter  $K$  is a soil erodibility factor, representing soil loss in mass/area. Technically, it represents soil loss (tons/ha) for a specific slope length and gradient, and specific land cover, which results in weird units. The dimensionless  $\text{LS}$ ,  $C$  and  $P$  parameters are then modifications on this coefficient depending on the different catchment characteristics.

In reality, each catchments sediment yield from a rain event will be represented by:

$$Y = 11.8(Q_T Q_{\text{max}})^{0.56} \alpha$$

where  $\alpha = K \times \text{LS} \times C \times P$  is a catchment specific coefficient for sediment yield.

Our  $K$  values will be found from the literature, and will then use  $\text{LS}$  parameter that is proportional to catchment area and slope. One example of  $\text{LS}$  is ([Chalov et al., 2017](#)):

$$\text{LS} = (m + 1) \left( \frac{A_c}{a_0} \right)^{0.4} \left( \frac{\sin \beta}{b_0} \right)^{1.6}$$

where  $A_c$  is catchment area/catchment width (units of m),  $\beta$  is the slope of the catchment,  $a_0$  and  $b_0$  are the normalised length and slopes used on the plots where  $K$  was measured. The land management factor  $C$  is proportional to vegetation. More vegetation results in a lower factor.

So basically,  $? = ?_{\text{max}} Q$  (??? missing variable names). We then need the area and width of each catchment, and its slope. Then the tephra loss  $Y$  into the river (provided there is still some present) is proportional to

$$c Q^{??} \left( \frac{??}{??} \right)^{??} (??)^{??} = c(i) Q(i)^{1.12},$$

for catchment  $i$ , where  $A$  is the catchment area,  $w$  its width,  $b$  its slope, and  $c$  is a constant depending on vegetation. Allowing for tephra presence,

$$Y_{\text{tostream}} = \min(Y, T),$$

where  $T$  is the current subaerial tephra in the catchment.

## A.5 Streamflow Segments/Catchments

How many segments? (1 per catchment). **Stuart**

Catchments have centroid (for tephra/rain location), area, and anything else which affects sediment mobilization (average slope, vegetation type?)

Stream segments. Discrete or continuous states? Need to track bed height (cross section), sediment load, water cross section. Is flow rate a function of water height? Cross section (i.e. stopbank height) is a static parameter

Need relationship for how fast a 'wave' propagates downstream.

## A.6 Flood Nodes

One per stream segment. Infrastructure is either in or not. Or perhaps a probability?

## A.7 Infrastructure List

From ME (type, locations)

## A.8 Dams

Matahina – special case

Landslide / DA / lava dams – potential failure at overtop (calculated from dam height, flow rate). Otherwise becomes a weir.

Dam failure – generate lahar with dam volume + water. How to model? **Stuart**

## A.9 Landslides

Track only specific landslide nodes (can either hit infrastructure or dam a stream).

Static properties – slope, material (volume distribution / runout)

Keep track of water content (precipitation decay). Landslide becomes possible with excessive saturation OR moderate saturation PLUS external trigger (e.g. earthquake)

## A.10 Earthquakes

**David**

## **B Miscellaneous Thoughts – Probably Irrelevant**

### **B.1 Physical and Statistical Models**

1. Physical natural hazard models tend to focus on a specific hazard: earthquakes, floods, landslides, volcanic eruption, storms, etc.
2. Physical models tend to focus on the physics of the process with little or no attention to stochastic uncertainty.
3. Many of these physical processes have also been described by statistical models, with considerably less computational effort.
4. The statistical models are generally based on observed empirical relationships in historical data. Ultimately, one wants the observed empirical relationships to have a physical explanation, and the physical models to produce simulated data that are consistent with observed historical data. Hence both approaches support the development of the other.
5. The above leads into the notions of epistemic and aleatory uncertainty.

### **B.2 Quantifying Uncertainty in Forecasts**

1. The purpose of the statistical model is to retain credibility with the known physics of the process, but to also provide a measure of the associated uncertainty. Physical models without an uncertainty component will only produce average (expected) behaviour.
2. We present a stochastic framework for the modelling of multiple hazards, in particular, where infrastructure may be already damaged due to a previous event.
3. Our model treats the recurrence of each individual natural hazard, and the effect of cascading multiple hazards with probability distributions.
4. Forecast probability distributions (empirical) can be determined by simulating the model multiple times.

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## Unprocessed References

- 1.

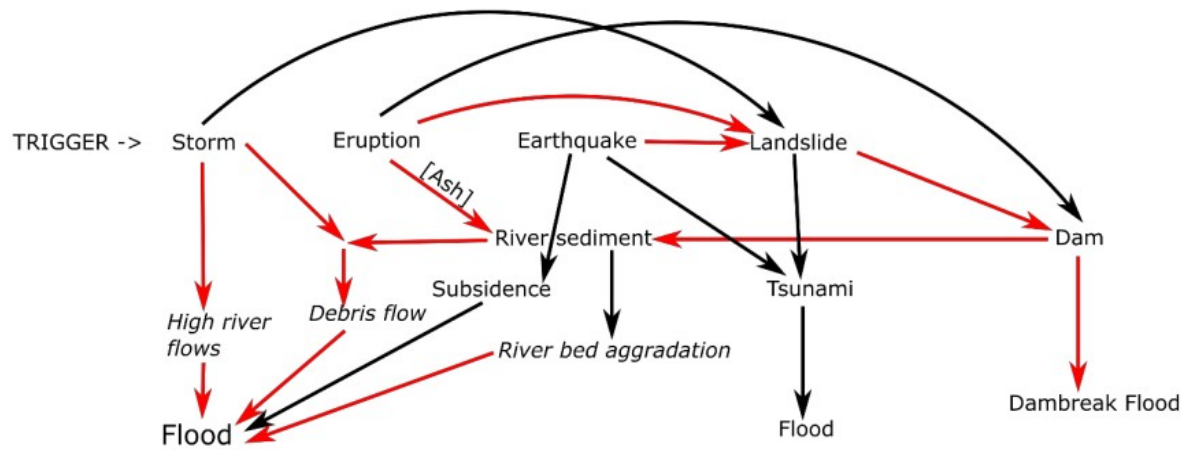


Figure 1: Potential hazard cascades leading to flooding: red arrows indicate preferred cascades scenario for the Tarawera/Rangitaiki case study (Davies et al., 2020).

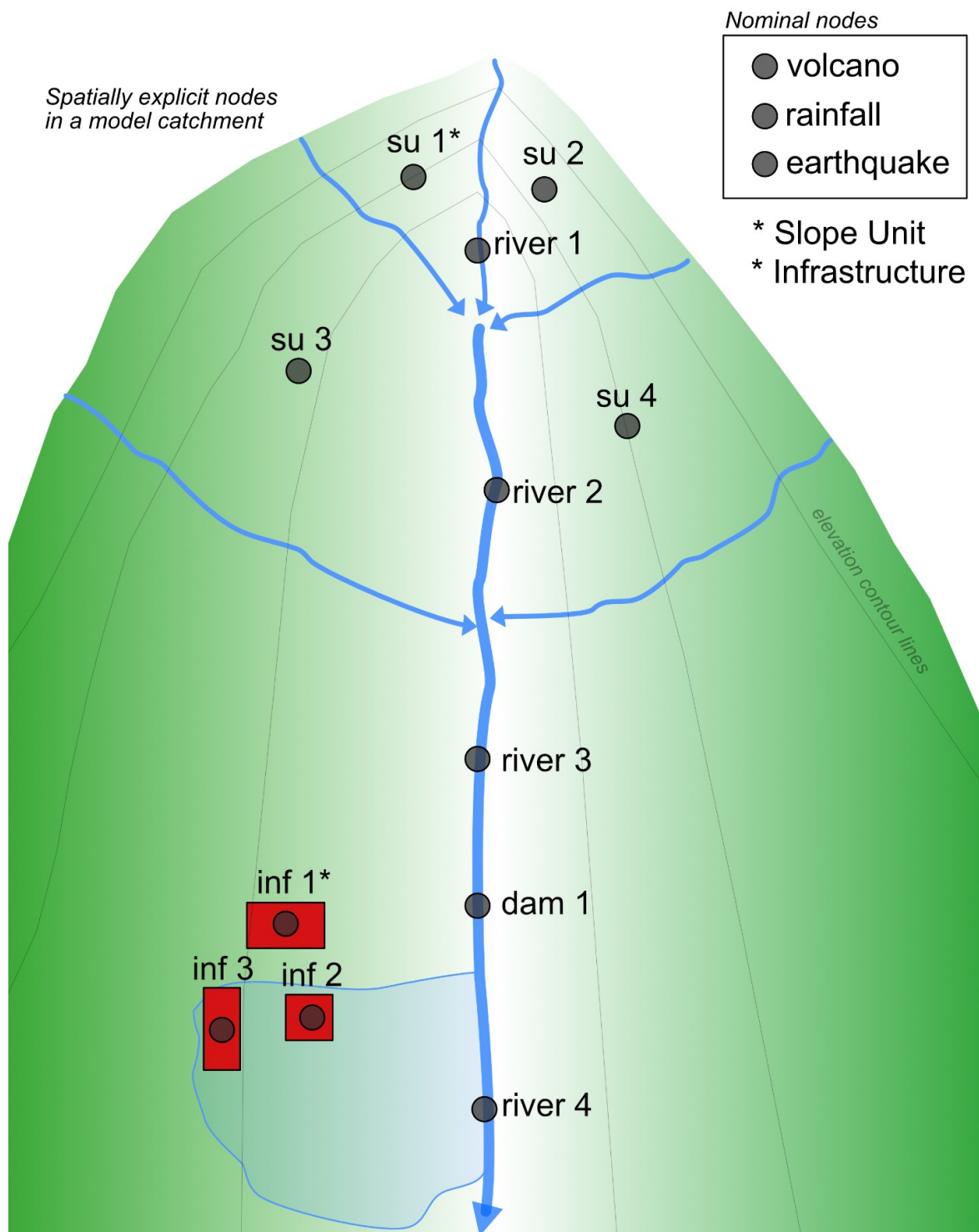


Figure 2: Node proposed locations in a virtual catchment. The blue lines represent river streams of different orders. The blue transparent outline represents potential flooding.

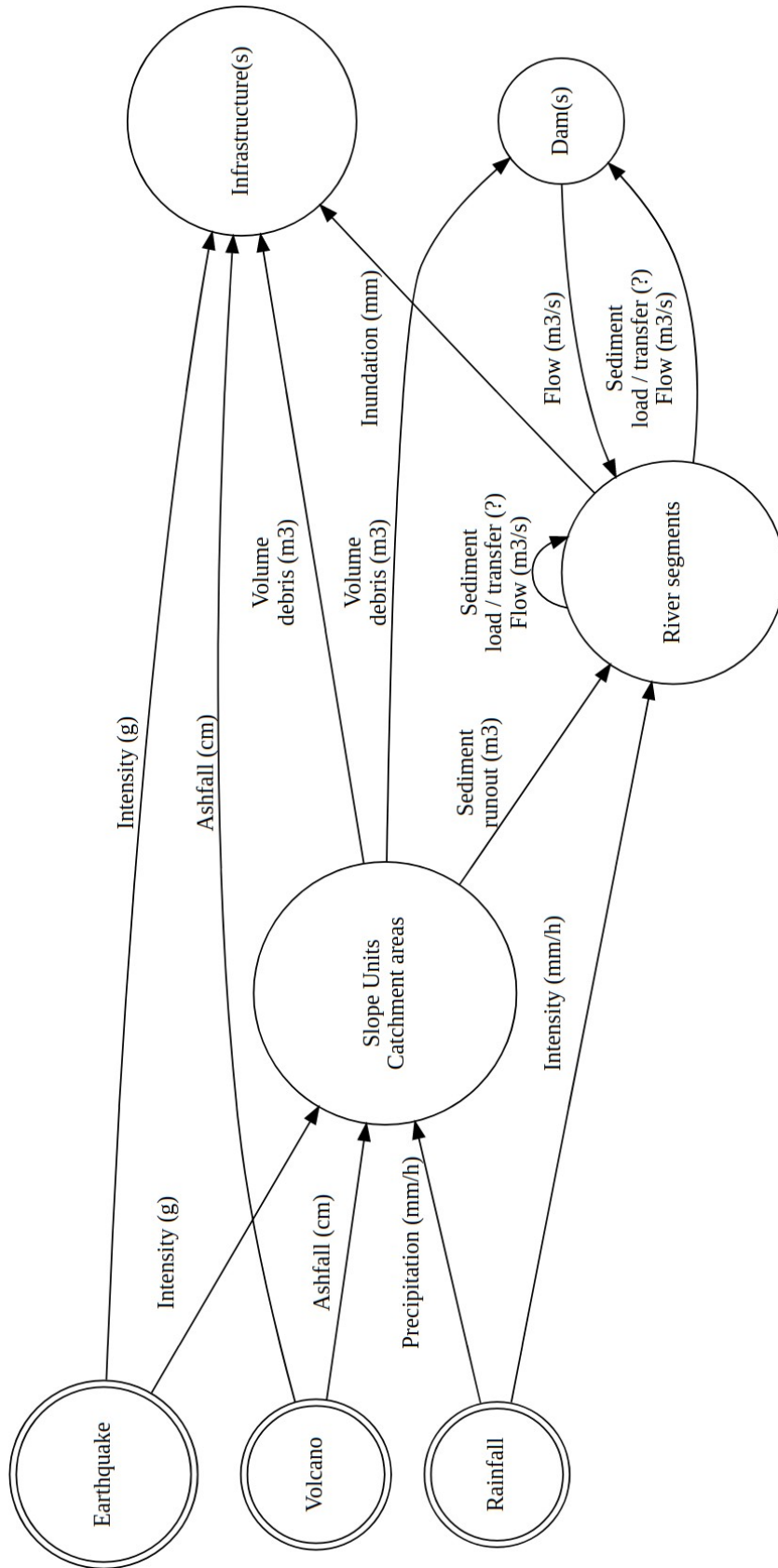


Figure 3: Connections between the various components types of the multi-hazard system (the double circles represent the nominal nodes). In this diagram, dams refer to natural dams and man-made dams. Dam failure is also a possibility, particularly for natural dams.

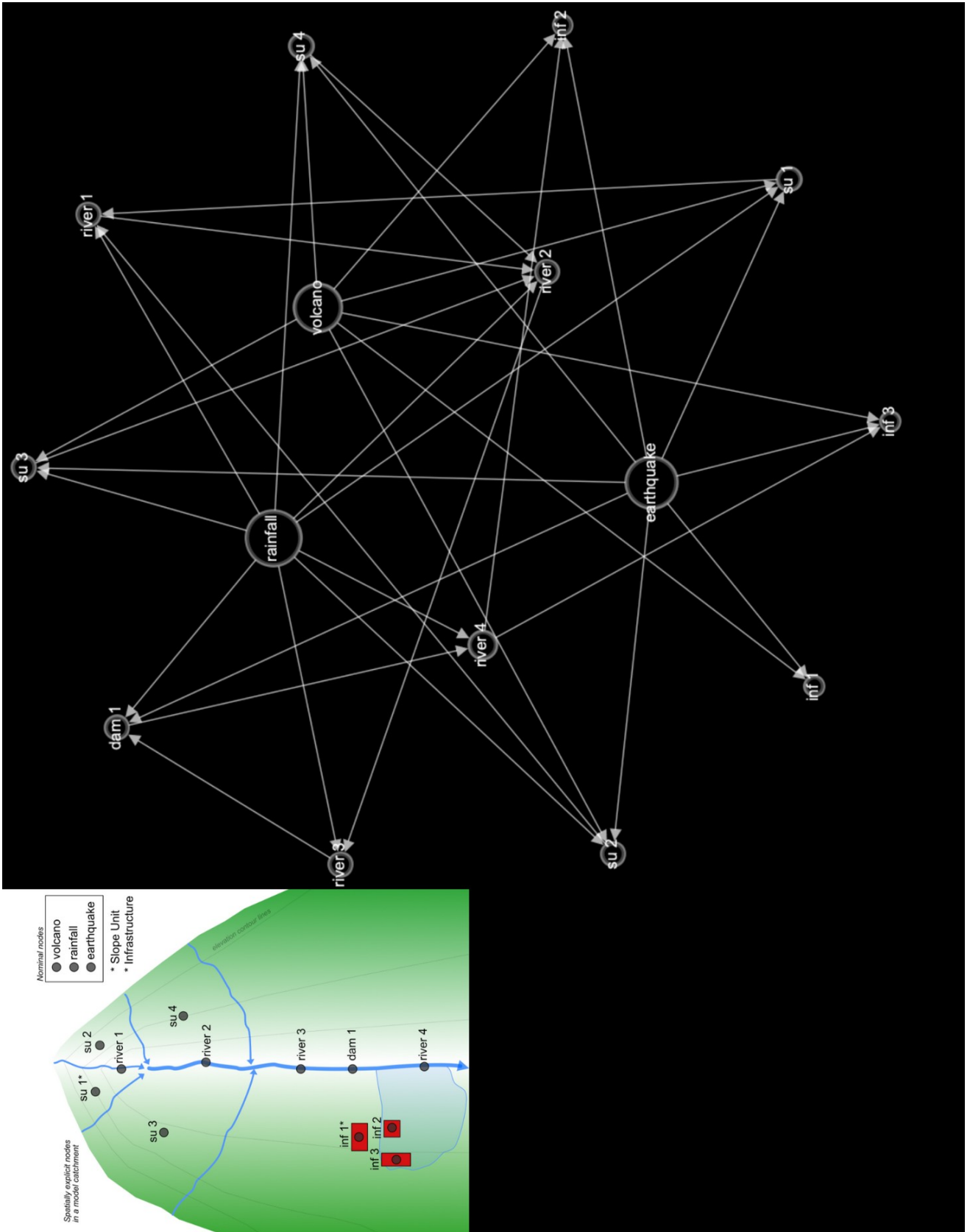


Figure 4: Spatial connections between the various nodes presented Fig 2.

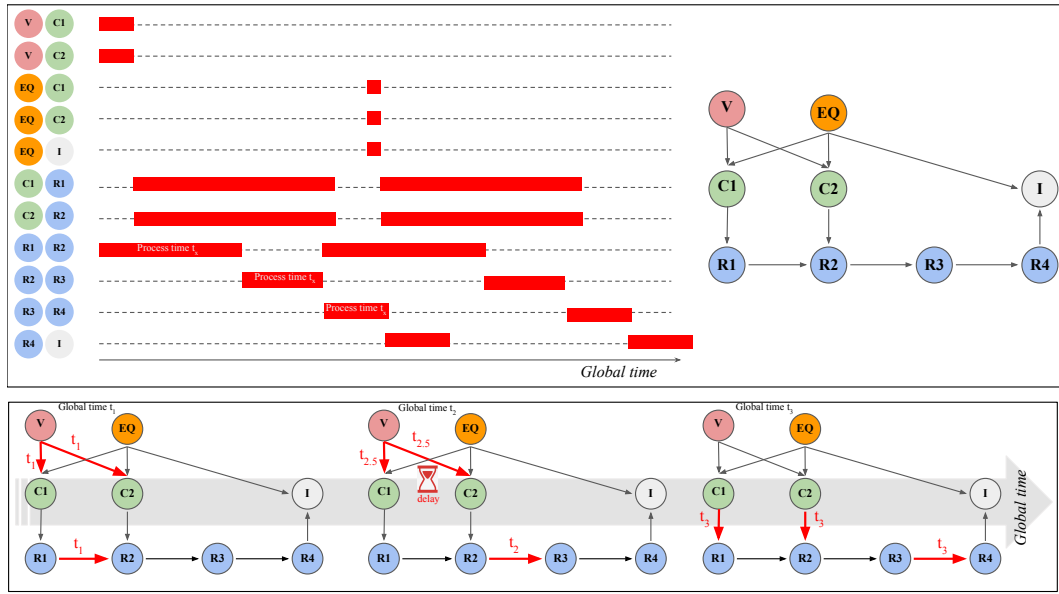


Figure 5: Taking time into account.

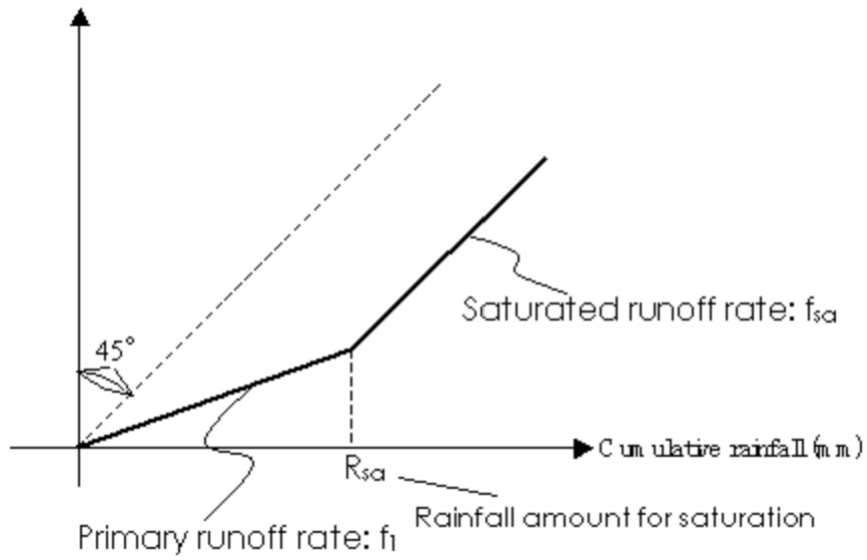


Figure 6: Sediment Runoff.

Source/Origin	Target/Destination	Requirements
Volcano	Slope Unit	Cumulative volume of ash deposited per unit of time over the destination area
Volcano	Infrastructure	Cumulative volume of ash deposited per unit of time. Would require vulnerability function for ash loading (?)Also direct effects - lava, debris avalanche, PDC etc.
Volcano	River segment	Lava or debris avalanche blocking river (natural dam)
Storm/rainfall	Slope Unit	Precipitation over the slope unit area of the catchment per unit of time. The phenomenon driving the runout mechanics should be the one to use. Is it mean, maximum per hour / per 24hours? Answer to be sought from sediment teams)
Storm	Infrastructure	Wind and or rain (including surface flooding). May include debris? Need fragility/vulnerability functions.
<b>ONLY VIA SLOPE UNIT</b>		
Slope Unit	River segment	Runout volume to the river normalized to the dimension of the river segment. Must be linked to the edge rainfall-slope unit and volcano-slope unit for coherency
Slope Unit	(River segment) Landslide dams	Dam nodes could be river nodes originally. Volume of landslides to block rivers and create natural dam(s).
Slope Unit Only specific landslide nodes. General landslides will be included in erosion. We're only interested in landslides that can hit something (infrastructure or a waterway)	Infrastructure	Volume/extent of landsliding (as runout distance is volume dependant) - used Flow-R in past publications. Infrastructure fragility function to landslides (those are rare so a binary impact could be an option)

Table 3: Modelling requirements for the multi-hazard model.



Source/Origin	Target/Destination	Requirements
River segment / Dam (manmade or landslide)	River segment / Dam (manmade or landslide)	Delayed (damped) relation between flow from one river segment to another. Also the sediment loads from one river segment to the next. The dams can probably be treated in the same fashion to reflect the buffer effect on the flow and quantity of sediment load as well as the impact on the infrastructure vulnerability itself. Includes dam collapse. Note potential for management decisions here.
River segment	Infrastructure	Would require a relationship between flow and inundation depth at the exposed locations Pre-generate inundations for each river segment under certain flow/failure conditions – make distribution. Note, inundation is _deterministic_ not random, given where and how much water. Randomness may enter into where in the river segment length the breach occurs  Bridges etc. will need fragility functions, flow depth and speed.
Earthquake	Infrastructure	PGA value at exposed location. Fragility / vulnerability function for each exposed location type for different types of hazards
Earthquake	Slope Unit (or landslide node)	Coseismic landslide initiation threshold. Susceptibility map for the AOI without Earthquake related parameters to allow dynamic assessment (so only slow onset parameters e.g. slope, aspect, distance to rivers etc.) Assume that most slope units have too low a slope to fail, and that ones that can't impact anything are treated as erosion.
Earthquake	Dam (manmade or natural)	PGA value at exposed location. Fragility / vulnerability function leading to dam failure or potential failure
Dam (manmade or landslide)	River segment	Controlled transfer (or uncontrolled – dam failure, potential failure)

Table 3 continued from page 24.